Firm-to-Firm Trade:

Imports, Exports, and the Labor Market[1]

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Abstract

Customs data and firm-level production data reveal both the heterogeneity and the granularity of individual buyers, and sellers. We seek to capture these firm-level features in a general equilibrium model that is also consistent with observations at the aggregate level. Our model is one of product trade through random meetings. Buyers, who may be households looking for final products or firms looking for inputs, connect with sellers randomly. At the firm level, the model generates predictions for imports, exports, and the share of labor in production broadly consistent with observations on French manufacturers. At the aggregate level, firm-to-firm trade determines bilateral trade shares as well as labor's share of output in each country.

1 Introduction

International economists have begun to exploit data generated by customs records, which describe the finest unit of trade transactions. These records expose the activity of individual buyers and sellers that underlie the aggregate trade flows, which had been the object of earlier quantitative analysis in international trade.

Some striking regularities emerge. One that has received attention previously (e.g., Eaton, Kortum, and Kramarz (2011), Eaton, Kortum, and Sotelo (2013)) is the tight connection between market size, market share, and the number of individual exporters. Figures 2 and 3 illustrate this relationship for French manufacturing exports to other members of the European Union (EU). Figure 1 reports a destination's market size, as measured by its manufacturing absorption, on the x-axis, and the number of French manufacturing firms selling there, on the y-axis. The slope of 0.52 (standard error 0.064) is well above 0 but also well below 1. Figure 2 repeats the exercise, only dividing the number of exporters by French market share in that destination. The relationship is tighter, with a slope of around 0.49 (standard error 0.045).

While previous work has documented regularities among exporters, the data reveal some interesting patterns among importers as well. Figure 3 reports the average number of buyers per French exporter across the other EU members, again with market size on the x-axis. The relationship is positive, but also with a slope of only 0.20 (standard error 0.051).

While international trade theory has now incorporated exporter heterogeneity, most analysis has continued to treat demand as monolithic. But, as Figure 4 reveals, the average exporter has only a small number of buyers. Moreover, there is a lot of heterogeneity across exporters in terms of their number of buyers. Table 0 reports on the customers of French exporters in 4 EU destinations of diverse size. Note that the modal number remains below 5 even in Germany, the largest EU market, but numbers at the top end soar into the hundreds.

The theory has also taken a monolithic approach to modeling technology, with all firms in a sector employing factors and intermediate inputs in the same way. But the data reveal substantial heterogeneity with respect to inputs as well. Figure 1 portrays the distribution of the total labor share and unskilled labor share in total costs across French manufacturing firms.

We seek to capture both the heterogeneity and the granularity in individual buyer-seller relationships in a general equilibrium model that is also consistent with observations at the aggregate level. Our model is one of product trade through random meetings. Buyers, who may be households looking for final products or firms looking for inputs, connect with sellers randomly. At the firm level, the model generates predictions for imports, exports, and the share of labor in production broadly consistent with observations on French manufacturers. At the aggregate level, firm-to-firm trade determines bilateral trade shares as well as labor's share of output in each country.

In contrast to standard production theory, we model a firm's technology as combining a set of tasks. Each task can be performed by labor, which can be of different types appropriate for different tasks. But labor competes with intermediate goods produced by other firms which can also perform these tasks. Firms may thus look very different from one another in terms of their production structure, depending on the sellers of intermediate goods that they happen to encounter. A firm's cost in a market thus depends not only on its underlying efficiency, but also on the costs of its suppliers. An implication is that an aggregate change, such as a reduction in trade barriers, can reduce the share of labor in production by exposing producers to more and cheaper sources of supply.

Our model is complementary to recent work of Oberfield (2013) in which a producer's cost depends not only on its own efficiency but the efficiencies of its upstream suppliers. It is also complementary to recent work of Chaney (2014) and Eaton, Eslava, Jinkins, Krizan, and Tybout (2014), with trade the consequence of individual links formed between buyers and sellers over time. In order to embed the framework into general equilibrium, however, our analysis here remains static.

Our model also relates to Garetto (2013), in that firms and workers compete directly to provide inputs for firms.

Our work relates to several other strands in the literature. Recent papers looking at exports and labor markets (although not at imports) include Hummels, Jorgenson, Munch, and Xiang (2011), Felbermayr, Prat, and Schmerer (2008), Egger and Kreickemeier (2009), Helpman, Itskhoki, and Redding (2010), and Caliendo and Rossi-Hansberg (2012). In addition to Oberfield (2013), other theories of networks or input-output interactions include Lucas (2010), Acemoglu and Autor (2011), Luttmer (2013), and Acemoglu and Carvalho (2012). Quantitative work on exports, imports, and labor markets includes Irarrazabal, Moxnes, and Ulltveit-Moe (2010), Klein, Moser, and Urban (2010), Frias, Kaplan, and Verhoogen (2010), Kramarz (2009), Caliendo, Monte, and Rossi-Hansberg (2013), and Helpman, Itskhoki, Muendler, and Redding (2013).

We proceed as follows. Section 2 develops our model. Section 3 analyzes its theoretical and quantitative implications for aggregate outcomes such as the distribution of wages.

2 A Model of Production through Random Encounters

Consider a world with a set of $i = 1, 2, ..., \mathcal{N}$ countries. Each country has an endowment of L_i^l workers of type $l = 1, 2, ..., L$.

2.1 Technology

A producer j in country i can make a quantity of output $Q_i(j)$ by combining a set of $k = 1, ..., K$ *tasks* according to the production function

$$Q_i(j) = z_i(j) \prod_{k=1}^{K} b_k^{-1} \left(\frac{m_{k,i}(j)}{\beta_k} \right)^{\beta_k}$$

where $z_i(j)$ is the overall efficiency of producer j, $m_{k,i}(j)$ is the input of task k, b_k is a constant, and β_k is the Cobb-Douglas share of task k in production. The Cobb-Douglas parameters satisfy $\beta_k > 0$ and

$$\sum_{k=1}^{K} \beta_k = 1.$$

A task can be performed either by one type of labor appropriate for that task, denoted $l(k)$, or with an input produced by a firm. We restrict $K \geq L$, so that one type of labor might be able to perform several different tasks. We denote the set of tasks that labor of type l can perform as Ω_l.

Worker productivity performing a task for a given firm is $q_{k,i}(j)$. If the firm hires labor it pays the wage for workers of type $l(k)$. The producer also is in contact with a set of suppliers of an intermediate good that can also perform the task. From producer j's perspective, labor and the available inputs are perfect substitutes for performing the task. Hence it chooses whatever performs the task at lowest cost.

We assume that producers can hire labor in a standard Walrasian labor market at the market wage $w_{k,i} = w_i^{l(k)}$. In finding intermediates, however, buyers match with only an integer number of potential suppliers, either because of search frictions or because only a handful of producers make an input appropriate for this particular firm. We could make various assumptions about the price at which the intermediate is available. Because it yields the simplest set of results, we assume Nash bargaining in which the buyer has all the bargaining power, so that the price is pushed down to unit cost.[1]

Let $c_{k,i}^{\min}(j)$ denote the lowest price available to firm j for an intermediate to perform task k. The price it pays to perform task k is thus:

$$c_{k,i}(j) = \min\left\{\frac{w_{k,i}}{q_{k,i}(j)}, c_{k,i}^{\min}(j)\right\}$$

and the firm's unit cost of delivering a unit of its output to destination n is:

$$c_{ni}(j) = \frac{d_{ni}}{z_i(j)} \prod_{k=1}^{K} \left(\frac{c_{k,i}(j)^{\beta_k}}{b_k}\right). \tag{1}$$

where $d_{ni} \geq 1$ is the iceberg transport cost of delivering a unit of output from source i to destination n, with $d_{ii} = 1$ for all i. In order to derive a closed form solution we impose specific distributions for producer efficiency, the efficiency of labor in performing a task, and the distribution of the prices of intermediate inputs.

First, following Melitz (2003) and Chaney (2008), each country has a measure potential

[1] An implication is that there are no variable profits. Our model thus cannot accommodate fixed costs, either of market entry as in Melitz (2003) or in accessing markets for inputs, as in Antras et al. (2014). An alternative which would allow for variable profits and hence fixed costs is Bertrand pricing. While we found this alternative analytically tractable, we deemed the added complexity not worth the benefit.

producers. The measure of potential producers in country i with efficiency $Z \geq z$ is:

$$\mu_i^Z(z) = T_i z^{-\theta}. \tag{2}$$

Second, worker productivity performing a task for a given producer Q is drawn from the distribution:

$$F(q) = \Pr[Q \leq q] = e^{-q^{-\phi}}. \tag{3}$$

Third, the measure of producers who can supply country i at a unit cost below c is given by:

$$\mu_i(c) = \Upsilon_i c^{\theta}, \tag{4}$$

where $\theta > 0$ and $\Upsilon_i \geq 0$. These suppliers could be located in country i or anywhere else.

Our specification of the distribution of producer efficiency z given in (2) and the distribution of labor productivity q given in (3) are primitives of the model, with T_i, θ, and ϕ exogenous parameters. We show below, however, that the distribution of unit costs c given in (4) arises endogenously from our other assumptions, with Υ_i determined by underlying technology, labor market conditions, and access to intermediates in different countries of the world, as well as to trade barriers between countries.

