Mortgage Origination and the Rise of Securitization:
An Incomplete-Contracts Model

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Abstract: Recent empirical studies find that securitised mortgages yield higher default rates than those which are originated and held by the same party, raising a number of theoretical puzzles which we address with a model based on incomplete contracts. Our model that combines the features of diversion (along the lines of Hart and Moore 1989, 1998), with asymmetric information regarding borrowers default risk (as in Stiglitz and Weiss 1981) and soft-information screening by lenders. We show that securitisation weakens the incentive to screen compared with bank originated-and-held-loans, and that securitisation is more prevalent (inter-alia) with rising house prices, lower interest rates, reduced liquidation costs and higher bank regulation costs. Extending our basic model to the case of stochastic prices allows us to analyse strategic default: Enforcement of recourse loans or policies to encourage renegotiation reduces repayments and default rates in our model. In a final extension, we assume a rise in the volume of mortgage-default properties raises the depreciation rate of such properties in the second-hand market. This systemic factor introduces multiple equilibria, i.e. either securitised or bank-held mortgages (not both at the same time) exist within a region. Securitised mortgages can generate significantly lower welfare if depreciation is sufficiently high so that mortgage markets in this region are fragile if external factors can shift the equilibrium from banking (with its low default rate) to securitisation (with its higher rate).

Journal of Economic Literature codes: G21, D86, D43, D82, L14, G38

Keywords: securitization, mortgage, debt, banking, incomplete contracts

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1. Introduction

Recent empirical studies of the period prior to the financial crisis found that securitized mortgages held by investors had significantly higher default rates than loans originated and held by the same institution.¹ This presents some significant puzzles for economists: Why should default rates be affected by the identity of the parties holding foreclosure rights, e.g. banks vs. outside investors? Put another way, why weren’t originators subject to the same incentive contracts regardless of who might be the final holder of mortgages? Even in the absence of such incentive contracts, why is it that investors were willing to purchase low-quality mortgage-backed securities? Relatedly, why is it that excessive credit, rather than credit rationing, became a feature of the lending market? And why is it that we experienced such a rapid expansion of securitization?²

Linked to these questions are some broader issues: Would different loan forms or renegotiation improve default rates? What are the welfare impacts of securitization, and how does it affect mortgage market stability?

To address these questions, among others, we develop a new theory of mortgage borrowing, based on incomplete contracts. The key notion is that methods of finance which entail long-term contracting are costly. Accordingly, we endogenize the mode of finance chosen by intermediaries who originate mortgages in our benchmark model: Finance is either through mortgage-backed deposits held over the long term, a costly form of funding—i.e. via banking, or by immediate sale of the loan to investors, a cheaper form of funding—i.e. via securitization. In our model under banking, loans—foreclosure rights—are held by the bank over the long term. Under securitization, these rights are promptly transferred to investors after origination.

The incentives imbedded in foreclosure rights are important in our theory because of moral hazard in the origination process. Regardless of the mode of finance, intermediaries in our framework can commit to a screening decision. Screening reveals to them whether a customer is a good or a bad type. In equilibrium, only banking provides an incentive to screen, as long-term holding of foreclosure rights makes a bank the residual claimant of any funds left over after deposits have been repaid. Securitization avoids the costs of long-term contracting, but it leaves the intermediary with no incentive to screen.

Our theory therefore delivers the following tradeoff when comparing equilibrium outcomes: Banking is a long run activity which entails high financing costs due to screening and long-term contracting, but

¹See, for example, Berndt and Gupta (2009), Keys et al. (2010), Krainer and Laderman (2009), Mian and Sufi (2009) and Nadauld and Sherlund (2009).

²In particular, during the period from 2001 through 2007 CITE HERE.
which yields high quality loans with low default rates. Origination with immediate securitization—which we refer to as ‘broking’ for ease of expression—is a short-run activity that entails low financing costs due to the absence of long-term contracting and screening, but yields lower quality loans, since both good and bad customer types are served. Investors are willing to hold mortgage-backed securities despite their lower quality, precisely because origination costs are low, increasing their return. We are thus able to explain three of the puzzles outlined above: the higher default rates observed with securitized mortgages, the willingness of investors to hold them, and ‘excessive credit’ (i.e. the extension of loans to bad as well as good customers)—all as equilibrium phenomena that arise in the presence of incomplete contracts.

Our comparative-statics results yield insights into the rapid expansion of securitization over the decade prior to the financial crisis. Our benchmark model predicts that there will be more securitization in housing markets with steeply rising prices, lower foreclosure/liquidation costs, lower interest rates, higher costs of banking-finance and higher expected incomes. These results accord well with recent empirical research on mortgage lending.\(^3\) We also use the benchmark model to analyze the comparative statics of default rates. An exogenous switch from banking to broking increases the frequency of foreclosure.\(^4\) A rise in the quality of a borrower (analogous to a rise in a FICO score) reduces default rates but for a discrete upwards jump at an intermediate level. This striking result provides a theoretical basis for the finding by Keys et al (2010) of a discrete rise in the rate of default at some threshold level of quality.\(^5\) At the threshold in our model, the mortgage originator switches from bank to broker and the jump occurs because banks screen and brokers do not, thus granting loans to both good and bad borrower types.

We extend the benchmark model by introducing stochastic future house prices, in order to analyze the impact of the collapse of house prices experienced before the financial crisis, and possible policy measures to address such events. Under the US practice of no-recourse loans, whereby borrowers are not liable in default beyond losing their home, this allows borrowers to strategically default when the repayment exceeds the price of a new home (i.e. ‘underwater loans’). We find that strategic default by high income customers occurs in equilibrium, which leads to increased repayments, lower expected welfare and possibly lower loan volumes. Full-recourse loans, whereby borrower’s future assets can be seized in lieu of repayment, recovers

\(^3\)Using zip-code level data on mortgage lending, Mian and Sufi (2009) find that rising house prices, lower interest rates, lower origination costs, and lender moral hazard are each associated with increased securitization. Nadauld and Sherlund (2009) similarly find an association between rising house prices and securitization.

\(^4\)Identifying the causal effect of switching from banking to broking on default rates is difficult for reasons we discuss in the paper. Attempting to control for other factors, Berndt and Gupta (2009) find that loans that were originated to be distributed had higher default rates than those that were originated to be held. Similar results using different identification strategies are found in Krainer and Laderman (2009), Mian and Sufi (2009), Nadauld and Sherlund (2009), and Keys et al. (2010).

\(^5\)They find that 620 is a threshold FICO credit score. Below 620 there is discretely more bank lending and above it more broking and securitization. Default rates fall with the FICO score everywhere except at the threshold 620, where there is the discrete rise, congruent with our theory.
the outcomes of the non-stochastic benchmark case. The introduction of stochastic house prices allows us to study the impact of renegotiation, which we assume is harder under securitization than banking, due to the diffusion of investors in the former case. We find that renegotiation expands the set of parameters where banking is an equilibrium, reduces repayments and increases welfare.

Our most important extension is to introduce a key systemic factor into the benchmark model, and a continuum of borrowers, in order to capture important general-equilibrium type effects that are absent in the benchmark case. We assume that the greater is the rate of default, the higher are the ‘frictional’ or depreciation losses from liquidation. In practice, outside of our model, this arises because increased default leads to a larger stock of empty homes in a quantity-clearing housing market. Securitization now imposes a negative externality, altering the equilibrium, because in a banking equilibrium a broker which deviates by entering ends up serves all borrowers including lemons, leading to increased market-wide liquidation costs. Accordingly, the systemic factor allows us to succinctly capture the social trade-off between securitization and traditional banking: The welfare benefit of securitization is that loans are extended to parties who would not otherwise get them. The welfare cost is due to a larger measure of defaults, and the larger liquidation costs due greater depreciation of stock which this entails.

Perhaps the most important welfare feature we study is market stability. The introduction of a market liquidation cost which increases with the rate of default, makes the mortgage market unstable in the following sense: Rising liquidation costs introduces a region of multiple equilibria in which either all loans are securitized by a broker, or all are originated and held by a bank. This allows us to compare welfare across actual equilibria rather than between one equilibrium and some counter-factual equilibrium. A parameter shift such as a fall in the interest rate may now lead to a shift between banking and broking equilibria. The consequence would be a discrete jump in default rates and hence a discrete jump in liquidation costs and a discrete fall in welfare: The mortgage market is unstable due to the possibility of securitization. We apply the idea of ‘evolutionarily stable equilibria’ to argue that introducing securitizing brokers to the banking equilibrium should tend to drive banks out, but introducing banks to a broking equilibrium has no such effect.

Before we turn to the relationship between our work and earlier literature, it is worth emphasising that our focus is on what we see as one of the defining features of securitization, that of finance by a broader pool of ‘outside’ investors, compared to finance through traditional bank deposits. Securitization, of course, has other important features from which we abstract in this paper. Asset-backed securities are pooled, tranched and divided in a myriad of ways to provide their holders with useful risk-management features.

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6Depreciation due to idleness is a feature of the housing market. As Karl Case (2008) argues “Home prices are subject to inertia and are sticky downward. Housing markets have traditionally been quantity clearing markets, with excess inventories absorbed only as new households are formed.”
They are evaluated by ratings agencies for risk. Many important issues pertaining to these features remain as important future topics of research.

2. Literature

The classical literature on mortgage lending, begun by Stiglitz and Weiss (1981), and developed by Bester (1985, 1987), Besanko and Thakor (1987) and many others, is focused on why credit may be rationed in equilibrium. To study current events, however, the focus needs to be on the apparently excessive extension of credit. The core assumptions of the credit-rationing literature are (a) that borrowers have private information on their credit-worthiness, and (b) that some form of single crossing condition holds, which allows for contractual screening through menus consisting of a repayment along with some combination of collateral and (probabilistic) ex-ante rationing. While we retain the assumption of private information, we assume limited income (to focus on the lower end of the mortgage market) and rule out ex-post but not ex-ante public randomization. These assumptions obviate contractual screening in our model and (endogenously) yield simple debt, without ex-ante random rationing, as the optimal contractual form.

A more recent strand of literature focused on the moral hazard problem due to loan sales and is closer in spirit to the topic of this paper—securitization—than the credit-rationing literature. The basic tenant, as pointed out earlier in Diamond (1984), is that diminished incentives to screen means that marketed loans will be tend to be worthless. To explain the growing trend toward loan sales in the 80s and 90s, Gorton and Pennacchi (1995) assumed that fractions of loans with limited implicit guarantees could be sold, but that full transfer is prevented by regulation. However, this restriction has gradually eroded over time, and with the Gramm–Leach–Bliley Act of 1999, such restrictions on banks were eased. As pointed out in the first paragraph of this paper, such easing did not appear to result in improved screening, as would be predicted by theory if loans were sold which incorporated optimal incentive schemes for bank monitoring.

The two literatures were neither aimed at nor do they resolve one of the central puzzles our theory seeks to address: i.e. why comprehensive contracts were not used to address incentive problems, thus yielding the same default rates regardless of the holder. Aghion and Bolton (1992), and Hart and Moore (1989, 1998) emphasise the importance of diversion, an extreme form of costly contract enforcement, as well as private non-pecuniary payoffs, in explaining incomplete financial contracts. However, these papers are not directly

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7Ex-post randomization requires that a mortgage is able to be be liquidated at random regardless of whether or not the borrower repays. Some households would repay and be liquidated, others would not repay and remain in their properties. We rule out this possibility. See Appendix C.

8See the discussion on the necessary illiquidity of intermediary assets on p410 of this article.

9The Glass–Steagall Act of 1933 limited banks activities in the securities markets, effectively preventing the full transfer or securitization of loans.
applicable to the problem of mortgage lending and securitization for several reasons. First, they are based on symmetric information. Asymmetric information, assumed in our model, is a vital ingredient of such lending. Second, the contracts written by creditors in their models could be sold without altering outcomes. In other words, these theories provide insights for the questions of this paper, but are not directly adaptable to the issue of securitization.

Our model combines features of the credit-rationing, moral hazard and incomplete contracts literature on debt into a carefully developed theory of mortgage securitization. Borrowers have asymmetric information. Intermediaries can uncover this (soft) information at a cost. Long-term contract payments are subject to enforcement costs by payees, allowing us to endogenously explain banking and securitization. Simple debt is optimal. As well as addressing important theoretical puzzles, our model has empirical relevance, uncovers a key welfare trade-off, and examines some of the policy solutions suggested in the literature.

One additional paper, Greenbaum and Thakor (1987), hereafter GT, requires closer attention because it is one of the few examples of a paper in the contractual-screening literature with implications for securitization. Deposit funding in GT involves depositors’ active participation in the loan. They pay a screening cost, becoming fully informed about the borrower’s type, allowing them to sign the efficient, full-information loan contract, which uses all the risk-neutral intermediary’s capital to insure risk-averse depositors in case the borrower defaults. With securitization, investors’ involvement is more arm’s-length. Instead of screening, investors infer the borrower’s type from the loan’s terms, in particular from the percentage of the loan the intermediary “backstops” for investors in case of default.

Our approach has several advantages relative to GT. First, our analysis does not involve the considerable complication of signaling contracts and so is simpler at the same time it allows us to be more rigorous. Rather than making behavioral assumptions, we provide foundations based on the theory of incomplete contracts from which we derive optimal behavior for all agents. For example, rather than assuming that the intermediary always screens as in GT, this is a central question to be studied in our analysis. GT’s implication that depositors pay more attention to the quality of mortgages issued by their banks than investors in mortgage-backed securities seems counterfactual. Even if one argues that investors in mortgage-backed securities should have been more vigilant about quality of the mortgages involved in the run-up to the crisis, it is hard to argue that they paid less attention than the typical depositor, whose deposits are government-insured and who does not veto individual loans of the thousands originated by his or her bank. In our model, neither depositors nor investors screen.
3. Model

We model the mortgage market as a game involving two periods, 1 and 2, and three groups of risk-neutral players: consumers, who require financing for a house; investors, who supply financial capital, and intermediaries, who link demanders and suppliers of financial capital by originating mortgages. We first describe the contractual setting, then characterize the players and the market in which they operate, and conclude with discussion of the timing diagram, which will serve to recapitulate the model’s main elements.

3.1. Contractual Environment

We begin with a discussion of the contractual environment, highlighting two key features: diversion and moral hazard. These features will lead the equilibrium mortgage contract and intermediaries to take on simple and realistic forms. The optimal mortgage contract will turn out to be simple debt. The optimal form of intermediary will turn out to be either a screening bank that holds the mortgages it originates or a non-screening broker that securitizes its mortgages, one or the other optimal depending on conditions. In order to progress to the main results quickly, after discussing the contractual environment at a conceptual level in this subsection, we will take the form of mortgage contracts and financial intermediaries as exogenously given until Section 9, where we prove the optimality of these forms.

Diversion is a key feature of the environment leading to a high degree of contractual incompleteness. Specifically, all players are assumed to be able to divert any funds which they hold from one period to the next for private uses. The simplest alternative, following Hart and Moore (1989, 1998), is to imagine that players consume the diverted funds. Another alternative is to imagine that the funds are put in a risky investment that, holding constant its expected return, provides a diminishing probability of a nonzero return that is increasingly high. It can be seen that either alternative would limit players’ ability to commit to making a certain payment in the future.

Two exceptions constrain diversion. First, players can subject themselves to costly monitoring. We will assume that funds are fungible across different accounts held by the same player, so that for monitoring to be effective all of the accounts on a player’s balance sheet must be thus monitored. The cost is formalized below as being proportional to the size of the monitored players’ combined accounts. Our analysis will ultimately show that monitoring will only be used with screening banks in equilibrium. It will be natural to interpret monitoring as a form of prudential regulation that allows banks to raise deposits by committing them to repaying the principal and interest on the deposits. However, because we are interested in endogenizing the form of intermediation, we will allow all players (including banks, brokers, or any other form of intermediary as well as investors) to have access to the same monitoring technology. The second exception to diversion
follows from Hart and Moore’s (1998) idea that physical assets cannot be diverted and are therefore subject to a threat of seizure. The relevant physical asset here is the house: the threat that the house will otherwise be taken incentivizes the borrower to make mortgage payments.

Diversion helps narrow down the set of feasible contracts considerably. Proposition ??, stated and proved in Section 9, implies that the combination of diversion with the ability of lenders to seize the physical home leads optimal mortgage contracts to have the form of simple debt. That is, the mortgage specifies a repayment in period 2 for the amount borrowed in period 1. The amount borrowed exactly equals the amount needed to finance the house. If the borrower fails to make the repayment, the lender can seize the house from the borrower. If the borrower makes the repayment, it can continue to enjoy the property through period 2.

The second key feature of the contractual environment is a moral-hazard problem between investors (as principal) and the intermediary (as agent), which is exacerbated by the high degree of contractual incompleteness. At a cost, the intermediary can screen out consumer types who are unlikely to be able to make their repayments. However, investors have limited contractual means to induce the intermediary to screen because any promised payments between them can be diverted.

As we will show in Section 9, organizing the intermediary as a bank turns out to be the optimal way to induce screening. The bank uses its deposits as a source of funds to hold the mortgage lien as an asset on its own balance sheet until period 2. As the mortgage lien holder, the bank becomes the residual claimant of the repayment stream. We show that no contract can improve on the screening incentives provided by residual claimancy or further economize on the costs of monitoring the bank’s accounts required to prevent diversion. The only strategy for intermediation with any prospect of outcompeting a bank takes the opposite approach of forgoing screening entirely. Because the intermediary does not screen and thus need not receive period-2 incentive payments, the cost of monitoring accounts to ensure incentive payments are made can be avoided by having the intermediary transfer the mortgage lien to investors immediately upon origination. We label this form of intermediary a broker and the spot transfer of the mortgage to investors securitization. Whether bank or broker intermediation is optimal will depend on the costs of screening and monitoring relative to the benefits.

3.2. Consumer

We next characterize the players in the model. A single consumer has the opportunity of purchasing a new house in period 1, providing utility $u_1$ from the consumption of housing services in period 1 and $u_2$ if he remains in the house through period 2. His utility function is additively separable in housing services and income. He has no initial wealth, so needs to obtain a mortgage to purchase a home. The consumer makes
mortgage repayments using his period-2 income, which depends on his type, \( \theta \), a Bernoulli random variable.

The “good” consumer type (corresponding to \( \theta = 1 \)) has probability \( h \in (0, 1) \) of earning high income and complementary probability \( (1-h) \) of earning low income in period 2. Let \( y > 0 \) be the high income level and normalize low income to 0. The “bad” type (corresponding to \( \theta = 0 \)) earns no income in period 2 with certainty. Nature draws the good type with probability \( \gamma \in (0, 1) \) and the bad type with complementary probability \( 1-\gamma \). The specific realization of the consumer’s type is his private information, but the distribution of types (i.e., \( \gamma \)) and all other parameters of the model are public information.

The consumer should be envisaged as being part of a segment of the market sharing attributes \( h, y, \gamma, u_1 \) and \( u_2 \) containing both good and bad types. As explained further below, the consumer’s market segment represents the “hard” information about him and type \( \theta \) is thought of as “soft” information. In Section 9, allow many consumers across many market segments to be served simultaneously, but until then for simplicity we will consider one representative consumer in just this one market segment.

### 3.3. Investors

Investors are the only source of financial capital in the model. They provide a perfectly elastic supply of credit. Given that they are risk neutral, this means they are willing to sign any contract providing an expected return greater than or equal to principal plus the risk-free interest rate, \( r \geq 0 \).

