The Risk Sharing Benefit versus the Collateral Cost: The Formation of the Inter-Dealer Network in Over-the-Counter Trading

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Abstract:

The decentralized over-the-counter (OTC) market generates a trading network among dealers. In this paper, I model the driver behind the formation of this inter-dealer network (the selling network in particular) as the need for dealers to share risk. The trade-off between the benefit of risk-sharing and the funding cost of collateral determines the shape of the inter-dealer network. In equilibrium, dealers' markups and trading volumes increase with the number of links they have to other dealers, whereas dealers' inventory risks decrease as they form links. In addition, when capacity of providing liquidity differentiates dealers, the network formed exhibits the empirically observed core-periphery structure. Specifically, dealers with large capacity comprise the core of the network, connecting them to all other dealers, while dealers who have small capacity operate at the periphery. My model matches recent empirical findings on the negative relationship between order sizes and markups. More importantly, I show that there may be structural breaks in this negative relationship as variations in order sizes may alter the inter-dealer network. These results suggest that empirical studies on OTC markets should control for the stability of an inter-dealer network to avoid model misspecification.

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1. Introduction

Over-the-counter (OTC) markets have grown exponentially in the last decade. As OTC markets grow, researchers have increasingly investigated the trading structure of these markets. Research interest is further stimulated by the recent financial crisis, which has brought attention to the OTC market for subprime mortgage derivatives. However, studies on OTC markets overlook an important element — inter-dealer trading. Since dealers act as intermediaries in OTC markets, inter-dealer trades should affect trades between dealers and other market participants, and hence affect the entire market. To provide new insights into how inter-dealer trades influence OTC trading, I study an important aspect of inter-dealer trades, the dealers' trading network. Specifically, I ask several questions. How does such a network form? What determines a dealer's position within the network? How does the network affect price determination in an OTC market?

To address these questions, I construct a theoretical model to study how dealers strategically form an inter-dealer network (the selling network in particular), and I then examine how such an inter-dealer network affects other aspects of an OTC market.[2] In my model, OTC dealers form an inter-dealer network and trade through the network to share their inventory risks. The more links a dealer has, the more benefits the dealer obtains from risk-sharing. But the more links a dealer has, the greater are the costs he has to bear for maintaining his links. A major part of this linking cost comes from the funding constraint of collateral. For example, a seller in the CDS markets is usually required to post collateral as a protection in case he fails to deliver his commitment.[3]

[2] I focus on the selling network because dealers are more likely to be net sellers in OTC derivative markets. Empirical research by Peltonen, Scheicher, and Vuillemey (2014) shows that dealers are net sellers in the CDS markets.
[3] Duffie, Scheicher, and Vuillemey (2014) provide detail discussion on collateral requirements in the CDS markets.

Preparing collateral could be costly because of funding liquidity. A dealer with many links faces a larger linking cost, as he has to prepare a larger collateral pool in the event of selling in the inter-dealer market.

The trade-off between the benefit of risk-sharing and the funding cost of collateral determine the shape of the inter-dealer network. At one extreme, when the collateral cost is trivial compared with the risk-sharing benefit, the inter-dealer network is a complete network. In a complete network, all dealers are connected. At the other extreme, when the collateral cost is overwhelming, the inter-dealer network is an empty network, one in which no dealer is connected with any other dealer. Between these two extremes, inter-dealer networks can exhibit connectedness to varying degrees depending on the risk-sharing benefit and the collateral cost.

Inter-dealer networks affect OTC trading insofar as the number of links a dealer has influences his markup (the difference between the price for which a dealer buys a security and the price at which he sells it), trading volume, and inventory risk. In a more connected network, a dealer has more links, which leads to a higher markup, higher volume, and lower inventory risk. In such a network, having more links gives a dealer greater market power in the inter-dealer market, which enables the dealer to sell at a higher price to other dealers. Since a markup is proportional to its corresponding inter-dealer price, this highly connected dealer charges a higher markup. Having more links also provides a dealer with more opportunities to trade in the inter-dealer market. As a result, the dealer completes more trades and manages his inventory risk more effectively.

My model resonates with recent empirical studies which show that inter-dealer networks have a significant influence on OTC trading. Hollifield, Neklyudov, and Spatt (2012) study the inter-dealer network of securitization markets (e.g., asset-backed securities, collateral debt obligations, commercial mortgage-backed securities, and collateral mortgage obligations) and Li and Schürhoff (2012) study the inter-dealer network operating in the municipal bond markets. Both studies document that the structure of the inter-dealer network correlates with dealers' markups in OTC trading. Moreover, they show that inter-dealer networks across OTC markets exhibit structural similarity in spite of trading distinct classes of assets. This common structure is the core-periphery structure. That is, some dealers are closer to the center of a network than others.

In the abovementioned empirical studies, inter-dealer networks are treated as exogenously determined. This limits the capacity of the analyses to explain why inter-dealer networks form the observed core-periphery structure, and how the core-periphery network is related to prices in OTC trading. In principle, inter-dealer networks should be jointly determined with prices and trading volumes in equilibrium, since these are outcomes based on dealers' decisions. This suggests that theoretical models are needed to explain the formation of inter-dealer networks. More importantly, such theoretical models should generate new empirical implications by treating inter-dealer networks as endogenously determined rather than exogenously determined as in past empirical studies. The theoretical model I construct in this paper satisfies these conditions.

Using differences in dealers' capacity of providing liquidity, my model explains the core-periphery feature of an inter-dealer network. Large-capacity dealers who can accommodate large orders comprise the core, while small-capacity dealers who only accommodate small orders become the periphery. This gives a novel testable empirical prediction regarding a dealer's location in a network: a dealer's capacity of liquidity provision positively determines his centrality (a measure that captures how central a dealer is in a network).

In addition, I show that the unconditional relationship between investors' trading prices and dealers' centrality is ambiguous. Dealers with high centrality do not necessarily offer better prices to investors than dealers with low centrality. However, this relationship is determined when it is conditioned on the size of the investor order. On orders with the same size, high-centrality dealers offer investors more favorable prices than low-centrality dealers. This conditional relationship between investors' trading prices and dealers' centrality is consistent with empirical findings in Hollifield, Neklyudov, and Spatt (2012). The above suggests that the order size is an important control variable in determining how centrality is related to investors' trading prices.

Another novel empirical implication arising from my model involves potential structural breaks in the price-size and price-volatility relationships in OTC markets. Changes in order sizes or volatility can alter the fundamental structure of an economy, which in the setup of this work is the inter-dealer network. As a result, sudden structural jumps emerge in these relationships. Based on this result, I suggest that empirical studies examining OTC markets should control for the stability of an inter-dealer network in order to avoid model misspecification. Empirical research should, for example, include a measure of a network's connectedness as an additional control variable interacting with other control variables in the regression model.

To the best of my knowledge, my study is the first to study strategic formation of an inter-dealer network arising from dealers' risk-sharing needs. My model not only confirms existing empirical findings, but also provides new empirical implications pertaining to OTC markets. Malamud and Rostek (2013) have also studied dealers who share risks through inter-dealer networks. While they focus on dealers' strategic interactions in simultaneous trading on the network, I emphasize the formation process of the network. Although I model the rise of an inter-dealer network from a risk-sharing perspective, I do not rule out other possible forces that may generate such a network. For example, sharing information is a possible incentive for building a dealers' network.

My study is also the first to apply the risk-sharing idea of the network formation literature to a specific type of financial market, the OTC market. This approach provides the advantage of identifying the relationship between agents' payoffs and primitive parameters, e.g., order sizes and volatility, as the trading protocol and needs are concrete and specific. As a result, I can explore issues that have not yet received much attention. For example, I consider how order sizes and volatility contribute to determining a network as well as how they affect equilibrium outcomes such as prices and quantities traded through the network.

In the next section, I review the related literature. Section 3 presents the benchmark model and Section 4 analyses the equilibrium results. In Section 5, I extend the benchmark model to a case in which dealers' capacity of providing liquidity varies and show the core-periphery network that emerges in equilibrium. Section 6 discusses the implications of the model for "hot potato" trading, which involves trades that occur between successive dealers. The empirical implications are summarized in Section 7. Finally, I conclude in Section 8.

2. Literature Review

Inter-dealer trading has been an important subject in market microstructure studies for a long time. Ho and Stoll (1983) point out that inter-dealer trading benefits dealers, since dealers are better able to manage their inventory risks by trading among themselves instead of filling an investor order with uncertain arrival. Viswanathan and Wang (2004) show that inter-dealer trading also benefits investors. In their model, an investor prefers trading with one dealer and letting that dealer unwind his extra inventory later in the inter-dealer market rather than splitting up the order and trading with multiple dealers. Thus, inter-dealer trading is beneficial to both dealers and investors. Both papers model the incentive for inter-dealer trading as the sharing of inventory risks. Based on this risk sharing idea**,** others build models to study issues such as price formation, information transmission, and transparency in multi-dealer markets (see Biais (1993), Lyons (1997), Naik, Neuberger, and Viswanathan (1999), de Frutos and Manzano (2002), Yin (2005), and Cao, Evans, and Lyons (2006)). Empirical evidence supports risk-sharing as the main driver behind inter-dealer trading. Reiss and Werner (1998) and Hansch, Naik, and Viswanathan (1998) find that dealers on the London Stock Exchange use the inter-dealer market primarily to share their inventory risks. In the foreign exchange market, Lyons (1995) finds that dealers control risk by systematically laying off inventory to other dealers.

Another thread of literature to which this work contributes, studies price determination in OTC markets. Duffie, Garleanu, and Pedersen (2005, 2007) study how search and bargaining determine prices in OTC markets.[4] Spulber (1996), employing an alternative type of search model, shows that prices in decentralized markets (OTC markets) are determined by dealers' transaction costs.[5] In addition, dealers' transaction costs also affect OTC market structure. Atkeson, Eisfeldt, and Weill (2013) show that market entry costs help to determine the structure of OTC trading, and thereby prices charged in OTC trading. Past studies also show that dealers' strategies influence price determination. For example, Zhu (2012) shows that repeated visits to the same dealer results in a less favorable price for the trader. Empirically, the price of an asset in OTC trading seems to depend on order sizes and transparency of the market environment. Green,

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[4] Vayanos and Wang (2007), Vayanos and Weill (2008), and Weill (2008) extend the original model to study OTC markets with multiple assets. Lagos and Rocheteau (2009) relax the assumption on constraint asset holdings in the original model, which enables market participants to accommodate trading frictions by adjusting their asset positions.

[5] This type of search model also receives extended treatment in the literature. Rust and Hall (2003) extend the original model by introducing a centralized market to compete with decentralized markets. Zhong (2013) incorporates Knightian uncertainty into the search process to study the impact of transparency on OTC markets.

Hollifield, and Schürhoff (2007) find that a dealer earns smaller markups on larger trades in municipal bond markets. This negative relationship between order sizes and markups is also found in corporate bond markets by Schultz (2001) and Randall (2013). Bessembinder, Maxwell, and Venkataraman (2006), Goldstein, Hotchkiss, and Sirri (2007), and Edwards, Harris, and Piwowar (2007) estimate the bid-ask spread in the OTC market for corporate bonds, finding that more transparent bonds have smaller bid-ask spreads. Recently, new empirical studies (Li and Schürhoff (2012) and Hollifield, Neklyudov, and Spatt (2012)) have discovered a new factor that affects prices in OTC markets, namely the inter-dealer network.

Finally, my study adds to the growing literature on network studies in financial markets. Compared with the rich applications of network theory that have been made to other areas in economics, the application of network theory to financial markets has only just begun.[6] Blume et al. (2009) and Gale and Kariv (2007) study how a network intermediates trades in a decentralized market. Gofman (2011) assesses the efficiency of resource allocation through the trading network in an OTC market. Malamud and Rostek (2013) develop a general framework for studying dealers' strategic interactions in decentralized markets. The decentralized market in their model is represented by a hypergraph (an abstract network, loosely speaking). Breton and Vuillemey (2014) perform a numerical analysis on the network of credit exposures in OTC derivative markets to examine impacts from different regulatory collateral and clearing requirements. Many past studies also focus on information acquisition from a network and its impact on financial markets. Han and Yang (2012) extend the rational expectation equilibrium model to study the information network in a financial market. Babus and Kondor (2012) study

[6] Economic research on networks has tapped into various fields, such as job hunting in labor economics, decentralized market trading in microeconomics, and international alliance and trading agreements in macroeconomics. Jackson (2008) and Easley and Kleinberg (2010) provide excellent surveys of network applications in economic research.

information transmission through inter-dealer networks in OTC markets by extending the model in Vives (2011) to games in networks. In addition to using network models to study OTC markets, others apply network models to the inter-bank market to analyze the contagion risk in the banking system (see Leitner (2005), Babus (2013), Blume et al. (2013), and Elliott, Golub, and Jackson (2013)). There is also a growing body of empirical studies that explore networks' implications on a variety of topics ranging from return predictability to CEOs' wages (see Cohen and Frazzini (2008), Cohen, Frazzini, and Malloy (2008), and Engelberg, Gao, and Parsons (2012)).

3. The Model

3.1 The Environment

Suppose there are $N \geq 2$ dealers in an OTC market. All dealers have the same mean-variance utility function over their wealth W, and all dealers have the same risk-aversion parameter $\rho > 0$. That is,

$$U(W) = E(W) - \frac{\rho}{2} Var(W). \tag{1}$$

The initial endowment, consisting of a portfolio of m units of a risk-free asset and I units of a risky asset, is identical for all dealers. In this initial endowment, the risk-free asset has a constant value of 1, while the risky asset has a random value v following a normal distribution $\boldsymbol{N}(\bar{v}, \sigma^2)$.

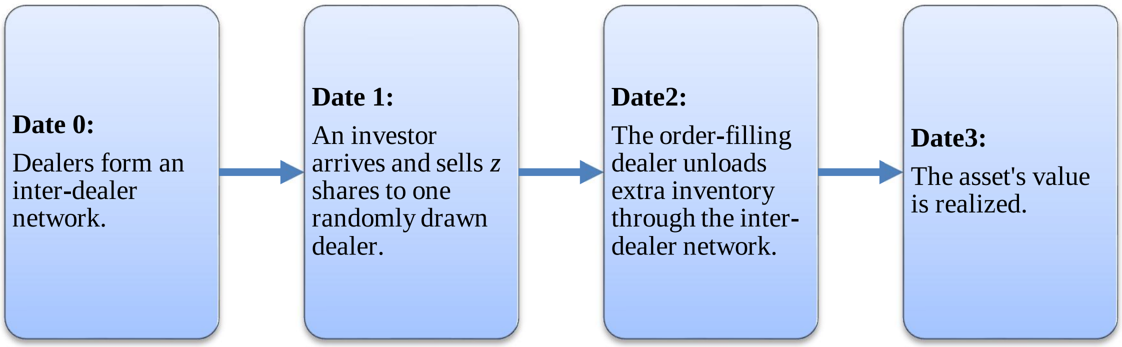

Figure 1: The Timeline

Figure 1 illustrates the timeline within the model. The timeline goes as follows. At date 0, dealers strategically form an inter-dealer network by building or severing links between each other. At date 1, an investor arrives and wants to trade an order of size z. Only one dealer in the network meets this investor, with a probability of meeting of $\frac{1}{N}$, which is the matching rate. Assuming that the matching rate is $\frac{1}{N}$ implies that the arriving investor meets and trades with a dealer with probability one. The price of the investor-dealer transaction is p_1. At date 2, the dealer who fills the investor's order at date 1 re-trades with other dealers to adjust his inventory risk. However, this order-filling dealer can trade only with those dealers who are connected to him. In this inter-dealer trade, the order-filling dealer solicits bids from his connected dealers, and then chooses the price that clears the market. To differentiate that price from the investor-dealer price p_1, I denote the price in the inter-dealer market as p_2. Finally, at date 3, the value of the risky asset is realized. In Section 6, I extend the model to consider multiple rounds of inter-dealer trading before the value of the asset is realized. By doing so, I am able to generate implications on “hot potato” trading in inter-dealer markets.

