Political Capital*

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November 15, 2013

Abstract

Decision making inside organizations often requires aggregating dispersed information through communication. This paper considers a mechanism in which an expert sacrifices future participation in decision making to influence the current period's decision in favor of his preferred project. This mechanism captures a notion often described as "political capital" whereby an individual is able to achieve their own prefered decision in the current period at the expense of being able to exert influence in future decisions ("spending political capital"). I show that this mechanism facilitates communication in environments where information is neither verifiable ex ante or ex post. I show that first best decision making is possible when agents value future participation sufficiently well. When the first best is not possible, the decision rule increases the influence of the expert relative to the first best. In a multi-expert setting a finite team size is found to be optimal.

Keywords: Information Disclosure, Cheap Talk, Political Capital
JEL codes: D82, D83, D23, L23

*I thank Florian Ederer, Mihai Manea [...] as well as seminar participants at [...] for helpful discussions and suggestions.

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1 Introduction

A key task an organization undertakes when making decisions is aggregating dispersed information via communication. A challenge for aggregating information is inducing truthful communication from individuals whose interests are not perfectly aligned with an organization's. The strategic limits of communication are thus an important issue for understanding the internal functioning of organizations. The main contribution of this paper is to analyze how the value of the relationship between a decision maker (principal/her) and a better informed expert (agent/he) can be used to facilitate communication. In the mechanism the expert's value of being involved in the relationship with the decision maker serves as a form of depletable capital. The expert is able to make his recommendations credible by making his ability to participate in future decisions contingent on the decision today. We refer to this as spending "political capital" because of its similarity to the phenomenon it refers to in popular usage.

Consider a politician consulting an advisor (economics, education, foreign policy, etc.) over a decision where the politician has a known option in hand, say the status quo, but knows that the advisor has better knowledge of an alternative course of action. There is a regular demand for advice as new issues arrive over time. In this setting, the advisor's preferences over the decision may be correlated with the politicians but not perfectly so. The information that is provided is potentially unobservable not only ex ante, but also ex post after a decision is made. For example, the economy may respond to policy decisions with a considerable lag. Or the numerous other unobserved factors which affect the economy make its performance a very noisy signal of the accuracy of the recommendation of the expert. Under these conditions mechanisms based on ex post observable measures may be particularly blunt and entirely ineffectual for facilitating communication. However, as we show, in these situations the value of being involved in future decisions can facilitate communication. This setting is also typical of many situations in organizations where an individual higher up in the organizational hierarchy has the authority to take a decision, but in part relies on the advice of individuals lower down in the hierarchy with better information.

The premise that individuals have a stock of an intangible form of capital, which can be used to influence decision making, is consistent with observations of decision making in organizations. The term "political capital" is often used to refer to this type of intangible asset. For instance Chait, Ryan and Taylor (2005 p150-151) "Political capital connotes, in shorthand, the influence and leverage that people within an organization acquire and deploy [...] to promote one solution over another." Other examples include Evert and Van Deuren (2013, page 11) who discuss the experiences of superintendents of school districts leading up to departure "Superintendents must make choices about what to support and advocate for and what to leave alone for another day. Depending on the issue, the level of controversy, and the commitment of the superintendent to achieve a desired outcome, the superintendent may end up spending personal political capital in addressing the issue. Over time, this expenditure of political capital (whether with the board, internal audiences, or the community) can result in weakened effectiveness as a leader." Diokno (2013) discussing Phillipino politics "A cautious, risk-averse President may choose to sit comfortably on his political capital and do nothing. An audacious, forward-looking President may use it to open up the economy and reform the tax system." Geri and McNabb (2011) discussing the challenges of crafting new policy "Political capital [...] will be needed to pass the combination of tax increases, policy changes, and spending cuts needed to stabilize the budget. There will be less political capital left over to devote to other policy challenges." Finally and perhaps the most frequently cited instance of the use of the term "political capital" is by President George W. Bush after winning reelection in 2004 "I earned capital in the campaign, political capital, and now, I intend to spend it." Common across all these examples is the idea that each of the relevant actors has a depletable source of capital which can be used to influence decision making. The model in this paper focuses on a setting where such a mechanism arises to facilitate communication between an expert and decision maker.

This paper considers a non-transferable utility setting where decisions are made repeatedly over time and an expert's future participation in decision making is conditional on the information he provides in the current period. In each period there is a known outside option for the decision maker. Its value is drawn at the start of each period from a standard uniform distribution, $\phi_t \in U[0,1]$. The value of the expert's preferred decision is independently drawn from another standard uniform distribution, $v_t \in U[0,1]$, and is privately known to the expert. The decision maker must choose which project to implement. The central tension, which inhibits communication, is that the expert only cares about his own project (and so always prefers that it is chosen) whereas the decision maker cares about both and thus prefers that the project with the higher value be chosen.[1] In a one-shot situation it is not possible to elicit any information from the expert about the value of his preferred project because the expert will always recommend that his project is undertaken. However, in a repeated setting it is possible for the decision maker to elicit some information by making it less likely that the expert participates in future periods of decision making when the expert reports a higher value in the current period. This places a shadow cost on the reported value that the expert provides to the decision maker and induces the expert to reveal information. An expert may achieve his preferred decision in the current period at the expense of being able to exert influence in future decisions. The value of participating in future decision making is the expert's "political capital." In any given period an expert may probabilistically spend this capital to convince the decision maker to choose his preferred project. Furthermore the expert must spend more capital to convince a decision maker to

[1] Implicit in our setup is the idea that an expert is useful for the decision maker because he has a potentially better project than her outside option. Hence the source of "political capital" in our model is the private information of the expert.

ignore better outside options. We show that such a mechanism facilitates communication in environments where information is unobservable ex ante and ex post and where the one-shot setting has no informative equilibria. We find that the first-best is possible when the expert is sufficiently patient. When the first-best is not possible, the second-best involves increasing the expert's influence (i.e., the range of reported values for which the decision maker chooses the expert's preferred decision) relative to the first-best. That is, the decision maker chooses the expert's preferred project more frequently in the second-best than in the first-best. The reason for doing so is that it increases the value of the relationship for the expert which facilitates better communication and improved project choice for the decision maker. In some extensions we find that when the value of projects is positively correlated over time a larger set of discount rates may achieve the first-best and in a setting with multiple experts a finite team size is optimal.

2 Related Literature

Economics has long recognized that truthful communication is challenging in environments where experts' objectives are not perfectly aligned and information is not readily verifiable. The particular setting we are interested in, where a decision maker must make a decision and an expert holds relevant information but has different preferences, has also received considerable attention. The two basic approaches taken to this setting have been to assume either transferable or non-transferable utility. This paper makes the latter assumption. In the latter literature which assumes that utility is non-transferable some papers assume that the decision maker is able to commit to undertake certain actions as a function of the expert's report while others assume the decision maker has no commitment power. We consider both of these assumptions in our paper, and find that the qualitative results of our model are robust under both assumptions.

The non-transferable utility literature which confers the decision maker with the power to commit to a decision rule conditional on the information provided by the expert began with Holmstrom (1977, 1984). Since then a significant body of scholarly contributions has considered the optimal design of decision rules in a variety of environments (for instance Armstrong and Vickers (2010), Aghion and Tirole (1992), Dessein (2002)). Predominantly, this literature has focused on single decision settings. An exception is Alonso and Matouschek (2007) who consider a repeated setting of delegated decision making. Similar to the current paper, the ongoing relationship between the decision maker and the agent plays a role in improving efficiency. The relationship serves to discipline the behavior of the decision maker in Alonso and Matouchek (2007) whereas in our model the value of the relationship gives the expert's recommendations credibility. In Alonso and Matouschek (2007) the commitment power of the decision maker comes from relational concerns with the expert. In the event that a decision maker does not follow a specified decision rule, the equilibrium reverts to a cheap talk equilibrium. Thus the "penalty" to the decision maker from deviating away from a decision rule is the lower payoff she receives in the future from playing the cheap talk equilibrium. That is to say, the decision maker can commit to implement certain decision rules due to the threat of damaging the future relationship with the expert if she deviates from the rule in any period. In contrast, the role of the relationship between the decision maker and the expert in the current paper is very different. The value of being involved in future decision making allows an expert to credibly communicate information about the current decision. As we show, when the expert's participation in future decisions is not a function of the information he communicates there is no information conveyed in equilibrium.

The literature on cheap talk, beginning with Crawford and Sobel (1982), assumes that the decision maker has no commitment power. A strand of this literature considers the effect that an expert's concern for his reputation has on communication (e.g. Sobel (1985), Prendergast (1993), Prendergast and Stole (1996), Ottaviani and Sorenson (2006)). A related paper in this literature is Morris (2001) who considers a repeated cheap talk setting where an expert may be unbiased or biased. In this setting the reputational concern of an expert may distort truthful communication of an unbiased expert. Similarly to the current paper it is the expert's desire to influence future decisions which impacts his communication in the current period. This aside, the focus and results are very different. The current paper shows a concern for the future allows a biased expert (where it is known that agent is biased) to reveal information whereas Morris (2001) shows that a concern for the future, through an expert's reputation (where it is unknown whether an agent is biased or unbiased), leads an unbiased expert to not reveal information.

Some papers have shown how various elements of the environment may improve communication more than that achieved in the classic cheap talk setting of Crawford and Sobel (1982). Krishna and Morgan (2004) show that allowing multiple rounds of cheap talk involving both the expert and decision maker may lead to equilibria where more information is communicated. The current paper shows that despite a severe conflict of interest between the sender and receiver in a binary choice model, such that no information can be communicated in a one-shot setting with one or more rounds of cheap talk, communication is improved in a repeated setting even though the information communicated by the expert is never verifiable ex ante or ex post. Chakraborty and Harbaugh (2007, 2010) show in a multi-dimensional setting that common interest across dimensions between the sender and receiver may allow informative cheap talk despite large conflicts of interest within each dimension independently. The mechanism in this paper has some similarity to this in that it utilizes an agent's indifference between getting their preferred decision today and participation in future decision-making. However the nature of our results, in terms of the influence of the decision-maker and expert, optimal team size, and the effect of positive correlation over time of decisions, are quite distinct from the focus of their work.

3 Model

Time t is discrete in the model $t = 1, 2, 3...$ During each period the value of the decision maker's outside option ϕ_t is an i.i.d. draw from a uniform distribution $\phi_t \sim U[0,1]$. The outside option is observed by both the expert and the decision maker at the start of the period. The value of the expert's preferred project v_t is also an i.i.d. draw from a uniform distribution $v_t \sim U[0,1]$. This is privately observed by the expert. The decision maker values both the outside option and the expert's preferred decision. Her utility in period t, U_t^{dm}, is given by

$$U_t^{dm} = \left\{ \begin{array}{c} \phi_t \text{ if the outside option is chosen} \\ v_t \text{ if the expert's project is chosen} \end{array} \right\} \tag{1}$$

Hence the decision maker, conditional on knowing v_t prefers to choose the decision with the higher value $\max\{\phi_t, v_t\}$. The expert values his own decision but not the decision maker's outside option. His utility in period t, U_t^e, is given by

$$U_t^{e} = \left\{ \begin{array}{c} 0 \text{ if the outside option is chosen} \\ v_t \text{ if the expert's project is chosen} \end{array} \right\} \tag{2}$$

The value of the outside option is worth 0 to the expert. Thus, the expert always prefers that his own preferred action is taken. The difference in how the decision maker and the expert evaluate the two alternatives is the central tension in the model that makes truthful communication non-trivial. We assume that the realization of v_t is never observed by the decision maker, neither ex ante nor ex post, including after taking the expert's preferred decision. In the event that the expert's preferred decision is implemented, this assumption precludes the decision maker from designing a mechanism based on the accuracy of the expert's report relative to the realized value of the decision. We feel this is a realistic assumption in many of the circumstances we are interested in where the actual realization of a decision is either very noisy (e.g., v_t is the expected value of a distribution with large variance) or only observed with significant delay, if at all (e.g., v_t is only observed T periods after the decision where T is large). In both these cases it is intuitive that incentive schemes based on these realizations are likely to be particularly blunt and ineffective. Furthermore, one of the interesting features of our political capital mechanism is that it is not necessary that the decision maker ever needs to verify an expert's recommendation. We thus maintain this assumption throughout the paper.

