A Better Specified Asset Pricing Model to Explain the Cross-section and Time-Series of Commodity Returns*

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Abstract

We show that a model featuring an average commodity factor, a commodity carry factor, and a commodity momentum factor is capable of describing both the cross-sectional and time-series variation of commodity returns. Going beyond extant studies, our empirical results indicate that more parsimonious one- and two-factor models that feature the average and/or carry factors are rejected and the momentum factor contains additional information beyond that conveyed by the carry factor. Furthermore, we find that additional factors (e.g., value or volatility) are statistically and economically insignificant. Together, the three factors appear to forecast real economic activity, returns of bonds, equities, and commodity currencies, and they correlate with economic fundamentals. We explore economic interpretations of our findings and the sources of model performance.

KEY WORDS: Commodity asset pricing models, commodity futures returns, backwardation, contango, average returns, carry strategy, momentum strategy, investment opportunity set, macroeconomy.

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1. Introduction

The interaction of storage and convenience yield, and the hedging motives of producers, consumers, speculators has led to the development of theories about the behavior of commodity futures prices and has spearheaded efforts to understand the evolution of futures prices over time and across maturities.[1] Yet there is a paucity of tractable commodity asset pricing models, whose stochastic discount factor is capable of reconciling the stylized patterns in the cross-section and time-series of commodity futures returns.

Despite substantial headway, several other key questions remain unresolved. Which risk factors provide a parsimonious characterization of both the cross-sectional and time-series variation in commodity returns? Are the risk factors also able to forecast developments in the real economic activity, bond, equity, and currency markets, as argued by adherents of asset pricing theories (e.g., as summarized in Cochrane (2005, Chapter 20))? How do these risk factors correlate with the macroeconomy? Our aim is to fill in the aforementioned gaps in a market that has grown tremendously with the advent of financialization.

Extending the analysis in Szymanowska, de Roon, Nijman, and Goorbergh (2013), we consider both time-series as well as cross-sectional tests to discriminate among commodity asset pricing models. Importantly, we reject the model in Szymanowska, de Roon, Nijman, and Goorbergh (2013) that features carry as the only factor. We also extend the analysis in Yang (2013), by showing that the three-factor model that incorporates the momentum factor appears better aligned with the data compared to a two-factor nested counterpart that contains the average and the carry factor. The predominant finding is that the momentum factor contains additional information beyond that conveyed by the carry factor.

In the context of the three-factor model, our analysis highlights three additional findings: (i) the Hansen and Jagannathan (1997) distance test does not reject correct model pricing; (ii) the average pricing errors are not statistically different from zero, when standard errors are computed using the Newey and West (1987) procedure, with and without the Shanken (1992) correction; and (iii) the risk factors have low correlations. We also elaborate on the joint pricing ability of the model using time-series regressions and

[1] The commodity literature has evolved considerably since Keynes (1930), Hicks (1939), Kaldor (1939), and Samuelson (1965). For a partial list of empirical treatments, we mention Chang (1985), Fama and French (1988a), Bessembinder (1992, 1993), Bessembinder and Chan (1992), Deaton and Laroque (1992), de Roon, Nijman, and Veld (2000), Erb and Harvey (2006), Gorton and Rouwenhorst (2006), Gorton, Hayashi, and Rouwenhorst (2013), Yang (2013), and Szymanowska, de Roon, Nijman, and Goorbergh (2013). These studies mainly focus on the economic nature of commodity risk premia and their statistical attributes. Complementing the empirical work, a strand of theoretical research has centered around characterizing the shape of the futures curve. Such studies include Hirshleifer (1988, 1990), Litzenberger and Rabinowitz (1995), Routledge, Seppi, and Spatt (2000), Carlson, Khokher, and Titman (2007), Kogan, Livdan, and Yaron (2009), and Acharya, Lochstoer, and Ramadorai (2012). However, these contributions do not cater to the structure of the cross-sectional relations and are silent about the connections between the slope of the futures curves, average commodity returns, and commodity momentum. The review articles by Till (2006) and Basu and Miffre (2012) provide a historical perspective.

find that statistical tests do not reject model adequacy based on the implied alphas. Taken all together, our approach shows some promise in reconciling the large average returns to investing in backwardated commodities and high momentum commodities.

With three factors, the commodity pricing model is parsimonious. Our specification tests show that incorporating an additional value factor (along the lines of Asness, Moskowitz, and Pedersen (2013)) or a volatility factor (along the lines of Lustig, Roussanov, and Verdelhan (2011) and Menkhoff, Sarno, Schmeling, and Schrimpf (2012a)) fails to improve pricing ability across our test assets. Additionally, we find that conditional pricing models that allow for state-dependence in the sensitivity of the stochastic discount factor to the risk factors can often outperform their unconditional counterparts.

Considerable attention is devoted to understanding the economic underpinnings of the model. This portion of our analysis is inspired by the evolving literature that investigates the reasons behind the explanatory power of the size, value, and momentum factors for the cross-section of equity returns. We follow the lead of Fama (1991) and Campbell (1996) and show that the commodity risk factors capture state variables that forecast changes in the investment opportunity set. Specifically, our predictive regressions support the view that the commodity factors can forecast real GDP growth across the G7 economies (our inference is throughout based on the conservative Hodrick (1992) 1B covariance estimator).

Going further, we also uncover that the factors, predominantly the average and carry factors, can forecast bond and equity returns up to 12 months, with predictive slope coefficients that are mutually compatible across GDP growth, bond returns, and equity returns. For instance, an increase in the carry factor is associated with a future economic slowdown, higher bond returns, and lower equity returns. Testifying to the global economic nature of the commodity factors, we also show that some of the factors can forecast the returns of commodity currencies, extending the analysis of Chen, Rogoff, and Rossi (2010). Finally, we consider a set of economic fundamentals and show that the factors are correlated with the macroeconomy.

Our efforts complement a growing literature that strives to comprehend the behavior of commodity returns. Specifically, our work can be differentiated from Erb and Harvey (2006), Gorton and Rouwenhorst (2006), and Gorton, Hayashi, and Rouwenhorst (2013) in that our focus is on unconditional and conditional commodity asset pricing models and their ability to price the cross-section and time-series of commodity returns. Our study also departs from Hong and Yogo (2012), who provide a horse race among alternative predictors of commodity futures returns but do not investigate cross-sectional implications. We also differ from Asness, Moskowitz, and Pedersen (2013) and Koijen, Pedersen, Moskowitz, and Vrugt (2012), whose focal point is to construct an empirically viable global asset pricing model. Finally, we provide evidence that deviates from the core conclusions in Yang (2013) and Szymanowska, de Roon, Nijman, and Goorbergh (2013), and we offer a further distinction by exploring the connection of the three factors to changes in the investment opportunity set (as gauged by real economic activity and returns of bonds, equities, and commodity currencies).

Our empirical work also elaborates on the disparity in the cross-section of commodity futures returns, providing some distinction from the approaches in Deaton and Laroque (1992), Litzenberger and Rabinowitz (1995), Hirshleifer (1988), Routledge, Seppi, and Spatt (2000), Casassus and Collin-Dufresne (2005), and Kogan, Livdan, and Yaron (2009). The compatibility of the three-factor model with the documented commodity return patterns has possible implications for investment theory and practice, which transcends the scope of commodity investments.

2. Data description and commodity futures returns

Our commodity futures returns are constructed from end-of-day data provided by the Chicago Mercantile Exchange (CME). For each commodity and maturity available, the database contains, at the daily frequency, a record of the open, low, high, and closing prices, along with information on open interest and trading volume. Our analysis centers on 29 commodity futures contracts covering four major categories, namely, agriculture, energy, livestock, and metal.

In line with Asness, Moskowitz, and Pedersen (2013), Gorton, Hayashi, and Rouwenhorst (2013), and Hong and Yogo (2012), we do not impose explicit data filters except to detect recording errors. We take the start (end) date for our commodity futures sample to be January 1970 (September 2011). Starting the sample in January 1970 allows us to construct carry and momentum portfolios that contain at least three commodities. The number of commodities available ranges from a minimum of 15 in 1970 to a maximum of 28 in July 1994.

An important element to the calculation of monthly futures returns is the treatment of the first notice day, which varies across commodities (as can be seen from Table Online-I). For each commodity, we take a position in the futures contract with the shortest maturity at the end of month t, while guaranteeing that its first notice day is *after* the end of month $t+1$. We follow this treatment because, if the first notice day occurs before a long (short) position is closed, the investor may face a physical delivery (delivery demand) from the counterparty.

Consider our return calculation in the context of crude oil futures between the end of February and March 2011. Let $F_t^{(0)}$ be the price of the front-month futures contract and $F_t^{(1)}$ the price of the next maturity futures contract, both observed at the end of month t. Among the available contracts at the end of February 2011, we take a position in the May 2011 contract (i.e., $F_t^{(1)}$), as its first notice day falls in the middle of April. We do not invest in the April 2011 contract (i.e., $F_t^{(0)}$) because its first notice day falls in the middle of March 2011. The position in the May 2011 contract is closed at the end of March at price $F_{t+1}^{(1)}$. In the same vein, we switch to the June 2011 contract at the end of March 2011.

We calculate the returns of the long and short futures positions as

$$r_{t+1}^{\text{long}} = \frac{1}{F_t^{(1)}}\left(F_{t+1}^{(1)} - F_t^{(1)}\right) + r_t^f \qquad \text{and} \qquad r_{t+1}^{\text{short}} = -\frac{1}{F_t^{(1)}}\left(F_{t+1}^{(1)} - F_t^{(1)}\right) + r_t^f, \tag{1}$$

where r_t^f reflects the interest earned on the fully collateralized futures position (e.g., Gorton, Hayashi, and Rouwenhorst (2013, equation (14))). Define

$$\text{er}_{t+1}^{\text{long}} \equiv r_{t+1}^{\text{long}} - r_t^f \qquad \text{and} \qquad \text{er}_{t+1}^{\text{short}} \equiv r_{t+1}^{\text{short}} - r_t^f, \tag{2}$$

as the excess return of a long and short futures position between the end of month t and $t+1$, respectively.

Our procedure for constructing futures returns, which accounts for the first notice day, deviates from Shwayder and James (2011), but is broadly consistent with Gorton, Hayashi, and Rouwenhorst (2013) and Hong and Yogo (2012). We note that $F_t^{(0)}$ never enters our return calculation because of the way that the first notice day calendar interacts with our returns that are based on end of month observations.

Several checks are performed to safeguard the integrity of our futures returns data. First, we extract the monthly composite futures quotes from Bloomberg and find that the correlation with our returns series is high. Moreover, there is a substantial overlap in the extreme returns of the two return series, pointing to the broader reliability of our futures return observations.[2]

The summary statistics tabulated in Table Appendix-I show that 20 out of 29 commodities have Sharpe ratios below 0.25, indicating that stand-alone investments in commodities are not attractive. Among other salient features, the commodity returns are serially uncorrelated (the absolute first-order autocorrelations

[2]Some data limitations are addressed in the following manner. First, when there is a missing observation (for example, palladium on a few occasions), we fill in the corresponding return from Bloomberg to maintain a complete time-series. Second, if there is no recorded futures price for a commodity on the last business day of a given month, we use prices from the second-to-last business day. For example, because there is no trading record for crude oil, natural gas, gasoline, and heating oil on Monday, May 31, 2010, we employ prices from Friday, May 28, 2010.

are below 0.1 for 22 commodities) and typically positively skewed. Corn is the most liquid futures contract, as measured by its open interest, and propane is the least liquid.

Our data offers flexibility in two additional ways. First, the availability of daily futures returns allows us to construct monthly realized volatilities for each commodity and, hence, an average volatility factor that could be used in asset pricing tests (e.g., Menkhoff, Sarno, Schmeling, and Schrimpf (2012a)). Next, futures prices at multiple maturities help to identify whether a commodity is in backwardation or contango.

Inspection of Table Appendix-I indicates that (i) the fraction of the months in which a commodity is in contango is often greater than when it is in backwardation, and (ii) a predominant portion of the commodities exhibit contango on average. Overall, the magnitudes reported in Table Appendix-I appear aligned with the corresponding ones in others, for example, Erb and Harvey (2006, Table 4) and Gorton, Hayashi, and Rouwenhorst (2013, Table I). The goal of this paper is to explain the cross-sectional and time-series patterns in commodity returns.

3. Asset pricing approach and methodology

To outline our approach and empirical tests, we denote the time $t+1$ excess return of a commodity portfolio i by er^i_{t+1} and collect the returns on all test assets in a vector $\mathbf{er}_{t+1}$. No-arbitrage implies the existence of a candidate stochastic discount factor (SDF) m_{t+1} such that (Cochrane (2005, Chapter 12)):

$$E\left[m_{t+1}\,\mathbf{er}_{t+1}\right] = \mathbf{0}, \qquad \text{with} \qquad m_{t+1} = 1 \,-\, \mathbf{b}'(\mathbf{f}_{t+1} \,-\, \boldsymbol{\mu}), \tag{3}$$

where $\mathbf{f}_{t+1}$ is a vector of risk factors and $\boldsymbol{\mu}$ are the factor means.

In our setup, the parameter vector $\mathbf{b}$ is estimated in the system:

$$E\begin{bmatrix} (1 \,-\, \mathbf{b}'(\mathbf{f}_{t+1} - \boldsymbol{\mu})) \otimes \mathbf{er}_{t+1} \\ \mathbf{f}_{t+1} - \boldsymbol{\mu} \\ \mathrm{vec}\left((\mathbf{f}_{t+1} - \boldsymbol{\mu})(\mathbf{f}_{t+1} - \boldsymbol{\mu})\right)' - \mathrm{vec}\left(\Sigma_{\mathbf{f}}\right) \end{bmatrix} \;=\; \mathbf{0}, \tag{4}$$

using the generalized method of moments of Hansen (1982), where $\Sigma_{\mathbf{f}}$ is the variance-covariance matrix of $\mathbf{f}_{t+1}$. Our formulation follows that of Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011) and Menkhoff, Sarno, Schmeling, and Schrimpf (2012a, equation (A4)) in that our estimates incorporate the uncertainty associated with estimating the means and covariances of $\mathbf{f}_{t+1}$.

The specification of the SDF in equation (3) implies a beta representation, where the expected excess returns of each asset depend on the vector of factor risk premia $\boldsymbol{\lambda}$, which is common to all assets, and the vector of risk loadings $\boldsymbol{\beta}_i$, which is asset-specific. More formally,

$$E\left[\mathrm{er}^i\right] \;=\; \boldsymbol{\lambda}'\boldsymbol{\beta}_i, \qquad \text{where} \qquad \boldsymbol{\lambda} = \Sigma_{\mathbf{f}}\,\mathbf{b}. \tag{5}$$

Our baseline implementation uses 12 test assets and 3 factors. We therefore estimate 12 parameters using 21 moment conditions. As in Cochrane (1996), we first focus on a one-step GMM that uses the identity matrix as a weighting matrix, but we also report results for a two-step GMM that uses the optimal weighting matrix. The standard errors are based on the Newey and West (1987) procedure with lags selected automatically according to Newey and West (1994).

Additionally, we provide estimates of $\boldsymbol{\lambda}$ using the cross-sectional regression methodology of Fama and MacBeth (1973). The standard errors of $\boldsymbol{\lambda}$ are computed using the Newey and West (1987) procedure with automatic lag selection with and without the correction of Shanken (1992). We employ the framework in equations (3)-(5) to compare alternative models of commodity returns.

4. Motivating a model with average, carry, and momentum factors

In this section, we propose an asset pricing model for commodities that incorporates an average factor, a carry factor, and a momentum factor. The three-factor model is capable of explaining both the cross-section and time-series variation in expected commodity returns.

We also compare, statistically and economically, the performance of the three-factor model to alternative specifications that include a value and a volatility factor, and we explore conditional models that relax the assumption of constant loadings on the stochastic discount factor. Our evidence suggests that the one-factor model in Szymanowska, de Roon, Nijman, and Goorbergh (2013) and the two-factor model in Yang (2013) do not appear to adequately characterize commodity returns.

Our empirical analysis centers on the following specification of the SDF:

$$m_{t+1} = 1 - b_{\mathrm{AVG}}\left(\mathrm{AVG}_{t+1} - \mu_{\mathrm{AVG}}\right) - b_{\mathrm{CARRY}}\left(\mathrm{CARRY}_{t+1} - \mu_{\mathrm{CARRY}}\right) - b_{\mathrm{CMOM}}\left(\mathrm{CMOM}_{t+1} - \mu_{\mathrm{CMOM}}\right), \tag{6}$$

implying that the expected excess returns of a commodity portfolio is a function of its exposure to three factors.

The average factor, denoted by AVG_{t+1}, is the excess return of a long position in all available commodity futures, (see equation (2)). AVG_{t+1} is required, because models that do not incorporate this average factor fail to explain the time-series variation in commodity returns (see Section 5.2).

The commodity carry factor, denoted by CARRY_{t+1}, and the momentum factor, denoted by CMOM_{t+1} deserve further comments, since these factors can be constructed in a variety of ways and depend on the implementation of the underlying carry and momentum strategies. Define the slope of the futures curve by $y_t \equiv F_t^{(1)}/F_t^{(0)}$. As detailed in Appendix A and the captions of Table Online-II and Table 1, we construct CARRY_{t+1} as the return on a portfolio that is long in the five commodities that are most backwardated (i.e., the lowest $\ln(y_t) < 0$) and short the ones that are most in contango (i.e., the highest $\ln(y_t) > 0$).

Several salient features are worth mentioning. At the outset we emphasize that the carry strategy based on the short-end of the futures curve maximizes the number of observations, the number of commodities included, and is associated with the highest open-interest (as noted in Table Online-II). In contrast, the carry factor of Yang (2013, equation (3)) is obtained by sorting commodities based on the log difference between the twelve months and the one month futures prices. Our carry factor is also different from the one in Szymanowska, de Roon, Nijman, and Goorbergh (2013) in that (i) they construct the factor by going long in an equally weighted portfolio of the 10 commodities that are most backwardated and short in an equally weighted portfolio of the 10 commodities that are most in contango (Section IV-B.1), and (ii) their study is based on bi-monthly returns (Section II.A).

As noted in Panel A of Table 1, over the past 42 years the carry factor (strategy C5) has been economically profitable, with an average annualized return of 16.34%, three to five times larger than the returns generated by investing in commodity indexes (see Table Appendix-II). The average return of the carry factor is statistically different from zero as indicated by the bootstrap confidence intervals. The 95% confidence intervals, denoted as PW, lower CI and upper CI, are based on a stationary bootstrap with 10,000 iterations, where the block size is based on the algorithm of Politis and White (2004).

In our analysis, CMOM_{t+1} is constructed as the return on a portfolio that is long in the five commodities with the highest returns over the previous six months and short the ones with the lowest returns over the previous six months. In this sense our momentum factor differs from that of Gorton, Hayashi, and Rouwenhorst (2013, Table VII) and Szymanowska, de Roon, Nijman, and Goorbergh (2013, Appendix B) as they construct it using the prior 12-month futures excess returns. Note, however, that this choice does not appear to have a significant impact on the performance of the momentum factor, as discussed in Appendix B. The average return of the momentum factor is 16.11% (see strategy M5 in Panel C of Table 1), and is statistically significant.

The joint inclusion of carry and momentum in the SDF as distinct factors can be defended on several grounds. First, the top and middle panels of Figure 1 reveal the distinct time-series behavior of the carry and momentum factors, with shaded areas representing NBER recessions. The bottom panel plots the return differential between the carry and the momentum factors, that is, $\text{er}_t^{cm} \equiv \text{CARRY}_t - \text{CMOM}_t$, and shows that the standard deviation of er_t^{cm} is 8.5%, the minimum is -40.1% and the maximum is 39.4%. As seen, the factors may be loading differently on economic conditions (see Table 14 for further discussion). For example, consider March 1980, where the momentum (carry) factor delivered a return of -28.2% (11.2%). The poor performance of momentum over this month was caused by the sharp decline in silver prices, but such decline had no impact on the returns to carry. Additionally, CARRY_t and CMOM_t *do not share* the same sign in 41% of the months, which further helps to dichotomize between the two factors.

[Fig. 1 about here.]

To assess whether the two strategies are *conditionally* loading on the same set of commodities, we first compute the commodity overlap in the long and short legs of the carry and momentum strategies separately, and then sum the two overlaps. The results, reported in Figure 2, indicate that the two strategies are largely decoupled, with the third quartile (Q3) of the overlap distribution equal to one.

[Fig. 2 about here.]

Later we also show that *unconditionally* the two strategies have little overlap in their set of constituent commodities (see Table 15).

[Fig. 3 about here.]

The notion that the two strategies are distinct can be further analyzed by computing their conditional correlations. To do so, we estimate the time-varying correlation between the returns generated by the carry and momentum factors using a dynamic conditional correlation model (Engle (2002)), in which the dynamics of carry and momentum returns are modeled using a bivariate GARCH (1,1) model. The results, reported in Figure 3, indicate that the returns correlation between the two strategies has not increased or decreased over time. Furthermore, their unconditional returns correlation is only 0.27.

We report the descriptive statistics of these risk factors in Table Appendix-II. Importantly, additional tests show that none of the factors display seasonality.

Several other considerations motivate an SDF driven by three-factors. First, we conduct a principal component analysis with our test assets, which reveals that AVG_{t+1} is highly correlated to the first principal component, CMOM_{t+1} is highly correlated with the second and third principal components, and CARRY_{t+1} loads only, but substantially, on the third principal component. Importantly, the first three components explain 71% of the variation in the data, which is non-trivial, given that we have 12 portfolios and, hence, 12 potential principal components.

