A Regime Switching Skew-normal Model for Measuring Financial Crisis and Contagion

Joshua C.C. Chan[ˆ,%], Renée A. Fry-McKibbin[%,#] and Cody Yu-Ling Hsiao[%,#*]

[%]Centre for Applied Macroeconomic Analysis (CAMA)
[#]Crawford School of Public Policy, Australian National University
[ˆ]Research School of Economics, Australian National University

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Abstract

A regime switching skew-normal model for financial crisis and contagion is proposed in which we develop a new class of multiple-channel crisis and contagion tests. Crisis channels are measured through changes in 'own' moments of the mean, variance and skewness, while contagion is through changes in the covariance and co-skewness of the joint distribution of asset returns. In this framework: i) linear and non-linear dependence is allowed; ii) transmission channels are simultaneously examined; iii) crisis and contagion are distinguished and individually modeled; iv) the market that a crisis originates is endogenous; and v) the timing of a crisis is endogenous. In an empirical application, we apply the proposed model to equity markets during the Great Recession using Bayesian model comparison techniques to assess the multiple channels of crisis and contagion. The results generally show that crisis and contagion are pervasive across Europe and the US. The second moment channels of crisis and contagion are systematically more evident than the first and third moment channels.

Keywords: Great Recession, Crisis tests, Contagion tests, Co-skewness, Regime switching skew-normal model, Gibbs sampling, Bayesian model comparison

JEL Classification: C11, C34, G15

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1 Introduction

The US subprime mortgage delinquencies in the late 2000's formed the foundations of the Great Recession in the US. Although the crisis began in the US, the shock spread globally to a wide range of markets and countries, with a clear channel of the shock transmission arising through the subsequent impact on the interbank markets. The popular term first coined by Goldstein (1998) describing the phenomenon of the transmission of a crisis from a crisis affected market to others is financial market contagion. Contagion models are based on identifying significant changes in the dependence structures between financial asset returns during financial crisis compared to non-crisis times. Using a Bayesian approach, this paper builds on Hamilton (1989) by developing a regime-switching skew-normal (RSSN) model of crisis and contagion, mainly by relaxing the assumption of the error term which is assumed here to be a multivariate skew-normal distribution. The framework is able to simultaneously measure five different linear and non-linear channels of financial market crisis and contagion transmission, and nine different joint channels of financial market crisis and contagion, and the paper includes an illustrative application to the Great Recession period.

The RSSN framework for detecting financial market crisis and contagion is able to circumvent several econometric problems evident in the contagion literature. First, the framework avoids the sole use of conventional dependence measures such as the Pearson correlation coefficient or adjusted correlation coefficients in testing for contagion as is the case in the earliest literature (King and Wadhwani, 1990; Forbes and Rigobon, 2002). The correlation coefficient method compares an exogenously defined non-crisis period correlation with an exogenously defined crisis period correlation to determine significant changes in the dependence (namely, contagion) between markets.[1] In the framework of multivariate Gaussian distributions, correlation provides an appropriate linear dependence structure. However, it is well accepted that financial market returns are not normally distributed. This class of contagion tests may deliver partial or limited information on the actual underlying dependence of asset returns during extremely bad events (Embrechts et al. 2001b). The framework of the RSSN model allows for both linear and non-linear dependence.

[1] As Dungey and Zhumabekova (2001) and Billio and Pelizon (2003) emphasize, the power of test statistics of contagion can be seriously affected by large difference in sample sizes between tranquil and crisis period, and statistics for contagion are usually sensitive to the definition of crisis periods in particular.

There do exist a range of alternative approaches of testing for contagion that go beyond the linear approach. Several papers focus on higher order co-moments of the returns distribution (i.e. non-linear parameters) instead of using a multivariate Gaussian distribution with a constant linear correlation structure.[2] For example: Longin and Solnik (2001) derive a distribution of extreme correlation based on extreme value theory; Favero and Giavazzi (2002) test for non-linearities in the propagation of devaluation expectations; Bae et al. (2003) use extreme value theory to model the joint behavior of extremal realizations (co-exceedances); Pesaran and Pick (2007) identify outliers in a structural model to examine the threshold tests of contagion; Fry et al. (2010) develop a portfolio model of higher order moments with asymmetric dependence (co-skewness); and Hsiao (2012) explores portfolio choice with extremal dependence (co-kurtosis and co-volatility). Although these works extend the Forbes and Rigobon linear model by including non-linear co-moments (asymmetric, extremal and tail dependencies), none of them simultaneously test for a range of linear and non-linear channels in the same model. This is the second contribution of this paper.

There are a handful of papers which use regime switching models with second order moments and co-moments to measure market relationships. Ang and Bekaert (2002) explore time-varying correlations and volatilities for portfolio choice, Pelletier (2006) allows switching in the correlation structure, and Gravelle, Kichian and Morley (2006) include time varying volatility; Billio et al. (2005) and Kasch and Caporin (2013) include time-varying correlation; Guo et al. (2011) allow simultaneous mean and variance shifts. The regime switching models of these papers analyze only the case of normality of the distribution of the error term, which for the case of financial market crisis and contagion, may ignore potentially important dimensions of financial market data and contagion arising through non-linear dependence. Few papers do specify a regime switching model with higher order moments. Ang and Timmermann (2011) is one example, but they do not focus on contagion. Perhaps the most related in concept to this paper is Rodriguez (2007) who identify time-varying higher order moment and co-moments such as tail dependence using copulas.

A third contribution of this paper is that the non-linear RSSN model provides a general framework for examining different types of transmission channels of financial

[2] Non-linear dependence estimates measure the probability of the worst event occurring in one market given that a worst event occurs in another market, whereas linear dependence, which is weighted by small and large returns, is not appropriate to evaluate the differential impact of large versus small changes in returns (Bae et al., 2003; Garcia and Tsafack, 2007).

market crisis and contagion through changes in the model parameters including of the mean, variance, skewness, covariance and co-skewness.[3] We are careful to distinguish between a crisis where 'own' moments experience a shift in a regime-specific parameter of the RSSN model during a crisis regime, and a channel of contagion where a 'cross-market' moment (co-moment) experiences a shift in a regime-specific parameter of the model during the crisis period. The crisis is captured in the mean-shift, variance-shift and skewness-shift parameters of an asset. Contagion is captured through the covariance-shift, and co-skewness-shift of an asset.

The fourth and fifth contributions of the paper are that the use of the RSSN model allows for endogenous determination of many features of the model which are often not possible in models of crisis and contagion. In particular, the model chooses whether a country is deemed to be in crisis, and the crisis period is endogenously chosen by the model. Most models of contagion do not allow an 'own' crisis to be endogenous, and the crisis duration usually relies on researcher choice.

The final contribution is the empirical application of the RSSN model to European equity returns and US banking equity returns over a period which contains the Great Recession. The empirical results show that the identification of a crisis (shift in "own" parameters) is pervasive across equity markets through the variance-shift and skewness-shift channels. The mean-shift channel is not operational at all. The contagion tests through the covariance-shift and co-skewness-shift channels are significant in almost every case, indicating the importance of changes in market dependence during a crisis. The multivariate framework adopted allows the conduct of multiple-channel crisis and contagion tests. These tests also show strong statistical evidence of the joint channels during the Great Recession. The second order channels of both crisis and contagion (i.e. volatility-shift and covariance-shift) are more evident than the first and third order channels of mean-shift, skewness-shift and co-skewness-shift.

The paper proceeds as follows. Section 2 presents a RSSN model in which five types of crisis and contagion channels are developed. Section 3 documents the Bayesian estimation approach of the RSSN model including the Markov Chain Monte Carlo (MCMC) sampling scheme for estimation of the model, along with the Bayesian model comparison tools used to implement the tests. Section 4 outlines the tests for crisis and contagion for each of the five channels. Section 5 presents the empirical analysis,

[3] The regime switching feature of the model means that the model parameters are allowed to differ which deals with the heteroskedasticity problem arising in several methods caused by a volatility increase during a crisis.

and Section 6 provides some concluding comments.

2 Modeling Crisis and Contagion in the RSSN Framework

A RSSN model is constructed in this section to provide a framework to analyze five types of transmission channels of financial market crisis and contagion. Section 2.1 introduces the underlying multivariate skew-normal distribution and details its properties. Section 2.2 develops the RSSN model, while Section 2.3 extends the model to include the channels of crisis and contagion.

2.1 The Skew-normal Distribution

This section builds on the skew-normal distribution developed by Sahu et al. (2003) to provide a general framework to model crisis and contagion using a RSSN model. The skew-normal distribution has the following latent variable representation:

$$
\begin{aligned}
y_t &= \mu + \Delta Z_t + \varepsilon_t, && (1)\\
\varepsilon_t &\overset{iid}{\sim} N(0, \Sigma), && (2)\\
Z_t &\overset{iid}{\sim} N(c1_m, I_m)\, 1(Z_{jt} > c, j = 1, \ldots, m), && (3)
\end{aligned}
$$

where $Z_t = (Z_{1t}, \ldots, Z_{mt})'$ is an m-dimensional random vector with $t = 1, \ldots, T$. 1_m is an $m \times 1$ column of ones, I_m is the identity matrix and $1(\cdot)$ is the indicator function. The inclusion of the vector of latent variables Z_t induces skewness in the distribution, which enriches the dependence structure between the components of y_t. Sahu et al. (2003) assume that Δ is a diagonal matrix. However, this assumption is restrictive in the context of modeling crisis and contagion since the assumption of a diagonal matrix does not allow for non-linear relationships between the components of y_t. An asymmetric dependence structure for y_t is introduced here by relaxing the assumption that Δ is diagonal. Specifically, $\Delta = (\delta_{ij})$ is a full $m \times m$ co-skewness matrix with $i, j = 1, \ldots, m$. The off-diagonal elements of Δ are the co-skewness parameters which control the asymmetric dependence structure between the components of y_t.

The probability density function of y_t marginally of Z_t is

$$
f_{SN}(y_t; \mu, \Sigma, \Delta) = \frac{2^m}{\det(\Sigma + \Delta^2)^{1/2}} f_N\left(\left(\Sigma + \Delta^2\right)^{-\frac{1}{2}} (y_t - \mu) \right) \Pr(V > 0), \quad (4)
$$

where

$$V \sim N\left(\Delta\left(\Sigma+\Delta^2\right)^{-1}(y_t-\mu), I_m-\Delta\left(\Sigma+\Delta^2\right)^{-1}\Delta\right). \tag{5}$$

$f_N(y_t)$ is the density function of the standard multivariate normal distribution with mean 0 and identity covariance matrix I_m evaluated at y_t. If $\Delta = 0$, then the skew-normal distribution in equations (1) to (3) reduces to the usual multivariate normal specification with the density given by

$$f_N(y_t;\mu,\Sigma) = \frac{1}{\det(\Sigma)^{1/2}} f_N\left(\Sigma^{-\frac{1}{2}}(y_t-\mu)\right). \tag{6}$$

The dependence structure of y_t, for different values of the parameters governing the skew-normal distribution in equation (6) is illustrated in Figure 1. The Figure plots the contours of the bivariate skew-normal density in equation (4) with zero mean ($\mu = 0$), identity scale matrix (Σ=I_2) and various patterns of asymmetric dependence ($\Delta = (\delta_{ij})$, $i, j = 1, 2$). The center panel of Figure 1 illustrates the case of a symmetric bivariate normal distribution with $\delta_{11} = \delta_{22} = \delta_{12} = \delta_{21} = 0$. The remaining panels show how the parameters δ_{ij} influence the dependence structure of y_t, reminiscent of the relationships expected in high frequency financial market data.

