Bond Market Exposures to Macroeconomic and Monetary Policy Risks

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November 2014

Abstract

I document evidence of structural changes in key moments of the yield curve and the correlation between bond-stock returns and consumption growth-inflation. I estimate an equilibrium model that features regime shifts in monetary policy aggressiveness and the conditional covariance of consumption and inflation that generate endogenous regime-switching inflation and bond price dynamics. The shifts in the conditional covariance process affect the dynamics of the yield curve and asset prices, while policy changes mostly influence their second moments. The model accounts for several bond market features, including the presence of unspanned macroeconomic factors and changes in the bond-stock return correlation.

Keywords: Regime-Switching Taylor Rule; Time Varying Consumption Growth-Inflation Covariations; Bond-Stock Return Correlation; Regime Uncertainty; Risk Premiums.

JEL classification: E43, G12

I am immensely grateful to my advisors Francis X. Diebold, Frank Schorfheide, Ivan Shaliastovich, and Amir Yaron for their invaluable guidance in writing this paper. I thank Ravi Bansal, Francesco Bianchi, Luigi Bocola, Mikhail Chernov, Francis DiTraglia, Gregory Duffee, Jesus Fernandez-Villaverde, Campbell Harvey, Christian Opp, Minchul Shin, Nikolai Roussanov, Ram Yamarthy, and seminar participants at the Board of Governors, Boston College, Chicago Booth, Duke Fuqua, the Kansas City Fed, Notre Dame, the New York Fed, Penn, UCLA Anderson, and UIUC.

1 Introduction

There is mounting evidence that the U.S. Treasury yield curve and macroeconomic fundamentals, that is, consumption growth and inflation, have undergone structural changes over the past decade. For example, recent empirical studies have come to understand that U.S. Treasury bonds have served as a hedge to stock market risks in the last decade.[1] In sharp contrast to the 1980s, during which both bond and stock returns were low and tended to co-move positively, the bond-stock return correlation has turned strongly negative in the 2000s. Several other aspects of bond markets have changed over the years between 1998 and 2011. Among them are a flattening of the yield curve and a substantial drop in the degree of time variation in excess bond returns. The striking feature is that the correlation between consumption growth and inflation has also changed from negative to positive in the same period.[2] In this paper, I study the role of structural changes in macroeconomic fundamentals as well as in the aggressiveness of monetary policy in explaining the bond market changes over the last decade. The central contributions of this paper are to investigate whether the bond market changes are brought about by external shocks, by monetary policy, or by both, and to quantify and characterize bond market price exposures to macroeconomic and monetary policy risks.

I develop a state-space model to capture the joint dynamics of consumption growth, inflation, and asset returns. The real side of the model builds on the long-run risks (LRR) model of Bansal and Yaron (2004) and assumes that consumption growth contains a small predictable component (i.e., long-run growth), which, in conjunction with investors' preference for early resolution of uncertainty, determines the price of real assets. The nominal side of the model extends Gallmeyer, Hollifield, Palomino, and Zin (2007) in that inflation dynamics are derived endogenously from the monetary policy rule, and the nominal assets inherit the properties of monetary policy. My model distinguishes itself from the existing literature in two important dimensions. First, it allows for changes in the monetary policy rule, both in the inflation target and in the

[1]See Baele, Bekaert, and Inghelbrecht (2010); Campbell, Pflueger, and Viceira (2013); Campbell, Sunderam, and Viceira (2013); and David and Veronesi (2013).

[2]See Table 1 for descriptive statistics and Figure 1 for impulse responses.

stabilization rule (i.e., the central bank's response to deviations of actual inflation from the inflation target and to fluctuations in consumption). The regime-switches in stabilization policy coefficients are modeled through a Markov process. Second, I allow for a channel that breaks the long-run dichotomy between the nominal and real sides of the economy. I assume that the fluctuations in the long-run growth component are not just driven by its own innovation process but also by the innovation to the inflation target of the central bank. I add flexibility to this channel by allowing for both positive-negative fluctuations.[3] In essence, there is a regime-switching Markov process that captures the sign-switching behavior of the conditional covariance between long-run growth and the inflation target.

As a consequence of my model features, asset prices and macroeconomic aggregates are affected through two distinct channels: (1), changes in the conditional covariance between the inflation target and long-run growth, and (2), changes in the stabilization policy rule. This leads to endogenous inflation dynamics, and the resulting nominal bond market prices are differentially affected through both channels. In order to empirically assess the relative strengths of the two channels and examine the quantitative fit of the model, I apply a Bayesian approach to the estimation. I embed a particle-filter-based likelihood approximation into a Metropolis-Hastings algorithm to generate parameter draws from the posterior distribution and to solve a nonlinear filtering problem. The estimation incorporates mixed-frequency data to maximize the use of data. The idea is to combine data from a quarterly survey of professional forecasters with monthly macroeconomic fundamentals and bond yields to identify hidden regimes, latent conditional means, and conditional variance dynamics of growth and inflation. All of these procedures are done simultaneously in a nonlinear state-space framework.

The estimation of the model delivers several important empirical findings. First, the estimation results suggest that the economic environment involves two regimes with

[3]The economic reasoning behind the changes in the covariance structure shares the New Keynesian view that there are periods in which the inflation target is above the so-called *desirable rate of inflation*, e.g., the rate at which prices can be changed without costs. Any positive shock to the inflation target during such periods creates distortions by reducing long-run growth. When the inflation target is assumed to be lower than the *desirable* one, positive shocks to the inflation target will actually remove distortions and generate positive long-run growth movements. See Aruoba and Schorfheide (2011) for more.

different conditional covariance dynamics: one with a negative covariance between the inflation target and long-run growth (*countercyclical* inflation) and one with a positive covariance (*procyclical* inflation). The relative magnitude of the conditional heteroscedasticity present is larger in the countercyclical inflation regime. In each inflation regime the central bank either increases interest rates more than one-for-one with inflation (*active* monetary policy) or does not (*passive* monetary policy). Overall, a total of 4 different regimes affect the comovement of bond prices and macroeconomic aggregates. Second, the changes in the conditional covariance between the inflation target and long-run growth alter the dynamics of the long-run components and have a persistent effect on bond markets. On the other hand, the changes in the conduct of monetary policy are more targeted to affecting the short-run volatility dynamics of inflation, and therefore, their effect on bond markets is relatively short-lived. Therefore, the model predicts the changes in the conditional covariance dynamics to be the main driver of structural changes in bond markets, such as sign changes in the stock-bond return correlation and the time variations in risk premiums.

Third, each regime carries distinctly different risk prices, and uncertainty about moving across regimes poses additional risks to bond markets. The risk channels can be broadly classified into two types: "*within*-regime" and "*across*-regime" risks. For the purpose of explanation, I decompose the long-term bond yields into the expected sum of future short rates (the expectations component) and the term premium (risk compensation for long-term bonds). Risks associated with the countercyclical inflation regime raise both the expectations component and the term premium.[4] Risks for the procyclical inflation regime work in the opposite direction. With regard to monetary policy risks, the effect is mostly on the expectations component, but its directional influence depends on the inflation regime. When the policy stance is active, monetary policy works to lower the inflation expectation and produces a downward shift in the level of the term structure (i.e., lowers the expectations component). With passive monetary policy and a countercyclical inflation regime, agents understand that the central bank is less effective in stabilizing the economy (raising the expectations component) and they demand a greater inflation premium, leading to the steepest term structure. With

[4]Note that this is how Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2013) generate the inflation premium.

passive monetary policy and a procyclical inflation regime, the inherent instability associated with the passive monetary policy will amplify the "procyclicality" (lower the expectations component). The across-regime risks imply that the risk properties of alternative regimes are incorporated, since agents are aware of the possibility of moving across regimes. This is a prominent feature of the model that generates an upward-sloping yield curve even when the economy is in the procyclical inflation regime. When in the procyclical inflation regime, which otherwise would lead to a downward sloping yield curve, the risks of the monetary authority not responding aggressive by enough to inflation fluctuations and the risks of transitioning to the countercyclical inflation regime generate an upward sloping yield curve.

Fourth, the time variations within a regime and risks associated with moving across regimes give rise to time variations in risk premia, which provide testable implications for the expectations hypothesis (EH). The estimated model as a whole overwhelmingly rejects the EH and provides strong empirical evidence of time variations in expected excess bond returns. The evidence is supported by the model-implied term spread regression of Campbell and Shiller (1991) and the excess bond return predictability regression of Cochrane and Piazzesi (2005). I find that the degree of violation of the EH is least apparent within the procyclical inflation and passive monetary policy regime, which is also supported in the data.

Finally, due to the nonlinearities created by regime switching, the model is not affine in yields. This model feature allows me to analyze issues related to unspanned macroeconomic risks (see Joslin, Priebsch, and Singleton (2014) for example). In the context of linear predictive regression analysis, model simulations reveal that current macroeconomic variables, i.e., consumption growth and inflation, are informative about future values of macroeconomic variables, the level of bond yields, and risk premiums after controlling for the current cross-section of bond yields. My model provides strong evidence of macroeconomic risks in consumption growth and inflation that are unspanned by the cross-section of the current bond yields. However, consumption growth and inflation become insignificant once I condition on the model state variables. This finding implies that the information set of the model state variables encompasses those of macroeconomic variables and of the cross-section of bond yields. This finding further indicates that the model state variables contain (nearly) all risks in consumption growth and inflation that are priced in the bond market. I then ask how much of the variation in model state variables is spanned by the cross-section of yields. The R^2 values from the linear regression for predicting the real state variables (i.e., long-run growth and its volatility component) are about one third of those from predicting the nominal ones. The evidence is consistent with the findings in Joslin, Priebsch, and Singleton (2014) that the portfolio of risks that shape real economic growth is not spanned by the cross-section of bond yields.

Related Literature. This paper is related to several strands of the literature. My work is related to a number of recent papers that study the changes in the bond-stock return correlation. Baele, Bekaert, and Inghelbrecht (2010) utilize a dynamic factor model in which stock and bond returns depend on a number of economic state variables, e.g., macroeconomic, volatility, and liquidity factors. The authors attribute the changes in the bond-stock return correlation to liquidity factors. Campbell, Sunderam, and Viceira (2013) embed time-varying bond-stock return covariance in a quadratic term-structure model and argue that the root cause is due to changes in nominal risks in bond markets. What distinguishes my work from these reduced-form studies is that it builds on a consumption-based equilibrium model with monetary policy to identify the driving forces behind the bond-stock return correlation changes.

The works that come closest to my paper are Burkhardt and Hasseltoft (2012), Campbell, Pflueger, and Viceira (2013) and David and Veronesi (2013). Burkhardt and Hasseltoft (2012) find an inverse relation between bond-stock correlations and correlations of growth and inflation. Burkhardt and Hasseltoft (2012) rationalize their finding in a long-run risks model with regime shifting (in the conditional dynamics of macroeconomic fundamentals) calibrated to data on fundamentals and asset returns. Campbell, Pflueger, and Viceira (2013) examine the role of monetary policy in nominal bond risks using a New Keynesian model. Using macroeconomic fundamentals and asset prices, Campbell, Pflueger, and Viceira (2013) estimate the model separately over three different subsamples. From the counterfactual analysis, the authors claim that the change in monetary policy parameters is the main driver of bond risks. David and Veronesi (2013) estimate an equilibrium model of learning about inflation news and show that market participants' variation in their beliefs about inflation regimes strongly affects the direction of bond-stock comovement.

Here are a few of the important distinctions between my paper and the existing ones. First, the structural changes in the economy are identified from macroeconomic fundamentals and asset prices without imposing (sometimes ad hoc) assumptions, e.g., known break points as in Burkhardt and Hasseltoft (2012) and Campbell, Pflueger, and Viceira (2013). Second, I explicitly account for the role of market participants' beliefs about regime switches in the bond market. I find strong empirical evidence in the data that the anticipation of moving across regimes is one of the most important risk factors priced in the bond market. For example, ignoring the role of beliefs overstates (understates) the implications of a passive (active) monetary policy regime or countercyclical (procyclical) inflation regime because the risk properties of alternative regimes are not incorporated. Campbell, Pflueger, and Viceira (2013) do not allow for a beliefs channel to operate. Third, my model exhibits a richer structure than that of David and Veronesi (2013). By accounting for time variations in all elements (beyond diagonal components) of the covariance matrix of macroeconomic innovations and in monetary policy parameters, I am able to provide extensive descriptions of the bond market transmission mechanism of monetary policy and macroeconomic shocks. In this regard, my model complements Burkhardt and Hasseltoft (2012), Campbell, Pflueger, and Viceira (2013), and David and Veronesi (2013).

By investigating the time variation in the stance of monetary policy, my work also contributes to the monetary policy literature, e.g., Clarida, Gali, and Gertler (2000), Coibon and Gorodnichenko (2011), Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez (2010), Lubik and Schorfheide (2004), Schorfheide (2005), and Sims and Zha (2006).[5] While most of these papers study the impact of changes in monetary policy on macroeconomic aggregates, Ang, Boivin, Dong, and Loo-Kung (2011) and Bikbov and Chernov (2013) focus on their bond market implications (using reduced-form modeling frameworks). My work distinguishes itself from these last two papers, since I focus on a fully specified economic model and characterize time-varying bond market exposures to monetary policy risks.

[5] Note that I am including those that explicitly account for changes in monetary policy.

In terms of modeling the term structure with recursive preferences, this paper is closely related to those of Bansal and Shaliastovich (2013), Le and Singleton (2010), Doh (2012), and Piazzesi and Schneider (2006), who work in an endowment economy setting, and, van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012), who study a production-based economy. My work generalizes the first four by endogenizing inflation dynamics from the monetary policy rule. While van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012) allow for endogenous capital and labor supply and analyze their interaction with the yield curve, which are ignored in my analysis, they do not allow for time variations in volatilities and in monetary policy stance, both of which are key risk factors in my analysis.

There is a growing and voluminous literature in macro and finance that highlights the importance of volatility for understanding the macroeconomy and financial markets (see Bansal, Kiku, and Yaron (2012); Bansal, Kiku, Shaliastovich, and Yaron (2013); Bloom (2009); and Fernández-Villaverde and Rubio-Ramírez (2011)). This paper further contributes to the literature by incorporating a time-varying conditional covariance process. Finally, the estimation algorithm builds on Schorfheide, Song, and Yaron (2013), yet further develops to accommodate Markov-switching processes (see Kim and Nelson (1999) for a comprehensive overview of estimation methods for the Markov switching models) and efficiently implements Bayesian inference using particle filtering in combination with a Markov chain Monte Carlo (MCMC) algorithm.

The remainder of the paper is organized as follows. Section 2 provides empirical evidence on structural changes in the U.S. economy. Section 3 introduces the model environment and describes the model solution. Section 4 presents the empirical state-space model and describes the estimation procedure. Section 5 discusses the empirical findings, and Section 6 provides concluding remarks.

2 Empirical Evidence on Structural Changes

Here, I provide empirical evidence on structural changes in macroeconomic fundamentals, in the Treasury yield curve, and in the correlation between bond and stock returns.

2.1 Changes in Macroeconomic Fundamentals

A recurrent theme of macro-finance term structure models that underlies risk premiums is that inflation uncertainty makes nominal bonds risky.[6] A common pattern to model inflation uncertainty, which Piazzesi and Schneider (2006) provide empirically and which is found by many others, is to assume that inflation is a carrier of bad news to consumption growth. By construction, inflation erodes the value of nominal bonds precisely at times when consumption growth is low (or marginal utility is high). However, this pattern does not appear to be robust over different sub-samples.

Following Piazzesi and Schneider (2006), I assume that the vector of inflation (π) and consumption growth (Δc) has the following state-space representation

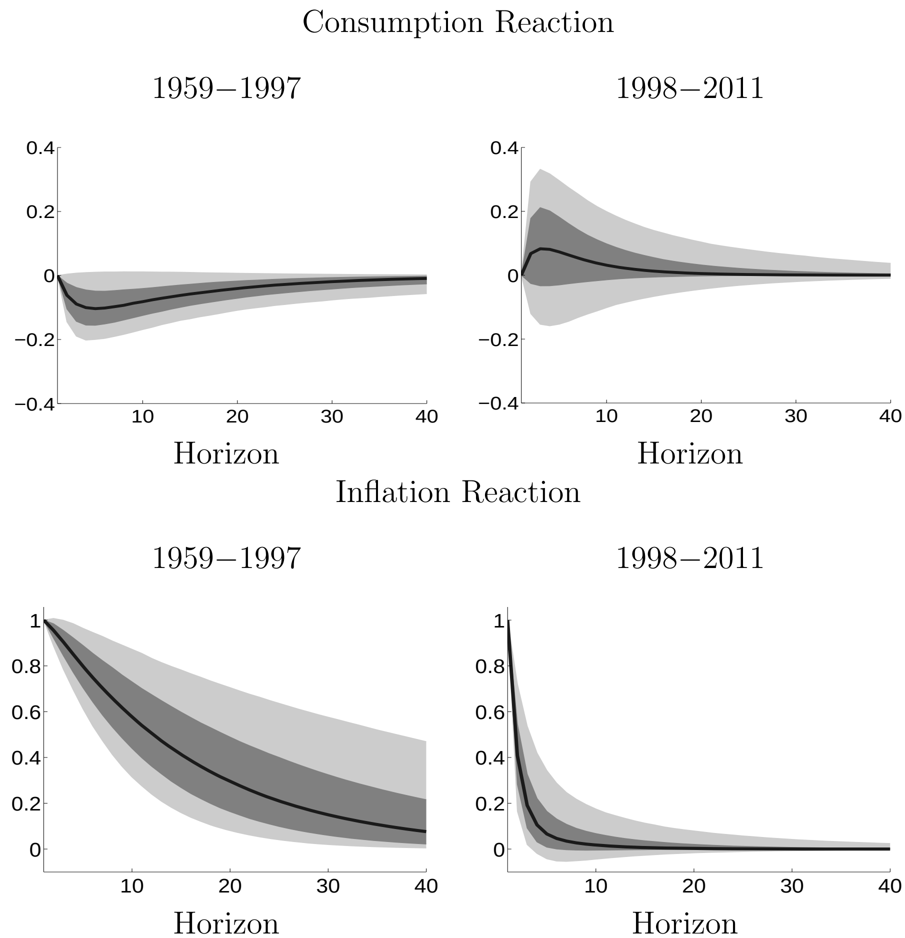
$$\begin{aligned} z_t &= s_{t-1} + \varepsilon_t, \quad z_t = [\pi_t, \Delta c_t]' \\ s_t &= \phi s_{t-1} + \phi K \varepsilon_t, \quad \varepsilon_t \sim N(0, \Omega). \end{aligned} \tag{1}$$

The state vector s_t is 2-dimensional and contains expected inflation and consumption, ϕ is the 2×2 autoregressive matrix, and K is the 2×2 gain matrix. I estimate this system with data on quarterly consumption and inflation that span 1959 to 2011 using Bayesian methods. Details (priors and posterior estimates) are provided in the Online Appendix. The estimation sample is split into two parts. One is from 1959 to 1997 and the other spans the period 1998 to 2011. In order to understand the key properties of the estimated dynamics, I report the impulse responses of the system in Figure 1. Each response represents either the change in consumption or inflation forecasts following a 1 percent inflation shock $\varepsilon_{\pi,t}$.

The findings on the left (the top and bottom left plots) are consistent with what Piazzesi and Schneider (2006) report. Two aspects of the results on the right are noteworthy. First, the sign of consumption's reaction to an inflation shock changed from negative (the top left plot) to positive (the top right plot) over the last fifteen years. A 1 percent inflation surprise predicts that consumption growth will be higher

[6] Macro-finance term structure models refer to models in which the pricing kernel comes directly from a utility-maximization problem. Gürkaynak and Wright (2012) provide a nice overview of macro-finance term structure models.

Figure 1: Consumption and Inflation Reaction to 1 Percentage Point Inflation Surprises



Notes: Black lines represent posterior median reactions to 1 percentage point surprises in inflation. Light (dark) gray-shaded areas correspond to 90 (60)% credible intervals. x-axis represents the impulse response horizon (in quarters).

by approximately 10 basis points in the next year.[7] Second, the own-shock responses for inflation decayed much faster over the last fifteen years. The impact of a 1 percent inflation surprise on itself completely dies out over the next 1-2 years (the bottom right plot). This is mainly due to a large decline in the persistence of the expected inflation process, e.g., the autoregressive coefficient for inflation dropped from 0.96 to 0.41 (see

[7]Table 1 also reports the unconditional correlation between different measures of real growth and inflation. The sign-switching pattern is robust to the choice of growth and inflation variables and to different sampling frequencies.

Table 1: Descriptive Statistics

	Pre-1998	Post-1998	Full Sample
	Annualized Average Bond Yields		
Mean (y_{3m})	6.07	2.64	5.16
Mean (y_{1y})	6.51	2.88	5.55
Mean (y_{3y})	6.87	3.35	5.94
Mean (y_{5y})	7.05	3.78	6.19
Mean (y_{10y})	7.35	4.38	6.57
	Correlation between Growth and Inflation		
Corr($\Delta c, \pi$)	-0.19	0.02	-0.11
Corr$(\Delta c, \pi)^Q$	-0.36	0.18	-0.16
Corr$(\Delta \text{gdp}, \pi)^Q$	-0.26	0.33	-0.13
Corr$(\mathbb{E}\Delta \text{gdp}, \mathbb{E}\pi)^Q$	-0.43	0.19	-0.31
	Correlation between Stock and Bond Returns		
Corr(r_m, r_{2y})	0.16	-0.13	0.09
Corr(r_m, r_{3y})	0.21	-0.14	0.13
Corr(r_m, r_{4y})	0.22	-0.14	0.14
Corr(r_m, r_{5y})	0.24	-0.14	0.15
	Term Spread Regression, Slope Coefficient		
$r_{2y,t+1y}$ onto spread$_{2y,t}$	-0.95	0.89	-0.62
$r_{3y,t+1y}$ onto spread$_{3y,t}$	-1.37	0.43	-1.00
$r_{4y,t+1y}$ onto spread$_{4y,t}$	-1.77	0.02	-1.40
$r_{5y,t+1y}$ onto spread$_{5y,t}$	-1.69	-0.28	-1.41
	Excess Bond Return Predictability, R^2		
$rx_{2y,t+1y}$ onto forward$_t$	34.34	13.60	20.68
$rx_{3y,t+1y}$ onto forward$_t$	35.29	13.92	21.54
$rx_{4y,t+1y}$ onto forward$_t$	37.72	15.79	24.38
$rx_{5y,t+1y}$ onto forward$_t$	34.49	19.15	22.32

Notes: The top three panels report descriptive statistics for aggregate consumption growth (Δc), gross domestic product (GDP) growth (Δgdp), expected GDP growth ($\mathbb{E}\Delta$gdp), consumer price index (CPI) inflation (π), expected inflation ($\mathbb{E}\pi$), log returns of the aggregate stock market (r_m), the log bond yields (y_n), log bond returns (r_n), and log bond excess returns (rx_n) where $n \in \{3m, 1y, 2y, 3y, 4y, 5y, 10y\}$. It shows mean (Mean) and pairwise correlation (Corr) between growth and inflation and market and bond returns. Measures of expected GDP growth ($\mathbb{E}\Delta$gdp) and expected inflation ($\mathbb{E}\pi$) are based on the Survey of Professional Forecasters historical forecasts, which are available from 1968 to 2011. The remaining variables are available from 1959 to 2011. The numbers in the table are derived from monthly frequency data except for those with the superscript "Q"; those numbers are derived from quarterly frequency data. The fourth panel provides the slope coefficient from the term spread regression of Campbell and Shiller (1991). The "spread$_{n,t}$" is the difference between an n year yield and a 1-year yield. I focus on a 1-year return horizon. r_n (rx_n) denotes return (excess return) on an n year bond. The last panel provides R^2 values (in percent) from the excess bond return predictability regression found in Cochrane and Piazzesi (2005). "forward$_t$" includes a constant term, a 1-year bond yield, and four forward rates.

the Online Appendix for details).

