Learning about the Neighborbood: A Model of Housing Cycles*

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Abstract

This paper develops a tractable model to analyze information aggregation and learning in housing markets. In the presence of informational frictions, households face a realistic problem in learning about the quality of a neighborhood and housing prices serve as important signals. Our model highlights how the learning by households interacts with local housing supply and demand characteristics and affects housing price dynamics. These learning effects are particularly strong when supply elasticity is in an intermediate range, and can cause short-run price momentum even when shocks to both housing supply and demand mean-revert over time.

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People buy a house not just for shelter but also for the neighborhood to which the house belongs. There are many characteristics that affect the living conditions in a neighborhood, such as public safety, quality of local schools, and availability of local amenities like restaurants and public parks. In wealthier neighborhoods, the greater demand for public safety, high quality schools, and nice restaurants tends to attract a greater supply of these amenities, which in turn makes housing in these neighborhoods more desirable. The economic literature has recognized the importance of neighborhood effects in determining housing demand (e.g., Ioannides and Zabel (2003)) and a host of other urban issues (e.g., Durlauf (2004); and Glaeser, Sacerdote, and Scheinkman (2003)).

It is important to recognize that the quality of a neighborhood, which is ultimately driven by the financial health of its residents and the strength of the local economy, is often not directly observable to potential home buyers. Home buyers also face other unobservable factors in housing markets, such as local housing supply conditions. In the presence of these important informational frictions, local housing markets provide a useful platform for aggregating information. This fundamental aspect of housing markets, however, has received little attention in the academic literature. Furthermore, while the existing literature has emphasized the importance of accounting for home buyers' expectations (and in particular extrapolative expectations) in understanding dramatic housing boom and bust cycles (e.g., Case and Shiller (2003); Glaeser, Gyourko, and Saiz (2008); and Piazzesi and Schneider (2009)), much of the analysis and discussions are made in the absence of a systematic framework that anchors home buyers' expectations on their information aggregation and learning process. In this paper, we aim to fill this gap by developing a model to analyze information aggregation and learning in housing markets and its implications for housing cycles.

Our paper integrates the standard framework of Grossman and Stiglitz (1980) and Hellwig (1980) for information aggregation in asset markets with a housing market in a local neighborhood. This setting allows us to extend the insights of market microstructure analysis to explore the real consequences of informational frictions in housing markets. In particular, our model allows us to analyze how agents form expectations in housing markets, how these expectations interact with characteristics endemic to a neighborhood, and how these expectations feed into housing prices.

We first present a static setting to highlight the basic information aggregation mechanism with perfectly rational households, and then extend the model to a dynamic setting to characterize the implications of learning about the neighborhood for housing cycles. The model presented herein features a continuum of households in each generation in a closed neighborhood. Each household specializes in producing a consumption good, which can be broadly interpreted as local services such as health care, restaurant services, or plumbing, and needs to trade his good for goods produced by other households in the neighborhood for consumption. The households have a Cobb-Douglas utility function over consumption of their own good, goods produced by other households, and housing. In choosing their housing demand, the households face uncertainty regarding the aggregate productivity of other households, a key aspect of the quality of the neighborhood, which ultimately determines the demand for each household's good and its own housing demand. In more desirable neighborhoods, households are more productive and there is a greater demand for each household's good, which in turn makes each household more wealthy and thus have a greater demand for housing consumption. In this way, the complementarity in the households' goods consumption leads to a complementarity in their housing demand.

In our model, each household possesses a private signal regarding the aggregate productivity of the households in the neighborhood. By aggregating each household's housing demand, the housing price aggregates their private signals. The presence of unobservable housing supply shocks, however, prevents the housing price from perfectly revealing the quality of the neighborhood and acts as a source of informational noise in the housing price. In this way, characteristics endemic to local supply and demand determine the informational content of the housing price and affect households' learning from the housing price.

Despite each household's housing demand being non-linear, the Law of Large Numbers allows us to aggregate their housing demand in closed-form and to derive a unique log-linear equilibrium for the housing market. In this equilibrium, the housing price is a log-linear function of the unobservable quality of the neighborhood and the housing supply shock, and each household's housing demand is a log-linear function of its private signal and the housing price. In the equilibrium, a higher housing price does not simply represent a larger cost of housing but also gives a positive signal about the neighborhood. Through this learning channel, supply shocks have a larger negative impact, and demand shocks a smaller positive impact, on the equilibrium housing price than they would in an otherwise identical economy without informational frictions. This is because informational frictions prevent households from fully separating supply shocks from demand shocks and instead attribute a high housing price partially to a strong aggregate household productivity and partially to a weak supply. This inference in turn amplifies the price impact of supply shocks and attenuates the impact of demand shocks.

Our analysis particularly highlights that supply elasticity can play a subtle role in households' learning—the learning effect is most pronounced in neighborhoods with an intermediate supply elasticity. In neighborhoods with very elastic supply, prices are uninformative about the households' aggregate productivity, while in neighborhoods with very inelastic supply, they are so revealing that households face little uncertainty about the aggregate productivity. Netting out these two forces leads to the strongest reaction in housing demand to prices because of learning in neighborhoods with an intermediate supply elasticity. This non-monotonic effect of supply elasticity on household learning is in sharp contrast to the common wisdom that supply elasticity monotonically attenuates housing cycles. It also provides a new insight for explaining a puzzling phenomenon summarized by Glaeser (2013) and Gao (2013) that, during the recent U.S. housing cycle in the 2000s, areas with relatively elastic supply like Phoenix and Las Vegas experienced dramatic boom and bust cycles similar to inelastic areas like New York and Los Angeles.

To characterize the dynamic implications of the learning effect for housing cycles, we also extend the model into a dynamic setting with overlapping generations of households where each generation is modeled in a manner that closely resembles the static setting. We are particularly interested in examining whether the learning effect can help explain the patterns of short-run momentum and long-run reversals observed in housing prices (e.g., Case and Shiller (1989); Glaesar and Gyourko (2006)). In our setting, the housing price in each period is determined by the households' expectation of the current period aggregate productivity and housing supply. As shocks to both of them tend to mean-revert over time, the mean-reversion, on one hand, provides a natural explanation to the long-run reversals observed in housing prices, and, on the other hand, makes it more challenging to explain the short-run momentum. A commonly held perception is that the housing price momentum is related to home buyers' extrapolative expectations.

Interestingly, in the presence of informational frictions and persistence in shocks to both housing supply and demand, households in each generation use not only their private signals and the current period housing price but also the housing price of the previous period to learn about the current aggregate household productivity. Despite this more elaborate dynamic learning process than in the static model, we manage to maintain the tractable log-linear equilibrium in the housing market. The households' learning from the previous period's price can lead to housing price momentum under certain conditions, even when shocks to both supply and demand mean-revert over time and household learning is perfectly rational.

To dissect the mechanism, it is useful to discuss how the relative persistence of shocks to aggregate productivity and housing supply affects the households' learning from the previous period's price. If shocks to aggregate productivity are sufficiently more persistent than shocks to supply, a higher price in the previous period signals a stronger aggregate productivity, which is likely to persist into the current period, and thus cause the households to hold a higher expectation of the current period aggregate productivity. When this learning effect is sufficiently strong—stronger than the opposing force from the inherent mean reversion of the shocks—the housing price exhibits short-run momentum. On the other hand, if shocks to housing supply are sufficiently more persistent than shocks to aggregate productivity, a higher price in the previous period induces learning in the opposite direction. That is, households perceive the supply in the prior period to be weak and believe the weak supply is likely to persist into the current period, which, conditional on the current period housing price, causes the households to have a lower expectation of the current period aggregate productivity. This inference causes stronger housing price reversals than those caused by the mean reversion of the shocks.

Besides the relative persistence of the shocks, the households' learning effect also depends on several other local characteristics. For example, as mentioned already, housing supply elasticity has a non-monotonic effect on the magnitude of the households' learning from the housing prices and thus a non-monotonic effect on the price momentum induced by household learning. Furthermore, a higher degree of complementarity in households' housing demand causes each household to put a greater weight on the publicly observed housing prices and a smaller weight on its own private signal. The former tends to strengthen the short-run price momentum induced by the learning effect, while the latter makes housing prices less informative and exacerbates the informational frictions faced by the households. Taken together, our model provides a rich set of patterns in housing price dynamics, which crucially depends on the interactions of household learning with local characteristics of housing supply and demand. These patterns are potentially testable in the cross-section of local housing markets.

The mechanism for driving housing cycles in our model is different from the slow information diffusion model of Hong and Stein (1999), which generates momentum and reversals in asset prices through a slow diffusion of information among a population of investors. In their model, investors ignore the information revealed by the prices of traded assets, while in our model, learning from the current and past housing prices plays a central role in driving housing price cycles. Neither does our model rely on extrapolative expectations, even though extrapolative expectations, if incorporated into our learning framework, are likely to induce even more pronounced and interesting learning effects. As a result of the nature of rational learning, our model also differs from Burnside, Eichenbaum, and Rebelo (2013), which offers a model of housing market booms and busts based on the epidemic spreading of optimistic or pessimistic beliefs among home buyers through their social interactions.

Our paper is related to the growing literature that studies the informational feedback effects from asset prices to real world activities. The early contribution includes Bray (1981) and Subrahmanyam and Titman (2001). More recently, Morris and Shin (2002) points out that such feedback effects are particularly strong in the presence of strategic complementarity in agents' actions. A series of recent work, e.g., Ozdenoren, and Yuan (2008), Angeletos, Lorenzoni and Pavan (2010), Goldstein, Ozdenoren, and Yuan (2011, 2012), and Sockin and Xiong (2013), analyze specific feedback effects from stock prices to firm capital investment decisions, from exchange rates to policy choices of central banks, and from commodity prices to commodity demand. In particular, the tractable log-linear equilibrium derived in our model for housing markets resembles a similar log-linear equilibrium derived by Sockin and Xiong (2013) for commodity markets.

Finally, by focusing on information aggregation and learning of symmetrically informed households with dispersed private information, our study differs in emphasis from those that analyze the presence of information asymmetry between buyers and sellers of homes, such as Garmaise and Moskowitz (2004) and Kurlat and Stroebel (2014).

The paper is organized as follows. Section 1 presents a static model to highlight the key mechanism of information aggregation and learning from prices in housing markets. Section 2 extends the model to a dynamic setting with many periods to characterize the effects of household learning on housing cycles. Finally, we conclude in Section 3. The technical proofs of propositions are provided in the Appendix.

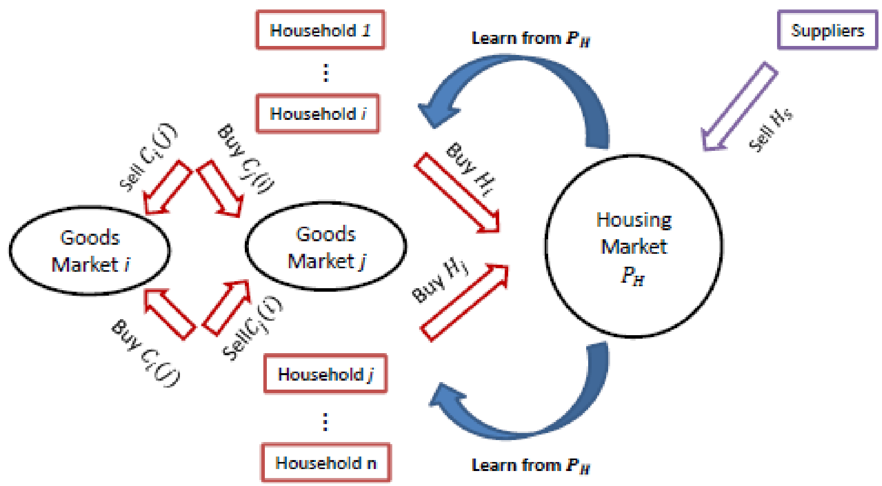

Figure 1: Structure of the Static Model

1 A Static Model

In this section we develop a static model with two dates $t = 1, 2$ to analyze the effects of informational frictions on the housing market equilibrium in a closed neighborhood. A key feature of the static model is that the housing market is not only a place for households to trade housing but also a platform to aggregate private information about the unobservable quality of the neighborhood. This static model also serves as a basic module in the dynamic model that we present in the next section to analyze housing cycles.

1.1 Model setting

Figure 1 illustrates the structure of the static model. There are two types of agents in the economy: households looking to buy housing in a neighborhood and home builders. Suppose that the neighborhood is new and all households need to purchase housing at the same time.[1] Each household cares about the quality of the neighborhood in which it lives, as it has to trade produced goods and services, such as health care, restaurants, luxury stores and schools, with other households in the same neighborhood. For simplicity, we assume that the neighborhood is closed and that each household is specialized in producing one good and can trade its good for other goods only with other households in the same neighborhood.

[1] For simplicity, we do not consider the endogenous decision of households to choose their neighborhood, and instead take the pool of households in the neighborhood as given. See Van Nieuwerburgh and Weill (2010) for a systematic treatment of moving decisions by households across neighborhoods.

The quality of this closed neighborhood is thus reflected by the aggregate productivity of the households in the neighborhood. A strong aggregate productivity implies a higher output for all households, which in turn leads to a stronger demand for each household's good.

In addition to consuming the produced goods, each household also wishes to consume housing. Through the trading of produced goods with each other, a strong aggregate productivity increases the demand for each household's good and its income, which in turn motivates each household to demand more housing consumption. This gives rise to strategic complementarity in each household's housing demand.[2] In the presence of realistic informational frictions in gauging the quality of the neighborhood, the housing market provides an important platform for aggregating information. The resulting housing price, in turn, serves as a useful signal about the quality of the neighborhood.

At $t = 1$, households purchase houses from home builders in a centralized market and decide how much of their goods to produce for trading with other households at $t = 2$. Each household will choose to purchase a bigger house in the first period if it expects to produce more goods in the second.

1.1.1 Households

There is a continuum of households, indexed by $i \in [0, 1]$. Household i has a Cobb-Douglas utility function over housing H_i and a continuum of goods $\{C_j\}_{j\in[0,1]}$, given by

$$U\left(H_i, \{C_j(i)\}_{j\in[0,1]}\right) = \left(\frac{H_i}{1-\eta_H}\right)^{1-\eta_H} \left\{\frac{1}{\eta_H}\left(\frac{C_i(i)}{1-\eta_c}\right)^{1-\eta_c}\left(\frac{\int_{[0,1]/i} C_j(i)\,dj}{\eta_c}\right)^{\eta_c}\right\}^{\eta_H}, \quad (1)$$

where $C_j(i)$ is good j consumed by household i.[3] The parameters $\eta_H \in (0, 1)$ and $\eta_c \in (0, 1)$ measure the weights of different consumption components in the utility function. A higher η_H means a stronger complementarity between housing consumption and goods consumption, while a higher η_c means a stronger complementarity between consumption of the good produced by household i itself and consumption of the composite good $\int_{[0,1]/i} C_j(i)\,dj$ produced by the other households in the neighborhood.

The production function of household i is $e^{A_i} l_i$,where l_i is the household's labor choice and A_i is its productivity. A_i is composed of a component A common to all households in

[2]There are other types of social interactions between households living in a neighborhood, which are explored, for instance, in Durlauf (2004) and Glaeser, Sacerdote, and Scheinkman (2010).

[3]Our modeling choice of non-separable preferences for housing and consumption is similar to the CES specification of Piazzesi, Schneider, and Tuzel (2007).

the neighborhood and an idiosyncratic component ε_i:

$$A_i = A + \varepsilon_i,$$

where $A \sim \mathcal{N}\left(\bar{A}, \tau_A^{-1}\right)$ and $\varepsilon_i \sim \mathcal{N}\left(0, \tau_\varepsilon^{-1}\right)$ are both normally distributed. The common productivity A represents the quality of the neighborhood, as a higher A implies a more productive neighborhood.

Due to realistic informational frictions, neither A nor A_i is observable to the households. Instead, each household observes a noisy private signal about A at $t = 1$. Specifically, household i observes

$$s_i = A + \nu_i$$

where $\nu_i \sim \mathcal{N}\left(0, \tau_s^{-1}\right)$ is signal noise independent across households.

We assume that each household experiences a disutility for labor $\frac{l_i^{1+\psi}}{1+\psi}$ when producing its good, and that it maximizes its utility subject to its budget constraint

$$\max_{\left\{H_i, \{C_i\}_{i\in[0,1]}, l_i\right\}} E\left[U\left(H_i, \{C_j(i)\}_{j\in[0,1]}\right) - \frac{l_i^{1+\psi}}{1+\psi}\middle|\, \mathcal{I}_i\right] \quad (2)$$

$$\text{such that} \quad : \quad P_H H_i + \int_0^1 P_j C_j(i)\, dj = P_i e^{A_i} l_i + w_i,$$

where P_i is the price of the good it produces and w_i is the income from building the house. We assume for simplicity that the home builder for household i is part of the household, so that $w_i = P_H H_i$ in equilibrium. The choice of goods consumption is made at $t = 2$, while the choice of labor and housing is made at $t = 1$ subject to each household's information set $\mathcal{I}_i = \{s_i, P_H\}$, which includes its private signal s_i and the housing price P_H.

1.1.2 Builders

Home builders face a convex labor cost

$$\frac{k}{1+k} e^{-\xi} H_S^{\frac{1+k}{k}}$$

in supplying housing, where H_S is the quantity of housing supplied, $k \in (0, 1)$ is a constant parameter, and ξ represents a supply shock. We assume that ξ is observed by builders but not households, and that from the perspective of households $\xi \sim \mathcal{N}\left(\bar{\xi}, \tau_\xi^{-1}\right)$, i.e., a normal distribution with $\bar{\xi}$ as the mean and τ_ξ^{-1} as the variance. In the housing market equilibrium, the supply shock ξ acts as information noise for the households when they try to learn about the common productivity A from the housing price P_H.

