The Macroeconomics of Shadow Banking

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ABSTRACT

We propose a dynamic macroeconomic model of financial intermediation in an environment where households demand liquid assets. In contrast to the literature, intermediaries can issue equity without any friction. In normal times, intermediaries maximize liquidity creation by levering up the collateral value of their assets, a process we call shadow banking. A rise in uncertainty causes investors to demand liquidity in bad states. Intermediaries respond by simultaneously deleveraging and substituting toward safe liabilities; shadow banking effectively shuts down, prices fall and investment plummets. The model generates amplification via endogenous collateral runs, as well as flight to quality effects. The model's macroeconomic effects are consistent with a slow economic recovery even when—and especially when—intermediary capital is high. We analyze the impact of several policy interventions in the areas of unconventional monetary policy and regulatory reform.

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1. Introduction

Economic performance over the last ten years is the story of a boom, a bust, and a sluggish recovery. Recent research has emphasized the role of the financial sector in promoting the boom of 2003—2006 and amplifying the bust of 2007—2008. Yet the prolonged low-growth, high-unemployment slump that began in 2009 presents a challenge to the macro-finance storyline: Under the leading framework of Bernanke, Gertler, and Gilchrist (1999), financial restraints on growth should have lifted as corporations are awash with cash and household balance sheets have been repaired.[1] The more recent intermediary-centric models of He and Krishnamurthy (2012) and Brunnermeier and Sannikov (2013) also predict a return to normal as financial institutions are well-capitalized and their balance sheets are highly liquid.[2] The question arises, does finance play a continuing role in the slow recovery?

We propose a macroeconomic model that reproduces key features of the full 2003—2013 economic cycle—including the buildup of risk in the boom and the slow recovery in the bust—as a result of the rise and fall of shadow banking. We interpret shadow banking as the process of creating securities that are money-like in normal times but become illiquid in a crash. We call such securities shadow money; repurchase agreements and asset-backed

[1]According to the Financial Stability Oversight Council's (2013) Annual Report, "Corporate balance sheets remained strong in 2012. The ratio of liquid assets to total assets of the sector was near its highest level in more than 20 years, and debt has declined significantly relative to total assets over the past few years," and "Household deleveraging, low interest rates, and modest increases in employment and income reduced the household debt service ratio (the ratio of debt service payments to disposable personal income) to a historic low."

[2]From Bernanke (2013), "The comparison of today's bank capital levels with those at the time of the SCAP [May 2009] is particularly striking. Over the past four years, the aggregate tier 1 common equity ratio of the 18 firms that underwent the recent [stress] tests has more than doubled, from 5.6 percent of risk-weighted assets at the end of 2008 to 11.3 percent at the end of 2012–in absolute terms, a net gain of nearly $400 billion in tier 1 common equity, to almost $800 billion at the end of 2012.... The broader banking system–including both larger and smaller banks–has generally improved its liquidity position relative to pre-crisis levels" The tier 1 ratio stood at 8% over 2003—2006.

commercial paper provide useful examples. In our model investors use liquid assets to insure against shocks that force them to quickly liquidate part of their wealth. Shadow banking transforms illiquid productive assets into liquid securities that meet this demand.

A prolonged quiet period like the Great Moderation leads investors to anticipate low economic uncertainty. As uncertainty falls, collateral transformation becomes ever more profitable, the shadow banking sector enters a boom, and liquidity becomes abundant. Greater liquidity encourages households to save, raising asset prices, investment and growth. The effects are greatest for productive but risky assets whose collateral value is low. As a result, in quiet periods the financial sector drifts toward shadow banking and the economy drifts toward a riskier though more productive capital stock. Over time, this process builds up economic fragility.

A series of uncertainty shocks like the ones that began in July 2007 and culminated in September 2008 set off a chain of events in securities and asset markets.[3] Faced with greater uncertainty, investors demand securities that remain liquid in all circumstances. We call such securities money, in contrast to shadow money. Intermediaries respond by retiring shadow money from their balance sheets and replacing it with money, which requires deleveraging. Shadow banking markets dry up. The supply of liquidity falls, discount rates rise, and aggregate prices fall. A flight to quality effect pushes up the prices of safe assets as collateral becomes scarcer.

[3] In July 2007, two Bear Stearns hedge funds failed. We think of this episode as exemplifying the uncertainty shocks in our model. The Financial Crisis Inquiry Report (2011) refers to it as "a canary in the mineshaft," borrowing the phrase from a Wall Street insider. At the FOMC meeting in June, Boston Fed President Cathy Minehan also expressed a sense of uncertainty, "While the Bear Stearns hedge fund issue may well not have legs, the concerns regarding valuation of the underlying instruments do give one pause. Can markets adequately arrive at prices for some of the more exotic CDO tranches? What happens when the bottom falls out and positions thought to be at least somewhat liquid become illiquid? Is there a potential for this to spread and become a systemic problem?" (Federal Open Market Committee 2007).

Our model generates slow recoveries as the occurrence of a crash increases the perceived likelihood of future crashes due to learning.[4] As uncertainty grows, good collateral becomes ever scarcer and the economy's optimal capital mix swings towards safety. Transition to the new target is slowed by technological illiquidity. Investment in productive but risky projects plummets, depressing growth. Intermediaries hoard unproductive safe assets, issuing money to meet demand for crash-proof liquidity and equity to back it up; high levels of capital accompany low growth, not high. Even a highly liquid economy is slow-growing when liquidity is produced directly from safe assets rather than through collateral transformation of productive assets. A "fortress balance sheet", whether on the assets or liabilities side, is a sign of low liquidity transformation and results in low growth.

In contrast to the existing macro-finance literature, our model does not draw any distinction between inside and outside sources of funding. Intermediaries are free to change their capital structures at no cost as long as they have the assets to back their promises. They do so in response to investor demand for liquidity as reflected in funding spreads between money, shadow money, and equity. The financial sector is competitive so funding costs alone drive today's issuance and investment decisions which determine the economy's risk profile tomorrow. Asset prices capitalize the future path of the supply of liquidity. In short, intermediaries in our model are not special, intermediation is.

Our model generates amplification via endogenous "collateral runs", positive reinforcement between prices and haircuts (Brunnermeier and Pedersen 2009). Uncertainty shocks cause shadow banking to contract, reducing liquidity and raising discount rates. A collateral

[4]In our model, uncertainty jumps most from a low level, in the spirit of a "Minsky moment" (Minsky Hyman 1986). Since the collapse of Lehman Brothers in September 2008, there have been persistent fears of aftershocks, including European sovereign defaults, a break-up of the Euro currency, the U.S. debt ceiling crisis, and a slow-down in emerging markets.

run occurs when this rise in discount rates is accelerating so that as an asset's price today falls, its worst-case price tomorrow falls even more. This happens near the top of the credit cycle when uncertainty shocks get progressively larger yet prices remain exposed so that the near-term worst-case scenario for valuations is deteriorating; i.e. "the bottom is dropping out." In our model collateral runs are the result of shadow banking.

Central to our analysis is the endogenous evolution of the economy's sensitivity to uncertainty shocks, which arises from the liquidity transformation decisions of intermediaries. Shadow banking increases liquidity provision in low-uncertainty states, boosting asset prices and setting the economy on a path to a riskier but more productive capital mix. The increased liquidity and risky capital, however, cannot be supported when uncertainty rises and investor demand turns to crash-proof liquidity, even as intermediaries strive to absorb this demand by crash-proofing their balance sheets. By their actions, intermediaries cause prices and investment to increase at the top of the credit cycle while cushioning their fall at the bottom.

We use the model to examine the implications for liquidity provision of various policy interventions. Our analysis shows that large-scale asset purchases (LSAPs) can help speed the transition to a new equilibrium, stabilizing markets. A mis-timed intervention, an early exit, or talk of an early exit (a.k.a. "tapering"), can each undo the effectiveness of LSAPs. Other policies like "Operation Twist" can be counter-productive as they syphon off long-duration safe assets that act as uncertainty hedges on intermediary balance sheets. Regulatory reform in the spirit of the "Volcker rule" or the Glass-Steagall Act, which we interpret as the mandatory segregation of safe and risky assets, can reduce efficiency as flight-to-quality effects create complementarities between assets in the production of liquidity services. Stricter capital requirements like those proposed by the Basel Committee also reduce the supply of liquidity, and we study their ex-ante announcement effects.

The rest of the paper is organized as follow: Section 2 reviews the literature, section 3 presents the model, section 4 presents results from a numerical implementation, section 5 contains policy analysis, and section 6 concludes.

2. Related literature

Our paper lies at the intersection of the traditional banking literature and the growing macro-finance literature. Our starting point are the insights of Diamond and Dybvig (1983) and Gorton and Pennacchi (1990). As in Diamond and Dybvig (1983), financial intermediaries add value only to the extent that their liabilities are more liquid than their assets. As in Gorton and Pennacchi (1990), liabilities are liquid only if their valuations are insensitive to information.Our principal contribution is to incorporate this liquidity provision view of banking into a macro-finance framework.

So far, the emphasis in the macro-finance literature has been on the scarcity of entrepreneurial or intermediary capital in amplifying and propagating fundamental shocks (for a recent survey, see Brunnermeier, Eisenbach, and Sannikov 2012). In Bernanke and Gertler (1989) and Bernanke, Gertler, and Gilchrist (1999), funding costs are decreasing in entrepreneurial net worth. In our paper, equity markets are frictionless so net worth plays no role, but issuing liquid debt lowers funding costs as a result of household demand for liquidity. The two approaches produce opposite results: funding costs in our model are minimized when equity is low, not high.

In Kiyotaki and Moore (1997), external funding is limited by the availability of collateral. Similar constraints arise in Geanakoplos (2003), Gertler and Kiyotaki (2010), Gertler and Kiyotaki (2013), Rampini and Viswanathan (2012), Brunnermeier and Sannikov (2013), Sannikov (2013), Maggiori (2013), and Simsek (2013). In these papers, collateral values limit debt and equity issuance alike, whereas here they only limit issuance of liquid debt. Our collateral constraint therefore affects funding costs rather than funding amounts, and it allows us to make equity issuance costless.

The collateral constraint (i.e. limited liability), the household demand for liquidity, and the either-or nature of the liquidity attribute (a claim is either liquid or not) break the assumptions of Modigliani-Miller (Modigliani and Miller 1958) and give intermediaries a role in tranching assets.[5] DeAngelo and Stulz (2013) applies this same logic to show that high intermediary leverage can be socially valuable. In contrast, the macro-finance literature generally relies on differences in preferences (Geanakoplos 2003, e.g.), expertise (e.g. Kiyotaki and Moore 1997), or on limited participation (He and Krishnamurthy 2012, e.g.). We view our framework as best suited specifically for analyzing the liquidity transformation role of banks.

Our model delivers a fully dynamic and endogenous version of the collateral run amplification mechanism in Brunnermeier and Pedersen (2009). We find that collateral runs occur only after a shadow banking boom, when liquidity provision is maximally stretched relative to the availability of collateral, making prices highly exposed to uncertainty shocks.

Our interpretation of shadow banking is related to Allen and Gale (1998) who show that

[5] Limited liability must also apply to households' direct capital holdings (though not their securities holdings) so they cannot short capital to synthetically create more assets for intermediaries to tranche.

aggregate uncertainty can make state-contingent default on deposits efficient by increasing the supply of liquidity ex ante. Whereas Allen and Gale (1998) focus on a mixing equilibrium in which some depositors are made whole, we find our distinction between crash-sensitive and crash-proof debt to have a close mapping to the modern commercial and shadow banking sectors.

Recent macro models based on the information sensitivity view of liquidity (Gorton and Pennacchi 1990) include Gorton and Ordoñez (2012) and Dang, Gorton, and Holmström (2010), which feature rich learning dynamics and focus on amplification during crises.[6] Instead of incorporating learning which creates time variation in the liquidity of a given asset, we invoke uncertainty shocks which cause demand shifts across assets with different state-contingent liquidity profiles. This allows us to derive a broad set of results across financial markets.

By introducing money into a macro-finance framework, our paper relates to the strand of the monetary economics literature that studies the origins of the demand for money. This literature has a long tradition and it is beyond our scope to discuss it here. Williamson and Wright (2010) offer a perspective on the renewed interest in the microfoundations of the demand for money.

Robin, Hanson, and Stein (2012), Krishnamurthy and Vissing-Jorgensen (2012b), Caballero and Farhi (2013), and Bansal, Coleman, and Lundblad (2010) also study environments in which households have preferences for safe or liquid securities supplied by financial intermediaries.[7] On the methodological side, these papers employ two-period frameworks

[6]Kurlat (2012) and Bigio (2013) study a related form of amplification driven by asymmetric information.

[7]Holmström and Tirole (1998) and Kiyotaki and Moore (2012) model the liquidity needs of the production sector by building liquidity into the production function. Our model is easily modified to take this perspective.

that do not speak to the dynamic relation between liquidity, uncertainty, and pledgability that is the focus here. On the conceptual side, we distinguish between fully-safe (money) and near-safe (shadow money) securities and focus on the trade-offs between them.

On the policy side, the unprecedented actions of central banks in the wake of the 2008 financial crisis have sparked research interest in unconventional monetary policy and financial stability. Gertler and Karadi (2011) interpret large-scale asset purchases as a second-best intermediation service provided by the central bank. In Kiyotaki and Moore (2012), entrepreneurs are constrained by their money holdings so that greater liquidity increases investment. We analyze these interventions from the perspective of the liquidity transformation role of the financial sector. Our model is consistent with the finding in Krishnamurthy and Vissing-Jorgensen (2013) that central bank purchases of risky assets are more effective than purchases of long-dated government debt.

Recent empirical work has examined the cyclicality of financial sector leverage. Adrian and Shin (2010) find that broker-dealer leverage is pro-cyclical, whereas the opposite seems to be the case for commercial banks (He, Khang, and Krishnamurthy 2010). Gorton and Metrick (2011) show that margins in repo markets increased sharply during the financial crisis. In our model, as shadow banking markets dry up in a downturn, institutions like broker-dealers that depend on them for funding are forced to de-lever. This effect is muted for money-only issuers whose funding is stable (e.g. commercial banks).

In our model as in Adrian and Boyarchenko (2012), aggregate financial sector leverage is pro-cyclical, whereas in models where the key role is played by intermediary net worth, the opposite is true.[8] Consistent with the view that leverage is low in bad times, Adrian,

[8]Leverage is typically pro-cyclical in models with VaR-type constraints and counter-cyclical volatility like

Etula, and Muir (2011) show that exposure to a leverage growth factor carries a positive risk premium in the cross section of equities. Our results are also consistent with the evidence in Adrian, Moench, and Shin (2010) that periods of high leverage growth are followed by lower expected returns.

Our model where liquid assets trade at a premium generates a securities market line that flattens out to the right, as in the data (Frazzini and Pedersen 2010). Our model predicts further that this "low-risk anomaly" should strengthen during periods of high uncertainty when liquidity is scarce, but we are not aware of empirical evidence along these lines. Baker and Wurgler (2013) point out that as a result of the low-risk anomaly, capital requirements can potentially increase the funding costs of banks. Our model allows us to quantify the total effect of capital requirements, taking into account the level shift of the securities market line across equilibria in addition to the slope.

Our model is also consistent with the result in Krishnamurthy and Vissing-Jorgensen (2012a) that the supply of government debt affects the spread between money (Treasurys) and shadow money (AAA-rated commercial paper), and with the result in Krishnamurthy and Vissing-Jorgensen (2012b) that government debt crowds out private money. As documented empirically by Krishnamurthy and Vissing-Jorgensen (2012b), in our model financial crises occur when the supply of shadow money is high because it is only then that cash flow shocks have a substantial impact on asset prices.

Brunnermeier and Pedersen (2009).

3. Model

We develop a dynamic model of financial intermediation built on two frictions. Households experience liquidity events during which they perceive a higher marginal value of consumption, but their spending is constrained by their liquid holdings. This creates a precautionary demand for liquid assets but a limited liability constraint on the part of intermediaries limits their supply. Absent either friction, the economy exhibits constant asset prices and high levels of investment and growth.

3.1. Technology

Consider an economy set in continuous time $t \geq 0$. Agents are endowed with production technologies that embed a tradeoff between productivity and risk. Technology A is high-productivity but risky, technology B is low-productivity but safe. Let k^a and k^b denote the efficiency units of capital devoted to each technology and write

$$dk_t^a = \mu\left(k_t^a + k_t^b\right) + k_t^a\left[\phi\left(\iota_t^a\right) - \delta\right]dt - k_t^a\kappa^a\left(dN_t - \lambda_t dt\right) \tag{1}$$

$$dk_t^b = \mu\left(k_t^a + k_t^b\right) + k_t^b\left[\phi\left(\iota_t^b\right) - \delta\right]dt - k_t^b\kappa^b\left(dN_t - \lambda_t dt\right), \tag{2}$$

where ι^a and ι^b are investment rates (as a proportion of the stock of each type of capital), μ is a constant level inflow of capital that does not require investment, ϕ is an investment adjustment cost function, κ^a and κ^b reflect cash flow risk, and δ is depreciation. The level inflow term $\mu\left(k_t^a + k_t^b\right)$ serves a technical purpose.[9] To keep things simple, we assume this

[9]Specifically, this term makes the boundaries of our capital mix state variable reflecting rather than absorbing. The effect is a more gradual change in prices along the capital mix dimension. We keep μ small

inflow accrues to the aggregate capital stock but not inside the portfolios of investors in our model. For example, it can stand in for the arrival of new workers with productivity drawn from a fixed distribution. Output is given by

$$y_t = \gamma^a k_t^a + \gamma^b k_t^b, \tag{3}$$

where $\gamma^a > \gamma^b$ reflect the productivity of type-A and type-B capital. Since households are risk-neutral, first-best investment favors technology A over B.