Lewellen, Nagel, and Shanken (2010) observe that when the underlying test portfolios admit a certain factor structure, even misspecified asset pricing models could deliver high R^2 and low pricing errors. We allay such concerns by considering an expanded set of test assets that broadly represent the commodity market: (i) the four carry portfolios, (ii) the five momentum portfolios, and (iii) three category portfolios i.e., agriculture, livestock, and metal. We exclude energy as a test asset because of its shorter time-series. By jointly pricing all the 12 commodity portfolios, we reasonably challenge the asset pricing model. As an additional robustness check, we also present the results from a randomization exercise (Lustig, Roussanov, and Verdelhan (2011)) and results from portfolios constructed on variance (guided by Menkhoff, Sarno, Schmeling, and Schrimpf (2012a)).

Next, we conduct specification tests and show that more parsimonious models are rejected in the data, while additional commodity factors, such as the value and volatility factors, are statistically insignificant.

Complementing our statistical analysis, we show that some of the factors forecast economic growth and asset returns and therefore capture shifts in the investment opportunity set. This evidence appears aligned with the interpretation (Cochrane (2005, Chapter 20)) that variables able to forecast economic conditions could be viewed as risk factors, exposure to which can help characterize the cross-sectional variation in average returns. Strengthening this notion, we further establish that our factors are contemporaneously correlated with certain economy-wide fundamentals, alluding to a possible risk-based interpretation.

5. Understanding the cross-section and time-series of commodity returns

In this section we study the ability of our model to jointly explain the cross-section and time-series of commodity returns. We highlight our incremental contribution relative to the extant literature, notably Szymanowska, de Roon, Nijman, and Goorbergh (2013), Yang (2013), and Asness, Moskowitz, and Pedersen (2013).

5.1. AVG_t, CARRY_t, and CMOM_t summarize the cross-section of commodity returns

Can the three-factor model explain the cross-section of commodity returns? Are carry and momentum priced risk factors? Could these factors rationalize the documented average returns across our test portfolios? To answer these questions, we report in Panel A of Table 2 the GMM estimates of the factor risk premia $\boldsymbol{\lambda}$, the loadings on the SDF $\mathbf{b}$, and the Hansen and Jagannathan (1997) distance measure. The Newey and West (Shanken) p-values are reported in parentheses (curly brackets).

The estimated risk premia of both carry and momentum are positive, implying that portfolios that covary more with CARRY_{t+1} and CMOM_{t+1} earn extra compensation. In particular, the estimate of 0.018 (0.012) for λ_{CARRY} (λ_{CMOM}) amounts to an annualized risk premium of 21.6% (14.4%) for CARRY_{t+1} (CMOM_{t+1}). The p-values attest to the statistical significance of both factors in pricing the test portfolios.

Note that the average factor helps to describe the cross-section of commodity returns, and such finding contrasts the corresponding one from the equity market (Fama and French (1992)) and the currency market (Lustig, Roussanov, and Verdelhan (2011, Table 4) and Menkhoff, Sarno, Schmeling, and Schrimpf (2012a, Table 2)). However, the estimated annualized risk premium for the average factor is about 6%, below the risk premia for the other two commodity risk factors. In this sense, carry and momentum risk premia reflect concerns that may be of fundamental importance to futures market participants.

Displayed in the column "HJ-Dist." is the Hansen and Jagannathan (1997) distance measure, which quantifies the normalized maximum pricing errors. For the three-factor model, the distance measure is 0.006, with a p-value of 0.21. Consequently, we do not reject correct pricing.

The estimates of $\boldsymbol{\lambda}$ obtained using the Fama-MacBeth procedure are identical, by construction, to those obtained using the one-step GMM. The p-values based on Newey-West and Shanken are in agreement and establish the statistical significance of the three risk premia, even after accounting for the fact that the $\boldsymbol{\beta}_i$'s are estimated. Overall, our evidence strongly supports the presence of priced risk factors.

With a GLS cross-sectional uncentered R^2 of 96.3% (Lewellen, Nagel, and Shanken (2010, Prescription 3)) and OLS uncentered R^2 of 93.9%, the three factors capture a large fraction of the cross-sectional variation of the commodity portfolios. Furthermore, the χ^2 tests for the null hypothesis that the pricing errors are zero, have p-values equal to 0.22 and 0.25 for Newey and West (1987) and Shanken (1992), respectively, indicating that the asset pricing model cannot be rejected. The model pricing errors, displayed in Panel A of Figure 4, as measured by the deviation from the 45-degree line, reveal that the unexplained returns are small. The results from our cross-sectional regressions point to a risk based explanation.

[Fig. 4 about here.]

How can one quantify the contribution of each factor in explaining the returns cross-section? The issue of a possibly redundant factor is addressed from two different perspectives. First, Panels B and C of Table 2 reports results for restricted versions of the baseline model that exclude, alternatively, the momentum or the carry factor. In this regard, the χ^2_{SH} tests of the pricing errors show that both restricted models are rejected, with p-values equal to 0.01 for the model that excludes CMOM_{t+1} and 0.00 for the model that excludes CARRY_{t+1}. In particular, omitting the carry (momentum) factor worsens the model performance, as GLS R^2 drops to 67.7% (82.9%), and the p-value for the HJ-Dist. drops to 0.00 (0.15). In sum, the three-factor generalization provides a better characterization of commodity returns compared to its nested counterparts.

Building on our analysis, we also perform a two-step GMM estimation based on an optimal weighting matrix (e.g., Cochrane (1996, Table 1) and Lustig, Roussanov, and Verdelhan (2011, row GMM_2 in Table 4)). We first test three exclusion restrictions on the SDF in equation (6): (i) $b_{\text{CARRY}} \equiv 0$, (ii) $b_{\text{CMOM}} \equiv 0$, and (iii) $b_{\text{CARRY}} = b_{\text{CMOM}} \equiv 0$, and report the results below:

$b_{\text{CARRY}} \equiv 0$	$b_{\text{CMOM}} \equiv 0$	$b_{\text{CARRY}} = b_{\text{CMOM}} \equiv 0$
$\chi^2(1) = 14.18$, p-val.=0.00	$\chi^2(1) = 5.30$, p-val.=0.02	$\chi^2(2) = 30.13$, p-val.=0.00

All the restrictions are rejected, illustrating that both the carry and the momentum factors have loadings on the SDF that are statistically different from zero and, hence, provide additional pricing flexibility. Second, we perform the Hansen (1982) test of over-identifying restrictions, which is $\chi^2(9)$-distributed, and find that it has a p-value of 0.167, reinforcing the conclusion that the three-factor model cannot be rejected.

Could other economically relevant factors drive out the explanatory ability of our commodity factors? To investigate additional models, we augment the three-factor model in equation (6) with either (i) a commodity value factor (in the spirit of Asness, Moskowitz, and Pedersen (2013)) or (ii) a commodity volatility factor (in the spirit of Menkhoff, Sarno, Schmeling, and Schrimpf (2012a)). As reported in Panel A of Table 3, the Newey-West p-values for the null hypothesis of $\lambda_{\text{VALUE}} = 0$ and $\lambda_{\Delta\text{VOL}} = 0$ are, respectively, 0.21 and 0.94, indicating that these additional risks are not priced. Next, to examine the relevance of each additional factor in the SDF specification, we also perform a χ^2 exclusion restriction test in the context of a two-step GMM. The p-values for the $\chi^2(1)$ statistics are 0.51 (0.41) for the value (volatility) factor. Thus, our tests seem to favor a more parsimonious three-factor model specification.[3]

[3] We also assess whether a common set of factors prices both commodity and equity portfolios by replacing the three commodity factors with the four Fama-French equity factors. Our statistical tests reject correct model pricing, implying that the commodity and equity markets may be segmented (Daskalaki, Kostakis, and Skiadopoulos (2012)).

Are our conclusions robust to an alternative set of test portfolios? For this purpose, we conduct cross-sectional tests on the four portfolios double-sorted first by momentum and then carry, as described in Appendix B. Here we find that the χ^2_{NW} test for the model that excludes the carry (momentum) factor is rejected with a p-value of 0.07 (0.00). On the other hand, the model that includes both is not rejected. For instance, the χ^2_{NW} has a p-value of 0.38, affirming that the pricing errors are statistically indistinguishable from zero. Furthermore, the magnitudes of the factor risk premia are consistent with those in Table 2. The three-factor model offers considerable flexibility in pricing the various test assets.

How sensitive are our results to the randomization procedure advocated in Lustig, Roussanov, and Verdelhan (2011)? Following their approach, in the first step we construct the average, carry, and momentum factors based on commodities whose ticker symbol starts with the letter A through the letter M. In the second step we construct two carry, two momentum, and three category portfolios based on commodities whose ticker symbol starts with the letter N through the letter Z. Finally, we use the factors constructed on the first set of commodities to price portfolios formed on the second set of commodities and we also conduct the reverse exercise. Such procedure poses a higher hurdle for the pricing models, given that the commodities included in the pricing factors are different from the ones included in the priced portfolios. Together, the evidence from Panels A and B of Table 4 provides justification for the inclusion of both carry and momentum to price the cross-section of commodity returns. In fact, the three factor model is not rejected on the first set of commodities and is borderline rejected on the second. On the the other hand, the two-factor that features the average and the carry factor is rejected in both cases. The model that features the average and the momentum factors is not rejected in the first sub-sample, but is rejected on the second.

Additionally, we augment the composition of the test portfolios to include five portfolios that are sorted based on the volatility of commodity returns computed using daily returns over the past month, as in Menkhoff, Sarno, Schmeling, and Schrimpf (2012a). The results, reported in Table 5, show that the three-factor model continues to perform well while the two-factor models are rejected according to both the χ^2_{NW} and χ^2_{SH} tests.

Finally, motivated by Tang and Xiong (2013) and Henderson, Pearson, and Wang (2012), we also study whether the financialization of commodities had an impact on the performance of the model. To address this issue, we compare model ability over 1970:07 to 2003:12 and 2004:01 to 2011:09, and obtain a p-value of 0.15 and 0.24, respectively, for the χ^2_{NW} test of zero pricing errors. Our evidence of correct model pricing on both sub-samples shows that the financialization of commodities does not affect model performance.

5.2. Time-series regressions: the hypothesis of zero alphas is not rejected

To assess how the three-factor model fares in capturing the time-series dimension of commodity returns, we perform the following regressions for test assets indexed by $i = 1, \ldots, 12$:

$$\mathrm{er}_t^i = \alpha^i + \beta^i_{\mathrm{AVG}} \mathrm{AVG}_t + \beta^i_{\mathrm{CARRY}} \mathrm{CARRY}_t + \beta^i_{\mathrm{CMOM}} \mathrm{CMOM}_t + \varepsilon_t^i \qquad \text{for} \quad t = 1, \ldots, T. \tag{7}$$

We gauge model adequacy by testing the joint hypothesis that $\boldsymbol{\alpha} = \mathbf{0}$, where $\boldsymbol{\alpha} = [\alpha^1, \ldots, \alpha^{12}]'$. This hypothesis of zero pricing errors is tested by constructing the statistic $\hat{\boldsymbol{\alpha}}' \mathbf{var}(\hat{\boldsymbol{\alpha}})^{-1} \hat{\boldsymbol{\alpha}}$ in a GMM setting, which is asymptotically distributed as $\chi^2(df)$, where df is the dimensionality of $\boldsymbol{\alpha}$ (see Cochrane (2005, page 234)). We also test the individual significance of α^i for $i = 1, \ldots, 12$.

As can be inferred from Table 6, the three-factor model aptly describes the time-series returns of the test portfolios. For example, with a p-value of 0.11, we are unable to reject the null hypothesis that $\boldsymbol{\alpha} = \mathbf{0}$. Furthermore, ten out of the 12 α estimates have p-values that exceed 0.05. The largest (absolute) α has an annualized value of 4.8%, while the majority of α's are below 2.4%, implying that the departures from the model are economically small.

All the β_{AVG} coefficients are uniformly positive and statistically significant and range from 0.659 to 1.165. The implication of this finding is that the returns of commodity portfolios manifest a strong commodity market component.

As nine (ten) out of 12 p-values on β^i_{CMOM} are below 0.05 (0.1), we also conclude that CMOM_t explains the return dynamics of the commodity portfolios. We further note that the β^i_{CARRY} coefficients are statistically significant for carry portfolios. However, CARRY_t does not seem to impact the returns of momentum and commodity category portfolios in a statistically significant manner.

Overall, the portfolio returns display differential sensitivity to the carry and momentum factors. Specifically, the extreme contango portfolio (i.e., P4) serves to hedge CMOM_t risks, offering some justification for its low average returns (see Table 1 and the discussion in Appendix B). Our findings also enable the insight that commodities linked to the livestock can hedge momentum related risks, while the returns of commodity categories are largely detached from the carry factor. The adjusted R^2's range from 22.3% for Livestock to 84.5% for the momentum portfolio Q1, implying that the factors help to track the time-series behavior of commodity portfolios. A later analysis is directed at understanding the economic sources of model performance.

There is some disagreement about the number of factors needed to characterize the time-series of commodity returns. Szymanowska, de Roon, Nijman, and Goorbergh (2013, Tables IX, and X) provide tests suggesting that only carry is needed, while Yang (2013) makes the case for a two-factor model featuring the average and the carry factors. We shed light on this issue by reporting, in Table 7, the performance of the one and the two-factor models. From a statistical standpoint, the R^2's associated with the one-factor model, driven by the carry factor alone, are negative for five out of the 12 portfolios and are significantly lower than the ones associated with the three-factor model. This model is fundamentally mis-specified with five out of 12 α's significantly different from zero and a $\chi^2(12)$ statistic indicating model inadequacy.[4] The two-factor model featuring the average and the carry factor is also rejected according to the $\chi^2(12)$ statistic indicating that momentum has explanatory power beyond the average and carry factor and may be needed to characterize the variation in commodity returns.

Table 8, which presents the performance of the three-factor model under the randomization approach further reinforces the relevance of the momentum factor. The main message from combining the results from Table 6 through Table 8 is that generalizing the two factor model in Yang (2013) can help to further explain the dynamic behavior of commodity returns.

5.3. The conditional versions provides some additional flexibility

Cochrane (1996), Lettau and Ludvigson (2001), Jagannathan and Wang (2002), and Nagel and Singleton (2011), among others, have compared the performance of conditional and unconditional asset pricing models. One finding is that conditional models can enhance performance if the loadings on the risk factors affecting the SDF are not constant but fluctuate with the state of the economy.

Following the literature on equity returns, we modify the SDF in equation (6) and set it to $m_{t+1} = 1 - \mathbf{b}'[z_t]\,(\mathbf{f}_{t+1} - \boldsymbol{\mu})$. We further assume that $\mathbf{b}[z_t]$ is a linear function of a state variable z_t. In the context of the three-factor model, the SDF becomes:

$$\begin{aligned} m_{t+1} \;=\;& 1 - b_{\text{AVG}}\left(\text{AVG}_{t+1} - \mu_{\text{AVG}}\right) - b_{\text{CARRY}}\left(\text{CARRY}_{t+1} - \mu_{\text{CARRY}}\right) - b_{\text{CMOM}}\left(\text{CMOM}_{t+1} - \mu_{\text{CMOM}}\right) \\ &- b_{\text{AVG},z}\left(\text{AVG}_{t+1} - \mu_{\text{AVG}}\right)(z_t - \mu_z) - b_{\text{CARRY},z}\left(\text{CARRY}_{t+1} - \mu_{\text{CARRY}}\right)(z_t - \mu_z) \\ &- b_{\text{CMOM},z}\left(\text{CMOM}_{t+1} - \mu_{\text{CMOM}}\right)(z_t - \mu_z). \end{aligned} \tag{8}$$

[4]The departure in our results from Szymanowska, de Roon, Nijman, and Goorbergh (2013, Tables IX, and X) could be presumably attributed to (i) our longer sample, (ii) our use of monthly sampled observations versus their bi-monthly sampled observations, and (iii) our use of 12 test portfolios versus their use of four test portfolios.

This model reflects the additional interaction terms between AVG_{t+1}, CARRY_{t+1}, CMOM_{t+1} and z_t.

Choosing a conditioning variable is critical because, in principle, it should contain all the information available to investors, which is unobservable to the econometrician. Because theory offers little guidance, we assess the performance of a number of alternative z_t's (defined in Appendix C): (i) open interest growth, (ii) change in commodity volatility ΔVOL_t, (iii) log dividend yield of the US equity index, (iv) the currency returns (US dollar index, FX|USD), (v) industrial production growth (G7 countries), (vi) the US term spread, (vii) the average slope of the futures curve for the commodities in backwardation (i.e., $\ln(y_t) < 0$), and (viii) the average slope of the futures curve for the commodities in contango (i.e., $\ln(y_t) > 0$).

Our choices of z_t are meant to capture the state of the economy, and broadly reflect developments in real economic activity, equity, bond, currency, and commodity markets. For example, the aggregate open interest is a slight variation of the one used in Hong and Yogo (2012), while the relation between the commodity risk premia, currency returns, the slope of the futures curve, and macroeconomic variables have been studied by Bailey and Chan (1993, Table 1), and Chen, Rogoff, and Rossi (2010, Table IV).

For each conditioning variable z_t, Table 9 presents the results from the Hansen and Jagannathan (1997) distance test as an overall measure of fit, as well as exclusion tests that analyze in what respects the conditional models differ from the unconditional model featured in equation (6).

In all cases, we are unable to reject the null that the distance measure is equal to zero, as all the p-values are greater than 0.1 (see Panel A of Table 9). This implies that more elaborate parameterizations do not substantially worsen model performance.

Additional evidence of the effect of conditioning can be gathered from evaluating the null hypothesis that all coefficients representing the interaction between the factors and z_t are jointly equal to zero, i.e., $H_0 : b_{\text{AVG},z} = b_{\text{CARRY},z} = b_{\text{CMOM},z} \equiv 0$, against the alternative that at least one is different from zero. Five out of the eight p-values for the $\chi^2(3)$ tests in Panel B of Table 9 are less than 0.1. Such findings indicate that the combined effect of allowing time-varying coefficients $\mathbf{b}[z_t]$ could produce a better pricing model.

Complementing the above results, Panel C of Table 9 reports the $\chi^2(1)$ exclusion tests on the individual interaction terms between z_t and each of the factors. The number of statistically significant interaction terms varies from three to five among the three factors, reinforcing the possible need to model time-varying $\mathbf{b}[z_t]$.

Finally, Panels B through D of Figure 4 illustrate that incorporating conditioning can better align the predicted and realized returns along the 45-degree line. This can be gauged by the slightly higher uncentered R^2 and the lower mean absolute pricing errors. The takeaway is that conditional models that admit time-varying loadings on the SDF can be expected to improve the pricing of certain commodity portfolios.

To summarize, we find that the three-factor model offers versatility in describing the cross-section of commodity returns and establish the relevance of the momentum factor. The estimates of the factor risk premia appear reasonable and are statistically significant, the χ^2-tests are supportive of zero pricing errors, the model yields a high GLS R^2, and the χ^2 exclusion tests do not support the importance of value or volatility as additional factors. At the same time, allowing for time-varying loadings on the average, carry, and momentum factors could deliver slightly better asset pricing.

5.4. Comparisons with the extant literature

Our treatment builds on Yang (2013), who uses an average factor and a carry factor to price the cross-section and time-series of commodity returns. Recall that Yang (2013)'s approach to portfolio formation involves constructing seven test portfolios with commodities sorted on the slope of the futures curve that is inferred from log-difference of the 12-month and one-month futures prices. Given that, at any point in time, the number of commodities available can be as low as 15, this approach may be subject to the limitation that the number of commodities included in each portfolio can be as low as two or three, giving rise to potentially large estimation error. This feature can be problematic as evidenced by the cross-sectional tests in Yang (2013, Page 169), where a model that contains the average factor as the *only* factor cannot be rejected. Moreover, in the time-series tests (Table 3, Page 170), none of the seven portfolios has significant α when the average factor is used as the *only* factor.

Relative to Yang (2013), our novelty lies in establishing the importance of the commodity momentum factor. In the time-series dimension, we show that the momentum factor can price the carry portfolios, but the carry factor is not able to price the momentum portfolios. In the cross-sectional dimension, we show that a two-factor model that does not include the momentum factor has difficulties pricing the cross-section of commodity returns, when confronted with a larger set of test portfolios. The essential point is that the three-factor model outperforms the nested two-factor models in capturing the documented patterns in commodity returns.

The focus of Szymanowska, de Roon, Nijman, and Goorbergh (2013) is on decomposing commodity futures risk premia into spot premia and term premia. One feature highlighted in their analysis is that the carry factor alone can provide an acceptable description of the time-series variation in commodity spot premia. Guided by their analysis, we investigated in Table 7 the ability of the carry factor to explain the time-series behavior of our test portfolios. The main finding to be gleaned from our exercises is that the omission of the average factor leads to a rather poor performance of the model in terms of adjusted R^2's. We surmise that the mis-specification associated with the omission of the average factor could potentially induce large standard errors and this could possibly explain the failure to reject the null that the α's are jointly equal to zero, when carry is used as the *only* factor. A distinctive finding in our setting is that a commodity asset pricing model driven by the carry factor alone is strongly rejected in the data.

Going beyond Szymanowska, de Roon, Nijman, and Goorbergh (2013), we also perform cross-sectional tests and show that the momentum factor contains additional information beyond that conveyed by the carry factor.

Besides, while accommodating the first-notice-day convention, we employ the second-nearest maturity futures contract sampled at the *monthly* frequency. This is in contrast with Szymanowska, de Roon, Nijman, and Goorbergh (2013) as they construct futures returns at the *bi-monthly* frequency. Furthermore, our dataset features 16 more years of data (it starts in 1970 rather than 1986 and ends in 2011 rather than 2010) and contains 29 commodities rather than 21. The combination of the higher data frequency and the longer sample translates into a larger number of time-series observations (approximately 500 compared to approximately 150) and that could result into a higher precision of our estimates. Our results indicate that a three factor model is needed to explain both the cross-sectional and time-series variation in commodity returns.