Builders at $t = 1$ maximize their profit subject to their noisy supply curve

$$\Pi\left(H_S\right) = \max_{H_S} P_H H_S - \frac{k}{1+k} e^{-\xi} H_S^{\frac{1+k}{k}}, \tag{3}$$

It is easy to determine the builders' optimal supply curve:

$$H_S = P_H^k e^{k\xi}, \tag{4}$$

where k measures the price elasticity of housing supply.

1.1.3 Equilibrium

Our model features a noisy rational expectations equilibrium, which requires clearing of the goods and housing markets that are consistent with the optimal behavior of both households and home builders:

- Household optimization: $\left\{H_i, \{C_i\}_{i\in[0,1]}, l_i\right\}$ solves each household's maximization problem in (2).
- Builder optimization: H_S solves the builders' maximization problem in (3).
- At $t = 2$, the market for each household's good clears:

$$\int_0^1 C_i\left(j\right) dj = e^{A_i} l_i, \quad \forall\ i \in [0,1]\,.$$

- At $t = 1$, the housing market clears:

$$\int_{-\infty}^{\infty} H_i\left(s_i, P_H\right) d\Phi\left(v_i\right) = P_H^k e^{k\xi},$$

  where each household's housing demand $H_i\left(s_i, P_H\right)$ depends on its private signal s_i and the housing price P_H. The demand from the households is integrated over the idiosyncratic component of their private signals $\{\nu_i\}_{i\in[0,1]}$.

1.2 The equilibrium

1.2.1 Goods market equilibrium

We begin our analysis of the equilibrium with the goods markets at $t = 2$. Household i has $e^{A_i} l_i$ units of good i for consumption and trading with the other households. It maximizes its utility function given in (2). The following proposition describes the goods market equilibrium for each good.

Proposition 1 *Households i's optimal goods consumption is*

$$C_i(i) = (1-\eta_c) e^{A_i} l_i, \; C_j(i) = \eta_c e^{A_j} l_j.$$

The price of the good produced by household i is

$$P_i = \left(\frac{\eta_H}{1-\eta_H} H_i\right)^{1-\eta_H} \left(e^{A_i} l_i\right)^{(1-\eta_c)\eta_H - 1} \left(\int_{[0,1]/i} e^{A_j} l_j dj\right)^{\eta_c \eta_H}. \tag{5}$$

Proposition 1 shows that each household divides its consumption of goods between its own good and those produced by other households with fractions $1-\eta_c$ and η_c, respectively. When $\eta_c = 1/2$, the household consumes its own good and the goods of its neighbors equally. The price of each good is determined by its choice of housing and its goods output relative to that of the rest of the neighborhood. One household's good is more valuable when the rest of the neighborhood produces more, and thus each household needs to take into account the labor decisions of the other households in its neighborhood when making its own decision.

1.2.2 Housing market equilibrium

We now turn our attention to the housing market equilibrium at $t = 1$. We first solve for the optimal labor and housing choices for a household given its utility function and budget constraint in (2), as well as its optimal choices for consumption of goods at $t = 2$ characterized in Proposition 1.

Proposition 2 *Households i's optimal labor choice depends on its total housing expenditure*

$$l_i = \left(\frac{\eta_H}{1-\eta_H} P_H H_i\right)^{\frac{1}{1+\psi}},$$

and its demand for housing is

$$H_i = \left(\frac{\eta_H}{1-\eta_H}\right)^{-\frac{\psi}{\psi+\eta_c}} \left\{ e^{(\eta_H - 1 - \psi)\log P_H} E\left[e^{A_i (1-\eta_c)\eta_H} \left(\int_{[0,1]/i} e^{A_j} H_j^{\frac{1}{1+\psi}} dj\right)^{\eta_c \eta_H} \middle| \mathcal{I}_i \right]^{(1+\psi)} \right\}^{\frac{1}{(\psi+\eta_c)\eta_H}}. \tag{6}$$

Proposition 2 demonstrates that the labor chosen by a household toward producing its good is determined by its housing expenditure $P_H H_i$, and that its housing demand is determined by not only its own productivity A_i but also the aggregate productivity of the rest of the neighborhood. This latter component arises from the social complementarity in the utility function of the household, and captures the key idea that the household cares about the neighborhood as a result of its need to exchange goods with its neighbors.

By clearing the aggregate housing demand of the households with the supply from the builders, we derive the housing market equilibrium. Despite the nonlinearity in each household's demand and in the supply from builders, we obtain a tractable unique log-linear equilibrium. The following proposition summarizes the housing price and each household's housing demand in this equilibrium.

Proposition 3 *At $t = 1$, the housing market has a unique log-linear equilibrium: 1) The housing price is a log-linear function of A and ξ:*

$$\log P_H = p_A A + p_\xi \xi + p_0, \tag{7}$$

with the coefficients p_A and p_ξ given by

$$p_A = \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1} - \frac{(\psi+\eta_c)\, kb}{\psi k + \frac{1+\psi}{\eta_H} - 1}\tau_s^{-1}\tau_A > 0, \tag{8}$$

$$p_\xi = -\frac{\psi k}{\psi k + \frac{1+\psi}{\eta_H} - 1} - \frac{1+\psi+\eta_c kb}{\psi k + \frac{1+\psi}{\eta_H} - 1}\frac{b\tau_\xi}{\tau_A + \tau_s + b^2\tau_\xi} < 0, \tag{9}$$

where $b \in \left[0, \frac{1+\psi}{k}\frac{\tau_s}{(\psi+\eta_c)\tau_A + \psi\tau_s}\right]$ is the unique positive, real root of equation (36), and p_0 is given in equation (41).

2) The housing demand of household i is a log-linear function of its private signal s_i and $\log P_H$:

$$\log H_i = h_s s_i + h_P \log P_H + h_0, \tag{10}$$

with the coefficients h_s and h_P given by

$$h_s = kb > 0, \tag{11}$$

$$h_P = -\frac{1-\eta_H+\psi}{\psi\eta_H} + \frac{b^2\tau_\xi}{\psi p_A}\frac{(1+\psi+\eta_c kb)}{(\tau_A+\tau_s+b^2\tau_\xi)} \leq k, \tag{12}$$

and h_0 given by equation (30).

Proposition 3 establishes that the housing price P_H is a log-linear function of the common productivity of households A and the housing supply shock ξ, and that each household's housing demand is a log-linear function of its private signal s_i and the log housing price $\log P_H$. Similar to Hellwig (1980), the housing price aggregates the households' dispersed private information to partially reveal A. The price does not depend on the idiosyncratic noise in any individual household's signal because of the Law of Large Numbers. This last observation is key to the tractability of our model, and ensures that the housing demand from the households retains a log-normal distribution after aggregation.

In the presence of informational frictions, the housing supply shock ξ serves the same role as noise trading in standard models of asset market trading with dispersed information. This feature highlights an important channel for supply shocks to affect the expectations of potential home buyers that is not well-appreciated in the housing literature. Since households cannot perfectly disentangle changes in housing prices caused by supply shocks from those brought about by shocks to demand, they partially confuse a housing price change caused by a supply shock to be a signal about the quality of the neighborhood.

To facilitate our discussion of the impact of learning, it will be useful to introduce a perfect-information benchmark in which all households perfectly observe the common productivity of the neighborhood A. The following proposition characterizes this benchmark equilibrium.

Proposition 4 *Consider a benchmark setting, in which households perfectly observe A (i.e., $s_i = A$, $\forall i$.) There is also a log-linear equilibrium, in which the housing price is*

$$\begin{aligned}\log P_H \;=\;& \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1} A - \frac{\psi k}{\psi k + \frac{1+\psi}{\eta_H} - 1}\xi + \frac{\psi}{k\psi + \frac{1+\psi}{\eta_H} - 1}\log\left(\frac{1-\eta_H}{\eta_H}\right) \\ &+ \frac{1+\psi}{2\left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)}\left(\eta_H\left(1-\eta_c\right)^2 + \eta_c\right)\tau_\varepsilon^{-1},\end{aligned}$$

and all households have the same housing demand

$$\log H = \frac{1+\psi}{\psi} A - \frac{1-\eta_H+\psi}{\psi\eta_H}\log P_H + \log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{1+\psi}{2\psi}\left(\eta_H\left(1-\eta_c\right)^2 + \eta_c\right)\tau_\varepsilon^{-1}.$$

Furthermore, the housing market equilibrium with information frictions characterized in Proposition 3 converges to this benchmark equilibrium as $\tau_s \nearrow \infty$.

In this perfect-information benchmark, the housing price is also a log-linear function of the common productivity A and the supply shock ξ, and each household's identical demand is a log-linear function of its private signal (the perfectly observed A) and the housing price $\log P_H$. Consistent with the standard intuition, a higher common productivity A increases both the housing price and aggregate housing demand, while a larger supply shock ξ reduces the housing price but increases aggregate housing demand. It is also easy to see that in this benchmark setting, as the supply elasticity k rises from zero to infinity, the weight of A (demand side) in the housing price decreases while the weight of ξ (supply side) increases.

1.3 Impact of learning

In the presence of informational frictions about the quality of the neighborhood A, each household needs to use its private signal s_i and the publicly observed housing price $\log P_H$ to learn about A. As the housing price $\log P_H$ is a linear combination of A and the housing supply shock ξ, the supply shock interferes with this learning process. A larger supply shock ξ, by depressing the housing price, will have an additional effect of reducing the households' expectations of A. This, in turn, reduces their housing demand and consequently further depresses the housing price. Thus the learning effect causes supply shocks to have a larger negative effect on the equilibrium housing price than they would in the perfect-information benchmark. Similarly, this learning effect also causes the quality of the neighborhood A to have a smaller positive effect than in the perfect-information benchmark because informational frictions cause households to partially discount the value of A. The following proposition formally establishes this learning effect on the housing price.

Proposition 5 *In the presence of informational frictions, coefficients $p_A > 0$ and $p_\xi < 0$ derived in Proposition 3 are both lower than their corresponding values in the perfect-information benchmark. Furthermore, p_A is monotonically increasing in the precision τ_s of each household's private signal and decreasing in the degree of complementarity in households' goods consumption η_c.*

The precision of the households' private information τ_s determines the informational frictions they face. Proposition 5 shows that an increase in τ_s mitigates the informational frictions and brings coefficient p_A closer to its value in the perfect-information benchmark. In fact, as τ_s goes to infinity, the housing market equilibrium converges to the perfect-information benchmark (Proposition 4).

Each household's housing demand also reveals how the households learn from the housing price. In the presence of informational frictions about A, the housing price is not only the cost of acquiring shelter but also a signal of A. The housing demand of each household derived in (10) reflects both of these two effects. Specifically, we can decompose the price elasticity of each household's housing demand h_P in equation (12) into two components: The first component $-\frac{1-\eta_H+\psi}{\psi\eta_H}$ is negative and represents the standard cost effect (i.e., downward sloping demand curve) in the perfect-information benchmark in Proposition 4, and the second component $\frac{b^2\tau_\xi}{\psi p_A}\frac{(1+\psi+\eta_c kb)}{\left(\tau_A+\tau_s+b^2\tau_\xi\right)}$ is positive and represents the information effect. A higher housing price raises the household's expectation of A and induces it to consume more housing through two related but distinct learning channels. First, a higher A implies a higher productivity for the household itself. Second, a higher A also implies that other households are more productive, which in turn leads to greater demand for the household's good. As a reflection of this complementarity effect, the second component in the price elasticity of housing demand increases with η_c, the degree of complementarity in the household's consumption of its own good and goods produced by other households. Both channels cause the household to expect a higher wealth at $t = 2$ and thus to demand more housing consumption at $t = 1$. As a result, under certain sufficient conditions, the price elasticity of housing demand may even become positive.

As a result of the presence of the complementarity channel, η_c also affects the impact of learning on the housing price. As η_c increases, each household puts a greater weight on the housing price in its learning of A and a smaller weight on its own private signal. This in turn makes the housing price less informative of A. In this way, a larger η_c exacerbates the informational frictions faced by the households. Indeed, Proposition 5 shows that the loading of $\log P_H$ on A is decreasing with η_c.

Housing supply elasticity k also plays an important role in determining the informational frictions faced by the households, in addition to its standard supply effect. To illustrate this learning effect of supply elasticity, we consider two limiting economies as k goes to 0 and ∞, which are characterized in the following proposition.

Proposition 6 *As $k \to \infty$, the price and demand for housing converge to*

$$\log P_H = -\xi,$$

and

$$\log H_i = (1+\psi)\left(\psi + \frac{\eta_c\tau_A}{\tau_A+\tau_\varepsilon}\right)^{-1}\frac{\tau_\varepsilon}{\tau_A+\tau_\varepsilon}s_i - \frac{1}{\psi}\left(\frac{1+\psi}{\eta_H}-1\right)\log P_H + h_0.$$

As $k \to 0$, the price and demand for housing converge to

$$\log P_H = \frac{1+\psi}{\frac{1+\psi}{\eta_H}-1}A + \left(\frac{1+\psi}{\eta_H}-1\right)^{-1}\left(\psi\log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{1+\psi}{2}\left(\eta_c + (1-\eta_c)^2\eta_H\right)\tau_\varepsilon^{-1}\right),$$

and $\log H_i = 0$. *Furthermore, under the necessary and sufficient condition that*

$$\frac{\tau_s}{\psi + \eta_c} \leq \frac{1}{\psi}\left(\frac{1}{4}\left(\frac{(1+\psi)\,\eta_H}{1+\psi-\eta_H}\right)^2 \tau_\xi - \tau_A\right), \tag{13}$$

there exists an intermediate range for k *such that* h_P*, the price elasticity of the households' housing demand, is positive.*

As supply elasticity goes to infinity, the housing price is completely driven by the supply shock ξ and contains no information for households about A. As a result, they do not learn from the price, and the price elasticity of their housing demand h_P equals its value in the perfect-information benchmark. As supply elasticity goes to zero, the housing price is completely driven by A and fully reveals it. In this case, there is a significant learning effect, which exactly offsets the cost effect in the households' housing demand to make h_P zero.

The magnitude of the learning effect for an intermediate supply elasticity depends on the information frictions faced by the households. Proposition 6 shows that when the informational frictions are sufficiently severe (i.e., τ_s lower than the critical level given in (13)), h_P can become positive for an intermediate range of supply elasticity. This means that the learning effect is non-monotonic with respect to k and becomes so strong in the intermediate range of k that it dominates the cost effect. In this case, a higher housing price induces households to demand more, rather than less, housing. This non-monotonicity arises from two offsetting forces. On one hand, a higher k makes the housing price less informative of the common productivity A; on the other, households face a greater uncertainty about A and thus have a greater incentive to learn from the price.

This non-monotonic effect of supply elasticity on the households' learning about the neighborhood has been largely ignored by the literature. We will further explore this effect of supply elasticity in the dynamic setting.

The condition given in (13) also sheds some light on how the degree of complementarity in households' goods consumption η_c interacts with informational frictions. All else equal, a higher η_c is equivalent to a lower τ_s, so that a greater complementarity between households is indistinguishable from having noisier private information. This is because a greater complementarity causes households to pay less attention to their private signals. The quantity $\frac{\tau_s}{\psi+\eta_c}$ acts as a sufficient statistic for τ_s and η_c in affecting housing price and demand in the equilibrium, with the exceptions of the constants h_0 and p_0. A similar insight will also appear in our discussion of the dynamic model.[4]

2 A Dynamic Model

We now extend our model to multiple periods with overlapping generations of households to characterize how learning affects the dynamics of housing prices. In particular, we show that in the presence of informational frictions, learning can lead to short-run price momentum despite that shocks to both aggregate productivity and housing supply mean-revert over time. We also highlight the important roles played by the persistence of the shocks and supply elasticity in affecting the households' learning, and in determining the price effects of this learning.

2.1 Model setting

We consider a setting with infinitely many periods: $t = 0, 1, 2, ...$ In each period, a generation of households arrives to the neighborhood. Households of each generation first make their housing and labor decisions to produce their respective goods when they arrive, and then trade their goods with each other in the same generation and consume in the next period. To simplify the model, we avoid trading of either housing or goods across generations. Consider households that arrive at τ. They acquire housing from home builders of their own generation at τ and then trade goods only with households of their own generation at $\tau + 1$. This simplification allows us to keep the same setting in our static model for each generation of households. The link between different generations goes through the persistence in the households's aggregate productivity and housing supply. Due to such persistence, housing prices from the past periods contain useful information about the current period aggregate productivity and housing supply. This setting thus allows us to focus on the housing dynamics driven by the households' learning.[5]

[4]This insight is not immediately apparent from the equations in Proposition 3. From equation (36), $\frac{\tau_s}{\psi+\eta_c}$ is a sufficient statistic for τ_s and η_c for b. Since $p_\xi = -\frac{1}{b}p_A$, and $\frac{\tau_s}{\psi+\eta_c}$ is a sufficient statistic for τ_s and η_c in p_A, it is also for p_ξ. Finally, substitution with equation (39) for h_P yields

$$h_P = -\frac{1-\eta_H+\psi}{\psi\eta_H} + \frac{kb^3\tau_\xi}{\psi p_A}\left(\frac{\tau_s}{\psi+\eta_c}\right)^{-1}.$$

[5]We implicitly assume that housing units last only for the use of one generation. As a result, speculators cannot accumulate vacant housing units in anticipation of a future demand increase. While such trading is present in reality, the cost is substantially higher than that in speculation of financial assets. In the absence

Specifically, the generation-τ households have the same utility function as specified in equation (1) for consumption of their own good, goods produced by others, and housing. We denote their aggregate productivity by A_τ, which follows an $AR(1)$ process:

$$A_\tau = \rho_A A_{\tau-1} + Z_\tau^A,$$

where $Z_\tau^A \sim \mathcal{N}\left(0, \tau_A^{-1}\right)$ is a random shock to the aggregate productivity in this period, and $\rho_A \in [0,1]$ is a coefficient that determines the persistence of the productivity. When $\rho_A = 1$, the aggregate productivity is perfectly persistent (i.e., it follows a random walk). When $\rho_A < 1$, the aggregate productivity mean-reverts over time. A_τ is unobservable, and each household of this generation observes a private signal about A_τ:

$$s_{i\tau} = A_\tau + \nu_{i\tau},$$

where $\nu_{i\tau} \sim \mathcal{N}\left(0, \tau_s^{-1}\right)$ is independent and identically distributed signal noise.

