Commodity Prices, Long-Run Growth and Fiscal Vulnerability

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Abstract

Long-lasting commodity price declines are often associated with abrupt tax revenue shortfalls in commodity-exporting countries. Therefore, reliance on the tax base of the commodity-exporting sector makes the country's fiscal stance vulnerable to exogenous variations in commodity prices—*fiscal vulnerability*. In this paper, we study the short- and long-run effects of commodity price changes and how fiscal policy interacts with the amplification and propagation of external shocks to these prices. To this aim, we develop a Schumpeterian small open economy (SOE) model of endogenous growth that does not exhibit the scale effect. Because of the sterilization of the scale effect, commodity prices have level effects on economic activity but no steady-state growth effects. A general implication of our analysis is that the economy dynamic response to commodity price changes depends both on the structure of the tax code in place and on the policy response necessary to balance the government budget. We show that asset income taxation has negative steady-state growth effects. Furthermore, a positive tax rate on asset income acts as an automatic amplifier of external shocks to commodity prices and makes the effects of these shocks more persistent.

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1 Introduction

Oil prices and more generally commodity prices exhibit long-lasting declines and prolonged periods of rises. Figure 1 depicts the annual series of the real oil price (solid line) from 1861 to 2011 with a smooth trend (dotted line) estimated by fitting a second-order polynomial to the series.[1] The figure shows a striking U-shape pattern in the trend, which captures a period of roughly 100 years of declining prices replaced by 40 years of price increases. Notice also the 20 years pattern of steadily price declines from 1980 to 2000, which reverts roughly 10 years of price increases from 1970 to 1980. Similar patterns emerge in several other commodities, e.g., agricultural raw materials, metals, food and beverage. This figure is suggestive of long waves of price declines and raises. Overall, the evidence suggests that commodity price movements are large both at high and low frequency.[2]

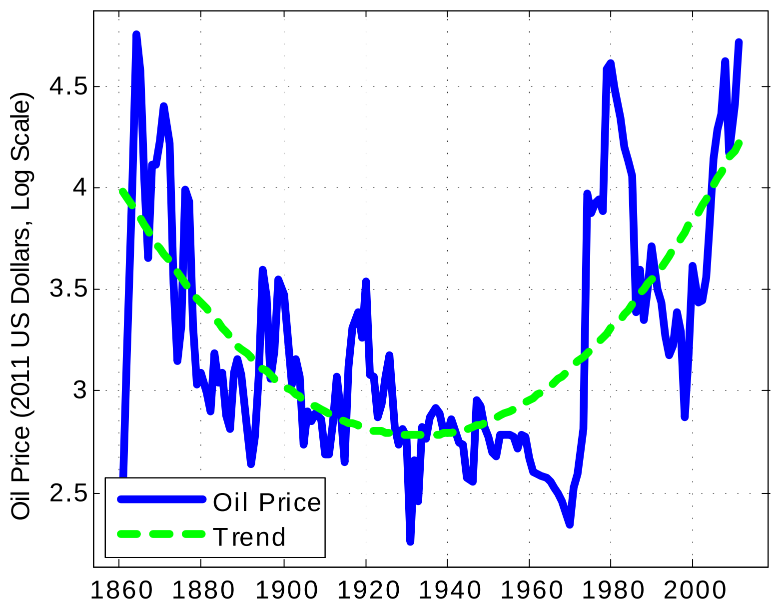

Figure 1: Real oil price and trend, 1861-2011

It is also widely recognized that many real world economies, both developing and developed, depend for their exporting sector income on a narrow range of commodities. For these countries, long periods of commodity price declines are often associated with large tax revenue shortfalls and deteriorating public finances. Only few examples are the fiscal

[1] Data source: British Petroleum Statistical Review of World Energy Report 2012.

[2] See Jacks (2013) for an extensive treatment of long-run trends, medium-run cycles, and short-run boom/bust episodes in commodity prices.

crisis occurred in Nigeria in 1991 and Kenya in the early 1980's when the prices of oil and coffee reverted to their previous levels prior to respectively the oil price boom at the time of the Iraqi invasion of Kuwait, and the 1976-1979 coffee price boom. Overall, the existing evidence suggests that extensive reliance on the tax base of the commodity-exporting sector makes the country's fiscal stance particularly vulnerable to variations in world commodity prices.[3] We think of this phenomenon as *fiscal vulnerability*. The issue of fiscal dependence on commodity-linked revenues in commodity-rich countries has long been and still is a relevant matter for policy making. Quoting the 2013 Nigeria Economic Report by the World Bank, *"As oil revenues comprise 75 percent of budgetary revenues and 95 percent of exports in Nigeria, the effective management of the country's oil wealth is critical to stability and fiscal sustainability in the country."* Hence, the current debate points to the *interaction* of fiscal policy with conditions in the commodity market as a mechanism particularly relevant for the economics of commodity-rich countries. Motivated by the empirical observations above and the renewed interest in the policy debate, this paper tackles, from a theoretical prospective, the following questions: absent fiscal considerations, how do commodity-exporting economies respond to commodity price changes? How does the economy dynamic response depend on the tax code in place? And how should governments react to world commodity price changes in the wake of tax revenue shortfalls? To answer these questions, we develop a Schumpeterian small open economy (SOE) model of endogenous growth. We study both the short- and long-run effects of commodity price changes and how fiscal policy interacts with the amplification and propagation of external shocks to these prices. In the spirit of the SOE tradition, we assume that commodity prices are taken parametrically by agents inside our model and determined in the world commodity market.

The paper studies the *joint* role of commodity prices and distortionary taxation in an environment where technological change is endogenous. We think endogenous growth theory is the natural framework to study these issues for two main reasons: first, there is now a large literature on the "curse of natural resources" which hints at very long-run effects of natural resources. However, as exemplified by the title of the *Journal of Economic Literature* survey paper by Van der Ploeg (2011), "Natural Resources: Curse or Blessing?", the outcome of this research effort is far from conclusive, with mixed empirical evidence.[4] In this latter regard, a model where steady-state growth is the endogenous equilibrium outcome

[3]See Sinnott (2009) for a detailed discussion of fiscal dependence on hydrocarbon revenues in Latin America and the Caribbean.

[4]On a similar note see Gelb (1988).

of activities undertaken by economic agents, allows us to take seriosuly the notion of the curse of natural resources and provide theoretical predictions about conditions in the commodity market (and/or natural resources) and economic growth. Second, since historically governments have reacted differently in the wake of commodity prices booms and busts, we are interested in studying the effects of different tax policies, with a special focus on the differences between short- and long-run effects. Ultimately, our analysis provides insights on how to design welfare-enhancing tax policies for commodity-exporting countries.

Specifically, our theoretical framework is a Schumpeterian model of endogenous growth featuring both horizontal (expanding variety) and vertical (quality upgrading and/or cost reducing) innovation. Market structure is endogenous in that both firm *size* and the *mass* of firms are jointly determined in equilibrium. In fact, it is the interaction of the entry and quality margin of innovation—*a variety-quality frontier*—that drives the equilibrium dynamics of the model. A key property of our theoretical structure is that the model economy features different growth regimes, with transitions from one regime to the other being endogenous. We believe this is an appealing feature of the model since it allows us to derive predictions for economies that are at different stages of economic development. We provide results for both the transitional dynamics and the steady state of the model economy. A distinctive feature of the model is that long-run growth is independent of the scale of economic activity, i.e., there is no scale effect.[5] This feature is essential for the purpose of the paper for at least two reasons. First, commodity price changes interact with the scale of the economy by the induced income effect. This implies that sterilization of the scale effect is needed to have a balanced growth path consistent with varying commodity prices. Second, analyses of fiscal policy under scale effects are subject to the Stokey and Rebelo (1995)'s critique. Models of endogenous growth exhibiting scale effects predict effects of fiscal policy that are too large compared to the available empirical evidence. Few examples of this evidence are Easterly and Rebelo (1993) and Mendoza et al. (1997) for cross-country growth regressions, Easterly et al. (1993) and Jones (1995) for time series evidence.

The results of the paper can be summarized as follows. 1) Commodity prices affect the short-run equilibrium dynamics of the model economy. Spending on manufacturing goods is increasing in the commodity price if the domestic demand for the commodity is inelastic. It is instead decreasing if the demand is elastic. The persistence of these short-run effects depends on the parameters of the model that determine the speed of reversion to the steady

[5] See Peretto (1998) for a detailed analysis of the mechanism driving the sterilization of the scale effect in this class of models.

state. 2) Commodity price changes have no long-run growth effects, i.e., the steady-state growth rate of the model economy is fully insulated from the conditions in the commodity market, that is, from commodity prices and endowments. This happens because endogenous entry makes the steady-state growth rate independent of the size of the manufacturing sector. It is exactly in this latter regard that the co-existence of the *entry* and *quality* margin of innovation plays a crucial role: the process of entry induces product proliferation which fragments the aggregate market into sub-markets whose size does not increase with the size of the manufacturing sector. The sterilization of this *market size effect* results in the sterilization of the *scale effect*, that is, the sterilization of the steady-state growth effects of parameters that drive the size of the economy. Finally, 3) the economy dynamic response to commodity price changes depends on both the structure of the tax code in place and on the policy response necessary to balance the government budget. Specifically, a distortionary tax on asset income (dividends plus capital gains) *amplifies* the effects of a given commodity price change and *slows down* the reversion to the steady state after a commodity price shock. Furthermore, if the government raises the tax rate on asset income in response to a resource revenue shortfall, then the commodity price has an indirect adverse effect on the steady-state growth rate of the model economy. This happens because tax rates affecting the equilibrium rate of return to cost reduction and entry have steady-state growth effects.[6] Notice that, in this latter case, the negative steady-state growth effects are to be exclusively imputed to the (misguided) reaction of the government to a commodity price decrease, which, by changing the tax rate on asset income, it is distorting the effective return to innovation.

The paper is organized as follows. In Section 2, we discuss the setup of the model. In Section 3, we discuss the equilibrium of the market economy. Section 4 discusses the effects of commodity price changes. In Section 5, we introduce distortionay taxation into the model. Section 6 contains a simple numerical exercise. Section 7 concludes.

2 The Model

2.1 Overview

We consider a small open economy (SOE) populated by a representative household that supplies labor services inelastically in a competitive labor market. The household faces a

[6] See Peretto (2003) for a previous discussion of level and steady-state growth effects of taxation in models of endogenous growth without scale effects.

standard expenditure-saving decision such that it optimally chooses the path of expenditures (home and foreign goods) and savings by freely borrowing and lending in a competitive market for financial assets at the prevailing interest rate.[7] The household's income consist of returns on asset holdings, labor income, profits and resource income. Resource income is the (constant) commodity endowment valued at the world commodity price.

The production side of the economy consists of three sectors: 1) consumption goods; 2) intermediate goods or manufacturing; and 3) materials. The consumption goods sector consists of a representative competitive firm which combines differentiated intermediate goods to produce an homogeneous final good. Upon entry, manufacturing firms combine labor services and materials to produce differentiated intermediate goods. They also engage in activities aimed to reduce their costs of production, and consequently, improve efficiency. Entry requires the payment of a sunk cost. Finally, materials are supplied by a separate competitive sector which demands as inputs labor services and the commodity paying the world commodity price. At this stage, there is no government sector, which we introduce into the model later in Section 5.