The stochastic exposure of capital is driven by a Poisson jump process N_t with time-varying perceived intensity λ_t modeled in the next section. We require jumps in order to generate a wedge between the current value and the collateral value of an asset. In a crash, $dN_t = 1$, a proportion κ^a of type-A capital and κ^b of type-B capital are destroyed. Type-A capital is riskier, $\kappa^a > \kappa^b$. The crash exposures are compensated so that higher crash risk λ_t does not affect expected growth (see Brunnermeier and Sannikov 2013, for a similar approach). Once again, since households are risk-neutral, first-best capital prices are independent of λ_t.

We think of technology A as representing new, high-potential but untested investment projects, versus the storage-like safe but unproductive technology B.

The capital evolution dynamics (1) and (2) feature two forms of technological illiquidity: capital cannot be directly re-deployed across technologies and investment is subject to adjustment costs. The former generates persistence: today's investment affects tomorrow's asset mix. This will result in build-ups of risky capital during prolonged booms and

in calibrations.

retrenchment at the expense of growth during downturns.

The latter form of technological illiquidity, adjustment costs, formally follows Bernanke, Gertler, and Gilchrist (1999). As always, it leads to variation in Tobin's q and by extension asset prices, but its ultimate role here is different. In Bernanke, Gertler, and Gilchrist (1999), asset prices amplify shocks to net worth, creating a static feedback loop. Here, net worth is unimportant as equity issuance is costless. Instead, the potential for even lower asset prices in the future depresses collateral values today, generating dynamic feedback.

As a result of technological illiquidity, the economy's asset mix becomes a state variable. Let $\chi = k^a / \left(k^a + k^b\right)$ and apply Ito's Lemma using (1) and (2):

$$\begin{aligned} d\chi_t &= \mu \left(1 - 2\chi_t\right) dt + \chi_t \left(1 - \chi_t\right) \left[\phi\left(\iota_t^a\right) - \phi\left(\iota_t^b\right) + \lambda_t \left(\kappa^a - \kappa^b\right)\right] dt \\ &\quad -\chi_t \left(1 - \chi_t\right) \left[\frac{\kappa^a - \kappa^b}{\chi_t \left(1 - \kappa^a\right) + \left(1 - \chi_t\right)\left(1 - \kappa^b\right)}\right] dN_t. \end{aligned} \tag{4}$$

Absent a crash, the risky asset share χ drifts according to relative investment in the two technologies, adjusted by the level-inflow and crash-compensating terms.[10] During a crash, the risky asset share falls as more of the risky capital is wiped out. In other words, crashes shift the economy's capital stock towards safety. This produces a dampening effect: uncertainty shocks absent a crash have a stronger impact on asset prices.[11]

[10] Notice that the level-inflow term pulls χ away from the boundaries, making it better-behaved.

[11] There are several ways to remove the dampening effect: (1) introduce a capital composition shock that converts some fraction of the safe capital into risky capital when a crash hits; and (2) a crash need not destroy any capital in the aggregate, it is enough that it destroy some units of capital at the expense of others (a dispersion shock) as long as investors cannot hold fully diversified portfolios. We resort to the latter approach in sections 4.4 (Flight to quality) and 5.3 (Operation Twist).

3.2. Uncertainty

As noted earlier, risk neutrality and the compensation for cash-flow risk imply that crash intensity λ_t is irrelevant in a frictionless economy; any role for λ_t is the result of the frictions that we introduce, specifically its effect on collateral values. We think of λ_t as a measure of uncertainty, and we have constructed the model so as to emphasize the interplay between uncertainty and financial frictions.

To provide more structure, we interpret λ_t as the outcome of a filtering problem: Suppose that the jump process N_t has a true, latent intensity $\tilde{\lambda}_t$ that follows a two-state Markov chain,

$$\tilde{\lambda} \in \{\lambda_L, \lambda_H\} \tag{5}$$

with infinitesimal generator

$$q = \begin{bmatrix} -q_H & q_H \\ q_L & -q_L \end{bmatrix}. \tag{6}$$

Households learn about $\tilde{\lambda}$ from the occurrence of crashes as well as from an additional diffusive signal with precision $1/\sigma_e$. In Appendix A we show how to compute the innovations of the filtered process $\lambda_t = E_t\left[\tilde{\lambda}_t\right]$:

$$\frac{d\lambda_t}{(\lambda^H - \lambda_t)(\lambda_t - \lambda^L)} = \left(-\frac{q^H}{\lambda^H - \lambda_t} + \frac{q^L}{\lambda_t - \lambda^L} - 1\right) dt + \frac{1}{\sigma_e} dB_t + \frac{1}{\lambda_t} dN_t. \tag{7}$$

We note three properties of the filtered crash intensity. First, it tends to drift down, so crashes are less likely following a prolonged quiet period. This leads to endogenous build-ups of leverage and asset risk (the volatility paradox in the language of Brunnermeier and Sannikov (2013)). Second, crash realizations cause λ_t to jump up. As a result, crashes are followed by persistent de-leveraging, a shift towards safe capital, and low growth. Third, λ jumps most from low levels (in particular, from the neighborhood around $\sqrt{\lambda^L \lambda^H} < \frac{1}{2}\left(\lambda^L + \lambda^H\right)$). It follows that crashes after prolonged booms have the strongest effects on funding costs, the optimal mix of securities, asset prices, and investment.[12]

3.3. Households

The economy is populated by households with risk-neutral preferences over consumption who are subject to liquidity shocks of Diamond and Dybvig (1983) variety. Specifically, agents maximize

$$V_t = \max E_t \left[\int_t^\infty e^{-\rho(s-t)} W_s \left(c_s ds + \psi C_s dN_s^h \right) \right], \tag{8}$$

where N_t^h is a Poisson jump process with constant intensity h that is uncorrelated with N_t. A realization $dN_t^h = 1$ signifies the onset of a liquidity event, defined as an opportunity to consume a fraction of wealth C_t up to a limit ξ_t at a high marginal value $\psi > 1$. To generate a concave demand for liquidity, we assume ξ_t is random, specifically i.i.d. exponential with mean $1/\eta$. For example, a household may have to make a large payment whose size is

[12] An alternative way to model uncertainty shocks would be to vary crash size instead of frequency. In this case, collateral values would also fall following a rise in uncertainty, not endogenously via prices, but directly via cash-flow risk. We choose not to pursue this route for three reasons. First, learning gives us a natural connection between the occurrence of a crash and crash intensity. Second, by making capital scarcer following a crash, a higher crash size would disproportionately reduce the issuance of fully-safe securities instead of near-safe securities, contrary to the evidence (e.g. Gorton and Metrick 2011). Third, we are particularly interested in endogenous risk.

uncertain ahead of time.

Liquidity-event consumption is further constrained by the household's liquid asset holdings w_t^l[13], which we discuss in detail once we discuss the security markets

$$C_t \leq \min\left\{\xi_t, w_t^l\right\}. \tag{9}$$

A household experiencing a liquidity event must quickly sell off a substantial part of its portfolio. As emphasized by Gorton and Pennacchi (1990), only certain securities, in particular those with a low sensitivity to private information, can be sold quickly and in large quantities without incurring the costs of price impact. We designate which securities qualify as liquid and under what conditions as we introduce them.

A household facing a liquidity event experiences a loss with probability $1-e^{-\tau\lambda_t}$, in which case some of the securities in the household's portfolio lose their liquidity attribute. We call such securities shadow money in contrast to money which never ceases to be liquid. Both are formally defined below.

We have linked the probability of a loss during a liquidity event to the probability of an aggregate crash. This link is important for our results: it generates a fall in the demand for shadow money as uncertainty rises. To motivate it, we interpret a liquidity event as in fact lasting τ periods, with a burst of consumption coming at the end. The probability that a crash occurs during τ periods would indeed be $1-e^{-\tau\lambda}$ if λ were to remain constant.

[13] We are ruling out a market for individual liquidity insurance. Households must self-insure by amassing a precautionary buffer of liquid assets. A form of liquidity insurance can be introduced via collateral rehypothecation.

Since in practice it does not remain constant, our formulation represents a simplification that preserves the intuition of a positive-duration liquidity event while maintaining tractability. This allows us to study the interaction between liquidity demand and uncertainty.

3.4. *Securities and asset markets*

There are two aggregate shocks in our economy and three financial securities used to span them. We call these money, shadow money, and equity in order to convey their risk and liquidity profiles. We will show that the optimal provision of liquidity generally requires all three securities, and that the three are indeed sufficient.

We write the innovations to the return processes of money, shadow money, and equity as

$$dm_t = \mu_{m,t} dt \tag{10}$$

$$dr_t = \mu_{r,t} dt - \kappa_{r,t} \left(dN_t - \lambda_t dt\right) \tag{11}$$

$$dR_t = \mu_{R,t} dt - \sigma_{R,t} dB_t - \kappa_{R,t} \left(dN_t - \lambda_t dt\right), \tag{12}$$

respectively. All drifts and loadings are determined endogenously. Money is fully safe. Shadow money is safe at all times except in a crash when it takes a loss. Equity absorbs all normal-times risk and the remaining crash risk.

Given risk neutrality, volatility is only relevant insofar as it relates to liquidity. Guided by the information-sensitivity literature (Gorton and Pennacchi 1990), we call a security liquid if the sensitivity of its expected return with respect to private information does not exceed a fixed upper bound that we think of as the cost of information. Formally,

Definition 1. *An asset with return process dS_t is liquid if and only if*

$$\frac{1}{dt}\left(\frac{E_t\left[dS_t\middle|\tilde{\lambda}_t\right]-E_t\left[dS_t\middle|\lambda_t\right]}{\tilde{\lambda}_t-\lambda_t}\right) \;\; \leq \;\; \overline{\kappa}, \tag{13}$$

where $\overline{\kappa} < 1$ is a fixed constant.

In Appendix B, we derive microfoundations for this definition by examining the information acquisition problem of an arbitrageur. Intuitively, a high sensitivity to private information gives agents— households and intermediaries alike—a strong incentive to become privately informed. If this incentive exceeds the cost of information, an adverse selection problem can arise in either the secondary or primary securities markets. For example, an informed intermediary could time its issuance to earn abnormal profits, or an informed household could feign a liquidity event in an attempt to offload overpriced securities. Adverse selection would limit market liquidity, making it difficult to quickly exit a position during a genuine liquidity event.

It is clear given our definition that money is liquid. For shadow money to be liquid, substituting (11) into (13), it must be the case that

$$\kappa_{r,t} \;\; \leq \;\; \overline{\kappa}. \tag{14}$$

To be liquid ex ante, shadow money must not suffer too large a loss ex post. Indeed, intermediaries will choose to respect this cap in order to take advantage of the liquidity premium that households are willing to pay. The cap will also bind from below as intermediaries strive to maximize liquidity provision.

We will show that equity is not liquid within our framework. Normal-times risk $\sigma_{R,t} > 0$ makes the collateral cost of "liquid equity" exceed that of shadow money: applied to equity, (13) becomes $\sigma_{R,t} + \kappa_{R,t} \leq \overline{\kappa}$. This means that intermediaries can always increase liquidity provision by offloading all normal-times risk (and some crash risk as necessary) to equity and issuing more shadow money. We will also show that $\kappa_R = 1$ in equilibrium so equity is always wiped out in a crash.

We assume that shadow money ceases to be liquid following a loss.[14] For example, recovery may be prolonged due to a "failure to deliver", extended legal proceedings, or the defaulted security may be subject to greater adverse selection. Letting w_t^m and w_t^r be the weights of money and shadow money in a household's portfolio, the household's liquid assets are thus $w_t^l = w_t^m + w_t^r$ absent a loss (probability $e^{-\tau\lambda_t}$) and $w_t^l = w_t^m$ given a loss (probability $1 - e^{-\tau\lambda_t}$).

Finally, there is also a frictionless market for existing capital, with price dynamics

$$\frac{d\pi_t^a}{\pi_t^a} = \mu_{\pi,t}^a dt - \sigma_{\pi,t}^a dB_t - \kappa_{\pi,t}^a \left(dN_t - \lambda_t dt\right) \tag{15}$$

$$\frac{d\pi_t^b}{\pi_t^b} = \mu_{\pi,t}^b dt - \sigma_{\pi,t}^b dB_t - \kappa_{\pi,t}^b \left(dN_t - \lambda_t dt\right). \tag{16}$$

We solve for π^a and π^b in equilibrium, and they determine investment and growth.

Although there are no differences in expertise or preferences between households and intermediaries, holding capital directly is not efficient in this economy. Here, intermediaries tranche, and tranching increases the supply of liquidity and hence growth and welfare. As

[14] This assumption drives a wedge between the amount of collateral firms must use to back shadow money and the amount of liquidity households derive from it. If instead the recovered amount remains liquid, the leverage advantage of shadow money over money always exceeds the relative valuations of households. This means that money is not produced except in states where equity is not required in the first place.

a stark example, even a cash-flow safe asset (e.g. $\kappa^b = 0$) cannot directly back money as fluctuations in household demand for liquidity introduce volatility into its price ($\sigma^b_{\pi,t}, \kappa^b_{\pi,t} > 0$). More broadly, we will show that tranching is always necessary to achieve the optimal provision of liquidity.

3.5. Intermediaries

Intermediaries buy capital, set investment, and issue securities to maximize profits. Although we have combined the investment and liquidity provision functions under one roof, these can easily be separated as investment here is static.[15] We think of the intermediaries as financial institutions since their key role is the tranching of assets.

In contrast to the literature, in this model intermediaries can issue and repurchase equity without any frictions. To formulate the intermediary's maximization problem, we distinguish between existing and new equity. Specifically, the intermediary maximizes the value of existing equity:

$$\rho V_t dt = \max_{k^a_t, k^b_t, \iota^a_t, \iota^b_t, w^m_t, w^r_t} \left[\left(\gamma^a - \iota^a_t\right) k^a_t + \left(\gamma^b - \iota^b_t\right) k^b_t \right] dt + E_t \left[dV_t \right], \tag{17}$$

where k^a_t and k^b_t are units of capital, ι^a_t, and ι^b_t are investment rates, and w^m_t and w^r_t are the ratios of money and shadow money liabilities to assets on the intermediary's balance sheet. Neither can be negative, $w^m_t, w^r_t \geq 0$.

The value of purchased capital $A_t = \pi^a_t k^a_t + \pi^b_t k^b_t$ pins down the size of the intermediary's

[15] Indeed we do so in section 5.3 (Operation Twist) when introducing public debt where the "entrepreneur" is the government.

balance sheet. Any discrepancy between A_t and the intermediary's money, shadow-money, and existing equity must be made up with new equity, which is compensated at the equilibrium rate of return. The evolution of existing equity V_t is given by

$$dV_t = dA_t - A_t\left(w_t^m dm_t + w_t^r dr_t\right) - \left[A_t - A_t\left(w_t^m + w_t^r\right) - V_t\right] dR_t. \tag{18}$$

The change in the value of existing equity is equal to the change in the value of total assets minus the payouts to money, shadow money, and new equity.

The intermediary faces a limited liability constraint,

$$A_t - A_t\left(w_t^m + w_t^r\right) \geq -\left[dA_t - A_t\left(w_t^m dm_t + w_t^r dr_t\right)\right]. \tag{19}$$

The left side is firm equity, old plus new (assets minus debt), and the right side is the loss (minus the gain) of firm equity. New equity cannot be used to pay for previously incurred losses; assets must be sufficient to cover existing liabilities after any write-offs.

The limited liability constraint prevents firms from issuing unlimited amounts of money and shadow money backed by future outside cash infusions. It can be microfounded with lack of commitment: an infusion into an insolvent firm is not optimal ex post (there are no reputation or distress costs).

The limited liability constraint (19) must hold state by state. Specifically, it must hold both during normal times and in a crash. Let $\kappa_{A,t}$ be the fraction of asset value lost in a crash. Rearranging (19), we have

$$w_t^m + w_t^r \leq 1 \tag{20}$$

$$w_t^m + w_t^r \leq 1 - \kappa_{A,t} + w_t^r \kappa_{r,t}. \tag{21}$$

By writing (20) as a weak inequality, we are implicitly allowing ex post cash infusions on the order of the normal-times shocks dB_t.[16]

We call $1 - \kappa_{A,t}$ the collateral value of the assets. Equation (21) shows that one dollar of collateral can back one dollar of money or $1/(1 - \kappa_{r,t}) > 1$ dollars of shadow money. Applying Ito's Lemma to A_t,

$$1 - \kappa_{A,t} = \frac{\pi_t^a k_t^a}{\pi_t^a k_t^a + \pi_t^b k_t^b} \left(1 - \kappa_{\pi,t}^a\right)(1 - \kappa^a) + \frac{\pi_t^b k_t^b}{\pi_t^a k_t^a + \pi_t^b k_t^b} \left(1 - \kappa_{\pi,t}^b\right)\left(1 - \kappa^b\right). \tag{22}$$

Collateral values are driven by cash-flow risk which changes with the asset mix, and by price risk which arises endogenously.

3.6. *Households and the demand for liquidity*

Households use liquid securities to self-insure against liquidity events, whose random size generates a concave demand for overall liquidity. As crashes become more likely, households seek securities that remain liquid in all states. Demand for money rises and demand for shadow money falls, a flight to quality effect.

