Reference Dependence and Labor-Market Fluctuations[*]

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Abstract

We incorporate reference-dependent worker behavior into a search-matching model of the labor market, in which firms have all the bargaining power and productivity follows a log-linear AR(1) process. Motivated by Akerlof (1982) and Bewley (1999), we assume that existing workers' output falls stochastically from its normal level when their wage falls below a "reference point", which (following Kőszegi and Rabin (2006)) is equal to their lagged-expected wage. We formulate the model game-theoretically and show that it has a unique subgame perfect equilibrium that exhibits the following properties: existing workers experience downward wage rigidity, as well as destruction of output following negative shocks due to layoffs or loss of morale; newly hired workers earn relatively flexible wages, but not as much as in the benchmark without reference dependence; market tightness is more volatile than under this benchmark. We relate these findings to the debate over the "Shimer puzzle" (Shimer (2005)).

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1 Introduction

Economists have long pondered over the observation that wages display downward rigidity and do not fall in recessions as much as one might expect on the basis of supply-and-demand analysis. An idea with a long pedigree, going back to Keynes (1936), Solow (1979), Akerlof (1982), Kahneman et al. (1986), Falk and Fehr (1999) and many others, is that reciprocal-fairness considerations deter employers from cutting wages during recessions. Specifically, the theory is that the labor contract's inherent incompleteness forces employers to rely to some extent on workers' intrinsic motivation. When workers feel that they have been treated unfairly, their intrinsic motivation is dampened and their output declines. According to this "morale hazard" theory, wage cuts relative to a "reference point" have such an effect, which is why employers try to avoid them.

Blinder and Choi (1990) and Bewley (1999) surveyed personnel managers and other labor-market actors, and found overwhelming support for the morale theory. As Bewley (1999) puts it:

> "My findings support none of the existing economic theories of wage rigid-
> ity, except those emphasizing the impact of pay cuts on morale. Other
> theories fail in part because they are based on the unrealistic psychological
> assumptions that people's abilities do not depend on their state of mind and
> that they are rational in the simplistic sense that they maximize a utility
> that depends only on their own consumption and working conditions..."

At the very least, the evidence from survey data suggests that the "morale hazard" theory is intuitive and thus worth exploring theoretically. Fehr et al. (2009) review a large body of research on experimental labor markets that corroborates this view.

In this paper we incorporate a reference-dependent account of the labor relation into a search-and-matching (S&M) model of the labor market in which "productivity" fluctuates according to a log-linear AR(1) process, and explore its theoretical implications for equilibrium wage and unemployment fluctuations. Following Akerlof (1982) and Akerlof and Yellen (1990), our main departure from the standard S&M model in the Mortensen-Pissarides tradition (see Pissarides (2000) and Shimer (2010) for textbook treatments) lies in the assumption that the labor contract is incomplete, such that the worker's normal productivity relies to some extent on "intrinsic motivation". When the worker's wage falls below a "reference point", he becomes less motivated and his output falls below the normal level by a random fraction which captures the importance of "morale" in the production function.

How is the reference point determined? We assume that a formerly unemployed worker enters his first employment period only with the "aspiration" to be paid the lowest admissible wage (normalized to zero). At the end of his first period of employment, when the worker has developed a relationship with his employer, he cultivates an aspiration to earn the expected equilibrium wage of existing workers (conditional on his current information). This aspiration will constitute the worker's reference point at the next period. Thus, the reference wage of an existing worker at period t is equal to his expected wage, calculated according to his "rational" expectations at period $t-1$. In Appendix B, we present a slightly different formulation of the reference point, which *endogenizes* this distinction between newly hired and existing workers; our main results are robust to this variation.

The "lagged expectations" approach to reference-point formation follows an influential model due to Kőszegi and Rabin (2006). The justification for the expectation-based specification is that a given wage offer may be greeted as a pleasant surprise or as a demoralizing disappointment, depending on how it compares with the worker's former expectations. For instance, if the worker expected a big salary raise, failure to meet this expectation may hurt his morale, even if his current wage is higher than yesterday's wage. The justification for the "lagged" aspect is that it takes the reference point some time to adapt to changing circumstances, just as it takes people time to change a habit. This delayed adaptation will be the source of wage rigidity in our model.[1]

Before giving an overview of our results, we wish to comment on our methodology. We follow a microeconomic-theory approach, seeking a complete analytical characterization of dynamic equilibria and highlighting their qualitative features. This has several implications. First, we focus exclusively on the labor market (consumption and capital are left out). Second, while the standard Mortensen-Pissarides model mixes non-cooperative game-theoretic modeling with the "cooperative" Nash bargaining solution, we formulate the model as an extensive-form non-cooperative game with moves of Nature and study its subgame perfect equilibria (SPE), as in Rubinstein and Wolinsky (1985). Third, we eliminate two degrees of freedom in the standard S&M model: workers have no bargaining power (firms make take-it-or-leave-it wage offers), and their non-market payoff is proportional to productivity. Doing so not only simplifies the analysis, but also ensures that all wage-rigidity effects are due to the novel behavioral element. We do not add any new parameters, and our equilibrium char-

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[1] Another example of reference-point "stickiness" is the reluctance of homeowners to lower their asking price when a boom in the real-estate market is followed by a downturn (Genesove and Mayer (2001)).

acterization is presented for virtually arbitrary reference-dependent output functions. Finally, for most of the paper, we impose a two-period exogenous separation process, which is innocuous in the reference-independent benchmark but facilitates analysis under reference dependence.[2]

As long as the magnitude of productivity shocks is not too large, our model generates a unique SPE, which displays the following features.

Wage rigidity and destruction of output. Equilibrium wage for existing workers displays downward rigidity w.r.t current productivity shocks. Specifically, for intermediate noise realizations, the firm offers the reference wage, and therefore does not respond to local productivity fluctuations. At high noise realizations, the firm pays the current outside option (which we assumed to be proportional to current productivity). In certain special cases of the model, when the output loss due to loss of morale is large, the wage is entirely rigid. At low noise realizations, the firm either lays off existing workers or pays them their outside option (in which case, the workers' output declines), depending on the realized importance of "morale" in the production function. Thus, existing workers experience layoffs or demoralization in equilibrium following bad shocks.

History dependence. The fraction of existing workers' output that is destroyed as a result of wage rigidity is purely a function of the current productivity and morale shocks. Since both shocks are drawn from stationary distributions, this fraction is history-independent. This means that existing workers' observed output depends on both productivity *levels* and productivity *changes*. In particular, for a given productivity level, we may observe recession symptoms (layoffs, reduced worker output) if this level follows a negative shock.

Entry-level wages. Newly matched workers are always hired in equilibrium and paid a wage below existing workers' wage. The entry-level wage is not rigid; it fluctuates with current productivity, although to a lesser extent than in the benchmark model without reference dependence. Unlike existing workers, the equilibrium wage of new hires is purely a function of current productivity.

Increased volatility of market tightness. As in the standard S&M model, free entry implies that market tightness is determined by the firms' hiring incentive. We show that the elasticity of tightness with respect to productivity is higher than in the reference-independent benchmark. This effect is strong for intermediate values of the AR(1)

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[2]Kuang and Wang (2010) conduct a quantitative analysis of an S&M model with a reduced-form fair-wage equation, which includes past wages as some of the independent variables. Dufwenberg and Kirchstegier (2000) study a static model with one firm and two workers, in which firms refrain from exploiting competition between workers to cut wages due to reciprocal-fairness considerations.

autocorrelation coefficient. The reason for this volatility effect is that existing workers' output destruction due to reference-dependence increases the weight of newly hired workers' output in the determination of the value of a vacancy. This raises the sensitivity of this value to initial conditions, since the stochastic process that governs productivity is mean-reverting.

In an influential paper, Shimer (2005) argued that the S&M model has shortcomings in accounting for real-life labor-market fluctuations, in the sense that the wage volatility it predicts is too large and the unemployment volatility it predicts is too small. A fast-growing literature ensued. One research direction, suggested by Shimer (2005) and Hall (2005), and challenged by Pissarides (2009), Kudlyak (2009) and Haefke et al. (2012), has centered around the hypothetical role of wage stickiness in addressing Shimer's puzzle.

Our results can be viewed in light of this debate. Since our paper follows a purely theoretical and qualitative approach, it cannot be viewed as an attempt to resolve Shimer's puzzle, which is quantitative in nature. However, we believe it helps understanding the questions that the puzzle has raised. First, the volatility effects our model generates are in the "right" direction. Second, as we show in Section 4, our model synthesizes the arguments raised by the two sides in the debate, showing they are not mutually contradictory after all. Finally, the model provides a behavioral foundation for the association between wage rigidity and enhanced tightness volatility.

2   A Model

Consider the following complete-information, infinite-horizon game. There is a continuum of players: a measure one of workers and an unbounded measure of firms (the latter assumption captures free entry among firms). We break the description into the following components: search and matching, separation, wage and output determination, the agents' information and their preferences.

Search and matching
Time is discrete. At each period t, firms and workers are matched according to the following process. An unemployed worker (including workers who lost their job at the beginning of period t, as described below) is automatically in the search pool. (That is, we abstract from questions of labor market participation.) An unmatched firm (including firms that dismissed workers at the beginning of the period, as described

below) decides whether to be in the search pool, i.e., post a vacancy.[3]

If there are U_t unemployed workers and V_t open vacancies at this stage, then a measure $m(U_t, V_t) \leq \min\{U_t, V_t\}$ of unemployed workers are matched to vacancies at the beginning of period $t + 1$. The matching function m satisfies the standard assumptions: it is continuous, strictly increasing in each of its arguments and exhibits constant returns to scale.

The matching probabilities for workers and firms at period t are thus $q_t = m(U_t, V_t)/U_t$ and $p_t = m(U_t, V_t)/V_t$, respectively. Note that $\lim_{V \to \infty} m(U, V)/V = 0$. We assume that if all firms post vacancies, then $p = 0$. Define *market tightness* at t as the ratio

$$\eta_t = \frac{V_t}{U_t} = \frac{q_t}{p_t}$$

Since m exhibits constant returns to scale, it is easy to verify that q is a strictly decreasing function of p, given by the implicit function,

$$m(\frac{p}{q}, 1) = p \tag{1}$$

Thus, η_t is a strictly decreasing function of p_t, and a strictly increasing function of q_t. From now on we will be primarily interested in market tightness as an indicator of the state of unemployment, and we will suppress U and V.[4]

Separation and wage determination

Consider a worker who, at the end of period t, completes a tenure of $i \geq 1$ consecutive periods of employment at the same firm. We say that the worker is of type i at period t. With probability $s(i)$, the two parties will be separated by the beginning of period $t + 1$ for some unspecified exogenous reason. With probability $1 - s(i)$, the match will survive into the beginning of period $t + 1$, and the worker will turn into type $i + 1$.

When the two parties are matched at the beginning of period t, the firm first chooses whether to employ the worker. We use $r_{i,t} \in \{0, 1\}$ to denote the firm's endogenous separation decision when facing a worker of type i, where $r_{1,t} = 1$ means that the firm chooses to employ the worker at t, and $r_{i,t} = 0$ means that the firm chooses to dismiss him. Conditional on employing a worker of type i at period t, the firm makes a take-it-or-leave-it, flat-wage offer $w_{i,t} \geq 0$. This is a "spot" contract that covers period t only (put differently, the firm can renegotiate the labor contract at the beginning of

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[3]For expositional simplicity, we assume that each firm can post at most one vacancy. This entails no loss of generality, as long as production is separable across vacancies.