The intermediate goods is the key sector of our model economy in that it is the engine of endogenous growth. Precisely, the economy starts out with a given range of intermediate goods, each supplied by one firm. Entrepreneurs compare the present value of profits from introducing a new good to the entry cost. They only target new product lines because entering an existing product line in Bertrand competition with the existing supplier leads to losses. Once in the market, firms devote labor to cost-reducing (or, equivalently, productivity enhancing) projects. As each firm strives to figure out how to improve efficiency, it contributes to the pool of public knowledge that benefits the future cost reduction activity of all firms. This allows the economy to grow at a constant rate in steady state, which is reached when entry stops and the economy settles into a stable industrial structure.

2.2 Households

The representative household maximizes lifetime utility

$$U(t) = \int_t^\infty e^{-\rho(s-t)} \log u\,(s)\,ds, \quad \rho > 0 \tag{1}$$

[7] It is possible to think of our model economy as taking the world interest rate parametrically. Since the model has the property that the domestic interest rate jumps to its steady-state level, given by the domestic discount rate, as long the SOE has the same discount rate as the rest of the world, the equilibrium discussed in the paper displays the same properties as an equilibrium with free financial flows.

where

$$\log u = \varphi \log \left(\frac{Y_H}{P_H L} \right) + (1-\varphi) \log \left(\frac{Y_F}{P_F L} \right), \quad 0 < \varphi < 1 \tag{2}$$

subject to the flow budget constraint

$$\dot{A} = rA + WL + \Pi_H + \Pi_M + p\Omega - Y_H - Y_F, \quad \Omega > 0 \tag{3}$$

where ρ is the discount rate, φ controls the degree of home bias in preferences, A is assets holding, r is the rate of return on financial assets, W is the wage, L is population size, which equals labor supply since there is no preference for leisure, Y_H is expenditure on a home consumption good whose price is P_H, and Y_F is expenditure on a foreign consumption good whose price is P_F. In addition to asset and labor income, the household receives the dividends paid out by the producers of the home consumption good, Π_H, the dividends paid out by firms in the material sector, Π_M, and the revenues from sales of the domestic endowment of the commodity, Ω, at the world commodity price p. The solution to this problem consists of the optimal consumption-expenditure allocation rule

$$\varphi Y_F = (1-\varphi) Y_H, \tag{4}$$

and the Euler equation governing saving behavior

$$r = r_A \equiv \rho + \frac{\dot{Y}_H}{Y_H} = \rho + \frac{\dot{Y}_F}{Y_F}. \tag{5}$$

2.3 Trade Structure

The foreign good is imported at the constant world price P_F. The economy can be either an importer or an exporter of the commodity. In the first case, it sells the home consumption good to buy the commodity in the world market; in the second case, it accepts the foreign consumption good as payment for its commodity exports. Only final goods and the commodity are tradable. The balanced trade condition, which is also the market clearing condition for the consumption good market, is $Y_H + Y_F + p(O - \Omega) = Y$, where Y is the aggregate value of production of the home consumption good. Using the consumption expenditure allocation rule (4), we can rewrite the balance trade condition as,

$$\frac{1}{\varphi} Y_H + p(O - \Omega) = Y, \tag{6}$$

where O is home use of the commodity.

2.4 The Consumption Good Sector

The home homogeneoeus consumption good is produced by a representative competitive firm with the technology

$$C_H = N^{\chi} \left[\int_0^N \frac{1}{N} X_i^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \quad \chi > 0, \ \epsilon > 1 \tag{7}$$

where ϵ is the elasticity of product substitution, X_i is the quantity of the non-durable intermediate good i, and N is the mass of goods. We follow Ethier (1982) and separate the elasticity of substitution between intermediate goods from the degree of increasing returns to the variety of intermediate goods, χ. The final good producer maximizes,

$$\Pi_H = P_H C_H - \int_0^N P_i X_i di$$

subject to (7). This structure yields the demand curve for each intermediate good as

$$X_i = Y \frac{P_i^{-\epsilon}}{\int_0^N P_i^{1-\epsilon} di}, \tag{8}$$

where $Y = P_H C_H$. Because this sector is perfectly competitive, $\Pi_H = 0$.

2.5 The Intermediate Goods Sector

The typical firm produces one differentiated good with the technology

$$X_i = Z_i^{\theta} \cdot F\left(L_{X_i} - \phi, M_i\right), \quad 0 < \theta < 1, \ \phi > 0 \tag{9}$$

where X_i is output, L_{X_i} is production employment, ϕ is a fixed labor cost, M_i is use of materials, and Z_i^{θ} is the firm's total factor productivity (TFP), a function of the stock of firm-specific knowledge Z_i. The function $F(\cdot)$ is a standard production function homogeneous of degree one in its arguments. The associated total cost is,

$$W\phi + C_X(W, P_M) Z_i^{-\theta} \cdot X_i, \tag{10}$$

where $C_X(\cdot)$ is a standard unit-cost function homogeneous of degree one in its arguments. Hicks-neutral technological change internal to the firm shifts this function down. The elasticity of unit cost reduction with respect to firm-specific knowledge is the constant θ.

The firm accumulates knowledge according to the technology

$$\dot{Z}_i = \alpha K L_{Z_i}, \quad \alpha > 0 \tag{11}$$

where $\dot{Z}_i$ is the flow of firm-specific knowledge generated by a project employing L_{Z_i} units of labor for an interval of time dt, and αK is the productivity of labor in such a project as determined by the exogenous parameter α and by the stock of public knowledge, K. Public knowledge accumulates as a result of spillovers.

When a firm generates a new idea to improve the production process, it also generates general-purpose knowledge which is not excludable and that other firms can exploit in their own research efforts. Firms appropriate the economic returns from firm-specific knowledge but cannot prevent others from using the general-purpose knowledge that spills over into the public domain. Formally, a project that produces $\dot{Z}_i$ units of proprietary knowledge also generates $\dot{Z}_i$ units of public knowledge. The productivity of research is determined by some combination of all the different sources of knowledge. A simple way of capturing this notion is to write

$$K = \int_0^N \frac{1}{N} Z_i di,$$

which says that the knowledge frontier is determined by the average knowledge of all firms.[8]

2.6 Materials

A representative competitive firm combines labor services, L_M, and commodities, O, to produce materials M, used as inputs in the manufacturing sector. The technology is $M = G(L_M, O)$, where $G(\cdot)$ is a standard production function homogeneous of degree one in its arguments. The associated total cost is

$$C_M(W, p) M, \tag{12}$$

where $C_M(\cdot)$ is a standard unit-cost function homogeneous of degree one in the wage W and the commodity price p. This is the simplest way to model the materials sector for the

[8] For a detailed discussion of a spillovers function of this class, see Peretto and Smulders (2002).

purposes of this paper. Materials are produced with labor and the commodity purchased or sold at a given price in the world commodity market. The sector for materials competes for labor with the manufacturing sector. This captures the fundamental inter-sectoral allocation problem faced by this economy.

3 Agents' Behavior and Equilibrium Dynamics

This section constructs the equilibrium of the manufacturing sector. It then characterizes the equilibrium of the sector producing materials. Finally, it imposes general equilibrium conditions to determine the aggregate dynamics of the model economy.

3.1 The Manufacturing Sector

The typical intermediate firm maximizes the present discounted value of net cash flows,

$$V_i\left(t\right) = \int_t^{\infty} e^{-\int_t^s [r(v)+\delta]dv}\Pi_i(s)ds, \quad \delta > 0$$

where δ is a death shock. Using the cost function (10), instantaneous profits are

$$\Pi_i = [P_i - C_X(W, P_M)Z_i^{-\theta}]X_i - W\phi - WL_{Z_i},$$

where L_{Z_i} is labor devoted to cost-reducing projects. V_i is the value of the firm, the price of the ownership share of an equity holder. The firm maximizes V_i subject to the cost-reduction technology (11), the demand schedule (8), $Z_i(t) > 0$ (the initial knowledge stock is given), $Z_j(t')$ for $t' \geq t$ and $j \neq i$ (the firm takes as given the rivals' knowledge accumulation paths), and $Z_j(t') \geq 0$ for $t' \geq t$ (knowledge accumulation is irreversible). The solution of this problem yields the (maximized) value of the firm given the time path of the number of firms.

To characterize entry, we follow Peretto and Connolly (2007) and assume that upon payment of a sunk cost $(\beta Y/N) \cdot W$, an entrepreneur can create a new firm that starts out its activity with productivity equal to the industry average.[9] Once in the market, the new firm solves a problem identical to the one outlined above for the incumbent firm. A free entry equilibrium, therefore, requires $V_i = W \cdot (\beta Y/N)$.

The Appendix shows that the equilibrium thus defined is symmetric and is characterized

[9]See Peretto and Connolly (2007) for an interpretation of this assumption.

by the factor demands:

$$WL_X = Y\frac{\epsilon - 1}{\epsilon}S_X^L + W\phi N; \tag{13}$$

$$P_M M = Y\frac{\epsilon - 1}{\epsilon}S_X^M, \tag{14}$$

where the shares of the firm's variable costs due to labor and materials are respectively:

$$S_X^L \equiv \frac{WL_{X_i}}{C_X(W, P_M)Z_i^{-\theta}X_i} = \frac{\partial \log C_X(W, P_M)}{\partial \log W};$$

$$S_X^M \equiv \frac{P_M M_i}{C_X(W, P_M)Z_i^{-\theta}X_i} = \frac{\partial \log C_X(W, P_M)}{\partial \log P_M}.$$

Note that $S_X^L + S_X^M = 1$. Associated to these factor demands are the rates of return to cost reduction and entry, respectively:

$$r = r_Z \equiv \frac{\alpha}{W}\left[\frac{Y}{\epsilon N}\theta(\epsilon - 1) - W\frac{L_Z}{N}\right] + \frac{\dot{W}}{W} - \delta; \tag{15}$$

$$r = r_N \equiv \frac{N}{W\beta Y}\left[\frac{Y}{\epsilon N} - W\phi - W\frac{L_Z}{N}\right] + \frac{\dot{W}}{W} - \delta + \frac{\dot{Y}}{Y} - \frac{\dot{N}}{N}. \tag{16}$$

Neither the return to cost reduction in (15) nor the return to entry in (16) depend on factors related to the commodity market. Why is this the case? The production technology (9) yields a unit-cost function that depends only on input prices and is independent of the quantity produced and thus of inputs use. Since the optimal pricing rule features a constant markup over unit cost, the firm's gross-profit flow (revenues minus variable costs), $Y/\epsilon N$, is independent of input prices. Equations (15) and (16), then, capture the idea that investment decisions by incumbents and entrants do not respond directly to conditions in the commodity market because they are guided by the gross-profit flow. Conditions in the commodity market have an indirect effect through aggregate spending on intermediate goods, Y.

3.2 Materials

Given the cost function (12), competitive materials producers that purchase commodities at the given world price p operate along the infinitely elastic supply curve

$$P_M = C_M\left(W, p\right). \tag{17}$$

In equilibrium, then, materials production is given by (14) evaluated at the price P_M. Defining the commodity share in material costs as

$$S_M^O \equiv \frac{pO}{C_M\left(W,p\right)M} = \frac{\partial \log C_M(W,p)}{\partial \log p},$$

we can write the associated demands for labor and commodity as:

$$WL_M = M\frac{\partial C_M(W,p)}{\partial W} = Y\frac{\epsilon-1}{\epsilon}S_X^M\left(1-S_M^O\right); \tag{18}$$

and

$$pO = M\frac{\partial C_M(W,p)}{\partial p} = Y\frac{\epsilon-1}{\epsilon}S_X^M S_M^O. \tag{19}$$

3.3 General Equilibrium

The model consists of the returns to saving (5), to cost reduction (15), and to entry (16), the labor demands in the manufacturing sector (13), materials (18), and the household's budget constraint (3).[10] Assets market equilibrium requires equalization of all rates of return, $r = r_A = r_Z = r_N$, and that the value of the household's portfolio equal the value of the securities issued by firms, $A = NV = \beta WY$. We choose labor as the numeraire, i.e., $W \equiv 1$. A convenient implication of this normalization is that all expenditure terms are constant.