### 3.4. Intermediaries

The consumer cannot access investors’ financial capital directly but must go through an intermediary. Intermediaries are indexed by \( i = 1, \ldots, N \), where \( N \geq 2 \). They compete to provide mortgages to consumers by posting contract terms.

As will be shown in Section 9, mortgages optimally have the form of simple debt. This implies that the posted contractual terms simply consist of the single variable \( R_i \), representing the repayment from the borrower at the start of period 2 required to avoid foreclosure. The repayment is made in return for the loan of \( p_1 \), the amount needed to buy a house in period 1. The household observes all \( R_i \) posted by active intermediaries and chooses to apply to one or none of them. Upon receiving an application, intermediary \( i \) decides whether or not to screen the applicant, with \( S_i = 1 \) indicating screening and \( S_i = 0 \) indicating no screening. If the intermediary screens, it incurs a non-pecuniary cost \( k_i > 0 \) and learns the applicant’s type \( \theta \). After screening, the intermediary decides whether or not to offer a mortgage to the consumer.

We make the following assumptions about the information available to intermediaries and other players. All the parameters of the model aside from the consumer’s type, \( \theta \), are public information. We equate
knowledge of these parameters with the sort of “hard” information embodied in FICO and other commercially available credit scores. The model makes this information costless to acquire for simplicity. We equate the borrower’s type \( \theta \) with additional “soft” information on the household’s income prospects which can only be uncovered through costly screening. Rajan quote XXX. While mortgage repayment \( R_i \) is posted and thus verifiable, the screening decision \( S_i \) and the soft information revealed are unverifiable. More specifically, assume \( S_i \) and \( \theta \) are never observable to investors; \( S_i \) is only observable to the consumer after he applies for a mortgage. The unverifiability of screening and the soft information revealed will prevent investors and intermediaries from conditioning contracts on screening, leading screening to be a source of moral hazard.

Intermediaries and the incentive contracts they sign with investors can take myriad forms. To get to the main results as quickly as possible, we will focus on the two forms shown in Figure 1, banks and brokers, taking the forms as given and seeing which of the two emerges in equilibrium under different conditions. Banks raise deposits that allow them to hold the mortgages that they originate in period 1 on their own accounts through period 2. Banks incur a monitoring cost to assure depositors that the promised principal and interest is repaid in period 2. On the other hand, brokers forgo deposits, originating mortgages which are immediately securitized to investors.

We defer to Section ?? the formal analysis showing that our focus on these two forms is without loss of generality as they are weakly more efficient than any other intermediary or contractual form. Here we just provide the intuition for the result. Banks are a candidate for the optimal intermediary form because they have powerful incentives to screen, being residual claimants of the return stream from the mortgage. The broker model involves the opposite approach, relinquishing screening incentives, but avoiding the associated screening costs and monitoring costs. These are the only two candidates for the equilibrium intermediary form. We show in Section ?? that intermediate approaches in which investors hold the mortgages but try to incentive the intermediary to screen through a performance contract tying the intermediaries payment to the performance of the loan provide less powerful screening incentives than banks have without economizing on monitoring costs required for the incentive payments to be credible. This is why screening banks and non-screening brokers are the only candidates for equilibrium intermediary forms.

The intermediary forms have different opportunity costs of funds. As discussed above, for banks to credibly commit not to divert deposits requires monitoring, assumed to cost \( d \in (0, 1) \) per dollar of deposits taken. To fund a loan of size \( p_1 \) (as we will see, the period-1 price of a house), a bank has to raise \( p_1/(1-d) \) in deposits. Accounting for the interest rate \( r \) required by investors on their deposits, the bank’s opportunity cost of funds for this loan is

\[
\frac{(1+r)p_1}{1-d}.
\]
Because brokers do not raise deposits, they can avoid the monitoring cost $d$, allowing them a loan with a lower opportunity cost of funds. To fund a loan of size $p_1$, the broker’s opportunity cost of funds is simply

$$ (1+r)p_1. $$  \hspace{1cm} (2)

Figure 1 provides a schematic diagram of the two intermediary forms. It simplifies the presentation to assume that intermediaries can only take these two forms and analyze competition among these two forms on the mortgage market. It should be kept in mind that the results are the same if we allow intermediaries to enter the market as *tabulae rasae* and adopt the efficient form in equilibrium.

### 3.5. Housing Market

New houses are available in perfectly elastic supply at a price of $p_1$ in period 1 and $p_2$ in period 2. A house which was occupied in period 1 can be resold on the market in period 2. An owner-occupier selling his house does not involve unusual frictions. For simplicity we will assume that there are in fact no frictions, so the owner-occupier would obtain the full $p_2$ from the sale. By contrast we will assume that sale by an outside party involves additional frictions. These frictions are present whether the outside party holds a vacant property during period 1 and sells it in period 2 or whether it seizes the property from one consumer in foreclosure and resells to another. Both processes involve degredation of the property which we capture by assuming that an outside party selling in period 2 only obtains a fraction of the sale price, $\lambda p_2$, where $\lambda \in (0,1)$.

We provide a simple general-equilibrium model of the housing market in Section 9. Until then, for simplicity, we will take $p_1$, $p_2$, and $\lambda$ to be exogenous, imposing two weak conditions on these otherwise free parameters. First we assume

$$ (1+r)p_1 > \lambda p_2. $$  \hspace{1cm} (3)

Condition (3) would follow from a general-equilibrium model. If it were violated, investors could profit from buying houses at $p_1$ in period 1 and reselling for a return of $\lambda p_2$ in period 2, even if the houses were left vacant in the interim. This behavior would drive up $p_1$ until (3) was reestablished. Condition (3) is not crucial for the analysis, but it does eliminate trivial cases in which screening is worthless because lenders profit from serving the bad type, who is guaranteed to default. Second, we assume

$$ p_2 \leq u_2. $$  \hspace{1cm} (4)
Condition (4) would also follow from a general-equilibrium model. If it were violated, consumers would all sell their houses, driving $p_2$ down until (4) was reestablished. Condition (4) is again not crucial for the analysis but ensures that the consumer prefers to stay in the home in period 2 if possible rather than selling.

### 3.6. Summary of Timing

Figure 2 summarizes the timing of the game. Events related to the consumer appear above the timeline and those related to intermediaries appear below. At the start of period 1, nature draws the consumer’s type $\theta \in \{0, 1\}$. This is private information for the consumer. Competing intermediaries $i = 1, \ldots, N$ simultaneously post the repayment terms $R_i$ associated with their mortgage contracts. The consumer observes the contracts and chooses one, $i$, or none of them. Intermediary $i$ chooses whether or not to screen the applicant, indicated by $S_i$, expending $k_i$ if $S_i = 1$. Intermediary $i$ then obtains funds from investors. If it is a bank, $i$ raises deposits and if it is a broker, $i$ securitizes the loan to investors. The consumer uses the loan to purchase the house at price $p_1$ and derives utility $u_1$ from its services.

In period 2, the consumer’s income is realized, either $y$ or 0. The consumer next decides whether or not to repay $R_i$. If the consumer defaults, the lien holder—investors if the mortgage was securitized, the bank if not—forecloses on the house, obtaining liquidation value $\lambda p_2$. If the consumer repays, he stays in the house and obtains utility $u_2$ from its services.

### 4. Equilibrium

#### 4.1. Existence and Characterization

In this section we solve for the perfect Bayesian equilibrium of this sequential game of incomplete information. Our focus will be on finding the terms of the equilibrium intermediary form, repayment $R^*$, and screening decision $S^*$ emerging from competition among the intermediaries.

Several insights help pin down their strategies. First, the profit from serving the bad consumer type is negative. To see this, note that from expressions (1) and (2), the opportunity cost of the funds for the mortgage is at least $(1+r)p_1$. The bad type can never repay in period 2 because it earns no income, so the return on the mortgage comes from the liquidation proceeds $\lambda p_2$. By (3), however, the liquidation proceeds cannot cover the cost of funds. Thus, competition among intermediaries is directed toward maximizing the good type’s payoff. Because intermediaries would never deny a loan to a good type, and the direct screening cost imposed on the consumer is assumed to be negligible, the good type’s payoff from a mortgage contract does not depend (directly at least) on the screening decision $S_i$ but only on the repayment $R_i$. The good type obviously prefers lower values of $R_i$. Hence competition among intermediaries is of the familiar Bertrand
form, generating the lowest repayment $R_i$, subject to the intermediary’s breaking even.

Before solving for equilibrium value of the continuous variable $R^*$, we will characterize other elements of the behavior of intermediaries. As discussed in Section 3.4, without loss of generality the equilibrium intermediary will either be a bank or a broker. Other forms and incentive contracts are weakly less efficient. Thus the intermediary form is a binary outcome. Whether intermediary $i$ screens or not is also a binary outcome, leading to four combinations of form and screening. It will help fix ideas to elaborate on why only two of the four combinations are relevant in equilibrium: screening brokers and non-screening banks can be ruled out, leaving screening banks and non-screening brokers as the only possibilities.

To rule out screening brokers, note that a broker securitizes the mortgage, directly transferring the lien to investors after origination. Because it is not a residual claimant on the repayment stream, the broker has no incentive to screen the consumer unless it has signed an incentive contract with investors. But as we will prove in Appendix C, for the promised payments in an incentive contract to be credible, the same or higher monitoring costs as associated with bank lending need to be expended. A bank has first-best screening incentives as a residual claimants on the loan repayment. Hence a broker induced to screen through an incentive contract is weakly less efficient than a bank and could never undercut a bank in Bertrand competition, allowing us to rule out screening brokers without loss of generality.

To rule out non-screening banks, note that raising deposits involves a higher opportunity cost of funds than securitizing, (1) rather than (2), due to the additional monitoring cost. A non-screening bank will always be undercut by a broker in Bertrand competition. The only way a bank could be observed in equilibrium is if it used deposits to pursue a strategy not open to brokers, that is, to give itself an incentive to screen the consumer. Hence non-screening banks are ruled out in the benchmark model.\textsuperscript{10} Furthermore, there is no reason for the bank to pay the enforcement and screening costs if it does not reject the bad type. Thus, if a bank is observed in equilibrium, it must screen and reject the bad type.

Bertrand competition ends up driving the equilibrium repayment down to the intermediary’s zero-profit level. Let $R^{SBK}$ be the zero-profit repayment for a screening bank and $R^{NBR}$ be the zero-profit repayment for a non-screening broker. We will solve for $R^{SBK}$ and $R^{NBR}$ and compare them to determine which form of intermediary “wins” in Bertrand competition.

Before solving for $R^{SBK}$ and $R^{NBR}$, we have to address the existence of multiple equilibria in the model. As argued above, if a bank operates in equilibrium, it must screen and reject the bad type. Given that he would certainly be rejected, the bad type is indifferent between applying to a screening bank and not. The

\textsuperscript{10}In the extension with renegotiation in Section 6, the ability to renegotiate is an additional advantage of banks over brokers. If this advantage is sufficiently great, banks may be observed in equilibrium even if they do not screen.
behavior of the bad type when indifferent in this situation is what can generate multiple equilibria. We will assume throughout the text that the bad type applies to a screening bank with probability 1 when indifferent. This assumption simplifies the analysis and produces an equilibrium with certain robustness properties. Appendix B generalizes the bad type’s behavior, placing no exogenous restrictions on the probability $a \in [0, 1]$ of applying to a screening bank. The appendix provides a full characterization of the expanded set of equilibria, showing its boundaries and its comparative-statics properties are virtually identical to those derived in under the restriction in the text.

Return to the analysis of Bertrand competition among intermediaries, first considering the competitive behavior of screening banks. To compute $R_{SBK}$, assume for the moment that the good type accepts the screening bank’s mortgage contract. Further, assume that he is willing and able to repay the posted repayment, $R_i$, when his income is high, i.e. $y$. (We will investigate the conditions under which these assumptions hold shortly.) The profit for a representative screening bank then is

$$
\gamma \left[hR_i + (1-h)(1+r)p_1 - \frac{(1+r)p_1}{1-d} - \lambda p_2 - k_I \right] - k_I. 
$$

The consumer only receives a mortgage if he is the good type, which occurs with probability $\gamma$. Conditional on being a good type, with probability $h$, the consumer earns high income and repays the bank $R_i$. With probability $1-h$, the consumer cannot repay; the bank forecloses and earns proceeds $\lambda p_2$. The bank’s costs include the opportunity cost of funds from (1), which reflect the monitoring costs to ensure deposits are repaid. The bank’s costs also include the screening cost $k_I$. Since bad as well as good types apply to the bank for mortgages in this equilibrium, $k_I$ is paid with certainty. Setting (5) to zero and solving yields the zero-profit repayment for screening banks:

$$
R_{SBK} = \lambda p_2 + \frac{1}{h} \left[ \frac{(1+r)p_1}{1-d} - \lambda p_2 + \frac{k_I}{\gamma} \right].
$$

Next, consider non-screening brokers. We argue in a series of steps that if a broker is active in equilibrium, this broker must be the only active intermediary and must serve all consumers of all types. Note first that if brokers are active, all bad types end up being served by a broker. The bad type strictly benefits from obtaining a mortgage because he gains at least the utility $u_1$ from period-1 housing services. Because brokers do not screen, they have no information to exclude the bad type. Let $i$ index an active broker serving a bad type with some probability. Note second that because bad types generate losses $i$ must have some positive probability of serving a good type as well. Moreover, it can be shown that this probability must be 1 in equilibrium. If this probability were less than 1, the good type must be indifferent between $i$ and some
other intermediary, implying that this other intermediary must be offering the same repayment $R_i$ as does $i$. But $i$ then has a strictly profitable deviation: $i$ can obtain a discrete jump in the probability of attracting the good type to 1 with an infinitesimal reduction in $R_i$. Hence any intermediary having any probability of serving a bad type must serve all good types, so there cannot be any other active intermediaries, because there are only loss-making bad types left to serve. Note finally that since $i$ is the only active intermediary, by our previous argument all bad types will apply and be served by $i$. In sum, if a broker is active in equilibrium, it alone must serve all consumers of all types.

Because brokers immediately securitize mortgages, broker $i$’s profit must come in the form of a fixed origination fee, denoted $F_i$. This fee must be low enough to ensure that investors’ ex ante expected return net of the fee is sufficient to cover their opportunity cost of funds:

$$\gamma h R_i + (1 - \gamma h)\lambda p_2 - (1 + r)p_1 - F_i \geq 0. \quad (7)$$

Because $i$ serves all consumers of all types, the probability that the borrower makes the repayment $R_i$ is $\gamma h$, the unconditional probability of being a good type times the probability that the good type earns high income. (Again we are making the implicit assumption that the high-income consumer is willing and able to make the repayment, an assumption we will investigate shortly.) With probability $1 - \gamma h$, the consumer earns no income and thus defaults. Investors then earn foreclosure proceeds $\lambda p_2$. Subtracting the investors’ opportunity cost of funds in the unregulated case given by (2) and $i$’s origination fee leaves (7). No screening costs need to be subtracted because neither $i$ nor the investors screen. Bertrand competition among intermediaries results in the zero-profit origination fee $F_i = 0$ and the zero-profit repayment, which can be found by substituting $F_i = 0$ into (7), treated as an equality, and solving:

$$R_{NBR}^{NBR} = \lambda p_2 + \frac{1}{\gamma h} [(1 + r)p_1 - \lambda p_2]. \quad (8)$$

We argued above that Bertrand competition among intermediaries selects the mode of lending that provides the good type with a higher expected payoff at the repayment levels calculated above, which by construction are feasible for the lenders to offer. The good type’s expected payoff from an arbitrary mortgage contract involving repayment $R_i$ is

$$u_1 + h(u_2 + y - R_i), \quad (9)$$

consisting of the utility in period 1 from housing consumption and, if he earns positive income, which happens with probability $h$, the utility from period-2 housing consumption and the income $y - R_i$ left over
after the mortgage repayment. Because (9) is decreasing in \( R_i \), the good type chooses the intermediary with the lower repayment. Equilibrium involves a screening bank if \( R^{SBK} < R^{NBR} \) and a non-screening broker if \( R^{NBR} < R^{SBK} \).

The preceding analysis took for granted that the consumer would accept either contract and would repay if he earned positive income. We need to tie up this loose end by deriving conditions under which the consumer behaves this way. In particular, given an arbitrary mortgage contract specifying repayment \( R_i \), we will derive a constraint ensuring that the good consumer type accepts the contract (participation constraint) and a constraint ensuring that the high-income consumer repays (repayment constraint).

Begin with the repayment constraint. Clearly a consumer with no period-2 income will always default regardless of type. It remains to see when a consumer with positive income would repay or default. As can be seen with the help of the timeline in Figure 2, in the continuation game following realization of high income, incomplete information plays no material role. The last decision in the continuation game is the lien holder’s choice of whether or not to foreclose. Foreclosing is a dominant strategy if the consumer defaults (regardless of any private consumer information) because the payoff is \( \lambda p^2 \) from doing so rather than nothing. Anticipating this, the high-income consumer will repay (regardless of any private information) if his utility from staying in the house exceeds the payoff from the alternatives. Repayment yields the consumer \( u_2 + y - R_i \), default and repurchase yields \( u_2 + y - p^2 \), and default without repurchase yields \( y \). Comparing these payoffs, the consumer weakly prefers repayment when

\[
R_i \leq \min\{y, p^2, u_2\} \equiv m, \tag{10}
\]

and defaults otherwise.

The consumer’s repayment decision constrains the set of feasible values of the zero-profit repayments. For example, if \( R^{SBK} > m \), screening banks are ruled out in equilibrium because the consumer never repays the loan to a screening bank: the intermediary earns just the foreclosure proceeds \( \lambda p^2 \), which by (3), cannot cover its cost of funds. Similar reasoning implies that non-screening brokers are ruled out in equilibrium if \( R^{NBR} > m \). Repayment of \( R^{SBK} \) or \( R^{NBR} \) by the consumer is therefore feasible whenever (10) holds at these values.

Next consider the consumer’s participation constraint. The good type weakly prefers to accept an offer

---

11 Note that (9) implicitly assumes that the consumer stays in the home in period 2 if possible rather than selling; this follows from (4).
from an intermediary if its payoff in (9) is non-negative. Rearranging provides a bound on the repayment:

$$R_i \leq y + u_2 + \frac{u_1}{h}.$$  \hspace{1cm} (11)

The right-hand side of (11) obviously exceeds $m$. Thus we can ignore the participation constraint because it is automatically satisfied if the repayment constraint (10) holds.

Summarizing the preceding analysis, the equilibrium involves lending by the intermediary with the lower of $R^{NBR}$ and $R^{SBK}$ unless both exceed $m$, in which case no repayment can simultaneously satisfy the repayment constraint and allow intermediaries to break even. The following proposition states these results formally for reference.

**Proposition 1.** Assume (3) holds. Assume the bad type applies for a mortgage when indifferent between applying and not. The equilibrium falls into one of the following three cases.

(i) If $R^{NBR} < \min\{R^{SBK}, m\}$, then brokers originate all mortgages in equilibrium, securitizing them immediately. The equilibrium repayment is $R^* = R^{NBR}$. Both good and bad consumer types apply for mortgages. The broker does not screen so serves both types.

(ii) If $R^{SBK} < \min\{R^{NBR}, m\}$ then banks originate all mortgages by raising deposits from investors and hold the mortgages for both periods. The equilibrium repayment is $R^* = R^{SBK}$. All consumers apply for mortgages. The bank screens with probability 1, offering a mortgage to the good type, rejecting the bad type.