I assume that the risk-aversion parameter, the initial endowment, the distribution of the risky asset’s value, the matching rate, and the cost of adding links are common knowledge to all dealers. Further, I assume that the arriving investor is a seller. Another interpretation of this assumption is that the order-filling dealer (at date 1) faces a positive order imbalance that he has to sell in the inter-dealer market to balance his inventory. An example of this assumption is the AIG, whose book consisting almost solely of sold protection before the crisis.

Since the equilibrium is solved by backward induction, I discuss the equilibrium at each date in a backward sequence in the following sections.

3.2 The Inter-Dealer Trade at Date 2

In an inter-dealer market, a given dealer is able to contact several other dealers to explore their interest in trading through inter-dealer brokers. Typically, a dealer who has filled an investor’s order solicits bids from other dealers. Then, as soon as the order-filling dealer receives quotes from interested dealers he chooses the price to clear the market. Past studies use search-theoretic models to capture such an inter-dealer trade (see Duffie, Garleanu, and Pedersen (2005, 2007) and Lagos and Rocheteau (2009)). Those studies postulate that an order-filling dealer sequentially searches for another dealer with whom to conduct a bilateral trade. Recently, empirical studies by Saunders, Srinivasan, and Walter (2002), Dunne, Hau, and Moore (2010), and Hendershott and Madhavan (2013) suggest that inter-dealer trading in OTC markets has become more like multilateral trading than bilateral trading. Services from inter-dealer brokers and the evolution of inter-dealer markets into limit-order book alike systems enable the order-filling dealer to approach other dealers at the same time rather than searching sequentially among dealers. To capture this multilateral feature of inter-dealer trading, I model the inter-dealer trade as an auction of shares, as in Viswanathan and Wang (2004).[7]

To reflect that an order-filling dealer trades only through his inter-dealer network, I modify the model in Viswanathan and Wang (2004) by restricting the order-filling dealer to soliciting bids only from his connected dealers. Specifically, if dealer i fills the investor's order at date 1, he then announces an auction at date 2 to all his connected dealers. In the auction, dealer i's connected dealers submit their demand schedules, which are combinations of prices and quantities, to dealer i. After dealer i collects those demand schedules, he chooses the price and quantity to clear the market.

Following Viswanathan and Wang (2004), in such an inter-dealer trade auction, dealer i's equilibrium strategy is

$$\left\{\left(\underbrace{I + z - X^{A,i}}_{\text{quantity supply}}, \underbrace{p_2^i}_{\text{the inter-dealer price}}\right) : X^{A,i} = (\overline{v} - p_2^i)\gamma + \frac{I + z}{n_i}\right\}, \tag{2}$$

where $X^{A,i}$ is dealer i's risky holding after the inter-dealer trade, n_i is the number of links dealer i has, and $\gamma = (n_i - 1)/(n_i \rho \sigma^2)$. Let dealer j be a dealer who is linked to dealer i; dealer j's equilibrium strategy is

$$\left\{\left(\underbrace{X^{B,j}}_{\text{quantity demand}}, \underbrace{p_2^i}_{\text{the inter-dealer price}}\right) : X^{B,j} = (\overline{v} - p_2^i)\gamma - \frac{n_i - 1}{n_i} I\right\}, \tag{3}$$

where $X^{B,j}$ is the quantity demanded by dealer j. The market-clearing condition, which requires that $I + z - X^{A,i} = \sum_{j:j \text{ is linked with } i} X^{B,j}$, indicates that the inter-dealer price is

$$p_2^i = \overline{v} - \rho\sigma^2 \left(I + \frac{z}{n_i + 1}\right). \tag{4}$$

[7] The share auction is also called a uniform-price double auction. In such an auction, each player (the dealer in my model) bids for his residual supply and the market-clearing condition determines the price. This trading structure is used extensively in the literature to study the impact of strategic player interactions on asset prices (e.g., Kyle (1989), Vives (2011), and Malamud and Rostek (2013)).

Viswanathan and Wang (2004) prove that the above strategies (Eq.(2), Eq.(3)) and price (Eq.(4)) constitute a unique linear equilibrium in the inter-dealer trade. In the linear equilibrium, dealer i's risky holding after the inter-dealer trade is

$$X^{A,i} = I + \frac{2z}{n_i + 1}, \tag{5}$$

and dealer j receives

$$X^{B,j} = \frac{n_i - 1}{n_i}\frac{z}{n_i + 1}, \tag{6}$$

shares of the risky asset from the inter-dealer trade. Dealers who are not connected with dealer i maintain their risky holdings as before. Eq.(6) indicates that the minimum number of links for ensuring that the inter-dealer trade occurs is two, since $n_i < 2$ implies that $X^{B,j} \leq 0$. In other words, if the order-filling dealer connects to only one other dealer, no inter-dealer trade occurs.

Both dealer i and dealer j benefit from the inter-dealer trade. For dealer i, his welfare increases by

$$Eu\left(W\left(X^{A,i}\right)\right) - Eu\left(W(I + z)\right) = \frac{\rho\sigma^2 z^2}{2}\frac{n_i - 1}{n_i + 1} \geq 0. \tag{7}$$

And for dealer j, his welfare increases by

$$Eu\left(W\left(I + X^{B,j}\right)\right) - Eu\left(W(I)\right) = \frac{\rho\sigma^2 z^2}{2}\frac{n_i - 1}{n_i^2(n_i + 1)} \geq 0. \tag{8}$$

These benefits become more prominent when the risk increases (that is, increases in ρ, σ, and z), which reinforces the idea that the inter-dealer trade is a channel through which dealers share inventory risks. The benefit for dealer i increases with the number of links he has, whereas the benefit for dealer j decreases with the number of dealer i's links. This reflects the fact that dealer i uses his market power to extract more benefits from risk-sharing, since he can increase his selling price by exerting his market power (see Eq.(4)).

3.3 The Investor-Dealer Trade at Date 1

In an OTC market, direct trades between investors are rare, since each investor has his unique needs. In most cases, investors trade with OTC dealers. Having said that, it should be noted that investors cannot trade with multiple OTC dealers simultaneously. The lack of a centralized venue where dealers and investors can post their quotes implies that investors and dealers must search their counterparties for trades in OTC markets.[8] As a result, even though inter-dealer trades have evolved into multilateral trading, trades between investors and dealers remain bilateral.

Following precedent in the literature, I use a search-and-bargaining model to characterize the bilateral trading relationship between investors and dealers. To emphasize the influence of the inter-dealer network, I simplify the search problem. In particular, the probability that a dealer is matched with an incoming investor equals his matching rate $\frac{1}{N}$. The matching rate measures the intensity of a dealer's search for an investor.

When an investor meets a dealer, they bargain over the price. Following Nash (1950), the price is the solution of the following bargaining problem

$$\max_{p_1^i} \left(Eu\left(W\left(X^{A,i}\right)\right) - Eu\left(W(I)\right)\right)^q \left(z\left(p_1^i - M_0 + M_1\sigma^2\right)\right)^{1-q} \tag{9}$$

where q represents the dealer's bargaining power and $z(M_0 - M_1\sigma^2)$ is the investor's reservation value of holding the asset. Hence, $p_1^i - M_0 + M_1\sigma^2$ is the per unit utility gain for the investor if he sells. The investor's gains from the trade can arise from aspects such as the search cost, his information about the asset, his risk aversion, and so on.[9] To ensure that the investor is willing to

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[8] Although some inter-dealer markets have adopted limit-order book systems in which dealers can post their quotes, those systems are usually not accessible to investors.

[9] One can replace this reduced-form assumption by explicitly modeling the seller's decision; e.g., a seller having a

sell, I assume that $0 < M_0 - M_1\sigma^2 < \bar{v} - \rho\sigma^2\left(I + \frac{z}{2}\right)$.

If there exists an inter-dealer trade at date 2, then the order-filling dealer's gains from trade is $Eu\left(W\left(X^{A,i}\right)\right) - Eu\left(W(I)\right)$, which equals $z\left(p_2^i - p_1^i\right)$. p_2^i is the inter-dealer price from Eq.(4). Hence, the solution to the bargaining problem involving an inter-dealer trade is

$$p_1^i = (1-q)p_2^i + q(M_0 - M_1\sigma^2). \tag{10}$$

Eq.(10) implies that the investor-dealer's price, p_1^i, is proportional to the inter-dealer's price, p_2^i. In other words, when dealer i realizes that he can unload the extra inventory at a higher price in the inter-dealer market, he is more inclined to fill the investor's order at a higher price.

If there is no inter-dealer trade at date 2, then the order-filling dealer's final risky holding is $I + z$. This indicates that the order-filling dealer obtains $Eu\left(W(I+z)\right) - Eu\left(W(I)\right)$ in gains from trade, which equals $z\left(\bar{v} - \rho\sigma^2\left(I + \frac{z}{2}\right) - p_1^i\right)$. Under this case, the solution of the bargaining problem is

$$p_1^i = (1-q)\left(\bar{v} - \rho\sigma^2\left(I + \frac{z}{2}\right)\right) + q(M_0 - M_1\sigma^2). \tag{11}$$

In all,

$$p_1^i = \tag{12}$$

$$\begin{cases} (1-q)\left(\overline{v} - \rho\sigma^2\left(I + \dfrac{z}{n_i+1}\right)\right) + q(M_0 - M_1\sigma^2), & \text{when there exists inter-dealer trading,} \\ (1-q)\left(\overline{v} - \rho\sigma^2\left(I + \dfrac{z}{2}\right)\right) + q(M_0 - M_1\sigma^2), & \text{when there is no inter-dealer trading.} \end{cases}$$

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liquidity shock maximizes his mean-variance preference. This setting does not change the result of the model, but it adds considerable complexity and introduces more parameters.

3.4 Network Formation at Date 0

In Sections 3.2 and 3.3, I show that the inter-dealer price, the investor-dealer price, and trading volume of an inter-dealer trade depends on the equilibrium number of a dealer's links. In this section, I show that the equilibrium network determines the equilibrium number of a dealer's links. In particular, I demonstrate how the trade-off between the risk-sharing benefit and the funding cost of collateral determines the equilibrium network, and hence prices and trading volume in OTC trading.

At date 0, dealers strategically form and sever links with each other. For every link the dealer adds, he incurs larger linking costs. In reality, the predominant part of the linking cost comes from the funding cost of preparing collateral. Many OTC traded products are credit derivatives, which usually impose collateral requirements on the seller of the credit product. When a dealer adds more links to his network, he sells more in the inter-dealer market. (Recall that a dealer sells $I + z - X^{A,i} = z\left(1 - \frac{2}{n_i+1}\right)$ in his inter-dealer trading.) This means the dealer has to arrange more collateral in the event of selling in the inter-dealer market. Since the preparation of extra collateral is costly, the dealer incurs larger funding cost on collateral when he increases his links (see Brunnermeier and Pedersen (2008) for more discussion on funding liquidity in trading).

The funding cost of collateral is

$$\Pr\{\text{sell in inter-dealer trading}\} \times \underbrace{\sigma\, \overbrace{z\frac{n_i - 1}{n_i + 1}}^{\text{shares sold}}}_{\text{the risk of shares sold}} \times m. \tag{13}$$

Eq.(13) shows that the funding cost of collateral consists of three parts. The first part is the probability that collateral is needed in inter-dealer trading. That happens when the order-filling dealer needs to sell parts of his investor order in the inter-dealer market, since only the seller needs to post collateral in OTC trading. The second part measures the risk of shares sold, which is the standard deviation of the value of shares sold. The last part, m, is an arbitrary multiplier capturing the funding constraint. The larger the m, the higher the funding cost. This constraint could relate to the margin requirement or the aggregate stress of obtaining funding.

The value of adding links to a dealer is his risk-sharing benefit from inter-dealer trading as shown in Eq.(7) and Eq.(8). With the benefit and the cost specified, I can solve for the equilibrium network at date 0. A natural approach to modeling network formation is defining a non-cooperative game among dealers, and such a non-cooperative game generates an equilibrium outcome as a graph. An equilibrium network is such a graph, consisting of a set of nodes and pairs of links that connect those nodes. Hence, the equilibrium network G is written as $(\mathcal{N}, \mathcal{E})$. $\mathcal{N}$ is the set of all dealers, i.e., $\mathcal{N} = \{1, 2, 3, \dots, N\}$, and $\mathcal{E}$ is the set of all links among those dealers, i.e., $\mathcal{E} = \{ij\text{: for some } i, j \in \mathcal{N}\}$.

Although it is appealing to study network formation within a game-theoretical framework, there are problems. There are, for example, various ways to specify such a game, such as the simultaneous link-announcement game in Myerson (1977) and the sequential link-announcement game in Aumann and Myerson (1988). In addition, as pointed out by Jackson and Wolinsky (1996), some standard game-theoretic equilibrium notions are not suitable for the study of network formation, since those notions do not reflect communication and coordination in the formation of networks.

To circumvent the abovementioned problems, network theorists study properties of networks that are of interest to them and can be satisfied in the equilibria of some network-formation games. In this spirit, I define an equilibrium inter-dealer network formed at date 0 using the strong stability concept from Jackson and van den Nouweland (2005):

Definition 1

A network G' is obtainable from G via deviation by $\mathcal{N}' \subset \mathcal{N}$ if

i) $ij \in G'$ *and* $ij \notin G$ *implies* $\{i, j\} \subset \mathcal{N}'$*, and*

ii) $ij \in G$ *and* $ij \notin G'$ *implies* $\{i, j\} \cap \mathcal{N}' \neq \emptyset$.

In the above, $ij \in G$ *means that* i *and* j *are linked in network* G*, whereas* $ij \notin G$ *means that* i *and* j *are not linked in network* G. $\mathcal{N}$ *is the set of all nodes and* $\mathcal{N}'$ *is the subset of nodes.*

Definition 1 says that changes in a network can be made by a coalition $\mathcal{N}'$ without the consent of any dealers outside of $\mathcal{N}'$. Specifically, i) indicates that any new links that are built involve only dealers in $\mathcal{N}'$; ii) indicates that at least one dealer involved in any deleted link is in $\mathcal{N}'$.

Definition 2

Let $U_i(G)$ *be the payoff for dealer* i *in network* G. *Network* G *is strongly stable if, for any* $\mathcal{N}' \subset \mathcal{N}$, G' *is obtainable from* G *via deviations by* $\mathcal{N}'$*, and* $i \in \mathcal{N}'$ *such that* $U_i(G') > U_i(G)$*, there exists* $j \in \mathcal{N}'$ *such that* $U_j(G') < U_j(G)$.

Definition 2 states that one cannot find a coalitional deviation from a strongly stable network in which all relevant dealers are better off and with some are strictly better off. Strong stability requires that the network formed be coalition-proof. That is, a coalitional move from any subset of dealers cannot make all of them better off without hurting some dealers in this subset.

Requiring that a network exhibit strong stability imposes a requirement that is stricter than most other network stability requirements, since a strongly stable network makes tighter predictions due to coalitional considerations. Thus, strong stability is more robust than other definitions of an equilibrium network. In addition, the concept of being coalition-proof, which is used for cases in which players can communicate before they play a game, is particularly applicable to describing the equilibrium of an inter-dealer network. In an inter-dealer market, communications among dealers are almost inevitable.

Another appealing feature of strong stability is that a strongly stable network is the outcome of a pure strategy Nash equilibrium from Myerson's (1977) simultaneous link-announcement game. More importantly, such a strongly stable network is the Pareto-efficient outcome of this simultaneous link-announcement game (see Jackson and van den Nouweland (2005) and Jackson (2008)).

