A Formal Framework for Strategic Representations and Conceptual Reorganization

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Abstract

In this paper, we introduce a formal language for modeling the structure of strategic representations and operations that conceptualize change on basis of them. Strategic representations are lower dimensional representations of the world that underlie the understanding of what business environments are, how they may change, and attempts to shape them. We start from discussing known strategic representations like Porter's five forces model or the strategy canvas. We elicit the conceptual structure underlying these representations by capturing them in our formal language.

We demonstrate that our formal language can express operations of conceptual change of strategies such as stretching (the extension of value ranges), lifting (deleting dimensions), extending (adding dimensions), amalgamation (enabling new combinations of features by amalgamating different domains), and transferring structure (exploring analogies). These operations can be the basis for strategizing: for seeing possible reorganizations of strategies and even to become aware of new opportunities. We apply these operations to explain classical business cases, including a detailed study of the conceptual structure underling Steve Jobs' digital hub concept.

Our formal language is, to our knowledge, the first attempt to capture the variety of conceptual operations underlying strategic change using one comprehensive model.

INTRODUCTION

A fundamental departure of the field of organization and strategy science from standard economics is the insight that decision-makers don't rely on a complete description of an outside world, but create a selective representation of it. Such selective representations underlie the understanding of what business environments are, how they may change, and how decision-makers attempt to shape them. Indeed, it can be argued that an adequate representation of an environment lies at the very heart of a successful strategy (Gavetti 2012, Brandenburger and Vinokurova 2012). One of the founding fathers of strategy, K. R. Andrews, has defined corporate strategy as "a kind of Everyman's conceptual scheme". B. Henderson has famously said that strategy is "the business of selling powerful oversimplifications", adding that "given that decision making is necessarily a complex process, the most useful frame of reference is the concept"(Henderson 1979). Conceptual thinking is the skeleton or the framework on which all other choices are sorted out. The distinction of a conceptual framework from a model is at the core of Porter's landmark approach to competitive advantage. ”Frameworks identify the relevant variables and the questions that the user must answer in order to develop conclusions tailored to a particular industry and company”. Recently, Gavetti (2012) has identified the ability to shape new conceptual representations as the fundamental source of competitive advantage for firms.

Representations are simplifications, as they reduce the high dimensionality and (more general) complexity of the real world into a relatively simple structure (Gavetti and Levinthal, 2000). This structure is conceived as the basis for strategizing: it can be used for making inferences about the environment, simulating possible events, and for searching new action possibilities. It can even be manipulated to generate new representations that make new opportunities visible. Over the years the strategy field, both in academic life and in practice, has been generating a large number of conceptual representations. Yet, relatively little attention has been devoted to analyze the structure of strategy representations and their implications. Students of strategic decision making have sometimes tried to make an explicit representation of strategist’s mental models using tools such as cause maps or scripts (Porac et al. 1989, Huff and Jenkins 2002, Axelrod 1976). While extremely interesting, such tools lack of a coherent formal structure and do not provide support to modelling the architecture of strategy representations. Furthermore, they lack the ability to express the basic operations that underlie conceptual reorganizations which are at the heart of the generation of new strategies.

This paper has the ambitious goal of providing a formal language for modelling the structure of strategy representations and the possible arrangements of choices on basis of them. The tools that we introduce allow to express a decision-maker’s consideration of how different components of a business strategy relate, and how structures that are at work in components of a business strategy can be reiterated and applied to further areas.

By strategic representation we mean generically a simplified model of the task environment of a strategic decision maker (or of a group of decision makers) and of the feasible combinations of its different features. While the notion of a representation can evoke a mentalistic stance (Craik 1943, Johnson Laird 1983), a representation can also be a social artefact, embedded in some shared linguistic or material support (Hutchins 1995). As we have already said, in this paper we will be mostly concerned with the *structural* aspects of such representations - we will thus abstain from debating in detail the cognitive nature of strategists’ mental models, or whether representations are mental or material, individual or collective.

Our approach departs from the tradition in the strategy literature in that we try to capture fundamental common structural elements of different strategic representations, and express them in a language which is both general - it applies to different types of representations and enables to relate them - and expressive - it allows to describe in fine grain specific instances of representation. Despite being quite abstract (or rather due to its abstraction) the language allows to bring mathematical thinking into domains that have traditionally been considered quite resistant to formalization. At the same time, we think that our approach has a potential to go beyond unifying representations and providing powerful descriptive tools. It also suggests how strategy representations can be manipulated and transformed to generate new ones and disclose new action possibilities.

Our utilization of mathematical structures in strategy science is not only technically, but also conceptually novel. Modern mathematics consists of three different domains or research lines. The first is the quantitative one, as represented by analysis. In this domain, one may for instance derive a precise numerical value for a certain quantity by solving a differential equation governing the time course of that quantity. Or one may derive an expectation value by applying a sophisticated statistical scheme. The second is the algorithmic aspect, building upon techniques and concepts from algebra and embodied, for instance, in computer programming. Here, we utilize neither the quantitative nor the algorithmic aspect, but rather the third, the structural or qualitative one, as currently represented in particular by the domain of geometry. While the fields of geometry, like topology, constitute some of the most vibrant fields of contemporary mathematical research, their enormous potential for applications outside of mathematics remains largely unexplored. Modern geometry has vastly transcended the investigation of two- or three-dimensional spaces and rather develops abstract tools for a qualitative investigation and understanding of all kinds of high-dimensional, complicated or abstract configurations. And business schemes may involve many variables, that is, many dimensions, they may link the values of those variables in an intricate, that is, potentially complicated, manner, and their analysis and development might wish to abstract from those details of a specific business plan that do not reveal a general pattern. Therefore, modern geometry could offer useful tools for elucidating general structures of strategy across specific domains. In the present paper, we want to provide such tools and show their usefulness for understanding strategic representations.

We proceed by steps. First, we introduce the basic "Cartesian view" that underlies the space in which classical strategic representations are couched . We argue that such view fails to capture the fundamental correlations that provide structure to such space, and that it further falls prey to a 'curse of dimensionality'. We introduce a richer geometric structure, based on bundles and (pre)sheaves, and introduce on its ground a definition of local and global strategic coherence. We further define "change operators" that allow to manipulate strategic representation in order to allow new coherent strategic opportunities to emerge. This enables us to introduce more rigorous concepts of amalgamation and structure-preserving mapping that formalize important aspects of conceptual combination and analogy in strategic decision making.

A detailed reconstruction of the structure of Apple's 'digital hub' concept provides a concrete application of such formal tools. The 'digital hub' example further allows us to look at how an original strategy concept can be extended and generalized to new businesses.

Beyond its main goal, which is the definition of a formal language for strategic representations and their "grammar of transformation", the paper demonstrates the fruitfulness of its approach by providing a series of contributions to the strategy field: first, we offer a notion of strategic coherence that is compatible with - but also extends beyond - the notion offered by Porter and Siggelkow (2008). We express not only the strategic coherence in a set of choices, but also provide rigorous expressions of how these constraints are preserved if we extend a strategy to new domains or reorganize its structure. Second, the concept of amalgamation of strategic domains provides a clear and rigorous definition of a major source of complementarities, filling an important gap in the literature. Third, we offer a constructive approach to analogies in strategy-making. Using the concept of commutative diagrams and structure-preserving mappings, our approach allows to shift the focus from the detection of similarity between domains to the construction of similarity, i.e. how analogy can be used to construct new strategic concepts by using pre-exisiting strategic concepts as structural moulds.

THE CARTESIAN STRATEGIST

Since the early years of strategic thinking, the basic concepts of strategy have been couched in a language with a very strong spatial orientation: positioning, target, landscapes, etc. These concepts presume an underlying space within which firms can position themselves, exert their strengths, etc. This space is usually made of ”qualities” or ”domains” that define the support of the strategy representations. A strong Cartesian imprinting characterizes the way these spaces are represented, i.e. as ”product spaces” of independent dimensions. Specific firms or business units can be represented as points in such space, while regions of such space can be used to capture categories of strategic positions, such as cash cows or strategic groups. Many examples from early strategy research suggest that the construction of a Cartesian space, within which different businesses can be represented and compared, is a major aspect of strategy (it defines its ”scope”, in Hofer and Schendel's 1978 terminology). The so-called portfolio matrices, such as the BCG matrix, provide a canonical example of such spaces. The BCG matrix (compare figure 1) is a three dimensional space - the Cartesian product of industry growth, relative market share, and strategic business unit (SBU) sales) -visually packed into two-dimensions by representing the third dimension (each SBU's sales) by the diameter of the circles. Categories of SBUs are quadrants of the square.

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The canonical BCG example also suggests the ”curse of dimensionality” implied by such kind of Cartesian representations of the space of strategy: the need to compress such representations in two or maximally three dimensions (representations that allow to derive three dimensions from two, such as the BCG matrix, have been labeled as ”2 1/2 sketches” in theories of vision: Marr 1982). While these 2D Cartesian representations match remarkably our visual processing abilities, they obviously reduce enormously our ability to represent multi-dimensional concepts.