(iii) If $m < \min\{R^{NBR}, R^{SBK}\}$, then there is no mortgage lending in equilibrium.

There are no other equilibria when the stated inequalities are strict. For each of the cases (i)–(iii), the stated outcome is also an equilibrium if any of the strict inequalities holds as an equality.

The proof, provided in Appendix A, verifies the existence of the posited equilibria and proves uniqueness by ruling out an exhaustive set of alternatives. Appendix B characterizes the expanded set of equilibria allowing a measure of bad types not to apply to screening banks when indifferent between applying and not. The appendix shows that the set of equilibria is virtually unchanged from that stated in Proposition 1.

Figure 3 depicts the equilibrium. The $R^{SBK} = R^{NBR}$ line is the good type’s indifference curve. Ignoring the repayment constraint for the moment, this line by itself delineates when there is broking versus banking in equilibrium. Above this line—i.e., for high values of $\gamma$ and $k_I$—non-screening brokers undercut screening banks. The reverse is true below this line. To see why these regions are positioned as they are, it is obvious that non-screening brokers are more efficient at supplying mortgages to the good type for sufficiently high $\gamma$. The only advantage of banks is in their holding of mortgages, which incentivizes them to screen out loss-making bad types. If the share of good types is sufficiently high (implying that the share of bad types is
sufficiently low), the advantage of banks disappears. What remains is the brokers’ advantage in economizing on the costs of regulation and screening. Obviously non-screening brokers are more efficient than screening banks if the screening cost $k_I$ is sufficiently high.

The repayment constraint (10) for broking is represented in Figure 3 by the vertical line $R^{NBR} = m$. It is vertical because $R^{NBR}$ is independent of screening costs. To the left of this line, the share of good types $\gamma$ is too low for broking to be feasible; but to the right, $\gamma$ is sufficiently high. The repayment constraint for banking is represented by the $R^{SBK} = m$ line. The figure shows all three lines intersecting at the same point, which is easy to show is true in general. The $R^{SBK} = m$ line slopes up because the higher is $\gamma$, the lower the probability that the applicant is a bad type that forces the bank to incur a screening cost for no positive return. Thus banks are feasible for a larger range of $k_I$ the higher is $\gamma$. Above the $R^{SBK} = m$ line, screening banks are not feasible because screening cost $k_I$ is too high. Below this line, screening banks are feasible. Both modes of lending are infeasible in the unshaded region, corresponding to case (iii) in Proposition 1. No mortgages are signed in this region. The share of bad types is too high for non-screening brokers to be viable, and the cost of screening is too high relative to the chance of revealing a good type for screening banks to be viable. A whole section below (Section 4.2) is devoted to a more detailed discussion of comparative-statics results with respect to these parameters as well as for parameters not plotted on the axes, changes in which can be represented by shifting the curves in the figure.

Proposition 1 implies that only one form of intermediary is active in the market. This prediction may seem unrealistic at first glance. However, the model can easily be extended to the case in which many market segments, each with different observable characteristics (reflected in parameters $\gamma$, $h$, $y$, $p_1$, etc.), exist simultaneously, each of which can be served by the intermediary form specified by the proposition. We provide this extension in Section 9.

4.2. Comparative Statics

The model yields sharp comparative-statics results, which as discussed in the introduction are consistent with key aspects of the history of securitized mortgage lending in the United States over the last several decades. We divide the comparative-statics analysis into three parts. First, we consider how parameter changes affect the prevalence of mortgage lending of any form. Second, we examine the impact of parameter changes on the prevalence of securitized versus bank-held mortgages. Finally, we examine the impact of parameter changes on mortgage prices, quantities, and default rates. The analysis of default rates will shed some light

\footnote{Exactly on the $R^{SBK} = R^{NBR}$ segment, there is an equilibrium with screening banks and another with non-screening brokers. There are no equilibria in which both forms of intermediary are active in the market together. This follows from the argument in the text leading up to Proposition 1 that, if a broker is active, it must be the sole active intermediary.}
on the stability of the mortgage market even in our partial-equilibrium setting.

The next proposition provides comparative-statics results for overall mortgage lending, whether by screening banks or securitizing brokers.

**Proposition 2.** *There is (weakly) more mortgage contracting the greater are $y$, $u_2$, $p_2$, $h$, $\lambda$ and $\gamma$ and the lower are $p_1$, $r$, $d$ and $k_I$.***

These results are intuitive: the parameter changes all serve to relax the repayment constraint (10) for one or both lending modes.

The proposition can be easily proved using Figure 3. Whether or not there is mortgage lending depends only on the size of the unshaded “no contract” region, not on the $R_{SBK} = R_{NBR}$ line, which delineates the intermediary form rather than the existence of a mortgage. Consider an increase in $y$. As can be seen from the formulae (6), (8), and (10), an increase in $y$ has no effect on $R_{SBK}$ or $R_{NBR}$ but increases $m$. A consumer with higher income can afford a greater range of repayments. This relaxes the repayment constraints for both banking and broking, represented by an upward pivot in the $R_{SBK} = m$ line and a leftward shift in the $R_{NBR} = m$ line, shrinking the size of the unshaded region. Thus there is lending for a larger set of parameters besides $y$. An increase in $u_2$ has a similar effect, relaxing the repayment constraints in this case by increasing the consumer’s attachment to his house, making him less likely to strategically default.

A rise in $p_2$ also makes repayment more likely, but through two distinct channels. One channel is through a reduction in strategic defaults. Defaulting and purchasing an equivalent home becomes more expensive with $p_2$, represented mathematically by an increase in $m$ on the right-hand side of (10). The other channel is through an increase in foreclosure proceeds, which allows intermediaries to reduce the break-even repayment levels $R_{SBK}$ and $R_{NBR}$. One can see this mathematically by differentiating (6) and (8): 
$$\frac{\partial R_{SBK}}{\partial p_2} = -\lambda(1-h)/h < 0 \quad \text{and} \quad \frac{\partial R_{NBR}}{\partial p_2} = -\lambda(1-\gamma h)/\gamma h < 0.$$ 
An increase in $\lambda$ has this same effect on foreclosure proceeds, thus also reducing the zero-profit repayments, relaxing the repayment constraints. A rise in $h$ also reduces the zero-profit repayments, in this case by increasing the probability that the consumer can afford the repayment.

Reductions in $r$ and $p_1$ reduce the loan amount, therefore also relaxing the repayment constraints by reducing $R_{SBK}$ and $R_{NBR}$, pivoting the $R_{SBK} = m$ line up and shifting the $R_{NBR} = m$ line left, shrinking the unshaded region. A reduction in $d$ reduces the bank’s operating costs, reducing $R_{SBK}$ ($R_{NBR}$ is left unchanged because brokers do not pay regulatory costs), thus shrinking the unshaded region.

The effect of a rise in $\gamma$ or a fall in $k_I$ are easy to see in Figure 3 because rather than shifting the lines, these variables appear directly on the axes. Because the unshaded region is in the upper left of the graph, either parameter change may cause a point in the unshaded region to move outside but would never move a
point outside back in. Turning from the graph to the underlying economic intuition, when $\gamma$ rises, reducing the share of bad types, $R_{SBK}$ falls because the probability that the bank pays the screening cost only to find the consumer has to be rejected falls. $R_{NBR}$ falls because there is a smaller measure of loss-making bad types served by the broker. When the screening cost $k_I$ falls, $R_{NBR}$ is left unchanged but $R_{SBK}$ falls.

The next proposition examines the impact of parameter changes on the prevalence of broking (equivalently securitization). The broking region in Figure 3 depends on the position of the $R_{SBK} = R_{NBR}$ line, representing the good type’s indifference between the two intermediary forms, as well as the $R_{NBR} = m$ line, representing the repayment constraint for broking, which we have already analyzed. To understand how the parameters affect the position of the $R_{SBK} = R_{NBR}$ line, substitute the expressions for the zero-profit repayments, (6) and (8), into the equation $R_{SBK} = R_{NBR}$ and rearrange, giving the following equation:

$$k_I = (1 + r)p_1 - \lambda p_2 - \gamma \left[ \frac{(1 + r)p_1}{1 - d} - \lambda p_2 \right].$$

(12)

This is a downward-sloping line with intercept $(1 + r)p_1 - \lambda p_2$ and slope given by the negative of the factor in brackets. It is straightforward to see how parameter changes shift this line.

**Proposition 3.** There is more broking/securitization the greater is $y$, $u_2$, $p_2$, $h$, $\lambda$, $\gamma$, $d$ and $k_I$ and the lower is $p_1$ and $r$.

Before we prove these results, it is worth emphasizing that they are consistent with the changes in economic conditions that accompanied the rise in securitization over the last several decades. The housing boom prior to the crisis with its sharply rising prices can be captured by an increase in $p_2$ holding $p_1$ constant. Proposition 3 implies that this parameter change would lead to more securitization. The proposition also predicts that a low interest rate spurs securitization. Some argue that Federal Reserve Board policy kept interest rates artificially low prior to the crisis. Securitization is also spurred by general boom in economic conditions, as captured by various parameters in the model including a higher income $y$, a higher probability of high income $h$, or higher probability that a consumer is a good type $\gamma$. An increase in the cost of traditional banking—as captured by $d$ and $k_I$—also spurs securitization.

Turning to a proof of Proposition 3, by (12), increases in $y$, $u_2$ and $h$ do not affect the position of the $R_{SBK} = R_{NBR}$ line in Figure 3. However, by Proposition 2, increases in these variables relax the repayment constraint for broking, shifting the $R_{NBR} = m$ line left, thereby increasing securitization. An increase in $p_2$ also relaxes the repayment constraint for broking, shifting the $R_{NBR} = m$ line left. It also shifts the $R_{SBK} = R_{NBR}$ line down by inducing a larger reduction in $R_{NBR}$ than $R_{SBK}$. This is because foreclosure proceeds, which depend on $p_2$, matter more with broking than banking. Brokers serve bad as well as good
types, and so their mortgages have a higher associated default rate \textit{ceteris paribus}. A rise in \(\lambda\) or \(\gamma\) also benefit broking contracts relatively more than banking contracts, shifting the \(R_{SBK} = R_{NBR}\) line down, while also relaxing the repayment constraints for broker mortgages. Finally, higher \(d\) or \(k_I\) increases \(R_{SBK}\) but leaves \(R_{NBR}\) constant, shifting the \(R_{SBK} = R_{NBR}\) line down, expanding the set of cases in which brokers can undercut bankers.

The last two comparative-statics results in Proposition 3 involve a fall in \(p_1\) or \(r\), which together comprise the opportunity cost of the loan to investors, \((1+r)p_1\). This reduction in the opportunity cost reduces \(R_{NBR}\), shifting the \(R_{NBR} = m\) line left, expanding the set of parameters for which broking is feasible. This reduction also reduces \(R_{SBK}\), so it is not at first obvious which way the \(R_{SBK} = R_{NBR}\) line shifts. We will argue that the line shifts down, so that the expansion of the broking region is unambiguous. The easiest way to see this point is to treat \((1+r)p_1\) as the numeraire by which we divide all the dollar values in \((12)\). Then it is clear that a reduction in \((1+r)p_1\) has two real effects. One is a rise in the real screening cost \(k_I/(1+r)p_1\), decreasing the attractiveness of banking relative to broking. The other is a rise in the real foreclosure proceeds \(\lambda p_2/(1+r)p_1\), increasing the attractiveness of broking relative to banking. Both effects contribute to downward shift in the \(R_{SBK} = R_{NBR}\) line, expanding the broking region.

So far, the comparative-statics results have touched on the size of the no-contract and broking regions in Figure 3. For completeness, it is worth mentioning the effect of parameter changes on the remaining—banking—region. For most of the parameters, the effect on banking is ambiguous. For example, consider \(p_1\) and \(r\). As just mentioned, a decrease in these expands the broking region. But because repayments \(R_{NBR}\) and \(R_{SBK}\) fall, the no-contract region shrinks. The net effect on the size of the banking region is ambiguous. For most of the other parameters, the same is true: whatever expands the broking region contracts the no-contract region, and whatever contracts the broking region expands the no-contract region, generating an ambiguous effect on the residual, banking, region. We can be more concrete only about the effects of \(d\) and \(k_I\). Reductions in these parameters make banking more efficient relative to banking and no contract, unambiguously expanding the banking region.

The comparative-statics results so far have effectively analyzed each region of Figure 3 in isolation. However, there is a relationship among the regions. The banking region appears as a buffer between the no-contract region, in which the environment is inhospitable to lending, and the broking region, in which conditions are ripe for efficient lending. In the banking region, extra costs can be expended to make lending feasible when it would not be otherwise.

\textbf{Proposition 4.} Consider an increase in one of the parameters of the model from the lowest to the highest value on its support, holding others parameters constant. There are cases in which the banking interval is
Proposition 4 is a corollary of a richer comparative-statics result, Theorem 1, which catalogs how equilibrium outcomes change with increases in the value of every parameter in the model over its entire range. Theorem 1 is stated and proved in Appendix A.

Additional comparative-statics results provide a rigorous theoretical explanation for recent empirical work by Keyes et al. (2010). The authors assert that in the U.S. mortgage market, before the crisis, a threshold FICO score of 620 emerged as an industry standard or “rule of thumb”: it was relatively easy for originators to securitize mortgages for borrowers rated above this threshold and difficult for them to securitize loans with scores below. Mortgages below the threshold thus tended to be held by the originator as they are by screening banks in our model. Keyes et al. also found a discontinuous increase in default risk as the FICO score moved from just below the threshold to just above. As we will show below, this predicted by our theory: banks use additional soft information for (retained) mortgages just below some threshold, screening out the bad consumer type and moderating the default risk otherwise embodied in such loans. Consumers above the threshold are not screened, implying that mortgage liens consist of both the good and bad consumer type. The consequence is a discrete upwards jump in default risk.

Three parameters in the model, \( h \), \( \gamma \), and \( y \), are related to a consumer’s FICO score. Figure 4 illustrates how one of these, \( \gamma \), affects equilibrium mortgage quantity and quality. The figure is drawn for the interesting case in which each of the three possibilities—no contract, banking, broking—arises for certain values of \( \gamma \). In particular, there is no contract below \( \gamma' \), banking between \( \gamma' \) and \( \gamma'' \), and broking above \( \gamma'' \).

As the first panel in Figure 4 shows, equilibrium mortgage quantity (captured in this representative consumer model by the probability that the consumer obtains a mortgage) is weakly increasing in \( \gamma \). In the initial no-contract interval, quantity equals 0. In the next banking interval, all good types receive mortgages. Since their measure increases in \( \gamma \), this segment slopes up. At the threshold \( \gamma'' \) between the banking and broking intervals, quantity jumps as now all consumers receive mortgages. As shown in the lower panel, default risk also jumps at the margin between banking and broking. Technically, default risk jumps from \( 1-h \) to \( 1-\gamma'' h \) at \( \gamma = \gamma'' \). We expect this jump to be substantial in practice since it captures the bank’s use of soft information to screen borrowers who have FICO scores around the margin where securitization becomes feasible. The model thus provides a rigorous explanation of the discontinuity in default risk for

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\(^{13}\) Other cases are possible, including that one or two of the three intervals is empty. Proposition 4 rules out the possibility that broking is observed between no-contract and banking intervals. See Theorem 1 in Appendix A for details. The other credit-worthiness parameters, \( y \) and \( h \), have only two associated intervals. Holding constant all other parameters, only one of the two forms of intermediary arises in equilibrium for all values of \( y \) and similarly for \( h \).
securitized and non-securitized mortgages at a FICO of 620 along with other comparative statics in Keyes’ et al. (2010).

For reference, the next proposition formalizes the most important of the comparative-statics results for \( \gamma \) as well as the other parameters related to credit-worthiness.

**Proposition 5.** Consider the consumer’s credit-worthiness parameters \( h, \gamma, \) or \( y \). An increase in any of these parameters weakly increases mortgage volume (captured by the probability that the consumer receives a mortgage). An increase any of these parameters weakly decreases the default probability over the parameter’s whole range except for an upward jump at the margin between banking and broking (whenever this margin exists).

Proposition 5 is also a corollary of the more general comparative-statics result, Theorem 1, stated and proved in Appendix A.

### 4.3. Welfare

Propositions 1–5 make positive predictions regarding equilibrium. Here, we answer normative questions, comparing social welfare in the equilibrium under broking to that under banking.

Because intermediaries and investors earn zero expected profit in equilibrium, social welfare is identical to consumer surplus. In the continuation equilibrium with banking, consumer surplus is given by the good type’s payoff alone because the bad type is screened and does not receive a mortgage. Substituting \( R^\text{SBK} \) from (6) into the expression for the good type’s payoff, (9), yields expected social welfare with screening banks

\[
W^\text{SBK} = \gamma [u_1 + h(u_2 + y - R^\text{SBK})].
\]  

The expression is pre-multiplied by the probability of a good type, \( \gamma \). In the continuation equilibrium with broking, both good and bad types obtain a positive payoff. The expected payoff across these types is

\[
W^\text{NBR} = \gamma [u_1 + h(u_2 + y - R^\text{NBR})] + (1 - \gamma)u_1.
\]  

With probability \( \gamma \), the consumer is the good type, obtaining a payoff found by substituting \( R^\text{NBR} \) into expression (9). With probability \( (1 - \gamma) \), the consumer is the bad type, and obtains \( u_1 \): after consuming housing services in period 1, he is always forced out of the house because he never has the income to repay.

Subtracting (13) from (14) yields

\[
W^\text{NBR} - W^\text{SBK} = \gamma h(R^\text{SBK} - R^\text{NBR}) + (1 - \gamma)u_1.
\]  

22
The right-hand side has two terms. The first captures the good type’s payoff from broking relative to banking. This is the same as the condition determining the form of intermediary emerging in equilibrium. In particular, it equals 0 when \( R_{SBK} = R_{NBR} \) so that the good type is indifferent between broking and banking.

The second term, \((1 - \gamma)u_1\), reflects the bad type’s payoff from a mortgage. It represents a wedge between the equilibrium form of intermediary and the efficient form. The bad type’s payoff contributes to social welfare, but is not valued by lenders in equilibrium as they are unable to finance loss-making bad types. This positive externality associated with broking leads to insufficient broking with securitization and excessive banking with screening in equilibrium. Formally, we have the following proposition.

**Proposition 6.** If equilibrium entails securitizing brokers, this is the socially optimal form of intermediary. There is a set of parameters for which equilibrium entails screening banks but the continuation equilibrium with a securitizing broker would be socially more efficient.

The formal proof is almost immediate from (15). If equilibrium involves broking, then \( R_{NBR} \leq R_{SBK} \) by of Proposition 1, implying that the first term of (15) is non-negative. Given the second term is positive, \( W_{NBR} > W_{SBK} \). One can easily generate cases in which equilibrium involves inefficient banking. Take any parameters for which both forms of intermediary are feasible but banking emerges in equilibrium. In the limit as \( u_1 \to \infty \), we have \( W_{NBR} > W_{SBK} \).

5. **Equilibrium Strategic Default**

The benchmark model allowed for the possibility of strategic default. In particular, if the contractual repayment satisfied \( R_i > p_2 \), then the consumer would prefer to default on the mortgage and purchase a different home at price \( p_2 \), thus saving \( R_i - p_2 \). Still, strategic default never occurred in equilibrium. If \( R_i > p_2 \), the consumer always defaults, and the bank only obtains the foreclosure proceeds, which by (3) are insufficient to cover the principal and interest on the initial loan. Thus, for any case where there would be strategic default, there simply would be no mortgage lending.