Communication in the model occurs through a direct revelation mechanism. We focus on stationary mechanisms where the decision maker specifies $\{d(\phi, \hat{v}), \theta(\phi, \hat{v})\}$ at the beginning of the game. This menu specifies the probability an expert is allocated the decision in any period $d(\phi, \hat{v}) : [0,1] \times [0,1] \to [0,1]$ and the probability that the expert is replaced $\theta(\phi, \hat{v}) : [0,1] \times [0,1] \to [0,1]$ as a function of the outside option that period ϕ and the message sent by the expert $\widehat{v}$. We note that the mechanism allows the decision rule and replacement rule to depend on the realized value of the outside option ϕ. Thus the decision rule and replacement rule used in each period is different for different values of the outside option.

We assume that the decision maker chooses a mechanism which maximizes her expected per period payoff from the stationary mechanism:

$$EU^{dm} = \frac{1}{2} + \int_0^1 \int_0^1 (v - \phi)\, d(\phi, v)\, dvd\phi \tag{3}$$

where the expected value of the outside option is $\frac{1}{2}$ and $[v - \phi]\, d(\phi, v)$ is the change in the objective when the expert's preferred project is chosen over the outside option. In the usual way, the mechanism must satisfy the incentive compatibility constraint of the expert to report truthfully $\hat{v}_t = v_t$. Important for our analysis of the mechanism is the feasibility constraint that the maximum replacement probability for an expert is 100%. In the model this constraint may limit the scope for informative communication. The timing during each period is as follows:

1. The values ϕ_t and v_t are realized. Both the decision maker and the expert observe ϕ_t, but only the expert observes v_t.
2. The expert chooses a message $\widehat{v}_t \in [0, 1]$ to send to the decision maker.
3. The expert's preferred action is implemented with probability $d(\phi_t, \widehat{v}_t)$. Otherwise the decision maker takes the outside option.
4. The decision maker replaces the expert with probability $\theta(\phi_t, \widehat{v}_t)$.
5. The decision maker and expert discontinue the relationship with exogenous probability $1 - p$ and the expert is replaced.

The continuation value of the expert from participating in the mechanism in the future is V. The continuation value V is determined by the implicit relationship:

$$V = \frac{1}{1-p} \int_0^1 \int_0^1 [vd(\phi, v) - \theta(\phi, v)\, pV]\, dvd\phi \tag{4}$$

The incentive constraint for the expert is:

$$v = \arg\max_{\widehat{v}} vd(\phi, \widehat{v}) - \theta(\phi, \widehat{v})\, pV + pV \;\; \forall v, \phi \tag{5}$$

We assume that the decision maker maximizes the expected value of the per period decision value. The maximization problem facing the decision maker is:

$$\max_{d(\phi,v),\theta(\phi,v)} EU^{dm} = \frac{1}{2} + \int_0^1 \int_0^1 [v - \phi]\, d(\phi, v)\, dvd\phi \tag{6}$$

subject to the global incentive compatibility constraint and two feasibility constraints

$$v = \arg\max_{\widehat{v}} vd(\phi, \widehat{v}) + [1 - \theta(\phi, \widehat{v})] pV \tag{7}$$

$$0 \leq d(\phi, v) \leq 1 \tag{8}$$

$$0 \leq \theta(\phi, v) \leq 1 \tag{9}$$

where

$$V = \frac{1}{1-p} \int_0^1 \int_0^1 [vd(\phi, v) - \theta(\phi, v) pV] \, dvd\phi \tag{10}$$

We replace the global incentive compatibility constraint with a monotonicity and a local incentive compatibility constraint:

$$\frac{dd(\phi, v)}{dv} \geq 0 \tag{11}$$

and

$$pV \frac{d\theta(\phi, v)}{dv} = v \frac{dd(\phi, v)}{dv} \tag{12}$$

Remark 1 *When it is more likely that an expert's project is chosen then it is also more likely that the expert is replaced,* $sign \frac{\partial\theta(\phi,v)}{\partial v} = sign \frac{dd(\phi,v)}{dv}$.

At this point one can see how the notion of spending "political capital" to influence the current period's decision arises in our model. It is embodied in the local incentive compatibility constraint where sacrificing (in expectation) future participation (increasing the likelihood of being replaced θ) correponds to increasing the likelihood that an expert's preferred project is chosen in the current period (increasing d). Furthermore, along with the monotonicity constraint, this also implies that conditional on the outside option, an expert with a better project is more likely to see his project implemented, higher d, and therefore spends more "political capital," higher θ. Finally if there is no chance that the expert's project is chosen $d(\phi, \widehat{v}) = 0$ then there is also no chance that the expert is replaced endogenously $\theta(\phi, \widehat{v}) = 0$.[2][3]

Integrating equation 12 by parts in the usual way we find:

$$\theta(\phi, v) pV = vd(\phi, v) - \int_0^v d(\phi, s) \, ds \tag{13}$$

and when this is substituted into the relationship for the continuation value V we find:

$$V = \frac{1}{1-p} \int_0^1 \int_0^1 \int_0^v d(\phi, s) \, dsdvd\phi \tag{14}$$

[2] Also there is always an exogenous probability p that the expert is replaced.

[3] It is straightforward to show that a mechanism where $d(\phi, \widehat{v}) = 0$ and $\theta(\phi, \widehat{v}) > 0$ for a positive measure of $(\phi, \widehat{v})$ can not be an improvement over one where $d(\phi, \widehat{v}) = 0$ and $\theta(\phi, \widehat{v}) = 0$.

Further integration by parts yields

$$V = \frac{1}{1-p} \int_0^1 \int_0^1 d(\phi, v)[1-v]\, dv d\phi \tag{15}$$

Finally, we substitute equations (13) and (15) into the upper bound of the feasibility constraint for $\theta(\phi, v)$ (9) to find the following modified feasibility condition

$$vd(\phi, v) - \int_0^v d(\phi, s)\, ds \leq \frac{p}{1-p} \int_0^1 \int_0^1 d(\phi, v)[1-v]\, dv d\phi \tag{16}$$

The decision maker's objective function is maximized subject to the feasibility constraint for $d(\phi, v)$ (8), the monotonicity constraint (11), and the modified feasibility constraint for for $\theta(\phi, v)$ (16).

4 Analysis

In this section we first show that a mechanism where an expert's future participation is contingent on his report improves communication and the expected value of the decision. The improvement of the decision is relative to a benchmark setting where an expert's future participation is unrelated to the message he sends and no informative communication is possible. The second result characterizes the set of values of p for which the first best is possible. The main result of the section is the third result which shows that when the first best is not achievable, that the optimal mechanism shifts influence towards the expert.

The first result is that there exists a mechanism which allows the decision maker to achieve a higher payoff than what she could obtain in a mechanism where the probability of being replaced θ does not depend on the report of the expert. In this case the expert will always report the value of v which results in the highest probability of his preferred decision being taken. This report does not depend on the realized value of v and so there cannot be any informative communication. The best the decision maker can do in this case is to make the decision herself if her outside option is better than the expected value of the expert's preferred decision ($\phi \geq \frac{1}{2}$) and allocate the decision to the expert when her outside option is worse than the expected value of the expert's decision ($\phi < \frac{1}{2}$).

Proposition 1 *For $p < 1$ there exists a mechanism with a strictly higher payoff for the decision maker than what she could obtain without informative communication. The decision maker's objective function under this mechanism is weakly increasing in the discount factor of the expert.*

This proposition establishes that it is always possible for the decision maker to increase the per period expected value of the project by implementing a mechanism with informative communication. For any positive p the expert values his ability to influence future decisions.

The decision maker is able to take advantage of the expert's concern for future influence to extract useful information for making an informed decision in the current period. We now find necessary and sufficient conditions for the first best outcome to be possible. The first best decision rule d^{FB} is

$$d^{FB}(\phi, \widehat{v}) = \begin{cases} 1 \text{ if } \widehat{v} \geq \phi \\ 0 \text{ if } \widehat{v} < \phi \end{cases} \tag{17}$$

and the replacement probability is

$$\theta^{FB}(\phi, \widehat{v}) = \begin{cases} \frac{\phi}{pV} \text{ if } \widehat{v} \geq \phi \\ 0 \text{ if } \widehat{v} < \phi \end{cases} \tag{18}$$

The following lemma shows that the most difficult feasibility constraint to satisfy to achieve the first best is for $\phi = 1^{-}$.

Lemma 1 *For a given mechanism $\{d(v, \theta), \theta(v, \theta)\}$ with continuation value V, if the first best decision rule is feasible for a given outside option $\phi' \in (0, 1)$ then it is also feasible for all less valuable outside options $\phi'' < \phi'$.*

The lemma shows that the most "difficult" feasibility constraints to satisfy are those for important decisions; those decisions where the value of the outside option is greatest. There is a slight caveat that the feasibility constraint is trivially satisfied for the highest value of the outside option since the decision maker always chooses the outside option in this case. The reason that the incentive constraints for important decisions (high values of ϕ) are the most difficult to satisfy in the first best is that in the first best decision rule experts with values slightly less than the outside option must be willing to forego the opportunity to implement their preferred action. This requires that the "cost" imposed on the expert (i.e., the probability that the expert is replaced if he reports that his value is greater than the outside option) needs to be large. The maximum cost the decision maker can impose is to replace the expert with certainty, $\theta = 1$. Thus, there is an upper bound on the maximum cost the decision maker can impose. This means that it may not be possible to always implement the first best. In these instances this occurs because under the first best decision rule it is not possible to satisfy the modified feasibility constraint for higher values of the outside option.

Necessary and sufficient conditions for the first best to be possible are pinned down by satisfying the feasibility constraint for $\phi = 1^{-}$. The condition for the first best to be possible is given in the following proposition.

Proposition 2 *The first best decision rule is feasible iff $p \geq \frac{1}{7}$.*

If the relationship between the decision maker and the expert is not too likely to dissolve for exogeneous reasons, such that the expert places sufficient value on the relationship in the future, then the first best is achievable. The first best requires that efficient decisions are made for all ϕ. The "hardest" feasibility constraint to satisfy are the constraints in the limit as ϕ approaches 1. First best decision making requires that $\phi \leq pV$ for all $\phi \in [0,1)$. At $\phi = 1$ of course the decision maker always takes the decision herself, thus the binding constraints are actually those slightly below 1. The discount factor at which the first best decision rule achieves a discounted continuation value exactly equal to 1 is $\frac{1}{7}$ which defines the bound on first best decision making. We now proceed to analyze the case $p < \frac{1}{7}$ where the first best is not achievable. The main result is that in this second best setting the optimal mechanism results in the expert's project being chosen more often than in the first best.