2.2 Matching Buyers and Sellers

In contrast with standard Walrasian models, we assume that matching between buyers and sellers is random. Even though there are a continuum of possible sellers and buyers, an individual seller matches with only an integer number of buyers and an individual buyer matches with only an integer number of sellers. Specifically, the intensity with which a buyer in country i seeking to fulfill task k encounters a seller with cost c is:

$$e_{k,i}(c) = \lambda_{k,i} c^{-\varphi}, \tag{5}$$

where $0 \leq \varphi < \theta$ captures the extent to which buyers are more aware of lower-cost sellers. The key new parameter is $\lambda_{k,i}$, which governs how easy it is for a seller to come into contact with a buyer for task k.[2]

Aggregating across the measure of potential suppliers with different costs, the number of suppliers that a buyer encounters with unit cost below c for task k is distributed Poisson with parameter

$$\begin{aligned}
\rho_{k,i}(c) &= \int_0^c e_{k,i}(x) d\mu_i(x) \\
&= \int_0^c \lambda_{k,i} x^{-\varphi} \theta \Upsilon_i x^{\theta-1} dx \\
&= \frac{\theta}{\theta - \varphi} \lambda_{k,i} \Upsilon_i c^{\theta-\varphi}. \qquad (6)
\end{aligned}$$

Note that this Poisson parameter grows arbitrarily large with c, so that many potential suppliers are available to serve any given buyer.

The firm can perform task k at a cost below c_k unless the cost of hiring workers directly and the cost of the best supplier both exceed c_k. From the Poisson density, we know that with probability $\exp\left[-\rho_{k,i}(c_k)\right]$ the buyer will encounter no suppliers of intermediates for task k with cost below c_k. The option of hiring workers to perform the task will cost more than c_k if $w_{k,i}/Q > c_k$, which occurs with probability $F(w_{k,i}/c_k)$. The distribution of the lowest cost

[2] Matching in our framework can be interpreted literally as coming into contact with each other, but it also could relate to the appropriateness of a seller's product for the buyer's purpose. In this sense we can think of products as differentiated not only by seller, but by user as well.

to fulfill task k is thus:

$$G_{k,i}(c_k) = 1 - F(w_{k,i}/c_k)e^{-\rho_{k,i}(c_k)}.$$

To work out the implications of this distribution for the resulting distribution of production costs, we restrict

$$\theta - \varphi = \phi.$$

With this restriction, the parameter governing heterogeneity in the distribution of costs of intermediates is the same as the parameter governing heterogeneity in the distribution of worker efficiency (3) at a given task for a given buyer. In particular, the distribution of the cost to the buyer of fulfilling task k becomes:

$$G_{k,i}(c_k) = 1 - e^{-\Xi_{k,i}c_k^{\phi}}, \tag{7}$$

where

$$\Xi_{k,i} = \nu_{k,i} + w_{k,i}^{-\phi} \tag{8}$$

and

$$\nu_{k,i} = \frac{\theta}{\phi}\lambda_{k,i}\Upsilon_i. \tag{9}$$

With probability $\upsilon_{k,i} = w_{k,i}^{-\phi}/\Xi_{k,i}$ the buyer hires workers to perform task k while with probability $1 - \upsilon_{k,i} = \nu_{k,i}/\Xi_{k,i}$ it purchases an intermediate from the lowest-cost supplier. Notice that these probabilities are independent of the unit cost c.

While $\upsilon_{k,i}$ is the probability that task k is performed by labor in country i, since there are a continuum of producers, it is also the aggregate share of labor in performing task k in country i.[3] The aggregate share of labor of type l in total production costs is consequently:

$$\beta_i^l = \sum_{k \in \Omega_l} \beta_k \upsilon_{k,i}$$

and the overall labor share in production costs is:

$$\beta_i^L = \sum_l \beta_i^l.$$

Note that, even though our basic technology is Cobb-Douglas, the labor share depends on wages and other factors.

We proceed by showing first how the cost measure (4) arises from our model of firm-to-firm trade. We then turn to consumer demand and then to intermediate demand before closing the model in general equilibrium.

2.3 Deriving the Cost Distribution

Each c_k is distributed independently according to (7). From (2) and (1), the measure of potential producers from source i that can deliver to destination n at a unit cost below c is:

$$\begin{aligned}
\mu_{ni}(c) &= T_i d_{ni}^{-\theta} c^{\theta} \prod_k \int_0^{\infty} b_k^{\theta} c_k^{-\theta\beta_k} dG_{k,i}(c_k) \\
&= T_i d_{ni}^{-\theta} c^{\theta} \prod_k \int_0^{\infty} b_k^{\theta} c_k^{-\theta\beta_k} \phi \Xi_{k,i} c_k^{\phi-1} \exp\left(-\Xi_{k,i} c_k^{\phi}\right) dc_k \\
&= T_i d_{ni}^{-\theta} c^{\theta} \prod_k \Xi_{k,i}^{\tilde{\beta}_k} \\
&= T_i \Xi_i d_{ni}^{-\theta} c^{\theta} \qquad (10)
\end{aligned}$$

where:

$$\tilde{\beta}_k = \frac{\theta}{\phi} \beta_k,$$

[3] Similarly, in Eaton and Kortum (2002) the probability π_{ni} that destination n buys a good from a source i is also source i's share in destination n's spending.

$$\Xi_i = \prod_{k=1}^{K} (\Xi_{k,i})^{\tilde{\beta}_k},$$

and we have defined:

$$b_k = \left[\Gamma(1 - \widetilde{\beta}_k)\right]^{-1/\theta}.$$

to eliminate the multiplicative constant emerging from integration. We require that parameter values satisfy $\tilde{\beta}_k < 1$.

Aggregating across all sources of supply, the measure of potential producers that can deliver a good to market n at a cost below c is:

$$\mu_n(c) = \sum_{i=1}^{\mathcal{N}} \mu_{ni}(c) = \Upsilon_n c^{\theta}$$

where:

$$\Upsilon_n = \sum_i T_i \Xi_i d_{ni}^{-\theta}, \tag{11}$$

showing how the parameter Υ_n posited in (4) relates to deeper parameters of technology, search, and trade costs, as well as to wages, to which we turn below.

Substituting in (9), we can solve for the vector of Υ_n from the system of equations:

$$\Upsilon_n = \sum_i T_i d_{ni}^{-\theta} \prod_k \left(\frac{\theta}{\phi} \lambda_{k,i} \Upsilon_i + w_{k,i}^{-\phi} \right)^{\tilde{\beta}_k} \tag{12}$$

for $n = 1, 2, ..., \mathcal{N}$. Given wages and exogenous parameters of the model, the Υ_n are thus the solution to the set of equations (12). The Appendix provides sufficient conditions for a unique solution to the Υ_n's and an iterative procedure to compute them.

The measure of potential producers from source i with unit cost below c in destination n is $T_i \Xi_i d_{ni}^{-\theta} c^{\theta}$. The total measure of potential producers with unit cost below c in n is $\Upsilon_n c^{\theta}$. Hence the probability that a potential producer selling in n with unit cost below c is from i

is just:

$$\pi_{ni} = \frac{T_i \Xi_i d_{ni}^{-\theta}}{\sum_{i'} T_{i'} \Xi_{i'} d_{ni'}^{-\theta}} \tag{13}$$

regardless of c. Just as in Eaton and Kortum (2002), with our continuum of producers, in the aggregate π_{ni} is the share of source i in the purchases of destination n.

Inspection of (12) reveals that, together, the Υ_n's are homogeneous of degree one in all of the T_i's and all of the $1/\lambda_{k,i}$'s.[4] An implication is that proportional increases in all T_i's and $1/\lambda_{k,i}$'s do not affect the share of labor in gross production or bilateral trade shares. The intuition is that an across-the-board improvement in technology raises the measure of potential suppliers in proportion (through the Υ_n's). But if the ability to access these suppliers falls in the same proportion, there is no effect on other outcomes.

2.4 The Aggregate Production Function

Before finishing our specification of the model and turning to its solution, we take a moment to show how our assumptions about technology are consistent with a standard aggregate production function of the form:

$$Q_i = \prod_{k=1}^{K} \left[\tilde{\gamma} \left(L_{k,i} \right)^{\phi/(\phi+1)} + (1-\tilde{\gamma}) \left(I_{k,i} \right)^{\phi/(\phi+1)} \right]^{\beta_k (\phi+1)/\phi}, \tag{14}$$

where Q_i is aggregate output, $L_{k,i}$ is the labor force employed in performing task k, $I_{k.i}$ are intermediates used for task k, and:

$$\tilde{\gamma} = \frac{1}{1+\gamma^{\phi/(1+\phi)}},$$

[4] To see this result, start from a set of Υ_n's that satisfy (12). Imagine, holding fixed all wages, scaling the technology endowment T_i of each country by the factor s while scaling all search parameters $\lambda_{k,i}$ by the factor $1/s$. Given that we started at a solution, we will remain at a solution if each Υ_n increases by the factor s.

where:

$$\gamma = \Gamma(1 + 1/\phi).$$

To see this implication, note that, since the distribution of the price for an intermediate to perform task k in country i is:

$$H_{k,i}(p) = 1 - e^{-\nu_{k,i}p^{\phi}},$$

the average of such prices across firms in i is:

$$\begin{aligned}
\bar{p}_{k,i} &= \int_0^\infty p dH_{k,i}(p) \\
&= \int_0^\infty p\phi\nu_{k,i}e^{-\nu_{k,i}p^{\phi}}p^{\phi-1}dp \\
&= \int_0^\infty \left(\frac{x}{\nu_{k,i}}\right)^{1/\phi} e^{-x}dx \\
&= \gamma\left(\nu_{k,i}\right)^{-1/\phi},
\end{aligned}$$

We can then write the share in total production costs of type k labor in performing task k as:

$$\begin{aligned}
\beta^{L,k} &= \beta_k \upsilon_{k,i} \\
&= \beta_k \frac{w_{k,i}^{-\phi}}{\nu_{k,i} + w_{k,i}^{-\phi}} \\
&= \beta_k \frac{w_{k,i}^{-\phi}}{(\bar{p}_{k,i}/\gamma)^{-\phi} + w_{k,i}^{-\phi}} \qquad (15)
\end{aligned}$$

For each task k the representative firm can hire labor $L_{k,i}$ at wage $w_{k,i}$ and purchase a composite intermediate $I_{k,i}$ at price $\bar{p}_{k,i}$.

