Deposits and Bank Capital Structure[*]

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Abstract

In a model with bankruptcy costs and segmented deposit and equity markets, we endogenize the choice of bank and firm capital structure and the cost of equity and deposit finance. Despite risk neutrality, equity capital is more costly than deposits. When banks directly finance risky investments, they hold positive capital and diversify. When they make risky loans to firms, banks trade off the high cost of equity with the diversification benefits from a lower bankruptcy probability. When bankruptcy costs are high, banks use no capital and only lend to one sector. When these are low, banks hold capital and diversify.

JEL Codes: G21, G32, G33

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1 Introduction

There is a growing literature on the role of equity in bank capital structure focusing on equity as a buffer, liquidity, agency costs and various other frictions.[1] One important feature of these analyses is that they involve partial equilibrium models that do not consider the role of equity in non-financial firms and usually take the cost of equity capital as given. The standard assumption is that equity capital for banks is a more expensive form of financing than deposits.[2] However, there is no clear theoretical foundation for this assumption in this literature and many papers have questioned whether this is justified. Risky equity usually has a higher expected return than debt but, as in Modigliani and Miller (1958), this does not necessarily mean that it is more costly on a risk adjusted basis (e.g., Miller (1995), Brealey (2006), and Admati, DeMarzo, Hellwig and Pfleiderer (2010)).

We develop a general equilibrium model of bank and firm financing based on three main elements. First, banks differ from non-financial firms in that they raise funds using deposits. Second, the markets for deposits and equity are segmented. Third, banks and firms incur bankruptcy costs. Our aim is to determine the optimal bank and firm capital structures and the implications of these for the pricing of equity, deposits and loans. In this framework the main role of equity is to reduce bankrutpcy costs and we analyze its interaction with diversification, which is an alternative way to achieve this.

The use of deposits distinguishes the funding of banks from the corporate finance of other firms. While both banks and firms use equity and bonds, only banks use deposits. Although their role has varied over time, deposits remain an important source of funds for banks in all countries. Figure 1 shows deposits as a proportion of bank liabilities for a number of countries from 1990-2009. In all these countries deposits are the major form

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[1] See, e.g., Diamond and Rajan (2000), Hellmann, Murdock and Stiglitz (2000), Gale (2004), Repullo (2004), Morrison and White (2005), Allen, Carletti and Marquez (2011), Acharya, Mehran and Thakor (2012).

[2] See also Berger, Herring and Szego (1995) and the survey by Gorton and Winton (2003) for a discussion of this issue.

of bank finance. Deposits also play an important role in the aggregate funding structure of the economy, as shown in Figure 2 where the ratio between deposits and GDP in the period 1990-2009 is illustrated.

Despite its empirical importance, deposit finance has played a relatively small role in the theory of bank funding. It is usually simply treated as another form of debt. However, there is considerable evidence that the market for deposits is significantly segmented from other markets. While most people in developed countries have bank accounts, with the exception of the U.S., relatively few people own stocks, bonds or other types of financial assets either directly or indirectly (see, e.g., Guiso, Haliassos and Jappelli (2002) and Guiso and Sodini (2013)). In addition to deposits held by households, considerable amounts are held in this form by businesses. These amounts are held for transaction purposes and reserves. In most cases there are limited substitution possibilities with other assets, particularly equity.

The other important foundation of our analysis is the significance of bankruptcy costs. There is considerable empirical evidence that these are substantial for both banks and non-financial firms. For example, James (1991) finds that when banks are liquidated, bankruptcy costs are 30 cents on the dollar. In a sample of non-financial firms, Andrade and Kaplan (1998) and Korteweg (2010) find a range of 10-23 per cent for the ex post bankruptcy costs and 15-30% for firms in or near bankruptcy, respectively. There are a number of issues that arise with the measurement of bankruptcy costs that suggest they are in fact higher than these estimates (see., e.g., Almeida and Philippon (2007), Acharya, Bharath and Srinivasan (2007) and Glover (2012)).

In our model banks finance themselves with equity capital and deposits and invest in risky assets. The providers of equity capital can invest directly in the risky assets, while the providers of deposits only have a storage alternative opportunity with a return of one. For simplicity, both groups are risk neutral. There is a fixed supply of equity capital and deposits in the economy. We distinguish four cases described below that differ in terms of the assets that banks can invest in.