Households in generation τ buy housing from home builders of their own generation, who have the supply curve derived in (4), $H_S = P_H^k(\tau)\, e^{k\xi_\tau}$, with k as the supply elasticity and an unobservable variable ξ_τ that determines the housing supply. The supply follows an $AR(1)$ process:

$$\xi_\tau = \rho_\xi \xi_{\tau-1} + Z_\tau^\xi$$

where $Z_\tau^\xi \sim \mathcal{N}\left(0, \tau_\xi^{-1}\right)$ is a random shock to the housing supply in this period. The supply shock is independent of other shocks in the economy. The parameter $\rho_\xi \in [0,1]$ determines the persistence of the housing supply over time.

To simplify the learning dynamics in this dynamic economy with the persistent aggregate productivity and housing supply, we suppose that the aggregate productivity and housing supply are always revealed to the public after two periods. That is, at time τ, the values of $A_{\tau-2}$ and $\xi_{\tau-2}$ become publicly observable to generation-τ households. This assumption avoids the infinite regress problem highlighted by Townsend (1983) in dynamic models with dispersed asymmetric information and infinite horizons, and is commonly employed in theoretical modeling, for instance, in Singleton (1987). Thus, at τ, in learning about A_τ each

of sufficient demand from the current households, a housing speculator does not receive any rent in holding a vacant home and at the same time has to pay financing cost and property taxes. In contrast, a stock speculator receives dividends while holding a stock. As a result, we do not expect housing speculation to smooth over housing prices across generations as one would expect for prices of financial assets. Instead, housing prices are determined by the current period demand rather than anticipation of future demand. Similar settings are commonly used in the housing literature (e.g., Glaeser, Gyourko, and Saiz, 2008; Piazzesi and Schneider, 2009; Glaeser, Gottlieb, and Gyourko, 2010).

household of generation τ observes its private signal $s_{i\tau}$, the current period housing price $P_H(\tau)$, the previous period housing price $P_H(\tau-1)$, and the history of the realized A_u and ξ_u up until $\tau-2$: $\{A_u,\xi_u\}_{u=1}^{\tau-2}$. We summarize this information set as[6]

$$\mathcal{I}_{i\tau}=\left\{\{A_u,\xi_u\}_{u=1}^{\tau-2},P_H(\tau-1),P_H(\tau),s_{i\tau}\right\}.$$

We denote $\hat{A}_\tau^i=E\left[A_\tau|\mathcal{I}_{i\tau}\right]$ as the conditional expectation of household i regarding A_τ.

In the information set $\mathcal{I}_{i\tau}$, only the signal $s_{i\tau}$ is private to the individual household, while the other information is public. We denote the set of public information by

$$\mathcal{I}_{c\tau}=\left\{\{A_u,\xi_u\}_{u=1}^{\tau-2},P_H(\tau-1),P_H(\tau)\right\}.$$

It is useful to note that, by the law of iterated expectations, the average of the households' conditional expectations $\left\{\hat{A}_\tau^i\right\}_i$ is equal to the conditional expectation of A_τ based on the public information $\mathcal{I}_{c\tau}$:

$$E\left[\hat{A}_\tau^i|\mathcal{I}_{c\tau}\right]=E\left[A_\tau|\mathcal{I}_{c\tau}\right],$$

which we denote by $\hat{A}_\tau^c=E\left[A_\tau|\mathcal{I}_{c\tau}\right]$.

2.2 The equilibrium

To derive the housing market equilibrium, we first analyze each household's learning of the aggregate productivity based on a conjectured housing price function and then derive the equilibrium housing price based on each household's housing demand and housing market clearing.

At τ, the households observe $A_{\tau-2}$, but face uncertainty in $A_{\tau-1}$ and A_τ. The prior belief of each household regarding $A_{\tau-1}$ is

$$A_{\tau-1}\left|\{A_u\}_{u=1}^{\tau-2}\right.\sim\mathcal{N}\left(\rho_A A_{\tau-2},\tau_A^{-1}\right).$$

Similarly, as

$$A_\tau=\rho_A A_{\tau-1}+Z_\tau^A=\rho_A^2 A_{\tau-2}+\rho_A Z_{\tau-1}^A+Z_\tau^A,$$

its prior of A_τ is

$$A_\tau\left|\{A_u\}_{u=1}^{\tau-2}\right.\sim\mathcal{N}\left(\rho_A^2 A_{\tau-2},\left(1+\rho_A^2\right)\tau_A^{-1}\right).$$

The housing prices at $\tau-1$ and τ provide useful information, in addition to the households' private signals about A_τ.

[6] Note that $\{A_u,\xi_u\}_{u=1}^{\tau-2}$ fully replicates the information contained in the past housing prices $\{P_H(u)\}_{u=1}^{\tau-2}$.

To derive the equilibrium, we start by conjecturing the following log-linear form for the housing price in each period t:

$$\log P_H(t) = p_A A_t + p_\xi \xi_t + p_{Ac} \hat{A}_t^c + p_0. \tag{14}$$

In this conjectured form, the log housing price aggregates the information revealed through the households' housing demand to reflect the aggregate productivity A_t (the fundamental of the demand side), the housing supply ξ_t, and the average expectation of all households $\hat{A}_t^c$ about the aggregate productivity.[7] Their coefficients are p_A, p_ξ, and p_{Ac}, respectively. There is also a constant term p_0. By letting these coefficients be constant, we focus on a stationary equilibrium.

To facilitate our analysis, we define

$$\begin{aligned} q_\tau &\equiv \frac{\log P_H(\tau) - p_{Ac}\hat{A}_\tau^c - p_\xi \rho_\xi^2 \xi_{\tau-2} - p_0}{p_A} \\ &= A_\tau + \frac{p_\xi}{p_A}\left(Z_\tau^\xi + \rho_\xi Z_{\tau-1}^\xi\right), \end{aligned}$$

as a summary statistic of the informational content of $\log P_H(\tau)$ regarding A_τ with the supply shocks Z_τ^ξ and $Z_{\tau-1}^\xi$ as the informational noise. Similarly, we define

$$\begin{aligned} q_{\tau-1} &\equiv \rho_A \frac{\log P_H(\tau-1) - p_{Ac}\hat{A}_{\tau-1}^c - p_\xi \rho_\xi \xi_{\tau-2} - p_0}{p_A} \\ &= \rho_A \left(A_{\tau-1} + \frac{p_\xi}{p_A} Z_{\tau-1}^\xi\right) \end{aligned} \tag{15}$$

as a summary statistic of the informational content of $\log P_H(\tau-1)$.

The following proposition derives the average expectation of the households about A_τ based on $\mathcal{I}_{c\tau}$ and each household's conditional expectation based on $\mathcal{I}_{i\tau}$.

Proposition 7 *At time τ, the expectation of A_τ conditional on the set of public information*

[7] Note that the conjectured price includes the households' average expectation of the aggregate productivity $\hat{A}_t^c$ but not their average expectation of the housing supply (i.e., $\widehat{\xi}_\tau^c = E[\xi_\tau | \mathcal{I}_{c\tau}]$). This is because under the conjectured price form, $\widehat{\xi}_\tau^c$ is replicable by $\log P_H(t)$ and $\hat{A}_t^c$: by taking the conditional expectation of the two sides of (14) based on $\mathcal{I}_{c\tau}$, we have $\log P_H(t) = (p_A + p_{Ac})\hat{A}_t^c + p_\xi \widehat{\xi}_\tau^c + p_0$, which implies that $\widehat{\xi}_\tau^c = \frac{1}{p_\xi}\left[\log P_H(t) - (p_A + p_{Ac})\hat{A}_t^c - p_0\right]$.

$\mathcal{I}_{c\tau}$ *is given by*

$$\hat{A}^c_\tau = \frac{\alpha}{\hat{\tau}^c_A}\frac{\tau_A}{1+\rho_A^2}\left(2\rho_A + \left(\rho_A\rho_\xi\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1} - \tau_A^{-1}\right)\frac{2\rho_A - \left(1+\rho_A^2\right)\rho_\xi}{\tau_A^{-1} + \left(1+\left(\rho_A-\rho_\xi\right)\rho_A\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}\right)\rho_A A_{\tau-2}$$
$$+\frac{\alpha}{\hat{\tau}^c_A}\left[\frac{p_A^2}{p_\xi^2}\tau_\xi q_\tau + \frac{2\rho_A - \left(1+\rho_A^2\right)\rho_\xi}{\tau_A^{-1} + \left(1+\left(\rho_A-\rho_\xi\right)\rho_A\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}\frac{1}{\rho_A}q_{\tau-1}\right], \tag{16}$$

where

$$\alpha = \left(2 + \rho_\xi\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}\frac{2\rho_A - \left(1+\rho_A^2\right)\rho_\xi}{\tau_A^{-1} + \left(1+\left(\rho_A-\rho_\xi\right)\rho_A\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}\right)^{-1},$$

and $\hat{\tau}^c_A$ *is the precision of the conditional expectation:*

$$\hat{\tau}^c_A = \alpha\left(\frac{p_A^2}{p_\xi^2}\tau_\xi + \frac{2\tau_A}{1+\rho_A^2} + \frac{\rho_A + \rho_\xi\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}\tau_A}{1+\rho_A^2}\frac{2\rho_A - \left(1+\rho_A^2\right)\rho_\xi}{\tau_A^{-1} + \left(1+\left(\rho_A-\rho_\xi\right)\rho_A\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}\right).$$

The expectation of A_τ *conditional on household* i*'s information set* $\mathcal{I}_{i\tau}$ *is given by*

$$\hat{A}^i_\tau = \hat{A}^c_\tau + \frac{\hat{\tau}^c_A}{\hat{\tau}^i_A}\left(s_{i\tau} - \hat{A}^c_\tau\right), \tag{17}$$

where the precision of this conditional expectation is

$$\hat{\tau}^i_A = \hat{\tau}^c_A + \alpha\frac{2\tau_A^{-1} + \left(1+\rho_A^2\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}{\tau_A^{-1} + \left(1+\left(\rho_A-\rho_\xi\right)\rho_A\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}\tau_s.$$

The expectations derived in Proposition 7 follow directly from the standard Bayes rule. In particular, in equation (17) each household's private signal $s_{i\tau}$ affects its expectation $\hat{A}^i_\tau$ linearly based on the "surprise" in the signal relative to the average expectation of all households $\hat{A}^c_\tau$.

According to the conjectured housing price in (14), the households' average expectation $\hat{A}^c_\tau$ is a key factor. Indeed, its dynamics derived in (16) is central to our later analysis of housing price momentum. Note that the coefficient of $q_{\tau-1}$ in (16) can be either positive, zero, or negative, depending on the sign of $2\rho_A - \left(1+\rho_A^2\right)\rho_\xi$. That is, a higher housing price in the previous period can cause the average expectation of the households about the current period aggregate productivity to increase, stay flat, or decrease. To clarify the intuition, let us consider three cases.

First, suppose that $\rho_A = 1$ and $\rho_\xi = 0$, which imply that the shock to the aggregate productivity (i.e., demand shock) is perfectly persistent, while the supply shock is perfectly transitory. In this case, it follows directly from (16) that $\hat{A}^c_\tau$ is increasing with $q_{\tau-1}$. Note from (15) that

$$q_{\tau-1} = \rho_A \left(A_{\tau-2} + Z^A_{\tau-1} + \frac{p_\xi}{p_A} Z^\xi_{\tau-1} \right) \tag{18}$$

is a noisy signal for the prior period productivity shock $Z^A_{\tau-1}$ with the prior period supply shock $Z^\xi_{\tau-1}$ as informational noise. As $Z^A_{\tau-1}$ persists into the current period while $Z^\xi_{\tau-1}$ does not, a higher housing price in the prior period signals a larger productivity shock in the previous period and thus a higher aggregate productivity in this period as well. Consequently, the households' expectation of the current period productivity $\hat{A}^c_\tau$ increases with $q_{\tau-1}$. More generally, according to (16), this intuition holds true if $\frac{2\rho_A}{1+\rho_A^2} > \rho_\xi$.

Next, we consider the opposite case when $\rho_A = 0$ and $\rho_\xi = 1$, which imply that the productivity shock is perfectly transitory, while the supply shock is perfectly persistent. Then, it follows from (16) that $\hat{A}^c_\tau$ is decreasing with $q_{\tau-1}$. In this case, (18) shows that $q_{\tau-1}$ is a noisy signal of the prior period supply shock $Z^\xi_{\tau-1}$ with the prior period productivity shock $Z^A_{\tau-1}$ as the informational noise. As $Z^\xi_{\tau-1}$ persists into the current period while $Z^A_{\tau-1}$ does not, a higher housing price in the prior period signals a more negative supply shock. As a result of the persistence of supply shocks, the current period supply is likely to be tight as well, which in turn implies that the current period productivity is weaker, conditional on the current period housing price. Thus, the households' expectation of the current period productivity A_τ decreases with $q_{\tau-1}$. More generally, according to (16), this intuition holds if $\frac{2\rho_A}{1+\rho_A^2} < \rho_\xi$.

Finally, consider a balanced case that $\rho_A = 1$ and $\rho_\xi = 1$. In this case, shocks to both aggregate productivity and housing supply are perfectly persistent, and $\hat{A}^c_\tau$ does not change with $q_{\tau-1}$. This is because according to (18), $q_{\tau-1}$ is a noisy signal for both shocks $Z^A_{\tau-1}$ and $Z^\xi_{\tau-1}$ in the prior period. As both shocks are perfectly perisistent, the information in $q_{\tau-1}$ about $Z^A_{\tau-1}$ exactly offsets the information about $Z^\xi_{\tau-1}$ in the households' learning about the current period demand A_τ.[8]

[8] More precisely, note that in this case $q_{\tau-1} - A_{\tau-2} = Z^A_{\tau-1} + \frac{p_\xi}{p_A} Z^\xi_{\tau-1}$, and $q_\tau - A_{\tau-2} = Z^A_{\tau-1} + \frac{p_\xi}{p_A} Z^\xi_{\tau-1} + Z^A_\tau + \frac{p_\xi}{p_A} Z^\xi_\tau$. Thus, conditional on both $q_{\tau-1}$ and q_τ, the best estimator for $A_\tau = A_{\tau-2} + Z^A_{\tau-1} + Z^A_\tau$ is $A_{\tau-2} + \frac{\frac{p_A^2}{p_\xi^2}\tau_\xi}{\tau_A + \frac{p_A^2}{p_\xi^2}\tau_\xi} \left(q_\tau - A_{\tau-2} \right)$, which is independent of $q_{\tau-1}$.

Taken together, the relative persistence of the shocks to aggregate productivity and housing supply plays a critical role in determining the direction of the households' reaction to the previous period's housing price. If the productivity shocks are sufficiently more persistent than the supply shocks, the households' average expectation of the current period aggregate productivity rises with the previous period's housing price. On the other hand, if supply shocks are sufficiently more persistent than productivity shocks, the households' average expectation falls with the previous period's housing price. If the persistence of productivity shocks balances that of supply shocks, the households' average expectation does not change with the previous period's housing price. This intuition is useful in understanding housing price dynamics in equilibrium.

With the households' learning processes derived, we are now ready to analyze the housing market equilibrium. It is important to note that the setting for each generation of households resembles the static model analyzed in the previous section, except that their learning processes are more elaborate and now depend on the previous period's housing price. As a result, the housing and labor choices of each household follow directly from Proposition 2. By substituting each household's expectation from Proposition 7 into its housing demand and equating the aggregate housing demand to home builders' supply, we can derive the housing market equilibrium in each period as summarized by the following proposition.

Proposition 8 *The housing market equilibrium at τ is characterized by the following features: 1) The housing demand by household i is a log-linear function of its conditional expectations $\hat{A}_\tau^i$ and $\hat{A}_\tau^c$, and $\log P_H(\tau)$:*

$$\log H_i(\tau) = \frac{1+\psi}{\psi}\hat{A}_\tau^c + \frac{1+\psi}{\psi + \eta_c \hat{\tau}_A^c / \hat{\tau}_A^i}\left(\hat{A}_\tau^i - \hat{A}_\tau^c\right) - \frac{1+\psi-\eta_H}{\psi\eta_H}\log P_H(\tau) + h_0, \tag{19}$$

where h_0 is given by equation (49).