The first-order-conditions for cost minimization deliver:

$$\frac{L_{k,i}}{I_{k,i}} = \left(\frac{(1-\tilde{\gamma})w_{k,i}}{\tilde{\gamma}\bar{p}_{k,i}}\right)^{-(1+\phi)} = \left(\gamma^{\phi/(1+\phi)}\right)^{-(1+\phi)}\left(\frac{w_{k,i}}{\bar{p}_{k,i}}\right)^{-(1+\phi)}.$$

Hence

$$\frac{w_{k,i}L_{k,i}}{\bar{p}_{k,i}I_{k,i}} = \left(\gamma^{\phi/(1+\phi)}\right)^{-(1+\phi)}\left(\frac{w_{k,i}}{\bar{p}_{k,i}}\right)^{-\phi} = \left(\gamma\frac{w_{k,i}}{\bar{p}_{k,i}}\right)^{-\phi}$$

Thus the share in total production costs of labor of type k in performing task k in country i is:

$$\begin{aligned}
\beta^{L,k} &= \beta_k \frac{w_{k,i}L_{k,i}}{w_{k,i}L_{k,i} + \bar{p}_{k,i}I_{k,i}} \\
&= \beta_k \frac{\left(\gamma\frac{w_{k,i}}{\bar{p}_{k,i}}\right)^{-\phi}}{\left(\gamma\frac{w_{k,i}}{\bar{p}_{k,i}}\right)^{-\phi} + 1} \\
&= \beta_k \frac{(w_{k,i})^{-\phi}}{(w_{k,i})^{-\phi} + (\bar{p}_{k,i}/\gamma)^{-\phi}},
\end{aligned}$$

just as above.

2.5 Preferences

Final demand is by different types of workers spending their wage income (since there are no profits in our model). We model their preferences in parallel to our assumptions about production. Consumers have an integer number K of needs, with each need having a Cobb-Douglas share α_k in preferences, with $\alpha_k > 0$ and

$$\sum_{k=1}^{K} \alpha_k = 1.$$

In parallel with the tasks of a producer, need k of consumer j can be satisfied either directly with the services of an appropriate type of labor $l(k)$ at wage $w_{k,i} = w_i^{l(k)}$ with efficiency Q drawn from the distribution (3) or with a good produced by a firm. Final buyers match with potential sellers with the same intensity as firms, as given by (5).

Proceeding as above, a consumer faces a distribution of costs for fulfilling need k given by (7). The probability that need k is fulfilled by labor is again $\upsilon_{k,i}$, which, with our continuum of consumers, is the share of labor in fulfilling need k. The share of labor of type l used by consumers in their total spending is thus:

$$\alpha_i^l = \sum_{k \in \Omega_l} \alpha_k \upsilon_{k,i}$$

and the share of labor in consumer spending in country i is:

$$\alpha_i^L = \sum_l \alpha_i^l.$$

As with the share of labor in production costs, the share of labor in final spending depends on wages and other factors.

When a consumer in country n fulfills a need by purchasing a good, the probability that the good come from country i is given by π_{ni} in expression (13). With our continuum of consumers π_{ni} thus represents the share of country i in country n's final spending.

2.6 Consumer Welfare

Two worker's with the same income won't typically have the same level of utility as they encounter different goods and worker productivities in satisfying their needs. We can write the indirect utility of a consumer j in n spending $y_n(j) = y$ and facing costs of performing each need k given by $\boldsymbol{c(j)} = (c_1, c_2, ..., c_K)$ as:

$$V(j) = V(y(j), \boldsymbol{c(j)}) = \frac{y(j)}{\prod_{k=1}^{K} c_k^{\alpha_k} / a_k}.$$

where a_k is a constant that will be chosen to eliminate the effect of K on utility. The expenditure $Y(V)$ needed to obtain expected utility V in market n is thus:

$$Y(V) = V\prod_{k=1}^{K}\left(\frac{1}{a_k}\int_0^\infty c_k^{\alpha_k} dG_{k,n}(c_k)\right).$$

In parallel to the derivation of the cost distribution, the term in parentheses above can be expressed as:

$$\begin{aligned} & \frac{1}{a_k}\int_0^\infty (c_k)^{\alpha_k}\, dG_{k,n}(c_k) \\ = & \frac{1}{a_k}\int_0^\infty c_k^{\alpha_k}\phi\Xi_{k,n}c_k^{\phi-1}\exp\left(-\Xi_{k,n}c_k^{\phi}\right)dc_k \\ = & \frac{1}{a_k}\int_0^\infty \left(\frac{x}{\Xi_{k,n}}\right)^{\tilde{\alpha}_k} e^{-x}dx \\ = & (\Xi_{k,n})^{-\tilde{\alpha}_k} \end{aligned}$$

where:

$$\tilde{\alpha}_k = \frac{1}{\phi}\alpha_k.$$

and $a_k = \Gamma\left(1+\tilde{\alpha}_k\right)$.

The expected expenditure function is thus:

$$Y(V) = V\prod_{k=1}^{K}(\Xi_{k,n})^{-\tilde{\alpha}_k}.$$

We can write the result more compactly as:

$$Y(V) = P_n^C V,$$

where

$$P_n^C = \prod_{k=1}^{K}(\Xi_{k,n})^{-\tilde{\alpha}_k}$$

is the consumer price index.

3 Aggregate Equilibrium

We now have in place the assumptions that we need to solve for the aggregate equilibrium. We first solve for equilibrium in the production of intermediates, given wages, and then to labor-market equilibrium, which determines those wages.

3.1 Production Equilibrium

With balanced trade, total final spending X_n^C is labor income:

$$X_n^C = \sum_{l=1}^{L} w_n^l L_n^l = \sum_{k=1}^{K} w_{k,n} L_{k,n}. \tag{16}$$

Total production in country i equals total revenue in supplying consumption goods and intermediates around the world:

$$Y_i = \sum_{n=1}^{\mathcal{N}} \pi_{ni} \left[\Phi_n^C X_n^C + \Phi_n^I Y_n \right]$$

where $\Phi_n^C = 1 - \alpha_n^L$ and $\Phi_n^I = 1 - \beta_n^L$, the shares of goods in final spending and in production spending, respectively.

We can write this result in matrix form as:

$$\boldsymbol{Y} = \boldsymbol{\Pi} \left(\boldsymbol{\Phi}^C \mathbf{X}^C + \boldsymbol{\Phi}^I \boldsymbol{Y} \right)$$

where:

$$\boldsymbol{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ \cdot \\ Y_{\mathcal{N}} \end{bmatrix}, \quad \boldsymbol{X}^C = \begin{bmatrix} X_1^C \\ X_2^C \\ \cdot \\ \cdot \\ \cdot \\ X_{\mathcal{N}}^C \end{bmatrix}$$

$$\mathbf{\Phi}^j = \begin{bmatrix} \Phi_1^j & 0 & \dots & 0 & 0 \\ 0 & \Phi_2^j & \dots & 0 & 0 \\ . & . & . \; . \; . & . & . \\ . & . & . \; . \; . & . & . \\ . & . & . \; . \; . & . & . \\ 0 & 0 & \dots & \Phi_{\mathcal{N}-1}^j & 0 \\ 0 & 0 & \dots & 0 & \Phi_{\mathcal{N}}^j \end{bmatrix} \quad j = C, I$$

and:

$$\mathbf{\Pi} = \begin{bmatrix} \pi_{11} & \pi_{21} & \dots & \pi_{\mathcal{N}-1,1} & \pi_{\mathcal{N}1} \\ \pi_{12} & \pi_{22} & \dots & \pi_{\mathcal{N}-1,2} & \pi_{\mathcal{N}2} \\ . & . & . \; . \; . & . & . \\ . & . & . \; . \; . & . & . \\ . & . & . \; . \; . & . & . \\ \pi_{1,\mathcal{N}-1} & \pi_{2,\mathcal{N}-1} & \dots & \pi_{\mathcal{N}-1,\mathcal{N}-1} & \pi_{\mathcal{N},\mathcal{N}-1} \\ \pi_{1\mathcal{N}} & \pi_{2\mathcal{N}} & \dots & \pi_{\mathcal{N}-1,\mathcal{N}} & \pi_{\mathcal{N}\mathcal{N}} \end{bmatrix}$$

We can then solve for $\boldsymbol{Y}$:

$$\boldsymbol{Y} = (\boldsymbol{I}_{\mathcal{N}} - \mathbf{\Pi}\mathbf{\Phi}^I)^{-1}\mathbf{\Pi}\mathbf{\Phi}^C\mathbf{X}^C$$

where $\boldsymbol{I}_{\mathcal{N}}$ is the $\mathcal{N} \times \mathcal{N}$ identity matrix.

3.2 Labor-Market Equilibrium

With balanced trade, final spending on manufactures in country i, X_i^C is given by (16). Equilibrium in the market for labor of type l in country i solves the expression:

$$w_i^l L_i^l = \alpha_i^l X_i^C + \beta_i^l Y_i.$$

where the first term on the right-hand side corresponds to labor demanded directly by households and the second term to labor demanded by firms. These sets of equations, for each type of labor l in each country i, determine the wages w_i^l.

3.3 Some Quantitative Aggregate Implications

We can now investigate some quantitative implications of the model for aggregate outcomes. Table 1 provides a parameterization with two types of labor, which we call nonproduction and production. The labor force in each country is divided identically into nonproduction workers (60 percent) and production workers (40 percent). Nonproduction workers can perform 4 tasks or fulfill 4 needs each with Cobb-Douglas shares $\beta^N = \alpha^N = .1$. Production workers can perform 12 tasks each with $\beta^P = \alpha^P = .05$. In our base case the iceberg cost is $d_{ni} = 1.2$ for all $i, n,\ i \neq n$. Finally $\lambda^N = 0$ for each nonproduction tasks and $\lambda^P = 0.2$ for production tasks. The world labor force normalized at 1 is divided into 6 countries with the sizes given along the top of table 2. The countries are identical to each other except for the sizes of their labor forces.