[4]The requirement that m exhibits constant returns to scale is not only sufficient, but also *necessary* for the one-to-one correspondence between p and q.

every period).

The two parties are endogenously separated at period t if the firm dismisses the worker, or if the worker rejects the firm's wage offer. In this case (as well as following an exogenous separation), the worker joins the search pool of period t, while the firm chooses whether to be in the search pool of period t.

Reference-dependent output

Conditional on accepting a wage offer w_t at period t, an employed worker of type i produces an output level given by

$$y_{i,t} = \begin{cases} \theta_t & if \quad w_{i,t} \geq e_{i,t} \\ \gamma_t \theta_t & if \quad w_{i,t} < e_{i,t} \end{cases} \tag{2}$$

where:

- θ_t is the level of productivity that characterizes the economy at period t. We assume that θ_t follows a log-linear AR(1) process with a long-run mean of 1, i.e. $\theta_t = \theta_{t-1}^\rho \varepsilon_t$, where $\rho \in (0,1)$ is the autocorrelation coefficient, and ε_t is $i.i.d$ according to a continuous, strictly increasing *cdf* $F[\frac{1}{\xi}, \xi]$, where $\xi > 1$. Finally, $F(\varepsilon) \equiv 1 - F(\frac{1}{\varepsilon})$ - that is, $\ln(\varepsilon)$ is symmetrically distributed around zero.

- $e_{i,t}$ is the worker's reference wage. We assume that a worker enters his first period of employment at a given firm with "modest aspirations", in the sense that his reference point $e_{1,t}$ equals the lowest possible wage, which is zero. On the other hand, existing workers, who were employed by the same firm at period $t-1$, enter period t with a reference point equal to the wage they expected to earn at t conditional on being retained. Thus, at any period t, $e_{1,t} = 0$; and for every $i > 1$, $e_{i,t}$ is the expectation of $w_{i,t}$ conditional on being retained at t, given the worker's information at the end of period $t-1$ and the continuation strategies followed by all agents.

- $\gamma_t \in [0,1]$ is a random parameter representing the fraction of output loss due to worker demoralization when their wage falls below the reference point. It captures the effect of wage disappointment on workers' output (and implicitly, the extent to which the labor contract is incomplete; this interpretation is substantiated in Appendix C). We assume that γ_t is $i.i.d$ according to a cdf G that has no mass point in $[0,1)$. We also assume that $G(\gamma) < 1$ for every $\gamma < 1$.

Information

In each period $t \geq 1$, every agent observes the realizations of all exogenous random variables up to (and including) period t. In particular, ε_t and γ_t are common knowledge at the time the firm chooses its wage offer w_t. The agent also observes his own private history. Finally, whenever a firm and a worker interact, they observe the history of wage offers since they were matched. They do not observe the negotiation history in other firm-worker matches.

Preferences

All agents in the model maximize their expected discounted sum of payoffs, using the same constant discount factor δ. The payoff flow for firms at each period is as follows. A firm outside the labor market earns zero. A firm in the search pool earns $-c$, where $c > 0$ is the cost of posting a vacancy. A firm in a relationship with a worker earns a payoff that equals output minus the wage paid. An unemployed worker at period t receives a non-market payoff of $b\theta_t$, where $b \in (0,1)$. An employed type-i worker gets a payoff of $w_{i,t}$. The assumption that the outside option is proportional to current productivity is made not only for simplicity, but also to ensure that in the reference-independent benchmark, equilibrium wages will be fully flexible, such that all rigidity effects will arise from the novel behavioral element.

Discussion of reference dependence

Reference dependence of output in our model is interpreted in terms of worker motivation. This suggests that the phenomenon can be traced to the workers' preferences. Indeed, Akerlof (1982) formulated his model of the labor relation in terms of reference-dependent worker preferences that dictate their choice of unobserved effort, such that when their wage falls below the reference point, their subjective cost of effort increases. In Appendix C we show how to derive our model from a more elaborate model with reference-dependent worker preferences.

Formula (2) captures what Fehr et al. (2009) call "negative reciprocity", in the sense that a worker's motivation diminishes when he is disappointed by the firm's wage offer. It does not give room to "positive reciprocity", namely increased motivation following a wage offer above the reference point. This asymmetry reflects findings in the literature: "Whereas the positive effects of fair treatment on behavior are usually small, the negative impact of unfair behavior is often large" (Fehr et al. (2009, p. 366)). It is also in the spirit of Prospect Theory (Kahneman and Tversky (1979)): losses relative to the reference point loom significantly larger than gains.

2.1 The Reference-Independent Benchmark

Let us first consider the benchmark model in which $\gamma = 1$ with probability one, where output is reference-independent. In this case, our model reduces to a standard S&M model in which firms have all the bargaining power.

Proposition 1 *Let $\gamma = 1$. There is a unique SPE, in which firms choose $(r_t, w_t) = (1, b\theta_t)$ at every t and regardless of the worker's type, and workers accept any wage offer weakly above $b\theta_t$.*

Equilibrium in the reference-independent benchmark exhibits several noteworthy features. First, equilibrium behavior is Markovian in a narrow sense: hiring/retention and wages at any period t are purely a function of θ_t. Second, wages are entirely flexible, in the sense that they are proportional to productivity. Third, there is no behavioral distinction between newly matched and existing workers. Finally, there are no layoffs.

Proposition 1 determines equilibrium market tightness via a free-entry property. A firm's expected discounted benefit from posting a vacancy at period t, conditional on finding a new match at the beginning of $t+1$, is equal to the expected discounted sum of the firm's payoffs over the duration of the employment relation. Formally, it is a function of the state at t, defined as follows:

$$\Pi(\theta_t) = (1 - b) \sum_{i=1}^{\infty} \delta^i \left(\prod_{0 < j < i} (1 - s(j)) \right) \mathbb{E}(\theta_{t+i} \mid \theta_t) \tag{3}$$

Note that Π is an increasing function. If $c > \Pi(\theta_t)$, then in SPE no firm posts a vacancy at t, and market tightness is infinite. If $c \leq \Pi(\theta_t)$, then in equilibrium firms will be indifferent between searching and not searching. The probability p_t that a searching firm will find a match at the beginning of $t + 1$ will be set such that $c = p_t \Pi(\theta_t)$. Market tightness is derived from p_t according to (1). Hence, equilibrium tightness at t is purely a function of θ_t as well.

3 Equilibrium under a Two-Period Separation Process

We now analyze SPE in our model, under the following restriction on the exogenous job separation process: $s(1) = 0$ and $s(2) = 1$. That is, the employment relation lasts at most two periods. This could approximate industries in which firm-specific human

capital depletes quickly as a result of rapid technological changes. However, we assume it mainly for tractability. We briefly discuss more complex finite-horizon separation processes at the end of this section.[5]

It is useful to make two preliminary observations. First, in SPE, all newly matched workers at any given period are treated identically; similarly, all existing workers at any given period are treated identically. The reason is that all agents on each side of the market are identical, and no firm-worker pair gets to observe the history of any pairwise interaction prior to their own match, thus preventing history-dependent asymmetries from emerging. In what follows we often refer to the way "the worker" or "the firm" behave at a given history, with the understanding that this pertains to all firms and all workers of the same type at the same period.

Second, we can think about an equilibrium wage offer in terms of whether it satisfies a worker's "individual rationality" (IR) and "morale hazard" (MH) constraints, in analogy to IR/IC constraints in contract theory. Fix a history h following a wage offer. An SPE satisfies the *IR constraint* at h if the worker is weakly better off than if he rejects the firm's wage offer and sticks to his equilibrium strategy thereafter. An SPE satisfies the *MH constraint* at h if the wage offer at h is weakly higher than the worker's reference wage at that history.[6]

By assumption, the MH constraint coincides with the constraint that wages are non-negative, as far as newly matched workers are concerned. Therefore, the MH constraint is only relevant for existing workers. According to the one-deviation property of SPE, the IR constraint always holds in equilibrium, and the only question is at which histories it is binding. Note that in SPE, if the IR constraint holds with slack at h, the MH constraint must be binding. The reason is simple: if the MH constraint is violated or holds with slack, the firm can slightly lower its wage without changing the set of constraints it satisfies.

The following results characterizes wage and retention policies in SPE, under a mild condition on the magnitude of the business cycle.

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[5] We have been able to derive analytic solutions for a stationary infinite-horizon separation process when $\rho = 0$ or $\rho = 1$. However, since these cases are degenerate in terms of the effect of wage rigidity on tightness volatility, we chose not to include this analysis in the paper.

[6] Note that unlike the standard way of modeling IC constraints, we take a reduced-form approach to modeling worker output, and do not incorporate explicit effort decisions. As mentioned earlier, Appendix C derives this reduced-form model from a more elaborate model in which the worker decides whether or not to exert unobservable effort.

Proposition 2 *Let* $\xi \leq \frac{1}{2}(1+\sqrt{5})$. *Then, the game has a unique SPE outcome, which has the following properties.*[7]

(i) An existing worker's period-t reference point is

$$e_{2,t} = \phi \cdot b\theta_{t-1}^{\rho}$$

where the coefficient $\phi \in [\mathbb{E}(\varepsilon), \xi]$ *is uniquely determined by the following equations:*

$$\phi = \frac{\int_0^1 \int_{\varepsilon > \varepsilon^*(\gamma)} \max\{\phi, \varepsilon\} dF(\varepsilon) dG(\gamma) + \int_b^1 \int_{\varepsilon < \varepsilon^*(\gamma)} \varepsilon dF(\varepsilon) dG(\gamma)}{1 - G(b)F(\phi b)} \tag{4}$$

$$\varepsilon^*(\gamma) = \frac{b\phi}{1 - \max\{0, \gamma - b\}} \tag{5}$$

(ii) An existing worker is dismissed at period t if and only if $\gamma_t < b$ *and* $\varepsilon_t < \phi b$. *Conditional on being retained at t, his wage is*

$$w_2(\theta_{t-1}, \theta_t) = \begin{cases} \max\{e_{2,t}, b\theta_t\} & if & \varepsilon_t > \varepsilon^*(\gamma_t) \\ b\theta_t & if & \gamma_t > b \text{ and } \varepsilon_t < \varepsilon^*(\gamma_t) \end{cases} \tag{6}$$

(iii) A newly matched worker at period t is always hired; his wage at period t is

$$w_1(\theta_t) = b\left[\theta_t - \delta\theta_t^{\rho} \int_0^1 \int_{\varepsilon^*(\gamma)}^{\phi} (\phi - \varepsilon) dF(\varepsilon) dG(\gamma)\right] \tag{7}$$

Qualitative features of the SPE outcome

Wage rigidity and endogenous output destruction. Existing workers may experience wage rigidity, layoffs or loss of morale, depending on the realizations ε_t, γ_t. When $\varepsilon_t \in (\varepsilon^*(\gamma_t), \phi)$, existing workers at period t are retained and paid their reference wage $e_{2,t}$, which is purely a function of θ_{t-1} and therefore rigid in the sense that it is not responsive to the productivity shock in the range $(\varepsilon^*(\gamma_t), \phi)$. When $\varepsilon_t > \phi$, existing workers receive their participation wage, which lies above the reference wage, and therefore produce normal output. When $\varepsilon_t < \varepsilon^*(\gamma_t)$, existing workers experience destruction of output: either $\gamma < b$, in which case they are fired; or $\gamma > b$, in which case they are kept at their participation wage which lies below their reference wage,

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[7]We ignore the firm's behavior at zero-probability cutoff events.

and thus produce sub-normal output due to loss of morale. Because the destruction of output experienced by an existing worker is purely a function of ε_t, γ_t, the expected output that a newly hired worker at period t believes he will produce at $t + 1$ is $\lambda\theta_t^\rho$, where the constant λ is given by

$$\lambda = \int_0^1 \int_{\varepsilon > \varepsilon^*(\gamma)} \varepsilon dF(\varepsilon)dG(\gamma) + \int_b^1 \int_{\varepsilon < \varepsilon^*(\gamma)} \gamma\varepsilon dF(\varepsilon)dG(\gamma) \tag{8}$$

History dependence. The equilibrium treatment of existing workers at period t is Markovian with respect to an extended state $(\theta_t, \theta_{t-1}, \gamma_t)$ (or, equivalently, $(\theta_t, \varepsilon_t, \gamma_t)$). Their reference wage is purely a function of θ_{t-1}. Whether they receive it or the participation wage $b\theta_t$ (which is purely a function of θ_t) depends entirely on the realizations ε_t, γ_t; and so does their retention policy. Therefore, existing workers' layoff rate at any period is $G(b)F(\phi b)$, independently of the history up to period $t - 1$. Newly hired workers' wage is purely a function of θ_t.