The key aspects of the data are that the inflation dynamics have changed substantially over time and there are periods in which inflation an surprise can be good news for consumption growth.[8]

2.2 Changes in the Treasury Bond Yields and Asset Returns

The top panel of Table 1 shows that yields with longer maturities are on average higher than those with shorter maturities. This evidence is typically known as un upward sloping yield curve. What is puzzling from the perspective of existing term structure models is that the Treasury yield curve still slopes upward during those periods in which inflation is a carrier of good news to consumption growth. It is interesting to observe that several other aspects surrounding the Treasury bond markets have also changed during this period (see the bottom three panels of Table 1). The correlation between bond and stock returns changed from positive to negative, and the degree of violation of the expectations hypothesis (explained in detail later) and the excess nominal bond return predictability (risk premiums) appear to be much lower over the last fifteen years.

Overall, the evidence in Table 1 is interesting not only because it shows the limitations of the existing approaches, but also because it implies that the sources of risk behind the yield curve might have changed over time. There is an important reason to believe that the yield curve and inflation dynamics are sensitive to monetary policy shifts or changes in the distribution of economic shocks. In fact, a large literature in macroeconomics supports frequent shifts in the Federal Reserve's inflation policy action and in the distribution of fundamental shocks (see Clarida, Gali, and Gertler (2000), Coibon and Gorodnichenko (2011), Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez (2010), Justiniano and Primiceri (2008), Schorfheide (2005), and Sims and Zha (2006)). Such empirical facts posit the need to look at the data from a broader perspective, which calls for a more flexible approach to the joint modeling of macroeconomic fundamentals, monetary policy, and stock and bond asset prices.

[8] This evidence is also documented in David and Veronesi (2013).

3 The LRR Model with Monetary Policy

In response to the previous empirical facts, I develop an asset pricing framework that embeds risks through regime changes in the monetary policy action as well as in the covariance matrix of nominal inflation and real growth innovations. These shifts, which give rise to endogenous regime switching inflation and bond price dynamics, are considered to be a potential source of risk variations that can explain several bond market features, including the upward sloping yield curve.

3.1 Preferences and Cash-Flow Dynamics

I consider an endowment economy with a representative agent who maximizes her lifetime utility,

$$V_t = \max_{C_t} \left[(1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \big(\mathbb{E}_t [V_{t+1}^{1-\gamma}] \big)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}},$$

subject to the budget constraint

$$W_{t+1} = (W_t - C_t) R_{c,t+1},$$

where W_t is the wealth of the agent, $R_{c,t+1}$ is the return on all invested wealth, γ is risk aversion, $\theta = \frac{1-\gamma}{1-1/\psi}$, and ψ is the intertemporal elasticity of substitution (IES).

Following Bansal and Yaron (2004), consumption growth, $g_{c,t+1}$, is decomposed into a (persistent) long-run growth component, $x_{c,t+1}$, and a (transitory) short-run component, $\bar{\sigma}_c \eta_{c,t+1}$. The persistent long-run growth component is modeled as an AR(1) process with two fundamental shocks: a shock to growth, $\sigma_{c,t} e_{c,t+1}$, and a shock to the inflation target, $\sigma_{\pi,t} e_{\pi,t+1}$ (both with stochastic volatilities). The inflation target is modeled by an AR(1) process with its own stochastic volatilities and the persistence is allowed to switch regimes. The persistence of the long-run growth, $\rho_c(S_{t+1})$, and its exposure to an inflation target shock, which is captured by $\chi_{c,\pi}(S_{t+1})$, are subject to regime changes, where S_{t+1} denotes the regime indicator variable. The value of $\chi_{c,\pi}(S_{t+1})$ can be either negative or positive. The economic reasoning behind this follows the view that there are periods in which the inflation target is above the so-called desirable rate of inflation,[9] and that any positive shock to the inflation target during those periods creates distortions and hampers long-run growth. The negative $\chi_{c,\pi}(S_{t+1})$ values correspond to these periods. The periods with positive $\chi_{c,\pi}(S_{t+1})$ values depict periods during which the inflation target is assumed to be lower than the desirable one, and a positive shock to the inflation target removes distortions and facilitates long-run growth. Dividend streams, $g_{d,t+1}$, have levered exposures to both $x_{c,t+1}$ and $\bar{\sigma}_c\eta_{c,t+1}$, whose magnitudes are governed by the parameters ϕ_x and ϕ_η, respectively. I allow $\bar{\sigma}_d\eta_{d,t+1}$ to capture the idiosyncratic movements in dividend streams. Overall, the joint dynamics for the cash flows are

$$\begin{aligned}
\begin{bmatrix} g_{c,t+1} \\ g_{d,t+1} \end{bmatrix} &= \begin{bmatrix} \mu_c \\ \mu_d \end{bmatrix} + \begin{bmatrix} 1 \\ \phi_x \end{bmatrix} x_{c,t+1} + \begin{bmatrix} 1 & 0 \\ \phi_\eta & 1 \end{bmatrix} \begin{bmatrix} \bar{\sigma}_c\eta_{c,t+1} \\ \bar{\sigma}_d\eta_{d,t+1} \end{bmatrix} \qquad (2) \\
x_{c,t+1} &= \rho_c(S_{t+1})x_{c,t} + \sigma_{c,t}e_{c,t+1} + \chi_{c,\pi}(S_{t+1})\sigma_{\pi,t}e_{\pi,t+1}, \\
x_{\pi,t+1} &= \rho_\pi(S_{t+1})x_{\pi,t} + \sigma_{\pi,t}e_{\pi,t+1}
\end{aligned}$$

where the stochastic volatilities evolve according to

$$\sigma_{j,t} = \varphi_j\bar{\sigma}_c\exp(h_{j,t}), \quad h_{j,t+1} = \nu_j h_{j,t} + \sigma_{h_j}\sqrt{1-\nu_j^2}w_{j,t+1}, \quad j = \{c,\pi\}, \qquad (3)$$

and the shocks are assumed to be

$$\eta_{i,t+1}, e_{j,t+1} \sim N(0,1), \quad i \in \{c,d\}.$$

Following Schorfheide, Song, and Yaron (2013), the logarithm of the volatility process is assumed to be normal, which ensures that the standard deviation of the shocks remains positive at every point in time.

[9] In a New Keynesian model, the desirable rate of inflation would be the rate at which prices can be changed without costs. See Aruoba and Schorfheide (2011) for a more detailed discussion.

3.2 Monetary Policy

Monetary policy consists of two components: stabilization and a time-varying inflation target. Stabilization policy is "active" or "passive" depending on its responsiveness to the consumption gap and inflation fluctuations relative to the target. The monetary policy shock, $x_{m,t}$, is also modeled as an AR(1) process. In sum, monetary policy follows a regime-switching Taylor rule,

$$\begin{aligned} i_t &= \mu_i^{MP}(S_t) + \underbrace{\tau_c(S_t)(g_{c,t} - \mu_c)}_{\text{consumption gap}} + \underbrace{\tau_\pi(S_t)(\pi_t - x_{\pi,t})}_{\text{short-run inflation}} + x_{\pi,t} + x_{m,t}, \qquad (4) \\ &= \mu_i^{MP}(S_t) + \big[\tau_c(S_t), 1 - \tau_\pi(S_t), 1, \tau_c(S_t)\big] X_t^B + \tau_\pi(S_t)\pi_t, \quad X_t^B = [x_{c,t}, x_{\pi,t}, x_{m,t}, \eta_{c,t}]', \end{aligned}$$

where $\tau_c(S_t)$ and $\tau_\pi(S_t)$ capture the central bank's reaction to the consumption gap and to the variation in short-run inflation, respectively. To recap, the dynamics of the inflation target and monetary policy shocks are

$$x_{\pi,t+1} = \rho_\pi(S_{t+1})x_{\pi,t} + \sigma_{\pi,t}e_{\pi,t+1}, \quad x_{m,t+1} = \rho_m x_{m,t} + \sigma_m e_{m,t+1}.$$

Observe that several important modifications have been made in (4). To begin with, the role of interest rate smoothing is assumed to be absent. While (4) may look quite restrictive in its form, it yields a much simpler expression in that the current short rate is affine with respect to the "current" state variables, X_t^B, and "realized" inflation, π_t, without any "lagged" term. Moreover, given the argument posited in Rudebusch (2002), it seems sensible to consider the monetary policy rule without an interest rate smoothing motive in order to study the term structure.[10] More important, however, (4) assumes that the central bank makes informed decisions with respect to inflation fluctuations at different frequencies. While the central bank attempts to steer actual inflation toward the inflation target at low frequencies, it aims to stabilize inflation fluctuations relative to its target at high frequencies. Furthermore, in the context of the term structure models, it is very important to consider an explicit role for the target inflation since it behaves similarly to a level factor of the nominal term structure. The specification of

[10]Based on the term structure evidence, Rudebusch (2002) shows that monetary policy inertia is not due to the smoothing motive but is due to persistent shocks.

(4) resembles specifications in which the level factor of the term structure directly enters into the monetary policy rule (see Rudebusch and Wu (2008), for example).[11] Finally, (4) assumes that the strength with which the central bank tries to pursue its goal—a stabilization policy—changes over time along the lines explored in Clarida, Gali, and Gertler (2000).

3.3 Endogenous Inflation Dynamics

Inflation dynamics can be determined endogenously from the monetary policy rule (4) and a Fisher-type asset-pricing equation, which is given below,

$$\begin{aligned} i_t &= -\mathbb{E}_t\left[m_{t+1}-\pi_{t+1}\right]-\frac{1}{2}\mathbb{V}_t\left[m_{t+1}-\pi_{t+1}\right] \qquad (5)\\ &\approx \mu_i^{AP}(S_t)+\Big[\frac{1}{\psi}\mathbb{E}_t[\rho_c(S_{t+1})],0,0,0\Big]X_t^B+E_t\left[\pi_{t+1}\right], \quad X_t^B=[x_{c,t},x_{\pi,t},x_{m,t},\eta_{c,t}]'. \end{aligned}$$

(See Cochrane (2011) and Backus, Chernov, and Zin (2013) for a similar discussion.) The approximation is exact if the short rate contains no risk premium.[12] Substituting the asset-pricing equation (5) into the monetary policy rule (4), the system reduces to a single regime-dependent equation

$$\tau_\pi(S_t)\pi_t = \mathbb{E}_t\left[\pi_{t+1}\right]+\Lambda(S_t)X_t^B, \qquad (6)$$

where $\Lambda(S_t)=\left[\frac{1}{\psi}\mathbb{E}_t[\rho_c(S_{t+1})],0,0,0\right]-\left[\tau_c(S_t),1-\tau_\pi(S_t),1,\tau_c(S_t)\right]$.[13] In the Online Appendix, I show that the equilibrium inflation dynamics can be expressed as

$$\pi_t = \Gamma(S_t)X_t^B, \quad \text{where} \quad \Gamma(S_t)=[\underbrace{\Gamma_{x,c}(S_t),\Gamma_{x,\pi}(S_t),\Gamma_{x,m}(S_t)}_{\Gamma_x(S_t)},\Gamma_\eta(S_t)]. \qquad (7)$$

[11]Note also that incorporating a time-varying inflation target is quite common in the monetary policy literature (see Coibon and Gorodnichenko (2011) and Aruoba and Schorfheide (2011)).

[12]This assumption is not unreasonable given the results of the variance decomposition of the short rate in the subsequent section; see Table 6. Also, Campbell, Pflueger, and Viceira (2013) apply a similar assumption.

[13]Equation (6) holds true if $\mu_i^{MP}(S_t)=\mu_i^{AP}(S_t)$.

3.4 Markov Chain

In order to achieve flexibility while maintaining parsimony,[14] I assume that the model parameters evolve according to a four-state Markov chain $S_t = (S_t^X, S_t^M)$ (i.e., that the regime switching is not synchronized). It can be further decomposed into two independent two-state Markov chains, S_t^X, S_t^M,

$$\mathbb{P}_X = \begin{bmatrix} p_{X_1} & 1-p_{X_1} \\ 1-p_{X_2} & p_{X_2} \end{bmatrix}, \quad \mathbb{P}_M = \begin{bmatrix} p_{M_1} & 1-p_{M_1} \\ 1-p_{M_2} & p_{M_2} \end{bmatrix}$$

where X_i and M_i are indicator variables for correlation and monetary policy regimes, $i = 1, 2$. Define

$$S_t = \begin{cases} 1 & \text{if} \quad S_t^X = X_1 \quad \text{and} \quad S_t^M = M_1 \\ 2 & \text{if} \quad S_t^X = X_1 \quad \text{and} \quad S_t^M = M_2 \\ 3 & \text{if} \quad S_t^X = X_2 \quad \text{and} \quad S_t^M = M_1 \\ 4 & \text{if} \quad S_t^X = X_2 \quad \text{and} \quad S_t^M = M_2, \end{cases}$$

from which I construct the transition probability $\mathbb{P} = \mathbb{P}_X \otimes \mathbb{P}_M$.

3.5 Solution

The first-order condition of the agent's expected utility maximization problem yields the Euler equations

$$\mathbb{E}_t \left[\exp\left(m_{t+1} + r_{k,t+1}\right)\right] = 1, \quad k \in \{c, m\}, \quad \text{(Real Assets)} \tag{8}$$

$$p_{n,t} = \log \mathbb{E}_t[\exp(m_{t+1} - \pi_{t+1} + p_{n-1,t+1})], \quad \text{(Nominal Assets)} \tag{9}$$

where $m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} g_{c,t+1} + (\theta - 1) r_{c,t+1}$ is the log of the real stochastic discount factor (SDF), $r_{c,t+1}$ is the log return on the consumption claim, $r_{m,t+1}$ is the log market return, and $p_{n,t}$ is the log price of an n-month zero-coupon bond.

[14] There is no reason to assume *a priori* that the coefficient, $\chi_{c,\pi}$, and the monetary policy parameters, τ_c, τ_π, switch simultaneously.

The solutions to (8) and (9) depend on the joint dynamics of consumption, dividend growth, and inflation, which can be conveniently broken up into three parts and rewritten as:

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$$\begin{bmatrix} g_{c,t+1} \\ g_{d,t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \mu_c \\ \mu_d \\ \mu_\pi \end{bmatrix} + \begin{bmatrix} e_1 \\ \phi_x e_1 \\ \Gamma_x(S^X_{t+1}, S^M_{t+1}) \end{bmatrix} X_{t+1} + \begin{bmatrix} 1 & 0 & 0 \\ \phi_\eta & 1 & 0 \\ \Gamma_\eta(S^X_{t+1}, S^M_{t+1}) & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{\sigma}_c \eta_{c,t+1} \\ \bar{\sigma}_d \eta_{d,t+1} \\ \bar{\sigma}_\pi \eta_{\pi,t+1} \end{bmatrix}$$

The Conditional Mean and Volatility Dynamics

$$\underbrace{\begin{bmatrix} x_{c,t+1} \\ x_{\pi,t+1} \\ x_{m,t+1} \end{bmatrix}}_{X_{t+1}} = \underbrace{\begin{bmatrix} \rho_c(S^X_{t+1}) & 0 & 0 \\ 0 & \rho_\pi(S^X_{t+1}) & 0 \\ 0 & 0 & \rho_m \end{bmatrix}}_{\Upsilon(S^X_{t+1})} \underbrace{\begin{bmatrix} x_{c,t} \\ x_{\pi,t} \\ x_{m,t} \end{bmatrix}}_{X_t} + \underbrace{\begin{bmatrix} 1 & \chi_{c,\pi}(S^X_{t+1}) & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\Omega(S^X_{t+1})} \underbrace{\begin{bmatrix} \sigma_{c,t} e_{c,t+1} \\ \sigma_{\pi,t} e_{\pi,t+1} \\ \sigma_m e_{m,t+1} \end{bmatrix}}_{E_{t+1}}$$

$$\underbrace{\begin{bmatrix} \sigma^2_{c,t+1} \\ \sigma^2_{\pi,t+1} \end{bmatrix}}_{\Sigma_{t+1}} = \underbrace{\begin{bmatrix} (1-\nu_c)(\varphi_c \bar{\sigma})^2 \\ (1-\nu_\pi)(\varphi_\pi \bar{\sigma})^2 \end{bmatrix}}_{\Phi_\mu} + \underbrace{\begin{bmatrix} \nu_c & 0 \\ 0 & \nu_\pi \end{bmatrix}}_{\Phi_\nu} \underbrace{\begin{bmatrix} \sigma^2_{c,t} \\ \sigma^2_{\pi,t} \end{bmatrix}}_{\Sigma_t} + \underbrace{\begin{bmatrix} \sigma_{w_c} w_{c,t+1} \\ \sigma_{w_\pi} w_{\pi,t+1} \end{bmatrix}}_{W_{t+1}}.$$

In the above, derivations of $\Gamma_x(S^X_{t+1}, S^M_{t+1}), \Gamma_\eta(S^X_{t+1}, S^M_{t+1})$ are provided in (7), $e_1 = [1, 0, 0]$, and the shocks follow $\eta_{j,t+1}, e_{k,t+1}, w_{l,t+1} \sim N(0,1)$ for $j \in \{c, d, \pi\}$, $k \in \{c, \pi, m\}$, and $l \in \{c, \pi\}$, and $W_{t+1} \sim N(0, \Phi_w)$. I approximate the exponential Gaussian volatility process by linear Gaussian processes such that the standard analytical solution techniques that have been widely used in the LRR literature can be applied. The approximation of the exponential volatility process is used only to derive the solution coefficients in the law of motion of the asset prices. $\{S_{t+1}, X_{t+1}, \Sigma_{t+1}\}$ are sufficient statistics for the evolution of the fundamental macroeconomic aggregates.

3.5.1 Real Equity Asset Solutions

Real asset prices are determined from the approximate analytical solution described in Bansal and Zhou (2002) and Schorfheide, Song, and Yaron (2013). Let I_t denote the current information set $\left\{S^X_t, X_t, \Sigma_t\right\}$ and define I_{t+1}=$I_t \cup \{S^X_{t+1}\}$ that includes information regarding S_{t+1}^X in addition to I_t.[15] Suppose $S_t^X = i$ for i=1,2. The derivation of (8) follows Bansal and Zhou (2002), who make repeated use of the law of iterated expectations and log-linearization, and Schorfheide, Song, and Yaron (2013), who utilize a log-linear approximation for returns and for volatilities

$$\begin{aligned}
1 &= \mathbb{E}\Bigg(\mathbb{E}\left[\exp\left(m_{t+1}+r_{m,t+1}\right) \mid I_{t+1}\right] \mid I_t\Bigg) \\
&= \sum_{j=1}^{2} \mathbb{P}_{X_{ij}} \mathbb{E}\Bigg(\exp\left(m_{t+1}+r_{m,t+1}\right) \mid S_{t+1}^X = j, X_t, \Sigma_t\Bigg) \\
0 &= \sum_{j=1}^{2} \mathbb{P}_{X_{ij}} \underbrace{\left(\mathbb{E}\left[m_{t+1}+r_{m,t+1} \mid S_{t+1}^X = j, X_t, \Sigma_t\right] + \frac{1}{2}\mathbb{V}\left[m_{t+1}+r_{m,t+1} \mid S_{t+1}^X = j, X_t, \Sigma_t\right]\right)}_{B}.
\end{aligned}$$

The first line uses the law of iterated expectations, the second line uses the definition of Markov chain, and the third line applies log-linearization (i.e., $\exp(B) - 1 \approx B$), a log-normality assumption, and log-linearization for returns and for volatilities.

The state-contingent solution to the log price to consumption ratio follows

$$z_t(i) = A_0(i) + A_1(i) X_t + A_2(i)\Sigma_t,$$

where

$$\begin{aligned}
\left[\begin{array}{cc} A_1(1) & A_1(2) \end{array}\right] &= (1-\frac{1}{\psi})e_1 \left[\begin{array}{cc} p_{X_1}\Upsilon(1) + (1-p_{X_1})\Upsilon(2) & (1-p_{X_2})\Upsilon(1) + p_{X_2}\Upsilon(2) \end{array}\right] \\
&\quad \times \left[\begin{array}{cc} \mathbb{I}_2 - p_{X_1}\kappa_{1,c}\Upsilon(1) & -(1-p_{X_2})\kappa_{1,c}\Upsilon(1) \\ -(1-p_{X_1})\kappa_{1,c}\Upsilon(2) & \mathbb{I}_2 - p_{X_2}\kappa_{1,c}\Upsilon(2) \end{array}\right]^{-1} \\
\left[\begin{array}{c} A_{2,c}(1) \\ A_{2,c}(2) \end{array}\right] &= \frac{\theta}{2}\left[\begin{array}{c} \mathbb{I}_2 - \kappa_{1,c}\nu_c\mathbb{P}_X \end{array}\right]^{-1} \times \mathbb{P}_X \times \left[\begin{array}{c} \left\{\left((1-\frac{1}{\psi})e_1 + \kappa_{1,c}A_1(1)\right)\cdot\Omega(1)e_1'\right\}^2 \\ \left\{\left((1-\frac{1}{\psi})e_1 + \kappa_{1,c}A_1(2)\right)\cdot\Omega(2)e_1'\right\}^2 \end{array}\right] \\
\left[\begin{array}{c} A_{2,\pi}(1) \\ A_{2,\pi}(2) \end{array}\right] &= \frac{\theta}{2}\left[\begin{array}{c} \mathbb{I}_2 - \kappa_{1,c}\nu_\pi\mathbb{P}_X \end{array}\right]^{-1} \times \mathbb{P}_X \times \left[\begin{array}{c} \left\{\left((1-\frac{1}{\psi})e_1 + \kappa_{1,c}A_1(1)\right)\cdot\Omega(1)e_2'\right\}^2 \\ \left\{\left((1-\frac{1}{\psi})e_1 + \kappa_{1,c}A_1(2)\right)\cdot\Omega(2)e_2'\right\}^2 \end{array}\right].
\end{aligned}$$

[15] Note that regime information on S_t^M is irrelevant for real equity asset solutions.

The log price to consumption ratio is loaded with respect to long-run growth and $A_{1,c}(i)$ will be positive whenever the IES, ψ, is greater than 1. The loadings on the inflation target, $A_{1,\pi}(i)$, and on the monetary policy shock, $A_{1,m}(i)$, are zero. The sign of the responses of the log price to consumption ratio to long-run growth and inflation target volatilities, $A_{2,c}(i)$ and $A_{2,\pi}(i)$, will be negative if $\theta < 0$ (i.e., $\gamma > 1$ and $\psi > 1$).

3.5.2 Nominal Bond Asset Solutions

Similar to the previous case, the approximate analytical expressions for the state-contingent log bond price coefficients $p_{n,t} = C_{n,0}(i) + C_{n,1}(i)X_t + C_{n,2}(i)\Sigma_t$ are derived by exploiting the law of iterated expectations and log-linearization,

$$p_{n,t} \approx \sum_{j=1}^{4} \mathbb{P}_{ij} \log \Big(\mathbb{E}[\exp(m_{t+1} - \pi_{t+1} + p_{n-1,t+1}) | S_{t+1} = j, S_t = i] \Big),$$

where

$$\begin{aligned}
C_{n,1}(i) &= \sum_{j=1}^{4} \mathbb{P}_{ij} \Big(C_{n-1,1}(j) - \frac{1}{\psi} e_1 - \Gamma_x(j) \Big) \Upsilon(j) \\
C_{n,2}(i) &= \sum_{j=1}^{4} \mathbb{P}_{ij} \Big(C_{n-1,2}(j)\Phi_\nu + (\theta - 1) \left\{ \kappa_{1,c} A_2(j)\Phi_\nu - A_2(i) \right\} \\
&+ \frac{1}{2} \begin{bmatrix} \{(C_{n-1,1}(j) - \gamma e_1 - \Gamma_x(j) + (\theta - 1)\kappa_{1,c} A_1(j)) \cdot \Omega(j) e_1'\}^2 \\ \{(C_{n-1,1}(j) - \gamma e_1 - \Gamma_x(j) + (\theta - 1)\kappa_{1,c} A_1(j)) \cdot \Omega(j) e_2'\}^2 \end{bmatrix}' \Big)
\end{aligned}$$

with the initial conditions $C_{0,1}(i) = [0, 0, 0]$ and $C_{0,2}(i) = [0, 0]$ for i=1,. . . ,4. Because of the regime-switching feature, the coefficients are not easy to interpret. However, it is relatively easy to verify that bond prices will respond negatively to positive shocks to long-run growth and the inflation target when $n = 1$.