This section extends the model to the case of stochastic period-2 house prices, \( \tilde{p}_2 \). This extension allows for the possibility of strategic default in equilibrium—for low realizations of \( \tilde{p}_2 \)—yet for lending to still be feasible—as repayments can be made for higher realizations of \( \tilde{p}_2 \). Besides allowing the possibility of equilibrium strategic default, the model with stochastic house prices is interesting in its own right because it allows for an analysis of the economic impact of volatile house prices, an important recent issue in the mortgage market. A further use of the stochastic-price model is that it will allow us to analyze renegotiation between the consumer and bank, which may happen if a buyer can threaten to purchase a new home at a lower price than the contractual repayment. We analyze renegotiation in Section 6.
5.1. Model with Stochastic Prices

Let the second-period house price be a Bernoulli variable \( \hat{p}_2 \) taking on high value \( \bar{p}_2 \) with probability \( \psi \) and low value \( p_2 \) with probability \( 1 - \psi \). For ease of comparison, assume the mean of \( \hat{p}_2 \) is the same as in the benchmark model:

\[
p_2 = \psi \bar{p}_2 + (1 - \psi) p_2. \tag{16}
\]

Several additional parameter restrictions will help streamline the analysis. First, we will consider positive values of \( p_2 \) in a neighborhood of 0 (indicated with some abuse of notation using the limit notation \( p_2 \to 0 \)). By making \( p_2 \) as low as possible, this restriction represents extreme volatility, thus making the contrast with the benchmark model as stark as possible. It also pins down the consumers’ behavior in the low-price state, ensuring he strategically defaults if he does not default for other reasons (such as if he has no income). Second, we restrict attention to sufficiently high values of \( y \) and \( u_2 \) (indicated using the limit notation \( y, u_2 \to \infty \)) so that these parameters do not constrain feasibility. This eliminates all the channels of consumer default except for the focus of this section: the consumer defaulting when the second-period house price is so low that it is cheaper to buy a different house than to repay the existing mortgage. After taking limits, repayment constraints \( R^{\text{NBR}}, R^{\text{SBK}} \leq m = \min\{y, p_2, u_2\} \) in the benchmark model would reduce to \( R^{\text{NBR}}, R^{\text{SBK}} \leq p_2 \). Finally, we restrict attention to values of \( p_2 \) sufficiently high that both forms of lending would be feasible in the benchmark model: \( R^{\text{NBR}}, R^{\text{SBK}} < p_2 \). As we will show, under this restriction, both modes of lending will also be feasible under stochastic prices and the equilibrium form of lending will remain the same when moving from deterministic to stochastic prices.

5.2. Equilibrium with Stochastic Prices

As in the deterministic-price model, the consumer defaults if he has low income. The new outcome with stochastic prices is that the consumer now also defaults in the high-income, low-price state. We know the consumer strategically defaults in this state because the limit value \( p_2 \to 0 \) of the low price is less than any finite repayment, so the consumer always finds it cheaper abandon his current house, saving the mortgage repayment, and buying a different house. Thus the only state in which the consumer repays is when he earns positive income and the house price is high. Bertrand competition drives this repayment to the zero-profit level, denoted \( \hat{R}^{\text{NBR}} \) for a mortgage originated by a broker and \( \hat{R}^{\text{SBK}} \) for a mortgage originated by a screening bank.
Repayment $\hat{R}^{SBK}$ satisfies the screening bank’s zero profit condition

$$
\gamma \left[ h \psi \hat{R}^{SBK} + (1-h) \lambda p_2 + (1-h) \psi \lambda \bar{p}_2 - \frac{(1+r)p_1}{1-d} \right] - k_I = 0.
$$

(17)

The bank only makes the loan to the good type, which nature selects with probability $\gamma$. Conditional on this, it receives repayment $\hat{R}^{SBK}$ only in the high-income, high-price state, which occurs with probability $h\psi$. The bank obtains foreclosure proceeds $\lambda \bar{p}_2$ in the low-price state regardless of the borrower’s income (i.e. with probability $1-\psi$) and $\lambda \bar{p}_2$ in the low-income, high-price state with probability $(1-h)\psi$. The last two terms on the left-hand side of (17) capture the bank’s cost of making the loan.

Repayment $\hat{R}^{NBR}$ yields zero profits for investors in the securitized mortgage given zero originating fee for the broker:

$$
\gamma h \psi \hat{R}^{NBR} + (1-h) \lambda p_2 + (1-\gamma h) \psi \lambda \bar{p}_2 - (1+r)p_1 = 0.
$$

(18)

Investors only receive the repayment when the consumer is the good type, has high income, and the house price is high. When the price is low, the consumer—regardless of its type—defaults, and the broker obtains $\lambda p_2$. Whenever price is high and the consumer is the good type but without high income—which happens with probability $(1-\gamma h)\psi$—the broker receives the high liquidation value $\lambda \bar{p}_2$. The last term on the left-hand side of (18) captures the investors’ opportunity cost of funds for the loan amount.

Using the formulas for $R^{SBK}$ and $R^{NBR}$ from (6) and (8) and rearranging, (17) and (18) give

$$
R^{SBK} = \psi \hat{R}^{SBK} + (1-\psi) \lambda p_2
$$

(19)

$$
R^{NBR} = \psi \hat{R}^{NBR} + (1-\psi) \lambda p_2.
$$

(20)

These equations are intuitive. Because the lender’s costs are the same regardless of whether house prices are deterministic or stochastic, the lender’s expected revenue with stochastic prices must be the same as the expected revenue generating zero profits with deterministic prices (although the repayments, of course, will be different). Further, the components of expected revenue only differ between the stochastic and deterministic cases in the high-consumer-income state; in the other states of nature the consumer always defaults, leaving the intermediary with the liquidation revenue $\lambda p_2$. When house price is deterministic, revenue in the high-consumer-income state is simply the repayment given by the left-hand sides of (19) and (20). When house price is stochastic, the high-income consumer only repays when house price is high, which happens with probability $\psi$. With probability $1-\psi$ price is low, the consumer strategically defaults and the intermediary receives only the liquidation value $\lambda p_2$. 

25
Using the formula for $R_{SBK}$ together with condition (3) (ensuring that foreclosure proceeds do not cover the loan cost) we obtain $R_{SBK} > \lambda p_2$. But then (19) implies $\hat{R}_{SBK} > R_{SBK}$ and similarly (20) yields $\hat{R}_{NBR} > R_{NBR}$. Thus mortgage repayments must increase when prices are stochastic. Intuitively, mortgage repayments must cover the additional contingency of strategic default in the stochastic case, along with default caused by low income which happens in both cases.

We have seen that Bertrand competition in the deterministic-price model leads to the form of lending with the lower of the two repayments $R_{NBR}, R_{SBK}$. Similar logic implies that Bertrand competition in the stochastic-price model leads to the form of lending with the lower of the two repayments $\hat{R}_{NBR}, \hat{R}_{SBK}$. But (19) and (20) imply that $\hat{R}_{NBR}$ and $\hat{R}_{SBK}$ preserve whatever inequality exists between $R_{NBR}$ and $R_{SBK}$ because the same constant is added in both equations. One can check that if we substitute the parameter restrictions we have adopted in this section ($y, u_2 \to \infty$, and $R_{NBR}, R_{SBK} < p_2$) back into Proposition 1, there is always lending in equilibrium in the deterministic model. The repayment constraints $R_i \leq m$ do not bound. To prove that the equilibrium form of lending is the same in the deterministic- and stochastic-price model requires one more step. We need to demonstrate that $\hat{R}_{NBR}$ and $\hat{R}_{SBK}$ satisfy their respective repayment constraints. Because we have taken $y, u_2 \to \infty$, the last step reduces to showing that $\hat{R}_{SBK}$ and $\hat{R}_{NBR}$ are not so high as to induce strategic default by the high-income consumer in the high-price state: i.e., $\hat{R}_{SBK}, \hat{R}_{NBR} < \hat{p}_2$. This fact is established in the proof of Proposition 7 in Appendix A. We have the following.

**Proposition 7.** Impose the parameter restrictions $y, u_2 \to \infty$, $p_2 \to 0$, $R_{NBR}, R_{SBK} < p_2$, and equation (16).

(i) In the benchmark, deterministic-price model, a mortgage contract is always signed in equilibrium. Let $R^*$ denote the equilibrium repayment. If $R_{NBR} < R_{SBK}$, the contract is offered by a securitizing broker, specifying repayment $R^* = R_{NBR}$. If $R_{SBK} < R_{NBR}$, the contract is offered by a screening bank, specifying repayment $R^* = R_{SBK}$.

(ii) In the stochastic-price model, a mortgage contract is also always signed in equilibrium, offered by the same form of lender as in the benchmark model. The good type of consumer strategically defaults in the high-income, low-price state. The repayment $R^*$ is higher than in benchmark model, satisfying $R^* = \psi R^* + (1-\psi) \lambda p_2$. Expected welfare is lower than in the benchmark model by $\gamma h(1-\psi)(1-\lambda) p_2$.

The expression for the welfare loss in moving from deterministic to stochastic prices can be derived by direct calculation. Intuitively, the welfare loss stems from the new possibility of strategic default: the borrower strategically defaults in the high-income, low-price state, which arises with probability $\gamma h(1-\psi)$ (from an ex-ante perspective, before the consumer’s type is drawn). The welfare loss is the friction $(1-\lambda) p_2$ associated with foreclosure in the low-price state.
5.3. Non-Recourse Mortgages

The model of stochastic prices can be used to derive policy implications for the relative efficiency of recourse and non-recourse mortgages. A recourse mortgage allows the lien holder to seize assets beyond the original house to recover mortgage debts. A non-recourse loan prohibits recovery beyond the original house. The analysis so far has implicitly focused on non-recourse mortgages because they best embody our stark assumptions on the borrower’s ability to divert all assets except for the original house. However, a slight modification of the model allows some scope for recourse mortgages. While maintaining the assumption that the lender cannot seize the borrower’s income directly in our model, we will now assume that a recourse loan allows the lender to seize a new property.

There is no difference between recourse and non-recourse mortgages in the low-income state because the borrower has no assets to seize. Nor is there a difference when the borrower makes the required repayment. The only possible difference arises when the borrower is tempted to strategically default on the first house and buy a second one for a price less than the repayment. As we have seen, this sort of strategic default arises in equilibrium with non-recourse mortgages. The threat of seizure of the new property provided by a recourse loan eliminates this sort of strategic default.

We saw in Proposition 7 that with non-recourse mortgages, the possibility of strategic default arising in equilibrium in the stochastic-price model (case (ii)) causes repayments to rise and welfare to fall relative to the equilibrium in the deterministic-price model (case (i)). With recourse mortgages, this is no longer true. Recourse mortgages eliminate strategic default, returning the equilibrium to case (i) of the proposition. The policy implication of the analysis is that if recourse loans were to replace the typical non-recourse style in the United States, mortgage repayments would fall and welfare would rise.\textsuperscript{14}

6. Renegotiation

In this section, we analyze another possible benefit to banks from holding mortgages in addition to screening: a bank is in a good position to renegotiate mortgages that would go into default in the face of an adverse housing market. As argued by Hart and Zingales (2008), diffuse investors who hold securitized mortgages originated by brokers would have much greater difficulty renegotiating with borrowers threatening to default.

Renegotiation avoids cases in which the borrower strategically defaults if forced to make the contractual repayment but would be willing to stay in the current house at a reduced repayment. This benefits the

\textsuperscript{14}As a matter of law, only six of the 50 U.S. states explicitly prohibit recourse mortgages. In practice, however, lenders generally regard the costs of judicial foreclosure to be too high to make such recovery worthwhile, so that non-recourse loans are the standard in the United States. See Ellis (2008).
lender if the reduced repayment still exceeds the foreclosure proceeds. Such cases arise in our model with stochastic prices. In particular, in the state with high income but low price \( p_2 \), the consumer strategically defaults. The lender obtains \( \lambda p_2 \) from foreclosure, but the borrower would be willing to pay as much as \( p_2 \) to stay in the home.

We will thus adopt the previous section’s stochastic-price model for our analysis of renegotiation. To focus on the cases of most interest, we maintain the parameter restrictions from the previous section. We will make the stark assumption that banks can costlessly renegotiate the loans they hold, but securitized loans cannot be renegotiated. The bargaining process is also simplified by giving the bank all the bargaining power vis-à-vis the borrower in the event of renegotiation.\(^{15}\)

The continuation equilibria for each form of lending are similar to those found in the stochastic-price model. Indeed, the outcome with a broker is identical because securitized mortgages cannot be renegotiated when second-period price is low: consumers continue to strategically default. With banking, the only difference is that instead of allowing strategic default in the high-income, low-price state, the bank renegotiates the mortgage. Since the bank has all the bargaining power, the renegotiated repayment is set at the highest amount that the borrower would be willing to pay instead of moving, i.e., the price of a different house, \( p_2 \).

Note that this is more than the liquidation value \( \lambda p_2 \) the bank would obtain if it simply foreclosed. The revenue increase which renegotiation allows alters the zero-profit repayment under renegotiation, which we denote as \( \hat{R}_{SBK} \):

\[
\gamma \left[ h\psi\hat{R}_{SBK} + h(1-\psi) + (1-h)\lambda p_2 - \frac{(1+r)p_1}{1-d} \right] - k_l = 0. \tag{21}
\]

Using the formula for \( R_{SBK} \) from (6) and rearranging, (21) gives

\[
R_{SBK} = \psi\hat{R}_{SBK} + (1-\psi)p_2. \tag{22}
\]

A comparison of (19) with (22) shows that \( \hat{R}_{SBK} < \hat{R}_{SBK} \), implying that renegotiation reduces the contractual repayment relative to the case with stochastic prices but no renegotiation. Renegotiation thus gives an additional advantage to banks relative to brokers in addition to screening. A natural question is whether the renegotiation effect could be substantial enough that banks emerge as the equilibrium lending form even if screening is prohibitively expensive. That is, is it possible to observe non-screening banks? In general, the

\(^{15}\)The model abstracts from some obvious drawbacks of renegotiation. If the borrower had private information about \( \tilde{p}_2 \), there would be scope for him to claim that the price is \( p_2 \) rather than \( \tilde{p}_2 \) to receive a reduced repayment. A lender might gain from committing not to renegotiate to avoid giving away information rent to the borrower. This benefit from committing not to renegotiate is absent from our model because information about the state of prices is symmetric. The assumption of symmetric information is realistic in the post-crisis economy, in which it is widely known that house prices had fallen substantially. It may be less realistic in periods in which local conditions are responsible for most of the fluctuations in house prices.
answer is yes. If the cost of screening and the benefit of renegotiation are sufficiently high, non-screening banks may be more efficient than either screening banks or brokers.\footnote{The full analysis for larger values of $p_2$ in which non-screening banks can emerge in equilibrium is available from the authors on request. One can show that two conditions are sufficient for equilibrium to involve non-screening banking:}

\[ p_1(1+r) \left( \frac{d}{1-d} \right) < \gamma h(1-\psi)(1-\lambda)p_2 \]

\[ (1-\gamma) \left[ p_1(1+r) - \lambda p_2 \right] < k_1. \]

Under the maintained parameter restriction $p_2 \to 0$, however, the renegotiation benefit is vanishingly small, so non-screening banks do not emerge in equilibrium.

\begin{proposition}
In the stochastic-price model, maintain the parameter restrictions $y,u_2 \to \infty$, $p_2 \to 0$, $R^{NBR},R^{SBK} < p_2$, and equation (16). Renegotiation enlarges the set of parameters for which banking is the equilibrium form of lending, emerging in equilibrium when

\[ R^{SBK} < R^{NBR} + (1-\psi)(1-\lambda)p_2. \] (23)

Renegotiation reduces the equilibrium contractual repayment offered by a bank: $\hat{R}^{SBK} < \hat{R}^{SBK}$. There is weakly more banking than is socially efficient even with renegotiation.
\end{proposition}

The proof in Appendix A derives condition (23). One might think that the social benefit of renegotiation—avoiding foreclosure waste $(1-\lambda)p_2$—might tip the balance of social welfare toward banking. While this is a social benefit, it is also a private benefit and so does not change the wedge between private and social preference toward banking. The positive externality associated with lending to bad types is still present with broking, and so equilibrium involves socially too little broking whether or not there is renegotiation.

\section{7. Systemic Factors}

To this point, we have conducted partial-equilibrium analysis. The representative-consumer model we used was simple to analyze but left no role for correlation in shocks in income, house prices, and foreclosure losses across consumers. Many commentators point to these systemic factors as causing a cascade of problems that precipitated the global financial crisis. This section shows how to extend the basic model to allow the analysis of such systemic factors.

\subsection{7.1. Model}

For concreteness, we will focus on one simple systemic factor, allowing foreclosure frictions to increase in the number of borrowers in foreclosure. Because securitized mortgages serve bad types and thus have

higher default rates than bank mortgages, securitization can exert a negative externality on the market. This negative externality raises the possibility of excessive securitization in equilibrium, contrasting our finding in the absence of systemic factors that there could only be socially too little securitization (see Proposition 6).

Return to the model in Section 3 with deterministic price $p_2$. Rather than a representative consumer, assume now that the market has a unit mass of ex ante identical consumers. Let $\gamma$ be the measure of good types and $1 - \gamma$ the measure of bad types. Instead of taking the fraction of period-2 house value that can be recovered in foreclosure to be a constant $\lambda$, we will now assume that it is a decreasing function $\lambda(\delta)$ of the number of period-2 defaults in the market, $\delta \in [0, 1]$. We will sometimes refer to the original model as the exogenous-$\lambda$ model and the current extension the endogenous-$\lambda(\delta)$ model.

### 7.2. Equilibria

With some additional notation, we can characterize equilibrium in a way quite similar to what we have seen in Proposition 1. Consider an equilibrium in which screening banks are active. Previous arguments can be used to show that no brokers are active, that all good types are served, that only good types are served, and that the borrower repays if and only if his income is high. Thus the measure of defaults in a banking equilibrium is $\gamma(1 - h)$. Let $\lambda^{SBK} = \lambda(\gamma(1 - h))$ be the fraction of period-2 house value recovered in this equilibrium. Consider an equilibrium in which brokers are active. Previous arguments can be used to show that no banks are active, that all consumers are served, but that only the high-income, good type repays. Thus the measure of defaults is $1 - \gamma h$ in this equilibrium. Let $\lambda^{NBR} = \lambda(1 - \gamma h)$ be the fraction of period-2 house value recovered in this equilibrium. Notice $\lambda^{SBK} > \lambda^{NBR}$ because $\gamma(1 - h) < 1 - \gamma h$ and $\lambda(\delta)$ is a decreasing function. A further piece of notation, it will be useful in the analysis to emphasize the dependence of the zero-profit repayments on $\lambda$ alone by writing the left-hand side of (6) as $R^{SBK}(\lambda)$ and the left-hand side of (8) as $R^{NBR}(\lambda)$, recognizing that in fact they are potentially functions of all of the underlying parameters.

With this notation in hand, notice the similarity between the following characterization of equilibrium and Proposition 1.

**Proposition 9.** Assume (3) holds. Assume the bad type applies for a mortgage when indifferent between applying and not. Equilibrium of the endogenous-$\lambda(\delta)$ model falls into one of the following three cases.