Proposition 3 *When the first best decision rule is not feasible, $p < \frac{1}{7}$, the optimal decision d^* rule allows the expert to take more decisions than under the first best rule:*

$$\int_0^1 \int_0^1 d^* (\phi, v) \, dv d\phi > \int_0^1 \int_0^1 d^{FB} (\phi, v) \, dv d\phi = \frac{1}{2}.$$

In the second-best mechanism the modified feasibility constraint binds. In other words the amount of information an expert may credibly communicate is limited by his continuation value. This requires that the decision rule is distorted away from the first best. Some intuition for the source of the distortion can be gained through the following thought experiment. For an exogenously given continuation value X and a model which simply required that the mechanism satisfy the feasibility constraint:

$$vd(\phi, v) - \int_0^1 d(\phi, s) \, ds \leq pX \tag{19}$$

then it can be shown (see proof in the appendix) that the optimal decision mechanism results in the expert making the decision exactly half the time, $\int_0^1 \int_0^1 d(\phi, v) \, dv d\phi = \frac{1}{2}$. When the continuation value is endogenously determined, as it is in our model, then it is possible to increase d for values where the distortionary impact on the objective is small/zero relative to the beneficial impact through increasing the continuation value thereby improving communciation. The shift of influence towards the expert arises to increase the ongoing value of the relationship and thereby facilitate better communication.

Another intuition for the result comes from auction literature. The continuation value of the expert is conceptually similar, during a given period, to a budget constraint in an auction model. In our model there is effectively an upper bound (being replaced with certainty) on the "price" an expert can pay for having his preferred project implemented. In an auction a bidder's budget limits the efficiency of the allocation of a good. In our setting low continuation value acts in much the same way. In the second best setting making the expert more influential is a way to increase the "budget" available to the expert and it is through this channel that the efficiency of the choice made by the decision maker is improved.

4.1 Serial Correlation

In some circumstances the importance of decisions may exhibit serial correlation. For instance, an economic advisor may be required to make recommendations during a financial crisis, or managers in organizations may be required to make recommendations during a period of restructuring, contraction, or expansion in response to changes in market conditions. In a model where the state of the world follows a Markov process, we show that more positive serial correlation increases the range of separation probabilities for which the first best decision making rule is feasible.

To study serial correlation we augment the earlier model. In each period the state of the world $\alpha = h, l$ is either high or low and it is observed by both the decision maker and the expert. In the first period, each state is equally likely. Thereafter the state evolves according to a Markov process where the probability that the state remains the same in the next period is $\frac{1}{2} \leq \beta < 1$ and switches with probability $1-\beta$. In each state the outside option and the expert's preferred action are i.i.d. drawn from $U[0, \psi_\alpha]$ where $\psi_h > \psi_l = 1$ and we have normalized the upper bound of the low state to 1. We maintain the assumption of a stationary mechanism subject to the state of the world. The direct revelation mechanism, $\{d(\phi, v, \alpha), \theta(\phi, v, \alpha)\} : [0,1] \times [0,1] \times \{l, h\} \rightarrow [0,1] \times [0,1]$, is a function of the realized state of the world α. The continuation value is now a function of the current state of the world. Thus, there are now two continuation values V_h and V_l depending on whether the state is high or low. These are defined jointly by the following condition:

$$\begin{bmatrix} V_h \\ V_l \end{bmatrix} = \begin{bmatrix} \beta & 1-\beta \\ 1-\beta & \beta \end{bmatrix} \begin{bmatrix} \int_0^{\psi_h} \int_0^{\psi_h} \left[1 - \frac{v}{\psi_h}\right] \frac{1}{\psi_h} d(\phi, v, h)\, dv d\phi \\ \int_0^1 \int_0^1 [1-v]\, d(\phi, v, l)\, dv d\phi \end{bmatrix} + \begin{bmatrix} p\beta & p(1-\beta) \\ p(1-\beta) & p\beta \end{bmatrix} \begin{bmatrix} V_h \\ V_l \end{bmatrix} \tag{20}$$

As before the first best decision rule is

$$d^{FB} = \begin{cases} 1 \text{ if } v \geq \phi \\ 0 \text{ if } v < \phi \end{cases} \tag{21}$$

Define the minimum separation probability for which the first best decision is feasible for a given β by $p^{FB}(\beta)$. The following proposition characterizes the comparative statics of the first best set of separation probabilities.

Proposition 4 *More positive serial correlation increases the range of discount factors for which the first best decision rule is feasible,* $\frac{dp^{FB}(\beta)}{d\beta} < 0$.

Here, we find that positive serial correlation increases the set of separation probabilities for which the first best is feasible. The intuition for the result is relatively straightforward. The most difficult feasibility constraint to satisfy in the first best is the constraint for $\phi_t = \psi_h^-$ for $\alpha = h$. This requires that the continuation value of being in the high state satisfies $pV_h \geq \psi_h$. In the case where there is no serial correlation $V_h = V_l$ all constraints in the state $\alpha = l$ are slack because $pV_h = pV_l \geq \psi_h > 1$. Introducing positive serial correlation increases the continuation value of being in the high state and reduces the continuation value of being in the low state. The ratio $\frac{V_h}{V_l}$ is increasing and approaches ψ_h as serial correlation becomes perfect, $\beta \to 1$.

4.2 Lack of Commitment to Ex-post Inefficient Decisions

We now return to our original assumption that the state of the world is constant where ϕ_t, v_t are both i.i.d. draws from $U[0,1]$. In the earlier sections we assumed that the decision maker can commit to a direct revelation mechanism. This assumption is consistent with the literature which follows Holmstrom (1977, 1984). The literature on cheap talk, following Crawford and Sobel (1982), is also concerned with similar issues of communication and decision-making but under the assumption that the decision maker has no ability to take a decision that is not in her own interests conditional on the information sent by the expert. In this section, we consider the model under the assumption that the decision maker must take decisions which are ex-post in her self interest. We find similar results to Propositions 2 and 3 in the best perfect Bayesian equilibrium which satisfies the constraint that the decision makes ex-post efficient decisions conditional on the information provided by the expert.

In this section we allow the expert to send a message $m_t \in [0,1]$ to the decision maker during each period. The decision maker specifies a message rule $m(\phi, v) : [0,1] \times [0,1] \to [0,1]$, decision rule $d(\phi, m) : [0,1] \times [0,1] \to [0,1]$ and probability of replacing the expert $\theta(\phi, m) : [0,1] \times [0,1] \to [0,1]$. The message sent by the expert is a function of the status quo and the valuation of the expert's project. The decision and replacement functions are conditional on the value of the status quo ϕ and the message sent by the expert. In a perfect Bayesian equilibrium the lack of commitment to ex post inefficient decisions means that the decision rule satisfies:

$$d(\phi_t, m_t) \in \begin{cases} \{1\} \text{ if } E[v_t|m_t, \phi_t] > \phi_t \\ \{0\} \text{ if } E[v_t|m_t, \phi_t] < \phi_t \\ [0,1] \text{ if } E[v_t|m_t, \phi_t] = \phi_t \end{cases} \tag{22}$$

where the expectations are calculated using Bayes rule. We require the expert's message function to satisfy the incentive constraint for each type of expert, that is

$$m(\phi_t, v_t) \in \arg\max_{m \in [0,1]} v_t d(\phi_t, m) - \theta(\phi_t, m) pV \text{ for all } \phi, v \tag{23}$$

We replace this constraint by a local incentive compatibility constraint

$$pV\theta\left(\phi, m\left(\phi, v\right)\right) = vd\left(\phi, m\left(\phi, v\right)\right) - \int_0^v d\left(\phi, m\left(\phi, s\right)\right) ds \tag{24}$$

and a monotonicity constraint

$$d\left(\phi, m\left(\phi, v'\right)\right) \geq d\left(\phi, m\left(\phi, v''\right)\right) \text{ for } v' > v'' \tag{25}$$

The expert's continuation value is also found in a similar way as earlier

$$V = \frac{1}{1-p}\int\int d\left(\phi, m\left(\phi, v\right)\right)\left[1-v\right] dvd\phi \tag{26}$$

and finally we use equations 24, 26 and the feasibility constraint $\theta\left(\phi, m\right) \leq 1$ to derive a modified feasibility constraint

$$vd\left(\phi, m\left(\phi, v\right)\right) - \int_0^v d\left(\phi, m\left(\phi, s\right)\right) ds \leq \frac{p}{1-p}\int\int d\left(\phi, m\left(\phi, v\right)\right)\left[1-v\right] dvd\phi \tag{27}$$

We note that the decision maker is indifferent between keeping and replacing an expert between periods and so faces no commitment issues to implement any replacement policy $\theta\left(\phi, v\right)$. The decision maker's problem is to choose the decision rule and message rule to maximize the expected per period value of the project chosen:

$$\max_{d(\phi,m(\phi,v)),m(\phi,v)} \frac{1}{2} + \int_0^1\int_0^1 \left[v-\phi\right] d\left(\phi, m\left(\phi, v\right)\right) dvd\phi \tag{28}$$

subject to constraints 22, 25, 27 and $0 \leq d\left(\phi, m\right) \leq 1$. We refer to this as the decision maker's problem. Note that we assume any message $m \in [0,1]$ not used on-equilibrium (not in the range of $m\left(\phi, v\right)$ for each value of ϕ) results in $\theta\left(\phi, v\right) = 1$ and $d\left(\phi, v\right) = 0$. This guarantees that these messages are never a strict best response to equation 23. Hence we require that any decision rule and replacement rule (mechanism) is both a perfect Bayesian equilibrium of the model, as before, and, in addition, satisfies the commitment constraint 22.

When the first-best mechanism is feasible, there exists a perfect Bayesian equilibrium where experts send one of two messages $m \in \{0,1\}$ where $m\left(\phi, v\right) = 1$ if $v \geq \phi$ and $m\left(\phi, v\right) = 0$ if $v < \phi$ and the first best decision rule d^{FB} is implemented. This satisfies the feasibility and incentive constraints as in the earlier analysis and furthermore satisfies the commitment constraints because $E\left[v|m=1\right] \geq \phi$ and $E\left[v|m=0\right] < \phi$. Thus, we achieve a similar result to Proposition 2 in a setting without commitment.

We now turn to our analysis of the second best when the first best is not possible for $p < \frac{1}{7}$. We first simplify the problem by noting that the best perfect Bayesian equilibrium subject to the commitment constraint may be characterized by a decision rule involving two messages.

Lemma 2 *For any solution* $\{d^*(\phi, m^*(\phi,v)), m^*(\phi,v)\}$ *to the decision maker's problem let* $\delta^*(\phi,v) = d^*(\phi, m^*(\phi,v))$, *then there exists another solution* $\{d^{**}(\phi, m^{**}(\phi,v)), m^{**}(\phi,v)\}$ *where* $\delta^*(\phi,v) = d^{**}(\phi, m^*(\phi,v))$ *and* $m^{**}(\phi,v) \in \{0,1\}$.

This lemma allows us to restrict our attention to perfect Bayesian equilibria where only one of two messages are ever sent on the equilibrium path. The next proposition establishes a qualitatively similar result to Proposition 3 under the assumption that the decision maker can not commit to make ex-post inefficient decisions conditional on the information communicated by the expert.

Proposition 5 *When the first best decision rule is not feasible,* $p < \frac{1}{7}$, *the optimal decision* d^* *rule without commitment allows the expert to take more decisions than under the first best rule:*

$$\int_0^1 \int_0^1 d^*(\phi,v)\, dv d\phi > \int_0^1 \int_0^1 d^{FB}(\phi,v)\, dv d\phi = \frac{1}{2}.$$

Under an assumption of no commitment power we find that the second best mechanism involves shifting influence towards the expert. The rationale is exactly the same as under the commitment setting. Allowing the expert to have greater influence increases the value of the relationship which in turn helps facilitate improved communication and decision-making. The proposition also highlights that the robustness of our earlier qualitative insight, that allowing the expert's project to be chosen more frequently, is optimal in a second best mechanism. It also serves to highlight how the value of a relationship between a sender and receiver in a cheap talk setting may help to improve the amount of information which can be communicated. Predominantly the focus of the existing literature, when there is an ongoing relationship, has has been on the reputational concerns of the agent. Our model highlights that an ongoing relationship may also serve as a form of capital which may help to facilitate the communication of information.