4 State-Space Representation of the LRR Model

To facilitate estimation, it is convenient to cast the LRR model of Section 3 into a state-space form. The state-space representation consists of a measurement equation that relates the observables to underlying state variables and a transition equation that describes the law of motion of the state variables. I use the superscript o to distinguish observed variables from model-implied ones. The regime-contingent measurement equation can be written as

$$y^o_{t+1} = A_{t+1}\bigg(D(S_{t+1})+F(S_{t+1})f_{t+1}+F^v(S_{t+1})f^v_{t+1}+\Sigma^\varepsilon\varepsilon_{t+1}\bigg), \quad \varepsilon_{t+1}\sim iidN(0,I). \tag{10}$$

The vector of observables, y^o_{t+1}, contains consumption growth, dividend growth, the log price to dividend ratio, inflation, U.S. Treasury bills with maturities of one and three months, U.S. Treasury bonds with maturities of between one and five years, as well as bonds with maturity of ten years, and measures of one-quarter-ahead forecasts for real growth from the historical forecasts taken from the Survey of Professional Forecasters (SPF). The vector f_{t+1} stacks state variables that characterize the level of fundamentals. The vector f^v_{t+1} is a function of the log volatilities of long-run growth and the inflation target, h_t and h_{t+1}, in (3). Finally, ε_{t+1} is a vector of measurement errors, and A_{t+1} is a selection matrix that accounts for deterministic changes in the data availability.

The solution of the LRR model sketched in Section 3.5 provides the link between the state variables and the observables y^o_{t+1}. The state variables themselves follow regime-contingent vector autoregressive processes of the form

$$f_{t+1} = \Phi(S_{t+1})f_t + v_{t+1}(S_{t+1})(h_t), \quad h_{t+1} = \Psi h_t + \Sigma_h w_{t+1}, \quad w_{t+1}\sim iidN(0,I), \tag{11}$$

where $v_{t+1}(S_{t+1})$ is an innovation process with a variance that is a function of the log volatility process h_t, and w_{t+1} is the innovation of the stochastic volatility process. Roughly speaking, the vector f_{t+1} consists of the long-run components $x_{c,t}$, $x_{\pi,t}$, and $x_{m,t}$ in Section 3. In order to express the observables y^o_{t+1} as a linear function of f_{t+1} and to account for potentially missing observations it is necessary to augment f_{t+1} by lags of $x_{c,t}, x_{\pi,t}, x_{m,t}$ as well as the innovations for the fundamentals.

The novelty in the estimation is that the state-space representation is set up in a way that incorporates the measurement error modeling of consumption growth outlined in Schorfheide, Song, and Yaron (2013). The authors show that post-1959 monthly consumption series are subject to sizeable measurement errors and argue that accounting for measurement errors is crucial in identifying the predictable component in consumption growth. In addition, the state-space representation exploits the SPF measures that are released in a different (quarterly) frequency. As argued in Bansal and Shaliastovich (2013), survey-based expected measures provide the most accurate forecasts of future growth, which is why bringing this information into the estimation will sharpen the inference on expected terms. For purposes of illustration, I represent the monthly time subscript t as $t = 3(j-1)+m$, where $m = 1, 2, 3$. Here j indexes the quarter and m the month within the quarter. The formulae below summarize the implementation of measurement error modeling of consumption and exploitation of the SPF measures:

1. A Measurement Equation for Consumption

$$
\begin{aligned}
g^o_{c,3(j-1)+1} &= g_{c,3(j-1)+1} + \sigma_\epsilon\big(\epsilon_{3(j-1)+1} - \epsilon_{3(j-2)+3}\big) - \frac{1}{3}\sum_{m=1}^{3}\sigma_\epsilon\big(\epsilon_{3(j-1)+m} - \epsilon_{3(j-2)+m}\big) \\
&\quad + \sigma^q_\epsilon\big(\epsilon^q_{(j)} - \epsilon^q_{(j-1)}\big) \\
g^o_{c,3(j-1)+m} &= g_{c,3(j-1)+m} + \sigma_\epsilon\big(\epsilon_{3(j-1)+m} - \epsilon_{3(j-1)+m-1}\big), \quad m = 2, 3,
\end{aligned}
$$

   the monthly and quarterly measurement errors follow $\epsilon_{3(j-1)+m}, \epsilon^q_{(j)} \sim N(0,1)$.

2. Exploiting the SPF Measures

$$
x^{q,o}_{c,(j)} = \sum_{\tau=1}^{5}\left(\frac{3-|\tau-3|}{3}\right) x_{c,3j-\tau+1} + \sigma^q_{x,\epsilon}\epsilon^q_{x,(j)},
$$

   where $x^{q,o}_{c,(j)}$ denotes the j^{th} quarter median SPF forecasts for real growth measured at $j-1^{th}$ quarter, and the measurement error follows $\epsilon^q_{x,(j)} \sim N(0,1)$.

4.1 Bayesian Inference

The system to be estimated consists of equations (10) and (11), whose coefficient matrices are functions of the parameter vector

$$
\begin{aligned}
\Theta_0 &= \big(\delta, \psi, \gamma\big) \\
\Theta_1 &= \left(\{\varphi_k, \bar{\sigma}_k, \mu_k, \nu_k, \sigma_{w_k}\}_{k=c}^{\pi}, \mu_d, \varphi_d, \phi_x, \phi_\eta, \sigma_\epsilon, \rho_m, \sigma_m, \left\{\rho_c^{(i)}, \rho_\pi^{(i)}, \chi_{c,\pi}^{(i)}, \tau_c^{(j)}, \tau_\pi^{(j)}\right\}_{i,j=1}^{2} \right) \\
\Theta_2 &= \big(\mathbb{P}_{X_1}, \mathbb{P}_{X_2}, \mathbb{P}_{M_1}, \mathbb{P}_{M_2}\big).
\end{aligned}
\tag{12}
$$

I will use a Bayesian approach to make inferences about $\Theta = \{\Theta_0, \Theta_1, \Theta_2\}$ and the latent state vector S and study the implications of the model. Bayesian inference requires the specification of prior distributions $p(\Theta)$ and $p(S|\Theta_2)$ and the evaluation of the likelihood function $p(Y^o|\Theta, S)$.

The posterior can be expressed as

$$p(\Theta, S|Y^o) = \frac{p(Y^o|\Theta, S)p(S|\Theta_2)p(\Theta)}{p(Y^o)}, \tag{13}$$

which can be factorized as

$$p(\Theta, S|Y^o) = p(\Theta|Y^o)p(S|\Theta, Y^o). \tag{14}$$

The practical difficulty is to generate draws from $p(\Theta|Y^o)$, since it requires a numerical evaluation of the prior density and the likelihood function $p(Y^o|\Theta)$. Due to the presence of the volatility states and the regime-switching processes, the computation of the likelihood function relies on a sequential Monte Carlo procedure also known as a particle filter. To obtain a computationally efficient filter, I extend the algorithm developed in Schorfheide, Song, and Yaron (2013), in which they exploit the partially linear structure of the state-space model conditional on the volatility states and derive a very efficient particle filter. The key feature of my state-space model is that it is still nonliner conditional on the volatility states. However, conditional on the volatility states, I can apply Kim's filter in Kim and Nelson (1999) (i.e., an extension of the Kalman filter with a collapsing procedure that is proposed for handling Markov-switching models) to evaluate the likelihood. In essence, I use a swarm of particles to represent the distribution of volatilities and employ Kim's filter for each particle (i.e., volatility). After a resampling step (i.e., eliminating particles with low weights), the filter produces a sequence of likelihood approximations. I embed the likelihood approximation in a fairly standard random-walk Metropolis algorithm and draw the parameter vector $\{\Theta^{(m)}\}_{m=1}^{n_{sim}}$. Conditional on the parameter vector, $\{\Theta^{(m)}\}_{m=1}^{n_{sim}}$, I use Kim's smoothing algorithm in Kim and Nelson (1999) to generate draws from the history of latent states, $\{S^{(m)}\}_{m=1}^{n_{sim}}$.

5 Empirical Results

The data set used in the empirical analysis is described in Section 5.1.

5.1 Data

Monthly consumption data represent per capita series of real consumption expenditure on nondurables and services from the National Income and Product Accounts (NIPA) tables available from the Bureau of Economic Analysis. Aggregate stock market data consist of monthly observations of returns, dividends, and prices of the CRSP value-weighted portfolio of all stocks traded on the NYSE, AMEX, and NASDAQ. Price and dividend series are constructed on a per share basis as in Campbell and Shiller (1988b) and Hodrick (1992). Market data are converted to real data using the consumer price index (CPI) from the Bureau of Labor Statistics. Growth rates of consumption and dividends are constructed by taking the first difference of the corresponding log series. Inflation represents the log difference of the CPI. Monthly observations of U.S. Treasury bills and bonds with maturities at one month, three months, one to five years, and ten years are from CRSP. The time series span of the monthly data is 1959:M1 to 2011:M12.[16] The quarterly SPF forecasts are from the Federal Reserve Bank of Philadelphia. I use the median survey forecast values for GDP growth that span the period 1968:Q4 to 2011:Q4. The descriptive data statistics are provided in Table 2.

[16]Monthly consumption growth is available from 1959:M2.

Table 2: Descriptive Statistics - Data Moments

(a) Quarterly Frequency: 1968:Q4–2011:Q4

	Δc	Δgdp	$\mathbb{E}\Delta$gdp	π
Mean	0.43	0.68	0.58	1.08
StdDev	0.44	0.86	0.58	0.80
AC1	0.54	0.33	0.71	0.74

(b) Monthly Frequency: 1959:M1–2011:M12

	Δc	Δd	π	r_m	pd	y_{1m}	y_{3m}	y_{1y}	y_{2y}	y_{3y}	y_{4y}	y_{5y}	y_{10y}
Mean	0.16	0.11	0.32	0.43	3.57	0.40	0.43	0.46	0.48	0.50	0.51	0.52	0.55
StdDev	0.34	1.26	0.32	4.55	0.39	0.24	0.25	0.25	0.24	0.24	0.23	0.22	0.22
AC1	-0.16	-0.01	0.63	0.10	0.99	0.97	0.98	0.98	0.99	0.99	0.99	0.99	0.99

Notes: I report descriptive statistics for aggregate consumption growth (Δc), gross domestic product (GDP) growth (Δgdp), expected GDP growth ($\mathbb{E}\Delta$gdp), consumer price index (CPI) inflation (π), dividend growth (Δd), log returns of the aggregate stock market (r_m), log price to dividend ratio (pd), and U.S. Treasury yields (y_n) with maturity $n \in \{1m, 3m, 1y, 2y, 3y, 4y, 5y, 10y\}$. The table shows mean, standard deviation, and sample first-order autocorrelation. Means and standard deviations are expressed in percentage terms.

5.2 Prior and Posterior Summaries

I begin by estimating the state-space model described in Section 4.

Prior Distribution. This section provides a brief discussion of the prior distribution. Percentiles for marginal prior distributions are reported in Table 3. The prior distribution for the preference parameters that affect the asset pricing implications of the model are the same as those used in Schorfheide, Song, and Yaron (2013). Thus, I focus on the parameters of the fundamental processes specified in (2) and (3).

The prior 90% credible intervals for average annualized consumption and dividend growth and inflation are fairly wide and agnostic and range from approximately -7% to +7%. The priors for ϕ_x and ϕ_η, parameters that determine the comovement of consumption and dividend growth, are centered at zero and have large variances. $\bar{\sigma}_c$ and $\bar{\sigma}_\pi$ are the average standard deviation of the *iid* component of consumption growth and inflation whose 90% prior intervals range from 1.2% to 7.2% at an annualized rate.

The parameters φ_d, φ_c, and φ_π capture the magnitude of innovations to dividend growth and the long-run growth and inflation target component relative to the magnitude of consumption growth innovations. The prior for φ_d covers the interval 0.2 to 12, whereas the priors for φ_c, and φ_π cover the interval 0 to 0.11. Finally, the prior interval for the persistence of the volatility processes ranges from -0.1 to 0.97 and the prior for the standard deviation of the volatility process implies that the volatility may fluctuate either relatively little, within the range of 0.67 to 1.5 times the average volatility, or substantially, within the range of 0.1 to 7 times the average volatility.

The prior distribution for the persistence of the long-run growth, inflation target, and monetary policy shock $x_{c,t}$, $x_{\pi,t}$, $x_{m,t}$ is a normal distribution centered at 0.9 with a standard deviation of 0.5, truncated to the interval $(-1, 1)$. The corresponding 90% credible interval ranges from -0.1 to 0.97, encompassing values that imply *iid* dynamics as well as very persistent local levels. The prior distribution for the parameter that captures the contemporaneous correlation between the long-run growth and inflation target shocks is a normal distribution centered at zero with a relatively large standard deviation of 0.5. Sign restrictions are imposed to identify two different correlation regimes: one is truncated below zero, and the other is truncated above zero. The prior intervals for the standard deviation of the monetary policy shock cover the range from 0 to 0.001.

The priors for the monetary policy rule coefficients are normal distributions that range between ± 4.28, but those for the inflation components are truncated above zero, reflecting the view that the central bank *raises* rather than *lowers* the interest rate in response to positive inflation fluctuations. Finally, I employ beta priors for the Markov-chain transition probabilities that cover 0.38 to 1.00.

Posterior Distribution. Percentiles for the posterior distribution are also reported in Table 3. The estimated parameters for preferences and dividend growth (first panel) are, by and large, similar to those reported in Schorfheide, Song, and Yaron (2013). Those for the consumption and inflation process (second panel) are consistent with the sample mean and standard deviation reported in Table 2. One interesting feature is that the unconditional standard deviation of long-run growth is substantially smaller than that of the inflation target, 0.07% versus 0.29% at annualized rates. The estimation

Table 3: Posterior Estimates

	Distr.	Prior 5%	Prior 95%	Posterior 5%	Posterior 50%	Posterior 95%		Distr.	Prior 5%	Prior 95%	Posterior 5%	Posterior 50%	Posterior 95%
Preferences							Dividend Process						
δ	B	[.9951	.9999]	.9985	.9989	.9991	μ_d	N	[-.007	.006]	-	.0010	-
ψ	G	[0.31	3.45]	1.80	1.81	1.82	ϕ_x	N	[-13.1	13.4]	2.39	2.51	2.67
γ	G	[2.74	15.45]	10.82	10.99	11.17	ϕ_η	N	[-1.68	1.63]	1.09	1.10	1.13
							φ_d	G	[0.22	11.90]	4.74	5.01	5.19
Consumption Process							Inflation Process						
μ_c	N	[-.006	.006]	-	.0016	-	μ_π	N	[-.007	.006]	.0027	.0029	.0030
$\bar{\sigma}_c$	IG	[.001	.006]	.0020	.0021	.0021	$\bar{\sigma}_\pi$	N	[.001	.006]	.0015	.0015	.0016
φ_c	G	[0.00	0.11]	.026	.031	.033	φ_π	G	[0.00	0.11]	0.11	0.12	0.12
ν_c	N^T	[-0.08	0.97]	.9906	.9952	.9959	ν_π	N^T	[-0.08	0.97]	.9915	.9928	.9937
σ_{w_c}	IG	[0.22	1.03]	0.30	0.31	0.34	σ_{w_π}	IG	[0.22	1.03]	0.43	0.45	0.46
Regime-Switching VAR Coefficients													
Countercyclical Inflation Regime							Procyclical Inflation Regime						
ρ_c	N^T	[-0.08	0.97]	.9957	.9957	.9958	ρ_c	N^T	[-0.08	0.97]	.951	.953	.957
ρ_π	N^T	[-0.08	0.97]	.9957	.9959	.9961	ρ_π	N^T	[-0.08	0.97]	.980	.980	.981
$\chi_{c,\pi}$	N	[-0.80	0.80]	-.40	-.40	-.41	$\chi_{c,\pi}$	N	[-0.80	0.80]	.150	.155	.162
ρ_m	N^T	[-0.08	0.97]	.9906	.9916	.9929	ρ_m	N^T	[-0.08	0.97]	.9906	.9916	.9929
σ_m	IG	[.000	.001]	.0001	.0002	.0003	σ_m	IG	[.000	.001]	.0001	.0002	.0003
Regime-Switching Monetary Policy Coefficients													
Active Monetary Policy Regime							Passive Monetary Policy Regime						
τ_c	N	[-4.28	4.28]	.9540	.9543	.9545	τ_c	N	[-4.28	4.28]	.548	.550	.551
τ_π	N^T	[0.00	4.28]	3.09	3.10	3.11	τ_π	N^T	[0.00	4.28]	.960	.960	.961
Markov-Chain Transition Probabilities													
Inflation Regime							Monetary Policy Regime						
$\mathbb{P}_{X_1}$	B	[0.38	1.00]	.989	.992	.995	$\mathbb{P}_{M_1}$	B	[0.38	1.00]	.987	.990	.991
$\mathbb{P}_{X_2}$	B	[0.38	1.00]	.938	.941	.945	$\mathbb{P}_{M_2}$	B	[0.38	1.00]	.974	.975	.979

Notes: The estimation results are based on monthly data from 1959:M1 to 2011:M12 with the exception that the consumption series starts only in 1959:M2. For consumption I adopt the measurement error model of Schorfheide, Song, and Yaron (2013) with the modification that the statistical agency uses the proxy series to distribute quarterly (instead of annual) consumption growth over the three months of the quarter (instead of the twelve months of a year). I fix $\mu_c = 0.0016$ and $\mu_d = 0.0010$ in the estimation. B, N, N^T, G, and IG denote beta, normal, truncated (outside of the interval $(-1, 1)$) normal, gamma, and inverse gamma distributions, respectively.

results also provide strong evidence for a stochastic variation in the long-run growth and inflation target. According to the posteriors reported in Table 3, all $\sigma_{c,t}$ and $\sigma_{\pi,t}$ exhibit significant time variation. The posterior medians of ν_c and ν_π are .9952 and

.9928, respectively, and the unconditional volatility standard deviations σ_{w_c} and σ_{w_π} are around 0.31 and 0.45.

The most important results for the subsequent analysis are provided in the third and fourth panels of Table 3. First, there is strong evidence of parameter instability in the VAR dynamics of the long-run components. Most prominently, the posterior median estimate of $\chi_{c,\pi}$, which captures the contemporaneous correlation between the long-run growth and inflation target shocks, is -0.40 in the first regime and 0.15 in the second regime. Another notable difference between the two regimes is the drop in the persistence of the long-run growth and inflation target components. Unlike in their appearance, the process half-life is very different between the two regimes: the process half-life for the long-run growth (inflation target) component in the first regime is about 12 (12) years, while that in the second regime is about 1 (3) year(s). The values of persistence and the standard deviation of the monetary policy shock are 0.9916 and 0.0002, and are assumed to be identical across regimes. In general, the magnitude of the differentials between the two VAR coefficient regimes is small, but the sign change in the correlation structure is notable. Since the groups of estimates distinguish themselves as ones that generates a negative correlation between long-run growth and inflation target shocks and ones that do not, I label the first regime as the "countercyclical" inflation regime and the second regime as the "procyclical" inflation regime.

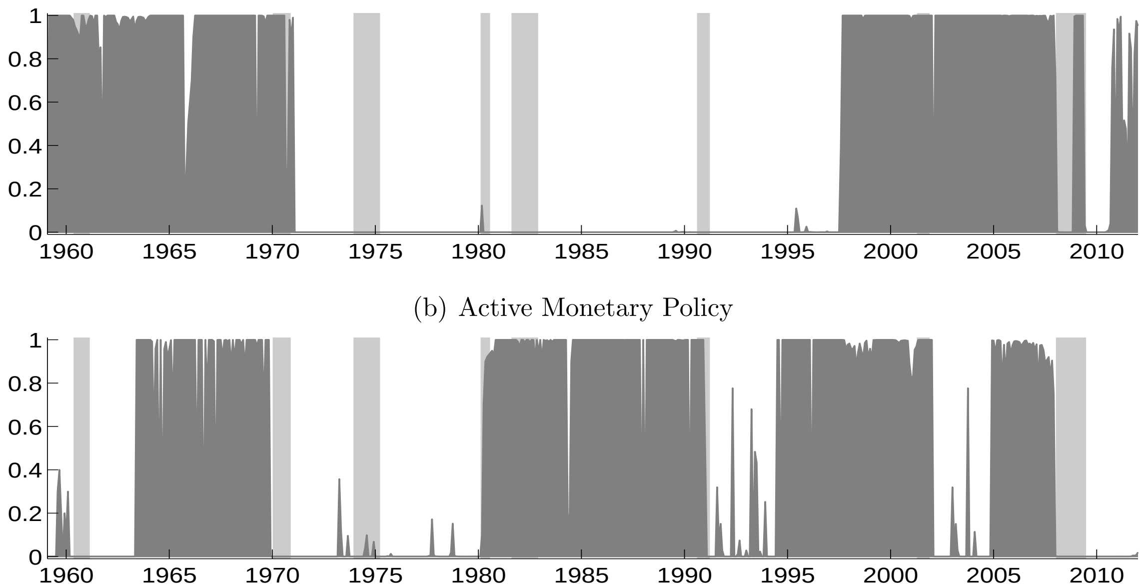
Second, two very different posterior estimates of the monetary policy rule in the fourth panel of Table 3 support the view of Clarida, Gali, and Gertler (2000) that there has been a substantial change in the way monetary policy is conducted. One regime is associated with larger monetary policy rule coefficients, which implies that the central bank will respond more aggressively to consumption gap, short-run, and long-run inflation fluctuations. The other regime is characterized by a less responsive monetary policy rule, in which I find much lower loadings on consumption gap and short-run inflation fluctuations. In particular, the magnitude of the loading on short-run inflation fluctuations τ_π is one-third of that in the former regime and is below one. Following the convention in the monetary policy literature, I distinguish the regimes by which has an "active" central bank, and which has a "passive" central bank.

Finally, the bottom panel of Table 3 reports posterior estimates of the Markov-chain

Figure 2: Smoothed Probabilities for Transitions between Regimes

(a) Procyclical Inflation

(b) Active Monetary Policy



Notes: Dark gray shaded areas represent posterior medians of smoothed regime probabilities. Light gray shaded bars indicate NBER recession dates. Figure 2(a) displays the smoothed probabilities of the procyclical inflation regime, while Figure 2(b) shows the smoothed probabilities of the active monetary policy regime.

transition probabilities. The countercyclical inflation regime is most persistent: The probability that it will continue is 99.2%. The procyclical inflation regime, on the contrary, is the less persistent one: Its duration is one-fourteenth of the countercyclical inflation regime. This result can be interpreted as indicating that the "risks" of falling back to the countercyclical inflation regime are substantial. The transition probability of the active monetary policy regime is around 0.99, which implies that agents expect its average duration to be about 9 years. For the passive monetary policy regime, the same result is about 3-4 years. Given the posterior transition probabilities, it is interesting to look at the smoothed probabilities for transitions between regimes.

