

Bad Bets: Excessive Risk-Taking, Convex Incentives, and Performance*

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Abstract

Managerial incentives influence risk-taking as well as effort. Theoretical research has long considered inefficient risk-taking to be a potential side effect of incentive pay, but empirical analysis that examines effort and risk-taking simultaneously has been limited. This paper uses exogenous variation in incentives to separate how convex incentive schemes influence performance and risk-taking. We first establish that when hedge fund managers fall below their threshold, beyond which they earn performance fees, risk-taking increases and performance drops. On average, risk-taking increases 50% and performance falls 2.3 percentage points when a hedge fund is below the incentive threshold. We then examine the link between performance and risk-taking explicitly. First, we separately identify the shirking and risk-taking effects of being below the incentive threshold, and show that excessive risk-taking, rather than shirking, causes much of the performance declines. Second, we show that risk-taking behavior is non-monotonic; managers who are significantly below the threshold reduce risk-taking relative to those who are moderately below. These results suggest that risk-taking, due to convex incentives, is not only inefficient—inappropriately choosing from the efficient risk-return frontier given the principal’s objectives—but excessive—in taking risk that are dominated and lead to absolute performance declines. The importance of risk-taking to performance adds to the debate about the impact of incentives on behavior. Whether increasing incentive intensity is justified by moral hazard considerations, or concerns about adverse selection, it is crucial to recognize how incentive-induced risk-taking will influence organizational performance.

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1 Introduction

Managerial incentives fundamentally influence risk-taking and effort and are therefore crucial to understanding how compensation schemes affect the performance and behavior of managers and firms. Effort has been the primary focus of theoretical and empirical work on incentives. But as the Great Recession demonstrated, risk-taking by managers can have drastic consequences not just for their own firms but also for the global economy.

At least since Jensen and Meckling (1976), scholars have noted the potential for performance incentives to induce inefficient risk-taking as a moral hazard response rather than shirking, when performance pay is convex with respect to output. The risk-taking and performance implications of convex compensation may have important economic consequences given that performance pay comprises the majority of executive compensation¹ and that stock options are extensively used in non-executive compensation.² In addition, common non-stock incentive schemes have convex features (See, e.g. Downs and Rocke 1994, Larkin 2012, Oyer 1998, Weinstein 1998).³ In this paper we build on and extend the literature on incentive contracts and moral hazard by examining the risk-taking and performance implications of convex incentives in the hedge fund industry, where such incentive schemes are ubiquitous. The features of this industry, where variation in incentives is plausibly exogenous and measure of risk and performance are well-defined, allow us not only to separately analyze risk-taking and performance but also to show that these are not distinct outcomes but are closely linked. In particular, we disentangle shirking and risk-taking mechanisms and provide evidence that risk-taking, as opposed to shirking, is an important driver of performance declines.

Few studies examine risk-taking and performance consequences of incentives together. While there has been considerable theoretical attention on the effects of performance pay on risk-taking behavior (e.g., Jensen and Meckling, 1976; Hall and Murphy, 2000, 2002; Vereshchagina and Hopenhayn, 2009; Panageas and Westerfield, 2009),⁴ there have been significant challenges in estimating the causal effects of incentives on risk-taking and performance for four main reasons. First, most studies do not examine both risk-taking and performance, let alone disentangle effort and risk-taking mechanisms for performance changes. Perhaps closest to this work, Shue and Townsend (2013) use patterns in option grant contracts to show risk-taking responses, but they find mixed performance implications and do not disentangle risk-taking and performance. Yet, as we demonstrate, understanding the

¹Anderson and Muslu (2011) estimate that half of executive compensation is from options, and an additional 30% is from bonuses and long term incentive plans.

²“Taking Stock: Are Employee Options Good for Business?”
<http://knowledge.wpcarey.asu.edu/article.cfm?cid=8&aid=26>

³For example, sales people often face convex incentives when they are rewarded by either meeting quotas (Oyer, 1998) or increasing marginal commissions (Larkin, 2012), a system that sales people appear to deliberately game at considerable cost to their employers. Profit-sharing contracts, such as those used in the movie industry, are also convex, where payouts are accelerated above certain thresholds (Weinstein, 1998). Entrepreneurs with debt financing face an implicit convex incentive scheme from limited liability. Convex compensation even extends to the political realm; Downs and Rocke (1994) argue that political leaders face a form of limited liability, similar to entrepreneurs, where the threshold reflects the approval necessary to remain in office.

⁴There is also a larger literature about agents responding to, or gaming incentives, which this paper builds on. See, for a recent survey, Oyer and Schaefer (2011.)

performance consequences of risk-taking is crucial to properly evaluate the normative implications of incentive contracts. Second, in examining risk-taking or performance many studies ignore the endogeneity of compensation schemes, which are presumably matched selectively to particular types of managers and firms. In many contexts, including CEO compensation, the compensation structure is set based on the manager's skills, risk attitudes, and characteristics as well as the firm's risk exposure, opportunities and objectives. Typical cross-sectional comparisons of executives' compensation, such as Wright et al. (2007) or Carpenter et al. (2003), do not distinguish between the effects of incentives and the decision to offer incentive compensation, even though it seems likely that riskier firms give more convex and option-like compensation. This endogenous matching problem is further confounded by the fact that many of these compensation contracts may be altered ex post. For example, CEO options are often set to new strike prices when the stock price falls. A third issue is that good measures of incentives for risk-taking are often limited. Many studies use option counts as a crude measure of employee incentives, but options are difficult to compare across firms. Further, more options may not imply more incentive to take risk as they may actually induce risk-aversion (e.g. Carpenter, 2000; Malmendier and Zheng, 2012). Even Chevalier and Ellison's (1997) path-breaking empirical research on mutual fund managers could only impute a proxy for implicit incentives, as explicit incentive measures were not available in their setting. Others, such as Brown, Harlow and Starks (1996), focus on incentives from market-derived relative "tournaments". Finally, measuring risk-taking is also difficult. Common measures in the literature such as merger and acquisition behavior and financing decisions (e.g. Devers et al., 2009, Eisenmann, 2002, Sanders and Hambrick 2007) are hard to interpret from the framework of an agency problem because they are measures over which the principal (the board) has direct control. Further, firms may have different opportunities. For example, the same acquisition target may provide different outcomes to acquirers with different resources.

The two trillion dollar hedge fund industry is a fertile setting in which to empirically investigate the impact of non-linear incentive contracts on both risk-taking and performance for a number of reasons. First, hedge funds have explicit convex compensation schemes provided by the fee structure. Almost all hedge funds charge both a management fee, which is calculated as a fixed percentage of assets, and a performance fee, which is calculated as a percentage of positive returns to investors. The convexity in hedge fund fees is generated because most performance fee terms contain a threshold known as a "high-water mark". The high-water mark is the highest value of assets for which performance fees have previously been paid or the initial value of the investments if none have been paid. Performance fees are only paid when the value of the assets are above the high-water mark. This fee structure implies that hedge fund managers face an incentive threshold, as hedge fund revenues are kinked at the high-water mark. Below the high-water mark performance fees are zero, above the high-water mark performance fees are a substantial percentage of the fund's value in excess of the threshold—typically 20%. Second, incentive contracts in hedge funds are generally fixed ex ante. This eliminates endogeneity problems in two ways: it removes the possibility of ex post readjustment to contracts seen in many settings, and it facilitates a research design where fund fixed-effects control for endogenous contracting choices. Third, and crucially, market movements and industry level asset flows, neither of which are subject to the control of a given manager, provide exogenous variation in the effective incentives of the fixed contract.

Finally, measurement of both performance and risk-taking are relatively straightforward in hedge funds given that asset returns are directly observable, interpretable, and comparable. This measurement advantage is reinforced by the fact that the decisions of the agent, the hedge fund manager, are not subject to veto or review by the principals who set incentives, which is unlike many measures used to examine risk-taking of executives.

To study the effect of convex (or specifically, kinked) incentives on risk-taking in our setting, we develop a model of a manager’s decision-making when facing a threshold incentive. In the model the manager chooses both how much costly effort to exert, where effort improves outcomes on average, and a risk level, where higher risk spreads the distribution of outcomes but may also have a performance cost. The model predicts that as managers become more distant from the threshold for positive performance compensation their risk-taking increases and performance declines from both reduced effort and excessive risk-taking.⁵ However, the model also reveals an additional testable implication of kinked compensation arrangements; while the excessive risk-taking result holds when managers are not “too far” from their incentive threshold, when managers are “very far” below their threshold they stop taking additional risks even though their incentives for effort continue to decrease monotonically. The intuition behind this prediction is that when managers are far from the threshold, despite the same set of choices, they have little to gain from taking more risk but face the same costs of risk-taking as a manager closer to the threshold.

We test the predictions of the theory using data on 3,845 hedge funds from 1994 to 2006. Each fund is categorized into one of thirty-four self-identified investment “strategies,” which describe the underlying assets the funds trade and the way they are traded. Variation in the measure of a fund’s distance to its high-water mark, an intuitive and explicit measure of the fund’s effective incentive compensation scheme, allows us to examine how both changes in performance and risk-taking as a function of variation in the incentive scheme across and within funds. Furthermore, we are able to exploit plausibly exogenous variation in the fund’s distance from its high-water mark to approximate random assignment of the intensity of incentive arrangements. While distance to the high-water mark is generally endogenous – a function of past manager success, among other things – market movements, particularly downturns, provide an exogenous movement in the distance of the fund below their threshold. To compute the exogenous shift parameter, our instrument, we use the return of all of the funds in each strategy to estimate strategy-level exposures to a set of market indexes that are commonly used to explain the performance of hedge funds and other financial assets (Fama and French, 1993; Carhart, 1997; and Fung and Hsieh, 2004). Our approach uses strategy-level variation in the measure of how exposed any given fund is to a composite of market indices. Since downturns in the indexes affect strategies differently, the instrument provides exogenous cross-sectional variation in funds’ distance to the threshold. In conjunction with fund fixed effects the instrumental variable approach allows us to identify the causal effect of incentives on both performance and risk-taking outcomes.

The results show that risk-taking increases as a function of distance from the incentive

⁵In contrast to much of the literature, risk-aversion plays no role in our model. Excessive risk-taking here is not simply an inefficient level of risk for the risk aversion of the parties; it is risk-taking that provides lower expected returns, *ceteris paribus*. We use the term “excessive” risk-taking to mean risk-taking that reduces expected returns and “inefficient” risk-taking as in the literature that examines optimal risk-levels given risk-aversion.

threshold (i.e., the high-water mark); managers respond to being farther below the threshold by increasing their risk exposure. Performance also declines, which we show is driven by the structure of the incentive contract itself rather than implicit incentives from aspirations, performance targets (as in March and Shapira, 1987), reference point behavior, loss aversion (as in Wiseman and Gomez-Mejia, 1998), or relative performance contests. Moreover, the effects are economically significant: moving a fund just 15% below its threshold (the average in our data conditional on being below the threshold), leads to 50% greater risk while returns drop by 2.3 percentage points on average. Moreover, we use variation in fee structures to show that this behavior is driven by the explicit incentive contracts rather than implicit tournaments, investor behavior, or reference dependent behavior and to provide important intra-strategy variation in our instrument. Together, our baseline results provide well-identified evidence that convex incentive schemes cause increased risk-taking and a decline in performance.

Indeed, the main contributions of the paper stem from linking convex incentives, risk-taking, and performance conceptually and empirically. In the canonical risk-return paradigm, higher risk implies higher returns in equilibrium. Yet, we find that underwater managers, those below their high-water marks, take more risk but generate lower returns. We explain this result by building on the idea that convex incentives can lead to excessive risk-taking, that is, risk-taking which reduces returns. Developing this idea further leads to three main contributions.

First, we raise and answer the question of whether performance declines are caused by reduced effort, excessive risk-taking, or both. Our model allows us to separate these two potential sources of performance declines empirically and show that independent of effort effects, excessive risk-taking causes a statistically and economically significant decline in performance. In fact, given the baseline assumptions of our empirical model our estimates suggest that about four fifths of the performance drop observed by managers who are not very far below their threshold is due to the performance costs of risk-taking and only a fifth of the performance decline is due to effort reduction.

A second contribution of our paper is to provide evidence that the effect of high-powered incentives on excessive risk-taking is non-monotonic with respect to the distance from the incentive threshold. Conceptually the non-monotonicity results from a feasible risk-return frontier that is initially upward sloping, but eventually turns downward (see, for example, Palomino and Prat 2003). In this context, agents who are below their incentive threshold will choose excessively high risk levels even though they reduce the level of expected returns. However, agents very far from their thresholds are not as willing to sacrifice return for risk. Empirically, although the average treatment effect on risk-taking is positive, we also find that managers that are very far below their thresholds take less risk and perform better than managers who are more moderate distances below their thresholds. At first glance, this finding is consistent with Chevalier and Ellison (1997) who also note non-monotonicities in risk-taking in mutual funds. That said, there is an important difference in the mechanism. In Chevalier and Ellison (1997), the non-monotonicities are driven by the shape of the elasticity of demand (i.e., the elasticity of fund flows) with respect to performance. Because the demand function is concave and then convex, this generates non-monotonic incentives to take risk. In other words, non-monotonicities in the returns to risk lead to non-monotonicities in risk-taking behavior. In our case, we show that even when incentives are weakly convex everywhere, the incentive to take risk is a non-monotonic function of distance from the

threshold. Close to the threshold, the incentive to take risk is increasing in distance, but once the distance becomes “large” the benefit in terms of increased incentive fees is outweighed by the fact that risk-taking comes at the cost of a reduction in expected performance. When managers are very far from the threshold, therefore, risk-taking decreases.