2) The housing price is a log-linear function of A_τ, $\hat{A}_\tau^c$, and ξ_τ:

$$\log P_H(\tau) = \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1}A_\tau + \frac{1+\psi-\psi k x}{\psi k + \frac{1+\psi}{\eta_H} - 1}\left(\hat{A}_\tau^c - A_\tau\right) - \frac{\psi k}{\psi k + \frac{1+\psi}{\eta_H} - 1}\xi_\tau + p_0, \tag{20}$$

where p_0 given by equation (57) and $x \in \left[0, \frac{1+\psi}{\psi k}\right]$ is a root of equation (60), which always exists and is unique under a sufficient condition given in equation (61).

3) As $\tau_s \nearrow \infty$, the housing equilibrium converges to the perfect-information benchmark characterized in Proposition 4.

Proposition 8 shows that our dynamic model maintains a tractable log-linear equilibrium as in the static model. Note several interesting features of this dynamic equilibrium. First, as each household's private signal $s_{i\tau}$ becomes infinitely precise, the informational frictions disappear and the housing market equilibrium in each period converges to the perfect-information benchmark characterized for the static economy.

Second, in the presence of informational frictions, equation (19) shows that the housing demand of individual households shares two common components, $\frac{1+\psi}{\psi}\hat{A}^c_\tau$ (determined by the households' average expectation) and $-\frac{1+\psi-\eta_H}{\psi\eta_H}\log P_H(\tau)+h_0$ (the common downward sloping demand curve). Individual demand differs in the term $\frac{1+\psi}{\psi+\eta_c\hat{\tau}^c_A/\hat{\tau}^i_A}\left(\hat{A}^i_\tau-\hat{A}^c_\tau\right)$, which is determined by the deviation of each household's expectation from the average expectation as a result of each household's private information. Then, it is straightforward to see that the aggregate housing demand is determined by the households' average expectation.

Finally, equation (20) shows that the housing price differs from that in the perfect-information benchmark only by the term $\frac{1+\psi-\psi kx}{\psi k+\frac{1+\psi}{\eta_H}-1}\left(\hat{A}^c_\tau-A_\tau\right)$, which is determined by the deviation of the households' average expectation from the actual value of A_τ. That is, the learning error in the average expectation determines the housing price dynamics. As we will show, in the presence of informational frictions, several factors such as supply elasticity, persistence of supply and productivity shocks, and the degree of complementarity in the households' goods consumption can affect the housing price dynamics, beyond their standard effects through the supply and demand of housing, by affecting the dynamics of the households' average expectation.

2.3 Housing cycles

The housing literature, e.g., Case and Shiller (1989) and Glaeser and Gyourko (2006), has documented a rich pattern of short-run momentum and long-run reversals in housing prices. More precisely, there are positive auto-correlations of housing price changes at one year frequencies and mean-reversion over longer periods. The presence of mean-reversion in both supply and demand shocks makes it easy to explain the long-run mean-reversion in housing prices, but makes it even more challenging to understand the presence of short-run price momentum. Indeed, Glaeser and Gyourko (2006) also find it difficult to explain housing price momentum in their calibration of a dynamic model of housing in a spatial equilibrium. We now examine whether learning can help resolve this challenge. In particular, we focus on analyzing whether learning from past housing prices can lead to price momentum.

To facilitate our analysis, we define

$$R_H(t_1, t_2) = \log P_H(t_2) - \log P_H(t_1)$$

as the housing return from t_1 to t_2. We evaluate short-run price momentum by measuring the auto-correlation between housing returns $R_H(\tau - 1, \tau)$ and $R_H(\tau, \tau + 1)$ over two consecutive periods:

$$\gamma_1 = \frac{Cov\left[R_H(\tau-1,\tau), R_H(\tau,\tau+1) \mid \mathcal{I}_{\text{ex},\tau}\right]}{Var\left[R_H(\tau-1,\tau) | \mathcal{I}_{\text{ex},\tau}\right]^{1/2} Var\left[R_H(\tau,\tau+1) | \mathcal{I}_{\text{ex},\tau}\right]^{1/2}}.$$

This auto-correlation is computed based on the ex ante information set $\mathcal{I}_{\text{ex},\tau} = \left\{\{A_t, \xi_t\}_{t \leq \tau-2}\right\}$, which includes the revealed aggregate productivity and housing supply up to $\tau - 2$ and does not include any household's private signal or the housing prices at $\tau - 1$ and τ. To examine long-run price reversals, we measure the auto-correlation between housing returns $R_H(\tau - 1, \tau)$ and $R_H(\tau + 1, \tau + 3)$:

$$\gamma_3 = \frac{Cov\left[R_H(\tau-1,\tau), R_H(\tau+1,\tau+3) \mid \mathcal{I}_{\text{ex},\tau}\right]}{Var\left[R_H(\tau-1,\tau) | \mathcal{I}_{\text{ex},\tau}\right]^{1/2} Var\left[R_H(\tau+1,\tau+3) | \mathcal{I}_{\text{ex},\tau}\right]^{1/2}}.$$

In γ_3, we skip the period from τ to $\tau + 1$, which is right after $R_H(\tau - 1, \tau)$, to isolate the longer run auto-correlation from being contaminated by the potential short-run price momentum in $R_H(\tau, \tau + 1)$.[9]

To evaluate the role of household learning in driving price momentum and reversals, it is useful to establish two sets of benchmark, one with both housing supply and aggregate productivity as random walks and the other without any informational frictions.

Random-walk benchmark We first present a benchmark, in which both housing supply and aggregate productivity are random walks (i.e., $\rho_A = 1$ and $\rho_\xi = 1$). By analyzing the housing price derived in Proposition 8, we prove in the following proposition that the housing price also follows a random walk even in the presence of informational frictions:

Proposition 9 *If $\rho_A = 1$ and $\rho_\xi = 1$, $Cov\left[R_H(\tau-1,\tau), R_H(\tau,\tau+n) \mid \mathcal{I}_{ex,\tau}\right] = 0$ for any $n > 0$ even if $\tau_s < \infty$.*

[9] The choice of the length of the second period from $\tau + 1$ to $\tau + 3$ is innocuous. We have also explored a longer period from $\tau + 1$ to $\tau + n$ with $n > 3$ and obtained qualitatively similar results as letting $n = 3$.

According to (20), the log housing price $\log P_H$ is a linear combination of A_τ, ξ_τ, and $\hat{A}_\tau^c$. If A_τ is a random walk, then $\hat{A}_\tau^c = E\left[A_\tau | \mathcal{I}_{c\tau}\right]$, as a conditional expectation of A_τ, is also a random walk and is uncorrelated with the past housing prices (which are in the set of conditioning information). It then follows that $\log P_H$ is a random walk too.

This random-walk benchmark shows that housing price momentum cannot arise when shocks to both housing supply and aggregate productivity are perfectly persistent. Can price momentum arise when these shocks are transitory? This seems less likely as the mean-reversion of supply and productivity gives a natural force for the housing price to reverse itself over time. Next, we illustrate this force for price reversals in another benchmark without any informational frictions.

Perfect-information benchmark Suppose that in each period τ, both A_τ and ξ_τ are perfectly observable to the households. Then, Proposition 4 gives the log housing price in each period:

$$\begin{aligned}\log P_H(\tau) \quad = \quad & \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1} A_\tau - \frac{\psi k}{\psi k + \frac{1+\psi}{\eta_H} - 1} \xi_\tau + \frac{\psi}{k\psi + \frac{1+\psi}{\eta_H} - 1} \log\left(\frac{1-\eta_H}{\eta_H}\right) \\ & + \frac{1+\psi}{2\left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)} \left(\eta_H (1-\eta_c)^2 + \eta_c\right) \tau_\varepsilon^{-1},\end{aligned}$$

which is a linear combination of the aggregate productivity and housing supply (i.e., A_τ and ξ_τ). As a consequence, the auto-correlations of $\log P_H(\tau)$ are directly determined by the auto-correlations of A_τ and ξ_τ. It is intuitive that if either of these variables displays mean-reversion, there will be mean-reversion in the housing price across all horizons. That is, both $\gamma_1 < 0$ and $\gamma_3 < 0$ if either $\rho_A < 1$ or $\rho_\xi < 1$. Consistent with the random-walk benchmark, $\gamma_1 = 0$ and $\gamma_3 = 0$ if $\rho_A = 1$ and $\rho_\xi = 1$. In this perfect-information benchmark, there is no price momentum in any horizon and the mean-reversion of either housing supply or aggregate productivity leads to housing price reversals across all horizons.

We obtain this perfect-information benchmark as the precision of the households' private signal $\tau_s \nearrow \infty$ (Proposition 8). Thus, by the continuity of the housing market equilibrium with respect to τ_s, we expect the properties of the perfect-information benchmark to hold when τ_s is sufficiently large. In this region, the mean-reversion of both housing supply and aggregate productivity gives a natural force for housing price reversals. This benchmark property makes it even more curious whether and how informational frictions can lead to

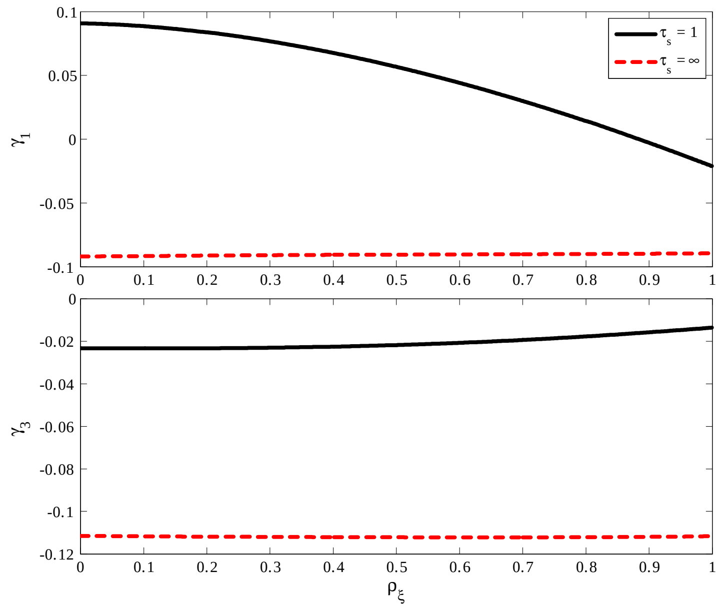

Figure 2: Short-run and long-run auto-correlations in housing price versus persistence of supply shocks. This figure is based on the following parameter values: $\rho_A = .9$, $k = 0.5$, $\eta_c = 0.5$, $\psi = \eta_H = 0.5$, $\tau_A = 10$, and $\tau_\xi = 100$.

price momentum when shocks to both housing supply and aggregate productivity mean-revert over time.

To address this issue, we use a series of numerical examples to illustrate whether housing price momentum can arise in equilibrium and how different model parameters contribute to the price momentum, by plotting γ_1 and γ_3 against several key parameters. We are particularly interested in the roles played by τ_s (precision of the households' private information), ρ_A (the persistence of aggregate productivity), ρ_ξ (the persistence of housing supply), k (supply elasticity), and η_c (degree of complementarity in households' goods consumption, which determines complementarity in households' housing demand). In this illustration, we use the following baseline values for the key parameters: $\tau_s = 1$, $\rho_A = .9$, $\rho_\xi = 0.75$, $k = 0.5$, and $\eta_c = 0.5$, as well the following value for the other parameters: $\psi = \eta_H = 0.5$, $\tau_A = 10$, and $\tau_\xi = 100$.

Persistence of shocks In Figure 2, we plot γ_1 and γ_3 against ρ_ξ (the persistence of housing supply), which goes from zero to one, for two levels of $\tau_s = 1$ and ∞ (plotted in solid and dashed lines, respectively). When $\tau_s = \infty$, we obtain the perfect-information benchmark. In this benchmark, both γ_1 and γ_3 are negative due to the mean-reversion of supply and demand shocks. The short-run auto-correlation γ_1 stays around -0.09 and displays a minimal, increasing relationship with ρ_ξ. The longer run auto-correlation γ_3 stays around -0.11 and is also insensitive to ρ_ξ.

When $\tau_s = 1$, the households face informational frictions in observing the current period A_τ and ξ_τ. Interestingly, γ_1 displays a substantial, decreasing relationship with ρ_ξ and, in particular, is positive when ρ_ξ is below a critical level around 0.8. That is, there is short-run price momentum. The mechanism for this short-run price momentum follows from our earlier discussion about the reaction of the households' average expectation $\hat{A}^c_\tau$ to the previous period housing price $q_{\tau-1}$. When the shocks to aggregate productivity are sufficiently more persistent than the shocks to housing supply, a higher housing price in the previous period signals a strong aggregate productivity in the current period. As a result, the households demand more housing and bid up the current housing price. This learning effect thus gives a force for short-run price momentum. When this momentum force is sufficiently strong to offset the reversal force from the mean-reversion of supply and productivity shocks, we observe price momentum. Figure 2 shows that this learning-based momentum mechanism dominates the reversal force when there is sufficient informational frictions (i.e., τ_s not too high) and the persistence of supply shocks ρ_ξ is sufficiently lower than the persistence of aggregate productivity shocks.[10]

Figure 2 also shows that when $\tau_s = 1$, the longer run auto-correlation γ_3 remains negative, even though its value is raised to a higher level around -0.02. This increase also reflects the force for price momentum induced by households' learning at a longer horizon.

Taken together, Figure 2 not only shows that household learning can lead to short-run housing price momentum, but also illustrates a critical role played by the relative persistence of housing supply and aggregate productivity shocks in affecting the households' learning from the previous period's housing price and thus generating price momentum. This learning effect of persistence of productivity and supply shocks is novel and, as far as we know, has not received much attention in the literature.

[10] As a smaller ρ_ξ also increases the mean-reversion of the supply shocks and thus strengthens the force for price reversals, the short-run price momentum may appear only in an intermediate range of ρ_ξ under certain parameter sets different from the set used in Figure 2.

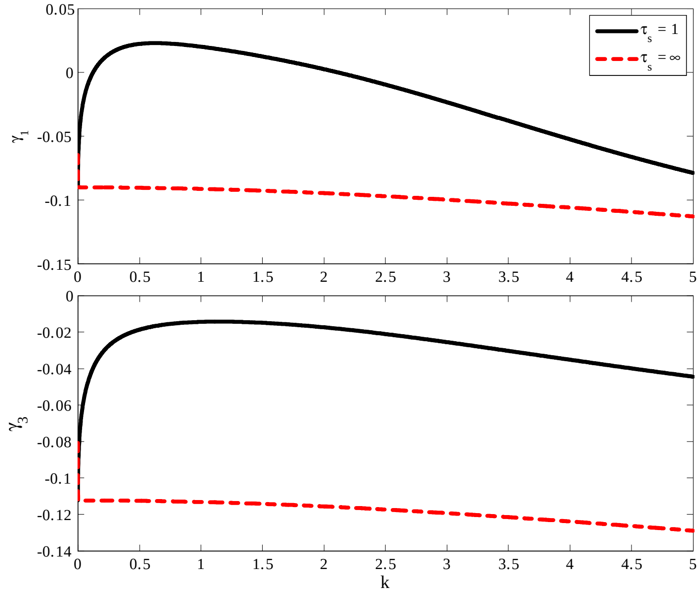

Figure 3: Short-run and long-run auto-correlations in housing price versus supply elasticity. This figure is based on the following parameter values: $\rho_A = .9$, $\rho_\xi = 0.75$, $\eta_c = 0.5$, $\psi = \eta_H = 0.5$, $\tau_A = 10$, and $\tau_\xi = 100$.

Supply elasticity In Figure 3, we plot γ_1 and γ_3 against k (supply elasticity), which goes from zero to five, and again for two levels of $\tau_s = 1$ and ∞. The dashed lines show that in the perfect-information benchmark, both γ_1 and γ_3 are negative and display a modest, decreasing relationship with k. Interestingly, in the presence of informational frictions ($\tau_s = 1$), the solid line shows that γ_1 exhibits a pronounced humped shape with k. Specifically, as k goes from 0 to around 0.5, γ_1 initially increases from a negative value of around -0.09 to a positive value of 0.02, which again illustrates the occurrence of short-run price momentum. As k goes further up above 0.5, γ_1 starts to decline and eventually becomes negative. As discussed before, the presence of short-run price momentum reflects the households' learning from the previous period's housing price. Thus, this humped shape in the plot of γ_1 shows that the households' learning from the previous period's price is particularly strong when supply elasticity is in an intermediate range around 0.5.

This outcome echoes our earlier discussion of the learning effect in the static model. As shown by Proposition 6, when the informational frictions faced by the households are sufficiently severe, there is a non-monotonic relationship between the learning effect and supply elasticity k. In particular, the learning effect is particularly strong in an intermediate range of k.

Figure 3 also shows that in the presence of informational frictions, while the longer run auto-correlation γ_3 stays negative, it also exhibits a humped shape with supply elasticity k. This humped shape again reflects household learning from the previous period's housing price, even though this learning effect is not sufficiently strong to offset the force for price reversals at this horizon.

