Inspecting the Mechanism:

Leverage and the Great Recession in the Eurozone*

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PRELIMINARY and INCOMPLETE

Abstract

Economists disagree about the nature of the eurozone crisis. Some see the crisis as driven by fiscal indiscipline, others by external imbalances and sudden stops, others by excessive private leverage. Motivated by the comparison of the Great Recessions in the U.S. and the Eurozone, we analyze the role of private and public leverage and ask: (i) What are the main drivers of the recession? (ii) To what extent can private and public leverage movements explain macroeconomic outcomes in the different Eurozone countries? (iii) What are the respective roles of sudden stops of foreign capital and fiscal policies? We find that, from 2008 to 2012, the difference in fate across Eurozone countries is well explained by the dynamics of household leverage. The dynamics of the Eurozone are similar to those of the U.S. In both currency areas, regions that experience the largest increases in household leverage during the credit boom (2000-2008) experience the largest declines in output and employment during the bust (2008-2010). After 2010, however, we find a distinct role for sudden stops and sovereign risk in the Eurozone, but not in the United States. To study these questions, we develop a model of open economies within a monetary union where macroeconomic dynamics are driven by private and public leverage decisions. We analyze how our model can explain the cross-sectional variance in employment, output and consumption in the Eurozone countries.

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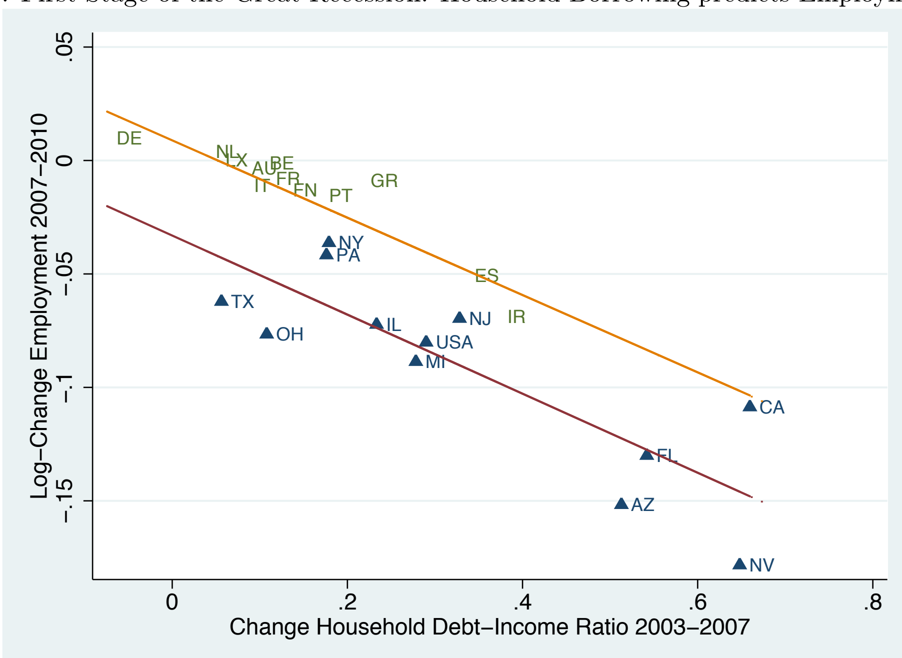
A striking feature of the European crisis is the lack of consensus about its causes, even among economists who normally share a common analytical framework. Economists profoundly disagree about causes and solutions. Some see the crisis as driven by fiscal indiscipline, others by external imbalances and sudden stops, others by private deleveraging.[1]

A salient feature of the great recession is that regions that have experienced the largest swings in household borrowing have also experienced the largest declines in employment and output. Figure 1 illustrates this feature of the data, by plotting the change in employment during the credit crunch (2007-2010) against the change in household debt-to-income ratios during the preceding boom (2003-2007) for US states[2] and Eurozone countries.

Figure 1: First Stage of the Great Recession: Household Borrowing predicts Employment Bust



The American and European cross-sectional experiences look strikingly similar in this respect on the period 2007-2010. This suggests that the shock faced by these two economies were similar in nature. Moreover this suggests that the structural parameters that govern the way the economy reacts to a deleveraging shock may also be similar in the two monetary zones. As noted in Midrigan and Philippon (2010), this pattern is at odds with the predictions of standard models of financing frictions. Such models predict that a tightening of borrowing constraints at the household level leads to a decline in consumption but, due to wealth effects, to

[1]In addition, there is disagreement about the role of multiple equilibria but this is not our focus here.

[2]State level household debt for the US comes from the Federal Reserve Bank of New York, see Midrigan and Philippon (2010).

an increase in the supply of labor. In this paper, we analyze a model where borrowing limits on "impatient" agents drive consumption, income, the saving decisions of "patient" agents and employment in small open economies belonging to a monetary union. We show that such a model with a first stage of leveraging up of "impatient" agents and then deleveraging can broadly reproduce what was observed across Eurozone countries during the recession starting in 2008 for employment, nominal GDP, consumption and wages. The nominal credit shock starting in 2008 generates a decline in borrowing thus leading to a fall in consumption spending and nominal income. "Patient" agents are affected because of the nominal income fall and reduce their saving as a consequence of the shock. We show that this crucially depends on the share of patient agents, on the openness of the economy and also on the reaction of the government. We indeed show that if the government can increase public nominal debt in reaction to the private credit shock, it can stabilize the economy. We introduce nominal wage rigidities which translate the decline of nominal expenditures into employment.

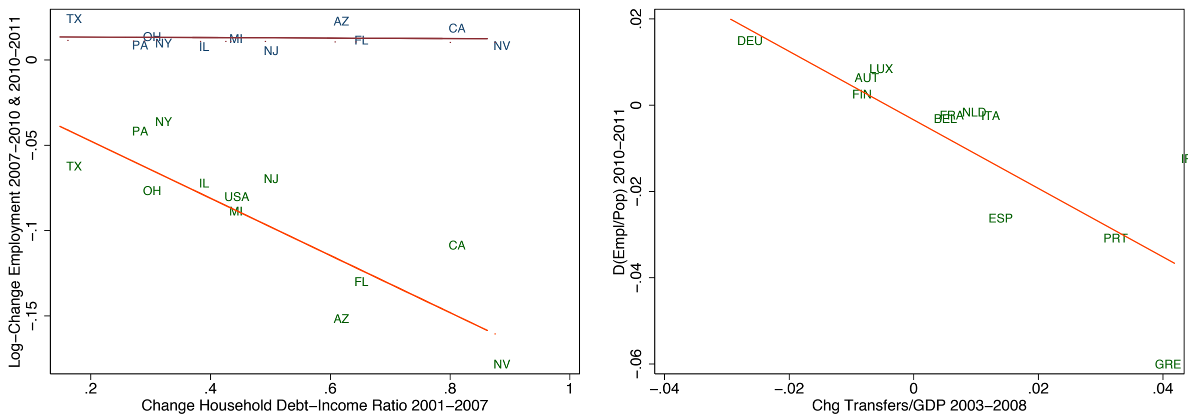
Figure 2 illustrates the differences between the American and European experiences during the later stage of the recession. Starting in the Spring of 2010, sovereign spreads widen and several European countries find it difficult to borrow on financial markets. The US and EZ experiences then start to diverge. While US states grow (slowly) together, EZ countries experience drastically different growth rates and employment. But the state variable that correlates with labor markets performance in 2010-2011 in the Eurozone is not household leverage anymore, it is social transfers during the boom. EZ countries where spending on transfers increased the most from 2003 to 2008 are the ones that are now experiencing severe recessions in the last stage.

We present a simple model that can shed light on the mechanisms at work in a monetary union and where we concentrate on the impact of private leverage, fiscal policy and sudden stops. Our model allows us to decompose the role of these various factors in the building up of macro-imbalances during the boom as well as the dynamics of each country during the crisis.

Relation to the literature

Our paper is related to three lines of research: (i) macroeconomic models with credit frictions, (ii) monetary economics, (iii) sudden stops and sovereign defaults. We discuss the connections of our paper to each topic. Following Bernanke and Gertler (1989), many macroeconomic papers introduce credit constraints at the entrepreneur level (Kiyotaki and Moore (1997), Bernanke et al. (1999), or Cooley et al. (2004)). In all these models, the availability of credit limits *corporate* investment. As a result, credit constraints affect the economy by affecting the size of the capital stock. Curdia and Woodford (2009) analyze the implication for

Figure 2: Second stage:



monetary policy of imperfect intermediation between borrowers and lenders. Gertler and Kiyotaki (2010) study a model where shocks that hit the financial intermediation sector lead to tighter borrowing constraints for entrepreneurs. We model shocks in a similar way. The difference is that our borrowers are households, not entrepreneurs, and, we argue, this makes a difference for the model's cross-sectional implications. Models that emphasize firm-level frictions cannot reproduce the strong correlation between household-leverage and employment at the micro-level, unless the banking sector is island-specific, as in the small open economy "Sudden Stop" literature (Chari et al. (2005), Mendoza (2010)). This "local lending channel" does not appear to be operative across U.S. states, however, presumably because business lending is not very localized[3].

Papers in the sudden stop literature have aimed at reproducing the stylized facts of these crises in emerging markets. According to Korinek and Mendoza (2013) the key characteristics of a sudden stop are 1) a sharp, sudden reversal in international capital flows, which is typically measured as a sudden increase in the current account 2) a deep recessions and 3) sharp changes in relative prices, including exchange rate depreciations. The eurozone crisis shares the two first characteristics even if the pace of current account adjustment in the euro area is slower than for non euro area countries (such as Bulgaria, Latvia and Lithuania) and past experiences of emerging markets crises (see Merler and Pisani-Ferry (2012) for a discussion). Substitution of private-capital inflows by public inflows, especially Eurosystem financing, partly explains this difference. The third characteristic of an emerging market sudden stop has been absent in the eurozone crisis: there has been (so far) no currency redenomination with a large depreciation and no sudden and large change in goods relative prices between countries hit at different degrees by a sudden stop (Greece, Spain, Ireland,

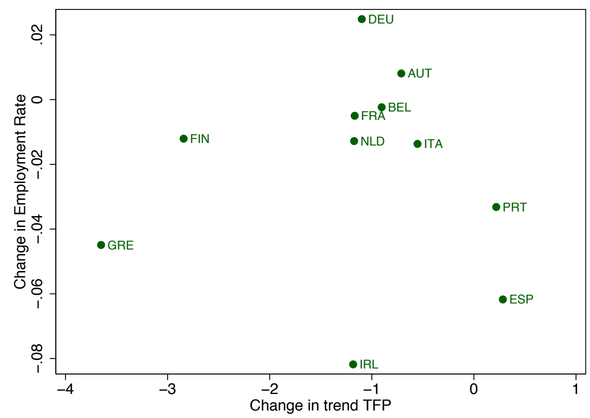
[3]For instance, Mian and Sufi (2010) find that the predictive power of household borrowing remains the same in counties dominated by national banks. It is also well known that businesses entered the recession with historically strong balanced sheets and were able to draw on existing credit lines Ivashina and Scharfstein (2008).

Portugal and Italy) and the rest of the eurozone. That these countries belong to a monetary union means the eurozone sudden stop stands apart. In fact, we do not know of any historical example of a sudden stop among countries or states inside a monetary union although sudden stops have been frequent in the 19th and 20th centuries (see Accominotti and Eichengreen (2013)). These differences are important for the choice of modeling approach. The sudden stop literature on emerging markets (see Mendoza (2010) and Korinek and Mendoza (2013) for example) has focused on a Fisherian amplification mechanism where debts are denominated in different units than incomes and collateral. This is not the case in our model as we study countries that belong to a monetary union. Another difference is that the sudden stop literature in emerging markets has focused on the sudden imposition of an external credit constraint (see Mendoza and Smith (2006) and Christiano and Roldos (2004) for example) or on transaction costs on international financial markets with multiple equilibria (see Martin and Rey (2006)). Our model integrates, for the first time to our knowledge, both a domestic credit crunch and a sudden stop produced by a spike in interest rate so that we can compare the impact of both on macroeconomic aggregates. The role of interest rates in our model relates our work to the paper of Neumeyer and Perri (2005). In their paper, as in ours, the economy is subject to interest rate shocks that generate a sudden stop in the form of a current account reversal. However, the mechanism is very different. In Neumeyer and Perri (2005), real interest rates movements either exogenous or induced by productivity shocks amplify the effect of the latter on production because they induce a working capital shortage. In our model, the increase in interest rate generates a demand shock through a fall in consumption.

Even if the bulk of the literature on sudden stops has put credit constraints at the center of the story, Gopinath (2004) and Aguiar and Gopinath (2007) have focused on an alternative explanation with TFP shocks taking center stage. Gopinath (2004) proposes a model with a search friction to generate asymmetric responses to symmetric shocks. A search friction in foreign investors entry decision into emerging markets creates an asymmetry in the adjustment process of the economy: An increase in traded sector productivity raises GDP on impact and it continues to grow to a higher long-run level. On the other hand, a decline in traded sector productivity causes GDP to contract in the short run by more than it does in the long-run. A related approach is the possibility of growth shocks as explored in Aguiar and Gopinath (2007). Because of the income effect, a negative shock leads to a fall in consumption and an increase in the trade balance. Aguiar and Gopinath (2007) do not study the response of the labor market but it is well known that income effects tend move consumption and hours in opposite directions.

Shocks to trend TFP growth might be important in emerging markets, but they do not seem to explain

Figure 3: TFP growth differential (2008-2012/2000-2007) and employment during bust (2008-2012)



the dynamics of euro area countries over the past 5 years. With the exception of Greece, countries that were hit by a sudden stop (Greece, Ireland, Italy, Spain, Portugal) are not those for which the reversal in TFP growth is the largest[4] Moreover, as illustrated by figure 3, there is no correlation between the differential in TFP growth (between the periods 2008-2012 and 2000-2007) and employment growth during the bust (2008-2012). Spain is a prime example. It is the only eurozone country that experienced an acceleration of its TFP growth during the crisis. Sanguinetti and Fuentes (2012) show that *"The Spanish economy experienced significantly weaker labour productivity growth than other OECD economies and failed to catch up with the most advanced economies in the period 1996-2007... The relatively weak performance largely reflects the low growth of total factor productivity within a wide range of sectors, with very limited impact of composition effects, while the capital stock and educational attainment of the workforce have grown relatively strongly."* We conclude that we need to look somewhere else for an explanation to the business cycles and to sudden stops in the eurozone. We focus on leverage and fiscal policy dynamics.