(i) If $R^{NBR}(\lambda^{NBR}) < \min\{R^{SBK}(\lambda^{NBR}), m\}$, then brokers originate all mortgages in equilibrium, securitizing them immediately. The equilibrium mortgage repayment is $R^* = R^{NBR}(\lambda^{NBR})$. Both good and bad consumer types apply for mortgages. The broker does not screen and serves both types.

(ii) If $R^{SBK}(\lambda^{SBK}) < \min\{R^{NBR}(\lambda^{NBR}), m\}$ then banks originate all mortgages by raising deposits from investors and hold the mortgages for both periods. The equilibrium mortgage repayment is $R^* = R^{SBK}(\lambda^{SBK})$. All consumers apply for a mortgage. The bank screens with probability 1, offering a mortgage to the good type, rejecting the bad type.
(iii) If \( m < \min\{R^{NBR}(\lambda_{NBR}), R^{SBK}(\lambda_{SBK})\} \), then there is no mortgage lending in equilibrium.

There are no other equilibria when the stated inequalities are strict. For each of the cases (i)–(iii), the stated outcome is also an equilibrium if any of the strict inequalities holds as an equality.

The only difference with Proposition 1 is that the relevant default rates at which the zero-profit repayments are evaluated differ across cases. In case (i), the broking equilibrium is stable unless a screening bank deviates by entering. If it enters, existing brokers are serving all consumers, so all consumers will continue to be served after the deviation, resulting in \( 1 - \gamma h \) defaults. Thus \( \lambda_{NBR}^{NBR} \) is the relevant value of \( \lambda \) at which to evaluate both the equilibrium repayment \( R^{NBR}(\lambda_{NBR}) \) and the deviating repayment \( R^{SBK}(\lambda_{NBR}) \). In case (ii), deviating entry by a broker changes the lending environment from one in which only good types are served, so that \( \lambda_{SBK}^{SBK} \) would be the relevant value of \( \lambda \), to one in which all types are served, so that \( \lambda_{NBR}^{NBR} \) becomes the relevant value of \( \lambda \). Thus there is an asymmetry between cases (i) and (ii). In case (i), the deviating bank’s entry does not change the overall quality of loans offered on the market, which are extended to good and bad types in any event. In case (ii), the deviating broker’s entry reduces the overall quality of loans offered, thus making deviation less profitable than in the case in which \( \lambda \) was fixed.

While the conditions for broking or banking were mutually exclusive when \( \lambda \) was exogenous (except on a boundary), with endogenous \( \lambda(\delta) \) there can be a region of overlap where multiple forms of intermediary arise in different equilibria. This can be seen in Figure 5. Line \( l_1 \) delineates indifference between banking and broking when the market default rate is \( \lambda_{NBR}^{NBR} \), i.e., delineates the set of parameters for which \( R^{NBR}(\lambda_{NBR}) = R^{SBK}(\lambda_{NBR}) \). This is the same as the \( R^{SBK} = R^{NBR} \) line in Figure 3 if we fix the exogenous \( \lambda \) behind Figure 3 so that it equals \( \lambda_{NBR}^{NBR} \). Line \( l_2 \) delineates the set of parameters for which \( R^{SBK}(\lambda_{SBK}) = R^{NBR}(\lambda_{NBR}) \). In the region between \( l_1 \) and \( l_2 \), the parameters can support both a banking equilibrium and a broking equilibrium.\(^{17}\)

### 7.3. Welfare

The addition of systemic factors allows us to enrich the welfare analysis from Section 4.3 in several dimensions. First, by restricting attention to parameters for which either lending mode can arise in an equilibrium, we can compare welfare across actual equilibria rather than comparing equilibrium welfare to welfare in a counterfactual outcome. More importantly, securitization only had a positive externality in the previous analysis, leading to the stark conclusion that securitization would never be socially excessive. Introducing systemic factors introduces the possibility of negative externalities associated with securitization. Whether

\(^{17}\)The fact that this region is non-empty is an implication of Proposition 10. The proposition provides a tight bound on the welfare difference between banking and broking equilibria, implying that a sequence of parameter vectors exist approaching the bound. But then a banking and a broking equilibrium exists for each of the parameter vectors in the sequence.
there is too much or two little securitization involves an interesting theoretical tradeoff that is also of practical interest, embodying concerns raised by commentators about the recent wave of securitization.

We will focus on parameters for which there are a banking and a broking equilibrium, i.e., the region between lines \( l_1 \) and \( l_2 \) in Figure 5. To be strictly in this region, the conditions behind both case (i) and case (ii) of Proposition 9 must hold. Combining these conditions,

\[
R^{SBK}(\lambda^{SBK}) < R^{NBR}(\lambda^{NBR}) < \min\{R^{SBK}(\lambda^{NBR}), m\}. \tag{24}
\]

Following the logic of Section 4.3, we can derive the difference between welfare in the broking and banking equilibria as

\[
W^{NBR} - W^{SBK} = \gamma h[R^{SBK}(\lambda^{SBK}) - R^{NBR}(\lambda^{NBR})] + (1 - \gamma)u_1. \tag{25}
\]

This is the same as equation (15) except the equilibrium repayments, \( R^{SBK}(\lambda^{SBK}) \) and \( R^{NBR}(\lambda^{NBR}) \), now reflect endogenous foreclosure frictions. The first term on the right-hand side of (25) is negative in the multiple-equilibrium region by (24). This is the negative externality associated with securitization, worsening market foreclosure frictions. The second term on the right-hand side of (25) is the positive externality associated with securitization, increasing homeownership among bad types, familiar from the analysis with fixed \( \lambda \).

We have already seen cases in which there is socially too little securitization. Proposition 6 found this result in the absence of systemic factors. By continuity, the result continues to hold in the presence of systemic factors if they are not too important; i.e., if \( \lambda(\delta) \) is fairly inelastic. The next proposition fleshes out the opposite possibility, examining whether there are cases in which there can be too socially much securitization and, if so, how much welfare can possibly be lost moving from a banking to a broking equilibrium. The answer turns out to be yes, there can be socially too much securitization; the proposition provides a formula for the maximum welfare loss from securitization.

**Proposition 10.** Consider any fixed values \( \gamma \in (0, 1) \), \( h \in (0, 1) \), \( u_1 > 0 \), and \( p_2 > 0 \). The welfare loss in moving from a banking to a broking equilibrium, \( W^{SBK} - W^{NBR} \), can be no greater than

\[
\gamma^2 h(1 - h)p_2 - (1 - \gamma)u_1 \tag{26}
\]

for any values of the other parameters and for any decreasing \( \lambda(\delta) \) on \([0, 1]\) that support both a banking and a broking equilibrium. Bound (26) is tight in that there exist values of the other parameters and \( \lambda(\delta) \) in the multiple-equilibrium set for which the welfare loss from securitization can be made arbitrarily close to (26).

The proof, which amounts to solving a concave-programming problem with many variables and constraints
via the Kuhn-Tucker method, is provided in Appendix A.

Proposition 10 has a number of relevant implications. It implies that in the multiple-equilibrium region, the broking equilibrium can be socially less efficient than then banking one. This can be seen by substituting a high value of $p_2$ and low value of $u_1$ into (26). It also implies that the loss from securitization can be arbitrarily high, as can be seen by substituting increasingly high values of $p_2$ into (26).

In the presence of systemic factors, renegotiation and non-recourse loans have additional social benefits. We already saw that renegotiation and non-recourse loans reduce strategic default (see Sections 5.3 and 6), a social benefit that is fully internalized by the contracting parties. With systemic factors, this reduction in strategic default reduces the overall default rate, which has a positive external benefit of reducing market-wide foreclosure frictions.

Securitization can be seen to increase the fragility of the financial system along several dimensions in the model. One dimension can be understood by looking more carefully at the proof of Proposition 10. The proof shows that the bound in (26) is approached as the parameters in the region in which either form of intermediary is possible approach the boundary with the banking-only region, line $l_1$ in Figure 5. But this implies that a small increase in what one might otherwise consider a beneficial parameter such as $\gamma$ can cause a discontinuous fall in social welfare by introducing the possibility of socially inferior broking equilibria. The fact that a slight improvement in the lending environment, by shifting the market toward securitization, can cause a large fall in welfare can be interpreted as a fragility in the financial system. This sort of fragility did not arise in the absence of systemic factors because expression for the difference in social welfare (15) was everywhere continuous in the parameters, even at the boundary between banking and broking regions. It should be emphasized that a parameter change shifting the market from the banking region to the region in which either form is possible does not automatically shift the equilibrium to broking—we have not yet provided a theory of equilibrium selection in the presence of multiple equilibria—but admits broking as a new possibility.

8. Endogenizing House Prices

In this section, we move from assuming house prices are exogenous parameters $p_1$ and $p_2$ to endogenizing them as stemming from equilibrium of a competitive housing market each period. Each variant, exogenous prices versus endogenous prices, has its virtues. The virtue of the exogenous-price variant is that it is able to capture shocks that enter the market from outside the model, including shocks to housing supply, to macroeconomic variables, and to other aspects of the economy that the model does not capture. It allows housing prices to take on arbitrary positive values (aside from the weak condition (3)) and is simple to
analyze. The virtue of the endogenous-price variant, on the other hand, is that it is able to link house prices across periods, to link house prices to financial conditions including the equilibrium forms of intermediary, and to account for demand shocks. It is more complicated than the exogenous-price variant, explaining why we deferred the analysis to this later section, yet the analysis is still tractable and preserves most of the insights from the previous analysis. At the end of the section we will highlight some new results in the endogenous-price variant that help rationalize additional facts about the mortgage market in practice, in particular why there appeared to be a concentration of brokering in the subprime market serving borrowers with the worst credit scores.

8.1. Model

Assume a fixed mass $H$ of houses is constructed prior to period 1. The builders supply this stock inelastically in period 1. Rather than a representative consumer, assume now there is a mass $N$ of consumers that are homogeneous in all parameters except one, $\gamma$. We focus on $\gamma$ because it was found to be the most useful index of credit worthiness in 4.2 to be a useful index of credit worthiness. Assume $\gamma$ be continuously distributed in the population with density function $f$ and distribution function $F$. Let $\bar{F} = 1 - F$ denote the complementary distribution function and $\mu_\gamma = \int_0^1 \gamma f(\gamma) d\gamma$ the mean of $\gamma$ in the consumer population.

The equilibrium price is set each period so that there is neither excess supply (meaning there is no supplier left willing to sell at a lower price) nor excess demand (meaning there is no buyer left willing to purchase at a higher price). We will solve for equilibrium prices using backward induction starting with the period-2 price.

Assume that suppliers face the same losses if they leave the house unoccupied.

8.2. Period-2 Price

Consumers obtain their income realizations in period 2. A mass $N \int_0^1 \gamma h f(\gamma) d\gamma = Nh\mu_\gamma$ of them end up with positive income. These are the only consumers who can possibly have retained a house from period 1. If $H > Nh\mu_\gamma$, at least some houses must be in the hands of suppliers who would accept any positive price for them, either the original builders who did not manage to sell in period 1 or mortgage lien holders who seized the house in foreclosure. Either group does not obtain directly obtain utility from housing and so would accept any positive price. Thus if $H > Nh\mu_\gamma$ there is excess supply at any positive price, implying the equilibrium price is $p^*_2 = 0$.

If the reverse inequality holds, i.e., $H < Nh\mu_\gamma$, then there must be some consumers with positive income in period 2 who were not able to buy in period 1. In fact, there is an excess of them over the supply $H$
for any price up to \( \min \{ y, u_2 \} \). They are unwilling to pay a period-2 price greater than \( u_2 \) an unable to pay a period-2 price greater than \( y \). Thus, if \( H < Nh_{\mu, \gamma} \), \( p_2^* = \min \{ y, u_2 \} \). Recall that in the exogenous-price variant, we had defined \( m = \min \{ y, u_2, p_2 \} \). In the endogenous-price variant, if period-2 price is positive, \( \min \{ y, u_2, p_2^* \} = \min \{ y, u_2 \} \). Thus there is little abuse of notation in redefining \( m = \min \{ y, u_2 \} \) in the present, endogenous-price variant of the model, allowing us to economize on notation.

In sum, equilibrium in the period-2 housing market is quite simple to characterize. Besides a knife-edged case (when the strict equality \( H = Nh_{\mu, \gamma} \) holds, a case which we will ignore), there are only two values for equilibrium price. If \( H > Nh_{\mu, \gamma} \) then \( p_2^* = 0 \). If \( H < Nh_{\mu, \gamma} \), then \( p_2^* = m \).

### 8.3. Period-1 Price

Next, we fold the game back to determine equilibrium period-1 house price. According to equation (10), the feasible repayments must satisfy \( R_i \leq \min \{ y, u_2, p_2^* \} \). If \( p_2^* = 0 \), then \( R_i \leq 0 \), implying that no positive price can be financed. The only possible equilibrium price is \( p_1^* = 0 \). Hence it is immediate that if \( H > Nh_{\mu, \gamma} \), \( p_1^* = p_2^* = 0 \).

If \( H < Nh_{\mu, \gamma} \), then prices can be positive in both periods. Assume suppliers put the entire stock \( H \) on the market rather than delaying the sales of some vacant homes until period 2. Assume further that buyers prefer to buy immediately rather than delaying until period 2. We will show both assumptions are justified in equilibrium. Under these assumptions, it is straightforward to compute \( p_1^* \). We array consumers in descending order of the maximum loan amount that each can finance. The mass \( H \) of them that can finance the highest amount end up buying houses at a price determined by the marginal consumer’s maximum loan amount.

The consumer’s maximum loan amount depends on his type \( \gamma \) and the intermediary to which he applies. Suppose type \( \gamma \) applies to a screening bank. The maximum loan amount can be found by the maximum \( p_1 \) for which the screening bank’s profit in equation (5) is still non-negative after substituting the highest possible repayment. By (10), the highest feasible repayment is \( \bar{R} = \min \{ y, u_2, p_2^* \} \). Since \( H < Nh_{\mu, \gamma} \), we have \( p_2^* = \min \{ y, u_2 \} = m \), implying \( \bar{R} = m \). Substituting \( \bar{R} = p_2^* = m \) into (5) and rearranging yields

\[
p_1^{SBK}(\gamma) = \frac{1-d}{1+r} \left[ hm + (1-h)\lambda m - \frac{k_1}{\gamma} \right],
\]

where \( p_1^{SBK}(\gamma) \) denotes the highest loan amount for type \( \gamma \) that can be financed with a screening-bank mortgage. Similarly, we can compute \( p_1^{SBK}(\gamma) \), the maximum loan amount for type \( \gamma \) that can be financed
with a broker-originated mortgage. Substituting $\bar{R} = p_2^* = m$ and $F_i = 0$ into (7) and rearranging yields

$$p_1^{NBR}(\gamma) = \frac{1}{1+r} \left[ \gamma hm + (1-\gamma-h)\lambda m \right].$$  \hspace{1cm} (28)

The consumer’s maximum loan amount is the upper envelope of the two: $p_1^{MAX}(\gamma) = \max\{p_1^{SBK}(\gamma), p_1^{NBR}(\gamma)\}$.

Because $p_1^{SBK}(\gamma)$ and $p_1^{NBR}(\gamma)$ are strictly increasing in $\gamma$, $p_1^{MAX}(\gamma)$ is as well. Rather than graphing $p_1^{MAX}$ as a function of $\gamma$ on the horizontal axis, Figure 6 graphs it versus $N\bar{F}(\gamma)$, the mass of consumers with that type or higher, for values of $\gamma$ in reverse order from 1 (at which value $N\bar{F}(1) = 0$) to 0 (at which value $N\bar{F}(0) = N$). Because $p_1^{MAX}(\gamma)$ is increasing in $\gamma$ and $N\bar{F}(\gamma)$ is decreasing in $\gamma$, the graph of $p_1^{MAX}(\gamma)$ versus $N\bar{F}(\gamma)$ is downward sloping, shown as the bold grey curve. This is the period-1 demand curve for housing.

The supply curve is vertical line at a quantity of $H$. The intersection of demand and supply gives $p_1^*$. Figure 6 shows $p_1^{SBK}(\gamma)$ and $p_1^{NBR}(\gamma)$ intersecting in two places. These values of $\gamma$ for which these curves intersect can be found by equating 27 and 28, yielding a quadratic equation in $\gamma$ with roots

$$\frac{(1-d)[h+(1-h)\lambda]-\lambda \pm \sqrt{(1-d)[h+(1-h)\lambda]-\lambda}^2-4(1-\lambda)(1-d)hk_i/m}{2(1-\lambda)h}. \hspace{1cm} (29)$$

Let $\gamma^+$ and $\gamma^-$ be the roots produced by, respectively, adding and subtracting the radical term. If the roots are complex, then, referring to Figure 6, $p_1^{SBK}(\gamma)$ never rises above $p_1^{NBR}(\gamma)$, in which case all buyers are financed with mortgages originated by brokers. If the roots are real, then there can be as many as three intervals of consumers, with the highest types—$\gamma > \gamma^+$—served by non-screening brokers, intermediate types—$\gamma \in (\gamma^-, \gamma^+)$—served by banks, and the lowest types—$\gamma < \gamma^-$—again served by non-screening brokers. Whether one, two, or three of these intervals are observed in equilibrium depends on whether the supply of houses is exhausted before lower-type consumers are served. This analysis is summarized in the following proposition.

**Proposition 11.** Suppose $H > Nh\mu_\gamma$. Then $p_1^* = p_2^* = 0$. Consumers do not need financing to buy the free houses. Suppose $H < Nh\mu_\gamma$. In period 1, all $H$ houses are sold. The marginal consumer type who buys is given by $\gamma^* = \bar{F}^{-1}(H/N)$. He and all consumers with higher types buy. Several intermediary forms may be active depending on the case.

(i) If $\gamma^+$ is complex or $\gamma^+ > \gamma^+$, then all consumers obtain a mortgage from brokers. The equilibrium price is $p_1^* = p_1^{NBR}(\gamma^*)$.

(ii) If $\gamma^* \in (\gamma^-, \gamma^+)$, then consumer types in $[\gamma^+, \gamma^+]$ obtain financing from banks and in $(\gamma^+, 1]$ from brokers. The equilibrium price is $p_1^* = p_1^{SBK}(\gamma^*)$.

(iii) If $\gamma^* < \gamma^-$, then consumer types in $[\gamma^+, \gamma^-)$ obtain financing from brokers, in $[\gamma^-, \gamma^+)$ from banks, and in $(\gamma^+, 1]$ from brokers. The equilibrium price is $p_1^* = p_1^{NBR}(\gamma^*)$. 

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In period 2, foreclosed houses are resold to consumers with types less than $\gamma^*$, so who did not buy in period 1, who have obtained positive income realizations.

The proof, provided in Appendix A, checks the claims that all suppliers strictly prefer to sell and all buyers strictly prefer to buy in period 1 rather waiting to period 2.

Figure 6 illustrates case (ii) of the proposition in which there are two intervals of consumer types, the highest types obtaining financing from brokers and the lowest types from banks. This case delivers similar comparative statics to those found earlier for exogenous-price model. Consider the lower panel of Figure 4 showing how the default probability varies with $\gamma$. We obtain the same picture in the endogenous-price model except now all $\gamma$ types are served together on the same market. At the margin between banking and broking (in Figure 4 denoted $\gamma'$, here denoted $\gamma^+$), the default probability jumps when moving from the less to more credit-worthy consumers as indexed by $\gamma$. This is the same result from Keyes et al. (2010), except now found for a cross section of consumers served on the same market.