Smoothed Posterior Regime Probabilities. Figure 2 depicts the smoothed posterior probabilities of the procyclical inflation and active monetary policy regimes. Figure 2(a) is consistent with the evidence provided in Table 1 that procyclical inflation regimes were prevalent after the late 1990s. It also suggests that the switch is not a permanent event, but rather, an occasional one.[17] Figure 2(b) provides the historical paths of the monetary policy stance: Active monetary policy appeared in the mid-1960s but was largely dormant during the 1970s; it became active after the appointment of Paul Volcker as Chairman of the Federal Reserve in 1979 and remained active for 20 years (except for short periods in the early 1990s). After that, in response to the economic crisis triggered by the 9/11 attacks in 2001, the central bank lowered interest rates and took a passive stance for 3-4 years. Around the mid-2000s, it switched back to a more active stance until the Great Recession started. Finally, the post-2008 periods are characterized by the passive regime.[18]

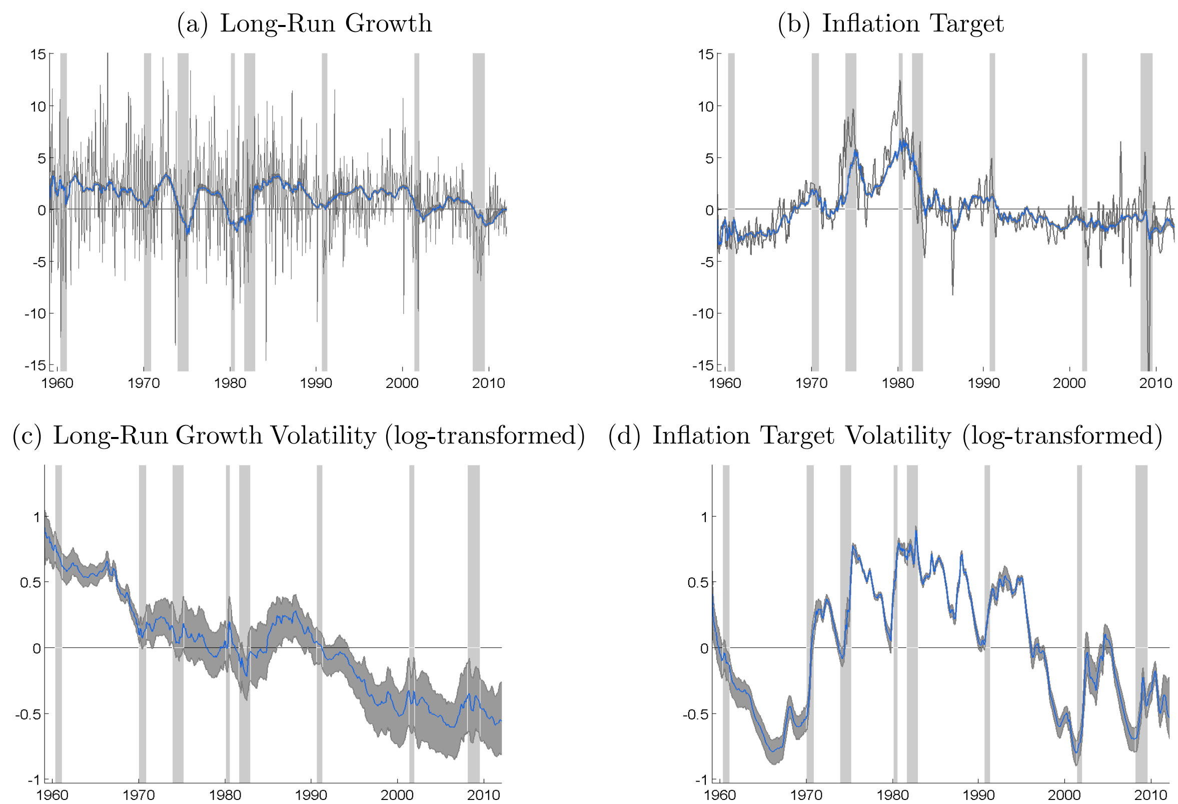
Smoothed Mean and Volatility States. The top panel of Figure 3 depicts smoothed estimates of long-run growth $x_{c,t}$ and the inflation target $x_{\pi,t}$, which are overlaid with monthly consumption growth and inflation, respectively.[19] $x_{c,t}$ tends to fall in recessions (indicated by the shaded bars in Figure 3) but periods of falling $x_{c,t}$ also occur during expansions; the pattern is broadly similar to the one reported in Schorfheide, Song, and Yaron (2013). $x_{\pi,t}$ reaches its peak during the Great Inflation periods and substantially decreases afterward. It is interesting to note that during the 1970s and 1980s, recessions were accompanied by increases in the inflation target. The pattern clearly reverses starting in the late 1990s. The smoothed volatility processes are plotted below. Recall that my model has two independent volatility processes, $h_{c,t}$ and $h_{\pi,t}$, which are associated with the innovations to long-run growth and the inflation target, respectively. The most notable feature of $h_{c,t}$ is that it captures a drop in growth volatility that occurred in the 1980s, also known as the Great Moderation. The stochastic volatility process for the inflation target displays different properties: It jumps around 1970 and remains high for 25 years, and features wide fluctuations at the beginning of the 2000s that are not apparent in $h_{c,t}$. Overall, the smoothed $h_{\pi,t}$ seems to exhibit more medium and high-frequency movements than $h_{c,t}$. Also, due to the inclusion of a greater amount of data on bond yields, $h_{\pi,t}$ is more precisely estimated than $h_{c,t}$, indicated by tighter credible intervals.

[17]This evidence is also supported by David and Veronesi (2013).

[18]The smoothed paths for monetary policy are broadly consistent with those found in Clarida, Gali, and Gertler (2000), Ang, Boivin, Dong, and Loo-Kung (2011), Bikbov and Chernov (2013), and Coibon and Gorodnichenko (2011).

[19]Figure C-1 in the Online Appendix provides the path of the estimated monetary policy shock.

Figure 3: Smoothed Mean and Volatility States



Notes: Blue lines represent posterior medians of smoothed states and the dark gray shaded area corresponds to 90% credible intervals. Light gray shaded bars indicate NBER recession dates. In the top panel, I overlay the smoothed states with monthly consumption growth and inflation (gray solid lines).

5.3 Implications for Macro Aggregates and Asset Prices

It is instructive to examine the extent to which sample moments implied by the estimated state-space model mimic the sample moments computed from the actual data set. To do this, I conduct a posterior predictive check (see Geweke (2005) for a textbook treatment). I use previously generated draws $\Theta^{(s)}, S^{(s)}$, $s = 1, \ldots, n_{sim}$, from the posterior distribution of the model parameters $p(\Theta, S|Y^o)$ and for each $\Theta^{(s)}, S^{(s)}$ I simulate the model for 636 periods, which corresponds to the number of monthly observations in the estimation sample.[20] This leads to n_{sim} simulated trajectories, which I denote

[20]To generate the simulated data, I also draw measurement errors.

Table 4: Model-Generated Correlations between Consumption and Inflation

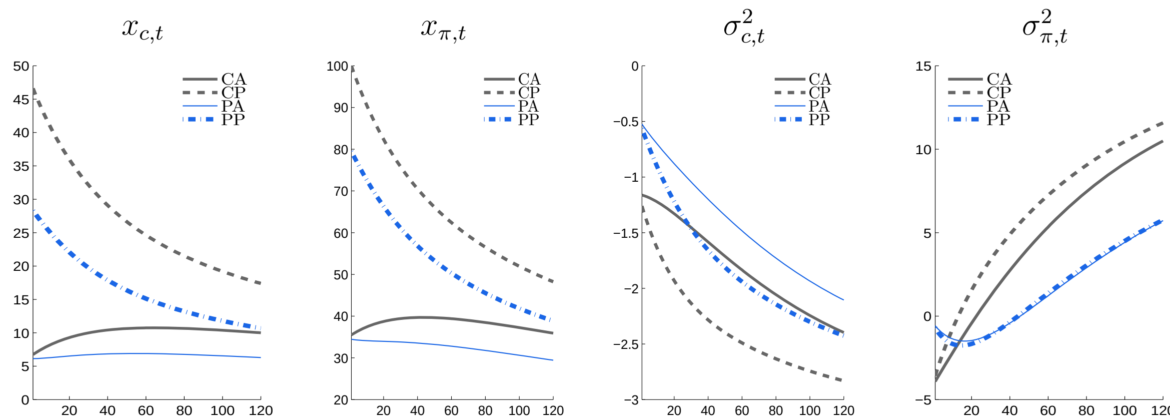
	Data	Model					
	$\text{corr}(\Delta c_t, \pi_t)$	$\text{corr}(\Delta c_t, \pi_t)$			$\text{corr}(\mathbb{E}\Delta c_{t+1}, \mathbb{E}\pi_{t+1})$		
Regime	Estimate	Median	5%	95%	Median	5%	95%
CA	-0.24	-0.58	[-0.80,	-0.22]	-0.93	[-0.99,	-0.64]
CP	-0.09	-0.48	[-0.78,	0.02]	-0.74	[-0.95,	-0.15]
PA	0.01	0.17	[-0.13,	0.42]	0.59	[0.27,	0.80]
PP	0.03	0.19	[-0.14,	0.47]	0.27	[0.44,	0.84]

Notes: "CA" stands for the countercyclical inflation and the active monetary policy regimes, while "PP" stands for the procyclical inflation and the passive monetary policy regimes. "CP" and "PA" indicate the remaining combinations of regimes. Data estimates are based on monthly consumption growth and inflation series.

by $Y^{(s,o)}$. For each of these trajectories, I compute various sample moments, such as means, standard deviations, and cross correlations. Suppose I denote such statistics generically by $\mathcal{S}(Y^{(s,o)})$. The simulations provide a characterization of the posterior predictive distribution $p(\mathcal{S}(Y^{(s,o)})|Y^o)$.

Matching Moments of the Macroeconomic Aggregates and Stock Price. To save space, the model-implied distributions for the first and second moments of the macroeconomic aggregates and stock price are provided in Table C-3 and Table C-4 in the Online Appendix. In sum, the first and second moments for consumption and dividend growth, log price to dividend ratio, and inflation implied by the model replicate the actual counterparts well. Since monetary policy does not affect cash flows, the sample moments for consumption and dividend growth and the log price to dividend ratio do not differ across monetary policy regimes (i.e., column-wise comparisons). Yet the sample moments across inflation regimes (i.e., row-wise comparisons) are quite different: Those in the countercyclical inflation regime are much more volatile. This finding is consistent with the near unit-root estimates of long-run growth and inflation target persistence in the countercyclical inflation regime (see Table 3). The sample correlation between consumption and inflation is provided in Table 4. While the model-implied numbers are somewhat larger than their data estimates, the model performs well in terms of matching the sign-switching patterns. One notable feature is that monetary

Figure 4: Equilibrium Nominal Bond Yield Loadings



Notes: Model-implied nominal bond yield loadings on long-run growth ($x_{c,t}$), the inflation target ($x_{\pi,t}$), long-run growth volatility ($\sigma^2_{c,t}$), and inflation target volatility ($\sigma^2_{\pi,t}$) are provided. "CA" stands for the countercyclical inflation and the active monetary policy regimes, while "PP" stands for the procyclical inflation and the passive monetary policy regimes. "CP" and "PA" indicate the remaining combinations of regimes. Maturity on the x-axis is in months. Numbers are displayed in percent.

policy does seem to matter for the correlation of expected values: Passive monetary policy lowers the correlation of expected values particularly more during the procyclical inflation regime. Overall, I find that $\chi_{c,\pi}$ is the key model ingredient for capturing the sign-switching patterns, and that monetary policy influences the correlation of expected consumption growth and inflation but on its own cannot change the sign.

Equilibrium Nominal Bond Yield Loadings. It is also instructive to understand the equilibrium bond yield loadings first before looking at the model-implied yield curve. Figure 4 shows the regime-contingent bond yield loadings on long-run growth, the inflation target, and long-run growth and the inflation target volatilities based on the median posterior coefficient estimates.[21] To ease exposition, I use abbreviations for each regime: "CA" stands for the countercyclical inflation and the active monetary policy regimes, while "PP" stands for the procyclical inflation and the passive monetary policy regimes; "CP" and "PA" indicate the remaining combinations of regimes. The CP loading on the inflation target for a bond with a maturity of 1 month is normalized

[21]I do not present the graph for monetary policy since its influence on bond yields is very small compared to these variables.

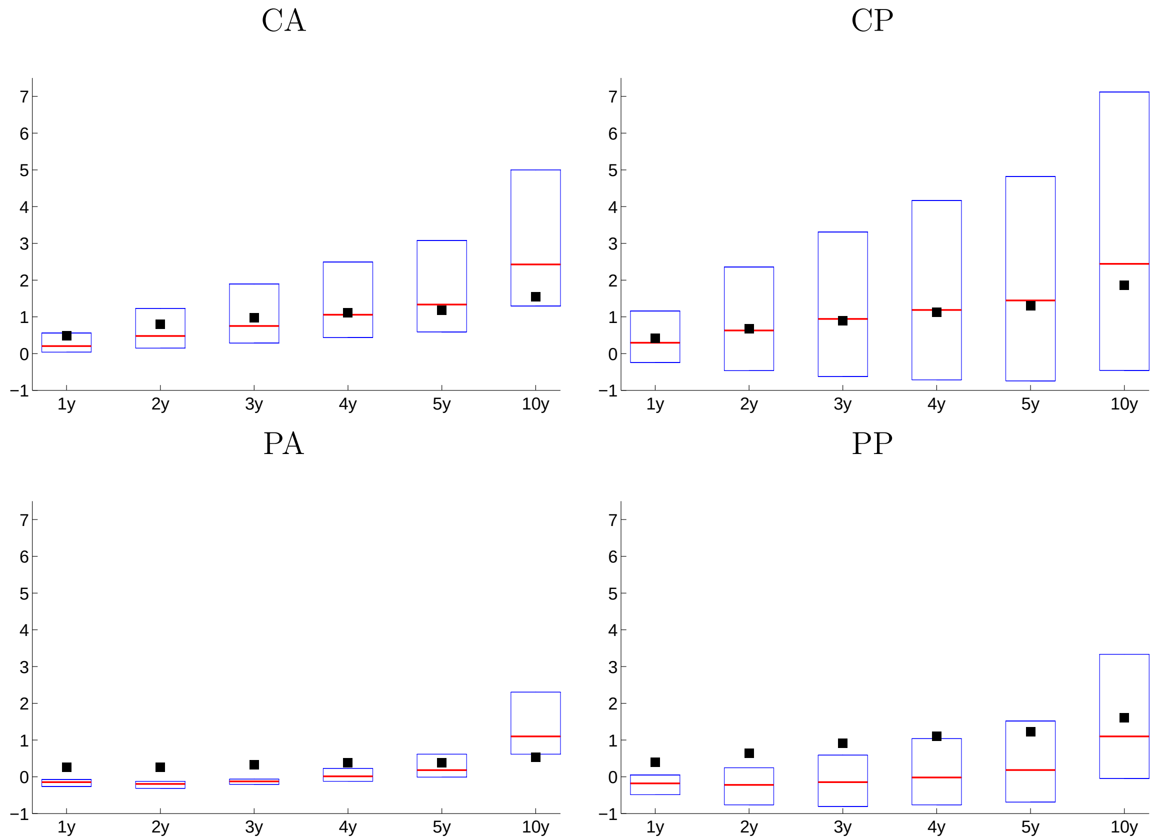
to 100% to bring all of the loadings into proportion with one another.[22] It is evident from Figure 4 that the inflation target is the most important factor in the term structure analysis. Note that loadings on inflation target volatility increase over maturities and become the second most important factor for longer maturity yields. In terms of patterns of the loadings, I find that they are broadly in line with those found in Bansal and Shaliastovich (2013). The loadings on long-run growth and inflation targets are positive; the loading on long-run growth volatility has a negative decreasing slope; and the loading on inflation target volatility is mostly positive and rises with maturities. However, the loadings across regimes have very different implications. Let us focus on monetary policy regimes. For example, while a positive shock to the inflation target induces an essentially parallel shift in the entire yield curve (loadings are nearly flat across maturities) in the active monetary policy regime, it has disproportionately larger effects on yields with short maturities (loadings decrease substantially over maturities) in the passive case. It seems that in the active monetary policy regime, the inflation target behaves like a level factor, but in the passive cases it becomes a slope factor.[23] Moreover, the magnitude of the loadings in the passive monetary policy stance almost doubles. With regard to inflation regimes, the loadings on all model state variables will be uniformly shifted out in the countercyclical inflation regime, implying that the risks associated with the countercyclical inflation regime are much larger than those in the procyclical case.

Matching Moments of the Yield Spread. The estimated model is quite successful at fitting Treasury yields over the entire sample—the yield prediction errors at different maturities are generally quite small over the entire sample. To save space, the evidence is provided in Figure C-5 in the Online Appendix. Now, in order to evaluate whether the model can reproduce key patterns in the data, I focus on the posterior predictive assessment in the main text. Distributions generated from the LRR model using the posterior estimates are graphically provided in Figure 5. The top and bottom ends of the boxes correspond to the 5th and 95th percentiles, respectively, of the posterior distribution, and the horizontal lines signify the medians. The first row of Figure 5 is

[22] An easier way to interpret this is to fix one regime and compare loadings across the model state variables. By focusing on one state variable, you can move across regimes to compare their magnitudes.

[23] Readers are referred to Figure 1 in Rudebusch and Wu (2008).

Figure 5: Model-Generated Yield Spread



Notes: "Spread" is the difference between 3m yield and yields with maturity at 1y–10y. Black squares indicate values from actual data. Figure also depicts medians (red lines) and 90% credible intervals (top and bottom lines of boxes) of the distribution of yield spreads obtained with model-generated data. "CA" stands for the countercyclical inflation and the active monetary policy regimes, while "PP" stands for the procyclical inflation and the passive monetary policy regimes. "CP" and "PA" indicate the remaining combinations of regimes. Numbers are displayed in percent (annualized).

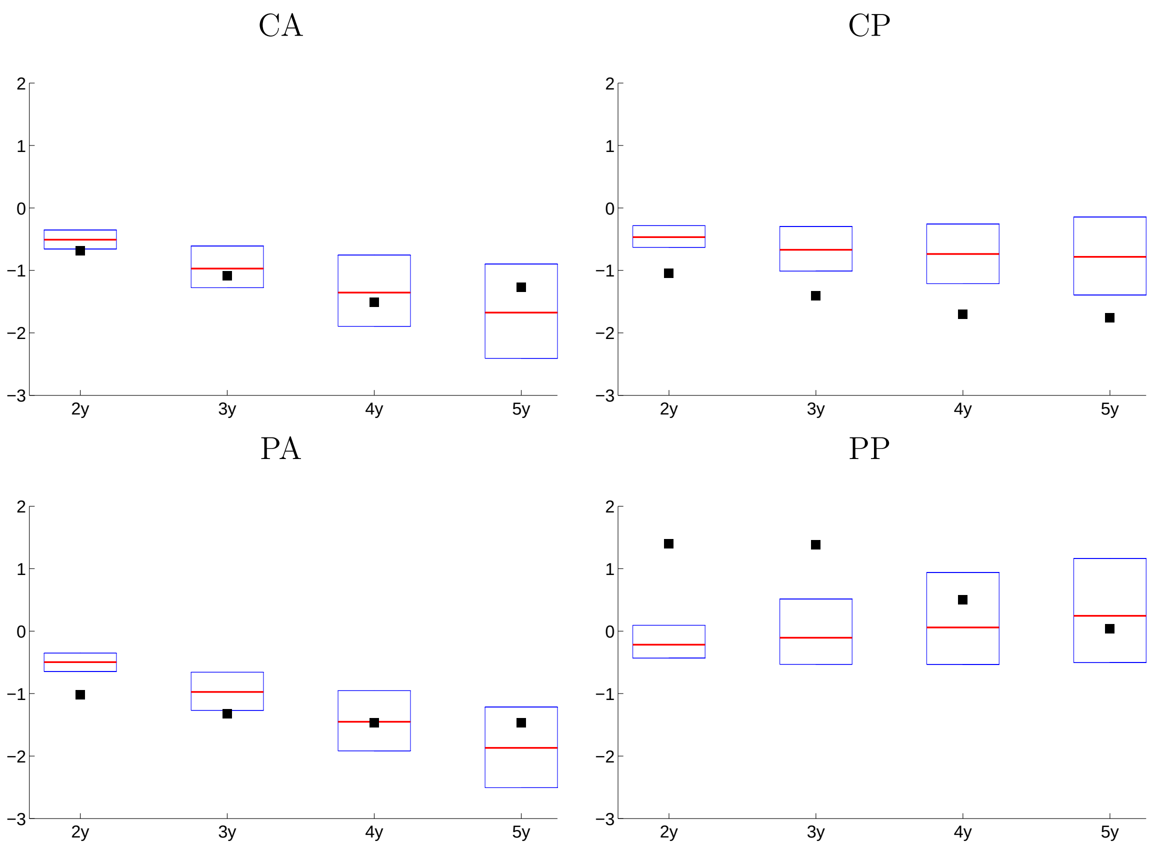
simulated conditional on the countercyclical inflation regime, while the second row in Figure 5 is generated from the procyclical inflation one. For each row, the figure on the left conditions on the active monetary policy regime, while the one on the right does the same on the passive monetary policy regime. The figure also depicts the same moments computed from U.S. data (black squares). "Actual" sample moments that fall far into the tails of the posterior predictive distribution provide evidence for model deficiencies. Roughly speaking, the model performs well along this dimension since the model-implied median values are fairly close to their data estimates. Yet important distinctions arise across regimes. Going from left to right (CA to CP or PA to PP), I find that yield spread distributions are more dispersed. The 90% credible intervals in the latter, right-hand figures (CP or PP) are approximately twice as large as those in the left-hand column (CA or PA). This is consistent with the impulse response functions shown in Online Appendix Figure C-2, in that the passive monetary policy leads to more unstable economic dynamics. From top to bottom (CA to PA or CP to PP), I find that the 10y-3m yield spreads in the countercyclical inflation regime are roughly 150 basis points (annualized) higher than those in the procyclical inflation regime. This implies that agents will demand higher yields as compensation for the risks associated with the countercyclical inflation regimes. An interesting feature of the model is that even in the procyclical inflation regime, which otherwise would lead to a downward sloping yield curve, the risks of the monetary authority not responding aggressive by enough to inflation fluctuations and the risks of transitioning to the countercyclical inflation regime give rise to an upward sloping yield curve. The second moment for the yield spread implied by the model is provided in Figure C-6 in the Online Appendix. The model performs well along this dimension and the model-implied patterns are very similar to the first moment case.

Bond Risk Premia Implications. Under the expectations hypothesis (EH), the expected holding returns from long-term and short-term bonds should be the same (strong form) or should only differ by a constant (weak form). However, even the weak form has been consistently rejected by empirical researchers. For example, Campbell and Shiller (1991), Dai and Singleton (2002), Cochrane and Piazzesi (2005), and Bansal and Shaliastovich (2013) all argue that the EH neglects the risks inherent in bonds and provide strong empirical evidence for predictable changes in future excess returns.

The presence of stochastic volatilities and regime-switching loadings in my model gives rise to time variations in risk premia, which has testable implications for the EH.[24] First, I focus on the term spread regression of Campbell and Shiller (1991) to examine the validity of the EH. The excess log return on buying an n month bond at t and selling

[24]My model extends Bansal and Shaliastovich (2013) by allowing regime-switching bond yield loadings that provide additional channels for time variations in risk premia.

Figure 6: Term Spread Regression



Notes: The model-implied 90% distributions for the slope coefficient, β_n, from the regression below are provided.

$$y_{t+12,n-12} - y_{t,n} \quad = \quad \alpha_n + \beta_n \left((y_{t,n} - y_{t,12}) \frac{12}{n-12} \right) + \epsilon_{t+12}, \quad n \in \{24, 36, 48, 60\}.$$

Medians are depicted by red lines. Black squares indicate estimates from actual data. "CA" stands for the countercyclical inflation and the active monetary policy regimes, while "PP" stands for the procyclical inflation and the passive monetary policy regimes. "CP" and "PA" indicate the remaining combinations of regimes.

it as an $n-12$ month bond at $t+12$ is denoted by

$$rx_{t+12,n} = (n)y_{t,n} - (n-12)y_{t+12,n-12} - 12y_{t+12}.$$

Under the weak form of the EH, the expected excess bond returns are constant, which implies that the theoretical slope coefficient β_n value (below) predicted by the EH is equal to unity for all n

$$y_{t+12,n-12} - y_{t,n} \quad = \quad \alpha_n + \beta_n \left((y_{t,n} - y_{t,12}) \frac{12}{n-12} \right) + \epsilon_{t+12}. \tag{15}$$

Bansal and Shaliastovich (2013)[25] show that the population value for β_n can be expressed by

$$\beta_n = 1 - \frac{cov(\mathbb{E}_t rx_{t+12,n}, y_{t,n} - y_{t,12})}{var(y_{t,n} - y_{t,12})}. \tag{16}$$

This means that downward deviation from unity, equivalent to $cov(\mathbb{E}_t rx_{t+12,n}, y_{t,n} - y_{t,12}) > 0$, implies that the term spread contains information about the expected excess bond returns. Put differently, the predictability of excess bond returns (by the term spread) reflects time variations in the expected risk premium.