Finally, our results contribute to the scholarly debate on the role of incentive contracts. Oyer and Schaefer (2011), highlight that there are still fundamental empirical questions as to the role of incentive contracts in labor markets, particularly for executives, managers, and others with complex jobs. As they point out, the importance of incentives as a motivator to managers to improve their output is an open empirical question. They argue that in the standard moral hazard model, high-powered incentives are needed to induce effort by the agent. Yet, most high-level managers and executives appear willing to work long hours without regard to any obvious high-powered incentives. This lacuna and the lack of well identified empirical evidence has led scholars to explore alternative avenues for incentives to operate that do not depend on moral hazard. For example, employers may use incentive contracts to induce selection effects and compensate for outside options. We offer well-identified evidence suggesting that moral hazard in response to explicit contractual incentives matters economically. However, in a twist, we show that moral hazard is important not merely because of effort, as emphasized by much of the extant literature, but because of excessive and inefficient risk-taking, which has implications for firms where employees have incentive contracts and the wherewithal to influence the riskiness of the firm’s operations. Indeed, these results may help explain part of the risk-taking and performance declines associated with financial crises.⁶

The paper proceeds as follows. In the next section we lay out the conceptual framework, model and predictions. Section 3 describes the data and institutional context. Section 4 describes the empirical approach used to estimate the risk and average return consequences of being below the incentive threshold. Section 5 provides the empirical results examining the average response of managers including threats to identification, those very far from their thresholds, predictions on the differential effects of fees, and finally discusses robustness concerns. The last section concludes.

⁶More specifically, our results are consistent with the idea that managerial effort does not change with incentives. The same calculation that disentangles risk-taking and effort’s impact on performance suggests that if risk-taking and its performance impact were removed from the agent’s choice set, we would find no statistically significant impact on performance. Further, if we were to observe the effort of the managers in our sample directly, we would likely find insignificant changes in how much they work based on changes in distance to their thresholds. However, our results do show that moral hazard is important for these managers—if not in how hard agents work then in how agents take risks that reduce return. In other words, the observation that effort does not increase does not imply that moral hazard is less important for high-level managers; instead, it may be the case that incentives address (or create) moral hazard issues but through a mechanism other than effort, including the return reducing managerial risk-taking we focus on here. Of course, if the only effects of incentive schemes are those evaluated in this paper it would be difficult to justify them. A simple flat wage could induce similar effort without the risk-taking consequences. Because we estimate within manager variation in risk and performance and do not include investor portfolios, despite the observed risk-increases and performance declines, the funds may still provide positive returns and valuable diversification. That is, the observed incentive structure may have been designed for reasons we do not observe, for example as a screening contract to induce selection on ability or risk-attitudes when employees are hired.

2 Conceptual Framework

Our research focuses on how convex incentives generally and kinked incentives specifically influence risk-taking and effort in combination. These are compensation schemes in which total compensation varies little, if at all, with performance when below a performance threshold but varies significantly with performance above the threshold. Figure 1 shows an example of a compensation scheme of a manager. In this example, the intercept of the compensation scheme is the manager’s base compensation. The low initial slope represents the impact of equity holdings on total compensation as firm value increases. The steeper region represents the realized value of option holdings where the exact threshold is determined by the exercise price of those options. Drawing on and extending existing theory,⁷ we develop a simple model of the decision a manager makes about effort and risk when facing threshold incentives. The model is intended to provide intuition and clarity for the empirical work that follows and therefore is simple and highly stylized.

In the model, a manager simultaneously chooses an effort level and a risk level. Greater effort increases the mean of the distribution of outcomes, but is costly to the manager.⁸ In choosing a risk level, the manager selects a position on a risk-return frontier. The risk-return frontier follows Palomino and Prat (2003) who generalize the standard assumption of the classical Capital Asset Pricing Model (CAPM) (Sharpe, 1964). Specifically, we assume that the risk-return frontier is single peaked; in other words, there is a level of risk-taking beyond which additional risk-taking reduces expected return.⁹

To formalize this stylization, the agent chooses an effort level $e \in [0, \infty)$ at cost $c(e)$ where $c(0) = 0, c'(\cdot) > 0, c''(\cdot) > 0$. The agent also chooses a risk level r that yields return $q(r)$.

⁷Classic incentive theory has linked options as an effective way to align managers and principals (e.g. Haugen and Senbet 1981) but at the cost of inefficient risk allocation (Jensen and Meckling 1976). More recently Vereshchagina and Hopenhayn (2009) argue that entrepreneurs facing a convex incentive scheme will choose higher risk projects. Hall and Murphy (2000 and 2002) study the price at which options should be granted, but do not consider risk-taking as a consequence. Other research has looked at how incentives influence risk-taking but do not consider the implications of the incentive threshold. Panageas and Westerfield (2009) focus on the dynamic ratcheting of thresholds in hedge funds and show that the value of future periods reduces risk-taking. In our setting incentives are fixed, thus, unlike Hermalin and Katz (2001) risk-taking does not influence incentives. In this section we develop some predictions on the consequence of the distance to the threshold on changes in the manager’s risk-taking and performance.

⁸Effort in this setting simply raises the risk-return frontier. For any given level of risk more effort yields higher return. In some settings there might be low return to effort, for example, if market prices are perfectly efficient, but in others managers may be able to change the performance of their firm separately from its risk exposure.

⁹As Palomino and Prat argue, standard depictions of the risk-return frontier simply stop at this peak because no risk-neutral or risk-averse agent with linear incentives (including those normally facing those making investment decisions for themselves) would choose assets beyond this frontier. Indeed, investment opportunities that have high risk and return below the peak of the frontier certainly exist (e.g. gambling, dominated trading strategies, and paying managers to trade with no information). In financial settings, the exact location of the peak may also be affected by the availability of leverage. Where opportunities for leverage exist, the peak of the levered frontier would not represent the peak of the underlying investment opportunities. Instead it would reflect the point where market frictions (e.g. liquidity, convex borrowing costs and limitations) make the cost of additional risk lower than the return to that underlying risk. See Albagli, Hellwig, and Tsyvinski (2013) for one such characterization. Agent’s who wish to choose risk levels beyond this peak face reduced returns for additional risk as pointed out by Palomino and Prat (2003).

We assume that $q(\cdot)$ is single peaked with maximum at $r^{max} > 0$ and weakly concave.¹⁰ An agent's decision therefore is a pair (e, r) , and yields outcome x that is normally distributed with mean $e + q(r)$ and variance r , $x \sim \mathcal{N}(e + q(r), r)$.

The manager is opportunistic and risk-neutral, and the manager's compensation scheme is an exogenous, convex, two part linear contract. The contract pays the manager a share of the performance of the project (base rate) and a share of the performance of the project above a threshold (performance rate). For an executive the base rate would reflect his equity holdings and the performance rate would reflect options.¹¹ Formally, the compensation of the manager for a realized outcome x is:

$$\pi(x) = bx + \max\{0, p(x - d)\}$$

Where $b > 0$ is the base rate, $p > 0$ is the performance rate, and d is the manager's distance below the threshold. As such the expected utility of the manager is:

$$\begin{aligned} \Pi(e, r) = & b(e + q(r)) \\ & + p \left(1 - \Phi \left(\frac{d - e - q(r)}{\sqrt{r}} \right) \right) \left(-d + e + q(r) + \frac{\sqrt{r}\phi \left(\frac{d - e - q(r)}{\sqrt{r}} \right)}{1 - \Phi \left(\frac{d - e - q(r)}{\sqrt{r}} \right)} \right) - c(e) \end{aligned}$$

where the first term is the base payment for the expected mean, the second term is the expected performance fee given the distance from the threshold, and the final term is the private cost of effort.

The manager then chooses e, r to maximize her welfare. The first order conditions yield the first insight:

Lemma 1. *Risk taking is on the strictly downward sloping part of the curve $q(r)$.*

Proof. See appendix for all proofs. □

Lemma 1 indicates that in general, agents will take more risk than that which maximizes expected return, even though they are risk neutral. Indeed, a risk neutral agent facing a linear, increasing incentive curve would always choose a level of risk that would maximize the expected return, or in other words, would be at the peak of the risk-return frontier. However, here the convexity of the incentive scheme implies that the manager would always take some

¹⁰That is, there exists $r^{max} > 0$ such that on range $[0, r^{max})$ $q'(r) \geq 0$ and on range (r^{max}, ∞) $q'(r) < 0$ and $q'' \leq 0$. These assumptions generalize the risk-reward frontier of the CAPM to describe return-dominated, but riskier parts of the frontier, and can be derived from basic assumptions about the risk-return set. The concavity of q follows directly from an agent's ability to allocate assets to a combination of underlying assets. $r^{max} > 0$ follows from the existence of some asset with return above the risk-free rate. As discussed above, since opportunities that provide higher risk and lower return than the opportunity that provides the peak return, the curve is downward sloping after the peak. If there are limits to leverage, than the levered version of the frontier retains this shape because of the costs of leverage and/or levering of investment opportunities beyond the peak.

¹¹In a one period game the level of fixed compensation (base rate times starting assets) provides no incentives so we ignore it. However, the base rate incents returns because compensation increases as assets increase.

measure of risk above the peak. While the set-up of the problem does not assume that risk-taking is inherently bad for returns, these managers always choose risk levels that are high enough so that the marginal return to risk is negative. Moreover this result combined with the convexity of the cost of effort, $b > 0$, and $p > 0$, implies that the solution is interior in both effort and risk-taking.

To further develop predictions about the interaction between risk-taking and effort, we also assume the cost of effort is sufficiently convex. Specifically, we have

Assumption $c''(\cdot) \geq \frac{p}{\sqrt{2\pi}r^{max}}$.

This technical assumption means that either r^{max} is high enough or c is sufficiently curved. It is a sufficient condition to ensure that the maximization problem we have is concave over all possible d , and $r \geq r^{max}$. This allows us to establish the uniqueness of the solution to the agent's maximization problem.

Lemma 2. *There exists a single solution to the maximization problem for any triple (d, b, p) . Moreover, it is the unique local maximum.*

Lemma 2 allows use of the Implicit Function Theorem to characterize comparative statics everywhere by ensuring that there is only one local maxima in the optimization problem.¹² Since close to the threshold effort is most valuable when risk is low there is the possibility of both a low-risk high-effort and a high-risk low-effort local maxima. The proof uses the assumption above to preclude the low-risk, high effort local maxima. With the above results, the empirical predictions of this model on effort and risk-taking as distance below the thresholds follow from the implicit function theorem.

Proposition 1. *Effort is decreasing in distance below the threshold.*

The intuition behind Proposition 1 is that increasing the distance to the incentive threshold decreases the probability that outcomes will be above the threshold. The effort level undertaken is driven by the marginal return to improving outcomes, which is the average slope of the incentive curve that the manager expects outcomes to reach. Increasing the distance decreases the probability the manager will be in the high marginal incentive region, so the manager reduces effort.

Proposition 2. *Risk-taking has the following comparative statics with respect to distance from the threshold:*

- (i) *Risk-taking is increasing with distance "near" or above the threshold;*
- (ii) *Risk-taking is decreasing in distance "far" below the threshold; and*
- (iii) *Risk-taking is single-peaked if the marginal cost of effort is sufficiently high. A sufficient condition is $c'(e) > p \exp\left(\frac{-1}{2}\right) / \sqrt{2\pi}$.*

The intuition behind these results is nuanced. Increasing the distance below the threshold decreases the marginal return to risk-taking because the kink in the compensation scheme is

¹²Since the implicit function describes changes in the local maximum around a solution knowing that there is always one local maximum ensures that these comparative statics are meaningful beyond local results. That is, the global maximum does not jump between one local maximum to another as parameters change.

farther away. However, increasing the distance below the threshold also decreases the cost of the risk-taking, because it is more likely that the marginal movement of outcomes is in the low marginal incentive region than the high incentive region. At first, the second effect dominates and risk-taking increases. However, when the manager is “far” from the threshold, increasing risk-taking only has second order effect because the compensation scheme the manager faces is nearly linear. But the manager still faces first order costs of risk, so risk-taking decreases. Further, an assumption of sufficiently costly effort ensures that these two regions meet.

While the above propositions describe effort and risk-taking, empirically, we do not observe effort directly. Instead, we observe performance, $e - q(r)$. How performance changes with effort and risk-taking depends on the relative cost of risk and value of effort. In particular, from the two Propositions, we have

Corollary 1. *When the threshold is near, increasing distance unequivocally reduces performance.*

and

Corollary 2. *When the threshold is far, the performance consequence of increasing distance depends on the cost of risk-taking relative to the cost of effort. If the cost of risk-taking is high relative to the cost of effort, performance improves; it reduces otherwise.*

Effectively, these results imply that managers that are far from their performance threshold can be used to tease out the importance of effort and risk-taking on performance.

Figure 2 displays the predictions of the model on risk-taking, performance, and unobservable effort. In both panels, the green effort line is monotonically decreasing, per Proposition 1. Similarly, in both panels, risk-taking is increasing and then decreasing, per Proposition 2. The difference in the two panels reflects the different effects these two results may have on performance. In panel (a), the cost of effort relative to risk is high, so performance is first decreasing and then increasing. When the cost of effort is relatively low, as in panel (b) the non-monotonicity in performance is not observed.

Finally, the interactive effects of fees are also important predictions of the model. The intuition behind these effects are simple. Consider a manager with a very small performance fee. This manager is going to respond to distance very little because her compensation scheme is nearly linear. On the other hand, a manager with a very small base fee will respond quite sharply to distance because she faces no other incentives.

Proposition 3. *If the threshold is near, and the cost of effort is sufficiently convex,*

i) the rate of decrease in effort with distance is increasing in the ratio of the performance fee to the base fee; and

ii) the rate of increase in risk-taking with distance is increasing in the ratio of the performance fee to the base fee.

3 Industry and Context

The setting for this study is the hedge-fund industry. In this industry, hedge fund managers are paid fees to make investments with investor’s assets. Each hedge fund is a standalone

private investment vehicle with hedge fund management firms as general partners and high net worth individuals and institutional investors as limited partners. Hedge funds face minimal regulatory constraints and managers are free, unlike other asset managers such as those who manage mutual funds, to make almost any investments, including derivatives, short sales, leveraging and private transactions.¹³ Hedge funds identify an investment strategy that broadly identifies the sort of assets the fund will invest in, the sort of profit opportunities that the manager will pursue, and the risk exposure that the fund will accept. In the context of our analysis, we view these categorizations as similar to industry classifications; they identify that, within a strategy, firms face similar exogenous factors that influence performance.