The second column of Figure 1 illustrates the effect of changing the level of skewness (δ_{11} and δ_{22}) in y_1 and y_2. The negative skewness values ($\delta_{11} = \delta_{22} = -1.5$) in the top panel generates left skewness in comparison to the bivariate normal distribution in the center. The positive skewness values ($\delta_{11} = \delta_{22} = 1.5$) in the bottom panel generates right skewness. The first and third columns present the contour plots when the values of the off-diagonal elements of the co-skewness matrix δ_{12} and δ_{21} are allowed to vary. These parameters control the level of asymmetry between y_1 and y_2. The first column presents three distributions for a negative value of co-skewness ($\delta_{12} = -1.2$). The top panel presents the interaction of the co-skewness term with negative skewness in both assets ($\delta_{ii} < 0$). The second panel presents the case with no skewness ($\delta_{ii} = 0$), while the bottom panel presents the case with co-skewness and positive skewness. The third column presents the symmetric contour plots for the case when coskewness is positive. The non-center panels of the figure emphasize the skewness and heavy tails generated compared to the bivariate distribution in the center panel as the structure of skewness and co-skewness within the distribution interact.

2.2 The Regime Switching Skew-normal Model

The RSSN model is built on the regime switching model of Hamilton (1989) where under each regime y_t is assumed to have a multivariate skew-normal distribution. This extension is useful for analyzing financial time series data as it captures the stylized behavior of asset returns including asymmetry, heavy tails, heteroskedasticity, time-varying linear and non-linear co-moments among asset markets, with controlling parameters which are allowed to differ across states.

Consider the multivariate skew-normal distribution of a set of asset returns, y_t, of Section 2.1, but allowing for the model parameters to be state dependent as follows

$$
\begin{aligned}
y_t &= \mu_{s_t} + \Delta_{s_t} Z_t + \varepsilon_t, & (7)\\
\varepsilon_t &\overset{iid}{\sim} N\left(0, \Sigma_{s_t}\right), & (8)\\
Z_t &\overset{iid}{\sim} N\left(c1_m, I_m\right) 1\left(Z_{jt} > c, j = 1, \ldots, m\right). & (9)
\end{aligned}
$$

The regime s_t at time t is a binary variable that takes the values of 0 or 1, i.e., $s_t \in \{0, 1\}$. To focus the discussion on modeling crisis and contagion, the state $s_t = 0$ is called a non-crisis period and $s_t = 1$ a crisis period. In other words, there are two sets of regime-dependent parameters: $(\mu_0, \Delta_0, \Sigma_0)$ and $(\mu_1, \Delta_1, \Sigma_1)$. To emphasize the regime, the set of parameters $(\mu_l, \Delta_l, \Sigma_l)$ is sometimes written as $(\mu_{s_t=l}, \Delta_{s_t=l}, \Sigma_{s_t=l})$ for $l = 0, 1$.

The parameters of the model including the means, μ_{s_t}, co-skewness, Δ_{s_t}, and the error cross-covariances, Σ_{s_t}, are subject to change in different regimes identified by the RSSN model. Equations (7) to (9) show that the regime parameters are given by $\mu_{s_t=0}$, $\Delta_{s_t=0}$ and $\Sigma_{s_t=0}$ under regime 0, and are $\mu_{s_t=1}$, $\Delta_{s_t=1}$ and $\Sigma_{s_t=1}$ under regime 1.

For estimation purposes, equations (7) to (9) are rewritten as

$$
\begin{aligned}
y_t &= X_t \beta_{s_t} + \varepsilon_t, & (10)\\
\varepsilon_t &\overset{iid}{\sim} N\left(0, \Sigma_{s_t}\right), & (11)
\end{aligned}
$$

where

$$X_t = \left(I_m, I_m \otimes Z_t'\right), \quad \beta_{s_t} = \left(\mu_{s_t}', \delta_{s_t}'\right)', \quad \delta_{s_t} = \operatorname{vec}\left(\Delta_{s_t}'\right).$$

Here $y_t = (y_{1t}, \ldots, y_{mt})'$ is an m-dimensional random vector with $t = 1, \ldots, T$, depending on the latent variables, $Z_t = (Z_{1t}, \ldots, Z_{mt})'$, the error terms, ε_t, and the regime process, s_t. Note that the dimensions of μ_{s_t}, δ_{s_t} and β_{s_t} are m, k and $(m + k)$ respectively with $k = m^2$.

To complete the model, the process governing the underlying state of the regime, s_t, needs to be specified. To keep the model estimation tractable the regime process is assumed to be independent of its own past history

$$\Pr(s_t = 1 | s_{t-1} = 0) = \Pr(s_t = 1 | s_{t-1} = 1) = p_t, \tag{12}$$

where the probability p_t is a fixed constant that varies with time. The parameters of the RSSN model are

$$\Theta = (\beta_0, \beta_1, \Sigma_0, \Sigma_1). \tag{13}$$

For later reference, stack $y = (y_1', \ldots, y_T')'$, $Z = (Z_1', \ldots, Z_T')'$ and $s = (s_1, \ldots, s_T)'$.[4] For convenience, let $\mu_{i,l}$ denote the i-th element of μ_l, $l = 0, 1$, and similarly define $\Sigma_{ij,l}$ and $\Delta_{ij,l}$.

2.3 Channels of Crisis and Contagion

The flexibility of the RSSN model specified in Section 2.2 allows the identification of five potentially important channels of crisis and contagion through changes in each parameter of the model during financial crisis ($s_t = 1$) compared to non-crisis periods ($s_t = 0$). The channels are as follows: i) a mean-shift crisis; ii) a variance-shift crisis; iii) a skewness-shift crisis; iv) covariance-shift contagion; and v) co-skewness-shift contagion. The difference between a "crisis" and "contagion" is that a crisis occurs through parameter shifts within an own asset in a crisis regime compared to a non-crisis regime, while contagion captures parameter shifts through the cross asset market linkages. These types of crisis and contagion channels have been defined and analyzed in the literature, but are often considered in isolation. Here, we have developed a framework to analyze the five channels simultaneously. Each of the channels is defined below.

2.3.1 Mean-shift Crisis

The first type of crisis channel is represented by a change in the mean of returns for asset market i in the crisis period ($s_t = 1$) compared with the non-crisis period ($s_t = 0$), given by

$$\mu_{i,s_t=1} \neq \mu_{i,s_t=0}.$$

[4] A alternative regime process for s_t may be a homogenous first-order Markov chain. This specification is common in the literature, but to ensure tractability of the current model is not assumed here.

This is similar to the mean-shift contagion proposed by Baur (2003). He uses the mean parameter to capture an additional effect of a crisis after controlling for common global shocks and country-specific shocks in a particular crisis period for a particular asset market. In contrast to Baur (2003), we define this channel in terms of a crisis rather than contagion because there are no crisis linkages between markets which are controlled by this parameter.

2.3.2 Variance-shift Crisis and Covariance-shift Contagion

The second and third types of crisis and contagion channels are the variance-shift crisis and covariance-shift contagion. These channels are associated with the second order moments and co-moments of the variance and covariance. These channels are respectively

$$\begin{aligned} \Sigma_{ii,s_t=1} &\neq \Sigma_{ii,s_t=0}, \\ \Sigma_{ij,s_t=1} &\neq \Sigma_{ij,s_t=0},\ i \neq j. \end{aligned}$$

The variance-shift crisis is interpreted as a change in return volatility of asset market i during the crisis period ($s_t = 1$) compared to when $s_t = 0$.[5]

The third type of channel is covariance-shift contagion, which is a common focus in the literature. It is described as a significant change in the linear co-movement of asset returns between market i and j during a crisis period ($s_t = 1$) compared to a non-crisis period ($s_t = 0$). This type of contagion is consistent with Forbes and Rigobon (2002) and Corsetti, Pericoli and Sbracia (2005) who define contagion as a significant change in correlation not related to changes in market fundamentals during a period of financial turmoil.

The variance-shift crisis is different from the existing literature on volatility spillovers (Edwards, 1998) and volatility contagion (Baur, 2003; King and Wadhwani, 1990; Diebold and Yilmaz, 2009; and Chiang and Wang, 2011), which examine volatility spillovers across markets in some form. These two works are closely related to the contagion test through the co-volatility channel of Hsiao (2012), which in the framework of this paper would be a test of contagion though the fourth order comoments which are not considered here.

[5] Recent works focusing on volatility contagion are undertaken by King and Wadhwani (1990), Baur (2003), Diebold and Yilmaz (2009), and Chiang and Wang (2011).

2.3.3 Skewness-shift Crisis and Co-skewness-shift Contagion

The last two types of crisis and contagion channels are the skewness-shift crisis and co-skewness-shift contagion. These channels are respectively given by

$$\begin{aligned}\delta_{ii,s_{t=0}} &\neq \delta_{ii,s_{t=1}},\\ \delta_{ij,s_{t=0}} &\neq \delta_{ij,s_{t=1}},\ i \neq j.\end{aligned}$$

A skewness-shift crisis manifests itself through a significant change in the tail behavior of returns in asset market i during the non-crisis period ($s_t = 0$) compared with the crisis period ($s_t = 1$). Yuan (2005) and Hsiao (2012) focus on skewness after a crash with the former finding that the skewness of asset price distributions increases with borrowing constraints and the latter finding that change in skewness are attributed to volatility skew and smile effects.

Co-skewness-shift contagion is interpreted as a significant change in the asymmetric dependence of returns between asset markets i and j between $s_t = 0$ and $s_t = 1$. This type of contagion channel is examined in Fry, Martin and Tang (2010) and Fry, Hsiao and Tang (2011).[6]

3 Bayesian Estimation of the RSSN Model

A Bayesian approach is used to estimate the states and the model parameters. More specifically, MCMC methods are used to obtain draws from the posterior distribution required for the analysis as documented in Sections 3.1 to 3.2. Section 3.3 outlines the Bayesian model comparison techniques which form the basis of testing for crisis and contagion.

[6] Co-skewness can take two forms. The first form is $(1/T)\sum(r_i - \mu_i)(r_j - \mu_j)^2$ where $(r_i - \mu_i)$ is the (demeaned) level of returns of the asset market i and $(r_j - \mu_j)^2$ is the variance of returns of asset market j. The second form is $(1/T)\sum(r_i - \mu_i)^2(r_j - \mu_j)$. In this paper, the co-skewness matrix is restricted to be a symmetric matrix, which means that coskewness is given by $\frac{(1/T)\sum(r_i-\mu_i)(r_j-\mu_j)^2+(1/T)\sum(r_i-\mu_i)^2(r_j-\mu_j)}{2}$.