In the eurozone, financial integration has also - until the most recent crisis - led to large cross-border lending. Most closely connected to our paper is the work of Midrigan and Philippon (2010), Guerrieri and Lorenzoni (2010) and Eggertsson and Krugman (2011) who also study the responses of an economy to a household-level credit crunch. Consistent with our results, Mian and Sufi (2012) show that differences in the debt overhang of households across U.S. counties partly explain why unemployment is higher in some regions than others. Schmitt-Grohe and Uribe (2012) emphasize the role of downward wage rigidity in the Eurozone recession. Our paper is also related to the literature on sovereign default (see Eaton and Gersovitz (1982), Arellano (2008) and Mendoza and Yue (2012)) that models default as a strategic decision

[4] If the differential of TFP is computed on the period 2010-2012, this conclusion remains unchanged.

with a tradeoff between gains from forgone repayment and the costs of exclusion from international credit markets. The objective of our paper however is to analyze how the sovereign default risk can affect the real economy through the impact it can have on liquidity available to households. The paper by Corsetti et al. (2011) considers a "sovereign risk channel," through which sovereign default risk spills over to the rest of the economy, raising funding costs in the private sector.

Finally the paper is related to the recent research on fiscal multipliers at the regional level (Nakamura and Steinsson (2014), Farhi and Werning (2013)).

In Section 1 we present the model. In 3, we conduct a series of quantitative experiments with leverage dynamics, fiscal policy and sudden stops. Section 4 concludes.

1 Model

Our model can be interpreted as a large country with a collection of regions (e.g., USA), or a monetary union with a collection of states (e.g., EZ). The key assumption are that these regions share a common currency, and that agents live and work in only one region.

1.1 Description of the model

We consider an open economy that trades with other regions of the currency area. Each region$j$ produces a tradable domestic good and is populated by households who consume the domestic good and an aggregate of foreign goods. Following Mankiw (2000) and more recently Eggertsson and Krugman (2011), we assume that households are heterogenous in their degree of time preferences. More precisely, in region j, there is a fraction χ_j of impatient households, and $1-\chi_j$ of patient ones. Patient households (indexed by $i=s$ for savers) have a higher discount factor than borrowers (indexed by $i=b$ for borrowers): $\beta \equiv \beta_s > \beta_b$. Saving and borrowing are in units of the currency of the monetary union.

Within period trade and production. Consider household i in region j at time t. Within period, all households have the same log preferences over the consumption of home (h), foreign goods (f), and labor supply n:

$$u_{i,j,t} = \alpha_j \log\left(\frac{c^h_{j,t}}{\alpha_j}\right) + (1-\alpha_j)\log\left(\frac{c^f_{j,t}}{1-\alpha_j}\right) - \nu\left(n_{i,j,t}\right)$$

With these preferences, households of region j spend a fraction α_j of their income on home goods, and $1-\alpha_j$ on foreign goods. The parameter α_j measures how closed the economy is, because of home bias in preferences or specialization in trade. The demand functions are then:

$$\begin{aligned} p^h_{j,t} c^h_{i,j,t} &= \alpha_j x_{i,j,t}, \\ p^f_t c^f_{i,j,t} &= (1-\alpha_j)\, x_{i,j,t}. \end{aligned}$$

where $x_{i,j,t} \equiv p^h_{j,t} c^h_{i,j,t} + p^f_t c^f_{i,j,t}$ measures the spending of household i in region j in period t, $p^h_{j,t}$ is the price of home goods in country j and p^f_t is the price index of foreign goods. This gives the indirect utility

$$U(x_{i,j,t}, p_{ijt}) = \log(x_{i,j,t}) - \log \bar{p}_{j,t} - \nu(n_{i,j,t}),$$

where the CPI of country j is $\log \bar{p}_{j,t} = \alpha_j \log p^h_{j,t} + (1-\alpha_j) \log p^f_{j,t}$, the PPI is $p^h_{j,t}$, and the terms of trade are $\frac{p^f_t}{p^h_{j,t}}$. From the perspective of country j, foreign demand for home good $\bar{x}_{f,t}$ is exogenous, and we assume a unit elasticity with respect to export price $p^h_{j,t}$. Production is linear in labor $n_{j,t}$, and competitive, so $p^h_{j,t} = w_{j,t}$. Market clearing in real terms requires:

$$n_{jt} = \chi_j c^h_{b,j,t} + (1-\chi_j)\, c^h_{s,j,t} + \frac{\bar{x}_{f,t}}{p^h_{j,t}} + g_{j,t},$$

where $g_{j,t}$ are government expenditures. Note that we assume that the government spends only on domestic goods. We can also write the market clearing condition in nominal terms

$$y_{jt} = \chi_j \alpha_j x_{b,j,t} + (1-\chi_j)\, \alpha_j x_{s,j,t} + \bar{x}_{f,t} + p^h_{j,t} g_{j,t}.$$

Labor income and transfers. Each household supplies labor at the prevailing wage and receives wage income net of taxes $(1-\tau_{j,t})\, w_{j,t} n_{i,j,t}$. They also receive transfers from the government $T_{j,t}$. We assume that wages are sticky and we ration the labor market uniformly across households. This assumption simplifies the analysis because we do not need to keep track separately of the labor income of patient and impatient households within a country. Not much changes if we relax this assumption, except that we loose some tractability.[5]

[5] In response to a negative shock, impatient households would try to work more. The prediction that hours increase more for credit constrained households appears to be counter-factual however. One can fix this by assuming a low elasticity of labor supply, which essentially boils down to assuming that hours worked are rationed uniformly in response to slack in the labor market. Assuming that the elasticity of labor supply is small (near zero) also means that the natural rate does not depend on

Borrowing, lending and budget constraints. Let $b_{j,t}$ be the amount borrowed by impatient households. It is the face value of the debt issued in $t-1$ and due in period t. The budget constraint of impatient households in country j is:

$$\frac{b_{j,t+1}}{1+r_{b,j,t}} + (1-\tau_{j,t})\, w_{j,t} n_{j,t} + T_{j,t} = x_{b,j,t} + b_{j,t}, \tag{1}$$

where $x_{b,j,t} \equiv p^h_{j,t} c^h_{b,j,t} + p^f_t c^f_{b,t}$ is spending, as defined above and $r_{b,j,t}$ is the nominal borrowing rate between t and $t+1$. Borrowing is subject to the exogenous limit $\bar{b}_{j,t}$:

$$b_{j,t} \leq \bar{b}_{j,t}. \tag{2}$$

Savers save at the same nominal rate r_t and their budget constraint is:

$$s_{j,t} + (1-\tau_{j,t})\, w_{j,t} n_{j,t} + T_{j,t} = x_{s,j,t} + \frac{s_{j,t+1}}{1+r_{s,j,t}}, \tag{3}$$

so their Euler equation is

$$\frac{1}{x_{s,j,t}} = \mathbb{E}_t \left[\frac{\beta\,(1+r_{s,j,t})}{x_{s,j,t+1}} \right]. \tag{4}$$

We will analyze the case where r is common to all countries in the monetary union and the case where r differs across countries. We will also discuss the issue of risk premia and of differences between the borrowing and lending rates. For now we keep the notations general by using a country and agent specific rate. The government budget constraint is

$$\frac{b^g_{j,t+1}}{1+r_{g,j,t}} + \tau_{j,t} w_{j,t} n_{j,t} = p^h_{j,t} g_{j,t} + T_{j,t} + b^g_{j,t}, \tag{5}$$

where $b^g_{j,t}$ is public debt issued by government j at time t. We define nominal domestic product as

$$y_{j,t} \equiv p^h_{j,t} n_{j,t}$$

fiscal policy. In an extension we study the case where the natural rate is defined by the labor supply condition in the pseudo-steady state $\nu'\left(n^\star_i\right) = (1-\tau_j)\,\frac{w_j}{x_{i,j}}$. We can then ration the labor market relative to their natural rate: $n_{i,j,t} = \frac{n^\star_i(\tau)}{\sum_i n^\star_i(\tau)} n_{j,t}$ where $n^\star_i(\tau)$ is the natural rate for household i in country. This ensures consistency and convergence to the correct long run equilibrium. Steady state changes in the natural rate are quantitatively small, however, so the dynamics that we study are virtually unchanged. See Midrigan and Philippon (2010) for a discussion.

Nominal exports are $\bar{x}_f$, nominal imports are $(1-\chi_j)\, p^f_{j,t} c^f_{s,j,t} + \chi_j p^f_{j,t} c^f_{b,j,t}$ since the government does not buy imported goods. So net exports are:

$$e_{j,t} = \bar{x}_{f,t} - (1-\alpha_j)\left(\chi_j x_{b,j,t} + (1-\chi_j)\, x_{s,j,t}\right). \tag{6}$$

The net foreign asset position of the country is at the *end of period* t, measured in market value, is

$$a_{j,t} \equiv (1-\chi_j)\frac{s_{j,t+1}}{1+r_{s,j,t}} - \chi_j \frac{b_{j,t+1}}{1+r_{b,j,t}} - \frac{b^g_{j,t+1}}{1+r_{g,j,t}} \tag{7}$$

and the current account is

$$ca_t = e_{j,t} + \bar{r}_{j,t-1} a_{j,t},$$

where $\bar{r}_{j,t}$ is defined as the weighted average interest rate. The closed economy corresponds to $a = 0$ and endogenous interest rate r. The small open region corresponds to exogenous r and endogenous a.

It will be convenient to define disposable income (after tax and transfers but before interest payments) as

$$\tilde{y}_{j,t} \equiv (1-\tau_{j,t})\, y_{j,t} + T_{j,t}.$$

With this notation, we can write the budget constraint of the impatient agent as

$$x_{b,j,t} = \frac{b_{j,t+1}}{1+r_{b,j,t}} + \tilde{y}_{j,t} - b_{j,t},$$

and similarly for the savers: $x_{s,j,t} = s_{j,t} + \tilde{y}_{j,t} - \frac{s_{j,t+1}}{1+r_{s,j,t}}$. The first equilibrium condition can simply be obtained by combining market clearing at time t with the budget constraints of the borrowers, the savers, and the government:

$$(1-\alpha_j)\,\tilde{y}_{j,t} = \alpha_j \chi_j \left(\frac{b_{j,t+1}}{1+r_{b,j,t}} - b_{j,t}\right) - \alpha_j (1-\chi_j)\left(\frac{s_{j,t+1}}{1+r_{s,j,t}} - s_{j,t}\right) + \bar{x}_{f,t} + \frac{b^g_{j,t+1}}{1+r_{g,j,t}} - b^g_{j,t} \tag{8}$$

This equation shows the four sources of nominal demand: impatient borrowers, patient savers, foreign demand and public expenditures. It holds even if interest rates vary over time and across agents. Equivalently this equations says that the current account equals the change in net foreign assets:

$$ca_{j,t} = a_{j,t+1} - a_{j,t}.$$

1.2 Pseudo-Steady State

We consider a steady state with constant interest rates equal to the rate of time preference of savers, i.e. $\beta(1+r) = 1$. The borrowing limit $\bar{b}_j$ is exogenous and we consider equilibria where the borrowing constraint 2 binds. Our notion of steady state is complicated by the fact that savings $s_{j,t}$ are history-dependent. We define a steady state as the long run equilibrium of an economy with initial savings s_j and government debt b_j^g, subject to no further shocks, constant government spending and constant government debt. All nominal quantities are constant and employment is at its natural rate $n^\star$.[6] The long-run equilibrium conditions are

$$
\begin{aligned}
p_j^h n^\star &= \chi_j \alpha_j x_{b,j} + (1-\chi_j)\,\alpha_j x_{s,j} + \bar{x}_f + p_j^h g_j \\
x_{b,j} &= \tilde{y}_j - \frac{r}{1+r}\bar{b}_j \\
x_{s,j} &= \tilde{y}_j + \frac{r}{1+r}s_j \\
\tau_j p_j^h n^\star &= p_j^h g_j + T_j + \frac{r}{1+r}b_j^g
\end{aligned}
$$

Nominal output (the price of home goods) is pinned down by

$$p_j^h n^\star = \alpha_j\left((1-\tau_j)\,p_j^h n^\star + T_j\right) - \chi_j\alpha_j\frac{r}{1+r}\bar{b}_j + (1-\chi_j)\,\alpha_j\frac{r}{1+r}s_j + \bar{x}_f + p_j^h g_j.$$

There are several ways to specify government policy. Here we assume that the policy is to keep government debt and nominal spending $p_j^h g_j$ constant. Long run nominal output is then given by

$$p_j^h n^\star = \frac{\alpha_j}{1-\alpha_j}\frac{r}{1+r}a_j + \frac{\bar{x}_f}{1-\alpha_j} + p_j^h g_j \tag{9}$$

where recall that we have defined net foreign assets as $a_j \equiv (1-\chi_j)\,s_j - \chi_j b_j - b_j^g$. This equation shows the determinants of the long run price level. The long run price level depends on the exogenous components of spending: net asset income, foreign demand, and government spending. All these are inflationary. For a given tax rate τ_j, transfers are then chosen to satisfy the government's budget constraint:

$$T_j = \tau_j p_j^h n^\star - p_j^h g_j - \frac{r}{1+r}b_j^g.$$

[6] We consider here the case where labor supply is inelastic, so $n^\star$ is effectively exogenous. The case of elastic labor supply is presented in the Appendix. The differences are entirely predictable: $n^\star$ depends on τ and is not the same for borrowers and for savers. These effects are rather small and essentially irrelevant for the dynamics that we study, so we leave them out of the main body of the paper.

1.3 Employment and Inflation

The system above completely pins down the dynamics of nominal variables: $y_{j,t}, x_{i,j,t}, \ldots$. Employment (real output) is given by

$$n_{j,t} = \frac{y_{j,t}}{p^h_{j,t}}.$$

We need to specify the dynamics of inflation. Letting $n^\star$ denote the natural rate of unemployment, we assume the following Phillips curve:

$$\frac{p^h_{j,t} - p^h_{j,t-1}}{p^h_{j,t-1}} = \kappa \left(n_{j,t} - n^\star\right) \tag{10}$$

$$\frac{\left(p^h_{j,t}\right)^2}{p^h_{j,t-1}} + \left(\kappa n^\star - 1\right) p^h_{j,t} - \kappa y_{j,t} = 0$$

Defining $\Delta \equiv \left(\kappa n^\star - 1\right)^2 + 4\kappa \frac{y_{j,t}}{p^h_{j,t-1}}$, we find that

$$\frac{p^h_{j,t}}{p^h_{j,t-1}} = \frac{1 - \kappa n^\star + \sqrt{\Delta}}{2}$$

Note that if $\frac{y_{j,t}}{p^h_{j,t-1}} = n^\star$, then $\Delta = \left(\kappa n^\star + 1\right)^2$, and $\frac{p^h_{j,t}}{p^h_{j,t-1}} = 1$. We also experiment with asymmetric wage rigidity where wages are more flexible upward (when the output gap is positive) than downward (when the output gap is negative)[7]: $\kappa = \kappa_d 1_{n_{j,t} < n^\star} + \kappa_u 1_{n_{j,t} > n^\star}$ where $\kappa_d < \kappa_u$.