Hedge fund management firms earn revenue from fees paid from the assets of investors. These fees are composed of a management fee and a performance fee. The management fee pays the manager a percentage of fund assets each year. Management fees are usually between 1 and 2%. On average, the performance fee pays the manager a substantially larger share of the profits the fund makes than the management fee. The most common performance fee rate is 20%. Because the performance fee is calculated on profits, often above a benchmark rate, the benchmark is a threshold in a kinked incentive structure. The details of this performance fee are central to the analysis and we discuss it in detail in the context of the empirical approach. While the internal organization of the management firms vary, all are known for high powered incentives that tie compensation of the individuals in the firm quite closely to fees and performance. Indeed, funds generally have a single manager who is responsible for the ultimate investment decisions and is a defacto residual claimant on fees.¹⁴

Our work builds on an extant recent literature on hedge fund risk-taking and performance which provides both descriptive and cross-sectional results. Ackermann, McEnally, and Ravenscraft (1999) describe hedge fund risk, return and fee profiles. Agarwal and Naik (2004) focus on identifying market factors that are relevant to explaining hedge fund performance and risk exposures. Agarwal, Daniel, and Naik (2009) correlate a measure of marginal fees with expected outcomes, focusing on issues of effort and performance. Kouwenberg and Ziemba (2007) correlate fee rates and various measures of risk-taking. Like Chevalier and Ellison (1997), Brown, Goetzmann, and Park (2001) focus on intra-year changes in risk-taking following good absolute and relative performance, but do not consider the contracts explicitly. Similarly, Holland, Kazemi, and Li (2010) correlate performance in the first half of the year and changes in risk-taking. Smith (2011) looks at how investors respond to idiosyncratic risk-taking by managers.

Our data set includes monthly assets under management and returns for approximately 9,000 hedge funds from 1994 through 2006. This data set was compiled by merging data on hedge funds from Lipper-TASS and Hedge Fund Research. Each of these data sets retains “graveyard” funds, funds that have closed or otherwise stopped reporting to the data vendor. While exact measures do not exist, these data sets are together estimated to include about

¹³See Stulz (2007) for more comparison to the more familiar mutual fund.

¹⁴Because the data does not include information about the decisions of the manager separate from the ultimate actions of the management firm one cannot distinguish between actions of the individual or of the firm. However, the strong internal incentives suggest that in this context the two are closely aligned. These results can be fairly interpreted as either about the individuals’ decisions or the firms’ response.

a quarter of the entire hedge fund industry. The data also contain fund fee structures classification of the funds into 34 investment categories. While the data are self-reported, which raises potential selection concerns, the results are robust to selection corrections as discussed more fully in Section 5.2.

4 Empirical strategy

To understand how managerial incentives influence risk-taking and performance our empirical approach focuses on hedge fund performance fees and high-water marks, where the high-water mark offers a useful kink, or threshold, in the performance fee calculation. At the end of each year hedge fund managers are paid any performance fees and the high-water mark is adjusted for this payment. Figure 3 illustrates how high-water marks are adjusted over time. The red line identifies the cumulative return of a hypothetical hedge fund. At the end of 1994 this fund is 8% below its high-water mark, and, therefore, is not paid a performance fee. At the end of 1995, however, the fund's returns exceed the previous high-water mark. Thus, the fund is paid a performance fee, and its high-water mark ratchets up. In practice high-water marks are tracked individually for each investment into the fund, so each vintage of assets may have a different high-water mark. Because managers make investment decisions for the fund, rather than separate investment decisions by vintage, we use an asset weighted average of the distance to the threshold. In the example illustrated in Figure 3, if the fund at the end of 1994 was composed equally of two vintages of equal size from the end of 1993 and the end of 1994, we would take the average distance by vintage and treat the fund as if it is 4% below its high-water mark.

The implementation of this calculation depends on the returns the fund experiences as well as the flow of assets in and out of the funds. Returns and gross assets are directly reported in the data, but funds do not report asset flows. Therefore, we use gross assets to impute net asset flows treating net inflows as new vintages and allocating net outflows proportionally across previous vintages.¹⁵ Potential measurement error concerns related to

¹⁵The formal calculation of distance, how far a fund is from its threshold, is as follows. Let r_{it} represent the return of fund i in period (month) t . Representing the initial period of fund i as t_{0i} , let $CR_{it} = \prod_{j=t_{0i}}^t (1 + r_{ij})$ be the cumulative return to t . The high-water mark of investments of vintage v in month t is $HWM_{ivt} = \max(CR_{iv}, CR_{iY(v)}, CR_{iY(v+1)}, \dots, CR_{iy(t)})$ where $y(x)$ and $Y(x)$ note the last month of the calendar year ending before, respectively after, month x and if $y(t) < v$ this is understood to equal CR_{iv} . To aggregate each vintage's high-water mark, we measure how far it is from the threshold as $Distance_{ivt} = (HWM_{ivt} - CR_{it}) / (CR_{it})$ or what percentage growth is needed to bring that vintage to its high-water mark. To aggregate vintages, we weight each vintage by its share of assets. Let In_{iv} be the dollar inflows in month v . Let Out_{iv} be the outflows in period v as a percent of assets. So the assets remaining in vintage v at time t is $A_{ivt} = In_{iv} \frac{CR_{it}}{CR_{iv}} (\prod_{j=v+1}^t (1 - Out_{ij}))$. Note that by construction the assets of fund i at time t is $A_{it} = \sum_{v=t_{0i}}^t A_{ivt}$. Then, weighting vintages by assets: $Distance_{it} = \sum_{v=t_{01}}^t (Distance_{ivt} * \frac{A_{ivt}}{A_i})$. How this depends on the underlying data is clearer with some expansion:

$$Distance_{it} = \sum_{v=t_{01}}^t \left(\frac{\max(CR_{iv}, CR_{iY(v)}, CR_{iY(v+1)}, \dots, CR_{iy(t)}) - CR_{it}}{CR_{it}} * \frac{In_{iv} CR_{it} (\prod_{j=v+1}^t (1 - Out_{ij}))}{CR_{iv} A_i} \right) \quad (4.1)$$

this approach are discussed in Section 5.2.

One of the key outcome variables, risk-taking, is measured as the realized variance of the fund’s monthly net returns over the prior year.¹⁶ A year is the natural length of time over which to measure risk-taking because that is the period used to compute performance fees, but the results are robust to shorter and longer measurement periods (e.g., 6 months or 24 months).

We use a fixed-effects estimator as the basic test of the relationship between *Distance* and risk-taking of the form:

$$Risk_{iy+1} = \beta_1 Distance_{iy} + \lambda X_{iy} + \delta_i + \gamma_y + \epsilon_{iy+1}, \quad (4.2)$$

where the vector X_{iy} are controls for assets and age, δ_i are fund fixed effects, γ_y are time fixed effects, and each period is one year.¹⁷ Empirically, the fund fixed effects are important as they absorb time-invariant fund-specific sources of heterogeneity that might be correlated with both distance and risk-taking. To control for convexities in the effects of fund age and size we include both linear and curvature terms for these controls.

To examine the relationship between return consequences, we use reported returns as the dependent variable and the same regressors as in expression (2) above:

$$Return_{iy+1} = \beta_1 Distance_{iy} + \lambda X_{iy} + \delta_i + \gamma_y + \epsilon_{iy+1}. \quad (4.3)$$

While fund fixed effects control for time-invariant sources of fund-specific heterogeneity one may be concerned about time-varying factors correlated with both *Distance* and the dependent variables of interest. For example, if a fund changes to a higher return, higher volatility risk profile in period 2 it will increase the probability that it will be below its high-water mark at the end of period 2 and increase its expected distance below the high-water mark. At the same time the change in strategy also increases the fund’s realized volatility and performance in period 3, which lead to an overstatement of the causal relationship between risk-taking and *Distance*. The opposite problem is probably even more prevalent. If managers borrow to buy an asset which is mispriced. Until the price corrects, the current returns will reflect that the manager is moving farther away from the threshold, but the manager is not changing their average risk or return choices because they understand that the accounting valuations do not reflect the true values of their endogenous choices. This will lead to an understatement of the causal effect in regressions where *Distance* is endogenous.

To address concerns about endogenous time-varying contractual changes, we use an instrumental variable approach where the instrument for $Distance_{iy}$ is the average distance of a representative fund in the same strategy, $\overline{Distance_{iy}}$. $\overline{Distance_{iy}}$ satisfies the exclusion restriction because is based on a synthetic high-water mark that does not depend on the choices of the fund and should not influence risk-taking or performance directly, except through its correlation with $Distance_{iy}$.

¹⁶This measure of risk-taking measures realized risk and not intended risk directly.

¹⁷An alternate specification would calculate the fund’s incentive to take risk as the average of the incentives for risk-taking provided by each vintage. However, to implement the alternative approach one would need to make explicit functional form assumptions. The approach we take here makes weaker assumptions that depend only on non-monotonicity in incentives.

There are two key inputs into the measure $\overline{Distance}_{iy}$: representative returns and representative fund flows (in and out). Representative return is calculated in two steps. The first step uses the performance of 15 market “factors” to estimate average strategy-level risk-adjusted return and risk factor loadings, where the factors represent the returns to indices of pertinent bundles of securities. We use standard factors from the asset pricing literature, which reflect both the performance of equity markets (Fama and French, 1993) as well as additional factors found to be important in explaining the returns of mutual funds (Carhart, 1997) and hedge funds (Fung and Hsieh, 2004). For each strategy we regress the monthly return of the funds in that strategy on the monthly market factors. That is, indexing months by m , we estimate:

$$Return_{im} = \alpha_s + \beta_{sj} Factor_{jm} + \epsilon_{im} \quad (4.4)$$

In the second step of the representative return calculation we use the estimates $\widehat{\alpha}_s$ and $\widehat{\beta}_{sj}$ from the regression in (4) above to calculate the return of strategy s in time m :

$$r_{sm} \equiv \widehat{\alpha}_s + \widehat{\beta}_{sj} Factor_{jm} \quad (4.5)$$

The computed values from expression (5) capture the monthly returns of a representative “passive” hedge fund for each strategy category.

The second input to the instrument are representative fund flows.

Rather than using fund-level flows, which may be endogenous,¹⁸ we use the flows into and out of funds that identify themselves as “Fund of Funds”, which are excluded from the analysis otherwise, to capture the general availability of funds to the industry that are not a consequence of the beliefs of investors about the future performance of any particular fund or of its strategy.¹⁹ Using the average of all fund of funds we calculate the representative percentage inflows In_{FoFm} and outflows Out_{FoFm} to the industry in each month. Combining the exogenous inputs, strategy returns and Fund of Funds flows, we calculate the instrument $\overline{Distance}_{im}$ using the formula in expression (1).²⁰

¹⁸For example, suppose that the flows a particular fund experiences reflect investors’ beliefs about the future performance of the fund. Also, suppose that investors believe that a fund with recent poor performance will experience low risk returns in the next period. If investors add funds to this fund at the end of this period then it will be less underwater than it would have been and, if those beliefs were correct, realized risk would be lower. Thus, the correct beliefs would produce a correlation between distance to the threshold and realized risk.

¹⁹Using aggregate flows observed in the data as a measure of industry flows produces similar results.

²⁰The exact formula is:

$$\overline{Distance}_{im} = \sum_{v=t_0}^m \left(\frac{\max(CR_{sv}, CR_{sY(v)}, CR_{sY(v+1)}, \dots, CR_{sy(t)}) - CR_{sm}}{CR_{sm}} * \frac{In_{FoFv} CR_{sm} (\prod_{j=v+1}^m (1 - Out_{FoFj}))}{v} \right) / \left(CR_{sv} \sum_{k=t_0}^m \left(\frac{In_{FoFk} CR_{sk}}{CR_{sk}} (\prod_{j=v}^m (1 - Out_{FoFj})) \right) \right) \quad (4.6)$$

where subscript s denotes representative returns, risk, inflows and outflows. Despite the apparent complexity of this formula it has a simple interpretation. It is the Distance of a fund that had the same initial founding date as the fund, experienced the same average flows of funds of funds, and had the same returns exposure to market factors.

Using $\overline{Distance_{iy}}$, we estimate the annual first stage regression:

$$Distance_{iy} = \beta_1 \overline{Distance_{iy}} + \lambda X_{iy} + \delta_i + \gamma_y + \epsilon_{iy+1} \quad (4.7)$$

Which yields $\widehat{Distance_{it}}$ as the predicted value to be used in the second stage regressions:

$$Risk_{iy+1} = \beta_1 \widehat{Distance_{iy}} + \lambda X_{iy} + \delta_i + \gamma_y + \epsilon_{iy+1} \quad (4.8)$$

$$Return_{iy+1} = \beta_1 \widehat{Distance_{iy}} + \lambda X_{iy} + \delta_i + \gamma_y + \epsilon_{iy+1} \quad (4.9)$$

It is illustrative to consider how the instrument might be positive, or below the representative high-water mark. Managed Futures and Global Macro strategies were below their threshold high-water marks in 1994, presumably, because of the spike in interest rates.²¹ Similarly, in 1998 emerging market funds were below their thresholds because of the crash in emerging market returns, however, some regional emerging market strategies were much more affected than others. The technology crash in 2000 and the market wide downturn in 2002 were also significant downward shocks that caused strategies to be below their thresholds. Despite operating at the category level, the instrument can be expected to be strong in the first stage because different strategy-level exposures provide meaningful variation in how far different funds are from their thresholds. Another advantage of using a category-level instrumental variable approach is that, by definition, in the second stage it will break the endogenous relationship between funds that have planned for the downturn and their investment approach, which allows us to make valid inferences about the causal effect of incentives on risk and return. Finally, the category-level IV approach removes the influence of implicit relative performance incentives from our measures of explicit incentives.