3.1 Likelihood Function and Priors

The (complete-data) likelihood function of the RSSN model in equations (10) to (11) is given by

$$f\left(y|Z,\Theta,s\right)=(2\pi)^{-\frac{mT}{2}}\prod_{t=1}^{T}\left|\Sigma_{s_t}\right|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2}\sum_{t=1}^{T}\left[y_t-X_t\beta_{s_t}\right]'\Sigma_{s_t}^{-1}\left[y_t-X_t\beta_{s_t}\right]\right\}, \tag{14}$$

where $\Theta=(\beta_0,\beta_1,\Sigma_0,\Sigma_1)$ and $s_t\in\{0,1\}$.

The priors for the model parameters are specified as

$$\begin{aligned}\beta_{s_t} &\sim N\left(\underline{\beta},\underline{V}_{\beta}\right), && (15)\\ \Sigma_{s_t} &\sim IW\left(\underline{\tau}_{\Sigma},\underline{S}_{\Sigma}\right), && (16)\\ \Pr\left(s_t=1\right) &= p_t, \quad \Pr\left(s_t=0\right)=1-p_t, && (17)\end{aligned}$$

where $IW\left(\underline{\tau}_{\Sigma},\underline{S}_{\Sigma}\right)$ denotes the inverse-Wishart distribution with degree of freedom $\underline{\tau}_{\Sigma}$ and scale matrix $\underline{S}_{\Sigma}$. The prior mean for β_{s_t} is set to $\underline{\beta}=\left(\underline{\mu}',\underline{\delta}'\right)'$, and the prior covariance matrix for β_{s_t} is set to $\underline{V}_{\beta}=\begin{bmatrix}\phi_{\mu}I_m & 0\\ 0 & \phi_{\delta}I_k\end{bmatrix}$, where $k=m^2$.

3.2 Posterior Analysis

This section describes the Gibbs sampler used for estimating the RSSN model. It follows from the Bayes rule that the joint posterior distribution is proportional to the product of the (complete-data) likelihood function and the joint prior density, written as

$$\pi\left(\Theta,Z,s|y\right)\propto f\left(y|Z,\Theta,s\right)f\left(Z\right)f\left(s|\Theta\right)\pi\left(\Theta\right), \tag{18}$$

where $f\left(Z\right)$ and $f\left(s|\Theta\right)$ are given in equations (9) and (17) respectively. Note that the notation π denotes the prior and posterior density functions. The likelihood function $f\left(y|Z,\Theta,s\right)$ is given in equation (14). By assuming prior independence between β and Σ, the joint prior density is given by

$$\pi\left(\Theta\right)=\pi(\beta_0)\pi(\beta_1)\pi(\Sigma_0)\pi(\Sigma_1). \tag{19}$$

Posterior draws from the joint posterior distribution can be obtained via the following Gibbs sampler:

- Step 1: Specify starting values for $\Theta^{(0)}=\left(\beta_0^{(0)},\beta_1^{(0)},\Sigma_0^{(0)},\Sigma_1^{(0)}\right)$ and $Z^{(0)}$, where $\beta_l^{(0)}=\left(\mu_l^{(0)\prime},\delta_l^{(0)\prime}\right)'$ with $l=0,1$. Set counter $loop=1,...n$.

- Step 2: Generate $s^{(loop)}$ from $\pi\left(s|y, Z^{(loop-1)}, \Theta^{(loop-1)}\right)$.
- Step 3: Generate $\beta_l^{(loop)}$ from $\pi\left(\beta_l|y, Z^{(loop-1)}, \Sigma_l^{(loop-1)}, s^{(loop)}\right)$.
- Step 4: Generate $\Sigma_l^{(loop)}$ from $\pi\left(\Sigma_l|y, Z^{(loop-1)}, \beta_l^{(loop)}, s^{(loop)}\right)$.
- Step 5: Generate $Z^{(loop)}$ from $\pi\left(Z|y, \Theta^{(loop)}, s^{(loop)}\right)$.
- Step 6: Set $loop = loop + 1$ and go to Step 2.

The number of iterations set for Steps 2 to 5 is n. The first n_0 of these are discarded as "burn-in" draws, and the remaining n_1 are retained to compute the parameter estimates, where $n = n_0 + n_1$.

The full conditional distributions are given below and their derivations are presented in Appendix A.1.

The posterior distribution for $\beta_l, l = 0, 1$, conditional on y, Z, Σ_0, Σ_1 and s is an m-variate normal distribution given by

$$(\beta_l|y, Z, \Sigma_l, s) \sim N_m\left(\widehat{\beta}_l, D_{\beta_l}\right), \ l = 0, 1, \tag{20}$$

where $D_{\beta_l} = \left(\underline{V}_\beta^{-1} + \sum_{t=1}^T 1(s_t = l) X_t' \Sigma_{s_t}^{-1} X_t\right)^{-1}$ and $\widehat{\beta}_l = D_{\beta_l}\left[\underline{V}_\beta^{-1}\underline{\beta} + \sum_{t=1}^T 1(s_t = l) X_t' \Sigma_{s_t}^{-1} y_t\right]$.

The posterior distribution for $\Sigma_l, l = 0, 1$, conditional on y, Z, β_0, β_1 and s has an inverse-Wishart distribution

$$(\Sigma_l|y, Z, \beta_l, s) \sim IW\left(\tau_{\Sigma_l}, S_{\Sigma_l}\right), \tag{21}$$

where $\tau_{\Sigma_l} = \underline{\tau}_\Sigma + \sum_{t=1}^T 1(s_t = l)$ and $S_{\Sigma_l} = \underline{S}_\Sigma + \sum_{t=1}^T 1(s_t = l)\left(y_t - X_t\beta_{s_t}\right)\left(y_t - X_t\beta_{s_t}\right)'$.

Next, the latent variables $Z_1, \ldots, Z_T$ are conditionally independent given $y, \beta_0, \beta_1, \Sigma_0, \Sigma_1$ and s. In fact, each Z_t has an independent truncated multivariate normal distribution

$$(Z_t|y, \Theta, s) \overset{ind}{\sim} N\left(\widehat{Z}_t, D_{Z_t}\right) 1\left(Z_{jt} > c, j = 1, \ldots, m\right), \tag{22}$$

where $D_{Z_t} = \left(I_m + \delta_{s_t}' \Sigma_{s_t}^{-1} \delta_{s_t}\right)^{-1}$ and $\widehat{Z}_t = D_{Z_t}\left(c 1_m + \delta_{s_t}' \Sigma_{s_t}^{-1}\left(y_t - \mu_{s_t}\right)\right)$. A feasible sampling approach to obtain draws from the above truncated multivariate normal distribution is to draw Z_t component by component, where each component follows a truncated univariate normal distribution given all other components. Draws from a truncated univariate normal distribution can be generated by using the inverse transform method (Kroese et al. 2011, p.45).

To generate the regime variable s_t, the multi-move Gibbs sampling method is used. Since the regime variable s_t evolves independently of its own past values, the regimes $s_1, \ldots, s_T$ are conditionally independent of each other given the data and other parameters:

$$\pi\left(s|y, Z, \Theta\right) = \prod_{t=1}^{T} \pi\left(s_t|y, Z, \Theta\right), \tag{23}$$

where the success probability can be calculated as

$$\Pr\left(s_t = 1|y, Z, \Theta\right) = \frac{\pi\left(s_t = 1|y, Z, \Theta\right)}{\pi\left(s_t = 0|y, Z, \Theta\right) + \pi\left(s_t = 1|y, Z, \Theta\right)}. \tag{24}$$

Once the above probability is calculated, a random number from a uniform distribution between 0 and 1 is generated to compare with the calculated value of $\Pr\left(s_t = 1|y, Z, \Theta\right)$. If the probability $\Pr\left(s_t = 1|y, Z, \Theta\right)$ is greater than the generated number, the regime variable s_t is set to 1; otherwise, s_t is set to 0.

3.3 Bayesian Model Comparison

Bayesian model comparison provides a unified approach for comparing non-nested models, and can be used as an alternative to classical hypothesis testing. Consider comparing two models, namely, M_r and M_u. Evidence in favor of model M_r can be measured by the Bayes factor, defined as

$$BF_{ru} = \frac{p\left(y|M_r\right)}{p\left(y|M_u\right)}, \tag{25}$$

where $p\left(y|M_r\right)$ and $p\left(y|M_u\right)$ are the marginal likelihoods of the data under models M_r and M_u respectively. Intuitively, the marginal likelihood $p(y|M_r)$ is simply the marginal distribution of the observables under model M_r evaluated at the actual data. If the data are "improbable" under model M_r, the marginal likelihood would be "small" and vice versa. Hence, the Bayes factor BF_{ru}, which is the ratio of the marginal likelihoods under two models, assesses which model better predicts the data.

Furthermore, the posterior odds ratio for model M_r against model M_u is related to their Bayes factor as follows

$$PO_{ru} = \frac{\pi\left(M_r\right)}{\pi\left(M_u\right)} BF_{ru}, \tag{26}$$

where $\pi(M_r)$ and $\pi(M_u)$ are the prior probabilities of models M_r and M_u. Clearly, if both models are equally likely a priori, then the Bayes factor is also the posterior odds ratio of the two models.

Obtaining the Bayes factor generally involves computation of the marginal likelihoods, which is often a difficult task. Two popular methods for calculating the marginal likelihood in the literature are those of Gelfand and Dey (1994) and Chib (1995). However, if the two models under comparison are nested, i.e., if one model is a restricted version of the other, then their Bayes factor can be calculated using the Savage-Dickey density ratio (Verdinelli and Wasserman, 1995), which is often much simpler to compute. Since hypothesis testing can be framed as comparing nested models, the density ratio can be used to compute the relevant Bayes factor. The details of Savage-Dickey density ratio are contained in Appendix A.2. Both approaches—computing the Bayes factor via marginal likelihoods and Savage-Dickey density ratio—are adopted in this paper. In each model comparison exercise, the more convenient approach and numerically stable approach is used.

To compare two models, model M_r is chosen over than model M_u if the Bayes factor in favor of M_r (BF_{ru}) is sufficiently large. The choice of threshold on which this decision is made is based on the scale of evidence for model selection proposed by Jeffreys (1961) as shown in Table 1. Table 1 shows the scale of the natural logarithm of the Bayes factor, $\ln(BF_{ru})$, which is used here to determine the strength of evidence for selecting model M_r over model M_u.

4 Testing for crisis and Contagion

The tests for crisis and contagion using the Bayesian model comparison techniques introduced in Section 3.3 are formally set out in this section. Sections 4.1 to 4.4 set out the hypotheses for the different forms of crisis and contagion tests. Namely: i) the mean-shift crisis test; ii) the variance-shift crisis test and the covariance-shift contagion test; iii) the skewness-shift crisis test and the co-skewness-shift contagion test; and finally; iv) the joint crisis and contagion tests which consider multiple channels of crisis and contagion.