Note from Table 2 that there are a number of systematic differences across countries. Least surprisingly, the import share declines as country size increases. Because less has to be imported, goods prices are on average cheaper in larger countries. Hence more purposes are fulfilled with goods rather than labor. Since production labor competes with goods in fulfilling purposes, production workers earn relatively lower wages in larger countries, so that the "skill premium" (defined as the ratio of the wage of nonproduction to the wage of production workers) increases with size. Even though prices are lower and welfare higher in large countries, the real wage of production workers declines with size.

Table 3 reports the results of varying the iceberg trade costs for the second smallest and second largest countries. At d of 10 is nearly prohibitive. A decline in trade costs, making goods more competitive with production workers, leads to a decline in the relative and real wage of production workers, even though total welfare rises.

4 Implications for Individual Producers

While our analysis so far has allowed us to investigate the implications of various changes in exogenous variables on equilibrium aggregate outcomes, we have more work to do to find out what happens to individual producers. We have not yet solved for the measure of active producers or sellers in an economy or for the distributions of the number of final and intermediate customers a firm has.

We first examine what our model implies about the distribution of buyers per firm, and then for the measure of firms selling and producing in a market. We conclude by examining what it predicts about the distribution of firm size.

4.1 Distribution of Buyers

How many buyers a firm has depends not only on its efficiency z, but on its luck in finding low-cost suppliers and its luck in running into buyers who don't have better alternatives.

We start with a firm's contacts with final buyers. Consider a supplier with unit cost c in market n and final buyers for need k. The number of such customers it connects with is distributed Poisson with parameter:

$$e_{k,n}(c)L_n = \lambda_{k,n}c^{-\varphi}L_n.$$

Having met a final buyer, this supplier will make the sale if and only if the buyer hasn't found a lower-cost means of fulfilling the need, either with a cheaper intermediate or with labor. The probability that a seller with cost c is the probability that there is no cheaper source, which is $e^{-\Xi_{k,n}c^{\phi}}$. Combining these two results the number of final consumers in n buying from a supplier with unit cost c for need k is distributed Poisson with parameter $\eta^{C}_{k,n}(c)$, given by:

$$\eta^{C}_{k,n}(c) = \lambda_{k,n} L_n c^{-\varphi} e^{-\Xi_{k,n}c^{\phi}},$$

where, recall,

$$\Xi_{k,n} = \frac{\theta}{\phi}\lambda_{k,n}\Upsilon_n + w_{k,n}^{-\phi}.$$

Note that $\eta^{C}_{k,n}(c)$ is decreasing in the producer's unit cost c for two reasons. First, as long as $\varphi > 0$, a low-cost producer typically finds more potential customers. Second, each potential customer is more likely to have no better option. Note also that, given Υ_n and $w_{k,n}$, the Poisson parameter is at first increasing and then decreasing in $\lambda_{k,n}$. If it's impossible to meet customers ($\lambda_{k,n} = 0$) then it's impossible to make a sale. Thus, starting from 0, an increase in $\lambda_{k,n}$ increases the likelihood of a sale. But an increase in $\lambda_{k,n}$ also means that a potential buyer is more likely to have found another seller with a lower cost. At some point (which is earlier for a firm with a high c) as $\lambda_{k,n}$ rises, this second effect dominates, so that further increases reduce expected sales.

Since purchases are independent across k, the number of total purchases by consumers in n from a producer with unit cost c is distributed Poisson with parameter:

$$\eta^{C}_{n}(c) = \sum_{k=1}^{K} \eta^{C}_{k,n}(c).$$

By the properties of the Poisson distribution, $\eta^{C}_{n}(c)$ is also the expected number of customers for a potential producer selling a product at unit cost c in market n.

In the case of final sales the set of potential customers in a market is exogenously given by the set of workers. For intermediate demand, however, the set of customers is given by the endogenous measure of local firms that actually make a sale. Let M_n denote the measure of active producers in country n, the determination of which we turn to below. Analogous to our reasoning above, a supplier in country n with unit cost c encounters a number of buyers wanting to perform task k that is distributed Poisson with parameter:

$$e_{k,n}(c)M_n = \lambda_{k,n}c^{-\varphi}M_n.$$

and its number of sales is distributed Poisson with parameter:

$$\eta^I_{k,n}(c) = \lambda_{k,n}M_n c^{-\varphi}e^{-\Xi_{k,n}c^{\phi}}.$$

Summing across tasks, the total number of sales by a seller with unit cost c in country i is distributed Poisson with parameter:

$$\eta^I_n(c) = \sum_{k=1}^{K}\eta^I_{k,n}(c).$$

By the properties of the Poisson distribution, $\eta^I_n(c)$ is also the expected number of customers for a potential producer selling an intermediate at unit cost c in market n.

Combining these results, the number of buyers for a firm selling in n at cost c is distributed Poisson with parameter:

$$\eta_n(c) = \eta^C_n(c) + \eta^I_n(c) = (L_n + M_n)\, c^{-\varphi}\sum_{k=1}^{K}\lambda_{k,n}e^{-\Xi_{k,n}c^{\phi}}.$$

Now consider worldwide sales of a producer in country i with local cost c. Its unit cost in country n is cd_{ni}. The total number of customers around the world for this producer is distributed Poisson with parameter:

$$\begin{aligned}
\eta^W_i(c) &= \sum_{n=1}^{\mathcal{N}}\eta_n(cd_{ni}) \\
&= \sum_{n=1}^{\mathcal{N}}(L_n + M_n)\,(d_{ni})^{-\varphi}\,c^{-\varphi}\sum_{k=1}^{K}\lambda_{k,n}e^{-\Xi_{k,n}(d_{ni})^{-\phi}c^{\phi}}.
\end{aligned}$$

So far we've considered only the distribution of a seller's customers in market n conditional on its c there, Let S_n be the integer-valued random variable for the number of customers in n that a firm sells to. From the Poisson distribution, the probability that a firm with cost c has s customers is

$$\Pr[S_n = s|c] = \frac{e^{-\eta_n(c)}\left[\eta_n(c)\right]^s}{s!},$$

for $s = 0, 1, ...$ Let N_n denote the measure of active sellers in n. We can integrate over the cost distribution and condition on $S_n > 0$ (since if $S_n = 0$ the firm would not be among those observed to sell in n) to get

$$\begin{aligned}\Pr[S_n &= s|S_n > 0] = \frac{1}{N_n}\int_0^\infty \frac{e^{-\eta_n(c)}\left[\eta_n(c)\right]^s}{s!}d\mu_n(c) \\ &= \frac{\Upsilon_n}{N_n s!}\int_0^\infty e^{-\eta_n(c)}\left[\eta_n(c)\right]^s \theta c^{\theta-1}dc, \end{aligned} \tag{17}$$

for $s = 1, 2, ...$

The expected number of buyers per active firm is thus simply:

$$\begin{aligned} E\left[S_n|S_n > 0\right] &= \frac{1}{N_n}\int_0^\infty \eta_n(c)d\mu_n(c) \\ &= \frac{L_n + M_n}{N_n}\int_0^\infty c^{-\varphi}\left(\sum_{k=1}^K \lambda_{k,n}e^{-\Xi_{k,n}c^\phi}\right)\theta\Upsilon_n c^{\theta-1}dc \\ &= \frac{L_n + M_n}{N_n}\sum_{k=1}^K \frac{\nu_{k,n}}{\Xi_{k,n}} \end{aligned}$$

Since $\nu_{k,n}/\Xi_{k,n}$ is the probability that a potential customer purchases a good for a purpose (rather than hiring labor), the summation on the right hand side is then expected purchases per potential customer. Thus, expected sales per firm is the product of the measure of potential customers, $L_n + M_n$, in market n and the expected number of goods purchased per potential customer, all divided by the measure of sellers in that market.

4.2 The Measures of Producers and Sellers

In an open economy the measure of firms making sales in country n, denoted N_n is not the same as the set actually producing there, denoted M_n.

To appear as a firm a seller has to sell somewhere. The probability that a potential producer from source i with unit cost c fails to make a sale anywhere is $\exp(-\eta_i^W(c))$. Integrating over the cost distribution of potential producers in source i (those from i that can deliver to i at cost c):

$$\begin{aligned} M_i &= \int_0^\infty (1 - e^{-\eta_i^W(c)}) d\mu_{ii}(c) \\ &= T_i \Xi_i \int_0^\infty (1 - e^{-\eta_i^W(c)}) \theta c^{\theta-1} dc. \end{aligned} \tag{18}$$

Since $\eta_i^W(c)$ itself depends on the measure of customers for intermediates M_n in each market n, we need to iterate to find a solution for all the M_i's.

Having solved for the M_i's, the measure of firms selling in n can be calculated as

$$\begin{aligned} N_n &= \int_0^\infty (1 - e^{-\eta_n(c)}) d\mu_n(c) \\ &= \Upsilon_n \int_0^\infty (1 - e^{-\eta_n(c)}) \theta c^{\theta-1} dc. \end{aligned} \tag{19}$$

We can evaluate this integral numerically to determine the relationship between entry N_n and market size, $L_n + M_n$.

The measure of firms from i exporting to n is

$$N_{ni} = \pi_{ni} N_n = \int_0^\infty (1 - e^{-\eta_n(c)}) d\mu_{ni}(c). \tag{20}$$

Thus the fraction of firms from i that export to n is N_{ni}/M_i. The fraction of firms from i that sell domestically is N_{ii}/M_i.