We depart in our focus from Asness, Moskowitz, and Pedersen (2013) who employ commodities in their construction of value and momentum factors across countries and asset classes. While their intent is to study the performance of a global asset pricing model, our goal is to explain the cross-section and time-series of commodity returns. The common denominator between our studies lies in establishing the importance of momentum.

Finally, our approach has the flavor of the currency studies of Menkhoff, Sarno, Schmeling, and Schrimpf (2012a), Menkhoff, Sarno, Schmeling, and Schrimpf (2012b) and Lustig, Roussanov, and Verdelhan (2011) in that we construct the commodity factors from the space of commodity returns to price commodity portfolios. Overall, having a richer commodity asset pricing model could improve our understanding of commodity price dynamics and could refine the single-factor setup considered by Cortazar, Kovacevic, and Schwartz (2013).

6. An economic rationale for the factors

A key feature to reconcile is that the commodity factors are useful for characterizing the cross-sectional and time-series properties of commodity returns. In search of an interpretation of our findings, we further address two questions. First, do the factors help to forecast changes in the investment opportunity set? Second, what real macroeconomic risks underpin the performance of the three-factor model?

The motivation for our forecasting exercises stems from the approaches in Campbell (1996) and Ferson and Harvey (1999) and is also guided by an analogy in Cochrane (2005, page 445): *"Though Merton's (1971, 1973) theory says that variables which predict market returns should show up as factors which explain cross-sectional variation in average returns, surprisingly few papers have actually tried to see whether this is true, now that we do have variables that we think forecast the market return."* In what follows, we assume that changes in the investment opportunity set could be summarized by (i) GDP growth, (ii) Treasury bond returns, (iii) equity returns, and (iv) returns of commodity currencies.

6.1. The factors have joint predictive content for aggregate economic activity

Motivated by Liew and Vassalou (2000, Table 5), Groen and Pesenti (2009, Section 2), Cespedes and Velasco (2012, Table 1), and Caballero, Farhi, and Gourinchas (2012, Table 9), among others, we consider whether the commodity factors can forecast growth of economic activity. Our framework uses the real GDP growth of the G7 countries as a measure of economic activity and the following linear model:

$$\ln\left(\mathrm{GDP}_{t+k}/\mathrm{GDP}_t\right) \;=\; \theta_0 \;+\; \boldsymbol{\theta}'\mathbf{f}_t \;+\; \varepsilon_{t+k}, \qquad\qquad k \in \{1,2,3,4\}, \tag{9}$$

where $\mathbf{f}_t \equiv [\mathrm{AVG}_t\ \mathrm{CARRY}_t\ \mathrm{CMOM}_t]'$. Our goal is to investigate whether the predictive slope coefficients contained in $\boldsymbol{\theta}$ are individually and jointly statistically significant. We draw inference based on the Newey and West (1987) estimator, with lags automatically selected according to Newey and West (1994), and the Hodrick (1992) 1B covariance estimator under the null of no predictability.

Three features of our results reported in Table 10 deserve discussion. First, a higher AVG_t forecasts stronger global economic growth, while higher CARRY_t and CMOM_t forecast lower economic growth. Second, the statistical significance of the commodity factors is preserved at all horizons, with the exception of AVG_t which loses its significance at four quarters. Both the Newey and West p-values, denoted by NW[p], and the Hodrick p-values, denoted by H[p], are in agreement. The factors are also relevant from an economic perspective. For example, a one standard deviation increase in the carry factor (i.e., 22%; see Table Appendix-II) is associated with a decline of 88 basis points of annualized GDP in the subsequent quarter. Third, the *p*-values corresponding to the null hypothesis that none of the predictors are statistically significant, i.e., $\boldsymbol{\theta} = \mathbf{0}$, are consistently below 0.02 (reported as $J[p]$). These results affirm the ability of the commodity factors to jointly predict future economic conditions across industrialized countries.

We conduct three additional exercises to demonstrate the reliability of the above findings. First, the commodity factors forecast the G7 and US industrial production growth as well as the US GDP growth (not reported), indicating robustness to alternative measures of economic activity. Next, we build on Liew and Vassalou (2000, Tables 5 and 6) and include the SMB_t and HML_t factors of Fama and French (1996) in our specification (9). Our Wald statistic points to the lack of joint statistical significance of these two equity factors for US GDP growth. Finally, we account for the possible effect of serial correlation in the GDP growth by implementing an ARMAX model. We first select the best model for GDP growth, using the Bayesian information criterion, and find it to be an ARMA(1,1). We then include the commodity factors in an ARMAX specification, and find that they maintain their joint statistical and economic significance. In sum, our results extend beyond the linear forecasting model in equation (9).

While policymakers have often relied on futures markets to derive price forecasts of commodities (e.g, Ben Bernanke, June 9, 2008), our evidence on the information content of commodity factors for economic activity adds to the extant literature in two new ways. First, the ability of the commodity factors to forecast output growth is not yet recognized. Second, our evidence transcends the predictive role of oil prices on macroeconomic fluctuations (e.g., Hamilton (1983) and the follow-up studies). Specifically, when the growth of oil prices is added as an additional predictor in equation (9), the slope coefficient on oil is insignificant, while the factors remain strongly statistically significant (see Table Online-III).

6.2. The commodity factors predict returns of Treasury bonds and equities

The preceding evidence emphasizes that the commodity factors can forecast one dimension of changes in the investment opportunity set, namely, growth of real economic activity. Still, the commodity factors could also forecast other dimensions of the time variation in the investment opportunity set related to bond returns and equity returns, suggesting a multifaceted explanation for the documented findings.

Following Fama and French (1988b) and Hodrick (1992), consider the following predictive regressions with overlapping observations for Treasury bonds and equity returns for $K \in \{1,3,6,9,12\}$:

$$\sum_{k=1}^{K} \mathrm{er}_{t+k}^{\mathrm{bond}} = \xi_0 + \boldsymbol{\xi}'\mathbf{f}_t + \varepsilon_{t+K}, \qquad \text{with} \qquad \mathrm{er}_{t+1}^{\mathrm{bond}} \equiv \ln(1+\mathrm{r}_{t+1}^{\mathrm{bond}}) - \ln(1+\mathrm{r}_t^f), \tag{10}$$

$$\sum_{k=1}^{K} \mathrm{er}_{t+k}^{\mathrm{equity}} = \delta_0 + \boldsymbol{\delta}'\mathbf{f}_t + \varepsilon_{t+K}, \qquad \text{with} \qquad \mathrm{er}_{t+1}^{\mathrm{equity}} \equiv \ln(1+\mathrm{r}_{t+1}^{\mathrm{equity}}) - \ln(1+\mathrm{r}_t^f). \tag{11}$$

We consider both one-year and 30-year US Treasury bond returns (source: CRSP) and US value-weighted equity returns. The results are presented in Tables 11 and 12.

Examining the behaviors of bond and equity returns serves an important purpose. For instance, if the factors predict equity returns, then they should either predict discount rate uncertainty or cash flow uncertainty. On the other hand, Treasury bonds embed fixed cash flows (e.g., Ilmanen (1995, Table 1)). Thus, the predictability of the default-free component of the discount rates, when viewed in conjunction with equity returns, could help trace the economic nature of the commodity factors.

There are three points to glean from the reported slope coefficients. First, the average factor has predictive content for one-year bond returns at all horizons and for 30-year bonds for horizons of one-month and three-months, but is generally uncorrelated with future equity returns. When statistically significant, the average factor exerts a negative effect on future bond returns, which intuitively agrees with our findings in Table 10 that the average factor positively predicts output growth.

Next, our results with respect to the carry factor indicate that it tends to predict positively (negatively) bond (equity) returns. In particular, the estimated coefficients δ_{CARRY} decline with the return horizon, and the effect of the carry factor on future equity returns is statistically significant. The forecasting ability of the carry factor also holds in univariate regressions (i.e., when $\delta_{\mathrm{AVG}} = \delta_{\mathrm{CMOM}} = 0$), with δ_{CARRY} estimates that lie between -0.06 and -0.32. Additionally, the 12-month horizon adjusted R^2 is equal to 2.5%, in line with other equity market predictors (e.g., Cochrane and Piazzesi (2005, Table 3)).[5]

The negative slope coefficients with GDP growth and equity returns, and the positive slope coefficients with bond returns, impart a countercyclical attribute to the carry factor. In particular, the relative spread between the returns of commodities in backwardation versus those in contango contains information for aggregate future cash flows, as measured by the GDP growth, and for equity returns, but our evidence also illustrates the predictive link to bond returns.

[5] Appreciate, in addition, that the carry factor is not persistent (see column ρ_1 in Table Appendix-II), offering a certain deviation from other predictors of bond and equity returns, such as the term premium and the dividend yield, which are instead highly persistent (see Campbell (2001)).

Observe that the effect of the momentum factor is inconspicuous and statistically insignificant for equity returns and 30-year bond returns but predictability surfaces for one-year bond returns at horizons ranging from six to 12 months. Viewed together, the commodity momentum factor is negatively related to GDP growth at all horizons, and positively related to short-term bond returns at horizons equal to and longer than six months.

Overall, our analysis points to the need to develop theoretical models that recognize the inter-linkages among the expected returns across different asset classes.

6.3. Relation between the factors and the future behavior of commodity currencies

Chen, Rogoff, and Rossi (2010, Table IV, Panel B) show that commodity returns have some predictive power for the returns of commodity currencies. Extending their analysis, we consider the predictive regressions:

$$\ln\left(\mathrm{FX}_{t+k}/\mathrm{FX}_t\right) = \pi_0 + \boldsymbol{\pi}'\mathbf{f}_t + \varepsilon_{t+k} \qquad \text{and} \qquad k \in \{1,2,3,4\}, \tag{12}$$

where $\ln\left(\mathrm{FX}_{t+k}/\mathrm{FX}_t\right)$ represents currency returns, obtained by equally weighting returns across the commodity currencies (i.e., Australia, Canada, Chile, Norway, New Zealand, and South Africa; see Labuszewski (2012)). Following convention, we maintain the US dollar as the reference currency. We assess the commodity factors to see if they reflect certain global risks, which would translate into a statistically significant $\boldsymbol{\pi}$.

Table 13 reveals three prominent findings. First, an increase in the average factor predicts the depreciation of the US dollar, but the effect weakens after one quarter and becomes statistically insignificant. Next, increases in the carry and momentum factors imply an appreciation of the dollar, with six out of eight $\mathrm{H}[p]$ below 0.1. Analogous to our findings from Tables 10 through 12, the signs of the predictive slope coefficients π_{AVG} are opposite to those of π_{CARRY} and π_{CMOM}, sharpening the distinction between the economic nature of the three factors.

At the same time, our results appear economically sensible. For example, a rise in commodity carry returns forecasts future appreciation of the US dollar, which coincides with a forecast of an economic downturn, and is accompanied by falling equity prices. Such downturns often witness the unwinding of the long-leg of the currency carry trades, caused by the unraveling of positions in high interest rate currencies (e.g., Australian dollar, New Zealand dollar, and South African rand). This piece of evidence implies that the commodities factors are likely aligned with certain risks that have global orientations.

In summary, our findings are in the spirit of Ilmanen (1995), Cochrane (2005, pages 442–445), and Hong and Yogo (2012) and add to a growing body of studies that suggest the inter-linkages between equity returns, bond returns, currency returns, and now, commodity returns. Our view is also shaped by Fama (1991, page 1610): "*In the end, I think we can hope for a coherent story that (1) relates the cross-section properties of expected returns to the variation of expected returns through time, and (2) relates the behavior of expected returns to the real economy in a rather detailed way. Or we can hope to convince ourselves that no such story is possible.*" We provide some resolution to this ongoing debate by corroborating the connection between the commodity factors and economic variables that proxy for changes in the investment opportunity set.

6.4. Quantifying the exposure of the factors to economic fundamentals

To broadly explore the role of the macroeconomy, we follow Bailey and Chan (1993), Fama and French (1989), Driesprong, Jacobsen, and Maat (2008), Menkhoff, Sarno, Schmeling, and Schrimpf (2012b), and Asness, Moskowitz, and Pedersen (2013), among others, and run the following univariate regressions:

$$\mathrm{AVG}_t = \phi_0 + \phi X_t + e_t, \quad \mathrm{CARRY}_t = \phi_0 + \phi X_t + e_t, \quad \mathrm{CMOM}_t = \phi_0 + \phi X_t + e_t, \tag{13}$$

where X_t proxies for a source of risk. We take X_t to be variables related to real economic activity, equity markets, bond markets, currency markets, and liquidity. The central proposition is whether X_t influences risk premia in the commodity market, as gauged by a statistically significant ϕ.

Table 14 presents the ϕ estimates and the corresponding Newey and West (1987) p-values, with lags automatically selected according to Newey and West (1994). We normalize ϕ so that the coefficients correspond to a percentage change in the commodity risk premia for one standard deviation change in X_t.

There is a strong association between the average factor and the macroeconomy, whereby five (six) out of 14 ϕ's have p-values lower than 0.05 (0.1). For example, the ϕ estimate is positive for output growth, equity risk premium, and open interest growth. On the other hand, there is a negative association with the term spread and equity volatility, and average commodity returns are higher with the depreciation of the US dollar. These findings convey the cyclical nature of AVG_t.

The carry factor exhibits markedly different exposures, whereby the carry strategies tend to perform well during periods of broad-based equity market declines and when the commodity market experiences high volatility. The commodity carry is profitable when value stocks outperform growth stocks and high momentum stocks outperform low momentum stocks.

Our findings also point to the elusive nature of commodity momentum returns, with 12 out of 14 ϕ estimates statistically insignificant. An exception is that commodity momentum returns are significantly higher in periods of high value spreads and momentum spreads, reinforcing the view that they load on certain risks relevant to equity investors. Our observations hint to possible feedback effects between commodity returns and equity returns, as argued also by Buyuksahin and Robe (2012).

Even though commodity carry and momentum strategies appear to provide some protection against equity market declines, the negative effect on expected returns is offset by extra compensation, due to the positive exposures with respect to the equity value and equity momentum factors. Thus, the search for a plausible explanation of higher average returns to both carry and momentum factors is challenging and depends on countervailing effects.

Importantly, open interest growth exerts a positive, and statistically significant, effect on both the average factor and the momentum factor. Our evidence, thus, supports the view in Hong and Yogo (2012) that the evolution of open interest reflects developments affecting the commodity markets.

Combining the various parts of our analysis, we also find that the commodity factors are seemingly detached from our bond market variables (five out of six ϕ's are insignificant). Furthermore, we find no discernible link between any of the commodity factors and a measure of liquidity, i.e., the TED spreads.

In contrast to Lustig, Roussanov, and Verdelhan (2011), who emphasize the information content of currency carry returns and their connection to global risks, the factors are decoupled from currency carry trade returns. Moreover, the factors do not significantly respond to changes in currency volatility.

Overall, the commodity factors display differential exposures to our universe of economic fundamentals. Moreover, these exposures, when viewed in conjunction with the forecasting exercises, offer some insights as to why the identified factors help to describe the cross-sectional and time-series variation in commodity returns. Our arguments rely on the premise that the factors synthesize the multidimensional aspects of the macroeconomy and, at the same time, track variations in the investment opportunity set.[6]

[6]Table Online-IV shows that the returns of long and short legs of carry and momentum display correlations with the average factor that are equal in magnitude but of opposite sign and, hence, the combined strategies are commodity market neutral. However, the central result is that the combined positions isolate risks that correlate with future output growth and asset returns.

6.5. Composition of the long and short legs of the carry and momentum strategies

Returning to Table 1, Table Appendix-III, and the discussion in Appendix B, we note that the average returns to a long position in a dynamically re-balanced set of backwardated (or, high momentum) commodities is high, whereas the average returns to a short position in contangoed (or, low momentum) commodities is low. To probe this rather puzzling feature, we analyze the composition of the long and short legs of each strategy. Specifically, we compute the number of months in which each commodity enters the long and short legs of C5 and M5, respectively, and report the findings in Table 15.

To elaborate on Table 15, let G_i be the number of months a commodity i enters in the long (short) leg of the M5 portfolio, and let H_i be the number of months the same commodity enters in the long (or short) leg of the C5 portfolio. For example, Soybean oil appears 126 times in the long leg of M5 and 65 times in the long leg of C5. We perform two OLS estimations: $G_i = \eta_0 + \eta H_i + \varepsilon_i$, where $i = 1, \ldots, 29$ and report the η estimates and the p-values (based on White's standard errors) below:

	η	p-val.	R^2 (%)
Aligning long legs of momentum and carry	0.08	0.31	1.7
Aligning short legs of momentum and carry	0.23	0.02	19.3

These regression results allow for new insights into the return variation that underlie the carry and momentum strategies. For one, the slope coefficient η for the long leg is insignificant, implying that the outperformance of the long legs of carry and momentum do not emanate from a similar set of commodities. In contrast, a unit increase in the short leg of the carry strategy membership is associated with a 0.23 increase in the momentum strategy membership, and the effect is statistically significant. The R^2 obtained for the short leg implies that 19.3% of the variation in momentum strategy membership can be explained by variation in the carry strategy membership.

The above finding furnishes additional information content, namely, that the distribution of membership in the long legs is not skewed toward a few economically sensitive commodities. One could ascribe the outperformance of the long legs to their ability to rotate across a set of commodities that reflect the prevailing state of the macroeconomy. For instance, a predominant portion of the profitability of the long leg of the momentum strategies during the financial crisis was inherited from the run-up in gold, which co-moved negatively with the stock market and real economic activity.

Our analysis provides an intriguing portrayal of the membership structure of the carry and momentum strategies and how these strategies may be adapting to macroeconomic conditions.

7. Concluding remarks

This paper studies cross-sectional and time-series patterns of commodity futures returns over a 42-year period, from 1970 to 2011. The big picture is that the three-factor model driven by an average factor, a carry factor, and a momentum factor is not rejected in the data, whereas a two-factor model (as featured in Yang (2013)) or a one-factor model (as featured in Szymanowska, de Roon, Nijman, and Goorbergh (2013)) furnish pricing errors that are statistically different from zero. Thus, our results indicate that the momentum factor is incrementally important in explaining the cross-section and time-series of commodity returns.

Our exercises establish that the commodity factors capture variations in the future investor opportunity set. Specifically, we show that they predict, in isolation or together, measures of output growth, bond returns, equity returns, and returns of commodity currencies. Additionally, the returns of carry and momentum strategies correlate with the macroeconomy. For example, backwardated commodities outperform cantangoed commodities during periods when equities are doing poorly, commodity volatility is high, and value outperforms growth. The carry strategy provides higher returns during economic downturns.

There are avenues to extend our work. For one, we need tractable dynamic stochastic general equilibrium models in which commodities differ in characteristics, such as carry and momentum, and which furnish an economically identified link between commodity characteristics and expected futures returns. Such models should provide a parsimonious description of the commodity returns and a framework for interpreting the documented empirical regularities.

References

Acharya, V., Lochstoer, L., Ramadorai, T., 2012. Limits to arbitrage and hedging: Evidence from commodity markets. Journal of Financial Economics (forthcoming).

Asness, C., Moskowitz, T., Pedersen, L., 2013. Value and momentum everywhere. Journal of Finance 68, 929–985.

Bailey, W., Chan, K., 1993. Macroeconomic influences and the variability of the commodity futures basis. Journal of Finance 48, 555–573.

Basu, D., Miffre, J., 2012. Capturing the risk premium of commodity futures: The role of hedging pressure. Unpublished working paper. EDHEC Business School.

Bessembinder, H., 1992. Systematic risk, hedging pressure, and risk premiums in futures markets. Review of Financial Studies 5, 637–667.

Bessembinder, H., 1993. An empirical analysis of risk premia in futures markets. Journal of Futures Markets 13, 611–630.

Bessembinder, H., Chan, K., 1992. Time-varying risk premia and forecastable returns in futures markets. Journal of Financial Economics 32, 169–193.

Burnside, C., Eichenbaum, M., Kleshchelski, I., Rebelo, S., 2011. Do peso problems explain the returns to the carry trade?. Review of Financial Studies 24, 853–891.

Buyuksahin, B., Robe, M., 2012. Speculators, commodities and cross-market linkages. Unpublished working paper. American University.

Caballero, R., Farhi, E., Gourinchas, P., 2012. Financial crash, commodity prices and global imbalances. Unpublished working paper. NBER#14521.

Campbell, J., 1996. Understanding risk and return. Journal of Political Economy 104, 298–345.

Campbell, J., 2001. Why long horizons? A study of power against persistent alternatives. Journal of Empirical Finance 8, 459–491.

Carlson, M., Khokher, Z., Titman, S., 2007. Equilibrium exhaustible resource price dynamics. Journal of Finance 62, 1663–1703.

Casassus, J., Collin-Dufresne, P., 2005. Stochastic convenience yield implied from commodity futures and interest rates. Journal of Finance 60, 2283–2331.

Cespedes, L., Velasco, A., 2012. Macroeconomic performance during commodity price booms and busts. Unpublished working paper. NBER#18569.

Chang, E., 1985. Returns to speculators and the theory of normal backwardation. Journal of Finance 40, 193–208.

Chen, Y., Rogoff, K., Rossi, B., 2010. Can exchange rates forecast commodity prices. Quarterly Journal of Economics 125, 1145–1194.

Cochrane, J., 1996. A cross-sectional test of an investment-based asset pricing model. Journal of Political Economy 104, 572–621.

Cochrane, J., 2005. Asset Pricing. Princeton University Press, Princeton, NJ.

Cochrane, J., Piazzesi, M., 2005. Bond risk premia. American Economic Review 95, 138–160.

Cortazar, G., Kovacevic, I., Schwartz, E., 2013. Commodity and asset pricing models: An integration. Unpublished working paper. NBER#19167.