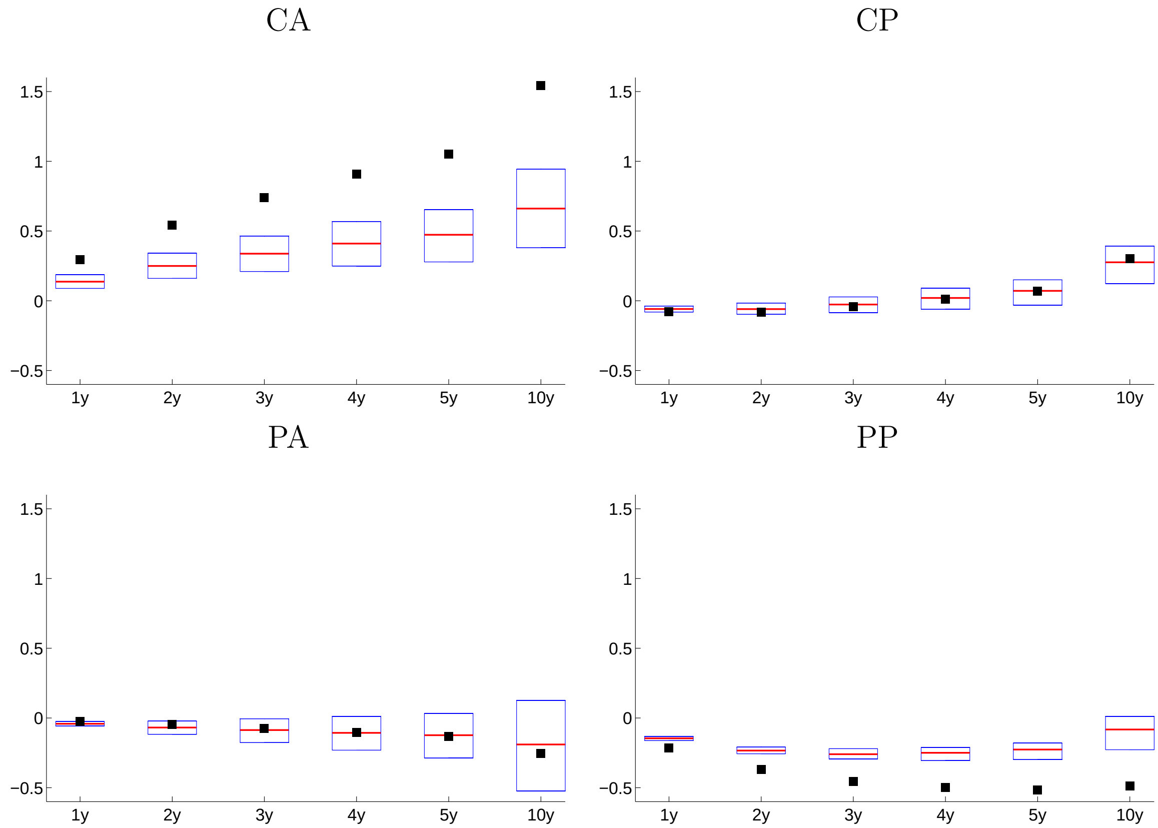
Figure 6 compares model-implied distributions for the slope coefficient, β_n, to the corresponding data estimates. The first thing to note is that the model generates very comparable results. Roughly speaking, the model produces β_ns that are significantly lower than unity and whose absolute magnitudes rise over maturities, as in the data. Second, it is important to understand that the violations of the EH or deviations from unity are less apparent in the passive monetary policy regimes. In particular, the model-implied distributions for β_ns in the PP regime are close to or even greater than zero. The striking feature is that the data estimates for β_n in the PP regime are all greater than zero and even close to unity for maturities of two and three years. It can be deduced from (16) that either the term spread contains much less information about the expected excess bond returns, or the variance of the term spread is much larger in the passive monetary policy regime.

In order to understand this feature, I decompose the bond yields into the component implied by the EH, the expected sum of future short rates, and the term premium,

$$y_{t,n} = \underbrace{\frac{1}{n}\sum_{i=0}^{n-1} \mathbb{E}_t(y_{t+i,1})}_{\text{short-rate expectations}} + \text{term premium}_{t,n}. \tag{17}$$

[25] The earlier version of their paper considered this explanation.

Figure 7: Term Premia



Notes: The model-implied 90% distributions for the term premium$_{t,n} = y_{t,n} - \frac{1}{n}\sum_{i=0}^{n-1}\mathbb{E}_t(y_{t+i,1})$ are provided, $n \in \{12, 24, 36, 48, 60, 120\}$. Medians are depicted by red lines. Black squares indicate estimates from actual data. "CA" stands for the countercyclical inflation and the active monetary policy regimes, while "PP" stands for the procyclical inflation and the passive monetary policy regimes. "CP" and "PA" indicate the remaining combinations of regimes.

Let us focus on the monetary policy regimes and assume that we are in the countercyclical inflation regime. Here are two possible channels through which the passive monetary policy stance can affect bond yields. In order to generate results that are consistent with Figure 5, we would expect to see an increase either in the expected sum of future short rates or in the term premium.

Figure 7 compares the model-implied distributions for the term premium to the corresponding data estimates (black squares). Data estimates are within-regime averages from Figure C-7 in the Online Appendix where the time-series of the estimated term premia for bonds with maturities of 1–10 years are depicted. It is very interesting to observe that the term premia in the passive monetary policy regime are actually smaller than those in the active regimes (both in the data and model-implied estimates). This implies that the effect of monetary policy is mostly on the expectations component (without affecting the term premium component), which further implies an increase in the variance of the current period's term spread. From (16), an increase in the term spread variance will bring the slope coefficient, β_n, closer to 1. The underlying economic intuition is that the future yields will incorporate the expected increase into future inflation rates, as the passive monetary policy stance is more prone to large inflation, which is predicted by the EH. While the estimated model is successful in generating these patterns, it falls short of data estimates found in the CA regime. The model is not able to capture the substantive increase in term premiums as in the data.

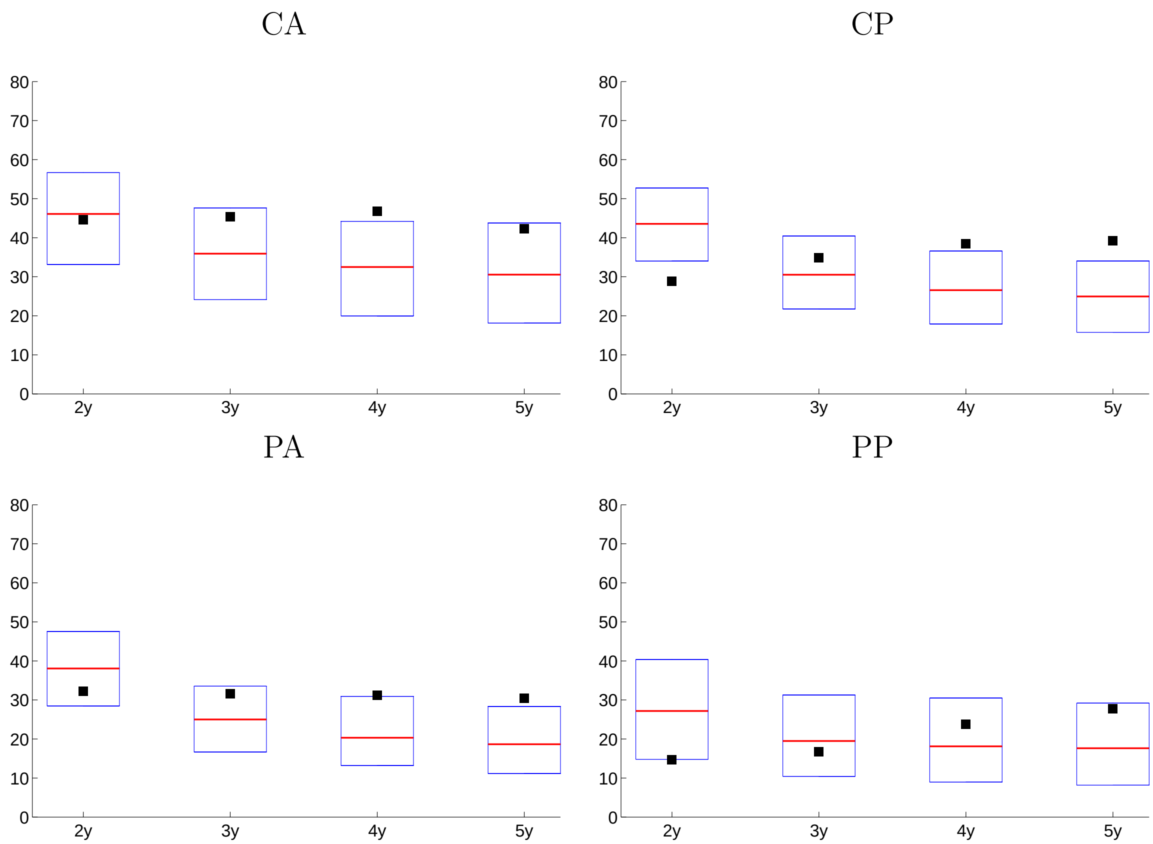
Similar logic can be applied when the inflation regime is procyclical. The directional influence of the passive monetary policy stance on the expectations component is ambiguous because, on the one hand, the procyclicality will lower expected inflation, but on the other hand, the risks of falling back to the countercyclical inflation regime will increase expected inflation. However, the inherent instability associated with the passive monetary policy stance will increase the relative weight on the expectations component, which brings the bond market closer to what the EH predicts.

In contrast to monetary policy, the countercyclical inflation regime affects both terms. It is clear from the row-to-row comparison of Figure 7 that the risks associated with the countercyclical inflation regime increase the term premiums, which are on average 50 basis points higher for 10-year bonds.[26]

Another exercise consists of running regressions that predict excess bond returns. Following Cochrane and Piazzesi (2005), I focus on regressing the excess bond return of an n year bond over the 1-year bond on a linear combination of forward rates that includes a constant term, a 1-year bond yield, and four forwards rates with maturities of 2 to 5 years. The model-implied 90% distributions for R^2 values (in percents) from

[26]Note that the differences are modest because the term premia are generated from the unconditional distributions. Once I condition on different levels of volatilities (the relative magnitude of the conditional heteroscedasticity present is larger in the countercyclical inflation regime), the results will change.

Figure 8: Excess Bond Return Predictive Regression by Cochrane and Piazzesi (2005)



Notes: The model-implied 90% distributions for R^2 values (in percents) from the excess bond return predictability regression by Cochrane and Piazzesi (2005) are provided. Medians are depicted by red lines. Black squares indicate estimates from actual data. I focus on regressing the excess bond return of an n year bond over the 1-year bond on a linear combination of forward rates that includes a constant term, a 1-year bond yield, and four forward rates with maturities of 2 to 5 years. "CA" stands for the countercyclical inflation and the active monetary policy regimes, while "PP" stands for the procyclical inflation and the passive monetary policy regimes. "CP" and "PA" indicate the remaining combinations of regimes.

the regression are provided in Figure 8. Consistent with previous findings, the expected excess returns are less predictable (indicated by R^2 values that are about 5% lower) in the passive monetary policy stance.[27] This is due to the relative decrease in the role played by the risks channel (term premium) in the passive monetary policy regime. Also, I find that the procyclical inflation regimes (PA and PP) deliver, on average, R^2

[27] Again, the differences are modest since they are generated from the unconditional distributions.

values that are 5–10% lower (see the bottom panel in Table 1).

Unspanned Macroeconomic Risks. The standard implication of the macro-finance affine term structure model is that the state vector is closed under affine transformation of yields. This theoretical spanning condition often implies that macroeconomic risk factors, e.g., expected consumption growth and expected inflation (see Ang and Piazzesi (2003)), can be fully spanned by yields. Equivalently put, the information set of current bond yields encompasses that of current macroeconomic variables. This implies that macroeconomic variables ought to be uninformative about future values of macroeconomic variables or bond yields after controlling for the current yield curve.

To illustrate this point further, I run the following univariate predictive regression

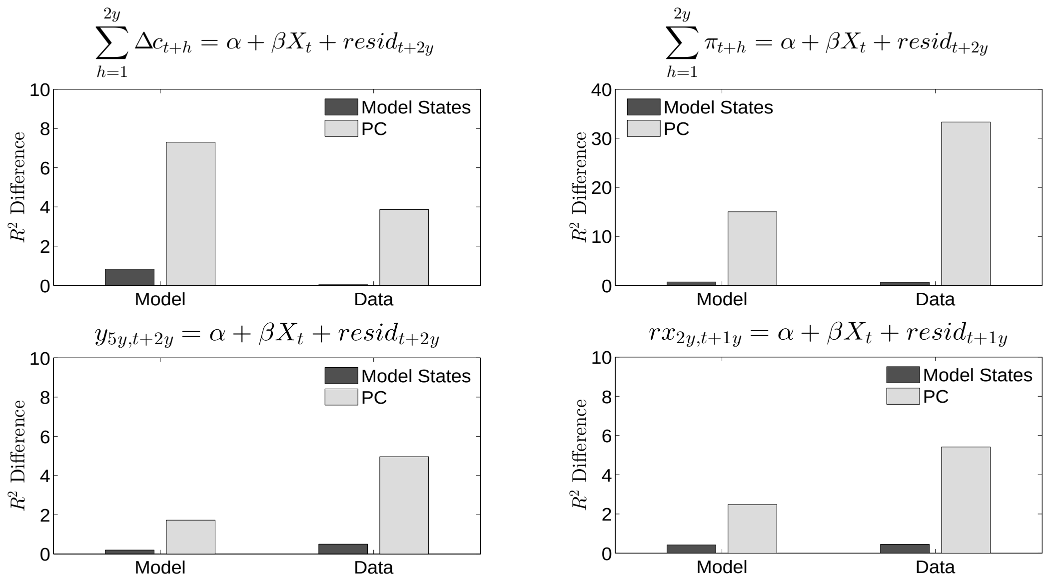
$$z_{t+H} \quad = \quad \alpha + \beta X_t + \text{resid}_{t+H}, \qquad z_{t+H} = \begin{cases} (1) \quad \sum_{h=1}^{H} \Delta c_{t+h}, & (2) \quad \sum_{h=1}^{H} \pi_{t+h}, \\ (3) \quad y_{5y,t+H}, & (4) \quad rx_{H,t+12}. \end{cases} \tag{18}$$

Consider four cases where z_{t+H} is (1) cumulative consumption growth, (2) cumulative consumer price index inflation, (3) log bond yield with maturity at five years, and (4) the excess (log) bond return of an H year bond over the 1-year bond, respectively. Suppose X_t is a vector of the (linearly transformed) current cross-section of bond yields. Then, the theoretical spanning condition implies that the incremental predictive power for the addition of macroeconomic variables is marginal. However, as emphasized by Joslin, Priebsch, and Singleton (2014), the data run strongly counter to what theory tells us (see Ludvigson and Ng (2009) and Stock and Watson (2003), for example). The evidence of unspanned macroeconomic risks poses a significant challenge to standard affine term structure models.

Due to the nonlinearities created from regime switching, the model is not affine in bond yields. This model feature allows me to analyze issues related to unspanned macroeconomic risks. For ease of exposition, define $macro_t = \{\Delta c_t, \pi_t\}$, $state_t = \{x_{c,t}, x_{\pi,t}, x_{m,t}, \sigma^2_{c,t}, \sigma^2_{\pi,t}\}$, and pc_t= first as the five principal components of yields with maturities at one month, three months, one to five years, and ten years.[28] I simulate

[28] I set the dimension of principal components to be identical to that of the model state variables for concreteness. Nonetheless, my finding is robust to the number of principal components.

Figure 9: The Incremental Gain in the R-squared due to Macroeconomic Fundamentals



Notes: Dark (light) gray bars represent the increment in adjusted R^2 due to macroeconomic fundamentals as predictors of z_{t+2y} after controlling for model state variables (principal components of bond yields). First, I run the following univariate predictive regression, $z_{t+2y} = \alpha + \beta X_t + \text{resid}_{t+2y}$. I consider four cases where z_{t+2y} is (1) cumulative consumption growth ($\sum_{h=1}^{2y} \Delta c_{t+h}$), (2) cumulative consumer price index inflation ($\sum_{h=1}^{2y} \pi_{t+h}$), (3) log bond yield with maturity at five years ($y_{5y,t+2y}$), and (4) the excess (log) bond return of a 2-year bond over the 1-year bond ($rx_{2y,t+1y}$), respectively. Define $macro_t = \{\Delta c_t, \pi_t\}$. Second, I obtain adjusted R^2 values (in percents) from the predictive regression when $X_t = X_{\setminus macro_t} \cup macro_t$ and subtract the R^2 values obtained from the regression in which $X_t = X_{\setminus macro_t}$. $X_{\setminus macro_t}$ is either "Model States" $\{x_{c,t}, x_{\pi,t}, x_{m,t}, \sigma^2_{c,t}, \sigma^2_{\pi,t}\}$ or "PC" first five principal components of yields with maturities at one month, three months, one to five years, and ten years. I set the dimension of principal components to be identical to that of the model state variables for concreteness. I compare the findings from the model-generated data ("Model") to those from the actual data ("Data"). Data R^2 estimates are obtained by taking the median *estimated* model state variables as predictors in the predictive regression. To facilitate comparison with data estimates, the R^2 differences are calculated based on the median adjusted R^2 values from model-generated predictive regressions.

Interquartile Range	Model States				PC			
	Δc	π	y_{5y}	rx_{2y}	Δc	π	y_{5y}	rx_{2y}
80%	2.33	0.68	0.46	0.50	11.53	24.85	3.52	4.81
20%	0.05	-0.02	-0.07	-0.20	1.21	4.76	0.01	-0.08

I report the posterior 60% interquartile range of R^2 differences. Except for Δc, all intervals reported above include zero for "Model States." For "PC" all intervals except rx_{2y} exclude zero.

$state_t$ and $macro_t$ forward and construct bond yields with $state_t$ and regime-switching equilibrium yield loadings. Note that I am also simulating measurement errors for bond yields. Based on the estimated regime probabilities, the simulated economy occasionally switches regimes. Consider two cases where the predictor vector $X_{\setminus macro_t}$ is $state_t$ and pc_t, respectively. For each $X_{\setminus macro_t}$, define $X_t = X_{\setminus macro_t} \cup macro_t$. I set $H = 2$ years and run equation (18) twice by changing the predictor vector from X_t to $X_{\setminus macro_t}$ to examine the incremental predictive power of $macro_t$. Differences in adjusted R^2 values are provided in Figure 9.

First, the evidence for unspanned macroeconomic risks is strongly supported by both the model-generated and the actual data. Even after controlling for principal components, the improvements in R^2 values are not small (see light gray bars). For model-generated (actual) data, the increase in the predictive power for two-year-ahead cumulative consumption growth, cumulative inflation, yield with maturity at five years, and one-year excess holding period return on the 2-year bond over the 1-year bond is 7(4)%, 15(33)%, 2(5)%, and 2.5(5)%, respectively. Figure 9 also provides the posterior 60% interquartile range of R^2 differences for the model-generated data. All intervals except the excess return of a 2-year bond exclude zero. Second, the incremental predictive power of $macro_t$ is very little after controlling for the model state variables. All dark gray bars are on average less than 0.5%. Except for cumulative consumption growth, all 60% interquartile ranges reported in Figure 9 include zero. The findings imply that the null hypothesis of no improvement cannot be rejected.

A more natural question to ask is how much of the variation in $state_t$ is spanned by pc_t. In order to identify the role played by the regime-switching dynamics, I consider the following regression

$$state_t^{(i)} = \alpha + \beta pc_t^{(i)} + \mathrm{resid}_t^{(i)}, \quad (i) \in \{\text{regime switching, fixed regime}\}. \tag{19}$$

(i) denotes whether the economy is subject to regime switching. Resulting R^2 values are provided in Table 5. With the exception of $\sigma^2_{c,t}$, the predictive power of pc_t substantially drops for $x_{c,t}$,$x_{\pi,t}$, $x_{m,t}$, $\sigma^2_{\pi,t}$ when the economy switches regimes. This finding implies that nonlinearity generated from the regime-switching dynamics has a substantial impact

Table 5: R^2 Values from State Predictive Regression: $state_t = \alpha + \beta pc_t + resid_t$

	Data	Model								
		Regime-Switching			CA			CP		
State	Estimate	Median	5%	95%	Median	5%	95%	Median	5%	95%
x_c	0.30	0.21	[0.01,	0.66]	0.60	[0.16,	0.91]	0.53	[0.06,	0.84]
x_π	0.69	0.58	[0.19,	0.86]	0.80	[0.41,	0.96]	0.69	[0.30,	0.91]
x_m	0.40	0.54	[0.17,	0.86]	0.32	[0.04,	0.65]	0.54	[0.10,	0.83]
σ_c^2	0.16	0.18	[0.03,	0.52]	0.17	[0.00,	0.51]	0.16	[0.00,	0.51]
σ_π^2	0.60	0.60	[0.15,	0.94]	0.84	[0.44,	0.98]	0.75	[0.24,	0.96]
					PA			PP		
State					Median	5%	95%	Median	5%	95%
x_c					0.21	[0.01,	0.54]	0.42	[0.20,	0.65]
x_π					0.41	[0.05,	0.79]	0.74	[0.51,	0.88]
x_m					0.71	[0.25,	0.95]	0.92	[0.66,	0.98]
σ_c^2					0.17	[0.02,	0.48]	0.17	[0.02,	0.52]
σ_π^2					0.60	[0.19,	0.94]	0.49	[0.13,	0.89]

Notes: In order to understand whether the model state variables ($state_t = \{x_{c,t}, x_{\pi,t}, x_{m,t}, \sigma^2_{c,t}, \sigma^2_{\pi,t}\}$) are spanned by the principal components of the yield curve (pc_t= eight principal components of $\{y_{1m,t}, y_{3m,t}, y_{1y,t}, y_{2y,t}, y_{3y,t}, y_{4y,t}, y_{5y,t}, y_{10y,t}\}$), R^2 values from the OLS regression, $state_t = \alpha + \beta pc_t + resid_t$, are provided. Data R^2 estimates are obtained by replacing $state_t$ with median *estimated* state variables, $\hat{state}_t$ in the OLS regression. The table provides medians and 90% credible intervals of distributions of R^2 values obtained with model-generated data. "CA" stands for the countercyclical inflation and the active monetary policy regimes, while "PP" stands for the procyclical inflation and the passive monetary policy regimes. "CP" and "PA" indicate the remaining combinations of regimes.

(lowers) on the spanning ability of pc_t. When the economy switches regimes, I find that the R^2 values for $x_{c,t}$ and $\sigma^2_{c,t}$ are around 20% (see Table 5). This evidence is consistent with the findings in Joslin, Priebsch, and Singleton (2014) that the portfolio of risks that shape real economic growth is not spanned by the principal components of bond yields. The finding is also supported by the actual data. On the other hand, the R^2 values for nominal state variables, $x_{\pi,t}, x_{m,t}$, and $\sigma^2_{\pi,t}$, are larger by a factor of 3 (around 60%).