To simplify the notation, let $m^* = \frac{2m}{\rho\sigma z}$ be the effective margin. Given a network G, dealer i's payoff $U_i(G)$ is

$$U_i(G) = \overbrace{\frac{1}{N}Eu\big(W(X^{A,i})\big)}^{i \text{ fills the order}} + \overbrace{\sum_{j:ij\in G}\frac{1}{N}Eu\big(W(X^{B,j} + I)\big)}^{i'\text{s connected dealer fills the order}} \tag{14}$$

$$+ \overbrace{\left(1 - \frac{1}{N} - \sum_{j:ij\in G}\frac{1}{N}\right)Eu\big(W(I)\big)}^{\text{neither } i \text{ nor his connected dealers fill the order}}$$

$$- \overbrace{\frac{1}{N}\sigma z\frac{n_i - 1}{n_i + 1}m}^{\text{total cost of links}}$$

$$= \begin{cases} \dfrac{\rho\sigma^2 z^2}{N}\left((q - m^*)\dfrac{n_i - 1}{2(n_i + 1)} + \displaystyle\sum_{j:ij\in G}\frac{1}{2}\frac{n_j - 1}{n_j^2(n_j + 1)}\mathbf{1}_{[n_j\ge 2]}\right) + U_0, & n_i \ge 1 \\ U_0, & n_i = 0 \end{cases},$$

where n_i is the number of links dealer i has in network G, $\mathbf{1}_{[n_j\ge 2]}$ is an indicator function that takes 1 when $n_j \geq 2$ and 0 otherwise, and U_0 is the payoff when dealer i has no link,

$$U_0 = \frac{qz(\overline{v} - \rho\sigma^2 I - M_0 + M_1\sigma^2)}{N} + Eu\big(W(I)\big) - \frac{\rho\sigma^2 qz^2}{2N}. \tag{15}$$

Since all dealers are identical ex-ante, a strongly stable network in equilibrium should be symmetric. That is, all dealers should obtain the same level of payoff. If not, then some dealers enjoy higher payoffs than others. In such cases, dealers with lower payoffs could deviate together with those connected to a higher payoff dealer to provide an improving deviation. Thus, the original network would not be strongly stable. **Proposition 1** formalizes this intuition.

<u>Proposition 1</u>

Let a connected component be a sub-graph in which any two nodes are either directly connected or indirectly connected through a path consisting of several links. In a strongly stable network, all dealers in the same connected component, which has more than one connection, have the same number of connections. If such a strongly stable network consists of more than one connected component, then dealers in distinct components obtain identical payoffs.

The symmetry of a strongly stable network suggests that the total number of dealers, N, affects the existence of such a network. For example, when N is 6, symmetric networks are those in which every dealer has 2, 3, or 5 links. Any discontinuity between links in a symmetric network implies that no strongly stable network involving those links exists. In the above case, when N is 6, there is no strongly stable network in which every dealer has 4 links. To avoid such discontinuities, I assume that N equals 2^k, where k is an integer greater than one. Under this assumption, a symmetric network can have links the number of which equals any integer between 2 and $2^k - 1$.

In **Proposition 2**, I characterize a strongly stable network in equilibrium. Together with

Eq.(2), Eq.(3) and Eq.(4), which characterize the inter-dealer equilibrium, and Eq.(10), which characterizes the price of the investor-dealer trade, **Proposition 2** describes the equilibrium of the model.

Proposition 2

The following describes a strongly stable network in equilibrium.

a) If $m^* > q + 1$, then the strongly stable network is an empty network.

b) If $m^* < q + \frac{2N-1}{(N-1)^2} - \frac{1}{2}$, then the strongly stable network is a complete network.

c) If $q + \frac{2N-1}{(N-1)^2} - \frac{1}{2} \leq m^* \leq q + 1$, the strongly stable network is such that all dealers have the same number of links, and the number equals $\left[\!\left[\frac{1+\sqrt{2-2q+2m^*}}{1-2q+2m^*}\right]\!\right]$.[10]

In equilibrium, risky asset holdings, the inter-dealer price, and the investor-dealer price depend on the number of dealers' links. Specifically, under a), there is no inter-dealer network. Hence, there is no inter-dealer trading. The investor-dealer price is

$$p_1^{a)} = (1-q)\left(\bar{v} - \rho\sigma^2\left(I + \frac{Z}{2}\right)\right) + q(M_0 - M_1\sigma^2). \tag{16}$$

Under b), the price in the inter-dealer trade is

$$p_2^{b)} = \bar{v} - \rho\sigma^2\left(I + \frac{Z}{N}\right), \tag{17}$$

and the price in the investor-dealer trade is

$$p_1^{b)} = (1-q)p_2^{b)} + q(M_0 - M_1\sigma^2). \tag{18}$$

Under c), the inter-dealer price is

[10] $\left[\!\left[\frac{1+\sqrt{2-2q+2m^*}}{1-2q+2m^*}\right]\!\right] = \arg\max_{n\in\left\{\left\lfloor\frac{1+\sqrt{2-2q+2m^*}}{1-2q+2m^*}\right\rfloor, \left\lfloor\frac{1+\sqrt{2-2q+2m^*}}{1-2q+2m^*}\right\rfloor+1\right\}} \left(q - m^* + \frac{1}{n}\right)\frac{n-1}{n+1}$, where $\left\lfloor\frac{1+\sqrt{2-2q+2m^*}}{1-2q+2m^*}\right\rfloor$ represents the largest integer no larger than $\frac{1+\sqrt{2-2q+2m^*}}{1-2q+2m^*}$.

$$p_2^{c)} = \overline{v} - \rho\sigma^2 \left(I + \frac{z}{\left[\!\left[\frac{1 + \sqrt{2 - 2q + 2m^*}}{1 - 2q + 2m^*} \right]\!\right] + 1} \right), \tag{19}$$

and the price in the investor-dealer trade is

$$p_1^{c)} = (1 - q)p_2^{c)} + q(M_0 - M_1\sigma^2). \tag{20}$$

Figure 2 shows a complete network as an equilibrium network corresponding to **Proposition 2.b)**, while Figure 3 shows a 4-link symmetric network as an equilibrium network, which corresponds to **Proposition 2.c)**. The total number of dealers in Figure 2 and 3 is eight.

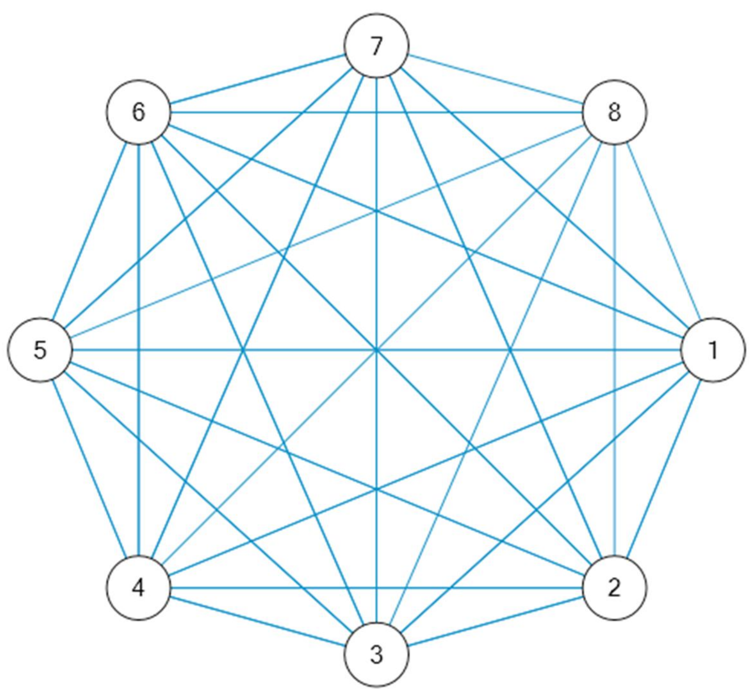


Figure 2: A Strongly Stable Network that is Complete

The above figure shows an equilibrium network that is complete. In the complete network all dealers are connected. Every dealer has seven links.

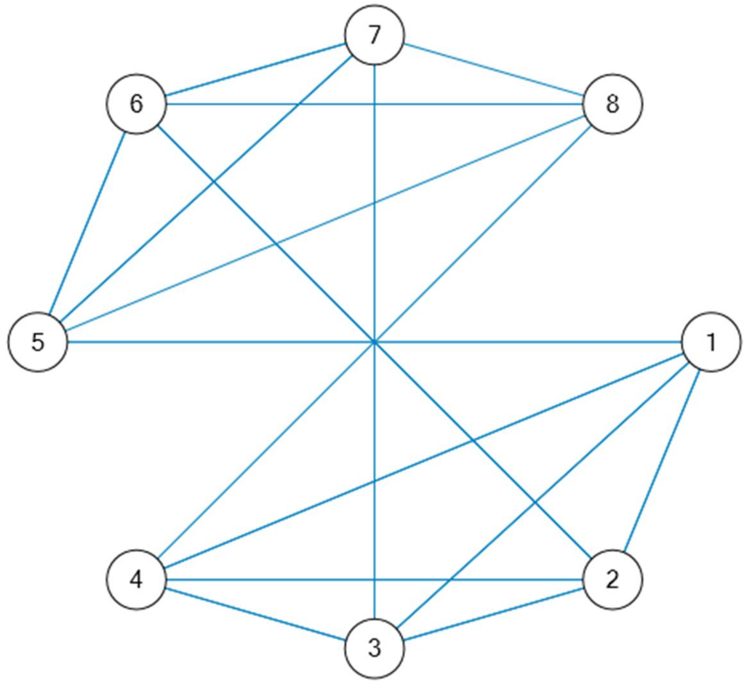


Figure 3: A Strongly Stable Network with Four Links

The above figure shows an equilibrium network in which every dealer has four links.

Proposition 2 indicates that the trade-off between the collateral cost and the risk-sharing benefit determines the equilibrium of a network. A dealer becomes more connected when the benefit from risk-sharing increases or when the collateral cost decreases. The following proposition formalizes this statement.

Proposition 3

The number of links made by a dealer increases when the effective margin m^* decreases, that is, a) when the order size increases, ceteris paribus; b) when volatility increases, ceteris paribus; or c) when the funding constraint (m) loses, ceteris paribus.

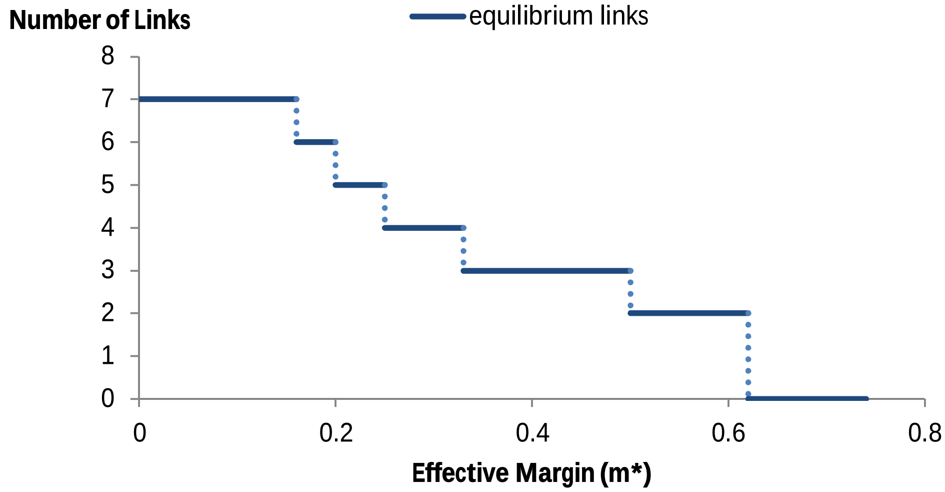


Figure 4: The Equilibrium Number of Links and the Effective Margin

Figure 4 depicts the relationship between the equilibrium number of links and the effective margin. The effective margin is $m^* = \frac{2m}{\rho\sigma z}$. *Parameters chosen are* $N = 8, \rho = 1, \sigma = 1, z = 1, q = 0.5$ *and* $m^* \in [0,0.8]$.

Figure 4 shows the negative relationship between the number of links and the effective margin as stated in **Proposition 3**. **Proposition 3** implies that larger orders give rise to more connected inter-dealer markets. This seems to be consistent with anecdotal evidence from dealer markets with tightly connected dealers. For example, in the foreign exchange market, the bulk of the trading volume comes from inter-dealer trades, and those trades usually consist of larger orders. In the past, stock trading in the upstairs market, where broker-dealer firms trade with each other, almost exclusively carries out block trades. **Proposition 3** provides a testable empirical prediction pertaining to the inter-dealer network of an OTC market. The connectedness of an inter-dealer network is positively related to order sizes and volatility in an OTC market.

4. Comparative Statics Analysis of an Inter-Dealer Network

Proposition 3 suggests two ways, or layers, in which primitives such as order sizes and volatility can affect equilibrium. At the first layer, primitives change equilibrium outcomes when an equilibrium network does not change. At the second layer, primitives change the equilibrium network, which then changes equilibrium outcomes. I refer to the first layer as the local property and the second layer as the global property. In the following sections, I first show the results of a comparative analysis of the local property and then illustrate results regarding the global property. Finally, I discuss the connection between local and global properties.

4.1 The Local Property of an Inter-Dealer Network

To investigate the local property of an equilibrium network, I fix the equilibrium network and then investigate how order sizes and volatility affect equilibrium prices. An important equilibrium price is the markup for an order-filling dealer. The markup measures the order-filling dealer's profitability in making the market for investors. The markup is the price difference between the price at which the order-filling dealer buys an asset from an investor and the price at which he sells it to other dealers. That is,

$$\text{markup}^i = p_2^i - p_1^i = q\left(p_2^i - M_0 + M_1\sigma^2\right). \tag{21}$$

<u>Proposition 4</u>

Given an equilibrium network, the inter-dealer price, the investor-dealer price, and the markup decrease with the order size.

When the order size increases, inventory risk also increases. Meanwhile, the order-filling dealer's risk-sharing ability is fixed insofar as the network is fixed. To unload extra inventory, the order-filling dealer has to sell it at a lower price in the inter-dealer market. The lower inter-dealer price reduces the investor-dealer price. The order-filling dealer decreases his price when buying an asset from an investor in the anticipation of a lower price for off-loading a large order in the inter-dealer market. However, due to bargaining, the order-filling dealer is not able to transfer completely the decrease in the inter-dealer price to the investor. This reduces the order-filling dealer's profitability because he must accept a smaller markup.

The negative relationship between markups and order sizes conforms to empirical findings for corporate and municipal bond markets (see Randall (2013) and Green, Hollifield, and Schürhoff (2007)). More importantly, my model offers an alternative explanation to those offered in past studies. Past studies argue that larger orders are from sophisticated investors who have greater bargaining power and hence lead to smaller markups for dealers. I show that, even if dealers have the same bargaining power as investors (when $q = \frac{1}{2}$), the negative relationship between markups and order sizes persists because of the increasing cost to dealers of unloading large inventory volume in the inter-dealer market. That said, my explanation for this negative relationship does not contradict the explanation based on bargaining power. Eq.(21), makes it obvious that a decrease in dealers' bargaining power, q, decreases the markup. Thus, a larger order associated with a smaller dealer's bargaining power decreases the markup.

<u>Proposition 5</u>

The inter-dealer price and the investor-dealer price decrease with volatility. If $M_1 > \rho\left(I + \frac{z}{n_i+1}\right)$, then the markup increases with volatility; otherwise the markup decreases with volatility.

As the previous discussion of the relationship between inter-dealer prices and order sizes suggests, when volatility increases, a traded asset becomes more risky, which intensifies the order-filling dealer's risk-sharing need. Consequently, the inter-dealer price decreases, which leads to a decrease in the investor-dealer price. However, the impact of volatility on the markup is different from the impact of an order size on the markup. Besides affecting the markup from the dealer side, volatility also affects the markup from the investor's side. Specifically, when volatility increases the investor's utility for holding the asset $M_0 - M_1\sigma^2$ decreases, which implies that the investor is more willing to sell the asset. This results in a further decrease in the investor-dealer price. When the investor's willingness to sell is relatively strong (when $M_1 > \rho\left(I + \frac{z}{n_i+1}\right)$), the drop in the investor-dealer price exceeds the drop in the inter-dealer price, and hence the markup increases. The relationship between price markups and volatility depends on the investor's altitude towards risk.