Daskalaki, C., Kostakis, A., Skiadopoulos, G., 2012. Are there common factors in commodity futures returns?. Unpublished working paper. University of Piraeus.

de Roon, F., Nijman, T., Veld, C., 2000. Stochastic convenience yield implied from commodity futures and interest rates. Journal of Finance 55, 1437–1456.

Deaton, A., Laroque, G., 1992. On the behaviour of commodity prices. Review of Economic Studies 59, 1–23.

Driesprong, G., Jacobsen, B., Maat, B., 2008. Striking oil: Another puzzle?. Journal of Financial Economics 89, 307–327.

Engle, R., 2002. Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. Journal of Business & Economic Statistics 20, 339–350.

Erb, C., Harvey, C., 2006. The strategic and tactical value of commodity futures. Financial Analysts Journal 62, 69–97.

Fama, E., 1991. Efficient capital markets: II. Journal of Finance 46, 1575–1617.

Fama, E., French, K., 1988a. Business cycles and the behavior of metals prices. Journal of Finance 43, 1075–1093.

Fama, E., French, K., 1988b. Dividend yields and expected stock returns. Journal of Financial Economics 22, 3–25.

Fama, E., French, K., 1989. Business conditions and expected returns on stocks and bonds. Journal of Financial Economics 25, 23–89.

Fama, E., French, K., 1992. The cross-section of expected stock returns. Journal of Finance 47, 427–465.

Fama, E., French, K., 1996. Multifactor explanation of asset pricing anomalies. Journal of Finance 51, 55–84.

Fama, E., MacBeth, J., 1973. Risk, return and equilibrium: Empirical tests. Journal of Political Economy 81, 607–636.

Ferson, W., Harvey, C., 1999. Conditioning variables and cross-section of stock returns. Journal of Finance 54, 1325–1360.

Gorton, G., Hayashi, F., Rouwenhorst, G., 2013. The fundamentals of commodity futures returns. Review of Finance 17, 35–105.

Gorton, G., Rouwenhorst, G., 2006. Facts and fantasies about commodity futures. Financial Analyst Journal 62, 47–68.

Groen, J., Pesenti, P., 2009. Commodity prices, commodity currencies, and global economic developments. Unpublished working paper. NBER#15743.

Hamilton, J., 1983. Oil and the macroeconomy since World War II. Journal of Political Economy 91, 228–248.

Hansen, L., 1982. Large sample properties of generalized methods of moments estimators. Econometrica 50, 1029–1054.

Hansen, L., Jagannathan, R., 1997. Assessing specification errors in stochastic discount factor models. Journal of Finance 52, 557–590.

Henderson, B., Pearson, N., Wang, L., 2012. New evidence on the financialization of commodity markets. Unpublished working paper. University of Illinois at Urbana-Champaign.

Hicks, J., 1939. Value and Capital. Oxford University Press, London.

Hirshleifer, D., 1988. Residual risk, trading costs, and commodity futures risk premia. Review of Financial Studies 1, 173–193.

Hirshleifer, D., 1990. Hedging pressure and futures price movements in a general equilibrium model. Econometrica 58, 411–428.

Hodrick, R., 1992. Dividend yields and expected stock returns: alternative procedures for inference and measurement. Review of Financial Studies 5, 357–386.

Hong, H., Yogo, M., 2012. What does futures market interest tell us about the macroeconomy and asset prices?. Journal of Financial Economics 105, 473–490.

Ilmanen, A., 1995. Time-varying expected returns in international bond markets. Journal of Finance 50, 481–506.

Jagannathan, R., Wang, Z., 2002. The conditional CAPM and the cross-section of expected returns. Journal of Finance 51, 3–53.

Jegadeesh, N., Titman, S., 2001. Profitability of momentum strategies: An evaluation of alternative explanations. Journal of Finance 56, 699–720.

Kaldor, N., 1939. Speculation and economic stability. Review of Economic Studies 7, 1–27.

Keynes, J., 1930. A Treatise on Money. Macmillan, London.

Kogan, L., Livdan, D., Yaron, A., 2009. Oil futures prices in a production economy with investment constraints. Journal of Finance 64, 1345–1375.

Koijen, R., Pedersen, L., Moskowitz, T., Vrugt, E., 2012. Carry. Unpublished working paper. University of Chicago.

Labuszewski, J., 2012. Commodities as a currency driver. Unpublished working paper. Chicago Mercentile Exchange (http://www.cmegroup.com/education/files/commodities-as-a-currency-driver-2012-09-20.pdf).

Lettau, M., Ludvigson, S., 2001. Resurrecting the (C)CAPM: a cross-sectional test when risk premia are time-varying. Journal of Political Economy 109, 1238–1287.

Lewellen, J., Nagel, S., Shanken, J., 2010. A skeptical appraisal of asset pricing tests. Journal of Financial Economics 96, 175–204.

Liew, J., Vassalou, M., 2000. Can book-to-market, size and momentum be risk factors that predict economic growth?. Journal of Financial Economics 57, 221–245.

Litzenberger, R., Rabinowitz, N., 1995. Backwardation in oil futures markets: Theory and empirical evidence. Journal of Finance 50, 1517–1545.

Lustig, H., Roussanov, N., Verdelhan, A., 2011. Common risk factors in currency markets. Review of Financial Studies 24, 3731–3777.

Menkhoff, L., Sarno, L., Schmeling, M., Schrimpf, A., 2012a. Carry trades and global foreign exchange volatility. Journal of Finance 67, 681–718.

Menkhoff, L., Sarno, L., Schmeling, M., Schrimpf, A., 2012b. Currency momentum strategies. Journal of Financial Economics 106, 660–684.

Nagel, S., Singleton, K., 2011. Estimation and evaluation of conditional asset pricing models. Journal of Finance 66, 873–909.

Newey, W., West, K., 1987. A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. Econometrica 55, 703–708.

Newey, W., West, K., 1994. Automatic lag selection in covariance matrix estimation. Review of Economic Studies 61, 631–653.

Patton, A., Timmermann, A., 2010. Monotonicity in asset returns: New tests with applications to the term structure, the CAPM and portfolio sorts. Journal of Financial Economics 98, 605–625.

Politis, D., White, H., 2004. Automatic block-length selection for the dependent bootstrap. Econometric Reviews 23, 53–70.

Routledge, B., Seppi, D., Spatt, C., 2000. Equilibrium forward curves for commodities. Journal of Finance 55, 1297–1338.

Samuelson, P., 1965. Proof that properly anticipated prices fluctuate randomly. Industrial Management Review 6, 41–49.

Shanken, J., 1992. On the estimation of Beta-pricing models. Review of Financial Studies 5, 1–33.

Shwayder, A., James, A., 2011. Returns to the commodity carry trade. Unpublished working paper. Northwestern University.

Szymanowska, M., de Roon, F., Nijman, T., Goorbergh, R. V. D., 2013. An anatomy of commodity futures risk premia. Journal of Finance [forthcoming] (http://ssrn.com/abstract=1343809).

Tang, K., Xiong, W., 2013. Index investment and financialization of commodities. Financial Analyst Journal 68, 54–74.

Till, H., 2006. A long-term perspective on commodity futures returns. Unpublished working paper. EDHEC Business School.

Yang, F., 2013. Investment shocks and the commodity basis spread. Journal of Financial Economics 110, 164–184.

Appendix A: Carry and momentum strategies in commodity markets

The simplest carry strategy entails a long futures position in a commodity that is in backwardation and a short futures position in a commodity that is in contango. One may express the n-period excess return on such strategy as:

$$\mathrm{er}_{t+n} \;=\; \begin{cases} \frac{1}{F_t^{(n)}}\left(S_{t+n}-F_t^{(n)}\right) & \text{Long commodity if } \frac{F_t^{(n)}}{S_t}<1 \\ -\frac{1}{F_t^{(n)}}\left(S_{t+n}-F_t^{(n)}\right) & \text{Short commodity if } \frac{F_t^{(n)}}{S_t}>1, \end{cases} \tag{A1}$$

where the n-period futures price is denoted by $F_t^{(n)}$.

To implement a carry strategy, we cast backwardation and contango in terms of the log price ratio of the two nearest maturity futures. Define $y_t \equiv F_t^{(1)}/F_t^{(0)}$. Then the condition $\ln(y_t)<0$ ($\ln(y_t)>0$) maps to a downward (upward) sloping commodity futures curve and, hence, captures backwardation (contango).

We compute

$$\mathrm{er}_{t+1}^{\mathrm{carry}_k} \;=\; \frac{\mathrm{er}_{t+1}^{\mathrm{long}_k} \,+\, \mathrm{er}_{t+1}^{\mathrm{short}_k}}{k}, \qquad\qquad k=1,\ldots,5, \tag{A2}$$

whereby $\mathrm{er}_{t+1}^{\mathrm{carry}_k}$ represents the excess return generated by a carry strategy that consists of k long futures positions in the commodities with the k lowest $\ln(y_t)<0$ and k short futures positions in the commodities with the k highest $\ln(y_t)>0$. The strategy is dynamic, as commodities enter and exit a portfolio based on the slope of their futures curve.

Each month t, we also divide the commodity universe into two backwardation portfolios (ranked in ascending order of $\ln(y_t)<0$) and two contango portfolios (ranked in ascending order of $\ln(y_t)>0$), and compute their equally-weighted returns over the following month. Our separation of commodities into two backwardation portfolios and two contango portfolios differentiates our approach from the one adopted in Koijen, Pedersen, Moskowitz, and Vrugt (2012, Table 2) and Yang (2013, Table 2).

We also consider commodity momentum strategies. The design of momentum strategies often relies on the number of commodity futures bought and shorted, the weight given to each commodity, and the re-balancing frequency. Here we focus on equal weights, monthly re-balancing, and ranking determined by a commodity's prior J month performance:

$$\overline{\mathrm{er}}_{t,J} \;=\; \left(\Pi_{j=1}^{J}\left(1 \,+\, \mathrm{er}_{t-j}\right)\right)^{\frac{1}{J}} \,-\, 1, \qquad\qquad J=1,\ldots,12. \tag{A3}$$

Accordingly, the momentum strategy corresponds to k long futures positions in commodities with the k highest $\overline{\text{er}}_{t,J}$ (i.e., winners) and k short futures positions in commodities with the k lowest $\overline{\text{er}}_{t,J}$ (i.e., losers). The excess return is

$$\text{er}_{t+1}^{\text{momentum}_k} = \frac{\text{er}_{t+1}^{\text{long}_k} + \text{er}_{t+1}^{\text{short}_k}}{k}, \qquad k = 1, \ldots, 5. \tag{A4}$$

Each month t, we also rank commodities based on their $\overline{\text{er}}_{t,J}$ (in line with Gorton, Hayashi, and Rouwenhorst (2013, Table VII)) into quintiles and then compute their equally-weighted return over month $t+1$.

Appendix B: Returns of carry and momentum strategies

Here we summarize the return patterns generated by carry and momentum strategies in commodity markets. We also establish the statistical significance of the returns using a stationary bootstrap, and we emphasize the distinction between the long and short legs of the strategies.

Panels A and C of Table 1 present the descriptive statistics of the excess returns generated by commodity carry strategies (denoted by C1 through C5) and momentum strategies (denoted by M1 through M5), respectively. Symmetrically, Panels B and D of Table 1 present the results from carry portfolios (denoted by P1 and P2 for backwardation and P3 and P4 for contango) and momentum portfolios (denoted by Q1 through Q5). While we feature momentum strategies based on a formation period of six months, our conclusions do not appear to be sensitive to this particular choice (as shown in Figure 5).

The carry and momentum strategies are lucrative. For example, the carry strategy C5 that buys the 5 commodities with the lowest $\ln(y_t) < 0$ and shorts the 5 commodities with the highest $\ln(y_t) > 0$ delivers *annualized* average monthly excess returns of 16.34%. The momentum strategy M5 generates average returns equal to 16.11%. Four out of the five carry strategies (C2 through C5) and four out of the five momentum strategies (M2 through M5) exhibit average returns that are statistically different from zero, as indicated by the bootstrap confidence intervals. The 95% confidence intervals, denoted as PW, lower CI and upper CI, are based on a stationary bootstrap with 10,000 iterations, where the block size is based on the algorithm of Politis and White (2004).

How do these strategies fare relative to futures-based commodity indexes? Table Appendix-II shows that, over the same sample period, the Goldman Sachs Commodity Index (GSCI) and the Commodity Research Bureau (CRB) index deliver annualized average monthly excess returns equal to 5.43% and 3.53%, respectively. Our dynamic strategies, therefore, deliver returns that are about three to five times larger compared to benchmarks that invest in a basket of commodity futures.

Exploiting the conditional strategies has considerable wealth implications. Specifically, a $1 investment in July 1970 in the C5 (M5) strategy cumulates to $300.16 ($185.49) in September 2011, which sharply contrasts the $4.10 generated by the GSCI over the same period. The large disparity between the average returns of carry and momentum strategies versus the average returns of commodity indexes is puzzling, and motivates our search for asset pricing explanations.

The magnitudes of the Sharpe ratios further convey the attractiveness of commodity carry and momentum. Specifically, the Sharpe ratios (denoted by SR) of the C5 and M5 strategies are 0.73 and 0.61 – more than twice the ones associated with the GSCI and CRB index.

Further, note that the commodity carry and momentum returns are generated, at least in part, by a higher percentage of positive return realizations compared to the GSCI, as reflected in the reported $1_{\text{er}>0}$. The $1_{\text{er}>0}$ for carry strategies increases from 53.13% for C1 to 56.77% for C5. At the same time, the return skewness of the conditional strategies is essentially equal to zero (and similar to that of the GSCI), indicating that none of them generate high returns by loading excessively on possible crash risk.

One unresolved question is whether the profitability of the strategies can be traced to their long or short legs. Table Appendix-III isolates the contribution of each leg of the carry and momentum strategies and provides three additional insights. First, the profitability emanates from the long legs and not from the short legs. In particular, our bootstrap analysis indicates that the excess returns of the short legs are uniformly not statistically different from zero. Our momentum results therefore deviate from the corresponding strategy in the equity market where both the long and short legs are equally profitable (e.g., Jegadeesh and Titman (2001, Table 1)). Second, given that the standard deviations of the short and long legs are comparable, the Sharpe ratios of the short legs are overshadowed by those of the long legs. Finally, the long (short) leg of the strategies consistently produce positively (negatively) skewed returns, indicating that the long legs are not generating high returns by loading on possible crash risk. The message is that the profitability of the carry and momentum strategies stems from the long component.

Changing our focus, Panels B and D of Table 1 report the descriptive statistics corresponding to the carry and momentum portfolios. Our results show that the returns of the carry portfolios are declining when the commodities are sorted in ascending order of $\ln(y_t)$: the two backwardation portfolios yield positive and statistically significant average returns, whereas the contango portfolios yield negative, but statistically insignificant, average returns.[7] On the other hand, the average returns generated by the momentum

[7] Since the majority of the commodities are in contango most of the time, our results could partially explain the disappointing

portfolios are increasing when the commodities are sorted on the basis of their past performance. In fact, the monotonicity test of Patton and Timmermann (2010) is supportive of a monotonic pattern in the average returns of carry and momentum portfolios. Specifically, we reject the null of a non-monotonic relation between the carry (momentum) portfolios in favor of the alternative of a monotonically decreasing (increasing) relation with a p-value of 0.000 (0.079). Finally, a strategy that is long commodities with pronounced backwardation (high momentum), that is, P1 (Q5), and shorts commodities with pronounced contango (low momentum), that is, P4 (Q1), generates economically large average spreads of 16.85% (16.00%).

How profitable is carry, controlling for momentum? To investigate this question, we perform a two-way dependent sort whereby commodities are divided first into two portfolios by momentum, and then each group is divided into two carry portfolios. We then compute the next-month returns on each of the four portfolios. Below we report the average returns, with bootstrap lower and upper CI in square brackets:

	Backwardation	Contango
Low momentum	10.29	-0.66
	[4.80 15.72]	[-5.76 4.80]
High momentum	16.29	5.81
	[8.64 25.08]	[0.72 10.80]

The main takeaway, in relation to Tables 1 and Appendix-III, is threefold. First, backwardation is highly profitable across both low and high momentum commodities, highlighting the working of the carry strategy. Second, among commodities that are in backwardation, high and low momentum are equally profitable, whereas among commodities that are in contango, the high momentum commodities dominate the low momentum counterparts. Finally, a long position in commodities featuring high momentum and backwardation and a short position in commodities featuring low momentum and contango produces a return of 16.95% (i.e., 16.29% plus 0.66%). Overall, these results reiterate the distinct nature of the two strategies.

[Fig. 5 about here.]

Closing, we ask how sensitive momentum returns are to the formation period J in equation (A3). To address this concern, Figure 5 presents the average annualized returns (top panel) and the Sharpe ratios (bottom panel) for momentum strategy M5, as J varies from one to 12 months. We observe consistently high average annualized returns, ranging from 12.50% to 19.03%, and Sharpe ratios, ranging from 0.42 to 0.62, thereby affirming the profitability of momentum strategies beyond our Tables 1 and Appendix-III.

performance of the commodity indexes as an asset class.

Appendix C: Definition of macroeconomic fundamentals and the correlation among the risk factors

We adopt the following series as economy-wide fundamentals or conditioning variables:

- **IP growth**: The growth rate of industrial production for the G7 countries (source: Datastream);
- **Oil growth**: The growth rate of West Texas intermediate oil price (source: Federal Reserve Bank–St. Louis);
- **Equity risk premium**: Returns of the US value-weighted index, in excess of the risk-free return (source: Kenneth French's website);
- **Size factor**: Returns of small capitalization stocks minus the returns of large capitalization stocks (source: Kenneth French's website);
- **Value factor**: Returns of value stocks minus the returns of growth stocks (source: Kenneth French's website);
- **Momentum factor**: Returns of high momentum stocks minus the returns of low momentum stocks (source: Kenneth French's website);
- **Equity variance**: The monthly sum of squared daily returns of the MSCI G7 equity index (source: Datastream);
- **Log dividend yield**: This series is taken from Michael Robert's website;
- **Term spread**: Difference in yields between the ten-year Treasury note and the three-month Treasury bill rate (source: CRSP);
- **Default spread**: The BAA-rated corporate bond yield minus the ten-year Treasury yield (from Morningstar);
- **Currency returns**: The monthly log change of the US dollar index, expressed as FX|USD. A rise in the index implies US dollar appreciation against major foreign currencies (source: Federal Reserve Board);
- **Currency variance**: The monthly sum of squared daily log changes of the US dollar index, where the index is expressed as FX|USD;
- **Currency carry returns**: We rank G10 currencies (USD|FX, so FX is the reference) based on $fd \equiv \ln(\text{forward/spot})$. Consistent with Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011, equation (2)), we assume a long position in foreign currencies with $fd < 0$ and a short position in foreign currencies with $fd > 0$;

- **Commodity (cross-sectional) volatility** (VOL_t): At the end of each month, we compute the monthly sum of absolute daily log returns for each commodity. The cross-sectional volatility is the average across all commodities (Menkhoff, Sarno, Schmeling, and Schrimpf (2012a, equation (4)));
- **Open interest growth**: We first construct the end-of-the-month cross-sectional average dollar open interest (i.e., the open interest multiplied by the contract price) corresponding to the second nearest maturity futures contract (i.e., $F_t^{(1)}$). The open interest growth is the log change of the dollar open interest, which deviates, in certain respects, from Hong and Yogo (2012);
- **TED spread**: The three-month dollar LIBOR rate minus the three-month US Treasury-bill rate (source: Datastream).

We analyze two additional risk factors that are constructed as follows:

(i) **Commodity value factor** (VALUE_t): At the end of each month t, we rank all the commodities by the ratio of $F_{t-60}^{(1)}$ to their time t futures price $F_t^{(1)}$. Then we divide the commodities into five groups and compute the next-month return of each commodity portfolio. VALUE_t is the return spread between the top and bottom quintiles (we deviate from Asness, Moskowitz, and Pedersen (2013, Section I.B), who construct three portfolios);

(ii) **Commodity volatility factor** (ΔVOL_t): We construct this factor as the monthly change in the commodity cross-sectional volatility, that is, $\Delta\text{VOL}_t \equiv \text{VOL}_t - \text{VOL}_{t-1}$.

As seen from the correlations reported below, the commodity momentum and commodity value factors display a negative correlation of -0.39, which mimics its counterpart across asset classes in Asness, Moskowitz, and Pedersen (2013, Panel A of Table II).

	AVG_t	CARRY_t	CMOM_t	VALUE_t
CARRY_t	0.09			
CMOM_t	0.11	0.27		
VALUE_t	-0.23	-0.18	-0.39	
ΔVOL_t	-0.06	0.15	0.05	0.08

Importantly, the correlation between the average factor and the carry (momentum) factor is 0.09 (0.11), while the correlation between carry and momentum is 0.27, implying that the three factors adopted in our SDF specification (6) are mildly correlated. The correlations of ΔVOL_t with other risk factors are small in absolute values.