5 Team of Experts

Teams are frequently charged with solving problems and undertaking various tasks within organizations. In these settings a decision maker aggregates information from multiple sources before coming to a decision. Similarly an individual in a position of authority often utilizes a set of advisors to whom she consults before making decisions. There is a large body of literature which studies committee design and decision making in teams (Lipman and Seppi (1995), Feddersen and Pesendorfer (1997), Glazer and Rubinstein (1998), Banerjee and Somanathan (2001), Ottoviani and Sorenson (2001), Persico (2003)). Much of the previous literature has considered a single decision setting. However teams often face a regular flow of decisions over time and we would like to know what additional insights can be gained in a dynamic setting. In this section we consider our inherently dynamic "political capital" mechanism in a setting with multiple experts. In this setting the political capital mechanism also offers scope for informative communication even though none would be possible in a single decision setting. Furthermore we find that limiting the size of a team of experts increases the continuation value of being part of the team and thereby facilitates better communication through the "political captial" mechanism. The optimal team size is thus finite.

In this section we focus on communication between a decision maker and a set of experts $i = 1, ..., n$. For simplicity we assume that the decision maker's outside option is 0 and she is simply interested in aggregating the information of the experts. The one-shot setting again offers no scope for informative communication since we assume each expert is only interested in having his preferred project chosen. However, we show that in a repeated setting it is possible to sustain informative communication through a mechanism which allows experts to effectively communicate better information about their own project in the current period at the expense of participation in decision making in future periods.

Time is again discrete and at the start of each period each expert i draws the value of his preferred decision v_t^i in that period. Each expert's draw is i.i.d. from $U[0,1]$ and is private information. He derives utility v_t^i if his preferred project is chosen and zero otherwise. The decision maker cares about all projects and derives utility v_t^i if she chooses the preferred project of expert i in period t. Again we focus on a stationary direct revelation mechanism and assume that the decision maker is able to commit to implement the mechanism. During each period, each expert chooses a message $\hat{v}_t^i$ to send to the decision maker. The decision maker specifies a mechanism conditional on the set of reports received $\hat{v}_t = \left\{\hat{v}_t^1., \hat{v}_t^n\right\}$ where $\{d(m), \theta(m)\} : [0,1]^n \rightarrow [0,1]^n \times [0,1]^n$ specifies the probability for each expert that his preferred decision is implemented $d_i(m)$ and the probability $\theta_i(m)$ that he is replaced. We also restrict our attention to symmetric mechanisms. In our context this means that the project depends on the value reported by an expert, but does not depend on the identity of the individual.[4]

The objective for the decision maker is to maximize the expected per period value of the project she chooses through the mechanism EU^{dm}. This is simply the sum of the expected utility of each team member (prior to learning v^i) during the period, which in a symmetric mechanism is given by:

$$EU^{dm} = n \int_0^1 v_i E_{v_{-i}} d_i(v_i, v_{-i})\, dv_i \tag{29}$$

Each expert's incentive constraint is

$$\arg\max_{\widehat{v}_i} E_{v_{-i}} \left[v_i d_i(\widehat{v}_i, v_{-i}) - \theta_i(\widehat{v}_i, v_{-i})\, pV\right] \tag{30}$$

[4] This means that if $\sigma\cdot$ is a permutation operator on n elements then the mechanism $\{d(m), \theta(m)\}$ satisfies $\{d(\sigma \cdot m), \theta(\sigma \cdot m)\} = \{\sigma \cdot d(m), \sigma \cdot \theta(m)\}$.

We replace this constraint with a local incentive compatability constraint and monotonicity constraint:

$$v_i \frac{\partial}{\partial \widehat{v}_i} E_{v_{-i}} d_i \left(\widehat{v}_i, v_{-i}\right) = pV \frac{\partial}{\partial \widehat{v}_i} E_{v_{-i}} \theta_i \left(\widehat{v}_i, v_{-i}\right) \tag{31}$$

and

$$\frac{dd_i \left(\widehat{v}_i, v_{-i}\right)}{d\widehat{v}_i} \geq 0 \tag{32}$$

The local incentive constraint can then be used in a similar way to Section 3 to find the modified feasibility constraint:

$$v_i E_{v_{-i}} d_i \left(v_i, v_{-i}\right) - \int_0^{v_i} E_{v_{-i}} d_i \left(s, v_{-i}\right) ds \leq pV \tag{33}$$

where the continuation value V is given by:

$$V = \frac{1}{1-p} \int_0^1 E_{v_{-i}} d_i \left(v_i, v_{-i}\right) \left[1 - v_i\right] dv_i \tag{34}$$

and

$$\sum_i d_i \left(\hat{v}\right) = 1 \text{ for all } \hat{v} \in [0,1]^n \tag{35}$$

We first note that any decision rule $d^* (m)$ can be implemented using a replacement rule $\theta (\hat{v})$ where the probability of replacing expert i, θ_i, is independent of the reports of all the other agents. It need only be a function of the expert's own report thus one may write $\theta (\hat{v}) = \{\theta_1 (\hat{v}_1) ... \theta_n (\hat{v}_n)\}$.

Remark 2 *Any decision rule $d^* (m)$ that is feasible can be implemented in a mechanism where each expert's probability of being replaced is independent of the reports of other agents $\theta_i \left(\hat{v}_i, \hat{v}'_{-i}\right) = \theta_i \left(\hat{v}_i, \hat{v}''_{-i}\right) \; \forall \hat{v}'_{-i}, \hat{v}''_{-i}$.*

The reasoning is straightforward. An expert's incentive constraint only depends on the agent's expected probability of replacement given his report $\widehat{v}$, $E_{v_{-i}} \theta_i \left(\widehat{v}_i, v_{-i}\right)$. It is always feasible to implement a given $E_{v_{-i}} \theta_i \left(\widehat{v}_i, v_{-i}\right)$ by choosing $\theta_i \left(\widehat{v}_i, v_{-i}\right) = E_{v_{-i}} \theta_i \left(\widehat{v}_i, v_{-i}\right)$. The incentive constraint for an expert, with a given valuation, only depends on the expert's expected probability of being replaced. It is possible to reduce the variability of an experts' probabilities of being replaced when there is the upper bound of replacing the expert with certainty. Thus, the optimal direct revelation mechanism may always take the form whereby an expert which reports a particular valuation will be replaced with a fixed probability independent of the valuations of other experts and whether the expert's preferred decision is taken. Here the continuation value acts like a budget in an auction setting. An analogous type of result is given in Maskin (2000) where it is shown that an all pay auction is optimal for efficiently allocating an object subject to budget/liquidity constraints.

The second result is the main result of this section. It shows that the optimal team size is finite due to the tradeoff between improving the available projects through having more experts and the detriment to communication that this induces in the political capital mechanism.

Proposition 6 *For $0 < p < 1$ the optimal mechanism, amongst symmetric mechanisms, involves a finite number of individuals $n \geq 2$.*

This proposition shows that the decision maker faces a trade-off when increasing the set of experts she consults. The benefit of an additional expert is that this expert may have a better decision at hand. The cost of increasing the number of experts is that this reduces the value of the relationship because it is less likely any single one of them will be able to influence the decision. This reduces the scope for truthful communication and in the limit of large team size no informative communication is possible. The optimal team size is thus finite and balances the beneficial impact of potentially having a better project available against the cost of reducing the scope for communication.

The rationale for a limited team size in our setting is related to some other settings where members of a team face a moral hazard/hold up problem repeatedly over time (for instance Board (2011) Barron (2013)). The rationale for limiting team size is that it divides the value of the team amongst fewer members and hence may allow improved mitigation of the moral hazard and or hold up concern during each period.[5] Our setting has a similar property, however, unlike these other papers the loss of continuation value is not a punishment for a “bad” action or signal, rather it is useful for experts for the purposes of communicating information absent any observation by principal/decision maker.

6 Conclusion

In this paper we have shown how a mechanism that resembles “political capital” may arise to overcome communication frictions between a principal and an expert. The setting we consider allows no scope for informative communication between a decision maker and an expert in a single decision setting due to their mis-aligned interests. In a repeated decision-making setting an expert may credibly communicate information about his current project by making his future participation in decision making conditional on whether it is implemented in the current period. In this sense an agent “spends political capital” in order to influence decision making in the current period. Our analysis of this mechanism finds that when the expert does not place too much weight on the future (resulting in the first best decision rule not being feasible) then the optimal mechanism increases the likelihood relative to the first best that the expert’s project is chosen. We find that this holds both under the assumption that the decision maker can and can not commit to taking ex-post inefficient project choices. In a team setting we find that a finite team size is optimal. Here

[5] On the other hand when experts preferences are not aligned introducing an expert with very difference preferences may in fact provide additional incentives for effort (e.g. Che and Kartik (2009), Dewatripont and Tirole (1999)).

there is a trade-off between improving the available projects by having a larger team and reducing the scope for informative communication in larger teams.

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A Appendix

A.1 Proof of Proposition 1

Proof. The decision maker may either choose the status quo or delegate the decision to the expert. Absent communication the optimal delegation rule is to choose the status quo if $\phi \geq E[v] = \frac{1}{2}$ and delegation if $\phi < \frac{1}{2}$. The expected value of the decision is then $\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{3}{4} = \frac{5}{8}$. The continuation value the expert has from the relationship is $V = \frac{1}{1-p}\frac{1}{2}E[v] = \frac{1}{1-p}\frac{1}{4}$.
Now consider the following mechanism for an exogenous X:
For $\phi \leq pX$

$$\begin{aligned} d(\phi, v) &= \begin{cases} 1 \text{ if } v \geq \phi \\ 0 \text{ if } v < \phi \end{cases} \\ \theta(\phi, v) &= \begin{cases} \frac{v}{pX} \text{ if } v \geq \phi \\ 0 \text{ if } v < \phi \end{cases} \end{aligned}$$

For $pX \leq \phi < \frac{1+pX}{2}$

$$\begin{aligned} d(\phi, v) &= \begin{cases} 1 \text{ if } v \geq pX \\ 0 \text{ if } v < \phi \end{cases} \\ \theta(\phi, v) &= \begin{cases} 1 \text{ if } v \geq pX \\ 0 \text{ if } v < \phi \end{cases} \end{aligned}$$

For $\phi \geq \frac{1+pX}{2}$

$$d(\phi, v) = \theta(\phi, v) = 0$$

This is a feasible mechanism if $X \leq V$. We note that for $X = 0$ the continuation value of the agent $V(0) > 0$ and $V(X)$ is continuous in X. Hence there exists an $X > 0$ such that $X < V(X)$. We now show that for any $X > 0$ this mechanism results in a better expected decision than the no communication decision. The expected value of the decision is:

$$\begin{aligned}
&\left(1-\frac{1+pX}{2}\right)\frac{1+\frac{1+pX}{2}}{2} \\
&+\int_{pX}^{\frac{1+pX}{2}} pXs+(1-pX)\frac{1+pX}{2}ds \\
&+\int_{0}^{pX} s^2+(1-s)\frac{1+s}{2}ds
\end{aligned}$$

evaluating this one finds:

$$\begin{aligned}
&= \frac{1}{2}\left(1-\left(\frac{1+pX}{2}\right)^2\right)+\left(\frac{1+pX}{2}-pX\right)\frac{1-(pX)^2}{2} \\
&\quad +pX\left[\frac{s^2}{2}\right]_{pX}^{\frac{1+pX}{2}} \\
&\quad +\frac{(pX)^3}{3}+\frac{pX}{2}-\frac{(pX)^3}{6} \\
&= \frac{3}{8}-\frac{1}{2}\left(\frac{pX}{2}+\frac{(pX)^2}{4}\right)+\frac{1-pX-(pX)^2+(pX)^3}{4} \\
&\quad +pX\left[\frac{(1+pX)^2}{8}-\frac{pX}{2}\right] \\
&\quad +\frac{(pX)^3}{6}+\frac{pX}{2} \\
&= \frac{5}{8}-\frac{pX}{2}\left[1+\frac{3pX}{4}+\frac{(pX)^2}{2}\right] \\
&\quad +\frac{pX}{2}\left[\frac{1-2pX+(pX)^2}{4}\right] \\
&\quad +\frac{pX}{2}\left(1+\frac{(pX)^2}{3}\right) \\
&= \frac{5}{8}+\frac{pX}{2}\left[\frac{1-pX}{4}+\frac{(pX)^2}{12}\right] > \frac{5}{8}
\end{aligned}$$