4.2 The Global Property of an Inter-Dealer Network

In Section 4.1, I discussed relationships between equilibrium outcomes and order sizes and relationships between equilibrium outcomes and volatility within a fixed equilibrium network. In this section, I consider the global property of an equilibrium network. In other words, I examine what happens to equilibrium outcomes such as prices and trading volumes when the equilibrium network changes.

Proposition 6

If the number of links that an order-filling dealer has increases, he sells at a higher inter-dealer price, buys at a higher investor-dealer price, and earns a larger markup.

In an inter-dealer trade, the order-filling dealer solicits bids from his connected dealers. If the network becomes more connected, the order-filling dealer links to more dealers. The bidding competition becomes more intense, and hence drives the inter-dealer price in favor of the order-filling dealer. Consequently, the order-filling dealer is willing to buy at a higher price from the seller. However, the order-filling dealer increases the investor-dealer price only to the extent that his profit still increases. That is, his markup goes up.

Trading volume for a dealer involve two parts. The first part is his trading volume when he is an order-filling dealer; the second part is his trading volume when one of his connected dealers is an order-filling dealer. Specifically, dealer i's expected number of trades is

$$\frac{1}{N} + \sum_{j:ij\in G} \frac{1}{N} = \frac{n_i + 1}{N}. \tag{22}$$

And dealer i's expected trading volume is

$$\frac{1}{N}\left(I + z - X^{A,i}\right) + \sum_{j:ij\in G} \frac{1}{N} X^{B,j} = \frac{2}{N}\left(1 - \frac{2}{n_i + 1}\right)z. \tag{23}$$

In the above, both equalities are obtained as $n_i = n_j$ because dealer i and dealer j have the same number of links when they are connected in an equilibrium network (see **Proposition 1**).

Proposition 7

The more links a dealer has in an inter-dealer network, the more trades he makes and the greater is his trading volume.

Proposition 7 states that more trades take place when the network becomes more connected. This is not surprising, as more links increase a dealer's probability of participating in risk-sharing trades with other dealers.

To see if a more connected network improves risk-sharing, I examine the risk of a dealer's inventory in equilibrium. The expected risky holding for dealer i is

$$EX^i = \frac{1}{N} X^{A,i} + \sum_{j:ij\in G} \frac{1}{N}\left(I + X^{B,j}\right) + \left(1 - \frac{1}{N} - \sum_{j:ij\in G} \frac{1}{N}\right) I \tag{24}$$

And the variance of the risky holding is

$$Var\left(X^i\right) = \frac{1}{N} {X^{A,i}}^2 + \sum_{j:ij\in G} \frac{1}{N}\left(I + X^{B,j}\right)^2 + \left(1 - \frac{1}{N} - \sum_{j:ij\in G} \frac{1}{N}\right) I^2 - \left(EX^i\right)^2 \tag{25}$$

Proposition 8

The variance of a risky holding decreases as the number of links a dealer has increases.

Based on **Proposition 8**, a more connected network reduces dealers' inventory risks. Together, **Proposition 7** and **8** imply that a more connected network achieves better risk-sharing among dealers, which accompanies higher trading volumes in the inter-dealer market. The positive relationship between a dealer's connectedness and his trading volume and the negative relationship between a dealer's connectedness and his inventory risks yield two testable empirical predictions from my model.

4.3 The Connection between Local Properties and Global Properties

As discussed at the beginning of Section 4, changes in primitives have two layers of impacts on equilibrium. One affects equilibrium outcomes directly, while the other exerts influence through changing the equilibrium network's structure. Because of the second impact, the local property of the network is not stable. In other words, relationships between prices and order sizes, or between prices and volatility, can exhibit structural breaks as variations in order sizes and volatility can also change the structure of the equilibrium network.

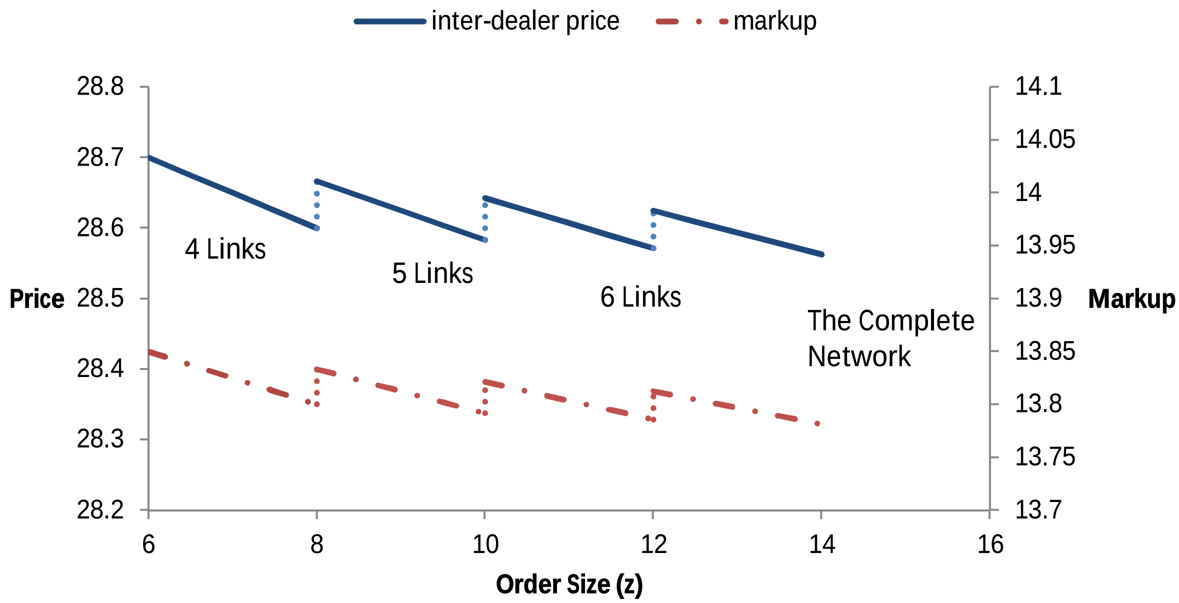


Figure 5: The Relationship between Prices and Order Sizes

Figure 5 depicts structural breaks in the negative relationship between inter-dealer prices and order sizes, and the negative relationship between markups and order sizes. Parameters chosen are $m = 0.5, N = 8, \rho = 1, \sigma = 0.5, I = 1, \overline{v} = 30, q = 0.5, M_0 = 0, M_1 = 4$, *and* $z \in [6, 14]$. *The structural break occurs at* $z = 8{,}10$, *and* 12.

Figure 5 shows that the negative relationship between markups and order sizes exhibits jumps as the order size increases. Such jumps occur when the network becomes more connected, i.e., the number of a dealer's links increases. As shown in **Proposition 6**, the markup and the inter-dealer price increase when the network becomes more connected, and the jumps shown in Figure 5 reflect this increase. The same pattern exists in relationships between prices and volatility (see Figure 6).

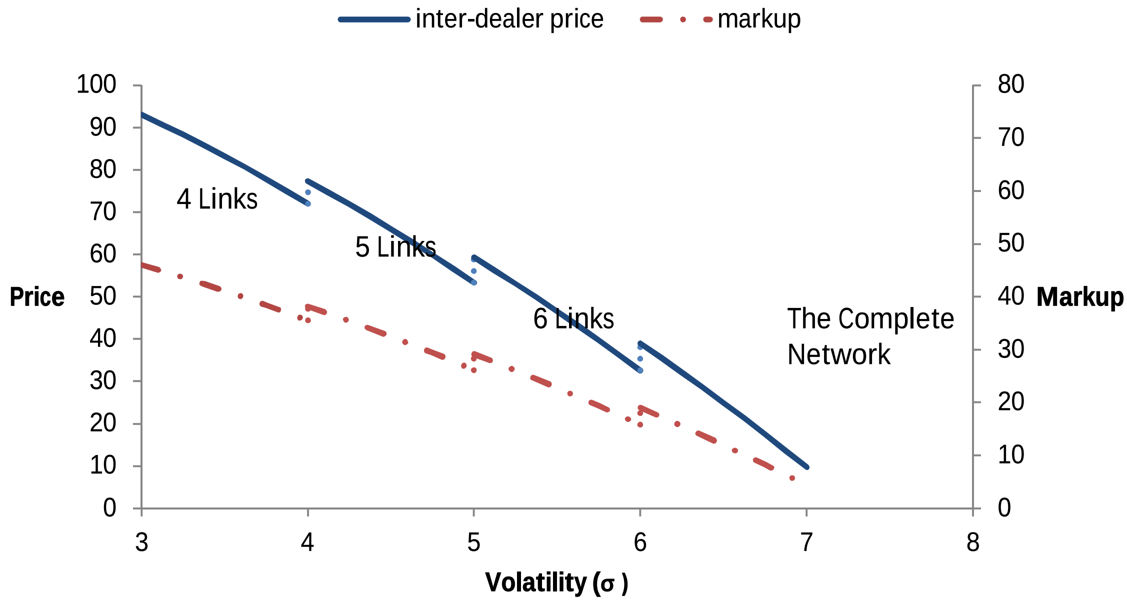


Figure 6: The Relationship between Prices and Volatility

Figure 6 depicts structural breaks in the negative relationship between inter-dealer prices and volatility, and the negative relationship between markups and volatility. Parameters chosen are $m = 0.5, N = 8, \rho = 1, z = 10, I = 1, \overline{v} = 120, q = 0.5, M_0 = 1, M_1 = 0$*, and* $\sigma \in [3,7]$*. The structural break occurs at* $\sigma = 4, 5,$ *and* 6*.*

The above discussion suggests that empirical research on OTC markets should take into account the stability of the underlying network. Otherwise, the regression model used runs the risk of model misspecification, since the regression model may suffer from structural breaks. For example, empirical research should include a measure of a network's connectedness as an additional control variable interacting with other important explanatory variables in a regression model. In Section 7, I discuss this empirical implication more thoroughly, together with other implications of the model.

5. Core-Periphery Inter-Dealer Networks

In the previous model I assume that dealers are homogeneous. This assumption reduces the model's complexity. In the model dealers have to decide only how many links to make, but they do not have to decide with whom they should connect, since all dealers are the same ex-ante. In this section, I introduce heterogeneity among dealers into the model. Dealers are different in their capacity of providing liquidity to investors. Specifically, there are three types of dealers. The first type consists of dealers with small capacity $s_S = \underline{z}$. Those dealers are small or regional banks who can only accommodate retail-sized orders, i.e., the size of the order is no larger than $\underline{z}$. The second type consists of dealers with medium capacity $\underline{z} + s_M$. The third type dealer has large capacity $\underline{z} + s_L$ and $s_M < s_L \leq 1$. Large-capacity dealers are those big banks who are able to provide liquidity to both retail investors (with small orders) and institutional investors (with huge orders).

In addition to introducing differences in dealers' capacity of liquidity provision, I relax the assumption that the size of the investor order is constant. I assume that the size of the investor order is random and follows a uniform distribution.[11] This assumption together with the above assumption that dealers have different capacity determines a dealer's probability of trading with an investor. Specifically, at date 1, an investor arrives and wants to trade an order of size $z \sim \mathrm{Uniform}(\underline{z}, \underline{z} + 1)$. The investor meets with one dealer in the network with probability $\frac{1}{N}$. If the order size z is smaller than the chosen dealer's capacity, then the dealer fills the investor order. Otherwise, no investor-dealer trade occurs. Hence, for a large-capacity dealer, his probability of trading with an investor equals $\frac{1}{N} \times \Pr[z \leq \underline{z} + s_L] = \frac{s_L}{N}$; for a medium-capacity dealer, his probability of trading with an investor equals $\frac{s_M}{N}$; for a low-capacity dealer, his

[11] The assumption that the order size follows a uniform distribution does not affect any implication in the model. For any distribution, large capacity dealers always have the highest probability of trading with an investor, since large capacity dealers are able to accommodate any orders that medium or small capacity dealers accommodate. The probability of trading is the key driver that gives rise to the asymmetric equilibrium network (such as the core-periphery network). That being said, using the uniform distribution significantly reduces redundancy in the mathematical derivation.

probability of trading with an investor is zero.

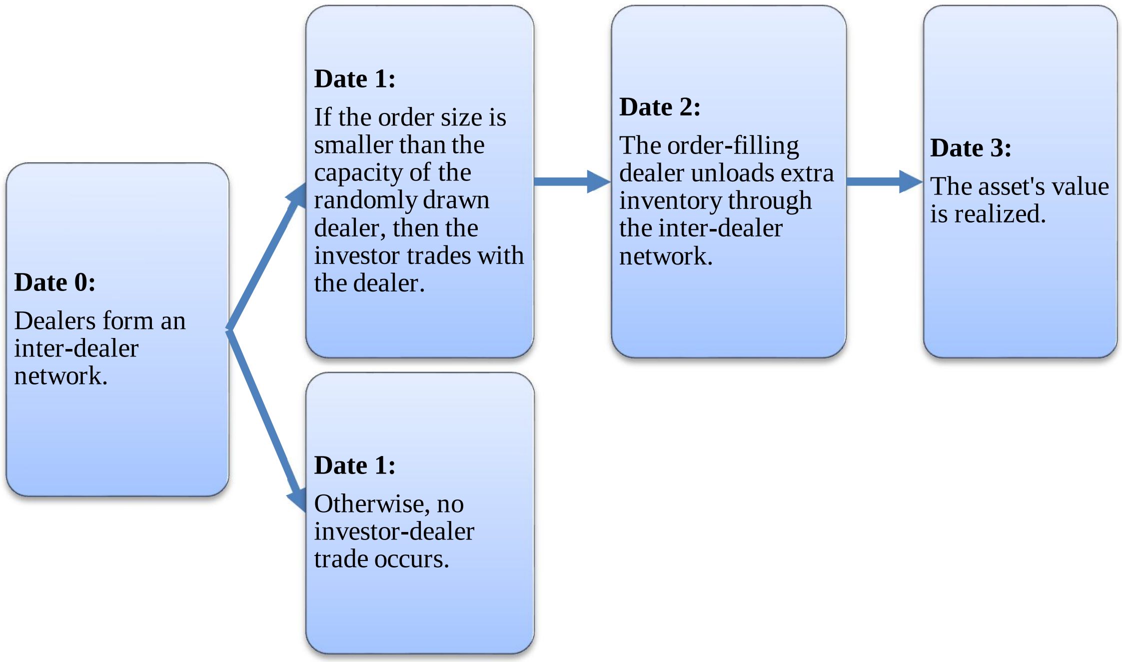


Figure 7: The Timeline of the Extended Model

Figure 7 gives the timeline of the extended model. At date 0, dealers form an inter-dealer network. At date 1, a randomly selected dealer meets with the investor. They trade if the order size is smaller than the dealer's capacity. Otherwise, they don't trade. At date 2, the dealer who fills the order at date 1 starts to re-trade through his inter-dealer network. At date 3, the asset's value is realized.

Figure 7 gives the timeline of this extended model. It is similar to the model in Section 3 except for two differences. The first difference is that the size of the investor order is random, and it follows a uniform distribution. The second difference is that at date 1, an investor-dealer trade occurs if the order size is smaller than the capacity of the selected dealer. Otherwise, no investor-dealer trade occurs. All the rest is the same as the benchmark model (see Figure 1).

With capacity as the only device of heterogeneity that differentiates dealers, I show that the equilibrium network is asymmetric. An asymmetric network means that dealers do not have the same number of links. The core-periphery structure is a special case of this asymmetric network. Additionally, I show that differences in capacity create a vacillating relationship between investor-dealer prices and dealers' centrality (measured by the number of a dealers' links).