Proposition 1. *At any point in time, the value of home manufacturing production and the balanced trade condition, respectively, are:*

$$Y\left(p\right) = \frac{L}{1-\xi\left(p\right)-\rho\beta}, \quad \textit{with} \quad \xi\left(p\right) \equiv \frac{\epsilon-1}{\epsilon}S_X^M\left(p\right)S_M^O\left(p\right); \tag{20}$$

$$\frac{1}{\varphi}Y_H\left(p\right) - p\Omega = Y\left(p\right)\left(1-\xi\left(p\right)\right). \tag{21}$$

The associated expenditures on the home and foreign consumption goods, respectively, are:

$$Y_H\left(p\right) = \varphi\left[\frac{L\left(1-\xi\left(p\right)\right)}{1-\xi\left(p\right)-\rho\beta} + p\Omega\right]; \tag{22}$$

[10] The household's budget constraint (3) and the balance trade condition (6) imply the labor market clearing condition $L = L_N + L_X + L_Z + L_M$, where L_N is aggregate employment in entrepreneurial activity, $L_X + L_Z$ is aggregate employment in production and cost-reducing operations of existing firms, L_M is aggregate employment in materials producing firms. See the Appendix at the end of the paper for the derivations.

$$Y_F(p) = (1-\varphi)\left[\frac{L(1-\xi(p))}{1-\xi(p)-\rho\beta} + p\Omega\right]. \tag{23}$$

Because $Y_H(p)$ and $Y_F(p)$ are constant, the interest rate is $r = \rho$ at all times.

Proof. See the Appendix. □

Given this structure of expenditures, the equilibrium dynamics are as follows.

Proposition 2. *Let $x \equiv Y/\epsilon N$ denote the gross profit rate. The general equilibrium of the model reduces to the following piece-wise linear differential equation in the gross profit flow:*

$$\dot{x} = \begin{cases} \frac{\delta L\epsilon/N_0}{(1-\xi(p))-\frac{1}{\epsilon}} & \text{if} \quad \phi \le x \le x_N \\ \frac{\phi}{\beta\epsilon} - \left[\frac{1}{\beta\epsilon} - (\rho+\delta)\right]x & \text{if} \quad x_N < x \le x_Z \\ \frac{\phi - \frac{\rho+\delta}{\alpha}}{\beta\epsilon} - \left[\frac{1-\theta(\epsilon-1)}{\beta\epsilon} - (\rho+\delta)\right]x & \text{if} \quad x > x_Z. \end{cases}$$

Assuming

$$\frac{\phi-(\rho+\delta)/\alpha}{1-\theta(\epsilon-1)-\beta\epsilon(\rho+\delta)} > \frac{\rho+\delta}{\alpha\theta(\epsilon-1)},$$

The economy converges to:

$$x^* = \frac{\phi-(\rho+\delta)/\alpha}{1-\theta(\epsilon-1)-\beta\epsilon(\rho+\delta)}. \tag{24}$$

The associated steady-state rate of cost-reduction is

$$\hat{Z}^* = \frac{(\phi\alpha-\rho-\delta)\,\theta(\epsilon-1)}{1-\theta(\epsilon-1)-\beta\epsilon(\rho+\delta)} - (\rho+\delta). \tag{25}$$

Proof. See the Appendix. □

Figure 2 illustrates the dynamics. Proposition 2 states a strong **"long-run commodity price super-neutrality"** result: the steady-state growth rate of the economy is independent of conditions in the commodity market and therefore of the commodity price p. In other words, the long-run economic *growth* performance of the model economy is insulated from external commodity price shocks. This happens because the sterilization of the market size effect through entry implies that steady-state growth does not depend on the size of the manufacturing sector and therefore on the inter-sectoral allocation of labor. The resulting sterilization of the scale effect is a key property of the model coming from the interaction of

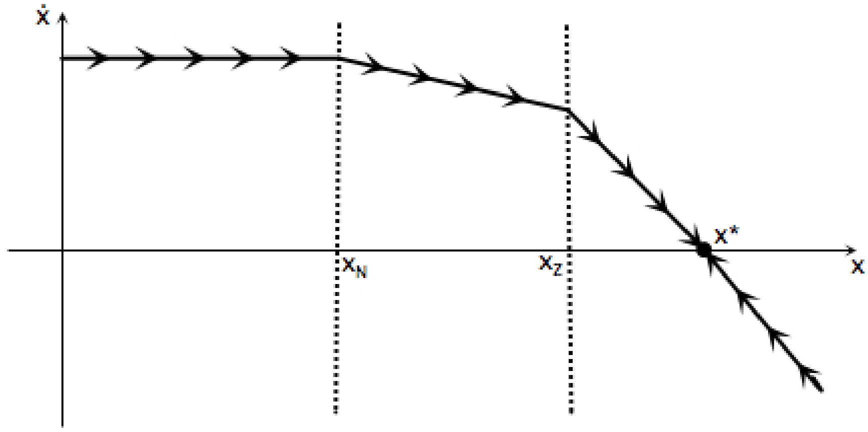

Figure 2: Global equilibrium dynamics

the entry and quality margins of innovation. As new firms enter, the expansion in product variety fragments the aggregate market in sub-markets whose size does not increase with the size of the manufacturing sector. It is worth stressing that the same forces that yield the sterilization of the scale effect insulate the steady-state growth rate of the model economy from the commodity price.

3.4 Productivity, Utility and Welfare

Since the home consumption good sector is competitive, $P_H(p) = P_Y(p)$. Accordingly,

$$P_H(p) = N^{-\chi}\left[\frac{1}{N}\int_0^N (P_j(p))^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}} = N^{-\chi} Z^{-\theta}\left(\frac{\epsilon}{\epsilon-1}\right) C_X(1, C_M(1,p)),$$

where to simplify the notation, we define $c(p) \equiv C_X(1, C_M(1,p))$. We also define aggregate total factor productivity (TFP) as

$$T = N^{\chi} Z^{\theta}. \tag{26}$$

Accordingly,

$$\hat{T}(t) = \chi \hat{N}(t) + \theta \hat{Z}(t).$$

Using (25) in steady state this gives

$$\hat{T}^* = \theta \hat{Z}^* = \theta\left[\frac{(\phi\alpha-\rho-\delta)\theta(\epsilon-1)}{1-\theta(\epsilon-1)-\beta\epsilon(\rho+\delta)} - (\rho+\delta)\right] \equiv g. \tag{27}$$

Observe how g is independent of conditions in the commodity market and of population size. In steady state, $x \equiv Y/\epsilon N$ is invariant to the commodity price p. We study the economy in the region $x(t) > x_Z$ and write the differential equation for x as

$$\dot{x} = \nu (x^* - x),$$

where

$$\nu \equiv \frac{1 - \theta(\epsilon - 1) - \beta\epsilon(\rho + \delta)}{\beta\epsilon} \quad \text{and} \quad x^* \equiv \frac{\phi - (\rho + \delta)/\alpha}{1 - \theta(\epsilon - 1) - \beta\epsilon(\rho + \delta)}.$$

We thus work with the solution

$$x(t) = x_0 e^{-\nu t} + x^* \left(1 - e^{-\nu t}\right), \tag{28}$$

where x_0 is the initial condition. The following states the key result.

Proposition 3. *Consider an economy starting at time $t = 0$ with initial condition x_0. At any time $t > 0$ the log of TFP is*

$$\log T(t) = \log\left(Z_0^\theta N_0^\chi\right) + gt + \left(\frac{\gamma}{\nu} + \chi\right)\Delta\left(1 - e^{-\nu t}\right), \tag{29}$$

where

$$\Delta \equiv \frac{x_0}{x^*} - 1.$$

The instantaneous utility flow is

$$\log u(t) = \log\varphi\left(\frac{1 - \xi(p)}{1 - \xi(p) - \rho\beta} + \frac{p\Omega}{L}\right) - \varphi\log c(p) + \varphi g t + \varphi\left(\frac{\gamma}{\nu} + \chi\right)\Delta\left(1 - e^{-\nu t}\right). \tag{30}$$

The resulting level of welfare is

$$U(0) = \frac{1}{\rho}\left[\log\varphi\left(\frac{1 - \xi(p)}{1 - \xi(p) - \rho\beta} + \frac{p\Omega}{L}\right) - \varphi\log c(p) + \frac{\varphi g}{\rho} + \frac{\varphi\left(\frac{\gamma}{\nu} + \chi\right)}{\rho + \nu}\Delta\right]. \tag{31}$$

Proof. See the Appendix. □

This structure identifies three main effects: 1) the *windfall effect* through $p\Omega$; 2) the *cost of living effect* through $c(p)$; and 3) the *curse or blessing effect* through g and the transitional dynamics associated to Δ, the initial displacement from the steady state. These effects drive welfare as follows. The first two terms in (31) capture the role of steady-state utility calculated holding technology, T, constant; the third term captures the role of steady-state growth, g; the fourth terms is the contribution from the acceleration/deceleration of TFP growth along the transition.

The first two *static* components capture forces that the literature has discussed at length. An economy with a positive endowment of a commodity that sells for a higher price experiences a windfall. In our model this shows up as a rise in commodity income, which, given our assumptions, is formally equivalent to a lump-sum transfer from abroad. The cost of living effect is due to the fact that the economy uses the commodity for home production and, therefore, an increase in the world commodity price works its way through the home vertical structure of production – from upstream materials production to downstream manufacturing – and shows up as a higher price of the home consumption good.

The last two *dynamic* components capture forces that are the focus of modern endogenous growth theory. While the role of steady-state growth is well understood, this model allows us to investigate in detail the less studied role of the transitional dynamics. The reason is that we have a closed-form solution for the model's dynamics. Specifically, the fourth effect runs through the TFP operator in (29), which has two transitional components: the first is the cumulated gain/loss from the acceleration/deceleration of the rate of cost reduction; the second is the cumulated gain/loss from the acceleration/deceleration of product variety expansion. What these accelerations/decelerations do, is amplify the change in manufacturing expenditure due to a change in the commodity price. We discuss this mechanism in the next section.

4 The Dynamic Effects of World Price Shocks

In this section, we analyze the effects of commodity price changes on the transitional dynamics of the model economy. In Section 3.3 above, we argued that the steady-state growth rate is independent of the conditions in the commodity market. In a nutshell, the long-run economic *growth* performance of the model economy is insulated from external price shocks. However, conditions in the commodity market still matter for the short-run and the transition to the steady state.

An important building block of our theory is that the commodity, which is a fixed endowment of the economy, is used as input into production of materials. Hence, the demand of the commodity is endogenous and it responds to variations in the exogenous price p. This implies that the status of commodity importer or exporter is determined within the model as a function of the endowment Ω, price p, technological properties subsumed in the term $\xi(p)$, and other relevant parameters of the model. The following proposition characterizes the commodity-exporting or commodity-importing regions, and the effects of commodity price changes on manufacturing expenditure. We begin with the effect on manufacturing activity.