■

A.2 Proof of Lemma 1

Proof. The feasiblity constraint for a given (ϕ, v) is:

$$vd(\phi, v) - \int_0^v d(\phi, s)\, ds \leq \frac{p}{1-p}\int_0^1\int_0^1 d(\phi, v)[1-v]\, dvd\phi$$

The right-hand side is the discounted continuation value and integrates across all values of ϕ, hence it is the same for all ϕ. The first best decision rule requires that the left-hand side is equal to ϕ for all (ϕ, v) where $v \geq \phi$ and 0 for $v < \phi$. It is thus immediate that if the constraint is satisfied for a given ϕ' then it is also satisfied for all $\phi'' < \phi'$. ∎

A.3 Proof of Proposition 2

Proof. The continuation value under the first best decision rule is given by

$$\begin{aligned}
V &= \frac{1}{1-p}\int_0^1\int_0^1 d^{FB}(\phi, v)[1-v]\, dvd\phi \\
&= \frac{1}{1-p}\int_0^1\int_\phi^1 [1-v]\, dvd\phi \\
&= \frac{1}{1-p}\left[\phi - \frac{\phi^2}{2} - \frac{\phi - \frac{\phi^3}{3}}{2}\right]_0^1 \\
&= \frac{1}{1-p}\frac{1}{6}
\end{aligned}$$

From lemma 1 we need only check the case for $\phi = 1:$

$$\begin{aligned}
\frac{p}{1-p}\frac{1}{6} &\geq 1 \\
p &\geq \frac{1}{7}
\end{aligned}$$

∎

A.4 Proof of Proposition 3

Proof. We first show that if the optimal decision rule in the second best results in the expert's decision weakly less than $\frac{1}{2}$ the time then the discounted continuation value is less than 1. ∎

Lemma 3 *For $p < \frac{1}{7}$ any optimal $\int_0^1\int_0^1 d^*(\phi, v) \leq \frac{1}{2}$ implies that $V < 1$.*

Proof. If not then $d = 1$ is optimal for any $v \geq \phi$ which immediately implies $\int_0^1\int_0^1 d^*(\phi, v) > \frac{1}{2}$ since the first best decision rule $d^{FB} = 1$ if $v \geq \phi$ $d^{FB} = 0$ if $v < \phi$ gives $\int_0^1\int_0^1 d^{FB}(\phi, v) = \frac{1}{2}$ but is not feasible. ∎

Given Lemma 3 we proceed with the proof under the condition that the continuation value is less than one.

First consider the following problem for an exogenously given X (we will refer to it as Problem X):

$$\max_{d(\phi,v)} \frac{1}{2} + \int_0^1 \int_0^1 [v-\phi]\, d(\phi,v)\, dv d\phi$$

subject to

$$vd(\phi,v) - \int_0^v d(\phi,s)\, ds \leq X \text{ for all } v,\phi.$$

$$0 \leq d(\phi,v) \leq 1$$

For $X \geq 1$ we are able to implement the first best decision rule in this problem. However, for $X < 1$ it is not possible. In this problem, where X is exogenous, it suffices to maximize the objective for each ϕ. For a fixed ϕ

$$\max_{d(\phi,v)} \frac{1}{2} + \int_0^1 [v-\phi]\, d(\phi,v)\, dv$$

subject to

$$vd(\phi,v) - \int_0^v d(\phi,s)\, ds \leq X \text{ for all } v.$$

$$0 \leq d(\phi,v) \leq 1$$

If $X \geq \phi$ the optimal decision rule is the first best $d(\phi,v) = 1$ for $v \geq \phi$ and $d(\phi,v) = 0$ for $v < \phi$. If $X < \phi$, then this first best is not feasible. It is immediate that $d(\phi,v)$ is chosen to be its maximum feasible value for $v > \phi$ because this relaxes the feasability constraint for higher values of v and improves the objective. This also means that $\frac{dd(\phi,v)}{dv} = 0$ and $d(\phi,v) = \overline{d}(X,\phi) = \frac{X+\int_0^\phi d(\phi,v)dv}{\phi}$ for all values of $v \geq \phi$.

Lemma 4 *The optimal decision rule $d^X(\phi,v)$ for Problem X takes the following form, $d^X(\phi,v) = \overline{d}$ for all values $v \geq \overline{v}$ and $d^X(\phi,v) = 0$ for $v < \overline{v}$.*

Proof. Consider a different scheme where $\exists [v_1, v_2]$ such that $d(\phi,v) = \overline{d}$ for $v > v_2$, $0 < d(\phi,v) < \overline{d}$ for $v \in [v_1,v_2]$ and $d(\phi,v) = 0$ for $v < v_1$. Let $D = \int_{v_1}^{v_2} d(\phi,v)\, dv$. A feasible scheme is $d(\phi,v) = \overline{d}$ for $v \geq v_2 - \frac{\overline{d}}{D}$, and $d(\phi,v) = 0$ for $v < v_2 - \frac{\overline{d}}{D}$. This scheme improves the objective because it places more weight on higher values of v. ■

We now characterize the optimal scheme for Problem X. We find the level $\overline{d}(X,\phi)$ and the cutoff $\overline{v}(X,\phi)$ for each value of ϕ as a function of X.

Note that we may maximize the objective of Problem X pointwise. For values of $\phi \leq X$ we may implement the first best rule where $d = 1$ for all $v \geq \phi$ and $d = 0$ for $v < \phi$. The objective, when $\phi > X$ may be written as

$$
\begin{aligned}
& \max_{\overline{v}} \frac{1}{2} + \overline{d} \int_{\overline{v}}^{1} (v - \phi)\, dv \\
= \; & \max_{\overline{v}} \frac{1}{2} + \overline{d}\,(1 - \overline{v}) \left[\frac{1 + \overline{v}}{2} - \phi \right]
\end{aligned}
$$

FOC using $\overline{d} = \frac{X}{\overline{v}}$

$$
\begin{aligned}
-\frac{1}{\overline{v}^2} \left[\frac{1 + \overline{v}}{2} - \phi \right] + \frac{1}{2} \left[\frac{1}{\overline{v}} - 1 \right] &= 0 \\
-\frac{\frac{1}{2} - \phi}{v^2} &= \frac{1}{2} \\
\frac{\overline{v}}{2} (1 - \overline{v}) &= \frac{1 + \overline{v}}{2} - \phi \\
\overline{v}^2 &= 2\phi - 1
\end{aligned}
$$

SOC

$$
\begin{aligned}
& \partial \frac{-\frac{1}{\overline{v}^2} \left[\frac{1+\overline{v}}{2} - \phi \right] + \frac{1}{2} \left[\frac{1}{\overline{v}} - 1 \right]}{\partial \overline{v}} \\
= \; & \frac{2}{v^3} \left[\frac{1 + \overline{v}}{2} - \phi \right] - \frac{1}{v^2} \\
= \; & \frac{1}{v^3} [1 - 2\phi] < 0 \text{ for } \phi > \frac{1}{2}
\end{aligned}
$$

Hence for $X < 1$ the decision rule d^X is characterised by the following $\overline{d}(\phi)$ and $\overline{v}(\phi)$:

$$
\begin{aligned}
\overline{v}(\phi) &= \begin{cases} \sqrt{2\phi - 1} \text{ if } \frac{X^2+1}{2} < \phi \\ X \text{ if } \phi > X \geq \sqrt{2\phi - 1} \\ \phi \text{ if } \phi > X \end{cases} \\
\overline{d}(\phi) &= \begin{cases} \frac{X}{\sqrt{2\phi - 1}} \text{ for } \phi > \frac{X^2+1}{2} \\ 1 \text{ for } \phi \leq \frac{X^2+1}{2} \end{cases}
\end{aligned}
$$

Lemma 5 *The probability of a decision by the expert in the optimal decision rule for Problem X is* $\int_0^1 \int_0^1 d^X(\phi, v)\, dv d\phi = \frac{1}{2}$.

Proof. The probability of a decision by the expert is:

$$\begin{aligned}
&\int_0^1 \int_{\overline{v}(\phi)}^1 \overline{d}(\phi)\, dv d\phi \\
= &\int_0^X \int_{\phi}^1 1 dv d\phi + \int_X^{\frac{X^2+1}{2}} \int_X^1 1 dv d\phi + \int_{\frac{X^2+1}{2}}^1 \int_{\sqrt{2\phi-1}}^1 \frac{X}{\sqrt{2\phi-1}} dv d\phi \\
= &X\left[1-\frac{X}{2}\right] + [1-X]\left[\frac{X^2+1}{2} - X\right] + X\int_{\frac{X^2+1}{2}}^1 \left(\frac{1}{\sqrt{2\phi-1}} - 1\right) d\phi \\
= &X - \frac{X^2}{2} + \frac{[1-X]^3}{2} + X\left(\left[\sqrt{2\phi-1}\right]_{\frac{X^2+1}{2}}^1 - \left[1 - \frac{X^2+1}{2}\right]\right) \\
= &X - \frac{X^2}{2} + \frac{[1-X]^3}{2} + X\left([1-X] - \left[1-\frac{X^2+1}{2}\right]\right) \\
= &X - \frac{X^2}{2} + \frac{[1-X]^3}{2} + X\left(\frac{[1-X]^2}{2}\right) \\
= &X - \frac{X^2}{2} + \frac{[1-X]^2}{2} = X + \frac{1-2X}{2} = \frac{1}{2}
\end{aligned}$$

■

Now return to the proof of the proposition. Define the continuation value of the optimal decision rule by V^*.