[7] For empirical evidence on downward nominal wage rigidity, see for example, Schmitt-Grohe and Uribe (2012) on periphery countries in Europe.

2 Nominal Dynamics

2.1 Savers

The main challenge is to pin down the behavior of savers. The problem of the savers is[8]

$$\max \textstyle\sum_{t\geq 0} \beta^t \log\left(x_{s,j,t}\right)$$
$$s.t.$$
$$x_{s,j,t} + \tfrac{s_{j,t+1}}{1+r_{s,j,t}} = s_{j,t} + \tilde{y}_{j,t}$$
$$s_{j,t} > -\bar{b}_{j,t}$$

where $\tilde{y}_{j,t}$ and $r_{s,j,t}$ are both random variables and $\bar{b}_{j,t}$ is the ad-hoc borrowing limit in country j at time t (it is of course binding for impatient households). This is a well-studied problem but we will not do justice to all of its interesting aspects. In particular, we will neglect (for now) the role of precautionary savings and use a certainty equivalent approach by linearizing the Euler equation.

Instead, our focus is on the budget constraint, and its Ricardian properties. Let us first define the k-period discount rate from the savers' perspective as

$$R_{j,t,k} \equiv \left(1 + r_{s,j,t}\right) .. \left(1 + r_{s,j,t+k-1}\right).$$

with the convention $R_{j,t,0} = 1$. Define also the financing spreads ϕ for private borrowers and for the government as

$$\begin{aligned} 1 - \phi_{b,j,t} &\equiv \frac{1 + r_{s,j,t}}{1 + r_{b,j,t}} \\ 1 - \phi_{g,j,t} &\equiv \frac{1 + r_{s,j,t}}{1 + r_{g,j,t}} \end{aligned}$$

Then we have the following Lemma

[8] The complete objective function is of course

$$\sum_{t\geq 0} \beta^t \left(\log\left(x_{s,j,t}\right) - \log\left(\bar{p}_{j,t}\right) - \nu\left(n_{j,t}\right)\right).$$

It is simplified by log preferences (so prices are additively separable) and by our raitonning rule for labor. Because of precautionary savings, we know that the interest rate consistent with finite savings must be such that $\beta(1+r) < 1$. We consider the limit where aggregate shocks are small and $\beta(1+r)$ is close to one.

Lemma 1. *The intertemporal current account condition is*

$$(1-\alpha_j)\left((1-\chi_j)\, s_{j,t} - \chi_j b_{j,t} + \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\tilde{y}_{j,t+k}}{R_{j,t,k-1}}\right) = (1-\chi_j)\, s_{j,t} - \chi_j b_{j,t} - b^g_{j,t} + \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\bar{x}_{f,t+k}}{R_{j,t,k}} - \Phi_{j,t},$$

where Φ_t is the net present value of financing costs

$$\Phi_{j,t} \equiv \alpha_j \chi_j \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\phi_{b,j,t+k-1}}{R_{j,t,k}} b_{j,t+k} + \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\phi_{g,j,t+k-1}}{R_{j,t,k}} b^g_{j,t+k}.$$

On the left we have private wealth (discounted at the savers's rate) and $(1-\alpha_j)$ is the share wealth spent on imports. On the right we have net foreign assets plus the value of exports. From the intertemporal budget constraint of savers

$$\mathbb{E}_t \sum_{k=0}^{\infty} \frac{x_{s,j,t+k}}{R_{j,t,k}} = s_{j,t} + \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\tilde{y}_{j,t+k}}{R_{j,t,k}}$$

we then get the following proposition

Proposition 1. *When interest rates are the same within a country ($\Phi_{j,t} = 0$) the nominal spending of patient agents $x_{s,j}$ does not react to credit expansion or to fiscal policy.*

Proposition 1 clarifies the behavior of savers. In particular, their spending reacts neither to $g_{j,t}$ nor to $T_{j,t}$. This result is related to – but also different from – Ricardian equivalence. To understand it, one needs to focus on the budget constraint of the patient households. Clearly, shocks to interest rates will affect this budget constraint and also the Euler equation, so we know that they will affect spending $x_{s,j,t}$. What is surprising is that, even though changes in the borrowing constraints of impatient agents or changes in fiscal policy have a direct impact on disposable income $\tilde{y}_{j,t}$, savers do not react. The reason is that patient agents know that higher spending today – which increases output – means higher interest payments in the future – which decreases spending and output. The key result is that the *net present value* of disposable income does not change as long as rates are the same within the country. Changes in $\bar{b}, g, T, \tau$ have no effect on the permanent income of patient agents. Shocks to foreign demand, on the other hand, affect consumption expenditures of patient households because they affect their permanent income. Of course, even when expenditures remain constant, this does not mean that real consumption remains constant. In fact real consumption always changes because prices (wages) always react to changes in aggregate spending.

These results are related to Cole and Obstfeld (1991) and Farhi and Werning (2013). TBC.

2.2 Shocks and Policies

We consider an economy subject to four series of shocks: the borrowing limit of the impatient households $\bar{b}_{j,t}$, foreign demand $\bar{x}_{f,t}$, interest rates, and fiscal policy. We assume that interest rates are time varying and country-specific, but they are the same for all agents within a country: $r_{b,j,t} = r_{s,j,t} = r_{g,j,t}$. We define $\rho_{j,t}$ as the deviation of the interest rate from its long run value:

$$1 + r_{j,t} \equiv (1 + r)(1 + \rho_{j,t}).$$

We make the following assumptions about the shocks:

Assumption A1.

- *Exogenous shocks are such that* $\mathbb{E}_t\left[\bar{b}_{j,t+2}\right] = \bar{b}_{j,t+1}$, $\mathbb{E}_t\left[\bar{x}_{f,t+1}\right] = \bar{x}_{f,t}$, *and* $\mathbb{E}_t\left[\rho_{j,t+1}\right] = 0$
- *Fiscal policy is such that* $\mathbb{E}_t\left[b^g_{j,t+2}\right] = b^g_{j,t+1}$
- *The variance of interest rates and foreign demand is small, and* β_b *is small enough that* $b_{j,t} = \bar{b}_{j,t}$ *at all times.*

The first point says that the shocks are permanent and that interest rates are iid. The second point defines a class of fiscal policies. These assumptions are important for the expectations of the agents in the model. The last point is purely technical. It allows us to linearize Euler equations. Notice that we do not need to assume that shocks to $\bar{b}_{j,t}$ are small or that fiscal policy shocks are small. We assume that the shocks are small enough that impatient households find it optimal to borrow up to the constraint (this is a joint restriction on the discount factor and the size of the shocks).

We now look for decision rules for the savers $\{s_{j,t}\}_{t=1,2...}$ and the other variables of the model, $y_{j,t}$, $p^h_{j,t}$, etc. The following Lemma is used repeatedly

Lemma 2. *Under A1,* $\mathbb{E}_t\left[s_{j,t+2}\right] = s_{j,t+1}$.

In fact, we show in the Appendix that

$$\frac{s_{j,t+1}}{1+\rho_{j,t}} - s_{j,t} = \frac{\alpha_j \chi_j}{1-\alpha_j \chi_j}\left(\frac{\bar{b}_{j,t+1}}{1+\rho_{j,t}} - \bar{b}_{j,t}\right) + \frac{1}{1-\alpha_j \chi_j}\left(\frac{b^g_{j,t+1}}{1+\rho_{j,t}} - b^g_{j,t} + \frac{\rho_{j,t}}{1+\rho_{j,t}}\frac{\bar{x}_{f,t}}{1-\alpha_j}\right) \tag{11}$$

Savings inherit the dynamic properties of $\bar{b}_{j,t}$ and $b^g_{j,t}$. Therefore this validates our conjecture that $\mathbb{E}_t\left[s_{j,t+2}\right] = s_{j,t+1}$.

Proposition 1 becomes

Corollary 1. *Under A1, nominal spending of patient agents follows*

$$\Delta x_{s,j,t} = \frac{\Delta\left[\bar{x}_{f,t}\right]}{1-\alpha_j} + \Omega_{j,t}\left(\rho_{j,t}, \rho_{j,t-1}\right)$$

where $\Omega_{j,t}\left(0,0\right) = 0$.

This result is important because it says that in response to shocks to $\bar{b}_{j,t}$ or shocks to fiscal policy – including shocks to $p^h_{j,t} g_{j,t}$ – we not only have $x_{s,j,t} = \mathbb{E}_t\left[x_{s,j,t+1}\right]$ but in fact $x_{s,j,t} = x_{s,j,t-1}$. As explained earlier, the reason is that patient agents know that higher spending today – which increases output – means higher interest payments in the future – which decreases spending and output. The key result is that the *net present value* of disposable income does not change. Since on average $\beta\left(1+r\right) = 1$, savers optimally choose to keep $x_{s,j,t}$ exactly constant.

2.3 Simple Solutions with Constant Interest Rates

The Appendix presents the solution with stochastic interest rates, but much can be learned from the dynamics of a small open economy with a constant nominal rate such that $\beta\left(1+r\right) = 1$. This will help us understand the properties of the model. We need to determine the dynamics of $x_{j,t}$ and $s_{j,t}$. We know from the previous Lemma that $\mathbb{E}_t\left[s_{j,t+1}\right] = s_{j,t}$. From time $t+1$ onward, the expected budget constraint of savers is:

$$\mathbb{E}_t\left[x_{s,j,t+1}\right] = \mathbb{E}_t\left[\tilde{y}_{j,t+1}\right] + \frac{r}{1+r} s_{j,t+1}.$$

We can combine this last equation with the linearized Euler equation of the savers $x_{s,j,t} = \mathbb{E}_t\left[x_{s,j,t+1}\right]$ and the date-t budget constraint, to obtain the optimal path of saving for patient households:

$$s_{j,t+1} - s_{j,t} = \tilde{y}_{j,t} - \mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]. \tag{12}$$

The conditions says that savers optimally choose to keep their expected disposable wealth $\tilde{y}_{j,t} + s_{j,t}$ constant.

We can rewrite the market clearing condition contained in equation (8) at time $t+1$, and use our conjecture for s_t, b_t and $\bar{x}_{f,t}$ to get:

$$(1-\alpha_j)\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right] = \alpha_j \frac{r}{1+r}\left(\left(1-\chi_j\right) s_{j,t+1} - \chi b_{j,t+1}\right) + \bar{x}_{f,t} + \mathbb{E}_t\left[\frac{b^g_{j,t+2}}{1+r} - b^g_{j,t+1}\right]. \tag{13}$$

From Equation (13) it is clear that government debt dynamics play a crucial role. To be consistent with our assumptions, we assume that government debt also follows a random walk: $\mathbb{E}_t\left[b^g_{j,t+1}\right] = b^g_{j,t}$. This assumption is quite flexible. We will see that several government policy rules are consistent with it, including complete or partial stabilization of nominal GDP.

The three equilibrium conditions are then given by equations (8), (12), and (13). Our conjecture requires that disposable income follows a random walk: for all $h > 1$, we need to verify that $\mathbb{E}_t\left[\tilde{y}_{j,t+h}\right] = \mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]$. Combining (8), (12) and (13) we can then find savings dynamics as a function of the borrowing by impatient agents and the government:

$$s_{j,t+1} - s_{j,t} = \frac{\alpha_j\chi_j}{1-\alpha_j\chi_j}\left(b_{j,t+1} - b_{j,t}\right) + \frac{1}{1-\alpha_j\chi_j}\left(b^g_{j,t+1} - b^g_{j,t}\right) \tag{14}$$

This shows that savings inherit the dynamics of borrowing by impatient agents and the government. The reason is that borrowing affects total spending and therefore nominal income dynamics to which patient agents react optimally by adjusting their savings. The coefficient $\left(1-\alpha_j\chi_j\right)^{-1}$ is an aggregate multiplier that depends on the share of impatient agents χ_j and the share of home goods in consumption α_j. For a given amount of borrowing, the impact on spending and income is larger if the share of impatient is larger and the economy is more closed. Government borrowing – which is also the change in nominal government spending net of tax receipts – has a larger than private borrowing impact because government expenditures are spent entirely on domestic goods. For GDP, we show in the Appendix that

$$\left(1-\tau_j\right)\Delta\left[y_{j,t}\right] = \frac{1}{1-\alpha_j\chi_j}\left(\alpha_j\chi_j\left(\Delta\left[\frac{b_{j,t+1}}{1+r}\right] - \Delta\left[b_{j,t}\right]\right) + \Delta\left[\frac{b^g_{j,t+1}}{1+r}\right] - \Delta\left[b^g_{j,t}\right]\right) - \Delta\left[T_{j,t}\right] + \frac{\Delta\left[\bar{x}_{f,t}\right]}{1-\alpha_j} \tag{15}$$

where Δ denote first differences, i.e., $\Delta\left[y_{j,t}\right] = y_{j,t} - y_{j,t-1}$. Changes in transfers enter negatively because they have a smaller impact on output than direct spending. The reason is that transfers can be saved by the patient agents and that government expenditures are assumed to have 100% home bias. The GDP multiplier for government spending is $\frac{1}{1-\alpha_j\chi_j}$. Note that it does not take into account the impact of spending on GDP and therefore on government revenues. Assuming that the tax rate does not change at the time of the shock and constant levels of transfers, the multiplier for government debt is $\frac{1}{1-\tau_j}\frac{1}{1-\alpha_j\chi_j}$ (measured in current period currency, so dropping the $\frac{1}{1+r}$ term). The multiplier from private debt is $\frac{1}{1-\tau_j}\frac{\alpha_j\chi_j}{1-\alpha_j\chi_j}$.