Usually, the geometry of the strategy space is enriched with a more complex structure than its pure Cartesian one – typically that one of a force field within which the forces exerted by firms on their competitive environment can be represented. Indeed, forces are another fundamental conceptual building block of classical strategy representations, from military maps to Porter's ”five forces”, and they are usually represented through geometric objects such as vectors or arrows. A good example is the notion of strategic positioning (Hofer and Schendel, 1978; Porter 1980) - the location of a business in a competitive domain where the forces of competition interact with the forces exerted by the firms resources - extending to the domain of business the classical definition of general Ulysses Grant: ”Strategy is the deployment of one's resources in a manner which is more likely to defeat the enemy” (from Mintzberg 1987). Even outside the classical positioning framework, vectors are also commonly used to express motion and directionality in the strategy domains, as in Mintzberg's (1985) ”emergent strategies” concept.

The strategy Cartesian space, even when enriched with force vectors, hardly captures the rich structure of strategy representations, as we will see. Still it provides a clear base space for strategic thinking, as witnessed by the huge success of such type of representations in both academia and practice. Furthermore, sometimes, relevant strategic innovations can arise from reconsidering such dimensions and restructuring the representation space. Thus, the Cartesian perspective suggests some very simple ways to manipulate strategic representations. For example, a very popular book on strategic innovation suggests that the fundamental strategy challenge is 'creating uncontested market space that make competition irrelevant' (Kim and Mauborgne, 2005, p.x). In order to create such space, strategists have to consider and represent the different 'factors' of competition (dimensions in our terminology). A firm's position is a point in such space of 'factors'. In order to create new market spaces, firms have to change the space into which they search a position, instead of just moving their point in the prevailing representation of the strategy space. This implies deleting some dimensions, adding new ones, and changing the relative weight of pre-existing dimensions. For example, the Cirque du Soleil has innovated competition in the declining circus industry by doing away some factors and introducing new ones. So animal shows, costly and increasingly targeted by animalists, were removed, while new dimensions such as a story-line inherited from theatre were introduced. This allowed to shift from children audience to an adult one (Kim and Mauborgne, 2005). In a different vein but making a related argument, Gavetti and Levinthal (2000) represent limited strategic cognition in NK-landscapes as lower dimensional representation of a multidimensional fitness landscape – and analyze the advantages as well as the losses that can derive from shifts in the dimensions of the representation. Similarly, Csaszar and Levinthal (2013) equate managers' mental representations to sets of dimensions, and discuss the relative advantages of modifying such set of dimensions versus exploring new solutions within a given space. In a broader context, many recognize that "the breakthroughs built into Porter's framework was that it emphasized 'extended competition' for value rather than just competition between existing rivals" (Ghemawat 2002) - a large shift in the dimensionality of the representation of competition.

BEYOND THE CARTESIAN VIEW: INTRODUCING CORRELATIONS IN STRATEGY SPACES

While the Cartesian representation of a strategy space with its independent dimensions is an important structural component of a strategy representation, its simple structure is not sufficient to characterize the latter. A strategy representation is far more than the product space of its conceptual dimensions. A second fundamental component of strategy representations captures correlations and constraints among such dimensions - how values in one dimension relate to values in other dimensions.

The emphasis on the "linkages" across the different dimensions of a strategy space is recurrent in strategic discourse. For example, Kim and Mauborgne's "strategy canvas" (see figure 2) introduces multiple strategic factors together with value curves that capture values on each of these factors.

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Taking into account such relations across dimensions requires introducing some new mathematical language, that one of bundles and presheaves. Despite its apparent exotic nature, this language really captures fundamental intuitions about strategic thinking. We thus have to ask the reader for some patience in dealing with the following definitions, which we try to keep as intuitive and simple as possible.

First of all, we have to introduce some structure in the set of dimensions which constitute a strategy space. The first notion that we need is that one of a ”bundle”. (The notions introduced here are illustrated in figure 3.) A bundle is basically a smart way to represent a space of multiple features or domains. From now onwards, we will use the term ”observable” to denote each dimension, feature, attribute or quality which is bundled. Just like a product can be imagined as a ”bundle of characteristics” (Lancaster 1966), a strategy representation can be characterized by its bundle of observables. The bundle is made of two components: a base space that indexes the different observables, and a set of observables, which are containers of values (e.g. real numbers if an observable can be represented by a line, but also discrete values if an observable represents a domain of discrete entities). The fact that each observable can have a different internal structure gives the bundle concept a huge representational flexibility. The ”Cartesian view” of the strategy space can be considered as a special case of this more general structure.

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Let us look first at the *base space*, the space whose points are the ”indexes” of the different observables. In general, such a space does not need to be a traditional Euclidean space. Indeed, it does not need to be a metric space at all. It is sufficient for our purpose to assume that it is endowed with the inclusion order between subsets of points. We will see later why considering such a relational structure over subsets will be very important to define concepts such as ”strategic coherence”.

To each observable index we may associate a set of values (once more, think of the different values of each factor in the strategy canvas). As already mentioned, some of these sets of values can have some internal structure (such as a complete order) but they can also be just mere collections of elements with no internal structure. Once more, the concept is very flexible and general. The collection of all observables is often called the *global space*. We can now assemble the global space and the base space by representing how the global space of observables S maps onto the index base space I (the mapping π is a *projection*).

The structure resulting from the mapping of the set of observables onto the base space is called a *bundle*. The concept of a bundle is the first new concept that we add to the representation of the strategy conceptual space, so it is worth spending some more time on it. Bundles are ubiquitous conceptual structures in management and in strategy in particular, and they are already more familiar to the reader than the mathematical language may suggest. We have already mentioned the way observables are represented in the strategy canvas of Kim and Mauborgne (see fig. 2). Another particularly important example of a bundle is the value chain (Porter 1985). The value chain consists of a base space of discrete activity types, to each of which a set of specific activities and functions is associated, from which competitive advantage may arise. For example, to the activity type ”outbound logistics” correspond activities such as ”finished goods warehousing, material handling, delivery vehicle operations, order processing, and scheduling” (Porter 1985). In general, all activities of an SBU are expected to be mapped onto the value chain basic types. Thus, a value chain is nothing but a bundle, with the important qualification that observables’ elements are functions (activities) rather than single values of a variable, as in the strategy canvas example. This is an important qualification, as it shows an important point: that observables can be more complex structures than point sets – in this case they can be sets of functions. One can also say that a value chain projects the global space of a Business Unit's activities onto a base space of few activity types – which shows that a bundle is a kind of abstract sorting structure. The value chain example also shows that the base space may have a more complex structure than just that one induced by the inclusion order: in this case some additional sequential order in the internal partitions of classes of activities. How value chains can be understood as bundles of activities, which is illustrated in figure 4.

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What makes bundles really useful, however, is that they provide a structure upon which it is natural to represent how different observables are related. To stay with the same examples, in a strategy canvas value curves capture how a firm or its competitors combine the different competitive factors that make the canvas. In a value chain, the fundamental problem is to analyze the linkages across different activities types – how the single functions performed in an activity type interact and interlock with other functions in other activity types.

In geometric terminology, such a horizontal linkage across selected observables is called a *local section*.[1] In a supermarket, there is an implication between a central location and the cost of space, that is, a central location implies a constraint on available space. This link is expressed by a local section that combines the value 'central' for location and 'expensive' or 'constrained' for available space. More interestingly, the concept of a shop clerk as the sole person with whom a customer interacts combines the values 'handled by the clerk' for product handling, 'cash over the counter' for payment, 'costly' for the time of the clerk paid as an employee with a fixed salary, 'information through personal discussion' for the mode of information acquisition by the customer, 'politeness and good manners, product knowledge, etc.' as the qualifications required for the clerk, and so on. In that sense, the concept of the clerk yields a local section combining the values of various observables. Porter (1985, 1996) provides many examples of local sections in the value chain. For example, better scheduling are associated to reduced vehicle time, or procurement processes may affect manufacturing quality and cost. No-frills airlines provide clear examples of local sections. Point-to-point routes with no connections allow more frequent flies in a day and thus correlate with higher aircraft utilization. On-line check-in and limited ground services allow leaner, more productive ground and gate crews. Of course there will be in general multiple local sections in a bundle, corresponding to multiple linkages among different values of each observable. So, for example, within same industry, multiple "activity systems" can coexist (Porter 1996; Siggelkow 2002). The flight industry, with its coexistence of no frills carriers, low fare/high value carriers, value carriers etc. is once more a natural example.

FROM LOCAL TO GLOBAL COHERENCE

While linkages can be locally compatible, they may fail to do so globally. As one shifts focus from a set of observables to another one, incompatibilities may emerge – linkages will disagree on relevant intersections of different subsets of the base space. This is so because we tend to

[1] In this paper, we introduce deterministic local sections, and we assume that they can be determined piecemeal. Introducing additional structure, it is possible to model probabilistic sections as well as local sections that can only be jointly present, e.g. because one cannot exist without the other one. For the sake of simplicity, we avoid here such technical complications.

perceive and conceive interactions between the values of different observables only a few at a time (just like we don't see objects from all sides simultaneously). As these smaller sets of observables are integrated, compatibility may be lost. In other words, locally compatible sets of linkages do not necessarily aggregate into larger compatible sets of linkages. So, for example, if one considers together small airports and frequent flights they are compatible. Small airports and the coordination of connected flights are compatible as well. But small airports, the coordination of connected flights and frequent flights are not compatible. The reverse, however, is different: if small airports, frequents flights and and short routes are compatible, then any pairwise subset of these elements will be compatible.

In order to make the concept of coherence more precise we introduce more formally the definition of a presheaf. Presheaves are exactly those structure that allow to express the interaction of local compatibility and global coherence, leaving room to overall incoherent sets of linkages.