Denote $m_i^{**} = \frac{3m}{\rho\sigma s_i}$, m_i^{**} is dealer i's effective margin. Given a network G, the payoff for dealer i is,

$$U_i(G) = \int_{\underline{z}}^{\underline{z}+1} \overbrace{\frac{1}{N}\mathbf{1}_{[z\le \underline{z}+s_i]} Eu\left(W\left(X^{A,i}\right)\right)}^{i \text{ fills the order}} \tag{26}$$

$$+ \overbrace{\sum_{j:ij\in G} \frac{1}{N}\mathbf{1}_{[z\le \underline{z}+s_j]} Eu\left(W\left(X^{B,j} + I\right)\right)}^{i'\text{s connected dealer fills the order}} dz$$

$$+ \overbrace{\left(1 - \frac{s_i}{N} - \sum_{j:ij\in G} \frac{s_j}{N}\right) Eu\left(W(I)\right)}^{\text{neither } i \text{ nor his connected dealers fill the order}}$$

$$- \int_{\underline{z}}^{\underline{z}+1} \overbrace{\frac{1}{N}\mathbf{1}_{[z\le \underline{z}+s_i]} \sigma z \frac{n_i - 1}{n_i + 1} m}^{\text{total cost of links}} dz$$

$$= \begin{cases} \dfrac{\rho\sigma^2}{3N}\left(s_i^3\left(q - m_i^{**}\right)\dfrac{n_i - 1}{2(n_i + 1)} + \displaystyle\sum_{j:ij\in G} \frac{1}{2} s_j^3 \frac{n_j - 1}{n_j^2\left(n_j + 1\right)} \mathbf{1}_{[n_j\ge 2]}\right) + U_0, & n_i \ge 1 \\ U_0, & n_i = 0 \end{cases},$$

and where U_0 is dealer i's payoff when he has no link. U_0 is defined as follows,

$$U_0 = \frac{s_i^2}{2N} q\left(\overline{v} - \rho\sigma^2 I - M_0 + M_1\sigma^2\right) + Eu\left(W(I)\right) - \frac{\rho\sigma^2 q s_i^3}{6N}. \tag{27}$$

<u>Proposition 9</u>

Let n_{s_L}, n_{s_M}, and n_{s_S} be the number of links for large-capacity dealers, medium-capacity dealers, and small-capacity dealers, respectively. Then, in a strongly stable network,

$$n_{s_L} \ge n_{s_M} \ge n_{s_S}. \tag{28}$$

Proposition 9 indicates that the equilibrium network when dealers have different capacity in providing liquidity is asymmetric. Some dealers have more links than others. I show that centrality measured by the number of links a dealer has is positively determined by the dealer's capacity. A dealer who has larger capacity and is more capable of accommodating investors' orders has more links. The dealer with large capacity has greater risk-sharing needs, since he has a greater likelihood of facing a liquidity shock. Such a liquidity shock occurs if the dealer fills the order from an incoming investor. As a result, the large-capacity dealer is inclined to build more links. At the same time, connecting with the large-capacity dealer implies more chances for other dealers to participate in risk-sharing activities, which means greater benefits. Hence, other types of dealers are also inclined to connect to the large-capacity dealer. This mutual consent leads to the equilibrium in which the large-capacity dealer has the greatest number of links.

Since the core-periphery network is a special case of the asymmetric network, **Proposition 9** explains the core-periphery structure of the inter-dealer network found in empirical studies (Hollifield, Neklyudov, and Spatt (2012) and Li and Schürhoff (2012)). In a core-periphery network, some dealers operate at the core of the network, connecting to all dealers, while peripheral dealers connect to no one but those at the core. Consequently, core dealers have more links than peripheral dealers. **Proposition 9** suggests that large-capacity dealers comprise the core and have more links than peripheral dealers, who are those small-capacity dealers.

As a large-capacity dealer has a higher probability of trading than other dealers, **Proposition 9** also justifies the model in Neklyudov (2012). In that paper, the author studies the impact of the core-periphery structure using a dealer's matching rate, which is essentially a dealer's probability of trading, as the proxy for a dealer's centrality in the network. My model supports this idea of approximating a dealer's centrality with his matching rate. I show that dealers with high matching rates have higher centrality than dealers with low matching rates, which is an equilibrium consequence of strategic network formation.

To focus on the core-periphery network and illustrate the vacillating relationship between investor-dealer prices and dealers' centrality, I assume that $s_M < \frac{3m}{\rho\sigma q} < s_L$. This implies that $m_{s_L}^{**} < q < m_{s_M}^{**}$. In addition, I assume that $s_L^3 \frac{N-2}{q(N-1)^2 N} > s_M^2 \left(\frac{3m}{\rho\sigma q} - s_M\right)$. Let N_{s_L} be the total number of large-capacity dealers, $N_{s_M} = 2^k$ $(k > 1)$ be the total number of medium-capacity dealers, and $N_{s_M} > N_{s_L} > 2$, I characterize the core-periphery equilibrium network as follows.

<u>Proposition 10</u>

When dealers have varying capacity s_L, s_M, and s_S, a strongly stable network in equilibrium is as follows. Dealers with the large capacity s_L form the core of the network and connect to all dealers; dealers with the small capacity s_S form the periphery and connect only to those at the core; dealers with the medium capacity s_M connect to all large-capacity dealers and other $n_{s_M}^* - N_{s_L}$ medium-capacity dealers. $n_{s_M}^*$ is

$$n_{s_M}^* = \arg\max_{n_{s_M} \in \mathbb{N}} \left(q - m_{s_M}^{**} + \frac{n_{s_M} - N_{s_L}}{n_{s_M}^2}\right) \frac{n_{s_M} - 1}{n_{s_M} + 1}. \tag{29}$$

Proposition 10 shows the equilibrium network that exhibits the core-periphery structure as found in empirical studies. Figure 8 gives an example of this core-periphery network. In Figure 8, there are 20 dealers (3 large-capacity dealers, 8 medium-capacity dealers, and 9 small-capacity dealers). Only large-capacity dealers operate at the core of the network, while small-capacity dealers are the periphery of the network.

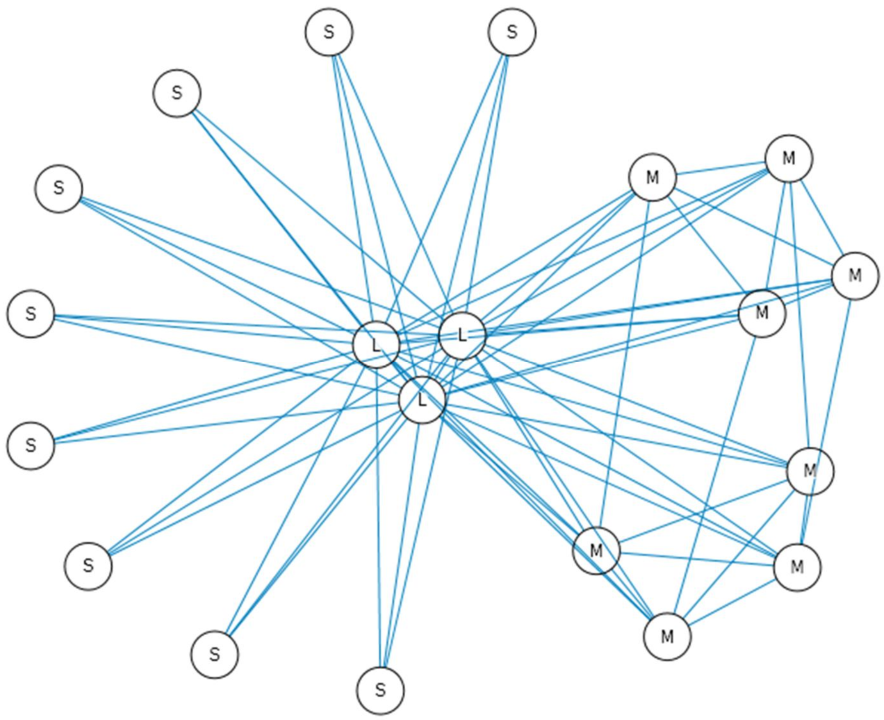


Figure 8: A Core-Periphery Network

Figure 8 shows a core-periphery network in which large-capacity dealers comprise the core of the network and small-capacity dealers become the periphery. L represents the large-capacity dealer, M represents the medium-capacity dealer, and S represents the small-capacity dealer. In equilibrium, each L has 19 links, each M has 7 links, and each S has only 3 links.

In the core-periphery network, core dealers do not necessarily offer more favorable prices to investors. Two opposite forces affect the investor-dealer price that a core dealer offers. On one side, a core dealer has more links, thereby greater market power in inter-dealer trading. Greater market power in the inter-dealer market enables the core dealer to sell at a higher price, and hence to buy from an investor at a higher price. On the other side, a dealer becomes the core because of his large capacity, which implies he fills larger orders than other dealers. Larger orders overburden the dealer's inventory rebalancing in inter-dealer trading, and hence worsen the dealer's price in the inter-dealer market. Consequently, the large-capacity dealer buys from an investor at a lower price. In short, the cross-sectional relationship between investor-dealer prices and dealers' centrality is ambiguous. **Proposition 11** illustrates this undetermined relationship.

Proposition 11

Denote $\bar{p}^0_{s_L}$ *and* $\bar{p}^0_{s_M}$ *as the average investor-dealer price from large-capacity dealers and medium-capacity dealers, respectively. If* $\left(\bar{v} - \rho\sigma^2 I + \frac{q}{1-q}\left(M_0 - M_1\sigma^2\right)\right)(s_L - s_M) \geq \frac{\rho\sigma^2}{2}\left(\frac{s_L^2}{N} - \frac{s_M^2}{n^*_{s_M}+1}\right)$*, then* $\bar{p}^0_{s_L} \geq \bar{p}^0_{s_M}$*. Otherwise,* $\bar{p}^0_{s_L} < \bar{p}^0_{s_M}$*. In the above,* $n^*_{s_M}$ *is defined in* ***Proposition 10*** *and* N *is the total number of dealers.*

Proposition 11 gives the condition under which large-capacity dealers buy from investors at higher prices, and under which large-capacity dealers buy at lower prices. Since a dealer's capacity positively determines his centrality, **Proposition 11** suggests that the relationship between investor-dealer prices and dealers' centrality vacillates between positive and negative. Figure 9 further illustrates this ambiguous relationship between investor-dealer prices and dealers' centrality by following the example in Figure 8. In the figure, a medium-capacity dealer has 7 links and a large-capacity dealer has 19 links in equilibrium. The large-capacity dealer has higher centrality than the medium-capacity dealer. In the upper panel of the figure, the relationship between investor-dealer prices and centrality is positive. This occurs when the difference in capacity between high centrality dealers and low centrality dealers is small. That is, $\left(\bar{v} - \rho\sigma^2 I + \frac{q}{1-q}\left(M_0 - M_1\sigma^2\right)\right)(s_L - s_M) \geq \frac{\rho\sigma^2}{2}\left(\frac{s_L^2}{N} - \frac{s_M^2}{n^*_{s_M}+1}\right)$. However, when the difference in capacity is big, the relationship becomes negative, which is illustrated in the bottom panel of the figure. This occurs when $\left(\bar{v} - \rho\sigma^2 I + \frac{q}{1-q}\left(M_0 - M_1\sigma^2\right)\right)(s_L - s_M) < \frac{\rho\sigma^2}{2}\left(\frac{s_L^2}{N} - \frac{s_M^2}{n^*_{s_M}+1}\right)$.

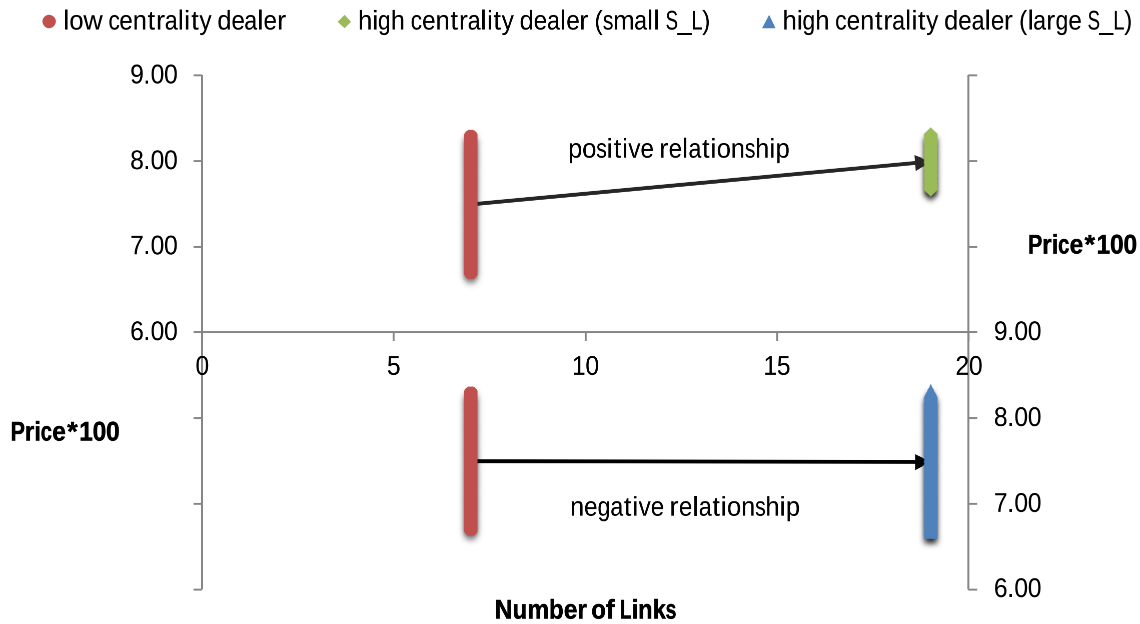


Figure 9: The Relationship between Investor-Dealer Prices and Centrality

Figure 9 shows that the relationship between average investor-dealer prices and dealers' centrality (which measures how central a dealer is in the network) is undetermined. In the upper panel of the figure, the relationship is positive. This occurs when the difference in capacity between high centrality dealers and low centrality dealers is small. That is, $\left(\overline{v} - \rho\sigma^2 I + \frac{q}{1-q}\left(M_0 - M_1\sigma^2\right)\right)(s_L - s_M) \geq \frac{\rho\sigma^2}{2}\left(\frac{s_L^2}{N} - \frac{s_M^2}{n_{s_M}^*+1}\right)$*. However, when the difference in capacity is big, the relationship between investor-dealer prices and centrality is negative. This occurs when* $\left(\overline{v} - \rho\sigma^2 I + \frac{q}{1-q}\left(M_0 - M_1\sigma^2\right)\right)(s_L - s_M) < \frac{\rho\sigma^2}{2}\left(\frac{s_L^2}{N} - \frac{s_M^2}{n_{s_M}^*+1}\right)$ *Parameters chosen are* $N = 20, N_{s_L} = 3, N_{s_M} = 8, m = 0.45,$ $q = \frac{2}{3}, \rho = 1, I = 2, \overline{v} = 26.25, M_0 = 0, M_1 = 0.02, \sigma = 5$, $z \sim \text{Uniform}(0,1), s_M = 0.4$, $s_L(\mathit{small}) = 0.405$ *and* $s_L(\mathit{large}) = 1$. *Based on* ***Proposition 10****, this set of parameters implies that* $n_{s_M}^* = 7$.

Though the relationship between investor-dealer prices and dealers' centrality is undetermined, the conditional relationship between them is determined. When conditioning on the size of the investor order, high-centrality dealers offer better prices than low-centrality dealers. That is, when z_i is fixed, $p_1^i = (1-q)\left(\overline{v} - \rho\sigma^2\left(I + \frac{z_i}{n_i+1}\right)\right) + q(M_0 - M_1\sigma^2)$ is positively determined by n_i. This is consistent with Hollifield, Neklyudov, and Spatt (2012), which shows that investors get more favorable prices when trading with core dealers. The above suggests that the size of the investor order is an important control variable in determining how dealers' centrality is related to investor-dealer prices.