It is straightforward to calculate two important controls: the return and variance of a passive hedge fund in each strategy. We use these to control for changes in the opportunity set of investments available, including, for example cyclicity in strategy returns. Additionally, we use this to estimate the risk increases that a passive fund would experience. If there is persistence or cyclicity in the performance and risk characteristics of underlying assets beyond that absorbed by time fixed effects, as for example Carhart (1997) demonstrated, then using the variation driven by these factors makes controlling for the risk and return that is driven by market factors particularly more important.

5 Primary Results

The most basic prediction of the model, is that the farther managers are from their threshold, the more risk they will take. Figure 4 shows how exogenous changes in the distance below the high-water mark (horizontal axis) influences risk-taking (left vertical axis, solid line) and performance (right vertical axis, dashed line), restricted to managers that are not “far” from their thresholds. The line plots the non-parametric fitted values of risk and return after adjusting for fund and year fixed effects and controls for age, age-squared, assets under

²¹http://money.cnn.com/magazines/fortune/fortune_archive/1994/10/17/79850/index.htm

management, and log assets under management. The figure illustrates the baseline prediction graphically— the farther a manager is from the threshold the more they increase risk.

Table 2 regresses risk (columns 1-4) and return (columns 5-8) on the (endogenous) distance a fund manager is below their high-water mark. Columns (1) and (2) are OLS regressions, with the fund and time fixed effects and controls for age, age-squared, assets under management, and log assets under management. Robust standard errors are clustered at the fund-level. Column (2) includes controls for the return and variance of the passive comparison. With fund fixed effects the interpretation of columns (1) and (2) is that for funds with assets that fall from the high-water mark to half of the high-water mark in one year, and thus increase distance to the threshold by 100%, risk-taking increases by 40 percent-squared/year, an amount equal to the average variance, compared to a fund that stays at its high-water market. However, distance rarely increases so quickly. The mean distance for funds that are below their thresholds is 15.8%,²² implying that the average below-threshold fund increases risk-taking by about 16%.²³

While the correlations reported in the OLS regressions are suggestive of a relationship between changes in *Distance* and in risk-taking, *Distance* is clearly endogenous, making causal inference fraught. Columns (3) and (4) implement the instrumental variables strategy described in section 4, where the endogenous measure *Distance* is replaced with an exogenously determined strategy-specific instrument. In the presence of fund fixed effects, the instrument captures the change in *Distance* the fund is expected to be experience due of the performance of its strategy class, independent of own choices. The second stage of the two stage least squares (2SLS) instrumental variable (IV) approach is reported in column (3).²⁴ When instrumenting for *Distance* the estimated increase in risk from a 100% increase in *Distance* is 130 percent-squared/year, or more than three times the average variance.²⁵

One potential concern with the identification strategy is that the variation in the instrument, which depends on past strategy-level performance, might be correlated with current fund-level risk-taking and performance through current strategy-level risk and return (i.e., rather than only influencing outcomes through the incentives captured in the variable *Distance*), which would violate the exclusion restriction. One way of addressing this concern is to control for contemporaneous strategy-level risk and performance directly. Controlling for the variance of the strategy in the same period in column (4) reduces the point estimate to 71 percent-squared/year. The interpretation is that the average underwater fund increases risk-taking by 28% due to the impact of non-linear incentives.²⁶

²²Because the specifications include fund fixed effects, they can be interpreted as differences in differences estimates. Because all funds are sometimes at their threshold (a distance of zero), the mean distance for funds that are below their threshold is the average change and useful to discuss the economic magnitudes of these effects.

²³These specifications include all managers, even those that are very far from their thresholds. If the non-monotonic prediction of Proposition 2 holds, which we show below, it suggests that these results underestimate the impact for the typical managers.

²⁴The first stage is quite strong, as the F-statistic (cluster-robust) is 46.46 (column (3)).

²⁵The coefficient estimates from the 2SLS specifications are 2-3 times larger than the OLS estimates. One explanation for this difference is that the endogeneity in the difference between valuations and accounting valuations used to calculate *Distance* causes some managers to appear to be below their threshold, but they act like they are at it.

²⁶The results are robust to aggregating the data to the strategy-year or also clustering at the strategy-year

The second basic prediction we test is that the farther managers are from their thresholds the worse they will perform, as illustrated by the dashed line in Figure 4, and displayed more concretely in the next four columns of Table 2. The OLS results in Columns (5) and (6) show insignificant increases in performance before addressing endogeneity concerns, but after instrumenting for Distance in column (7) we see an economically meaningful decrease in expected returns when funds are 100% below the high-water mark of 6.5 percentage points/year. However, the point estimate is only on the margin of statistical significance. Once one controls for the performance of the strategy directly in column (8), the effect of being 100% underwater is substantially larger at 14.1 percentage points/year, and precisely estimated. ²⁷The interpretation of the result in column (8)²⁸ is that annual returns are 2.3 percentage points lower for the mean below threshold fund.²⁹ While this effect is large economically, it is actually somewhat smaller than other findings in the literature. For example, Agarwal, Daniel, and Naik (2009) find a cross-sectional correlation between hedge fund managers' marginal incentives including those provided by performance fees and future returns of 12 percentage point of return for a 1 standard deviation change in incentives. A one standard deviation increase in distance yields a 5.3 percentage point drop in returns.³⁰

A rough calculation of the size of the risk and performance effects in dollars, assuming contracts are always readjusted to keep funds at the threshold, but that this adjustment does not affect performance when the manager is at the threshold, is equivalent to \$12 billion per year in risk-aversion costs and \$20 billion per year in average performance.³¹ Taken together the results in Table 2 suggest that when funds are below their high-water mark they tend to behave in ways that simultaneously increase risk and reduce their expected return. These results are consistent with managers being increasingly likely to take riskier, yet lower expected value projects, as they move farther from their threshold. The key novel testable implication of the model follows from Proposition 2, which predicts that managers with distant thresholds will behave differently than those closer, because distant managers

level.

²⁷The differences between the OLS and 2SLS estimates and the differences between specification (7) and (8) suggest, in addition to the other endogeneity concerns, there is mean-reversion at the strategy level.

²⁸Specification (8) is clearly preferred to specification (7) because it controls for the average performance of other funds in the same strategy class. An alternative approach to measuring performance would be to use the estimated alphas from a standard hedge fund asset pricing model. Replacing raw returns in specifications 4-8 produce comparable results. An alternative way of interpreting the variation in the specifications that include contemporaneous strategy risk and return, columns 2, 4, 6 and 8, is that the dependent variables are the actively managed outcomes while contemporaneous strategy risk and return control for the passive performance.

²⁹14.1 percentage points/year * 15.8% below the high-water mark, conditional on being underwater= 2.3

³⁰14.1 percentage points/year * 37.9% below the high-water mark (one standard deviation)= 5.3

³¹Valuing the risk requires assuming something about the utility function of investors. Suppose investors have a risk aversion coefficient of 1. If returns are normally distributed we can characterize investors' utility functions as mean-variance utility of $return - variance/2$. The average fund is 7.5% below the threshold (15.8% below the high-water mark, conditional on being underwater x 47.6% of funds are underwater at any given point in time), and using the coefficient estimate from column (3), which is in monthly variance and scaling to percentages, yields a cost of risk-taking of 0.0479 percentage points per month ($130/10000*7.5/2*100=0.0479$), which is 0.587 percentage points per year, which when applied to the \$2 trillion hedge fund industry is approximately \$12 billion per year. Similarly, a the average fund under-performs an average of 1.06 percentage points per year using the estimates column (8) ($7.5% * 14.1% = 1.06%$), which, when applied to the hedge fund industry, is approximately \$20 billion per year.

will not find risk-taking profitable given that the threshold is so unlikely to be surpassed. However, the incentives for exerting effort continue to decline as distance increases. Thus, performance should continue to decline because of decreased effort, but the performance cost of risk-taking will no longer magnify the decline. Figure 5 extends Figure 4, but now includes all managers, including those who are very distant from the threshold. One can see that, consistent with the Proposition 2, risk-taking appears to decrease with distance for managers far from their thresholds (solid line). These same managers perform better as they take less risk (dashed line), which is consistent with Corollary 2 that the performance cost of risk-taking is large.

To test this effect statistically, we estimate a separate response to distance for managers far from their thresholds. The measure of “far from their threshold” is admittedly somewhat arbitrary. Here we present results where “far” means one would need a return of 75% to reach the threshold, but the results are robust to other reasonable definitions of “far.” (Approximately 2% of funds are 75% below their high-water mark in at least one year.) Table 3 presents the 2SLS regression results on risk-taking and performance using a similar specifications as in Table 2, except the interaction of the “far from the threshold” dummy with *Distance* is included and instrumented for with the interaction of the “far from the threshold” dummy with the representative strategy-level distance as well.³² The interaction term reflects the difference between the main effect, the responsiveness of near managers, and the responsiveness of distant managers. The net effect on risk-taking in columns (1) and (2) are that distant managers take much less risk than managers moderately far from their thresholds. Indeed, while the point estimates suggest that distant managers take more risk than managers at their thresholds the estimate is not significantly different from zero. To put the effect sizes in perspective, the estimates in column (2) imply that a manager 50% from their threshold increases risk-taking by about twice the amount of a manager who is 100% below their high-watermark.³³

Columns (3) and (4) show the same pattern: managers far from their high-watermark perform better than managers who are moderate distances to their threshold. The estimates in Column (4) suggest that a manager 50% from the threshold reduces performance by almost three times as much as a manager 100% from the threshold.³⁴ These regressions facilitate an estimate of how much of the performance drop due to moral hazard associated with convex incentives for managers who are underwater, but not “far” from their thresholds, is due to reductions in effort, and how much is due to the performance cost of risk-taking. To do so we assume that the reduction in performance observed by managers far from their thresholds is entirely due to effort reduction, which allows one to net out this effect from other managers

³²Being far from the threshold is potentially endogenous as well. However, the bias works against us because the results suggest that risk-taking is lower and performance is better beyond this threshold, and the key endogeneity concern would be that funds with riskier and lower performance are more likely to be far from the threshold.

Alternative specifications (e.g., using a spline) and alternative instrumental variables approaches (e.g., interacting an exogenous measure of “far” with exogenous distance) yield similar point estimates. The specification in Table 3 parsimoniously allows us to capture the information in the sparsely populated tail of managers and facilitates a simple and direct comparison of those managers to the mass of managers closer to the threshold.

³³ $157.0 \cdot .5 = 78.5$ vs $157.0 \cdot 1 - 114.5 \cdot 1 = 42.5$

³⁴ $-36.9 \cdot .5 = 18.5$ vs $-36.9 \cdot 1 + 30.3 \cdot 1 = 6.6$

who are closer to the threshold.³⁵ We then take the difference between the performance effect of “far” managers and the performance effect of managers closer to the threshold to find the performance cost of risk-taking. The estimates in column (4), suggest that 82% of the performance drop observed in managers who are underwater, but not “far” underwater can be attributed to the performance cost of risk-taking and only 18% is due to reduced effort.³⁶

5.1 Fee Variation

The primary results show that moving below the threshold increases risk-taking and reduces performance, until the fund becomes “too far” underwater. To extend these results we explore whether there are heterogeneous treatment effects associated with the underlying performance contracts (i.e., fees), as predicted by the model. Proposition 3 says that when the ratio of the performance fee to base fee increases the incentive effects increase. This follows because the importance of the threshold, and thus the distance to it, is relative to the change in incentives provided by the performance fee over the base fee.

The heterogeneous treatment effect of the fee ratio is examined in Table 4. There is limited variation in the fees: 80% of the funds have performance fees of 20% and 80% of funds have base fees of 1, 1.5, or 2%. However, the results are consistent with the theory. Columns (1) and (2) show that funds with higher fee ratios increase their risk more with distance to the threshold. Evaluating the magnitudes of column (2) says that a fund with a mean fee ratio increases their risk-taking by 36% when below the threshold, while a fund with a standard deviation lower fee ratio increases their risk-taking by only 4%. Columns (3) and (4) show that these funds decrease their performance more with distance to the threshold. Again comparing a fund with a one standard deviation lower fee ratio, that fund has an average performance drop of only 1.4 percentage points compared with a fund with a mean fee ratio having a performance drop of 2.5 percentage points.³⁷

Taken together the heterogeneous responses of underwater funds to contractual fee structures provides several interesting results. First, consistent with the theory, the fee ratio results show that, ceteris paribus, bigger performance fees lead to more responsiveness to the threshold, while higher management fees serve to blunt the incentives to take extra risk for underwater funds. Further, these results address a potential threat to the identification strategy by using meaningful intra strategy-year variation and provide robustness to the main empirical findings in section 5, as they suggest that the results are being driven by explicit incentives. Indeed, the direct effect of *Distance* in Table 4 is a useful falsification test. It

³⁵As the point estimates in columns (1) and (2) suggest that risk-taking is still increasing slowly in managers far from their thresholds this may result in an underestimate of the performance cost of risk-taking.

³⁶The net performance decrease of “far” managers is $(-36.9+30.3)*.01=6.6$ basis points per 1% increase in distance. Assuming that far managers are no longer taking more risk and the linear functional form of the performance response this is the impact of decreased effort. The performance decrease of “near” managers is 36.9 basis points, which reflects both increased risk-taking and decreased effort. Thus the performance cost of risk-taking is 30.3 basis points per increase in distance.