Aggregate nominal expenditures is given by $X_{j,t} \equiv \chi_j x_{b,t} + (1-\chi_j)x_{s,t}$, the sum of expenditures by impatient and patient households. The reaction of patient households depends on the shocks. When interest rates, foreign demand and tax rates are constant, it is easy to see that patient spending remains exactly constant in response to shocks to borrowing by impatient agents or the government, as discussed in Proposition 1. Hence, changes in aggregate spending are driven by the movements that affect the consumption of impatient agents (their borrowing constraint and public borrowing) and those that affect the consumption of the patient agents (foreign demand):

$$\Delta X_{j,t} = \frac{\chi_j}{1-\alpha_j\chi_j}\left[\Delta\left(\frac{b_{j,t+1}+b^g_{j,t+1}}{1+r}\right) - \Delta\left(b_{j,t}+b^g_{j,t}\right)\right] + \frac{1-\chi_j}{1-\alpha_j}\Delta\left[\bar{x}_{f,t}\right]. \tag{16}$$

2.4 Impulse Responses

For simplicity, we assume that the marginal tax rate is τ_j is constant. The five equilibrium conditions of the model are

1. $s_{j,t+1} = (1+\rho_{j,t})\,(s_{j,t}+\tilde{y}_{j,t}) - \mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]$
2. $(1-\alpha_j)\,\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right] = \frac{r}{1+r}\left(\alpha_j\,(1-\chi_j)\,s_{j,t+1} - \alpha_j\chi_j\bar{b}_{j,t+1} - b^g_{j,t+1}\right) + \bar{x}_{f,t}$.
3. $(1-\alpha_j)\,\tilde{y}_{j,t} = \alpha_j\chi_j\left(\frac{\bar{b}_{j,t+1}}{1+r_{j,t}} - \bar{b}_{j,t}\right) + \alpha_j\,(1-\chi_j)\left(s_{j,t} - \frac{s_{j,t+1}}{1+r_{j,t}}\right) + \frac{b^g_{j,t+1}}{1+r_{j,t}} - b^g_{j,t} + \bar{x}_{f,t}$.
4. $p^h_{j,t}g_{j,t} + T_{j,t} - \tau_j y_{j,t} = \frac{b^g_{j,t+1}}{1+r_{j,t}} - b^g_{j,t}$
5. $\tilde{y}_{j,t} = (1-\tau_j)\,y_{j,t} + T_{j,t}$

The five unknown endogenous variables are $\tilde{y}_{j,t}$, $y_{j,t}$, $b^g_{j,t}$, $s_{j,t}$, and $\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]$. The exogenous shocks are $\bar{b}_{j,t}$, $\bar{x}_{f,t}$ and $r_{j,t}$. The policy shocks variables are $T_{j,t}$ and $g_{j,t}$. The state space of predetermined endogenous variables is $s_{j,t-1}$ and $b^g_{j,t-1}$.

Policy Rules We now turn to the policy function of the government. [9] We look for rules that are plausible, simple and deliver the property that the public debt follows a random walk: $\mathbb{E}_t\left[b^g_{j,t+2}\right] = b^g_{j,t+1}$. Automatic

[9]Here is a simple example, that we do not use in the quantitative experiments, but that can help understand the model. If the only shocks are changes in the private debt constraint $\bar{b}_{j,t}$, a government trying to stabilize its economy could react to these shocks with the following simple rule:

$$b^g_{j,t+1} - b^g_{j,t} = -\gamma\alpha_j\chi_j\left(\bar{b}_{j,t+1} - \bar{b}_{j,t}\right),$$

with $0 \leq \gamma \leq 1$. From equation (14), we then get

$$s_{j,t+1} - s_{j,t} = (1-\gamma)\frac{\alpha_j\chi_j}{1-\alpha_j\chi_j}\left(\bar{b}_{j,t+1} - \bar{b}_{j,t}\right).$$

Nominal GDP is a more volatile when the economy is more closed and when the share of impatient borrowers is larger. We see that perfect stabilization of $s_{j,t}$ is theoretically possible by choosing $\gamma = 1$. We can check that this rule also implies a constant disposable income.

stabilizers have become a characteristic of modern fiscal systems in all OECD countries and the income tax is a strong component of automatic stabilizers (see Fatas and Mihov (2012)). We assume that spending and transfers are predetermined. From the government budget constraint, this means that a recession at time t automatically increases government debt at time t. To maintain fiscal stability, we specify that transfers adjust from t to $t+1$ to keep public debt constant thereafter: $\mathbb{E}_t\left[b^g_{j,t+2}\right] = b^g_{j,t+1}$. More precisely, we specify the general policy rule as follows

1. Fiscal variables $p^h_{j,t}g_{j,t}$ and $T_{j,t}$ are pre-determined. Government debt $b^g_{j,t}$ is determined in equilibrium at time t.

2. Set transfers $T_{j,t+1}$ for next period so that $\mathbb{E}_t\left[b^g_{j,t+2}\right] = b^g_{j,t+1}$ assuming a martingale for $p^h_{j,t}g_{j,t}$.

    (a) The expected budget constraint is

$$\tau_j \mathbb{E}_t\left[y_{j,t+1}\right] = p^h_{j,t}g_{j,t} + T_{j,t+1} + \frac{r}{1+r}b^g_{j,t+1}$$

    (b) By definition we have

$$\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right] = \left(1-\tau_j\right)\mathbb{E}_t\left[y_{j,t+1}\right] + T_{j,t+1}$$

    (c) Therefore (recall that $\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]$ is part of our solution at time t), $T_{j,t+1}$ is given by

$$T_{j,t+1} = \tau_j \mathbb{E}_t\left[\tilde{y}_{j,t+1}\right] - \left(1-\tau_j\right)p^h_{j,t}g_{j,t} - \left(1-\tau_j\right)\frac{r}{1+r}b^g_{j,t+1}$$

We present in Figures (4), (5), (6) and (7) the simple impulse reaction functions[10] that illustrate the impact of shocks on household debt ($b_{j,t}$), public spending ($p^h_{j,t}g_{j,t}$), interest rates ($r_{j,t}$) and foreign demand ($\bar{x}_{f,t}$). An increase in household debt generates a boom in nominal GDP, employment, consumption of impatient households (but not of patient ones who increase their saving, as explained in Proposition 1) and imports. Public debt falls but the net foreign asset position deteriorates. A fiscal expansion has qualitatively similar effects except that in this case public debt increases, although it decreases in percentage of GDP. An increase in interest rates is very different because it induces patient households to save more so it reduces their expenditures and generates a recession (fall in nominal GDP and in employment) that obliges impatient

[10]For these impulse response functions, we use the following parameters: $\alpha = 0.75, \chi = 0.5, r = 0.05, \kappa = 0.3, \tau = 0.4$. Prices, wages and employment are normalized to unity at time $t = 0$. The debt to income ratio is set at 60% for impatient households at time $t = 0$, so that the household debt to income ratio is 30%. The government debt to GDP ratio is set at 50% and the net foreign asset position over GDP is set at zero at time $t = 0$.

households to reduce their spending. Imports fall and the net foreign asset position improves. Because of lower tax revenues, the recession increases public debt. Finally, an increase in foreign demand permanently increases nominal GDP which induces patient households to increase their saving. Consumption of both patient and impatient households increase. The net foreign asset position improves. Public debt falls because of higher tax revenues.

Figure 4: Credit Expansion

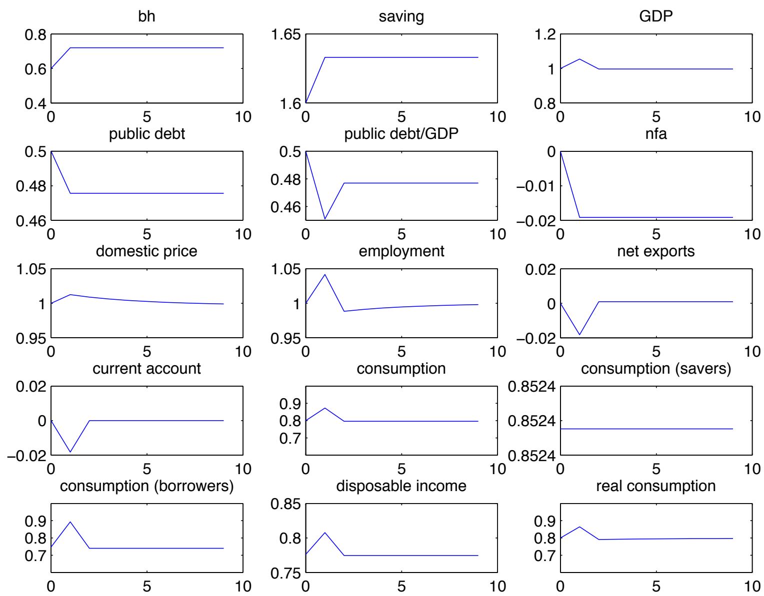


Figure 5: Fiscal Expansion

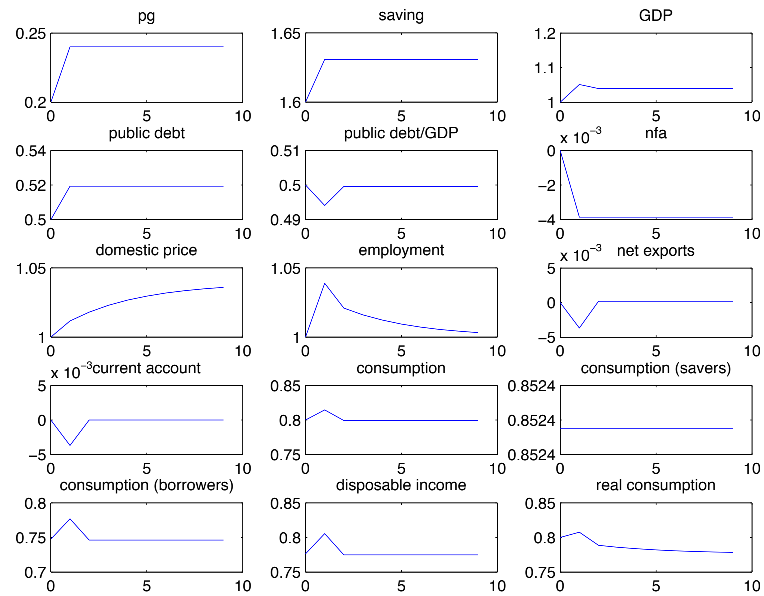


Figure 6: Interest rate shock

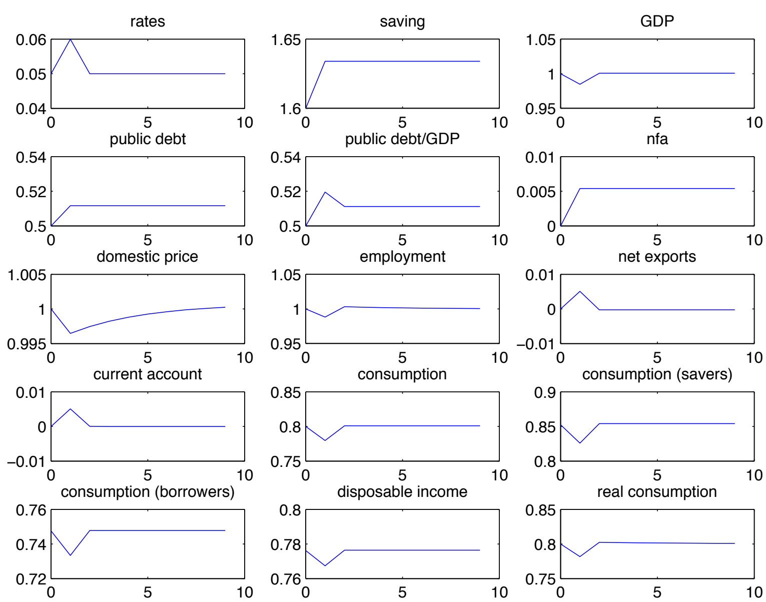
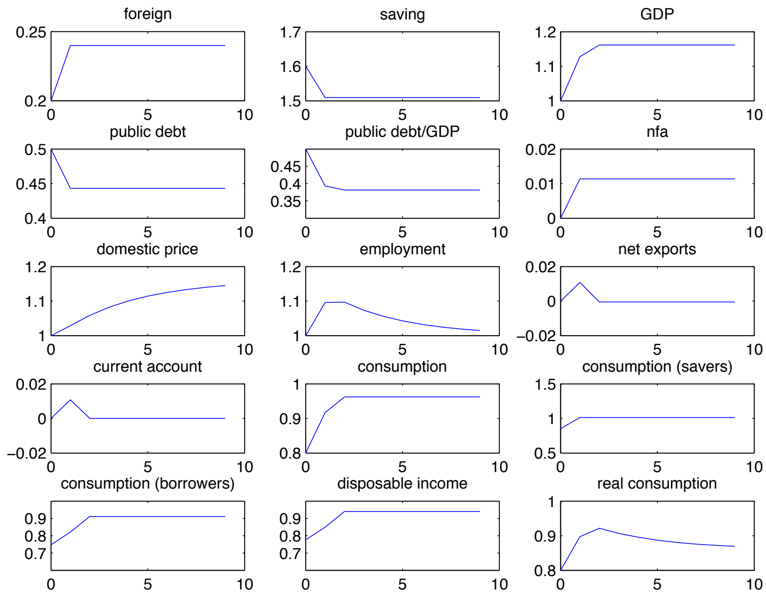


Figure 7: Foreign demand shock



3 Quantitative Experiments

We next analyze a cross-sectional experiment to compare the model predictions and the data. We describe the sources of the cross-sectional and aggregate data we use in the Appendix. We use data for 11 Eurozone countries: Austria, Belgium, Germany, Spain, Finland, France, Greece, Ireland, Italy, Netherlands and Portugal and calibrate the shocks on the observed data.

In order to map the observed data into the model we rebase the data in the following manner. We construct a benchmark level of GDP for each country and year: it is the GDP the country would have experienced if it had the same growth rate as the whole eurozone during the period 2001-2007. Hence, we assume no difference in TFP growth. Starting in 2008 this benchmark growth rate is the average of the eurozone crisis, around 1%. We compute a benchmark in the same manner for consumption, government spending, transfers and unit labor costs. The normalized data is the ratio of the observed data to this benchmark level. This enables us therefore to interpret these as deviations from the benchmark levels for each data series we observe. For both the household debt and the government debt, the rebased levels are the ratios of debt to the benchmark levels of GDP. Aggregate variables (employment, GDP, consumption, transfers, government spending and taxes and transfers...) are analyzed either in per capita terms or in ratios of GDP. For employment per capita, we take the deviation with respect to 2001 with an index of 1 for that year. 2001 is the base year for consumption and unit labor costs (index 1 in 2001). The normalized data are given in figures 15 and16 in the Appendix.

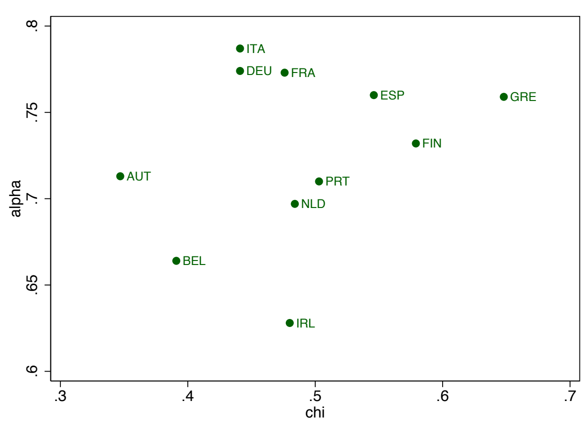
The parameters that serve in the simulations are given below.