Formally, we write FO for the sections over a base space O. When this base space consists solely of a single element o, then Fo simply is the observable over o, that is, the set of all the values that the observable that o stands for can assume. When O contains more than one element, then FO also reflects the compatibility conditions: As just explained, not every combination of the values for the different observables in O is feasible or compatible, and FO only contains the compatible value combinations. The larger O becomes, the more constraints for compatibility may arise. Therefore, it may not be possible to extend every local section over a subset $O' \subset O$ to a section over all of O. However, when in turn we have a section in FO, that is, it meets all constraints over the set O, then it will also meet all the constraints over any subset $O' \subset O$, and therefore restrict to a local section in FO'. In mathematical terminology, this is the definition of a *presheaf* FO of sections over a base O.

A somewhat different, and slightly more formal, but equivalent formulation of the preceding runs as follows. We consider observables $o, o', o'', \ldots$. Each observable o has a value range Fo. The possible values here could be discrete, e.g. binary ('present' vs. 'not present', 'small' vs. 'large', etc.), or continuous, like the outcomes of some measurement. In most cases of interest for us, they can assume only finitely many values. For instance the screen size of a computer is restricted by industry standards to a small number of possible values. Or the pixel resolution of a camera is discrete anyway. An object is characterized by its observables. For a computer, the observables would be storage capacity, processing speed, screen size, etc. In order to specify a realization of an object, all these observables need to attain specific values. Typically, not all value combinations for the different observables are compatible with each other, however. A coherent collection of values, one for each observable, then constitutes a global section (henceforth section tout court). The concept of a presheaf then stipulates the following condition: Over each set O of observables, we have a set of sections s.t. whenever $O' \subset O$ and $f \in FO$, then the restriction $f|O'$ of f to O' is in FO'. This simply expresses the fact that when we only look at a smaller collection of observables, the compatibility condition becomes easier to satisfy. Conversely, however, not every section $f' \in FO'$ can be extended to some section $f \in FO$. Thus, when we take an additional observable into account, not all previously coherent collections of observable values find a value of the new observable with which they are all compatible. As a special case, when we consider a single observable $o \in O$, the possible values of the observable o form the sections Fo. According to the preceding, not every element $\phi \in Fo$ admits an extension to a section over all of O, that is, to an element of FO.

Henceforth, we shall consider sets $O_1, O_2, O_3, \ldots$ of observables. Each such set can comprise one or several elements. The possible value combinations for the observables in a set O are denoted by FO.

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In figure 5 (a) we can see that the linkages between all three observables agree. In figure 5 (b), by contrast, we can see that the only possibility to link the left observable with the the right observable is to link the top dot in the left observable with the top dot in the right observable. This is not consistent, however, with the possibilities to link the left observable with the middle observable, and the right observable and the middle observable.

Porter's (1996) analysis of the failure of Continental Airlines to maintain a global coherence in its strategy offers a nice example. The attempt to maintain full service on some routes while introducing elements of the no-frill strategy produced all kind of incompatibilities. Thus, frequent flyer programs are compatible with full service but increase costs. Maintaining seat reservations is incompatible with reduced ground crews and may slow down boarding times. Many incompatibilities may creep in once local sections that are locally compatible have to be glued in a global picture.

The language of bundles and sections thus provides a simple criterion for defining the coherence of a strategy. A coherent strategy requires that local sections can be glued in a coherent global section. As discussed already, such a patching together of compatible local sections into a global one can find a formal expression within the mathematical concept of a *presheaf*. Formally, the requirement is that given two overlapping subsets of points in the base space (e.g. two overlapping sets of indices) the corresponding local section sets will agree on their intersection. Thus, in a presheaf, we have the set of compatible values of the different observables of a strategy representation, and we can check whether they can be patched together to yield globally consistent strategies.

Given bounded rationality, a representation may not necessarily include all feasible coherent sections. In some cases, these may be just combinations of values of the observables that have been unnoticed, maybe because they rely on correlations distant from those commonly observed. Discovering new coherent sections is a primary motor of innovation. For example, Zara operates by very quickly copying and producing new fashion designs that appear in public media, like the new dress of a movie star at an award ceremony. Zara consequently has a very frequent turnover of the clothes that it offers for sale in its stores. Zara realized two important links here. First, by quickly reproducing designs shown in public media, it needs only little or no advertisement of its own. The public media do this for Zara. Secondly, through a high turnover rate of what is offered in the stores, it induces more frequent visits of its customers that are curious to check what is new. This link between the frequency of display changes and of customer visits had apparently not been seen, or at least not been exploited before. In the Zara example, the issue of speed, that is, the reconceptualization of lead time, induces several new constraints between other observables, like the vertical organization of the supply chain or the design process, and the relations between the values of other variables like frequency of customer visits and advertising budget offered new possibilities. A new business model can thus consist in the systematic change of the values of several observables, but needs to respect new links and relations to be coherent. This idea resonates with Casadesus-Masanell and Ricart's (2007) ideas about competition through business models, and conception of business model interactions.

Our framework can easily include economic dimensions, such as revenues, market share, or profits. This can be done by literally include these dimensions as observables in the formal framework. The compatibility constraints expressed by local and global sections then implicitly describe the economic evaluation of different sections. Take the example of convenience shops. A local section describing convenience shops, as discussed, runs through 'handled by the clerk' for product handling, 'cash over the counter' for payment, 'costly' for the time of the clerk paid as an employee with a fixed salary, 'information through personal discussion' for the mode of information acquisition by the customer. However, this local section may not extend to positive values of the observable 'profit'. Competition by local supermarkets can drive the profitability of convenience shops down and finally force them to leave the market. If we add yet other observables, however, this can change. For instance, most supermarkets in big German cities such as Berlin close at 10pm. As there are enough consumers in German urban neighborhoods that are willing to pay premium prices if they can shop at night, convenience stores are profitable in various neighborhoods. Thus, if we add the observable, 'opening hours', the section describing German supermarkets may not run through night hours, but the section describing German convenience stores does. The section describing German convenience stores then runs through positive values of the observable 'profits'. If this section would not run through the value 'late opening hours', it couldn't be extended to a positive value of the observable 'profits'.

Of course, structures as general and abstract as bundles or presheaves may not be able to express all the dependencies, correlations and constraints that one might encounter in real business situations. Nevertheless, the framework that we have developed is flexible enough to permit modifications or extensions that can readily incorporate more complex kinds of dependencies. One such type that occurs quite frequently is when the possibility of some local section depends on the presence of a local section over a region somewhere else in the base space. That means simply that something depends on certain preconditions, like a sufficient amount of capital or suitable machine tools. In order to incorporate this without violating the presheaf condition, we can simply restrict the subsets of the base space for which we require that condition. More precisely, we declare a certain region U_0 in O as the *core set*; for instance, it could contain the observable "capital stock". We then consider only those subsets $U \subset O$ that contain the core U_0. Every local section over such a U then has to go through a sufficiently high value of the observable "capital stock". U_0 could, of course, also contain other observables, for instance corresponding to suitable machines or technical devices, and their values might have to be connected by suitable local sections. In any case, here we do not want to elaborate upon this point, but simply point out that our formal framework permits extension that can readily incorporate those and other phenomena.

CHANGE OPERATIONS

The framework we have shortly sketched does more than characterizing strategy representations and providing a clear notion of strategic coherence. It can also offer the definition of operators that can be used to manipulate such representations, generating new ones and supporting the discovery of new strategies.

The idea that representations can be manipulated in systematic, structured ways to generate new opportunities has repeatedly surfaced in the recent literature on strategy and decision making (Kim and Mauborgne 2005, Osterwalder and Pigneur 2010, Gavetti 2012, Csazar and Levinthal 2013). However, such attempts rely on an informal characterization of representation structures (as in Kim and Mauborgne 2005) , or reduce representations to Cartesian products of basic dimensions (as in Csazar and Levinthal 2013). In both cases, this limits the possibility to fully capture the generative potential of manipulating representations, as well as the structural constraints on such manipulations. In what follows, we define a few basic ”change operators” that underlie such process.

Stretching More generally, the discovery of new sections within a given representation may require the broadening of the range of some observables. New values in some observables may enable new coherent combinations of other values of other observables. Technological innovation may support such innovations. For example, progress in lithium batteries may enable new combinations of computing power and light weight in laptops. Stretching the time range of food preservation (e.g. via refrigeration) has enabled formidable innovation in the distribution and product strategies in the food industry. In general, stretching is the outcome of explorative activities that extend the standard limits of common observables, but it can be occasionally the result of serendipity.

Lifting Sometimes innovative strategies can emerge by reducing the dimensionality of a strategic representation - by lifting some observables. The discovery of new strategies can result from the fact that by lifting dimensions, constraints are removed as well, and new coherent sections can emerge. Dell’s decision to focus on the assembly process only has removed all the constraints arising from having to take care of other phases of the production process of PCs, generating freedom to organize the assembly process in more efficient and flexible ways. Strategic outsourcing is designed to lift companies from the burden of complexity arising form too many interdependencies and help them focus on few key linkages among ”core” dimensions (Rothermael et al. 2006).