A feature of the endogenous-price model that did not appear before is the possibility that brokers operate in separate intervals, serving both the highest and lowest quality borrowers as measured by FICO scores. When $p_1$ is exogenous, the loan amount is too high to finance for this interval of low-quality borrowers, so there is no lending for them. When $p_1$ is endogenous, if the supply is great enough for homes to be sold to consumers with low types, the price falls so the loan amount may feasibly be financed. The reason that brokers may be more efficient than banks are the very lowest end is that the screening cost, which is fixed, may become a large fraction of the loan value. With these lower loan amounts, it is more efficient to finance the bad credit risks without screening and accept the high default rates that come with them.

The finding that brokers may specialize in the lowest and highest ends of the market in terms of credit scores rationalizes the concentration of broker activity in the subprime market in the run-up to the financial crisis. For example, Rajan (XXXX) details the operation of New Century XXX.

The next proposition provides comparative-statics results on the prevalence of broking.

**Proposition 12.** In the endogenous-price model there is weakly more broking/securitization the greater is $\lambda, \delta$ and $k_I$ and the lower is $\gamma, u_2$, and $h$; $u_1$ and $r$ have no effect. The effect of $\gamma$ on broking/securitization is ambiguous, weakly increasing it for $\gamma$ near $\gamma^+$ but weakly decreasing it for $\gamma$ near $\gamma^-$.

The results for $\gamma$ are already stated in Proposition 11. The results for the rest of the parameters involve a straightforward comparison of (??) and (28). For example, consider an increase in $k_I$. Starting from a parameter configuration for which the equilibrium involves broking, this change reduces $p_1^{SBK}(\gamma)$ but has no effect on $p_1^{NBR}(\gamma)$. Hence the equilibrium must continue to involve broking. The proof for the rest of the parameters is similar and is provided in Appendix A.
Proposition 12 preserves many but not all of the comparative-statics results from the exogenous-price model. Increases in \(k_I, d,\) and \(\lambda\) increase broking in either model as does an increase in \(\gamma\) near the upper threshold \(\gamma^+\). Increases in \(y, u_2,\) and \(h\) have different effects in the exogenous and endogenous-price variants. Increases in these parameters increased broking in the exogenous-price variant because broking replaced no contract in some cases; these parameters had no effect on the banking/broking margin. These parameters do have an effect on the banking/broking margin in the endogenous-price variant. A decrease in \(y\) or \(u_2\) decrease \(m\), thus decreasing equilibrium housing prices, thus raising the real screening cost (as one can see by normalizing all dollar values by \(m\)), making broking relatively more efficient than banking. A decrease in \(h\) reduces the difference between good and bad types, reducing the value of screening, again making broking relatively more efficient than banking.

9. Endogenizing Contract and Intermediary Forms

10. Conclusion

Recent history has seen a rapid expansion in the securitization of mortgages and a substantially higher rate of default when compared with bank-held loans. The natural question that emerges is why should there be a difference in the quality of loans, depending on whether investors or banks hold the right to foreclose? We have argued that incomplete contracts must be part of the explanation, developing and analyzing a model which features significant contractual incompleteness due to the ability of parties to divert funds. Banks can commit to repay depositors because of regulation, whereas brokers cannot, financing loans through immediate sale of mortgage liens to investors—i.e. securitization. This breaks the link between brokers’ screening decision and the right to foreclose, reducing the incentive to undertake soft-information screening of consumers, leading to increased default rates for securitized mortgages.

Our results are consistent with recent empirical findings. As well as the basic link between increased default and securitization, we predict that rising house prices, lower liquidation costs, lower interest rates, higher regulatory costs and cheaper underwriting lead to increased securitization and default. We provide a theoretical basis for Keys et al’s (2010) finding of a discrete jump in default rates around the FICO threshold of 620.

While welfare analysis demonstrates that there is insufficient securitization, extending our model to the case of stochastic prices yields insights regarding the stability of mortgage markets under different funding modes. When house prices are low, high income consumers will be tempted to strategically default on a non-recourse loan, which are the standard type of mortgage in the United States. Enforcement of recourse
loans would reduce repayments as well as eliminate such strategic default and improve market stability. We analyze the impact of renegotiation, assuming that banks are able to do so and diffuse investors are not. Renegotiation also reduces repayments, reduces strategic default, and increases the incentive for bank origination and screening, thus improving mortgage market and hence macroeconomic stability. Policies to encourage renegotiation thus would provide some of the benefits of recourse mortgages loans, if such are not feasible.
Appendix A: Proofs of Propositions

The appendices make use of some simplifying notation. Define

\[ \phi(d) = \frac{(1+r)p_1}{1-d} - \lambda p_2, \]  
(A1)

representing the loss suffered by a screening bank from issuing a mortgage certain to be foreclosed on. The bank loans out \((1+r)p_1\), scaled up by the monitoring cost needed to safeguard the deposits used to fund the loan, receiving foreclosure proceeds \(\lambda p_2\) in period 2. By (3),

\[ \phi(d) > 0 \text{ for all } d \in [0,1]. \]  
(A2)

Further,

\[ \phi'(d) = \frac{(1+r)p_1}{(1-d)^2} < 0. \]  
(A3)

Equation (A1) can also be used to represent the loss suffered by a broker from issuing a mortgage certain to be foreclosed on: \(\phi(0) = (1+r)p_1 - \lambda p_2\). We have \(0 < \phi(0) < \phi(d)\), where the first inequality follows from (3) and the second from (A3). Using this new notation, screening bank profit from (5) can be written more compactly as

\[ \gamma [h(R_i - \lambda p_2) - \phi(d)] - k_i \]  
(A4)

and its associated zero-profit repayment from (6) as

\[ R_{SBK} = \lambda p_2 + \frac{1}{h} \left[ \phi(d) + \frac{1}{\gamma} \right], \]  
(A5)

The profit of the combination of broker and investors in its securitized mortgages from (7) can be written more compactly as

\[ \gamma h(R_i - \lambda p_2) - \phi(0) \]  
(A6)

where \(F_i\) is ignored here because it is a transfer between the parties whose combined profit we are considering. The associated zero-profit repayment from (8) can be written more compactly as

\[ R_{NBR} = \lambda p_2 + \frac{\phi(0)}{\gamma h}. \]  
(A7)

Proof of Proposition 1

The proof is divided into a number of subsections. For each case of broking, banking, and no contract, the one subsection establishes existence and the following one establishes uniqueness. The proof concludes with an analysis of the borders between cases.

Existence in Case (i). Maintain the following conditions behind case (i):

\[ R_{NBR} < R_{SBK} \]  
(A8)

\[ R_{NBR} < m. \]  
(A9)

Posit the following equilibrium outcome. Two brokers are the only active intermediaries, offering contracts specifying repayment \(R_{NBR}\), not screening, and charging no origination fee. The good type chooses the contract with the lower repayment. Along the equilibrium path, the bad type applies to the broker with the lowest repayment if any are active and, if not, then to the bank with the lowest repayment. Off the equilibrium path, the bad type applies to an intermediary offering a non-equilibrium repayment. We will show that no player strictly gains from deviating.

Consider deviations by a consumer. The bad type’s equilibrium payoff is \(u_1\)—he occupies the house in period 1 after which his house is foreclosed—greater than his payoff (zero) from rejecting the contract. The good type’s expected equilibrium payoff is

\[ u_1 + h(y + u_2 - R_{NBR}). \]

This is positive because \(R_{NBR} < m \leq y\), where the first inequality holds by (A9) and the second from the definition \(m \equiv \min \{y, p_2, u_2\}\). Thus the good type prefers not to reject.

Next, consider deviations by intermediaries. Inactive intermediaries earn zero profit. Active brokers earn zero profit because they charge no origination fee. Investors in the securitized mortgages originated by active brokers earn zero profit from contract by construction of \(R_{NBR}\). This can be verified by substituting for \(R_{NBR}\) from (8) into (7). It remains to show that there is no deviation by an intermediary that will generate positive profit. If an inactive broker enters and matches the active brokers’ repayment \(R_{NBR}\), it (and its investors if any) earn the same (zero) profit as the original active lenders.

Suppose an intermediary (active or not) deviates to repayment \(R_i > R_{NBR}\). Whether the intermediary screens or not, this deviation will not attract good types and so will not generate positive profit.

Deviation by a broker to repayment \(R_i < R_{NBR}\) will attract all consumers, generating expected profit for the combination of the broker and investors

\[ \gamma h(R_i - \lambda p_2) - \phi(0) \]  
\[ < \gamma h(R_{NBR} - \lambda p_2) - \phi(0) \]  
\[ = 0. \]

The first line follows from (A6). The second line holds because \(R_i < R_{NBR}\) and profit is increasing in the repayment. The third line follows from the construction of \(R_{NBR}\), as can be seen by substituting from (A7). Hence the deviation is unprofitable for at least one party in the broker-investor combination.

Deviation by a bank to repayment \(R_i < R_{NBR}\) will attract all consumers, generating expected profit

\[ \gamma h(R_i - \lambda p_2) - \phi(d) \]  
\[ < \gamma h(R_{NBR} - \lambda p_2) - \phi(d) \]  
\[ < k_i \]  
\[ = 0. \]

The first line follows from (A4). To see the second line, note \(R_i < R_{NBR} < R_{SBK}\) by (A8). The line then follows because profit is increasing in the repayment. The third line follows from the construction of \(R_{SBK}\), as can be seen by substituting from (A5). Hence the deviation is unprofitable for at least one party in the broker-investor combination.

The preceding arguments also hold if the inequality in (??) is weak. Q.E.D.

Uniqueness in Case (i). Maintain conditions (A8) and (A9) behind case (i). We first show that there must be lending in equilibrium, and the lowest posted repayment must satisfy \(R_i \leq R_{NBR}\). Suppose that either \(R_i > R_{NBR}\) or there is no lending. This outcome cannot be stable because a broker could enter by posting a contract with repayment \(R_j \in (R_{NBR}, R_i)\) and earn profit

\[ \gamma h(R_j - \lambda p_2) - \phi(0) \]  
\[ > \gamma h(R_{NBR} - \lambda p_2) - \phi(0) \]  
\[ = 0, \]

a strictly profitable deviation.
We next argue that active intermediaries cannot offer different repayment levels. If they did offer different repayment levels, intermediaries offering other than the lowest repayment level would attract no good types. The intermediary would either be inactive or serve only bad types. The profit from serving a bad type is $-\phi(0)$ for a broker and $-\phi(d)$ for a bank, negative in either case.

We next show that screening banks cannot operate in equilibrium. Consider an outcome in which measure $\gamma'$ of good types and $\beta'$ of bad types are served by banks. Because borrowers obtain positive surplus, if any are served in equilibrium, all must be served. Letting $\gamma''$ be the measure of good types and $\beta''$ of bad types served by brokers, the fact that all borrowers are served implies

$$\gamma' + \gamma'' = \gamma, \quad \beta' + \beta'' = \beta.$$  

(A10) (A11)

This specification allows for the possibility that brokers do not operate: $\gamma'' = \beta'' = 0$.

For this outcome to be stable, the combination of brokers and their investors together cannot earn negative profit:

$$0 \leq \gamma''[hR_i + (1-h)\lambda_p - (1+r)p_1] + \beta''[\lambda_p - (1+r)p_1] \quad (A12)$$

$$= \gamma''[hR_i - \lambda_p - (\gamma'' + \beta'')\phi(0)] \quad (A13)$$

$$\leq \gamma''[h(R_{NBR} - \lambda_p) - (\gamma'' + \beta'')\phi(0)] \quad (A14)$$

$$= \left[\frac{\gamma''}{\gamma} - (\gamma'' + \beta'')\right] \phi(0). \quad (A15)$$

To see (A12), note the measure $\gamma''$ of good types repay $R_i$ with probability $h$ and are foreclosed on otherwise. The cost of the securitized loan to them is $(1+r)p_1$. The measure $\beta''$ of bad types never repay, so the net return from them is $\lambda_p - (1+r)p_1$. Equation (A13) follows from substituting from (A1) for $d$. Condition (A14) follows because $R_i \leq R_{NBR}$. Equation (A15) follows from substituting from (A7) and rearranging. Conditions (A12)–(A15) together imply $\gamma''/\gamma \geq \gamma'' + \beta''$, implying

$$\frac{\gamma'}{\gamma} \leq \gamma' + \beta'$$

(A16)

using (A10) and (A11).

Total profit across screening banks is

$$\gamma' \left[ hR_i + (1-h)\lambda_p - \frac{(1+r)p_1}{1-d} - k_1 \right] - \beta'k_1 \quad (A17)$$

$$= \gamma'[hR_i - \lambda_p - \phi(d)] - (\gamma' + \beta')k_1 \quad (A18)$$

$$< \gamma'[h(R_{SBK} - \lambda_p) - \phi(d)] - (\gamma' + \beta')k_1 \quad (A19)$$

$$= \left[\frac{\gamma'}{\gamma} - (\gamma' + \beta')\right] k_1 \quad (A20)$$

$$\leq 0. \quad (A21)$$

To see (A17), note the measure $\gamma'$ of good types repay $R_i$ with probability $h$ and are foreclosed on otherwise. The cost of a loan to them financed with deposits, requiring a monitoring cost to raise, is $(1+r)p_1/(1-d)$. Since the bank screens all borrowers, the good type needs to be screened at cost $k_1$. The measure $\beta'$ of bad types are screened at cost $k_1$ but not given loans. Equation (A18) follows from substituting from (A1). Condition (A19) follows because $R_i \leq R_{NBR} < R_{SBK}$, where the last inequality holds by (A8). Equation (A20) follows from substituting from (A5) and rearranging. Condition (A21) follows from (A16). This shows that screening banks earn negative profit if brokers do not, proving banks cannot be active in equilibrium unless the posited conditions.

We have ruled out all possibilities except active brokers offering mortgages at repayment $R_{NBR}$, establishing uniqueness. Q.E.D.

Existence and Uniqueness in Case (ii). The arguments are similar to those for case (i) and are omitted for brevity.

Existence in Case (iii). Maintain the following condition behind case (i):

$$m < \min\{R_{SBK}, R_{NBR}\}.$$  

(A22)

Posit the equilibrium outcome that no lenders offer mortgages. All borrowers apply to any entering intermediary. Given that all lenders are inactive, they earn zero profit. To establish existence, we need to check that there are no lending opportunities providing positive profits.

Suppose an intermediary deviates by entering, offering a contract with repayment $R_i$. If $R_i > m$, as argued in Section the borrower will not make the repayment, either because the borrower cannot afford the payment $(R_i > \gamma)$, prefers to keep the money over the utility from period-2 housing services $(R_i > r_2)$, or strategically defaults, using the money to buy a different house $(R_i > p_2)$. The combination of intermediary and investors earns $-\phi(d)$ (if the loan is financed with deposits) or $-\phi(0)$ (if the loan is financed through securitization), negative in either case.

Suppose instead $R_i \leq m$. If the deviating intermediary is a screening bank, it earns

$$\gamma'h(R_i - \lambda_p - \phi(d)) - k_1$$

$$\leq \gamma'[h(m - \lambda_p) - \phi(d)] - k_1$$

$$< \gamma'[h(R_{SBK} - \lambda_p) - \phi(d)] - k_1 = 0.$$  

If the deviating intermediary is a screening bank, it earns

$$\gamma'h(R_i - \lambda_p - \phi(d)) - k_1$$

$$\leq \gamma'[h(m - \lambda_p) - \phi(0)]$$

$$< \gamma'[h(R_{SBK} - \lambda_p) - \phi(0)]$$

$$= 0.$$  

In each set of calculations, the first inequality holds by $R_i \leq m$, the second by (A22), and the last by construction of, respectively, $R_{SBK}$ and $R_{NBR}$. This rules out deviations by intermediaries. Q.E.D.

Uniqueness in Case (iii). Maintain condition (A22). We will show there cannot be lending in equilibrium. Consider an outcome with some active intermediaries. As argued in the proof of uniqueness in case (i), all active intermediaries must post the same repayment, say $R_i$. This repayment must be feasible: $R_i \leq m$. We will show that if the combination of brokers and their investors do not earn negative profit, there must be active banks who do earn negative profit, ruling the outcome out as an equilibrium.

As in the proof of uniqueness in case (i), consider an outcome in which measure $\gamma'$ of good types and $\beta'$ of bad types are served by banks and in which measure $\gamma''$ of good types and $\beta''$ of bad types are served by brokers. This specification allows for the possibility that one form of intermediary does not operate, allowing for either $\gamma' = \beta' = 0$ or $\gamma'' = \beta'' = 0$. As we argued, all borrowers are served implies (A10) and (A11) must hold. We can repeat the steps in (A12)–(A15). The only modification to the argument is that to establish (A14), one must notice that $R_i \leq m < R_{NBR}$, where the first inequality follows from feasibility the second from (A22). Conditions (A12)–(A15) prove (A16). But then repeating steps (A17)–(A21) show that a bank must be operating that earns negative profit, ruling out the outcome as an equilibrium. Again,
the slight modification in the argument needed to establish (A19) is that \( R_{r} \leq m < R_{\text{SBK}} \), where the first inequality follows from feasibility and the second from (A22). Q.E.D.

**Boundary Cases.** The remaining cases lie on the boundary between the regions in Figure 3. Consider the boundary between the banking and no-contract regions. We can repeat steps (A12)–(A21) to show that there cannot be bank lending alone or alongside broker lending on this boundary. The reason these arguments still apply to this boundary case is that (A19) continues to be strict there because \( R_{\text{SBK}} > R_{\text{NBR}} \) there. There only exist broking and no-contract equilibria on this boundary.

Similar arguments can establish that there cannot be active brokers on the boundary between the banking and no-contract regions. There only exist banking and no-contract equilibrium on this boundary. Q.E.D.

**Proof of Propositions 4 and 5**

These propositions are corollaries of a richer result, Theorem 1. We will state and prove Theorem 1 before returning to prove its corollaries.

**Theorem 1.** Enumerated are comparative-static exercises changing a single parameter, holding all other parameters constant. The thresholds bounding the intervals are functions of the other parameters, but these arguments are suppressed for brevity. All intervals can be empty unless otherwise noted. Parameter \( u_1 \) has no comparative-statics effects.

(i) Hold constant all parameters except \( y \). There exists threshold \( \gamma \) such that the equilibrium involves no contract for all \( y < \gamma \) and lending for all \( y > \gamma \). If there is banking in equilibrium for some \( y > \gamma \), there is a banking equilibrium for all \( y \in (\gamma, \infty) \). Conversely, if there is broking in equilibrium for some \( y > \gamma \), there is a broking equilibrium for all \( y > \gamma \). The no-contract interval \( (0, \gamma) \) is guaranteed to be non-empty. The preceding statements hold analogously for parameters \( u_2 \) and \( h \).

(ii) Hold constant all parameters except \( p_1 \). There exists thresholds \( p_1^1 \) and \( p_1^2 \geq p_1^1 \) such that the equilibrium involves broking for \( p_1 < p_1^1 \), banking for \( p_1 \in (p_1^1, p_1^2) \), and no contract for \( p_1 > p_1^2 \). The no-contract interval \( (p_1^2, \infty) \) is guaranteed to be non-empty. The preceding statements also hold analogously for parameter \( r \).