Lemma 6 *$d^*(\phi, v) \geq \overline{d}(V^*, \phi)$ for $v > \phi$ and $\int_0^\phi d^*(\phi, v)\, dv \geq \overline{d}(\rho V^*, \phi)(\phi - \overline{v}(V^*, \phi))$*

Proof. $d^*(\phi, v)$ can not be less than $\overline{d}(V^*, \phi)$ for $v > \phi$. If $\int_0^\phi d^*(\phi, v)\, dv \geq \overline{d}(\phi - \overline{v})$ then it is feasible to increase $d^*(\phi, v)$ to $\overline{d}(V^*, \phi)$ for $v > \phi$ and improve the objective function. If $\int_0^\phi d(\phi, v)\, dv < \overline{d}(\phi - \overline{v})$, let $\Delta = \overline{d}(\phi - \overline{v}) - \int_0^\phi d(\phi, v)\, dv$, then it is possible to increase $d^*(\phi, v)$ for $v \in [\overline{v}, \phi]$ so that $\int_0^\phi d^*(\phi, v) = \overline{d}(\phi - \overline{v})$ in which case it is also feasible to choose $d^* = \overline{d}$ for $v \geq \phi$. This improves the objective (and the continuation value), if not it would be a contradiction of $\overline{d}$ being the optimal solution to Problem X. ■

Lemma 6 establishes that $\int_0^1 \int_0^1 d^*(\phi, v)\, dv d\phi \geqslant \int_0^1 \int_0^1 d^{\rho V^*}(\phi, v)\, dv d\phi = \frac{1}{2}$ because $\int_0^1 \int_\phi^1 d^*(\phi, v)\, dv d\phi = \int_0^1 \int_\phi^1 d^{\rho V^*}(\phi, v)\, dv d\phi$ and $\int_0^1 \int_0^\phi d^*(\phi, v)\, dv d\phi \geqslant \int_0^1 \int_0^\phi d^{\rho V^*}(\phi, v)\, dv d\phi$. We now show that the second inequality is strict by showing that one can improve on any decision rule where $d^*(\phi, v) = d^X(\phi, v)$ for $v \geq \phi$ and $\int_0^\phi d^*(\phi, v)\, dv = \int_0^\phi d^X(\phi, v)\, dv$ for $v < \phi$. Consider the following decision rule, for $\varepsilon < \phi < \rho V^* < 1$:

$$\begin{aligned}
d^*(\phi, v) &= 1 \text{ for } v \geq \phi - \varepsilon \\
d^*(\phi, v) &= 0 \text{ for } v < \phi - \varepsilon
\end{aligned}$$

where $\varepsilon > 0$. First note that $d^{\rho V^*}(\phi, v) = 0$ for $v < \phi$ and $\phi < \rho V^*$. Hence this is a strict increase in the amount of decisions taken by the expert. Second note that this change is feasible. Third it reduces the objective function by a term $\frac{\varepsilon}{2} \times \varepsilon \times [V - \varepsilon] \approx \frac{\varepsilon^2}{2} V$ for small ε. Finally it increases the continuation value by $\frac{p}{1-p} \int_\varepsilon^V \int_{\phi-\varepsilon}^{\phi} [1-v]\, dv d\phi = \frac{p\varepsilon}{1-p} \left[(V-\varepsilon)\left(1+\frac{\varepsilon}{2}\right) - \frac{V^2-\varepsilon^2}{2} \right] \approx \frac{p\varepsilon}{1-p} V \left(1 - \frac{V}{2}\right)$ for small ε. This now makes it feasible to make a second change where d^* is increased to

$$\widetilde{d}(\phi, v) = \frac{V + \frac{p\varepsilon}{1-p} V \left(1 - \frac{V}{2}\right)}{\sqrt{2\phi - 1}} \text{ for } v \geq \sqrt{2\phi - 1} \text{ and } \phi \geq \frac{\left(V + \frac{p\varepsilon}{1-p} V \left(1 - \frac{V}{2}\right)\right)^2 + 1}{2}$$

This improves the objective function by

$$\begin{aligned}
& \frac{p\varepsilon}{1-p} V \left(1 - \frac{V}{2}\right) \int_{\frac{\left(V + \frac{p\varepsilon}{1-p} V \left(1 - \frac{V}{2}\right)\right)^2 + 1}{2}}^{1} \int_{\sqrt{2\phi-1}}^{1} \frac{[v - \phi]}{\sqrt{2\phi - 1}} dv d\phi \\
& \frac{p\varepsilon}{1-p} V \left(1 - \frac{V}{2}\right) \int_{\frac{\left(V + \frac{p\varepsilon}{1-p} V \left(1 - \frac{V}{2}\right)\right)^2 + 1}{2}}^{1} \left[\frac{\frac{v^2}{2} - \phi v}{\sqrt{2\phi - 1}} \right]_{\sqrt{2\phi-1}}^{1} d\phi \\
& \frac{p\varepsilon}{1-p} V \left(1 - \frac{V}{2}\right) \int_{\frac{\left(V + \frac{p\varepsilon}{1-p} V \left(1 - \frac{V}{2}\right)\right)^2 + 1}{2}}^{1} \left[\frac{(1-\phi) - \phi\left(1 - \sqrt{2\phi - 1}\right)}{\sqrt{2\phi - 1}} \right] d\phi \\
& \frac{p\varepsilon}{1-p} V \left(1 - \frac{V}{2}\right) \int_{\frac{\left(V + \frac{p\varepsilon}{1-p} V \left(1 - \frac{V}{2}\right)\right)^2 + 1}{2}}^{1} \left[\phi - \sqrt{2\phi - 1} \right] d\phi \\
& \frac{p\varepsilon}{1-p} V \left(1 - \frac{V}{2}\right) \left[\frac{\phi^2}{2} - \frac{1}{3} (2\phi - 1)^{\frac{3}{2}} \right]_{\frac{\left(V + \frac{p\varepsilon}{1-p} V \left(1 - \frac{V}{2}\right)\right)^2 + 1}{2}}^{1} \\
& \frac{p\varepsilon}{1-p} V \left(1 - \frac{V}{2}\right) \left[\frac{1 - \left(\frac{\left(V + \frac{p\varepsilon}{1-p} V \left(1 - \frac{V}{2}\right)\right)^2 + 1}{2} \right)^2}{2} - \frac{1}{3} \left[1 - \left(V + \frac{p\varepsilon}{1-p} V \left(1 - \frac{V}{2}\right) \right)^3 \right] \right] \\
\approx \; & \frac{p\varepsilon}{1-p} V \left(1 - \frac{V}{2}\right) \left[\frac{1 - \left(\frac{V^2+1}{2}\right)^2}{2} - \frac{1}{3}\left(1 - V^3\right) \right]
\end{aligned}$$

We see that the first change decreases the objective by a term which is $o\left(\varepsilon^2\right)$ and the second change increases it by a term which is $o(\varepsilon)$. Thus for small ε the net effect will be positive. Thus, we have strictly improved the decision rule and established that the optimal decision mechanism is one where $\int \int d^*(\phi, v)\, dv d\phi > \frac{1}{2}$.

A.5 Proof of Proposition 4

The continuation values are determined by:

$$\begin{bmatrix} V_h \\ V_l \end{bmatrix} = \begin{bmatrix} \beta & 1-\beta \\ 1-\beta & \beta \end{bmatrix} \begin{bmatrix} \int_0^{\psi_h} \int_0^{\psi_h} \left[1 - \frac{v}{\psi_h}\right] \frac{1}{\psi_h} d(\phi, v, h)\, dv d\phi + \beta p V_h + (1-\beta) p V_l \\ \int_0^1 \int_0^1 [1 - v]\, d(\phi, v, l)\, dv d\phi + (1-\beta) p V_h + \beta p V_l \end{bmatrix}$$

the continuation value using the first best decision rule

$$\begin{aligned}
&\begin{bmatrix} \int_0^{\psi_h}\int_0^{\psi_h}\left[1-\frac{v}{\psi_h}\right]\frac{1}{\psi_h}d\left(\phi,v,h\right)dvd\phi \\ \int_0^1\int_0^1\left[1-v\right]d\left(\phi,v,l\right)dvd\phi \end{bmatrix} \\
=&\begin{bmatrix} \int_0^{\psi_h}\int_\phi^{\psi_h}\left[1-\frac{v}{\psi_h}\right]\frac{1}{\psi_h}dvd\phi \\ \int_0^1\int_\phi^1\left[1-v\right]dvd\phi \end{bmatrix} \\
=&\begin{bmatrix} \frac{1}{\psi_h}\int_0^{\psi_h}\left[\psi_h-\phi-\frac{\psi_h^2-\phi^2}{2\psi_h}\right]d\phi \\ \frac{1}{6} \end{bmatrix} \\
=&\begin{bmatrix} \frac{1}{\psi_h}\left[\psi_h\phi-\frac{\phi^2}{2}-\frac{\phi\psi_h^2-\frac{\phi^3}{3}}{2\psi_h}\right]_0^{\psi_h} \\ \frac{1}{6} \end{bmatrix} \\
=&\begin{bmatrix} \frac{\psi_h}{6} \\ \frac{1}{6} \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
&\begin{bmatrix} 1-p\beta & -p\left(1-\beta\right) \\ -p\left(1-\beta\right) & 1-p\beta \end{bmatrix}^{-1}\begin{bmatrix} \beta & 1-\beta \\ 1-\beta & \beta \end{bmatrix} \\
&\frac{1}{\left(1-p\beta\right)^2-p^2\left(1-\beta\right)^2}\begin{bmatrix} 1-p\beta & p\left(1-\beta\right) \\ p\left(1-\beta\right) & 1-p\beta \end{bmatrix}\begin{bmatrix} \beta & 1-\beta \\ 1-\beta & \beta \end{bmatrix} \\
&\frac{1}{\left(1-p\beta\right)^2-p^2\left(1-\beta\right)^2}\begin{bmatrix} \beta\left(1-p\beta\right)+p\left(1-\beta\right)^2 & \left(1-\beta\right)\left(1-p\beta\right)+p\beta\left(1-\beta\right) \\ \left(1-\beta\right)\left(1-p\beta\right)+p\beta\left(1-\beta\right) & \beta\left(1-p\beta\right)+p\left(1-\beta\right)^2 \end{bmatrix} \\
&\frac{1}{\left(1-p\beta\right)^2-p^2\left(1-\beta\right)^2}\begin{bmatrix} \beta-p\left(1-2\beta\right) & 1-\beta \\ 1-\beta & \beta-p\left(1-2\beta\right) \end{bmatrix}
\end{aligned}$$

Evaluating

$$\begin{aligned}
\begin{matrix} V_h \\ V_l \end{matrix} &= \frac{1}{6}\frac{1}{\left(1-p\beta\right)^2-p^2\left(1-\beta\right)^2}\begin{bmatrix} \left(\beta-p\left(1-2\beta\right)\right)\psi_h+1-\beta \\ \psi_h\left(1-\beta\right)+\beta-p\left(1-2\beta\right) \end{bmatrix} \\
&= \frac{1}{6}\frac{1}{1-p^2-2\beta p\left(1-p\right)}\begin{bmatrix} \left(\beta-p\left(1-2\beta\right)\right)\psi_h+1-\beta \\ \psi_h\left(1-\beta\right)+\beta-p\left(1-2\beta\right) \end{bmatrix}
\end{aligned}$$

we see that V_h is increasing in β and V_l is decreasing in β. The ratio $\frac{V_h}{V_l}=\frac{(\beta-p(1-2\beta))\psi_h+1-\beta}{\psi_h(1-\beta)+\beta-p(1-2\beta)}$ is thus increasing in β and is equal to ψ_h at $\beta=1$.
The feasibility constraints are:

$$\phi d\left(\phi,\phi,\alpha\right)\leq pV_\alpha$$

The constraints which are the most difficult to satisfy are those where $\phi = \psi_h, 1$. These two constraints are then

$$\psi_h \leq pV_h \tag{36}$$

and

$$1 \leq pV_l \tag{37}$$

we note that if equation 36 is satisfied then this implies that equation 37 is also satisfied. It thus suffices to satisfy equation 36. The feasible region is $p \geq p^{FB}(\beta)$ where $p^{FB}(\beta)$ satisfies

$$\psi_h = p^{FB} V_h \left(\beta, p^{FB}\right)$$

Implicit differentiation gives:

$$\begin{aligned}
F(\beta, p) &= \frac{1}{6} \frac{p}{1 - p^2 - 2\beta p (1-p)} \left[(\beta - p(1-2\beta)) \psi_h + 1 - \beta \right] \\
\frac{\partial F}{\partial \beta} &= \frac{1}{6} \frac{p}{1 - p^2 - 2\beta p (1-p)} \left[(1+2p) \psi_h - 1 \right] \\
&\quad + \frac{p}{6} \left(\frac{2p(1-p)}{(1 - p^2 - 2\beta p (1-p))^2} \right) \left[(\beta - p(1-2\beta)) \psi_h + 1 - \beta \right] \\
&> 0 \\
\frac{\partial F}{\partial p} &= \frac{1}{6} \frac{2p(1-\beta) + 2\beta}{(1 - p^2 - 2\beta p (1-p))^2} p \left[(\beta - p(1-2\beta)) \psi_h + 1 - \beta \right] \\
&\quad + \left[\left[(\beta 2p (2\beta - 1)) \psi_h + 1 - \beta \right] \right] \frac{1}{6} \frac{1}{1 - p^2 - 2\beta p (1-p)} \\
&> 0
\end{aligned}$$

hence we conclude that:

$$\frac{dp^{FB}}{d\beta} = -\frac{\frac{\partial F}{\partial \beta}}{\frac{\partial F}{\partial p}} < 0$$