Extending Enriching the dimensionality of representations can be the key to finding new strategies. This seems to be for example the driving message of the ”blue ocean” idea (Kim and Mauborgne, 2005). When australian winemaker Casella Wines adds to low aging quality and low wine complexity the experience of ”drinking ease for everyone”, it looks at a larger space than the conventional wine features representation, and can add value and interest for new customers to a product that would be poorer under a narrower representation. Our framework allows however to qualify the nature of extending. Introducing new dimensions does not increase the number of consistent sections. Indeed, it introduces new constraints. If a section is consistent in an extended representation, it has to be consistent also in a narrower one. For example, a low aging quality and low wine complexity association is consistent even without considering its ”drinking experience” associations. Thus extending does not increase the number of consistent strategic combinations, it rather creates new, additional value for possibilities that would exist also in lower dimensionality but would be less valuable.

The essential conceptual moves underlying ’streching’, ’lifting’ and ’extending’ are depicted in figure 6 .

- - - INSERT FIGURE 6 ABOUT HERE - - -

Amalgamating Connecting separate but overlapping and compatible pre-sheaves is another fundamental strategy for generating new representations and opening new opportunities of coherent sections. Consider the way the value of a digital camcorder could be redefined by interactions with the PC. In a simplified representation, the camcorder has rich content capture possibilities (due to its portability and its interface with analog optical hardware) but poor image processing capabilities, due to its limited computing power and small screen. On the other side, the desktop PC has much higher computational capabilities and can afford a large screen, but has limited content capture possibilities due to its size and relative immobility. However, the two devices have an interface made possible by the digital language in which images can be stored and manipulated. As a result, connecting the two can produce an entirely different configuration of coherent features, that simultaneously leverages rich content capture and elevated processing capacity. This has been the revolution introduced by software businesses such as iMovie, that have allowed to redefine the interaction between computers and portable devices. In order to capture such examples we have to introduce a new formal notion, that one of amalgamation of two base spaces, implying a unification of their observables. We consider two presheaves F_1, F_2 with base sets O_1, O_2. We assume that they share one (or perhaps more, but this will not affect the idea) observable o, but that the possible value ranges $F_1 o, F_2 o$ in the two presheaves are different. For instance, $F_1 o$ might be larger than $F_2 o$. We then define the amalgamation of the two presheaves as the presheaf with base set $O_1 \cup O_2$, and with the value ranges of each observable $o \in O_1 \cap O_2$, that is, o occurs in both O_1 and O_2 as the screen size in the above example, being the union $F_1 o \cup F_2 o$ of the individual value ranges. Thus, the amalgamation of the PC and the videocam makes a larger screen size available for processing video data. And since, when as in our example $F_2 o$ is smaller than $F_1 o$, by this amalgamation the value range for $o \in O_2$ gets extended, then, as explained in the previous sections, new global sections could emerge. Amalgamation combines the virtues of stretching with the value-adding properties of extension. It constitutes a fundamental building block of the conceptual combination of two or more representations, as it allows to understand how two different structures can provide entirely new coherent sections and thus innovate the set of available opportunities. Of course these advantages do not come without new constraints: a point that we will illustrate more in depth in our subsequent case study.

Transferring structure In the case of amalgamation, the overlap between observables of two representations is the key architectural factor. In other cases, however, even if dimensions are different, similarities can be exploited to create connections between two representations. One example that has attracted much attention in the recent strategic literature is analogy. In analogy, a source representation is mapped into a target one, establishing two types of mappings: a mapping between the elements of the source representation and those of the target one, and a mapping between the structure that relates the elements within the source representation and the corresponding structure of the target one. While similarities can be just ”recognized” between existing representations, in some cases they can be constructively created by shaping a new target representation on basis of the source one. A classical example is the analogy created by the application of the so called razor and blade model to inkjet printers. In the original razor and blade source model of

Gillette, a durable good (the razor body) was sold at very low margins in order to favor its adoption by consumers. The razor body was associated to an element (the re-changeable blade) that due to its quick consumption cycle could be sold in large quantities. Once the razor was sold, consumer were locked in the use of Gillette blades. Blades were sold at high margins and were the source of profit, compensating the low margins of the razor. The analogical application of the model to inkjet printers is well known: a durable hardware (the printer) is sold at low prices but consumers are then locked in buying the high margins ink, which becomes the source of profit.

This kind of double mapping can be formalized through the notion of structure preserving maps (morphism). Given two sets of elements endowed with some internal structure (e.g. two representations) morphism transfer structure by creating a double mapping, one between the elements (e.g. the observables) of the two sets, the other one between their structural relations (e.g. between their sections).

For example, if O_A and O_B are the observables of two representations and α_A and α_B are their sections, the diagram in figure 7 expresses a structure preserving mappings between the two representations:

— INSERT FIGURE 7 about here. —

The expression on the left of the diagram says that the diagram "commutes", i.e. there is an equivalence between the two paths on the left and on the right of the expression. This means that if an element a_1 is related by α_A to a_2 in O_A, and b_1 is related by α_B to b_2 in O_A, then one can reach b_2 from a_1 either by mapping a_1 to b_1 via f and then mapping b_1 to b_2 via α_B , or by mapping a_1 to a_2 via α_A and the mapping a_2 to b_2 via f. As we will see in the following case study, the constructive use of such structure preserving mappings is a fundamental operation to generalize successful strategic representations by generating new businesses.

The presheaf property that we introduced above is a general property of our model. It underlies all change operations and expresses how constraints are preserved while strategies are reorganized.

A CASE STUDY: THE DIGITAL HUB

In one of his most famous keynote presentations, on January 9, 2001, Steve Jobs delineated a strategic concept, the digital hub, that proved fundamental for the spectacular ascent of Apple to prominence in the market for digital devices. In contrast with a dominant trend towards considering the PC waning, Jobs drew a vision of an explosion of a world of devices rotating around the PC, making it the "digital hub" of the emerging digital lifestyle. Due to its computational power, large and inexpensive storage, big screen, connectivity to the internet, the PC could add value to digital devices and interconnect them. A graphical depiction of the digital hub, taken from Steve Job's own presentation, is reproduced in figure 8. In what follows, we will analyze the conceptual structure underlying the digital hub using the concepts and formal tools we introduced above. Our analysis is based on Jobs'(2001) presentation and is an attempt to capture formally key aspects of such remarkable enunciation of a novel strategic vision.

- - - INSERT FIGURE 8 ABOUT HERE - - -

Amalgamation

The seed of the digital hub concept, in the words of Steve Jobs, can be found in the creation of the iMovie 2 application, that multiplied the functional possibilities associated to a camcorder and made apparent the potential advantages of integrating the Mac and mobile devices. We will thus start from representing the conceptual moves underlying the 'iMovie' innovation using our formal language. As we shall see, the concept of amalgamation will play here the key role. We understand the digital hub to be composed of different types of objects, that essentially correspond to the PC-hub and peripheral devices in Jobs' own presentation (figure 8). Figure 10 summarizes how we formalize objects of the digital hub and their relations. This figure will be explained in greater detail below. In what follows we will use the term "object" to refer to a subset of a base space that contains a set of observables.

We consider two presheaves F_1, F_2 with base sets O_1, O_2. O_1 represents the observables of the object 'PC' that matter for connecting it with video devices. O_2 represents the object camcorder ('V') and its observables. Figure 9 describes the observables in both 'PC' and 'Camcorder' (indicated by black labels). Concretely, PC contains the observables 'content transfer', 'screen size', and 'computing power'. Camcorder contains the observables 'mobility', 'screen size', and 'possibilities to edit content'. The observable 'screen size' appears in both PC and Camcorder and can be amalgamated when we consider the possibilities of PC and Camcorder together.

A PC of course permits a larger screen size than a Camcorder. We then define the amalgamation of the two presheaves describing the possibilities of the objects as the presheaf with base set $O_1 \cup O_2$, and with the value ranges of each observable $o \in O_1 \cap O_2$, that is, o occurs in both O_1 and O_2 as the screen size in the above example, being the union $F_1 o \cup F_2 o$ of the individual value ranges. Thus, the amalgamation of the PC and the Camcorder makes a larger screen size available for processing video data. And since, when as in our example $F_2\ screensize$ is smaller than $F_1\ screensize$, by this amalgamation the value range for $o \in O_2$ gets extended, then, as explained above, new global sections can emerge. In our example, the emerging global section expresses a coherent way to extend the possibilities to edit content captured by a camcorder.

- - - INSERT FIGURE 9 ABOUT HERE. - - -

Figure 9 illustrates the principle of amalgamation by combining objects, and how this affects the constraints of those objects. Consider the local section over O_2 in figure 9. The observable 'screen size' constraints the admissible values of the observable 'possibilities to edit content'. The section that runs through all three observables of O_2 does not run through 'quick and easy editing'. However, there is a local section that connects the observables 'mobility' and 'possibilities to edit content' which runs through 'quick and easy editing'. Local sections that don't extend to global sections are expressed by dotted blue lines. The possibility of quick and easy editing is given in principle, it is just ruled out by the constraints imposed by the observable 'screen size'. By means of amalgamation of 'screen size' in PC and Camcorder this constraint can be overcome. In $FO_1 \cup O_2$ the global section can now run through 'quick and easy editing'.

We now describe this example in formal terms. The section over O_1 in FO_1 is described as follows (it contains just one section here, but of course could contain more):

$$FO_1 = \{(\begin{array}{c}\text{Internet and Firewire}\\ \text{large screen size}\\ \text{large computing power}\end{array})\} \tag{1}$$

FO_2 is described as follows:

$$FO_2 = \{(\begin{array}{c}\text{high mobility}\\ \text{small screen size}\\ \text{difficult and inconvenient editing}\end{array})\} \tag{2}$$

These formal definitions correspond to the blue sections in figure 9.