[16] This assumption means that in the region of the state-space where all crash losses are optimally covered by shadow-money, equity can be reduced to zero instead of just to near-zero. It has no effect elsewhere.

The household optimization problem can be formulated as

$$\rho V_t dt = \max_{c_t, C_t, w_t^m, w_t^r} W_t \left(c_t dt + \psi E_t \left[C_t dN_t^h \right] \right) + E_t \left[dV_t \right] \tag{23}$$

subject to the dynamic budget constraint

$$\frac{dW_t}{W_t} = -\left(c_t dt + C_t dN_t^h \right) + dR_t + w_t^m \left(dm_t - dR_t \right) + w_t^r \left(dr_t - dR_t \right) \tag{24}$$

and the liquidity constraint

$$C_t \leq \min \left\{ \xi_t, w_t^l \right\}. \tag{25}$$

Risk neutrality imposes $V_t = W_t$. Non-liquidity consumption c_t is supplied elastically. Substituting and simplifying,

$$\begin{aligned} \rho dt &= \max_{c_t, C_t, w_t^m, w_t^r} c_t dt + \psi E_t \left[C_t dN_t^h \right] + E_t \left[\frac{dW_t}{W_t} \right] & (26) \\ &= \max_{C_t, w_t^m, w_t^r} \left(\psi - 1 \right) E_t \left[C_t dN_t^h \right] + \left[\mu_{R,t} + w_t^m \left(\mu_{m,t} - \mu_{R,t} \right) + w_t^r \left(\mu_{r,t} - \mu_{R,t} \right) \right] dt. & (27) \end{aligned}$$

Since $\psi > 1$, the liquidity constraint always binds, $C_t = \min \left\{ \xi_t, w_t^l \right\}$. Households always consume as much as possible when a high-value opportunity arises. Imposing the liquidity constraint and evaluating the expectation under the exponential distribution,

$$\begin{aligned} \rho = \max_{w_t^m, w_t^r} & \frac{h}{\eta} \left(\psi - 1 \right) \left[e^{-\tau \lambda_t} \left(1 - e^{-\eta (w_t^m + w_t^r)} \right) + \left(1 - e^{-\tau \lambda_t} \right) \left(1 - e^{-\eta w_t^m} \right) \right] & (28) \\ & + \mu_{R,t} + w_t^m \left(\mu_{m,t} - \mu_{R,t} \right) + w_t^r \left(\mu_{r,t} - \mu_{R,t} \right). \end{aligned}$$

The optimality conditions pin down spreads in security markets:

$$\mu_{R,t} - \mu_{m,t} = h(\psi - 1)\left[e^{-\tau\lambda_t} e^{-\eta(w_t^m + w_t^r)} + \left(1 - e^{-\tau\lambda_t}\right) e^{-\eta w_t^m}\right] \tag{29}$$

$$\mu_{R,t} - \mu_{r,t} = h(\psi - 1)\left[e^{-\tau\lambda_t} e^{-\eta(w_t^m + w_t^r)}\right]. \tag{30}$$

Households equate the marginal benefit of liquidity in all states with the premium for money and the marginal benefit of liquidity in normal times with the premium for shadow money. Clearly, both money and shadow money trade at a premium over equity, and money trades at a premium over shadow money:

$$\mu_{r,t} - \mu_{m,t} = h(\psi - 1)\left(1 - e^{-\tau\lambda_t}\right) e^{-\eta w_t^m}. \tag{31}$$

The safety spread $\mu_{r,t} - \mu_{m,t}$ reflects the marginal value of liquidity in a crash. All else equal, a rise in uncertainty λ_t increases the safety spread. In the next section, we will see that intermediaries respond by issuing more money and less shadow money, which requires de-leveraging.

Substituting the optimality conditions (29) and (30) into the HJB equation gives the cost of equity:

$$\begin{aligned} \mu_{R,t} = \rho - \frac{h}{\eta}(\psi - 1)\Big[1 - e^{-\tau\lambda_t}\left(1 + \eta\left(w_t^m + w_t^r\right)\right) e^{-\eta(w_t^m + w_t^r)} \\ - \left(1 - e^{-\tau\lambda_t}\right)\left(1 + \eta w_t^m\right) e^{-\eta w_t^m}\Big]. \end{aligned} \tag{32}$$

The cost of equity is decreasing in overall liquidity. Greater liquidity encourages saving by raising the opportunity for consumption during a liquidity event when it is most valuable. Greater savings in turn push down the cost of capital, including equity.

All else equal, a rise in uncertainty raises the cost of capital as liquidity becomes scarcer. Intermediaries respond by adjusting their capital structure. Nevertheless, higher discount rates result in lower prices, investment, and growth.

3.7. Intermediaries and the supply of liquidity

The intermediary's problem can be split into two parts: the capital structure decision for a given level of asset risk, and the capital purchase and investment decisions for a given funding policy. We characterize the liabilities side of the problem in this section followed by the asset side in the next section.

We can state the intermediary's capital structure problem more broadly as a liquidity provision problem, and then show how to implement the optimal liquidity provision policy using money, shadow-money, and equity. This allows us to proceed without assuming that equity is illiquid, but instead showing it as a result.

Consider an intermediary whose assets have collateral value $1 - \kappa_{A,t}$. Let l_t^n and l_t^c be the amount of liquidity the intermediary delivers in normal times and in a crash. The intermediary's liquidity provision problem can be formulated in terms of securities spreads which measure the marginal benefits of liquidity provision:

$$\max_{l_t^n, l_t^c} \; l_t^c \left(\mu_{R,t} - \mu_{m,t}\right) + \left(l_t^n - l_t^c\right)\left(\mu_{R,t} - \mu_{r,t}\right) \tag{33}$$

subject to the constraints

$$\begin{aligned} l_t^c &\geq 0, \quad & l_t^n &\leq 1-\kappa_{A,t}+\left(l_t^n-l_t^c\right)\kappa_{r,t} \\ l_t^n &\geq l_t^c, \quad & l_t^n &\leq 1. \end{aligned} \tag{34}$$

Crash-proof liquidity l_t^c receives a full funding cost reduction that includes the safety premium. The incremental liquidity supplied absent a crash $l_t^n - l_t^c$ does not. Crash-proof liquidity must be non-negative and it cannot exceed normal-times liquidity. In turn, normal-times liquidity cannot exceed asset value, and it is also constrained by the availability of collateral after write-offs.

We summarize the solution of the liquidity provision problem with the following proposition.

Proposition 1. *The intermediary optimally sets $\kappa_{r,t} = \overline{\kappa}$. For a given level of asset risk $\kappa_{A,t}$, the intermediary's optimal liquidity provision policy is characterized by*

i. $l_t^n = (1-\kappa_{A,t})/(1-\overline{\kappa})$ *and* $l_t^c = 0$ *if* $\kappa_{A,t} \geq \overline{\kappa}$ *and* $\kappa_{A,t} > 1 - \frac{1-\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$*;*

ii. $l_t^n = 1-\kappa_{A,t}$ *and* $l_t^c = 1-\kappa_{A,t}$ *if* $\lambda_t > -\frac{1}{\tau}\log\left(1-\overline{\kappa}\right)$*;*

iii. $l_t^n = 1$ *and* $l_t^c = 1-\kappa_{A,t}/\overline{\kappa}$ *if* $\kappa_{A,t} \leq \overline{\kappa}$*; and* $\kappa_{A,t} < \frac{\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$*; and*

iv. $l_t^n = 1-\kappa_{A,t}+\frac{\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$ *and* $l_t^c = 1-\kappa_{A,t}-\frac{1-\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$ *otherwise.*

Proof. See Appendix C. □

Intermediaries always make full use of their collateral in order to take full advantage of the premia households are willing to pay for liquid assets.

Case (i) occurs when uncertainty is low enough so that crash-proof liquidity is not worth its high collateral cost. In this case all collateral is pledged to deliver normal-times liquidity. By contrast, under case (ii) high uncertainty raises demand for crash-proof liquidity so there is no spare collateral to generate additional normal-times liquidity. Under case (iii), low asset risk allows intermediaries to deliver normal-times liquidity equal to the full value of their assets. Finally, in case (iv) the marginal rate of substitution between normal-times and crash-proof liquidity equals the leverage advantage of supplying normal-times liquidity over crash-proof liquidity.

We can now characterize the intermediary's optimal capital structure:

Proposition 2. *The intermediary's optimal liquidity provision policy is implemented by*

i. $w_t^m = 0$ *and* $w_t^r = (1-\kappa_{A,t})\,/\,(1-\overline{\kappa})$ *if* $\kappa_{A,t} \geq \overline{\kappa}$ *and* $\kappa_{A,t} > 1 - \frac{1-\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$*;*

ii. $w_t^m = 1-\kappa_{A,t}$ *and* $w_t^r = 0$ *if* $\lambda_t > -\frac{1}{\tau}\log(1-\overline{\kappa})$*;*

iii. $w_t^m = 1-\kappa_{A,t}/\overline{\kappa}$ *and* $w_t^r = \kappa_{A,t}/\overline{\kappa}$ *if* $\kappa_{A,t} \leq \overline{\kappa}$ *and* $\kappa_{A,t} < \frac{\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$*; and*

iv. $w_t^m = (1-\kappa_{A,t}) - \frac{1-\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$ *and* $w_t^r = \frac{1}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$ *otherwise.*

Moreover, in all cases $\kappa_{R,t} = 1$ *so equity is illiquid. Absent money and shadow money, liquidity provision is suboptimal in all cases. Absent only shadow money, liquidity provision is suboptimal in all cases except (ii).*

Proof. See Appendix C. □

Only money can deliver crash-proof liquidity since it experiences no losses, which gives $w_t^m = l_t^c$. It is always efficient to deliver the incremental normal-times liquidity using shadow money, so $w_t^r = l_t^n - l_t^c$. Clearly, when asset risk is too high, as in cases (i) and (iv), illiquid equity is required to make shadow money liquid. In case (iii), asset risk may in principle be low enough to support liquid equity but normal-times risk $\sigma_{A,t} = \frac{\pi_t^a k_t^a}{\pi_t^b k_t^b + \pi_t^b k_t^b}\sigma_{\pi,t}^a + \frac{\pi_t^b k_t^b}{\pi_t^b k_t^b + \pi_t^b k_t^b}\sigma_{\pi,t}^b > 0$ raises the recovery rate necessary to make equity liquid to above $\overline{\kappa}$. This means that even when liquid, equity cannot supply as much liquidity as shadow money. It is therefore optimal to leave equity illiquid and uncollateralized.

Note that in all regions of the state-space at least two of the three available securities are required for optimal liquidity provision. This means that tranching is always efficient. Shadow money is unnecessary only in high-uncertainty states, when crash-proof liquidity is valuable enough to take up all available collateral.

In sum, intermediaries trade off the leverage advantage of shadow money against the funding advantage of money. Every dollar of collateral can back one dollar of money or $1/(1-\overline{\kappa})$ dollars of shadow money. When uncertainty rises, the marginal rate of substitution between normal-times liquidity and crash-proof liquidity falls below the leverage advantage of shadow money, intermediaries substitute money and equity for shadow money in their liabilities mix. This leads to a quick winding-down of the shadow-money market and deleveraging.

The intermediary's total cost of capital is $\mu_{R,t} + w_t^m(\mu_{m,t} - \mu_{R,t}) + w_t^r(\mu_{r,t} - \mu_{R,t})$. Using the household optimality conditions (28), it equals

$$\rho - \frac{h}{\eta}(\psi - 1)\left(1 - \left[e^{-\tau\lambda_t}e^{-\eta(w_t^m + w_t^r)} + \left(1 - e^{-\tau\lambda_t}\right)e^{-\eta w_t^m}\right]\right). \tag{35}$$

The cost of capital is decreasing in the supply of liquidity. Greater uncertainty λ_t raises funding costs, even as intermediaries shift resources to the crash state. In the next section, we show how liquidity affects the dispersion in discount rates across assets.

3.8. Asset prices and investment

On the asset side of the balance sheet, intermediaries purchase capital and set investment. Let $A_t\theta_t^c$ and $A_t\theta_t^n$ be the LaGrange multipliers on the crash-state and normal-times limited liability constraints (20) and (21) of the intermediary. Their expressions are provided in Appendix C. Each additional dollar of assets relaxes the normal-times constraint by one dollar and the crash-state constraint by $1-\kappa_{A,t}$ dollars. The intermediary's asset-side problem is thus

$$\max_{k_t^a, k_t^b, \iota_t^a, \iota_t^b} \left[\left(\gamma^a - \iota_t^a\right) k_t^a + \left(\gamma^b - \iota_t^b\right) k_t^b\right] dt + E_t\left[dA_t - A_t dR_t\right] + A_t\left[\theta_t^n + \theta_t^c\left(1-\kappa_{A,t}\right)\right] dt \quad (36)$$

with $A_t = \pi_t^a k_t^a + \pi_t^b k_t^b$ and by Ito's Lemma,

$$\begin{aligned} E_t\left[dA_t\right] \quad = \quad & \Big(\pi_t^a k_t^a \left[\mu_{\pi,t}^a + \phi\left(\iota_t^a\right) - \delta + \kappa^a \kappa_{\pi,t}^a \lambda_t\right] \\ & + \pi_t^b k_t^b \left[\mu_{\pi,t}^b + \phi\left(\iota_t^b\right) - \delta + \kappa^b \kappa_{\pi,t}^b \lambda_t\right] \Big) dt. \end{aligned} \quad (37)$$

The optimal investment policy sets marginal q in each technology equal to one:

$$1 \quad = \quad \pi_t^a \phi'\left(\iota_t^a\right) \quad = \quad \pi_t^b \phi'\left(\iota_t^b\right). \quad (38)$$

Higher asset prices lead to higher investment and growth. Prices themselves are determined in the market for capital, where equilibrium requires

$$\mu_{R,t} - \theta_t^c \left(1 - \kappa^a\right)\left(1 - \kappa_{\pi,t}^a\right) - \theta_t^n = \frac{\gamma^a - \iota_t^a}{\pi_t^a} + \left[\mu_{\pi,t}^a + \phi\left(\iota_t^a\right) - \delta + \kappa^a \kappa_{\pi,t}^a \lambda_t\right] \quad (39)$$

$$\mu_{R,t} - \theta_t^c \left(1 - \kappa^b\right)\left(1 - \kappa_{\pi,t}^b\right) - \theta_t^n = \frac{\gamma^b - \iota_t^b}{\pi_t^b} + \left[\mu_{\pi,t}^b + \phi\left(\iota_t^b\right) - \delta + \kappa^b \kappa_{\pi,t}^b \lambda_t\right]. \quad (40)$$

The left side is the discount rate and the right side is the expected return of each asset. The return has a cash flow component which is higher for type-A capital since $\gamma^a > \gamma^b$, and an expected appreciation component that takes into account price growth, physical growth, and depreciation (there is also an adjustment coming from the crash-compensation term).

The discount rates vary across the two assets as a result of their differential ability to supply liquidity. Both assets are funded at a discount relative to the cost of equity, and this discount depends on their collateral values. Since type-A capital is riskier $\kappa^a > \kappa^b$, and since collateral is generally scarce, $\theta_t^c > 0$, its discount rate tends to be higher than that of type-B capital. Although it has higher cash flows, type-A capital can have a lower price than type-B capital if liquidity is sufficiently dear. As liquidity becomes more abundant, not only does the overall cost of capital fall, but the wedge in discount rates between the two types of capital shrinks. This means that greater liquidity provision raises investment overall, and investment in technology A in particular.

The investment and capital-price optimality conditions embed a feedback effect between prices and investment: lower prices reduce investment, which in turn reduces the return on capital causing prices to fall even further. This effect increases with the level of adjustment costs. Greater technological illiquidity thus amplifies the pro-cyclicality of investment.

The capital-price optimality conditions also embed the potential for a collateral run, a feedback effect between prices and collateral values (Brunnermeier and Pedersen 2009). We return to this discussion in section 4.3.

4. Results

In this section, we present the results of the model, focusing on its macroeconomic dynamics and financial markets effects. We solve the model by characterizing PDEs in $\pi_t^a = \pi^a(\lambda_t, \chi_t)$ and $\pi_t^b = \pi^b(\lambda_t, \chi_t)$, which we then solve numerically using projection methods. Appendix D provides details.

We pick an investment cost function that is consistent with quadratic adjustment costs, $\phi(\iota_t) = \frac{1}{\varphi}\left(\sqrt{1+2\varphi\iota_t} - 1\right)$. Our benchmark parameter values are available in Table I. We view these as an illustration rather than a calibration.

4.1. *Macroeconomic effects*

Figure 1 plots key macroeconomic quantities in our benchmark economy against the risky asset share χ and uncertainty λ. The top two panels show the prices of the high-productivity low-collateral type-A capital and the low-productivity high-collateral type-B capital. The price of type-A capital is on average higher, reflecting its greater productivity. However, it declines steeply with uncertainty and gradually with the riskiness of the asset mix. The price of type-B capital displays the opposite behavior and it can exceed that of type-A capital when uncertainty and asset risk are high.