³⁷These compare a fee ratio of 15.2 and 3.5 at a distance of 15.8%. This leads to changes in risk-taking increases of $(-14.1+6.9*15.2)*.158/40.0=36\%$ and $(-14.1+6.9*3.5)*.158/40.0=4\%$ respectively. And performance drops of $(6.6+0.6*15.2)*.158=2.5$ percentage points and $(6.6+0.6*3.5)*.158=1.4$ percentage points respectively.

estimates (assuming a linear heterogeneous treatment effect) what the responsiveness of a fund without a performance fee, and thus with a fee ratio of zero. These funds serve as a useful placebo, because they are exposed to the same market conditions and similar investor responses, but do not have a meaningful high-water mark. Here we see statistically insignificant and, in the case of columns (2) and (3), opposite signed estimates of the effect of distance for those funds. If these coefficients were large and significant, it would suggest that the funds were responding to something other than explicit incentives or we were missing important controls.³⁸ For example, if these behaviors were the result of reference points, we would not expect differential responses for different fee structures. Applied more broadly the results suggest that top management teams with many options are more likely to increase risk when they are out of the money.

5.2 Robustness Checks

The data used in this analysis is well suited to studying the impact of incentives on risk-taking and performance, but they do have some potential limitations. The first set of challenges arises because the data are self-reported and represent only a subset of the hedge fund industry. However, the funds included in this data set represent approximately one quarter of all hedge funds during this period and are believed to be broadly representative.

Beyond the question of coverage, there are two potential forms of self-selection bias discussed in the literature that might apply to these data. First, when a fund first begins to report to the data vendor, it generally reports not only current and future performance, but past performance as well. This “instant history” bias tends to include funds with particularly good initial performance. We can follow the standard approach to dealing with instant history bias by excluding the first two years of a fund’s data and find that doing so does not meaningfully change to the results reported. Additionally, for a subset of the data, we have access to information about when a fund first began reporting. This allows robustness checks of limiting only to funds that began reporting immediately or to more precisely exclude the instant history. Neither is a meaningful change. The second potential form of self-selection bias relates to a firm’s decision to exit the data set (i.e., to stop to reporting to the data vendors). One way to test for this bias is to restrict the analysis to a set of funds that as of a particular point are actively reporting, and only including those funds before that time. Again, the results are not qualitatively changed by this restriction. Additionally, for a subset of the funds that have exited the data set we know the reason the fund has left. While prior literature has assumed that funds exited the data because of extreme success and failure, the vast majority of fund exits in this data are due to fund liquidation (45%)

³⁸For example, in this paper we emphasize the role of effort and risk-taking in response to incentives, however, if a fund experiences significant net outflows it may change the composition of the fund as the fund sells liquid assets, which may be correlated with the effects of interest. Similarly, as a fund experiences inflows it may acquire liquid assets faster than illiquid assets. If illiquid and liquid assets have different risk and return profiles, these asset composition effects might be confounded with changes in distance. Empirically, because flows are potentially endogenous, it is not trivial to separate compositional effects from the effect of changes in the distance to the threshold. However, the results in Table 4 suggest that the effects are being driven by the contracting terms. For the results in Table 4 to be spurious there must be a correlation not only between flows and distance, but also differential correlations between those flows as the contracting terms. The first is plausible, the second less so.

or firm failure (18%). Less than one percent of exits are due to closures to new investments (a sign of success). 26% are voluntary decisions to stop reporting for an unspecified reason, and 5% are mergers into other funds. These last three groups would be the potential sources of reporting bias.³⁹

One potentially important source of measurement error arises from the possibility that definition of “return” may vary across funds, or within funds over time. For example, if different investment vintages have different fee structures, some funds may report net returns using the fees of oldest vintage, while other funds (properly) report a weighted average. A small number of funds report gross returns. Returns net of fees also depend on when fees are accounted for out of assets – returns are monthly, but fees are often accounted for quarterly and paid annually. None of our data report the method of calculating net returns though a subset of our data suggests that gross returns are reported by less than 2% of funds. None the less, the results are qualitatively similar and statistically indistinguishable if we treat the raw data as net returns, input gross returns, and analyze those. However, because we observe net flows rather than the full details of each vintage and use those to calculate distances as well, this adjustment produces an omitted variable bias because both the dependent and independent variables are correlated with the measurement error in the flows.

Hurdle rates complicate the calculation of high-water marks. A hurdle is a base rate of return that a fund must earn before earning a performance fee, which effectively moves the high-water mark by a specific amount every year regardless of performance. The hurdle rate may be specified as a fixed percentage or as a floating rate, such as the 3-month LIBOR rate. Though we know which funds have hurdle rates, we rarely observe the rate itself; however the results are robust to excluding funds with hurdles⁴⁰

Finally, the calculation of high-water marks depends on vintages of investments into funds. However, we do not observe actual flows, only net flows each month, which, because hedge funds were growing rapidly during the sample period biases the estimates of asset

³⁹Survivor bias is another concern in this style of research, but is not a limitation of this data. Funds are included in the data regardless of whether they have exited or continue to report to the data vendors.

⁴⁰There are, additionally, a variety of sources of measurement error in the data and in the calculation of high-water marks. We exclude returns before assets are reported, which tends to bias our regression estimates towards zero. Similarly, excluding any funds for which returns are not available from inception would produce the same bias. An untabulated robustness check restricting to funds for which assets are available at inception verifies that the results are not sensitive to this form of measurement error. Any funds which stop reporting assets but continue to report returns are treated as exits and are addressed in the robustness checks for voluntary reporting. For funds for which assets are not reported for some intermediate period, net flows over that period are distributed evenly over the period. Another source of measurement error is self-categorization into investment strategy types. If categories are too broad it will include funds with heterogeneous strategies. Relatedly, a fund may not properly self-identify in a way that allows it to be categorized with like funds. Both of these effects would weaken our instrument in the first stage by lowering the correlation between the representative “passive” distance measure, and a fund’s actual distance below their high-water mark. However, the first stage results are strong, which suggests that any bias from self-categorization is not meaningfully influencing the results. Serial correlation in returns is another potential source of measurement error. There are several sources of serial correlation. First, funds which hold illiquid assets may use valuation measures that induce serial correlation. Second, assets that the funds own may exhibit momentum. The second source is partially addressed by the inclusion of Carhart’s (1997) momentum factor for equity. However as we pursue a broader robustness check by estimating an AR(1) return process and using a measure of return which is net of this AR(1) process. Doing so does not meaningfully change the results.

vintages (i.e., it treats them as older vintages than they actually are). As older funds always have the highest high-water mark, our distance measure is, therefore, biased upward. However, this is only a problem of scale because the vintages do not matter unless a fund is some distance below its threshold. As soon as a fund is above its threshold all of the old vintages get shifted up. The scaling issue, however, is further complicated by not observing the vintage of the assets that flow out. Instead, we apply the outflows proportionally among all fund assets. An alternative assumption is to apply out flows on a first-in first-out basis that assumes that the funds that leave are the always the oldest vintages. Neither is a perfect representation of actual flows, but either approach generates similar results.⁴¹

6 Conclusion

This paper demonstrates how convex incentive schemes, such as increasing payouts for performance above a certain threshold, create moral hazard problems for organizations. Most importantly, we show that threshold incentives influence principals not only by inducing inefficient risk-taking, but also by inducing excessive risk-taking, when the principal fall below their threshold, This finding has implications for contract and incentive design.

A similar logic provides a further cautionary consequence of kinked incentives used to incent managers in many industries. Kinked incentives lead managers to increase risk and reduce expected performance following a negative shock. However, if there is negative macroeconomic shock, many managers will simultaneously increase their risk exposure in a manner that destroys value systematically across an industry, or even across an economy, essentially multiplying and sustaining the original shock. This magnification effect is important from a public policy perspective as it suggests that kinked incentives may have contributed to both the depth and duration of recent economic downturn.

Yet, the results do not suggest that convex incentive compensation should necessarily be avoided, but rather suggest the importance of setting thresholds correctly or designing alternative incentives that consider risk-taking explicitly. Interestingly hedge fund performance contracts are already being written to address the excessive risk-taking behavior we document. Whether these new contracts will be superior to the standard pre-crisis structures promises to be a rich area for future study.

⁴¹The uncertainty over net flows produces a form of multiplicative heteroscedasticity, which may bias the 2SLS estimates. The usual solution of a logarithmic specification is not appropriate in this setting because of the large number of funds at zero distance. In the reported specifications the weighting induces a bias towards zero in the reported coefficients, but unbiased average treatment effects. The average treatment effects are unbiased, since the same weighting is used in the calculation of average treatments.

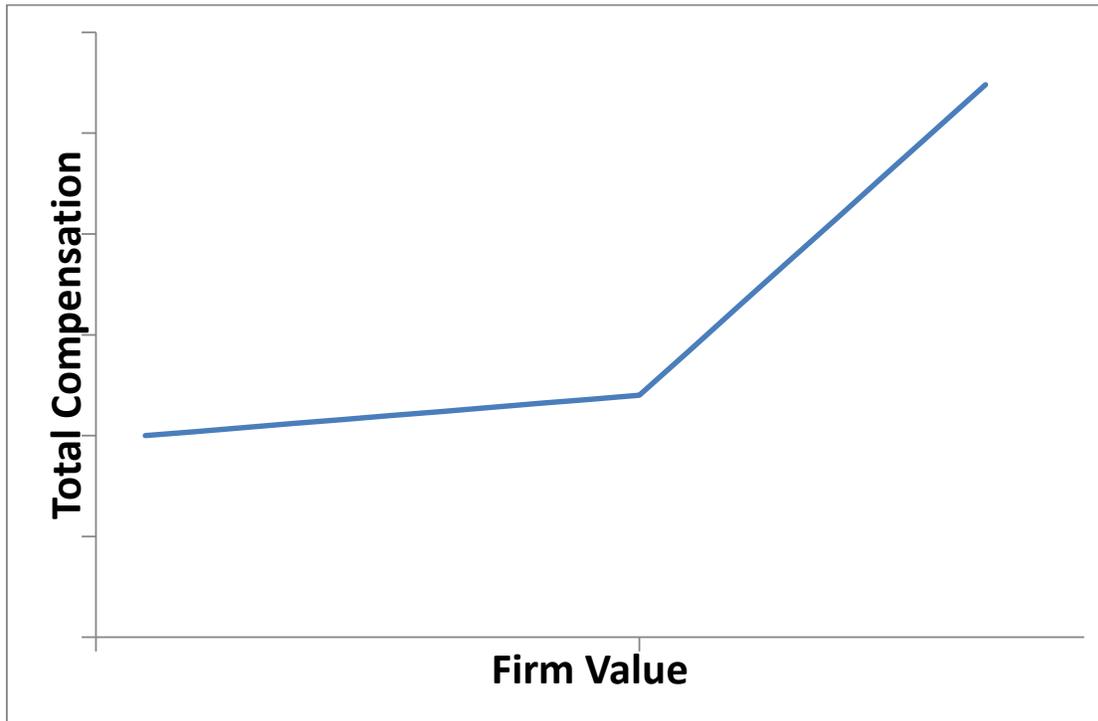
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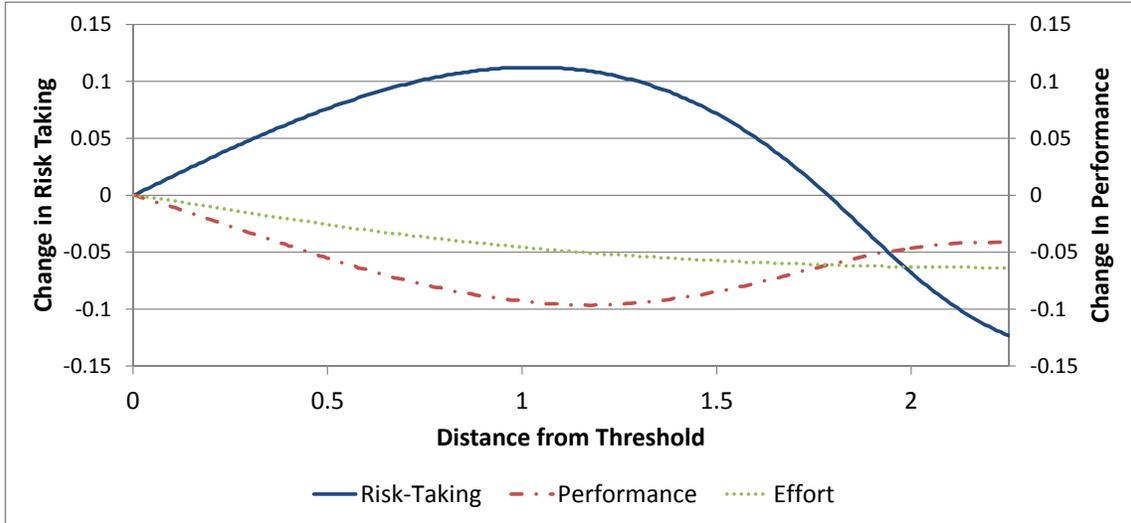
Figure 1: **Threshold Incentive Schemes**



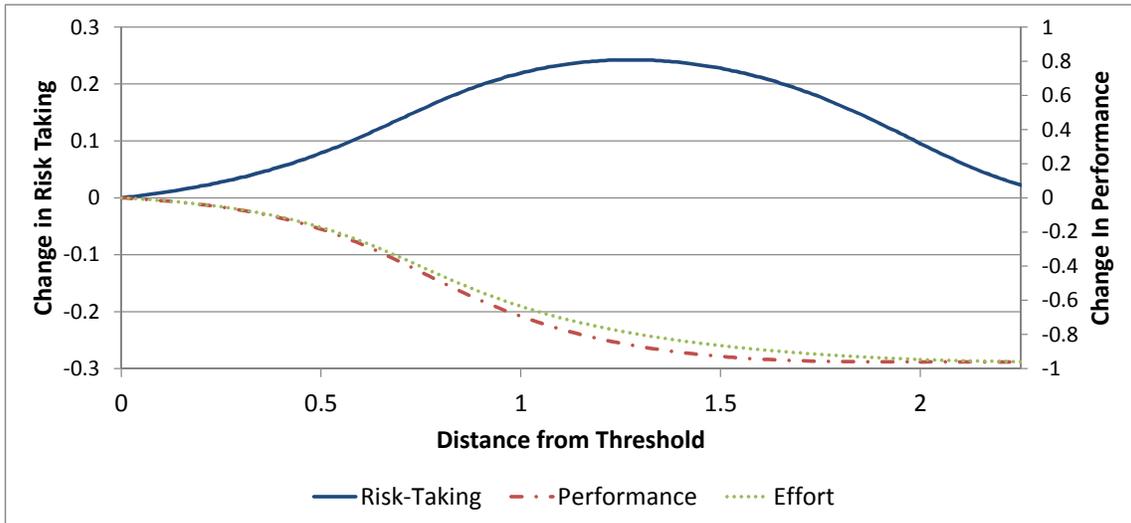
Note: The figure shows a generic threshold incentive scheme where total compensation is increasing at a slow rate below the performance threshold and at a higher rate after the performance threshold. In this example the threshold is measured in firm value, but could be sales, financial returns, or other context specific metrics.