Table 1

Excess returns of commodity carry and momentum strategies

This table presents the descriptive statistics of the excess returns generated by commodity carry and momentum strategies. Let $y_t \equiv F_t^{(1)}/F_t^{(0)}$, where $F_t^{(0)}$ is the price of the front-month futures contract and $F_t^{(1)}$ is the price of the next maturity futures contract, both observed at the end of month t. A commodity is in backwardation if $\ln(y_t) < 0$ and in contango if $\ln(y_t) > 0$. The carry strategy entails taking a long (short) futures position in a commodity that is in backwardation (contango) at the end of month t, and we compute the returns over the subsequent month. For example, carry strategy C5 (C2) contains an equally-weighted portfolio consisting of five (two) commodities with the most negative $\ln(y_t)$ and five (two) commodities with the most positive $\ln(y_t)$. In addition, each month t, we divide the commodity universe into two backwardation portfolios (P1 and P2) and two contango portfolios (P3 and P4), based on their respective rankings of $\ln(y_t)$, and then we compute the next-month returns. For the momentum strategies, the commodities are ranked on the basis of their past six-month performance. Analogously, the momentum strategy M5 (M2) contains an equally-weighted portfolio consisting of five (two) commodities with the highest past returns (winners) and five (two) commodities with the lowest past returns (losers). Ranking the commodities by the past six-month performance, the commodities are collected in quintile portfolios Q1 (lowest) through Q5 (highest). For each portfolio, we report the average annualized monthly return and its 95% confidence interval based on a stationary bootstrap (denoted by PW, lower CI and PW, upper CI) with 10,000 bootstrap iterations, where the block size is based on the algorithm of Politis and White (2004), the annualized monthly standard deviation (SD), the annualized Sharpe ratio (SR), and the monthly skewness. The percentage of months in which the excess return of a strategy is positive is recorded as $1_{\text{er}>0}$. There are 501 monthly observations in our sample from January 1970 to September 2011.

	Panel A: Carry Strategies Commodities long backwardation and short contango					*Panel B: Backwardation/Contango Portfolios* Commodities sorted based on				
						$\ln(y_t) < 0$		$\ln(y_t) > 0$		
	C1	C2	C3	C4	C5	P1	P2	P3	P4	P1-P4
Mean	9.31	10.08	12.14	14.27	16.34	16.32	13.50	4.63	-0.53	16.85
PW, lower CI	-2.04	0.60	2.04	6.48	9.36	8.28	7.44	-0.96	-6.00	9.48
PW, upper CI	22.32	21.24	22.56	21.84	23.40	26.16	19.56	10.20	5.40	26.16
SD	48.21	34.87	27.46	24.16	22.26	23.23	19.48	17.02	16.94	23.08
SR	0.19	0.29	0.44	0.59	0.73	0.70	0.69	0.27	-0.03	0.73
Skewness	-0.29	0.02	-0.05	-0.03	0.30	0.47	0.25	0.18	0.82	0.45
$1_{\text{er}>0}$	53.13	53.54	53.94	55.76	56.77	55.56	58.59	51.92	48.69	57.17

	Panel C: Momentum Strategies Commodities long winners and short losers					*Panel D: Momentum Portfolios* Commodities sorted based on past six-month performance					
	M1	M2	M3	M4	M5	Q1	Q2	Q3	Q4	Q5	Q5-Q1
Mean	10.75	17.04	13.88	14.70	16.11	-1.66	1.31	10.07	9.16	14.35	16.00
PW, lower CI	-4.32	5.64	6.00	6.96	9.36	-7.56	-4.08	3.72	3.84	6.60	7.92
PW, upper CI	29.40	30.36	22.92	22.92	22.92	4.44	7.20	17.04	14.76	24.00	24.24
SD	59.75	42.51	34.33	29.19	26.28	20.84	16.76	18.40	18.22	25.58	27.61
SR	0.18	0.40	0.40	0.50	0.61	-0.08	0.08	0.55	0.50	0.56	0.58
Skewness	0.14	0.09	0.11	0.06	0.34	0.53	0.43	1.53	0.45	0.44	0.34
$1_{\text{er}>0}$	52.53	54.14	53.54	55.56	57.37	46.67	50.51	55.56	54.55	57.78	55.56

Table 2
Cross-sectional asset pricing results with the average, carry, and momentum factors

Reported are the factor risk premia ($\boldsymbol{\lambda}$) and the SDF parameters ($\mathbf{b}$). The SDF specification in Panel A is of the form: $m_{t+1} = 1 - b_{\mathrm{AVG}}\,\mathrm{AVG}_{t+1} - b_{\mathrm{CARRY}}\,\mathrm{CARRY}_{t+1} - b_{\mathrm{CMOM}}\,\mathrm{CMOM}_{t+1}$, where AVG_{t+1} is the average factor (excess return obtained by holding all commodities available), CARRY_{t+1} is the carry factor (which corresponds to the returns of strategy C5), and CMOM_{t+1} is the momentum factor (which corresponds to the returns of strategy M5). Reported in Panels B and C are results for the restricted versions of the SDF that impose $b_{\mathrm{CMOM}} \equiv 0$ and $b_{\mathrm{CARRY}} \equiv 0$, respectively. In the row marked "GMM," the parameters are estimated based on the system (4) following a one-step GMM procedure, while those in the row "Fama-MacBeth" are based on a two-step cross-sectional regression. The p-values rely on the Newey and West (1987) procedure with lags selected automatically according to Newey and West (1994), and reported in parentheses. For the Fama and MacBeth procedure, the p-values are computed using both the Newey and West (1987) procedure without (with) the Shanken (1992) correction in parentheses (curly brackets). We report the GLS cross-sectional uncentered R^2 (Lewellen, Nagel, and Shanken (2010, Prescription 3)) and the OLS uncentered R^2 as $\lfloor . \rfloor$, and the χ^2 test corresponding to the null hypothesis that the pricing errors are zero, with p-values computed both based on the Newey-West and Shanken standard errors. The Hansen and Jagannathan (1997, equation (29)) distance measure (HJ-Dist.), and the associated p-value, is shown, which tests whether the distance measure is equal to zero.

	Factor risk premia			Loadings on the SDF				Pricing errors		
	λ_{AVG}	λ_{CARRY}	λ_{CMOM}	b_{AVG}	b_{CARRY}	b_{CMOM}	R^2_{GLS} $\lfloor R^2 \rfloor$	χ^2_{NW} (p-val.)	χ^2_{SH} $\{p$-val.$\}$	HJ-Dist. (p-val.)
Panel A: Three-factor model										
GMM	0.005	0.018	0.012	2.276	3.954	1.067				0.006
	(0.01)	(0.00)	(0.00)	(0.07)	(0.00)	(0.11)				(0.22)
Fama-MacBeth	0.005	0.018	0.012				96.3	11.91	11.40	
	(0.02)	(0.00)	(0.00)				$\lfloor 93.9 \rfloor$	(0.22)	{0.25}	
	{0.02}	{0.00}	{0.00}							
Panel B: Restricted model omitting a role for CMOM_{t+1}										
GMM	0.005	0.020		2.394	4.661					0.008
	(0.01)	(0.00)		(0.05)	(0.00)					(0.15)
Fama-MacBeth	0.005	0.020					82.9	23.87	22.70	
	(0.02)	(0.00)					$\lfloor 91.3 \rfloor$	(0.01)	{0.01}	
	{0.02}	{0.00}								
Panel C: Restricted model omitting a role for CARRY_{t+1}										
GMM	0.006		0.014	2.891		2.300				0.012
	(0.01)		(0.00)	(0.02)		(0.00)				(0.00)
Fama-MacBeth	0.006		0.014				67.7	29.67	28.65	
	(0.02)		(0.00)				$\lfloor 77.3 \rfloor$	(0.00)	{0.00}	
	{0.02}		{0.00}							

Table 3

Exclusion tests that evaluate whether commodity value and volatility are additional priced factors

Reported are the factor risk premia and exclusion tests for the validity of a four-factor asset pricing model, when we incrementally add the value factor and the volatility factor to the SDF specification in equation (6). Specifically, the SDF specification is of the form:

$$m_{t+1} = 1 - b_{\mathrm{AVG}}\,\mathrm{AVG}_{t+1} - b_{\mathrm{CARRY}}\,\mathrm{CARRY}_{t+1} - b_{\mathrm{CMOM}}\,\mathrm{CMOM}_{t+1} - b_{\mathrm{VALUE}}\,\mathrm{VALUE}_{t+1}$$

or,

$$m_{t+1} = 1 - b_{\mathrm{AVG}}\,\mathrm{AVG}_{t+1} - b_{\mathrm{CARRY}}\,\mathrm{CARRY}_{t+1} - b_{\mathrm{CMOM}}\,\mathrm{CMOM}_{t+1} - b_{\Delta\mathrm{VOL}}\,\Delta\mathrm{VOL}_{t+1}.$$

Our procedure for constructing the value factor, denoted by VALUE_{t+1}, is similar to that in Asness, Moskowitz, and Pedersen (2013), where in each month, we rank all the commodities by the ratio of the second nearest maturity futures price five years ago to its current price. We divide the commodities into five groups and compute the next month portfolio returns. VALUE_{t+1} is the return spread between the top and bottom quintiles (see Appendix C). The $\Delta\mathrm{VOL}_{t+1}$ corresponds to the innovation in commodity volatility (see Appendix C) and is computed following Menkhoff, Sarno, Schmeling, and Schrimpf (2012a, equation (4)). The estimates of the factor risk premia $\boldsymbol{\lambda}$ are based on a cross-sectional regression. The p-values are computed using both the Newey and West (1987) procedure and the Shanken (1992) correction (in curly brackets). The test of individual parameter restriction $b_{\mathrm{VALUE}} \equiv 0$ and $b_{\Delta\mathrm{VOL}} \equiv 0$ is based on the two-step GMM χ^2 test, where the p-values rely on the Newey and West (1987) procedure with lags selected automatically according to Newey and West (1994).

		Fama-MacBeth		GMM
		Value factor	Volatility factor	(two-step)
Panel A: Factor risk premia				
λ_{AVG}	Estimate	0.003	0.005	
	NW[p]	(0.11)	(0.02)	
	Shanken[p]	{0.11}	{0.02}	
λ_{CARRY}	Estimate	0.017	0.018	
	NW[p]	(0.00)	(0.00)	
	Shanken[p]	{0.00}	{0.00}	
λ_{CMOM}	Estimate	0.012	0.012	
	NW[p]	(0.00)	(0.00)	
	Shanken[p]	{0.00}	{0.00}	
λ_{VALUE}	Estimate	-0.014		
	NW[p]	(0.21)		
	Shanken[p]	{0.22}		
$\lambda_{\Delta\mathrm{VOL}}$	Estimate		0.000	
	NW[p]		(0.94)	
	Shanken[p]		{0.94}	
Panel B: Exclusion tests for the loadings on the SDF				
H_0: $b_{\mathrm{VALUE}} \equiv 0$	$\chi^2(1)$			0.43
	(p-val.)			(0.51)
H_0: $b_{\Delta\mathrm{VOL}} \equiv 0$	$\chi^2(1)$			0.68
	(p-val.)			(0.41)

Table 4

Cross-sectional asset pricing results with the average, carry, and momentum factors - alphabetical sorts A through M or N to Z based on the ticker symbol

In Panel A we conduct a randomization exercise in the style of Lustig, Roussanov, and Verdelhan (2011) where we construct the average, carry, and momentum factors based on commodities whose ticker symbol starts with the letter A through the letter M. Next we construct two carry, two momentum, and three category portfolios based on commodities whose ticker symbol starts with the letter N through the letter Z. Reported are the factor risk premia ($\boldsymbol{\lambda}$) and the SDF parameters ($\mathbf{b}$). The baseline SDF specification is of the form: $m_{t+1} = 1 - b_{\text{AVG}}\,\text{AVG}_{t+1} - b_{\text{CARRY}}\,\text{CARRY}_{t+1} - b_{\text{CMOM}}\,\text{CMOM}_{t+1}$. Also reported are results for the restricted versions of the SDF that impose $b_{\text{CMOM}} \equiv 0$ and $b_{\text{CARRY}} \equiv 0$, respectively. In the row marked "GMM," the parameters are estimated based on the system (8) following a one-step GMM procedure, while those in the row "Fama-MacBeth" are based on a two-step cross-sectional regression. The p-values rely on the Newey and West (1987) procedure with lags selected automatically according to Newey and West (1994), and reported in parentheses. For the Fama and MacBeth procedure, the p-values are computed using both the Newey and West (1987) procedure without (with) the Shanken (1992) correction in parentheses (curly brackets). We report the GLS cross-sectional uncentered R^2 (Lewellen, Nagel, and Shanken (2010, Prescription 3)) and the OLS uncentered R^2 as $\lfloor . \rfloor$, and the χ^2 test corresponding to the null hypothesis that the pricing errors are zero, with p-values computed both based on the Newey-West and Shanken standard errors. The Hansen and Jagannathan (1997, equation (29)) distance measure (HJ-Dist.), and the associated p-value, is shown, which tests whether the distance measure is equal to zero. In Panel B we repeat the exercise by constructing factors based on commodities whose ticker symbol starts with the letter N through the letter Z and test portfolios based on commodities whose ticker symbol starts with the letter A through the letter M.

Panel A. Alphabetical Sorting A through M

	Factor risk premia			Loadings on the SDF				Pricing errors		
	λ_{AVG}	λ_{CARRY}	λ_{CMOM}	b_{AVG}	b_{CARRY}	b_{CMOM}	R^2_{GLS} $\lfloor R^2 \rfloor$	χ^2_{NW} (p-val.)	χ^2_{SH} {p-val.}	HJ-Dist. (p-val.)
Three-factor model										
GMM	0.003 (0.54)	0.024 (0.19)	0.048 (0.03)	0.736 (0.79)	4.377 (0.42)	16.976 (0.02)				0.004 (0.61)
Fama-MacBeth	0.003 (0.43) {0.49}	0.024 (0.06) {0.14}	0.048 (0.00) {0.02}				50.8 ⌊92.6⌋	4.32 (0.37)	2.45 {0.65}	
Restricted model omitting a role for CMOM_{t+1}										
GMM	0.005 (0.08)	-0.004 (0.75)		3.601 (0.06)	-1.614 (0.70)					0.008 (0.04)
Fama-MacBeth	0.005 (0.00) {0.10}	-0.004 (0.75) {0.75}					35.7 ⌊68.4⌋	12.36 (0.03)	12.16 {0.03}	
Restricted model omitting a role for CARRY_{t+1}										
GMM	0.003 (0.45)		0.034 (0.04)	1.498 (0.52)		12.763 (0.04)				0.005 (0.48)
Fama-MacBeth	0.003 (0.42) {0.45}		0.034 (0.04) {0.08}				54.2 ⌊88.2⌋	5.41 (0.37)	3.98 {0.55}	

Panel B. Alphabetical Sorting N through Z

	Factor risk premia			Loadings on the SDF				Pricing errors		
	λ_{AVG}	λ_{CARRY}	λ_{CMOM}	b_{AVG}	b_{CARRY}	b_{CMOM}	R^2_{GLS} $\lfloor R^2 \rfloor$	χ^2_{NW} (p-val.)	χ^2_{SH} {p-val.}	HJ-Dist. (p-val.)
Three-factor model										
GMM	0.016 (0.01)	0.077 (0.04)	0.035 (0.22)	6.026 (0.01)	15.937 (0.03)	5.355 (0.35)				0.007 (0.04)
Fama-MacBeth	0.016 (0.00) {0.00}	0.077 (0.00) {0.02}	0.035 (0.03) {0.13}					63.2 ⌊81.8⌋	19.44 (0.00)	9.43 {0.05}
Restricted model omitting a role for CMOM_{t+1}										
GMM	0.014 (0.00)	0.049 (0.01)		5.303 (0.00)	10.780 (0.01)					0.008 (0.02)
Fama-MacBeth	0.014 (0.00) {0.00}	0.049 (0.00) {0.01}					60.4 ⌊78.7⌋	21.51 (0.00)	15.01 {0.01}	
Restricted model omitting a role for CARRY_{t+1}										
GMM	0.010 (0.02)		-0.014 (0.26)	3.953 (0.02)		-3.388 (0.24)				0.010 (0.00)
Fama-MacBeth	0.010 (0.01) {0.01}		-0.014 (0.22) {0.24}				41.3 ⌊66.0⌋	27.84 (0.00)	25.78 {0.00}	

Table 5

Cross-sectional asset pricing results with the average, carry, and momentum factors and expanded set of test portfolios

This Table assesses the performance of the three factor model, and its two-factor nested counterparts, when the number of test portfolios is expanded to additionally include five variance portfolios. The variance portfolios are constructed as follows. At the end of each month we compute the monthly sum of absolute daily log returns for each commodity following Menkhoff, Sarno, Schmeling, and Schrimpf (2012a, equation (4)). We then sort commodities in five equally weighted portfolios according to their variance and compute the next month returns for each portfolio. The SDF of the three-factor model is of the form: $m_{t+1} = 1 - b_{\mathrm{AVG}}\,\mathrm{AVG}_{t+1} - b_{\mathrm{CARRY}}\,\mathrm{CARRY}_{t+1} - b_{\mathrm{CMOM}}\,\mathrm{CMOM}_{t+1}$, where AVG_{t+1} is the average factor (excess return obtained by holding all commodities available), CARRY_{t+1} is the carry factor, and CMOM_{t+1} is the momentum factor. Reported are also results for the restricted versions of the SDF that impose $b_{\mathrm{CMOM}} \equiv 0$ and $b_{\mathrm{CARRY}} \equiv 0$, respectively. We report the χ^2 test corresponding to the null hypothesis that the pricing errors are zero, with p-values computed both based on the Newey-West and Shanken standard errors. The Hansen and Jagannathan (1997, equation (29)) distance measure (HJ-Dist.), and the associated p-value, is shown, which tests whether the distance measure is equal to zero.

	Three factor model			Omitting momentum			Omitting carry		
Portfolios	χ^2_{NW}	χ^2_{SH}	HJ-Dist.	χ^2_{NW}	χ^2_{SH}	HJ-Dist.	χ^2_{NW}	χ^2_{SH}	HJ-Dist.
Carry, Momentum, Industry, Variance	16.40	15.71	0.007	25.64	24.39	0.008	35.74	34.53	0.013
(17 portfolios)	(0.29)	{0.33}	(0.281)	(0.04)	{0.06}	(0.21)	(0.00)	{0.00}	(0.00)
Carry, Momentum, Variance	12.89	12.35	0.007	22.81	21.68	0.008	28.93	27.91	0.012
(14 portfolios)	(0.30)	{0.34}	(0.135)	(0.03)	{0.04}	(0.10)	(0.00)	{0.01}	(0.00)

Table 6

Time-series regressions based on the three-factor model

Results are based on the time-series regression: $\mathrm{er}_t^i = \alpha + \beta_{\mathrm{AVG}}^i \mathrm{AVG}_t + \beta_{\mathrm{CARRY}}^i \mathrm{CARRY}_t + \beta_{\mathrm{CMOM}}^i \mathrm{CMOM}_t + \varepsilon_t^i$, for $i = 1, \ldots, 12$. We report the coefficient estimates and the p-values in parentheses. The p-values are based on the Newey and West (1987) procedure with lags selected automatically according to Newey and West (1994). To test the null hypothesis that all intercepts are jointly equal to zero, we compute the statistic $\hat{\boldsymbol{\alpha}}' \mathbf{var}(\hat{\boldsymbol{\alpha}})^{-1} \hat{\boldsymbol{\alpha}}$ in a GMM setting (Cochrane (2005, page 234)), which is asymptotically distributed $\chi^2(12)$. The underlying test assets are the four carry portfolios, the five momentum portfolios, and the three category portfolios. We have excluded the energy category as a test asset in the time-series regressions because of its shorter time-series (the heating oil started in 1979:04, while crude oil started in 1983:04; see Table Online-I).

Commodity portfolio		α	β_{AVG}	β_{CARRY}	β_{CMOM}	$\overline{R}^2$ (%)	Joint test on α: $\chi^2(12)$	Joint test on α: p-val.
P1	Estimate	0.002	0.933	0.566	-0.041	66.4		
(backwardation, lowest y)	NW[p]	(0.36)	(0.00)	(0.00)	(0.22)			
P2	Estimate	0.004	0.882	0.284	-0.086	53.9		
	NW[p]	(0.01)	(0.00)	(0.00)	(0.00)			
P3	Estimate	-0.001	1.000	-0.063	0.041	70.9		
	NW[p]	(0.43)	(0.00)	(0.01)	(0.05)			
P4	Estimate	-0.001	1.016	-0.303	-0.038	82.9		
(contango, highest y)	NW[p]	(0.20)	(0.00)	(0.00)	(0.03)			
Q1	Estimate	-0.002	1.160	0.002	-0.443	84.5		
(momentum, lowest)	NW[p]	(0.15)	(0.00)	(0.96)	(0.00)			
Q2	Estimate	-0.002	0.874	-0.015	-0.091	54.5		
	NW[p]	(0.18)	(0.00)	(0.68)	(0.00)			
Q3	Estimate	0.004	0.950	0.000	-0.054	53.1		
	NW[p]	(0.01)	(0.00)	(0.99)	(0.05)			
Q4	Estimate	0.001	0.892	-0.009	0.146	55.9		
	NW[p]	(0.39)	(0.00)	(0.81)	(0.00)			
Q5	Estimate	-0.002	1.165	0.039	0.552	83.5		
(momentum, highest)	NW[p]	(0.15)	(0.00)	(0.15)	(0.00)			
Agriculture	Estimate	-0.001	0.983	-0.042	-0.022	69.1		
	NW[p]	(0.38)	(0.00)	(0.31)	(0.54)			
Livestock	Estimate	0.001	0.659	0.004	-0.094	22.3		
	NW[p]	(0.64)	(0.00)	(0.93)	(0.04)			
Metal	Estimate	0.000	0.985	-0.078	0.132	40.3		
	NW[p]	(0.93)	(0.00)	(0.18)	(0.08)			
All portfolios							18.20	0.11

Table 7
Time-series regressions with a two-factor model that excludes the momentum factor

In Panel A, the results are based on the time-series regression: $er_t^i = \alpha + \beta_{AVG}^i AVG_t + \beta_{CARRY}^i CARRY_t + \varepsilon_t^i$, for $i = 1, \ldots, 12$. In Panel B, the results are based on the time-series regression: $er_t^i = \alpha + \beta_{CARRY}^i CARRY_t + \varepsilon_t^i$, for $i = 1, \ldots, 12$. We report the coefficient estimates and the p-values in parentheses. The p-values are based on the Newey and West (1987) procedure with lags selected automatically according to Newey and West (1994). To test the null hypothesis that all intercepts are jointly equal to zero, we compute the statistic $\hat{\boldsymbol{\alpha}}' \mathbf{var}(\hat{\boldsymbol{\alpha}})^{-1} \hat{\boldsymbol{\alpha}}$ in a GMM setting (Cochrane (2005, page 234)), which is asymptotically distributed $\chi^2(12)$. The underlying test assets are the four carry portfolios, the five momentum portfolios, and the three category portfolios. We have excluded the energy category as a test asset in the time-series regressions because of its shorter time-series (the heating oil started in 1979:04, while crude oil started in 1983:04).