Lemma 1. *Let:*

$$\epsilon_X^M \equiv -\frac{\partial \log M}{\partial \log P_M} = 1 - \frac{\partial \log S_X^M}{\partial \log P_M} = 1 - \frac{\partial S_X^M}{\partial P_M}\frac{P_M}{S_X^M};$$

$$\epsilon_M^R \equiv -\frac{\partial \log R}{\partial \log p} = 1 - \frac{\partial \log S_M^R}{\partial \log p} = 1 - \frac{\partial S_M^R}{\partial p}\frac{p}{S_M^R}.$$

Then,

$$\xi'(p) = \frac{\epsilon - 1}{\epsilon}\frac{\partial \left(S_M^O(p)\, S_X^M(p)\right)}{\partial p} = \Gamma(p)\,\frac{\xi(p)}{p},$$

where

$$\Gamma(p) \equiv \left(1 - \epsilon_X^M(p)\right) S_M^O(p) + 1 - \epsilon_M^O(p)\,. \tag{32}$$

Proof. See the Appendix. □

The key object in this lemma is the function $\Gamma(p)$, which is the elasticity of $\xi(p) \equiv \frac{\epsilon-1}{\epsilon} S_M^O(p)\, S_X^M(p)$ with respect to p. According to (19), therefore, it is the elasticity of the home demand for the commodity with respect to the world price, holding constant manufacturing expenditure. It thus captures the partial equilibrium effects of price changes in the commodity and materials markets for given market size. Differentiating (20), rearranging terms and using (19) yields

$$\frac{d \log Y(p)}{dp} = \frac{\xi'(p)}{1 - \xi(p) - \beta\rho} = \Gamma(p)\,,$$

which says that the effect of changes in the resource price on expenditure on manufacturing goods depends on the overall pattern of substitution that is reflected in the price elasticities of materials and commodity demand and in the commodity share of materials production costs. The following proposition states the results formally.

Proposition 4. *Depending on the properties of the function $\Gamma(p)$, there are four cases.*

1. ***Global complementarity.*** *Suppose that $\Gamma(p) > 0$ for all p. Then, manufacturing expenditure $Y(p)$ in (20) is a monotonically increasing function of p.*

2. ***Cobb-Douglas-like economy***. *Suppose that $\Gamma(p) = 0$ for all p. This occurs when S_M^O and S_X^M are exogenous constants. Then, manufacturing expenditure $Y(p)$ in (20) is independent of p.*

3. ***Global substitution***. *Suppose that $\Gamma(p) < 0$ for all p. Then, manufacturing expenditure $Y(p)$ in (20) is a monotonically decreasing function of p.*

4. ***Endogenous switch from complementarity to substitution.*** *Suppose that there exists a price p^v where $\Gamma(p)$ changes sign, from positive to negative. Then, manufacturing expenditure $Y(p)$ in (20) is a hump-shaped function of p with a maximum at p^v.*

The Cobb-Douglas-like case is quite common in the literature as it occurs when both technologies are Cobb-Douglas with $\epsilon_X^M = \epsilon_M^O = 1$. We mention this case but not discuss it further since it is a knife-edge specification in which the world commodity price has no effect on home manufacturing activity. Proposition 4 says that the sign of the effect of the world commodity price on manufacturing activity depends on the substitution possibilities between labor and materials in manufacturing and between labor and the commodity in materials production. The most interesting case is when $\Gamma(p)$ changes sign at p^v and the model generates the endogenous switch from substitution to complementarity. We focus on this specification because it nests the two cases of monotonic effect of the price as we let $p^v \to 0$ or $p^v \to \infty$.

This analysis says that a commodity price boom raises home manufacturing activity when the economy exhibits overall complementarity between labor and the commodity. When the economy exhibits overall substitution, instead, a price boom results into a contraction of manufacturing activity. What about import/export behavior? We state the key result as follows.

Proposition 5. *The economy is an exporter of the commodity when*

$$\frac{\Omega}{L} > \frac{1}{1 - \xi(p) - \beta\rho} \frac{\xi(p)}{p}. \tag{33}$$

Proposition 5 contains several important results. 1) It provides an intuitive notion of **"commodity supply dependence"** that captures the traditional view that a country that imports a key commodity is *dependent on foreign supply* and thus subject to external shocks. In our model, for a given commodity price $p > 0$, there exists a threshold of the endowment ratio Ω/L such that for Ω/L below this threshold the economy is an *importer*, i.e., $O > \Omega$, and for Ω/L above it the economy is an *exporter*, i.e., $O < \Omega$. An extreme case of dependence is when $\Omega = 0$ such that by assumption the country must import the commodity. 2) Another way to see the link between the commodity price and the importer/exporter status is to note that, for a given relative endowment Ω/L, there exists a price threshold p^d such that for $p < p^d$ the economy is an importer whereas for $p > p^d$ the economy is an exporter. 3) The proposition also provides an intuitive notion of **"dynamic commodity vulnerability"** captured by the property that $\Gamma(p) \gtrless 0$ determines whether the economy gains or loses from its dynamic response to a higher commodity price. Specifically, if $\Gamma(p) > 0$ home manufacturing expenditure increases in response to a commodity price increase. If $\Gamma(p) < 0$, instead, home manufacturing expenditure decreases with a commodity price increase. How persistent these effects are depends on the parameters of the model. 4) As stated earlier, we focus on the case in which the upstream materials sector and the downstream manufacturing sector have opposite substitutability/complementarity properties. Specifically, we assume that materials production exhibits labor-commodity complementarity while manufacturing exhibits labor-materials substitution. Accordingly, there exists a threshold of the commodity price where the economy switches from overall complementarity to overall substitutability. More precisely, there exists a threshold price p^v such that $\Gamma(p) < 0$ for $p < p^v$ and $\Gamma(p) > 0$ for $p > p^v$. The reason is that when p is low the cost share $S_M^O(p)$ is small and $\Gamma(p)$ is dominated by the term $1-\epsilon_M^O(p)$, which is positive since complementarity implies $\epsilon_M^O(p) < 1$. In contrast, when p is high, the cost share $S_M^O(p)$ is large and $\Gamma(p)$ is dominated by the term $1 - \epsilon_X^M(p)$, which is negative since substitution implies $\epsilon_X^M(p) > 1$.

Definition 1. *An economy is dependent on the world commodity supply if $\Omega < O$, that is, if it consumes more of the commodity than it has. An economy is vulnerable to increases in the world commodity price p if its demand for the commodity is elastic.*

Notice that dependence and vulnerability are not the same. The reason is that home demand is endogenous and adjusts to the world price of the commodity. Figure 3 illustrates the determination of the threshold prices p^d and p^v. The threshold price p^d is increasing in the endowent ratio L/Ω and goes to infinity as $L/\Omega \to \infty$. Thus, economies with zero endowment, $\Omega = 0$, are dependent for all p. Interestingly, an economy with a positive

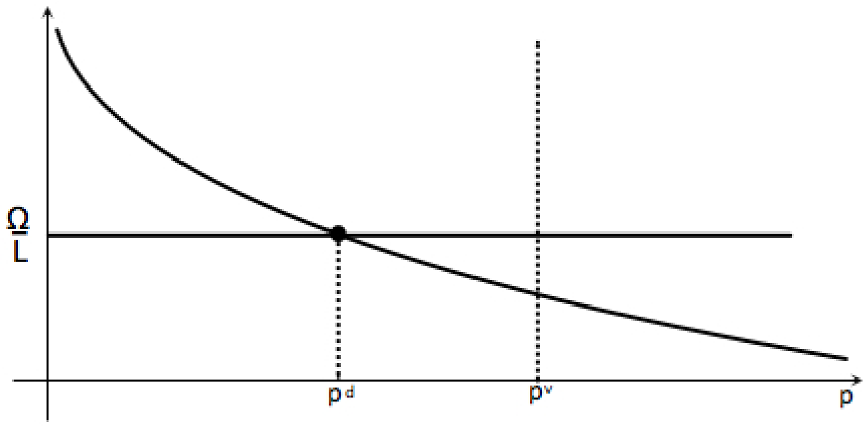

Figure 3: Dependence and vulnerability

endowment can gain from a higher world commodity price even if it is dependent. The reason is that the revenues from sales of the endowment Ω go up one-for-one with p while import costs go up less than linearly since home commodity consumption O responds negatively to p. Intuitively, this *specialization effect* is stronger the more elastic is home commodity demand. More importantly, however, what potentially matters most for welfare is not whether the country experiences an improvement in its commodity trade balance, but whether it is dynamically vulnerable in the sense defined above. And for dynamic vulnerability, elastic commodity demand is bad news. The reason is that the contraction of home commodity demand is just the other side of the contraction of manufacturing activity associated to the specialization effect. The Schumpeterian mechanism at the heart of the model amplifies such a contraction – the instantaneous fall in $Y(p)$ – into a deceleration of the rate of TFP growth. The economy eventually reverts to the steady-state growth rate g, but the temporary deceleration has a potentially substantial negative effect on welfare.

With these considerations in mind, now imagine a *permanent* change in the commodity price. For $p' > p$ we can write

$$\Delta \equiv \frac{x_0}{x^*} - 1 = \frac{Y(p')/\epsilon N(p)}{Y(p')/\epsilon N(p')} - 1.$$

This is the percentage displacement of the state variable x from its steady state that occurs at time 0 when the commodity price jumps up from p to p'. The numerator is the value of profitability holding constant the mass of firms; the denominator is the value of profitability at the end of the transition, when the mass of firms has fully adjusted to the new market size. Consider the case of a vulnerable exporter, that is, an exporter of the commodity with $\Gamma(p) < 0$.

Figure 4 illustrates the path of $\log u(t; p')$. On impact, technology T is pre-determined

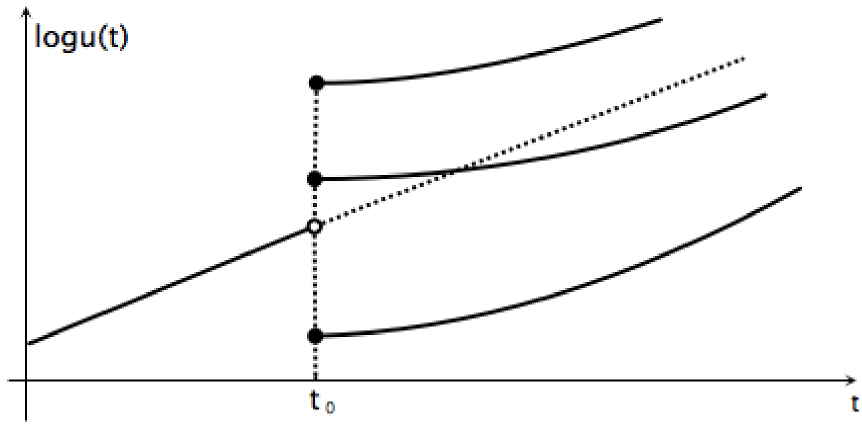

Figure 4: The transition path of utility after a commodity price boom in a vulnerable, exporting economy

and does not jump, while the price index spike and the windfall effect work in opposite directions and yield an initial jump in consumption that has an ambiguous sign. Thereafter, the transitional effects of endogenous TFP take over. The permanent fall in Y produces a slowdown of TFP growth due to a slowdown of entry and a reduction in cost-reducing activity internal to the firm. It is apparent then, that the world commodity price boom benefits this economy if and only if the windfall effect through $p\Omega$ is large enough to compensate for the *cost of living effect* through $c(p)$ and the *curse effect* through $\Delta < 0$. Our explicit solution (31) in Proposition 3 shows how the model's parameters determine the weights of these effects.