The middle panels of Figure 1 hint at the drivers of these price effects. When uncertainty is low, households are willing to hold shadow money to meet their liquidity needs. This is

Figure 1. Prices, issuance, growth, and liquidity

Asset prices, money and shadow money issuance, output growth, and liquidity services for the economy under the benchmark parameters in Table I. The state variables are the risky asset share χ and uncertainty λ. Prices solve 39 and 40. Money and shadow money issuance are characterized in Proposition 2. Output growth is the expected percentage growth of output $y_t = \gamma^a k_t^a + \gamma^b k_t^b$. Liquidity services are the flow rate of the net present value of liquidity-event consumption $(\psi - 1)\, E_t \left[dC_t dN_t^h \right]$ (see 28 for formula).

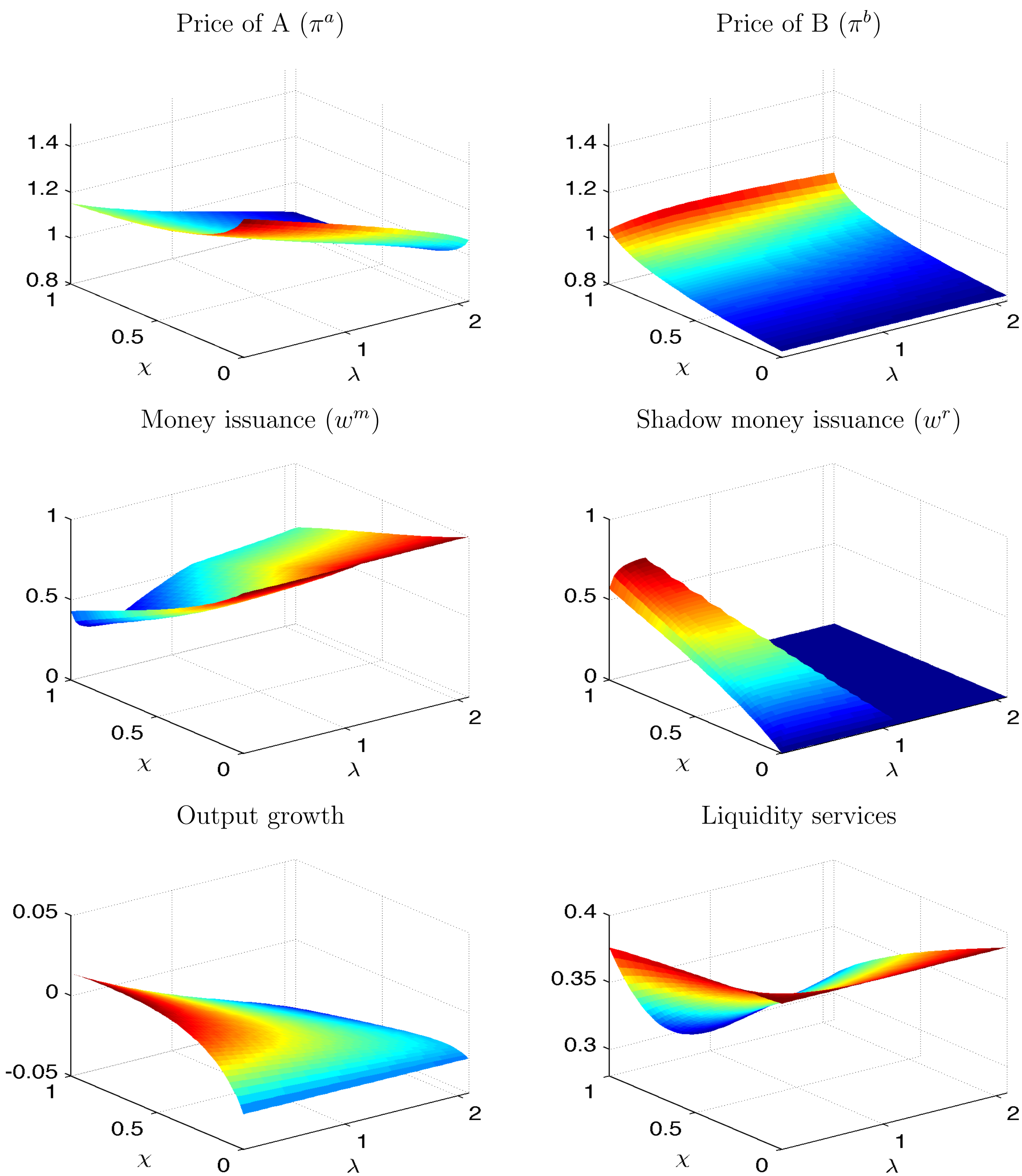


Table I. Benchmark parameter values

This table contains the values for the model parameters used in the benchmark results of the paper.

Description	Parameter	Value
Subjective discounting parameter	ρ	0.40
Depreciation	δ	0.03
Level-inflow	μ	0.01
Investment adjustment cost	φ	5
Type-A cash flow risk	κ^a	0.4
Type-B cash flow risk	κ^b	0.0
Type-A productivity	γ^a	0.08
Type-B productivity	γ^b	0.03
Low uncertainty state	λ^L	0.1
High uncertainty state	λ^H	2.1
Low uncertainty state exit rate	q^L	0.05
High uncertainty state exit rate	q^H	0.05
Uncertainty signal precision	$1/\sigma_e$	0.1
Liquid asset risk upper bound	$\overline{\kappa}$	0.7
Liquidity event duration	τ	1.032
Liquidity event intensity	h	0.1
Value of liquidity-event consumption	ψ	10
Mean liquidity shock size	$1/\eta$	0.5

because shadow money remains liquid as long as there is no crash and crashes are unlikely here. Intermediaries are eager to supply shadow money as it allows them to lever up the collateral value of their assets to create more liquidity and lower their funding costs (shadow money can sustain a limited loss in a crash and still remain liquid ex ante). As a result, shadow money displaces money at low levels of uncertainty and the total supply of liquidity, measured by the net present value of liquidity-event consumption, rises as the bottom right panel of Figure 1 shows. The effect is strongest when the capital mix is risky so that collateral is scarce.

Output growth is also highest when uncertainty is low and the productive type-A capital is abundant. By increasing the supply of liquidity for a given amount of collateral, shadow banking reduces discount rates and pushes up asset prices, investment, and growth. Note that a highly liquid economy is not coincident with a fast-growing one: liquidity services are very high when asset risk is low but the lack of the productive type-A asset actually causes the economy to shrink. In other words, shadow banking in particular enables the boom via liquidity transformation.

A rise in uncertainty sets off a contraction in liquidity transformation and ultimately a recession. Household demand shifts abruptly from shadow money to money as only money provides liquidity in a crash. Intermediaries cater to this demand by adjusting their liabilities mix. The shadow banking sector effectively shuts down within a narrow range of uncertainty. As an example, in the financial crisis of 2007—2008, the market for asset-backed commercial paper suffered a similarly abrupt collapse.

Issuing more money forces intermediaries to raise equity. The result is less liquidity provision and higher discount rates. The price of type-A capital falls sharply while type-B capital becomes more valuable as the liquidity shortage increases the premium for collateral. This effect is strongest when the asset mix is risky so collateral is scarce to begin with. The reversal of investment from productivity to safety can be characterized as cash-hoarding on the asset side of the balance sheet.

Importantly, it is neither a lack of capital nor a lack of demand for credit that causes intermediaries to stop investing in productive projects when uncertainty rises. In fact, equity is higher than ever and the investment opportunity set is unchanged. It is the low level of liquidity provision resulting from the contraction of shadow banking that produces the downturn.

4.2. Persistense

Our model generates persistence so booms and busts each last a while. First, uncertainty is persistent: it takes a few years absent a crash for λ to come down, and if there are aftershocks it takes even longer. Second, the capital mix χ is slow-moving due to technological illiquidity. These two sources of persistence interact so that a rise in uncertainty throws the economy off its target course, setting it on a long path to recovery.

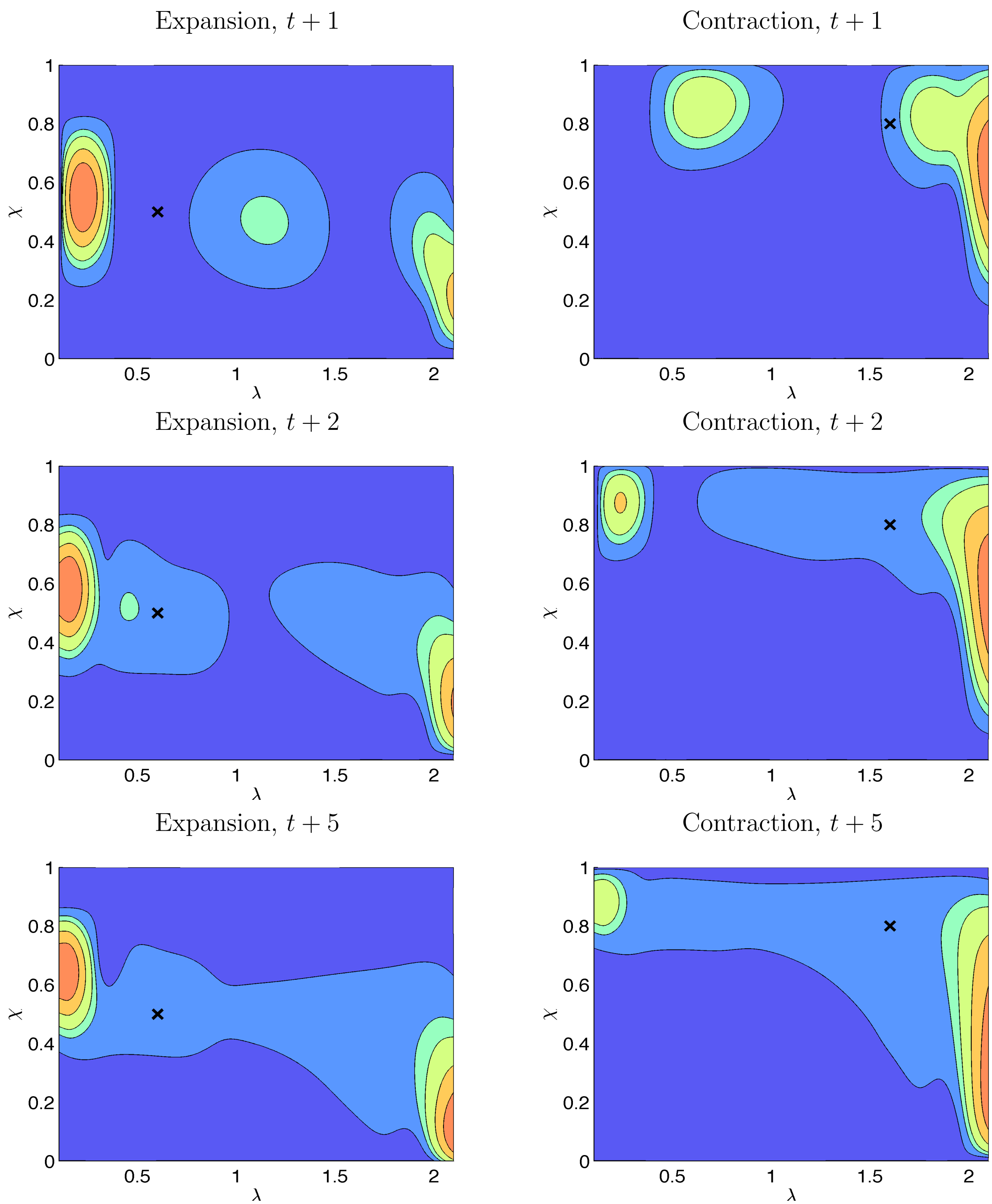
To illustrate the persistence of our model, Figure 2 plots the joint density of λ and χ at one, two, and five-year horizons, starting from an initial state of either expansion or contraction. These densities are obtained by solving the forward Kolmogorov equation of the system as described in Appendix E.

In the expansion specification on the left, the economy starts near $(\lambda, \chi) = (0.6, 0.5)$. At this low level of uncertainty, the price of type-A capital is high (see Figure 1), which spurs investment and growth. This causes its share χ to drift up so that it stands near 65% after five periods. Uncertainty λ drifts down as the absence of a crash leads households to update their beliefs, ending up near 0.15 after five periods. Thus, absent a crash mass shifts up and to the left. As crashes are rare here, the boom is expected to last a while.

With a small probability, however, the economy experiences a crash so a small amount of mass breaks off and reappears to the right and below the starting point. To the right because crashes raise the perceived probability of crashes, and below because they destroy more risky capital than safe capital. Following a crash, further shocks are much more likely, so the initial break-away mass quickly splits in two: one piece sails smoothly towards low uncertainty while the other is blown further off course where the same dynamic repeats with

Figure 2. Persistence

The joint density of the risky asset mix χ and uncertainty λ after one, two, and five periods, starting from a state of expansion or contraction. The densities are computed by iterating on the forward Kolmogorov equation for the system (see Appendix E). The initial density is a joint normal with standard deviation of 0.1 in each direction and its mean is marked with an $\times$. The contours are in the set $\{0.1, 1, 2, 5, 10\}$.



greater intensity.

The right set of panels in Figure 2 illustrate the prospects of an economy battered by a series of shocks after a long boom. The starting condition here is in the neighborhood of $(\lambda, \chi) = (1.6, 0.8)$. Here, aftershocks are likely, so only a few scenarios usher in a return to prosperity. After one, two, or five years, uncertainty most likely remains high and the risky capital mix is lower due to both a shift in investment and potentially further crashes. Even when the target capital mix is achieved, growth remains negative as safe capital is unproductive. The economy is mired in recession even though collateral values and liquidity provision are high. In the absence of shadow banking, which alone can fund productive capital via collateral transformation, the economy can be either liquid or productive, but not both.

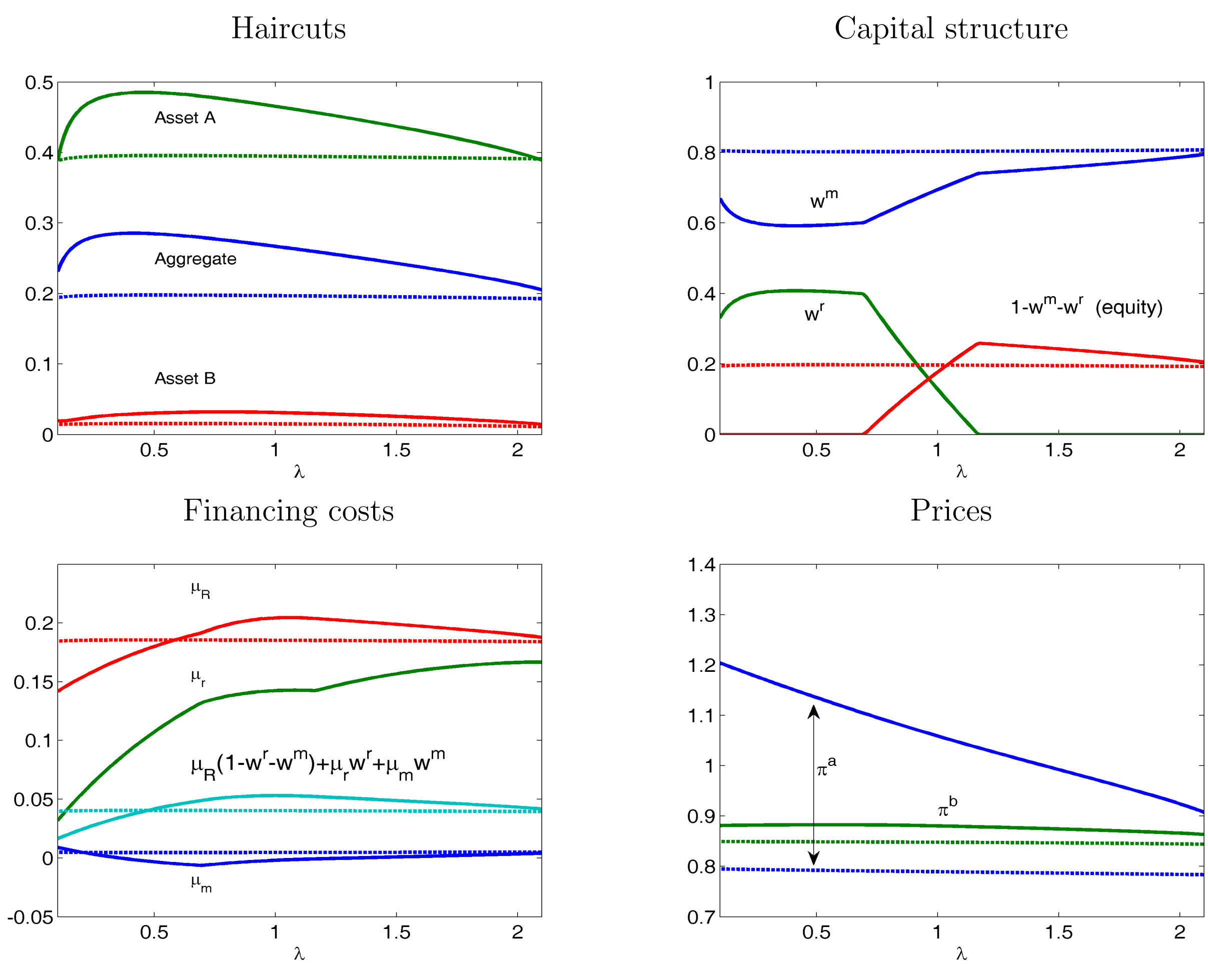
4.3. Collateral runs

Collateral runs are episodes during which liquidity creation requires progressively greater amounts of collateral, which one can also think of as a rise in haircuts. To be concrete, an asset i with collateral value $1 - \kappa$ has a haircut of κ. In our model haircuts have an exogenous cash flow component κ^i and an endogenous price component κ_π^i so that $1 - \kappa = (1 - \kappa^i)(1 - \kappa_\pi^i)$. An asset can have no cash-flow risk, $\kappa^i = 0$, yet still depreciate in a crash as discount rates rise, $\kappa_\pi^i > 0$. A collateral run occurs when $\kappa_\pi^i = 1 - \pi_+^i/\pi^i$ rises as π^i falls (pluses denote after-crash prices). This requires higher discount rates today to augur in the possibility of even higher discount rates tomorrow in the event of a crash.