Several results hold in all versions of the model provided that there are positive bankruptcy costs. First, equity capital has a higher expected return than investing directly in the risky asset. This in turn has a higher expected return than deposits. This implies that equity providers do not invest in the risky asset directly. Second, for low expected returns of the risky asset, deposits yield the same as the storage opportunity and deposit providers invest in both so there is limited financial inclusion. For high expected returns of the risky asset, deposits yield more than one and deposit providers only invest in banks.

The Modigliani and Miller results, of course, do not hold and bank capital structure depends on the investment opportunities of the banks.

- Case I: Banks invest in a single non-publicly traded sector, or equivalently in a line of business with a risky income like market making, underwriting, proprietary trading or fees from advisory services such as mergers and acquisitions. In this case, the optimal capital structure involves banks having positive equity in their capital structure to reduce bankruptcy costs.

- Case II: Banks make loans to firms operating in a single publicly traded productive sector and thus have perfectly correlated returns. The equilibrium entails that banks hold zero capital while firms hold a positive amount. All equity capital is used by firms. They hold the same capital that was held by banks in Case I. When banks hold zero capital, they are conduits that transfer firm payments on loans to depositors and their bankruptcy is aligned with that of the firms. This arrangement is privately and socially optimal because banks can go bankrupt only when firms do, so it is best to use equity to minimize firm bankruptcy and avoid unnecessary costs.

- Case III: Banks invest directly in two non-publicly traded productive sectors with independent returns, or equivalently in two lines of business with independent risky incomes. Banks hold positive amounts of capital and always diversify by investing in both opportunities to minimize bankruptcy costs.

- Case IV: Banks can make loans to firms in up to two publicly traded sectors with

independent returns. There is a trade-off in that equity capital is more costly than deposit finance but allows better diversification in two-sector banks. The benefit of diversification is higher the lower are bankruptcy costs. Thus, for low bankruptcy costs, banks diversify and lend to both sectors and both banks and firms use equity capital. For high bankruptcy costs, the higher cost of equity capital dominates the benefit of diversification. Banks specialize in lending to one sector of publicly traded firms and use zero capital as in Case II. All equity capital is held by firms. For intermediate bankruptcy costs, both diversified and undiversified banks coexist in equilibrium.

The paper contributes to the literature in a number of ways. First, it provides a theoretical foundation for why equity capital is costly relative to deposits, which, as explained above, is currently lacking in the literature.

The second contribution of our paper is to provide a theory of when banks should diversify across risky assets directly or across loans to firms operating in separate sectors with independent returns. The paper is related to Shaffer (1994), Wagner (2010), Allen, Babus, Carletti (2012) and Ibragimov, Jaffee and Walden (2011). These papers find that diversification is good for each bank individually but it can lead to greater systemic risk as banks' investments become more similar. In contrast, here diversification is not always optimal either privately or socially for individual banks. Diversification requires the use of costly equity capital and this may outweigh its benefit when bankruptcy costs are high enough.

Third, the paper provides a theory of the industrial organization of the banking sector and how this relates to the productive sector. In particular, it shows the forces that lead to banks diversifying or specializing in particular industries or regions and when these different types of banks can coexist. Previous theories have used partial equilibrium approaches and focus on asymmetric information, agency costs or efficiency arguments as the driving forces behind the banking industry (see, e.g., Dell'Ariccia (2001) and the survey by Mester (2008)).

There are relatively few empirical studies of bank capital structure. Some recent examples are Flannery and Rangan (2008), Gropp and Heider (2010) and Mehran and Thakor (2011). Flannery and Rangan (2008) document how US banks' capital ratios varied in the last decade. Gropp and Heider (2010) find that the determinants of bank capital structure are similar to those for non-financial firms. Mehran and Thakor (2011) document a positive relation between bank value and capital in the cross section. Each bank chooses an optimal capital structure and those with higher capital also have higher value. Our general equilibrium framework has many possible relationships depending on which bank investment possibility is relevant. None of these studies is designed to consider the interrelationship between asset and liability structures that is the focus of our model.

The paper proceeds as follows. Section 2 develops the basic model. The equilibrium of this is considered in Section 3. Section 4 considers what happens when firms are publicly traded and compete with banks for capital. The role of diversification when there are two non-publicly traded sectors is considered in Section 5, and when there are two publicly traded sectors in Section 6. Finally, Section 7 contains concluding remarks. All proofs are in the appendix.