Table 2 presents a summary of the tests conducted in this paper. The table presents the restricted model (M_r) for each test for the case of a single asset market, as well as for a group of asset markets. The RSSN model is the unrestricted model (M_u) with two sets of regime-specific parameters: the regime-specific mean vectors μ_0 and μ_1 (each of dimension $m \times 1$), covariance matrices Σ_0 and Σ_1 (each of dimension $m \times m$) and co-skewness matrices Δ_0 and Δ_1 (each of dimension $m \times m$). Recall that $\mu_{i,l}$ denotes the i-th element of μ_l, and similarly for $\Sigma_{ij,l}$ and $\Delta_{ij,l}$.

4.1 Mean-shift Crisis Test

Single market In examining the evidence for a crisis occurring in the mean-shift channel based on changes in average returns of an asset market i, across states $s_t = 0$ and $s_t = 1$, consider testing the hypothesis $\mu_{i,0} = \mu_{i,1}$. This can be recast as comparing the unrestricted model M_u to the restricted one M_r where $\mu_{i,0} = \mu_{i,1}$ is imposed. To elaborate, M_u is the unrestricted model where all regime-specific parameters are free to vary across the two periods of the non-crisis and crisis. M_r is the restricted model and features no shift in the mean of the asset market i between the two regimes. This implies that under the restricted model the average returns in the two periods remain the same. Clearly, M_r is nested within M_u by setting $\mu_{i,0} = \mu_{i,1}$. This is test $T1$ in Table 2.

The Bayes factor comparing M_r to M_u for evidence of the mean-shift crisis channel $T1$ can be computing using the Savage-Dickey density ratio

$$BF_{ru} = \frac{\pi\left(\mu_{i,1} - \mu_{i,0} = 0|y, M_u\right)}{\pi\left(\mu_{i,1} - \mu_{i,0} = 0|M_u\right)}, \tag{27}$$

where $\pi\left(\mu_{i,1} - \mu_{i,0} = 0|y, M_u\right)$ and $\pi\left(\mu_{i,1} - \mu_{i,0} = 0|M_u\right)$ are respectively the posterior and prior densities of $\mu_{i,1} - \mu_{i,0}$ evaluated at the point 0. Since the priors for $\mu_{i,0}$ and $\mu_{i,1}$ are assumed to be normal with mean zero and variance ϕ_μ (see equation (15)), the induced prior for $\mu_{i,1} - \mu_{i,0}$ is normal with mean zero and variance $2\phi_\mu$. Moreover, the quantity $\pi(\mu_{i,1} - \mu_{i,0} = 0|y, M_u)$ can be estimated by averaging $\pi(\mu_{i,1} - \mu_{i,0} = 0|y, \Sigma_0, \Sigma_1, \Delta_0, \Delta_1, s, Z, M_u)$ over the MCMC draws (which only involves evaluating normal densities at 0).

Group of markets The test for a crisis of the mean-shift form across several asset markets is also considered. The joint version of the test is denoted as $TG1$. In contrast to the test for a crisis in an individual market described above using the elements specific to market i ($\mu_{i,0}$ and $\mu_{i,1}$), the joint test for a mean-shift crisis for all m markets utilizes the whole mean vectors μ_1 and μ_0 in the model comparison. The joint test of a mean-shift crisis compares the restricted model M_r where the condition $\mu_1 = \mu_0$ is imposed with the unrestricted model M_u. Again, the relevant Bayes factor can be computed using the Savage-Dickey density ratio, which involves evaluating m-variate normal densities.

4.2 Variance-shift Crisis Test and Covariance-shift Contagion Test

Single market The single market tests for crisis and contagion through the second order channels are denoted as $T2$ for the variance-shift crisis test and $T4$ for the covariance-shift contagion test.

For the variance-shift crisis test for an asset market i, the restricted model is constructed by imposing the condition $\Sigma_{ii,0} = \Sigma_{ii,1}$. The Bayes factor comparing model M_r with the unrestricted model M_u for evidence of the variance-shift crisis channel $T2$ is given by

$$BF_{ru} = \frac{\pi(\Sigma_{ii,1} - \Sigma_{ii,0} = 0|y, M_u)}{\pi(\Sigma_{ii,1} - \Sigma_{ii,0} = 0|M_u)}, \tag{28}$$

where $\pi(\Sigma_{ii,1} - \Sigma_{ii,0} = 0|y, M_u)$ and $\pi(\Sigma_{ii,1} - \Sigma_{ii,0} = 0|M_u)$ are respectively the posterior and prior densities for $\Sigma_{ii,1} - \Sigma_{ii,0}$ evaluated at the point 0.

Equation (28) is slightly more difficult to evaluate. This is because although both $\pi(\Sigma_0|y, Z, \beta_0, s)$ and $\pi(\Sigma_1|y, Z, \beta_1, s)$ are inverse-Wishart densities (see (21)), $\pi(\Sigma_{ii,1} - \Sigma_{ii,0}|y, Z, \beta_0, \beta_1, s)$ is not a known density. However, using Gaussian kernel estimates to approximate the two quantities $\pi(\Sigma_{ii,1} - \Sigma_{ii,0} = 0|y, M_u)$ and $\pi(\Sigma_{ii,1} - \Sigma_{ii,0} = 0|M_u)$, the Bayes factor in equation (28) can still be estimated in a straightforward fashion. The details of the Gaussian kernel method for evaluating densities introduced by Geweke (2010) are discussed in Appendix A.3.

For the covariance-shift contagion test between asset markets i and j, the relevant restricted model is the one where the condition $\Sigma_{ij,0} = \Sigma_{ij,1}$, $i \neq j$, is imposed. This is a test of contagion through the covariance channel between asset markets i and j, which is test $T4$ in the table. The relevant Bayes factor can be computed using the Gaussian kernel estimates as discussed above.

Group of markets Tests for joint crisis and joint contagion across the m asset markets through the variance-shift crisis ($TG2$) and covariance-shift contagion ($TG4$) channels are also considered. In the first test, the relevant restriction is $\Sigma_{ii,0} = \Sigma_{ii,1}, i = 1, \ldots, m$. In this case, the Bayes factor comparing the restricted model M_r against the unrestricted one M_u can be estimated using the Savage-Dickey density ratio with

Gaussian kernel estimates. In the second test, the relevant restriction is $\Sigma_0 = \Sigma_1$. In this case, the marginal likelihoods—for the unrestricted model and the restricted version with $\Sigma_0 = \Sigma_1$ imposed—are first obtained in order to compute the Bayes factor.

4.3 Skewness-shift Crisis Test and Co–skewness-shift Contagion Test

Single market The single market tests for crisis and contagion through the third order channels are denoted as $T3$ for the skewness-shift crisis test and $T5$ for the co-skewness-shift contagion test. The restricted model for the crisis test through changes in return skewness is constructed by setting the i-th diagonal element of the co-skewness matrices between the non-crisis and crisis periods to be the same, i.e., $\delta_{ii,0} = \delta_{ii,1}$. The Bayes factor comparing models M_r and M_u is then

$$BF_{ru} = \frac{\pi\left(\delta_{ii,1} - \delta_{ii,0} = 0 | y, M_u\right)}{\pi\left(\delta_{ii,1} - \delta_{ii,0} = 0 | M_u\right)}. \tag{29}$$

The denominator of this expression can be calculated since the induced prior for $\delta_{ii,1} - \delta_{ii,0}$ is normal with mean zero and variance $2\phi_\delta$. The numerator of equation (29) can be estimated by averaging the quantity $\pi\left(\delta_{ii,1} - \delta_{ii,0} = 0 | y, Z, \Sigma_0, \Sigma_1, s\right)$ over the MCMC draws.

The co-skewness-shift contagion test ($T5$) uses the restricted model where $\delta_{ij,0} = \delta_{ij,1}$, $i \neq j$, and the rest follows as above.

Group of markets Two joint tests are also considered: test for crisis based on changes in return skewness ($TG3$) and test for contagion based on changes in co-skewness ($TG5$) across the m asset markets. In the first case, the restricted model is $\delta_{ii,0} = \delta_{ii,1}, i = 1, \ldots, m$, whereas for the second case, the restriction is $\delta_0 = \delta_1$. As before, the relevant Bayes factors are computed using the Savage-Dickey density ratio for the former, and marginal likelihoods for the latter.

4.4 Multiple Channels of Crisis and Contagion Tests

The flexibility of the RSSN model enables the testing of multiple channels of crisis and contagion. Eight types of joint tests of crisis and contagion are examined based on significant changes in the possible combinations of switching model parameters including the mean, variance, covariance, skewness and co-skewness during the crisis period compared to the non-crisis period. Four of these tests are to test for a crisis in asset market i based on identification of the joint crisis through: i) the mean- and variance-shift channels ($JT1$); ii) the mean- and skewness-shift channels ($JT2$); iii) the variance- and skewness-shift channels ($JT3$); and iv) the mean-, variance- and skewness-shift channels ($JT4$). There is one joint test of contagion only ($JT5$) which is to test for financial contagion between asset markets i and j through the covariance- and co-skewness-shift channels. The remaining tests for joint crisis and contagion in a group of m asset markets through: i) the mean-, variance- and covariance-shift channels ($JT6$); ii) the mean-, skewness- and co-skewness-shift channels ($JT7$); and iii) the mean-, variance-, covariance-, skewness- and co-skewness-shift channels ($JT8$) which represents a test of all channels operating.

In all the tests, the RSSN model is the unrestricted model with two sets of regime-specific parameters: μ_0 and μ_1, Σ_0 and Σ_1, and Δ_0 and Δ_1. The choice of restricted model M_r for use in the calculation of the Bayes factor depends on which parameters of the unrestricted model are constrained. For instance, the restricted model for $JT1$ is one with the restrictions $\mu_{i,0} = \mu_{i,1}$ and $\Sigma_{ii,0} = \Sigma_{ii,1}$, if the joint test of a crisis for asset market i through the mean and variance channels is conducted. The restrictions on the model parameters are summarized in Table 2.

5 Empirical Analysis

5.1 Data and Descriptive Statistics

The use of the RSSN model for measuring financial market crisis and contagion is illustrated using European equity and US banking data from January 4, 2005 to August 31, 2009 which includes the Great Recession crisis period. The sample period is chosen to end prior to the European debt crisis to avoid complications in the modeling framework through the existence of more than two regime possibilities. The data consist of daily percentage returns of equity market indexes of four European countries (France, Germany, Greece and Italy) and daily percentage returns of the US banking market index. The daily percentage returns are calculated as 100 times the log first difference of the value of the equity indices. There are $T = 1215$ observations for each series. All series are denominated in US dollars.[7]

[7] All data are collected from Datastream. The mneumonics are: France - CAC 40 price index (FRCAC40); Germany - MDAX Frankfurt price index (MDAXIDX); Greece - ATHEX Composite price index (GRAGENL); Italy - FTSE MIB price index (FTSEMIB), the US - US-DS Banks price index (BANKSUS).