IR and MH constraints. The IR constraint of newly matched workers is always binding, and consequently their continuation payoff at any period t is as if they earn $b\theta_{t'}$ at every $t' \geq t$. In contrast, existing workers' IR constraint holds with slack whenever $\varepsilon_t \in (\varepsilon^*(\gamma_t), \phi)$ - i.e., whenever they are retained and paid their reference wage, in which case their MH constraint is binding. When existing workers are retained at their participation wage, their IR constraint is binding while their MH constraint is violated (if $\gamma_t > b$ and $\varepsilon_t < \varepsilon^*(\gamma_t)$) or satisfied with slack (if $\varepsilon_t > \phi$).

The structure of entry-level wages. The equilibrium wage paid to new hires is both strictly positive and strictly increasing in θ_t (this is ensured by our restriction on ξ), albeit at a lower rate than in the $\gamma = 1$ benchmark. In this sense, entry-level wages are "partially flexible" w.r.t current productivity. Note that unlike the $\gamma = 1$ benchmark, equilibrium wages exhibit a "seniority premium": existing (newly matched) workers earn wages above (below) the current outside option.

Sketch of the proof of Proposition 2

First, we derive an upper bound on the rent that existing workers can get in equilibrium, which translates into a lower bound on newly hired workers' wage. This bound is above zero, such that wage offers to newly hired workers satisfies the MH constraint with slack. Hence, their IR constraint is always binding in equilibrium. (Here we rely on the assumption that $\xi < \frac{1}{2}(1 + \sqrt{5})$.) This in turn implies that newly matched workers must always be indifferent between accepting an equilibrium wage offer (and sticking to

their equilibrium strategy thereafter) and being permanently unemployed. Therefore, an existing worker at period t would accept any wage above $b\theta_t$. We have thus fixed existing workers' participation wage.

For any given reference wage $e_{2,t}$, we can check, for every realization of ε_t, γ_t, which of the following three courses of action maximizes the firm's profit: (i) dismiss an existing worker, (ii) keep him at his participation wage, (iii) keep him at his reference wage. This enables us to write down the expression for $e_{2,t}$, which is uniquely given by (4)-(5). The assumption that G assigns positive probability to any neighborhood of $\gamma = 1$ is instrumental in the uniqueness of the solution. Otherwise, it could be possible that existing workers' reference wage at t is strictly higher than $b\xi\theta_{t-1}^{\rho}$, namely the maximal outside option that is feasible given θ_{t-1}, and firms would always stick to the reference wage in order to avert loss of worker morale. When γ is very close to one, firms would not have an incentive to do so, and this prevents the reference wage from being equal to the lagged-expected wage. Alternative perturbations of the model can generate uniqueness (see Eliaz and Spiegler (2012)).

The cutoff $\varepsilon^*(\gamma)$ is the productivity shock for which the firm is indifferent between keeping the worker at his reference wage and dismissing / keeping him at his participation wage, depending on whether γ is below or above b. We have thus derived existing workers' equilibrium wage, and the firm's retention policy immediately follows from that. To obtain new hires' wage, we use their indifference to permanent unemployment, such that their equilibrium wage at t is equal to $b\theta_t$ minus the discounted rent they expect to receive as existing workers at $t + 1$.

Two special cases

First, revisit the reference-independent benchmark, by letting $G(\gamma) = 0$ for all $\gamma < 1$. Since $G(b) = 0$, existing workers are always retained. Applying formulas (4)-(5), we obtain $\phi = \mathbb{E}(\varepsilon) = \varepsilon^*(1)$; hence, an existing worker at t receives $b\theta_t$. Applying formula (7), we obtain that a newly hired worker receives the same wage. This reproduces Proposition 1 for the two-period separation process.

Second, consider the limit case $G(b) \to 1$. Observe that formula (4) collapses into

$$\phi = \mathbb{E}[\max\{\phi, \varepsilon\} \mid \varepsilon > \phi b]$$

The solution to this equation is $\phi = \xi$, which implies $\varepsilon^*(\cdot) = \xi b$ with probability one. Existing workers are thus retained and paid $w_{2,t} = \xi b\theta_{t-1}^{\rho}$ whenever $\varepsilon_t > \xi b$, and dismissed otherwise. Existing workers' output coefficient λ is given by a simple

formula:

$$\lambda = \mathbb{E}(\varepsilon \mid \varepsilon > \xi b)$$

Newly hired workers earn

$$w_{1,t} = b \left[\theta_t - \delta\theta_t^\rho \int_{\xi b}^{\xi} (\xi - \varepsilon)dF(\varepsilon) \right] \tag{9}$$

Existing workers' equilibrium wage in this case is absolutely rigid, in the sense that it is purely a function of productivity in the previous period. Wage rigidity here has a flavor of "grade inflation": in the $G(b) \to 1$ limit, existing workers' reference wage is the expectation of the maximum between the outside option and the reference wage itself. This means that the reference wage must always be greater than or equal to the expected outside option, which can only be true if the reference wage equals the highest possible value of the outside option. When $\gamma < b$, a firm would rather dismiss a worker than paying him a wage below his reference point. Thus, existing workers almost always get their reference wage conditionally on being retained.

3.1 Volatility of Market Tightness

In order to study the equilibrium volatility of market tightness, we follow the S&M literature, and assume in this subsection that the matching function takes the following form

$$m(U_t, V_t) = kU_t^\alpha V_t^{1-\alpha} \tag{10}$$

where $\alpha \in (0,1)$ and k is sufficiently small so that match probabilities are always well-defined. This allows us to get an explicit, closed-form expression for market tightness. Let us first establish that in SPE, tightness at any period t is purely a function of θ_t. The expected discounted profit generated by a vacancy opened in period t conditional on getting a new match at the beginning of period $t + 1$ is

$$\delta(1 - b) \left[\theta_t^\rho \mathbb{E}(\varepsilon) + \delta\lambda\theta_t^{\rho^2} \mathbb{E}(\varepsilon^\rho) \right]$$

where λ is given by (8). This expression is an increasing function of θ_t, and we denote it by $J(\theta_t)$. Note that in the $\gamma = 1$ benchmark, we have $\lambda = 1$, hence $J(\theta_t)$ is reduced to $\Pi(\theta_t)$, as given by (3).

Lemma 1 *In the SPE characterized by Proposition 2, η_t is a function of θ_t given by the following equation:*

$$\eta_t(\theta_t) = \sqrt[\alpha]{\frac{kJ(\theta_t)}{c}}$$

as long as $c/J(\theta_t) < 1$. Otherwise, market tightness is zero.

To understand why equilibrium market tightness is a well-defined function of current productivity, recall that η_t is a strictly decreasing function of p_t, the probability that a searching firm finds a match at t. Because of free entry, p_t itself is a function of $J(\theta_t)$. Thus, although some aspects of equilibrium behavior at t - specifically, the treatment of existing workers - depend on θ_{t-1}, tightness is only a function of θ_t.

To see how reference dependence affects tightness volatility, let us write down the expression for the elasticity of η_t w.r.t θ_t (omitting the subscript):

$$-\frac{\rho}{\alpha} \cdot \frac{\theta^\rho + \rho\delta\lambda\sigma\theta^{\rho^2}}{\theta^\rho + \delta\lambda\sigma\theta^{\rho^2}} \tag{11}$$

where $\sigma = \mathbb{E}(\varepsilon^\rho)/\mathbb{E}(\varepsilon)$. From our assumptions on the distribution of ε it follows that $\sigma < 1$.[8]

It is easy to verify that this expression decreases with λ in absolute terms. Recall that $\lambda = 1$ in the reference-independent benchmark, and that $\lambda < 1$ when reference dependence is introduced. We conclude that reference dependence increases tightness volatility. The intuition for this effect is as follows. In expectation, a constant fraction $1 - \lambda$ of existing workers' normal output is destroyed, independently of the history. Therefore, the constant λ acts like an additional discount factor between the worker's first and second periods of employment. The "extra discount factor" increases the weight that first-period output receives in the calculation of the value of the vacancy. Because $\rho < 1$, the worker's productivity in his second period of employment is less sensitive to the value of θ that prevailed at the time the firm originally posted the vacancy than his first-period productivity. Therefore, introducing the new term λ increases the sensitivity of the vacancy's value to the initial value of θ. This intuition clarifies why the volatility effect disappears when $\rho \to 1$.

For a closer look at the interplay between the effects of reference dependence and the persistence of the business cycle, suppose that productivity is at the long-run average,

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[8]Since $dF(\varepsilon) \equiv dF(\frac{1}{\varepsilon})$ and $\rho \in (0,1)$, it suffices to show that for $\varepsilon \in [\frac{1}{\xi}, \xi]$,the sum $\varepsilon^\rho + \varepsilon^{-\rho}$ decreases with ρ.

i.e. $\theta_t = 1$. At this point, the elasticity of tightness is

$$-\frac{\rho}{\alpha} \cdot \frac{1 + \rho\delta\lambda\sigma}{1 + \delta\lambda\sigma} \tag{12}$$

When ρ is high, the "standard" tightness volatility - i.e., the value of (12) for $\lambda = 1$ - is higher. However, at such values of ρ, the effect of reference dependence on tightness volatility vanishes. At the other extreme, when ρ is low, "standard" tightness volatility is low, but the effect of reference dependence is large. The derivative of (12) w.r.t λ is maximized at $\rho = \frac{1}{2}$. Thus, the effect of reference dependence on tightness volatility is maximized at intermediate levels of persistence.