2 A model of bank capital structure with a single non-publicly traded productive sector

In this section we develop a simple one-period model of financial intermediation where banks raise external funds through deposits and capital, and invest in a risky technology. This can either be interpreted as investment in non-publicly traded productive firms or as investment in a risky line of business such as market making, underwriting, proprietary trading or fees from advisory services such as mergers and acquisitions.

The risky technology is such that for each unit invested at date 0 there is a stochastic return r at date 1 uniformly distributed on the support $[0, R]$, with $Er = \frac{R}{2} > 1$.

Since there are constant returns to scale we normalize the size of every bank to 1. Each

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bank finances itself with an amount of capital k_B and an amount of deposits $1 - k_B$. The bank has limited liability. There are two groups of risk neutral investors, shareholders and depositors. The former supply capital to banks. The opportunity cost of capital in the bank equity market is ρ. Shareholders have an endowment of 1 each and also have the outside option of investing directly in the risky technology so that $\rho \geq R/2$. The latter supply deposits. The promised per unit rate from the bank is r_D and the opportunity cost of deposits in the bank deposit market is u. Depositors have an endowment of 1 each and also have a storage option with return 1 for each unit invested so that $u \geq 1$. The two markets are segmented in the sense that depositors do not have access to the equity market. The idea is that they have high participation costs that makes them unwilling to enter the equity market. The capital providers on the other hand have zero participation costs. The total supply of capital is denoted K. The total supply of deposits is D. The ratio of the two is

$$\frac{K}{D} = \eta > 0. \tag{1}$$

Since banks invest in a risky technology, deposits are risky. The bank repays the promised rate r_D if $r \geq \overline{r}_B$, where

$$\overline{r}_B = r_D(1 - k_B), \tag{2}$$

and it goes bankrupt otherwise. When it goes bankrupt, the proceeds from liquidation are $h_B r$ with $h_B \in [0, 1]$ and these are distributed pro rata to depositors. The bankruptcy costs are thus $(1 - h_B)r$.

Each bank maximizes its expected profits, which are distributed to the shareholders. There is free entry so that the banking sector is competitive. Depositors maximize their expected utility and deposit in the bank(s) that give them the highest expected return.

3 The equilibrium with a single non-traded productive sector

In this section we analyze the equilibrium of the model. This requires the following:

1. Banks choose k_B and r_D to maximize expected profits.

2. Capital providers maximize expected utility.

3. Depositors maximize expected utility.

4. Banks make zero expected profits in equilibrium.

5. The equity market clears.

6. The deposit market clears.

We start by considering the individual bank's optimization problem:

$$\max_{k_B, r_D} E\Pi_B = \int_{\overline{r}_B}^{R} \left(r - r_D(1 - k_B)\right) \frac{1}{R} dr - \rho k_B \tag{3}$$

subject to

$$EU_D = \int_{0}^{\overline{r}_B} \frac{h_B r}{1 - k_B} \frac{1}{R} dr + \int_{\overline{r}_B}^{R} r_D \frac{1}{R} dr \geq u \tag{4}$$

$$E\Pi_B \geq 0 \tag{5}$$

$$0 \leq k_B \leq 1, \tag{6}$$

where $\overline{r}_B$ is as in (2). The bank chooses k_B and r_D to maximize its expected profit net of the cost of funds. The first term in (3) is what the bank obtains from the investment after paying $r_D(1 - k_B)$ to the depositors. This is positive only when $r > \overline{r}_B$ and it is distributed

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to the shareholders. When $r < \bar{r}_B$, the bank goes bankrupt and obtains nothing. The second term ρk_B is the shareholders' opportunity cost of providing capital. Constraint (4) requires that the expected utility of depositors is at least equal to their opportunity cost u. The first term is the payoff when the bank goes bankrupt and each depositor receives a pro rata share $\frac{h_B r}{1-k_B}$ of the liquidation proceeds. The second term represents the payoff depositors receive when the bank remains solvent. Constraint (5) is the requirement that the shareholders obtain their opportunity cost from providing capital to the bank. The last constraint (6) is a feasibility constraint on the amount of capital.

In equilibrium, since there is free entry into the banking market, each bank's expected profit must be zero. This means that ρ adjusts so that $E\Pi_B = 0$. Capital providers can either supply equity to the banks for a return of ρ or invest in their outside option for a return $R/2$. The sum of these two investments must be equal to K for the equity market to clear. Capital providers will invest in bank equity alone if $\rho > R/2$. They will invest both in bank equity and in the outside option if $\rho = R/2$. In other words,

$$N_B k_B \leq K, \tag{7}$$

where N_B represents the number of banks and (7) holds with an equality when $\rho > R/2$.