Figure 3 compares haircuts, capital structures, funding costs, and asset prices in economies

Figure 3. Collateral runs

Equilibrium variables for economies with (solid lines) and without (dashed lines) a shadow-banking sector. The capital mix is fixed at $\chi = 0.5$. Haircuts are defined as one minus the collateral value of an asset. Capital structures consist of money w^m, shadow money w^r, and equity $1 - w^m - w^r$. The financing costs are the equity return μ_R, the yield on shadow money μ_r, the yield on money μ_m, and the total cost of capital $\mu_R (1 - w^r - w^m) + \mu_r w^r + \mu_m w^m$.



with and without a shadow banking sector.[17] We fix $\chi = 0.5$ and look across λ, our "fast-moving" state variable.

In a shadow-banking economy near the peak of the credit cycle (low λ), an uptick in uncertainty leads to a rise in both haircuts and discount rates. Higher haircuts raise funding costs by forcing a shift towards more expensive financing. As a result, prices fall further, producing a collateral run.

In a collateral run, money yields fall while shadow money yields rise, but since the economy is near its peak, shadow banking activity is very high and overall discount rates rise. The risky type-A capital suffers the greatest decline as its funding is most fragile. Even though the amplification created by collateral runs causes discount rates spike above their levels in the no-shadow-banking economy, the price of type-A capital is always higher with shadow banking than without because prices capitalize the higher levels of liquidity transformation in a shadow-banking-driven boom.

When λ is high and the economy is near the bottom, a fall in uncertainty leads to a rise in haircuts. Thus, as an economy recovers from a low level its exposure to uncertainty increases, which slows down the recovery of prices.

In our model, a collateral run is the result of a dynamic feedback between collateral values and discount rates. We can see this by inspecting the equilibrium discount rate from the pricing equations (39) and (40) of an asset i,

$$\mu_t^i \;=\; \mu_{R,t} - \theta_t^c \left(1 - \kappa_t^i\right)\left(1 - \kappa_{\pi,t}^i\right) - \theta_t^n. \tag{41}$$

[17] We implement a no-shadow banking economy with $\overline{\kappa} = 0$ which removes the leverage advantage of shadow money.

Holding aggregate financing conditions constant, a fall in asset i's collateral value raises its discount rate, lowering its price. Collateral values are determined by the difference between the current price and the near-term worst-case (after-crash) price, which itself depends on local discount rates. Therefore, when the rise of discount rates accelerates in uncertainty as it does when uncertainty is low, higher discount rates today produce lower collateral values resulting in a collateral run.

The shadow-value θ_t^c reflects the aggregate scarcity of collateral. When θ_t^c is high, the uncollateralized rate $\mu_{R,t}$ tends to rise. Equation (41) shows that collateral-poor assets see their discount rates rise the most, whereas collateral-rich assets can actually appreciate. We consider the latter case in the next section.

We note that a collateral run is distinct from a classic bank run. Agents in our model demand bigger haircuts when financial conditions become more sensitive to shocks. However, they do not face a first-come-first-served constraint as they do in (Diamond and Dybvig 1983). Such a constraint can occur if collateral is rehypothecated among investors. We hope to explore this possibility in future research.

4.4. Flight to quality

Our model generates flight to quality effects in both the securities and asset markets. A rise in uncertainty triggers a demand shift from shadow money to money. This causes the spread between shadow money and money to widen. As intermediaries struggle to absorb the excess demand for money, the premium for good collateral rises, causing the safe type-B capital to appreciate relative to the risky type-A capital. When these relative price changes dominate the overall rise in discount rates, the absolute yield of money falls and the price of safe capital rises, which we call flight to quality in the securities and asset markets.

Our benchmark parametrization produces modest flight to quality. Figure 1 shows that the price of type-B capital is increasing in λ at low levels of λ. In this section, we modify the model to strengthen this effect.

Intuitively, flight to quality is the result of a shortage of collateral. In our benchmark model however, crashes actually increase the relative supply of collateral as a higher proportion of the safe type-B capital survives unscathed. This increase in pledgability dampens the rise in the collateral premium after a crash.

To remove the dampening effect, we can replace the aggregate cash flow shock with a dispersion shock in equations (1) and (2). Let $k^a_{i,t}$ and $k^b_{i,t}$ be the capital holdings of intermediary i and suppose

$$dk^a_{i,t} = k^a_{i,t}\left[\phi\left(\iota^a_{i,t}\right) - \delta\right]dt - k^a_{i,t}\kappa^a_{i,t}\left(dN_t - \lambda_t dt\right)] \tag{42}$$

$$dk^b_{i,t} = k^b_{i,t}\left[\phi\left(\iota^b_{i,t}\right) - \delta\right]dt, \tag{43}$$

where $\kappa^a_{i,t} = \pm\kappa^a$ with probability 1/2 each. In addition to adding the dispersion shock, we have simplified our benchmark specification by setting $\mu = 0$, which further increases the scarcity of collateral at high levels of χ. We also increase η slightly to 2.8, which has the effect of reducing the elasticity of substitution between money and shadow money.

The dispersion shock makes type-A capital risky at the level of an individual intermediary but safe in the aggregate. This keeps pledgability low while eliminating aggregate cash flow risk, highlighting the fact that our model is about collateral rather than risk.

We are implicitly assuming that intermediaries cannot diversify the dispersion shock. As a possible example, it may be difficult to distinguish ex ante which assets are likely to co-move in the rare event of a crash, (e.g. mortgages in Miami and Las Vegas). Alternatively, individual intermediaries might develop special expertise in particular markets. Note that flight to quality does not require the dispersion shock but is enhanced by it.

The modified model is solved easily by simply altering the dynamics of χ in (4) to $d\chi_t = \chi_t(1-\chi_t)\left[\phi\left(\iota_t^a\right) - \phi\left(\iota_t^b\right)\right]dt$, while keeping the pricing PDEs (39) and (40) unchanged.

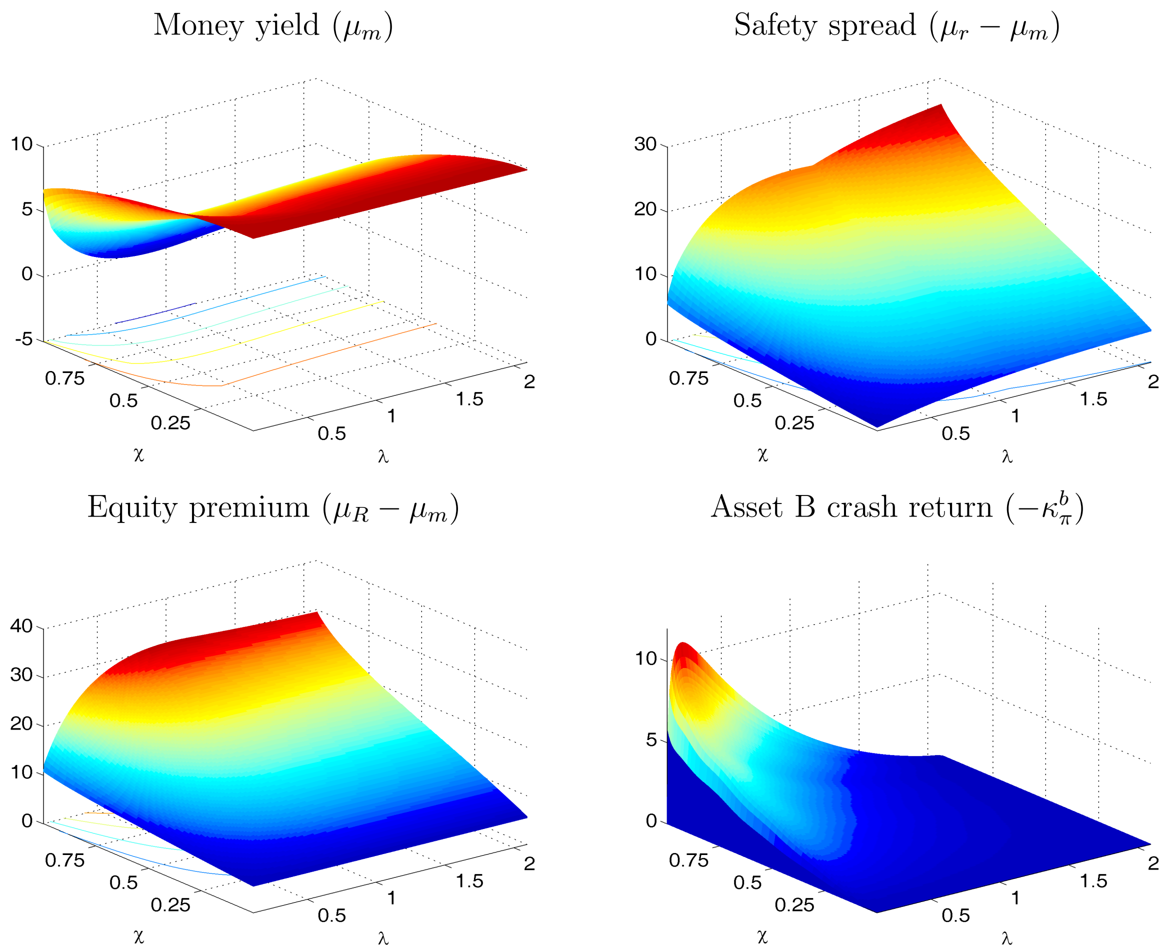
Figure 4 shows that the model with a dispersion shock produces strong flight to quality effects in securities markets. When uncertainty λ rises, the yield on money falls and the spread between shadow money and money (i.e. the safety spread in Krishnamurthy and Vissing-Jorgensen 2012a) opens up. The effect is strongest when the capital mix is risky so that intermediaries cannot fully absorb the excess demand for money. In this case overall liquidity falls, which causes discount rates to rise as reflected in the equity premium. These dynamics strongly resemble developments in U.S. markets after July 2007.

Figure 4 also shows strong flight to quality effects in asset markets. In the bottom right panel, the price of type-B capital rises the most in a crash when the economy is near the peak of the credit cycle; that is when uncertainty is low, liquidity transformation is high, and the capital mix is risky. U.S. long-term bonds similarly appreciated as the financial crisis unfolded in 2007—2008. Once uncertainty rises sufficiently (or the capital mix becomes safe enough) so that shadow banking shuts down, flight to quality disappears. Thus, flight to quality results from the acute shortage of collateral that occurs when uncertainty rises suddenly after a long boom.

Our model's learning dynamics tie together normal-times (dB) and crash (dN) flight to

Figure 4. Flight to quality

The yield on money, the spread between shadow money and money (the safety spread), the equity premium, and the return of type-B capital in a crash for the model with a dispersion shock (see discussion in section 4.4), $\mu = 0$ and $\eta = 2.8$. All quantities are $\times 10^2$.



quality (both raise λ). The two, however, are conceptually distinct as normal-times shocks are borne entirely by equity whereas crashes affect pledgability and the supply of liquidity. In our model only crash-driven flight to quality is important ex ante because equity markets are frictionless.[18] This observation suggests that differences in the pricing of instruments that act as normal-times versus crash-risk hedges can be used to assess the relative importance

[18]We explore the ex ante effects of flight to quality in sections 5.3 (Operation Twist) and 5.5 (Volcker rule).

of equity- and collateral-based frictions.

5. Policy interventions

In the aftermath of the 2007—2008 financial crisis central banks around the world and the U.S. Federal Reserve in particular have resorted to a wide variety of interventions broadly referred to as unconventional monetary policy. We use our model to analyze two of these interventions, the Large Scale Asset Purchase (LSAP) program of 2008—2010 and the Maturity Extension Program also known as "Operation Twist" of 2011—2012. Under LSAP, the FED purchased large amounts of mortgage-backed securities to support their prices.[19] Under Operation Twist, the FED purchased long-dated Treasurys and sold short-dated ones with the stated goal of reducing long-term interest rates.[20]

Alongside central banks, regulators have entertained a broad range of proposals and implemented a subset of them. We consider two of these: the so-called "Volcker Rule" which seeks to separate commercial banking and proprietary trading, and stricter capital requirements as have been adopted by the Basel III committee. In each case below we discuss our interpretation of these policies within the context of our model, and explore their impact on liquidity provision and the broader economy.

[19]The press release announcing the program reads, "Spreads of rates on GSE debt and on GSE-guaranteed mortgages have widened appreciably of late. This action is being taken to reduce the cost and increase the availability of credit for the purchase of houses, which in turn should support housing markets and foster improved conditions in financial markets more generally" (Federal Open Market Committee 2008).

[20]The program's announcement following the September 2011 FOMC meeting reads, "The Committee intends to purchase, by the end of June 2012, $400 billion of Treasury securities with remaining maturities of 6 years to 30 years and to sell an equal amount of Treasury securities with remaining maturities of 3 years or less. This program should put downward pressure on longer-term interest rates and help make broader financial conditions more accommodative" (Federal Open Market Committee 2011).

5.1. *Asset purchases*

We interpret LSAP as replacing risky type-A capital with an equal dollar amount of safe type-B capital. The basic effect of such a program is to increase aggregate pledgability, and by extension liquidity provision, prices, and investment. Importantly, since an anticipated intervention can have an effect even before it is implemented by raising collateral values, we consider both the ex ante and ex post effects of an LSAP program. To further contextualize our exercise within the active public debate, we model fiscal constraints by assuming that a new program can be initiated only after any previous balance sheet expansions have been reversed and we analyze the likelihood of reversal under the lens of the recent discussion of "tapering".

To be concrete, let $\zeta_t \in \{\zeta_E, \zeta_F\}$ denote the state of the central bank's balance sheet which is either "empty" ($\zeta_t = \zeta_E$) or "full" ($\zeta_t = \zeta_F$). An empty balance sheet is filled with probability $\beta_{LSAP}(\lambda_t, \chi_t)$ immediately after a crash in which case it becomes full.[21] The size of the program is given as a state-contingent fraction $\alpha(\lambda_t, \chi_t)$ of the outstanding supply of type-A capital. A full balance sheet is emptied ($\zeta_t = \zeta_E$) with intensity $\beta_{TAPER}(\lambda_t, \chi_t)$.

Appendix F shows how to solve the model with the new state variable ζ_t. The pricing equations (39) and (40) are modified to take into account the possibility for prices to switch between policy states.

Figure 5 implements a basic transitory but persistent policy intervention. We set $\alpha = 0.2$ so the risky asset share χ falls by 0.2, $\beta_{LSAP} = 0.5$ so half of all crashes trigger an LSAP program, and $\beta_{TAPER} = 0.1$ so the balance sheet expansion is expected to last about ten

[21] By tying interventions to cash flow shocks we avoid having to introduce separate policy jumps that may add additional binding constraints on intermediary balance sheets.

years.

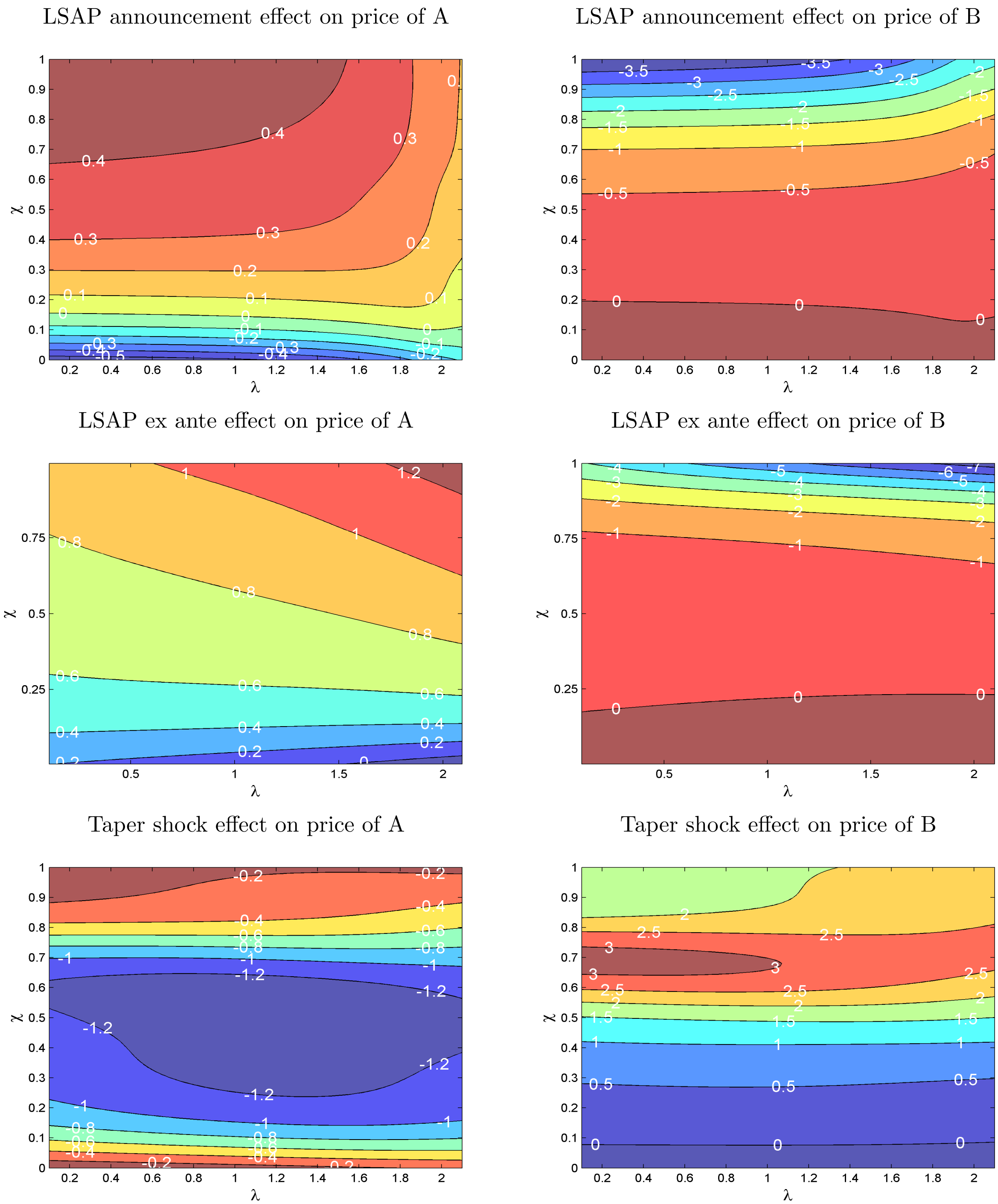
The top panels of Figure 5 show that in most of the state-space the LSAP program pushes the price of risky capital up and the price of safe capital down, with the effects for both assets being strongest in the high χ, moderately-high λ region. When χ is high, collateral is scarce and liquidity provision is low. By supplying safe assets, the central bank increases the pledgability of the capital stock, allowing for greater liquidity provision. The result is both a decline in overall discount rates which tends to push all prices up, and a decline in the collateral premium, which pushes the price of risky capital up and the price of safe capital down. The net effect is positive for the risky asset and negative for the safe asset.