5 Fiscal Policy

In this section, we introduce distortionary taxation into the model and study how the interaction of taxes and conditions in the commodity market shapes the dynamic response of the model economy to unexpected variations in the commodity price. The government taxes asset income, rA, at rate τ_A, commodity income, $p\Omega$, at rate τ_Ω and consumption expenditures, $Y_H + Y_F$, at rate τ_C. We consider the policy scenario in which all tax proceeds are rebated in lump-sum form. Hence, we abstract from income effects and focus exclusively on the pure distortions introduced by *ad-valorem* taxes. We compare both steady-state growth rates and transitional dynamics.

The household budget constraint now reads,

$$\dot{A} = (1-\tau_A)rA + WL + \Pi_H + \Pi_M + (1-\tau_\Omega)p\Omega - (1+\tau_C)(Y_H + Y_F) + R,$$

where $R = \tau_A rA + \tau_C(Y_H + Y_F) + \tau_\Omega p\Omega$ are the proceeds collected from income taxation.[11] Specifically, $\tau_A rA$ and $\tau_C(Y_H + Y_F)$ are revenues from taxing respectively asset income and consumption expenditures and $\tau_\Omega p\Omega$ are commodity-linked revenues. We abstract from taxation of profits. In our context, this choice is innocuous for two reasons: first, from the prospective of the government budget constraint, profits taxation does not generate any revenues. Since home consumption goods (H) and materials (M) sectors are both competitive, in equilibrium $\Pi_H = \Pi_M = 0$; second, in the current formulation, a positive tax rate on profits would have no distortionary effect on the equilibrium optimal allocations. We also abstract from labor income taxation since we assume labor services are inelastically supplied by the representative household. Taxing labor income would generate revenues to the government. However, this latter margin is irrelevant since we focus on the scenario in which tax revenues are lump-sum rebated to the household.

Because the tax rate on consumption expenditures τ_C is constant over time, the optimal expenditure sharing rule remains unaltered such that $\varphi Y_F = (1-\varphi)Y_H$. However, the Euler equation governing the intertemporal consumption-saving decision changes,

$$(1-\tau_A)r = r_A \equiv \rho + \frac{\dot{Y}_H}{Y_H} = \rho + \frac{\dot{Y}_F}{Y_F}. \tag{34}$$

Intuitively, the tax rate on asset income τ_A decreases the rate of return to savings. Since in equilibrium both $\dot{Y}_H/Y_H = \dot{Y}_F/Y_F = 0$, Equation (34) implies $(1-\tau_A)r = \rho$. For notational simplicity, let $\tilde{\rho} \equiv \rho/(1-\tau_A)$ denote the *effective* discount rate, and notice that an increase in the tax rate τ_A leads to an increase in $\tilde{\rho}$, such that the effective discount rate is actually increasing in the tax on asset income.

[11] In this formulation of the tax code, the tax base for τ_A is asset income, that is, the sum of dividends and capital gains. This is equivalent to tax dividends and capital gains at the same rate.

5.1 Growth Effects of Taxation

This section focuses on the effects of taxation on the steady-state growth rate and the transitional dynamics of the model.

Proposition 6. *Let $x \equiv Y/\epsilon N$ denote the gross profit rate. The general equilibrium of the model with taxes reduces to the following piece-wise linear differential equation in the gross profit flow:*

$$\dot{x} = \begin{cases} \frac{\delta L\epsilon/N_0}{(1-\xi(p))-\frac{1}{\epsilon}} & \text{if} \quad \phi \leq x \leq x_N \\ \frac{\phi}{\beta\epsilon} - \left[\frac{1}{\beta\epsilon} - (\tilde{\rho}+\delta)\right] x & \text{if} \quad x_N < x \leq x_Z \\ \frac{\phi - \frac{\tilde{\rho}+\delta}{\alpha}}{\beta\epsilon} - \left[\frac{1-\theta(\epsilon-1)}{\beta\epsilon} - (\tilde{\rho}+\delta)\right] x & \text{if} \quad x > x_Z. \end{cases}$$

Assuming

$$\frac{\phi - (\tilde{\rho}+\delta)/\alpha}{1-\theta(\epsilon-1) - \beta\epsilon(\tilde{\rho}+\delta)} > \frac{\tilde{\rho}+\delta}{\alpha\theta(\epsilon-1)},$$

the economy with taxes converges to

$$x^* = \frac{\phi - (\tilde{\rho}+\delta)/\alpha}{1-\theta(\epsilon-1) - \beta\epsilon(\tilde{\rho}+\delta)}.$$

The associated steady-state rate of cost-reduction is

$$\hat{Z}^* = \frac{(\phi\alpha - \tilde{\rho} - \delta)\,\theta(\epsilon-1)}{1-\theta(\epsilon-1) - \beta\epsilon(\tilde{\rho}+\delta)} - (\tilde{\rho}+\delta),$$

which is decrasing in the tax rate on asset income τ_A.

Proof. The proof follows the same steps of Proposition 2 with ρ replaced by $\tilde{\rho}$. The comparative statics with respect to τ_A follows by simply differentiating the steady-state growth rate of quality innovation with respect to $\tilde{\rho}$ and recognizing that $\tilde{\rho}$ is increasing in the tax rate τ_A. □

Proposition 6 provides two important results. 1) Asset income taxation has an adverse effect on the steady-state growth rate of quality innovation, which is the only driver of long-run growth in TFP. This happens because the asset income tax distorts the return to savings, and through the no arbitrage condition $r = r_A = r_Z = r_N$, it negatively affects the effective return to innovation. 2) Tax rates on consumption expenditures and commodity income have instead no effect on the steady-state growth rate of the model economy. This latter result is not special to the case with lump-sum tax rebates but it applies generally also with unproductive government spending.

The following lemma characterizes the effects of taxation on the dynamics of the gross profit flow x.

Lemma 2. *Let $x \equiv Y/\epsilon N$ denote the gross profit rate, and consider the solution to the general equilibrium of the model $x(t) = x_0 e^{-\tilde{\nu}t} + x^*(1-e^{-\tilde{\nu}t})$ where x_0 is the initial condition, x^* is the steady state value of x, and the eigenvalue of the differential equation for x is*

$$\widetilde{\nu} = \begin{cases} \frac{1}{\beta\epsilon} - (\widetilde{\rho} + \delta) \equiv \widetilde{\nu}_N & \text{if} \quad x_N < x \leq x_Z \\ \frac{1-\theta(\epsilon-1)}{\beta\epsilon} - (\tilde{\rho} + \delta) \equiv \widetilde{\nu}_Z & \text{if} \quad \quad x > x_Z. \end{cases}$$

The eigenvalue $\widetilde{\nu}$ is decreasing in the tax rate on asset income τ_A.

Proof. The comparative statics with respect to τ_A follows by simply differentiating the eigenvalue $\tilde{\nu}$ with respect to $\tilde{\rho}$ and realizing that $\tilde{\rho}$ is increasing in the tax rate τ_A. $\square$

Lemma 2 provides an important result. A positive tax rate on asset income slows down the transional dynamics towards the steady state. This happens because the tax rate on asset income τ_A reduces the eigenvalue of the dynamical system which is the only driver of the transitional dynamics of the model.

5.2 Level Effects of Taxation

This section focuses on the level effects of taxation. The following proposition characterizes the effects of taxation on the level of manufacturing expenditure.

Proposition 7. *Under lump-sum rebate of tax proceeds, $R = \tau_A r A + \tau_\Omega p \Omega + \tau_C (Y_H + Y_F)$, manufacturing expenditure is*

$$Y(p) = \frac{L}{1 - \xi(p) - \beta\tilde{\rho}}.$$

Manufacturing expenditure is increasing in the tax rate on asset income, τ_A, i.e.,

$$\partial \log Y(p)/\partial \tau_A > 0.$$

Moreover, a positive tax on asset income, $\tau_A > 0$, acts as an automatic amplifier of commodity price changes,

$$\frac{d^2 \log Y(p)}{dp d\tau_A} = \begin{cases} \frac{\rho\beta\xi'(p)/(1-\tau_A)^2}{[1-\xi(p)-\beta\tilde{\rho}]^2} \geq 0 & \text{if} \quad \xi'(p) \geq 0 \\ \frac{\rho\beta\xi'(p)/(1-\tau_A)^2}{[1-\xi(p)-\beta\tilde{\rho}]^2} \leq 0 & \text{if} \quad \xi'(p) \leq 0. \end{cases}$$

Proof. The proof of the proposition follows the same steps of Proposition 1. □

Proposition 7 contains three results. 1) Manufacturing expenditure is increasing in the tax rate on asset income τ_A. This happens because a higher tax on asset income leads a reallocation from savings towards consumption expenditures which in equilibrium translates into larger expenditure on manufacturing goods. 2) A positive tax rate on asset income, $\tau_A > 0$, is an *automatic amplifier* of commodity price changes. 3) Only the tax rate τ_A enters the determination of manufacturing expenditures. This happens because both tax rates on consumption expenditures and commodity income are, for different reasons, not distortionary. The tax rate τ_C is not distortionary because it is by assumption constant over time, as such it does not interfere with the intertemporal consumption allocations. Moreover, since we assume an inelastic supply of labor services, τ_C has also no intratemporal distortionary effect on the consumption-leisure optimal allocation. The tax rate levied on commodity income τ_Ω is not distortionary because the commodity is in fixed supply and the price is assumed constant and exogenous. Recall also that by focusing on the case of lump-sum rebates, we abstract from potential income effects of taxation.

5.3 Further Discussion

The combined of Propositions 6 and 7, and Lemma 2 conveys the main message of this section: asset income taxation has both *level* and *growth* effects. Furthermore, it acts as an *automatic amplifier* of commodity price changes and *slows down* the transitional dynamics of the model.

From a more general viewpoint, this section also suggests that in this class of models, the effects of taxation fall in two separate categories: tax instruments that have only *level* effects and no *growth effects*; tax instruments that have both *level* and *growth* effects.[12] By distorting the rate of return through the no arbitrage condition $r = r_A = r_Z = r_N$, the tax rate on asset income τ_A affects the equilibrium steady-state growth rate of the model economy because it alters the incentives to innovation. To this category belong not only taxes on asset income, but also any other tax that creates a wedge in the Euler equation, e.g., time-varying consumption taxes. Tax rates not interfering with the return to savings and innovation have no steady-state *growth* effects but have *level* effects on the endogenous variables of the model. To this category belong commmodity income taxes, constant consumption taxes, and labor income taxes when tax revenues are no longer lump-sum rebated to the household. In

[12] See Peretto (2003) for an early discussion of this argument.

our context, consumption and commodity income taxes have no level effects because their potential income effect is neutralized by the lump-sum transfer to the household.

6 Numerical Analysis

To further understand the mechanics of the model and how the tax code in place affects the dynamic properties of our model economy, we conduct a simple numerical exercise. We assign numerical values to the relevant parameters of the model and let the tax rate on asset income τ_A take values that range from 20 to 50 percent. Notably, this wide range of tax rates is consistent with the available evidence on cross-country capital income tax rates provided by Mendoza et al. (1994).

6.1 Calibration

One period is one year. Table 1 contains the baseline parameter values that are kept constant over the following analysis.