A.6 Proof of Lemma 2

Proof. The decision maker's problem is

$$\max_{d(\phi, m(\phi, v)), m(\phi, v)} \frac{1}{2} + \int_0^1 \int_0^1 [v - \phi] \, d(\phi, m(\phi, v)) \, dv d\phi$$

subject to

$$vd(\phi, m(\phi, v)) - \int_0^v d(\phi, m(\phi, s)) \, ds \leq \frac{p}{1-p} \int_0^1 \int_0^1 d(\phi, m(\phi, v)) [1 - v] \, dv d\phi$$

$$
\begin{aligned}
d(\phi_t, m_t) &= 1 \text{ if } E[v_t|m_t, \phi_t] \geq \phi_t \\
d(\phi_t, m_t) &= 0 \text{ if } E[v_t|m_t, \phi_t] \leq \phi_t \\
0 &\leq d(m_t) \leq 1 \text{ if } E[v_t|m_t, \phi_t] = \phi_t
\end{aligned}
$$

and monotonicity

$$
d\left(\phi, m\left(\phi, v'\right)\right) \geq d\left(\phi, m\left(\phi, v''\right)\right) \text{ for } v' > v''
$$

We first prove the following lemma. ■

Lemma 7 *Supppose* $\{d^*(\phi, m^*(\phi, v)), \theta^*(\phi, m^*(\phi, v)), m^*(\phi, v)\}$ *achieves the optimum and* $\exists v'', v'$ *such that* $m^*(\phi, v') = m^*(\phi, v'')$ *for* $v'' > v'$ *then* $\exists m^{**}(\phi, v)$ *such that* $m^{**}(\phi, s) = m(\phi, v') = m(\phi, v'')$ *for the interval* $v' \leq s \leq v''$ *and* $d^*(\phi, m^*(\phi, v)) = d^*(\phi, m^{**}(\phi, v))$ *is also an optimizer of the decision maker's problem.*

Proof. Suppose $\exists \tilde{v} : v' < \tilde{v} < v''$ such that $m(\phi, v') \neq m(\phi, \tilde{v})$. If $d(\phi, m(\phi, v')) = d(\phi, m(\phi, \tilde{v}))$ then the consequences of announcing either message are the same so we can simply relabel one of the messages such that $m(\phi, v') = m(\phi, \tilde{v})$ for all $\tilde{v} : v' < \tilde{v} < v''$. Otherwise if $d(\phi, m(\phi, v')) = d(\phi, m(\phi, v'')) \neq d(\phi, m(\phi, \tilde{v}))$ our mechanism violates the monotonicity constraint and cannot be a solution to the optimization.

The lemma implies that we can focus on intervals of valuations sending each message. ■

Lemma 8 *For a given level of* ϕ *the optimal decision rule takes at most three values of* $d \in \{0, \omega, 1\}$ *where* $0 < \omega < 1$.

Proof. Suppose that there were four or more messages, hence at least two messages correspond to different levels of mixing. However in this instance the commitment constraint requires that for each message that corresponds to the different mixing level of d that we have $E[v_t|m_t, \phi_t] = \phi_t$. This immediately implies that the monotonicity constraint is violated since valuations both above and below ϕ_t must send each message for this to be true. Thus the decision rule is not a solution to the decision maker's problem.

We note at this point that we have shown that there are three or fewer possible values of d which can be implemented for a given value of ϕ. It is thus possible to use three messages to implement such a mechanism where each message corresponds to a given value of d. We now proceed to show that an optimal decision rule involves only two possible values of d for each value of ϕ and may thus be implemented using only two messages. Suppose not, then for a given $\hat{\phi}$ there exists messages m_t', m_t'', m_t''' and cut offs v', v'' such that

$$
m\left(\hat{\phi}, v\right) = \begin{cases} m' \text{ for } v \in [0, v'] \\ m'' \text{ for } v \in (v', v''] \\ m''' \text{ for } v \in (v'', 1] \end{cases}
$$

$E\left[v_t|m_t'',\hat{\phi}\right] = \hat{\phi}$; $d\left(\hat{\phi},m_t'\right) = 0$; $0 < d\left(\hat{\phi},m_t''\right) < 1$; and $d\left(\hat{\phi},m_t'''\right) = 1$. Now consider increasing $d\left(\hat{\phi},m_t''\right)$ to 1. This change has no direct effect on the objective function because $E\left[v_t|m_t'',\phi_t\right] = \phi_t$; it does not violate the commitment constraint since the same set of values choose the message; it does not violate the monotonicity constraint since $d\left(\phi,m_t'''\right) = 1$. It has a number of effects on the feasibility and incentive constraint: it increases the right hand side for all values $(\phi,v) \in [0,1] \times [0,1]$; it has no effect on the left hand side for all (ϕ,v) where $\phi \neq \hat{\phi}$ or $v \leq v'$; it reduces the left handside for all $(\phi,v) \in \left\{\hat{\phi}\right\} \times (v'',1]$. Therefore in all these case this change continues to satisfy all the constraints and does not reduce the objective function. The remaining case to check is the modified feasibility constraint for $(\phi,v) \in \left\{\hat{\phi}\right\} \times (v',v'']$. We note that this constraint is slack when $d\left(\hat{\phi},m_t''\right) < 1$. To see this note that the modified feasibility constraint is satisfied for $v > v''$ and $\phi = \hat{\phi}$ where $m\left(\hat{\phi},v\right) = m_t'''$ and $d\left(\hat{\phi},m'''\right) = 1$; and

$$v - \int_0^v d\left(\hat{\phi},m\left(\hat{\phi},s\right)\right) ds \leq pV$$

We note that the left hand side of this constraint:

$$vd\left(\phi,m\left(\phi,v\right)\right) - \int_0^v d\left(\phi,m\left(\phi,s\right)\right) ds$$

is increasing in v and hence it is satisfied for any $v < v''$ where $d\left(\hat{\phi},m_t''\right) \leq 1$. We conclude that increasing is $d\left(\hat{\phi},m_t''\right)$ to 1 is possible without violating any of the constraints and without reducing the objective.

We now show that when we can make this change for a positive measure of ϕ, denote this set by Φ, then we can improve the objective function. When we make this change for a positive measure of ϕ then we increase the continuation value of the relationship V:

$$V = \frac{1}{1-p} \int \int d\left(\phi,m\left(v\right)\right)(1-v)\,dvd\phi$$

Denote the increase in the continuation value by Δ and write the new continuation value as $V + \Delta$. As argued above the prescribed change does not increase the feasibility and incentive constraint above V for any (ϕ,v). All of these constraints are thus slack. We can now improve the objective by choosing a small enough $\varepsilon > 0$ and changing the message sent for all $\phi \in \Phi$ and $v \in \left(v'\left(\phi\right), v'\left(\phi\right) + \varepsilon\right)$ from m'' to m' such that $d\left(\phi,m\left(\phi,v\right)\right) = 0$. This improves the objective because for small enough ε, $v'\left(\phi\right) + \varepsilon < \phi$. We have shown that any solution to the decision makers problem that has three different values of d for a positive measure of ϕ can be improved upon. Thus the solution to the decision makers problem involves at most two different value of d for each ϕ and there is therefore a solution involving only two messages being used. ■

A.7 Proof of Proposition 5

The previous lemma establishes that the optimal mechanism may be characterized by two messages, we denote these by $m = 0, 1$. Monotonicity of the decision rule then implies that there is a cutoff $\widehat{v}(\phi_t)$ such that $m(\phi_t, v) = 0$ for $v < \widehat{v}(\phi_t)$ and $m(\phi_t, v) = 1$ for $v \geq \widehat{v}(\phi_t)$, where we have arbitrarily assigned $m = 1$ to the values above the cutoff. We proceed by proving a sequence of lemmas.

Lemma 9 *The decision rule for the lower message results in the outside option being chosen* $d(\phi, 0) = 0$.

Proof. If not then $0 < d(\phi, 0) < 1$ and $d(\phi, 1) = 1$ (note if $m(\phi, v)$ is constant for all v then we assume $m = 1$). When this is case the commitment constraint requires that $E[v|\phi, m = 0] = \phi$ and it is feasible to increase $d(\phi, 0)$ to 1 without affecting the objective. This change strictly increasing the continuation value V. In this case one can then improve the objective function by setting the cutoff to some $\widehat{v}(\phi_t) = \varepsilon$ where $\phi_t > \varepsilon > 0$ and $d(\phi, 0) = 0, d(\phi, 1) = 1$. This improves the objective. It decreases the continuation value of the expert but is feasible provided that ε is chosen small enough such that the continuation value does not decrease below the level it was at prior to increasing $d(\phi, 0)$ to 1. ■

Lemma 10 *The second best decision rule does not result in both* $\int_0^1 \int_0^1 \widetilde{d}(\phi, m(\phi, v))\, dv d\phi \leq \frac{1}{2}$ *and* $pV \geq 1$.

Proof. Suppose $pV \geq 1$ then it is feasible to implement $d(\phi, 1) = 1$ for any choice of cutoff. If $d(\phi, 1) < 1$ for any ϕ then an improvement may be made to the objective function by first increasing it to 1.

This then means $pV > 1$ and the objective may be strictly imporved by increasing any cutoff $\hat{v}(\phi) < \phi$. Thus either we are in the first best where $\hat{v}(\phi) = \phi$ or $\int_0^1 \int_0^1 \widetilde{d}(\phi, m(\phi, v))\, dv d\phi > \frac{1}{2}$. ■

Lemma 11 *Suppose* $pV < 1$ *then for any message where* $0 < d(\phi, m) < 1$ *the feasibility constraint binds for this message.*

Proof. If not it is feasible to increase $d(\phi, m)$ to either 1 or until the feasibility constraint does bind at $\frac{pV}{\widehat{v}}$. This increases the continuation value. One can then improve the objective function in the following way:$\phi \in (pV, pV + \varepsilon]$ where $\varepsilon < \frac{1-pV}{2}$, $0 < \varepsilon < \Delta$, Δ is the improvement in the objective function due to increasing d, and (by assumption) $pV < 1$. For ϕ in this interval $\widehat{v}(\phi) < \phi$, since either $E[v|\phi, m] = \phi$ if $d(\phi, 1) < 1$ or if $d(\phi, 1) = 1$ then $\widehat{v}(\phi) = \phi$ does not satisfy the feasibility constraint. In either case $\exists \varepsilon'$ such that it is feasible to choose $d(\phi, 1) = 1$ and to increase the cutoff to $\widehat{v}(\phi) + \varepsilon'$. This results in a strict improvement in the objective function when ε' is chosen small enough since $\phi > pV > \widehat{v}(\phi)$ and hence we can choose $\varepsilon' > 0$ such that $\phi > \widehat{v}(\phi) + \varepsilon'$. ■

Lemma 12 *The second best does not result in* $\int_0^1 \int_0^1 \widetilde{d}(\phi, m(\phi, v))\, dv d\phi \leq \frac{1}{2}$ *and* $pV < 1$.