3.2   The Role of γ

So far, we compared SPE in our model for an arbitrary G to the reference-independent benchmark $\gamma = 1$. Let us extend this comparative-statics exercise and ask more generally how the equilibrium outcome changes as γ goes down, namely as output becomes more sensitive to wage drops below the worker's reference point. For simplicity, we focus on the limit case in which G assigns probability one to some particular value $\gamma < 1$. We already analyzed the case of $\gamma < b$, and saw that as long as we are restricted to this range, changes in γ have on impact on the equilibrium outcome.

Let $\gamma > b$. In this case, the expression for ϕ is reduced to

$$\phi = \mathbb{E}(\varepsilon) + \int_{m\phi}^{\phi} (\phi - \varepsilon) dF(\varepsilon)$$

where
$$m = \frac{\varepsilon^*(\gamma)}{\phi} = \frac{b}{1 - \gamma + b}$$

It is straightforward to check that as γ goes up, ϕ decreases while $\varepsilon^*(\gamma)$ rises. This

means that existing workers' reference wage, as well as the range of realizations of ε_t for which they are paid this wage, shrink. As a result, newly hired workers' wage goes up and approaches their outside option. When $\gamma \to 1$, the interval $[m\phi, \phi]$ vanishes, and equilibrium wage and retention policies converge to the reference-independent benchmark. The effect of raising γ on λ is ambiguous: on one hand, the probability of sub-normal output due to demoralization increases, but on the other hand, the output loss due to demoralization is lower because γ is higher. Therefore, in the range $\gamma > b$, it is not clear to us whether the effect of γ on tightness volatility is monotone in the

range $(b, 1)$.

3.3   An Exercise: The Effect of Payroll Tax

Suppose that we impose a payroll tax at a constant rate τ. In principle, tax incidence might matter through effects on the reference point - we ignore such considerations by assuming that the tax is imposed on firms, such that for workers there is no distinction between gross and net wages. For simplicity, consider the limit case $G(b) \to 1$, where firms pay existing workers' their reference wage conditional on retaining them. Existing workers' wage does not change, because their outside option at t continues to be $b\theta_t$, hence their wage continues to be $\xi b \theta_{t-1}$. However, as far as the firm is concerned, imposing the payroll tax is equivalent to raising the workers' outside option coefficient from b to $b/(1 - \tau)$, hence the layoff cutoff changes to $\varepsilon^* = \xi b/(1 - \tau)$. By (9), new workers' wage *increases*, because their expected future rent shrinks. The latter finding is surprising a priori. The rise in ε^* also implies that λ goes down, hence tightness volatility increases.

3.4   General Finite-Horizon Separation

Our analysis in this section was based on the assumption that $s(1) = 0$ an $s(2) = 1$. Let us consider a generalization of this exogenous separation process, in which $s(i) = 0$ for every $i = 1, ..., T - 1$, and $s(T) = 1$, where $T \geq 2$. Characterization of SPE would proceed along the same lines as in Proposition 2, with three key differences. First, the firing decision is more complex. In particular, a firm may prefer to retain a worker at a reference that exceeds his output because of a high continuation payoff. Second, in order to ensure that newly hired workers' wage is strictly positive, a stronger condition on the magnitude of the business cycle would be required. Finally, the expressions for the worker's wage as a function of his tenure would be more cumbersome.

For illustration, let us construct an SPE in which newly hired workers' IR constraint is always binding, under the following parametric restrictions: $T = 3$, $\rho = 1$, $\xi < \sqrt{2}$ and $G[b - (1 - b)\mathbb{E}(\varepsilon)] \to 1$ (the latter restriction implies $b > \frac{1}{2}$). Consider first a worker of type $i = 3$ at period t. This worker is essentially equivalent to a worker of type 2 in the two-period model: his wage equals the maximal outside option at period t conditional on θ_{t-1}, i.e., $w_{3,t} = e_{3,t} = \xi b \theta_{t-1}$, and he is retained if and only if $\varepsilon_t \geq \xi b$. Since $\gamma < b$ almost surely, the worker almost surely receives his reference wage conditional on being retained.

Next, consider a worker of type $i = 2$ in the same period t. His *participation* wage,

denoted $\bar{w}_{2,t}$, is the same as newly hired workers' equilibrium wage in the two-period model, i.e. $\bar{w}_{2,t} = \hat{b}\theta_t$, where

$$\hat{b} = b\left[1 - \delta \int_{\xi b}^{\xi} (\xi - \varepsilon)dF(\varepsilon)\right] < b$$

Let us guess that type 2 workers almost surely receive their reference wage conditional on being retained. Therefore, by the same reasoning as in the case of type 3 workers, we obtain $w_{2,t} = e_{2,t} = \xi\hat{b}\theta_{t-1}$. To confirm that the guess is correct, we need to verify that the firm's expected discounted sum of profits from keeping the worker at a wage below his reference point is almost surely negative, i.e.

$$\gamma_t\theta_t - \hat{b}\theta_t + \delta\theta_t \int_{\xi b}^{\xi} (\varepsilon - \xi b)dF(\varepsilon) < 0$$

for almost all realizations of γ_t. Our assumption on G ensures that this is the case. It follows that the firm retains type 2 workers at period t if and only if ε_t is above a cutoff ε^* that is given by

$$\theta_{t-1}\varepsilon^* - \xi\hat{b}\theta_{t-1} + \delta\theta_{t-1}\varepsilon^* \int_{\xi b}^{\xi} (\varepsilon - \xi b)dF(\varepsilon) = 0$$

hence

$$\varepsilon^* = \frac{\xi\hat{b}}{1 + \delta \int_{\xi b}^{\xi} (\varepsilon - \xi b)dF(\varepsilon)} < \xi b$$

It remains to derive the wage of type 1 workers at period t and verify that it is strictly positive. Since we are asserting a binding IR constraint for new hires, these workers should be indifferent to permanent unemployment. Therefore, their wage is equal to their outside option minus the expected discounted sum of rents they accumulate as existing workers:

$$w_{1,t} = b\theta_t - \delta \int_{\varepsilon^*}^{\xi} \left[\hat{b}\theta_t(\xi - \varepsilon_{t+1}) + \delta b\theta_t\varepsilon_{t+1} \int_{\xi b}^{\xi} (\xi - \varepsilon_{t+2})dF(\varepsilon_{t+2})\right] dF(\varepsilon_{t+1})$$

The assumption that $\xi < \sqrt{2}$ ensures that $w_{1,t} > 0$. Hence, newly hired workers' MH constraint holds with slack implying that their IR constraint is binding, as we have asserted.

Observe that in this equilibrium, all existing workers at t are paid a fully rigid wage conditional on being retained, which is purely a function of θ_{t-1}, whereas newly

hired workers' wage at t is proportional to θ_t. Wages exhibit a "seniority premium": $w_{i,t}$ increases with i (it is obvious that $w_{2,t} < w_{3,t}$; verifying that $w_{1,t} < w_{2,t}$ is less immediate, and ensured by the restriction that $b > \frac{1}{2}$). Finally, workers with a longer tenure are more likely to be dismissed.

4 Discussion

This section discusses two features of our model: its relation to the Shimer Puzzle, and the model's ability to capture persistent effects of productivity shocks on wages, inflows into unemployment and tightness.

4.1 Wage Rigidity and the Shimer Puzzle

The enhanced tightness volatility discussed in Section 3.1 relates our equilibrium characterization to Shimer's puzzle. Shimer himself suggested that incorporating wage rigidity into S&M models may be an appropriate response to his finding. Hall (2005) proposed an example of such a model, replacing the assumption that wages are determined by a Nash-Bargaining formula with the assumption that the wage is constant across all states of the economy, as long as it is in the bargaining set in each state. The latter is an IR requirement: wage-rigidity effects should not cause parties to turn down individually rational offers.

Two features of Hall's model are noteworthy in comparison to our model. First, Hall imposes wage rigidity a priori, without deriving it from explicit behavioral or institutional considerations. In contrast, our model generates wage rigidity from workers' reference-dependent behavior. Second, Hall's analysis does not distinguish between newly hired and existing workers; what he refers to as "the wage" applies to all workers, regardless of their tenure.

The latter feature was criticized by Pissarides (2009), Kudlyak (2009) and Haefke et al. (2012), who argue - echoing Bewley (1999) and Fehr et al. (2009) - that this distinction does seem to exist in reality. They claim that if one observes wage rigidity in aggregate data, one cannot infer anything about the wages of newly hired workers, since these are a small fraction of the stock of employed workers at any given point in time. In particular, Haefke et al. (2012) construct a time series for wages of new hires using micro-data on earnings and hours worked from the Current Population Survey (CPS) outgoing rotation groups. They find that the wage for newly hired workers is much more volatile than the aggregate wage and responds one-to-one to productivity.

If one wanted to reconcile Hall's model of wage determination with this critique, one would have to impose wage rigidity only on existing workers. However, Pissarides (2009) and Haefke et al. (2012) show that by doing so, one loses the modified S&M model's ability to generate increased tightness volatility. The reason is that in Hall's model, wage rigidity never causes the firm-worker relationship to break down. A newly matched pair fully incorporates all future rigidities into their negotiation, such that the agreed-upon wage offsets all future departures from the "normal" surplus-division rule. As a result, the firms' hiring incentives are unaffected by the anticipated rigidity of existing workers' wage.

How do our results fit into this interesting exchange? On one hand, our model respects the distinction between newly hired and existing workers, and derives rigid wages for the latter only. On the other hand, seemingly in contradiction to Pissarides (2009) and Haefke et al. (2012), it generates increased tightness volatility relative to the benchmark model. The key to resolving this apparent inconsistency is the incompleteness of the labor contract and the workers' changing reference point. The standard S&M model assumes complete contracts, and Hall (2005) shares this feature. When complete contracts are feasible, the rule for dividing the surplus does not affect the size of the surplus. This independence breaks down in our model. When a firm violates an existing worker's MH constraint by paying him a wage below his reference point, the bargaining set effectively shrinks due to the worker's loss of morale, potentially to the point where all gains from mutual agreement are dissipated. It follows that the value of a new firm-worker match is not neutral to anticipated wage rigidity.

Our model does respect Hall's desideratum that wage rigidity should not cause workers to turn down individually rational offers. Indeed, existing workers' equilibrium acceptance decisions are the same as in the benchmark model. However, the labor relation in our model involves events outside the scope of the labor contract, which are determined by the workers' changing reference point. The adverse effects of wage rigidity on the value of vacancies are traced to these incontractible events.

It is also useful to think about our theoretical argument in relation to another well-known response to Shimer's puzzle. Hagedorn and Manovskii (2008) argued that the Shimer puzzle can be resolved under a different calibration strategy that assigns more bargaining power to the firm and implies a less pro-cyclical outside option for workers. Note that while this approach can account for rigid wages, it does not distinguish between new hires and existing workers, and thus implies that the former earn rigid wages as well. Our model, of course, takes the hagedorn-Manovskii assumption regarding the distribution of bargaining power to the extreme. However, it shows that even if

the workers' outside option is maximally pro-cyclical, one can generate (qualitatively) patterns of rigid wages for existing workers and rather flexible wages for new hires.