Table 1: Parameters

Parameter Name		*Value*
Annual Discount Factor	β	0.95
Domestic share	α_j	country specific
Share of credit constrained	χ_j	country specific
Tax rate	τ	country specific
Asymmetric Phillips parameter	κ_d	0.3
Asymmetric Phillips parameters	κ_u	0.3

For the country specific domestic share of consumption α_j, we rely on the paper by Bussiere et al. (2011) who compute the total import content of expenditure components, including the value of indirect imports. For consumption expenditures and for our sample our countries the average implied domestic share in 2005 (the latest date in their study) is 27.3%. The lowest is 66.4% for Belgium and the highest is 78.7% for Italy. For the country specific share of credit constrained borrowers, we use the Eurosystem Household Finance and Consumption Survey (HFCS)[11]. As in Mendicino (2014), we use, for each country the fraction of household with liquid assets below two months of total households gross income to approximate the share of credit constrained households. The average for our set of countries is 48% with a maximum of 64.8% for Greece and a minimum of 34.7% for Austria. Ireland did not participate in the survey for this country we use the average of the eurozone. These two parameters for our panel of countries are shown on figure (8).

Figure 8: Share of credit constrained households (χ_j) and domestic share of consumption (α_j)



[11]The survey took place in 2010. In Greece and Spain, the data were collected in 2009 and 2008-09 respectively.

To transform observed aggregate household debt in the data into household debt $b_{j,t}$ in the model which is debt per impatient household (with share ψ_j), we take the household to benchmark income ratio for each country and then multiply it by the share of impatient in the country ψ_j. We use $t_0 = 2002$ as our base year.

1. Natural employment and prices are normalized to $n^\star = 1$ and $p^h_{j,t_0} = 1$ (so nominal GDP is normalized in the base year: $y_{j,t_0} = 1$)

2. Variables set to their observed values are: $\bar{b}_{j,t_0}$, T_{j,t_0}, $p^h_{j,t_0} g_{j,t_0}$, b^g_{j,t_0}, b^g_{j,t_0-1}, r_{j,t_0}. We then get $\bar{x}_{f,t_0}$, τ_j and $\tilde{y}_{j,t_0}$ from market clearing and budget constraints:

   (a) Foreign demand $\bar{x}_{f,t_0}$ is chosen to match net exports $e_{j,t_0} = \frac{1}{\alpha_j}\left(\bar{x}_{f,t_0} - (1-\alpha_j)\left(y_{j,t_0} - p^h_{j,t_0} g_{j,t_0}\right)\right)$

   (b) Get τ_j from the government budget constraint $p^h_{j,t_0} g_{j,t_0} + T_{j,t_0} - \tau_j y_{j,t_0} = \frac{b^g_{j,t_0+1}}{1+r_{j,t_0}} - b^g_{j,t_0}$

   (c) Disposable income at time t_0 is $\tilde{y}_{j,t_0} = (1-\tau_j)\, y_{j,t_0} + T_{j,t_0}$

3. Savers' assets s_{j,t_0} and s_{j,t_0-1} are chosen to solve the equilibrium conditions

   (a) $s_{j,t} = (1+\rho_{j,t})\,(s_{j,t-1} + \tilde{y}_{j,t}) - \mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]$

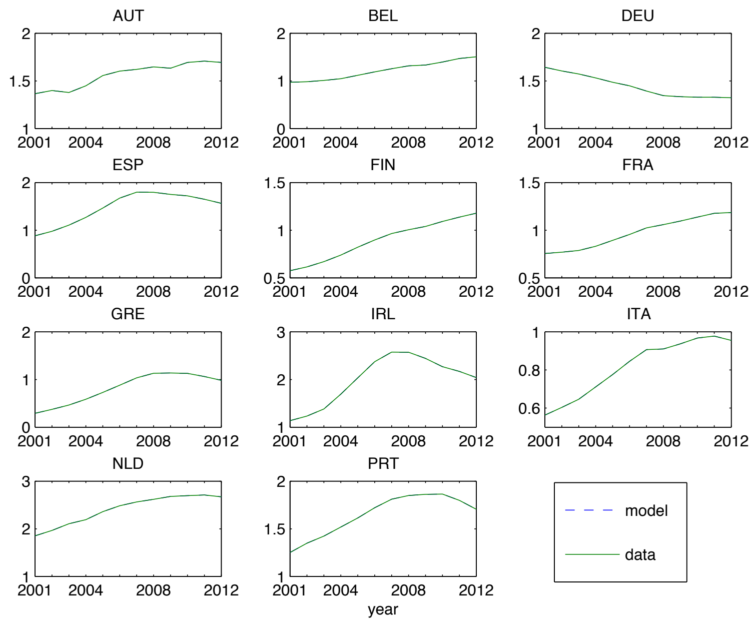
   (b) $(1-\alpha_j)\,\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right] = \frac{r}{1+r}\left(\alpha_j\,(1-\chi_j)\, s_{j,t+1} - \alpha_j \chi_j \bar{b}_{j,t+1} - b^g_{j,t+1}\right) + \bar{x}_{f,t}$.

   (c) $(1-\alpha_j)\,\tilde{y}_{j,t} = \alpha_j \chi_j \left(\frac{\bar{b}_{j,t+1}}{1+r_{j,t}} - \bar{b}_{j,t}\right) + \alpha_j\,(1-\chi_j)\left(s_{j,t} - \frac{s_{j,t+1}}{1+r_{j,t}}\right) + \frac{b^g_{j,t+1}}{1+r_{j,t}} - b^g_{j,t} + \bar{x}_{f,t}$.

We then feed exogenous processes for the different shocks (using rebased values) for observed household debt $\bar{b}_{j,t}$, fiscal policy $\tau_{j,t}$, $T_{j,t}$ and $p^h_{j,t} g_{j,t}$, and interest rate spreads $\rho_{j,t}$. For each country, we simulate the path between 2001 and 2012 of nominal GDP $y_{j,t}$, nominal consumption $x_{j,t}$, employment $n_{j,t}$, and public debt $b^g_{j,t}$.

The household debt (as a percentage of benchmark GDP) for each country is shown in Figure (9).

Figure 9: Household Debt



The simulated and observed paths of nominal GDP, employment, nominal consumption, unit labor costs, government debt for each country are respectively shown in Figures (10), (11), (12), (13), (14). Just to be clear, there is no degree of freedom in our simulations of nominal variables. There is *no* parameter which is set to match any moment in the data. The model is entirely constrained by observable micro estimates and by equilibrium conditions. The only parameter that we can adjust is the slope of the Phillips curve κ but it does not affect the GDP in euros, it only pins down the allocation of nominal GDP between prices (unit labor cost) and quantities (employment).

The model reproduces in particular the fact that countries that experienced the largest increase in household debt (Spain, Greece, Ireland, Italy and Portugal) in the boom period (from 2001 to 2008) also experienced the largest fall in nominal GDP, employment, consumption and to some extent wages in the bust period (up to 2012).

Figure 10: Simulated and observed nominal GDP

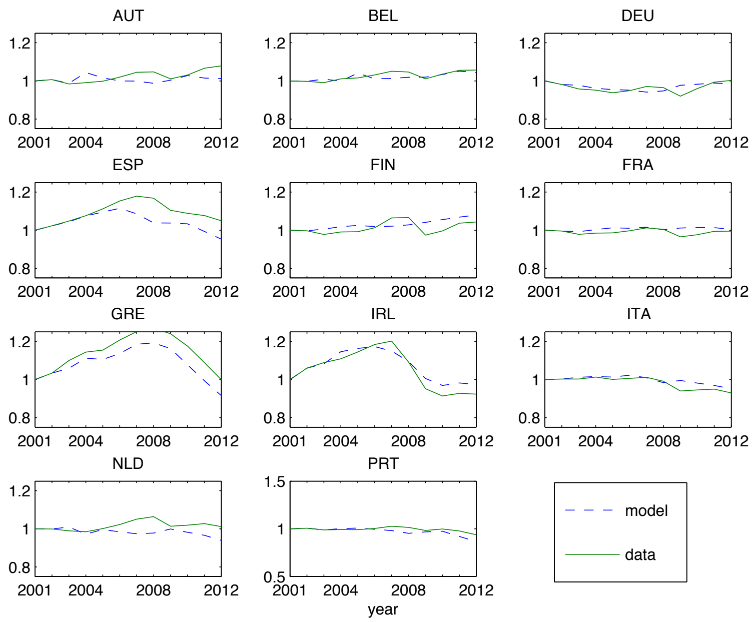


Figure 11: Simulated and observed employment

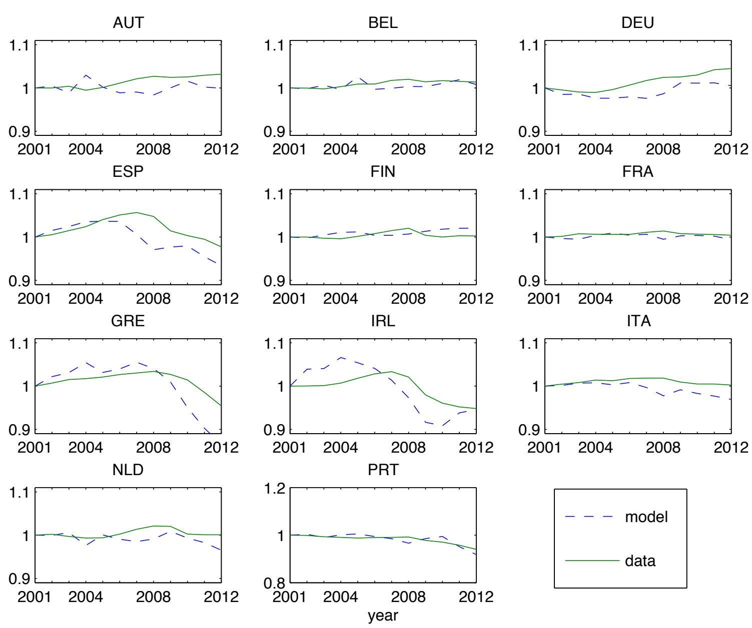


Figure 12: Simulated and observed nominal consumption

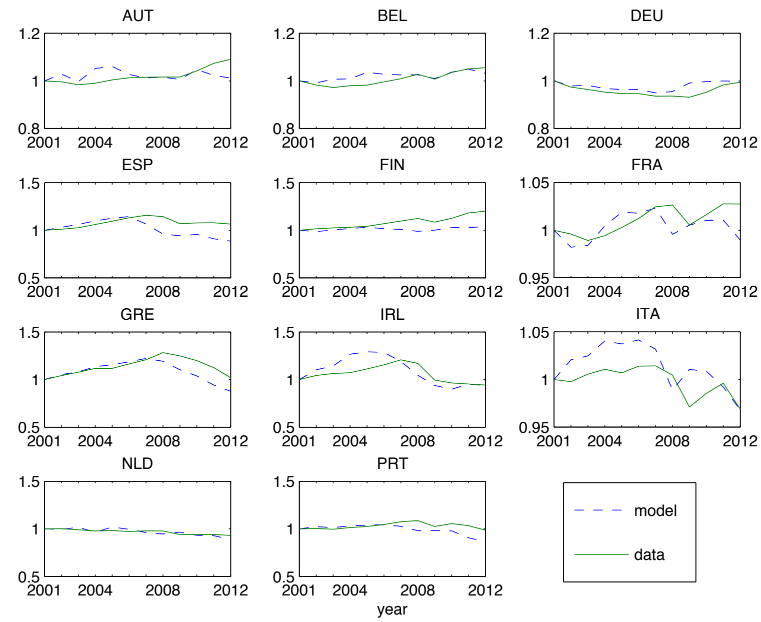


Figure 13: Simulated and observed unit labor costs

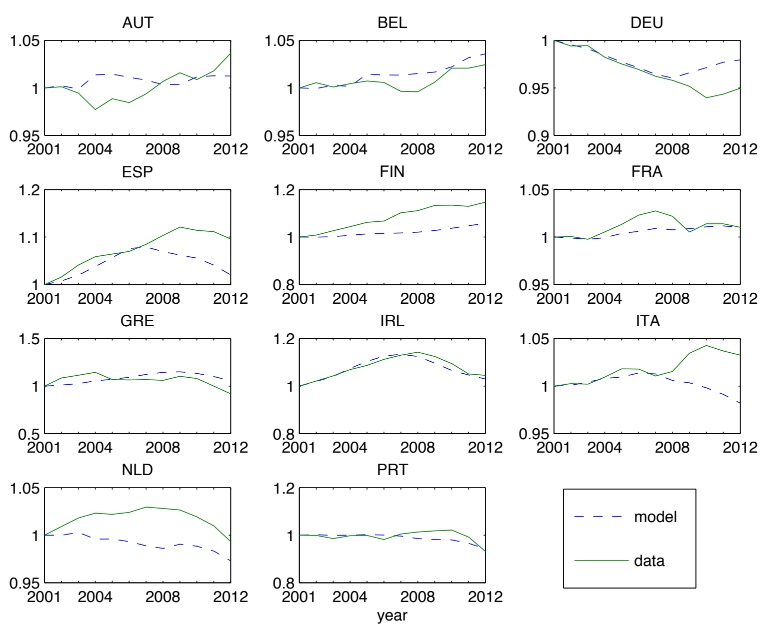
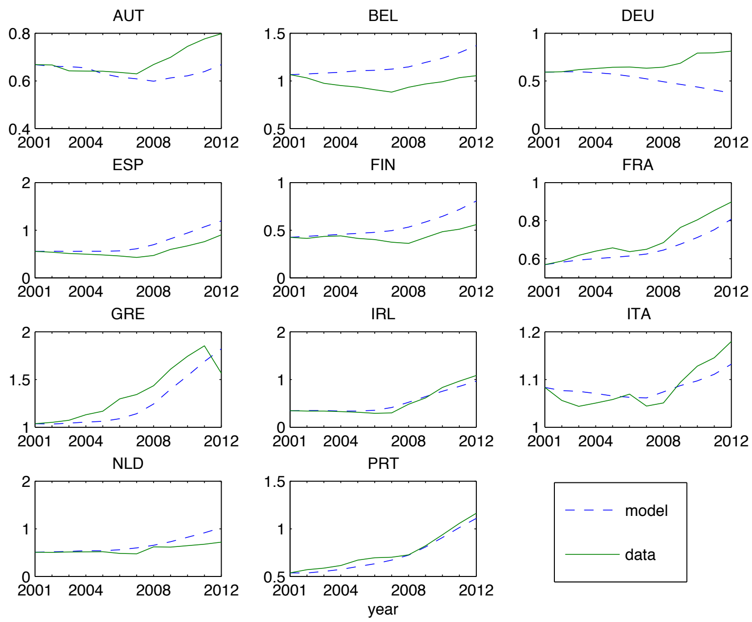


Figure 14: Simulated and observed government debt



4 Conclusion

To be completed.

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Appendix

A Model

A.1 Budget constraints.