We here also describe the presheaf over the two observables 'mobility' (which we denote o_2) and 'possibilities to edit content' (which we denote o_3). An available local section expresses that easy possibilities to edit content are available in principle; they are just constrained by the screen size:

$$F\{o_2, o_3\} = \{(\begin{array}{c}\text{high mobility}\\ \text{quick and easy editing}\end{array})\}$$

After amalgamation, we get the presheaf with base $O_1 \cup O_2$. Note that here those observables that are common to both O_1 and O_2 have been identified. In particular, there now is only a single observable screen size, with possible values *large* and *small*, whereas in O_2, only the value *small* had been permitted. Because of this fact, we now have a new section (we omit the combination of the original sections in 1 and 2 as they are no longer of interest):

$$FO_1 \cup O_2 \supset \tag{3}$$

$$\{(\begin{array}{c}\text{Internet and Firewire}\\ \text{large screen size}\\ \text{large computing power}\end{array}), (\begin{array}{c}\text{high mobility}\\ \sout{\text{small screen size}}\\ \text{quick and easy editing}\end{array})\}$$

In figure 9, we omitted the observable 'screen size' in Camcorder in $O_1 \cup O_2$, as this observable is amalgamated and already represented in PC in $O_1 \cup O_2$. Correspondingly, in 3, we crossed out the smaller value of the observable that has been amalgamated. Note that once value ranges are extended, it is not necessarily true that *any* new global section would be possible. The presheaf condition, which imposes a constraint preservation criterion, says that the section in $FO_1 \cup O_2$ needs to run through points of existing local sections in all dimensions whose value ranges have not been extended. As already discussed, in figure 9, the dotted line above O_2 indicates the local section $F\{o_2, o_3\}$. The section in $FO_1 \cup O_2$ runs through 'mobility is high' and 'quick and easy editing' just as the local section that we defined above runs through these values. Thus, the principles of constraint preservation and amalgamation are combined. Indeed, in any operation of amalgamation that leaves some dimensions as they were, the constraint preservation criterion also imposes constraints on possible emerging coherent value combinations.

Transferring structure

The digital hub is constructed via a series of analogies that expand the wheel and of integrations that glue together its pieces to the hub. (Figures 8 and 10 illustrate that the structure of the digital hub resembles a wheel.) For instance, 2001 was the year of the introduction of iTunes and the iPod. We will demonstrate that the iTunes strategy was structurally similar to the iMovie strategy and express this similarity formally. In our analysis, we again draw on examples from Job's (2001) presentation. To analyze the case formally, we first introduce some notation, which is summarized in figure 10.

- - - INSERT FIGURE 10 ABOUT HERE - - -

We denote sets of objects as $O_1, O_2, \ldots$. 'Sets of objects' contain objects of a strategy considered together. They can contain just one object (like 'camcorder') and its observables or a collection ('camcorder' and 'PC') and their observables.

$FO_1 \cup O_2$ contains the constraints of considering possibilities of PC and Camcorder ('V') together: $O_1 \cup O_2 = \{PC, V\}$. $FO'_1 \cup O'_2$ contains the constraints of considering possibilities of PC and Audio ('A') together: $O'_1 \cup O'_2 = \{PC, A\}$. $O_3 = O_1 \cup O'_1 \cup O_2 \cup O'_2$ contains the possibilities of the amalgams PC-Audio and PC-Video together: $O_3 = O_1 \cup O'_1 \cup O_2 \cup O'_2 = \{PC, V, A\}$. O'_1 contains the dimensions of PC that matter for connecting it with audio devices. O'_2 contains just the single object 'audio'. The definitions we make to analyze the structure underlying the digital hub are depicted in figure 10.

Structure preserving mappings allow us to express that constraints and possibilities that arise by linking observables in one domain apply to another domain as well. In this sense, structure preserving mapping express analogies between different areas of an overall strategy.

Consider the example of an analogy between the PC-Video and the PC-Audio spoke of the digital hub. By identifying specific values in the presheaf over the PC-Audio observables with specific values in the presheaf over the PC-Video observables, we can see that a pattern of constraints that characterizes the iMovie section also characterizes the iTunes section. Visually, this can be read from figure 11, by comparison with figure 9.

- - - INSERT FIGURE 11 ABOUT HERE - - -

As indicated in figure 11, there is one section contained in the presheaf over $FO'_1 \cup O'_2$ (we again omit the original sections in O'_1 and O'_2 as they are no longer of interest):

$$FO'_1 \cup O'_2 \supset \{(\begin{array}{c}\text{Burning CDs; Internet}\\ \text{storage capacity and screen size are large}\\ \text{computing power is large}\end{array}), \qquad (4)$$

$$(\begin{array}{c}\text{mobility is high}\\ \cancel{\text{storage capacity and screen size are small}}\\ \text{great flexibility to create playlists and easy search}\end{array})\}$$

By means of identifying observables in $O_1 \cup O_2$ with observables in $O'_1 \cup O'_2$, and values of observables in $O_1 \cup O_2$ with values of observables in $O'_1 \cup O'_2$, we can express that patterns of constraints among observables in $FO_1 \cup O_2$ match with patterns of constraints in $FO'_1 \cup O'_2$. In figure 12 we indicated which observables in $PC-iTunes$ are similar to observables in $PC-iMovie$. Moreover, we identify values of observables in $PC-iTunes$ with values of observables in $PC-iMovie$.

— INSERT FIGURE 12 ABOUT HERE —

Amalgamations in the iMovie spoke of the digital hub relate to amalgamations in the iTunes spoke of the digital hub. Above, we elaborated on the constraints in the iMovie spoke. For instance, it is a constraint for the local section in Camcorder in O_2 that 'possibilities to edit content' can only be filled with 'difficult and inconvenient editing', as the dimension 'screen size' takes the value 'small', and this constrains the value of 'possibilities to edit content'. By analogy, it is a constraint for the local section in MP3-Player in O'_2 that 'possibilities to edit and search' can only be filled with 'complicated and limited possibilities to create playlists', if the observable 'storage capacity and screen size' takes the value 'small'. However, if the constraint imposed by 'small storage capacity and screen size' is removed, the combination of the values 'high mobility' in music content and 'great flexibility to create playlists and easy search' in 'possibilities to edit and search' becomes admissible. That is expressed by the local section over 'mobility' and 'possibilities to edit and search' (blue dotted line), where the latter can take the value 'great flexibility to create playlists and easy search'. Thus, this possibility is in principle available, but the further dimension 'storage capacity and screen size' constrains it.

We will now express the deep match between constraints in the iMovie spoke and the iTunes spoke of the digital hub more formally. We express the deep match of constraints between the two domains using the commuting diagram 5 below. There is an important property of commuting diagrams, which is in fact the defining property of analogies. The whole diagram (compare diagram 5 below) expresses that there is a structural similarity between the constraints in a source domain (which here is the domain $O_1 \cup O_2$) and a target domain (which here is the domain $O'_1 \cup O'_2$): the patterns of constraints are similar. Just as the section in $O_1 \cup O_2$ in figure 9 and the section in $O'_1 \cup O'_2$ in 11 are visually similar, the structural constraints between different features in $FO_1 \cup O_2$ and $FO'_1 \cup O'_2$ are similar. One could derive information about a constraint in $O'_1 \cup O'_2$ by looking at an identified constraint in $O_1 \cup O_2$. Once observables and values in the source and target domain are identified (we did this in figure 12), the property of a commuting diagram tells us that we can derive information about the constraints among values in the target domain by looking at the source domain.

By identifying features and specific values in the presheaf over the PC-Audio observables with features and specific values in the presheaf over the PC-Video observables, we can see that a pattern of constraints that characterizes the iMovie section also characterizes the iTunes section. The analogy between the PC-Video spoke of the digital hub and the PC-Audio spoke of the digital hub is formally expressed in the following commuting diagram:

$$\begin{array}{ccc} O'_1 \cup O'_2 & \xrightarrow{h} & O_1 \cup O_2 \\ {\scriptstyle F'}\downarrow & & \downarrow{\scriptstyle F} \\ F'O'_1 \cup O'_2 & \xleftarrow{h^*} & FO_1 \cup O_2, \end{array} \tag{5}$$

This diagram describes in general terms how an analogy can be used to transfer structure from a well understood situation (base set O with subsets O_1, O_2, the Video setting in the example) to another one (base set O' with subsets O'_1, O'_2, the PC-Audio setting in the example) whose observables are similar to those of the former. In formal terms, we want to construct the presheaf F' from the presheaf F. The diagram explains how this works. We only need the map h, the upper horizontal arrow in the diagram, that identifies the observables from O' with those from O. In the case at hand, these identifications have been made in figure 12. Since F is already there, h induces the pullback map h^*, and we can therefore simply put

$$F' = h^* \circ F \circ h. \tag{6}$$

This identity is meant when we say that the diagram 5 commutes. It is important for the construction to work that the lower arrow, h^*, goes from right to left, that is, in the direction opposite to the arrow h. Figure 12 makes this concrete for the example at hand, that is, how the iMovie section yields the iTunes section (the realization of h^*), once the identification of the Audio with the Video observables has been made by the map h.