Table 1: Baseline parameters

Parameter	Interpretation	Value
$\epsilon/(\epsilon-1)$	Mfg price markup	1.3
θ	Mfg prod. function: $X_i = Z_i^{\theta} F(L_{X_i} - \phi, M_i)$	0.15
ρ	Discount rate	0.02
δ	Death rate	0.035
β	Mfg entry cost: $V_i = \beta \cdot \frac{WY}{N}$	1

We set $\epsilon = 4.33$ to match a price markup of 30 percent. Overall, the available evidence for the U.S. provides estimates of markups in value added data that range from 1.2 to 1.4.[13] Hence, we target a markup in the manufactuting sector of $\mu = \epsilon/(\epsilon-1) = 1.3$ that is at the middle of the available range of estimates. The condition for a symmetric equilibrium, $\theta(\epsilon-1) < 1$, imposes a restriction on the calibration of θ, i.e., $\theta \in (0, 1/(\epsilon-1))$.

[13] See Hall (1988), Morrison (1992), Norrbin (1993), Roeger (1995), and Basu and Fernald (1997, 2001).

Furthermore, given the calibrated value of $\epsilon = 4.33$, we have an upper bound on θ such that $\theta \in (0, 0.3)$. Since we have no reference value guiding our choice, we set $\theta = 0.15$ at the middle of the possible range. The death rate is set to $\delta = 0.035$ to match the average closing rate of establishments in the U.S. manufacturing sector for 1992-2012. Data for closing establishments are from the Business Employment Dynamics (BED) survey of the Bureau of Labor Statistics (BLS). The requirement of positive eigenvalues over all the state space of the model imposes a restriction on the calibration of the entry cost β. Specifically, $\widetilde{\nu}_Z > 0$ implies $\beta \in \left(0, \frac{1-\theta(\epsilon-1)}{\epsilon(\widetilde{\rho}+\delta)}\right)$. Notice that $\widetilde{\nu}_Z = \widetilde{\nu}_N - \theta(\epsilon-1)/\beta\epsilon < \widetilde{\nu}_N$ guarantees that the restriction on β is a sufficient condition to have both eigenvalues always greater than zero. We normalize the entry cost at $\beta = 1$, which is within the set identified by the above restrictions. Finally, the time discount rate is set to the conventional value of 2 percent.

In Figure 5 below, we consider a wide spectrum of tax rates that ranges from 20 to 50 percent. Importantly, such a variation in tax rates—20% to 50%— is consistent with empirical estimates of capital income tax rates. For example, look at the updated estimates of effective tax rates for a sample of seven OECD countries over the period 1965-1996, calculated with the method proposed in Mendoza et al. (1994) and available on Mendoza's website.

Table 2: Capital income tax rates by Mendoza et al. (1994)

Year	Canada	France	Germany	Italy	Japan	U.K.	U.S.
1980	37.60	27.31	32.11	20.02	35.98	64.32	46.88
1996	50.66	26.11	23.91	33.86	42.61	47.17	39.62

Table 2 reports the updated estimates of capital income tax rates for two sample years. In the data, capital income tax rates display large cross-country variation. In 1980, they range from a minimun of 20 percent for Italy to a maximum of 64.32 percent for the U.K. In 1996, from approximatively a 24 percent for Germany to 51 percent for Canada.

6.2 Dynamic Response to a "Profit Rate Shock"

In this section, we compute the dynamic response of the gross profit rate, $x \equiv Y(p)/\epsilon N(p)$, to a shock that temporarily displaces x from its steady-state value. In other words, we force the model to be in transition and study how the reversion to the original steady state depends on the value of the asset income tax τ_A. To this aim, we keep the relevant parameters of the model fixed and vary the tax rate on asset income. Recall that the gross profit rate x is the key state variable of the model regulating the incentives to innovate and hence driving the relevant equilibrium dynamics.

Figure 5 plots the time path of

$$\frac{x(t)}{x^*} - 1 = \Delta e^{-\widetilde{\nu}t}$$

where $x(t) = x_0 e^{-\widetilde{\nu}t} + x^* \left(1 - e^{-\widetilde{\nu}t}\right)$, the eigenvalue of the differential equation for x is

$$\widetilde{\nu} = \begin{cases} \frac{1}{\beta\epsilon} - \left(\frac{\rho}{1-\tau_A} + \delta\right) \equiv \widetilde{\nu}_N & \text{if} \quad x_N < x \leq x_Z \\ \frac{1-\theta(\epsilon-1)}{\beta\epsilon} - \left(\frac{\rho}{1-\tau_A} + \delta\right) \equiv \widetilde{\nu}_Z & \text{if} \quad x > x_Z \end{cases}$$

and the initial percentage displacement from the steady state is $\Delta = \frac{x_0}{x^*} - 1$—we call this a *"profit rate shock."* Given an initial displacement of $\Delta = 1\%$, we assess how the speed of reversion to the steady state differs at different levels of asset income taxation. Since the equilibrium gross profit flow x follows a linear differential equation, the speed of reversion to the steady state is governed by the magnitude of the eigenvalue $\widetilde{\nu}$, which depends on the tax rate τ_A and other parameters. Precisely, the eigenvalue $\widetilde{\nu}$ is decreasing in τ_A, i.e., higher asset income taxation leads to a slower reversion to the steady state for a given displacement Δ. Furthermore, $\widetilde{\nu}_Z = \widetilde{\nu}_N - \theta(\epsilon - 1)/\beta\epsilon < \widetilde{\nu}_N$ such that the dynamics in the "entry and quality regime" (i.e. for $x > x_Z$) are slower than those in the "entry only regime" (i.e. for $x_N < x \leq x_Z$).

Notice also that, as stated in Proposition 6, the steady-state values of x vary across taxation levels such that the dynamic responses in Figure 5 depict the reversion to the steady state associated with each tax rate τ_A. Moreover, we shock directly the state variable x and keep a displacement of $\Delta = 1\%$ in all cases we consider. In other words, we take the *impact response* of the economy to a commodity price shock as given, and focus exclusively on how asset income taxation affects the *speed of reversion* to the steady state due to the dynamics of the profit rate regardless of what causes its initial displacement. Later in this section, we discuss instead how the tax on asset income affects the impact response of the profit rate x to a commodity price shock.

Figure 5 contains three results. 1) The transitional dynamics are unambiguously slower in the "entry and quality regime," i.e. for $x > x_Z$, compared to the "entry only regime," i.e. for $x_N < x \leq x_Z$. This result holds irrespective of the specific value taken by the tax rate on asset income. Even in the benchmark case with no distortionary taxation, i.e. $\tau_A = 0$, the

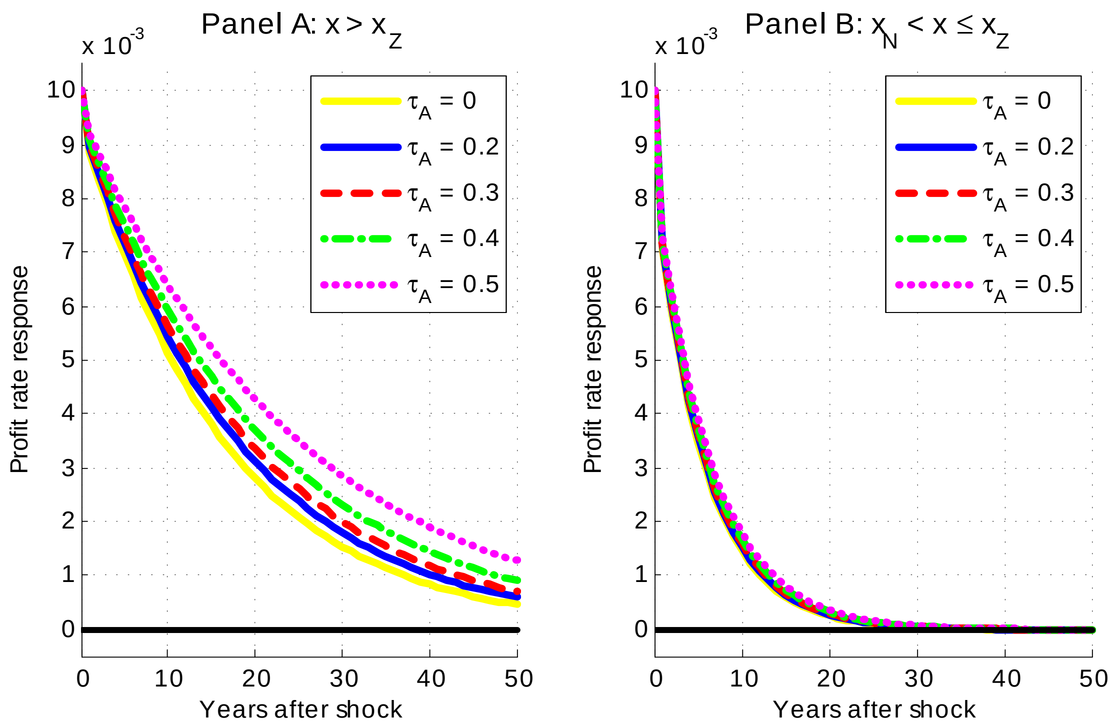

Figure 5: Dynamic response to a "profit rate shock"

difference in the speed of reversion to the steady state across regimes is quite striking. In the "entry only regime," the gap from the steady state is vitually closed 30 years after the shock. In the "entry and quality regime" instead, it takes more than 50 years to close the same initial gap of 1 percent. Importantly, the latter observations implicitly suggest that the persistence of the effects of unexpected commodity price changes greatly varies across growth regimes. 2) In each regime, the speed of reversion is decreasing in the tax rate τ_A. This is not surprising given the content of Lemma 2, which holds for all values that the asset income tax rate takes on the possible range, i.e., $\tau_A \in [0, 1)$. 3) Asset income taxation has larger effects in the "entry and quality regime" than in the "entry only regime." This latter result and the different persistence of shocks across the two regimes highlight a distinctive and appealing feature of our theoretical structure. The dynamics of the model and so the interaction of asset income taxation with commodity prices have quantitatively different effects at different stages of economic development.

Up to now, we studied the speed of reversion to the steady state taking as given the initial displacement in x. However, asset income taxation also affects the impact response of the profit rate x to a commodity price shock. As formally stated in Proposition 7, the change in manufacturing spending $Y(p)$ induced by a change in the commodity price p is increasing in the tax rate τ_A. This implies that for a given commodity price shock, the impact response of $x = Y(p)/\epsilon N(p)$ is larger at higher levels of asset income taxation. Hence, we next show that the initial displacement Δ induced by a commodity price change—*impact response*—is increasing in the tax rate on asset income τ_A. To understand why this is the case, lets consider a scenario in which there is a permanent fall in the commodity price, i.e, $p' < p$, and the economy operates under "global substitution," such that $\Delta_Y \equiv Y(p') - Y(p) > 0$ for all $p' < p$ (see Proposition 4 for more details on global substitutability). Recall that the "long-run commodity price super-neutrality" result stated in Proposition 2 implies that $x^*(p') = x^*(p)$. Moreover, Proposition 7 states that $\Delta_Y(\tau'_A) > \Delta_Y(\tau_A)$ for all $\tau'_A > \tau_A$. With these results in mind, lets write the initial displacement Δ as

$$\Delta \equiv \frac{x_0}{x^*} - 1 = \frac{Y(p')/\epsilon N(p)}{Y(p')/\epsilon N(p')} - 1 = \frac{N(p')}{N(p)} - 1 \equiv \Delta_N > 0. \tag{35}$$

The combined of Propositions 2, 4, and 7 then implies that $\Delta_N(\tau'_A) > \Delta_N(\tau_A)$ for all $\tau'_A > \tau_A$. Therefore, the impact response Δ to a commodity price shock is increasing in the asset income tax τ_A.