Another thing to point out is that even when there is no regime-switching in the data-generating process, none of $state_t$ is fully spanned by pc_t. The solution is affine in state variables, so this result is puzzling at first. However, I get near 100% R^2 values if I remove measurement errors for bond yields in the simulation. Even though the

Table 6: Variance Decomposition

Variable Name	Long-Run Growth & Inflation Target			Monetary Policy Shock			Long-Run Growth Vol. & Inflation Target Vol.		
	Median	5%	95%	Median	5%	95%	Median	5%	95%
log Price-Dividend Ratio	51.3	[43.5,	62.7]	-	[-	-]	49.7	[37.1,	57.2]
3-Month Bond Yield	94.5	[91.1,	97.4]	4.2	[2.1,	5.5]	0.2	[0.0,	0.3]
10-Year Bond Yield	80.7	[71.0,	94.3]	5.3	[3.3,	6.2]	14.2	[6.3,	23.7]

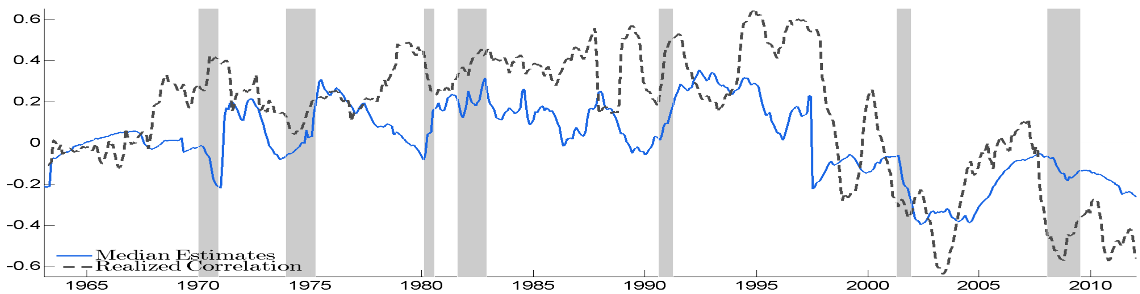
Notes: Fraction of volatility fluctuations (in percents) of the log price dividend ratio, the 3-month nominal bond yield, and the 10-year nominal bond yield that is due to long-run growth ($x_{c,t}$), the inflation target ($x_{\pi,t}$), the monetary policy shock ($x_{m,t}$), long-run growth volatility ($\sigma^2_{c,t}$), and inflation target volatility ($\sigma^2_{\pi,t}$), respectively. Note that due to measurement errors, the numbers do not sum to 100%.

measurement error variances are about 5% of the unconditional variance of bond yields, they significantly lowers the spanning ability of pc_t.

In sum, I have shown through various exercises that the model is capable of addressing the empirical regularities that often pose a serious challenge to affine term structure models.

Determinants of Asset Price Fluctuations. Table 6 provides the contribution of various risk factors, namely, the variation in long-run growth, the inflation target, the monetary policy shock, and the conditional volatility variations of long-run growth and the inflation target to asset price volatility. Given the posterior estimates of the state-space model I can compute smoothed estimates of the latent asset price volatilities. Moreover, I can also generate counterfactual volatilities by sequentially shutting down each risk factor. The ratio of the counterfactual volatility to the actual volatility measures the contribution of the non-omitted risk factors. If I subtract this ratio from one, I obtain the relative contribution of the omitted risk factor, which is shown in Table 6. I find that the key risk drivers of stock price variations are long-run growth, long-run growth volatility, and inflation target volatility. Since the shock to the inflation target moves long-run growth (captured by $\chi_{c,\pi}$), it becomes one of the major drivers of stock price variations. Bond yield variations are mostly driven by variations in the inflation target and in its volatility. Going from the short end to the long end of the yield curve, the importance of the inflation target volatility increases. My findings demonstrate that long-term rates are much more sensitive to fluctuations in inflation target volatility than

Figure 10: Estimated Stock-Bond Return Correlation



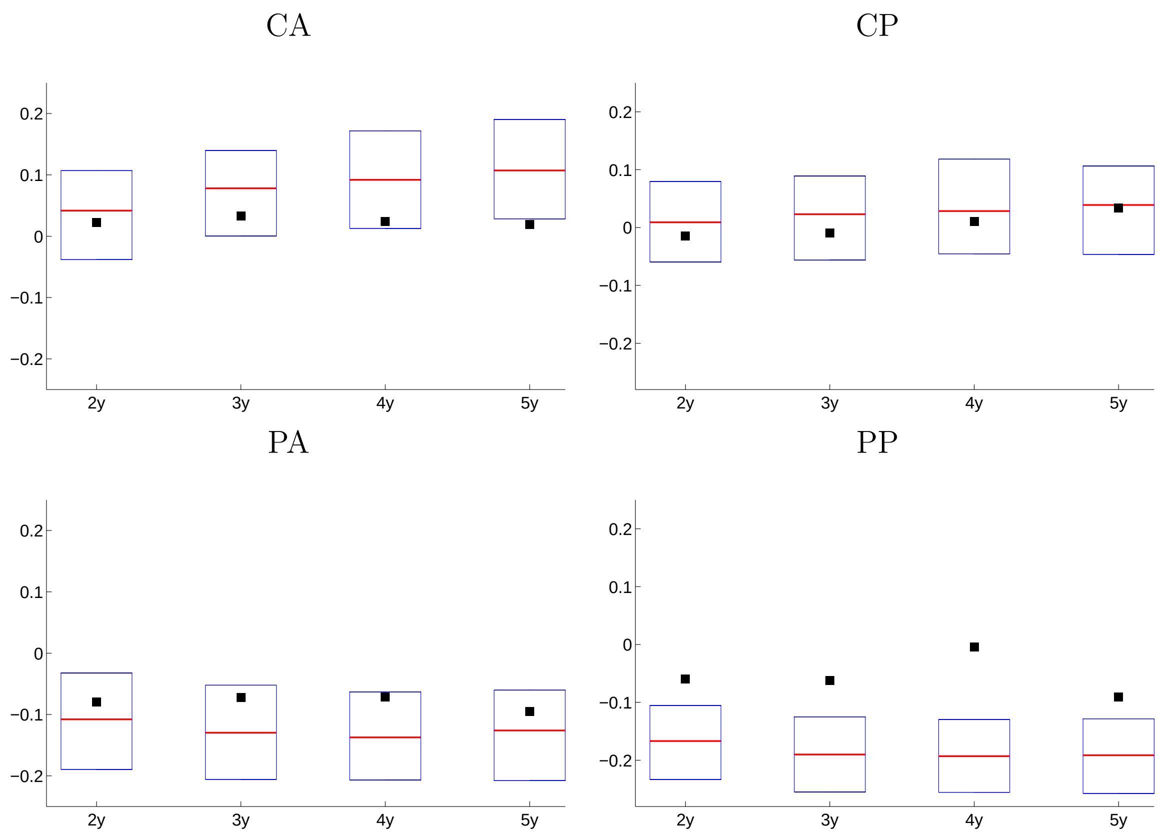
Notes: The correlation between stock market returns and bond returns for a 1-year holding period for maturity at 10 years is provided. Black dashed line depicts the monthly realized stock-bond correlation obtained from daily data. Blue solid line represents posterior median of correlations. Light gray shaded bars indicate NBER recession dates. The unconditional correlation between the two measures is about 0.68.

short-term rates. My model also shows that the variations in short-term rates are not driven by fluctuations in volatility. Hence, the assumption that the short rate contains no risk premium seems very plausible (see the Fisher-type asset-pricing equation in Section 3.3).

Understanding Stock-Bond Returns Comovement. An important feature of my estimation is that the likelihood also focuses on the conditional correlation between stock market returns and bond returns. Figure 10 displays the time-series of the estimated stock-bond correlation, which is overlaid with the monthly realized stock-bond correlation (dashed line). During the Great Inflation periods (1970s–1980s), returns on both assets were low, which resulted in positive comovements. The striking feature here is that in the beginning and toward the end of the estimation sample, the return performances decoupled, and stock and bond returns started to move in opposite directions. Through the estimation, I have identified that the economy faced changes in the covariance between the inflation target and long-run growth shocks (i.e., transition from the countercyclical inflation regime to the procyclical inflation regime). Hence, from an agent's perspective, positive shocks to the inflation target component are perceived as positive signals to long-run growth. Thus, stock returns, unlike bond returns, can respond positively to long-run inflation shocks.[29] The regime-switching covariance

[29]David and Veronesi (2013) support this evidence.

Figure 11: Stock-Bond Return Correlation



Notes: The estimated correlation between stock market returns and bond returns for a 1-year holding period for maturities of 2-5 years is provided. Black squares indicate regime-dependent sample correlations of actual data. "CA" stands for the countercyclical inflation and the active monetary policy regimes, while "PP" stands for the procyclical inflation and the passive monetary policy regimes. "CP" and "PA" indicate the remaining combinations of regimes.

coefficient in the model, $\chi_{c,\pi}$, is able to capture this data feature. Figure 11 displays the unconditional stock-bond correlation implied by the model. This experiment is useful because it disentangles the role of monetary policy in the stock-bond return correlation. I find that the active monetary policy stance tends to generate stronger positive stock-bond comovement, although the effect is small. My results are consistent with the findings in Campbell, Pflueger, and Viceira (2013), in which they argue that a more aggressive response of the central bank to inflation fluctuations will increase the stock-bond correlation. However, I find that changes in monetary policy stance alone cannot generate a sign-switch in the stock-bond return correlation.[30]

6 Conclusion

I developed an equilibrium term structure model incorporating monetary policy to address the issue of whether the structural changes in the U.S. Treasury yield curve are caused by changes in external shocks or in monetary policy. The model framework is general enough to encompass both Markov-switching coefficients and stochastic volatility processes. To estimate the model, I conditioned on the volatility states to achieve an efficient implementation of a particle Markov chain Monte Carlo algorithm and made inferences about the model parameters, volatility states, and Markov states. Through the estimation, I characterized bond market exposures to macroeconomic and monetary policy risks, and identified the changes in the conditional covariance dynamics of long-run growth and the inflation target as the main driver of structural changes in bond markets. I found that the changes in monetary policy affect the volatility of bond yields, while the changes in the correlation between growth and inflation affect both the level and the volatility of bond yields. Overall, the model is quite successful in explaining several bond market phenomena.

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[30] Campbell, Pflueger, and Viceira (2013) find similar results. However, they claim that changes in the persistence of monetary policy can generate sign-switches. Since I do not incorporate the "smoothing" motive into the monetary policy action, my results show a limited role for monetary policy.

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Online Appendix

Bond Market Exposures to Macroeconomic and Monetary Policy Risks

A Piazzesi and Schneider (2006) Revisited

Following Piazzesi and Schneider (2006), I assume that the vector of inflation and consumption growth has the following state space representation

$$\begin{aligned} z_t &= s_{t-1} + \varepsilon_t, \quad z_t = [\pi_t, \Delta c_t]' \\ s_t &= \phi s_{t-1} + \phi K \varepsilon_t, \quad \varepsilon_t \sim N(0, \Omega). \end{aligned} \tag{A.1}$$

The state vector s_t is 2-dimensional and contains expected inflation and consumption, ϕ is the 2×2 autoregressive matrix, and K is the 2×2 gain matrix. Denote

$$\phi = \begin{bmatrix} \phi_1 & \phi_{12} \\ \phi_{21} & \phi_2 \end{bmatrix}, \quad K = \begin{bmatrix} k_1 & k_{12} \\ k_{21} & k_2 \end{bmatrix}, \quad \Omega = \begin{bmatrix} \Omega_1 & \Omega_{12} \\ \Omega_{12} & \Omega_2 \end{bmatrix}.$$

I estimate this system with data on consumption and inflation using Bayesian method. Table A-1 provides details of parameter prior and posterior distributions. The complete estimation information in the tables can be difficult to absorb fully, however, so here I briefly present aspects of the results in a more revealing way. The parameters to be estimated are those in the transition equation ϕ, K and those in the covariance matrix Ω. Hence I simply display the estimated transition equation and the estimated Ω matrices.

1. From 1959:Q1 to 1997:Q4

$$
\begin{aligned}
s_t &= \begin{bmatrix} \underset{[0.92,0.98]}{0.96} & \underset{[0.03,0.25]}{0.14} \\ \underset{[-0.10,-0.02]}{-0.06} & \underset{[0.36,0.69]}{0.52} \end{bmatrix} s_{t-1} + \begin{bmatrix} \underset{[0.57,0.73]}{0.63} & \underset{[0.07,0.50]}{0.25} \\ \underset{[-0.22,-0.16]}{-0.21} & \underset{[0.11,0.57]}{0.27} \end{bmatrix} \epsilon_t \\
\epsilon_t &\sim N(0, \begin{bmatrix} \underset{[2.13,2.60]}{2.35} & \underset{[-0.21,-0.05]}{-0.14} \\ \underset{[-0.21,-0.05]}{-0.14} & \underset{[2.40,2.96]}{2.68} \end{bmatrix}), \quad var(\phi K \epsilon_t) = \begin{bmatrix} \underset{[0.68,2.10]}{1.06} & \underset{[-0.26,0.52]}{-0.14} \\ \underset{[-0.26,0.52]}{-0.14} & \underset{[0.14,1.02]}{0.32} \end{bmatrix}.
\end{aligned}
$$

2. From 1998:Q1 to 2011:Q4

$$
\begin{aligned}
s_t &= \begin{bmatrix} \underset{[0.28,0.55]}{0.41} & \underset{[0.12,0.39]}{0.26} \\ \underset{[-0.03,0.18]}{0.07} & \underset{[0.72,0.91]}{0.83} \end{bmatrix} s_{t-1} + \begin{bmatrix} \underset{[0.12,0.69]}{0.33} & \underset{[0.14,0.86]}{0.43} \\ \underset{[-0.20,0.24]}{-0.02} & \underset{[0.48,1.02]}{0.71} \end{bmatrix} \epsilon_t \\
\epsilon_t &\sim N(0, \begin{bmatrix} \underset{[4.63,6.37]}{5.42} & \underset{[-0.09,0.07]}{-0.01} \\ \underset{[-0.09,0.07]}{-0.01} & \underset{[0.93,1.30]}{1.10} \end{bmatrix}), \quad var(\phi K \epsilon_t) = \begin{bmatrix} \underset{[0.08,4.14]}{0.78} & \underset{[0.01,2.28]}{0.29} \\ \underset{[0.01,2.28]}{0.29} & \underset{[0.42,1.77]}{0.55} \end{bmatrix}.
\end{aligned}
$$

Many aspects of the results are noteworthy; here I simply mention a few. First, the autoregressive matrix ϕ estimates are quite different across the two periods. More specifically, I find a large decline in the persistence of the expected inflation process. Also, the lagged inflation used to predict negative future consumption, but in the last fifteen years it positively forecasts consumption. Second, the sign of the estimated covariance (in the reduced-form covariance matrix var($\phi K \varepsilon_t$)) changed from negative to positive during the recent periods.

Table A-1: Posterior Estimates

		Prior		1959:Q1 - 1997:Q4 Posterior			1998:Q1 - 2011:Q4 Posterior		
	Distr.	20%	80%	20%	50%	80%	20%	50%	80%
ϕ_1	N^T	[-.35	.99]	0.92	0.96	0.98	0.28	0.41	0.55
ϕ_{12}	N	[-.82	.82]	0.03	0.14	0.25	0.12	0.26	0.39
ϕ_{21}	N	[-.82	.82]	-0.10	-0.06	-0.02	-0.03	0.07	0.18
ϕ_2	N^T	[-.35	.99]	0.35	0.52	0.69	0.72	0.83	0.91
k_1	N	[.15	1.81]	0.63	0.71	0.80	0.53	0.87	1.26
k_{12}	N	[-.82	.82]	0.07	0.18	0.29	0.22	0.53	0.90
k_{21}	N	[-.82	.82]	-0.44	-0.32	-0.20	-0.26	-0.10	0.02
k_2	N	[.15	1.81]	0.33	0.55	0.83	0.68	0.81	0.95
Ω_1	IG	[0.80	5.78]	2.13	2.35	2.60	4.63	5.42	6.37
Ω_{12}	N	[-.82	.82]	-0.21	-0.14	-0.05	-0.09	-0.01	0.07
Ω_2	IG	[0.80	5.78]	2.40	2.68	2.96	0.93	1.10	1.30

Notes: The estimation results are based on (annualized) quarterly consumption growth data and inflation data from 1959:Q1 to 2011:Q4. N, N^T, and IG are normal, truncated (outside of the interval $(-1, 1)$) normal, and inverse gamma distributions, respectively.

B Solving the LRR Model

This section provides approximate analytical solutions for the equilibrium asset prices.

B.1 Exogenous Dynamics

The joint dynamics of consumption, dividend growth, and inflation are

$$\begin{bmatrix} g_{c,t+1} \\ g_{d,t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \mu_c \\ \mu_d \\ \mu_\pi \end{bmatrix} + \begin{bmatrix} e_1 \\ \phi_x e_1 \\ \Gamma_x(S^X_{t+1}, S^M_{t+1}) \end{bmatrix} X_{t+1} + \begin{bmatrix} 1 & 0 & 0 \\ \phi_\eta & 1 & 0 \\ \Gamma_\eta(S^X_{t+1}, S^M_{t+1}) & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{\sigma}_c \eta_{c,t+1} \\ \bar{\sigma}_d \eta_{d,t+1} \\ \bar{\sigma}_\pi \eta_{\pi,t+1} \end{bmatrix} \tag{A.2}$$

The conditional mean and volatility processes evolve according to

$$\begin{aligned} \underbrace{\begin{bmatrix} x_{c,t+1} \\ x_{\pi,t+1} \\ x_{m,t+1} \end{bmatrix}}_{X_{t+1}} &= \underbrace{\begin{bmatrix} \rho_c(S^X_{t+1}) & \rho_{c,\pi}(S^X_{t+1}) & \rho_{c,m}(S^X_{t+1}) \\ \rho_{\pi,c}(S^X_{t+1}) & \rho_\pi(S^X_{t+1}) & \rho_{\pi,m}(S^X_{t+1}) \\ 0 & 0 & \rho_m(S^X_{t+1}) \end{bmatrix}}_{\Upsilon(S^X_{t+1})} \underbrace{\begin{bmatrix} x_{c,t} \\ x_{\pi,t} \\ x_{m,t} \end{bmatrix}}_{X_t} \\ &+ \underbrace{\begin{bmatrix} 1 & \chi_{c,\pi}(S^X_{t+1}) & 0 \\ \chi_{\pi,c}(S^X_{t+1}) & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\Omega(S^X_{t+1})} \underbrace{\begin{bmatrix} \sigma_{c,t} e_{c,t+1} \\ \sigma_{\pi,t} e_{\pi,t+1} \\ \sigma_m e_{m,t+1} \end{bmatrix}}_{E_{t+1}} \\ \underbrace{\begin{bmatrix} \sigma^2_{c,t+1} \\ \sigma^2_{\pi,t+1} \end{bmatrix}}_{\Sigma_{t+1}} &= \underbrace{\begin{bmatrix} (1-\nu_c)(\varphi_c \bar{\sigma})^2 \\ (1-\nu_\pi)(\varphi_\pi \bar{\sigma})^2 \end{bmatrix}}_{\Phi_\mu} + \underbrace{\begin{bmatrix} \nu_c & 0 \\ 0 & \nu_\pi \end{bmatrix}}_{\Phi_\nu} \underbrace{\begin{bmatrix} \sigma^2_{c,t} \\ \sigma^2_{\pi,t} \end{bmatrix}}_{\Sigma_t} + \underbrace{\begin{bmatrix} \sigma_{w_c} w_{c,t+1} \\ \sigma_{w_\pi} w_{\pi,t+1} \end{bmatrix}}_{W_{t+1}}, \end{aligned} \tag{A.3}$$

where $\eta_{j,t+1}, e_{k,t+1}, w_{l,t+1} \sim N(0,1)$ for $j \in \{c,d,\pi\}$, $k \in \{c,\pi,m\}$, and $l \in \{c,\pi\}$ and $W_{t+1} \sim N(0, \Phi_w)$.

Note that the VAR dynamics are generalized to allow for intertemporal feedback effects (captured by off-diagonal coefficients) and that the inflation target can become correlated with long-run growth innovation. Furthermore, the channels through which monetary policy shock affects long-run growth or inflation target, are not restricted to zero as in the main text. (Of course, one could set them equal to zero.)

B.2 Derivation of Approximate Analytical Solutions

The Euler equation for the economy is

$$1 = E_t \left[\exp \left(m_{t+1} + r_{k,t+1} \right) \right], \quad k \in \{c, m\}, \tag{A.4}$$

where $m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{c,t+1}$ is the log stochastic discount factor, $r_{c,t+1}$ is the log return on the consumption claim, and $r_{m,t+1}$ is the log market return. All returns are given by the approximation of Campbell and Shiller (1988a):

$$\begin{aligned} r_{c,t+1} &= \kappa_{0,c} + \kappa_{1,c} z_{c,t+1} - z_{c,t} + g_{c,t+1} \\ r_{m,t+1} &= \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} - z_{m,t} + g_{d,t+1}. \end{aligned} \tag{A.5}$$

Let I_t denote the current information set $\left\{ S^X_{1:t}, X_t, \Sigma_t \right\}$ and define I_{t+1}=$I_t \cup \{S^X_{t+1}\}$ that includes information regarding S^X_{t+1} in addition to I_t. Suppose $S^X_t = i$ for i=1, 2. Derivation of (A.4) follows Bansal and Zhou (2002), who make repeated use of the law of iterated expectations and log-linearization, and Schorfheide, Song, and Yaron (2013) who utilize log-linear approximation for returns and for volatilities.

$$\begin{aligned} 1 &= \mathbb{E}\Big(\mathbb{E} \left[\exp \left(m_{t+1} + r_{m,t+1} \right) \mid I_{t+1} \right] \mid I_t \Big) \\ &= \sum_{j=1}^{4} \mathbb{P}_{ij} \mathbb{E}\Big(\exp \left(m_{t+1} + r_{m,t+1} \right) \mid S_{t+1} = j, X_t, \Sigma_t \Big) \\ 0 &= \sum_{j=1}^{4} \mathbb{P}_{ij} \underbrace{\left(\mathbb{E} \left[m_{t+1} + r_{m,t+1} \mid S_{t+1} = j \right] + \frac{1}{2} \mathbb{V} \left[m_{t+1} + r_{m,t+1} \mid S_{t+1} = j \right]. \right)}_{B} \end{aligned} \tag{A.6}$$

The first line uses the law of iterated expectations, second line uses the definition of Markov-chain; and the third line applies log-linearization, $\exp(B) - 1 \approx B$, log-normality assumption, and log-linearization for returns and for volatilities.

B.3 Real Consumption Claim

Conjecture that the price to consumption ratio follows

$$z_t(S_t^X) = A_0(S_t^X) + A_1(S_t^X)X_t + A_2(S_t^X)\Sigma_t, \tag{A.7}$$

$A_1(S_t^X) = \left[\begin{array}{ccc} A_{1,c}(S_t^X) & A_{1,\pi}(S_t^X) & A_{1,m}(S_t^X) \end{array}\right]$ and $A_2(S_t^X) = \left[\begin{array}{cc} A_{2,c}(S_t^X) & A_{2,\pi}(S_t^X) \end{array}\right]$.