Taken together, Figure 3 highlights an intriguing role played by local housing supply elasticity on household learning. While supply elasticity is widely-regarded in the housing literature as a critical variable for determining housing prices, the common wisdom is that it monotonically attenuates housing cycles. In sharp contrast to this common wisdom, our model shows that in the presence of informational frictions, supply elasticity can have a non-monotonic effect on the households' learning and thus on housing price dynamics. Interestingly, in the aftermath of the recent U.S. housing cycle in the 2000s, there is some evidence emerging to suggest that there might be a non-monotonic relationship between housing cycles and supply elasticity. In particular, Glaeser (2013) and Gao (2013) show that during this recent cycle, areas with relatively elastic supply like Las Vegas and Phoenix had nevertheless experienced dramatic boom and bust cycles similar to those in inelastic areas like New York and Los Angeles. Our model provides a new insight for understanding this puzzling phenomenon by highlighting the non-monotonic role played by supply elasticity in affecting household learning.[11]

Complementarity in housing demand Figure 4 plots γ_1 and γ_3 against η_c (the degree of complementarity in the households' goods consumption), which goes from zero to one, again for two levels of $\tau_s = 1$ and ∞. Note that η_c also determines the complementarity in the households' housing demand. When there is a greater complementarity in housing demand, each household puts a greater weight on the housing prices in both current and prior periods in its learning and a smaller weight on its own private signal, because the publicly observed housing prices also serve to coordinate the households' housing demand. Interestingly, Figure 4 shows that, in the presence of informational frictions ($\tau_s = 1$), γ_1 is positive and increases monotonically with η_c. This increasing relationship between γ_1 and η_c reflects the greater weight the households assign to the previous period's price in their learning.

[11] Nathanson and Zwick (2013) provide an explanation based on land speculation.

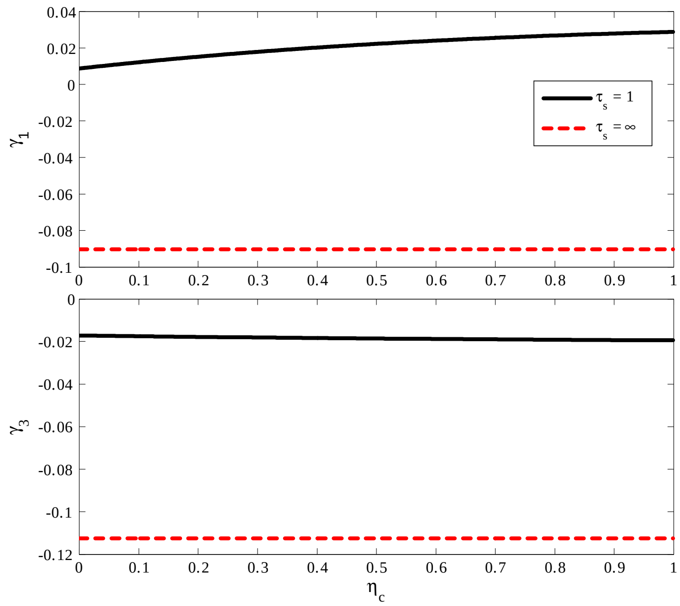

Figure 4: Short-run and long-run auto-correlations in housing price versus degree of complementarity of households' goods consumption. This figure is based on the following parameter values: $\rho_A = .9$, $\rho_\xi = 0.75$, $k = 0.5$, $\psi = \eta_H = 0.5$, $\tau_A = 10$, and $\tau_\xi = 100$.

At a deeper level, the greater tendency for households to follow the housing prices rather than their private signals also makes the housing prices less informative and thus indirectly increases the informational frictions faced by the households. In fact, one can see this effect directly from the equilibrium housing price derived in Proposition 8. In (20), the housing price $\log P_H(\tau)$ does not directly depend on either η_c or τ_s. Instead, η_c and τ_s affect the parameter x through equation (60), in which their roles are summarized by their joint appearance in the form of $\frac{\tau_s}{\psi+\eta_c}$. Through this form, a higher value of η_c offsets the effect of a higher value of τ_s. That is, a greater complementarity in housing demand exacerbates the effect of informational frictions.

3 Conclusion

This paper develops a tractable model to analyze information aggregation and learning in housing markets. In the presence of informational frictions regarding aggregate productivity and housing supply of a neighborhood, households face a realistic problem in learning about these fundamental variables with housing prices serving as important signals. Our model highlights how the households' learning interacts with characteristics endemic to local housing supply and demand to impact housing price dynamics. In particular, the learning effects are particularly strong when supply elasticity is in an intermediate range, and can cause short-run price momentum even when shocks to both housing supply and demand mean-revert over time.

Appendix Proofs of Propositions

A.1 Proof of Proposition 1

The first order conditions of household i's optimization problem in (2) respect to $C_i(i)$ and $C_j(i)$ at an interior point are

$$C_i(i) \quad : \quad \frac{\eta_H(1-\eta_c)}{C_i(i)} U\left(H_i, \{C_k(i)\}_{k\in[0,1]}\right) = \lambda_i P_i, \tag{21}$$

$$C_j(i) \quad : \quad \frac{\eta_H \eta_c}{C_j(i)} U\left(H_i, \{C_k(i)\}_{k\in[0,1]}\right) = \lambda_i P_j, \tag{22}$$

where λ_i is the Lagrange multiplier for the budget constraint. Dividing equations (21) and (22) leads to $\frac{\eta_c}{1-\eta_c}\frac{C_i(i)}{C_j(i)} = \frac{P_j}{P_i}$, which is equivalent to $P_j C_j(i) = \frac{\eta_c}{1-\eta_c} P_i C_i(i)$. By substituting this equation back to the household's budget constraint in (2), we obtain

$$C_i(i) = (1-\eta_c) e^{A_i} l_i.$$

The market clearing for the household's good requires that $\int_0^1 C_i(j)\,dj = e^{A_i} l_i$, which implies that $C_j(i) = \eta_c e^{A_i} l_i$. The symmetric problem of household j implies that $C_j(j) = (1-\eta_c) e^{A_j} l_j$, and the market clearing for its good implies $C_j(i) = \eta_c e^{A_j} l_j$.

The first order condition in equation (21) also gives the price of the good produced by household i. Since the household's budget constraint in (2) is entirely in nominal terms, the price system is only identified up to λ_i, the Lagrange multiplier. We therefore normalize λ_i to 1. Given that $C_i(i) = (1-\eta_c) e^{A_i} l_i$ and $C_j(i) = \eta_c e^{A_j} l_j$, it follows that

$$\begin{aligned} P_i &= \eta_H \frac{1-\eta_c}{C_i(i)} U\left(H_i, \{C_j(i)\}_{j\in[0,1]}\right) \\ &= \left(\frac{\eta_H}{1-\eta_H} H_i\right)^{1-\eta_H} \left(e^{A_i} l_i\right)^{(1-\eta_c)\eta_H - 1} \left(\int_{[0,1]/i} e^{A_j} l_j dj\right)^{\eta_c \eta_H}. \end{aligned}$$

A.2 Proof of Proposition 2

At $t = 0$, the first order conditions for household i's choices of H_i and l_i at an interior point are

$$H_i \;:\; \frac{1-\eta_H}{H_i} E\left[U\left(H_i, \{C_j(i)\}_{j\in[0,1]} \right) \middle| \mathcal{I}_i \right] = \lambda_i P_H, \tag{23}$$

$$l_i \;:\; l_i^{\psi} = \lambda_i E\left[P_i e^{A_i} \middle| \mathcal{I}_i \right]. \tag{24}$$

Taking expectations of equations (21) and (22) from Proposition 1, and imposing $\lambda_i = 1$ to equation (23), one arrives at

$$\eta_H (1-\eta_c) P_H H_i = (1-\eta_H) E\left[P_i C_i(i) \middle| \mathcal{I}_i \right] = (1-\eta_H)(1-\eta_c) E\left[P_i e^{A_i} l_i \middle| \mathcal{I}_i \right],$$

and therefore

$$P_H H_i = \frac{1-\eta_H}{\eta_H} l_i E\left[e^{A_i} P_i \middle| \mathcal{I}_i \right].$$

From equation (24), it follows that

$$l_i^{\psi} = E\left[e^{A_i} P_i \middle| \mathcal{I}_i \right],$$

from which we see that

$$l_i = \left(\frac{\eta_H}{1-\eta_H} P_H H_i \right)^{\frac{1}{1+\psi}}. \tag{25}$$

It follows then, from equations (5) and (25), that

$$\frac{\eta_H}{1-\eta_H} P_H H_i = \left(\frac{\eta_H}{1-\eta_H} \right)^{(1-\eta_H)\frac{1+\psi}{1+\psi-\eta_H}} E\left[e^{A_i(1-\eta_c)\eta_H} H_i^{1-\eta_H} \left(H_i^{-\frac{1}{1+\psi}} \int_{[0,1]/i} e^{A_j} H_j^{\frac{1}{1+\psi}} dj \right)^{\eta_c \eta_H} \middle| \mathcal{I}_i \right]^{\frac{1+\psi}{1+\psi-\eta_H}}.$$

Note that integrating over the continuum of other households' housing decisions is equivalent to taking an expectation with respect to another household's housing decision. We then obtain equation (6).

A.3 Proof of Proposition 3

We first conjecture that each household's housing purchasing and the housing price take the following log-linear forms:

$$\log H_i = h_P \log P_H + h_s s_i + h_0, \tag{26}$$

$$\log P_H = p_A A + p_\xi \xi + p_0, \tag{27}$$

where the coefficients h_0, h_P, h_s, p_0, p_A, and p_ξ will be determined by equilibrium conditions.

Given the conjectured functional form for H_i, we can expand equation (6). It follows that

$$\begin{aligned} & E\left[e^{A_i(1-\eta_c)\eta_H}\left(\int_{[0,1]/i} e^{A_j} H_j^{\frac{1}{1+\psi}} dj\right)^{\eta_c\eta_H} \middle| \mathcal{I}_i\right] \\ = \; & E\left[e^{A_i(1-\eta_c)\eta_H}\left(\int_{[0,1]/i} e^{A_j+\frac{1}{1+\psi}h_s s_j} e^{\frac{1}{1+\psi}(h_0+h_P\log P_H)} dj\right)^{\eta_c\eta_H} \middle| \mathcal{I}_i\right] \\ = \; & e^{\frac{\eta_c\eta_H}{1+\psi}(h_0+h_P\log P_H)} E\left[e^{A_i(1-\eta_c)\eta_H}\left(\int_{[0,1]/i} e^{A_j+\frac{1}{1+\psi}h_s s_j} dj\right)^{\eta_c\eta_H} \middle| \mathcal{I}_i\right] \\ = \; & e^{\frac{\eta_c\eta_H}{1+\psi}(h_0+h_P\log P_H)} e^{\frac{\eta_c\eta_H}{2}\left(\tau_\varepsilon^{-1}+\left(\frac{h_s}{1+\psi}\right)^2\tau_s^{-1}\right)} e^{\frac{1}{2}(1-\eta_c)^2\eta_H^2\tau_\varepsilon^{-1}} E\left[e^{A\eta_H\left(1+\eta_c\frac{h_s}{1+\psi}\right)} \middle| \mathcal{I}_i\right] \end{aligned}$$

where the last step uses the fact that A is independent of ε_j and the second step exploits the Law of Large Number for the continuum when integrating over households, which still holds if we subtract sets of measure 0 from the integral.

Define

$$q \equiv \frac{\log P_H - p_0 - p_\xi\bar{\xi}}{p_A} = A + \frac{p_\xi}{p_A}\left(\xi - \bar{\xi}\right),$$

which is a sufficient statistic of information contained in P_H. Then, conditional on observing its own signal s_i and the housing price P_H, household i's expectation of A is

$$E\left[A \mid s_i, \log P_H\right] = E\left[A \mid s_i, q\right] = \frac{1}{\tau_A + \tau_s + \frac{p_A^2}{p_\xi^2}\tau_\xi}\left(\tau_A\bar{A} + \tau_s s_i + \frac{p_A^2}{p_\xi^2}\tau_\xi q\right),$$

and its conditional variance of A is

$$Var\left[A \mid s_i, \log P_H\right] = \left(\tau_A + \tau_s + \frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}.$$

Therefore,

$$\begin{aligned} \log E\left[e^{A\eta_H\left(1+\eta_c\frac{h_A}{1+\psi}\right)} \middle| \mathcal{I}_i\right] = \; & \eta_H\left(1+\eta_c\frac{h_s}{1+\psi}\right)\left(\tau_A + \tau_s + \frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}\left(\tau_A\bar{A} + \tau_s s_i + \frac{p_A^2}{p_\xi^2}\tau_\xi q\right) \\ & + \frac{1}{2}\eta_H^2\left(1+\eta_c\frac{h_s}{1+\psi}\right)^2\left(\tau_A + \tau_s + \frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}. \end{aligned}$$

Then,

$$
\begin{aligned}
& E\left[e^{A_i(1-\eta_c)\eta_H}\left(\int_{[0,1]/i} e^{A_j} H_j^{\frac{1}{1+\psi}} dj\right)^{\eta_c\eta_H} \middle| \mathcal{I}_i\right] \\
= \; & \eta_H\left(1+\eta_c\frac{h_s}{1+\psi}\right)\left(\tau_A+\tau_s+\frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}\left(\tau_A\bar{A}+\tau_s s_i+\frac{p_A}{p_\xi^2}\tau_\xi\left(\log P_H-p_0-p_\xi\bar{\xi}\right)\right) \\
& +\frac{\eta_c\eta_H}{1+\psi}h_P\log P_H+\frac{\eta_c\eta_H}{1+\psi}h_0+\frac{1}{2}\left(\eta_c\eta_H\left(\tau_\varepsilon^{-1}+\left(\frac{h_s}{1+\psi}\right)^2\tau_s^{-1}\right)+(1-\eta_c)^2\eta_H^2\tau_\varepsilon^{-1}\right) \\
& +\frac{1}{2}\eta_H^2\left(1+\eta_c\frac{h_s}{1+\psi}\right)^2\left(\tau_A+\tau_s+\frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}.
\end{aligned}
$$

Substituting this expression into equation (6) and matching coefficients with the conjectured log-linear form in (26), it follows that

$$
h_s = \frac{1}{\psi+\eta_c}\left(1+\psi+\eta_c h_s\right)\left(\tau_A+\tau_s+\frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}\tau_s, \tag{28}
$$

$$
h_P = \frac{1}{\psi}\left(-\frac{1+\psi-\eta_H}{\eta_H}+\left(1+\psi+\eta_c h_s\right)\left(\tau_A+\tau_s+\frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}\frac{p_A}{p_\xi^2}\tau_\xi\right), \tag{29}
$$

$$
\begin{aligned}
h_0 = \; & \frac{1+\psi+\eta_c h_s}{\psi}\left(\tau_A+\tau_s+\frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}\left(\tau_A\bar{A}-\frac{p_A}{p_\xi^2}\tau_\xi\left(p_0+p_\xi\bar{\xi}\right)\right) \\
& +\log\left(\frac{1-\eta_H}{\eta_H}\right)+\frac{1}{2}\frac{1+\psi}{\psi}\left(\eta_c\left(\tau_\varepsilon^{-1}+\left(\frac{h_s}{1+\psi}\right)^2\tau_s^{-1}\right)+\eta_H(1-\eta_c)^2\tau_\varepsilon^{-1}\right) \\
& +\frac{1}{2}\frac{1+\psi}{\psi}\eta_H\left(1+\eta_c\frac{h_s}{1+\psi}\right)^2\left(\tau_A+\tau_s+\frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}.
\end{aligned} \tag{30}
$$

By aggregating households' housing demand and the builders' supply and imposing market clearing in the housing market, we have

$$
h_0+h_P\left(p_0+p_A A+p_\xi\xi\right)+h_s A+\frac{1}{2}h_s^2\tau_s^{-1}=k\left(\xi+p_0+p_A A+p_\xi\xi\right).
$$

Matching coefficients of the two sides of the equation leads to the following three conditions

$$
h_0+h_P p_0+\frac{1}{2}h_s^2\tau_s^{-1} = k p_0, \tag{31}
$$

$$
h_P p_A+h_s = k p_A, \tag{32}
$$

$$
h_P p_\xi = k+k p_\xi. \tag{33}
$$

It follows from equation (33) that

$$
p_\xi = -\frac{k}{k-h_P}, \tag{34}
$$

and further from equation (32) that

$$p_A = \frac{h_s}{k - h_P}. \tag{35}$$

Thus, by taking the ratio of equations (35) and (34), we arrive at

$$\frac{p_A}{p_\xi} = -\frac{h_A}{k}.$$

Substituting $\frac{p_A}{p_\xi} = -\frac{h_s}{k}$ into equation (28), and defining $b = -\frac{p_A}{p_\xi}$, we arrive at

$$kb^3\tau_\xi + \left(\tau_A + \frac{\psi}{\psi + \eta_c}\tau_s\right) kb - \frac{1+\psi}{\psi+\eta_c}\tau_s = 0. \tag{36}$$

We see from equation (36) that b has at most one positive root since the above 3rd order polynomial has only one sign change, by Descartes' Rule of Signs. By setting $b \to -b$, we see that there is no sign change, and therefore b has no negative root. Furthermore, by the Fundamental Theorem of Algebra, the roots of the polynomial (36) exist. Thus, it follows that equation (36) has only one real, nonnegative root $b \geq 0$ and 2 complex roots.[12]

Furthermore, by dropping the cubic term from equation (36), one arrives at an upper bound for b:

$$b \leq \frac{1+\psi}{k} \frac{\tau_s}{(\psi + \eta_c)\,\tau_A + \psi\tau_s}.$$

Since $\frac{h_s}{k} = -\frac{p_A}{p_\xi} = b$, we can recover $h_s = kb > 0$ and $p_\xi = -\frac{1}{b}p_A < 0$. From equation (29) and $b = -\frac{p_A}{p_\xi}$, it follows that

$$h_P = \frac{1}{\psi}\left(1 - \frac{1+\psi}{\eta_H} + \frac{b^2\tau_\xi}{\tau_A + \tau_s + b^2\tau_\xi}\left(1 + \psi + \eta_c kb\right)\frac{1}{p_A}\right). \tag{37}$$

From equation (33), one also has that $h_P = k\left(1 + p_\xi^{-1}\right)$. Since $p_\xi \leq 0$, it follows that $h_P < k$ whenever $k > 0$.