Interestingly, enacting a program when liquidity is abundant (when χ and λ are low) backfires and actually reduces the price of the risky asset. This is the result of the central bank's limited capacity: The good news (for liquidity) of the intervention is outweighed by the bad news that there will be no further interventions when the economy needs them. The economy becomes riskier because the central bank will be out of ammunition in the next crash.

Looking along the λ dimension, the price effects are strongest when uncertainty is moderately high. This is also the region where collateral runs push collateral values to their lowest levels. Here, an uncertainty shock generates a flight to quality in the securities markets that effectively forces the shadow banking sector to shut down, sending prices lower. The prospect of an intervention is most valuable when the economy is at its most vulnerable. In sum, we find that the effectiveness of an LSAP program is tied to conditions in the shadow banking sector.

Figure 5. Asset purchases (LSAP)

In an asset purchase (LSAP) program, the central bank buys risky capital and sells safe capital. In this example, an LSAP program shifts the risky asset share χ down by 20% (i.e. $\alpha = 0.2$), it arrives with 50% probability in the event of a crash ($\beta_{LSAP} = 0.5$), and it is withdrawn at a 10% intensity rising to 100% in the last two panels ($\beta_{TAPER} \in \{0.1, 1\}$). The announcement effect measures the difference in returns across crashes with and without an LSAP intervention. The ex ante effect compares prices in economies with and without the possibility of an LSAP intervention. The "taper" shock is an unanticipated increase in the intensity of policy withdrawal. See Appendix F for details on the calculations.



The middle two panels of Figure 5 consider the ex ante effect of an LSAP program by comparing prices across economies with and without the possibility of central bank intervention. Risky capital prices are higher and safe capital prices are lower in the LSAP economy throughout the state-space, as expected. Importantly, the ex ante effects are stronger than the ex post effects in the low uncertainty region. The LSAP program has a stabilizing effect ex post when uncertainty is high and in this way boosts collateral values ex ante when uncertainty is low. This second-round effect adds to the direct effect of the program at low levels of uncertainty, making risky capital more liquid.

Our results broadly suggest that asset purchases can be effective ex post by stabilizing prices on the way down and ex ante by boosting collateral values.

5.2. "Tapering"

We model tapering as an announcement that policy accommodation will be reversed sooner than anticipated. Specifically, the bottom two panels of Figure 5 show the effect of increasing the intensity of policy withdrawal from 10% to 100% (so the LSAP program lasts an average of one year instead of ten).

The change in expectations triggered by the tapering shock has a sharp effect on asset prices, one that is of the same magnitude as the initial effect of the LSAP intervention itself. Even though the balance sheet of the central bank is unchanged, financial markets anticipate that liquidity will become scarce sooner, pushing up collateral premia today. Even though liquidity premia in securities markets do not change today, collateral premia in the asset markets shoot up as prices capitalize the present value of future liquidity.

Our results suggest that a premature policy withdrawal can undermine the effectiveness of an asset purchase program.

5.3. *Operation Twist*

We model Operation Twist as a reduction in the duration of the economy's safe capital. Interestingly, in our framework this can reduce pledgability as long-term safe capital acts as a hedge in the presence of flight to quality. To demonstrate this effect, we use the parameters from the flight to quality section 4.4.

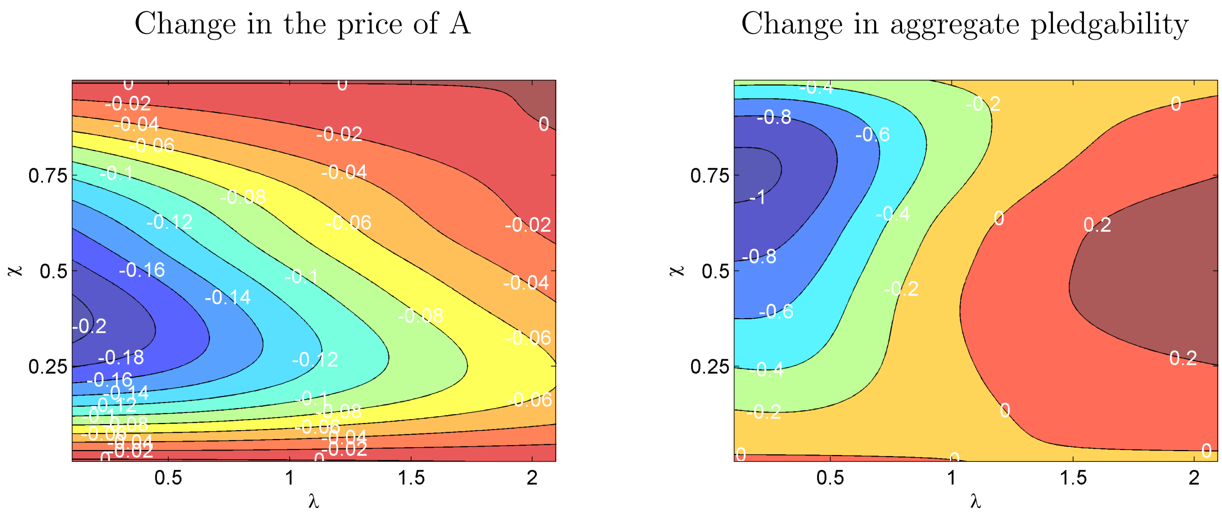
We map the safe asset B to government debt by assuming that the private sector cannot create it but that the government issues it at the same rate as in the baseline model.[22] To model a change in duration within a given economy, we split type B capital into two pieces: zero-duration floating-rate debt FB and long-duration safe bonds LB (so $k_t^b = k_t^{fb} + k_t^{lb}$). Floating debt pays the floating rate μ_m (the yield of money) and trades at par in equilibrium. Long bonds pay the fixed coupon γ^b as in the baseline model. We avoid introducing an additional state variable by assuming that the central bank sets the relative shares $\alpha_t = k_t^{fb}/k_t^b$ and $1 - \alpha_t = k_t^{lb}/k_t^b$ as a policy variable.

In an Operation Twist intervention, the central bank buys long-term bonds and sells floating rate bonds of equal dollar amounts at after-announcement prices. This changes their relative shares α_t and $1 - \alpha_t$.[23] While anticipated policies are easily implemented in our framework, we focus on a one-off unanticipated intervention in which the government

[22] Specifically, private investment ι_t^b is restricted to zero in the pricing equation (40). Existing long bonds depreciate on the balance sheet but the government controls their aggregate supply by setting issuance to $dk_t^b = \left[\phi\left(\iota_t^b\right) - \delta\right] dt$ absent a policy shock with $\pi_t^b \phi'\left(\iota_t^b\right) = 1$.

[23] It also changes the total number of bonds outstanding k_t^b but not their share of total wealth. The details are in Appendix H.

Figure 6. Operation Twist
This figure shows the price and pledgability effects of changing the mix of safe government bonds from fixed-coupon long-term bonds to floating-rate bonds ($\alpha_t = 0$ to $\alpha_t = 1$). The left plot shows the change in the price of the risky type-A capital π_t^a, while the right plot shows the change in aggregate pledgability $1 - \kappa_{A,t}$. This figure uses the parameter values from section 4.4 to generate strong flight to quality and $\mu = 0$ to keep growth invariant to the government's debt structure.



restructures its debt from long-term financed to fully floating, that is α_t rises from zero to one.

The left panel of Figure 6 shows that Operation Twist reduces the price of the risky type-A capital. This is due to a strong flight to quality effect: long-term debt appreciate in a crash. As such, it acts as a hedge for the crash risk of type-A capital, raising overall collateral values. Floating-rate debt always trades at par, so it cannot acts as a hedge. As a result, Operation Twist reduces overall pledgability as seen in the right panel of Figure 6, causing discount rates to rise and prices to fall.

The stated rationale for the FED's Maturity Extension Program is predicated on the view that risky productive assets, like long-term bonds, are exposed to duration risk so that reducing the supply of long-term bonds might free up capacity for intermediaries to undertake more risky investment. In our economy the opposite happens. Duration is actually a hedge when flight to quality is strong. This makes long-term bonds complements rather than substitutes for risky investment. We return to this discussion in section 5.5 in the context of the Volcker rule.

5.4. Capital requirements

In our model intermediaries add value by transforming illiquid assets into liquid securities. Capital requirements have the effect of reducing the amount of liquidity that intermediaries can produce per dollar of assets on the balance sheet. Lower liquidity provision raises funding costs and reduces asset pricing and investment.

We study the impact of an announcement that stricter capital requirements will be implemented at some future date. Specifically, we compare our benchmark economy to one where an equity capital requirement $\underline{e}$ goes into effect with an intensity of $\beta_{CAPITAL} > 0$. Capital requirements have a straight-forward effect on intermediary capital structure: the normal-times solvency constraint (20) is replaced by

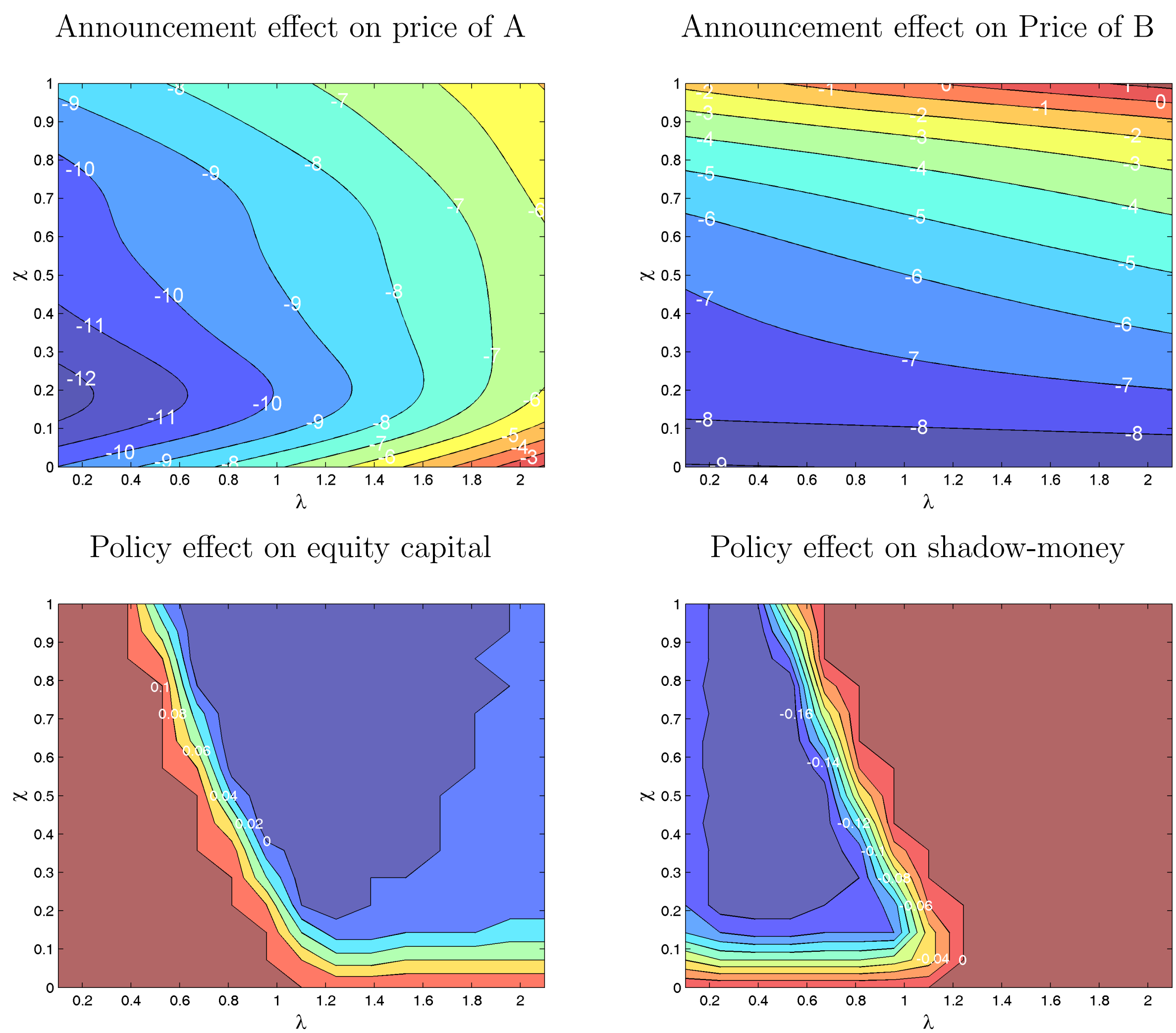
$$w_t^m + w_t^r \quad \leq \quad 1 - \underline{e}. \tag{44}$$

The resulting optimal capital structure is characterized in Appendix H. Given the new optimal capital structure, the rest of the model is identical to the benchmark setup. Thus, the direct effect of the capital requirement is to reduce the issuance of liquid liabilities in low-uncertainty states.

The indirect effect is through prices. Figure 7 shows the impact of an announcement that

Figure 7. Capital Requirements

This figure shows the effects of an announcement of higher future capital requirements on prices and intermediary capital structure. Specifically, each plot shows the change of a given quantity across two economies: an economy in which higher capital requirements are expected to be imposed within five years ($\beta_{CAPITAL} = 0.2$), and a benchmark economy. The anticipated capital requirement is set at $\underline{e} = 10\%$.



a capital requirement of $\underline{e} = 10\%$ (up from 0%) will likely be imposed in the next five years ($\beta_{CAPITAL} = 0.2$). The top panels plot prices and the bottom panels plot the intermediary capital structure once the capital requirement is in effect.

As a result of the announcement, prices drop in the aggregate, which ultimately results in lower growth. As discount rates and collateral premia rise simultaneously, the productive type-A capital is hit particularly hard whereas the safe type-B capital fares somewhat better.

The capital requirement has the effect of lowering shadow money creation by a disproportionally large amount compared to money. The reason is that the principal advantage of shadow money, the ability to lever up collateral, is curtailed. As the shadow banking sector is particularly sensitive to shocks, capital requirements have the effect of increasing "financial stability" in the sense of reducing asset price volatility. Note however, that stability is achieved by lowering risky capital prices in the boom without increasing them in the bust.

Figure 7 highlights the fact that capital requirements produce price drops even when they do not bind. Asset prices capitalize future funding costs, so higher funding costs when uncertainty is low propagate to lower prices when uncertainty is high.

5.5. *The Volcker rule*

We interpret the Volcker rule as imposing a segregation between intermediaries that hold risky and safe assets.[24] At this general level, our analysis could also refer to the reintroduction of the Glass-Steagall act. A full evaluation of the merits of these policies is beyond the scope of our paper. In particular, our framework lacks any notion of moral hazard which is often

[24]Note that liabilities-side segregation (e.g. between issuers of money and shadow money) has no effect in our model as the ability to issue equity costlessly allows capital to flow freely across intermediaries.

cited as a rationale for their implementation. Our aim here is to offer a perspective on the implications of risk-based balance sheet segregation for liquidity provision.

The key point is that our model features a complementarity between risky and safe asset holdings whenever flight to quality effects are present. The intuition is the same as in the discussion of Operation Twist: flight to quality makes safe capital act as a hedge for risky capital on the balance sheet, which raises pledgability. Under a Volcker rule, collateral is effectively wasted as a safe bank has too much and a risky bank has too little.