Similarly, depositors can either deposit their money in the banks for a promised return of r_D and an expected utility $EU = u \geq 1$, or use the storage option with a return of 1 and an expected utility $EU = 1$. The investments in deposits and in the storage option must sum to D for the deposit market to clear. The depositors will just deposit in banks and will not store if $u > 1$. They will both deposit and store if $u = 1$. It will be shown below that the form of the equilibrium depends on whether the constraint (4) binds with $u = 1$ or $u > 1$. In other words,

$$N_B(1 - k_B) \leq D,$$

where there is an equality when $u > 1$, and a strict inequality otherwise.

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3.1 Full bankruptcy costs in the banking sector $(h_B = 0)$

For simplicity, we assume $h_B = 0$ to start with. This corresponds to the case where the liquidation proceeds are zero and depositors obtain nothing in the case the bank goes bankrupt. We have the following result.

Proposition 1 *The unique equilibrium with $h_B = 0$ is as follows:*

i) For $R < \overline{R} = \frac{4(1+\eta)}{1+2\eta} < 4, k_B = \frac{4}{R} - 1 \in (0,1), r_D = \frac{R}{2}, \rho = \frac{2}{4-R} > \frac{R}{2}, E\Pi = 0, EU = u = 1, \overline{r}_B = R - 2, N_B k_B = K$ and $N_B(1 - k_B) < D$.

ii) For $R \geq \overline{R}, k_B = \frac{\eta}{1+\eta} \in (0,1), r_D = \frac{R}{2}, \rho = \frac{1+4\eta(1+\eta)}{4\eta(1+\eta)} \frac{R}{2} > \frac{R}{2}, u = \frac{1+2\eta}{2+2\eta} \frac{R}{2} \in [1, \frac{R}{2}), E\Pi = 0, EU = u > 1, \overline{r}_B = \frac{1}{(1+\eta)} \frac{R}{2}, N_B k_B = K$ and $N_B(1 - k_B) = D$.

The main result in the proposition is that $\rho > \frac{R}{2}$. Shareholders always obtain strictly more than their outside option. The reason for this is that equity allows bankruptcy costs to be reduced and its opportunity cost is bid up as a result. There is a trade-off in that equity is a relatively costly form of finance but has the advantage of reducing expected bankruptcy costs. There is then an optimal bank capital structure and each bank uses both capital and deposits to fund itself. The expected return on equity is strictly above the expected return on the risky technology because of market segmentation and bankruptcy costs. The bank can afford to pay $\rho > \frac{R}{2}$ for equity finance because the cost of deposit finance is $u < \frac{R}{2}$. If there was no market segmentation so that depositors could invest directly in equity, then ρ would be equal to $\frac{R}{2}$. As shown below, if there are no bankruptcy costs so that $h_B = 1$, equity has no value in reducing the bankruptcy costs so again $\rho = \frac{R}{2}$. Thus, both assumptions are necessary for the result. Since in equilibrium $\rho > \frac{R}{2}$, all the capital is absorbed in the banking sector and none is invested directly in the outside option.

The second result of the proposition is that, unlike capital, the opportunity cost of deposits u is not always bid up above the storage option. Deposit finance is cheaper than equity but introduces bankruptcy costs. The difference between the expected returns of the outside option of equity investors and the storage option of deposit providers is low when R is low. This means that deposits are not very attractive relative to equity given the

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bankruptcy costs they introduce. This is why for $R < \overline{R}$ deposits are only partly placed in the banking sector where they obtain $u = 1$, and the storage option is widely used. As R is increased, more deposits are used in the banking sector. At $R = \overline{R}$ all deposits are used there. For $R > \overline{R}$, the opportunity cost of deposits is bid up and $u > 1$. For $R = 2$ the proportion of deposit funds used in the banking sector, that is the degree of financial inclusion, is zero. As R increases to $\overline{R}$, the degree of financial inclusion increases to 1. It can be seen that $\partial \overline{R}/\partial \eta < 0$ so that full financial inclusion is reached at lower levels of R the greater is the amount of capital K in the economy for a given level of deposits D. This result on the relationship between financial inclusion and R holds in all cases below so we omit the explicit discussion on market clearing conditions in the following propositions.