Figure 2: Theoretical Risk-Taking, Performance, and Effort with Distance to the Threshold

(a) High Cost of Effort

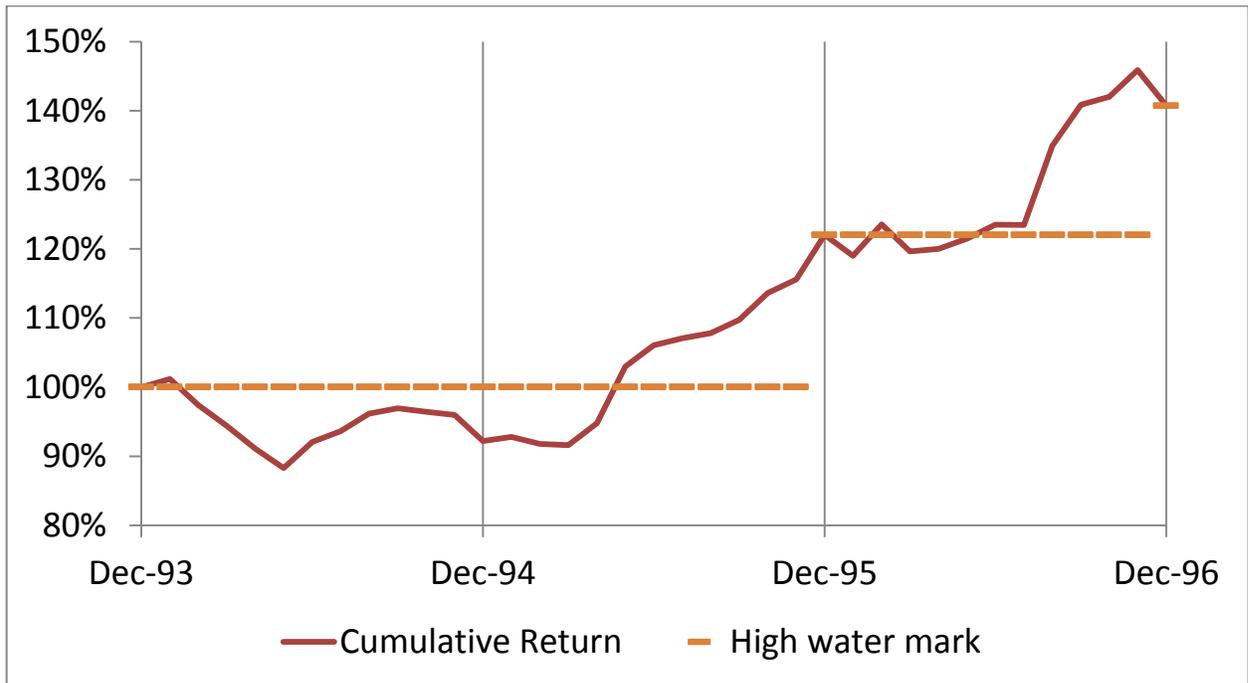


(b) Low Cost of Effort



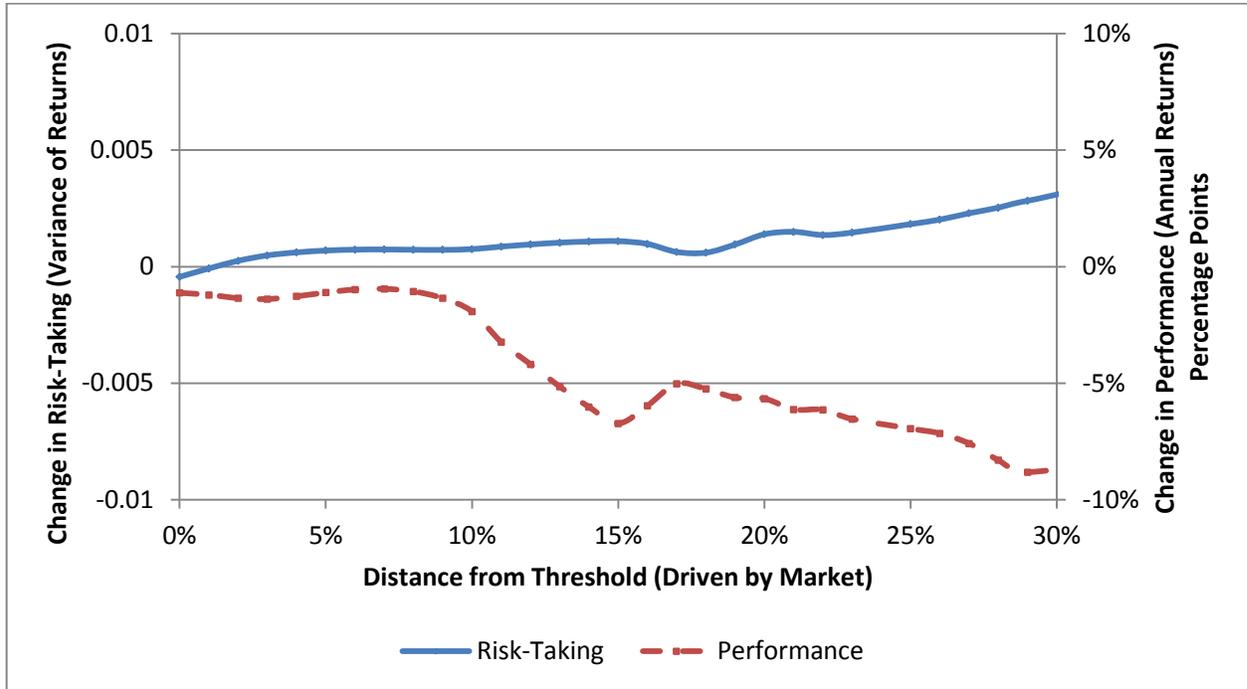
Note: Both figures show single peaked risk-taking and monotonically decreasing effort. Panel (a) displays non-monotonic performance when effort is relatively costly. Panel (b), in contrast, shows performance not deviating much from the decreasing effort line. In both panels, the solid blue line plots the change in risk-taking against the left vertical axis, the green dotted line plots the change in effort, and the red dashed line plots the change in performance, both against the right vertical axis, of the manager's solution to the maximization problem against distance below the threshold. Empirically, effort will be unobservable. $q(r) = r^2 - r$, $b = 2\%$, $p = 20\%$. For panel (a): $c(e) = e^2$. For panel (b): $c(e) = e^2/10$.

Figure 3: Calculation of High-Water Marks



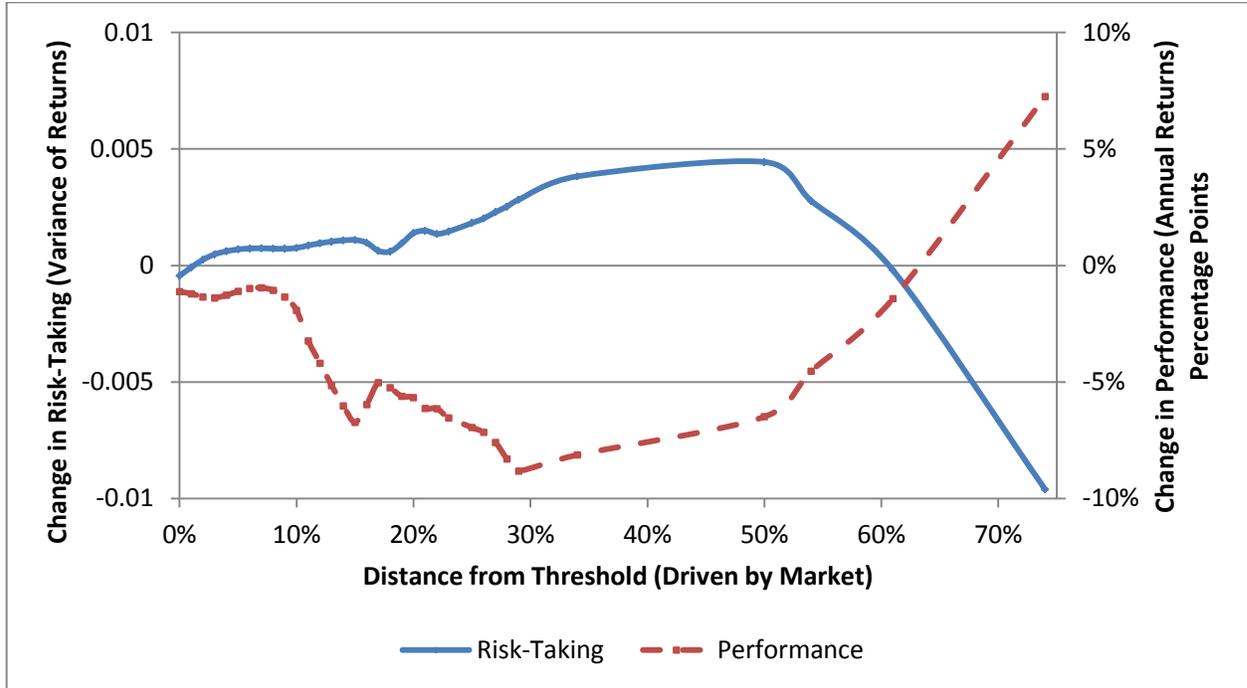
Note: The figure shows the calculation of high-water marks. The red line identifies the cumulative return of a hypothetical hedge fund. At the end of 1994 this fund is 8% below its high-water mark. At the end of 1995, this fund is paid a performance fee and its high-water mark ratchets up.

Figure 4: **Risk-Taking Increases and Performance Falls with Distance when the Threshold is Near.**



Note: The figure shows the relationship between exogenous changes in the distance to the high-water mark and changes in risk-taking and performance. The horizontal axis represents distance below the high-water mark due to exogenous market variation. The left vertical axis measures the variance of fund returns in the following year plotted in the increasing blue solid line. The right vertical axis measures the annual return of funds in the following year plotted in the decreasing red dashed line. Both lines are non-parametric fitted values after including fund and year fixed effects and controls from age, age-squared, assets under management, and log assets under management. Includes only market driven distances up to 30%.

Figure 5: Risk-Taking Declines and Performance Improves with Distance when the Threshold is Far.



Note: The figure includes extends Figure 4 to include managers far from the threshold and shows the non-monotonic relationships between between exogenous changes in the distance to the high-water mark and changes in risk-taking and performance. The horizontal axis represents distance below the high-water mark due to exogenous market variation. The left vertical axis measures the variance of fund returns in the following year plotted in the blue solid line. The right vertical axis measures the annual return of funds in the following year plotted in the red dashed line. Both lines are non-parametric fitted values after including fund and year fixed effects and controls from age, age-squared, assets under management, and log assets under management. Includes all distances.

Table 1: **Summary Statistics**

(a) **Fund-Year**

Variable	Mean	(Std. Dev.)	N
Variance	40.0	(411.3)	19750
Return	12.9	(24.9)	19750
Assets Under Management (Millions)	111.4	(252.7)	19750
Distance	0.075	(0.379)	19750
Below Threshold	0.476	(0.499)	19750
Distance if Below	0.158	(0.537)	9409
Age	4.339	(3.7)	19750

(b) **Funds**

Variable	Mean	(Std. Dev.)	Min.	Max.	N
Complete Years	5.14	(2.85)	2	11	3845
Inception Year	1998	(4.1)	1976	2006	3845
Performance Fee	18.85	(4.82)	0	50	3845
Base Fee	1.48	(0.84)	0.05	20	3845
Fee Ratio	15.19	(11.71)	0	250	3845
Has Hurdle	0.35	(0.48)	0	1	3845

Table 2: **Basic Results**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	OLS		2SLS		OLS		2SLS	
	Variance _{it}				Annual Return _{it}			
Distance	40.0*** [14.3]	43.2*** [14.4]	130.4*** [33.1]	71.1** [29.4]	2.0 [1.5]	0.8 [1.5]	-6.5* [3.7]	-14.1*** [3.6]
Strategy Return _t		-69.5 [49.2]		-79.0 [53.0]		65.4*** [3.6]		70.4*** [3.9]
Strategy Variance _t		2,914.8*** [366.9]		2,950.7*** [388.8]		12.3 [47.5]		-6.9 [46.6]
Observations	19,750	19,750	19,750	19,750	19,750	19,750	19,750	19,750
R-squared	0.002	0.004			0.138	0.172		
Number of Funds	3,845	3,845	3,845	3,845	3,845	3,845	3,845	3,845
Kleibergen-Paap Wald F Statistic on Excluded Variables			46.46	50.10			46.46	50.10

Robust standard Errors clustered by fund in brackets. *** p<0.01, ** p<0.05, * p<0.10.

All Specifications include Age, Age Squared, Assets Under Management, Log Assets Under Management, Time Fixed Effects, and Fund Fixed Effects. R-squared excludes fund fixed effects.

Distance measures distance below the threshold at the beginning of year t . Strategy Return_t and Strategy Variance_t are the performance of the passive strategy specific fund in year t . 2SLS specifications include "Distance" that reflects strategy performance.