To illustrate, suppose there is flight to quality so that the value of safe capital rises in a crash. Consider a Volcker rule economy with two (types of) intermediaries, a safe bank S and a risky bank R. Each bank's balance sheets must satisfy the solvency constraints (20) and (21):

$$w_S^m + w_S^r \leq \min\left\{1, \left(1-\kappa_\pi^b\right)\left(1-\kappa^b\right) + w_S^r\overline{\kappa}\right\} \tag{45}$$

$$w_R^m + w_R^r \leq \min\left\{1, \left(1-\kappa_\pi^a\right)\left(1-\kappa^a\right) + w_R^r\overline{\kappa}\right\}. \tag{46}$$

Flight to quality implies that the safe bank has excess collateral that allows it to issue 100% money, $w_S^m = 1$ and $w_S^r = 0$. The risky bank behaves as in our model, so its crash-solvency constraint binds, $w_R^m + w_R^r = \left(1-\kappa_\pi^a\right)\left(1-\kappa^a\right) + w_R^r\overline{\kappa}$. Let $x = \pi^a k^a / \left(\pi^a k^a + \pi^b k^b\right)$ represent the value-weighted share of type-A capital. Then aggregate liquidity under the Volcker rule satisfies

$$w^m + w^r = x\left(w_R^m + w_R^r\right) + (1-x)\left(w_S^m + w_S^r\right) \tag{47}$$

$$= x\left(1-\kappa_\pi^a\right)\left(1-\kappa^a\right) + (1-x) + w^r\overline{\kappa}. \tag{48}$$

In the benchmark economy, we have instead

$$w^m + w^r \;=\; x\left(1-\kappa_\pi^a\right)\left(1-\kappa^a\right) + (1-x)\left(1-\kappa_\pi^b\right)\left(1-\kappa^b\right) + w^r\overline{\kappa}. \tag{49}$$

Comparing (48) with (49) makes clear that flight to quality $\left(1-\kappa_\pi^b\right)\left(1-\kappa^b\right) > 1$ leads to lower liquidity provision $w^m + w^r$ in the Volcker-economy. In general both normal-times and crash-proof liquidity are lower even though the Volcker rule only reduces pledgability in a crash. The scarcity of collateral on the risky bank's balance sheet reduces its capacity to provide normal-times liquidity by levering up with shadow money. At the same time, the excess collateral on the safe bank's balance sheet goes unused.

We leave a more comprehensive analysis of the new equilibrium under a Volcker rule for future work while we believe the basic qualitative insights discussed here to be a robust feature of our model.

6. Conclusion

Reinhart and Rogoff (2009) uncover a common feature among financial crises: slow recoveries. A few years of lost growth can damage economic wellbeing far more than the initial shock that caused them. This fact motivates our paper which aims at the financial origins of slow recoveries. We find that financial intermediation can make the macroeconomic effects of uncertainty shocks greatly exceed those of level shocks. A persistent rise in uncertainty generates both a financial crisis (a quick drop in liquidity provision) and a prolonged aftermath. Reinhart and Reinhart (2010) find that indeed half of all economies that suffer a financial crisis are also subjected to follow-up crises.

Our focus is on the macroeconomic effects of shadow banking which we interpret as the creation of securities that are money-like most of the time but cease to be liquid in the event of a crisis. Our framework embeds an essential tradeoff: while shadow banking greatly increases the supply of liquidity when times are good, it uses up scarce collateral and crowds out crash-proof securities. The end result is a strongly pro-cyclical supply of liquidity, financial instability, and economic fragility. Nevertheless, if absent shadow banking the landing at the cycle's trough is not as hard, it is likely because the fall was from a lesser height.

Appendix

A. *Filtering*

Let $\mathcal{F}_t$ represent the public information filtration. Agents form beliefs

$$\lambda_t = E\left[\tilde{\lambda}_t \middle| \mathcal{F}_t\right]. \tag{50}$$

We assume agents learn about $\tilde{\lambda}$ from the realization of a jump (or lack thereof) and an additional noisy public signal. Specifically, the innovation to the household filtration $\mathcal{F}$ can be represented by the 2×1 signal

$$de_t = \begin{bmatrix} \left(\tilde{\lambda}_t - \lambda_t\right) dt + \sigma_e d\tilde{B}_t \\ dN_t - \lambda_t dt \end{bmatrix}, \tag{51}$$

where $\sigma_e > 0$ is the noise in the diffusive component of the signal and $\tilde{B}_t$ is a standard Brownian motion uncorrelated with all other fundamental shocks.

We seek to compute an innovations representation of the form

$$d\lambda_t = A_t dt + B_t' de_t. \tag{52}$$

Note that

$$\begin{aligned} d\lambda_t &= E\left[\tilde{\lambda}_{t+dt} \middle| \mathcal{F}_t, de_t\right] - E\left[\tilde{\lambda}_t \middle| \mathcal{F}_t\right] && (53)\\ &= E\left[\tilde{\lambda}_t + d\tilde{\lambda}_t \middle| \mathcal{F}_t, de_t\right] - E\left[\tilde{\lambda}_t \middle| \mathcal{F}_t\right] && (54)\\ &= \left[-\left(\lambda_t - \lambda^L\right) q^H + \left(\lambda^H - \lambda_t\right) q^L\right] dt + E\left[\tilde{\lambda}_t \middle| \mathcal{F}_t, de_t\right] - E\left[\tilde{\lambda}_t \middle| \mathcal{F}_t\right]. && (55) \end{aligned}$$

The last line follows from the fact that the jump and noisy signal innovations are uncorrelated with the process for $\tilde{\lambda}$. The innovation representation is therefore the conditional mean of the population regression

$$\tilde{\lambda}_t - \lambda_t = \left[-\left(\lambda_t - \lambda^L\right) q^H + \left(\lambda^H - \lambda_t\right) q^L\right] dt + B_t' de_t + \epsilon_t. \tag{56}$$

The orthogonality condition for ϵ gives

$$\begin{aligned} B_t &= E\left[de_t de_t' \middle| \mathcal{F}_t\right]^{-1} E\left[de_t \left(\tilde{\lambda}_t - \lambda_t\right) \middle| \mathcal{F}_t\right] && (57)\\ &= \begin{bmatrix} \sigma_e^2 dt & 0 \\ 0 & \lambda_t dt \end{bmatrix}^{-1} \begin{bmatrix} \left(\lambda^H - \lambda_t\right)\left(\lambda_t - \lambda^L\right) dt \\ \left(\lambda^H - \lambda_t\right)\left(\lambda_t - \lambda^L\right) dt \end{bmatrix} && (58)\\ &= \begin{bmatrix} \frac{1}{\sigma_e^2} \\ \frac{1}{\lambda_t} \end{bmatrix} \left(\lambda^H - \lambda_t\right)\left(\lambda_t - \lambda^L\right) && (59) \end{aligned}$$

Therefore, we can write the dynamics of the perceived jump intensity as

$$d\lambda_t = \left[-\left(\lambda_t - \lambda^L\right) q^H + \left(\lambda^H - \lambda_t\right) q^L\right] dt + \left(\lambda^H - \lambda_t\right)\left(\lambda_t - \lambda^L\right) \left[\begin{array}{cc} \frac{1}{\sigma_e^2} & \frac{1}{\lambda_t} \end{array}\right] de_t \quad (60)$$

$$= \left[-\left(\lambda_t - \lambda^L\right) q^H + \left(\lambda^H - \lambda_t\right) q^L - \left(\lambda^H - \lambda_t\right)\left(\lambda_t - \lambda^L\right)\right] dt + \frac{1}{\sigma_e} dB_t$$
$$+\frac{1}{\lambda_t}\left(\lambda^H - \lambda_t\right)\left(\lambda_t - \lambda^L\right) dN_t. \quad (61)$$

Rearranging,

$$\frac{d\lambda_t}{\left(\lambda^H - \lambda_t\right)\left(\lambda_t - \lambda^L\right)} = \left(-\frac{q^H}{\lambda^H - \lambda_t} + \frac{q^L}{\lambda_t - \lambda^L} - 1\right) dt + \frac{1}{\sigma_e} dB_t + \frac{1}{\lambda_t} dN_t. \quad (62)$$

This confirms (7).

B. *Learning and liquidity*

In this section we show how to motivate our definition of liquidity as a sufficient condition for avoiding information acquisition by arbitrageurs and the resulting adverse selection problem.

Consider an arbitrageur who runs a fund with assets under management normalized to one. For simplicity, assume that the fund is short-lived so its investment horizon has length dt. At the end of this period, the arbitrageur is compensated for outperforming a leverage-adjusted benchmark composed of the underlying assets.

We consider arbitrageurs in the equity and shadow money markets separately. This allows us to define the liquidity of one asset in isolation from the other. To motivate this assumption, one can think of hedge funds that adhere to specific investment styles.

An equity fund trades money and equity, yielding a return $dS_t = dm_t + w_t\left(dR_t - dm_t\right)$, where w_t is the portfolio weight in equity. On the other hand, her benchmark return is $dI_t = dm_t + \overline{w}_t\left(dR_t - dm_t\right)$, where $\overline{w}_t = E_t\left[w_t|\, \lambda_t\right]$. In order to obtain finite demand, suppose the fund faces a quadratic position cost $\frac{\alpha}{2}\left(w_t - \overline{w}_t\right)^2$. This means that an infinite position is prohibitively expensive. This could be due to some form of risk aversion or more broadly decreasing returns to scale at the trading strategy level.

Let ϑ_t be the LaGrange multiplier for the constraint $\overline{w}_t = E_t\left[w_t|\, \lambda_t\right]$. An informed arbitrageur thus maximizes

$$V_t dt = \max_{w_t} E_t\left[dS_t - dI_t|\, \tilde{\lambda}\right] - \frac{\alpha}{2}\left(w_t - \overline{w}_t\right)^2 dt + \vartheta_t\left(\overline{w}_t - E_t\left[w_t|\, \lambda_t\right]\right) dt \quad (63)$$

$$= \max_{w_t} E_t\left[\left(w_t - \overline{w}_t\right)\left(dR_t - dm_t\right)|\, \tilde{\lambda}\right] - \frac{\alpha}{2}\left(w_t - \overline{w}_t\right)^2 dt \quad (64)$$
$$+\vartheta_t\left(\overline{w}_t - E_t\left[w_t|\, \lambda_t\right]\right) dt.$$

The optimal policy is

$$w_t = \overline{w}_t + \frac{1}{\alpha}\left[\mu_{R,t} - \mu_{m,t} - \left(\sigma_{R,t} + \kappa_{R,t}\right)\left(\tilde{\lambda}_t - \lambda_t\right) - \vartheta_t\right]. \quad (65)$$

Averaging over $\tilde{\lambda}_t$ gives $\vartheta_t = \mu_{R,t} - \mu_{m,t}$. Therefore,

$$w_t = \overline{w}_t - \frac{1}{\alpha}\left(\sigma_{R,t} + \kappa_{R,t}\right)\left(\tilde{\lambda}_t - \lambda_t\right). \quad (66)$$

The resulting maximized objective is

$$V_t = \frac{1}{2\alpha}\left(\sigma_{R,t} + \kappa_{R,t}\right)^2\left(\tilde{\lambda}_t - \lambda_t\right)^2, \quad (67)$$

The ex ante (prior to the learning decision) maximized objective is

$$E_t\left[V_t \middle| \lambda_t\right] = \frac{1}{2\alpha}\left(\sigma_{R,t} + \kappa_{R,t}\right)^2 Var_t\left(\tilde{\lambda}_t \middle| \lambda_t\right). \quad (68)$$

Suppose the cost of learning accrues at a rate $fVar_t\left(\tilde{\lambda}_t \middle| \lambda_t\right)$. Then there will be no learning provided $\frac{1}{2\alpha}\left(\sigma_{R,t} + \kappa_{R,t}\right)^2 < f$ or simply

$$\sigma_{R,t} + \kappa_{R,t} < \sqrt{2\alpha f}. \quad (69)$$

When this restriction fails to hold, arbitrageurs find it optimal to acquire information. As capital flows into their funds, an adverse selection problem arises, reducing market liquidity. As a result, a household experiencing a liquidity event cannot unload large quantities of equity quickly without incurring substantial costs in the form of price impact or bid-ask spreads. Outside of a liquidity event, households can avoid these costs by trading more patiently or by simply buying and holding. It is with this interpretation in mind that we model the trading of equity in a liquidity event as prohibitive but at other times as costless.

By analogy, shadow-money remains liquid as long as

$$\kappa_{r,t} < \sqrt{2\alpha f}. \quad (70)$$

We can therefore equate $\sqrt{2\alpha f}$ with $\overline{\kappa}$ in the model. This condition ensures that households can liquidate their shadow money holdings quickly at no cost when the need arises as their trading partners can be confident that they are not dealing with privately-informed agents.

C. *Optimal capital structure*

Proof of Proposition 1. Consider an intermediary whose assets have collateral value $1 - \kappa_A$. Let l^n and l^c be the amount of liquidity the intermediary delivers in normal times and in a crash. The intermediary's liquidity provision problem is

$$\max_{l^n, l^c} \; l^c\left(\mu_R - \mu_m\right) + \left(l^n - l^c\right)\left(\mu_R - \mu_r\right) \quad (71)$$

subject to the constraints with associated multipliers

$$\begin{array}{rrcl} \theta_0^c: & l^c & \geq & 0 \\ \theta_0^n: & l^n & \geq & l^c \\ \theta^c: & l^n & \leq & 1-\kappa_A+(l^n-l^c)\,\kappa_r \\ \theta^n: & l^n & \leq & 1. \end{array} \tag{72}$$

Clearly $\kappa_r = \overline{\kappa}$ is optimal as it relaxes the collateral constraint. We substitute $\kappa_r = \overline{\kappa}$ from here on.

The LaGrangian is

$$\begin{aligned} \max_{l^n,l^c} \quad & l^c\left(\mu_R-\mu_m\right)+\left(l^n-l^c\right)\left(\mu_R-\mu_r\right)+\theta_0^c l^c+\theta_0^n\left(l^n-l^c\right) \\ & +\theta^c\left[1-\kappa_A+\left(l^n-l^c\right)\overline{\kappa}-l^n\right]+\theta^n\left(1-l^n\right). \end{aligned} \tag{73}$$

The optimality conditions are

$$\mu_R-\mu_m = \theta^c+\theta^n-\theta_0^c \tag{74}$$

$$\mu_R-\mu_r = \left(1-\overline{\kappa}\right)\theta^c+\theta^n-\theta_0^n. \tag{75}$$

Since $\mu_R-\mu_m > \mu_R-\mu_r$, we have $\overline{\kappa}\theta^c > \theta_0^c-\theta_0^n$.

Case (i): Suppose $\theta_0^c > 0$ so $l^c = 0$. We must have $\theta^c > 0$. (To see why, suppose $\theta^c = 0$. Then since $\mu_R-\mu_m > 0$, $\theta^n > 0$ but then $l^n = 1$ and so $\theta_n^0 = 0$. But then $\overline{\kappa}\theta^c > \theta_0^c-\theta_0^n$ cannot hold.) Therefore $l^n = 1-\kappa_A+\left(l^n-l^c\right)\overline{\kappa}$ and so $l^n = \left(1-\kappa_A\right)/\left(1-\overline{\kappa}\right)$. This requires $\kappa_A \geq \overline{\kappa}$. It follows that $\theta_0^n = \theta^n = 0$. Substituting for the spreads gives $\theta^c-\theta_0^c = h\left(\psi-1\right)\left[e^{-\tau\lambda}e^{-\eta\left(\frac{1-\kappa_A}{1-\overline{\kappa}}\right)}+\left(1-e^{-\tau\lambda}\right)\right]$ and $\left(1-\overline{\kappa}\right)\theta^c = h\left(\psi-1\right)e^{-\tau\lambda}e^{-\eta\left(\frac{1-\kappa_A}{1-\overline{\kappa}}\right)}$. Then $\theta_0^c = h\left(\psi-1\right)\left[\frac{\overline{\kappa}}{1-\overline{\kappa}}e^{-\tau\lambda}e^{-\eta\left(\frac{1-\kappa_A}{1-\overline{\kappa}}\right)}-\left(1-e^{-\tau\lambda}\right)\right]$ and $\theta^c = h\left(\psi-1\right)\frac{1}{1-\overline{\kappa}}e^{-\tau\lambda}e^{-\eta\left(\frac{1-\kappa_A}{1-\overline{\kappa}}\right)}$. Since $\theta_0^c > 0$, $\kappa_A > 1-\frac{1-\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda}}{1-e^{-\tau\lambda}}\right)$.

Case (ii): Suppose $\theta_0^n > 0$ so $l^n = l^c$. Since $\kappa_A > 0$, $l^n < 1$ so $\theta^n = 0$. Since $\mu_R-\mu_r > 0$, $\theta^c > 0$, so $l^n = l^c = 1-\kappa_A$. This implies that $\theta_0^c = 0$. Substituting for the spreads, $\theta^c = h\left(\psi-1\right)e^{-\eta\left(1-\kappa_A\right)}$ and $\left(1-\overline{\kappa}\right)\theta^c-\theta_n^0 = h\left(\psi-1\right)e^{-\tau\lambda}e^{-\eta\left(1-\kappa_A\right)}$. Therefore, $\theta_n^0 = h\left(\psi-1\right)\left[\left(1-\overline{\kappa}\right)-e^{-\tau\lambda}\right]e^{-\eta\left(1-\kappa_A\right)}$. This case requires $\lambda > -\frac{1}{\tau}\log\left(1-\overline{\kappa}\right)$.