For instance, if there is a section in $FO_1 \cup O_2$, as expressed in equation 3 above, the lower horizontal arrow expresses that there must be a corresponding section in $O'_1 \cup O'_2$. Indeed, the corresponding section exists and is expressed above in equation 4.

The properties expressed by commuting diagrams run even deeper. As we map from a presheaf to another, we implicitly describe that local sections in the target domain also agree with local sections in the source domain. As discussed above, this is the case in our example, as the two amalgamations are similar. In sum, commuting diagrams express that two areas of a strategy share the deep structure of constraints. A target domain is set into relation with a source domain by the identification of similar observables (the upper arrow in 5), with the aim of transferring structure of the source domain to the target domain (the lower arrow in 5).

A strategist who wants to utilize commuting diagrams either to generate an analogy to extend a strategy to a new domain, or to check if two areas of a strategy are similar, needs to identify observables and values in source and target domain, just as we did in figure 12. If a commuting diagram can be constructed, this means that the deep structure of constraints in source and target domain agree with each other.

Transferring structure and preserving constraints

If constructive analogy drives the expansion of the whole ”digital hub” structure, one should be aware of the fact that by increasing the number of amalgams that are merged in the digital hub, one creates a richer web of constraints.

This implication can be again expressed more formally by diagrammatic reasoning (see fig. 13).

$O_1 \cup O_2$ is the first amalgam (PC and camcorder). $O'_1 \cup O'_2$ is the second one constructed by analogy (the PC-audio amalgam). O_3 is the union of both amalgams, $O_1 \cup O_2$ and $O'_1 \cup O'_2$. $O'_1 \cup O'_2$ is constructed on the $O_1 \cup O_2$ template (f1). O_3 (a simple hub-with two-spokes) is the union of $O_1 \cup O_2$ and $O'_1 \cup O'_2$, thus the arrows from $O_1 \cup O_2$ to O_3 and $O'_1 \cup O'_2$ to O_3 represent the inclusion relation. The arrow $g_1 : O_1 \cup O_2 \rightarrow FO_1 \cup O_2$ is the first amalgam presheaf, and so for g_2 and g_3.

Notice the inversion of the direction of arrows h_1 and h_2 with regards to the direction of the inclusion ones. This inversion of direction simply represents the firing of constraints from the overall structure on its components. We have changed the stroke of g_3, h_1 and h_2 to stress that while the analogy supported by f1 and the union of the two amalgams are actively constructed by the decision maker, those constraints fire automatically. Once $O_1 \cup O_2$ and $O'_1 \cup O'_2$ are merged into O_3, the extension of the set of features is reflected in increased constraints over sections - a basic property of presheaves. In other words, once the PC-camcorder and the PC-audio amalgams are united, such union implies stronger constraints on the coherence of both (this is reflected in the emphasis in Jobs’ talk on the role of the operating system, codecs, firewire etc. in solving the problems of integration of multiple devices with the PC). Of course, the same logic applies as the embryonic ”digital hub” of O_3 is further expanded to include new devices.

DISCUSSION

Our first aim was to introduce a qualitative mathematical language to express formally some fundamental concepts in strategy representation. We have introduced notions such as bundles, (pre)sheaves and structure-preserving mappings and have shown how they provide a potential for unifying different concepts in strategy. Such formal tools also allow to provide a simple and rigorous context within which the somehow elusive notion of strategic coherence (Hofer and Schendel 1978, Porter 1996, Siggelkow 2002 and 2011) can be rigorously defined. Furthermore, once a basic formal structure has been established, it has been natural to define on its ground operations that can manipulate it, providing a sort of elementary grammar of change in strategy representations. We suggest that as long as strategic breakthroughs rely on new representations, the operators we have defined are fundamental building blocks of the strategic innovation process. To demonstrate the productivity of our approach we tried to apply it to the reconstruction of a specific case, by modelling the structure of a well-known keynote address in which Steve Jobs sketched in 2001 the fundamental concept of the digital hub, that drove the Apple strategy and the emergence of new product categories in the following decade. Our example shows that it is possible to evolve the case study genre towards more formal, mathematically grounded discourse. Of course, this is not substituting a more narrative approach, but provides strong complementarities with it and extends the possibilities of a theoretical use of case studies. Our effort to establish a formal language for strategic representations has offered a couple of useful side results. The first one is related to the concept of amalgamation. Amalgamation of two structures (pre-sheaves) provides a simple and rigorous definition of a fundamental mechanism generating complementarity, by enabling sections that would be otherwise unfeasible if the two structures were kept separated (notice that such enabling property derives directly from the definition of a pre-sheaf). While complementarity plays a key role in contemporary strategy thinking (Ghemawat and Levinthal 2008, Milgrom and Roberts 1990), most models of complementarity focus on its effects rather than on what generates it. Making those generative mechanisms apparent is in our view a sign of productivity of our structural approach. A second side result is that our formal language contributes to clarifying some aspects related to the use of analogy in strategy making, a theme broadly debated in recent years. In particular, the tool of commutative diagrams and the associated notion of structure-preserving mapping allow to model in abstract terms the notion of a deep analogy based on structural similarity - one that transfers the internal structure, not just the surface features, of the source analogue. Furthermore, our notion of change operators allows to distinguish the pure detection of similarity between existing objects from the constructive generation of similarity by using the source structure as a template to generate a new object - in this case a new strategy representation. Finally, when the transferred structure is characterized as a pre-sheaf, we show how analogy generates a contravariant direction of the structure mapping, that introduces new constraints over feasible sections, as the new structure has to be integrated with the source one. In more concrete words, the new possibilities conceived via analogy backfire as new constraints once the new business has to be integrated into the existing ones.

While in this paper we have explicitly eschewed the cognitive aspects of strategy representations, we believe that the formal concepts that we introduced may provide significant building blocks to articulate cognitive models of strategic decision making. Such concepts make it possible to model in general terms strategic representations capturing their internal structure (and not just their dimensions). This should allow to characterize important aspects of strategic thinking. For example, the notion of a section allows to express the correlation structure of features, underlying many inferential processes in decision making. Furthermore we expect that the change operations we sketched may be fundamental building blocks for process models of strategic change and innovation, offering the fundamental components that are combined in the search for new representations and the discovery of new opportunities. For example, the process of generating the representation of new opportunities by amalgamation of pre-existing concepts may be actually drawn from a combination of the extension and the stretching operations. As another example, one might expect that the constraints arising form the use of constructive analogy might drive the actual search processes for coherent solutions in implementing the original analogical insight into a viable configuration of features.

Mathematics is not just about calculation and proofs, it is also about recognizing common patterns in different objects, and extracting their underlying formal structure. We hope that such an approach may help shaping more rigorous concepts in domains traditionally resistant to formalization, and may contribute to broaden the formal toolkit of the strategy field.

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Figure 1: **The BCG Matrix**

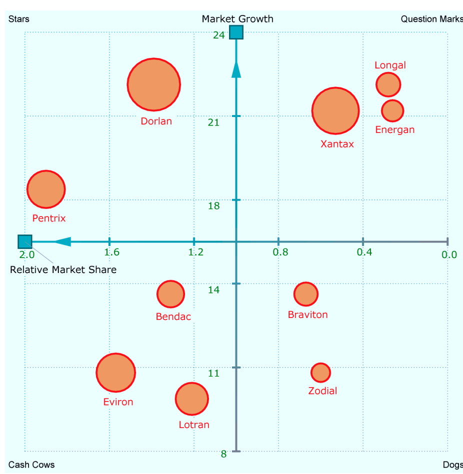

Figure 2: **The Strategy Canvas of the U.S. Wine Industry in the Late 1990s, from Kim and Mauborgne (2005)**