From $h_s = kb$ and equations (35) and (37), we arrive at

$$p_A = \left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)^{-1}\left(\psi kb + \frac{b^2\tau_\xi}{\tau_A + \tau_s + b^2\tau_\xi}\left(1 + \psi + \eta_c kb\right)\right) > 0. \tag{38}$$

One arrives at p_ξ from recognizing that $p_\xi = -\frac{1}{b}p_A$. Manipulating equation (36), we first we recognize that

$$1 + \psi + \eta_c kb = \left(\tau_A + \tau_s + b^2\tau_\xi\right)\left(\psi + \eta_c\right) kb\tau_s^{-1}. \tag{39}$$

[12] The uniqueness of the positive, real root also follows from the fact that the LHS of the polynomial equation is monotonically increasing in b.

Substituting equation (39) into equation (38), and invoking equation (36) to replace $k\tau_\xi b^3$, one arrives at

$$p_A = \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1} - \frac{(\psi + \eta_c)\, kb}{\psi k + \frac{1+\psi}{\eta_H} - 1} \tau_s^{-1} \tau_A. \tag{40}$$

From $h_A = kb$, $b = -\frac{p_A}{p_\xi}$, and equations (37), (31) and (30), one also finds that

$$\begin{aligned} p_0 \quad = \quad & \left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)^{-1} \left(\psi \log\left(\frac{1-\eta_H}{\eta_H}\right) + (1 + \psi + \eta_c kb) \frac{\tau_A \bar{A} + b\tau_\xi \bar{\xi}}{\tau_A + \tau_s + b^2 \tau_\xi} + \frac{\psi}{2} h_s^2 \tau_s^{-1} \right. \\ & + \frac{1+\psi}{2} \left(\eta_c \left(\tau_\varepsilon^{-1} + \left(\frac{kb}{1+\psi}\right)^2 \tau_s^{-1}\right) + \eta_H (1-\eta_c)^2 \tau_\varepsilon^{-1}\right) \\ & \left. + \frac{1+\psi}{2} \eta_H \left(1 + \frac{\eta_c kb}{1+\psi}\right)^2 \left(\tau_A + \tau_s + b^2 \tau_\xi\right)^{-1} \right). \end{aligned} \tag{41}$$

Given p_0, p_A, and $b = -\frac{p_A}{p_\xi}$, we can recover h_0 from equation (30).

Since we have explicit expressions for all other equilibrium objects as functions of b, and b exists and is unique, it follows that an equilibrium in the economy exists and is unique.

A.4 Proof of Proposition 4

When all households observe A directly, there are no longer information frictions in the economy. Since the households' idiosyncratic productivity components are unobservable, they are now symmetric. Then, it follows that $H_j = H_i = H$. Imposing this symmetry in equation (6), we see that each household's housing demand is then given by

$$\log H = \frac{1+\psi}{\psi} A - \frac{(1+\psi-\eta_H)}{\psi \eta_H} \log P_H + \log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{1+\psi}{2\psi} \left(\eta_H (1-\eta_c)^2 + \eta_c\right) \tau_\varepsilon^{-1}.$$

By market clearing, $\log H = k\left(\xi + \log P_H\right)$, it follows that

$$\begin{aligned} \log P_H \quad = \quad & \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1} A - \frac{\psi k}{\psi k + \frac{1+\psi}{\eta_H} - 1} \xi + \frac{\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1} \log\left(\frac{1-\eta_H}{\eta_H}\right) \\ & + \frac{1+\psi}{2\left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)} \left(\eta_H (1-\eta_c)^2 + \eta_c\right) \tau_\varepsilon^{-1}. \end{aligned}$$

This characterizes the economy in the limit as information frictions dissipate.

To see that the economy with information frictions (finite τ_s) converges to this perfect-information limit, we consider a sequence of τ_s that converges to ∞. From equation (36), it follows that, as $\tau_s \nearrow \infty$, $b \to \frac{1+\psi}{\psi k}$. Since $h_s = kb$, it follows that

$$h_s \to \frac{1+\psi}{\psi}.$$

Taking the limit $\tau_s \nearrow \infty$ in equation (38), recognizing that $h_s = kb$ remains finite in the limit, we see that

$$p_A \to \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1}.$$

Since $p_\xi = -\frac{1}{b} p_A$, it follows that

$$p_\xi \to -\frac{\psi k}{\psi k + \frac{1+\psi}{\eta_H} - 1}$$

In addition, from equation (37), we find that as $\tau_s \nearrow \infty$,

$$h_P \to \frac{1}{\psi}\left(1 - \frac{1+\psi}{\eta_H}\right)$$

Finally, from equations (41) and (30), it follows that

$$\begin{aligned} p_0 &\to \frac{\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1} \log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{1+\psi}{2\left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)} \left(\eta_H (1-\eta_c)^2 + \eta_c\right) \tau_\varepsilon^{-1}, \\ h_0 &\to \log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{1+\psi}{2\psi} \left(\eta_H (1-\eta_c)^2 + \eta_c\right) \tau_\varepsilon^{-1}. \end{aligned}$$

Thus, we see that the economy with information frictions converges to the perfect-information benchmark as $\tau_s \nearrow \infty$.

A.5 Proof of Proposition 5

From equation (40), it is clear that

$$p_A = \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1} - \frac{(\psi + \eta_c)\, kb}{\psi k + \frac{1+\psi}{\eta_H} - 1} \tau_s^{-1} \tau_A < \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1}.$$

Thus, it follows that p_A is always lower than its corresponding values in the perfect-information benchmark.

Similarly, since $p_\xi = -\frac{1}{b} p_A$, it follows with equation (38) that we can express p_ξ as

$$p_\xi = -\frac{\psi k}{\psi k + \frac{1+\psi}{\eta_H} - 1} - \frac{1}{\psi k + \frac{1+\psi}{\eta_H} - 1}\left(\psi k + \frac{b\tau_\xi}{\tau_A + \tau_s + b^2 \tau_\xi}(1 + \psi + \eta_c kb)\right) < -\frac{\psi k}{\psi k + \frac{1+\psi}{\eta_H} - 1},$$

which is the corresponding value of p_ξ in the perfect-information benchmark.

We now prove that p_A is increasing with τ_s and decreasing with η_c. Note that b is determined by the polynomial equation (36). We define the LHS of the equation as $G(b)$.

Comparative statics of b with respect to η_c reveal, by the Implicit Function Theorem and invoking equation (36), that

$$\frac{\partial b}{\partial \eta_c}=-\frac{\partial G/\partial \eta_c}{\partial G/\partial b}=-\frac{1+\psi-\psi kb}{3kb^2\tau_\xi+\left(\tau_A+\frac{\psi}{\psi+\eta_c}\tau_s\right)k}\frac{\tau_s}{\left(\psi+\eta_c\right)^2}=-\frac{1+\psi-\psi kb}{2kb^3\tau_\xi+\frac{1+\psi}{\psi+\eta_c}\tau_s}\frac{\tau_s b}{\left(\psi+\eta_c\right)^2}$$

Since, from Proposition 3, $0\le b\le \frac{1+\psi}{k}\frac{\tau_s}{(\psi+\eta_c)\tau_A+\psi\tau_s}$, it follows that

$$1+\psi-\psi kb\ge\left(1+\psi\right)\frac{\left(\psi+\eta_c\right)\tau_A}{\left(\psi+\eta_c\right)\tau_A+\psi\tau_s}>0,$$

Thus $\frac{\partial b}{\partial \eta_c}<0$. Similarly,

$$\frac{\partial b}{\partial \tau_s}=-\frac{\partial G/\partial \tau_s}{\partial G/\partial b}=\frac{1+\psi-\psi kb}{3kb^3\tau_\xi+\left(\tau_A+\frac{\psi}{\psi+\eta_c}\tau_s\right)kb}\frac{1}{\psi+\eta_c}=\frac{1+\psi-\psi kb}{2kb^3\tau_\xi+\frac{1+\psi}{\psi+\eta_c}\tau_s}\frac{b}{\psi+\eta_c}>0,$$

since $0\le b\le\frac{1+\psi}{\psi k}$.

From the expression for p_A in Proposition 3,

$$\frac{\partial p_A}{\partial \eta_c}=-\frac{k\tau_s^{-1}\tau_A}{\psi k+\frac{1+\psi}{\eta_H}-1}\left(b+\left(\psi+\eta_c\right)\frac{\partial b}{\partial \eta_c}\right)=-\frac{k\tau_s^{-1}\tau_A b}{\psi k+\frac{1+\psi}{\eta_H}-1}\left(\frac{2kb^3\tau_\xi\left(\psi+\eta_c\right)+\psi kb\tau_s}{2kb^3\tau_\xi\left(\psi+\eta_c\right)+\left(1+\psi\right)\tau_s}\right)<0.$$

Similarly, with respect to τ_s, we have

$$\frac{\partial p_A}{\partial \tau_s}=-\frac{\left(\psi+\eta_c\right)k\tau_A\tau_s^{-1}}{\psi k+\frac{1+\psi}{\eta_H}-1}\left(\frac{\partial b}{\partial \tau_s}-b\tau_s^{-1}\right)=\frac{\left(\psi+\eta_c\right)k\tau_A\tau_s^{-2}b}{\psi k+\frac{1+\psi}{\eta_H}-1}\frac{\psi kb\tau_s+2kb^3\tau_\xi\left(\psi+\eta_c\right)}{2kb^3\tau_\xi\left(\psi+\eta_c\right)+\left(1+\psi\right)\tau_s}>0.$$

A.6 Proof of Proposition 6

We first consider the limiting case for the economy as $k\to\infty$. From equation (36), it is apparent that as $k\to\infty$, the first and third terms dominate and either $b=0$ or $b=\pm i\sqrt{\tau_\xi^{-1}\left(\tau_A+\frac{\psi}{\psi+\eta_c}\tau_\varepsilon\right)}$. Thus, as $k\to\infty$, one has that $b\to 0$. Therefore, $p_A\to 0$ and the housing price is completely driven by the supply shock ξ. Given that $b=-\frac{p_A}{p_\xi}$, we can rewrite equation (37) as

$$h_P=\frac{1}{\psi}\left(1-\frac{1+\psi}{\eta_H}-\left(1+\psi+\eta_c kb\right)\frac{b\tau_\xi}{\tau_A+\tau_\varepsilon+b^2\tau_\xi}\frac{1}{p_\xi}\right).$$

Let us assume that p_ξ is bounded in the limit. Then, as $k\to\infty$ and $b\to 0$, one has that

$$h_P\to-\frac{1}{\psi}\left(\frac{1+\psi}{\eta_H}-1\right).$$

Then, it follows from equation (34) that $p_\xi\to -1$, since h_P remains bounded in the limit. This confirms the initial conjecture that p_ξ is bounded as $k\to\infty$.

From equation (28), it is straightforward to see that, as $k \to \infty$,

$$h_s \to (1+\psi)\left(\psi + \frac{\eta_c \tau_A}{\tau_A + \tau_\varepsilon}\right)^{-1} \frac{\tau_\varepsilon}{\tau_A + \tau_\varepsilon}.$$

Since h_A remains bounded in the limit, it is easy to see from equation (41) that $p_0 \to 0$ as $k \to \infty$. It further follows from equation (30) that in the limit

$$\begin{aligned} h_0 \quad \to \quad & \log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{1+\psi}{2\psi}\left(\eta_c\left(1+\frac{h_A}{1+\psi}\right)^2 + (1-\eta_c)^2 \eta_H\right)\tau_\varepsilon^{-1} \\ & + \frac{1+\psi}{2\psi}\eta_H\left(1+\eta_c\frac{h_A}{1+\psi}\right)^2 (\tau_A+\tau_s)^{-1} + \frac{1+\psi+\eta_c h_A}{\psi}\frac{\tau_A}{\tau_A+\tau_s}\bar{A}. \end{aligned} \tag{42}$$

To consider the case $k \to 0$, we first rewrite equation (36) as

$$k\tau_\xi + \left(\tau_A + \frac{\psi}{\psi+\eta_c}\tau_s\right)kb^{-2} - \frac{1+\psi}{\psi+\eta_c}\tau_s b^{-3} = 0.$$

We see that, as $k \to 0$, the last term of the equation dominates and $b \to \infty$. Thus as $k \to 0$, $b \to \infty$, and therefore $p_\xi \to 0$ and the demand shock A completely drives the housing price. From equation (28), it follows that as $k \to 0$ one has that $h_s \to 0$. Since h_A remains bounded in the limit, and $h_A = kb$, it follows from equation (38) that

$$p_A \to \frac{1+\psi}{\frac{1+\psi}{\eta_H} - 1}.$$

Since $h_s \to 0$, and h_A and p_A remain bounded as $k \to 0$, we also see from equation (32) that $h_P \to 0$. Since h_A remains bounded in the limit, it is easy to see from equation (41) that as $k \to 0$.

$$p_0 \to \left(\frac{1+\psi}{\eta_H} - 1\right)^{-1}\left(\psi\log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{1+\psi}{2}\left(\eta_c + \eta_H(1-\eta_c)^2\right)\tau_\varepsilon^{-1}\right). \tag{43}$$

It further follows from equation (30) that in the limit $h_0 \to 0$.

To find the conditions under which $h_P \geq 0$, we rewrite $h_P \geq 0$ with equations (37) and (38) as

$$b^2\tau_\xi(1+\psi+\eta_c kb)\frac{k\eta_H}{1+\psi-\eta_H} \geq k(\tau_A+\tau_s)b + kb^3\tau_\xi.$$

Then, substituting for $k\tau_\xi b^3$ with equation (36), the above condition reduces to

$$b \geq b^* = \sqrt{\frac{1+\psi-\eta_H}{k(\psi+\eta_c)\eta_H}\tau_\xi^{-1}\tau_s}.$$

It follows that, since $b^* \leq b$ and the LHS of equation (36) is monotonically increasing in b, that the LHS of equation (36) evaluated at b^* must be negative. Then,

$$\left(\sqrt{k}\right)^2 - \left(\frac{1+\psi}{2\left(\tau_A\left(\psi+\eta_c\right)+\psi\tau_s\right)}\sqrt{\frac{\left(\psi+\eta_c\right)\eta_H}{1+\psi-\eta_H}\tau_s\tau_\xi}\right)\sqrt{k} + \frac{1+\psi-\eta_H}{\eta_H}\frac{\tau_s}{\tau_A\left(\psi+\eta_c\right)+\psi\tau_s} \leq 0.$$

Thus, it follows that it is necessary and sufficient for $h_P \geq 0$ that

$$\sqrt{k} \in \left[\begin{array}{c} \max\left\{\frac{1+\psi}{2(\tau_A(\psi+\eta_c)+\psi\tau_s)}\sqrt{\frac{(\psi+\eta_c)\eta_H}{1+\psi-\eta_H}\tau_s\tau_\xi} - \gamma_\xi, 0\right\}, \\ \frac{1+\psi}{2(\tau_A(\psi+\eta_c)+\psi\tau_s)}\sqrt{\frac{(\psi+\eta_c)\eta_H}{1+\psi-\eta_H}\tau_s\tau_\xi} + \gamma_\xi \end{array}\right],$$

where

$$\gamma_\xi = \frac{\tau_s^{1/2}}{\tau_A\left(\psi+\eta_c\right)+\psi\tau_s}\sqrt{(1+\psi)^2\frac{\left(\psi+\eta_c\right)\eta_H}{1+\psi-\eta_H}\tau_\xi - 4\frac{1+\psi-\eta_H}{\eta_H}\left(\tau_A\left(\psi+\eta_c\right)+\psi\tau_s\right)},$$

provided that

$$\tau_s \leq \frac{\psi+\eta_c}{\psi}\left(\frac{1}{4}\left(\frac{(1+\psi)\eta_H}{1+\psi-\eta_H}\right)^2\tau_\xi - \tau_A\right).$$

Since $\gamma_\xi \geq 0$, the upper bound in the range of $\sqrt{k}$ over which $h_P \geq 0$ is positive, and therefore a range of k always exists for which $h_P \geq 0$ when τ_s is sufficiently small.