4.2 Comparison with other Classes of Models

In this sub-section we discuss our results in comparison with alternative S&M models of the labor market. We will demonstrate that the combination of effects that our model generates - wage rigidity for existing workers, flexible entry-level wages, a seniority premium, endogenous job destruction that is sensitive to changes in productivity, and enhanced volatility of market tightness - cannot be reproduced by these alternative models. This section is not intended to be a survey of recent attempts to resolve the Shimer puzzle. We focus on a small number of approaches that are straightforward to compare to ours, and a number of important works on the subject (such as Hall and Milgrom (2008) or Gertler and Trigari (2009)) are not mentioned because of the difficulty of comparison

4.2.1 Idiosyncratic Shocks and Endogenous Job Destruction

Since endogenous destruction of output plays a major part in our tightness volatility result, it is natural to ask whether other mechanisms of endogenous job destruction would generate similar patterns. The most well-known S&M model that exhibits endogenous job destruction, due to Mortensen and Pissarides (1994) - referred to as the MP model henceforth - generates this effect through idiosyncratic productivity shocks. To create an MP-like model that is as comparable to ours as possible, modify the benchmark model as follows. First, assume the same two-period separation process as in Section 3: $s(1) = 0$, $s(2) = 1$. Assume further that the output of an *existing* worker is subjected to a random idiosyncratic shock, such that an employed worker produces an output of $\theta_t \nu_t$, where v_t is *i.i.d* with $\mathbb{E}(v) = 1$ across firms and periods.

SPE wage offers in this model are exactly as in the benchmark: workers are always offered $b\theta_t$ when they are employed. Hiring and retention decisions are as follows: $r_t = 1$ if and only if $\nu_t \geq b$. Thus, in each period, a constant fraction of firms will choose to fire existing workers - just like our model. However, this variation *lowers* the volatility of market tightness - the exact opposite of our effect. To see why, note that we could reinterpret the benchmark model as an MP model in which firms make their hiring/retention decisions *before* learning the realization of their idiosyncratic shock. Because $\mathbb{E}(v) = 1 > b$, firms will always choose $r = 1$. When $v_t < b$, the firm's pre-commitment to play $r = 1$ is inefficient ex-post; if the firm could delay its decision

until *after* it has learned its idiosyncratic shock, it would efficiently fire the worker.

This is a simple value-of-information argument: enabling firms to move after learning their idiosyncratic shock increases expected profits. But this means that when we switch from the benchmark model to the MP-like model of this subsection, this is equivalent to introducing a *premium* factor $\lambda > 1$ to the firm's profit in the second period of its relationship with the worker. In other words, the MP-like variation *increases* the importance of the second period in determining the value of the vacancy, thereby *reducing* its sensitivity to initial conditions.

This comparison highlights the feature that endogenous separations in our model *destroy* value. The worker's changing reference point and the firm's inability to offer a complete labor contract imply that vacancies will be closed even though the two parties would have agreed ex-ante that it would be efficient to keep them. In contrast, vacancies in the MP model are closed if and only if it is efficient to do so. This difference translates to tightness volatility effects in opposite directions.

4.2.2 Moral Hazard and Efficiency Wages

Our model is essentially an efficiency-wage model: in equilibrium, firms pay (existing) workers a wage above their reservation value, in order to induce unobserved effort. The mechanism that generates this effect is based on reciprocal fairness considerations, but there could be others. Shapiro and Stiglitz (1984) assume that when a worker shirks, he is caught and fired with some probability. In order for the worker to have an incentive to exert effort, the firm must offer him a wage above his outside option.

Costain and Jansen (2010) and Malcolmson and Mavroeidis (2010) incorporated the Shapiro-Stiglitz efficiency wage model into an S&M model. To illustrate the similarities and differences between such a model and ours, we briefly analyze the following modification of the benchmark model. After a worker accepts a wage offer at period t, he makes an effort decision $x_t \in \{0, 1\}$ and products an output of $\theta_t[\gamma_t + (1 - \gamma_t)x_t]$. Assume $\gamma < b$, and suppose that the firm can observe x_t with probability α. An employed worker's payoff at period t is $w_t - \beta x_t$, where β is his cost of effort.

Since $\gamma < b$, the incentive constraint that induces workers to exert effort must hold in order for firms to earn positive profits. In SPE, both this constraint and the IR constraint will be binding. As a result, equilibrium wage at period t will be $b\theta_t + \beta/(1 - \alpha)$. Firms will therefore choose $r_t = 1$ if and only if $\theta_t \geq \beta/(1 - \alpha)(1 - b)$. This means that separation will be more frequent when productivity is low. As a result, efficiency wages will have an adverse effect on the incentive to hire new workers at low values of θ, such that the effect on tightness volatility will be roughly in the

same direction as in our model. However, the equilibrium wage is linear in θ, as in the benchmark model, which means that the model does not generate wage rigidity.

4.2.3 Long-Term Contracts and Consumption Smoothing

An alternative theory of wage rigidity is based on the idea (dating back to Azriadis (1975) and Beaudry and DiNardo (1989)) that employers can commit to long-term wage contracts which enable liquidity-constrained workers to smooth consumption across periods. When productivity fluctuates, a risk-neutral employer with no liquidity constraints can essentially offer insurance to a risk-averse worker with limited access to savings. By risk aversion, the worker would be willing to take a pay cut in return for a stream of flat wages. Thus, entry wages would fluctuate with productivity, whereas on-going wages would be rigid because of the long-term commitment to pay the same wage in each period.

To investigate the effect of risk sharing in our framework, let $\gamma = 1$, and assume that the worker is risk-averse and that the firm can commit to a two-period labor contract. For the sake of illustration, assume separable CARA utility from streams of wage earnings (an analogous argument would hold under CRRA). The risk premium that a new hire in period t would be willing to pay for a constant-wage scheme for periods t and $t + 1$ is independent of θ_t. Hence, a firm's expected discounted benefit from posting a vacancy at period t is equal to $\Pi(\theta_t)$ plus a constant. As in Section 3.2, assume the matching function is Cobb-Douglas. Lemma 1 implies that the ratio $\eta(\theta')/\eta(\theta)$ for $\theta' < \theta$ - and hence, volatility of market tightness - is *lower* than in the benchmark.[9]

4.3 Persistent Effects of Productivity Shocks

In our model, the SPE outcome at any period t is a function of the extended state (θ_{t-1}, θ_t) - or, equivalently, $(\theta_t, \varepsilon_t)$. Longer lags have no effect; and certain aspects of the equilibrium (new hires' wage, market tightness) are exclusively a function of θ_t. These are artefacts of two features of our model: (i) the stochastic process governing productivity is log-linear AR(1); (ii) the reference point at t is a function of workers' expectations at $t - 1$. In this sub-section we discuss how relaxing these assumptions

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[9]In a recent paper, Rudanko (2011) assumes that the employer is also risk-averse but has better access to capital markets than the employee. She then shows that the equilibrium generates higher tightness volatility compared to a benchmark in which employees can use the capital market to smooth their consumption. Recall that our model abstracts from consumption, thus implicitly assuming that workers spend their wage earnings instantaneously.

may lead to longer-lasting effects of productivity shocks.

Suppose that the process governing productivity is $\theta_t = \Psi(\theta_{t-1}, ..., \theta_{t-K}) \cdot \varepsilon_t$, where $K > 1$ is an integer, Ψ is some deterministic function, and ε_t is $i.i.d$ according to $F[1/\xi, \xi]$, as in our basic model. For simplicity, let us focus on the limit case $G(b) \to 1$. It can be shown that if ξ is sufficiently close to one, SPE characterization is essentially the same as in the basic model. In particular, the characterization of ϕ is the same, such that existing workers at period t earn $\xi b \Psi(\theta_{t-1}, ..., \theta_{t-K})$ and are retained as long as $\varepsilon_t > \xi b$. Existing workers' wage at t and market tightness are exclusively a funciton of θ_t. Lagged realizations of θ have no effect on the equilibrium outcome, once we control for θ_t, ε_t, as in the basic model.

Thus, it would appear that our model cannot generate long-lasting effects of shocks, even under more general productivity processes. However, this is a result of the "purely multiplicative" specification of our model: the process governing θ_t is log-linear, and the workers' output and outside option at t are proportional to θ_t. This specification was motivated by tractability and (especially in the case of the workers' outside option) other methodological considerations. It is not essential for the broad features of our analysis in Section 3, yet, as we saw, it rules out long-lasting effects of shocks.

To illustrate this point, suppose $\theta_t = \Psi(\theta_{t-1}, ..., \theta_{t-K}) + \varepsilon_t$, where Ψ is some deterministic function and ε_t is $i.i.d$ according some symmetric density over $[-\xi, \xi]$. Continue to assume $G(b) \to 1$. It can be shown that if ξ is small enough, there is a unique SPE outcome, which has the following properties. Existing workers are retained at t as long as $\varepsilon_t \geq b - (1-b)\Psi(\theta_{t-1}, ..., \theta_{t-K})$, and paid $b[\Psi(\theta_{t-1}, ..., \theta_{t-K}) + \xi]$. Note that this means layoff rates at t are sensitive to θ_{t-K}, even if we control for $\theta_t, ..., \theta_{t-K+1}$. In this sense, K-lagged productivity shocks have an effect of current inflows into unemployment, even after controlling for current productivity. Newly hired workers' equilibrium wage at t depends on the period-t outside option $b\theta_t$, as well as on the period-$(t+1)$ threshold, $b - (1-b)\Psi(\theta_t, ..., \theta_{t+1-K})$, below which the worker will be fired. Hence, past productivity shocks will also have a persistent effect on the equilibrium wage of new hires. Recall that tightness at t is determined by the value of a created vacancy conditional on filling it at the beginning of $t+1$. Since layoff rates at $t+2$ depend on $(\theta_{t+1}, ..., \theta_{t-K+1})$, it follows that tightness at t is a function of $(\theta_t, ..., \theta_{t-K+1})$. Thus, when we depart from the purely multiplicative specification of our model, long-lasting effects of productivity shocks emerge.

Long-lasting effects of productivity shocks can also be generated by assuming instead that existing workers' reference wage at t is defined as the expected wage conditional on the history at the end of period $t - K$. The treatment of existing workers will

then be sensitive to shocks at period $t - K$, even when we control for $\theta_t, ..., \theta_{t-K+1}$.

5   Concluding Remarks

Our objective in this paper was to formalize the idea that morale considerations affect the labor market's response to macroeconomic fluctuations, in the context of an S&M model. In our model, as in Akerlof (1982), workers' morale, and consequently their productivity are damaged when their wage falls below a reference point. Following Kőszegi and Rabin (2006), we assumed that existing workers' reference point is a function of their lagged wage expectations. The equilibrium predictions of the model are that existing workers' wages display downward rigidity with respect to macroeconomic shocks, while entry-level wages are lower and more flexible. The main open problem is to provide a complete characterization of SPE under general exogenous separation processes. Extending the model to other bargaining protocols is an additional interesting avenue for future research.

We believe that the model is capable of producing additional insights, some of which were made informally by Bewley (1999) on the basis of his survey. Here we make do with a brief description.

Part-time jobs. Suppose that a firm's hiring/retention decision is not binary, but any real number $r \in [0, 1]$, such that an interior r corresponds to a part-time job. Suppose further that wages are stated for full-time positions, such that an employed worker's total wage earnings are rw. It makes sense to assume that an existing worker's reference point will be based on w rather than on rw. This means that if a firm moves its worker from full- to part-time employment without cutting w below its lagged-expected value, this will not be construed as unfair behavior, and the worker will produce normal output. It follows that after a bad productivity shock, a firm may prefer this option to the alternative of keeping the worker at full-time employment while lowering his wage, even in circumstances where this would have been sub-optimal in a reference-independent model.