Table 3: Distant Managers

	(1)	(2)	(3)	(4)
	Variance 2SLS		Annual Return 2SLS	
Distance	337.6*** [85.6]	157.0** [74.7]	-30.9*** [11.5]	-36.9*** [9.6]
Distance X More than 75%	-271.3*** [77.8]	-114.5* [66.7]	32.0*** [10.5]	30.3*** [9.0]
Strategy Return _t		-71.2 [51.5]		68.3*** [3.9]
Strategy Variance _t		2,978.9*** [395.6]		-14.3 [47.1]
Observations	19,750	19,750	19,750	19,750
Number of Funds	3,845	3,845	3,845	3,845
Kleibergen-Paap Wald F Statistic on Excluded Variables	58.99	68.99	58.99	68.99

Robust standard Errors clustered by fund in brackets. *** p<0.01, ** p<0.05, * p<0.10.
See notes to Table 2. “More than 75%” indicates if the fund is more than 75% from the
threshold

Table 4: **Explicit Incentives**

	(1)	(2)	(3)	(4)
		Variance 2SLS	Annual Return 2SLS	
Distance	26.1 [28.0]	-14.1 [36.0]	4.8 [6.4]	-6.6 [5.5]
Distance X Fee Ratio	8.8*** [3.1]	6.9** [3.1]	-1.0** [0.4]	-0.6* [0.3]
Strategy Return _t		-74.0 [53.1]		70.0*** [3.9]
Strategy Variance _t		3,067.9*** [398.0]		-17.2 [46.2]
Observations	19,750	19,750	19,750	19,750
Number of Funds	3,845	3,845	3,845	3,845
Kleibergen-Paap Wald F Statistic on Excluded Variables	37.50	38.34	37.50	38.34

Robust standard Errors clustered by fund in brackets. *** p<0.01, ** p<0.05, * p<0.10
All Specifications include Age, Age Squared, Assets Under Management, Log Assets Under Management, Time Fixed Effects, and Fund Fixed Effects.
See notes to Table 2.

Table 5: **First Stages - Basic Regression**

	(1)	(2)
	Distance	
	OLS First Stage	
Market Distance	2.8*** (0.4)	2.9*** (0.4)
Strategy Return _t		0.3*** (0.1)
Strategy Variance _t		-3.1*** (0.6)
Observations	19,750	19,750
R-Squared	0.091	0.099
Number of Funds	3,845	3,845
Kleibergen-Paap Wald F Statistic on Excluded Variables	46.46	50.10

Robust standard Errors clustered by fund in brackets. *** p<0.01, ** p<0.05, * p<0.10.
See notes to Table 2.

Table 6: **First Stages - Distant Managers**

	(1)	(2)	(3)	(4)
	Distance	Distance X More than 75%	Distance	Distance X More than 75%
	OLS First Stage			
Market Distance	0.7*** (0.1)	-0.4*** (0.1)	0.9*** (0.1)	-0.2** (0.1)
Market Distance X More than 75%	8.9*** (2.3)	10.8*** (2.5)	8.8*** (2.3)	10.7*** (2.5)
Strategy Return _t			0.1* (0.1)	0.1 (0.1)
Strategy Variance _t			-3.2*** (0.6)	-2.1*** (0.6)
Observations	19,750	19,750	19,750	19,750
R-Squared	0.162	0.160	0.167	0.162
Number of Funds	3,845	3,845	3,845	3,845
Angrist-Pischke	52.04	27.76	70.83	20.96
multivariate F Statistic on Excluded Variables				

Robust standard Errors clustered by fund in brackets. *** p<0.01, ** p<0.05, * p<0.10.
See notes to Tables 2 and 3.

Table 7: **First Stages - Explicit Incentives**

	(1)	(2)	(3)	(4)
	Distance	Distance X Fee Ratio	Distance	Distance X Fee Ratio
	OLS First Stage			
Market Distance	4.6*** (1.5)	3.7 (7.7)	4.7*** (1.5)	5.9 (7.5)
Market Distance X Fee Ratio	-0.1 (0.1)	2.1*** (0.5)	-0.1 (0.1)	2.1*** (0.5)
Strategy Return _t			0.3*** (0.1)	2.8*** (0.7)
Strategy Variance _t			-3.2*** (0.6)	-54.0*** (10.0)
Observations	19,750	19,750	19,750	19,750
R-Squared	0.094	0.074	0.102	0.081
Number of Funds	3,845	3,845	3,845	3,845
Angrist-Pischke multivariate F Statistic on Excluded Variables	8.871	43.45	9.058	45.39

Robust standard Errors clustered by fund in brackets. *** p<0.01, ** p<0.05, * p<0.10.
See notes to Tables 2 and 4.

Appendix

A Proofs of Results

As a preliminary step to proving our results, we begin by characterizing the solution to the manager's problem. The (static) profit function of the manager is given by:

$$\begin{aligned} \Pi(e, r, d, b, p) &= b(e + q(r)) \\ &\quad + p \left(1 - \Phi \left(\frac{d - e - q(r)}{\sqrt{r}} \right) \right) \left(-d + e + q(r) + \frac{\sqrt{r} \phi \left(\frac{d - e - q(r)}{\sqrt{r}} \right)}{1 - \Phi \left(\frac{d - e - q(r)}{\sqrt{r}} \right)} \right) - c(e) \end{aligned}$$

which can be re-written:

$$\begin{aligned} \Pi(e, r, c, b, p) &= b(e + q(r)) + p\sqrt{r}\phi \left(\frac{d - e - q(r)}{\sqrt{r}} \right) \\ &\quad + p \left(1 - \Phi \left(\frac{d - e - q(r)}{\sqrt{r}} \right) \right) (-d + e + q(r)) - c(e) \end{aligned} \quad (\text{A.1})$$

Taking the derivative with respect to e and r yields the first-order conditions:

$$b + p \left(1 - \Phi \left(\frac{-m}{\sqrt{r}} \right) \right) - c'(e) = 0 \quad (\text{A.2})$$

$$p \left(\frac{1}{2\sqrt{r}} \right) \phi \left(\frac{-m}{\sqrt{r}} \right) + \left[b + p \left(1 - \Phi \left(\frac{-m}{\sqrt{r}} \right) \right) \right] q'(r) = 0 \quad (\text{A.3})$$

where $m = -d + e + q(r)$.

Lemma 3. *Risk taking is on the strictly downward sloping part of the curve $q(r)$.*

Proof. To prove this result we must show that $q'(r^*) < 0$. Given the Inada conditions on $c(e)$, we know the solution to the manager's problem is at an interior. At any interior effort level, from (A.2) we know that

$$b + p \left(1 - \Phi \left(\frac{-m}{\sqrt{r}} \right) \right) = c'(e) > 0$$

Examining risk-taking at its lowest bound, we also need to satisfy, from (A.3), that

$$p \left(\frac{1}{2\sqrt{r}} \right) \phi \left(\frac{-m}{\sqrt{r}} \right) + \left[b + p \left(1 - \Phi \left(\frac{-m}{\sqrt{r}} \right) \right) \right] q'(r) \leq 0$$

Since $p \left(\frac{1}{2\sqrt{r}} \right) \phi \left(\frac{-m}{\sqrt{r}} \right) > 0$ and $\left[b + p \left(1 - \Phi \left(\frac{-m}{\sqrt{r}} \right) \right) \right] > 0$, it must be that $q'(r^*) < 0$. \square

Lemma 4. *There exists a single solution to the maximization problem for any triple (d, b, p) . Moreover, it is the unique local maximum.*

Proof. For a solution to the manager's problem to be unique we must show: (i) $\frac{\partial^2 \Pi}{\partial^2 e} < 0$, (ii) $\frac{\partial^2 \Pi}{\partial^2 r} < 0$, and (iii) $\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e} > 0$.

For part (i) we take the derivative of the left hand side of (A.2). This yields

$$\frac{\partial^2 \Pi}{\partial^2 e} = p \left(\frac{1}{\sqrt{r}} \right) \left(\phi \left(\frac{-m}{\sqrt{r}} \right) \right) - c''(e) \quad (\text{A.4})$$

Note that $p \left(\frac{1}{\sqrt{r}} \right) \left(\phi \left(\frac{-m}{\sqrt{r}} \right) \right) \leq p \left(\frac{1}{\sqrt{r^{max}}} \right) (\phi(0))$ since $\phi(0)$ is the maximum of $\phi(\cdot)$. So a sufficient condition for this to hold is that $c''(\cdot) \geq \frac{p}{\sqrt{2\pi r^{max}}}$.

For parts (ii) and (iii) we must show that

$$\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e} > 0$$

Taking the derivative of (A.3) with respect to r and e gives

$$\begin{aligned} \frac{\partial^2 \Pi}{\partial^2 r} &= -p \left(\frac{1}{4r\sqrt{r}} \right) \phi \left(\frac{-m}{\sqrt{r}} \right) - p \left(\frac{-m}{2r} \right) \left(\frac{m}{2r\sqrt{r}} - \frac{q'(r)}{\sqrt{r}} \right) \phi \left(\frac{-m}{\sqrt{r}} \right) \\ &\quad + \left[b + p \left(1 - \Phi \left(\frac{-m}{\sqrt{r}} \right) \right) \right] q''(r) + \left[\frac{\partial^2 \Pi(e, r)}{\partial e \partial r} \right] q'(r) \end{aligned} \quad (\text{A.5})$$

and

$$\frac{\partial^2 \Pi(e, r)}{\partial e \partial r} = -p \left(\frac{m}{2r\sqrt{r}} - \frac{q'(r)}{\sqrt{r}} \right) \phi \left(\frac{-m}{\sqrt{r}} \right) \quad (\text{A.6})$$

Substituting (A.4), (A.5) and (A.6) and simplifying yields the result :

$$\begin{aligned} \frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e} &= \left[-\frac{\partial^2 \Pi}{\partial^2 e} \right] p \left(\frac{1}{4r\sqrt{r}} \right) \phi \left(\frac{-m}{\sqrt{r}} \right) \\ &\quad + c''(e) p \sqrt{r} \left(\frac{m}{2r\sqrt{r}} - \frac{q'(r)}{\sqrt{r}} \right)^2 \phi \left(\frac{-m}{\sqrt{r}} \right) \\ &\quad - \left[-\frac{\partial^2 \Pi}{\partial^2 e} \right] \left[b + p \left(1 - \Phi \left(\frac{-m}{\sqrt{r}} \right) \right) \right] q''(r) \end{aligned}$$

The first term is strictly positive if $\frac{\partial^2 \Pi}{\partial^2 e} < 0$. The second term is weakly positive if $c''(e) \geq 0$. The third term is strictly positive if $\frac{\partial^2 \Pi}{\partial^2 e} < 0$, $q''(r) < 0$ and weakly so if those hold weakly.

Taking these two together, $c''(\cdot) \geq \frac{p}{\sqrt{2\pi r^{max}}}$ and $q''(r) \leq 0$ are sufficient conditions for the Hessian to be negative semidefinite, and ensure that there is exactly one local maximum of the agent's problem. Because we also know that the solution is interior, that implies that there is a unique solution to the maximization problem for any set of triple (d, b, p) . \square

Proposition 4. *Effort is decreasing in distance below the threshold.*

Proof. Note that condition (iii) of Lemma 2 satisfies the full rank condition of the Implicit Function Theorem. Thus, by the IFT, we have

$$\begin{aligned}\frac{\partial e^*}{\partial d} &= \frac{-\frac{\partial^2 \Pi}{\partial^2 r} \frac{\partial^2 \Pi}{\partial e \partial d} + \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial d}}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} \\ \frac{\partial r^*}{\partial d} &= \frac{\frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial e \partial d} - \frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial r \partial d}}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}}\end{aligned}$$

We know that

$$-\frac{\partial^2 \Pi}{\partial^2 e} - c''(e) = \frac{\partial^2 \Pi}{\partial e \partial d}$$

and

$$-\frac{\partial^2 \Pi}{\partial e \partial r} = \frac{\partial^2 \Pi}{\partial r \partial d}$$

which implies

$$\frac{\partial e^*}{\partial d} = 1 + \frac{\frac{\partial^2 \Pi}{\partial^2 r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} \quad (\text{A.7})$$

$$\frac{\partial r^*}{\partial d} = \frac{-\frac{\partial^2 \Pi}{\partial e \partial r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} \quad (\text{A.8})$$

From (A.7) we must show

$$\frac{\partial^2 \Pi}{\partial^2 r} c''(e) < -\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} + \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}$$

Note that

$$\frac{\partial^2 \Pi}{\partial^2 r} c''(e) < -\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r}$$

is a sufficient condition since $\frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e} > 0$, which reduces to

$$c''(e) > -\frac{\partial^2 \Pi}{\partial^2 e}$$

Substituting (A.4) in the right hand side we have

$$\begin{aligned}c''(e) &> -p \left(\frac{1}{\sqrt{r}} \right) \phi \left(\frac{-m}{\sqrt{r}} \right) + c''(e) \\ &\quad -p \left(\frac{1}{\sqrt{r}} \right) \phi \left(\frac{-m}{\sqrt{r}} \right) < 0\end{aligned}$$

Which is satisfied since $p > 0$, $\frac{1}{\sqrt{r}} > 0$, and $\phi(\cdot) > 0$. Risk-taking has the following comparative statics with respect to distance from the threshold: \square

Proposition 5. (i) Risk-taking is increasing with distance “near” or above the threshold;
(ii) Risk-taking is decreasing in distance “far” below the threshold; and
(iii) Risk-taking is single-peaked if the marginal cost of effort is sufficiently high. A sufficient condition is $c'(e) > p \exp(-\frac{1}{2}) / \sqrt{2\pi}$.

Proof. Consider (A.8). It implies that the change in the agent’s optimal risk level has the same sign as $-\frac{\partial^2 \Pi}{\partial e \partial r} c''(e)$.