Let us first rewrite the budget constraints and market clearing conditions. For the sake of generality, let us allow for three different interest rates for borrowers, savers, and the government: $r_{b,j,t}$, $r_{s,j,t}$, and $r_{g,j,t}$. Using the market clearing condition, and competition $p^h_{j,t} = w_{j,t}$, we get

$$y_{j,t} = \alpha_j \left(\chi_j x_{b,j,t} + (1-\chi_j)\, x_{s,j,t}\right) + \bar{x}_{f,t} + p^h_{j,t} g_{j,t}.$$

Nominal exports are $\bar{x}_f$, nominal imports are $(1-\chi_j)\, p^f_{j,t} c^f_{s,j,t} + \chi_j p^f_{j,t} c^f_{b,j,t}$ since the government does not buy imported goods. So net exports are

$$e_{j,t} = \bar{x}_{f,t} - (1-\alpha_j)\left(\chi_j x_{b,j,t} + (1-\chi_j)\, x_{s,j,t}\right).$$

We define disposable (after-tax) income as

$$\tilde{y}_{j,t} \equiv (1-\tau_{j,t})\, y_{j,t} + T_{j,t}.$$

We can then write the system for *nominal variables*

- $x_{b,j,t} = \frac{b_{j,t+1}}{1+r_{b,j,t}} + \tilde{y}_{j,t} - b_{j,t}$ budget constraint of impatient agents
- $x_{s,j,t} = s_{j,t} + \tilde{y}_{j,t} - \frac{s_{j,t+1}}{1+r_{s,j,t}}$ budget constraint of patient agents
- $y_{j,t} = \alpha_j \left(\chi_j x_{b,j,t} + (1-\chi_j)\, x_{s,j,t}\right) + \bar{x}_{f,t} + p^h_{j,t} g_{j,t}$ market clearing
- $p^h_{j,t} g_{j,t} + T_{j,t} - \tau_{j,t} y_{j,t} = \frac{b^g_{j,t+1}}{1+r_{g,j,t}} - b^g_{j,t}$ budget constraint of the government
- $e_{j,t} = \frac{1}{\alpha_j}\left(\bar{x}_{f,t} - (1-\alpha_j)\left(y_{j,t} - p^h_{j,t} g_{j,t}\right)\right)$, definition of net exports

Combining the first four equations, we get market clearing at time t:

$$(1-\alpha_j)\,\tilde{y}_{j,t} = \alpha_j \chi_j \left(\frac{\bar{b}_{j,t+1}}{1+r_{b,j,t}} - \bar{b}_{j,t}\right) + \alpha_j (1-\chi_j)\left(s_{j,t} - \frac{s_{j,t+1}}{1+r_{s,j,t}}\right) + \frac{b^g_{j,t+1}}{1+r_{g,j,t}} - b^g_{j,t} + \bar{x}_{f,t}. \tag{17}$$

It will often be useful to obtain a recursive equation for nominal GDP. Taking the first difference of (17) we get

$$(1-\alpha_j)\,\Delta\left[\tilde{y}_{j,t}\right] = \chi_j \alpha_j \left(\beta \Delta\left[\frac{\bar{b}_{j,t+1}}{1+\rho_{j,t}}\right] - \Delta\left[\bar{b}_{j,t}\right]\right) - (1-\chi_j)\,\alpha_j \left(\beta\Delta\left[\frac{s_{j,t+1}}{1+\rho_{j,t}}\right] - \Delta\left[s_{j,t}\right]\right) + \beta\Delta\left[\frac{b^g_{j,t+1}}{1+\rho_{j,t}}\right] - \Delta\left[b^g_{j,t}\right] + \Delta\left[\bar{x}_{f,t}\right.$$

A.2 Euler Equation and Expected Income

We assume that interest rates are the same within a country: $r_{b,j,t} = r_{s,j,t} = r_{g,j,t}$. We define $\rho_{j,t}$ as the deviation of the interest rate from its steady state value:

$$1 + r_{j,t} \equiv (1+r)(1+\rho_{j,t}).$$

The crucial variable is now the response of savers' consumption. The Euler equation of savers is

$$\frac{1}{x_{s,j,t}} = (1+\rho_{j,t})\,\mathbb{E}_t\left[\frac{1}{x_{s,j,t+1}}\right].$$

and we use the linear approximation

$$\mathbb{E}_t\left[x_{s,j,t+1}\right] = (1+\rho_{j,t})\,x_{s,j,t}.$$

Consider the following experiment. Savers enter the period with a given level of savings. Then there is a shock to interest rates. For instance, starting from a steady state where $\rho = 0$, if the new rate is such that $\rho > 0$, savings jumps up, and if the new rate is such that $\rho < 0$, consumption jumps up. The budget constraint at time t is

$$x_{s,j,t} = s_{j,t} + \tilde{y}_{j,t} - \frac{s_{j,t+1}}{1+r_{j,t}}$$

and the expected budget constraint at time $t+1$ is

$$\mathbb{E}_t\left[\frac{s_{j,t+2}}{1+r_{j,t+1}}\right] = s_{j,t+1} + \mathbb{E}_t\left[\tilde{y}_{j,t+1} - x_{s,j,t+1}\right].$$

Combining the budget constraints and the linearized Euler equation, we get

$$(1+\beta)s_{j,t+1} + \mathbb{E}_t\left[\tilde{y}_{j,t+1}\right] - \beta\mathbb{E}_t\left[\frac{s_{j,t+2}}{1+\rho_{j,t+1}}\right] = (1+\rho_{j,t})\left(s_{j,t} + \tilde{y}_{j,t}\right). \tag{18}$$

since on average $\beta(1+r) = 1$.

A.3 Martingales and *iid* Interest Rates

We simplify the analysis by assuming martingales for the exogenous shocks $\bar{b}_{j,t}$ and $\bar{x}_{f,t}$: $\mathbb{E}_t\left[\bar{b}_{j,t+2}\right] = \bar{b}_{j,t+1}$ and $\mathbb{E}_t\left[\bar{x}_{f,t+1}\right] = \bar{x}_{f,t}$. We also assume that the policy function of the government is such that $\mathbb{E}_t\left[b^g_{j,t+2}\right] = b^g_{j,t+1}$. Finally we assume that interest rates are iid:

$$\mathbb{E}_t\left[\rho_{j,t+1}\right] = 0$$

We then have the following Lemma.

Lemma 3. $\mathbb{E}_t\left[s_{j,t+2}\right] = s_{t+1}$

We guess and verify that this is correct. Suppose the Lemma is true, then we obtain two important equations. Equation (18) becomes

$$s_{j,t} = (1+\rho_{j,t})(s_{j,t-1} + \tilde{y}_{j,t}) - \mathbb{E}_t[\tilde{y}_{j,t+1}], \tag{19}$$

and expected market clearing at $t+1$ is

$$(1-\alpha_j)\,\mathbb{E}_t[\tilde{y}_{j,t+1}] = \frac{r}{1+r}\left(\alpha_j(1-\chi_j)s_{j,t+1} - \alpha_j\chi\bar{b}_{j,t+1} - b^g_{j,t+1}\right) + \bar{x}_{f,t}. \tag{20}$$

Therefore

$$(1-\alpha_j)s_{j,t+1} = (1-\alpha_j)(1+\rho_{j,t})(s_{j,t}+\tilde{y}_{j,t}) - \frac{r}{1+r}\left(\alpha_j(1-\chi_j)s_{j,t+1} - \alpha_j\chi\bar{b}_{j,t+1} - b^g_{j,t+1}\right) - \frac{\bar{x}_{f,t}}{1-\alpha_j}.$$

Using market clearing at time t we get

$$\frac{s_{j,t+1}}{1+\rho_{j,t}} - s_{j,t} = \frac{\alpha_j\chi_j}{1-\alpha_j\chi_j}\left(\frac{\bar{b}_{j,t+1}}{1+\rho_{j,t}} - \bar{b}_{j,t}\right) + \frac{1}{1-\alpha_j\chi_j}\left(\frac{b^g_{j,t+1}}{1+\rho_{j,t}} - b^g_{j,t} + \frac{\rho_{j,t}}{1+\rho_{j,t}}\frac{\bar{x}_{f,t}}{1-\alpha_j}\right) \tag{21}$$

Savings inherit the dynamic properties of $\bar{b}_{j,t}$ and $b^g_{j,t}$. Therefore this validates our conjecture that $\mathbb{E}_t[s_{j,t+1}] = s_{j,t}$.

A.4 Equilibrium Conditions

The savers equation with iid shock is $s_{j,t+1} = \frac{1+r_{j,t}}{1+r}(s_{j,t}+\tilde{y}_{j,t}) - \mathbb{E}_t[\tilde{y}_{j,t+1}]$. The "1" comes from the fact that the shock lasts one period. If instead the expected persistence was $1/\lambda$ then the equation would be

$$s_{j,t+1} = \frac{\lambda + r_{j,t}}{\lambda + r}(s_{j,t}+\tilde{y}_{j,t}) - \mathbb{E}_t[\tilde{y}_{j,t+1}]$$

Thus $\lambda < 1$ makes savers more responsive to interest rate changes. Note that this is only about the expectations of savers, it does not change the resource constraints. In both cases, we need to make sure that

$$s_{j,t} > -\bar{b}_{j,t}$$

Equilibrium conditions

1. $s_{j,t+1} = \frac{\lambda+r_{j,t}}{\lambda+r}(s_{j,t}+\tilde{y}_{j,t}) - \mathbb{E}_t[\tilde{y}_{j,t+1}]$
2. $(1-\alpha_j)\,\mathbb{E}_t[\tilde{y}_{j,t+1}] = \frac{r}{1+r}\left(\alpha_j(1-\chi_j)s_{j,t+1} - \alpha_j\chi_j\bar{b}_{j,t+1} - b^g_{j,t+1}\right) + \bar{x}_{f,t}$.
3. $(1-\alpha_j)\tilde{y}_{j,t} = \alpha_j\chi_j\left(\frac{\bar{b}_{j,t+1}}{1+r_{j,t}} - \bar{b}_{j,t}\right) + \alpha_j(1-\chi_j)\left(s_{j,t} - \frac{s_{j,t+1}}{1+r_{j,t}}\right) + \frac{b^g_{j,t+1}}{1+r_{j,t}} - b^g_{j,t} + \bar{x}_{f,t}$.
4. $p^h_{j,t}g_{j,t} + T_{j,t} - \tau_j y_{j,t} = \frac{b^g_{j,t+1}}{1+r_{j,t}} - b^g_{j,t}$
5. $\tilde{y}_{j,t} = (1-\tau_j)y_{j,t} + T_{j,t}$

There are five equations and five unknowns: $\tilde{y}_{j,t}$, $y_{j,t}$, $b^g_{j,t+1}$, $s_{j,t+1}$, $\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]$. The exogenous shocks are $\bar{b}_{j,t}$, $\bar{x}_{f,t}$ and $r_{j,t}$. The policy shocks are $T_{j,t}$ and $g_{j,t}$ and τ_j is constant. The state space of predetermined endogenous variable is $s_{j,t}$ and $b^g_{j,t}$.

Combining 1 and 2 we can get rid of $\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]$ to get a system with 4 unknowns

$$\begin{aligned}
(1-\alpha_j)\, s_{j,t+1} &= (1-\alpha_j)\frac{\lambda+r_{j,t}}{\lambda+r}\left(s_{j,t}+\tilde{y}_{j,t}\right)-\frac{r}{1+r}\left(\alpha_j\left(1-\chi_j\right)s_{j,t+1}-\alpha_j\chi\bar{b}_{j,t+1}-b^g_{j,t+1}\right)-\bar{x}_{f,t}\\
(1-\alpha_j)\,\tilde{y}_{j,t} &= \alpha_j\chi_j\left(\frac{\bar{b}_{j,t+1}}{1+r_{j,t}}-\bar{b}_{j,t}\right)+\alpha_j\left(1-\chi_j\right)\left(s_{j,t}-\frac{s_{j,t+1}}{1+r_{j,t}}\right)+\frac{b^g_{j,t+1}}{1+r_{j,t}}-b^g_{j,t}+\bar{x}_{f,t}\\
\frac{b^g_{j,t+1}}{1+r_{j,t}}-b^g_{j,t} &= p^h_{j,t}g_{j,t}+T_{j,t}-\tau_j y_{j,t}\\
\tilde{y}_{j,t} &= (1-\tau_j)\,y_{j,t}+T_{j,t}
\end{aligned}$$

Policy Rule of Government We specify the general policy rule as follows

1. Fiscal variables $p^h_{j,t}g_{j,t}$ and $T_{j,t}$ are pre-determined. Solve the 4*4 system above.
2. Get $b^g_{j,t+1}$ from
$$\frac{b^g_{j,t+1}}{1+r_{j,t}}=b^g_{j,t}+p^h_{j,t}g_{j,t}+T_{j,t}-\tau_j y_{j,t}$$
3. Set transfers $T_{j,t+1}$ for next period so that $\mathbb{E}_t\left[b^g_{j,t+2}\right]=b^g_{j,t+1}$ assuming martingales for $p^h_{j,t}g_{j,t}$. Then the expected budget constraint is
$$\tau_j\mathbb{E}_t\left[y_{j,t+1}\right]=p^h_{j,t}g_{j,t}+T_{j,t+1}+\frac{r}{1+r}b^g_{j,t+1}$$
and $\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]$ is known from the 4*4 system
$$\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]=(1-\tau_j)\,\mathbb{E}_t\left[y_{j,t+1}\right]+T_{j,t+1}$$
so we can solve for $T_{j,t+1}$:
$$T_{j,t+1}=\tau_j\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]-(1-\tau_j)\,p^h_{j,t}g_{j,t}-(1-\tau_j)\frac{r}{1+r}b^g_{j,t+1}$$

A.5 The Case of Constant Interest Rates

Savers' Spending with Constant Interest Rates With constant interest rates, the Euler equation is simply $x_{s,j,t}=\mathbb{E}_t\left[x_{s,j,t+1}\right]$ and equation (19) becomes $s_{j,t+1}-s_{j,t}=\tilde{y}_{j,t}-\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]$. Combining market clearing at t and $t+1$, we get a simple equation

$$(1-\alpha_j)\left(\tilde{y}_{j,t}-\mathbb{E}_t\left[\tilde{y}_{j,t+1}\right]\right)=\alpha_j\chi_j\left(\bar{b}_{j,t+1}-\bar{b}_{j,t}\right)-\alpha_j\left(1-\chi_j\right)\left(s_{j,t+1}-s_{j,t}\right)+b^g_{j,t+1}-b^g_{j,t}$$

Equation (21) becomes

$$s_{j,t+1}-s_{j,t}=\frac{\alpha_j\chi_j}{1-\alpha_j\chi_j}\left(\bar{b}_{j,t+1}-\bar{b}_{j,t}\right)+\frac{1}{1-\alpha_j\chi_j}\left(b^g_{j,t+1}-b^g_{j,t}\right). \tag{22}$$