While equations (18) and (19) don't have closed form solutions, we can compute their solutions for numerical parameter values.

4.3 Some Quantitative Firm-Level Results

Using the same parameterization as in Table 1 and 2, we calculate the measures of sellers to each of our six hypothetical countries (labelled a through f, in increasing size) from country a, adjusting, as in Figure 3, with the actual data, for country a's market share in each destination. Note from Table 4 that the simulation mimics the patterns in distribution of buyers in Table 0. Figure 5 shows how we also capture the increasing but less than proportional relationship between market size and number of exporters, albeit with a somewhat greater slope of 0.77 (standard error 0.034).

Table 5 reports the effects of varying trade costs on the measures of active suppliers and producers in a market, with lower barriers tending to reduce each.

Finally, Figure 6 reports the average number of buyers per seller across our hypothetical markets. Notice that the pattern mimics that in Figure 4, with a similar slope of 0.34 (standard error 0.079).

5 Conclusion

Taking into account the granularity of individual buyer-seller relationships expands the scope for firm heterogeneity in a number of dimensions. Aside from differences in raw efficiency, firms experience different luck in finding cheap inputs. These two sources of heterogeneity combine to create differences in the firm's cost to deliver to different markets around the world. But within each market firms have different degrees of luck in connecting with buyers. We can thus explain why a firm may happen to sell in a small, remote market while skipping over a large one close by. It also explains why one firm may appear very successful in one market and sell very little in another, while another firm does just the opposite.

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6 Appendix: Computing Υ

We derive conditions under which there is a unique solution for Υ, given wages, that can be computed by simple iteration. To ensure a solution it helps to have a sufficient share of tasks in which outsourcing is not possible ($\lambda_k = 0$). Denote the set of such tasks as Ω^0 and its complement (among the set of all tasks $\{1, 2, ..., K\}$) as Ω^P and define:

$$\widetilde{\beta}^P = \sum_{k \in \Omega^P} \tilde{\beta}_k$$

We require $\widetilde{\beta}^P < 1$ which, in terms of primitives, implies:

$$\beta^P = \sum_{k \in \Omega^P} \beta_k < \frac{\phi}{\theta} = \frac{\theta - \varphi}{\theta}.$$

As a warm-up exercise, we start with the case of a single country ($\mathcal{N} = 1$), so that Υ is a scalar. We then turn to the general case with multiple countries, in which Υ is an $\mathcal{N} \times 1$ vector.

6.1 The Case of a Single Country

With a single country, the solution for Υ is a fixed point

$$\Upsilon = f(\Upsilon)$$

of the function f defined as:

$$f(x) = T \prod_{k=1}^{K} \left(\frac{\theta}{\phi} \lambda_k x + w_k^{-\phi} \right)^{\tilde{\beta}_k}.$$

Employing our assumption that $\lambda_k = 0$ for all tasks $k \in \Omega^0$, we can write:

$$f(x) = T \left(\prod_{k \in \Omega^0} (w_k)^{-\phi \tilde{\beta}_k} \right) \prod_{k \in \Omega^P} \left(\frac{\theta}{\phi} \lambda_k x + w_k^{-\phi} \right)^{\tilde{\beta}_k}.$$

It is convenient to work in logs. Thus $\ln \Upsilon$ is the fixed point

$$\ln \Upsilon = F(\ln \Upsilon)$$

of the function:

$$F(y) = A + \sum_{k \in \Omega^P} \tilde{\beta}_k \ln \left(u_k e^y + w_k^{-\phi} \right),$$

where

$$A = \ln T - \sum_{k \in \Omega^0} \phi \tilde{\beta}_k \ln w_k,$$

and

$$u_k = \frac{\theta}{\phi} \lambda_k$$

There exists a unique fixed point of F if it is a contraction. To show that it is, we can check Blackwell's sufficient conditions, monotonicity and discounting. For monotonicity, note that $x \leq y$ implies:

$$F(x) = A + \sum_{k \in \Omega^P} \tilde{\beta}_k \ln \left(u_k e^x + w_k^{-\phi} \right) \leq A + \sum_{k \in \Omega^k} \tilde{\beta}_k \ln \left(u_k e^y + w_k^{-\phi} \right) = F(y).$$

For discounting, $a > 0$ implies:

$$\begin{aligned} F(y+a) &= A + \sum_{k \in \Omega^P} \tilde{\beta}_k \ln \left(u_k e^{y+a} + w_k^{-\phi} \right) = A + \sum_{k \in \Omega^P} \tilde{\beta}_k \ln \left(e^a u_k e^y + w_k^{-\phi} \right) \\ &= A + \sum_{k \in \Omega^P} \tilde{\beta}_k \left[a + \ln \left(u_k e^y + e^{-a} w_k^{-\phi} \right) \right] = A + \beta a + \sum_{k \in \Omega^P} \tilde{\beta}_k \ln \left(u_k e^y + e^{-a} w_k^{-\phi} \right) \\ &\leq A + \sum_{k \in \Omega^P} \tilde{\beta}_k \ln \left(u_k e^y + w_k^{-\phi} \right) + \beta a = F(y) + \beta a. \end{aligned}$$

We can thus compute the fixed point by iterating on:

$$y^{(t)} = F(y^{(t-1)}),$$

starting with $y^{(0)} = 0$. This method is justified, since the contraction mapping theorem guarantees that:

$$\lim_{t\to\infty} y^{(t)} = \ln \Upsilon.$$

This result also give us the comparative statics. We see directly that $\ln \Upsilon$ is increasing in technology T, decreasing in any task-specific wage w_k, and increasing in any task-specific arrival of price quotes λ_k.

6.2 Multiple Countries

Consider generalizing the argument above to a world of many countries, trading intermediates and final goods with each other. Now Υ is an $\mathcal{N} \times 1$ vector satisfying

$$\Upsilon_n = \sum_i T_i d_{ni}^{-\theta} \prod_k \left(\frac{\theta}{\phi} \lambda_{k,i} \Upsilon_i + w_{k,i}^{-\phi} \right)^{\tilde{\beta}_k},$$

for $n = 1, ..., \mathcal{N}$.

Let $\ln \Upsilon$ be the corresponding vector with $\ln \Upsilon_n$ in place of Υ_n for $n = 1, ..., \mathcal{N}$. Thus $\ln \Upsilon$ is the fixed point

$$\ln \Upsilon = F(\ln \Upsilon)$$

of the mapping F, whose n'th element is:

$$F_n(y) = \ln \left[\sum_i \exp \left(A_{ni} + \sum_{k\in\Omega^k} \tilde{\beta}_k \ln \left(u_{k,i} e^{y_i} + w_{k,i}^{-\phi} \right) \right) \right],$$

where

$$A_{ni} = \ln \left(T_i d_{ni}^{-\theta} \right) - \sum_{k\in\Omega^0} \phi \tilde{\beta}_k \ln w_{k,i}$$

and

$$u_{k,i} = \frac{\theta}{\phi} \lambda_{k,i}.$$

We can check Blackwell's conditions again. For monotonicity, it is readily apparent that for a vector $x \leq y$ we have $F_n(x) \leq F_n(y)$ for all $n = 1, ..., \mathcal{N}$. For discounting, consider $a > 0$ so that

$$
\begin{aligned}
F_n(y+a) &= \ln\left[\sum_i \exp\left(A_{ni} + \sum_{k \in \Omega^k} \tilde{\beta}_k \ln\left(u_{k,i} e^{y_i + a} + w_{k,i}^{-\phi}\right)\right)\right] \\
&= \ln\left[\sum_i \exp\left(A_{ni} + \sum_{k \in \Omega^k} \tilde{\beta}_k \left[a + \ln\left(u_{k,i} e^{y_i} + e^{-a} w_{k,i}^{-\phi}\right)\right]\right)\right] \\
&= \ln\left[\sum_i \exp\left(A_{ni} + \beta a + \sum_{k \in \Omega^k} \tilde{\beta}_k \ln\left(u_{k,i} e^{y_i} + e^{-a} w_{k,i}^{-\phi}\right)\right)\right] \\
&\leq \ln\left[\sum_i \exp\left(A_{ni} + \beta a + \sum_{k \in \Omega^k} \tilde{\beta}_k \ln\left(u_{k,i} e^{y_i} + w_{k,i}^{-\phi}\right)\right)\right] \\
&= \ln\left[\sum_i \exp\left(A_{ni} + \sum_{k \in \Omega^k} \tilde{\beta}_k \ln\left(u_{k,i} e^{y_i} + w_{k,i}^{-\phi}\right)\right)\right] + \beta a \\
&= F_n(y) + \beta a.
\end{aligned}
$$

Thus, even with multiple countries, we can still compute the fixed point by iterating on:

$$y^{(t)} = F(y^{(t-1)}),$$

starting with an $\mathcal{N} \times 1$ vector $y^{(0)}$ (which could simply be a vector of zeros). This method is justified, since the contraction mapping theorem guarantees (just as in the scalar case) that:

$$\lim_{t \to \infty} y^{(t)} = \ln \Upsilon.$$

This result also give us the comparative statics. We see directly that each element of $\ln \Upsilon$ is increasing in technology anywhere T_i, decreasing in any task-specific wage $w_{k,i}$ in any country, and increasing in any task-specific arrival of price quotes $\lambda_{k,i}$ in any country. An important caveat, however, is that these comparative statics take task-specific wages as given, so do not predict general-equilibrium outcomes.

Customers per French Exporter

	Destination Market			
	Lithuania	Denmark	UK	Germany
Market Size ($billions)	18	94	882	1480
Customers per Exporter:				
Mean	4.2	7.1	17.9	24.9
Percentiles:				
25th	1	1	1	2
50th	2	2	3	4
75th	4	5	9	12
90th	9	12	25	35
95th	15	21	48	70
99th	40	77	224	329

Data are for 2005.