A.7 Proof of Proposition 7

The vector of household i's signals is

$$Q_{i\tau} = \left[\begin{array}{c} s_{i\tau} \\ q_\tau \\ q_{\tau-1} \end{array}\right] = \left[\begin{array}{c} A_\tau + \nu_{i\tau} \\ A_\tau + \frac{p_\xi}{p_A}\left(Z_\tau^\xi + \rho_\xi Z_{\tau-1}^\xi\right) \\ \rho_A\left(A_{\tau-1} + \frac{p_\xi}{p_A}Z_{\tau-1}^\xi\right) \end{array}\right].$$

We can normalize the noise variables in the signals to standard normal distribution:

$$Q_{i\tau} = \left[\begin{array}{c} A_\tau + \sqrt{b}u_{1\tau} \\ A_\tau + \sqrt{c}\left(u_{2\tau} + \rho_\xi u_{3\tau}\right) \\ \sqrt{\beta}A_{\tau-1} + \sqrt{\beta c}u_{3\tau} \end{array}\right]$$

where $\beta = \rho_A^2$, $b = \tau_s^{-1}$, $c = \frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}$, and $[u_{1\tau}, u_{2\tau}, u_{3\tau}]' \sim \mathcal{N}(0, I_3)$. Since the household believes that $A_\tau = \rho_A^2 A_{\tau-2} + \sqrt{\beta}Z_{\tau-1}^A + Z_\tau^A$, they have a Gaussian prior over A_τ:

$$A_\tau \sim \mathcal{N}\left(\rho_A^2 A_{\tau-2}, (1+\beta)a\right),$$

where $a = \tau_A^{-1}$ and $\tilde{Z}^A_{\tau-2}$ is the household's inference of $Z^A_{\tau-2}$ based on the observation of the history of $\{A_u\}_{u=1}^{\tau-2}$. Also note that the household also believes that $A_{\tau-1} = \rho_A A_{\tau-2} + Z^A_{\tau-1}$ and thus $E\left[A_{\tau-1} \left| \{A_u\}_{u=1}^{\tau-2}\right.\right] = \rho_A A_{\tau-2}$. Thus

$$E\left[Q_{i\tau} \left| \{A_u\}_{u=1}^{\tau-2}\right.\right] = \begin{bmatrix} \rho_A^2 A_{\tau-2} \\ \rho_A^2 A_{\tau-2} \\ \sqrt{\beta}\rho_A A_{\tau-2} \end{bmatrix}.$$

Then, conditional on observing the vector of signals $Q_{i\tau}$, household i arrives to its conditional belief $\hat{A}^i_\tau = E\left[A_\tau \mid \mathcal{I}_{i\tau}\right]$:

$$\hat{A}^i_\tau = A_{\tau-2} + Cov\left[A_\tau, Q_{i\tau} \left| \{A_u\}_{u=1}^{\tau-2}\right.\right]' Var\left[Q_{i\tau} \left| \{A_u\}_{u=1}^{\tau-2}\right.\right]^{-1} \left\{Q_{i\tau} - E\left[Q_{i\tau} \left| \{A_u\}_{u=1}^{\tau-2}\right.\right]\right\},$$

where

$$Var\left[Q_{i\tau}\right] = \begin{bmatrix} (1+\beta)a + b & (1+\beta)a & \beta a \\ (1+\beta)a & (1+\beta)a + 2c & \beta\left(a + \beta^{-1/2}\rho_\xi c\right) \\ \beta a & \beta\left(a + \beta^{-1/2}\rho_\xi c\right) & \beta(a+c) \end{bmatrix},$$

and

$$Cov\left[A_\tau, Q_{i\tau}\right]' = \begin{bmatrix} (1+\beta)a & (1+\beta)a & \beta a \end{bmatrix}.$$

The inverse of this matrix can be found using co-factors and the determinant simplified through a series of row manipulations. Following this approach, one can arrive at

$$\begin{aligned}
& Cov\left[A_\tau, Q_{i\tau}\right]' Var\left[Q_{i\tau}\right]^{-1} \\
= \; & \frac{\begin{bmatrix} \beta ac\left(2a + (1+\beta)c\right) & \beta ab\left(a + \left(1+\beta-\rho_\xi\sqrt{\beta}\right)c\right) & abc\left(2\beta - (1+\beta)\rho_\xi\sqrt{\beta}\right) \end{bmatrix}}{\beta ab\left(a + \left(1+\beta-\rho_\xi\sqrt{\beta}\right)c\right) - \beta abc\rho_\xi\sqrt{\beta} + 2\beta ac(a+b) + \beta c^2\left((1+\beta)a + b\right)} \\
= \; & \frac{1}{\frac{1}{b}\frac{2a+(1+\beta)c}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c} + \frac{1}{c} + \frac{2}{(1+\beta)a} + \frac{\beta a+\rho_\xi\sqrt{\beta}c}{(1+\beta)a}\frac{1}{\beta}\frac{2\beta-(1+\beta)\rho_\xi\sqrt{\beta}}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c}} \begin{bmatrix} \frac{1}{b}\frac{2a+(1+\beta)c}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c} & \frac{1}{c} & \frac{1}{\beta}\frac{2\beta-(1+\beta)\rho_\xi\sqrt{\beta}}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c} \end{bmatrix}.
\end{aligned}$$

Substituting this expression into $\hat{A}^i_\tau$, we obtain

$$\begin{aligned}
\hat{A}^i_\tau = \; & \frac{2 + \left(\left(\beta - \frac{(1+\beta)\sqrt{\beta}}{\rho_A}\right)a + \rho_\xi\sqrt{\beta}c\right)\frac{1}{\beta}\frac{2\beta-(1+\beta)\rho_\xi\sqrt{\beta}}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c}}{\frac{1}{b}\frac{2a+(1+\beta)c}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c} + \frac{1}{c} + \frac{2}{(1+\beta)a} + \frac{\beta a+\rho_\xi\sqrt{\beta}c}{(1+\beta)a}\frac{1}{\beta}\frac{2\beta-(1+\beta)\rho_\xi\sqrt{\beta}}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c}} \frac{\rho_A^2}{(1+\beta)a} A_{\tau-2} \\
& + \frac{\begin{bmatrix} \frac{1}{b}\frac{2a+(1+\beta)c}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c} & \frac{1}{c} & \frac{1}{\beta}\frac{2\beta-(1+\beta)\rho_\xi\sqrt{\beta}}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c} \end{bmatrix}}{\frac{1}{b}\frac{2a+(1+\beta)c}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c} + \frac{1}{c} + \frac{2}{(1+\beta)a} + \frac{\beta a+\rho_\xi\sqrt{\beta}c}{(1+\beta)a}\frac{1}{\beta}\frac{2\beta-(1+\beta)\rho_\xi\sqrt{\beta}}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c}} Q_{i\tau}.
\end{aligned}$$

Furthermore, the conditional precision of household $i's$ beliefs $\hat{\tau}^i_A$ is given by

$$\hat{\tau}^i_A = Var\left[A_\tau \mid \mathcal{I}_{i\tau}\right]^{-1} = \left((1+\beta)a - Cov\left[A_\tau, Q_{i\tau}\right]' Var\left[Q_{i\tau}\right]^{-1} Cov\left[A_\tau, Q_{i\tau}\right]\right)^{-1},$$

from which it is straightforward to see that

$$\hat{\tau}_A^i = \left(2+\rho_\xi c \frac{2\sqrt{\beta}-(1+\beta)\rho_\xi}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c}\right)^{-1} \cdot \left(\frac{1}{b}\frac{2a+(1+\beta)c}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c}+\frac{1}{c}+\frac{2}{(1+\beta)a}+\frac{\beta a+\rho_\xi\sqrt{\beta}c}{(1+\beta)a}\frac{1}{\beta}\frac{2\beta-(1+\beta)\rho_\xi\sqrt{\beta}}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c}\right).$$

We can similarly redefine the vector of public signals $Q_{c\tau}$ as

$$Q_{c\tau}=\begin{bmatrix} q_\tau \\ q_{\tau-1}\end{bmatrix}=\begin{bmatrix} A_\tau+\sqrt{c}\left(u_{2\tau}+u_{3\tau}\right) \\ \sqrt{\beta}A_{\tau-1}+\sqrt{\beta c}u_{3\tau}\end{bmatrix}.$$

The average belief of the community $\hat{A}_\tau^c = E\left[A_\tau \mid \mathcal{I}_{c\tau}\right]$ is given by

$$\hat{A}_\tau^c = A_{\tau-2}+Cov\left[A_\tau,Q_{c\tau}\right]' Var\left[Q_{c\tau}\right]^{-1}\left\{Q_{c\tau}-E\left[Q_{c\tau}\left|\{A_u\}_{u=1}^{\tau-2}\right.\right]\right\}.$$

Note that

$$Var\left[Q_{c\tau}\right]=\begin{bmatrix}(1+\beta)a+2c & \beta\left(a+\beta^{-1/2}\rho_\xi c\right) \\ \beta\left(a+\beta^{-1/2}\rho_\xi c\right) & \beta(a+c)\end{bmatrix},$$

and

$$Cov\left[A_\tau,Q_{c\tau}\right]'=\begin{bmatrix}(1+\beta)a & \beta a\end{bmatrix}.$$

It is straightforward to derive

$$\begin{aligned} & Cov\left[A_\tau,Q_{c\tau}\right]' Var\left[Q_{c\tau}\right]^{-1} \\ = & \frac{\begin{bmatrix}\beta a\left(a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c\right) & \beta a c\left[2-(1+\beta)\beta^{-1/2}\right]\end{bmatrix}}{\beta a\left(a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c\right)+\beta c\left(c+2a-a\sqrt{\beta}\right)} \\ = & \frac{1}{\frac{1}{c}+\frac{2}{(1+\beta)a}+\frac{\beta a+\rho_\xi\sqrt{\beta}c}{(1+\beta)a}\frac{1}{\beta}\frac{2\beta-(1+\beta)\rho_\xi\sqrt{\beta}}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c}}\begin{bmatrix}\frac{1}{c} & \frac{1}{\beta}\frac{2\beta-(1+\beta)\rho_\xi\sqrt{\beta}}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c}\end{bmatrix}.\end{aligned}$$

Substituting this expression into $\hat{A}_\tau^c$, we see that

$$\begin{aligned}\hat{A}_\tau^c = & \frac{2+\left(\left(\beta-\frac{(1+\beta)\sqrt{\beta}}{\rho_A}\right)a+\rho_\xi\sqrt{\beta}c\right)\frac{1}{\beta}\frac{2\beta-(1+\beta)\rho_\xi\sqrt{\beta}}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c}}{\frac{1}{c}+\frac{2}{(1+\beta)a}+\frac{\beta a+\rho_\xi\sqrt{\beta}c}{(1+\beta)a}\frac{1}{\beta}\frac{2\beta-(1+\beta)\rho_\xi\sqrt{\beta}}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c}}\frac{\rho_A^2}{(1+\beta)a}A_{\tau-2} \\ & +\frac{\begin{bmatrix}\frac{1}{c} & \frac{1}{\beta}\frac{2\beta-(1+\beta)\rho_\xi\sqrt{\beta}}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c}\end{bmatrix}}{\frac{1}{c}+\frac{2}{(1+\beta)a}+\frac{\beta a+\rho_\xi\sqrt{\beta}c}{(1+\beta)a}\frac{1}{\beta}\frac{2\beta-(1+\beta)\rho_\xi\sqrt{\beta}}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c}}Q_{c\tau},\end{aligned}$$

and similarly the conditional precision of the average community beliefs is

$$\hat{\tau}_A^c = \left(2+\rho_\xi c\frac{2\sqrt{\beta}-(1+\beta)\rho_\xi}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c}\right)^{-1}\left(\frac{1}{c}+\frac{2}{(1+\beta)a}+\frac{\beta a+\rho_\xi\sqrt{\beta}c}{(1+\beta)a}\frac{1}{\beta}\frac{2\beta-(1+\beta)\rho_\xi\sqrt{\beta}}{a+\left(1+\beta-\rho_\xi\sqrt{\beta}\right)c}\right).$$

By directly comparing the expressions for $\hat{A}_\tau^i$ and $\hat{A}_\tau^c$, one can write $\hat{A}_\tau^i$ as a linear expansion of $\hat{A}_\tau^c$ and $s_{i\tau}$ as stated in the proposition.

A.8 Proof of Proposition 8

We first conjecture that $\log P_H(\tau)$ and $\log H_i(\tau)$ take the following log-linear forms

$$\begin{aligned}\log P_H(\tau) &= p_A A_\tau + p_{Ac}\hat{A}^c_\tau + p_\xi \xi_\tau + p_0, \\ \log H_i(\tau) &= h_{Ai}\hat{A}^i_\tau + h_{Ac}\hat{A}^c_\tau + h_P \log P_H(\tau) + h_0.\end{aligned} \tag{44}$$

Under the conjectured form for $H_{i\tau}$, equation (6) allows us to expand each generation-τ household's housing demand. We first derive the expectation term in the equation:

$$\begin{aligned}
&\log E\left[e^{A_{i\tau}(1-\eta_c)\eta_H}\left(\int_{[0,1]/i} e^{A_{j\tau}} H_{j\tau}^{\frac{1}{1+\psi}} dj\right)^{\eta_c\eta_H}\middle| \mathcal{I}_{i\tau}\right] \\
= \; & \log E\left[e^{A_{i\tau}(1-\eta_c)\eta_H}\left(\int_{[0,1]/i} e^{A_{j\tau}+\frac{1}{1+\psi}h_{Ai}\hat{A}^j_\tau} e^{\frac{1}{1+\psi}\left(h_0+h_P\log P_H(\tau)+h_{Ac}\hat{A}^c_\tau\right)} dj\right)^{\eta_c\eta_H}\middle| \mathcal{I}_{i\tau}\right] \\
= \; & \frac{\eta_c\eta_H}{1+\psi}\left[h_0 + h_P\log P_H(\tau) + \left(h_{Ac} + h_{Ai}\hat{\tau}^c_A/\hat{\tau}^i_A\right)\hat{A}^c_\tau\right] \\
& + \frac{\eta_H}{2}\left((1-\eta_c)^2\eta_H/\tau_\varepsilon + \eta_c/\tau_\varepsilon + \eta_c\left(\frac{1}{1+\psi}\frac{h_{Ai}\alpha}{\hat{\tau}^i_A}\frac{2\tau_A^{-1}+(1+\rho_A^2)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}{\tau_A^{-1}+\left(1+\left(\rho_A-\rho_\xi\right)\rho_A\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}\right)^2\tau_s\right) \\
& + \eta_H\left(1+\eta_c\frac{1}{1+\psi}\frac{h_{Ai}\alpha}{\hat{\tau}^i_A}\frac{2\tau_A^{-1}+(1+\rho_A^2)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}{\tau_A^{-1}+\left(1+\left(\rho_A-\rho_\xi\right)\rho_A\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}\tau_s\right)\hat{A}^i_\tau \\
& + \frac{\eta_H^2}{2}\left(1+\eta_c\frac{1}{1+\psi}\frac{h_{Ai}\alpha}{\hat{\tau}^i_A}\frac{2\tau_A^{-1}+(1+\rho_A^2)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}{\tau_A^{-1}+\left(1+\left(\rho_A-\rho_\xi\right)\rho_A\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}\tau_s\right)^2\frac{1}{\hat{\tau}^i_A},
\end{aligned}$$

In this derivation, we have used Proposition 7 to substitute $\hat{A}^j_t$ by a linear combination of $\hat{A}^c_t$ and $s_{j\tau}$. By further substituting this derived expression into equation (6), and matching coefficients with the conjectured form for $\log H_i(\tau)$, it follows that

$$\begin{aligned}
h_0 = \; & \log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{\eta_H}{2}\frac{1+\psi}{\psi}\left(1+\eta_c\frac{1}{1+\psi}\frac{h_{Ai}\alpha}{\hat{\tau}^i_A}\frac{2\tau_A^{-1}+(1+\rho_A^2)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}{\tau_A^{-1}+\left(1+\left(\rho_A-\rho_\xi\right)\rho_A\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}\tau_s\right)^2\frac{1}{\hat{\tau}^i_A} \\
& + \frac{1+\psi}{2\psi}\left(\left(\eta_H(1-\eta_c)^2+\eta_c\right)\tau_\varepsilon^{-1} + \eta_c\left(\frac{1}{1+\psi}\frac{h_{Ai}\alpha}{\hat{\tau}^i_A}\frac{2\tau_A^{-1}+(1+\rho_A^2)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}{\tau_A^{-1}+\left(1+\left(\rho_A-\rho_\xi\right)\rho_A\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}\right)^2\tau_s\right),
\end{aligned} \tag{45}$$

and

$$
\begin{aligned}
h_{Ai} &= \frac{1+\psi}{\psi+\eta_c} + \frac{\eta_c}{\psi+\eta_c}\frac{\alpha}{\hat{\tau}_A^i}\frac{2\tau_A^{-1} + (1+\rho_A^2)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}{\tau_A^{-1} + \left(1+\left(\rho_A-\rho_\xi\right)\rho_A\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}\tau_s h_{Ai}, & (46)\\
h_P &= -\left(\frac{1+\psi-\eta_H}{\psi\eta_H}\right), & (47)\\
h_{Ac} &= \frac{\eta_c}{\psi}\frac{\hat{\tau}_A^c}{\hat{\tau}_A^i}h_{Ai}. & (48)
\end{aligned}
$$

By invoking Proposition 7, we can rewrite equation (45) as

$$
\begin{aligned}
h_0 &= \log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{\eta_H}{2}\frac{1+\psi}{\psi}\left[1+\eta_c\frac{h_{Ai}}{1+\psi}\left(1-\hat{\tau}_A^c/\hat{\tau}_A^i\right)\right]^2\frac{1}{\hat{\tau}_A^i}\\
&\quad+\frac{1+\psi}{2\psi}\left[\left(\eta_H\left(1-\eta_c\right)^2+\eta_c\right)\tau_\varepsilon^{-1}+\eta_c\left(\frac{h_{Ai}}{1+\psi}\left(1-\hat{\tau}_A^c/\hat{\tau}_A^i\right)\right)^2\tau_s^{-1}\right], \qquad (49)
\end{aligned}
$$