The first difference of the disposable income equation is

$$(1-\alpha_j)\,\Delta\left[\tilde{y}_{j,t}\right]=\chi_j\alpha_j\left(\frac{\Delta\left[\bar{b}_{j,t+1}\right]}{1+r}-\Delta\left[\bar{b}_{j,t}\right]\right)-(1-\chi_j)\,\alpha_j\left(\frac{\Delta\left[s_{j,t+1}\right]}{1+r}-\Delta\left[s_{j,t}\right]\right)+\frac{\Delta\left[b^g_{j,t+1}\right]}{1+r}-\Delta\left[b^g_{j,t}\right]+\Delta\left[\bar{x}_{f,t}\right]$$

Using equation (22), we then see that

$$\Delta\tilde{y}_{j,t}=\frac{\Delta s_{j,t+1}}{1+r}-\Delta s_{j,t}+\frac{\Delta\left[\bar{x}_{f,t}\right]}{1-\alpha_j}. \tag{23}$$

Equation (23) has an important implication. Using the budget constraint of the patient agents, we get

$$\begin{aligned}\Delta x_{s,j,t} &= \Delta s_{j,t}+\Delta\tilde{y}_{j,t}-\frac{\Delta s_{j,t+1}}{1+r}\\ &= \frac{\Delta\left[\bar{x}_{f,t}\right]}{1-\alpha_j}\end{aligned}$$

Therefore we have the following Lemma

Lemma 4. *The nominal spending of patient agents remains constant in response to any sequence of shocks to the debt of impatient agents and of the government.*

This is important because it means that our linear approximation for x is in fact exact for shocks to $\bar{b}_{j,t+1}$ and to $b^g_{j,t+1}$. Note that we can also express the evolution of disposable income as a function of exogenous shocks

$$\Delta\tilde{y}_{j,t}=\frac{\chi_j\alpha_j}{1-\alpha_j\chi_j}\left(\frac{\Delta\bar{b}_{j,t+1}}{1+r}-\Delta\bar{b}_{j,t}\right)+\frac{1}{1-\alpha_j\chi_j}\left(\frac{\Delta b^g_{j,t+1}}{1+r}-\Delta b^g_{j,t}\right)+\frac{\Delta\left[\bar{x}_{f,t}\right]}{1-\alpha_j}$$

And for GDP we get

$$\Delta\left[(1-\tau_{j,t})\,y_{j,t}\right]=\frac{\chi_j\alpha_j\left(\frac{\Delta\bar{b}_{j,t+1}}{1+r}-\Delta\bar{b}_{j,t}\right)+\frac{\Delta b^g_{j,t+1}}{1+r}-\Delta b^g_{j,t}}{1-\alpha_j\chi_j}-\Delta T_{j,t}+\frac{\Delta\left[\bar{x}_{f,t}\right]}{1-\alpha_j}$$

The fact that changes in transfers enter negatively reflect the fact that they have a smaller impact on output than direct spending. This is because transfers can be spent on foreign goods and can be saved by patient agents.

Simplest policy rule under constant interest rates In the model with constant interest rates, the simplest policy rule is

$$b^g_{j,t+1}-b^g_{j,t}=-\gamma\alpha_j\chi_j\left(\bar{b}_{j,t+1}-\bar{b}_{j,t}\right), \tag{24}$$

with $\gamma\leq 1$. From equation (14), we then get

$$s_{j,t+1}-s_{j,t}=(1-\gamma)\,\frac{\alpha_j\chi_j}{1-\alpha_j\chi_j}\left(\bar{b}_{j,t+1}-\bar{b}_{j,t}\right).$$

Perfect stabilization is possible with $\gamma = 1$. More generally we have

$$\tilde{y}_{j,t} - \tilde{y}_{j,t-1} = (1-\gamma)\frac{\chi_j \alpha_j}{1-\alpha_j \chi_j}\left(\frac{\Delta\left[\bar{b}_{j,t+1}\right]}{1+r} - \Delta\left[\bar{b}_{j,t}\right]\right)$$

so disposable income is partially stabilized or completeley if $\gamma = 1$. From the budget constraint of the government,

$$p^h_{j,t} g_{j,t} + T_{j,t} - \tau_{j,t} y_{j,t} = \frac{b^g_{j,t+1}}{1+r} - b^g_{j,t},$$

this policy also implies a path for nominal primary deficits. Disposable income and primary deficits are uniquely pinned down by the policy rule (24) irrespective of the composition of government spending. The composition of government spending matters for GDP, however, since

$$y_{j,t} = \frac{\tilde{y}_{j,t} - T_{j,t}}{1-\tau_{j,t}}$$

We get

$$\Delta\left[(1-\tau_{j,t})\, y_{j,t}\right] = (1-\gamma)\frac{\alpha_j \chi_j}{1-\alpha_j \chi_j}\left(\frac{\Delta\left[\bar{b}_{j,t+1}\right]}{1+r} - \Delta\left[\bar{b}_{j,t}\right]\right) - \Delta\left[T_{j,t}\right].$$

and if we assume a constant τ_j, we get

$$\mathbb{E}_t\left[y_{j,t+1}\right] - y_{j,t} = -\frac{s_{j,t+1} - s_{j,t}}{1-\tau_j} = -\frac{1-\gamma_j}{1-\tau_j}\frac{\alpha_j \chi_j}{1-\alpha_j \chi_j}\left(\bar{b}_{j,t+1} - \bar{b}_{j,t}\right)$$

These two equations show the tendency of mean reversion in this model. For net foreign assets, we get

$$\begin{aligned} a_{j,t} - a_{j,t-1} &= (1-\chi_j)\left(s_{j,t+1} - s_{j,t}\right) - \chi_j\left(\bar{b}_{j,t+1} - \bar{b}_{j,t}\right) - \left(b^g_{j,t+1} - b^g_{j,t}\right) \\ &= -\frac{(1-\alpha_j)\,\chi_j}{1-\alpha_j \chi_j}\left(\bar{b}_{j,t+1} - \bar{b}_{j,t} + b^g_{j,t+1} - b^g_{j,t}\right) = -\left(1-\alpha_j \chi_j \gamma_j\right)\frac{\chi_j\,(1-\alpha_j)}{1-\alpha_j \chi_j}\left(\bar{b}_{j,t+1} - \bar{b}_{j,t}\right) \end{aligned}$$

An increase in borrowing by impatient agents deteriorates the net foreign asset position of the country, but this deterioration is muted if the government conducts an active countercyclical fiscal policy with a high γ_j. Notice, however, that even when $\gamma_j = 1$, so that nominal GDP is perfectly stabilized, the net foreign position is not constant.

The evolution of net exports is given by:

$$e_{j,t} - e_{j,t-1} = \Delta\left[\bar{x}_{f,t}\right] - \frac{\chi_j\,(1-\alpha_j)}{1-\alpha_j \chi_j}\left[\frac{\Delta b_{j,t+1}}{1+r} - \Delta b_{j,t} + \frac{\Delta b^g_{j,t+1}}{1+r} - \Delta b^g_{j,t}\right] \tag{25}$$

With the government rule of equation (24) we can get:

$$e_{j,t} - e_{j,t-1} = \Delta\left[\bar{x}_{f,t}\right] - \left(1-\alpha_j \chi_j \gamma_j\right)\frac{\chi_j\,(1-\alpha_j)}{1-\alpha_j \chi_j}\left[\frac{b_{j,t+1} - b_{j,t}}{1+r} - \left(b_{j,t} - b_{j,t-1}\right)\right] \tag{26}$$

so that net exports decrease with borrowing of impatient agents even if the government conducts an active countercyclical fiscal policy with a high γ_j.

We can consider more general dynamics of the economy for arbitrary paths for transfers and spending T and g that are consistent with a martingale for government debt. The dynamic equations are

- $\Delta\left[\tilde{y}_{j,t}\right]=\frac{\chi_j\alpha_j}{1-\alpha_j\chi_j}\left(\frac{\Delta b_{j,t+1}}{1+r}-\Delta b_{j,t}\right)+\frac{1}{1-\alpha_j\chi_j}\left(\frac{\Delta b^g_{j,t+1}}{1+r}-\Delta b^g_{j,t}\right)+\frac{\Delta[\bar{x}_{f,t}]}{1-\alpha_j}$
- $\left(1-\tau_j\right)\Delta\left[y_{j,t}\right]=\frac{1}{1-\alpha_j\chi_j}\left(\alpha_j\chi_j\left(\Delta\left[\frac{b_{j,t+1}}{1+r}\right]-\Delta\left[b_{j,t}\right]\right)+\Delta\left[\frac{b^g_{j,t+1}}{1+r}\right]-\Delta\left[b^g_{j,t}\right]\right)-\Delta\left[T_{j,t}\right]+\frac{\Delta[\bar{x}_{f,t}]}{1-\alpha_j}$
- $\Delta\left[\frac{b^g_{j,t+1}}{1+r}\right]-\Delta\left[b^g_{j,t}\right]=\Delta\left[p^h_{j,t}g_{j,t}\right]+\Delta\left[T_{j,t}\right]-\tau_j\Delta\left[y_{j,t}\right]$

Combining we get

$$\left(1+\frac{\alpha_j\chi_j}{1-\alpha_j\chi_j}\tau_j\right)\Delta\left[y_{j,t}\right]=\frac{\alpha_j\chi_j}{1-\alpha_j\chi_j}\left(\Delta\left[\frac{\bar{b}_{j,t+1}}{1+r}\right]-\Delta\left[\bar{b}_{j,t}\right]\right)+\frac{1}{1-\alpha_j\chi_j}\Delta\left[p^h_{j,t}g_{j,t}\right]+\frac{\alpha_j\chi_j}{1-\alpha_j\chi_j}\Delta\left[T_{j,t}\right]+\frac{\Delta\left[\bar{x}_{f,t}\right]}{1-\alpha_j} \tag{27}$$

A.6 Saver spending lemma with stochastic rates

A.6.1 Proof of Proposition 1

Savers solve the following problem:

$$\begin{gathered}\max\mathbb{E}_t\sum_{t\geq0}\beta^t\log\left(x_{s,j,t}\right)\\ s.t.\\ x_{s,j,t}+\frac{s_{j,t+1}}{1+r_{s,j,t}}=s_{j,t}+\tilde{y}_{j,t}\\ s_{j,t}>-\bar{b}_{j,t}\end{gathered}$$

Let us integrate forward the budget constraint:

$$\sum_{k=0}^{K}\frac{x_{s,j,t+k}}{R_{j,t,k}}+\frac{s_{j,t+K+1}}{R_{j,t,K+1}}=s_{j,t}+\sum_{k=0}^{K}\frac{\tilde{y}_{j,t+k}}{R_{j,t,k}}$$

where the k-period ahead discount rate for $k\geq1$ from the savers' perspective

$$R_{j,t,k}\equiv\left(1+r_{s,j,t}\right)..\left(1+r_{s,j,t+k}\right),$$

and the convention $R_{j,t,0}=1$. The next step is to use the resource constraint

$$\left(1-\alpha_j\right)\tilde{y}_{j,t}=\alpha_j\chi_j\left(\frac{b_{j,t+1}}{1+r_{b,j,t}}-b_{j,t}\right)-\alpha_j\left(1-\chi_j\right)\left(\frac{s_{j,t+1}}{1+r_{s,j,t}}-s_{j,t}\right)+\bar{x}_{f,t}+\frac{b^g_{j,t+1}}{1+r_{g,j,t}}-b^g_{j,t}$$

Summing and rearranging the terms, we get

$$
\begin{aligned}
(1-\alpha_j)\left(\tilde{y}_{j,t}+\frac{\tilde{y}_{j,t+1}}{R_{j,t,1}}\right) &= \alpha_j\chi_j\left(\frac{b_{j,t+1}}{1+r_{b,j,t}}-\frac{b_{j,t+1}}{R_{j,t,1}}+\frac{1}{R_{j,t,1}}\frac{b_{j,t+2}}{1+r_{b,j,t+1}}-b_{j,t}\right)\\
&-\alpha_j(1-\chi_j)\left(-s_{j,t}+\frac{1}{R_{j,t,1}}\frac{s_{j,t+2}}{1+r_{s,j,t+1}}\right)+\bar{x}_{f,t}+\frac{\bar{x}_{f,t+1}}{R_{j,t,1}}\\
&+\frac{b^g_{j,t+1}}{1+r_{g,j,t}}-\frac{b^g_{j,t+1}}{R_{j,t,1}}+\frac{1}{R_{j,t,1}}\frac{b^g_{j,t+2}}{1+r_{g,j,t+1}}-b^g_{j,t}
\end{aligned}
$$

Then define

$$
\begin{aligned}
1-\phi_{b,j,t} &\equiv \frac{1+r_{s,j,t}}{1+r_{b,j,t}}\\
1-\phi_{g,j,t} &\equiv \frac{1+r_{s,j,t}}{1+r_{g,j,t}}
\end{aligned}
$$

to write

$$
\begin{aligned}
(1-\alpha_j)\left(\tilde{y}_{j,t}+\frac{\tilde{y}_{j,t+1}}{R_{j,t,1}}+\frac{\tilde{y}_{j,t+2}}{R_{j,t,2}}\right) &= -\alpha_j\chi_j\left(b_{j,t}+\frac{\phi_{b,j,t}}{R_{j,t,1}}b_{j,t+1}+\frac{\phi_{b,j,t+1}}{R_{j,t,2}}b_{j,t+2}-\frac{1}{R_{j,t,2}}\frac{b_{j,t+3}}{1+r_{b,j,t+2}}\right)\\
&+\alpha_j(1-\chi_j)\left(s_{j,t}-\frac{s_{j,t+3}}{R_{j,t,3}}\right)+\bar{x}_{f,t}+\frac{\bar{x}_{f,t+1}}{R_{j,t,1}}+\frac{\bar{x}_{f,t+2}}{R_{j,t,2}}\\
&-b^g_{j,t}-\frac{\phi_{g,j,t}}{R_{j,t,1}}b^g_{j,t+1}-\frac{\phi_{b,j,t+1}}{R_{j,t,2}}b^g_{j,t+2}+\frac{1}{R_{j,t,2}}\frac{b^g_{j,t+3}}{1+r_{g,j,t+2}}
\end{aligned}
$$

Therefore for a generic horizon K:

$$
\begin{aligned}
\sum_{k=0}^{K}\frac{(1-\alpha_j)\tilde{y}_{j,t+k}}{R_{j,t,k-1}} &= \alpha_j\left((1-\chi_j)s_{j,t}-\chi_j b_{j,t}\right)-b^g_{j,t}+\sum_{k=0}^{K}\frac{\bar{x}_{f,t+k}}{R_{j,t,k}}\\
&-\alpha_j\chi_j\sum_{k=1}^{K}\frac{\phi_{b,j,t+k-1}}{R_{j,t,k}}b_{j,t+k}-\sum_{k=1}^{K}\frac{\phi_{g,j,t+k-1}}{R_{j,t,k}}b^g_{j,t+k}\\
&-(1-\chi_j)\alpha_j\frac{s_{j,t+K+1}}{R_{j,t,K+1}}+\frac{1}{R_{j,t,K}}\left(\frac{\alpha_j\chi_j b_{j,t+K+1}}{1+r_{b,j,t+K}}+\frac{b^g_{j,t+K+1}}{1+r_{g,j,t+K}}\right)
\end{aligned}
$$