To summarize, after an unexpected fall in the commodity price—p to p'—output in the manufacturing sector $Y(p)$ spikes to the new steady-state level $Y(p')$. After the initial spike Δ, the reversion to the new steady state $x^* = Y(p')/\epsilon N(p')$ is governed by positive net entry of firms—$N(p)$ to $N(p')$ with $N(p') > N(p)$. The magnitude and duration of each phase is affected by the level of asset income taxation.

Overall, the model suggests that taxation of asset income affects the entire response to a commodity price shock, i.e., *impact response* and *steady-state reversion*. Specifically, higher levels of asset income taxation imply a larger response on impact with a slower reversion to the steady state. In this latter respect, the analysis points to the *interaction* of fiscal policy with conditions in the commodity market as an important mechanism through which external shocks to commodity prices are amplified and transmitted through the economy.

6.3 Policy Response to a Commodity Price Decline

In Section 6.2 above, we discussed the dynamic properties of the model in response to a commodity price permanent change keeping the level of asset income taxation fixed to a specified level. In this section instead, we discuss the scenario in which the government changes the tax rate on asset income in response to a decline in commodity prices.

Using equations (34) and (6), and the free-entry condition $A = NV = \beta WY$, we rewrite government revenues $R = \tau_A rA + \tau_\Omega p\Omega + \tau_C(Y_H + Y_F)$ as,

$$R(p) = \left[\frac{\rho\beta\tau_A}{1-\tau_A} + \tau_C\right] Y(p) + \tau_\Omega p\Omega + \tau_C p\,(\Omega - O)\,. \tag{36}$$

Lets consider the case in which $(\Omega - O) > 0$, i.e., the economy is a net commodity exporter. We believe this is the most relevant case for the current policy debate on commodity-rich countries. Equation (36) identifies three channels through which commodity prices affect government revenues: 1) the indirect effect through manufacturing expenditure $Y(p)$; 2) the direct *windfall effect* through taxation of commodity income $p\Omega$; and 3) the direct *expenditure effect*, which through the balance trade condition $Y_H + Y_F + p(O - \Omega) = Y$, manifests itself as taxation of exports $p(\Omega - O)$. Notice that the sign of the first effect depends on how manufacturing expenditure $Y(p)$ responds to commodity price changes. Specifically, a commodity price decline has a positive effect if the economy operates under "global substitution," a negative effect under "global complementarity," and it is neutral in a "Cobb-Douglas-like economy."[14] The sign of the second and third effect is instead unambiguous.

At this stage, the notion of *fiscal dependence* is operational: if commodity-linked revenues, $\tau_\Omega p\Omega$, represent a large fraction of total fiscal revenues R, then government revenues are vulnerable to commodity price movements.

Lets consider now a scenario in which there is a permanent fall in the commodity price, i.e., $p' < p$, and government revenues fall such that $R(p') < R(p)$. If the government let the lump-sum rebate to the household decrease accordingly then all the dynamics would be those described in Section 6.2. However, if the government raises the tax rate on asset income to $\tau'_A > \tau_A$ in response to the tax revenue shortfall then the dynamic response to the shock change along two dimensions: 1) according to the discussion of Section 6.2, the shock to the commodity price is further amplified and its effects made more persistent; and 2) according to Proposition 6, the steady-state growth rate of TFP decreases. Thus, the commodity price change has an *indirect* adverse effect on long-run growth. This happens exclusively because of the government's reaction to the tax revenue shortfall. Overall, the analysis suggests that the short- and long-run performance of commodity-rich economies depends on the policy response implemented in the aftermath of the commodity price decline.

[14] See Proposition 4 for more details on the properties of $Y(p)$.

7 Conclusions

In this paper, we develop a Schumpeterian small-open-economy model of endogenous growth. We focus on three policy relevant questions for commodity-exporting countries. How does the economy respond to external shocks to commmodity prices? How is the dynamic response to commodity price changes affected by the structure of the tax code in place? And, how should governments adjust taxation in response to declining commodity income? The model is analytically transparent in that we derive closed-form solutions for the transitional dynamics. This allows us to compute welfare and disentangle the short- and long-run effects of distortionary taxation.

The results can be summarized as follows. 1) *"Long-run commodity price super-neutrality."* Commodity price changes affect the transitional dynamics of the model but have no effect on the steady-state growth rate of the economy. This is an important results since it suggests that, absent fiscal considerations, the long-run growth performance of an economy like ours is completely insulated from the conditions in the commodity market. 2) An increase in the tax on asset income has a positive level effect on manufacturing expenditure but it has an adverse effect on the steady-state growth rate of the economy. This implies if the government endogenously raises the tax rate on asset income in response to a shortfall of resource revenues then commodity price changes can still have indirect adverse effects on the steady-state growth rate of the economy. Notice that these negative long-run effects are exclusively the result of the (misguided) government's response to changes in the economic environment. Finally, 3) a positive tax rate on asset income *amplifies* external shocks to the commodity price and *slows down* the reversion to the steady state after a commodity price shock. Overall, the theoratical analysis suggests that, in the aftermath of commodity price declines, the short- and long-run economic performance of a country is sensitive to the structure of the tax code in place and to the policy response implemented. In this sense, countries are *fiscally vulnerable.*

8 Appendix

8.1 Firms' Behavior and the Free-Entry Equilibrium

To characterize the typical firm's behavior, consider the Current Value Hamiltonian (CVH, henceforth)

$$CVH_i = [P_i - C_X(W, P_M)Z_i^{-\theta}]X_i - W\phi - WL_{Z_i} + z_i\alpha KL_{Z_i},$$

where the costate variable, z_i , is the value of the marginal unit of knowledge. The firm's knowledge stock, Z_i, is the state variable; effort in cost reduction, L_{Z_i}, and the product's price, P_i, are the control variables. Firms take the public knowledge stock, K, as given. Since the Hamiltonian is linear, one has three cases: 1) $W > z_i\alpha K$ implies that the value of the marginal unit of knowledge is lower than its cost. The firm, then, does not invest; 2) $W < z_i\alpha K$ implies that the value of the marginal unit of knowledge is higher than its cost. Since the firm demands an infinite amount of labor to employ in cost reduction, this case violates the general equilibrium conditions and is ruled out; 3) the first order conditions for the interior solution are given by equality between marginal revenue and marginal cost of knowledge, $W = z_i\alpha K$, the constraint on the state variable, (11), the terminal condition,

$$\lim_{s\to\infty} e^{-\int_t^s [r(v)+\delta]dv} z_i(s)Z_i(s) = 0,$$

and a differential equation in the costate variable,

$$r + \delta = \frac{\dot{z}_i}{z_i} + \theta C_X(W, P_M)Z_i^{-\theta-1}\frac{X_i}{z_i},$$

that defines the rate of return to cost reduction as the ratio between revenues from the knowledge stock and its shadow price plus (minus) the appreciation (depreciation) in the value of knowledge.

The revenue from the marginal unit of knowledge is given by the cost reduction it yields times the scale of production to which it applies. The price strategy is

$$P_i = C_X(W, P_M)Z_i^{-\theta}\frac{\epsilon}{\epsilon - 1}. \tag{37}$$

Peretto (1998) (Proposition 1) shows that under the restriction $1 > \theta(\epsilon - 1)$ the firm is always at the interior solution, where $W = z_i\alpha K$ holds, and equilibrium is symmetric. The cost function (10) gives rise to the conditional factor demands:

$$L_{X_i} = \frac{\partial C_X(W, P_M)}{\partial W} Z_i^{-\theta} X_i + \phi;$$

$$M_i = \frac{\partial C_X(W, P_M)}{\partial P_M} Z_i^{-\theta} X_i.$$

Then, the price strategy (37), symmetry and aggregation across firms yields (13) and (14). Also, in symmetric equilibrium $K = Z = Z_i$ yields $\dot{K}/K = \alpha L_Z/N$, where L_Z is aggregate effort in cost reduction. Taking logs and time derivatives of $W = z_i \alpha K$ and using the demand curve (8), the cost-reduction technology (11) and the price strategy (37), one reduces the first-order conditions to (15).

Taking logs and time-derivatives of V_i yields

$$r = \frac{\Pi_i}{V_i} + \frac{\dot{V}_i}{V_i} - \delta.$$

The cost of entry is $\beta WY/N$. The corresponding demand for labor in entry is L_N. The case $V > \beta WY/N$ yields an unbounded demand for labor in entry, $L_N = +\infty$, and is ruled out since it violates the general equilibrium conditions. The case $V < \beta WY/N$ yields $L_N = -\infty$, which means that the non-negativity constraint on L_N binds and $L_N = 0$. A free-entry equilibrium requires $V = \beta WY/N$. Using the price strategy (37), the rate of return to entry becomes (16).

8.2 Proof of Proposition 1

Since the sectors producing the home consumption good and energy are competitive, we have $\Pi_H = \Pi_M = 0$. The consumption expenditure allocation rule (4) and the choice of numeraire yield

$$\dot{A} = rA + L + p\Omega - \frac{1}{\varphi} Y_H.$$

Rewriting the domestic commodity demand (19) as

$$pO = Y \cdot \xi(p), \quad \xi(p) \equiv \frac{\epsilon - 1}{\epsilon} S_X^M(p) S_M^O(p),$$

allows us to rewrite the balanced trade condition as

$$\frac{1}{\varphi}Y_H - p\Omega = Y\left(1-\xi\left(p\right)\right).$$

Substituting the expressions for financial wealth, $A = \beta Y$, and the balanced trade condition in the household's budget constraint (3), and using the rate of return to saving in (5), yields

$$\begin{aligned}\frac{\dot{Y}}{Y} &= \rho + \frac{\dot{Y}_H}{Y_H} + \frac{L + p\Omega - \frac{1}{\varphi}Y_H}{\beta Y} \\ &= \rho + \frac{\dot{Y}_H}{Y_H} + \frac{L - Y\left(1-\xi\left(p\right)\right)}{\beta Y}.\end{aligned}$$

Differentiating the balanced trade condition yields

$$\frac{1}{\varphi}\dot{Y}_H = \dot{Y}\left(1-\xi\left(p\right)\right) \Rightarrow \frac{\dot{Y}_H}{Y_H} = \frac{\dot{Y}}{Y}\frac{Y}{Y_H}\varphi\left(1-\xi\left(p\right)\right) = \frac{\dot{Y}}{Y}\frac{Y\left(1-\xi\left(p\right)\right)}{Y\left(1-\xi\left(p\right)\right)+p\Omega}.$$

Substituting back in the budget constraint and rearranging terms yields

$$\frac{\dot{Y}}{Y} = \frac{Y\left(1-\xi\left(p\right)\right)+p\Omega}{p\Omega}\left[\rho + \frac{L - Y\left(1-\xi\left(p\right)\right)}{\beta Y}\right].$$

This differential equation has a unique positive steady-state value of manufacturing production:

$$Y\left(p\right) = \frac{L}{1-\xi\left(p\right)-\rho\beta}.$$

We ignore, for simplicity the issue of potential indeterminacy, assuming that Y jumps to this steady-state value. The associated expenditures on the home and foreign goods, respectively, are:

$$\begin{aligned}Y_H\left(p\right) &= \varphi\left[\frac{L\left(1-\xi\left(p\right)\right)}{1-\xi\left(p\right)-\rho\beta} + p\Omega\right]; \\ Y_F\left(p\right) &= \left(1-\varphi\right)\left[\frac{L\left(1-\xi\left(p\right)\right)}{1-\xi\left(p\right)-\rho\beta} + p\Omega\right].\end{aligned}$$

Since $Y_H\left(p\right)$ and $Y_F\left(p\right)$ are constant, the saving rule (5) yields that the interest rate is $r = \rho$ at all times.