(iii) Hold constant all parameters except \( r \). There exists thresholds \( \gamma^1 \) and \( \gamma^2 \geq \gamma^1 \) such that the equilibrium involves no contract for all \( r > \gamma^1 \), banking for \( r \in (\gamma^1, \gamma^2) \), and broking for \( r > \gamma^2 \). The no-contract interval \( (0, \gamma^1) \) is guaranteed to be non-empty. The preceding statements also hold analogously for parameter \( p_2 \). The preceding statements also hold for \( \lambda \) with the exception that the no-contract interval \( [0, \lambda] \) can be empty.

(iv) Hold constant all parameters except \( k_1 \). There exists threshold \( k_1^* \) such that the equilibrium involves banking for all \( k_1 < k_1^* \). If there is broking in equilibrium for some \( k_1 > k_1^* \), there is a broking equilibrium for all \( k_1 > k_1^* \). Conversely, if there is no contract in equilibrium for some \( k_1 > k_1^* \), there is a no-contract equilibrium for all \( k_1 > k_1^* \). The interval \( (k_1^*, \infty) \) in which there cannot be banking is guaranteed to be non-empty. The preceding statements hold analogously for parameter \( d \).

**Proof of Theorem 1**

We prove the theorem for case (i) and (ii) in turn. For each case, we provide a proof for one representative parameter. The proofs for the other parameters in these two cases, as well as the proofs for all parameters in cases (iii) and (iv), are similar and thus omitted.

**Case (i).** We will prove the claims for \( y \). Hold all the other parameters constant. We first show that there exists some income level \( y^* > 0 \) such that there is no lending in equilibrium. Because \( \phi(d) > 0 \) by (A2), \( R_{\text{NBR}} > 0 \) by (A5). Similarly, because \( \phi(0) > 0 \) by (A2), \( R_{\text{SBK}} > 0 \) by (A7). Thus there exists \( y^* \) such that \( 0 < y^* < \min\{R_{\text{NBR}}, R_{\text{SBK}}\} \), implying

\[
\min\{y^*, p_2, u_2\} \leq y^* \quad \text{(A23)}
\]

and

\[
< \min\{R_{\text{NBR}}, R_{\text{SBK}}\} \quad \text{(A24)}
\]

Hence, the condition for case (iii) of Proposition 1 is satisfied for income level \( y^* \).

It is easy to see that conditions (A23) and (A24) hold for all \( y \in [0, y^*) \) as well, so there can only be a no-contract equilibrium for all \( y \in [0, y^*) \).

Suppose there is some \( y' \) such that there exists a banking equilibrium at this income level. Then by Proposition 1,

\[
R_{\text{SBK}} < \min\{R_{\text{NBR}}, y', p_2, u_2\}.
\]

Because \( R_{\text{SBK}} \) and \( R_{\text{NBR}} \) are independent of \( y \) as inspection of (6) and (8), respectively, shows, (A25) is relaxed if \( y > y^* \) is substituted for \( y' \). Thus there exists a banking equilibrium for all \( y > y^* \). Similar arguments can be used to show that if there exists \( y' \) such that there exists a broking equilibrium at this income level, then there is also a broking equilibrium for all \( y > y^* \).

**Case (ii).** We will prove the statement for \( p_1 \). Hold all the other parameters constant. Write the zero-profit broking repayment as \( R_{\text{NBR}}(p_1) \) to emphasize its dependence on \( p_1 \). Inspection of (8) shows that \( \lim_{p_1 \to \infty} R_{\text{NBR}} = \infty \). Hence there exists \( p_1^* \) such that \( R_{\text{NBR}}(p_1^*) > \min\{y, p_2, u_2\} \). For this value of the first-period house price, the condition for a broking equilibrium in case (i) of Proposition 1 is violated even if treated as a weak inequality. Hence there is no broking in equilibrium. Similar analysis applies to banking. Thus there is no equilibrium with lending if the first-period house price is \( p_1^* \).

Next, suppose there is no lending in equilibrium for first-period house price \( p_1^* \). We will show there is no lending in equilibrium for all \( p_1 > p_1^* \). We have \( R_{\text{NBR}}(p_1) > R_{\text{NBR}}(p_1^*) > m \). The first inequality follows from

\[
\frac{dR_{\text{NBR}}}{dp_1} = \frac{1+r}{\gamma h} > 0.
\]

The second inequality follows because there is no lending in equilibrium, so no broking, so the condition in case (i) of Proposition 1 must be violated even as a weak inequality. But \( R_{\text{NBR}}(p_1) > m \) implies by Proposition 1 that there is no broking equilibrium when first-period house price is \( p_1 \). Similar analysis applies to banking. Thus there is no equilibrium with lending for all \( p_1 > p_1^* \).

Next, suppose there is an equilibrium with broking for first-period house price \( p_1^* \). We will show that there is an equilibrium with broking for all \( p_1 < p_1^* \). We have \( m \geq R_{\text{NBR}}(p_1^*) > R_{\text{NBR}}(p_1) \). The first inequality follows from case (i) of Proposition 1 and the fact that there is a broking equilibrium for \( p_1^* \). The second inequality follows from \( dR_{\text{NBR}}/dp_1 > 0 \). By case (i) of Proposition 1,
there exists a broking equilibrium for first-period house price $p_1$. Q.E.D.

**Completing Proof of Proposition 4**

We have thus proved Theorem 1. To complete the proof of Proposition 4, the reader can check parameter by parameter that if the broking interval borders a no-contract interval, there is no banking in equilibrium for any value of the parameter. It remains to demonstrate a case in which banking appears in an intermediate interval between no-contract and broking intervals. We will do this by fixing all the parameters except for $p_1$ and consider how the mode of lending changes as $p_1$ is increased starting from a lower bound of $\lambda p_2/(1+r)$; below this bound, (3) is violated.

Assume $p_2 < \min\{y,u_2\}$, implying $m = p_2$. Further assume $d + \gamma < 1$. Define

$$p_1' = \frac{1 - d}{(1 - d - \gamma)(1 + r)} [k_1 + (1 - \gamma) \lambda p_2]$$

$$p_1'' = \frac{1 - d}{1 + r} \left[ 1 + (1 - \lambda) h p_2 - \frac{k_1}{\gamma} \right].$$

Direct calculation verifies that $p_1'$ satisfies equation (12). Hence $R^{NBR} = R^{SBK}$ at $p_1'$. Substituting $p_1''$ into (6) shows that $R^{SBK}$, when evaluated at $p_1''$, equals $p_2$.

We have our case. For $p_1 < p_1'$, we have $R^{NBR} < R^{SBK}$ and $R^{NBR} < p_2 = m$. By case (i) of Proposition 1, the equilibrium involves banking. For $p_1 \in (p_1', p_1'')$, we have $R^{SBK} < R^{NBR}$ and $R^{SBK} < m$. By case (ii) of Proposition 1, the equilibrium involves broking. For $p_1 > p_1''$, $m < \min\{R^{NBR}, R^{SBK}\}$. By case (iii) of Proposition 1, there is no lending in equilibrium. Q.E.D.

**Completing Proof of Proposition 5**

We will analyze the comparative-statics effects of the credit-worthiness parameters $h, \gamma$, and $y$ on mortgage volume and default probability in turn.

**Mortgage Volume.** Consider an increase in $y$ holding all other parameters constant. Case (i) of Theorem 1 states that this parameter change moves the equilibrium from a no-contract to a banking to a broking interval. One or more of these intervals can be empty. The probability that the consumer receives a mortgage equals $0$ in the no-contract interval, $\gamma$ in the banking interval (because all good types and only good types are served), and $1$ in the broking interval (because all consumers are served). Thus mortgage volume is non-decreasing in $y$.

The analysis is similar for parameters $h$ and $\gamma$, except case (iii) of Theorem 1 is the relevant result.

**Default Probability.** Consider an increase in $y$ holding all other parameters constant. As discussed in the previous subsection, this parameter change moves the equilibrium from a no-contract to a banking to a broking interval. The probability of default conditional on a mortgage being signed is undefined in the no-contract interval. The conditional default probability is $1-h$ in the banking interval, which is the probability a good type earns low income. This is constant, and so non-increasing, over the banking interval. The conditional probability the consumer does not default equals $\gamma h$, the probability the consumer is the good type and earns high income. Thus the conditional probability of default is $1-\gamma h$. This is a constant, and so non-increasing, over the broking interval. The conditional default probability jumps from $1-h$ to $1-\gamma h$ at the boundary between banking and broking.

The analysis is similar for parameters $h$ and $\gamma$, except case (iii) of Theorem 1 is the relevant result. Q.E.D.

**Proof of Proposition 7**

Maintain the parameter restrictions stated in the proposition: $y, u_2 \rightarrow \infty$, $p_2 \rightarrow 0$, $R^{NBR}, R^{SBK} < p_2$, and equation (16). We will first show that bank lending is feasible with stochastic prices. As argued in the text, bank lending is feasible as long as the repayment does not induce strategic default in the high-income, high-price state: $R^{SBK} < \hat{p}_2$. We have

$$\psi R^{SBK} + (1 - \psi) \lambda p_2 = R^{SBK} < p_2$$

$$= \psi \hat{p}_2 + (1 - \psi) p_2,$$

where the first line follows from (19), the second line follows from the maintained assumption $R^{SBK} < p_2$, and the last line follows from (16). Rearranging,

$$\hat{R}^{SBK} < \hat{p}_2 + \left( \frac{1 - \psi}{\psi} \right) (1 - \lambda) p_2.$$

But by maintained assumption, $p_2 \rightarrow 0$, implying $\hat{R}^{SBK} < \hat{p}_2$.

Similar arguments show $\hat{R}^{NBR} < \hat{p}_2$, showing that broker mortgages are also feasible in the stochastic-price model under the maintained assumptions.

The last step in the proof is to compute the expected welfare loss in moving from deterministic to stochastic prices. Let $W^{SBK}$ be expected welfare with a bank loan and $W^{NBR}$ with a broker loan in the stochastic-price model. Because lenders and investors break even, welfare is given by the consumer’s utility. With a bank mortgage, $W^{SBK}$ equals

$$\gamma\{u_1 + h[\psi(u_2 + y - \hat{R}^{SBK}) + (1 - \psi)(u_2 + y - p_2)]\}$$

$$= W^{SBK} - \gamma h(1 - \psi)(1 - \lambda) p_2.$$

The equality follows from substituting for $\hat{R}^{SBK}$ from (19) and using the expression for $W^{SBK}$ from (13). Thus

$$W^{SBK} - \hat{W}^{SBK} = \gamma h(1 - \psi)(1 - \lambda) p_2.$$

Similarly, one can show

$$W^{NBR} - \hat{W}^{NBR} = \gamma h(1 - \psi)(1 - \lambda) p_2.$$

Q.E.D.

**Proof of Proposition 8**

Maintain the parameter restrictions stated in the proposition: $y, u_2 \rightarrow \infty$, $p_2 \rightarrow 0$, $R^{NBR}, R^{SBK} < p_2$, and equation (16). We will show that the good type obtains higher expected utility from the zero-profit banking than broking contract if (23) holds. The good type’s expected utility from a zero-profit banking contract is

$$u_1 + h [u_2 + y - \psi R^{NBR} - (1 - \psi) p_2]$$

(A26)

and from the zero-profit broking contract is

$$u_1 + h [u_2 + y - \psi \hat{R}^{SBK} - (1 - \psi) p_2].$$

(A27)

Expression (A26) exceeds (A27) if $\hat{R}^{SBK} < R^{NBR}$. Using (22) to substitute for $\hat{R}^{SBK}$, using (20) to substitute for $R^{NBR}$, and rearranging yields (23). Q.E.D.
Proof of Proposition 10

Fix any values of $\gamma \in (0, 1), h \in (0, 1), u_1 > 0$, and $p_2 > 0$. We will look for values of the other parameters and a function $\lambda(\delta)$ that together maximize $W^{SBK} - W^{NBR}$ subject to the constraint that both banking and broking equilibria exist in the case and $\lambda(\delta)$ is an admissible foreclosure friction function: i.e., that $\lambda(\delta)$ is a decreasing function on $[0, 1]$. One simplification is that the shape of $\lambda(\delta)$ over its range is immaterial except for the values it takes on at the equilibrium default rates, $\lambda^{NBR}$ and $\lambda^{SBK}$. Thus we will treat those as two additional variables in the maximization problem. For $\lambda(\delta)$ to be an admissible foreclosure function, $0 < \lambda^{SBK} < \lambda^{NBR} < 1$. We will treat these strict inequalities as equalities because we are looking for a supremum on the welfare difference and do not require that it will be reached by some parameter vector.

Using (25), the problem is to choose non-negative values of $\lambda^{NBR}, \lambda^{SBK}, d, y, r, u_2, p_1, \lambda$, and $k_\ell$ to maximize

$$
\gamma h [R^{NBR}(\lambda^{NBR}) - R^{SBK}(\lambda^{SBK})] - (1 - \gamma) u_1
$$

subject to

$$
\lambda^{NBR} \leq \lambda^{SBK}
$$

$$
d \leq 1
$$

$$
\max\{\lambda^{SBK}, \lambda^{NBR}\} p_2 \leq (1 + r)p_1
$$

$$
R^{SBK}(\lambda^{SBK}) \leq R^{NBR}(\lambda^{NBR})
$$

$$
R^{NBR}(\lambda^{NBR}) \leq R^{SBK}(\lambda^{SBK})
$$

$$
\max\{R^{SBK}(\lambda^{SBK}), R^{NBR}(\lambda^{NBR})\} \leq \gamma
$$

Constraint (A29) and (A30) were mentioned in the previous paragraph as ensuring the foreclosure function is admissible. Constraint (A31) ensures $d$ is a fraction as required. Constraint (A32) maintains the house-price assumption (3) for both $\lambda^{SBK}$ and $\lambda^{NBR}$. The final three constraints ensure that the conditions behind case (i) and (ii) of Proposition 9 are met so that the parameters are in the multiple-equilibrium region.

Several steps can help simplify the maximization problem. Given (A29) holds, (A32) reduces to $\lambda^{SBK} p_2 \leq (1 + r)p_1$. Given (A33) holds, (A35) reduces to $R^{NBR}(\lambda^{NBR}) \leq \gamma$. Constraints (A29), (A30), and (A33) can be ignored; we will show later that the solution satisfies them. The optimal values of $y$ and $u_2$ are easy to characterize. The variables only appear in (A35) through the dependence of $m$ on them. Increasing $y$ and $u_2$ weakly relaxes this constraint. Any values $y, u_2 \geq p_2$ are thus optimal. Thus $m = p_2$ at an optimum. Finally, because certain parameters occur in the problem in certain configurations, several changes of variable simplify the problem. Let

$$
\hat{p}_1 = (1 + r)p_1
$$

$$
\hat{k}_1 = k_\ell + \frac{(1 + r)p_1}{1 - d}.
$$

Substituting $\lambda^{SBK}$ into (6) to find $R^{SBK}(\lambda^{SBK})$, substituting $\lambda^{NBR}$ into (8) to find $R^{NBR}(\lambda^{NBR})$, substituting the resulting expressions for $R^{SBK}(\lambda^{SBK})$ and $R^{NBR}(\lambda^{NBR})$ into the previous maximization problem and making the changes indicated in the previous paragraph, the problem reduces to one of choosing $\lambda^{NBR}$, $\lambda^{SBK}$, $\hat{p}_1$, and $\hat{k}_1$ to maximize

$$
\gamma h \left[ \lambda^{NBR} p_2 + \frac{1}{\gamma h} (\hat{p}_1 - \lambda^{NBR} p_2) \right] - \lambda^{SBK} p_2 - \frac{1}{\gamma h} (\hat{k}_1 - \lambda^{SBK} p_2) - (1 - \gamma) u_1
$$

subject to

$$
\lambda^{SBK} p_2 \leq \hat{p}_1
$$

$$
\lambda^{NBR} p_2 + \frac{1}{\gamma h} (\hat{p}_1 - \lambda^{NBR} p_2) \leq \lambda^{NBR} p_2 + \frac{1}{h} (\hat{k}_1 - \lambda^{NBR} p_2)
$$

$$
\lambda^{NBR} p_2 + \frac{1}{\gamma h} (\hat{p}_1 - \lambda^{NBR} p_2) \leq p_2.
$$

The objective function (A36) is decreasing in $\hat{k}_1$. The only other place this variable appears is in constraint (A37), which obviously binds at an optimum. Treating it as an equality and rearranging,

$$
\hat{k}_1 = \frac{1}{\gamma} [\hat{p}_1 - (1 - \gamma) \lambda^{NBR} p_2].
$$

The objective function (A36) is increasing in $\lambda^{SBK}$. The only other place this variable appears is in constraint (A37), which obviously binds at an optimum, implying $\lambda^{SBK} = p_1 / p_2$.

Substituting this value of $\lambda^{SBK}$ and the value of $\hat{k}_1$ from (A40) into (A36) and rearranging, the problem reduces to one of choosing $\lambda^{NBR}$ and $\hat{p}_1$ to maximize

$$
\gamma (1 - h) (\hat{p}_1 - \lambda^{NBR} p_2) - (1 - \gamma) u_1
$$

subject to

$$
\lambda^{NBR} p_2 + \frac{1}{\gamma h} (\hat{p}_1 - \lambda^{NBR} p_2) \leq p_2.
$$

The objective function is increasing in $\hat{p}_1$. An increase in $\hat{p}_1$ tightens (A42), implying that this constraint binds at an optimum.

Solving (A42) as an equality for $\hat{p}_1$ and substituting this value of $\hat{p}_1$ into (A41), we are left with the unconstrained problem of choosing $\lambda^{NBR} \geq 0$ to maximize

$$
\gamma^2 h (1 - h) p_2 (1 - \lambda^{NBR}) - (1 - \gamma) u_1.
$$

The solution is $\lambda^{NBR} = 0$, which upon substituting into (A43), gives the bound in (26).

We have solved for the maximum welfare loss from securitization with weak inequality constraints. If any of the constraints are strict inequalities, the bound is a supremum that can be approached but perhaps not attained. Q.E.D.

Proof of Proposition 11

The arguments above the statement of Proposition 11 provide most of the proof. The only claim that needs to be checked is that suppliers and buyers strictly prefer to transact in period 1 than wait until period 2. First we will verify the supplier side. The revenue from selling in period is $p^1_{\text{MAX}}(\gamma^*) \geq p^1_{\text{NBR}}(\gamma^*) > \lambda m / (1 + r)$, where the second condition follows from (28). The present value of revenue from waiting to sell until period 2 is $\lambda p^2_{\text{NBR}} / (1 + r) = \lambda m / (1 + r)$, where the $\lambda$ factor reflects the assumed loss of value from leaving the house vacant for the period. Thus the present value from selling in period 1 exceeds that from selling in period 2.

Next we verify that consumers prefer to purchase in period 1 if they can obtain financing to do so. Of course bad types prefer to do so because they have no chance of buying a house in period 2 because they have no income then. To show that good types strictly prefer to purchase in period 1 requires more work. By equation (9), the good type’s expected payoff from buying in period 1 financed by a mortgage requiring repayment $R_1$ is $u_1 + h(u_2 + y - R_1)$. His expected payoff from waiting is $h(u_2 + y - p^2_{\text{NBR}}) = h(u_2 + y - m)$. His expected payoff from buying strictly exceeds that from waiting because $u_1 > 0$ and $R_1 \leq \min\{y, u_2, p^2_{\text{NBR}}\} = m$ for the repayment to be feasible. Q.E.D.
Proof of Proposition 12

Maintain the assumption $H < Nh_\mu_\gamma$ throughout the proof. We will argue that this assumption can be maintained without loss of generality. If $H > Nh_\mu_\gamma$, then $p_1^* = p_2^* = 0$ by Proposition 11. Changes in any of the parameters in the statement of Proposition 12 do not change the inequality $H > Nh_\mu_\gamma$, and so do not change the zero prices. We can trivially say that the zero prices are financed by brokers and so the parameter changes do not reduce the amount of broking. A reduction in $m$ makes the inequality $H > Nh_\mu_\gamma$ more likely to hold and again does not reduce the amount of broking if we say that zero prices are financed by broking. Thus we can maintain the assumption $H < Nh_\mu_\gamma$ without loss of generality.