A third result of the proposition concerns how the surplus generated from the banks' investments in the risky asset is split between the shareholders and the depositors. For $R < \overline{R}$, all the surplus is captured by the shareholders. As R increases in this interval, $\partial k_B/\partial R < 0$ and ρ rises. As the risky technology becomes more productive, it is increasingly profitable for banks to use deposits for funding. This makes capital more valuable because bankruptcy increases and ρ is bid up. For $R > \overline{R}$, all deposits are used and thus bank capital structure remains constant. As R increases beyond $\overline{R}$, the shareholders and depositors share the surplus with both u and ρ continuing to rise.

A fourth result of the proposition concerns how the variation in the ratio of total capital to total deposits, η, affects the equilibrium. For $\eta \to 0$, $\overline{R} \to 4$. In the first region deposits are abundant and this ensures that some depositors have to invest in their alternative opportunity so $u \to 1$. In the second region with $R > \overline{R}$, $k_B \to 0$, $\rho \to \infty$, and $u \to 1$. For $\eta \to \infty$, $\overline{R} \to 2$ and the first region in the proposition becomes empty. In the second region, as $\eta \to \infty$, $k_B \to 1$, $\rho \to \frac{R}{2}$ and $u \to \frac{R}{2}$. In other words, as capital becomes more abundant, banks use more and more equity finance, bankruptcy risk falls and both ρ and u tend to $\frac{R}{2}$.

3.2 Intermediate bankruptcy costs in the banking sector ($0 < h_B < 1$)

We now extend the basic model to include partial bankruptcy costs in the banking sector, i.e., $h_B \in (0,1)$, so that depositors obtain $\frac{h_B r}{1-k_B}$ when the bank goes bankrupt. We obtain the following result, which is similar in spirit to that in Proposition 1, but algebraically more complex.

Proposition 2 *The unique equilibrium with $h_B \in (0,1)$ is as follows:*

i) For $R \leq \overline{R} = \frac{2(1+\eta)-(1-h_B)\left(1-h_B+\sqrt{4\eta(1+\eta)+(1-h_B)^2}\right)}{(1+\eta)h_B}, k_B = \frac{(2-h_B R)(2(2-h_B)-R)}{2(1-h_B)^2 R} \in$

$(0,1), r_D = \frac{2(1-h_B)R}{2(2-h_B)-h_B R} < R, \rho = \frac{2-h_B R}{2(2-h_B)-R} > \frac{R}{2}, E\Pi = 0, EU = u = 1$ *and* $\overline{r}_B = \frac{R-2}{1-h_B}$.

ii) For $R > \overline{R}$, $k_B = \frac{\eta}{1+\eta} \in (0,1), r_D = \frac{2u(1-h_B)R}{2u(2-h_B)-h_B R} < R, \rho = \frac{u(2u-h_B R)}{2u(2-h_B)-R} >$

$\frac{R}{2}$, $E\Pi = 0, EU = u = \frac{2(1+\eta)-(1-h_B)\left(1-h_B-\sqrt{4\eta(1+\eta)+(1-h_B)^2}\right)}{2(1+\eta)(2-h_B)}\frac{R}{2} \in (1, \frac{R}{2})$ *and* $\overline{r}_B =$

$\frac{2u(1-h_B)R}{(1+\eta)(2u(2-h_B)-h_B R)}$.

The insights of Proposition 1 remain valid when $h_B \in (0,1)$. Capital is again costly in that $\rho > \frac{R}{2}$ and $u \geq 1$ depending on the value of R relative to $\overline{R}$. The surplus generated by the risky technology goes entirely to the shareholders for $R \leq \overline{R}$ while it is split among shareholders and depositors for $R > \overline{R}$. The abundancy of capital as measured by η affects the equilibrium in the same way as before.

The main difference from Proposition 1 is that banks' capital structure and the sharing of the surplus depend on the size of the bankruptcy proceeds as represented by h_B. For a given $R \leq \overline{R}$, the higher h_B the lower the amount of capital k_B at each bank and the higher the shareholders' return ρ. For a given $R > \overline{R}$, k_B remains constant as h_B increases, but both shareholders and depositors obtain higher returns ρ and u. The intuition is simple. As bankruptcy proceeds increase, capital becomes less necessary as a way to reduce bankruptcy costs and thus each bank uses less of it.