6. "Hot Potato" Trading in an Inter-Dealer Market

So far in this study, transactions among dealers take place only when an order-filling dealer initiates an auction in the inter-dealer market. This setup helps to demonstrate that risk-sharing drives network formation, since the sole role played by dealers in the inter-dealer market is risk-sharing. However, such one-shot trading limits the analysis of strategies that could be deployed by dealers, since dealers who connect to the order-filling dealer can only be end-users. In reality, one of the strategies deployed by dealers is intermediary or "hot potato" trading. "Hot potato" trading occurs when a dealer who has traded with the order-filling dealer continues to trade with other dealers who do not connect with the order-filling dealer. In so doing, this dealer serves as the intermediary between the order-filling dealer and dealers who are not in the order-filling dealer's network.

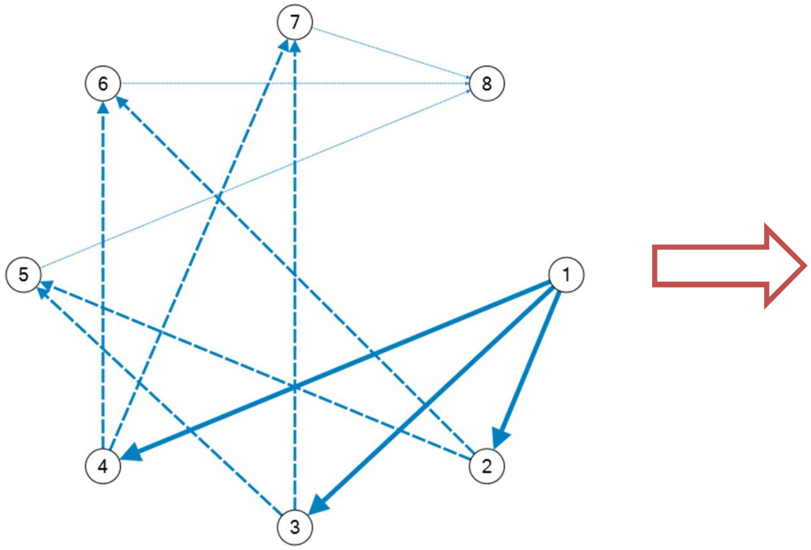
To analyze "hot potato" trading in an inter-dealer network, I relax the one-shot trading assumption, and allow dealers who trade with the order-filling dealer to also trade in their own networks simultaneously. Specifically, let dealer i be the order-filling dealer and dealer j be one of i's connected dealers. When dealer i starts an auction, dealer j not only submits his orders to i, he also solicits bids from his connected dealers j' (to focus sharply on "hot potato" trading, I consider only the case in which j' does not connect to dealer i). Similarly, dealer j' submits his orders to j, and in the meantime solicits bids from his connected dealers, and so forth. One can visualize this setup as consisting of multiple rounds of trading that occur instantaneously. That is, in a short period of time the order-filling dealer trades with his connected dealers in the first round, and then those order-filling-connected dealers trade in their own networks in the second round, and so forth.

Unlike the model with one-shot trading only, the above setup allows dealers to continue trading in an inter-dealer network. However, this general setup complicates the analysis of the equilibrium at date 2. Since trades continue through the network, a dealer's strategy depends not only on who he connects to (as in the one-shot setup) but also on who his connected dealers connect to and who his connected dealers connected dealers connect to, and so forth. Fortunately, the equilibrium at date 2 is still solvable, and it is characterized on another network derived from the inter-dealer network. Let us denote this derived network as the "trading-sets network." **Definition 3** and **4** show how a "trading-sets network" is derived from an inter-dealer network.

Definition 3

Given the flow of trades in an inter-dealer network, dealers in an inter-dealer network are grouped into various trading sets. In each trading set, there is a dealer, called the initiator, who trades in the previous round of trading, and there are other dealers, called participants, who do not participate in previous rounds but participate in the current round initiated by the initiator. Furthermore, if a trading set has only one participant who is not a participant in any other trading set, then the trading set is considered as an empty set. If this unique participant in the trading set is also the only participant in other trading sets, then these trading sets are grouped into one set consisting of only one participant but many initiators.

Definition 3 indicates that a trading set can take only two forms. One form includes many participants but a unique initiator; the other has many initiators but a unique participant. Figure 10 provides an example of the grouping for a symmetric network with three links. In those trading sets, numbers before the semicolons stand for initiators and numbers after the semicolons stand for participants. In Figure 10, the arrow on a link indicates the flow of trades.



Trading Set1 = {1; 2, 3, 4}

Trading Set2 = {2; 5, 6}

Trading Set3 = {3; 5, 7}

Trading Set4 = {4; 6, 7}

Trading Set5 = {5, 6, 7; 8}

Figure 10: Trading-Sets derived from a Symmetric Network with Three Links

Figure 10 provides an example of dealers in an inter-dealer network that is grouped into trading sets. In the figure, the arrow represents the direction of the flow of trades. The thickest line represents the first round of trading. The dashed line represents the second round of trading. The thinnest line represents the final round of trading. In the trading set brackets, the numbers before the semicolons stand for the initiator and the numbers after the semicolons stand for the participants.

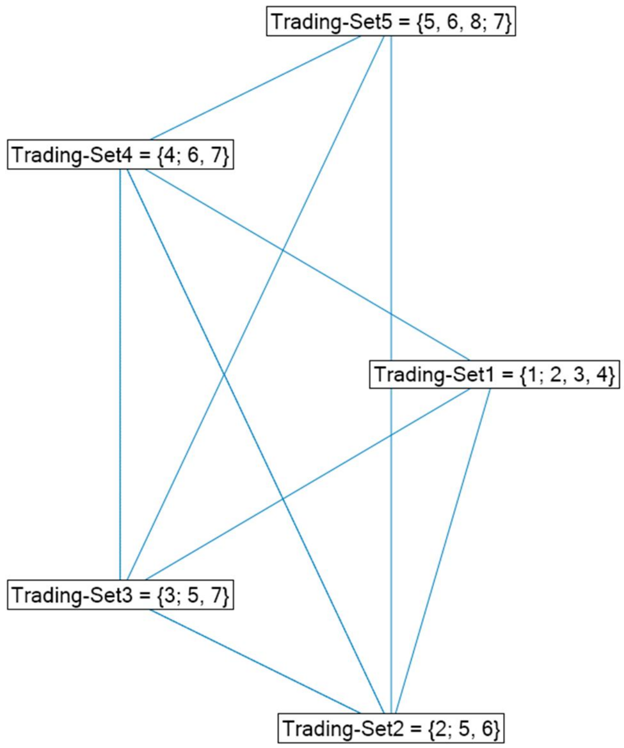


Figure 11: A "Trading-Sets Network" *derived from an Inter-Dealer Network*

Figure 11 shows a "trading-sets network" that is derived from the inter-dealer network shown in Figure 8. In Figure 8, trading set 1 connects to trading set 2 but not to trading set 5, since the intersection between trading set 1 and 2 contains a common dealer, dealer 2, but the intersection between trading set 1 and trading set 5 is empty.

Definition 4

Two trading sets are connected if their intersection is not empty.

While **Definition 3** defines how nodes (trading sets) in a "trading-sets network" are derived from an inter-dealer network, **Definition 4** defines how links in a "trading-sets network" are derived. Figure 11 gives an example of a "trading-sets network" derived from an inter-dealer network. It is obvious that with a given flow of trades in an inter-dealer network the grouping of trading sets is unique. Since links between trading sets are determined only by members of those sets, a "trading-sets network" is uniquely derived from an inter-dealer network through **Definition 3** and **Definition 4**. This means that characterizing the date 2 equilibrium when dealers continually trade along an inter-dealer network is equivalent to characterizing the equilibrium when dealers trade in the derived “trading-sets network.”

As in Section 3.2, initiators trade with participants strategically in each trading set. When the market in each trading set clears, it generates a unique price associated with the corresponding trading set. In other words, inter-dealer trading occurs in various fragmented markets (trading sets), and these fragmented markets are linked when they have common members (dealers).

In each of these trading sets dealers are divided into two classes, initiators and participants. This fits the model description in Malamud and Rostek (2013), in which exactly two classes of dealers trade in each trading set.[12] In fact, **Definition 3** and **Definition 4** map the dealers' network into the “trading-sets network” which is first studied by Malamud and Rostek (2013).

The equilibrium in inter-dealer trading is that every dealer (say dealer k) submits a vector of demand schedules (q^k) to all trading sets (say $\mathbb{M}^k$ sets) he belongs to, and his vector of demand schedules is

$$q^k(\Lambda^k) = \left(\rho\sigma^2 \mathbb{I}_{\mathbb{M}^k \times \mathbb{M}^k} + \Lambda^k\right)^{-1}\left(\bar{v}\mathbb{I}_{\mathbb{M}^k \times 1} - \mathbb{p}_{\mathbb{M}^k \times 1} - \rho\sigma^2 I_0 \mathbb{I}_{\mathbb{M}^k \times \mathbb{M}^k}\right),$$

where Λ^k is dealer k's price impact and $\mathbb{p}_{\mathbb{M}^k \times 1}$ is a vector of equilibrium prices in $\mathbb{M}^k$ sets.[13] To be more specific, Λ^k is the $\mathbb{M}^k \times \mathbb{M}^k$ Jacobian matrix $D_q\mathbb{p}$, in which entry (r, s) stands for the price change in set s caused by a demand change in set r. In equilibrium, Λ^k is determined by the market-clearing condition. Although solving the equilibrium is equivalent to finding every dealer's price impact, the actual work of solving for those price impacts (solving N matrices with $\mathbb{M}^k \times \mathbb{M}^k$ dimensions) is non-trivial, let along specifying how the network and Λ^k are jointly determined in the network formation process.

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[12] See **Example 1 (ii)** in Malamud and Rostek (2013).

[13] $\mathbb{I}_{\mathbb{M}^k \times \mathbb{M}^k}$ is an $\mathbb{M}^k$ by $\mathbb{M}^k$ matrix with all entries equal to 1.

To circumvent this difficulty, I focus on the property regarding “hot potato” trading that is persistent in any strongly stable network.

Proposition 12

Denote a dealer as a monopolistic dealer if his connected dealers belong to distinct connected components. In any strongly stable network, monopolistic dealers always buy and sell at different prices gaining non-zero markups in “hot potato” trading. In contrast, if a pair of dealers has more than two unconnected common neighbors, then this pair of dealers and all their common neighbors receive zero markups in “hot potato” trading.[14]

Proposition 12 identifies which dealer in the inter-dealer network receives non-zero markups for “hot potato” trading. Interestingly, the dealer with the most links does not necessarily enjoy non-zero markups. In fact, this dealer may receive zero markups for “hot potato” trading. For example, in Figure 12, dealer 6 has the highest number of links but he always receives zero markups since he has exactly two common neighbors with every dealer he connects to. The dealer who always receives non-zero markups is the one who reaches different groups of dealers, for example dealer 7 in Figure 12. This dealer is called the monopolistic dealer, as his ability to access unconnected parts of the inter-dealer network provides him with local monopoly power in “hot potato” trading.

[14] Neighbors of a dealer are his connected dealers.

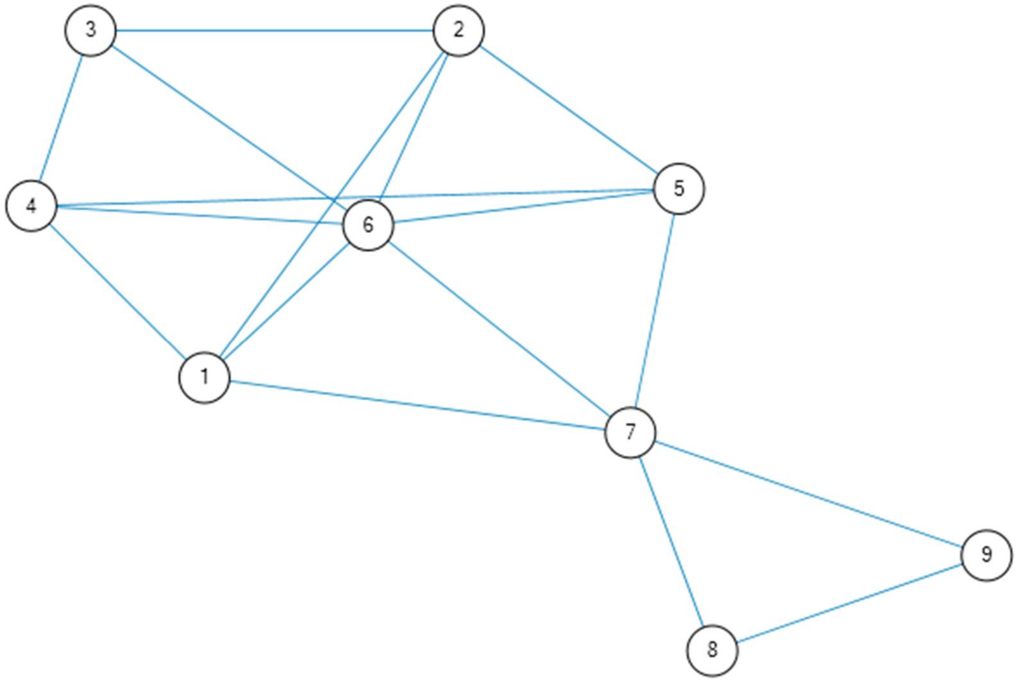

Figure 12: An Inter-Dealer Network in which Dealer 7 is the Monopolistic Dealer

In Figure 12, if there is "hot potato" trading, dealer 7 is the monopolistic dealer with non-zero markups, even though he has fewer links than dealer 6. In fact, dealer 6 gets zero markups in "hot potato" trading.

Proposition 12 is an extension of the study by Malamud and Rostek (2013). Their model implies that a dealer who acts as a "monopolistic bridge" in a "trading-sets network" is the one with non-zero markups. **Proposition 12** extends their results to identify the "monopolistic bridge" in an inter-dealer network. This helps empirical research to identify which dealer has local monopoly power in "hot potato" trading.

7. Empirical Implications

My model offers novel testable hypotheses in addition to confirming findings from past empirical studies. As an inter-dealer network is formed to share risks among dealers, the connectedness of the network is closely related to volatility and order sizes that characterize dealers' inventory risks.

Hypothesis 1:

An asset with high volatility has a more connected inter-dealer network than an asset with low volatility.

Hypothesis 2:

An asset traded in large order sizes has a more connected inter-dealer network than an asset traded in small order sizes.

The above hypotheses are novel empirical predictions obtained from endogenizing the formation of an inter-dealer network. However, the empirical design involved in testing those hypotheses requires statistics that measure the connectedness of an inter-dealer network. In the network literature, several statistics have been proposed to describe the connectedness of a network including average path length, cliquishness, a clustering coefficient, cohesiveness, etc.[15] In econometrics, Diebold and Yilmaz (2011) propose statistics based on variance decompositions to measure the connectedness of the network of financial firms.

Based on my model, the connectedness of an inter-dealer network determines prices and trading volume in an inter-dealer market. In a more connected inter-dealer market, dealers trade more and gain higher markups. This yields the following hypotheses:

Hypothesis 3:

In a more connected inter-dealer network, dealers generate larger trading volumes and face smaller inventory risks.

Hypothesis 4:

In a more connected inter-dealer network, dealers earn higher markups.

My model explains the observational finding regarding core-periphery networks in OTC

[15] See Jackson (2008) and Easley and Kleinberg (2010) for more details.

markets with varying capacity of providing liquidity among dealers. Dealers with large capacity of providing liquidity are more central in a network than dealers with small capacity. As a result, my model provides an additional testable implication:

Hypothesis 5:

A dealer with larger capacity of liquidity provision has higher centrality.