		Panel A: two-factor model that excludes the momentum factor				**Panel B: one-factor model with the carry factor**		
Commodity portfolio		α	β_{AVG}	β_{CARRY}	$\bar{R}^2$ (%)	α	β_{CARRY}	$\bar{R}^2$ (%)
P1	Estimate	0.00	0.927	0.553	66.3	0.005	0.609	33.9
(backwardation, lowest y)	NW[p]	(0.47)	(0.00)	(0.00)		(0.04)	(0.00)	
P2	Estimate	0.003	0.875	0.255	53.3	0.007	0.309	12.3
	NW[p]	(0.07)	(0.00)	(0.00)		(0.00)	(0.00)	
P3	Estimate	-0.001	1.008	-0.050	71.2	0.004	0.011	-0.2
	NW[p]	(0.56)	(0.00)	(0.03)		(0.14)	(0.79)	
P4	Estimate	-0.001	1.007	-0.315	82.8	0.003	-0.25	10.9
(contango, highest y)	NW[p]	(0.16)	(0.00)	(0.00)		(0.18)	(0.00)	
Q1	Estimate	-0.005	1.089	-0.135	55.8	0.000	-0.06	0.3
(momentum, lowest)	NW[p]	(0.00)	(0.00)	(0.00)		(0.87)	(0.24)	
Q2	Estimate	-0.003	0.859	-0.044	53.2	0.001	0.008	-0.2
	NW[p]	(0.09)	(0.00)	(0.22)		(0.68)	(0.87)	
Q3	Estimate	0.004	0.943	-0.017	53.4	0.008	0.040	0.0
	NW[p]	(0.02)	(0.00)	(0.64)		(0.01)	(0.49)	
Q4	Estimate	0.002	0.919	0.036	52.8	0.006	0.092	1.1
	NW[p]	(0.13)	(0.00)	(0.31)		(0.01)	(0.08)	
Q5	Estimate	0.003	1.245	0.212	54.2	0.008	0.287	6.1
(momentum, highest)	NW[p]	(0.24)	(0.00)	(0.00)		(0.01)	(0.00)	
Agriculture	Estimate	-0.001	0.983	-0.050	69.6	0.003	0.010	-0.2
	NW[p]	(0.30)	(0.00)	(0.16)		(0.22)	(0.87)	
Livestock	Estimate	0.001	0.612	-0.017	19.5	0.003	0.021	-0.1
	NW[p]	(0.79)	(0.00)	(0.76)		(0.18)	(0.71)	
Metal	Estimate	0.001	1.015	-0.038	39.5	0.005	0.024	-0.1
	NW[p]	(0.65)	(0.00)	(0.45)		(0.11)	(0.68)	
$\chi^2(12)$			28.94				31.28	
(p-val.)			(0.00)				(0.00)	

Table 8

Time-series regressions with a three-factor model - alphabetical sorts A through M or N through Z based on the ticker symbol

In panel A we conduct a randomization exercise in the style of Lustig, Roussanov, and Verdelhan (2011) where we construct the average, carry, and momentum factors based on commodities whose ticker symbol starts with the letter A through the letter M. Next we construct two carry, two momentum, and three category portfolios based on commodities whose ticker symbol starts with the letter N through the letter Z. Results are based on the time-series regression: $er_t^i = \alpha + \beta^i_{\text{AVG}}\,\text{AVG}_t + \beta^i_{\text{CARRY}}\,\text{CARRY}_t + \beta^i_{\text{CMOM}}\,\text{CMOM}_t + \varepsilon^i_t$, for $i = 1, \ldots, 7$. We report the coefficient estimates and the p-values in parentheses. The p-values are based on the Newey and West (1987) procedure with lags selected automatically according to Newey and West (1994). To test the null hypothesis that all intercepts are jointly equal to zero, we compute the statistic $\hat{\boldsymbol{\alpha}}'\mathbf{var}(\hat{\boldsymbol{\alpha}})^{-1}\hat{\boldsymbol{\alpha}}$ in a GMM setting (Cochrane (2005, page 234)), which is asymptotically distributed $\chi^2(7)$. We have excluded the energy category as a test asset in the time-series regressions because of its shorter time-series. In Panel B we repeat the exercise by constructing factors based on commodities whose ticker symbol starts with the letter N through the letter Z and test portfolios based on commodities whose ticker symbol starts with the letter A through the letter M.

Panel A. Alphabetical Sorting A through M

Commodity portfolio		α	β_{AVG}	β_{CARRY}	β_{CMOM}	$\bar{R}^2$ (%)	Joint test on α: $\chi^2(7)$	Joint test on α: p-val.
P1	Estimate	0.003	0.784	0.089	0.069	19.7		
(backwardation, lowest y)	NW[p]	(0.358)	(0.000)	(0.059)	(0.389)			
P2	Estimate	-0.001	0.939	-0.161	0.132	46.9		
(contango, highest y)	NW[p]	(0.553)	(0.000)	(0.000)	(0.001)			
Q1	Estimate	-0.004	0.960	-0.005	-0.004	39.1		
(momentum, lowest)	NW[p]	(0.043)	(0.000)	(0.926)	(0.937)			
Q2	Estimate	0.005	0.912	-0.197	0.256	34.0		
(momentum, highest)	NW[p]	(0.029)	(0.000)	(0.002)	(0.000)			
Agriculture	Estimate	-0.004	0.839	0.118	-0.080	9.5		
	NW[p]	(0.415)	(0.000)	(0.224)	(0.389)			
Livestock	Estimate	0.004	0.950	-0.292	0.178	22.2		
	NW[p]	(0.236)	(0.000)	(0.001)	(0.020)			
Metal	Estimate	-0.001	0.836	-0.094	0.087	31.6		
	NW[p]	(0.731)	(0.000)	(0.078)	(0.103)			
All portfolios							13.41	0.063

Panel B. Alphabetical Sorting N through Z

Commodity portfolio		α	β_{AVG}	β_{CARRY}	β_{CMOM}	$\bar{R}^2$ (%)	Joint test on α: $\chi^2(7)$	Joint test on α: p-val.
P1	Estimate	0.012	0.503	0.081	-0.122	23.2		
(backwardation, lowest y)	NW[p]	(0.000)	(0.000)	(0.014)	(0.009)			
P2	Estimate	-0.001	0.493	-0.075	-0.006	39.9		
(contango, highest y)	NW[p]	(0.360)	(0.000)	(0.001)	(0.877)			
Q1	Estimate	0.002	0.482	-0.008	-0.103	30.9		
(momentum, lowest)	NW[p]	(0.425)	(0.000)	(0.823)	(0.007)			
Q2	Estimate	0.006	0.572	-0.028	0.026	38.6		
(momentum, highest)	NW[p]	(0.001)	(0.000)	(0.321)	(0.419)			
Agriculture	Estimate	0.004	0.301	0.002	-0.096	10.3		
	NW[p]	(0.098)	(0.000)	(0.958)	(0.024)			
Livestock	Estimate	0.002	0.507	-0.138	0.133	23.6		
	NW[p]	(0.518)	(0.000)	(0.002)	(0.025)			
Metal	Estimate	0.001	0.562	0.018	-0.054	34.5		
	NW[p]	(0.747)	(0.000)	(0.525)	(0.250)			
All portfolios							34.11	0.000

Table 9
Performance of conditional asset pricing models

Here we test conditional asset pricing models, where the SDF takes the form:

$$\begin{aligned} m_{t+1} = \; & 1 - b_{\text{AVG}}\,(\text{AVG}_{t+1} - \mu_{\text{AVG}}) - b_{\text{CARRY}}\,(\text{CARRY}_{t+1} - \mu_{\text{CARRY}}) - b_{\text{CMOM}}\,(\text{CMOM}_{t+1} - \mu_{\text{CMOM}}) \\ & - b_{\text{AVG},z}\,(\text{AVG}_{t+1} - \mu_{\text{AVG}})\,(z_t - \mu_z) \\ & - b_{\text{CARRY},z}\,(\text{CARRY}_{t+1} - \mu_{\text{CARRY}})\,(z_t - \mu_z) \\ & - b_{\text{CMOM},z}\,(\text{CMOM}_{t+1} - \mu_{\text{CMOM}})\,(z_t - \mu_z), \end{aligned}$$

where z_t is a conditioning information variable. We first report, in Panel A, the Hansen and Jagannathan (1997, equation (29)) distance measure (HJ-Dist.), and the associated p-value, which assesses whether the distance measure is equal to zero. In Panel B, we test the null hypothesis $b_{\text{AVG},z} = b_{\text{CARRY},z} = b_{\text{CMOM},z} \equiv 0$ against the alternative that at least one is different from zero. Panel C tests the individual parameter restriction $b_{\text{AVG},z} \equiv 0$, $b_{\text{CARRY},z} \equiv 0$, and $b_{\text{CMOM},z} \equiv 0$. The p-values rely on the Newey and West (1987) procedure with lags selected automatically according to Newey and West (1994). In our two-step GMM implementation, we demean each factor and z_t. The conditioning variable $\ln(y_t)1_{\ln(y_t)<0}$ ($\ln(y_t)1_{\ln(y_t)>0}$) reflects the slope of the futures curves, averaged across all commodities in backwardation (contango).

	Panel A:		Panel B:		Panel C: Exclusion restrictions on individual interaction terms					
	HJ Dist. test		All $b_{\text{k},z} = 0$		$b_{\text{AVG},z} = 0$		$b_{\text{CARRY},z} = 0$		$b_{\text{CMOM},z} = 0$	
z_t	Dist.	p-val.	$\chi^2(3)$	NW[p]	$\chi^2(1)$	NW[p]	$\chi^2(1)$	NW[p]	$\chi^2(1)$	NW[p]
I. z_t reflects commodity market developments										
Open interest growth	0.006	0.12	9.35	0.03	5.91	0.02	9.36	0.00	5.97	0.02
ΔVol_t	0.003	0.98	12.43	0.01	4.98	0.03	0.26	0.61	4.43	0.04
II. z_t reflects business conditions										
Log dividend yield	0.006	0.31	7.57	0.06	5.81	0.02	0.28	0.60	5.75	0.02
Currency returns (FX|USD)	0.003	0.95	9.52	0.02	0.66	0.42	0.10	0.75	3.90	0.05
Industrial production growth	0.006	0.25	5.92	0.12	2.56	0.11	0.10	0.75	0.43	0.51
Term spread	0.007	0.14	6.22	0.10	1.72	0.19	3.53	0.06	2.82	0.09
III. z_t reflects dynamics of the commodity futures curves										
$\ln(y_t)1_{\ln(y_t)<0}$	0.006	0.15	3.97	0.27	0.84	0.36	3.81	0.05	1.00	0.32
$\ln(y_t)1_{\ln(y_t)>0}$	0.007	0.12	3.05	0.38	2.47	0.12	2.83	0.09	2.42	0.12

Table 10
Relation of the commodity factors to future real GDP growth

We report the coefficient estimates from the following predictive regressions with overlapping observations at the quarterly frequency:

$$\ln\left(\mathrm{GDP}_{t+k}/\mathrm{GDP}_t\right) = \theta_0 + \theta_{\mathrm{AVG}}\,\mathrm{AVG}_t + \theta_{\mathrm{CARRY}}\,\mathrm{CARRY}_t + \theta_{\mathrm{CMOM}}\,\mathrm{CMOM}_t + \varepsilon_{t+k} \quad \text{and} \quad k \in \{1,2,3,4\},$$

where GDP refers to the real GDP of the G7 countries (source: Datastream, ticker G7OCMP03D). The p-values based on the Newey and West (1987) covariance estimator, where lags are automatically selected according to Newey and West (1994)), are denoted by NW[p]. Additionally, the p-values based on the Hodrick (1992) 1B covariance estimator under the null of no predictability are denoted by H[p]. Adjusted R^2 is reported as $\overline{R}^2$ (in %), while the J[p] column reports the p-values for the null hypothesis that the slope coefficients are jointly equal to zero. The sample period is from 1970:Q1 to 2011:Q3.

Horizon		θ_0	Predictive slope coefficients θ_{AVG}	θ_{CARRY}	θ_{CMOM}	$\overline{R}^2$	J[p]
1 quarter	Estimate	0.01	0.03	−0.01	−0.01	13.8	
	NW[p]	(0.00)	(0.02)	(0.06)	(0.00)		(0.00)
	H[p]	⟨0.00⟩	⟨0.04⟩	⟨0.09⟩	⟨0.01⟩		⟨0.02⟩
2 quarters	Estimate	0.01	0.04	−0.02	−0.02	11.1	
	NW[p]	(0.00)	(0.03)	(0.06)	(0.00)		(0.02)
	H[p]	⟨0.00⟩	⟨0.03⟩	⟨0.00⟩	⟨0.00⟩		⟨0.00⟩
3 quarters	Estimate	0.02	0.04	−0.03	−0.02	9.3	
	NW[p]	(0.00)	(0.08)	(0.05)	(0.02)		(0.02)
	H[p]	⟨0.00⟩	⟨0.04⟩	⟨0.00⟩	⟨0.00⟩		⟨0.00⟩
4 quarters	Estimate	0.03	0.03	−0.03	−0.03	7.4	
	NW[p]	(0.00)	(0.23)	(0.08)	(0.00)		(0.01)
	H[p]	⟨0.00⟩	⟨0.16⟩	⟨0.01⟩	⟨0.00⟩		⟨0.00⟩

Table 11

Relation of the commodity factors to future excess Treasury bond returns

This table reports the coefficient estimates from the following predictive regressions with overlapping observations at the monthly frequency:

$$\sum_{k=1}^{K} \mathrm{er}_{t+k}^{\mathrm{bond}} = \xi_0 + \boldsymbol{\xi}'\mathbf{f}_t + \varepsilon_{t+K}, \qquad \text{with} \qquad \mathrm{er}_{t+1}^{\mathrm{bond}} \equiv \ln(1+\mathrm{r}_{t+1}^{\mathrm{bond}}) - \ln(1+\mathrm{r}_t^{f}) \qquad \text{and} \qquad K \in \{1,3,6,9,12\},$$

where $\mathbf{f}_t \equiv [\mathrm{AVG}_t\ \mathrm{CARRY}_t\ \mathrm{CMOM}_t]'$. We compute standard errors based on the Hodrick (1992) 1B covariance estimator under the null of no predictability and report the corresponding p-values, denoted by H[p]. Adjusted R^2 is reported as $\overline{R}^2$ (in %). The analysis is conducted for both one-year and 30-year Treasury bonds (source: CRSP) and the sample period is from January 1970 to September 2011. The inference based on the Newey-West p-values agrees with those from H[p], hence, the Newey-West p-values are not reported.

		Panel A: Bond returns, one-year maturity					Panel B: Bond returns, 30-year maturity				
Horizon		ξ_0	ξ_{AVG}	ξ_{CARRY}	ξ_{CMOM}	$\overline{R}^2$	ξ_0	ξ_{AVG}	ξ_{CARRY}	ξ_{CMOM}	$\overline{R}^2$
1 month	Estimate	0.00	-0.03	0.00	0.00	4.0	0.00	-0.20	-0.02	0.02	5.2
	H[p]	⟨0.00⟩	⟨0.02⟩	⟨0.57⟩	⟨0.50⟩		⟨0.12⟩	⟨0.00⟩	⟨0.53⟩	⟨0.48⟩	
3 months	Estimate	0.00	-0.04	0.02	-0.01	3.9	0.01	-0.30	0.03	0.03	3.1
	H[p]	⟨0.00⟩	⟨0.01⟩	⟨0.01⟩	⟨0.64⟩		⟨0.05⟩	⟨0.01⟩	⟨0.34⟩	⟨0.61⟩	
6 months	Estimate	0.01	-0.05	0.02	0.00	2.8	0.01	-0.22	0.05	0.01	0.5
	H[p]	⟨0.00⟩	⟨0.01⟩	⟨0.01⟩	⟨0.06⟩		⟨0.01⟩	⟨0.66⟩	⟨0.03⟩	⟨0.95⟩	
9 months	Estimate	0.01	-0.07	0.03	0.01	3.1	0.02	-0.21	0.15	-0.01	0.9
	H[p]	⟨0.00⟩	⟨0.00⟩	⟨0.00⟩	⟨0.00⟩		⟨0.00⟩	⟨0.39⟩	⟨0.00⟩	⟨0.91⟩	
12 months	Estimate	0.01	-0.10	0.02	0.01	3.9	0.03	-0.24	0.13	0.03	0.5
	H[p]	⟨0.00⟩	⟨0.00⟩	⟨0.00⟩	⟨0.00⟩		⟨0.00⟩	⟨0.12⟩	⟨0.00⟩	⟨0.48⟩	

Table 12
Relation of the commodity factors to future excess equity returns

This table reports the coefficient estimates from the following predictive regressions with overlapping observations at the monthly frequency:

$$\sum_{k=1}^{K} \mathrm{er}^{\mathrm{equity}}_{t+k} = \delta_0 + \boldsymbol{\delta}'\mathbf{f}_t + \varepsilon_{t+K}, \quad \text{with} \quad \mathrm{er}^{\mathrm{equity}}_{t+1} \equiv \ln(1+\mathrm{r}^{\mathrm{equity}}_{t+1}) - \ln(1+\mathrm{r}^{f}_{t}) \quad \text{and} \quad K \in \{1,3,6,9,12\},$$

where $\mathrm{er}^{\mathrm{equity}}_{t+1}$ is the excess return of the US value-weighted equity index over month t to $t+1$ and $\mathbf{f}_t \equiv [\mathrm{AVG}_t\ \mathrm{CARRY}_t\ \mathrm{CMOM}_t]'$. We compute standard errors based on the Hodrick (1992) 1B covariance estimator under the null of no predictability and report the corresponding p-values, denoted by H[p]. Also reported are the results of the restricted predictive regressions with $\delta_{\mathrm{AVG}} = \delta_{\mathrm{CMOM}} \equiv 0$. Adjusted R^2 is reported as $\overline{R}^2$ (%), and the sample period is from January 1970 to September 2011. The inference based on the Newey-West p-values agrees with those from H[p], hence, the Newey-West p-values are not reported.

Horizon	Unrestricted predictive regressions					Restricted regressions $\delta_{\mathrm{AVG}} = \delta_{\mathrm{CMOM}} \equiv 0$	
	δ_0	δ_{AVG}	δ_{CARRY}	δ_{CMOM}	$\overline{R}^2$	δ_{CARRY}	$\overline{R}^2$
1 month	0.01	-0.07	-0.06	0.02	0.4	-0.06	0.4
	⟨0.01⟩	⟨0.35⟩	⟨0.09⟩	⟨0.45⟩		⟨0.10⟩	
3 months	0.02	0.07	-0.15	0.02	0.7	-0.14	0.9
	⟨0.01⟩	⟨0.59⟩	⟨0.01⟩	⟨0.79⟩		⟨0.02⟩	
6 months	0.03	-0.08	-0.16	0.01	0.2	-0.16	0.5
	⟨0.01⟩	⟨0.67⟩	⟨0.09⟩	⟨0.89⟩		⟨0.08⟩	
9 months	0.05	-0.32	-0.27	0.07	1.6	-0.27	1.1
	⟨0.01⟩	⟨0.16⟩	⟨0.03⟩	⟨0.44⟩		⟨0.04⟩	
12 months	0.06	-0.51	-0.32	0.07	2.5	-0.33	1.4
	⟨0.01⟩	⟨0.04⟩	⟨0.04⟩	⟨0.43⟩		⟨0.04⟩	

Table 13
Relation of the commodity factors to the future returns of commodity currencies

This table reports the coefficient estimates from the following predictive regressions with overlapping observations at the quarterly frequency (for comparability with Chen, Rogoff, and Rossi (2010)):

$$\ln(\text{FX}_{t+k}/\text{FX}_t) \;=\; \pi_0 \,+\, \boldsymbol{\pi}'\mathbf{f}_t \,+\, \varepsilon_{t+k} \qquad \text{and} \qquad k \in \{1,2,3,4\},$$

where $\ln(\text{FX}_{t+k}/\text{FX}_t)$ represents the equally-weighted returns from a set of commodity currencies (i.e., Australia, Canada, Chile, Norway, New Zealand, and South Africa; see Labuszewski (2012)) with the US dollar as the reference currency (i.e., FX|USD) and $\mathbf{f}_t \equiv [\text{AVG}_t \ \text{CARRY}_t \ \text{CMOM}_t]'$. We compute standard errors based on the Hodrick (1992) 1B covariance estimator under the null of no predictability and report the corresponding p-values, denoted by H[p]. Adjusted R^2 is reported as $\overline{R}^2$ (in %). The sample period is from 1974:Q1 to 2011:Q3. The inference based on the Newey-West p-values agrees with those from H[p], hence, the Newey-West p-values are not reported.