Time series plots of the returns are contained in Figure 2. The period of financial market crisis is clearly shown in the Figure with the volatility of the equity returns changing dramatically in all markets during the period extending from July 2007 to August 2009. Table 3 summarizes the statistics of the returns of the five markets. Both France and Germany show positive mean daily returns, while Greece, Italy and the US banking sector have negative mean returns. The standard deviation of the US banking sector returns are the highest among the markets. In terms of the higher order moments, non-normality of all returns is apparent from the coefficients of skewness and kurtosis. For example, over the sample period, the value of skewness ranges from -0.376 for Greece to 0.221 for the US banking sector and the value of kurtosis ranges from 9.049 for Germany to 14.019 for the US banking sector, suggesting that all returns exhibit a fat-tailed and leptokurtic distribution. Finally, the null hypothesis of normality is strongly rejected based on the Jarque-Bera test for all equity returns.

5.2 Parameter Estimation

In line with the literature on models of contagion, market fundamentals and interdependences are controlled for by estimating a vector autoregressive model (VAR), and using the residuals of the VAR as the data y_t in equation (10). The lag length is set to $L = 5$ based on the selection criteria of the sequential modified log-likelihood ratio test statistic, Akaike's final prediction error and the Akaike Information Criterion. As is customary, prior hyperparameters in equations (15) to (17) are assumed to be known, and are set to be $\underline{\beta} = 0$, $\phi_\mu = 0.01$, $\phi_\delta = 1$, $\underline{\tau}_{\mathbf{\Sigma}} = 20 + m + 1$, $\underline{S}_{\mathbf{\Sigma}} = (\underline{\tau}_{\mathbf{\Sigma}} - m - 1) \times \mathbf{I}_m$ with $m = 5$. The prior variances are chosen to be relatively small, so that the prior distributions are proper and relatively informative.

This paper relaxes the need for strong assumptions usually adopted in the contagion literature about the dating of crisis periods and non-dogmatic beliefs about the likelihoods of crisis episodes occurring are incorporated formally via the prior probabilities $p_t = \Pr(s_t = 1) = 1 - \Pr(s_t = 0)$. Specifically, the initial value for the probability of being in regime 0 (non-crisis period) is set to $\Pr(s_t = 0) = 0.9999$ during the period from January 4, 2005 to June 30, 2007 and that of being in regime 0 is set to $\Pr(s_t = 0) = 0.0001$ during the period between March 3, 2008 and August 31, 2009. The probability of being in regime 0 decreases linearly from 0.9999 on July 1, 2007 to 0.0001 on August 31, 2009 with the margin of $\left(\frac{1}{177} \times (0.9999 - 0.0001)\right)$ per day. For instance, The probability of being in regime 0 on July 2, 2007 is $0.9999 - \left(\frac{1}{177} \times (0.9999 - 0.0001)\right)$

where there are 177 days between June 30, 2007 and March 3, 2008.

For estimation purposes, the co-skewness matrix Δ in equation (1) is restricted to be a symmetric matrix, which means that the dimension of δ reduces from $k = m^2$ to $k = m(m + 1)/2$. Furthermore, the constant term c in equation (3) is set to $-\sqrt{2/\pi}$ so that $E(Z_t) = 0$ and $V(Z_t) = (\pi - 2)/\pi$, and the inclusion of the latent variables Z_t does not affect the (unconditional) expectation of y_t. In the original specification of Sahu et al. (2003), c is set to be zero.

The procedure for Gibbs-sampling described in Section 3.2 is applied to the RSSN model. The first $10,000$ draws are discarded to allow the Markov Chain to converge to the stationary distribution. In order to reduce sample autocorrelation and to also avoid biased Monte Carlo standard errors, every 10 draws for the next $200,000$ iterations are recorded, for a total of $20,000$ draws used for posterior summaries. The criteria of choosing independent draws is based on the inefficiency factors with the details shown in Appendix A.4. The inefficiency factors of the switching parameters are reported in Table 4. The MCMC algorithm is well behaved in the RSSN model since the results of Table 4 indicate the low inefficiency factors for all of the switching parameters.

Table 5 presents the estimates (posterior means) of the regime-switching parameters when innovations are fitted to the RSSN model with two regimes for the five equity markets. The first panel of the table presents the results for regime $s_t = 0$ where there is no crisis, while the second panel presents the results for regime $s_t = 1$ where all markets are in crisis. Figure 3 presents the probability that the model is in a particular regime over the time period. A value of 0 represents a non-crisis regime, and a value of 1 represents a crisis regime. Inspection of the figure shows that apart from a brief period of a crisis regime in January 2006, the crisis regime began to become evident in January of 2007, and by February 2008 was consistently in the crisis regime.

As expected, in the non-crisis period average returns are positive for all markets, while they are negative or close to zero for all markets in the crisis period ($s_t = 1$). Furthermore, the variance and covariance between equity returns for all markets are much higher during the crisis regime compared with the non-crisis regime. For example, the variance of returns in the US banking sector which is the most dramatic, increases from nearly 0.60% when $s_t = 0$, to 15.80% when $s_t = 1$. European countries follow the US banking sector with variances of under 1% when $s_t = 0$, to that of over 3% when $s_t = 1$. The covariance between equity returns also show a dramatic increase during the crisis period, indicating that equity returns between the two markets are strongly correlated in the crisis period.

As for co-skewness structure, the table indicates that return skewness and asymmetric dependence between equity returns switch from being negative when $s_t = 0$, to positive when $s_t = 1$ in most cases. The return skewness ranges from -0.649 for Greece to 0.160 for the US banking sector when $s_t = 0$, and from 0.303 for Germany to 3.045 for the US banking sector when $s_t = 1$. In addition, the asymmetric dependence between equity returns increases or moves to the right during the crisis period for all cases except for the dependence relation between the US banking sector and European equity markets.

5.3 Crisis and Contagion in the Great Recession

The Bayes factors to evaluate the evidence for crisis and contagion through the first to third order channels are conducted in this Section on the sample of European and US equity market returns for the Great Recession period. Table 6 consists of three panels, presenting the results of the crisis tests, the contagion tests and the crisis and contagion tests respectively. The table is divided into a panel examining the evidence of crisis through mean-shift, variance-shift and skewness-shift channels; a panel examining the evidence of contagion from the US banking market returns through the European equity returns through covariance-shift and co-skewness-shift channels; and a final panel that considers joint tests of crisis and contagion. The first five columns present results for each country, while the last column presents the evidence considering the operation of each channel for all markets jointly. Refer to Table 1 for the evidence categories for the decision rule for the Bayesian model comparison to determine the strength of the evidence for the null hypothesis in each case.

5.3.1 Evidence of Crisis

The results of the tests for crisis reinforce the need to consider higher order moments when analyzing crisis period data. Inspection of the first panel of Table 6 shows the crisis channels during the Great Recession $(s_t = 1)$. The results for the mean-shift crisis tests show no evidence of a financial market crisis through the mean-shift channel in any of the own equity markets during the Great Recession (tests $T1$ in the Table). Further, the joint test of crisis through the mean-shift channel ($TG1$) also shows no evidence of crisis through the mean-shift channel where all markets are considered simultaneously. The higher order moment tests of crisis find more evidence. The variance-shift crisis channel shows decisive evidence of crisis for all markets (tests $T2$ in the Table), including for the joint test of crisis through the variance-shift channel ($TG2$). The third order own moment channel of a skewness-shift crisis gives mixed results with a crisis only decisively evident in the equity market in France and in the US banking market, and is strongly evident in Italy (see the results for test $T3$). Greece and Germany are not affected by a crisis through the skewness channel. The joint tests of the various combinations of mean-shift, variance-shift and skewness-shift crisis channels for the single markets (tests $JT1$ to $JT4$) and the group of markets considered jointly (tests $JTG1$ to $JTG4$) all indicate either decisive or strong evidence of crisis in the combined channels except for the combined mean-shift and skewness-shift channel for Greece (test $JT2$ for Greece in Table 6). Overall, the results indicate that it is the variance-shift channel which is most important for own country parameter changes, followed by the skewness-shift channel, with the mean-shift channel the least important.

5.3.2 Evidence of Contagion

The second panel of Table 6 shows which channels of contagion are important in explaining movements of the second and third order co-moments of the asset returns during the Great Recession period where $s_t = 1$, with the assumption that the relevant link to observe is that of contagion between the US and each of the European markets. The covariance-shift channel of contagion between the US banking equity returns and the European equity market returns (tests $T4$ in Table 6) shows decisive evidence of contagion with the value of the log of the Bayes factor $\ln(BF_{ru})$ ranging from -51.65 to -101.09.

Thematically, the results for the contagion channels are similar to those of the crisis channels in that evidence of contagion is stronger through the second order co-moments than through the third order co-moments. Inspection of the co-skewness-shift channel of contagion in test $T5$ reveals that the co-skewness-shift channel is not in operation between the US banking returns and the Greek equity returns during the Great Recession, with the log of the Bayes factor $\ln(BF_{ru})$ being -1.37. There is decisive evidence of contagion through all other channels, with the value of the log of the Bayes factor $\ln(BF_{ru})$ ranging from -5.32 to -7.88 for the other European countries markets. The bottom row of the second panel of Table 6 considers the possibility that contagion between the US and each of the European markets may occur through both the covariance-shift and the co-skewness-shift channels simultaneously (Test $JT5$ in the table). This is indeed the case, with decisive evidence of contagion through the two channels. When considering the tests $TG4$ and $TG5$ which examine the evidence for contagion amongst all five market simultaneously through the covariance-shift and the co-skewness-shift channels respectively, there is decisive evidence of contagion through each of these channels in both cases.

5.3.3 Evidence of Joint Crisis and Contagion

The third panel of Table 6 provides evidence on the operation of crisis and contagion channels simultaneously occurring for all markets. Although there is a multitude of combinations of crisis and contagion channels that can be considered, here three joint forms are evaluated to illustrate the possibility of the operation of multiple channels. The evidence of the operation of crisis and contagion through equity markets are pervasive across European equity markets and the US banking sector through the combinations of multiple channels of crisis and contagion during the Great Recession. More specifically, the crisis and contagion effects based on the $JT8$ where all channels of crisis and contagion are operating are almost double those of the $JT6$ and $JT7$ combinations, as the value of the natural logarithm of the Bayes factor ($\ln(BF_{ru})$) based on $JT8$ is -1033.20 and for $JT6$ and $JT7$ are -462.60 and -568.22 respectively. The results of this Section suggests the importance of examining changes in market linear and non-linear dependence during the Great Recession. The examination of just one channel of crisis and contagion (such as only focusing on correlation, or only focusing on co-skewness channels of contagion) is likely to be misleading in understanding the flow of a crisis.

6 Conclusions

This paper introduced a regime switching skew-normal model built upon the regime switching model of Hamilton (1989) but with relaxation of the assumption of the distribution of the error term which was assumed here to be a multivariate skew-normal distribution. This new approach provided a more general framework for developing five types of crisis and contagion channels based on identifying changes in the model parameters over the two regimes of the non-crisis and crisis periods. The framework also allowed for the development of a class of joint tests of crisis and contagion through changes in the possible combinations of regime switching parameters during a crisis compared with the non-crisis period.