For part (i), we will show that for all values of $m > \tilde{m}$ where $\tilde{m} < 0$, $\frac{\partial r^*}{\partial d} < 0$. From (A.6) we have

$$-\frac{\partial^2 \Pi}{\partial e \partial r} = p \left(\frac{m}{2r\sqrt{r}} - \frac{q'(r)}{\sqrt{r}} \right) \phi \left(\frac{-m}{\sqrt{r}} \right) c''(e).$$

Since $p\phi\left(\frac{-m}{\sqrt{r}}\right) c''(e) > 0$ we need to show that $\frac{m}{2r\sqrt{r}} - \frac{q'(r)}{\sqrt{r}} > 0$ for all $m > \tilde{m}$. Rearranging yields:

$$m > 2rq'(r)$$

By Lemma 1 the right hand side is strictly less than zero, which implies that there exists $\tilde{m} < 0$ such that for all $m > \tilde{m}$, the above is satisfied.

For part (ii), if d is large then from (A.6),

$$\frac{\partial^2 \Pi}{\partial e \partial r} \approx -p\phi\left(\frac{-m}{\sqrt{r}}\right) \frac{m}{2r^{\frac{3}{2}}}$$

because for d large $\left|\frac{m}{2r\sqrt{r}}\right| \gg \left|\frac{q'(r)}{\sqrt{r}}\right|$. Moreover, when d is large, this implies m is negative. So, as above, risk-taking has the same sign as:

$$-\frac{\partial^2 \Pi}{\partial e \partial r} \approx p\phi\left(\frac{-m}{\sqrt{r}}\right) \frac{m}{2r^{\frac{3}{2}}} < 0$$

For part (iii) we note that parts (i) and (ii) show that there is at least one sign change in $\frac{\partial r^*}{\partial d}$. To show that it is single peaked, it suffices to show that $m - 2rq'(r)$ is monotonic in d . Taking the derivative at the optimum:

$$\begin{aligned} \frac{\partial}{\partial d} (-d + e^* + q(r^*) + 2r^* q'(r^*)) &= -1 + 1 - \frac{\frac{\partial^2 \Pi}{\partial^2 r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} \\ &\quad + [q'(r^*) - 2q'(r^*) - 2r^* q''(r^*)] \\ &\quad \times \frac{-\frac{\partial^2 \Pi}{\partial e \partial r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} \end{aligned}$$

Which has the opposite sign as:

$$\frac{\partial^2 \Pi}{\partial^2 r} - [q'(r^*) + 2r^* q''(r^*)] \frac{\partial^2 \Pi}{\partial e \partial r}$$

Substituting in from A.5 and A.6, dropping the asterisks and simplifying yields:

$$\begin{aligned}
&= \left(c'(e) + \frac{m}{\sqrt{r}} p \phi \left(\frac{-m}{\sqrt{r}} \right) \right) \left(\frac{1}{2r} q'(r) + q''(r) \right) \\
&\quad + p \phi \left(\frac{-m}{\sqrt{r}} \right) \left(-\frac{m^2}{4r^2 \sqrt{r}} - 2\sqrt{r} q'(r) q''(r) \right)
\end{aligned}$$

The second line is always negative. The first line is also negative if

$$c'(e) + \frac{m}{\sqrt{r}} p \phi \left(\frac{-m}{\sqrt{r}} \right) > 0$$

Note that from the properties of the normal distribution $\frac{m}{\sqrt{r}} p \phi \left(\frac{-m}{\sqrt{r}} \right)$ has a minimum at $m = -\sqrt{r}$ equal to $-p \exp(-\frac{1}{2}) / \sqrt{2\pi}$. Thus $c'(e) > p \exp(-\frac{1}{2}) / \sqrt{2\pi}$ is a sufficient condition. \square

Proposition 6. *If the threshold is near, and the cost of effort is sufficiently convex,*

i) the rate of decrease in effort with distance is increasing in the ratio of the performance fee to the base fee, and

ii) the rate of increase in risk-taking with distance is increasing in the ratio of the performance fee to the base fee. We note that the maximization problem is written in the levels of the fees, not the ratios. However, without loss of generality, we can normalize such that $b = 1$ and interpret p as the ratio of performance to base fees.

Proof. Next, we extend the implicit function theorem to second order results for our specification in the standard way. From first order solution to the maximization problem we know that:

$$\begin{aligned}
\frac{\partial \Pi}{\partial e}(e^*, r^*, d, p) &= 0 \\
\frac{\partial \Pi}{\partial r}(e^*, r^*, d, p) &= 0
\end{aligned}$$

Taking the total derivative with respect to d and p and expanding yields:

$$\begin{aligned}
&\frac{\partial}{\partial p} \frac{\partial}{\partial d} e^* \frac{\partial^2 \Pi}{\partial^2 e} + \frac{\partial}{\partial p} \frac{\partial}{\partial d} r^* \frac{\partial^2 \Pi}{\partial r \partial e} \\
&+ \left(\frac{\partial}{\partial p} \frac{\partial}{\partial d} \frac{\partial \Pi}{\partial e} + \frac{\partial}{\partial d} e^* \frac{\partial}{\partial p} \frac{\partial^2 \Pi}{\partial^2 e} + \frac{\partial}{\partial d} r^* \frac{\partial}{\partial p} \frac{\partial^2 \Pi}{\partial r \partial e} \right) \\
&+ \left(\frac{\partial}{\partial e} \frac{\partial}{\partial d} \frac{\partial \Pi}{\partial e} + \frac{\partial}{\partial d} e^* \frac{\partial}{\partial e} \frac{\partial^2 \Pi}{\partial^2 e} + \frac{\partial}{\partial d} r^* \frac{\partial}{\partial e} \frac{\partial^2 \Pi}{\partial r \partial e} \right) \\
&+ \left(\frac{\partial}{\partial r} \frac{\partial}{\partial d} \frac{\partial \Pi}{\partial e} + \frac{\partial}{\partial d} e^* \frac{\partial}{\partial r} \frac{\partial^2 \Pi}{\partial^2 e} + \frac{\partial}{\partial d} r^* \frac{\partial}{\partial r} \frac{\partial^2 \Pi}{\partial r \partial e} \right) = 0
\end{aligned}$$

and

With those quantities identified, one can sign those values using substitution. In addition to the above numbered equations, we have:

$$\frac{\partial^3 \Pi}{\partial p \partial e \partial r} = - \left(\frac{m}{2r\sqrt{r}} + \frac{q'(r)}{\sqrt{r}} \right) \phi \left(\frac{-m}{\sqrt{r}} \right)$$

$$\frac{\partial}{\partial r} \frac{\partial}{\partial d} \frac{\partial \Pi}{\partial e} = - \frac{\partial}{\partial r} \frac{\partial}{\partial e} \frac{\partial \Pi}{\partial e}$$

So

$$\frac{\partial}{\partial r} \frac{\partial}{\partial d} \frac{\partial \Pi}{\partial e} + \frac{\partial}{\partial d} e^* \frac{\partial}{\partial r} \frac{\partial^2 \Pi}{\partial^2 e} = - \frac{\partial}{\partial r} \frac{\partial^2 \Pi}{\partial^2 e} + \frac{\partial}{\partial d} e^* \frac{\partial}{\partial r} \frac{\partial^2 \Pi}{\partial^2 e} = \frac{\frac{\partial^2 \Pi}{\partial^2 r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} \frac{\partial}{\partial r} \frac{\partial^2 \Pi}{\partial^2 e}$$

Suppose that m is small and negative, i.e. near the threshold. Then we have:

$$\frac{\partial^2 \Pi}{\partial e \partial r} = p \left(\frac{q'(r)}{\sqrt{r}} \right) \phi(0)$$

$$\frac{\partial^3 \Pi}{\partial p \partial e \partial r} = \left(\frac{q'(r)}{\sqrt{r}} \right) \phi(0)$$

$$\frac{\partial^3 \Pi}{\partial p \partial d \partial r} = - \left(\frac{q'(r)}{\sqrt{r}} \right) \phi(0)$$

$$\frac{\partial^2 \Pi}{\partial^2 e} = p \left(\frac{1}{\sqrt{r}} \right) (\phi(0)) - c''(e)$$

$$\frac{\partial^2 \Pi}{\partial^2 r} = -p \left(-\frac{(q'(r))^2}{\sqrt{r}} + \frac{1}{4r\sqrt{r}} \right) \phi(0) + [b + p/2] q''(r)$$

Since $\frac{\partial \phi(m)}{\partial m} = -m\phi(m)$, we have:

$$\frac{\partial^3 \Pi}{\partial r \partial^2 e} = 0$$

$$\frac{\partial^3 \Pi}{\partial d \partial^2 e} = 0$$

$$\frac{\partial^3 \Pi}{\partial p \partial^2 e} = \left(\frac{1}{\sqrt{r}} \right) (\phi(0))$$

$$\frac{\partial^3 \Pi}{\partial^3 e} = -c'''(e)$$

$$\frac{\partial^3 \Pi}{\partial r \partial e \partial d} = 0$$

$$\frac{\partial^3 \Pi}{\partial p \partial e \partial d} = - \left(\frac{1}{\sqrt{r}} \right) (\phi(0))$$

$$\frac{\partial^3 \Pi}{\partial e \partial^2 r} = p \left(\frac{q''(r)}{\sqrt{r}} - \frac{q'(r)}{2r\sqrt{r}} \right) \phi(0)$$

$$\frac{\partial^3 \Pi}{\partial d \partial^2 r} = -p \left(\frac{q''(r)}{\sqrt{r}} - \frac{q'(r)}{2r\sqrt{r}} \right) \phi(0)$$

$$\frac{\partial^3 \Pi}{\partial p \partial^2 r} = - \left(-\frac{(q'(r))^2}{\sqrt{r}} + \frac{1}{4r\sqrt{r}} \right) \phi(0) + \frac{1}{2} q''(r)$$

$$\frac{\partial^3 \Pi}{\partial^3 r} = -p \left(-\frac{2(q'(r))q''(r)}{\sqrt{r}} + \frac{(q'(r))^2}{2r\sqrt{r}} - \frac{3}{8r^2\sqrt{r}} \right) \phi(0) + [b + p/2] q'''(r)$$

Substituting those in, and simplifying we have:

$$\begin{aligned} & \frac{-\frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial e \partial r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} \left(- \left(-\frac{(q'(r))^2}{\sqrt{r}} + \frac{1}{4r\sqrt{r}} \right) \phi(0) + \frac{1}{2} q''(r) \right) \\ + & \frac{\frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial e \partial r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} p \left(-\frac{2(q'(r))q''(r)}{\sqrt{r}} + \frac{(q'(r))^2}{2r\sqrt{r}} - \frac{3}{8r^2\sqrt{r}} - \frac{q''(r)}{\sqrt{r}} + \frac{q'(r)}{2r\sqrt{r}} \right) \phi(0) \\ - & \frac{\frac{\partial^2 \Pi}{\partial^2 r} \frac{\partial^2 \Pi}{\partial^2 r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} \left(\frac{1}{\sqrt{r}} \phi(0) \right) - \frac{-\frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial e \partial r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} (b + p/2) q'''(r) \\ + & \frac{\frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial^2 r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} \left(-p \left(-\frac{q''(r)}{\sqrt{r}} + \frac{q'(r)}{2r\sqrt{r}} \right) + \frac{q'(r)}{\sqrt{r}} \right) 2\phi(0) \\ + & \frac{\partial^2 \Pi}{\partial^2 r} \left(1 + \frac{\frac{\partial^2 \Pi}{\partial^2 r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} \right) c'''(e) \end{aligned}$$

and

$$\begin{aligned}
& \frac{\partial^2 \Pi}{\partial^2 e} \frac{\frac{\partial^2 \Pi}{\partial e \partial r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} \left(- \left(- \frac{(q'(r))^2}{\sqrt{r}} + \frac{1}{4r\sqrt{r}} \right) \phi(0) + \frac{1}{2} q''(r) \right) \\
- & \frac{\partial^2 \Pi}{\partial^2 e} \frac{\frac{\partial^2 \Pi}{\partial e \partial r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} p \left(- \frac{2(q'(r))q''(r)}{\sqrt{r}} + \frac{(q'(r))^2}{2r\sqrt{r}} - \frac{3}{8r^2\sqrt{r}} - \frac{q''(r)}{\sqrt{r}} + \frac{q'(r)}{2r\sqrt{r}} \right) \phi(0) \\
& + \frac{\partial^2 \Pi}{\partial^2 e} \frac{\frac{\partial^2 \Pi}{\partial e \partial r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} (-[b + p/2] q'''(r)) \\
& + \frac{\partial^2 \Pi}{\partial^2 e} \frac{\frac{\partial^2 \Pi}{\partial^2 r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} \left(- \frac{q'(r)}{\sqrt{r}} + p \left(- \frac{q''(r)}{\sqrt{r}} + \frac{q'(r)}{2r\sqrt{r}} \right) \right) \phi(0) \\
& + \frac{\partial^2 \Pi}{\partial r \partial e} \frac{\frac{\partial^2 \Pi}{\partial^2 r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} \left(\left(\frac{1}{\sqrt{r}} \right) (\phi(0)) \right) \\
& + \frac{\partial^2 \Pi}{\partial r \partial e} \frac{\frac{\partial^2 \Pi}{\partial e \partial r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} \left(- \frac{q'(r)}{\sqrt{r}} + p \left(- \frac{q''(r)}{\sqrt{r}} + \frac{q'(r)}{2r\sqrt{r}} \right) \right) \phi(0) \\
& - \frac{\partial^2 \Pi}{\partial r \partial e} \left(1 + \frac{\frac{\partial^2 \Pi}{\partial^2 r} c''(e)}{\frac{\partial^2 \Pi}{\partial^2 e} \frac{\partial^2 \Pi}{\partial^2 r} - \frac{\partial^2 \Pi}{\partial e \partial r} \frac{\partial^2 \Pi}{\partial r \partial e}} \right) c'''(e)
\end{aligned}$$

Which, if $c'''(e)$ is sufficiently large, yield the signs of the proposition. \square