Horizon		π_0	Predictive slope coefficients			$\overline{R}^2$
			π_{AVG}	π_{CARRY}	π_{CMOM}	
1 quarter	Estimate	0.01	-0.12	0.03	0.06	6.0
	H[p]	$\langle 0.02\rangle$	$\langle 0.06\rangle$	$\langle 0.25\rangle$	$\langle 0.03\rangle$	
2 quarters	Estimate	0.02	-0.09	0.12	0.04	4.0
	H[p]	$\langle 0.02\rangle$	$\langle 0.27\rangle$	$\langle 0.01\rangle$	$\langle 0.25\rangle$	
3 quarters	Estimate	0.02	-0.06	0.17	0.07	6.0
	H[p]	$\langle 0.02\rangle$	$\langle 0.64\rangle$	$\langle 0.00\rangle$	$\langle 0.07\rangle$	
4 quarters	Estimate	0.03	-0.03	0.18	0.11	5.0
	H[p]	$\langle 0.02\rangle$	$\langle 0.81\rangle$	$\langle 0.00\rangle$	$\langle 0.01\rangle$	

Table 14
Contemporaneous association of the commodity factors with economic fundamentals

All results rely on the univariate regressions:

$$\text{AVG}_t = \phi_0 + \phi X_t + e_t, \quad \text{CARRY}_t = \phi_0 + \phi X_t + e_t, \quad \text{CMOM}_t = \phi_0 + \phi X_t + e_t,$$

where X_t represents a set of economy-wide fundamentals defined in Appendix C. We standardize X_t by its standard deviation, thus the reported ϕ's represent the exposure of each factor to a one standard deviation change in the economic fundamental. We report the Newey and West (1987) p-values (with lags automatically selected, as in Newey and West (1994)), and denote them by NW[p]. The regression intercepts are not reported to save space and the ϕ coefficients that are statistically significant are in bold. The regression coefficients have been multiplied by 100.

	Average factor		Carry factor		Momentum factor	
	ϕ	NW[p]	ϕ	NW[p]	ϕ	NW[p]
A. Economic activity						
IP growth	**0.73**	0.00	-0.04	0.88	-0.11	0.77
B. Equity market						
Equity premium	**0.66**	0.04	**-0.69**	0.03	-0.40	0.28
Size factor	0.30	0.14	-0.17	0.53	-0.27	0.39
Value factor	0.13	0.50	**0.51**	0.06	**0.64**	0.01
Momentum factor	-0.02	0.95	**0.68**	0.09	**1.44**	0.00
Equity variance	**-0.84**	0.02	-0.23	0.33	-0.01	0.98
C. Bond market						
Term spread	**-0.36**	0.10	0.09	0.73	0.24	0.32
Default spread	-0.44	0.16	0.11	0.75	-0.09	0.78
D. Commodity market						
Cross-sectional volatility	-0.25	0.57	**0.96**	0.01	0.40	0.27
Open interest growth	**0.93**	0.00	0.41	0.12	**1.53**	0.00
E. Currency market						
Currency returns (FX|USD)	**-0.80**	0.00	-0.05	0.86	-0.42	0.22
Currency variance	-0.40	0.27	0.01	0.97	-0.35	0.42
Currency carry returns	0.33	0.34	-0.18	0.63	0.21	0.46
F. Liquidity						
TED spread	-0.05	0.82	0.10	0.75	0.06	0.87

Table 15

Membership in the long and short components of the carry and momentum strategies

Entries under the column labeled "Long (Short)" depict how many months the respective commodity has entered the backwardation (contango) component of the carry strategy C5 (Panel A). The next four columns show how many months the commodity has entered the long and short components of the momentum strategy M5 (Panel B). For example, in 171 (126) months the live cattle (soybean oil) has been among the five highest backwardated (six-month momentum) commodities.

Panel A: Carry strategy, C5				Panel B: Momentum strategy, M5			
Long		Short		Long		Short	
Live cattle	171	Oats	225	Soybean oil	126	Sugar	139
Lean hog	168	Lumber	191	Corn	110	Pork belly	123
Pork belly	118	Lean hog	182	Cocoa	99	Lumber	121
Sugar	108	Wheat	165	Crude oil	97	Orange juice	112
Coffee	104	Sugar	158	Cotton	97	Cocoa	111
Oats	100	Corn	156	Feeder cattle	95	Oats	107
Lumber	93	Orange juice	103	Gold	90	Natural gas	102
Orange juice	91	Rough rice	94	Copper	89	Coffee	92
Cocoa	86	Cocoa	92	Heating oil	87	Corn	88
Unleaded gas	86	Cotton	92	Unleaded gas	83	Soybean oil	78
Cotton	85	Coffee	90	Coffee	80	Palladium	75
Feeder cattle	77	Live cattle	89	Lumber	79	Rough rice	72
Soybean meal	74	Natural gas	61	Barley	78	Silver	70
Wheat	69	Platinum	35	Live cattle	75	Lean hog	68
Heating oil	68	Pork belly	34	Lean hog	72	Wheat	68
Soybean oil	65	Soybean meal	32	Natural gas	67	Platinum	67
Copper	57	Feeder cattle	29	Oats	60	Natural gas	65
Crude oil	50	Unleaded gas	22	Orange juice	59	Soybean meal	59
Propane	37	Barley	22	Palladium	59	Cotton	56
Corn	36	Soybean oil	17	Pork belly	57	Heating oil	52
Natural gas	34	Silver	17	Platinum	50	Crude oil	45
Palladium	30	Copper	16	Propane	49	Silver	39
Soybeans	29	Palladium	16	RBOB gasoline	47	Gold	35
RBOB gasoline	28	Silver	13	Rough rice	44	Unleaded gas	30
Platinum	24	Crude oil	9	Soybeans	37	Live catttle	29
Rough rice	20	Heating oil	9	Sugar	32	Feeder cattle	27
Barley	6	RBOB gasoline	6	Silver	32	Propane	26
Silver	1	Propane	5	Soybean meal	25	Barley	15
Gold	0	Gold	0	Wheat	5	RBOB gasoline	9

Table Appendix-I

Descriptive statistics of commodity futures excess returns, frequency of contango, and open interest

The monthly excess returns are computed using equation (2), which takes into account the first notice day conventions and also incorporates the interest earned on a fully collateralized futures position. Displayed are the number of observations (N), the annualized mean, standard deviation (SD) and Sharpe ratio (SR), monthly skewness, and the first order autocorrelation (ρ_1). Also reported are (i) the fraction of the months in which a commodity is in contango, denoted by $1_{\ln(y_t)>0}$, where $y_t \equiv F_t^{(1)}/F_t^{(0)}$, $F_t^{(0)}$ is the price of the front-month futures contract and $F_t^{(1)}$ is the price of the next maturity futures contract; (ii) the mean of $\ln(y_t)$; and (iii) the end of the month open interest, as measured by the number of contracts. Our sample starts in January 1970 and ends in September 2011, and the futures data is constructed using end-of-day data provided by the CME.

Commodity futures	N	Mean	SD	SR	Skewness	ρ_1	$1_{\ln(y_t)>0}$	$\ln(y_t)$ (Mean)	Open interest (number)
Barley	107	-9.83	11.72	-0.84	0.34	-0.01	0.90	0.015	413
Cocoa	501	5.51	32.21	0.17	0.72	0.00	0.72	0.003	19,587
Coffee	457	7.81	37.46	0.21	1.18	-0.02	0.68	0.004	21,076
Corn	501	-0.76	25.91	-0.03	1.12	0.01	0.83	0.018	146,178
Cotton	501	4.67	25.81	0.18	0.58	0.10	0.69	0.005	21,925
Lumber	466	-6.42	27.87	-0.23	0.09	0.06	0.66	0.020	2,792
Oats	500	0.51	31.88	0.02	2.31	-0.04	0.72	0.018	4,967
Orange juice	501	3.55	31.53	0.11	1.83	-0.04	0.65	0.007	7,886
Rough rice	301	-2.41	28.70	-0.08	1.28	0.12	0.87	0.021	3,740
Soybeans	501	4.93	28.55	0.17	1.34	0.03	0.77	0.006	60,111
Soybean meal	501	8.22	33.12	0.25	2.18	0.06	0.62	0.001	23,806
Soybean oil	501	8.63	32.74	0.26	1.40	-0.04	0.75	0.001	30,674
Sugar	501	9.00	42.02	0.21	1.17	0.17	0.62	0.013	58,129
Wheat	501	0.42	27.10	0.02	0.73	0.07	0.76	0.014	52,989
Crude oil	342	11.62	33.47	0.35	0.42	0.19	0.46	-0.002	140,095
Heating oil	390	16.26	35.88	0.45	1.14	0.05	0.65	-0.005	5,452
Natural gas	257	-4.68	51.02	-0.09	0.60	0.10	0.77	0.018	69,613
Propane	230	29.31	64.88	0.45	7.01	-0.07	0.65	-0.008	142
RBOB gasoline	71	18.58	40.77	0.46	-0.59	0.22	0.46	-0.001	6,649
Unleaded gasoline	264	25.84	40.70	0.63	1.03	0.02	0.42	-0.012	2,044
Feeder cattle	452	2.68	16.28	0.16	-0.53	-0.01	0.45	0.000	4,102
Lean hogs	501	5.41	26.04	0.21	0.04	-0.03	0.53	0.013	17,334
Live cattle	501	5.34	17.51	0.31	-0.26	-0.01	0.49	-0.002	32,505
Pork belly	499	2.01	36.77	0.05	0.55	-0.07	0.45	0.003	3,309
Copper	501	7.91	27.94	0.28	0.33	0.17	0.69	0.000	11,504
Gold	441	2.11	19.57	0.11	0.49	0.02	1.00	0.008	44,645
Palladium	416	11.47	35.62	0.32	0.41	0.04	0.74	0.005	4,403
Platinum	501	6.46	27.84	0.23	0.47	0.00	0.76	0.008	7,703
Silver	501	6.41	34.35	0.19	1.47	0.09	0.98	0.009	19,380

Table Appendix-II

Excess returns of the commodity factors, the commodity indexes, and the four commodity categories

Panel A first reports the descriptive statistics for each of the factors. The average factor, denoted by AVG, is the excess return of a long position in all available commodity futures. The carry factor, denoted by CARRY, is the return on strategy C5, while the momentum factor, denoted by CMOM, is the return on strategy M5. Next, Panel B corresponds to the excess returns of the Goldman Sachs Commodity Index (GSCI, source: ticker GSCIEXR in Datasteam) and the Commodity Research Bureau index (CRB), while Panel C corresponds to the equally-weighted commodity returns across four categories. Panel D reports the summary statistics for two additional factors (details of the construction are in Appendix C). Our procedure for constructing the value factor, denoted by VALUE, is similar to that in Asness, Moskowitz, and Pedersen (2013), where in each month, we rank all the commodities by the ratio of the second nearest maturity futures price five years ago to its current price. We divide the commodities into five groups and compute the next month portfolio returns. The value factor is the return difference between the top and bottom quintiles. The ΔVOL corresponds to the innovation in commodity volatility and is computed following Menkhoff, Sarno, Schmeling, and Schrimpf (2012a, equation (4)). For AVG_t, CARRY_t and CMOM_t we investigate seasonality of the form: $f_t = \upsilon_0 + \sum_{j=2}^{12} \upsilon_j 1_{j,t} + \varepsilon_t$, where the 1_j's are dummy variables for the months February through December. We do not find evidence of seasonality in the factors.

	Mean	PW bootstrap CI lower	PW bootstrap CI upper	SD	SR	Skewness	ρ_1	$1_{er>0}$
Panel A: Commodity factors								
AVG	6.27	1.32	12.60	14.32	0.44	0.22	0.05	57.37
CARRY	16.34	9.36	23.40	22.26	0.73	0.30	0.09	56.77
CMOM	16.11	9.36	22.92	26.28	0.61	0.34	-0.01	57.37
Panel B: Commodity indexes								
GSCI	5.43	-0.84	11.52	20.04	0.27	0.06	0.16	54.75
CRB	3.53	-0.36	8.04	13.56	0.26	-0.06	0.08	52.53
Panel C: Commodity categories								
Agriculture	3.79	-2.88	11.28	16.78	0.23	0.79	0.00	50.71
Livestock	4.29	0.12	14.64	19.65	0.22	0.13	-0.01	52.73
Metal	6.98	-0.84	8.76	22.96	0.30	0.39	0.11	52.53
Energy	14.20	3.84	24.48	35.18	0.40	1.70	0.08	53.59
Panel D: Additional commodity factors								
VALUE	6.15	-2.16	14.28	28.17	0.22	-0.19	-0.01	52.61
ΔVOL	0.02	-0.12	0.12	0.68	-	0.24	-0.25	-

Table Appendix-III

Excess returns of long and short legs of the commodity carry and momentum strategies

This table presents the descriptive statistics of the excess returns generated by the long and short legs of the commodity carry and momentum strategies. Let $y_t \equiv F_t^{(1)}/F_t^{(0)}$, where $F_t^{(0)}$ is the price of the front-month futures contract and $F_t^{(1)}$ is the price of the next maturity futures contract, both observed at the end of month t. A commodity is in backwardation if $\ln(y_t) < 0$ and in contango if $\ln(y_t) > 0$. The carry strategy entails taking a long (short) futures position in a commodity that is in backwardation (contango) at the end of month t, and we compute the returns over the subsequent month. For example, the long (short) leg of the carry strategy C5 contains an equally-weighted portfolio consisting of five commodities with the most negative (positive) $\ln(y_t)$. For the momentum strategies, the commodities are ranked on the basis of their past six-month performance. Analogously, the long (short) leg of the momentum strategy M5 contains an equally-weighted portfolio consisting of five commodities with the highest (lowest) past returns. For each of the long and short legs of the strategies, we report the average annualized monthly return and its 95% confidence interval based on a stationary bootstrap (denoted by PW, lower CI and PW, upper CI) with 10,000 bootstrap iterations, where the block size is based on the algorithm of Politis and White (2004), the annualized monthly standard deviation (SD), the annualized Sharpe ratio (SR), and the monthly skewness. The percentage of months in which the excess return of a strategy is positive is recorded as $1_{er>0}$. There are 501 monthly observations in our sample from January 1970 to September 2011.

	Panel A: Long leg of carry strategy Commodities long backwardation					*Panel B: Short leg of carry strategy* Commodities short contango				
	C1	C2	C3	C4	C5	C1	C2	C3	C4	C5
Mean	15.34	10.47	11.38	12.93	14.14	-6.03	-0.40	0.76	1.34	2.20
PW, lower CI	2.76	-0.12	2.88	4.92	6.84	-14.16	-6.36	-6.72	-5.64	-4.20
PW, upper CI	30.72	22.44	22.68	21.84	23.16	1.68	6.24	8.04	7.44	8.28
SD	37.02	28.68	24.01	21.57	20.89	35.16	25.33	21.88	19.55	18.08
SR	0.41	0.37	0.47	0.60	0.68	-0.17	-0.02	0.03	0.07	0.12
Skewness	0.98	0.83	0.67	0.48	0.59	-1.56	-1.23	-1.44	-0.80	-0.48
$1_{er>0}$	50.91	52.12	54.95	57.58	55.35	49.29	51.31	50.10	51.31	52.93

	Panel C: Long leg of momentum strategy Commodities long winners based on past six-month performance					*Panel D: Short leg of momentum strategy* Commodities short losers based on past six-month performance				
	M1	M2	M3	M4	M5	M1	M2	M3	M4	M5
Mean	13.00	15.62	11.99	12.39	13.63	-2.25	1.42	1.89	2.31	2.48
PW, lower CI	-0.12	4.80	3.00	3.84	6.00	-11.04	-6.48	-6.00	-4.20	-3.48
PW, upper CI	27.60	28.80	22.44	23.04	22.08	6.48	9.00	9.12	8.64	8.64
SD	44.37	34.13	28.40	25.86	24.12	40.50	30.11	25.97	22.98	20.63
SR	0.29	0.46	0.42	0.48	0.56	-0.06	0.05	0.07	0.10	0.12
Skewness	0.84	0.82	0.45	0.16	0.19	-1.36	-1.39	-0.86	-0.53	-0.34
$1_{er>0}$	54.75	54.55	53.94	56.77	56.97	52.73	51.72	53.94	53.94	53.94

Table Online-I

First notice day conventions adopted by the CME and start and end dates for the commodity futures sample

This table describes the first notice day (denoted by FN) convention for each of the 29 commodities. FN is either specified in the contract specification or can be inferred from the product calendar (see, for instance, Shwayder and James (2011), Gorton, Hayashi, and Rouwenhorst (2013), Hong and Yogo (2012), and Yang (2013)). Our futures data is constructed using end-of-day data provided by the Chicago Mercantile Exchange (CME).

Category	Commodity futures	Start	End	First notice day convention
Agriculture	Barley	1994:07	2003:05	FN is usually the last business day of the month preceding the contract month
	Cocoa	1970:01	2011:09	FN is usually ten business days prior to first business day of contract month
	Coffee	1973:09	2011:09	FN is usually seven business days prior to first business day of contract month
	Corn	1970:01	2011:09	FN is usually the last business day of the month preceding the contract month
	Cotton	1970:01	2011:09	FN is usually around the 15th of the contract month
	Lumber	1972:12	2011:09	FN is usually around the 15th of the contract month
	Oats	1970:02	2011:09	FN is usually the last business day of the month preceding the contract month
	Orange juice	1970:01	2011:09	FN is usually the first business day of contract month
	Rough rice	1986:09	2011:09	FN is usually the last business day of the month preceding the contract month
	Soybeans	1970:01	2011:09	FN is usually the last business day of the month preceding the contract month
	Soybean meal	1970:01	2011:09	FN is usually the last business day of the month preceding the contract month
	Soybean oil	1970:01	2011:09	FN is usually the last business day of the month preceding the contract month
	Sugar	1970:01	2011:09	FN is usually the first business day of the contract month
	Wheat	1970:01	2011:09	FN is usually the last business day of the month preceding the contract month
Energy	Crude oil	1983:04	2011:09	FN is usually around the 25th calendar day of the month preceding the contract month
	Heating oil	1979:04	2011:09	FN is usually within the first two business days of the contract month
	Natural gas	1990:05	2011:09	FN is usually the last business day of the month preceding the contract month
	Propane	1987:09	2006:10	FN is usually within the first two business days of the contract month
	RBOB gasoline	2005:11	2011:09	FN is usually within the two business days of the contract month
	Unleaded gasoline	1985:01	2006:12	FN is usually within the first two business days of the contract month
Livestock	Feeder cattle	1973:10	2011:05	No first notice day, last trade day is usually the last Thursday of the contract month
	Lean hogs	1970:01	2011:09	No first notice day, last trade day is usually around 10th of the contract month
	Live cattle	1970:01	2011:09	FN is usually within the first four to ten days of the contract month
	Pork belly	1970:01	2011:07	FN is usually within the first four to ten days of the contract month
Metal	Copper	1970:01	2011:09	FN is usually the last business day of the month preceding the contract month
	Gold	1975:01	2011:09	FN is usually the last business day of the month preceding the contract month
	Palladium	1977:02	2011:09	FN is usually the last business day of the month preceding the contract month
	Platinum	1970:01	2011:09	FN is usually the last business day of the month preceding the contract month
	Silver	1970:01	2011:09	FN is usually the last business day of the month preceding the contract month

Table Online-II

Open interest and number of commodities across the futures curve

For a given commodity, let $F_t^{(0)}$ be the price of the front-month futures contract, $F_t^{(1)}$ be the price of the next maturity futures contract, and likewise $F_t^{(n)}$ be the price of the final futures contract available for trading, where n can vary across commodities. The futures prices $F_t^{(0)}$ through $F_t^{(n)}$ describe the futures curve for a given commodity. Tabulated below are the number of commodities, the open interest, and the number of observations across different points on the futures curve. For example, we have 29 commodities to construct the carry strategy if we use the first contract, while we have only 17 commodities available to construct the carry strategy if we use the sixth contract.

	Slope of the futures curve is based on:					
	$\ln\left(\frac{F^{(1)}}{F_t^{(0)}}\right)$	$\ln\left(\frac{F^{(2)}}{F_t^{(0)}}\right)$	$\ln\left(\frac{F^{(3)}}{F_t^{(0)}}\right)$	$\ln\left(\frac{F^{(4)}}{F_t^{(0)}}\right)$	$\ln\left(\frac{F^{(5)}}{F_t^{(0)}}\right)$	$\ln\left(\frac{F^{(6)}}{F_t^{(0)}}\right)$
Number of commodities	29	29	29	27	23	17
Open interest (end of month)	27,904	15,529	9,761	7,045	5,631	5,161
Number of observations	12,207	12,071	11,831	10,659	6,958	4,532

Table Online-III

Relation of the commodity factors to future real GDP growth, accounting for oil price growth

We report the coefficient estimates from the following predictive regressions with overlapping observations at the quarterly frequency:

$$\ln\left(\mathrm{GDP}_{t+k}/\mathrm{GDP}_t\right) = \theta_0 + \theta_{\mathrm{AVG}}\,\mathrm{AVG}_t + \theta_{\mathrm{CARRY}}\,\mathrm{CARRY}_t + \theta_{\mathrm{CMOM}}\,\mathrm{CMOM}_t + \mho_{\mathrm{OIL}}\,\mathrm{OIL\_growth}_t + \varepsilon_{t+k},$$

where $k \in \{1,2,3,4\}$, GDP refers to the real GDP of the G7 countries (source: Datastream, ticker G7OCMP03D), and $\mathrm{OIL\_growth}_t$ is the growth rate of West Texas intermediate oil price (Source: Federal Reserve Bank–St. Louis). The p-values based on the Newey and West (1987) covariance estimator, where lags are automatically selected according to Newey and West (1994)), are denoted by NW[p]. Additionally, the p-values based on the Hodrick (1992) 1B covariance estimator under the null of no predictability are denoted by H[p]. Adjusted R^2 is reported as $\overline{R}^2$ (in %). The sample period is from 1970:Q1 to 2011:Q3.