From equation (46), it follows that

$$
h_{Ai} = \left[\psi+\eta_c\frac{\alpha}{\hat{\tau}_A^i}\left(\alpha^{-1}\hat{\tau}_A^i - \frac{2\tau_A^{-1}+(1+\rho_A^2)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}{\tau_A^{-1}+\left(1+\left(\rho_A-\rho_\xi\right)\rho_A\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}\tau_s\right)\right]^{-1}(1+\psi),
$$

which, by invoking Proposition 7, reduces to

$$
h_{Ai} = \frac{1+\psi}{\psi+\eta_c\hat{\tau}_A^c/\hat{\tau}_A^i}. \qquad (50)
$$

Thus, it is immediate from equations (48) and (46) that

$$
h_{Ac} = \frac{1+\psi}{\psi} - h_{Ai}.
$$

By aggregating household demand, one has that

$$
\begin{aligned}
&\log\int H_i(\tau)\,di\\
=\;& h_{Ac}\hat{A}_\tau^c + h_P\left(p_A A_\tau + p_{Ac}\hat{A}_\tau^c + p_\xi\xi_t + p_0\right) + h_0 + \frac{\hat{\tau}_A^c}{\hat{\tau}_A^i}h_{Ai}\hat{A}_\tau^c\\
&+\frac{h_{Ai}\alpha}{\hat{\tau}_A^i}\frac{2\tau_A^{-1}+(1+\rho_A^2)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}{\tau_A^{-1}+\left(1+\left(\rho_A-\rho_\xi\right)\rho_A\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}\tau_s A_\tau + \frac{1}{2}\left(\frac{h_{Ai}\alpha}{\hat{\tau}_A^i}\frac{2\tau_A^{-1}+(1+\rho_A^2)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}{\tau_A^{-1}+\left(1+\left(\rho_A-\rho_\xi\right)\rho_A\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}\right)^2\tau_s,
\end{aligned}
$$

where we have invoked Proposition 7 to decompose $\hat{A}_t^j$ into $\hat{A}_t^c$ and $s_{j\tau}$. The market clearing condition implies that the aggregate household demand $\log\int H_i(\tau)\,di$ is equal to builders' supply

$$
k\log P_H + k\xi_\tau = k\left(\xi_\tau + p_A A_\tau + p_{Ac}\hat{A}_\tau^c + p_\xi\xi_\tau + p_0\right).
$$

Matching coefficients of the related variables in the market clearing condition gives rise to the following four conditions:

$$
\begin{aligned}
h_0 + h_P p_0 + \frac{1}{2}\left(\frac{h_{Ai}\alpha}{\hat{\tau}_A^i}\frac{2\tau_A^{-1} + (1+\rho_A^2)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}{\tau_A^{-1} + \left(1 + (\rho_A - \rho_\xi)\rho_A\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}\right)^2 \tau_s &= kp_0, & (51)\\
h_P p_A + \frac{h_{Ai}\alpha}{\hat{\tau}_A^i}\frac{2\tau_A^{-1} + \left(1 + (\rho_A+\phi)^2\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}{\tau_A^{-1} + \left(1 + (\rho_A + \phi - \rho_\xi)(\rho_A+\phi)\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}\tau_s &= kp_A, & (52)\\
h_{Ac} + h_P p_{Ac} + \frac{\hat{\tau}_A^c}{\hat{\tau}_A^i}h_{Ai} &= kp_{Ac}, & (53)\\
h_P p_\xi &= k\,(1 + p_\xi)\,. & (54)
\end{aligned}
$$

Being able to match the coefficients confirms the conjectured log-linear form of $P_H(\tau)$ and each household's housing demand.

From equations (47) and (54), it immediately follows that

$$p_\xi = -\frac{\psi k}{\psi k + \frac{1+\psi}{\eta_H} - 1}.$$

From equations (48), (47), and (53), one also has that

$$p_{Ac} = \frac{\psi + \eta_c}{\psi k + \frac{1+\psi}{\eta_H} - 1}\frac{\hat{\tau}_A^c}{\hat{\tau}_A^i}h_{Ai}. \tag{55}$$

In addition, from (51), it is easy to see that

$$p_0 = \left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)^{-1}\left[\psi h_0 + \frac{\psi}{2}\left(\frac{h_{Ai}\alpha}{\hat{\tau}_A^i}\frac{2\tau_A^{-1} + (1+\rho_A^2)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}{\tau_A^{-1} + \left(1 + (\rho_A - \rho_\xi)\rho_A\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}}\right)^2\tau_s\right]. \tag{56}$$

By invoking Proposition 7, it follows that we can rewrite equation (56) as

$$p_0 = \frac{\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1}\left[h_0 + \frac{1}{2}h_{Ai}^2\left(1 - \frac{\hat{\tau}_A^c}{\hat{\tau}_A^i}\right)^2\tau_s^{-1}\right].$$

Given equation (49), we can further rewrite p_0 as

$$
\begin{aligned}
p_0 \;=\;& \frac{\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1}\log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{1+\psi}{2\left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)}\left(\eta_H(1-\eta_c)^2 + \eta_c\right)\tau_\varepsilon^{-1}\\
&+\frac{\eta_H}{2}\frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1}\left(\frac{\psi + \eta_c}{\psi + \eta_c\hat{\tau}_A^c/\hat{\tau}_A^i}\right)^2\frac{1}{\hat{\tau}_A^i}\\
&+\frac{1}{2}\frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1}\left(\eta_c + \psi(1+\psi)\right)\left(\frac{1 - \hat{\tau}_A^c/\hat{\tau}_A^i}{\psi + \eta_c\hat{\tau}_A^c/\hat{\tau}_A^i}\right)^2\tau_s^{-1}. \qquad (57)
\end{aligned}
$$

Finally, from equations (47) and (52), and by invoking Proposition 7, it follows that

$$p_A = \frac{\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1}\left(1 - \frac{\hat{\tau}_A^c}{\hat{\tau}_A^i}\right) h_{Ai}. \tag{58}$$

Thus, it is immediate from equations (55) and (58) that

$$p_A + p_{Ac} = \frac{\psi + \eta_c \hat{\tau}_A^c / \hat{\tau}_A^i}{\psi k + \frac{1+\psi}{\eta_H} - 1} h_{Ai} = \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1},$$

and therefore

$$p_{Ac} = \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1} - p_A.$$

Defining $x = -\frac{p_A}{p_\xi}$ and invoking equations (50) and (58), as well as the definitions of τ_A^i and τ_A^c, it follows that

$$\begin{aligned} & \psi\left(2x^2\tau_A^{-1} + \left(1+\rho_A^2\right)\tau_\xi^{-1}\right)\tau_s kx + \left(\psi + \eta_c\right)\alpha^{-1}\hat{\tau}_A^c\left(x^2\tau_A^{-1} + \left(1 + \left(\rho_A - \rho_\xi\right)\rho_A\right)\tau_\xi^{-1}\right)kx \\ = \; & \left(1+\psi\right)\left(2x^2\tau_A^{-1} + \left(1+\rho_A^2\right)\tau_\xi^{-1}\right)\tau_s \end{aligned} \tag{59}$$

where

$$\alpha^{-1}\hat{\tau}_A^c = x^2\tau_\xi + \frac{2\tau_A}{1+\rho_A^2} + \frac{\rho_A x^2 + \rho_\xi \tau_\xi^{-1}\tau_A}{1+\rho_A^2}\frac{2\rho_A - \left(1+\rho_A^2\right)\rho_\xi}{x^2\tau_A^{-1} + \left(1 + \left(\rho_A - \rho_\xi\right)\rho_A\right)\tau_\xi^{-1}},$$

which implicitly defines x and p_A since

$$p_A = \frac{\psi k x}{\psi k + \frac{1+\psi}{\eta_h} - 1}.$$

By substituting $\hat{\tau}_A^c$ into equation (59), one arrives at a 5th-order polynomial

$$\begin{aligned} 0 = \; & \tau_A^{-1}\tau_\xi x^5 + \left(2\frac{\psi}{\psi+\eta_c}\tau_A^{-1}\tau_s + \frac{2 + \left(2\rho_A - \left(1+\rho_A^2\right)\rho_\xi\right)\rho_A}{1+\rho_A^2} + 1 + \left(\rho_A - \rho_\xi\right)\rho_A\right)x^3 \\ & -2\frac{1+\psi}{\psi+\eta_c}k^{-1}\tau_A^{-1}\tau_s x^2 - \frac{1+\psi}{\psi+\eta_c}\left(1+\rho_A^2\right)k^{-1}\tau_\xi^{-1}\tau_s \\ & + \left(\frac{2\left(1 + \left(\rho_A - \rho_\xi\right)\rho_A\right) + \rho_\xi\left(2\rho_A - \left(1+\rho_A^2\right)\rho_\xi\right)}{1+\rho_A^2}\tau_A + \frac{\psi}{\psi+\eta_c}\left(1+\rho_A^2\right)\tau_s\right)\tau_\xi^{-1}x. \end{aligned} \tag{60}$$

As we have explicit expressions for all other equilibrium objects as functions of x, the existence and uniqueness of the housing market equilibrium follow from the existence and uniqueness of a real root to equation (60). By Descartes' Rule of Signs, this equation has at most three positive roots since the 5th order polynomial on the right hand side has three sign changes. By setting $x \to -x$, we see that there are no sign changes, and therefore x has zero negative roots. Furthermore, by the Fundamental Theorem of Algebra, the roots of the polynomial (60) exist. Thus, it follows that x has at least one and at most 3 real, positive roots, and at least two complex roots. Since complex roots must occur in pairs, polynomial (60) has either one or three real solutions.

By taking the limit as $\tau_s \nearrow \infty$, equation (60) converges to

$$0 = \frac{1+\psi}{\psi+\eta_c}\left(2\tau_A^{-1}x^2 + \left(1+\rho_A^2\right)\tau_\xi^{-1}\right)\left(\frac{\psi k}{1+\psi}x - 1\right),$$

from which it is obvious that $x = \frac{1+\psi}{\psi k}$ is the only real root. We recognize that $\frac{1+\psi}{\psi k}$ is the value of x in the perfect-information benchmark economy from the static model. Similarly, it is straightforward to see that $\hat{\tau}_A^c/\hat{\tau}_A^i \to 0$ as $\tau_s \nearrow \infty$. Thus, it follows that $\hat{A}_\tau^i \to A_\tau$ and, since h_{Ai} remains bounded as $\tau_s \nearrow \infty$, that the housing price and housing converge to

$$\begin{aligned}
\log P_H(\tau) &= \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1}A_\tau - \frac{\psi k}{\psi k + \frac{1+\psi}{\eta_H} - 1}\xi_\tau + \frac{\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1}\log\left(\frac{1-\eta_H}{\eta_H}\right) \\
&\quad + \frac{1+\psi}{2\left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)}\left(\eta_H\left(1-\eta_c\right)^2 + \eta_c\right)\tau_\varepsilon^{-1}, \\
\log H_i(\tau) &= \frac{1+\psi}{\psi}A_\tau - \frac{1+\psi-\eta_H}{\psi\eta_H}\log P_H(\tau) + \log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{1+\psi}{2\psi}\left(\eta_H\left(1-\eta_c\right)^2 + \eta_c\right)\tau_\varepsilon^{-1},
\end{aligned}$$

which is the perfect-information benchmark of the static economy characterized in Proposition 4.

Furthermore, since all the terms that vanish in equation (60) as $\tau_s \nearrow \infty$ are positive, it follows that $\frac{1+\psi}{\psi k}$ is an upper bound for x. Thus $x \in \left[0, \frac{1+\psi}{\psi k}\right]$.

We now provide a sufficient condition to ensure that there exists a unique real root to equation (60). Define $G(x)$ to be the RHS of equation (60). Differentiating $G(x)$ with respect to x, one finds that

$$\begin{aligned}
\frac{dG(x)}{dx} &= 5\tau_A^{-1}\tau_\xi x^4 + 3\left(2\frac{\psi}{\psi+\eta_c}\tau_A^{-1}\tau_s + \frac{2+\left(2\rho_A - \left(1+\rho_A\right)^2\rho_\xi\right)\rho_A}{1+\rho_A^2} + 1 + \left(\rho_A - \rho_\xi\right)\rho_A\right)x^2 \\
&\quad + \left(\frac{2\left(1+\left(\rho_A-\rho_\xi\right)\rho_A\right) + \rho_\xi\left(2\rho_A - \left(1+\rho_A\right)^2\rho_\xi\right)}{1+\rho_A^2}\tau_A + \frac{\psi}{\psi+\eta_c}\left(1+\rho_A^2\right)\tau_s\right)\tau_\xi^{-1} \\
&\quad - 4\frac{1+\psi}{\psi+\eta_c}k^{-1}\tau_A^{-1}\tau_s x.
\end{aligned}$$

To ensure that the real root is unique, it is sufficient for $\frac{dG(x)}{dx} > 0$ for all $x \in \left[0, \frac{1+\psi}{\psi k}\right]$. Since x is bounded from above by $\frac{1+\psi}{\psi k}$, it is sufficient for $\frac{dG(x)}{dx} > 0$ that

$$k \geq 2\frac{1+\psi}{\psi}\sqrt{\frac{\tau_A^{-1}\tau_\xi\tau_s}{\frac{2\left(1+\left(\rho_A-\rho_\xi\right)\rho_A\right)+\rho_\xi\left(2\rho_A-\left(1+\rho_A\right)^2\rho_\xi\right)}{1+\rho_A^2}\frac{\psi+\eta_c}{\psi}\tau_A + \left(1+\rho_A^2\right)\tau_s}}. \tag{61}$$

This is a sufficient condition to ensure a unique real root of equation (60). In fact, our numerical exercises always find a unique real root even when k is outside this range.

A.9 Proof of Proposition 9

Recall the functional form of $\hat{A}^c_\tau$ from Proposition 7 and the expression for $q_{\tau-1}$ and q_τ. If $\rho_A = \rho_\xi = 1$, since $x = -\frac{p_A}{p_\xi}$, we can rewrite $\hat{A}^c_\tau$ as

$$\hat{A}^c_\tau = \frac{\alpha}{\hat{\tau}^c_A}\left(\tau_A + x^2\tau_\xi\right)A_{\tau-2} + x\frac{\alpha}{\hat{\tau}^c_A}\tau_\xi\left(xZ^A_\tau - Z^\xi_\tau\right) + x^2\frac{\alpha}{\hat{\tau}^c_A}\tau_\xi Z^A_{\tau-1} - x\frac{\alpha}{\hat{\tau}^c_A}\tau_\xi Z^\xi_{\tau-1}.$$

The change in beliefs $\hat{A}^c_\tau - \hat{A}^c_{\tau-1}$ is therefore given by

$$\hat{A}^c_\tau - \hat{A}^c_{\tau-1} = x\frac{\alpha}{\hat{\tau}^c_A}\tau_\xi\left(xZ^A_\tau - Z^\xi_\tau\right) + \frac{\alpha}{\hat{\tau}^c_A}\left(\tau_A Z^A_{\tau-2} + x\tau_\xi Z^\xi_{\tau-2}\right).$$

and consequently housing returns $R_H(\tau-1,\tau)$ and $R_H(\tau,\tau+n)$ take the form

$$\begin{aligned}
R_H(\tau-1,\tau) &= p_A Z^A_{\tau-1} + p_\xi Z^\xi_{\tau-1} + p_{Ac}x\frac{\alpha}{\hat{\tau}^c_A}\tau_\xi\left(xZ^A_\tau - Z^\xi_\tau\right) + p_{Ac}\frac{\alpha}{\hat{\tau}^c_A}\left(\tau_A Z^A_{\tau-2} + x\tau_\xi Z^\xi_{\tau-2}\right),\\
R_H(\tau,\tau+n) &= p_A\sum_{i=1}^{n} Z^A_{\tau+i} + p_\xi\sum_{i=1}^{n} Z^\xi_{\tau+i} + p_{Ac}x\frac{\alpha}{\hat{\tau}^c_A}\tau_\xi\sum_{i=1}^{n}\left(xZ^A_{\tau+i} - Z^\xi_{\tau+i}\right)\\
&\quad + p_{Ac}\frac{\alpha}{\hat{\tau}^c_A}\sum_{i=1}^{n}\left(\tau_A Z^A_{\tau+i-2} + x\tau_\xi Z^\xi_{\tau+i-1}\right).
\end{aligned}$$

Since all supply and demand shocks Z^A_τ and Z^ξ_τ are i.i.d. and $x = -\frac{p_A}{p_\xi}$, it follows that the covariance between $R_H(\tau-1,\tau)$ and $R_H(\tau,\tau+n)$ reduces to

$$\begin{aligned}
& Cov\left[R_H(\tau-1,\tau), R_H(\tau,\tau+n) \mid \mathcal{I}_{\text{ex},\tau}\right]\\
= & -p_\xi p_{Ac}\frac{\alpha}{\hat{\tau}^c_A}Cov\left[xZ^A_{\tau-1} - Z^\xi_{\tau-1}, \tau_A Z^A_{\tau-1} + x\tau_\xi Z^\xi_{\tau-1} \mid \mathcal{I}_{\text{ex},\tau}\right]\\
& + \left(p_{Ac}\frac{\alpha}{\hat{\tau}^c_A}\right)^2 x\tau_\xi Cov\left[xZ^A_\tau - Z^\xi_\tau, \tau_A Z^A_\tau + x\tau_\xi Z^\xi_\tau \mid \mathcal{I}_{\text{ex},\tau}\right].
\end{aligned}$$

Since both covariances are zero, it is apparent that for arbitrary $n \geq 1$

$$Cov\left[R_H(\tau-1,\tau), R_H(\tau,\tau+n) \mid \mathcal{I}_{\text{ex},\tau}\right] = 0.$$

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