From (A.2), (A.3), (A.5), and (A.7),

$$\begin{aligned} r_{c,t+1} &= \kappa_{0,c} + \kappa_{1,c}A_0(S_{t+1}^X) - A_0(S_t^X) + \mu_c + \kappa_{1,c}A_2(S_{t+1}^X)\Phi_\mu \qquad \text{(A.8)} \\ &+ \left\{(e_1 + \kappa_{1,c}A_1(S_{t+1}^X))\Upsilon(S_{t+1}^X) - A_1(S_t^X)\right\} X_t + \left\{\kappa_{1,c}A_2(S_{t+1}^X)\Phi_\nu - A_2(S_t^X)\right\} \Sigma_t \\ &+ \bar{\sigma}_c\eta_{t+1} + (e_1 + \kappa_{1,c}A_1(S_{t+1}^X))\Omega(S_{t+1}^X)E_{t+1} + \kappa_{1,c}A_2(S_{t+1}^X)W_{t+1} \end{aligned}$$

and from (A.2), (A.3), (A.5), (A.6), and (A.7)

$$\begin{aligned} m_{t+1} &= \theta\log\delta + (\theta-1)\left\{\kappa_{0,c} + \kappa_{1,c}A_0(S_{t+1}^X) - A_0(S_t^X) + \kappa_{1,c}A_2(S_{t+1}^X)\Phi_\mu\right\} - \gamma\mu \qquad \text{(A.9)} \\ &- \frac{1}{\psi}e_1\Upsilon(S_{t+1}^X)X_t + (\theta-1)\left\{((1-\frac{1}{\psi})e_1 + \kappa_{1,c}A_1(S_{t+1}^X))\Upsilon(S_{t+1}^X) - A_1(S_t^X)\right\} X_t \\ &+ (\theta-1)\left\{\kappa_{1,c}A_2(S_{t+1}^X)\Phi_\nu - A_2(S_t^X)\right\}\Sigma_t - \gamma\bar{\sigma}_c\eta_{c,t+1} \\ &+ \left\{-\gamma e_1 + (\theta-1)\kappa_{1,c}A_1(S_{t+1}^X)\right\}\Omega(S_{t+1}^X)E_{t+1} + (\theta-1)\kappa_{1,c}A_2(S_{t+1}^X)W_{t+1}. \end{aligned}$$

The solutions for As that describe the dynamics of the price-consumption ratio are determined from (A.6), and they are,

$$\begin{aligned} \left[\begin{array}{cc} A_1(1) & A_1(2) \end{array}\right] &= e_1\left[\begin{array}{cc} p_{X_1}\Upsilon(1) + (1-p_{X_1})\Upsilon(2) & (1-p_{X_2})\Upsilon(1) + p_{X_2}\Upsilon(2) \end{array}\right] \qquad \text{(A.10)} \\ &\quad \times(1-\frac{1}{\psi})\left[\begin{array}{cc} \mathbb{I}_2 - p_{X_1}\kappa_{1,c}\Upsilon(1) & -(1-p_{X_2})\kappa_{1,c}\Upsilon(1) \\ -(1-p_{X_1})\kappa_{1,c}\Upsilon(2) & \mathbb{I}_2 - p_{X_2}\kappa_{1,c}\Upsilon(2) \end{array}\right]^{-1} \\ \left[\begin{array}{c} A_{2,c}(1) \\ A_{2,c}(2) \end{array}\right] &= \frac{\theta}{2}\left[\begin{array}{c} \mathbb{I}_2 - \kappa_{1,c}\nu_c\mathbb{P}_X \end{array}\right]^{-1} \times \mathbb{P}_X \times \left[\begin{array}{c} \xi_c(1) \\ \xi_c(2) \end{array}\right] \end{aligned}$$

$$\begin{bmatrix} A_{2,\pi}(1) \\ A_{2,\pi}(2) \end{bmatrix} = \frac{\theta}{2}\left[\ \mathbb{I}_2 - \kappa_{1,c}\nu_\pi \mathbb{P}_X\ \right]^{-1} \times \mathbb{P}_X \times \begin{bmatrix} \xi_\pi(1) \\ \xi_\pi(2) \end{bmatrix}$$

$$\begin{bmatrix} A_0(1) \\ A_0(2) \end{bmatrix} = \left[\ \mathbb{I}_2 - \kappa_{1,c}\mathbb{P}_X\ \right]^{-1} \times \mathbb{P}_X \times \begin{bmatrix} \bar{A}_0 + \kappa_{1,c}A_2(1)\Phi_\mu + \frac{\theta}{2}\kappa_{1,c}^2 A_2(1)\Phi_w A_2(1)' + \frac{\theta}{2}\xi_m(1)\sigma_m^2(1) \\ \bar{A}_0 + \kappa_{1,c}A_2(2)\Phi_\mu + \frac{\theta}{2}\kappa_{1,c}^2 A_2(2)\Phi_w A_2(2)' + \frac{\theta}{2}\xi_m(2)\sigma_m^2(2) \end{bmatrix}$$

where $\bar{A}_0 = \log\delta + \kappa_{0,c} + \mu_c(1-\frac{1}{\psi}) + \frac{\theta}{2}\bar{\sigma}_c^2(1-\frac{1}{\psi})^2$ and

$$\xi_c(i) = \left\{\left((1-\frac{1}{\psi})e_1 + \kappa_{1,c}A_1(i)\right)\cdot\Omega(i)e_1'\right\}^2, \quad \xi_\pi(i) = \left\{\left((1-\frac{1}{\psi})e_1 + \kappa_{1,c}A_1(i)\right)\cdot\Omega(i)e_2'\right\}^2$$

$$\xi_m(i) = \left\{\left((1-\frac{1}{\psi})e_1 + \kappa_{1,c}A_1(i)\right)\cdot\Omega(i)e_3'\right\}^2, \quad i\in\{1,2\}.$$

B.4 Real Market Returns

Similarly, using the conjectured solution to the price-dividend ratio

$$z_{m,t}(S_t^X) = A_{0,m}(S_t^X) + A_{1,m}(S_t^X)X_t + A_{2,m}(S_t^X)\Sigma_t, \tag{A.11}$$

the market return equation can be expressed as

$$\begin{aligned} r_{m,t+1} &= \kappa_{0,m} + \kappa_{1,m}A_{0,m}(S_{t+1}^X) - A_{0,m}(S_t^X) + \mu_d + \kappa_{1,m}A_{2,m}(S_{t+1}^X)\Phi_\mu \\ &+ \left\{(\phi_x e_1 + \kappa_{1,m}A_{1,m}(S_{t+1}^X))\Upsilon(S_{t+1}^X) - A_{1,m}(S_t^X)\right\}X_t \\ &+ \left\{\kappa_{1,m}A_{2,m}(S_{t+1}^X)\Phi_\nu - A_{2,m}(S_t^X)\right\}\Sigma_t + \phi_\eta\bar{\sigma}_c\eta_{c,t+1} + \bar{\sigma}_d\eta_{d,t+1} \\ &+ (\phi_x e_1 + \kappa_{1,m}A_{1,m}(S_{t+1}^X))\Omega(S_{t+1}^X)E_{t+1} + \kappa_{1,m}A_{2,m}(S_{t+1}^X)W_{t+1}. \end{aligned} \tag{A.12}$$

From (A.2), (A.3), (A.5), and (A.11), the solutions for A_m-s that describe the dynamics of the price-dividend ratio are

$$\begin{aligned} \left[\ A_{1,m}(1) \quad A_{1,m}(2)\ \right] &= (\phi_x - \frac{1}{\psi})e_1\left[\ p_{X_1}\Upsilon(1) + (1-p_{X_1})\Upsilon(2) \quad (1-p_{X_2})\Upsilon(1) + p_{X_2}\Upsilon(2)\ \right] \\ &\times \begin{bmatrix} \mathbb{I}_2 - p_{X_1}\kappa_{1,m}\Upsilon(1) & -(1-p_{X_2})\kappa_{1,m}\Upsilon(1) \\ -(1-p_{X_1})\kappa_{1,m}\Upsilon(2) & \mathbb{I}_2 - p_{X_2}\kappa_{1,m}\Upsilon(2) \end{bmatrix}^{-1} \end{aligned}$$

$$
\begin{aligned}
\begin{bmatrix} A_{2,c,m}(1) \\ A_{2,c,m}(2) \end{bmatrix} &= \begin{bmatrix} \mathbb{I}_2 - \kappa_{1,m}\nu_c \mathbb{P}_X \end{bmatrix}^{-1} \left(\mathbb{P}_X \begin{bmatrix} (\theta-1)\kappa_{1,c}\nu_c A_{2,c}(1) + \frac{1}{2} f_c(1) \\ (\theta-1)\kappa_{1,c}\nu_c A_{2,c}(2) + \frac{1}{2} f_c(2) \end{bmatrix} - (\theta-1) \begin{bmatrix} A_{2,c}(1) \\ A_{2,c}(2) \end{bmatrix} \right) \\
f_c(i) &= \left((\phi_x - \gamma) e_1 \cdot \Omega(i) e_1' + \begin{bmatrix} A_1(i) \cdot \Omega(i) e_1' & A_{1,m}(i) \cdot \Omega(i) e_1' \end{bmatrix} \begin{bmatrix} (\theta-1)\kappa_{1,c} \\ \kappa_{1,m} \end{bmatrix} \right)^2, \\
\begin{bmatrix} A_{2,\pi,m}(1) \\ A_{2,\pi,m}(2) \end{bmatrix} &= \begin{bmatrix} \mathbb{I}_2 - \kappa_{1,m}\nu_\pi \mathbb{P}_X \end{bmatrix}^{-1} \left(\mathbb{P}_X \begin{bmatrix} (\theta-1)\kappa_{1,c}\nu_\pi A_{2,\pi}(1) + \frac{1}{2} f_\pi(1) \\ (\theta-1)\kappa_{1,c}\nu_\pi A_{2,\pi}(2) + \frac{1}{2} f_\pi(2) \end{bmatrix} - (\theta-1) \begin{bmatrix} A_{2,\pi}(1) \\ A_{2,\pi}(2) \end{bmatrix} \right) \\
f_\pi(i) &= \left((\phi_x - \gamma) e_1 \cdot \Omega(i) e_2' + \begin{bmatrix} A_1(i) \cdot \Omega(i) e_2' & A_{1,m}(i) \cdot \Omega(i) e_2' \end{bmatrix} \begin{bmatrix} (\theta-1)\kappa_{1,c} \\ \kappa_{1,m} \end{bmatrix} \right)^2, \\
\begin{bmatrix} A_{0,m}(1) \\ A_{0,m}(2) \end{bmatrix} &= \begin{bmatrix} \mathbb{I}_2 - \kappa_{1,m} \mathbb{P}_X \end{bmatrix}^{-1} \left(\mathbb{P}_X \begin{bmatrix} \bar{A}_{0,m} + f_0(1) \\ \bar{A}_{0,m} + f_0(2) \end{bmatrix} - (\theta-1) \begin{bmatrix} A_0(1) \\ A_0(2) \end{bmatrix} \right) \\
\bar{A}_{0,m} &= \theta \log \delta + (\theta-1)\kappa_{0,c} - \gamma\mu_c + \kappa_{0,m} + \mu_d + \frac{1}{2}\bar{\sigma}_d^2 + \frac{1}{2}\bar{\sigma}_c^2(\phi_\eta - \gamma)^2 \\
f_0(i) &= (\theta-1)\kappa_{1,c}\Big(A_0(i) + A_2(i)\Phi_\mu \Big) + \frac{\sigma_{w_c}^2}{2} \left(\begin{bmatrix} A_{2,c}(i) & A_{2,c,m}(i) \end{bmatrix} \begin{bmatrix} (\theta-1)\kappa_{1,c} \\ \kappa_{1,m} \end{bmatrix} \right)^2 \\
&+ \frac{\sigma_{w_\pi}^2}{2} \left(\begin{bmatrix} A_{2,\pi}(i) & A_{2,\pi,m}(i) \end{bmatrix} \begin{bmatrix} (\theta-1)\kappa_{1,c} \\ \kappa_{1,m} \end{bmatrix} \right)^2 + \kappa_{1,m} A_{2,m}(i) \Phi_\mu \\
&+ \frac{1}{2} \left((\phi_x - \gamma) e_1 \cdot \Omega(i) e_3' + \begin{bmatrix} A_1(i) \cdot \Omega(i) e_3' & A_{1,m}(i) \cdot \Omega(i) e_3' \end{bmatrix} \begin{bmatrix} (\theta-1)\kappa_{1,c} \\ \kappa_{1,m} \end{bmatrix} \right)^2 \sigma_m^2(i),
\end{aligned}
$$

for $i \in \{1, 2\}$.

B.5 Linearization Parameters

Let $\bar{p}_j = \frac{1-p_l}{2-p_l-p_j}$. For any asset, the linearization parameters are determined endogenously by the following system of equations

$$
\begin{aligned}
\bar{z}_i &= \sum_{j=1}^{2} \bar{p}_j \Big(A_{0,i}(j) + A_{2,c,i}(j)(\varphi_c \bar{\sigma})^2 + A_{2,\pi,i}(j)(\varphi_\pi \bar{\sigma})^2 \Big) \\
\kappa_{1,i} &= \frac{\exp(\bar{z}_i)}{1 + \exp(\bar{z}_i)} \\
\kappa_{0,i} &= \log(1 + \exp(\bar{z}_i)) - \kappa_{1,i}\bar{z}_i.
\end{aligned}
$$

The solution is determined numerically by iteration until reaching a fixed point of $\bar{z}_i$ for $i \in \{1, 2\}$.

B.6 Nominal Bond Prices

B.6.1 Endogenous Inflation Determination under a Regime-Switching Taylor Rule

I consider a version of the model where inflation is endogenous. The natural framework in which to this is a model where monetary policy is implemented by a central bank that follows a Taylor rule

$$
\begin{aligned}
i_t &= \mu_i^{MP}(S_t^M) + \tau_c(S_t^M)(g_{c,t} - \mu_c) + \tau_\pi(S_t^M)(\pi_t - x_{\pi,t}) + x_{\pi,t} + x_{m,t}, \qquad \text{(A.13)} \\
&= \mu_i^{MP}(S_t^M) + \left[\begin{array}{cccc} \tau_c(S_t^M) & 1 - \tau_\pi(S_t^M) & 1 & \tau_c(S_t^M) \end{array} \right] X_t^B + \tau_\pi(S_t^M)\pi_t,
\end{aligned}
$$

where $g_{c,t}$ is consumption growth, $x_{\pi,t}$ is the long-run inflation, and $x_{m,t}$ is the monetary policy shock. Assume for simplicity that π_t is "demeaned" inflation and $X_t^B = [x_{c,t}, x_{\pi,t}, x_{m,t}, \eta_{c,t}]'$.

The asset pricing equation for the short-rate is

$$
\begin{aligned}
i_t &= -E_t\left[m_{t+1}\right] + E_t\left[\pi_{t+1}\right] - \frac{1}{2}Var_t\left[m_{t+1}\right] - \frac{1}{2}Var_t\left[\pi_{t+1}\right] + Cov_t\left[m_{t+1}, \pi_{t+1}\right] \quad \text{(A.14)} \\
&= \tilde{\mu}_i^{AP}(S_t^X) + \alpha_{X^B}(S_t^X)X_t^B + \alpha_\Sigma(S_t^X)\Sigma_t \\
&\approx \tilde{\mu}_i^{AP}(S_t^X) + \alpha_{X^B}(S_t^X)X_t^B + \alpha_\Sigma(S_t^X)\bar{\Sigma} \\
&= \mu_i^{AP}(S_t^X) + \left[\frac{1}{\psi}\mathbb{E}_t[e_1\Upsilon(S_{t+1}^X)], 0\right]X_t^B + E_t\left[\pi_{t+1}\right].
\end{aligned}
$$

The first to second line uses the log normality assumption, the second to third line uses the fact that stochastic volatility contribute very little to the short-rate, and the third to fourth line rearranges parameter values such that the short-rate is expressed in terms of X_t^B and $E_t\left[\pi_{t+1}\right]$.

S_t^X and S_t^M are discrete-valued random variables that follow a two-state Markov chain,

$$\mathbb{P}_X = \begin{bmatrix} p_{X_1} & 1-p_{X_1} \\ 1-p_{X_2} & p_{X_2} \end{bmatrix}, \quad \mathbb{P}_M = \begin{bmatrix} p_{M_1} & 1-p_{M_1} \\ 1-p_{M_2} & p_{M_2} \end{bmatrix},$$

where X_1 (X_2) stands for negative (positive) correlation regime and M_1 (M_2) stands for active (passive) monetary policy regime. For notational convenience, define

$$S_t = \begin{cases} 1 & \text{if} \quad S_t^X = X_1 \quad \text{and} \quad S_t^M = M_1 \\ 2 & \text{if} \quad S_t^X = X_1 \quad \text{and} \quad S_t^M = M_2 \\ 3 & \text{if} \quad S_t^X = X_2 \quad \text{and} \quad S_t^M = M_1 \\ 4 & \text{if} \quad S_t^X = X_2 \quad \text{and} \quad S_t^M = M_2 \end{cases}$$

and $\mathbb{P} = \mathbb{P}_X \otimes \mathbb{P}_M$.

Joint restriction of (A.13) and (A.14) gives

$$\begin{aligned} \tau_\pi(S_t^M)\pi_t &= E_t\left[\pi_{t+1}\right] + \underbrace{\left([\frac{1}{\psi}\mathbb{E}_t[e_1\Upsilon(S_{t+1}^X)], 0] - \left[\tau_c(S_t^M), 1-\tau_\pi(S_t^M), 1, \tau_c(S_t^M)\right] \right)}_{\Lambda(S_t^X, S_t^M)} X_t^B \qquad \text{(A.15)} \\ &= E_t\left[\pi_{t+1}\right] + \Lambda(S_t^X, S_t^M)X_t^B, \end{aligned}$$

assuming $\mu_i^{MP}(S_t^M) = \mu_i^{AP}(S_t^X)$. Since (A.15) is satisfied for each current state, I can express them as

$$\text{Diag}\left(\begin{bmatrix} \tau_\pi(S_t=1) \\ \tau_\pi(S_t=2) \\ \tau_\pi(S_t=3) \\ \tau_\pi(S_t=4) \end{bmatrix} \right) \times \begin{bmatrix} \pi_t(S_t=1) \\ \pi_t(S_t=2) \\ \pi_t(S_t=3) \\ \pi_t(S_t=4) \end{bmatrix} = \begin{bmatrix} E\left[\pi_{t+1}|S_t=1\right] \\ E\left[\pi_{t+1}|S_t=2\right] \\ E\left[\pi_{t+1}|S_t=3\right] \\ E\left[\pi_{t+1}|S_t=4\right] \end{bmatrix} + \begin{bmatrix} \Lambda(S_t=1) \\ \Lambda(S_t=2) \\ \Lambda(S_t=3) \\ \Lambda(S_t=4) \end{bmatrix} X_t \qquad \text{(A.16)}$$

In a slight abuse of notation, I use (i) to denote the current state instead of $(S_t = i)$

for i=1,2,3,4. From (A.9), observe that

$$\begin{bmatrix} \Lambda(1) \\ \Lambda(2) \\ \Lambda(3) \\ \Lambda(4) \end{bmatrix} = \mathbb{P} \times \begin{bmatrix} \frac{1}{\psi}e_1\Upsilon(1) & 0 \\ \frac{1}{\psi}e_1\Upsilon(2) & 0 \\ \frac{1}{\psi}e_1\Upsilon(3) & 0 \\ \frac{1}{\psi}e_1\Upsilon(4) & 0 \end{bmatrix} - \begin{bmatrix} \tau_c(1) & 1-\tau_\pi(1) & 1 & \tau_c(1) \\ \tau_c(2) & 1-\tau_\pi(2) & 1 & \tau_c(2) \\ \tau_c(3) & 1-\tau_\pi(3) & 1 & \tau_c(3) \\ \tau_c(4) & 1-\tau_\pi(4) & 1 & \tau_c(4) \end{bmatrix}. \tag{A.17}$$

I posit regime-dependent linear solutions of the form as in Davig and Leeper (2007).

$$\begin{bmatrix} \pi_t(1) \\ \pi_t(2) \\ \pi_t(3) \\ \pi_t(4) \end{bmatrix} = \begin{bmatrix} \Gamma(1) \\ \Gamma(2) \\ \Gamma(3) \\ \Gamma(4) \end{bmatrix} X_t^B \tag{A.18}$$

where $\Xi(i) = \begin{bmatrix} \Gamma_{x,c}(i) & \Gamma_{x,\pi}(i) & \Gamma_{x,m}(i) & \Gamma_\eta(i) \end{bmatrix}$ for i=1,2,3,4.

Necessary and Sufficient Conditions for the Existence of a Unique Bounded Solution. According to Proposition 2 of Davig and Leeper (2007), there exists a unique bounded solution if the following conditions are satisfied:

1. $\tau_\pi(i) > 0$, for i=1,2,3,4,
2. All the eigenvalues of $\left(\begin{bmatrix} \tau_\pi(1) & 0 & 0 & 0 \\ 0 & \tau_\pi(2) & 0 & 0 \\ 0 & 0 & \tau_\pi(3) & 0 \\ 0 & 0 & 0 & \tau_\pi(4) \end{bmatrix}^{-1} \times \mathbb{P} \right)$ lie inside the unit circle.

Solution. Substituting (A.18) to (A.16) yields

$$\begin{bmatrix} \tau_\pi(1) & 0 & 0 & 0 \\ 0 & \tau_\pi(2) & 0 & 0 \\ 0 & 0 & \tau_\pi(3) & 0 \\ 0 & 0 & 0 & \tau_\pi(4) \end{bmatrix} \begin{bmatrix} \Gamma(1) \\ \Gamma(2) \\ \Gamma(3) \\ \Gamma(4) \end{bmatrix} X_t^B = \mathbb{P} \times \begin{bmatrix} \Gamma(1)\Upsilon(1) \\ \Gamma(2)\Upsilon(2) \\ \Gamma(3)\Upsilon(3) \\ \Gamma(4)\Upsilon(4) \end{bmatrix} X_t^B + \begin{bmatrix} \Lambda(1) \\ \Lambda(2) \\ \Lambda(3) \\ \Lambda(4) \end{bmatrix} X_t^B \tag{A.19}$$

Analytical expressions for $\Gamma(i)$s are quite difficult to interpret, but are easily obtained from solving (A.19).

B.6.2 Nominal Bond Prices

Define $m_{t+1}^{\$} = m_{t+1} - \pi_{t+1}$. Let $P_{n,t}$ be the price at date t of a nominal bond with n periods to maturity. Conjecture that $p_{n,t}$ depends on the regime S_t and the current state variables,

$$p_{n,t} = C_{n,0}(S_t) + C_{n,1}(S_t)X_t + C_{n,2}(S_t)\Sigma_t \tag{A.20}$$

where $C_{n,1}(S_t) = \begin{bmatrix} C_{n,1,c}(S_t) & C_{n,1,\pi}(S_t) & C_{n,1,m}(S_t) \end{bmatrix}$ and $C_{n,2}(S_t) = \begin{bmatrix} C_{n,2,c}(S_t) & C_{n,2,\pi}(S_t) \end{bmatrix}$.

Exploit the law of iterated expectations

$$P_{n,t} \quad = \quad E_t\bigg(E[\exp(m_{t+1}^{\$} + p_{n-1,t+1})|I_{t+1}]\bigg)$$

and log-linearization to solve for $p_{n,t}$

$$p_{n,t} \quad \approx \quad \sum_{j=1}^{4} \mathbb{P}_{ij} \log\bigg(E[\exp(m_{t+1}^{\$} + p_{n-1,t+1}^{\$})|S_t = i, S_{t+1} = j]\bigg).$$

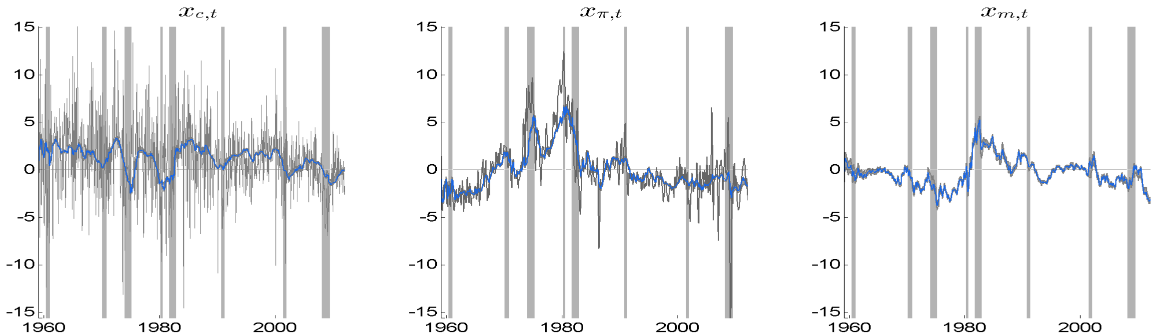
The solution to (A.20) is

$$
\begin{aligned}
C_{n,1}(i) &= \sum_{j=1}^{4} \mathbb{P}_{ij} \left(C_{n-1,1}(j) - \frac{1}{\psi} e_1 - \Gamma_x(j) \right) \Upsilon(j) \\
C_{n,2}(i) &= \sum_{j=1}^{4} \mathbb{P}_{ij} \Bigg(C_{n-1,2}(j)\Phi_\nu + (\theta - 1) \left\{ \kappa_{1,c} A_2(j) \Phi_\nu - A_2(i) \right\} \\
&+ \frac{1}{2} \begin{bmatrix} \{(C_{n-1,1}(j) - \gamma e_1 - \Gamma_x(j) + (\theta-1)\kappa_{1,c} A_1(j)) \cdot \Omega(j) e_1'\}^2 \\ \{(C_{n-1,1}(j) - \gamma e_1 - \Gamma_x(j) + (\theta-1)\kappa_{1,c} A_1(j)) \cdot \Omega(j) e_2'\}^2 \end{bmatrix}' \Bigg) \\
C_{n,0}(i) &= \sum_{j=1}^{4} \mathbb{P}_{ij} \Bigg(\theta \log \delta + (\theta - 1) \left\{ \kappa_{0,c} + \kappa_{1,c} A_0(j) + \kappa_{1,c} A_2(j) \Phi_\mu \right\} - (\theta - 1) A_0(i) - \gamma\mu - \mu_\pi \\
&+ C_{n-1,0}(j) + C_{n-1,2}(j) \Phi_\mu + \frac{1}{2} \bar{\sigma}_c^2 (\Gamma_\eta(j) + \gamma)^2 + \frac{1}{2} \bar{\sigma}_\pi^2 \\
&+ \frac{1}{2} \left\{ (C_{n-1,2,c}(j) + (\theta - 1)\kappa_{1,c} A_{2,c}(j)) \sigma_{w_c} \right\}^2 + \frac{1}{2} \left\{ (C_{n-1,2,\pi}(j) + (\theta - 1)\kappa_{1,c} A_{2,\pi}(j)) \sigma_{w_\pi} \right\}^2 \\
&+ \frac{1}{2} \left\{ (C_{n-1,1}(j) - \gamma e_1 - \Gamma_x(j) + (\theta - 1)\kappa_{1,c} A_1(j)) \cdot \Omega(j) e_3' \right\}^2 \sigma_m(j)^2 \Bigg),
\end{aligned}
$$

with initial conditions $C_{0,0}(i) = 0$, $C_{0,1}(i) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$, and $C_{0,2}(i) = \begin{bmatrix} 0 & 0 \end{bmatrix}$ for $i \in \{1, 2, 3, 4\}$.