Case (iii): Suppose $\theta^n > 0$ so $l^n = 1$. Since $\kappa_A > 0$, $\theta_n^0 = 0$. Then from $\overline{\kappa}\theta^c > \theta_0^c-\theta_0^n$, $\theta^c > 0$ so $l^c = 1-\kappa_A/\overline{\kappa}$, which requires $\kappa_A \leq \overline{\kappa}$ and gives $\theta_0^c = 0$. Substituting for the spreads, $\theta^c+\theta^n = h\left(\psi-1\right)\left[e^{-\tau\lambda}e^{-\eta}+\left(1-e^{-\tau\lambda}\right)e^{-\eta\left(1-\frac{\kappa_A}{\overline{\kappa}}\right)}\right]$ and $\left(1-\overline{\kappa}\right)\theta^c+\theta_n = h\left(\psi-1\right)e^{-\tau\lambda}e^{-\eta}$. Thus, $\theta^n = h\left(\psi-1\right)\left[e^{-\tau\lambda}e^{-\eta}+\left(1-\frac{1}{\overline{\kappa}}\right)\left(1-e^{-\tau\lambda}\right)e^{-\eta\left(1-\frac{\kappa_A}{\overline{\kappa}}\right)}\right]$ and $\theta^c = \frac{1}{\overline{\kappa}}h\left(\psi-1\right)\left(1-e^{-\tau\lambda}\right)e^{-\eta\left(1-\frac{\kappa_A}{\overline{\kappa}}\right)}$. This case also requires $\kappa_A < \frac{\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda}}{1-e^{-\tau\lambda}}\right)$.

Case (iv): Suppose $\theta_0^c = \theta_0^n = \theta^n = 0$. Then $\theta^c > 0$ and $l^n = 1-\kappa_A+\left(l^n-l^c\right)\overline{\kappa}$. Substituting for the spreads, $h\left(\psi-1\right)\left[e^{-\tau\lambda}e^{-\eta l^n}+\left(1-e^{-\tau\lambda}\right)e^{-\eta l^c}\right] = \left(1-\overline{\kappa}\right)h\left(\psi-1\right)\left[e^{-\tau\lambda}e^{-\eta l^n}\right]$. Solving for l^n and l^c gives $l^n = \left(1-\kappa_A\right)+\frac{\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda}}{1-e^{-\tau\lambda}}\right)$, $l^c = \left(1-\kappa_A\right)-\frac{1-\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda}}{1-e^{-\tau\lambda}}\right)$, and $\theta^c = h\left(\psi-1\right)e^{-\eta\left(1-\kappa_A\right)}\left(\frac{e^{-\tau\lambda}}{1-\overline{\kappa}}\right)^{1-\overline{\kappa}}\left(\frac{1-e^{-\tau\lambda}}{\overline{\kappa}}\right)^{\overline{\kappa}}$. $\square$

Proof of Proposition 2. Case (i): The optimal capital structure can be implemented with $w^m = 0$ and $w^r = (1-\kappa_A)/(1-\overline{\kappa})$. Therefore, equity is $1 - w^m - w^r = (\kappa_A - \overline{\kappa})/(1-\overline{\kappa})$. As all collateral is used to back shadow money, $\kappa_R = 1$ so equity is wiped out in a crash and is therefore illiquid. Optimal liquidity provision requires shadow money but not money.

Case (ii): The optimal liquidity supply can be implemented with $w^m = 1 - \kappa_A$, $w_r = 0$. Equity is $1 - w^m - w^r = \kappa_A$ and is thus wiped out in a crash, $\kappa_R = 1$. Since $\sigma_A \geq 0$ (asset value is decreasing in λ), equity is illiquid. The optimal policy can be implemented without shadow money but not without money or equity.

Case (iii): The optimal capital structure can be implemented with $w^m = 1 - \kappa_A/\overline{\kappa}$ and $w^r = \kappa_A/\overline{\kappa}$. Normal-times risk is concentrated on equity, $\sigma_{R,t} = \infty$. Without loss of generality, $\kappa_{R,t} = 1$. Equity is illiquid. Optimal liquidity provision requires all three securities.

Case (iv): The optimal capital structure can be implemented with $w^m = (1-\kappa_A) - \frac{1-\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda}}{1-e^{-\tau\lambda}}\right)$ and $w^r = \frac{1}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda}}{1-e^{-\tau\lambda}}\right)$. Since $w^m + (1-\overline{\kappa})\,w^r = 1-\kappa_A$, all collateral is pledged, so equity is again wiped out in a crash, $\kappa_R = 1$, so it cannot be liquid. $\square$

D. Numerical solution

Using the adjustment cost function $\phi(\iota) = \frac{1}{\varphi}\left(\sqrt{1+2\varphi\iota}-1\right)$, the optimality conditions for capital become

$$\mu_R - \theta\left(1-\kappa_\pi^a\right)\left(1-\kappa^a\right) - \theta^1 = \frac{\gamma^a + \frac{1}{2\varphi}}{\pi^a} + \frac{\pi^a}{2\varphi} + \mu_\pi^a - \frac{1}{\varphi} - \delta + \kappa^a\kappa_\pi^a\lambda \tag{76}$$

$$\mu_R - \theta\left(1-\kappa_\pi^b\right)\left(1-\kappa^b\right) - \theta^1 = \frac{\gamma^b + \frac{1}{2\varphi}}{\pi^b} + \frac{\pi^b}{2\varphi} + \mu_\pi^b - \frac{1}{\varphi} - \delta + \kappa^b\kappa_\pi^b\lambda. \tag{77}$$

Substituting for the investment cost function and the investment optimality conditions into (4), the dynamics of χ are

$$\begin{aligned} d\chi &= \mu\left(1-2\chi\right)dt + \chi\left(1-\chi\right)\left[\frac{\pi^a-\pi^b}{\varphi} + \lambda\left(\kappa^a-\kappa^b\right)\right]dt \\ &\quad - \chi\left(1-\chi\right)\left[\frac{\kappa^a-\kappa^b}{\chi\left(1-\kappa^a\right)+\left(1-\chi\right)\left(1-\kappa^b\right)}\right]dN. \end{aligned} \tag{78}$$

To get the dynamics of π^a and π^b, apply Ito's Lemma:

$$\begin{aligned}\frac{d\pi^a}{\pi^a} &= \left[\frac{\pi^a_\lambda}{\pi^a}\left(\lambda-\lambda^L\right)\left(\lambda^H-\lambda\right)\left(-\frac{q^H}{\lambda^H-\lambda}+\frac{q^L}{\lambda-\lambda^L}-1\right)\right. \\ &\quad +\frac{1}{2}\frac{\pi^a_{\lambda\lambda}}{\pi^a}\left(\lambda-\lambda^L\right)^2\left(\lambda^H-\lambda\right)^2\frac{1}{\sigma_e^2} \\ &\quad \left.+\frac{\pi^a_\chi}{\pi^a}\chi\left(1-\chi\right)\left(\mu\left(\frac{1}{\chi}-\frac{1}{1-\chi}\right)+\frac{1}{\varphi}\left(\pi^a-\pi^b\right)+\lambda\left(\kappa^a-\kappa^b\right)\right)\right]dt \\ &\quad +\frac{\pi^a_\lambda}{\pi^a}\left(\lambda-\lambda^L\right)\left(\lambda^H-\lambda\right)\frac{1}{\sigma_e}dB \\ &\quad -\left[1-\frac{\pi^a\left(\lambda+\frac{1}{\lambda}\left(\lambda-\lambda^L\right)\left(\lambda^H-\lambda\right),\chi-\frac{\chi(1-\chi)\left(\kappa^a-\kappa^b\right)}{\chi(1-\kappa^a)+(1-\chi)\left(1-\kappa^b\right)}\right)}{\pi^a}\right]dN\end{aligned} \tag{79}$$

and similarly for π^b. We solve for π^a and π^b using projection methods.

E. *State-space density*

Let $f_t(\lambda,\chi)$ be the joint density of λ and χ at date t given initial density $f_0(\lambda,\chi)$. We calculate f by solving the associated forward Kolmogorov equation (Hanson 2007, Theorem 7.7):

$$\begin{aligned}\frac{\partial f_t}{\partial t} &= -\frac{\partial}{\partial\lambda}\left(\mu_\lambda f_t\right)-\frac{\partial}{\partial\chi}\left(\mu_\chi f_t\right)+\frac{1}{2}\frac{\partial^2}{\partial\lambda^2}\left(\sigma_\lambda^2 f_t\right) \\ &\quad +\left[\lambda^- f_t\left(\lambda^-,\chi^-\right)\left|\frac{\partial\lambda^-}{\partial\lambda}\right|\left|\frac{\partial\chi^-}{\partial\chi}\right|-\lambda f_t\left(\lambda,\chi\right)\right].\end{aligned} \tag{80}$$

We have

$$\begin{aligned}\frac{\partial}{\partial\lambda}\left(\mu_\lambda f_t\right) &= \left[2\lambda-\left(q^H+q^L\right)-\left(\lambda^H+\lambda^L\right)\right]f_t \\ &\quad +\left[-q^H\left(\lambda-\lambda^L\right)+q^L\left(\lambda^H-\lambda\right)-\left(\lambda-\lambda^L\right)\left(\lambda^H-\lambda\right)\right]\frac{\partial f_t}{\partial\lambda}\end{aligned} \tag{81}$$

$$\begin{aligned}\frac{\partial}{\partial\chi}\left(\mu_\chi f_t\right) &= \left[\frac{1}{\varphi}\left(\pi^a-\pi^b\right)+\lambda\left(\kappa^a-\kappa^b\right)\right]\left[\left(1-2\chi\right)f_t+\chi\left(1-\chi\right)\frac{\partial f_t}{\partial\chi}\right] \\ &\quad +\mu\left[\left(1-2\chi\right)\frac{\partial f_t}{\partial\chi}-2f_t\right]+\frac{1}{\varphi}\left(\pi^a_\chi-\pi^b_\chi\right)\chi\left(1-\chi\right)f_t\end{aligned} \tag{82}$$

$$\frac{\partial^2}{\partial\lambda^2}\left(\sigma_\lambda^2 f_t\right) = 2\left[\left(\frac{\partial\sigma_\lambda}{\partial\lambda}\right)^2+\sigma_\lambda\left(\frac{\partial^2\sigma_\lambda}{\partial\lambda^2}\right)\right]f_t+4\sigma_\lambda\left(\frac{\partial\sigma_\lambda}{\partial\lambda}\right)\frac{\partial f_t}{\partial\lambda}+\sigma_\lambda^2\frac{\partial^2 f_t}{\partial\lambda^2}, \tag{83}$$

where $\sigma_\lambda = \left(\lambda - \lambda^L\right)\left(\lambda^H - \lambda\right)\frac{1}{\sigma_e}$, $\frac{\partial \sigma_\lambda}{\partial \lambda} = \left(\lambda^L + \lambda^H - 2\lambda\right)\frac{1}{\sigma_e}$, and $\frac{\partial^2 \sigma_\lambda}{\partial \lambda^2} = -\frac{2}{\sigma_e}$. The pre-jump state values λ^- and χ^- are given by

$$\lambda^- = \frac{\lambda^L \lambda^H}{\lambda^L + \lambda^H - \lambda} \tag{84}$$

$$\chi^- = \frac{1}{1 + \left(\frac{1-\kappa^a}{1-\kappa^b}\right)\left(\frac{1-\chi}{\chi}\right)} \tag{85}$$

with $\frac{\partial \lambda^-}{\partial \lambda} = \frac{\lambda^L \lambda^H}{\left(\lambda^L + \lambda^H - \lambda\right)^2}$ and $\frac{\partial \chi^-}{\partial \chi} = \frac{\left(\frac{1-\kappa^a}{1-\kappa^b}\right)\left(\frac{1}{\chi^2}\right)}{\left[1+\left(\frac{1-\kappa^a}{1-\kappa^b}\right)\left(\frac{1-\chi}{\chi}\right)\right]^2}$.

F. *Implementation of asset purchases*

We nest the permanent and transitory LSAP programs with $\beta_{TAPER}\left(\lambda_t, \chi_t\right) = 0$ under a permanent policy and $\beta_{TAPER}\left(\lambda_t, \chi_t\right) > 0$ under a transitory policy. We can express prices as $\pi_t^i = \pi^i\left(\lambda_t, \chi_t, \zeta_t\right)$ for $i = a, b$. The drift and pledgability of prices when $\zeta_t = \zeta_E$ can be written as

$$\mu_{\pi,\zeta_E}^i = \mu_{\pi,\zeta_0}^i - \lambda \beta_{LSAP}\left(\kappa_{\pi,LSAP}^i - \kappa_\pi^i\right) \tag{86}$$

$$1 - \kappa_{\pi,\zeta_E}^i = \min\left\{1 - \kappa_{\pi,LSAP}^i, 1 - \kappa_\pi^i\right\}, \tag{87}$$

for $i = a, b$ where μ_{π,ζ_0}^i is given in (79), $1 - \kappa_{\pi,LSAP}^i = \frac{\pi^i(\lambda_+, \chi_+ - \alpha, \zeta_F)}{\pi(\lambda, \chi, \zeta_E)}$ (LSAP shifts χ^+ to $\chi^+ - \alpha$), and $1 - \kappa_\pi^i = \frac{\pi(\lambda_+, \chi_+, \zeta_E)}{\pi(\lambda, \chi, \zeta_E)}$. These modified dynamics enter into (39) and (40). Under $\zeta_t = \zeta_F$, we have

$$\mu_{\pi,\zeta_F}^i = \mu_{\pi,\zeta_0}^i - \beta_{TAPER} \kappa_{\pi,TAPER}^i. \tag{88}$$

for $i = a, b$ with $1 - \kappa_{\pi,TAPER} = \frac{\pi(\lambda, \chi + \alpha, \zeta_E)}{\pi^i(\lambda, \chi, \zeta_F)}$. To keep things simple, we do not impose solvency constraints with respect to the reversal shock. This has the effect of understating the impact of policy withdrawal. In addition, the policy's entry and exit are generally not of equal size as the economy drifts in the meantime. In this case one can think of the central bank's balance sheet as retaining a residual position.

We measure the announcement effect of an LSAP program as the difference in the percentage change in crash returns with and without the intervention, $\kappa_\pi^i - \kappa_{\pi,LSAP}^i$. A policy reversal shock is measured analogously as $\kappa_{\pi,TAPER}$ though it is not concurrent with a crash. We measure ex-ante effects by $\frac{\pi^i(\lambda,\chi,\zeta_E)}{\pi^i(\lambda,\chi,\zeta_0)}$; the impact on prices of central bank capacity by $\frac{\pi^i(\lambda,\chi,\zeta_E)}{\pi^i(\lambda,\chi,\zeta_F)}$; and an unanticipated tapering shock that raises β_{TAPER}.

In the case of a permanent intervention, the economy at $\zeta_t = \zeta_F$ corresponds to the benchmark economy and we can solve backwards to obtain prices under ζ_E. When interventions are transitory, there is two-way flow between ζ_E and ζ_F and we solve for prices under the two regimes simultaneously.

G. Operation Twist

Operation twist changes the composition of government debt (α), while keeping its value constant. The change in the quantity of government debt is characterized as follows:

Let x_- and x_+ denote the pre- and post-intervention values of a given quantity x. For example π_+^{bl} is the post-intervention price of the long-term bond. Assuming that the government trades at post-announcement prices and that the intervention does not change the overall value of government liabilities, the pre- and post- quantities must respect

$$k_-^B(\alpha_-\pi_+^{bs} + (1-\alpha_-)\pi_+^{bl}) \;=\; k_+^B(\alpha_+\pi_+^{bs} + (1-\alpha_+)\pi_+^{bl}). \tag{89}$$

We solve for net new issuance $\frac{k_+^B}{k_-^B}$ as a function of equilibrium prices and the change in the debt maturity mix $\alpha_+ - \alpha_-$.

H. Capital requirements

The solution to the intermediary capital structure problem in the presence of capital requirements is characterized by

Proposition 3. *Let $\underline{e}$ be the minimum equity capital requirement, the intermediary's optimal liquidity provision policy is implemented by*

i. $w_t^m = 1 - \underline{e}$ *and* $w_t^r = 0$ *if* $\kappa_{A,t} \le \underline{e}$;

ii. $w_t^m = 1 - \underline{e} - \frac{\kappa_{A,t} - \underline{e}}{\overline{\kappa}}$ *and* $w_t^r = \frac{\kappa_{A,t} - \underline{e}}{\overline{\kappa}}$ *if* $\kappa_{A,t} \le \underline{e} + \overline{\kappa}$ *and* $\kappa_{A,t} < e + \frac{\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$;

iii. $w_t^m = 0$ *and* $w_t^r = (1-\kappa_{A,t})\,/\,(1-\overline{\kappa})$ *if* $\kappa_{A,t} > \overline{\kappa} + \underline{e} - \underline{e}\overline{\kappa}$ *and* $\kappa_{A,t} > 1 - \frac{1-\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$;

iv. $w_t^m = 1 - \kappa_{A,t}$ *and* $w_t^r = 0$ *if* $\lambda_t > -\frac{1}{\tau}\log(1-\overline{\kappa})$ *and* $\kappa_{A,t} \ge \underline{e}$;

v. $w_t^m = (1-\kappa_{A,t}) - \frac{1-\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$ *and* $w_t^r = \frac{1}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$ *otherwise.*

Proof of Proposition 3. The proof follows the recipe from Propositions 1 and 2 under the modified normal-times solvency constraint $w_t^m + w_t^r \le 1 - \underline{e}$. □

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