Measuring financial contagion within the RSSN model solved several econometric problems faced in the crisis and contagion literature. First, market dependence was fully captured by simultaneously considering both second and third order co-moments of asset returns, which detected the importance of changes in market linear and non-linear dependence during a crisis which was defined to be contagion. Second, alternative transmission channels of crisis and contagion were examined simultaneously. Third, the market in which crisis originated did not need to be defined and crisis and contagion were distinguished from each other. Fourth, the source crisis market was determined by the model rather than by the researcher, and finally, the timing of a financial crisis was endogenously determined by the model itself rather than being arbitrary chosen as is the case in the majority of work on financial market crisis. This occurred through the regime switching parameters which differed across regimes. This had the added advantage of dealing with the heteroskedasticity problem that arises in the simple correlation and co-skewness measures of contagion caused by volatility increases during a crisis.

This framework was applied to test for financial market crisis and contagion through single and multiple channels in each equity market over the period 2005 to 2009, which includes the Great Recession period. The empirical results showed strong evidence of a crisis channel across equity markets over Europe and the US through single channels of a variance-shift and skewness-shift. The exception was that there was no crisis evident in the mean-shift channel. The second finding was that the joint channel crisis and contagion tests showed strong statistical evidence of significant effects in both Europe and the US during the Great Recession period.

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A Appendix

A.1 The Gibbs Sampler for the RSSN Model

The details of the MCMC algorithm are described below. By assuming prior independence between β and Σ, the joint prior density $\pi(\Theta)$ is given by multiplying equations (15) and (16)

$$\begin{aligned}\pi(\Theta) &= \prod_{l=0}^{1} \pi(\beta_l) \times \pi(\Sigma_l) \qquad (30)\\ &\propto \prod_{l=0}^{1} (2\pi)^{-\frac{m}{2}} \left|\underline{V}_\beta\right|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\left(\beta_l - \underline{\beta}\right)' \underline{V}_\beta^{-1} \left(\beta_l - \underline{\beta}\right)\right\} \times \\ &\left(2^{-\frac{\underline{\tau}_\Sigma m}{2}} \Gamma_m\left(\frac{\underline{\tau}_\Sigma}{2}\right)\right)^{-1} \left|\underline{S}_\Sigma\right|^{\frac{\underline{\tau}_\Sigma}{2}} \left|\Sigma_l\right|^{-\frac{(\underline{\tau}_\Sigma + m + 1)}{2}} \exp\left(-\frac{tr\left(\underline{S}_\Sigma \Sigma_{s_t}^{-1}\right)}{2}\right).\end{aligned}$$

To calculate the posterior density, the complete-data likelihood function is combined with the joint prior density via Bayes rules. It is given by

$$\begin{aligned}
\pi(\Theta, Z, s|y) &\propto f(y|Z,\Theta,s)\, f(Z)\, f(s|\Theta)\, \pi(\Theta), \qquad (31)\\
&= (2\pi)^{-\frac{T}{2}} \prod_{t=1}^{T} |\Sigma_{s_t}|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\sum_{t=1}^{T}\left[y_t - X_t\beta_{s_t}\right]' \Sigma_{s_t}^{-1}\left[y_t - X_t\beta_{s_t}\right]\right\}\\
&\quad \times f(Z)\, f(s|\Theta)\, \pi(\Theta),
\end{aligned}$$

where $y = (y_1', \ldots, y_T')'$, $Z = (Z_1', \ldots, Z_T')'$ and $s = (s_1, \ldots, s_T)'$. $f(Z)$ and $f(s|\Theta)$ are provided in equations (9) and (17), respectively. Posterior draws can be obtained using the Gibbs sampler. Specifically, we sequentially draw from $\pi(\beta_0, \beta_1|y, Z, \Sigma_0, \Sigma_1, s)$, $\pi(Z|y, \beta_0, \beta_1, \Sigma_0, \Sigma_1, s)$, $\pi(\Sigma_0, \Sigma_1|y, Z, \beta_0, \beta_1, s)$ and $\pi(s|y, \beta_0, \beta_1, \Sigma_0, \Sigma_1, Z)$.

In the first step, $\pi(\beta_l|y, Z, \Sigma, s), l = 0, 1$, is a normal density. To see this, write

$$\begin{aligned}
\log \pi(\beta_l|y, Z, \Sigma_0, \Sigma_1, s) &= \log f(y|Z, \Theta, s) + \log \pi(\Theta) + \text{constant} \qquad (32)\\
&\propto -\frac{1}{2}\left(\beta_l - \underline{\beta}\right)' \underline{V}_\beta^{-1}\left(\beta_l - \underline{\beta}\right)\\
&\quad -\frac{1}{2}\sum_{t=1}^{T}\left[y_t - X_t\beta_{s_t}\right]' \Sigma_{s_t}^{-1}\left[y_t - X_t\beta_{s_t}\right],\\
&\propto -\frac{1}{2}\beta_l'\left(\underline{V}_\beta^{-1} + \sum_{t=1}^{T} 1(s_t = l) X_t'\Sigma_{s_t}^{-1} X_t\right)\beta_l\\
&\quad +\beta_{s_t}'\left[\underline{V}_\beta^{-1}\underline{\beta} + \sum_{t=1}^{T} 1(s_t = l) X_t'\Sigma_{s_t}^{-1} y_t\right],
\end{aligned}$$

where

$$D_{\beta_l} = \left(\underline{V}_\beta^{-1} + \sum_{t=1}^{T} 1(s_t = l) X_t'\Sigma_{s_t}^{-1} X_t\right)^{-1}, \quad \widehat{\beta}_l = D_{\beta_l}\left[\underline{V}_\beta^{-1}\underline{\beta} + \sum_{t=1}^{T} 1(s_t = l) X_t'\Sigma_{s_t}^{-1} y_t\right],$$

which is the kernel of an m-variate normal density with mean vector $\widehat{\beta}_l$ and covariance matrix D_{β_l}. In other words, $(\beta_l|y, Z, \Sigma_0, \Sigma_1, s) \sim N_m\left(\widehat{\beta}_l, D_{\beta_l}\right)$.

Next, following a similar argument, $\pi(Z|y, \Theta, s)$ is a normal density. To see this, using equations (14) and (9)

$$
\begin{aligned}
\log \pi\left(Z|y,\Theta,s\right) &= \log f\left(y|Z,\Theta,s\right)+\log f\left(Z\right)+\text{constant} &(33)\\
&\propto -\frac{1}{2}\sum_{t=1}^{T}(y_t-\mu_{s_t}-\Delta_{s_t}Z_t)'\Sigma_{s_t}^{-1}(y_t-\mu_{s_t}-\Delta_{s_t}Z_t)\\
&\quad -\frac{1}{2}\sum_{t=1}^{T}\left[Z_t-c1_m\right]' I_m^{-1}\left[Z_t-c1_m\right],\\
&\propto -\frac{1}{2}\sum_{t=1}^{T}Z_t'\left(\Delta_{s_t}'\Sigma_{s_t}^{-1}\Delta_{s_t}+I_m\right)Z_t\\
&\quad +\sum_{t=1}^{T}Z_t'\Delta_{s_t}'\Sigma_{s_t}^{-1}\left(y_t-\mu_{s_t}\right)+Z_t'c1_m,
\end{aligned}
$$

That is, $(Z_t|y,\Theta,s)\sim N_m\left(\widehat{Z}_t,D_{Z_t}\right)$ with

$$
D_{Z_t}=\left(I_m+\Delta_{s_t}'\Sigma_{s_t}^{-1}\Delta_{s_t}\right)^{-1},\quad \widehat{Z}_t=D_{Z_t}\left(c1_m+\Delta_{s_t}'\Sigma_{s_t}^{-1}\left(y_t-\mu_{s_t}\right)\right).
$$

Finally, the log conditional density $\pi\left(\Sigma_l|y,Z,\beta_0,\beta_1,s\right)$ is derived and given by

$$
\begin{aligned}
\log \pi\left(\Sigma_l|y,Z,\beta_0,\beta_1,s\right) &= \log f\left(y|Z,\Theta,s\right)+\log \pi\left(\Theta\right)+\text{constant} &(34)\\
&\propto -\frac{1}{2}\sum_{t=1}^{T}\left[y_t-X_t\beta_{s_t}\right]'\Sigma_{s_t}^{-1}\left[y_t-X_t\beta_{s_t}\right]-\frac{1}{2}\sum_{t=1}^{T}\log\left|\Sigma_{s_t}\right|\\
&\quad -\left(\frac{\underline{\tau}_{\boldsymbol{\Sigma}}+m+1}{2}\right)\log\left|\Sigma_l\right|-\frac{1}{2}tr\left(\underline{S}_{\Sigma}\Sigma_l^{-1}\right),\\
&\propto -\left(\frac{\underline{\tau}_{\boldsymbol{\Sigma}}+\sum_{t=1}^{T}1(s_t=l)+m+1}{2}\right)\log\left|\Sigma_l\right|\\
&\quad -\frac{1}{2}tr\left[\underline{S}_{\Sigma}+\sum_{t=1}^{T}1(s_t=l)\left(y_t-X_t\beta_{s_t}\right)\left(y_t-X_t\beta_{s_t}\right)'\right]\Sigma_l^{-1},
\end{aligned}
$$

which is the kernel of an inverse-Wishart distribution. In fact, $(\Sigma_l|y,Z,\beta_0,\beta_1,s)\sim IW\left(\tau_{\Sigma_l},S_{\Sigma_l}\right)$, where

$$
\tau_{\Sigma_l}=\underline{\tau}_{\Sigma}+\sum_{t=1}^{T}1(s_t=l),\quad S_{\Sigma_l}=\underline{S}_{\Sigma}+\sum_{t=1}^{T}1(s_t=l)\left(y_t-X_t\beta_{s_t}\right)\left(y_t-X_t\beta_{s_t}\right)'.
$$

A.2 The Savage-Dickey Density Ratio

The Savage-Dickey density ratio introduced by Dickey (1971) is a specific representation of the Bayes factor for comparing nested models.

Suppose $\theta=(\psi,\omega)$ is the vector of model parameters in the unrestricted model M_u. The likelihood and prior for this model are denoted as $f\left(y|\psi,\omega,M_u\right)$ and $\pi\left(\psi,\omega|M_u\right)$.

Suppose the restricted model M_r can be characterized as $\psi = \psi_0$, where ψ_0 is a constant vector, while the parameter vector ω free to vary. The likelihood and prior for the restricted model are then denoted as $f(y|\omega, M_r)$ and $\pi(\omega|M_r)$. Suppose the priors for the two models satisfy

$$\pi(\omega|\psi = \psi_0, M_u) = \pi(\omega|M_r). \tag{35}$$

Under this condition, Verdinelli and Wasserman (1995) show that the Bayes factor comparing M_r to M_u has the form

$$BF_{ru} = \frac{p(\psi = \psi_0|y, M_u)}{p(\psi = \psi_0|M_u)}, \tag{36}$$

where $p(\psi = \psi_0|y, M_u)$ and $p(\psi = \psi_0|M_u)$ are respectively the posterior and prior densities for ψ under the unrestricted model evaluated at the point ψ_0. Equation (36) is referred to as the Savage-Dickey density ratio.