Proof. The commitment constraint for the decision maker implies that $2\phi - 1 \leq \widehat{v}(\phi_t) \leq 2\phi_t$. Suppose the optimal mechanism, described by a threshold function $\widetilde{v}(\phi)$ exhibits $\int_0^1 \int_0^1 \widetilde{d}(\phi, m(\phi, v))\, dv d\phi < \frac{1}{2}$ and it generates a continuation value X. Now consider the following cutoffs:

$$\widehat{v}(\phi) = \begin{cases} \phi \text{ if } \phi \leq pX \\ pX \text{ if } pX < \phi \leq \frac{1+pX}{2} \\ 1 \text{ if } \frac{1+pX}{2} \leq \phi \end{cases}$$

and decision rule:

$$\begin{aligned} \widehat{d}(\phi, 1) &= 1 \\ \widehat{d}(\phi, 0) &= 0 \end{aligned}$$

These candidate cutoffs and decision rule satisfy the feasibility, mononicity and commitment constraints and result in $\int_0^1 \int_0^1 d(\phi, m(\phi, v))\, dv d\phi = \frac{1}{2}$. We will now show that for each value of ϕ this decision rule will either result in fewer decisions than $\widetilde{d}$ or would improve both the objective function and continuation value of the relationship relative to $\widetilde{d}$.

Case 1) For $\phi \leq pX$

If $\widetilde{v}(\phi) < \widehat{v}(\phi)$ then by lemma 11 $\widetilde{d}(\phi, 1) = 1$ because the feasibility constraint does not bind for any $d \leq 1$ when $\widetilde{v}(\phi) < \frac{pV+1}{2}$, thus $\int_0^1 \widetilde{d}(\phi, m(\phi, v))\, dv > \int_0^1 \widehat{d}(\phi, m(\phi, v))\, dv$. If $\widetilde{v}(\phi) > \widehat{v}(\phi)$ then it is immediate that the objective and the continuation value may be increased by switching to $\widehat{v}, \widehat{d}$.

Case 2) For $X < \phi < \frac{1+pX}{2}$

If $\widetilde{v}(\phi) < \widehat{v}(\phi)$ then by lemma 11 $\widetilde{d}(\phi, 1) = 1$ because the feasibility constraint does not bind for any $d \leq 1$ when $\widetilde{v}(\phi) < \frac{pV+1}{2}$, thus $\int_0^1 \widetilde{d}(\phi, m(\phi, v))\, dv > \int_0^1 \widehat{d}(\phi, m(\phi, v))\, dv$. It is not feasible to specify $\widetilde{v}(\phi) > \widehat{v}(\phi)$ because $E[v|m'', \phi] > \phi_t$ which implies that $d(\phi, m'') = 1$ which breaches the feasibility constraint because:

$$\widetilde{v}(\phi) > pX$$

Case 3) For $\phi \geq \frac{1+pX}{2}$

In this case $\int_0^1 \widehat{d}(\phi, m(\phi, v))\, dv = 0$, thus $\int_0^1 \widetilde{d}(\phi, m(\phi, v))\, dv \geq \int_0^1 \widehat{d}(\phi, m(\phi, v))\, dv$. Hence either $\int_0^1 \int_0^1 \widetilde{d}(\phi, m(\phi, v))\, dv d\phi \geq \frac{1}{2}$ or it is not optimal.

It remains to be shown that there exists a decision rule d^* that is better than any decision rule of the form of $\widehat{d}$ and where $\int_0^1 \int_0^1 d^*(\phi, m(\phi, v))\, dv d\phi > \frac{1}{2}$. Take any feasible decision rule of the form of $\widehat{d}$ where again we denote the contuation value by X. Consider changing the cutoffs for $\phi > \frac{pX+1}{2}$ from $\widehat{v} = 1$ to $\widehat{v} = 2\phi - 1$ and the decision rule to $d(\phi, 1) = \frac{pX}{\hat{v}}$. Note that $E[v|\phi, 1] = \phi$ hence the change has no effect on the objective function. It does improve the continuation value because the expert's preferred decision is now chosen in some cases, denote this improvement by Δ. There now exists $\omega > 0$ such that we can change the cutoff to $\widehat{v}(\phi) = \phi$ and the decision rule to $d(\phi, 1) = 1$ for $pX < \phi \leq pX + \omega$ such that the feasibility constraints are satisfied. This strictly improves the cutoff because it implements $\max(\phi, v)$ when $(\phi, v) \in [pX, pX + \Delta] \times [pX, pX + \omega]$ whereas the original decision rule implemented v over this set.

The proof of the proposition now follows directly from Lemmas 10 and 12. ■

A.8 Proof of Remark 2

Proof. Consider an optimal mechanism d^*, θ^* such that $\theta_i^*(v_i, v_{-i}) \neq \theta_i^*(v_i, \widetilde{v}_{-i})$ for some $v_i \in [0,1]$ and $v_{-i}, \widetilde{v}_{-i} \in [0,1]^{n-1}$. In this case $\theta_i^*(v_i, v_{-i}) \leq 1$ and $E_{v_{-i}}\theta_i^*(v_i, v_{-i}) \leq 1$. A mechanism $\widehat{\theta}_i(v_i, v_{-i}) = E_{v_{-i}}\theta_i^*(v_i, v_{-i})$ is feasible since $E_{v_{-i}}\theta_i^*(v_i, v_{-i}) \leq 1$ and $E_{v_{-i}}\widehat{\theta}_i(v_i, v_{-i}) = E_{v_{-i}}\theta_i^*(v_i, v_{-i})$. ■

A.9 Proof of Proposition 6

Proof. In expectation, prior to realization of project valuations, each expert is equally likely to have his project chosen, thus it is implemented $\frac{1}{n}$ fraction of the time. The maximum expected value of any project is 1. An upper bound on the continuation value when there are n experts is then $V(n) \leq \frac{1}{1-p}\frac{1}{n}$. Also a lower bound on $V(n)$ is 0, and hence $\lim_{n\to\infty} \frac{1}{1-p}\frac{1}{n} = 0 = \lim_{n\to\infty} V(n)$. Consider the feasibility constraint

$$v_i E_{v_{-i}} d_i(v_i, v_{-i}) - \int_0^{v_i} E_{v_{-i}} d_i(s, v_{-i})\, ds \leq pV(n)$$

this may be rearranged to get

$$E_{v_{-i}} d_i^n(v_i, v_{-i}) - E_s\left[E_{v_{-i}} d_i^n(s, v_{-i}) \,|s \leq v_i\right] \leq \frac{pV(n)}{v_i}$$

in the limit of large team size

$$\begin{aligned} 0 &\leq \lim_{n\to\infty} E_{v_{-i}} d_i^n(v_i, v_{-i}) - E_s\left[E_{v_{-i}} d_i^n(s, v_{-i}) \,|s \leq v_i\right] \leq \lim_{n\to\infty} \frac{pV(n)}{v_i} \\ &= 0 \text{ for all } v_i \end{aligned}$$

hence

$$\begin{aligned} \lim_{n\to\infty} E_{v_{-i}} d_i^n(1, v_{-i}) - E_{v_{-i}} d_i^n(0, v_{-i}) &= 0 \\ E_{v_{-i}} d_i^n(v_i, v_{-i}) &= \frac{1}{n} \text{ for all } v_i \end{aligned}$$

The final relationship follows from the previous line by also recalling the mononicity constraint. This is the equivalent of randomly choosing a project. The expected value of a randomly chosen project is $\frac{1}{2}$ hence we conclude that the objective function approaches $\frac{1}{1-p}\frac{1}{2}$ in the limit of large team size. The final step of the proof is to show that there exists a team size $2 \leq n < \infty$ where the objective function is greater than $\frac{1}{1-p}\frac{1}{2}$. We will show this for $n = 2$. Consider the following scheme for two experts:

$$d_i\left(v_i, v_{-i}\right) = \begin{cases} \frac{1}{2} + \varepsilon \text{ if } v_i \geq \alpha \text{ and } v_{-i} < \alpha \\ \frac{1}{2} \text{ if } v_i, v_{-i} \geq \alpha \text{ or } v_i, v_{-i} < \alpha \\ \frac{1}{2} - \varepsilon \text{ if } v_i < \alpha \text{ and } v_{-i} \geq \alpha \end{cases}$$

The expected value of this decision is:

$$\begin{aligned}
& 2\left(1-\alpha\right)\alpha\left[\left(\frac{1}{2}+\varepsilon\right)\frac{1+\alpha}{2} + \left(\frac{1}{2}-\varepsilon\right)\frac{\alpha}{2}\right] \\
& \left(1-\alpha\right)^2\frac{1+\alpha}{2} + \alpha^2\frac{\alpha}{2} \\
= \; & 2\left(1-\alpha\right)\alpha\left[\frac{1}{4}+\frac{\alpha}{2}+\frac{\varepsilon}{2}\right] + \alpha^2\frac{\alpha}{2} + \left(1-\alpha\right)^2\frac{1+\alpha}{2} \\
= \; & \frac{1}{2} + \varepsilon\left(1-\alpha\right)\alpha
\end{aligned}$$

Let $\alpha = \frac{1}{2}$, so this becomes $\frac{1}{2} + \frac{\varepsilon}{4}$. Now checking the feasibility constraint. The constraint is slack for all $v_i < \frac{1}{2}$ since

$$v_i E_{v_{-i}} d_i\left(v_i, v_{-i}\right) - \int_0^{v_i} E_{v_{-i}} d_i\left(s, v_{-i}\right) ds = 0 \text{ for } v_i < \frac{1}{2}$$

The left-hand side is equal to

$$\begin{aligned}
& \frac{1}{2}\left[\frac{1+\varepsilon}{2}\right] - \frac{1}{2}\left[\frac{1}{4}+\frac{1}{2}\left(\frac{1}{2}-\varepsilon\right)\right] \\
= \; & \frac{1}{2}\left[\frac{1+\varepsilon}{2}\right] - \frac{1}{2}\left[\frac{1-\varepsilon}{2}\right] \\
= \; & \frac{1}{2}2\varepsilon = \varepsilon
\end{aligned}$$

for all $v_i \geq \frac{1}{2}$.The continuation value is given by

$$\begin{aligned}
V &= \frac{1}{1-p}\int_0^1 E_{v_{-i}} d_i\left(v_i, v_{-i}\right)\left[1-v_i\right] dv_i \\
&= \frac{1}{1-p}\left[\frac{1-\varepsilon}{2}\int_0^{\frac{1}{2}}\left(1-v_i\right) dv_i + \frac{1+\varepsilon}{2}\int_{\frac{1}{2}}^1\left(1-v_i\right) dv_i\right] \\
&= \frac{1}{1-p}\left[\frac{1-\varepsilon}{2}\left[v_i - \frac{v_i^2}{2}\right]_0^{\frac{1}{2}} + \frac{1+\varepsilon}{2}\left[v_i - \frac{v_i^2}{2}\right]_{\frac{1}{2}}^1 dv_i\right] \\
&= \frac{1}{1-p}\left[\frac{1}{2} - \frac{1-\varepsilon}{2}\frac{1}{8} - \frac{1+\varepsilon}{2}\frac{3}{8}\right] \\
&= \frac{1}{1-p}\left[\frac{1}{4} - \frac{\varepsilon}{8}\right]
\end{aligned}$$

The objective function is increasing in ε. The maximum value of ε we can choose is

$$\begin{aligned}
\varepsilon &= \frac{p}{1-p}\left[\frac{1}{4} - \frac{\varepsilon}{8}\right] \\
\varepsilon &= \frac{\frac{1}{4}\frac{p}{1-p}}{\left(1 + \frac{p}{8(1-p)}\right)} \\
&= \frac{p}{4\left(1-p\right) + \frac{p}{2}} \\
&= \frac{2p}{8-7p}
\end{aligned}$$

which is bounded away from 0 for any $p > 0$. ■