As mentioned above, here the empirical design would entail constructing measures for each dealer's centrality in a network. Past studies in the network literature have used degree centrality, closeness centrality, betweenness centrality, eigenvector related measures, etc., to capture dealers' centrality.[16] One proxy for a dealer's capacity would be the size of the dealer. A large dealer is more likely to be capable of accommodating huge orders than a small dealer.

An important empirical implication of the model pertains to sudden jumps in relationships between prices and primitive parameters, e.g., volatility and order sizes. Such jumps occur as an inter-dealer network changes along with continuous changes in primitive parameters (see Figures 5 and 6). This implies that when an asset whose volatility or order sizes change over time is involved, empirical studies should consider testing for structural breaks since the corresponding inter-dealer network may have changed over time. The potential structural break in an inter-dealer network implies that time-series data on prices and trading volume may not be stationary. With respect to cross-sectional studies, my model implies that an inter-dealer network entails another layer of heterogeneity that should be controlled for. For example, assets traded in larger orders differ from assets traded in smaller orders not only in terms of the order size but also in terms of the structure of corresponding inter-dealer networks. Thus, statistics that describe inter-

[16] Hollifield, Neklyudov, and Spatt (2012) and Li and Schürhoff (2012) use degree centrality, closeness centrality, betweenness centrality, and eigenvector centrality to measure dealers' centrality in the inter-dealer network. Refer to Jackson (2008) for further elaboration of centrality measures in network studies.

dealer networks, e.g., the clustering coefficient, should be included as additional control variables in a regression model. In all, the structure of the inter-dealer network is an important state variable that should not be overlooked in empirical OTC studies.

8. Conclusion

In this paper, I investigate inter-dealer network formation in an OTC market. I assume that dealers form inter-dealer networks to share inventory risks. In equilibrium, the benefit from such risk-sharing and the funding cost of collateral determine the shape of a network. An equilibrium network pins down outcomes such as prices and trading volume. Furthermore, I show that differences in dealers' capacity of liquidity provision imply that dealers with large capacity have high centrality, whereas dealers with small capacity have low centrality. Hence, an equilibrium network exhibits the core-periphery structure. My model not only matches empirical findings in OTC markets, it also generates novel empirical implications. I demonstrate that empirical models that fail to control for the connectedness of an inter-dealer network may suffer from structural breaks.

In my model, dealers strategically form an inter-dealer network to share inventory risks. The inter-dealer network serves as the channel for dealers to rebalance their inventory. This feature differs from Babus and Kondor (2012), in which they assume that dealers use the network to share information. In reality, dealers are likely to use the inter-dealer network for both risk-sharing and information-sharing purposes. As a result, future research should emphasize the interaction between the inventory model and the information model in the formation of an inter-dealer network as well as the trading in this network market.

Appendix

Proof of Proposition 1

Proof:

Suppose G is a strongly stable network, in which dealers in the same component have uneven links. Let dealer j be the one with the maximal number of links in this component. Then there exists a pair of unconnected dealers i and i', both of whom are connected with j and at least one of whom has fewer links than j (let's say i has fewer links than j). Consider a deviation such that both i and i' cut their connections with j and then build a link between themselves. Denote the obtainable network via this deviation as G'. Since $1 < n_i < n_j$ and $1 < n_{i'} \leq n_j$,

$$U_i(G') - U_i(G) = \frac{\rho\sigma^2 z^2}{2N}\frac{n_{i'}}{n_{i'}+1}\left(\frac{1}{n_{i'}+1} - \frac{1}{n_{i'}+2}\right) - \frac{\rho\sigma^2 z^2}{2N}\frac{n_j}{n_j+1}\left(\frac{1}{n_j+1} - \frac{1}{n_j+2}\right) \geq 0 \tag{A1}$$

and

$$U_{i'}(G') - U_{i'}(G) = \frac{\rho\sigma^2 z^2}{2N}\frac{n_i}{n_i+1}\left(\frac{1}{n_i+1} - \frac{1}{n_i+2}\right) - \frac{\rho\sigma^2 z^2}{2N}\frac{n_j}{n_j+1}\left(\frac{1}{n_j+1} - \frac{1}{n_j+2}\right) > 0. \tag{A2}$$

The deviation is an improving deviation, since i' is strictly better off and i is weakly better off. Hence, G cannot be a strongly stable network.

The above shows that dealers in the same component have the same number of links. Now suppose that a strongly stable network $\tilde{G}$ has components of varying sizes and that, for i and i' from distinct components, $U_i(\tilde{G}) > U_{i'}(\tilde{G})$. Consider a deviation such that i' cuts all his links, all i's connected dealers (call them y) cut their links with i, and i' then builds links with those y dealers. The deviation replaces i's position in the network with i'. The new network is called $\tilde{G}'$. In $\tilde{G}'$, $U_y(\tilde{G}') = U_y(\tilde{G})$, since nothing is changed for them. But $U_{i'}(\tilde{G}') > U_{i'}(\tilde{G})$, since i' replaces i's position in the network and $U_i(\tilde{G}) > U_{i'}(\tilde{G})$. The deviation is an improvement, which contradicts to the proposition that $\tilde{G}$ is a strongly stable network.

Q.E.D.

<u>Proof of Proposition 2</u>

Proof:

The following lemma is useful in my proof of **Proposition 2**.

<u>Lemma A.1</u>

In a strongly stable network no connected dealer has exactly one link.

Proof:

Based on **Proposition 1**, if a connected network is strongly stable and one dealer has just one link, then the network consists of $\frac{N}{2}$ pairs. In this network, a dealer's payoff equals U_0 (from Eq.(15)). Thus, a dealer is indifferent between getting connected with one link or not. In such a case, I say the strongly stable network is the empty network, since an infinitesimal amount of cost could make the dealer prefers the empty network to the one link network.

Q.E.D.

Lemma A.1 implies that a connected strongly stable network is such that all connected dealers have the same number of links and this number is greater than one. If G is a strongly stable network, then $U_i(G)$ can be rewritten as

$$\begin{cases} \frac{\rho\sigma^2 z^2}{2N}\left(q - m^* + \frac{1}{n}\right)\left(1 - \frac{2}{n+1}\right) + U_0, & n_i \geq 2 \\ U_0, & n_i = 0 \end{cases}, \tag{A3}$$

which implies

$$U_i(G) - U_0 = \frac{\rho\sigma^2 z^2}{2N}\left(q - m^* + \frac{1}{n}\right)\left(1 - \frac{2}{n+1}\right). \tag{A4}$$

Based on Eq.(A4), for any n_i, when $m^* > q + 1$, $U_i(G) - U_0 < 0$. Thus, a strongly stable network should be an empty network when $m^* > q + 1$.

Now suppose the strongly stable network is an empty network when $m^* < q + 1$. Consider an obtainable deviation in virtue of which all dealers build two links. Then the change in a dealer's payoff is $\frac{\rho\sigma^2 z^2}{6N}\left(q - m^* + \frac{1}{2}\right) > 0$. Thus, the deviation is an improvement in those dealers' payoffs. The discussion above proves that when $m^* < q + 1$, the empty network cannot be strongly stable. Together with the above paragraph, I show that $m^* > q + 1$ is the sufficient and necessary condition for the strongly stable network being empty.

To find out the equilibrium number of links for each dealer in a connected network (when $m^* \leq q + 1$), we have to consider the following continuous function $F(n)$:

$$F(n) = \left(q - m^* + \frac{1}{n}\right)\left(1 - \frac{2}{n+1}\right), \tag{A5}$$

and $n \in [0, N-1]$. Taking the derivative of $F(n)$, we have

$$\frac{dF(n)}{dn} = \frac{1}{n^2(n+1)^2}\left(n^2(2q - 2m^* - 1) + 2n + 1\right). \tag{A6}$$

When $m^* > q - \frac{1}{2}$, $F(n)$ achieves the maximum at $n^* = \frac{1+\sqrt{2-2q+2m^*}}{1-2q+2m^*}$, $\frac{dF(n^{*-})}{n} > 0$, $\frac{dF(n^{*+})}{n} < 0$, and $\frac{dF(n^*)}{n} = 0$. When $n^* > N - 1$, that is, $m^* < q + \frac{2N-1}{(N-1)^2} - \frac{1}{2}$, then the strongly stable network is the complete network.

Based on the analysis of $F(n)$, when $q + \frac{2N-1}{(N-1)^2} - \frac{1}{2} \leq m^* \leq q + 1$, the strongly stable network is such that every dealer has $\left\lfloor\frac{1+\sqrt{2-2q+2m^*}}{1-2q+2m^*}\right\rfloor$ or $\left\lfloor\frac{1+\sqrt{2-2q+2m^*}}{1-2q+2m^*}\right\rfloor + 1$ links, whichever gives the dealer greater utility.[17] Formally speaking, the equilibrium number of links is

$$\left[\!\left[\frac{1+\sqrt{2-2q+2m^*}}{1-2q+2m^*}\right]\!\right] = \arg\max_{n\in\left\{\left\lfloor\frac{1+\sqrt{2-2q+2m^*}}{1-2q+2m^*}\right\rfloor, \left\lfloor\frac{1+\sqrt{2-2q+2m^*}}{1-2q+2m^*}\right\rfloor+1\right\}} \left(q - m^* + \frac{1}{n}\right)\frac{n-1}{n+1}. \tag{A7}$$

To close the proof, I summarize the shape of the strongly stable network and the corresponding condition as follows. If $m^* < q + \frac{2N-1}{(N-1)^2} - \frac{1}{2}$, then G is a complete network. If $q + \frac{2N-1}{(N-1)^2} - \frac{1}{2} \leq m^* \leq q + 1$, then the strongly stable network G has $\left[\!\left[\frac{1+\sqrt{2-2q+2m^*}}{1-2q+2m^*}\right]\!\right]$ links. If $m^* > q + 1$, then the strongly stable network is an empty network. If none of the above conditions is satisfied, there is no strongly stable network.

Q.E.D.

<u>Proof of Proposition 3</u>

Proof:

We first show that the equilibrium number of links weakly increases when the effective margin m^* decreases. From **Proposition 2**, we know when $m^* > q + 1$, the strongly stable network is an empty network. When m^* decreases such that $m^* < q + \frac{2N-1}{(N-1)^2} - \frac{1}{2}$, the strongly stable network is the complete network. When $q + \frac{2N-1}{(N-1)^2} - \frac{1}{2} \leq m^* \leq q + 1$, every dealer has $\left[\!\left[\frac{1+\sqrt{2-2q+2m^*}}{1-2q+2m^*}\right]\!\right]$ links. Thus, to show that the equilibrium number of links weakly increases when the effective margin decreases, we have to show only that $\left[\!\left[\frac{1+\sqrt{2-2q+2m^*}}{1-2q+2m^*}\right]\!\right]$ weakly increases when m^* decreases.

Since

[17] $\left\lfloor\frac{1+\sqrt{2-2q+2m^*}}{1-2q+2m^*}\right\rfloor$ is the largest integer no larger than $\frac{1+\sqrt{2-2q+2m^*}}{1-2q+2m^*}$.

$$\frac{dn^*}{dm^*} = -\frac{\sqrt{2-2q+2m^*} + \frac{3}{2} - 2q + 2m^*}{\sqrt{2-2q+2m^*}(1-2q+2m^*)^2} < 0, \tag{A8}$$

if $m_1^* < m_0^*$, then Eq.(A8) shows that $\left\lfloor\frac{1+\sqrt{2-2q+2m_1^*}}{1-2q+2m_1^*}\right\rfloor \geq \left\lfloor\frac{1+\sqrt{2-2q+2m_0^*}}{1-2q+2m_0^*}\right\rfloor$. Due to the definition of $[\![.]\!]$, $\left\lfloor\frac{1+\sqrt{2-2q+2m_1^*}}{1-2q+2m_1^*}\right\rfloor > \left\lfloor\frac{1+\sqrt{2-2q+2m_0^*}}{1-2q+2m_0^*}\right\rfloor$ implies $\left[\!\!\left[\frac{1+\sqrt{2-2q+2m_1^*}}{1-2q+2m_1^*}\right]\!\!\right] > \left[\!\!\left[\frac{1+\sqrt{2-2q+2m_0^*}}{1-2q+2m_0^*}\right]\!\!\right]$. Hence, we focus on the case, which $\left\lfloor\frac{1+\sqrt{2-2q+2m_1^*}}{1-2q+2m_1^*}\right\rfloor = \left\lfloor\frac{1+\sqrt{2-2q+2m_0^*}}{1-2q+2m_0^*}\right\rfloor$.

Now suppose that $m_1^* < m_0^*$ and $\left\lfloor\frac{1+\sqrt{2-2q+2m_1^*}}{1-2q+2m_1^*}\right\rfloor = \left\lfloor\frac{1+\sqrt{2-2q+2m_0^*}}{1-2q+2m_0^*}\right\rfloor$. Let $n_0^* = \left\lfloor\frac{1+\sqrt{2-2q+2m_0^*}}{1-2q+2m_0^*}\right\rfloor$. We will prove the claim by contradiction. Suppose that when $q + \frac{2N-1}{(N-1)^2} - \frac{1}{2} \leq m^* \leq q + 1$, the equilibrium number of links strictly increases in m^*. So $m_1^* < m_0^*$ implies $\left[\!\!\left[\frac{1+\sqrt{2-2q+2m_1^*}}{1-2q+2m_1^*}\right]\!\!\right] < \left[\!\!\left[\frac{1+\sqrt{2-2q+2m_0^*}}{1-2q+2m_0^*}\right]\!\!\right]$. Since $\left\lfloor\frac{1+\sqrt{2-2q+2m_1^*}}{1-2q+2m_1^*}\right\rfloor = \left\lfloor\frac{1+\sqrt{2-2q+2m_0^*}}{1-2q+2m_0^*}\right\rfloor = n_0^*$, it has to be the case that $\left[\!\!\left[\frac{1+\sqrt{2-2q+2m_1^*}}{1-2q+2m_1^*}\right]\!\!\right] = n_0^*$ and $\left[\!\!\left[\frac{1+\sqrt{2-2q+2m_0^*}}{1-2q+2m_0^*}\right]\!\!\right] = n_0^* + 1$. That is, the equilibrium number of links equals n_0^* when the effective margin is m_1^*, but it equals $n_0^* + 1$ when the effective margin increases to m_0^*. This implies that

$$\left(q - m_1^* + \frac{1}{n_0^*}\right)\frac{n_0^*-1}{n_0^*+1} > \left(q - m_1^* + \frac{1}{n_0^*+1}\right)\frac{n_0^*}{n_0^*+2} \tag{A9}$$

$$\Rightarrow m_1^* > q - 1 + \frac{2}{n_0^*},$$

and

$$\left(q - m_0^* + \frac{1}{n_0^*}\right)\frac{n_0^*-1}{n_0^*+1} < \left(q - m_0^* + \frac{1}{n_0^*+1}\right)\frac{n_0^*}{n_0^*+2} \tag{A10}$$

$$\Rightarrow m_0^* < q - 1 + \frac{2}{n_0^*}.$$

Eq.(A9) and Eq.(A10) imply that $m_1^* > q - 1 + \frac{2}{n_0^*} > m_0^*$ contradicting our set-up. Hence, when $m_1^* < m_0^*$, $\left[\!\!\left[\frac{1+\sqrt{2-2q+2m_1^*}}{1-2q+2m_1^*}\right]\!\!\right] > \left[\!\!\left[\frac{1+\sqrt{2-2q+2m_0^*}}{1-2q+2m_0^*}\right]\!\!\right]$.