The role of inflation. Discussions of wage rigidity often involve a distinction between real and nominal wages and the mitigating role of inflation. In a model with reference dependence, this distinction is traced to an assumption as to whether the reference point is formed in nominal or real terms. If the reference point is stated in terms of (lagged-expected) nominal wages, it should come as no surprise that unexpected inflation can have real (yet temporary) effects on the labor market, by lowering the reference

point in real terms, thereby making the MH constraint less likely to be binding.

We would like to conclude the paper with a discussion of alternative reference-point formation rules. In Appendix B, we examine a close variation on our model, in which the reference point of workers of *any* type is equal to their lagged-expected wage earnings, thus endogenizing the distinction between newly matched and existing workers. The main qualitative results of our model are reproduced.

Another variant would abandon the lagged-expectation component, and assume that an existing worker's reference point at period t is equal to his *actual* wage at period $t-1$. This alternative formulation could also generate persistent effects of productivity shocks: In a model with a long-horizon separation process, the treatment of existing workers would be sensitive to the value of θ when they were originally hired by the firm, even after controlling for the values of θ ever since.

The reference point that conditions the worker's effort decision could be a function of variables other than the worker's own (expected) wage. For instance, it could be the wage earned by his peers. Alternatively, the reference point could represent a fair share of his output. In fact, we have analyzed such a model, under the assumption that the worker considers receiving a fraction $\beta < b$ of lagged-expected output to be fair. The main results are qualitatively the same as the ones presented here.

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Appendix A: Proofs

Let us first introduce some notation that will serve us in several proofs. Fix an SPE.

Unemployed workers' payoff. Recall that for a given firm-worker pair, the only observable aspect of the history prior to their match is the sequence of realizations of θ. In particular, it does not matter whether the worker's unemployment at t is due to a matching failure, a firm's decision not to hire him, or his own decision to reject a wage offer. Therefore, we can denote an unemployed worker's equilibrium continuation payoff at t by $W_0(\theta_0, ..., \theta_t)$, without loss of generality.

Employed workers' payoff. Let h_t be the information set of a given firm-worker matched pair at period t, where the worker is of type i at t. Let (h_t, w_t) denote the immediate concatenation in which the firm hires/retains the worker and makes the offer w_t. Let $W_i(h_t, w_t)$ denote the worker's equilibrium continuation payoff at (h_t, w_t), where the subscript i clarifies the worker's type at t. Let $W_0(h_t)$ denote his reservation payoff at h_t, namely the continuation payoff if he rejects the wage offer that the firm makes and thus becomes unemployed at t. By definition, $W_i(h_t, w_t) \geq W_0(h_t)$.

Employed workers' rent. We define two types of rents. First, let $R_i(h_t, w_t) = W_i(h_t, w_t) - W_0(h_t)$ be the difference between the worker's equilibrium continuation payoff at (h_t, w_t) and his reservation payoff at this history. Second, let $B(\theta)$ denote a worker's continuation payoff from the strategy of rejecting all wage offers when the current state is θ, and define $Q_i(h_t, w_t) = W_i(h_t, w_t) - B(\theta_t)$. By revealed preferences, $Q_i(h_t, w_t) \geq R_i(h_t, w_t) \geq 0$. In addition, $Q(\cdot)$ is bounded from above because firms will never make offers that generate negative profits.

Proof of Proposition 1

Consider some SPE of the game. Define Q^* as the maximum of $Q_i(h, w)$ over all histories (h, w) and agent types i in this SPE. In general, the maximum need not be well-defined, and complete rigor demands it to be replaced with the sup. However, this would complicate our analysis in a way we find superfluous. Thus, to simplify exposition, we deal with the case in which Q^* is well-defined and attained in some finite history (h_t^*, w_t^*) by a worker of some type i^*.

If $Q^* = 0$ we are done, and so assume that $Q^* > 0$. Note that $Q^* = w_t^* - b\theta_t + \delta Q_{i^*}(h_{t+1}, w_{t+1})$. Suppose that $w_t^* = 0$ and i^* accepts the wage offer so that the non-negativity constraint is binding at (h_t^*, w_t^*). Since $w_t^* - b\theta_t < 0$ and $Q^* > 0$ we have that $Q_{i^*}(h_{t+1}, w_{t+1}) > Q^*$, a contradiction. Thus, the non-negativity constraint of a wage offer to worker i^* must hold with slack at (h_t^*, w_t^*). Similarly, it cannot be the case that worker i^* rejects the wage offer w_t^*. It follows that the IR constraint of i^*'s contract is binding at (h_t^*, w_t^*) - otherwise, the firm can slightly lower the worker's wage without changing his subsequent behavior.

By the definition of Q^*, $W_{i^*}(h_t^*, w_t^*) \geq W_1(h_t^*, w_t^*)$ and

$$W_1(h_t^*, \theta_{t+1}, w_{t+1}) - B(\theta_{t+1}) \leq W_{i^*}(h_t^*, w_t^*) - B(\theta_t) \tag{13}$$

for any realization of θ_{t+1} and a wage offer w_{t+1} made to a newly matched worker at

$t + 1$. Observe that

$$W_0(h_t^*) = b\theta_t + \delta[q_t \cdot \mathbb{E}W_1((h_t^*, \theta_{t+1}, w_{t+1}) \mid \theta_t) + (1 - q_t) \cdot \mathbb{E}W_0((h_t^*, \theta_{t+1}) \mid \theta_t)] \quad (14)$$

where q_t is the probability that an unemployed worker at t finds a match. The determinants of q_t are immaterial for our purposes. Since $W_0(h_t^*, \theta_{t+1}) \leq W_1(h_t^*, \theta_{t+1}, w_{t+1})$, we obtain from (14) that

$$W_0(h_t^*) \leq b\theta_t + \delta \mathbb{E}(W_1(h_t^*, \theta_{t+1}, w_{t+1}) \mid \theta_t)$$

Since the IR constraint of i^*'s contract is binding at (h_t^*, w_t^*), $W_{i^*}(h_t^*, w_t^*) = W_0(h_t^*)$. Using (13) we may therefore conclude that

$$W_{i^*}(h_t^*, w_t^*) = W_0(h_t^*) \leq b\theta_t + \delta W_{i^*}(h_t^*, w_t^*) + \delta \mathbb{E}B(\theta_{t+1} \mid \theta_t) - \delta B(\theta_t)$$

Since $b\theta_t + \delta \mathbb{E}B(\theta_{t+1} \mid \theta_t) = B(\theta_t)$, we have $W_{i^*}(h_t^*, w_t^*) \leq B(\theta_t)$, hence $Q^* = 0$.

By the definition of Q^*, it follows that for any worker type i and any (h_t, w_t) along the equilibrium path, $W_i(h_t, w_t) = B(\theta_t)$. Thus, if the worker accepts the wage offer, we have

$$W_i(h_t) = w_t + \delta \mathbb{E}(W_{i+1}(h_t, \theta_{t+1}, w_{t+1}) \mid \theta_t) = b\theta_t + \delta \mathbb{E}B(\theta_{t+1} \mid \theta_t)$$

and this implies $w_t = b\theta_t$. Finally, there cannot be a SPE in which a worker rejects an offer of $b\theta_t$ at some period t because the firm could profitably deviate by slightly raising the wage.

Proof of Proposition 2

We first prove a pair of lemmas that will serve us in several proofs. In particular, they hold for any γ. Define Q^{**} as the maximum of $Q(h, w)$ over all histories (h, w) in which a *newly matched worker* responds to a wage offer.

Lemma 2 *Let (h_t, w_t) be a history in which a newly matched worker responds to a wage offer, for which $Q(h_t, w_t) = Q^{**}$. If the IR constraint is binding at (h_t, w_t), then $Q^{**} = 0$.*

Proof. By the definition of Q^*, $W_1(h_t, \theta_{t+1}, w_{t+1}) - B(\theta_{t+1}) \leq W_1(h_t, w_t) - B(\theta_t)$. The proof that $Q^{**} = 0$ reproduces exactly the same steps that led us to conclude that $Q^* = 0$ in the proof of Proposition 1. (Note that here we simply assume that IR is binding at (h_t, w_t), rather than deriving this property.) ∎

Let $\bar{w}_i^t$ denote the participation wage of a worker of tenure $i = 1, 2$ at period t (implicitly, given the history) - that is, the highest wage offer they will accept given that all agents conform to their equilibrium continuation strategies. The next lemma shows that a worker's equilibrium wage at any history cannot exceed the highest participation wage he could get given the previous-period history.

Lemma 3 *In SPE,* $w_{i,t} \leq \max \bar{w}_i^t \mid h_{t-1}$.

Proof. Assume the contrary, i.e. $e_{2,t} > \max \bar{w}_i^t \mid h_{t-1}$. Since $e_{2,t}$ is a weighted average of $e_{2,t}$ and realizations of $\bar{w}_i^t$ that are feasible given h_{t-1}, it is equal to $e_{2,t}$ only if the firm pays $w_{2,t} = e_{2,t}$ with probability one, conditional on retaining the worker. However, since $e_{2,t} > \bar{w}_i^t$ with probability one, there exists a value of γ_t sufficiently close to one, such that for any h_t, $\theta_t - e_{2,t} < \gamma_t \theta_t - \bar{w}_i^t$, in which case the firm can profitably deviate from $w_{2,t} = e_{2,t}$ to $\bar{w}_i^t$. By assumption, such realizations of γ_t occur with positive probability, a contradiction. ∎

Lemma 4 *In SPE,* $w_{1,t} > 0$ *at any period* t.

Proof. Recall that W_0^t is independent of the worker's type at t, and that R_i^t is the rent (i.e., excess payoff above his reservation payoff) that a worker of type i gets at period t. If the worker is unemployed at t, we write $R_i^t = 0$. The following equations hold, by the definition of these objects:

$$\bar{w}_2^t + \delta \mathbb{E}(W_0^{t+1} \mid h_t) = W_0^t$$
$$\bar{w}_1^t + \delta \mathbb{E}(W_0^{t+1} \mid h_t) + \delta \mathbb{E}(R_2^{t+1} \mid h_t) = W_0^t$$

Therefore,

$$\bar{w}_1^t = \bar{w}_2^t - \delta \mathbb{E}(R_2^{t+1} \mid h_t) \tag{15}$$

Moreover, since

$$W_0^t = b\theta_t + \delta \mathbb{E}(W_0^{t+1} \mid h_t) + \delta q_t \mathbb{E}(R_1^{t+1} \mid h_t)$$

we obtain

$$\bar{w}_2^t = b\theta_t + \delta q_t \mathbb{E}(R_1^{t+1} \mid h_t) \tag{16}$$
$$\bar{w}_1^t = b\theta_t + \delta q_t \mathbb{E}(R_1^{t+1} \mid h_t) - \delta \mathbb{E}(R_2^{t+1} \mid h_t) \tag{17}$$

If the IR constraint of a wage offer to a newly matched worker is binding at t, then his period-t wage is equal to his period t reservation wage and $R_1^t = 0$. If his MH

constraint is binding at t, then the actual wage at t is zero, and $R_1^t = -\bar{w}_1^t$. If $\bar{w}_1^t < 0$ ($\bar{w}_1^t > 0$), then the MH (IR) constraint is binding. Therefore, $R_1^t = \max\{0, -\bar{w}_1^t\}$.