Suppose $H < Nh_\mu_\gamma$ and suppose that the parameters are such that the equilibrium involves broking for consumer type $\gamma$. Then $p_1^{SBK}(\gamma) > p_1^{NBR}(\gamma)$. Consider an increase in $k_i$. This reduces $p_1^{SBK}(\gamma)$ by (27) and leaves $p_1^{NBR}(\gamma)$ unchanged by (28). Thus the equilibrium continues to involve broking. An increase in $d$ has the same effect. To understand the effect of a reduction in $y$ or $u$, which appear in (27) and (28) only through $m$, normalize both equations by dividing by $m$. The parameter disappears from both prices except where it appears as a divisor of the $k_i$ term in (27).

A reduction in $m$ increases the magnitude of this term, reducing $p_1^{SBK}(\gamma)$, making broking more likely.

Before turning to the comparative-statics results for $h$ and $\lambda$, we show that $1 - d \leq \gamma$ implies $p_1^{SBK}(\gamma) < p_1^{NBR}(\gamma)$. Supposing $1 - d \leq \gamma$,

$$p_1^{SBK}(\gamma) = \frac{1-d}{1+r} \left[ \frac{hm + (1-h)\lambda m - k_j}{\gamma} \right]$$

$$\leq \left[ \frac{1-d}{1+r} \right] m \left[ h + (1-h)\lambda \right]$$

$$\leq \left[ \frac{1-d}{1+r} \right] \frac{m}{1+r} \left[ h + (1-h)\lambda \right]$$

$$\leq \frac{m}{1+r} \left[ h(1-\gamma)\lambda \right]$$

$$= p_1^{NBR}(\gamma).$$

The first line follows from (27), the second from $k_i > 0$, the third from $1 - d \leq \gamma$, the fourth from algebra, and the last from (28).

Differentiating (27) and (28) shows that a decrease in $h$ decreases $p_1^{SBK}(\gamma)$ by $(1-d)(1-\lambda)/1+r$ and $p_1^{NBR}(\gamma)$ by $\gamma(1-\lambda)/(1+r)$. The decrease in $p_1^{SBK}(\gamma)$ is greater unless $1-d \leq \gamma$. But arguments from the previous paragraph show that the equilibrium would involve broking for all $h$ if $1-d \leq \gamma$. Thus a decrease in $h$ weakly increases broking. Similar arguments can be used to show that an increase in $\lambda$ weakly increases broking. Q.E.D.

Appendix B: Generalizing Bad Type’s Application Behavior

The analysis up to this point narrowed the set of perfect Bayesian equilibria by adopting a particular restriction on the behavior of the bad type vis-à-vis screening banks. To understand the restriction more formally, let $a$ be the probability that a bad type applies to a screening bank if it is indifferent between applying and not. Note that if screening banks are the only active intermediaries, the bad type would indeed be indifferent between applying and not because he is always rejected if he applies. The text imposes the exogenous restriction $a = 1$. This appendix removes the exogenous restriction, allowing for any $a \in [0, 1]$.

We will show that the restriction to $a = 1$ in the text does not substantially impair the generality of the analysis. Under the restriction $a = 1$, equilibrium was characterized by Figure 3. Removing the restriction changes the characterization to that given in Appendix Figure 1. This is the same as Figure 3 except that there is a new region (the hatched triangle) in which there are multiple equilibria, some of which involve bank lending, whereas formerly no contract was possible for those parameters. There is no change to the broking region. Thus our central results on when securitization arises are robust to generalizing the bad type’s behavior.

Broker Region (i) Unchanged Across Figures. To prove that the broker region is the same in Appendix Figure 1 as it was in Figure 3, we will first show that there can be no banking in equilibrium in the broker region even after removing the restriction $a = 1$.

Consider an outcome in which a screening bank offers a mortgage involving repayment $R_i$ and is applied to with probability $a \in [0, 1]$ if the consumer is the bad type. If bank $i$ screens, its expected profit is

$$\gamma \left[ hR_i + (1-h)\lambda p_2 - \frac{(1+r)p_1}{1-d} - k_j \right] - (1-\gamma)a k_i. \quad (B1)$$

This is the same expression for expected profit as found in (5) except that with probability $(1-\gamma)(1-a)$, the bank saves on screening costs because the bad type does not bother to apply. Using the definition of $\lambda$ and rearranging, (B1) can be written more succinctly as

$$\gamma [h(R_i - \lambda p_2) - \phi(d)] - \gamma [1-\gamma]a k_i. \quad (B2)$$

If bank $i$ deviates from screening, its expected profit can be as high as

$$\gamma [hR_i + (1-h)\lambda p_2 - (1+r)p_1 + (1-\gamma)a \lambda p_2 - (1+r)p_1]. \quad (B3)$$

The bank saves on screening costs but must serve the bad types that apply with probability $(1-\gamma)a$. Implicit in (B3) is that the bank does not raise deposits to allow it to hold the mortgage across periods. The bank can earn more by securitizing the mortgage, saving monitoring costs associated with deposits. Thus $1 - d$ does not appear as a divisor of the loan amount in (B3). Using the definition of $\lambda$ and rearranging, (B3) can be written more succinctly as

$$\gamma h(R_i - \lambda p_2) - \gamma (1-\gamma)a \phi(0). \quad (B4)$$

To prevent the bank from deviating to not screening, (B2) must exceed (B4), or, after rearranging,

$$a[\phi(0) - k_i] \geq \frac{\gamma}{1-\gamma} \left[ d[\phi(d) + \lambda p_2] + k_i \right]. \quad (B5)$$

Additional necessary conditions ensure that $a$ is a well-defined probability:

$$a \geq 0 \quad (B6)$$

$$a \leq 1. \quad (B7)$$

Since the right-hand side of (B5) is positive, the inequality can only hold for $a$ satisfying (B6) if

$$\phi(0) - k_i > 0. \quad (B8)$$

Maintaining (B8), (B5) becomes, upon rearranging,

$$a \geq \frac{\gamma}{1-\gamma} \left\{ \frac{d[\phi(d) + \lambda p_2] + k_i}{\phi(0) - k_i} \right\}. \quad (B9)$$
Conditions (B8) and (B9) together imply (B6), so we can ignore (B6) because we will maintain the other assumptions. This leaves the three necessary conditions for the screening-bank equilibrium (B7), (B8), and (B9). For (B7) and (B9) not to be inconsistent, the right-hand side of (B9) must be less than or equal to 1, or upon rearranging,

\[ k_I \leq \phi(0) - \gamma \phi(d). \]  

(B10)

The upper bound on \( k_I \) implied by this condition is given by (12), which is the equation of the line \( R_{SBK} = R_{NBR} \). We have shown that a screening bank cannot operate above the \( R_{SBK} = R_{NBR} \) line even after relaxing the restriction \( a = 1 \), which was to be shown.

Some intuition can help explain why relaxing the restriction \( a = 1 \) does not lead the banking region to expand into the broking region as one might have though. It turns out that a bank’s deviation to not screening is more profitable than a broker’s to a repayment undercutting the screening bank’s. Deviation by a screening bank to not screening will allow it to achieve all of the economies of the broker, i.e., avoiding the monitoring costs of raising deposits and avoiding screening costs. An added benefit is that this deviation is not observed by customers, so the bad type continues to apply with probability \( a \leq 1 \). By contrast, undercutting is observable, allowing the bad type to respond to the deviation increasing its probability of applying to 1. This “punishment” helps deter deviation, allowing us to circumvent the most expensive set of equilibria possible, but such “punishment” cannot be used to deter a bank’s deviation to not screening. If (B8) holds, increasing \( a \) reduces the non-screening bank’s profit (B3) faster than the screening bank’s profit (B1). The deviation constraint is maximally relaxed by setting \( a \leq 1 \), the exogenous restriction imposed in the text. Q.E.D.

**Bank Region (ii) Unchanged Across Figures.** It is obvious that the banking region in Figure 3 remains solely a banking region after allowing the bad type to reduce its probability of applying to a screening bank, because this weakly increases the profitability of bank operations. Q.E.D.

**No-Contract Region (iii.a) Unchanged Across Figures.** Take any point \((\gamma', k_I')\) in the no-contract region (iii.a) in Appendix Figure 1. Let \((\gamma_N, k_I^N)\) be the “nexus” of the lines \( R_{SBK} = R_{NBR} \), \( R_{NBR} = m \), \( R_{SBK} = m \), indicated by the labeled dot in the figure. Solving these three equations (one is redundant, so there are only two independent equations) for the two unknowns \( \gamma \) and \( k_I \) yields

\[ \gamma_N = \frac{\phi(0)}{h(m - \lambda p_2)} \]  

(B11)

\[ k_I^N = \phi(0) - \gamma_N \phi(d). \]  

(B12)

where we have used (6) and (8) for the respective formulas for \( R_{SBK} \) and \( R_{NBR} \). Because region (iii.a) is in the upper left of the quadrant, we have \( \gamma' < \gamma_N \) and \( k_I' > k_I^N \).

We know there cannot be a broking equilibrium in region (iii.a) because \( \gamma' < \gamma_N \), implying that \( R_{NBR} > m \).

We are left to show there cannot be a screening-bank equilibrium either. Let \( R' \) be the repayment in an outcome involving screening-bank lending. To be feasible, \( R' \leq m \). Let \( a' \) be the probability that bad types apply to the screening bank. Condition (B9) implies

\[ a' \geq \frac{\gamma'}{1 - \gamma'} \left\{ \frac{\phi(d) + \lambda p_2}{\phi(0) - k_I'} \right\}. \]  

(B13)

We will show these conditions imply the bank must earn negative profit.

Following (B1), the bank’s profit is

\[ \gamma'(R' - \lambda p_2) - [\gamma' + (1 - \gamma') a'] \phi(0) \]

\[ \leq \gamma' \left[ h(R' - \lambda p_2) - \frac{\phi(d) k_I'}{\phi(0) - k_I'} \right] \]  

(B14)

\[ < \gamma_N \left[ h(R' - \lambda p_2) - \frac{\phi(d) k_I^N}{\phi(0) - k_I^N} \right] \]  

(B15)

\[ \leq \gamma_N \left[ h(m - \lambda p_2) - \frac{\phi(d) k_I^N}{\phi(0) - k_I^N} \right] \]  

(B16)

\[ = 0. \]  

(B17)

Equation (B14) follows from (B13) and some rearranging. Equation (B15) follows from \( \gamma' < \gamma_N \) and \( k_I' > k_I^N \). Equation (B16) follows from \( R' \leq m \) and some rearranging. Equation (B17) follows from (B11) and (B12). This shows that the bank makes negative profit, ruling out an equilibrium with a screening bank. Q.E.D.

**Banking and No-Contract Equilibria in Region (iii.b).** Take any point \((\gamma'', k_I'')\) in the hatched region (iii.b) in Appendix Figure 1. Existence of equilibria with no lending in region (iii.b) follows from Proposition 1. As shown in Figure 3, there is no contract in equilibria under the restriction \( a = 1 \) in that region.

We know there cannot be a broking equilibrium in region (iii.b) because \( \gamma'' < \gamma_N \), implying that \( R_{NBR} > m \).

We will construct a equilibrium with lending by a screening bank at point \((\gamma'', k_I'')\). Assume bad types apply to screening banks with probability

\[ d'' = \frac{\gamma''}{1 - \gamma''} \left\{ \frac{\phi(d) + \lambda p_2 + \gamma'' k_I''}{\phi(0) - k_I''} \right\}. \]  

(B18)

Because \( d'' \) satisfies (B9), there is no strict incentive for the screening bank to deviate to not screening.

Suppose two banks offer \( R' \) generating zero expected profit for the bank to which the borrower applies. We have

\[ 0 = \gamma'' \left[ h(R' - \lambda p_2) - \phi(d) \right] - [\gamma'' + (1 - \gamma'') a''] k_I'' \]  

(B19)

\[ = \gamma'' \left[ h(R' - \lambda p_2) - \phi(d) - \frac{\phi(d) k_I''}{\phi(0) - k_I''} \right] \]  

(B20)

\[ > \gamma'' \left[ h(R' - \lambda p_2) - \phi(d) - \frac{\phi(d) k_I^N}{\phi(0) - k_I^N} \right] \]  

(B21)

\[ = \gamma'' h(R' - m). \]  

(B22)

Equation (B19) follows from (B2). Equation (B20) follows from (B18) and some rearranging. Equation (B21) follows from \( k_I' > k_I^N \). Equation (B22) follows from (B11) and (B12). This proves \( R'' < m \). Hence the repayment is feasible. Because \( m < R_{NBR} \) and \( R'' < m \), \( R'' < R_{NBR} \), implying a broker cannot undercut \( R'' \). This establishes that the posited outcome with lending by a screening bank is an equilibrium at the point \((\gamma'', k_I'')\). Q.E.D.

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**Appendix C: Foundations of Simple Debt and Pooling Contracts**

The appendix provides two theorems that restrict the form of the mortgage contract in our model. Our aim is to show that the contractual form taken for granted in the analysis in the text is in fact
the optimal form, and indeed is optimal in a more general setting assumed in our model is essentially endogenous, and independent of some of the simplifying assumptions in the model, in particular the assumption of zero income. The first result demonstrates that contracts have the simple debt form. The theorem provides two sets of sufficient conditions for pooling, rather than sorting contracts to obtain in equilibrium. Notation and structure follow the basic model, with the following modifications:

- \( h_\theta \) is the probability that type \( \theta \in \{g, b\} \) (where \( g \) represents the good type, and \( b \) represents the bad type) has high period-2 income. \( 1 > h_\theta > h_g \geq 0 \) (\( h_b = 0 \) is assumed in basic model).
- \( \hat{\theta} \in \{g, b\} \) is an announcement of type.
- Period-2 income is \( y_L \) or \( y_H \). \( y_H > y_L \) (\( y_L = 0 \) in basic model). Following the paper, we assume \( y_L + w < \lambda P_2 \), i.e. liquidation yields higher revenue to the lender than the highest repayment the bad consumer can afford. The bad consumer is thus loss-making for lenders.
- \( \hat{I} \in \{H, L\} \) is an announcement of period-2 income, \( I \in \{H, L\} \) is the true type.
- \( w \geq 0 \) is period-1 wealth measured in period-2 dollars (assumed zero in the basic model).

**Theorem 2.** If there is no public randomizing device in period 2, period-2 income is low, and period-1 wealth is insufficient to fund borrowing ex-ante, then the only ex-post incentive-compatible contract has a simple-debt form.

**Proof.** Let \( (\alpha_I, R_I, w_I) \) be a general deterministic mechanism, where \( \alpha_I \in \{0, 1\} \) is an indicator function and \( R_I \) is a payment from the consumer to the lender, and \( w_I \leq w \) is a payment from the consumer’s initial wealth. The mechanism depends on the period-1 announcement \( \theta \), but we suppress this for notational simplicity. The consumer’s payoff under the mechanism is \( y_I + w + \alpha_I R_I - R_I \) if \( R_I \leq m_I \) and \( y_I + w \) if \( R_I > m_I \), where \( m_I = \min \{y_I, w_I, p_i, p_2, u_2\} \) is the consumer’s ex-post participation constraint. The IC constraint for the high income consumer is

\[
\alpha_H u_2 - R_H \geq \alpha_L u_2 - R_L \tag{C1}
\]

Period-1 feasibility for the lender implies that \( R_H > y_L + w \), i.e. payment of the consumer’s wealth plus the low level of income in both the high and low income states is not sufficient to fund borrowing. Any mechanism with \( R_H > y_L + w \) is always incentive compatible for the low-income consumer, who faces the IR constraint

\[
\alpha_L u_2 - R_L \geq 0 \tag{C2}
\]

Note that since \( R_H > y_L + w \geq R_L \), satisfaction of (C1) and (C2) is only possible if \( \alpha_H = 1, \alpha_L = 0, R_L = 0 \) and \( R_H \leq m_H \). This is the simple debt form. Q.E.D.

**Theorem 3.** Ex-ante incentive compatible contracts are pooling with both types accepting a loan in equilibrium if period-1 wealth \( w \) is sufficiently limited.

**Proof.** First note that it cannot be part of a Bertrand equilibrium with a separating contract with at least two lenders, for a loan to be given to a loss-making type: If both types are loss-making, then no contracts will be offered. If one type is loss making and the other is not, a lender will deviate from any putative equilibrium in which loans are given to both types, and only offer a contract that the profitable type will prefer, leaving others to offer the contract to the loss-making type. The only separating contract is some \( (x, R_H, R_L) \), where \( x \leq w \) is an up-front payment, which is accepted by the good type and rejected by the bad type. Noting by the last result that if low income is realized in period 0, the consumer is foreclosed, the good type will prefer this contract to no contract whenever

\[
u_1 + w - x + h_\theta (y_H + u_2 - R_H) + (1-h_\theta) y_L \geq w + h_\theta y_H + (1-h_\theta) y_L.
\]

The bad type will prefer no contract whenever

\[
u_1 + w - x + h_\theta (y_H + u_2 - R_H) + (1-h_\theta) y_L \leq w + h_\theta y_H + (1-h_\theta) y_L.
\]

These reduce, respectively, to

\[
u_1 + h_\theta (u_2 - R_H) \geq x
\]

and

\[
u_1 + h_\theta (u_2 - R_H) \leq x.
\]

If wealth is limited such that \( u_1 + h_\theta (u_2 - R_H) > w \), then a sorting contract is infeasible. Q.E.D.
References


Figure 1: Intermediary Forms
Figure 2: Timing of Model
Figure 3: Equilibrium Characterization

Region (i): Broker
Region (ii): Bank
Region (iii): No Contract

\( R_{\text{SER}} = m \)

\( R_{\text{SER}} = R_{\text{WAR}} \)
Figure 4: Effect of $\gamma$ on Mortgage Quantity and Quality

- Volume vs. $\gamma$: No Contract, $\gamma'$, Banking, $\gamma''$, Broking, 1
- Default probability vs. $\gamma$: No Contract, $\gamma'$, Banking, $\gamma''$, Broking, 1

Legend:
- n.a.: Not applicable

Graph shows the impact of $\gamma$ on mortgage volume and default probability across different contractual agreements.
Figure 5: Region in which Either Intermediary Form Can Arise in Model with Systemic Factors
Figure 6: Housing Supply and Demand in Period 1
Appendix Figure 1: Equilibrium Generalizing Bad Type’s Application Behavior
Figure 1: Intermediary Forms
Figure 2: Timing of Model
Figure 3: Equilibrium Characterization

Region (i): Broker
Region (ii): Bank
Region (iii): No Contract

\[ R_{\text{WAR}} = m \]

\[ R_{\text{SBR}} = R_{\text{WAR}} \]
Figure 4: Effect of $\gamma$ on Mortgage Quantity and Quality
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