A.3 A Gaussian Copula for Evaluating Probability Densities

The following approach of using a Gaussian copula for approximating a probability density function at a specified point is developed by Geweke (2010). Consider the random vector u with q components,

$$u = (u_1, \ldots, u_q). \tag{37}$$

Suppose $u^{(1)}, \ldots, u^{(B)}$ are independent and identically distributed draws from the probability density function $p(u)$. Then $p(u^0)$, the density function evaluated at the point u^0, can be approximated using the following steps:

Step 1: Using a Gaussian kernel to compute the approximations

$$p_i(u_i) = c^{-1}\frac{1}{B}\sum_{b=1}^{B}\phi\left(\frac{u_i - u_i^{(b)}}{c}\right), \quad P_i(u_i) = c^{-1}\frac{1}{B}\sum_{b=1}^{B}\Phi\left(\frac{u_i - u_i^{(b)}}{c}\right) \tag{38}$$

for $i = 1, \ldots, q$, where $\phi(\cdot)$ and $\Phi(\cdot)$ are respectively the probability density function and cumulative distribution function of the standard normal distribution. This approximation is computed at each draws, i.e., $u_i = u_i^{(b)}, b = 1, \ldots, B$.

Step 2: Using this approximation transform the sampled $u_i^{(b)}$ to the normal distribution $w_i^{(b)}$,

$$w_i^{(b)} = f_i\left(u_i^{(b)}\right), \tag{39}$$

where $f_i(\cdot) = \Phi^{-1}[P_i(\cdot)]$, and define

$$w^{(b)} = \left(w_1^{(b)}, \ldots, w_q^{(b)}\right), \tag{40}$$

where $l = 1, \ldots, L$.

Step 3: Approximate the variance as an $(q \times q)$ matrix,

$$\mathbf{\Sigma} = \frac{1}{B}\sum_{b=1}^{B} w^{(b)} w^{(b)\prime}. \tag{41}$$

since the mean vector $\frac{1}{B}\sum_{b=1}^{B} W^{(b)} \approx 0$.

Step 4: Estimate the value of function $f_i(\cdot)$ at the specified point u^0 similarly to step 2.

$$w_i^0 = f_i\left(u_i^0\right), f_i'\left(u_i^0\right) \tag{42}$$

for $i = 1, \ldots, q$.

Step 5: Finally, compute

$$p(u^0) = \phi\left(w^0; 0, \mathbf{\Sigma}\right) \prod_{i=1}^{q} f_i'\left(u_i^0\right). \tag{43}$$

A.4 Efficiency of the MCMC Algorithm

A common diagnostic of MCMC efficiency is the inefficiency factor, defined as

$$IF = 1 + 2\sum_{l=1}^{L} \rho(l),$$

where $\rho(l)$ is degree of correlation quantified by the autocorrelation function given by

$$\rho(l) = \frac{1}{T}\sum_{t=1}^{T} X_t X_{t-l},$$

where X_t is a sequence for dates $t = 1, \ldots, T$ and l represents the lags. L is chosen to be large enough so that the autocorrelation tapers off. To interpret the inefficiency factor, note that independent draws from the posterior would give an inefficiency factor of 1. Inefficiency factors indicate how many extra draws need to be taken to give results equivalent to independent draws. For instance, if $50,000$ draws of a parameter are taken and an inefficiency factor of 100 is found, then the draws are equivalent to 500 independent draws from the posterior.

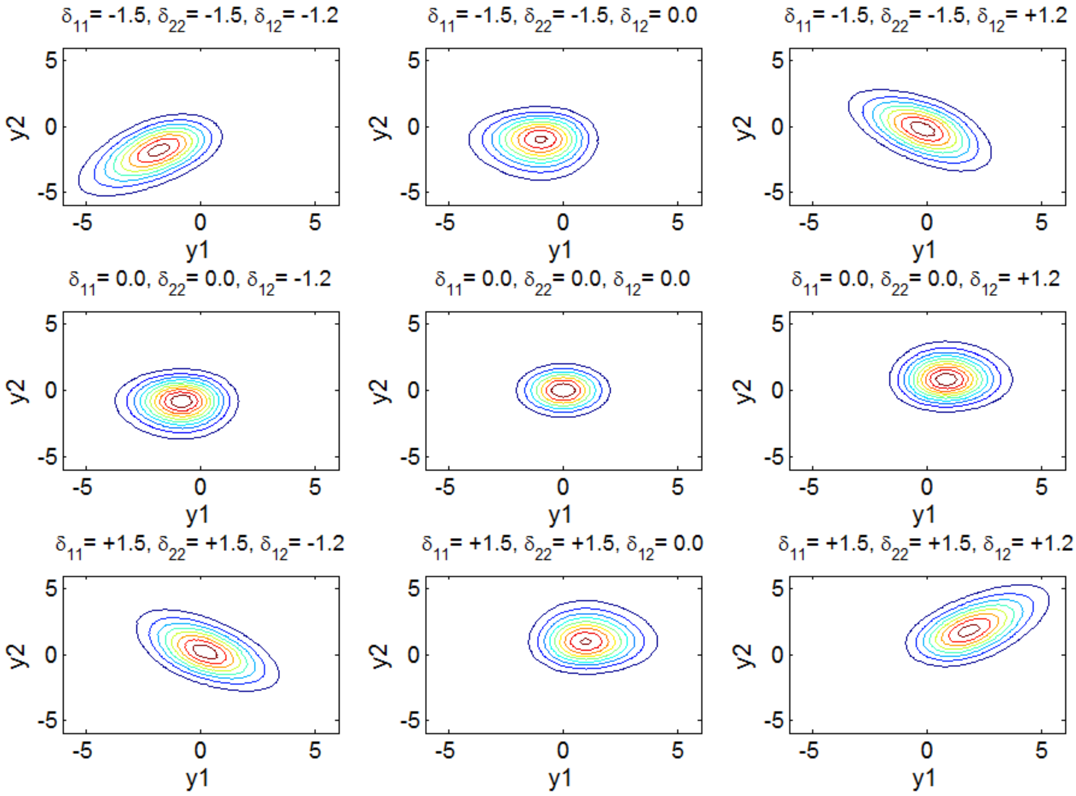

Figure 1: Contour plots of bivariate skew-normal density obtained from equation (4) with zeros mean ($\mu = 0$), identity scale matrix ($\Sigma = I_2$) and different values of asymmetric dependence δ_{ij}. The central panel corresponds to a symmetric multivariate normal distribution with $\delta_{11} = \delta_{22} = \delta_{12} = \delta_{21}$.

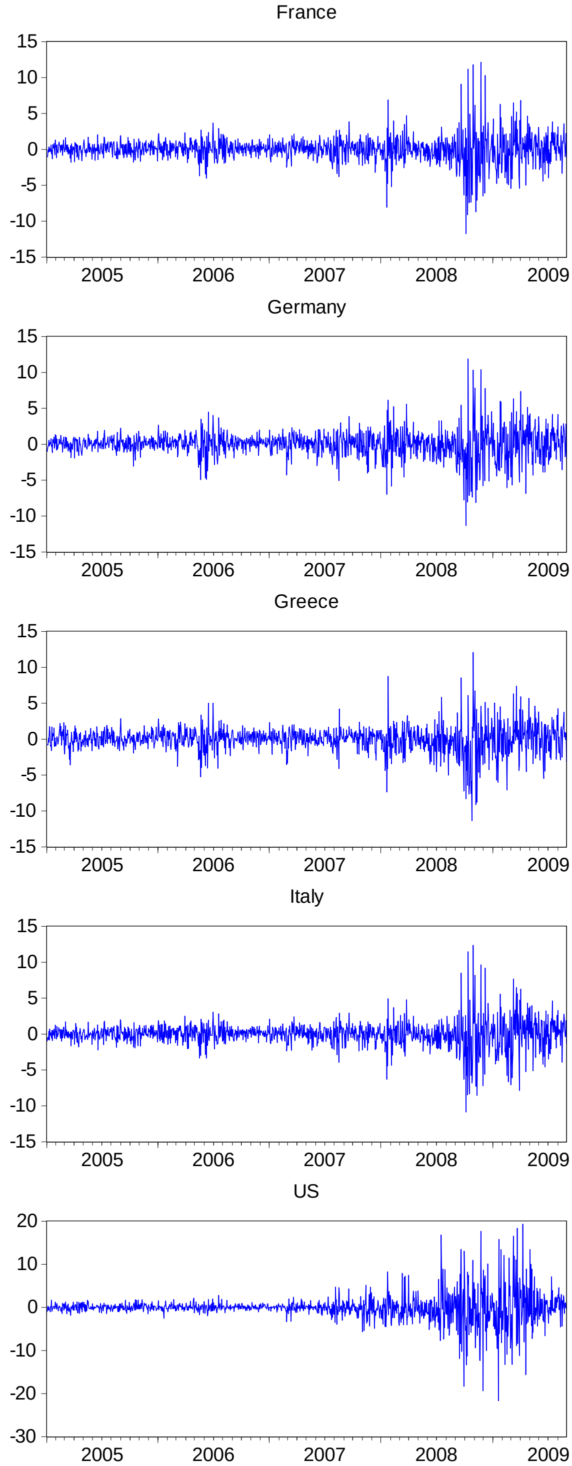

Figure 2: Daily percentage equity returns of France, Germany, Greece and Italy and banking equity returns for the US over the period January 1, 2005 to August 31, 2009.

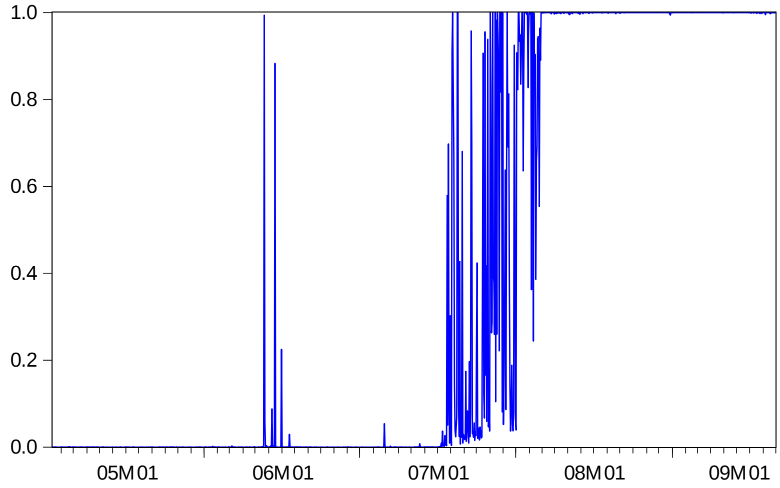

Figure 3: Probability of being in a non-crisis regime ($s_t = 0$) and a crisis regime ($s_t = 1$). A value of 0 represents the non-crisis regime and a value of 1 represents the crisis regime. The results come from the Gibbs-sampling.