Table 1: Baseline Parameter Settings for Simulation

Parameter	symbol	value
Pareto parameters:		
efficiency distribution	theta	5
price distribution	phi	3
Technology level per person	T_i/L_i	3.6
World labor force	L	1
Labor by type (fractions of labor force):	L^l	
service		0.6
production		0.4
Iceberg trade cost	d	1.2
Tasks, by type:		
service tasks:		
number of tasks	K	4
total share	beta	0.4
production tasks:		
number of tasks	K	12
total share	beta	0.6
Task shares in consumption (same as for production)	alpha	
Outsourcing parameters:	lambda	
service		0
production		0.2

Table 2: Aggregate Results of Simulation

	Country Size					
	L=0.001	L=0.009	L=0.09	L=0.2	L=0.3	L=0.4
Production value added:						
Share of GDP	0.133	0.133	0.134	0.135	0.135	0.135
Share of gross production	0.27	0.27	0.26	0.24	0.23	0.22
Fraction of production tasks outsourced:	0.55	0.55	0.57	0.60	0.62	0.64
Import share of production	1.00	0.98	0.79	0.61	0.48	0.39
Wage:						
service	0.91	0.91	0.96	1.01	1.06	1.11
production	0.93	0.93	0.92	0.91	0.91	0.90
Skill premium (service/production)	0.98	0.98	1.04	1.11	1.17	1.23
Real wage:						
service	1.39	1.40	1.46	1.54	1.61	1.67
production	1.42	1.42	1.41	1.39	1.37	1.36
Welfare (real per capita consumption)	1.40	1.41	1.44	1.48	1.51	1.54

1. Production value added does not include service tasks (i.e. purchased services)

2. Wage is normalized so that labor income of the World is 1

Table 3: Aggregate Results with Different Trade Costs

	Trade Cost (small country, L=.009)					Trade Cost (large country, L=0.3)				
	10.00	1.80	1.20	1.05	1.00	10.00	1.80	1.20	1.05	1.00
Production value added:										
Share of GDP	0.01	0.05	0.13	0.13	0.12	0.12	0.12	0.14	0.13	0.12
Share of gross production	0.59	0.50	0.27	0.17	0.14	0.34	0.32	0.23	0.16	0.14
Fraction of prod. tasks outsourced:	0.02	0.16	0.55	0.72	0.77	0.44	0.47	0.62	0.73	0.77
Import share of production	0.00	0.83	0.98	0.99	0.99	0.00	0.11	0.48	0.65	0.70
Wage:										
service	0.67	0.62	0.91	1.14	1.25	0.91	0.93	1.06	1.18	1.25
production	1.48	1.17	0.93	0.73	0.63	1.14	1.11	0.91	0.72	0.63
Skill premium (service/production)	0.46	0.53	0.98	1.56	1.97	0.79	0.84	1.17	1.63	1.97
Real wage:										
service	0.75	0.85	1.40	2.02	2.44	1.18	1.23	1.61	2.10	2.44
production	1.66	1.61	1.42	1.30	1.24	1.48	1.47	1.37	1.28	1.24
Welfare (real per capita cons.)	1.12	1.16	1.41	1.73	1.96	1.30	1.33	1.51	1.77	1.96

1. Production value added does not include service tasks (i.e. purchased services)

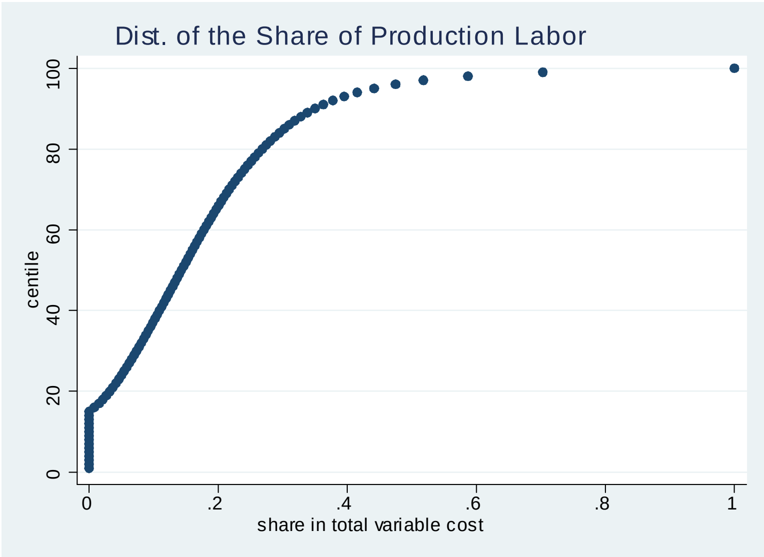
2. Wage is normalized so that labor income of the World is 1

Table 4: Firm-Level Results of Simulation

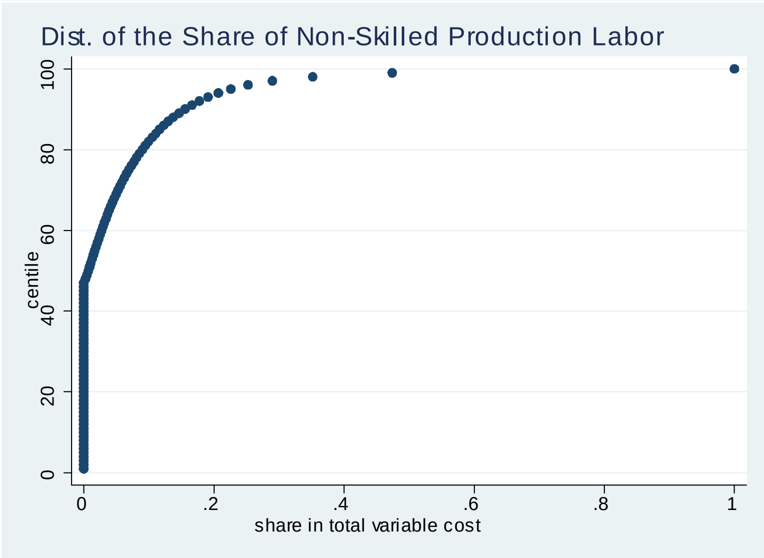
	Country Size					
	L=0.001	L=0.009	L=0.09	L=0.2	L=0.3	L=0.4
Measures of firms:						
producing	0.01	0.05	0.64	1.65	2.59	3.47
selling	0.04	0.34	2.21	3.72	4.62	5.27
Measures normalized by Labor:						
producing	5.7	5.8	7.1	8.2	8.6	8.7
selling	42.4	37.7	24.5	18.6	15.4	13.2
Fraction of firms:						
exporting						
selling domestically	0.02	0.15	0.71	0.88	0.92	0.93
Mean # customers per firm:	1.03	1.19	2.28	3.58	4.65	5.62
Size distribution (percentiles):						
25th	1	1	1	1	1	1
50th	1	1	1	2	2	2
75th	1	1	2	3	4	5
90th	1	2	4	7	10	12
95th	1	2	6	12	17	21
99th	2	4	15	30	43	55

Table 5: Firm-Level Results with Different Trade Costs

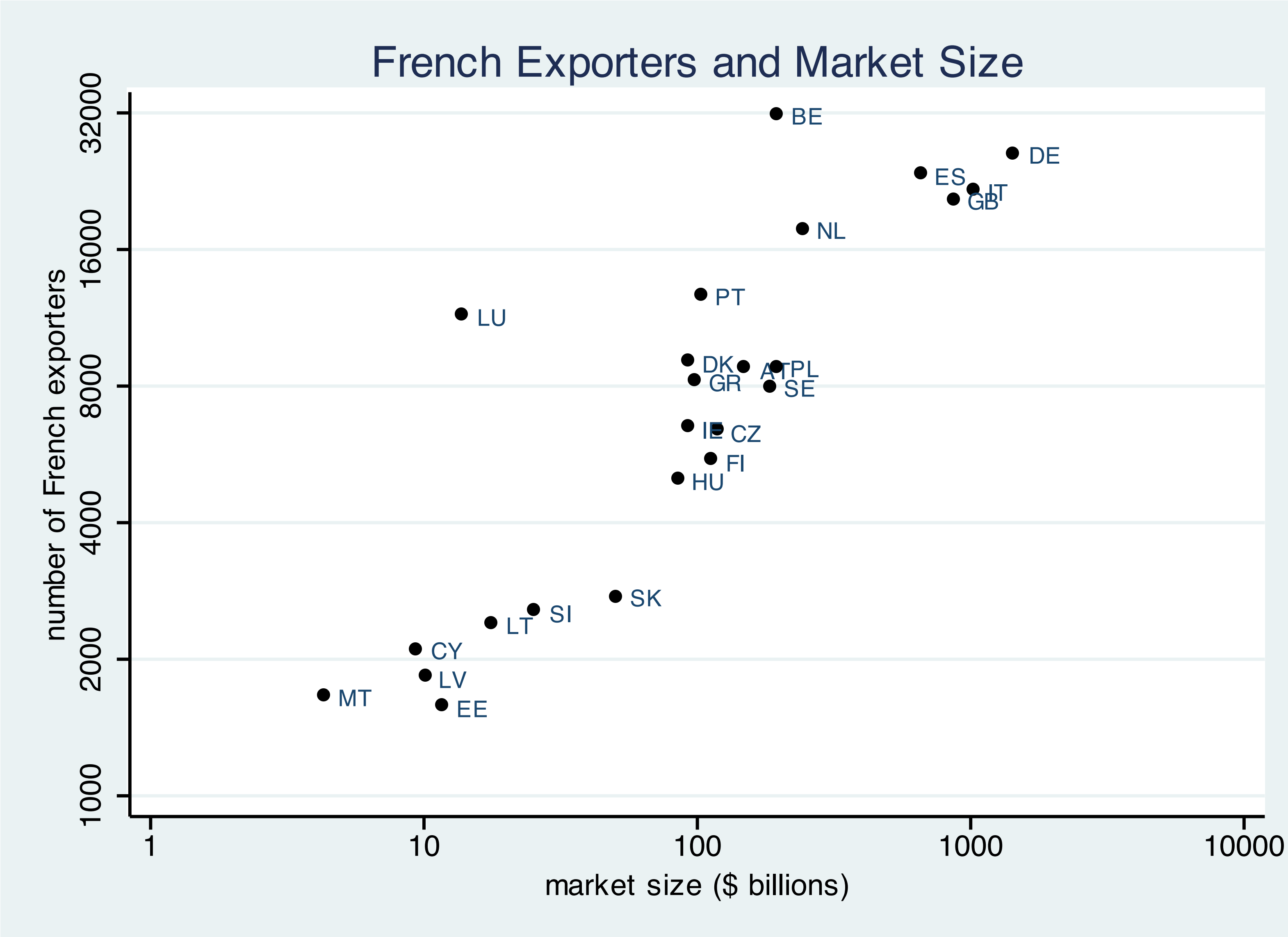
	Trade Cost (small country, L=.009)					Trade Cost (large country, L=0.3)				
	10.00	1.80	1.20	1.05	1.00	10.00	1.80	1.20	1.05	1.00
Measures of firms:										
producing	0.00	0.03	0.05	0.04	0.03	7.62	6.36	2.59	1.44	1.08
selling	0.00	0.07	0.34	0.33	0.28	7.62	7.16	4.62	2.98	2.28
Measures normalized by Labor:										
producing	0.4	3.4	5.8	4.5	3.6	25.4	21.2	8.6	4.8	3.6
selling	0.4	8.1	37.7	36.5	31.3	25.4	23.9	15.4	9.9	7.6
Fraction of firms:										
exporting										
selling domestically	1.00	0.41	0.15	0.09	0.08	1.00	1.00	0.92	0.73	0.63
Mean # customers per firm:	1.01	1.06	1.19	1.29	1.36	5.48	5.25	4.65	5.11	5.62
Size distribution (percentiles):										
25th	1	1	1	1	1	1	1	1	1	1
50th	1	1	1	1	1	2	2	2	2	2
75th	1	1	1	1	1	5	5	4	5	5
90th	1	1	2	2	2	12	12	10	11	12
95th	1	2	2	3	3	21	20	17	19	21
99th	2	2	4	5	5	53	50	43	49	55