8.3 Proof of Proposition 2

The return to entry (16) and the entry technology $\dot{N} = (N/\beta Y) \cdot L_N - \delta N$ yield

$$L_N = \frac{Y}{\epsilon x}\left[x - \left(\phi + \frac{L_Z}{N}\right)\right] - \rho\beta Y.$$

Taking into account the non-negativity constraint on L_Z, we solve (11) and (15) for

$$\frac{L_Z}{N} = \begin{cases} \theta(\epsilon - 1)x - (\rho + \delta)/\alpha & x > x_Z \equiv \frac{\rho+\delta}{\alpha\theta(\epsilon-1)} \\ 0 & x \leq x_Z \end{cases}. \tag{38}$$

Therefore,

$$L_N = \begin{cases} \frac{Y}{\epsilon}\left[1 - \theta(\epsilon - 1) - \frac{\phi-(\rho+\delta)/\alpha}{x}\right] - \rho\beta Y & x > x_Z \\ \frac{Y}{\epsilon}\left(1 - \frac{\phi}{x}\right) - \rho\beta Y & x \leq x_Z \end{cases}.$$

So we have

$$L_N > 0 \text{ for } \begin{cases} x > \frac{\phi-(\rho+\delta)/\alpha}{1-\theta(\epsilon-1)-\epsilon\rho\beta} & x > x_Z \\ x > \frac{\phi}{1-\epsilon\rho\beta} & x \leq x_Z \end{cases}.$$

We look at the case

$$\frac{\phi}{1 - \epsilon\rho\beta} \equiv x_N < \frac{\rho + \delta}{\alpha\theta(\epsilon - 1)} \equiv x_Z,$$

which yields that the threshold for gross entry x_N is smaller than the threshold for in-house innovation x_Z.[15]

To obtain the value of Y when $L_N = 0$, first note that

$$L_N = 0 \quad \text{for} \quad \frac{1}{\epsilon}\left(1 - \frac{\phi}{x}\right) \leq \rho\beta.$$

The household budget yields

$$0 = N\left(\frac{Y}{\epsilon N} - \phi\right) + L + \Omega p - \frac{1}{\varphi}Y_H.$$

Using the balanced trade condition and rearranging yields

$$Y = \frac{L - \phi N}{1 - \xi(p) - \frac{1}{\epsilon}}.$$

[15] The global dynamics are well defined also when this condition fails and $x_N > x_Z$. We consider only the case $x_N < x_Z$ to streamline the presentation since the qualitative result and, most importantly, the insight about the role of the commodity price remain essentially the same.

This equation holds for

$$x \leq x_N \equiv \frac{\phi}{1-\epsilon\rho\beta} \Leftrightarrow N \geq N_N \equiv \frac{\phi}{1-\epsilon\rho\beta}\frac{\epsilon}{Y}.$$

The interpretation is that with no effort in entry, there is net exit and thus saving of fixed costs. This shows up as aggregate efficiency gains as intermediate firms move down their average cost curves. Note that in this region,

$$Y(t) = \frac{L - \phi N_0 e^{-\delta t}}{1 - \xi(p) - \frac{1}{\epsilon}},$$

which shows that intermediate production grows in value as a result of net exit. The consolidation of the market results in growing profitability, that is,

$$\frac{\dot{x}}{x} = \frac{\delta L}{L - \phi N_0 e^{-\delta t}} \Rightarrow \dot{x} = \frac{\delta L/\epsilon N_0}{1 - \xi(p) - \frac{1}{\epsilon}}.$$

This saya that with the exit shock, the economy must enter the region where entry is positive because the very definition of steady state requires replacing firms that leave the market. Therefore, the only condition that we need to ensure convergence to the steady state with positive cost reduction is $x^* > x_Z$.

8.4 Proof of proposition 3

Taking logs of (26) yields

$$\log T(t) = \theta \log Z_0 + \theta \int_0^t \hat{Z}(s)\, ds + \chi \log N_0 + \chi \log\left(\frac{N(t)}{N_0}\right).$$

Using the expression for g in (27), and adding and subtracting $\hat{Z}^*$ from $\hat{Z}(t)$, we obtain

$$\log T(t) = \log\left(Z_0^\theta N_0^\chi\right) + gt + \theta \int_0^t \left[\hat{Z}(s) - \hat{Z}^*\right] ds + \chi \log\left(\frac{N(t)}{N_0}\right).$$

Using (38) and (28) we rewrite the third term as

$$\begin{aligned}\theta \int_0^t \left(\hat{Z}(s) - \hat{Z}^*\right) ds &= \alpha\theta^2 (\epsilon - 1) \int_0^t (x(s) - x^*) \, ds \\ &= \gamma \left(\frac{x_0}{x^*} - 1\right) \int_0^t e^{-\nu s} ds \\ &= \frac{\gamma}{\nu} \left(\frac{x_0}{x^*} - 1\right) \left(1 - e^{-\nu t}\right),\end{aligned}$$

where

$$\gamma \equiv \alpha\theta^2 (\epsilon - 1) \, x^*.$$

Observing that $N(t) = Y(p) / \epsilon x(t)$ yields $\dot{N}/N = -\dot{x}/x$, we use (28) to obtain

$$\frac{N(t)}{N_0} = \frac{1 + \left(\frac{N^*}{N_0} - 1\right)}{1 + \left(\frac{N^*}{N_0} - 1\right) e^{-\nu t}}.$$

We then rewrite the last term as

$$\begin{aligned}\chi \log \left(\frac{N(t)}{N_0}\right) &= \chi \log \frac{1 + \left(\frac{N^*}{N_0} - 1\right)}{1 + \left(\frac{N^*}{N_0} - 1\right) e^{-\nu t}} \\ &= \chi \log \left(1 + \left(\frac{N^*}{N_0} - 1\right)\right) - \chi \log \left(1 + \left(\frac{N^*}{N_0} - 1\right) e^{-\nu t}\right).\end{aligned}$$

Approximating the log terms, we can write

$$\begin{aligned}\chi \log \left(\frac{N(t)}{N_0}\right) &= \chi \left(\frac{N^*}{N_0} - 1\right) - \chi \left(\frac{N^*}{N_0} - 1\right) e^{-\nu t} \\ &= \chi \left(\frac{N^*}{N_0} - 1\right) \left(1 - e^{-\nu t}\right).\end{aligned}$$

Observing that

$$\frac{N^*}{N_0} - 1 = \frac{x_0}{x^*} - 1,$$

these results yield (29).

Now consider

$$\begin{aligned}
\log u &= \varphi \log\left(\frac{Y_H}{P_H L}\right) + (1-\varphi)\log\left(\frac{Y_F}{P_F L}\right) \\
&= \varphi \log\left(\frac{Y_H}{P_H L}\right) + (1-\varphi)\log\left(\frac{\frac{1-\varphi}{\varphi} Y_H}{P_F L}\right) \\
&= \log\left(\frac{Y_H}{L}\right) - \varphi \log P_H + (1-\varphi)\log\left(\frac{1-\varphi}{\varphi P_F}\right) \\
&= \log\left(\frac{Y_H}{L}\right) - \varphi \log c(p) + \varphi \log T - \varphi \log\left(\frac{\epsilon}{\epsilon-1}\right) + (1-\varphi)\log\left(\frac{1-\varphi}{\varphi P_F}\right).
\end{aligned}$$

To simplify the notation, and without loss of generality, we set

$$(1-\varphi)\log\left(\frac{1-\varphi}{\varphi P_F}\right) + \varphi \log\left(N_0^{\chi} Z_0^{\theta}\right) - \varphi \log\left(\frac{\epsilon}{\epsilon-1}\right) \equiv 0.$$

This is just a normalization that does not affect the results. We then substitute the expression derived above into (1) and write

$$\begin{aligned}
U(p) &= \int_0^{\infty} e^{-\rho t}\left[\log \varphi\left(\frac{1-\xi(p)}{1-\xi(p)-\rho\beta} + \frac{p\Omega}{L}\right) - \varphi \log(c(p)) + \varphi g t\right] dt \\
&\quad + \varphi\left(\frac{\gamma}{\nu} + \chi\right)\Delta \int_0^{\infty} e^{-\rho t}\left(1 - e^{-\nu t}\right) dt.
\end{aligned}$$

Integrating, we obtain (31).

8.5 Proof of Lemma 1

Observe that

$$\epsilon_X^M \equiv -\frac{\partial \log M}{\partial \log P_M} = 1 - \frac{\partial \log S_X^M}{\partial \log P_M} = 1 - \frac{\partial S_X^M}{\partial P_M}\frac{P_M}{S_X^M}$$

so that $\epsilon_X^M \leq 1$ if

$$\frac{\partial S_X^M}{\partial P_M} = \frac{\partial}{\partial P_M}\left(\frac{P_M M}{P_M M + L_X}\right) \geq 0.$$

This in turn is true if

$$\left(1 - S_X^M\right)\frac{\partial (P_M M)}{\partial P_M} - S_X^M \frac{\partial L_X}{\partial P_M} \geq 0.$$

Recall now that total cost is increasing in P_M so that

$$\frac{\partial (P_M M)}{\partial P_M} + \frac{\partial L_X}{\partial P_M} > 0 \Rightarrow \frac{\partial (P_M M)}{\partial P_M} > -\frac{\partial L_X}{\partial P_M}.$$

It follows that

$$\frac{\partial L_X}{\partial P_M} \leq 0$$

is a sufficient condition for $\epsilon_X^M \leq 1$ since it implies that both terms in the inequality above are positive. The proof for $\epsilon_M^O \leq 1$ is analogous.

8.6 Proof of Proposition 5

(19) and (20) yield

$$\Omega \gtrless O \Leftrightarrow \frac{\Omega}{L} \gtrless \frac{1}{1-\xi(p)-\beta\rho}\frac{\xi(p)}{p}.$$

Differentiating (20) yields

$$\frac{d\log Y(p)}{dp} = -\frac{d\log(1-\xi(p)-\beta\rho)}{dp} = \frac{\xi'(p)}{1-\xi(p)-\beta\rho}.$$

It is useful to write

$$\xi'(p) = \frac{\xi(p)}{p}\left[\left(1-\epsilon_X^M(p)\right)S_M^O(p) + 1 - \epsilon_M^O(p)\right],$$

which shows that the sign of $\xi'(p)$ depends on the upstream and downstream price elasticities of demand and on the overall contribution of the commodity to manufacturing cost. Assume for example that $1-\epsilon_X^M(p) < 0$ and $1-\epsilon_M^O(p) > 0$ because the upstream, materials technology exhibits labor-commodity complementarity and the downstream, manufacturing technology exhibits labor-materials substitution. Then there exists a price p^v such that

$$\xi'(p) = \frac{\xi(p)}{p}\left[\left(1-\epsilon_X^M(p)\right)S_M^O(p) + 1 - \epsilon_M^O(p)\right] = 0.$$

That is,

$$\left(\epsilon_X^M(p) - 1\right)S_M^O(p) = 1 - \epsilon_M^O(p).$$

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