We take the limit and we impose a No-Ponzi condition

$$
\begin{aligned}
\lim_{K\to\infty}\mathbb{E}_t\left[\frac{s_{j,t+K+1}}{R_{j,t,K+1}}\right] &= 0\\
\lim_{K\to\infty}\mathbb{E}_t\left[\frac{1}{R_{j,t,K}}\frac{b_{j,t+K+1}}{1+r_{b,j,t+K}}\right] &= 0\\
\lim_{K\to\infty}\mathbb{E}_t\left[\frac{1}{R_{j,t,K}}\frac{b^g_{j,t+K+1}}{1+r_{g,j,t+K}}\right] &= 0
\end{aligned}
$$

The inter-temporal current account condition is

$$
\begin{aligned}
(1-\alpha_j)\,\mathbb{E}_t \sum_{k=0}^{\infty} \frac{\tilde{y}_{j,t+k}}{R_{j,t,k}} &= \alpha_j\,(1-\chi_j)\, s_{j,t} - \alpha_j \chi_j b_{j,t} - b^g_{j,t} + \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\bar{x}_{f,t+k}}{R_{j,t,k}} \\
&\quad - \mathbb{E}_t \sum_{k=1}^{\infty} \frac{\alpha_j \chi_j \phi_{b,j,t+k-1} b_{j,t+k} + \phi_{g,j,t+k-1} b^g_{j,t+k}}{R_{j,t,k}}
\end{aligned}
$$

Define

$$
\Phi_t \equiv \mathbb{E}_t \sum_{k=1}^{\infty} \frac{\alpha_j \chi_j \phi_{b,j,t+k-1} b_{j,t+k} + \phi_{g,j,t+k-1} b^g_{j,t+k}}{R_{j,t,k}}.
$$

The intertemporal budget constraint is then

$$
(1-\alpha_j)\,\mathbb{E}_t \sum_{k=0}^{\infty} \frac{x_{s,j,t+k}}{R_{j,t,k}} = (1-\alpha_j\chi_j)\, s_{j,t} - \chi_j \alpha_j b_{j,t} - b^g_{j,t} + \mathbb{E}_t \sum_{k=0}^{\infty} \frac{\bar{x}_{f,t+k}}{R_{j,t,k}} - \Phi_t
$$

The net present value of savers' spending depends on beginning of period net foreign assets $(1-\alpha_j\chi_j)\, s_{j,t} - \chi_j \alpha_j b_{j,t} - b^g_{j,t}$, the net present value of exports, and the cost of debt when there are spreads.

A.6.2 Proof of Lemma 2

We define $\rho_{j,t}$ as the deviation of the interest rate from its steady state value:

$$
1 + r_{j,t} \equiv (1+r)\,(1+\rho_{j,t})\,.
$$

Note that we can rewrite equation (21) as

$$
(1-\alpha_j\chi_j)\,\Delta\left[\frac{s_{j,t+1}}{1+\rho_{j,t}} - s_{j,t}\right] - \Delta\left[\frac{\rho_{j,t}}{1+\rho_{j,t}} \frac{\bar{x}_{f,t}}{1-\alpha_j}\right] = \alpha_j\chi_j \Delta\left[\frac{\bar{b}_{j,t+1}}{1+\rho_{j,t}} - \bar{b}_{j,t}\right] + \Delta\left[\frac{b^g_{j,t+1}}{1+\rho_{j,t}} - b^g_{j,t}\right]
$$

or as

$$
\Delta\left[(1-\alpha_j\chi_j)\, s_{j,t+1} - b^g_{j,t+1} - \alpha_j\chi_j \bar{b}_{j,t+1}\right] = \rho_{j,t}\left(\frac{\bar{x}_{f,t}}{1-\alpha_j} + (1-\alpha_j\chi_j)\, s_{j,t} - \alpha_j\chi_j \bar{b}_{j,t} - b^g_{j,t}\right)
$$

We can write dy as

$$
(1-\alpha_j)\,\Delta\left[\tilde{y}_{j,t}\right] = \beta\left(\chi_j\alpha_j\Delta\left[\frac{\bar{b}_{j,t+1}}{1+\rho_{j,t}} - \bar{b}_{j,t}\right] + \Delta\left[\frac{b^g_{j,t+1}}{1+\rho_{j,t}} - b^g_{j,t}\right]\right) - (1-\beta)\,\Delta\left[\chi_j\alpha_j\bar{b}_{j,t} + b^g_{j,t}\right] + \Delta\left[\bar{x}_{f,t}\right] - (1-\chi_j)\,\alpha_j\left(\beta\Delta\left[\right.\right.
$$

Using the first we get

$$
(1-\alpha_j)\,\Delta\left[\tilde{y}_{j,t}\right] = \beta\,(1-\alpha_j)\,\Delta\left[\frac{s_{j,t+1}}{1+\rho_{j,t}} - s_{j,t}\right] - \beta\Delta\left[\frac{\rho_{j,t}}{1+\rho_{j,t}} \frac{\bar{x}_{f,t}}{1-\alpha_j}\right] - (1-\beta)\,\Delta\left[\chi_j\alpha_j\bar{b}_{j,t} + b^g_{j,t} - (1-\chi_j)\,\alpha_j s_{j,t}\right] + \Delta\left[\bar{x}_{f,t}\right]
$$

or

$$(1-\alpha_j)\,\Delta\left[\tilde{y}_{j,t}\right]=\beta\,(1-\alpha_j)\,\Delta\left[\frac{s_{j,t+1}}{1+\rho_{j,t}}\right]-(1-\alpha_j)\,\Delta\left[s_{j,t}\right]+\Delta\left[\bar{x}_{f,t}\right]-(1-\beta)\,\Delta\left[\chi_j\alpha_j\bar{b}_{j,t}+b^{g}_{j,t}-(1-\alpha_j\chi_j)\,s_{j,t}\right]-\beta\Delta\left[\frac{\rho_{j,t}}{1+\rho_j\ldots}\right.$$

or

$$(1-\alpha_j)\,\Delta\left[\tilde{y}_{j,t}\right]=\beta\,(1-\alpha_j)\,\Delta\left[\frac{s_{j,t+1}}{1+\rho_{j,t}}\right]-(1-\alpha_j)\,\Delta\left[s_{j,t}\right]+(1-\beta)\,\rho_{j,t}\left(\frac{\bar{x}_{f,t-1}}{1-\alpha_j}+(1-\alpha_j\chi_j)\,s_{j,t-1}-\alpha_j\chi_j\bar{b}_{j,t-1}-b^{g}_{j,t-1}\right)+\ldots$$

Therefore we get

$$\Delta\left[\tilde{y}_{j,t}\right]=\beta\Delta\left[\frac{s_{j,t+1}}{1+\rho_{j,t}}\right]-\Delta\left[s_{j,t}\right]+\frac{\Delta\left[\bar{x}_{f,t}\right]}{1-\alpha_j}+\Omega_{j,t}$$

where

$$\Omega_{j,t}\equiv\frac{(1-\beta)\,\rho_{j,t}\left(\frac{\bar{x}_{f,t-1}}{1-\alpha_j}+(1-\alpha_j\chi_j)\,s_{j,t-1}-\alpha_j\chi_j\bar{b}_{j,t-1}-b^{g}_{j,t-1}\right)-\beta\Delta\left[\frac{\rho_{j,t}}{1+\rho_{j,t}}\frac{\bar{x}_{f,t}}{1-\alpha_j}\right]}{1-\alpha_j}$$

where the key point is that Ω is zero if ρ is zero.

Using the budget constraint of the patient agents, we get

$$\Delta x_{s,j,t}=\Delta s_{j,t}+\Delta\tilde{y}_{j,t}-\beta\Delta\left[\frac{s_{j,t+1}}{1+\rho_{j,t}}\right]=\frac{\Delta\left[\bar{x}_{f,t}\right]}{1-\alpha_j}+\Omega_{j,t}$$

which proves the Lemma.

B Data

All economic data (employment, population, GDP, consumption, government debt, expenditures...) comes from Eurostat. The data on household debt comes from the BIS which itself compiled the data from national central banks. Credit covers all loans and debt securities and comes from both domestic and foreign lenders. We had to exclude Luxembourg for which data is available only starting in 2005. We also excluded eurozone countries that joined 2007 and later. We are left with 11 countries: Austria, Belgium, Germany, Spain, Finland, France, Greece, Ireland, Italy, Netherlands and Portugal.

In order to map the observed data into the model we rebase the data in the following manner. We construct a benchmark level of GDP for each country and year: it is the GDP the country would have if it had the same per-capita growth rate as the whole eurozone and its actual population growth. Define

- $Y_{j,t}$: GDP in euros of country j, and $\bar{Y}_t$ for the Eurozone
- $N_{j,t}$: population of country j, and $\bar{N}_t$ for the Eurozone
- Benchmark GDP for country j at time t:

$$\hat{Y}_{j,t}\equiv Y_{j,0}\frac{\bar{Y}_t}{\bar{Y}_0}\frac{N_{j,t}/\bar{N}_t}{N_{j,0}/\bar{N}_0}$$

We have experimented with two definitions of $\bar{Y}_t/\bar{Y}_0$: one is actual GDP, the other is the trend nominal growth before and after the crisis (starting in 2008 this benchmark growth rate is the average of the eurozone crisis, around 1%). Then we define the rebased GDP as

$$y_{j,t} \equiv \frac{Y_{j,t}}{\hat{Y}_{j,t}}$$

We compute rebased series for consumption, government spending by scaling by the benchmark GDP. For unit labor costs, we scale by the average unit labor cost in the eurozone. For employment per capita, we take the deviation with respect to 2001 with an index of 1 for that year. We define the rebased level as the ratio of debt to the benchmark levels of GDP:

$$b^g_{j,t} \equiv \frac{B^g_{j,t}}{\hat{Y}_{j,t}}.$$

Finally, when we map to our model, we must remember that we define $b_{j,t}$ as per capita debt, so in fact we have

$$\chi_j b_{j,t} \equiv \frac{B_{j,t}}{\hat{Y}_{j,t}}$$

The normalized data are given in figures 15 and16.

We need to treat carefully the debt accumulation equations:

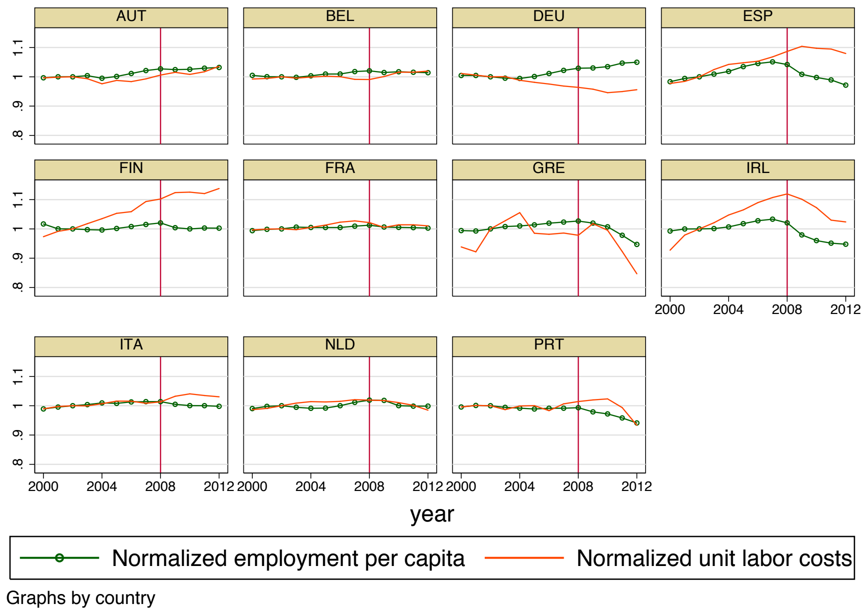
$$\frac{B^g_{j,t+1}}{1 + r_{j,t}} - B^g_{j,t} = p^h_{j,t} G_{j,t} + \top_{j,t} - \tau_{j,t} Y_{j,t}.$$

Therefore in terms of rebased series we have

$$\frac{\hat{Y}_{j,t+1}}{\hat{Y}_{j,t}} \frac{b^g_{j,t+1}}{1 + r_{j,t}} - b^g_{j,t} = p^h_{j,t} g_{j,t} + T_{j,t} - \tau_{j,t} y_{j,t},$$

The key point here is that the contribution of new debt to purchasing power at time t is $\frac{\hat{Y}_{j,t+1}}{\hat{Y}_{j,t}} \frac{b^g_{j,t+1}}{1+r_{j,t}} - b^g_{j,t}$.

Figure 15: Normalized data for comparison with model: employment, unit labor costs, consumption and GDP



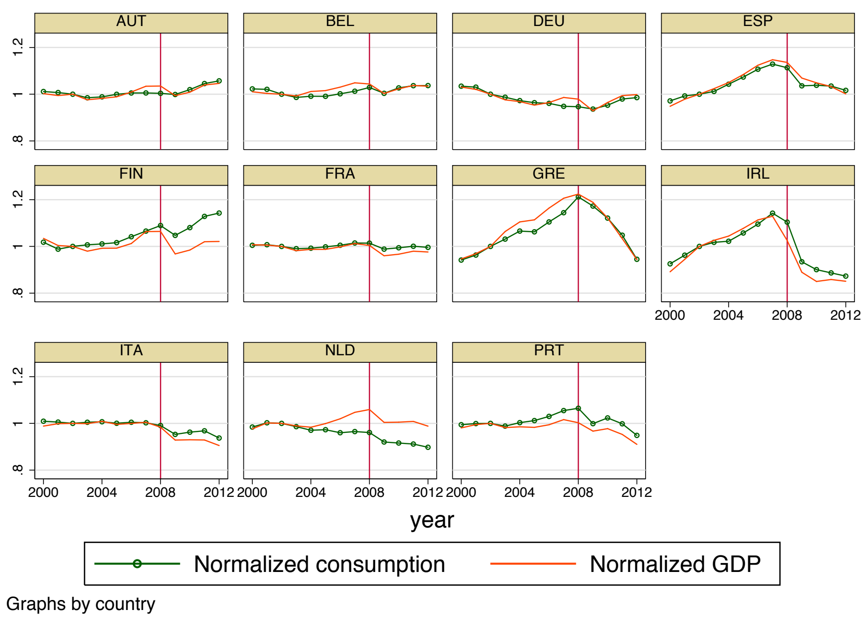


Figure 16: Normalized data for comparison with model: household and sovereign debt, govt. spending (adj. for bank recap) and social transfers