			Predictive slope coefficients				
Horizon		θ_0	θ_{AVG}	θ_{CARRY}	θ_{CMOM}	$\mho_{\mathrm{OIL}}$	$\overline{R}^2$
1 quarter	Estimate	0.01	0.03	-0.01	-0.01	0.00	13.3
	NW[p]	(0.00)	(0.01)	(0.07)	(0.01)	(0.86)	
	H[p]	⟨0.00⟩	⟨0.02⟩	⟨0.12⟩	⟨0.01⟩	⟨0.90⟩	
2 quarters	Estimate	0.01	0.05	-0.01	-0.02	0.00	10.7
	NW[p]	(0.00)	(0.02)	(0.05)	(0.01)	(0.72)	
	H[p]	⟨0.00⟩	⟨0.01⟩	⟨0.01⟩	⟨0.00⟩	⟨0.78⟩	
3 quarters	Estimate	0.02	0.04	-0.03	-0.02	0.00	8.9
	NW[p]	(0.00)	(0.06)	(0.06)	(0.02)	(0.61)	
	H[p]	⟨0.00⟩	⟨0.02⟩	⟨0.00⟩	⟨0.00⟩	⟨0.64⟩	
4 quarters	Estimate	0.03	0.04	-0.03	-0.03	-0.01	7.2
	NW[p]	(0.00)	(0.16)	(0.10)	(0.01)	(0.32)	
	H[p]	⟨0.00⟩	⟨0.08⟩	⟨0.02⟩	⟨0.00⟩	⟨0.35⟩	

Table Online-IV

Correlations between the long and short legs of the carry and momentum strategies

We report the correlation of the commodity factors and the long and short components of carry and momentum strategies. The long (short) leg of the carry trade returns is denoted by $\text{CARRY}_t^{\text{long}}$ ($\text{CARRY}_t^{\text{short}}$). Similarly, the long (short) leg of the momentum returns is denoted by $\text{CMOM}_t^{\text{long}}$ ($\text{CMOM}_t^{\text{short}}$). The calculation of the returns of the long and short legs of the strategies follows the procedure described in Appendix A and implemented in Appendix B.

	AVG_t	CARRY_t	CMOM_t	$\text{CARRY}_t^{\text{long}}$	$\text{CARRY}_t^{\text{short}}$	$\text{CMOM}_t^{\text{long}}$
$\text{CARRY}_t^{\text{long}}$	0.71	0.65	0.21			
$\text{CARRY}_t^{\text{short}}$	-0.71	0.48	0.10	-0.35		
$\text{CMOM}_t^{\text{long}}$	0.74	0.25	0.67	0.62	-0.41	
$\text{CMOM}_t^{\text{short}}$	-0.73	0.06	0.49	-0.46	0.61	-0.32

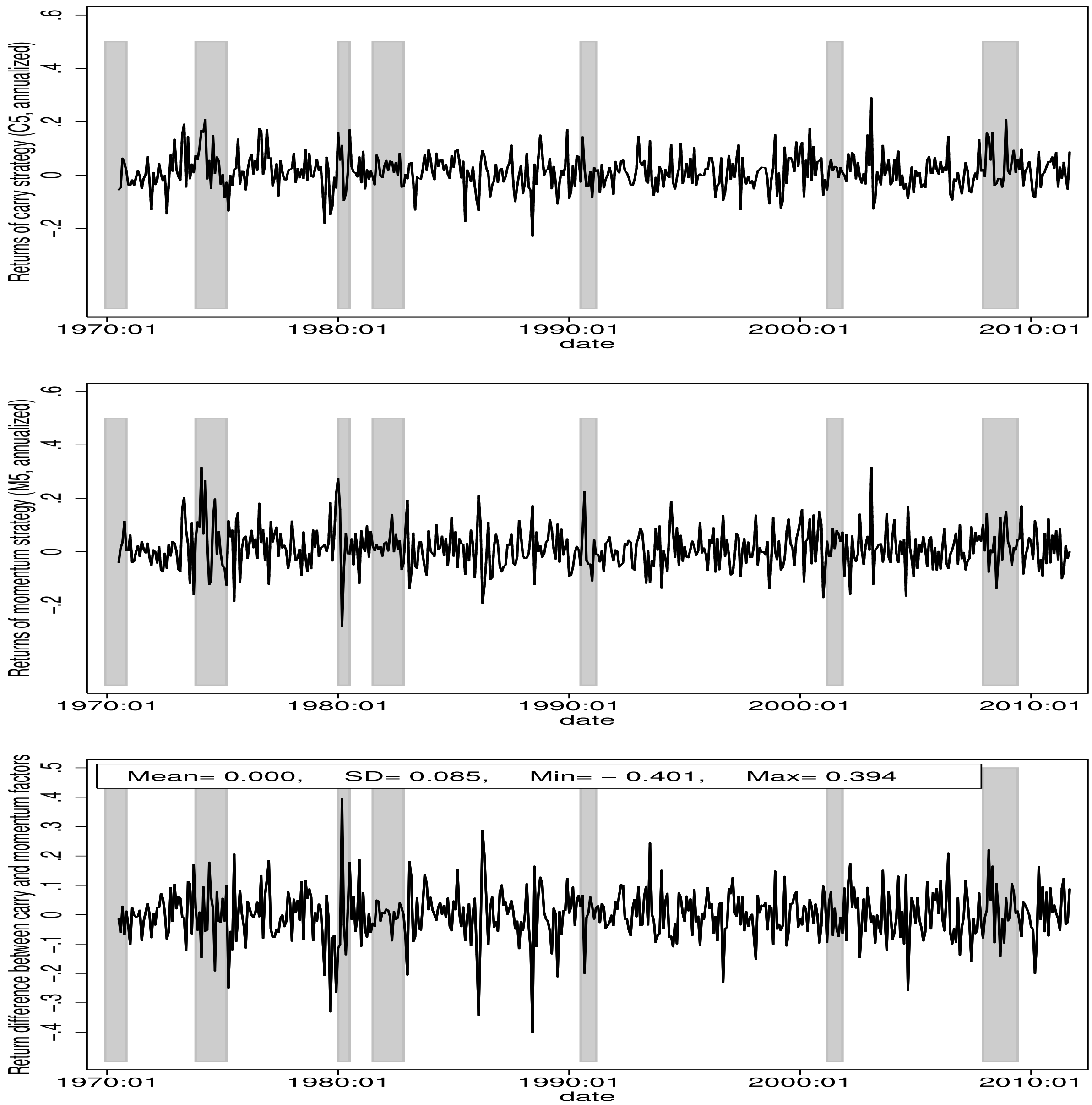

Fig. 1. **Returns of commodity carry and momentum strategies**

Plotted in the top (middle) panel is the time-series of the excess returns generated by the carry factor CARRY_t (the momentum factor CMOM_t). The bottom panel plots the time-series of $\mathrm{er}_t^{cm} \equiv \mathrm{CARRY}_t - \mathrm{CMOM}_t$, for $t = 1, \ldots, T$ and reports the average, standard deviation, minimum and maximum of the er_t^{cm} series. The shaded areas indicate NBER recessions. Let $y_t \equiv F_t^{(1)}/F_t^{(0)}$, whereby, at the end of month t, a commodity is in backwardation if $\ln(y_t) < 0$ and in contango if $\ln(y_t) > 0$. The carry strategy entails taking a long position in the five commodities with the lowest $\ln(y_t)$ and a short position in the five commodities with the highest $\ln(y_t)$. The momentum strategy entails taking a long position in the five commodities with the highest returns over the previous six months and a short position in the five commodities with the lowest returns over the previous six months. Our sample period is January 1970 to September 2011. The monthly returns of the carry and momentum strategies co-move with a correlation of 0.27.

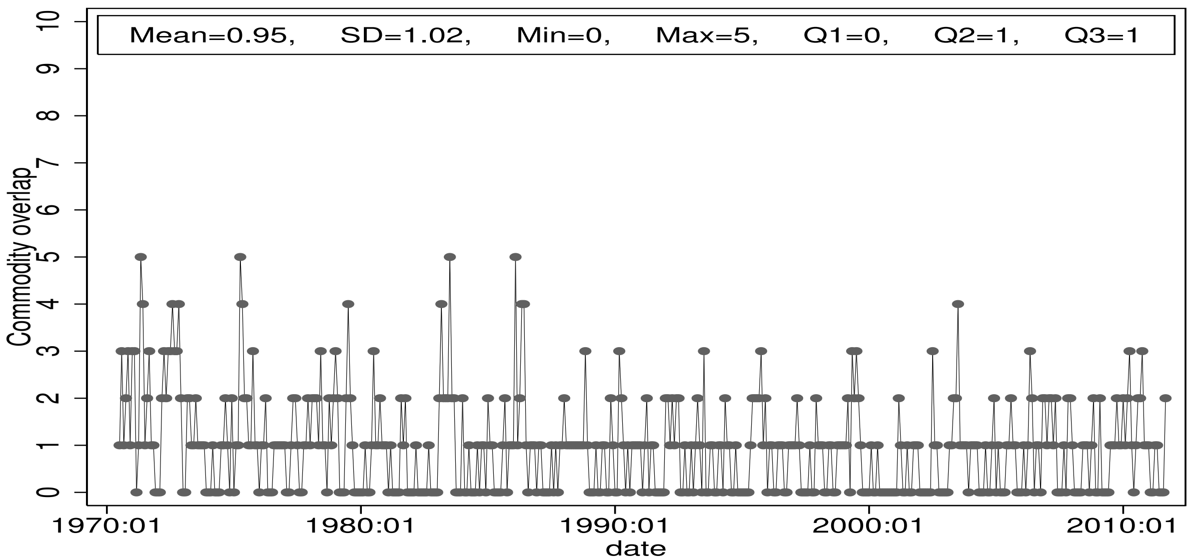

Fig. 2. **Overlap in the commodities selected by the carry and momentum factors**

Plotted is the time-series of the number of commodities selected by both the carry and the momentum factors. We compute this overlap in two steps. First, we identify the commodities in the long legs of the carry and momentum strategies each month and we compute the overlap (the maximum overlap is five). Second, we compute the same quantity for the short legs of the strategies (the maximum overlap is five). The plotted overlap is the sum of the overlaps of the long and short legs of the strategies. We report the average, standard deviation, minimum, maximum as well as the three quartiles (Q1, Q2, Q3) of the overlap distribution. The carry factor entails taking a long position in the five commodities with the lowest $\ln(y_t)$ and a short position in the five commodities with the highest $\ln(y_t)$. The momentum factor entails taking a long position in the five commodities with the highest returns over the previous six months and a short position in the five commodities with the lowest returns over the previous six months. Our sample period is January 1970 to September 2011.

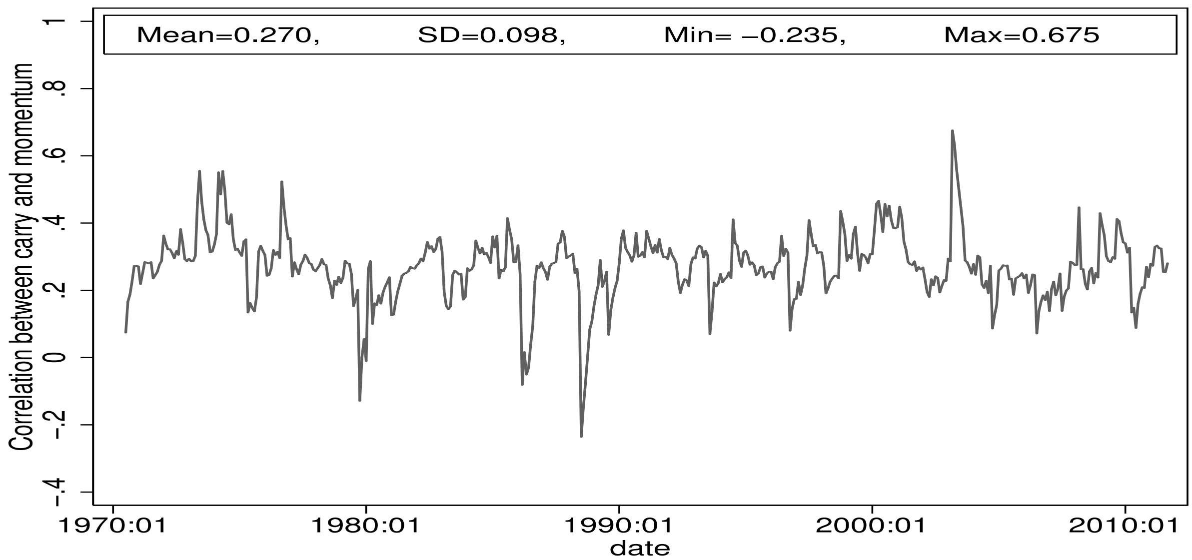

Fig. 3. **Dynamic correlation between the carry and momentum factors**

The plotted correlation between the carry and momentum factors is based on the dynamic conditional correlation model of Engle (2002). The estimated correlation relies on a bivariate GARCH (1,1) model for carry and momentum factors. We report the average, standard deviation, minimum, and maximum of the correlation series. The carry factor entails taking a long position in the five commodities with the lowest $\ln(y_t)$ and a short position in the five commodities with the highest $\ln(y_t)$. The momentum factor entails taking a long position in the five commodities with the highest returns over the previous six months and a short position in the five commodities with the lowest returns over the previous six months. Our sample period is January 1970 to September 2011.

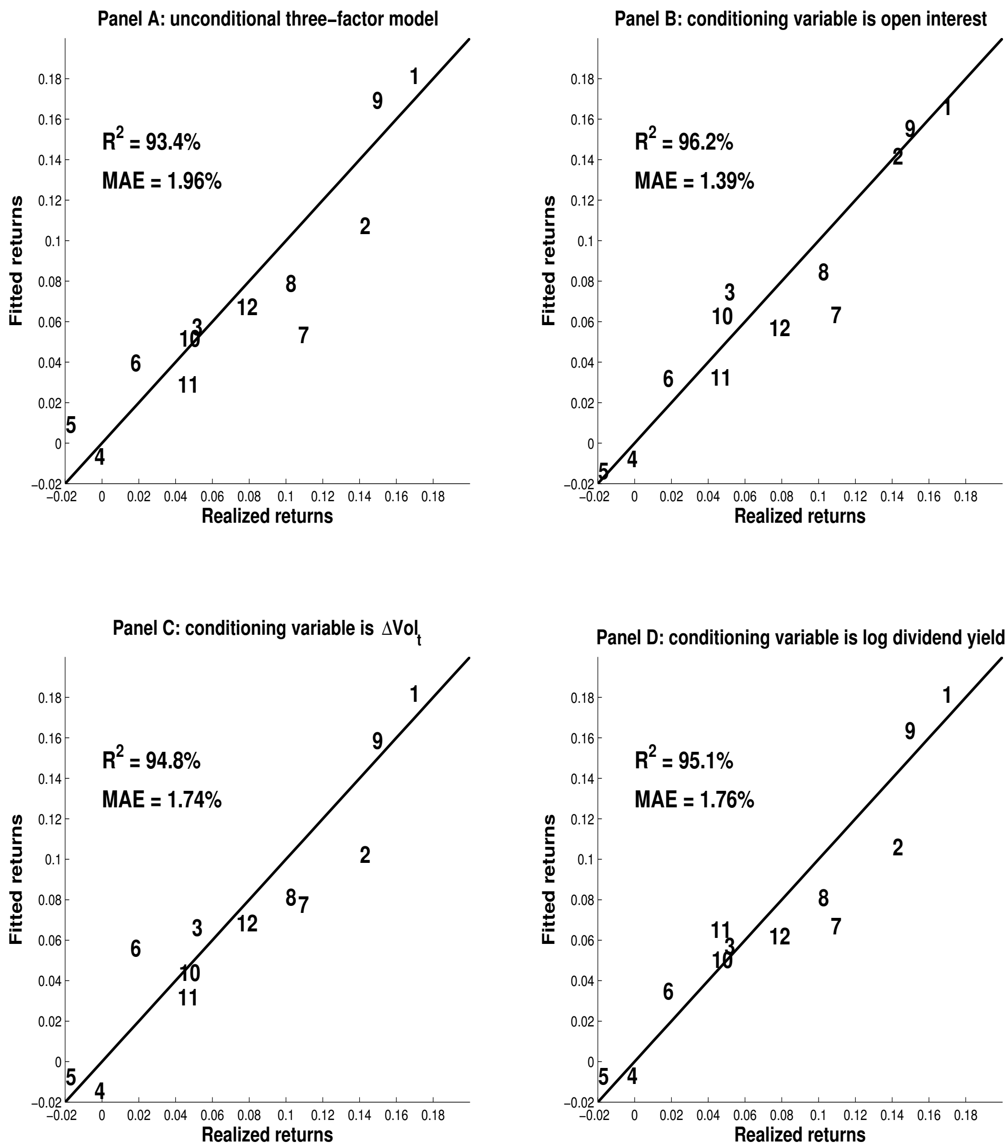

Fig. 4. **Realized versus fitted returns across the commodity portfolios**

Plotted are the realized returns (x-axis) and the fitted returns (y-axis) corresponding to the commodity portfolios indexed from 1 to 12 (see Table 6). We compare the performance of the unconditional three-factor model (equation (6)) against the one associated with three selected conditional pricing models (equation (8)). The conditional models are obtained by incorporating the following conditioning variables: (i) open interest growth, (ii) change in commodity volatility ΔVol_t, and (iii) log dividend yield. The fitted average returns are based on equation (5). We also display the uncentered R^2's and the mean absolute errors (denoted by MAE), as goodness-of-fit yardsticks. The MAE is computed as $(1/12)\sum_{i=1}^{12}|\text{Fitted}_i - \text{Realized}_i|$, in monthly percentage units.

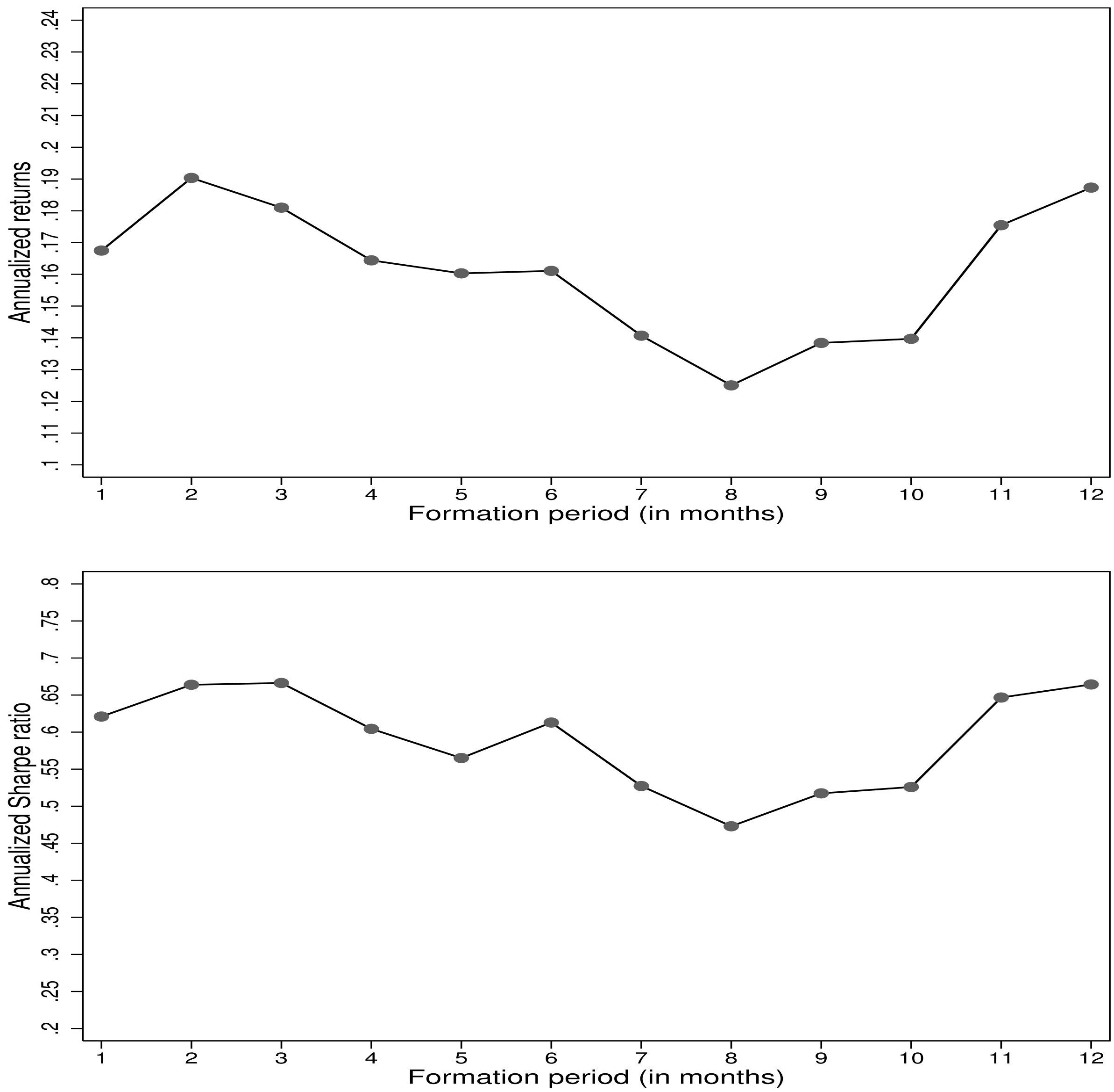

Fig. 5. **Average returns and Sharpe ratios of momentum strategies across formation periods**

Plotted are the annualized average monthly returns (top panel) and Sharpe ratios (bottom panel) of momentum strategies with formation periods *J* that range from 1 to 12 months. Specifically, we report the results for strategy M5 (see also the caption to Table 1), which entails taking a long position in the five commodities with the highest returns over the previous *J* months and a short position in the five commodities with the lowest returns over the previous *J* months. We measure past returns using the geometric average (equation (A3)). Our sample period is January 1970 to September 2011.