Let R^* and R_* denote the maximum and minimum values that R_1^t can attain at any t. By definition, $R_* \geq 0$. Assume that $R^* > 0$. Let w_* denote the minimum value that $\bar{w}_1^t$ may obtain at any t. Then $R^* = -w_*$, where $w_* < 0$. From (17) it follows that

$$\bar{w}_1^t = b\theta_t + \delta q_t \mathbb{E}(R_1^{t+1} \mid h_t) - \delta \mathbb{E}(R_2^{t+1} \mid h_t)$$

Observe that $R_2^{t+1} \mid \theta_t = w_{2,t+1} - \bar{w}_2^{t+1}$. By Lemma 3, $w_{2,t+1} \leq \max \bar{w}_2^{t+1} \mid h_t$. Therefore, $\delta \mathbb{E}(R_2^{t+1} \mid h_t)$ is smaller or equal to the sum

$$\delta[\max_{h_{t+1}|h_t} (b\theta_{t+1} + \delta q_{t+1} \mathbb{E}(R_1^{t+2} \mid h_{t+1})) - \mathbb{E}(b\theta_{t+1} + \delta q_{t+1} \mathbb{E}(R_1^{t+2} \mid h_{t+1}) \mid h_t)]$$

which in turn is lower or equal to

$$\delta b(\theta_t^\rho \xi - \theta_t^\rho \mathbb{E}(\varepsilon)) + \delta^2 \max_{h_{t+1}|h_t} [q_{t+1} \mathbb{E}(R_1^{t+2} \mid h_{t+1})] - \delta^2 \mathbb{E}[q_{t+1} \mathbb{E}(R_1^{t+2} \mid h_{t+1}) \mid h_t]$$

Note that

$$\max_{h_{t+1}|h_t} [q_{t+1} \mathbb{E}(R_1^{t+2} \mid h_{t+1})] \leq R^*$$
$$\mathbb{E}[q_{t+1} \mathbb{E}(R_1^{t+2} \mid h_{t+1}) \mid h_t] \geq 0$$
$$q_t \mathbb{E}(R_1^{t+1} \mid h_t) \geq 0$$
$$\theta_t^\rho \xi - \theta_t^\rho \mathbb{E}(\varepsilon) \leq \theta_t^\rho (\xi - 1)$$

Hence, for any t,

$$\bar{w}_1^t \geq b[\theta_t - \delta \theta_t^\rho (\xi - 1)] - \delta^2 R^* \tag{18}$$

Since $R^* = -w^*$, inequality (18) holds for every t only if it holds at the lowest possible value of $\bar{w}_1^t$, i.e., only if

$$w_* \geq b[\theta_t - \delta \theta_t^\rho (\xi - 1)] - \delta^2(-w_*)$$

which implies

$$w_* \geq \frac{b[\theta_t - \delta \theta_t^\rho (\xi - 1)]}{1 - \delta^2}$$

Recall that θ_t follows a log-linear AR(1) process where shocks take values in $[\frac{1}{\xi}, \xi]$. A simple calculation shows that since $\xi < \frac{1}{2}(1 + \sqrt{5})$, the numerator of the R.H.S is

strictly positive. But this contradicts our assumption that $w_* < 0$. It follows that $R^* = 0$, and this establishes the result. ∎

The rest of the proof proceeds in two steps. First, we use the above lemmas to derive the retention decision, reference point and equilibrium wages for existing workers. Second, we compute the hiring decision and equilibrium wages for newly matched workers. Since by assumption $e_{1,t} = 0$, Lemma 4 implies that the MH constraint of a wage offer to newly matched workers holds with slack after every history. Therefore, their IR constraint must be binding after every history.

Step 1: Existing workers

Let us first show that an existing worker at period t will accept a wage offer $w_{2,t}$ if and only if $w_{2,t} \geq b\theta_t$. This is his last period of employment. If he rejects the firm's offer, he will be unemployed and earn a payoff of $b\theta_t$ at t. We have seen that newly matched workers' IR is binding after every history. By Lemma 2, it follows that the worker's equilibrium continuation payoff from period $t+1$ onwards is the same as if he were to receive $b\theta_s$ in every period $s \geq t+1$. Therefore, the existing worker's participation constraint at t will be binding if he receives a payoff of $b\theta_t$.

It follows that if $b\theta_t \geq e_{2,t}$, the firm will choose $r_t = 1$ and $w_t = b\theta_t$ in equilibrium. Let us turn to the case of $b\theta_t < e_{2,t}$. Conditional on playing $r_t = 1$, the firm will offer $w_t \in \{e_{2,t}, b\theta_t\}$ because IR or MH are binding. Retaining the worker at $w_t = b\theta_t$ generates a profit of $\pi = \gamma_t \theta_t - b\theta_t$. If $\gamma_t < b$ ($\gamma_t > b$), $\pi < 0$ ($\pi > 0$); and since this is the last period of the worker's employment, the firm will choose $r_t = 0$ ($r_t = 1$). It follows that when $\gamma_t < b$, the firm will play $r_t = 0$ if $\theta_t - e_{2,t} < 0$ and $r_t = 1$, $w_t = e_{2,t}$ if $\theta_t - e_{2,t} > 0$. And when $\gamma_t > b$, the firm will play $r_t = 1$, and $w_t = e_{2,t}$ ($w_t = b\theta_t$) if $\theta_t - e_{2,t} > \gamma_t \theta_t - b\theta_t$ ($\theta_t - e_{2,t} < \gamma_t \theta_t - b\theta_t$).

We are now able to provide an expression for existing workers' reference wage at period t, which is equal to their expected wage conditional on being retained, according to their information at the end of period $t-1$. We use the abbreviated notation $e = e_{2,t}$, $\theta = \theta_{t-1}$:

$$e = \frac{G(b) \int_{\varepsilon > \varepsilon^*} \max\{e, b\theta^\rho \varepsilon\} dF(\varepsilon) + \int_b^1 \left[\int_{\varepsilon < \varepsilon^{**}} b\theta^\rho \varepsilon dF(\varepsilon) + \int_{\varepsilon > \varepsilon^{**}} \max\{e, b\theta^\rho \varepsilon\} dF(\varepsilon) \right] dG(\gamma)}{G(b)(1 - F(\varepsilon^*)) + 1 - G(b)}$$

(19)

where the productivity shock cutoffs ε^* and ε^{**} are given as follows:

$$\theta^\rho \varepsilon^* = e$$
$$\theta^\rho \varepsilon^{**} - e = \gamma \theta^\rho \varepsilon^{**} - b\theta^\rho \varepsilon^{**}$$

It is clear from (19) that $e_{2,t} \geq b\theta_{t-1}^{\rho}\mathbb{E}(\varepsilon)$. By Lemma 3, $e_{2,t} \leq b\xi\theta_{t-1}^{\rho}$, namely the highest outside option that is feasible at period t given θ_{t-1}. Our task now is to establish that the equation (19) has a unique solution e in the interval $[b\theta_{t-1}^{\rho}\mathbb{E}(\varepsilon), b\xi\theta_{t-1}^{\rho}]$. Rewrite the equation as $eB(e) - A(e) = 0$, where the functions A and B are the numerator and denominator of the R.H.S of (19), respectively. The L.H.S of this equation is a continuous function of e. Moreover, it is negative for $e = 0$ and positive for $e > b\xi\theta_{t-1}^{\rho}$. Differentiating w.r.t e, we obtain $[eB(e) - A(e)]' > 0$ for all e in the relevant domain. Therefore, (19) has a unique solution. Let us guess that the solution has the form $\phi \cdot b\theta_{t-1}^{\rho}$, where ϕ is a constant that is a function of F, G and b. Plugging this expression into (19) and simplifying, we obtain (4)-(5). In particular, $\varepsilon^* = \varepsilon^*(\gamma)$ for $\gamma < b$, and $\varepsilon^{**} = \varepsilon^*(\gamma)$ for $\gamma > b$. This system has a solution, by the same reasoning that ensured a solution for (19). Therefore, this solution gives us the unique solution for (19). We have thus fully characterized the equilibrium retention and wage policies for existing workers.

Step 2: Newly matched workers

A newly matched worker at period t expects to earn the discounted sum of payoffs in periods t and $t+1$:

$$w_{1,t} + \delta\mathbb{E}[r_{2,t+1}w_{2,t+1} + (1 - r_{2,t+1})b\theta_{t+1} \mid \theta_t] \tag{20}$$

We have already noted that a new worker's SPE continuation payoff is as if he receives $b\theta_t$ in every period t. Hence, in any SPE, the expected, discounted sum in (20) must equal $b\theta_t + \delta\mathbb{E}(b\theta_{t+1} \mid \theta_t)$. Expression (7) for $w_{1,t}$ thus follows from our characterization of $r_{2,t+1}$ and $w_{2,t+1}$. To see why $r_{1,t} = 1$ regardless of the history, note that in the second period of the interaction between the firm and the worker, the firm necessarily earns non-negative profits. The newly matched worker at t produces the normal output θ_t because as we saw, his MH constraint holds (with slack). Since he is paid at most $b\theta_t$, the firm earns strictly positive profits, and therefore would always prefer to hire the worker.

Appendix B: Endogenous Distinction between Worker Types

So far, we assumed a reference-point formation rule that imposed an exogenous distinction between newly matched and existing workers. One could argue that there are endogenous reasons for such a distinction. In particular, they have different employment prospects: the probability that a newly hired worker at t is employed at $t+1$

is a function of his employer's equilibrium retention policy, while the probability that an unemployed worker at t is employed at $t + 1$ is a function of market tightness at t and firms' hiring policy.

In this appendix, we modify the reference-point formation rule in order to capture this consideration and endogenize the distinction between the reference points of workers of different types. For simplicity, we restrict attention to the two-period exogenous separation process. Assume that at any period t and for any worker type $i = 1, 2$, the worker's reference point is equal to his *expected wage earnings* conditional on his information at the end of period $t - 1$. Specifically, the period-t reference points for newly matched and existing workers are

$$
\begin{aligned}
e_{1,t} &= q_{t-1} \cdot \mathbb{E}(r_{1,t} w_{1,t}) \\
e_{2,t} &= \mathbb{E}(r_{2,t} w_{2,t})
\end{aligned}
$$

where the expectation over $r_{i,t}, w_{i,t}$ is conditional on the worker's information set at the end of $t - 1$.

This reference point formation rule puts newly matched and existing workers on the same footing a priori. However, their different employment prospects translate into different reference points. In particular, if an unemployed worker at period $t - 1$ faces a low match probability q_{t-1}, his reference wage if matched at the beginning of t is close to zero.

Our main result in this appendix is that when the matching friction is sufficiently high and the magnitude of the business cycle is not too large, there exists an SPE that mimics the qualitative features of the unique SPE obtained in Section 3.

Proposition 3 *If*
$$
m(1, 1) < \min\left\{ \frac{c}{\Pi^0\left(\sqrt[1-\rho]{\xi}\right)}, 1 - \xi + \frac{1}{\xi} \right\} \tag{21}
$$
the game has a SPE with the following properties.