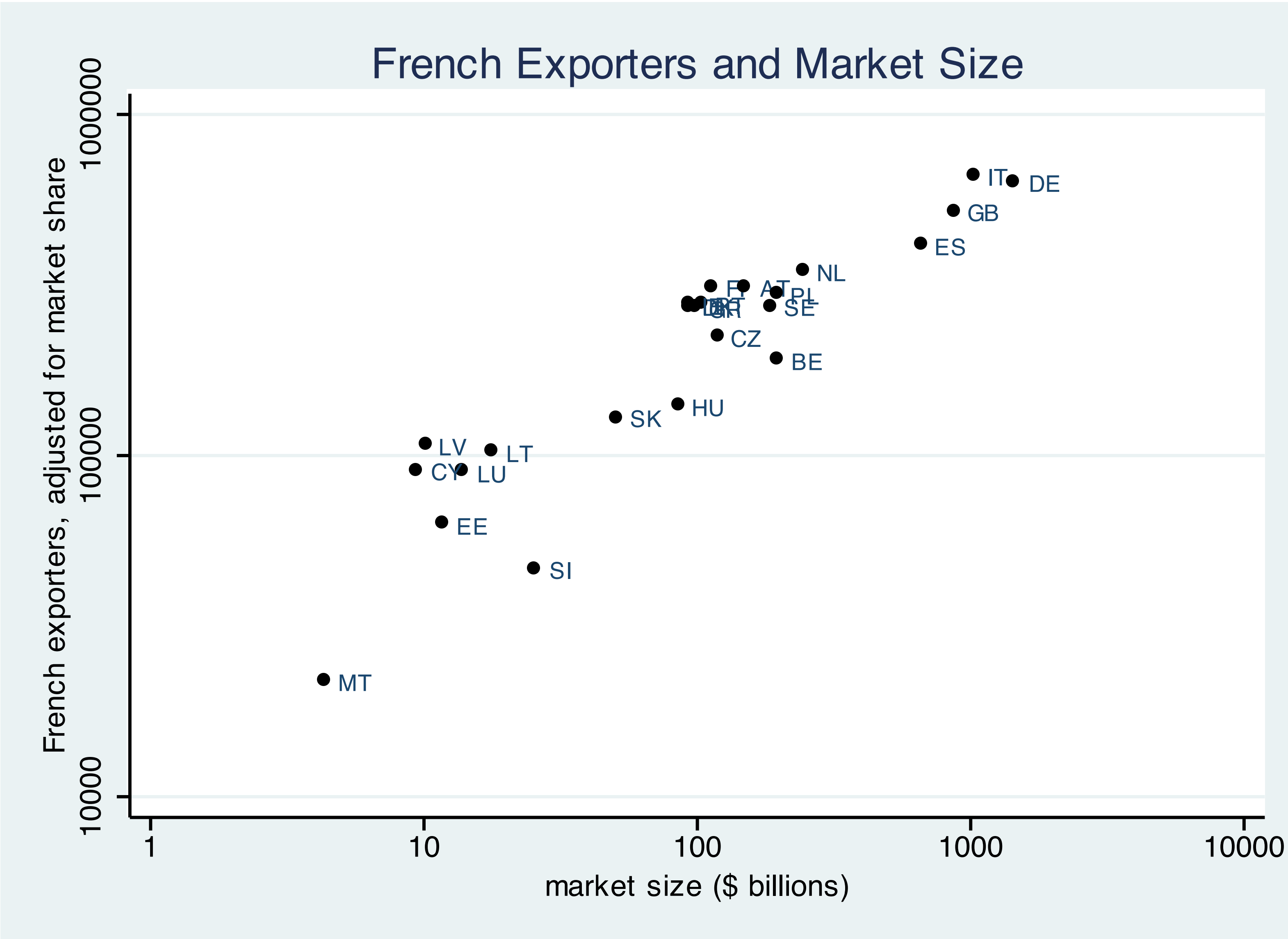


Dist. of the Share of Production Labor
centile
0
20
40
60
80
100
0
.2
.4
.6
.8
1
share in total variable cost