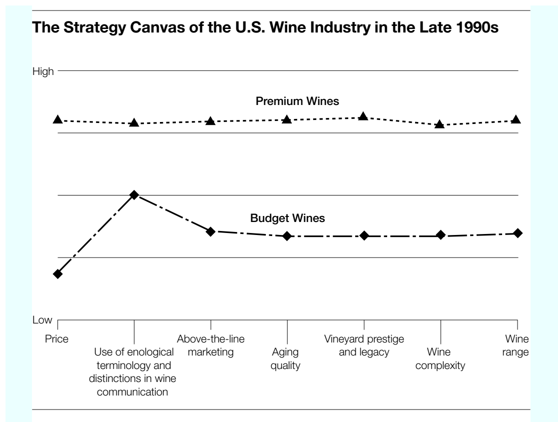

Figure 3: **A bundle**

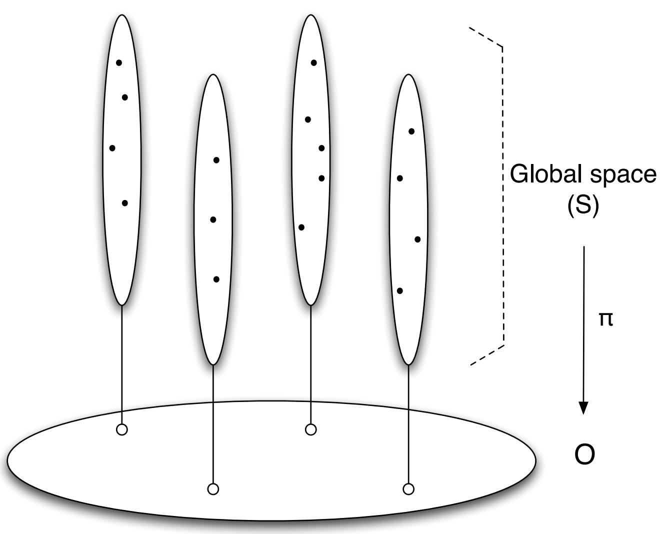

Figure 4: **The value chain as a bundle of activities**

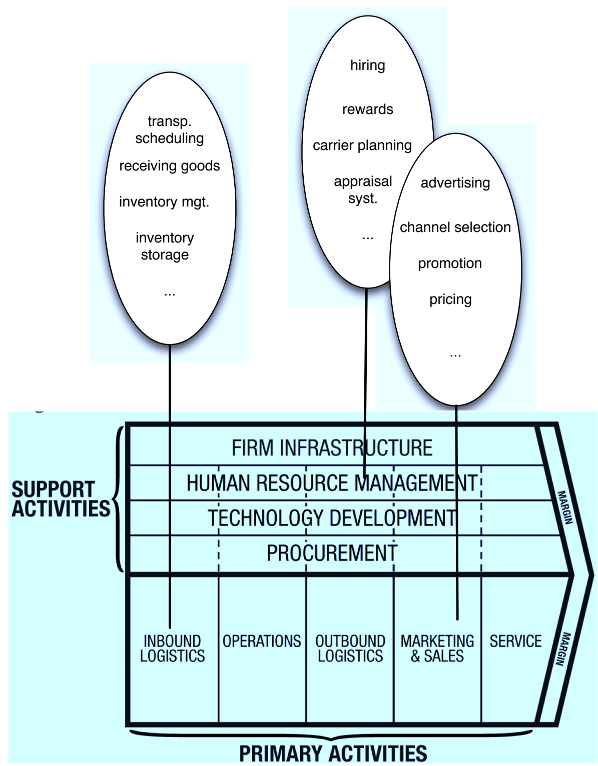

Figure 5: **a globally consistent section (a) and a globally inconsistent one (b)**

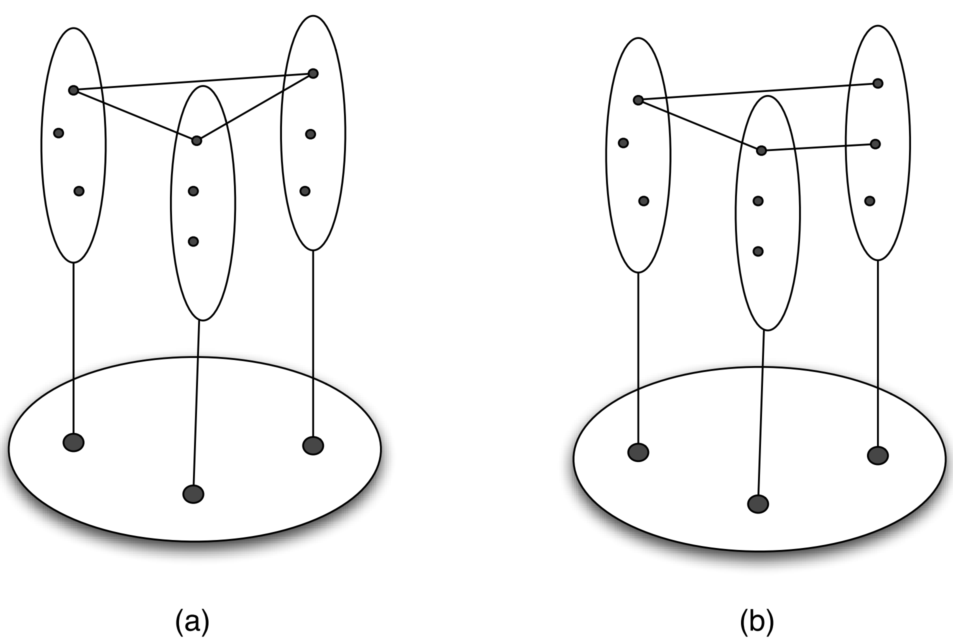

Figure 6: **Change operators**

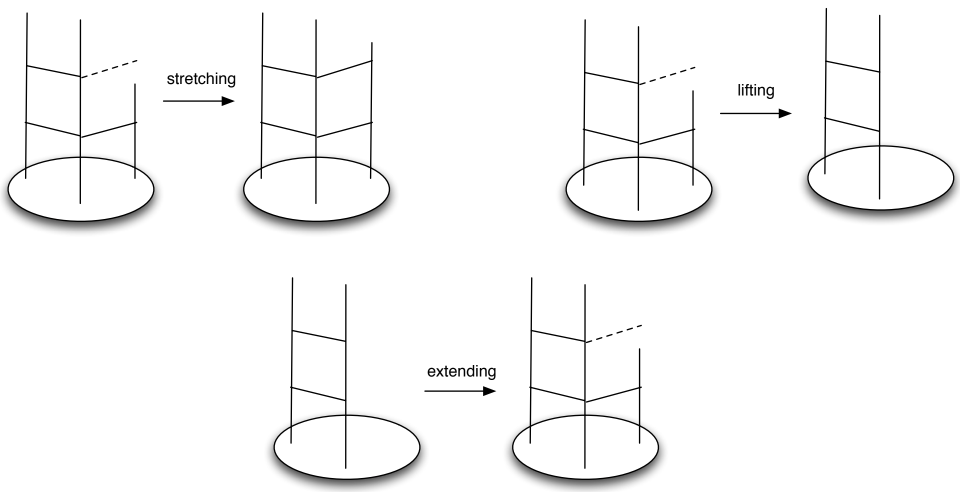

Figure 7: **Structure-preserving mapping**

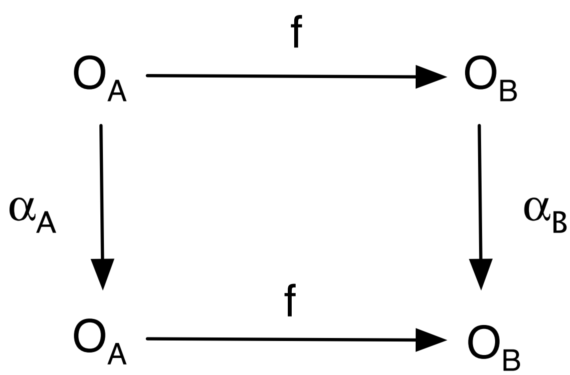

Figure 8: **Apple's "Digital hub" wheel**

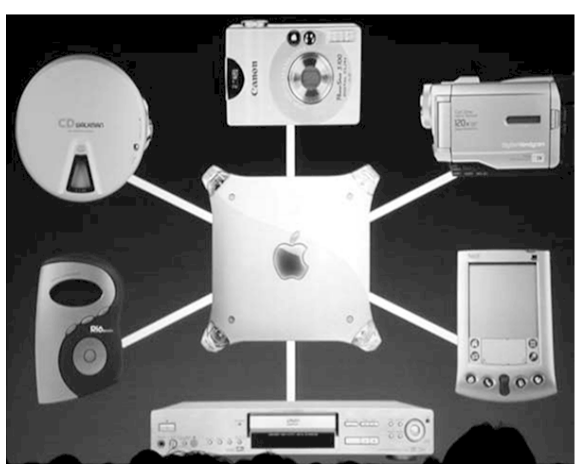

Figure 9: **Extending value ranges by amalgamation, and constraint preservation**