C Supplementary Figures

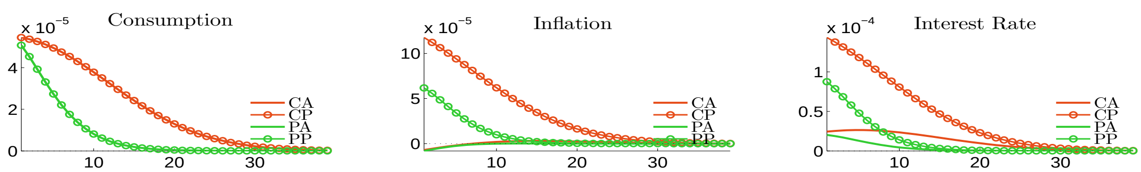
Figure C-1: Smoothed Mean States



Notes: Black lines represent posterior medians of smoothed states and the dark gray shaded area corresponds to 90% credible intervals. Light gray shaded bars indicate NBER recession dates. I overlay the smoothed long-run growth with monthly consumption growth and the smoothed long-run inflation with realized inflation (blue solid lines).

Figure C-2: Impulse Response Function

Growth Shock



Inflation Shock

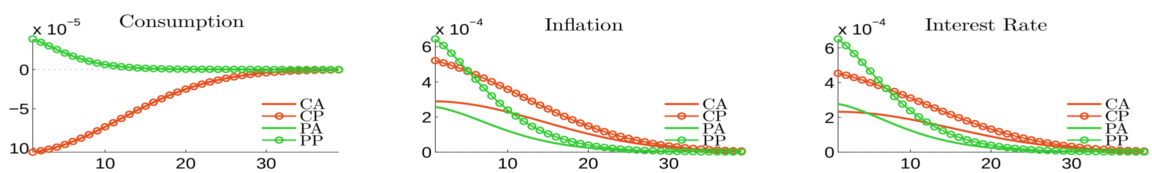


Figure C-3: Model-Generated Unconditional Mean

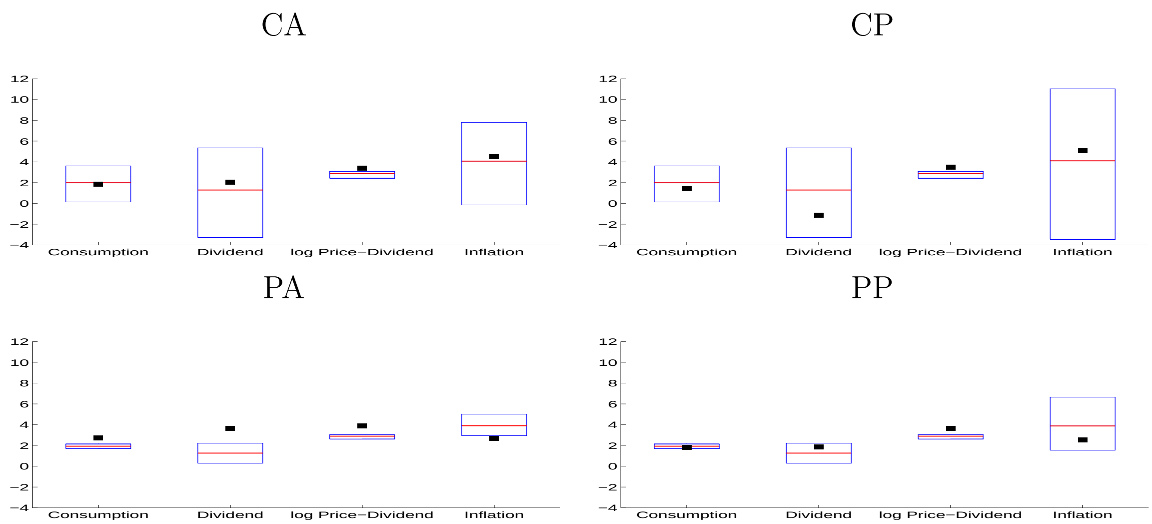
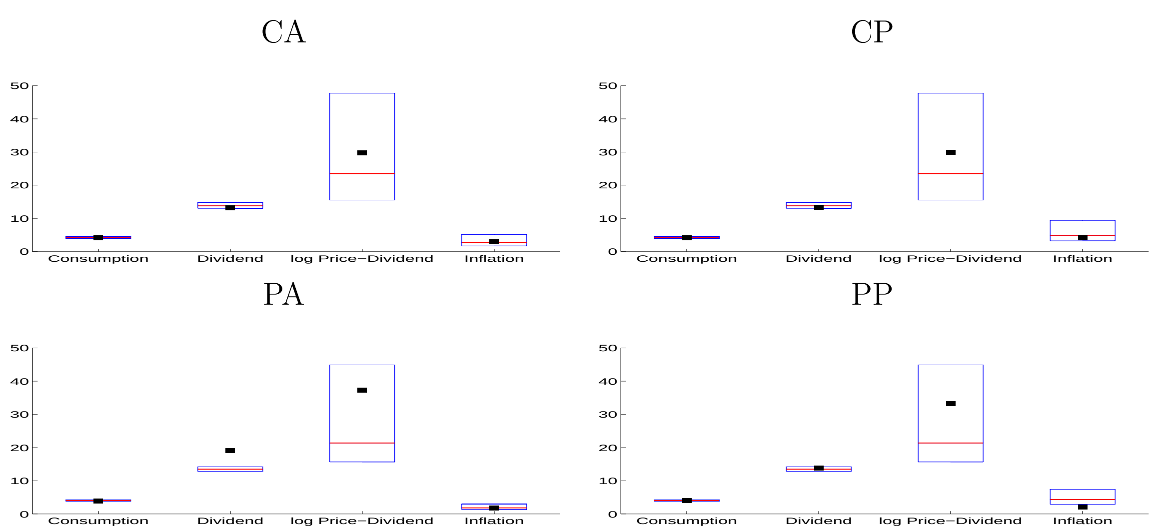
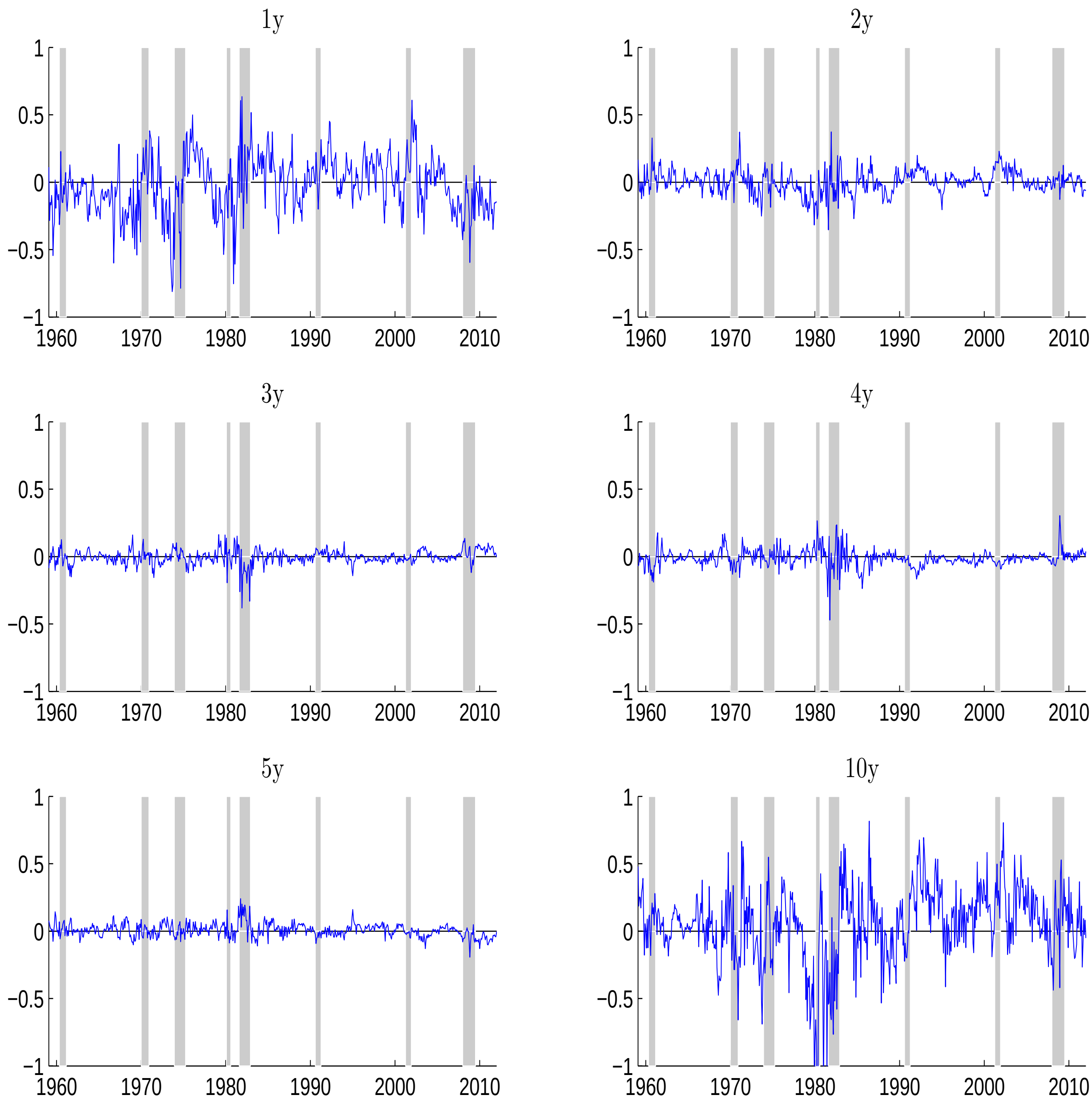


Figure C-4: Model-Generated Unconditional Standard Deviation



Notes: Black squares indicate values from actual data. The figure also depicts medians (red lines) and 90% credible intervals (top and bottom lines of boxes) of the distribution of yield spreads obtained with model-generated data. "CA" stands for the countercyclical inflation and the active monetary policy regimes while "PP" stands for the procyclical inflation and the passive monetary policy regimes. "CP" and "PA" indicate the remaining combinations of regimes. Numbers are displayed in percents (annualized).

Figure C-5: Yield Prediction Errors

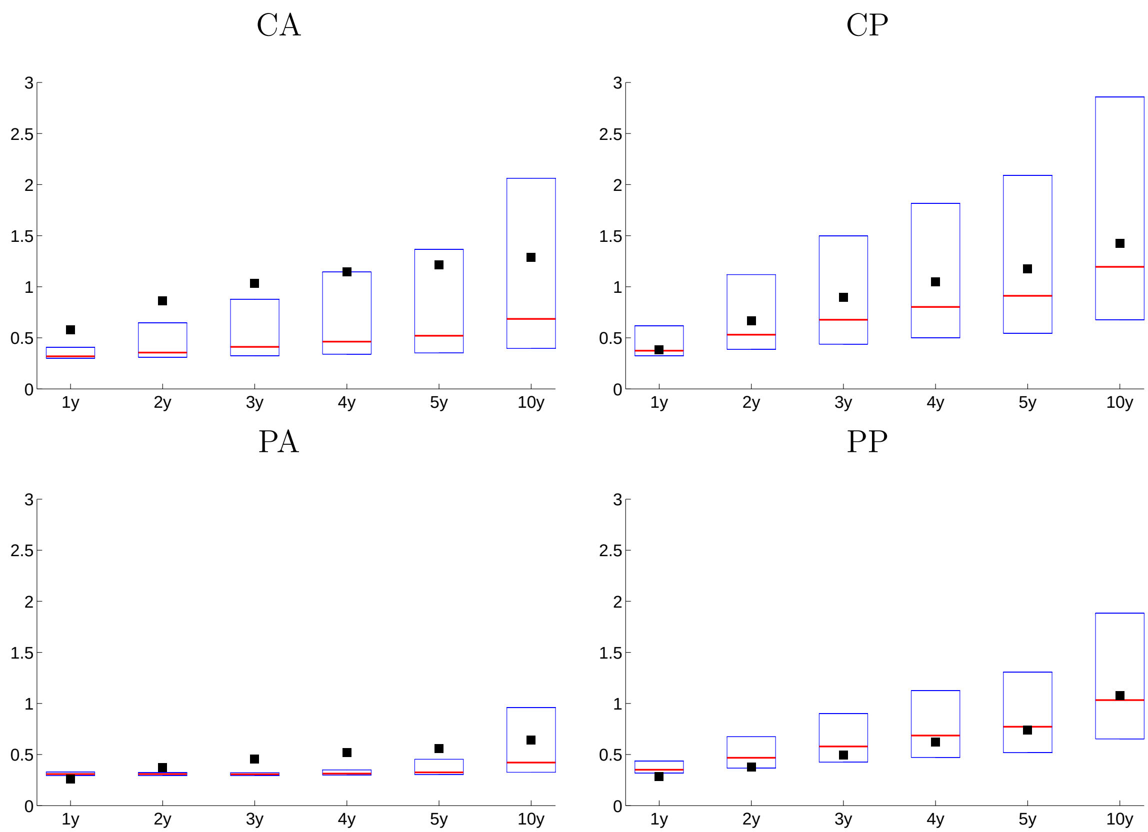


Note: Numbers are displayed in percents (annualized). In-sample RMSE numbers

1y	2y	3y	4y	5y	10y
21	8	5	7	5	28

are also provided in basis points (annualized).

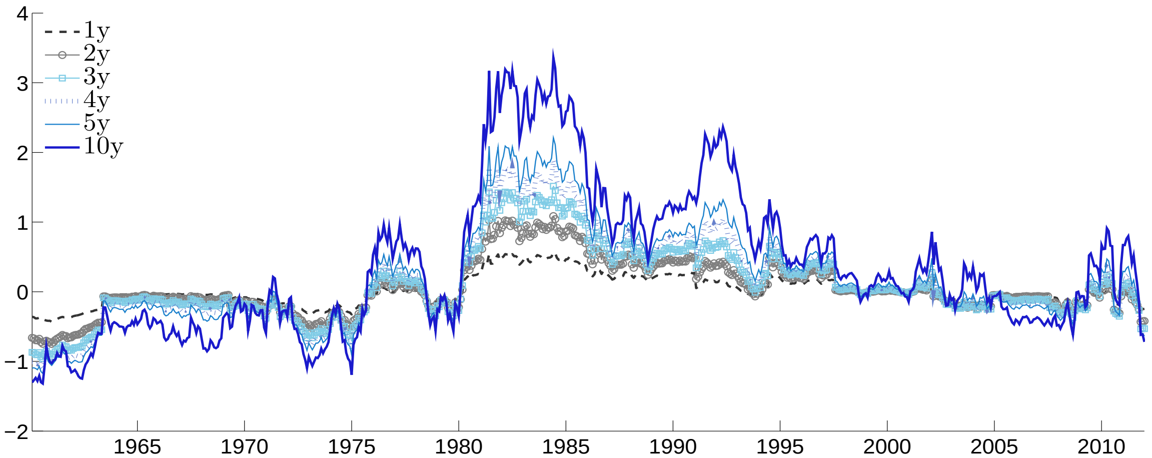
Figure C-6: Model-Generated Yield Spread: Unconditional Standard Deviation



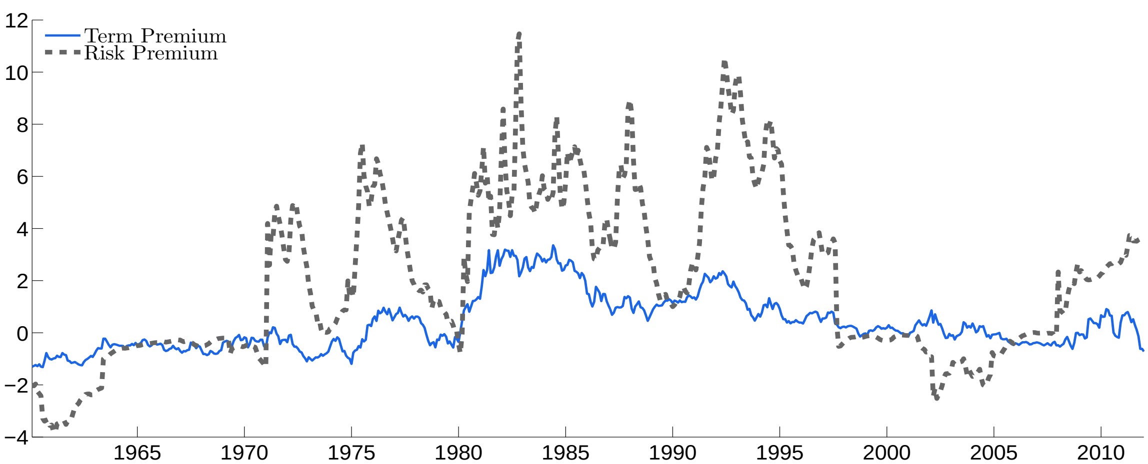
Notes: The "spread" is the difference between the 3m yield and yields with maturities of 1y–10y. Black squares indicate values from actual data. The figure also depicts medians (red lines) and 90% credible intervals (top and bottom lines of boxes) of the distribution of yield spreads obtained with model-generated data. "CA" stands for the countercyclical inflation and the active monetary policy regimes while "PP" stands for the procyclical inflation and the passive monetary policy regimes. "CP" and "PA" indicate the remaining combinations of regimes. Numbers are displayed in percents (annualized).

Figure C-7: Risk and Term Premia

Term Premia



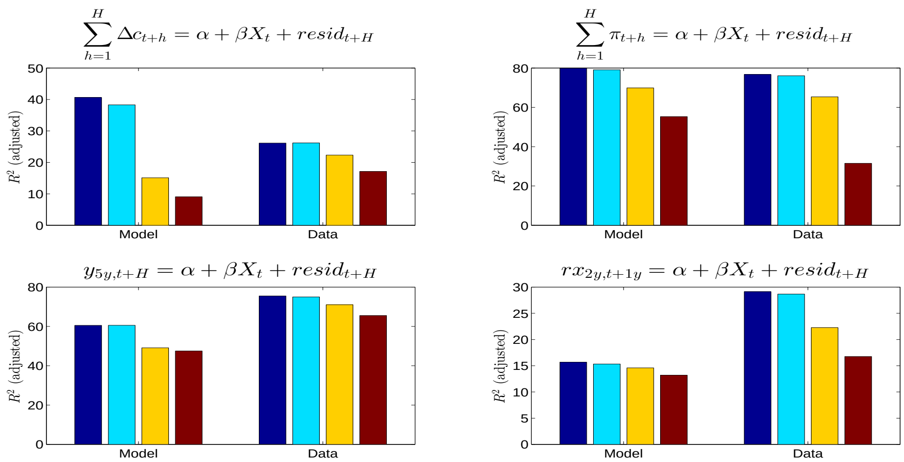
10-Year Bond Risk and Term Premia



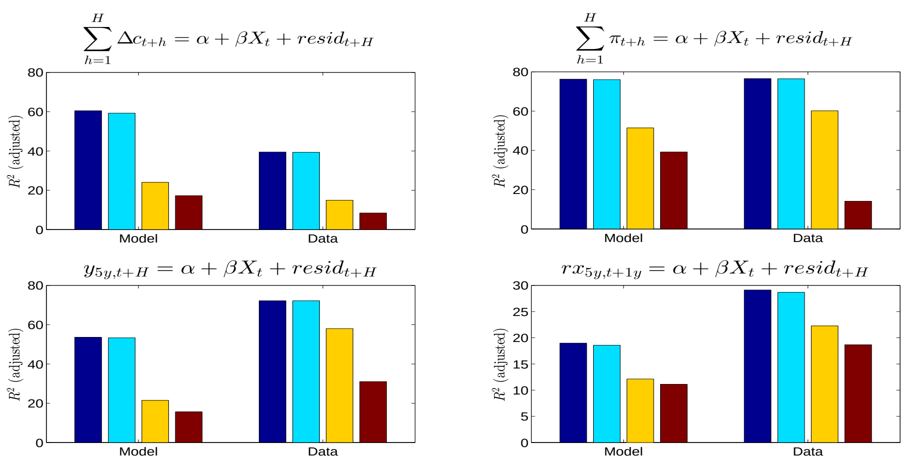
Note: The median estimates of term premium$_{t,n}$ $= y_{t,n} - \frac{1}{n}\sum_{i=0}^{n-1}\mathbb{E}_t(y_{t+i,1})$ and risk premium$_{t,n}$ $= -cov_t(m_{t+1} - \pi_{t+1}, rx_{t+1,n})$ are provided.

Figure C-8: Univariate Predictability Checks

H = 2-year



H = 5-year



Notes: Adjusted R^2 values (in percents) from the univariate predictive regressions of aggregate consumption growth (Δc_t), consumer price index inflation (π_t), log bond yield with maturity at five years ($y_{5y,t}$), and the excess (log) bond return of an n year bond over the 1 year bond ($xr_{n,t+12y}$) are provided. I regress each of them on X_t using OLS, $X_t \in \big\{\{macro_t, state_t\}, state_t, \{macro_t, pc_t\}, pc_t\big\}$ where $macro_t = \{\Delta c_t, \pi_t\}$, $state_t = \{x_{c,t}, x_{\pi,t}, x_{m,t}, \sigma^2_{c,t}, \sigma^2_{\pi,t}\}$, pc_t= first five principal components of $\{y_{1m,t}, y_{3m,t}, y_{1y,t}, y_{2y,t}, y_{3y,t}, y_{4y,t}, y_{5y,t}, y_{10y,t}\}$. Each bar represents the adjusted R^2 value from the OLS regression when X_t is $\{macro_t, state_t\}$ (blue), $state_t$ (light blue), $\{macro_t, pc_t\}$ (yellow), or pc_t (red), respectively. Data R^2 estimates are obtained by replacing $state_t$ with median *estimated* state variables, $\hat{state}_t$ in the OLS regression. To facilitate comparison with data estimates, median (adjusted) R^2 values from model-generated data regression are reported.