Table 1:
Evidence categories for the log of the Bayes Factor used for the model selection based on Jeffreys rule (Jeffreys, 1961). Note that $\ln(BF_{ru}) = \ln(p(y|M_r)) - \ln(p(y|M_u))$.

$\ln(BF_{ru})$	Evidence against the null hypothesis
$(0, \infty)$	Evidence supports model M_r
$(-1.15, 0)$	Very slight evidence supports model M_u
$(-2.30, -1.15)$	Slight evidence supports model M_u
$(-4.60, -2.30)$	Strong evidence supports model M_u
$(-\infty, -4.60)$	Decisive evidence supports model M_u

Table 2:
Summary of restrictions on the model parameters for tests of crisis and contagion used in the Bayesian model selection.

Channel	Single market: Test	Single market: Restrictions	Group of markets: Test	Group of markets: Restrictions
Panel A: Crisis tests				
Mean	T1	$\mu_{i,0} = \mu_{i,1}$	TG1	$\mu_0 = \mu_1$
Variance	T2	$\Sigma_{ii,0} = \Sigma_{ii,1}$	TG2	$\Sigma_{ii,0} = \Sigma_{ii,1}, \forall i$
Skewness	T3	$\delta_{ii,0} = \delta_{ii,1}$	TG3	$\delta_{ii,0} = \delta_{ii,1}, \forall i$
Mean, variance	JT1	$\mu_{i,0} = \mu_{i,1}, \Sigma_{ii,0} = \Sigma_{ii,1}$	JTG1	$\mu_0 = \mu_1, \Sigma_{ii,0} = \Sigma_{ii,1}, \forall i$
Mean, skewness	JT2	$\mu_{i,0} = \mu_{i,1}, \delta_{ii,0} = \delta_{ii,1}$	JTG2	$\mu_0 = \mu_1, \delta_{ii,0} = \delta_{ii,1}, \forall i$
Variance, skewness	JT3	$\Sigma_{ii,0} = \Sigma_{ii,1}, \delta_{ii,0} = \delta_{ii,1}$	JTG3	$\Sigma_{ii,0} = \Sigma_{ii,1}, \delta_{ii,0} = \delta_{ii,1}, \forall i$
Mean, variance, & skewness	JT4	$\mu_{i,0} = \mu_{i,1}, \Sigma_{ii,0} = \Sigma_{ii,1}, \delta_{ii,0} = \delta_{ii,1}$	JTG4	$\mu_0 = \mu_1, \Sigma_{ii,0} = \Sigma_{ii,1}, \delta_{ii,0} = \delta_{ii,1}, \forall i$
Panel B: Contagion tests for $i \neq j$				
Covariance	T4	$\Sigma_{ij,0} = \Sigma_{ij,1}$	TG4	$\Sigma_0 = \Sigma_1$
Co-skewness	T5	$\delta_{ij,0} = \delta_{ij,1}$	TG5	$\Delta_0 = \Delta_1$
Covariance, co-skewness	JT5	$\Sigma_{ij,0} = \Sigma_{ij,1}, \delta_{ij,0} = \delta_{ij,1}$	JTG5	$\Sigma_0 = \Sigma_1, \Delta_0 = \Delta_1$
Panel C: Crisis and contagion tests				
Mean, variance, covariance		n.a	JTG6	$\mu_0 = \mu_1, \Sigma_0 = \Sigma_1$
Mean, skewness, co-skewness		n.a	JTG7	$\mu_0 = \mu_1, \Delta_0 = \Delta_1$
Mean, variance, covariance, skewness, co-skewness		n.a	JTG8	$\mu_0 = \mu_1, \Sigma_0 = \Sigma_1, \Delta_0 = \Delta_1$

Table 3:
Summary statistics of the daily percentage equity returns of France, Germany, Greece and Italy and banking equity returns for the US over the period January 4, 2005 to August 31, 2009. The Jarque-Bera statistic is denoted by JB which under the null of normality is distributed as a $\chi^2(2)$. An * indicates that the JB statistic is significant at the 5% level.

Statistic	France		Germany		Greece		Italy		US Bank	
Mean	0.001		0.023		-0.006		-0.021		-0.063	
Minimum	-11.737		-11.326		-11.366		-10.864		-21.678	
Maximum	12.143		11.887		12.084		12.381		19.341	
Std. Dev.	1.800		1.886		1.825		1.792		3.215	
Skewness	0.104		-0.121		-0.376		0.026		0.221	
Kurtosis	12.724		9.049		9.450		11.961		14.019	
JB test	4788.658	*	1855.213	*	2134.998	*	4065.053	*	6157.082	*

Table 4:
Inefficiency factors of the parameters. The parameters are estimated based on the RSSN model with two regimes denoted by $s_t = 0$ and $s_t = 1$.

Parameters	μ_i		Σ_{ij}		δ_{ij}	
	$s_t = 0$	$s_t = 1$	$s_t = 0$	$s_t = 1$	$s_t = 0$	$s_t = 1$
$i = 1, j = 1$	1.00	1.00	1.20	1.00	1.33	1.26
$i = 2, j = 2$	1.00	1.00	1.22	1.00	1.17	1.43
$i = 3, j = 3$	1.00	1.00	1.53	1.00	2.51	2.24
$i = 4, j = 4$	1.00	1.00	1.16	1.00	1.24	1.21
$i = 5, j = 5$	1.00	1.00	1.11	1.00	1.28	1.00
$i = 1, j = 2$	-	-	1.22	1.00	1.40	1.48
$i = 1, j = 3$	-	-	1.44	1.00	2.65	1.44
$i = 1, j = 4$	-	-	1.18	1.00	1.33	1.25
$i = 1, j = 5$	-	-	1.10	1.00	1.39	1.00
$i = 2, j = 3$	-	-	1.42	1.00	1.95	1.76
$i = 2, j = 4$	-	-	1.21	1.00	1.45	1.27
$i = 2, j = 5$	-	-	1.10	1.00	1.29	1.00
$i = 3, j = 4$	-	-	1.37	1.00	2.37	1.00
$i = 3, j = 5$	-	-	1.20	1.00	2.26	1.00
$i = 4, j = 5$	-	-	1.10	1.00	1.65	1.00

Table 5:
Posterior means of switching parameters including mean, covariance and co-skewness. The results are estimated based on the RSSN model with two regimes of non-crisis and crisis using Bayesian Gibbs-sampling approach. Five equity markets including France, Germany, Greece, Italy and the US banking sector are examined during the period between January 10, 2005 to August 31, 2009.

Posterior means	Market	France	Germany	Greece	Italy	US
Regime $s_t = 0$ - non-crisis period						
Mean ($\mu_{s_t=0}$)		0.034	0.046	0.058	0.026	0.023
Covariance ($\Sigma_{s_t=0}$)	France	0.523	0.445	0.294	0.402	0.127
	Germany	0.445	0.638	0.350	0.375	0.122
	Greece	0.294	0.350	0.684	0.277	0.022
	Italy	0.402	0.375	0.277	0.479	0.099
	US	0.127	0.122	0.022	0.099	0.599
Co-skewness ($\Delta_{s_t=0}$)	France	-0.526	-0.568	-0.367	-0.473	-0.246
	Germany	-0.568	-0.619	-0.443	-0.509	-0.302
	Greece	-0.367	-0.443	-0.649	-0.329	-0.282
	Italy	-0.473	-0.509	-0.329	-0.426	-0.229
	US	-0.246	-0.302	-0.282	-0.229	0.160
Regime $s_t = 1$ - crisis period						
Mean ($\mu_{s_t=1}$)		-0.010	-0.028	-0.044	0.004	0.002
Covariance ($\Sigma_{s_t=1}$)	France	4.129	3.912	3.062	3.895	7.350
	Germany	3.912	4.578	3.299	3.833	7.548
	Greece	3.062	3.299	3.928	3.077	6.132
	Italy	3.895	3.833	3.077	4.292	7.357
	US	7.350	7.548	6.132	7.357	15.802
Co-skewness ($\Delta_{s_t=1}$)	France	0.417	0.473	1.058	0.463	-1.404
	Germany	0.473	0.303	1.033	0.552	-1.551
	Greece	1.058	1.033	0.361	1.048	-1.098
	Italy	0.463	0.552	1.048	0.493	-1.266
	US	-1.404	-1.551	-1.098	-1.266	3.045

Table 6:
Empirical results of the crisis and contagion tests for the equity market returns of France (FR), Germany (GE), Greece (GR), Italy (IT) and banking equity returns of the US (US). The tests are conducted using Bayesian model comparison methods via the natural logarithm of the Bayes factor $\ln(BF_{ru})$, and are interpreted using the scale in Jeffrey (1961) reported in Table 1. * denotes strong evidence of crisis and/or contagion, and ** denotes decisive evidence of crisis and/or contagion.

		Market												
	Test	FR		GE		GR		IT		US		Test	Group	
Crisis tests														
Mean	$T1$	0.24		-0.21		-0.66		0.43		0.03		$TG1$	3.12	
Variance	$T2$	-34.32	**	-27.14	**	-16.25	**	-36.98	**	-46.68	**	$TG2$	-69.79	**
Skewness	$T3$	-4.99	**	-2.57		-0.74		-3.73	*	-7.18	**	$TG3$	-41.32	**
Mean, variance	$JT1$	-36.05	**	-32.40	**	-19.75	**	-40.52	**	-46.79	**	$JTG1$	-84.53	**
Mean, skewness	$JT2$	-4.75	**	-2.78	*	-1.40		-3.29	*	-7.15	**	$JTG2$	-40.73	**
Variance, skewness	$JT3$	-42.73	**	-29.97	**	-16.35	**	-44.380	**	-51.92	**	$JTG3$	-137.06	**
Mean, variance, skewness	$JT4$	-41.35	**	-32.44	**	-21.86	**	-46.94	**	-57.60	**	$JTG4$	-133.94	**
Contagion tests														
Covariance	$T4$	-101.09	**	-95.66	**	-51.65	**	-99.37	**	n.a.		$TG4$	-418.64	**
Co-skewness	$T5$	-7.77	**	-7.88	**	-1.37		-5.32	**	n.a.		$TG5$	-570.65	**
covariance, co-skewness	$JT5$	-87.13	**	-91.07	**	-48.12	**	-98.95	**	n.a.		$JTG5$	-306.38	**
Crisis and contagion tests														
Mean, variance, covariance		n.a.		n.a.		n.a.		n.a.		n.a.		$JTG6$	-462.60	**
Mean, skewness, co-skewness		n.a.		n.a.		n.a.		n.a.		n.a.		$JTG7$	-568.22	**
Mean, variance, covariance skewness, co-skewness		n.a.		n.a.		n.a.		n.a.		n.a.		$JTG8$	-1033.20	**