(i) An existing worker's period-t reference point $e_{2,t}$ is

$$
e_{2,t} = \phi \cdot b\theta_{t-1}^{\rho}
$$

where ϕ is uniquely given by

$$\phi = \int_0^1 \int_{\varepsilon > \varepsilon^*(\gamma)} \max\{\phi, \varepsilon\} dF(\varepsilon) dG(\gamma) + \int_b^1 \int_{\varepsilon < \varepsilon^*(\gamma)} \varepsilon dF(\varepsilon) dG(\gamma)$$

$$\varepsilon^*(\gamma) = \frac{\phi b}{1 - \max\{0, \gamma - b\}}$$

(ii) An existing worker is dismissed at period t if and only if $\gamma_t < b$ and $\varepsilon_t < \phi b$. Conditional on being retained at t, his wage is

$$w_2(\theta_{t-1}, \theta_t) = \begin{cases} \max\{e_{2,t}, b\theta_t\} & if & \varepsilon_t > \varepsilon^*(\gamma_t) \\ b\theta_t & if & \gamma_t > b \ and \ \varepsilon_t < \varepsilon^*(\gamma_t) \end{cases} \tag{22}$$

(iii) A newly matched worker at period t is always hired; his wage at period t is

$$w_1(\theta_t) = b \left[\theta_t - \delta\theta_t^\rho \int_0^1 \int_{\varepsilon^*(\gamma)}^\phi (\phi - \varepsilon) dF(\varepsilon) dG(\gamma) \right] \tag{23}$$

Proof. Our method of proof is as follows. First, we construct a unique SPE under the assumption that $\bar{w}_{1,t} > e_{1,t}$ at any period t, regardless of the history - that is, newly matched workers' participation wage exceeds their reference wage. Then, we show that this assumption holds under (21). Many of the steps in the proof have analogues in the proof of Proposition 2, and are therefore described briefly.

Step 1: Existing workers

By assumption, newly matched workers' IR constraint is binding after every history in equilibrium. Therefore, existing workers' participation wage at any period t is exactly the same as in the basic model, namely $b\theta_t$. For a given reference wage $e_{2,t}$, the firm's retention and wage policy in SPE is the same as in the basic model. Specifically, when $\gamma_t < b$, the firm will retain an existing worker at period t if and only if $\theta_t \geq e_{2,t}$, and pay $w_{2,t} = \max\{e_{2,t}, b\theta_t\}$ conditional on retention. And if $\gamma_t > b$, the firm will always retain an existing worker, and pay him $w_{2,t} = e_{2,t}$ when $\theta_t - \gamma_t\theta_t \geq e_{2,t} - b\theta_t \geq 0$, and $w_{2,t} = b\theta_t$ otherwise. Therefore, an existing worker's reference wage at period t is given by the following equation:

$$e = G(b) \int_{\varepsilon > \varepsilon^*} \max\{e, b\theta^\rho\varepsilon\} dF(\varepsilon) + \int_b^1 \left[\int_{\varepsilon < \varepsilon^{**}} b\theta^\rho\varepsilon dF(\varepsilon) + \int_{\varepsilon > \varepsilon^{**}} \max\{e, b\theta^\rho\varepsilon\} dF(\varepsilon) \right] dG(\gamma)$$

where the productivity shock cutoffs ε^* and ε^{**} are given as follows:

$$\theta^\rho \varepsilon^* = e$$

$$\theta^\rho \varepsilon^{**} - e = \gamma \theta^\rho \varepsilon^{**} - b\theta^\rho \varepsilon^{**}$$

To establish existence of a solution to this equation, note first that the R.H.S is a continuous function of $e_{2,t}$. Second, the R.H.S cannot take values above $b\xi\theta^\rho$, hence we can view the R.H.S as a continuous mapping from $[0, b\xi\theta^\rho]$ to itself. By Brouwer's fixed-point theorem, this mapping has a fixed point. To see that this fixed point is unique, differentiate both sides of the equation w.r.t e. The derivative of the L.H.S w.r.t e is 1, while the derivative of the R.H.S w.r.t e is strictly below 1. Therefore, there can be at most one point in which the functions on the two sides of the equation intersect, hence precisely one fixed point.

Step 2. Newly matched workers

The derivation is exactly the same as in the basic model

Step 3: Verifying that newly matched workers' MH holds with slack

By the expression for newly matched workers' wage,

$$w_{1,t} > b[\theta_{t-1} - \theta_{t-1}^\rho(\xi - \mathbb{E}(\varepsilon))]$$

On the other hand, by the same expression and the definition of newly matched workers' reference point,

$$e_{1,t} \le q_{t-1} \cdot b\theta_{t-1}^\rho \mathbb{E}(\varepsilon)$$

In order to prove the result, it suffices to show that the lower bound on $w_{1,t}$ is always higher than the upper bound on $e_{1,t}$. A bit of algebra gives us the following sufficient condition (using the facts that $\mathbb{E}(\varepsilon) > 1$ and $\theta^{1-\rho} \ge \frac{1}{\xi}$):

$$q_{t-1} < 1 - \xi + \frac{1}{\xi}$$

The highest value that θ_t can get is $\sqrt[1-\rho]{\xi}$. By the free entry assumption, the following inequality holds in any equilibrium:

$$p_{t-1} \ge \frac{c}{\Pi^0(\sqrt[1-\rho]{\xi})}$$

By the assumption that m satisfies constant returns to scale, $p_{t-1} > m(1,1)$ if and only

if $q_{t-1} < m(1,1)$. Therefore, if $m(1,1)$ satisfies condition (21), newly matched workers' participation wage exceeds their reference wage after every history. ∎

Because the exact equation that describes the reference point for existing workers differs from its specification in the basic model, the SPE constructed here does not exactly replicate the SPE in the basic model. However, the qualitative features of firms' retention and wage policies for newly hired and existing workers are preserved. Note that the restriction on ξ required to obtain this result is more severe than in the basic model, but this difference vanishes as $m(1,1)$ gets closer to zero. Given the specification of ϕ, we are able to obtain the output constant λ just as in the basic model, and use it to replicate (qualitatively) the tightness volatility effect.

Appendix C: Reference-Dependent Worker Preferences

The model assumed that the workers' output function is reference-dependent. The justification for this assumption was that when a worker's wage falls below his reference point, his intrinsic motivation diminishes and therefore his productivity drops. The term "motivation" connotes a preference, and suggests that our model is a reduced form of a larger model, in which workers choose an effort level and their preferences are sensitive to the reference point. In this appendix we construct such a model, which can be viewed as a foundation for the reduced form, reference-dependent output function assumed in the main text.

The search, matching, separation and bargaining components, as well as the firms' preferences, are exactly the same as in the basic model. The only differences are in the description of workers' output and their preferences. Suppose that conditional on accepting an offer, an employed worker is committed to a minimal level of effort. On top of that, he chooses a level of discretionary effort $x_t \in \{0, 1\}$. We refer to $x = 1$ as "normal effort". This effort decision is not observed by the firm. The worker's output is $y_t = \theta_t[\gamma_t + (1 - \gamma_t)x_t]$. Under this formulation, γ_t is interpreted as an indicator of the *completeness of the labor contract*, such that $1 - \gamma_t$ captures the importance of discretionary effort in the output function.

Workers maximize expected discounted payoffs. Employed workers' payoff flow is modified as follows:

$$w_t - x_t \cdot \mathbf{1}[w_t < e_t] \tag{24}$$

whereas the basic model assumed only the first term. The interpretation is that when the worker's wage is below his reference point, he perceives this as unfair treatment; his intrinsic motivation is damaged, and he strictly prefers not to exert his normal effort.

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Otherwise, the worker is indifferent between $x = 0$ and $x = 1$, and we assume that he chooses the latter.

Given the assumption that the worker's discretionary effort is unobserved, the worker's choice of x is entirely myopic in any SPE. At any period t in which he accepts a wage offer w_t, he will play $x_t = 1$ if and only if $w_t \geq e_t$. As a result, the worker will respond to wage offers as if he maximizes the discounted sum of expected wage and non-market earnings, just as in the basic model. This is the reason that this larger model collapses to our basic model in equilibrium.

The value of recasting our model in terms of reference-dependent preferences is that it clarifies the interpretation of the random parameter γ_t. It also suggests new extensions and raises interpretational questions. We discuss some of them.

Positive vs. negative reciprocity. The preferences given by (24) capture what Fehr et al. (2009) call "negative reciprocity", but do not give room to "positive reciprocity" - namely, increased effort beyond the normal level following a wage offer that exceeds the reference point. This asymmetry reflects findings in the literature: "Whereas the positive effects of fair treatment on behavior are usually small, the negative impact of unfair behavior is often large" (Fehr et al. (2009, p. 366)). It is also in the spirit of Prospect Theory (Kahneman and Tversky (1979)): losses relative to the reference point loom significantly larger than gains.

The equilibrium concept. Because workers' preferences in this extended model depend on their expectations (both of the moves of Nature and of the players' strategies), this is not strictly speaking a conventional game, but rather an example of an extensive-form "psychological game" (after Geanakoplos et al. (1989)).[10] In general, extending standard game-theoretic solution concepts to this class of games may involve subtleties. However, in the present case, the standard concept of subgame perfect equilibrium (SPE) is defined and analyzed in a completely standard way, and we will follow this concept, which is appropriate for our setting.[11]

Contractual incompleteness. In our model, firms offer flat-wage contracts and do not observe workers' effort. The latter assumption may appear strange, because we assume that firms observe θ_t, γ_t, hence assuming that x_t is unobservable is tantamount to assuming that output is unobserved, which may seem odd. However, recall that

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[10]Rabin (1993) was the first to use the framework of psychological games to model reciprocity considerations.

[11]This is due to the fact that in our model, workers incorporate reciprocity considerations into their effort decision in a myopic way. Dufwenberg and Kirchsteiger (2004) and Battigalli and Dufwenberg (2009) develop tools to deal with more complicated dynamic settings, where reciprocity considerations may be sensitive to off-equilibrium events and higher-order beliefs.

although the model is presented in terms of one-to-one matching, this assumption is purely expositional and the entire analysis is valid for one-to-many matching where production is separable across vacancies. It is entirely realistic to assume that while the firm can only observe its aggregate output with some noise, it cannot monitor the contribution of any individual worker.

Even under this limited monitoring, one could argue that flat-wage contracts are too restrictive, and that firms could incentivize effort by conditioning the workers' compensation on the noisy signal, namely aggregate output. However, as the literature on moral hazard in teams has demonstrated (starting with Holmstrom (1982)), such incentives are limited in their ability to induce team effort. When these considerations are combined with reference-dependent worker preferences that exhibit loss aversion, the limitations are exacerbated and may lead firms to choose flat contracts (see Herweg et al. (2010), Herweg and Mierendorff (2011)).

Furthermore, incentivizing team performance may exacerbate morale problems for reasons that are not captured by our preference model, because it punishes individual workers for a drop in output which is due to chance or to other workers' effort decisions. Similar issues arise when the worker has multiple tasks and the firm can only monitor a subset of those (see Fehr et al. (2009)). Thus, morale considerations and limited monitoring of workers' effort complement each other in dissuading firms from elaborate incentive schemes toward flat-wage contracts, for reasons that can both be incorporated into our model and reasons that lie outside it.