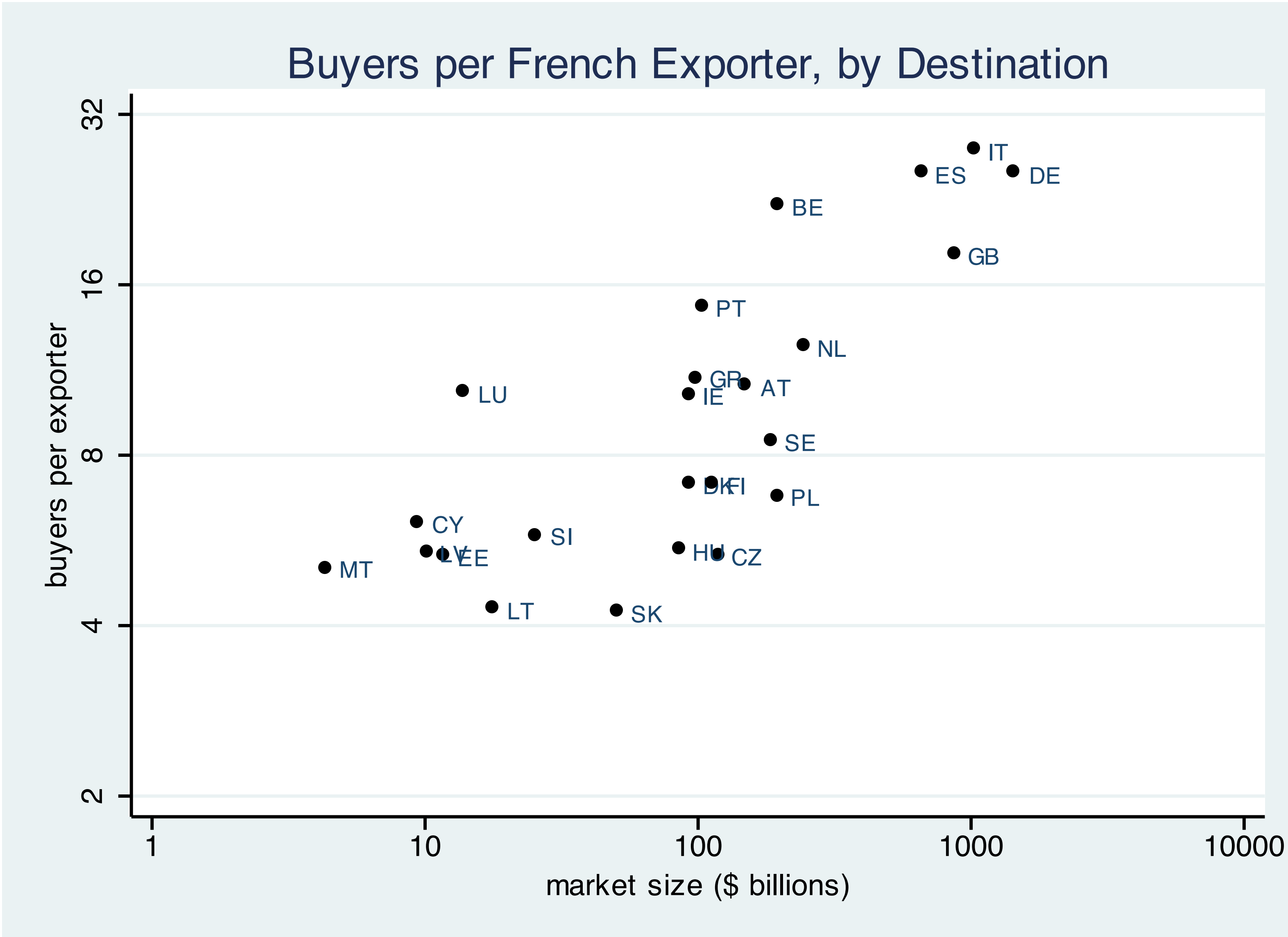


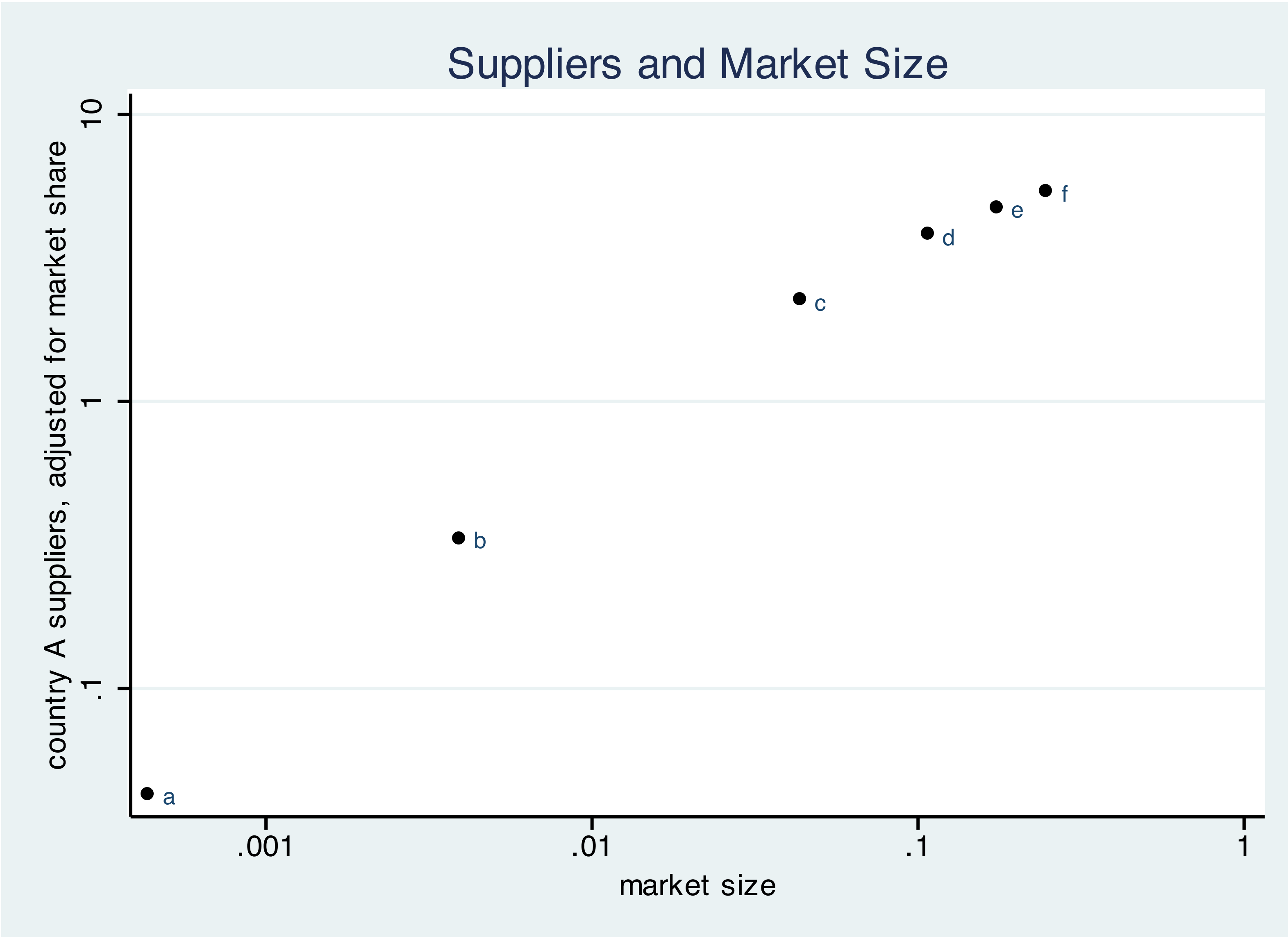
Dist. of the Share of Non-Skilled Production Labor
centile
0
20
40
60
80
100
0
.2
.4
.6
.8
1
share in total variable cost

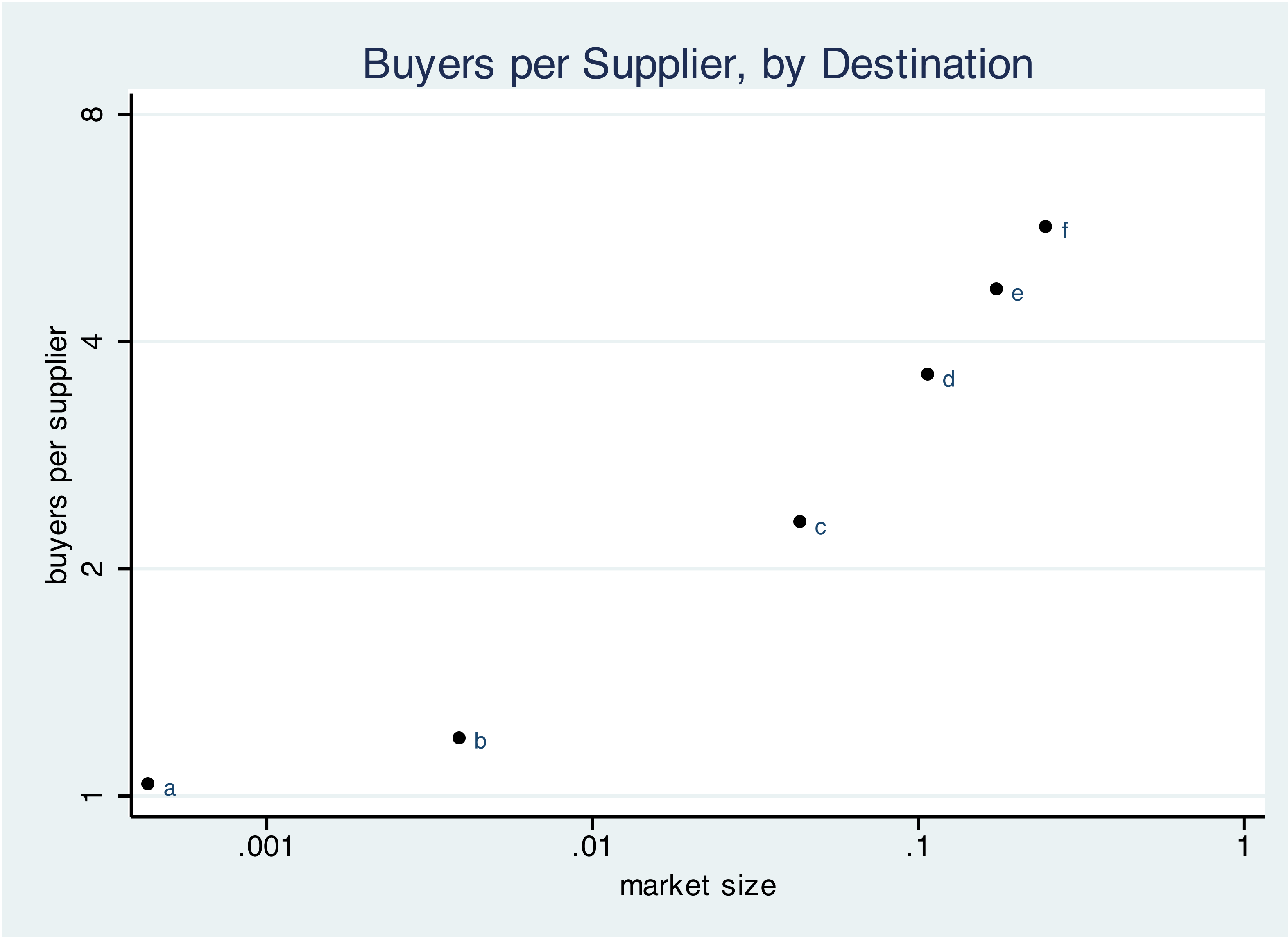

French Exporters and Market Size
number of French exporters
1000
2000
4000
8000
16000
32000
market size ($ billions)
1
10
100
1000
10000
BE
DE
ES
IT
GB
NL
PT
LU
DK
AT
PL
GR
SE
IE
CZ
FI
HU
SK
SI
LT
CY
LV
MT
EE



French Exporters and Market Size
French exporters, adjusted for market share
10000
100000
1000000
market size ($ billions)
1
10
100
1000
10000
IT
DE
GB
ES
NL
AT
PL
SE
CZ
BE
HU
SK
LV
LT
CY
LU
EE
SI
MT


Buyers per French Exporter, by Destination
buyers per exporter
market size ($ billions)
2
4
8
16
32
1
10
100
1000
10000
IT
ES
DE
BE
GB
PT
NL
LU
GR
AT
IE
SE
DK
FI
PL
CY
SI
LV
EE
MT
HU
CZ
LT
SK


Suppliers and Market Size
country A suppliers, adjusted for market share
10
1
.1
a
b
c
d
e
f
.001
.01
.1
1
market size


Buyers per Supplier, by Destination
buyers per supplier
8
4
2
1
.001
.01
.1
1
market size
a
b
c
d
e
f