Overall, **Proposition 2** implies that an equilibrium network becomes increasingly connected when m^* decreases. To prove **Proposition 3**, we have only to show that m^* decreases when σ or z increases, ceteris paribus, and m^* decreases when m increases, ceteris paribus. This is true, since

$$\frac{dm^*}{d\sigma} = -\frac{2m}{\rho\sigma^2 z} < 0, \tag{A11}$$

$$\frac{dm^*}{dz} = -\frac{2c}{\rho\sigma z^2} < 0, \tag{A12}$$

and

$$\frac{dm^*}{dm} = \frac{2}{\rho\sigma^2 z^2} > 0. \tag{A13}$$

Q.E.D.

Proof of Proposition 4

Proof:

Fixing an equilibrium network, we have

$$\frac{dp_2^i}{dz} = -\frac{\rho\sigma^2}{n_i + 1} < 0, \tag{A14}$$

$$\frac{dp_1^i}{dz} = -(1-q)\frac{\rho\sigma^2}{n_i + 1} < 0, \tag{A15}$$

$$\frac{d\text{markup}^i}{dz} = -q\frac{\rho\sigma^2}{n_i + 1} < 0. \tag{A16}$$

Q.E.D.

Proof of Proposition 5

Proof:

Given a fixed equilibrium network, we have

$$\frac{dp_2^i}{d\sigma} = -2\rho\sigma\left(I + \frac{z}{n_i + 1}\right) < 0, \tag{A17}$$

$$\frac{dp_1^i}{d\sigma} = -2(1-q)\rho\sigma\left(I + \frac{z}{n_i + 1}\right) < 0, \tag{A18}$$

$$\frac{d\text{markup}^i}{d\sigma} = -2q\rho\sigma\left(I + \frac{z}{n_i + 1}\right) + 2qM_1\sigma. \tag{A19}$$

If $M_1 \geq \rho\left(I + \frac{z}{n_i+1}\right)$, then $\frac{d\text{markup}^i}{d\sigma} \geq 0$. Otherwise, $\frac{d\text{markup}^i}{d\sigma} < 0$.

Q.E.D.

Proof of Proposition 6

Proof:

We want to show that p_1^i, p_2^i, and the markup^i increase in n_i. Without loss of generality, we let $n_i < n_i'$. Then

$$p_2^i(n'_i) - p_2^i(n_i) = \frac{\rho\sigma^2 z}{N}\left(\frac{1}{n_i + 1} - \frac{1}{n'_i + 1}\right) > 0 \Rightarrow p_2^i(n_i') > p_2^i(n_i). \tag{A20}$$

Since $p_1^i = (1 - q)p_2^i + q(M_0 - M_1\sigma^2)$, it is obvious that $p_1^i(n_i') > p_1^i(n_i)$. Similarly, as markup$^i = q\left(p_2^i - M_0 + M_1\sigma^2\right)$, markup$^i(n_i') >$ markup$^i(n_i)$.

Q.E.D.

Proof of Proposition 7

Proof:

The number of trades is

$$\frac{1}{N} + \sum_{j:ij\in G} \frac{1}{N} = \frac{n_i + 1}{N}, \tag{A21}$$

which obviously increases in n_i.

Since, in an equilibrium network, when dealer *i* and dealer *j* are linked they have the same number of links, the volume of trades is

$$\begin{aligned} &\frac{1}{N}\left(I + z - X^{A,i}\right) + \sum_{j:ij\in G} \frac{1}{N} X^{B,j} \\ &= \frac{1}{N}\left(1 - \frac{2}{n_i + 1}\right) z + \frac{n_i}{N}\frac{n_i - 1}{n_i}\frac{z}{n_i + 1} \\ &= \frac{2}{N}\left(1 - \frac{2}{n_i + 1}\right) z, \end{aligned} \tag{A22}$$

which also increases in n_i.

Q.E.D.

Proof of Proposition 8

Proof:

The expected risky holding is

$$EX^i = \frac{1}{N}X^{A,i} + \sum_{j:ij\in G}\frac{1}{N}\left(I + X^{B,j}\right) + \left(1 - \frac{1}{N} - \sum_{j:ij\in G}\frac{1}{N}\right)I \tag{A23}$$

$$= \frac{1}{N}\left(I + \frac{2z}{n_i + 1}\right)$$

$$+ \sum_{j:ij\in G}\frac{1}{N}\left(I + \frac{n_j - 1}{n_j}\frac{z}{n_j + 1}\right)$$

$$+ \left(1 - \frac{1}{N} - \sum_{j:ij\in G}\frac{1}{N}\right)I$$

$$= \frac{1}{N}z + I.$$

The variance if the risky holding is

$$Var\left(X^i\right) = E\left(X^i\right)^2 - \left(EX^i\right)^2 \tag{A24}$$

$$= \frac{1}{N}\left(I + \frac{2z}{n_i + 1}\right)^2$$

$$+ \sum_{j:ij\in G}\frac{1}{N}\left(I + \frac{n_j - 1}{n_j}\frac{z}{n_j + 1}\right)^2$$

$$+ \left(1 - \frac{1}{N} - \sum_{j:ij\in G}\frac{1}{N}\right)I^2 - \left(\frac{1}{N}z + I\right)^2$$

$$= \frac{1}{N}\left(I + \frac{2z}{n_i + 1}\right)^2 + n_i\left(I + \frac{n_j - 1}{n_j}\frac{z}{n_j + 1}\right)^2$$

$$+ \left(1 - \frac{1}{N} - \frac{n_i}{N}\right)I^2 - \left(\frac{1}{N}z + I\right)^2.$$

We have only to prove that the claim is true for $n_i$and $n_i + 1$, and then by induction we infer that the claim is true for any $n_i < n_j$.

$$Var\left(X^i(n_i)\right) - Var\left(X^i(n_i + 1)\right) \tag{A25}$$

$$= \frac{1}{N}\left(I + \frac{2}{n_i + 1}z\right)^2 - \frac{1}{N}\left(I + \frac{2}{n_i + 2}z\right)^2$$
$$+ \frac{n_i}{N}\left(\left(I + \frac{n_i - 1}{n_i}\frac{z}{n_i + 1}\right)^2 - \left(I + \frac{n_i}{n_i + 1}\frac{z}{n_i + 2}\right)^2\right)$$
$$- \frac{1}{N}\left(I + \frac{n_i}{n_i + 1}\frac{z}{n_i + 2}\right)^2 + \frac{1}{N}I^2.$$

Since

$$\frac{1}{N}\left(I + \frac{2}{n_i + 1}z\right)^2 - \frac{1}{N}\left(I + \frac{2}{n_i + 2}z\right)^2 = \frac{1}{N}z\frac{2}{(n_i + 1)(n_i + 2)}\left(2I + 2z\frac{2n_i + 3}{(n_i + 1)(n_i + 2)}\right), \tag{A26}$$

$$\frac{n_i}{N}\left(\left(I + \frac{n_i - 1}{n_i}\frac{z}{n_i + 1}\right)^2 - \left(I + \frac{n_i}{n_i + 1}\frac{z}{n_i + 2}\right)^2\right) = \frac{1}{N}z\frac{n_i - 2}{(n_i + 1)(n_i + 2)}\left(2I + z\frac{2n_i^2 + n_i - 2}{n_i(n_i + 1)(n_i + 2)}\right)' \tag{A27}$$

and

$$-\frac{1}{N}\left(I + \frac{n_i}{n_i + 1}\frac{z}{n_i + 2}\right)^2 + \frac{1}{N}I^2 = -\frac{1}{N}z\frac{n_i}{(n_i + 1)(n_i + 2)}\left(2I + z\frac{n_i}{(n_i + 1)(n_i + 2)}\right)' \tag{A28}$$

we have

$$\frac{Var\left(X^i(n_i)\right) - Var\left(X^i(n_i + 1)\right)}{\frac{z^2}{N}} = 4\frac{2n_i + 3}{(n_i + 1)^2(n_i + 2)^2} + \frac{(n_i - 2)(2n_i^2 + n_i - 2)}{n_i(n_i + 1)^2(n_i + 2)^2} - \frac{n_i^2}{(n_i + 1)^2(n_i + 2)^2} = \frac{n_i^3 + 5n_i^2 + 2n_i + 4}{n_i(n_i + 1)^2(n_i + 2)^2} > 0. \tag{A29}$$

Q.E.D.

Proof of Proposition 9

Proof:

Obviously, a small-capacity dealer never connects to his own kind, as there is no risk-sharing benefit. This implies that small-capacity dealers connect only to large-capacity or medium-capacity dealers. Hence, $n_{s_S} \leq n_{s_M}, n_{s_L}$.

Now suppose that G is a strongly stable network such that $n_{s_M} > n_{s_L} > 2$. Let dealer i be one of s_M's connected dealers who does not connect to s_L. Consider another network G' obtained via replacing the $s_M i$ link with the $s_L i$ link. Since $s_M < s_L$, which implies $m^{**}_{s_M} > m^{**}_{s_L}$, we have

$$U_i(G') - U_i(G) = \frac{\rho\sigma^2}{6N}\left(s_L^3 \frac{n_{s_L} - 1}{n_{s_L}^2(n_{s_L} + 1)} - s_M^3 \frac{n_{s_M} - 1}{n_{s_M}^2(n_{s_M} + 1)}\right) > 0, \tag{A30}$$

$$\begin{aligned} U_{s_L}(G') - U_{s_L}(G) &= \frac{\rho\sigma^2}{3N}\left(s_L^3(q - m^{**}_{s_L})\frac{n_{s_L}}{2(n_{s_L} + 2)} + \frac{1}{2}s_i^3 \frac{n_i - 1}{n_i^2(n_i + 1)}\right) \\ &\geq \frac{\rho\sigma^2}{3N}\left(s_M^3(q - m^{**}_{s_M})\frac{n_{s_M} - 1}{2(n_{s_M} + 1)} + \frac{1}{2}s_i^3 \frac{n_i - 1}{n_i^2(n_i + 1)}\right). \end{aligned} \tag{A31}$$

For G to be a strongly stable network it has to be the case that cutting the link between s_M and i cannot make s_M better off. This means that $\frac{\rho\sigma^2}{3N}\left(s_M^3(q - m^{**}_{s_M})\frac{n_{s_M}-1}{2(n_{s_M}+1)} + \frac{1}{2}s_i^3 \frac{n_i-1}{n_i^2(n_i+1)}\right) > 0$, which implies that $U_{s_L}(G') > U_{s_L}(G)$. Thus, G' makes both s_L and i better off, which means G cannot be a strongly stable network.

Q.E.D.

Proof of Proposition 10

Proof:

Since $m_{s_L}^{**} < q$, a large-capacity dealer always want to connect to an additional dealer. As a small-capacity dealer never has the chance to trade with an investor and hence never sells in the inter-dealer market, he does not pay any linking cost for preparing collateral. Based on these two arguments, a large-capacity dealer always connects to its own type and small-capacity dealers.

A small-capacity dealer does not connect to his own type, as there is no risk-sharing benefit.

For a medium-capacity dealer, he never connects to a small-capacity dealer because of $q < m_{s_M}^{**}$. If $s_L^3 \frac{N-2}{q(N-1)^2 N} > s_M^2 \left(\frac{3m}{\rho\sigma q} - s_M\right)$, then all medium-capacity dealer connect to all large-capacity dealer because this condition implies that for any $n_{s_M}, n_{s_L} \leq N-1$, $s_M^3\big(q - m_{s_M}^{**}\big)\frac{n_{s_M}-1}{2(n_{s_M}+1)} + \frac{1}{2}s_L^3 \frac{n_{s_L}-1}{n_{s_L}^2(n_{s_L}+1)} > 0$. That is, a medium-capacity dealer is always better off connecting to a large-capacity dealer.

Since any medium-capacity dealer connects to all high-capacity dealers, all medium-capacity dealers have at least N_{s_L} links. Since all medium-capacity dealers are identical, **Proposition 1** still applies. The payoff function for a medium-capacity dealer m is

$$U_m(G) = \frac{\rho\sigma^2 s_M^3}{6N}\left(q - m_{s_M}^{**} + \frac{n_{s_M} - N_{s_L}}{n_{s_M}^2}\right)\frac{n_{s_M}-1}{n_{s_M}+1}. \tag{A32}$$

Let

$$n_{s_M}^* = \arg\max_{n_{s_M}\in\mathbb{N}}\left(q - m_{s_M}^{**} + \frac{n_{s_M} - N_{s_L}}{n_{s_M}^2}\right)\frac{n_{s_M}-1}{n_{s_M}+1}, \tag{A33}$$

where $n_{s_M}^*$ is the number of links a medium-capacity dealer has in equilibrium.

Q.E.D.

Proof of Proposition 11

Proof:

Since

$$\bar{p}^0_{s_L} - \bar{p}^0_{s_M} = \int_{\underline{z}}^{\underline{z}+s_L} p^0_{s_L} dz - \int_{\underline{z}}^{\underline{z}+s_M} p^0_{s_M} dz, \tag{A34}$$

$$= (1-q)\left(\left(\overline{v} - \rho\sigma^2 I + \frac{q}{1-q}(M_0 - M_1\sigma^2)\right)(s_L - s_M) + \frac{\rho\sigma^2}{2}\left(\frac{s_M^2}{n^*_{s_M}+1} - \frac{s_L^2}{N}\right)\right),$$

it is obvious if $\left(\overline{v} - \rho\sigma^2 I + \frac{q}{1-q}\ (M_0 - M_1\sigma^2)\ )\right)(s_L - s_M) \geq \frac{\rho\sigma^2}{2}\left(\frac{s_L^2}{N} - \frac{s_M^2}{n^*_{s_M}+1}\right)$, then $\overline{p}^0_{s_L} \geq \overline{p}^0_{s_M}$. Otherwise, $\overline{p}^0_{s_L} < \overline{p}^0_{s_M}$.

Q.E.D.

Proof of Proposition 12

Proof:

Definition A.1

A cycle in a "trading-sets network" is a path consisting of more than two non-repeated trading sets and the starting set is the same as the ending set.

Lemma A.2 *[Theorem 4.2 in* Malamud and Rostek (2013)*]*

Any two trading sets in the "trading-sets network" have the same prices if and only if these two sets are on the same cycle.

Proof:

See Malamud and Rostek (2013).

Q.E.D.

<u>Definition A.2</u>

A link in a "trading-sets network" is a bridge if cutting it would cause its ending points to lie in separate components.

<u>Lemma A.3</u>

A monopolistic dealer in an inter-dealer network is a bridge in the "trading-sets network" derived from the inter-dealer network.

Proof:

Suppose a monopolistic dealer is not a bridge in the "trading-sets network." Then removing the monopolistic dealer does not increase the number of components in the "trading-sets network." This means that all trading sets that include the monopolistic dealer are still connected, even when the monopolistic dealer is removed. To see if dealers in those trading-sets are connected in the inter-dealer network, we do the following.

i) Label those trading-sets in sequence from 1 to m.

ii) Start from set 1, and then find its connected sets.

iii) Start from those connected sets identified in step 2, and then find their connected sets.

iv) Repeat step 3 until all trading sets are exhausted.

Based on **Definition 3** and **4**, the above algorithm shows that dealers in the same trading set are connected, and dealers in trading sets identified in step 3 are also connected. Since all those trading sets are connected, step 4 eventually ends, which means that all dealers in those trading sets are connected. This contradicts the definition of the monopolistic dealer, whose neighbors belong to separate components. Hence, a monopolistic dealer is a bridge in the "trading-sets network."

Q.E.D.

Lemma A.2 is used to prove the second half of **Proposition 12**. If a pair of dealers have more than two unconnected common neighbors, then in the “trading-sets network” this pair of dealers and their unconnected common neighbors construct a cycle. By **Lemma A.2**, prices along the cycle are the same, which means that any of those dealers buys and sells at the same price in distinct trading sets. That is, the markup for “hot potato” trading is zero. The first half of **Proposition 12** is proved by **Lemma A.3**, which states that a monopolistic dealer is a bridge in the “trading-sets network.” Hence, the monopolistic dealer can never be in a cycle. **Lemma A.2** then implies that the monopolistic dealer always charges non-zero markups.

Q.E.D.

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