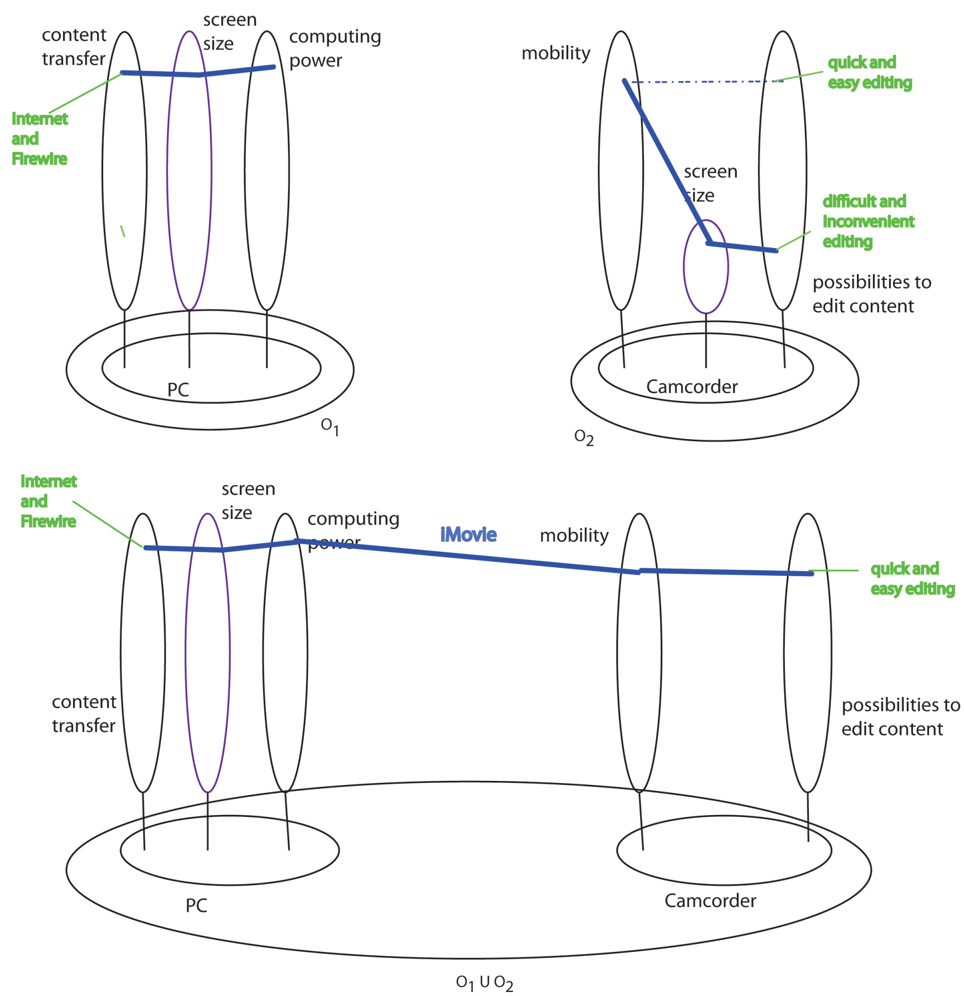

Figure 10: **The structure underlying Apple's "Digital hub" wheel**

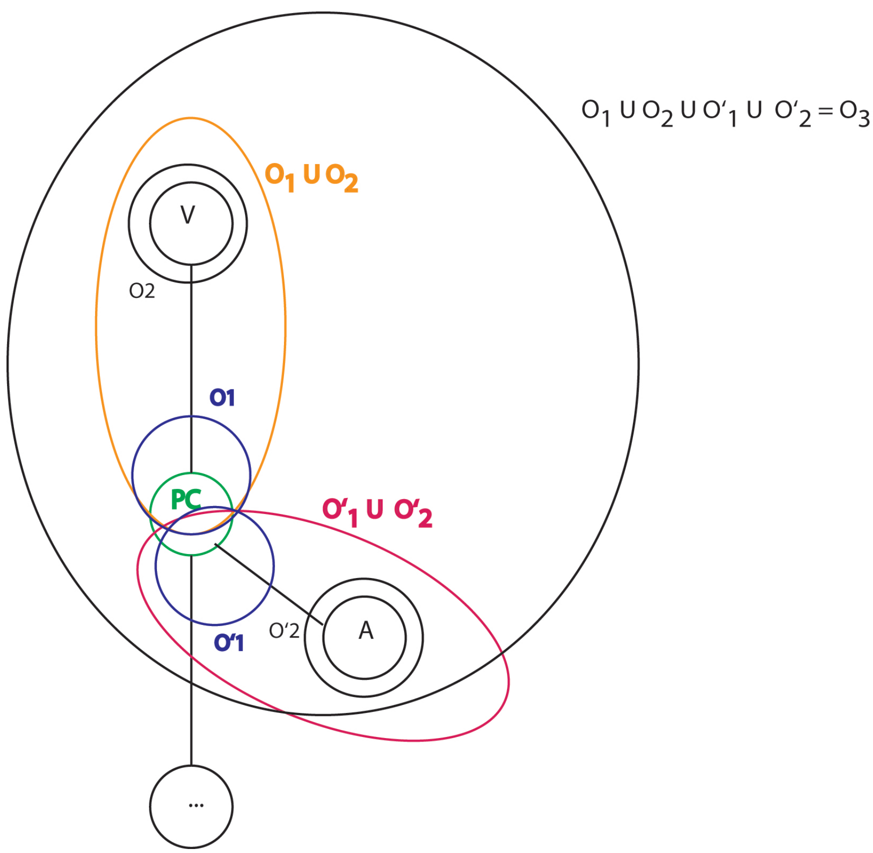

Figure 11: **The iTunes section**

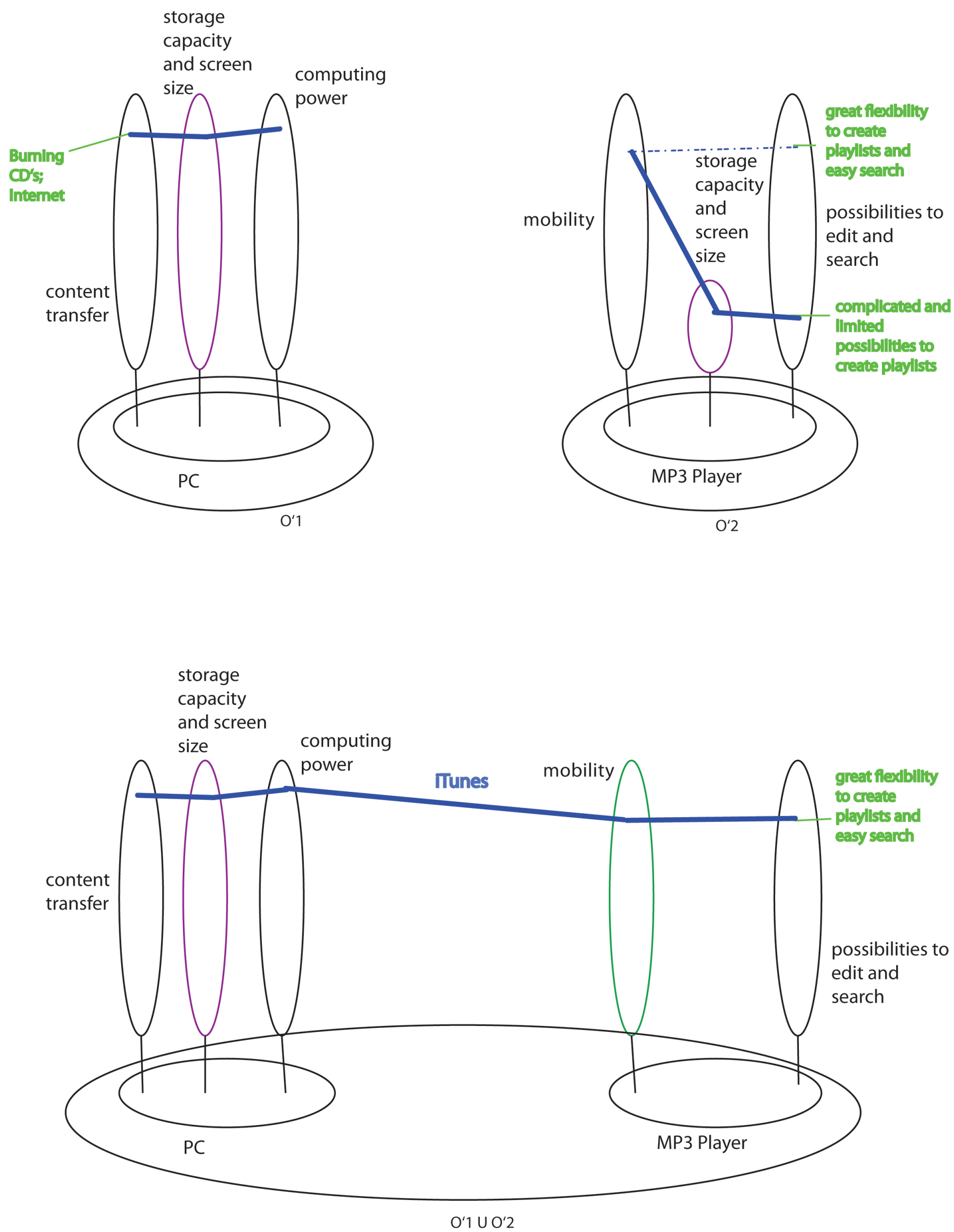

Figure 12: **Identification of similarities between $PC - iTunes$ and $PC - iMovie$**

Identifications observables in $PC - iTunes$ similar to observables in $PC - iMovie$:
content transfer (in PC in O'_1) $\rightarrow$ content transfer (in PC in O_1)
storage capacity and screen size (in PC in O'_1) $\rightarrow$ screen size (in PC in O_1)
computing power (in PC in O'_1) $\rightarrow$ computing power (in PC in O_1)
storage capacity and screen size (in MP3-Player in O'_2) $\rightarrow$ screen size (in Camcorder in O_2)
mobility (in MP3 Player in O'_2) $\rightarrow$ mobility (in Camcorder in O_2)
possibilities to edit and search (in MP3-Player in O'_2) $\rightarrow$ possibilities to edit content (in Camcorder in O_2)

Identification of values in observables in $PC - iTunes$ similar to values of observables in $PC - iMovie$:
'high' and 'low' or 'large' and 'small' values are identified in the obvious way ($high \rightarrow high$, $low \rightarrow low$; $large \rightarrow large$, $small \rightarrow small$)
'Burning CDs; Internet' (value of 'content transfer' in PC in O'_1) $\rightarrow$ 'Internet and Firewire' (value of 'content transfer' in PC in O_1)
'great flexibility to create playlists and easy search' (value of 'possibilities to edit and search' in 'MP3 Player' in O'_2) $\rightarrow$ 'quick and easy editing' (value of 'possibilities to edit content' in 'Camcorder' in O_2)
'complicated and limited possibilities to create playlists' (value of 'possibilities to edit and search' in MP3-Player in O_2) $\rightarrow$ 'difficult and inconvenient editing' (value of 'possibilities to edit content' in Camcorder in O_2)

Figure 13: **Expanding and integrating the hub**