3.3 No bankruptcy costs in the banking sector ($h_B = 1$)

We next consider the case where there are no bankruptcy costs so that $h_B = 1$. The difference is that depositors receive the full return r when the bank goes bankrupt. This

leads to the following result.

Proposition 3 *With $h_B = 1$, there are multiple equilibria. In any of these, $\rho = \frac{R}{2}$, $E\Pi = 0$, and $EU = u = \frac{R}{2}$. Banks can choose any level of $k_B \in [0, 1]$ and for $k_B < 1$, $r_D = \frac{(1-\sqrt{k_B})}{1-k_B} R$.*

When $h_B = 1$, both capital and deposits have a return $\rho = u = \frac{R}{2}$. This implies that capital can now be invested either in banks or directly in the risky technology, while all deposits are placed in the banking sector. This means that there are multiple equilibria depending on the proportion of capital invested in banks versus directly. This mix does not affect the real allocation.

Since the equilibrium is unique with $0 \leq h_B < 1$ but there are multiple equilibria with $h_B = 1$, we next consider the limit equilibrium as $h_B \to 1$.

Proposition 4 *As $h_B \to 1$, the unique equilibrium has $\overline{R} \to 2$, $k_B \to \frac{\eta}{1+\eta}$, $r_D \to \frac{1+\eta}{1+\eta+\sqrt{\eta(1+\eta)}} R$, $\rho \to \frac{R}{2}$, $E\Pi = 0$, and $EU \to \frac{R}{2}$. This limit equilibrium is one of the equilibria in Proposition 3.*

This result shows that the multiplicity with $h_B = 1$ is not robust to small changes in h_B. As soon as there are bankruptcy costs of any magnitude, bank capital structure matters. Only when there are no bankruptcy costs, does a Modigliani-Miller type of result hold and bank capital structure is irrelevant.

4 A single publicly traded productive sector

So far we have assumed that banks invest directly in the risky technology. We now consider the case where a continuum of publicly traded firms in a productive sector hold the risky technology with return $r \sim U[0, R]$ as before. Since it is a single sector, firms' returns are perfectly correlated. We analyze the case of multiple sectors with independent returns below in Section 6.

Each firm requires 1 unit of funds and finances this with equity k_F and loans from banks of $1 - k_F$. The opportunity cost of the capital in the firm is $\rho \geq \frac{R}{2}$. The promised per unit loan rate on bank loans is r_L. The firm is solvent if $r \geq \overline{r}_F$, where

$$\overline{r}_F = r_L(1 - k_F). \tag{8}$$

If $r < \overline{r}_F$, the firm goes bankrupt and the liquidation proceeds $h_F r$, with $h_F \in [0, 1]$, are distributed pro-rata to the banks providing the $1 - k_F$ in loans.

Banks raise equity k_B and take deposits $1 - k_B$. They pay $\rho \geq \frac{R}{2}$ to the capital providers and r_D to depositors. Each bank lends a total of 1 unit to firms. If $r \geq \overline{r}_F$ firms are solvent and the bank obtains the per unit loan rate r_L. The bank is then also solvent and repays $r_D(1 - k_B)$ to its depositors. If $r < \overline{r}_F$, firms go bankrupt and banks receive $h_F r$ for each $1 - k_F$ loaned out so that each bank receives $\frac{h_F r}{1 - k_F}$ per unit loaned. If $\frac{h_F r}{1 - k_F} \geq r_D(1 - k_B)$ the bank remains solvent and pays each of its $1 - k_B$ depositors the promised repayment r_D, but if $\frac{h_F r}{1 - k_F} < r_D(1 - k_B)$ the bank will itself go bankrupt and each depositor obtains only $\frac{h_F h_B r}{(1 - k_F)(1 - k_B)}$. This implies that when the firm goes bankrupt the bank can either remain solvent for $\frac{r_D(1 - k_B)(1 - k_F)}{h_F} < r < \overline{r}_F$ or go bankrupt with it for $r < \overline{r}_F$. Formally, the bank goes bankrupt for any $r < \overline{r}_B$, where

$$\overline{r}_B = \min\left(\frac{r_D(1 - k_B)(1 - k_F)}{h_F}, \; \overline{r}_F\right). \tag{9}$$

For simplicity, it is assumed that banks have the ability to impose loan covenants that specify a firm's k_F and r_L.

All the rest of the model remains the same.

4.1 The equilibrium with a single publicly traded productive sector

In addition to conditions 1-6 in Section 3, the equilibrium requires that

      7. Banks choose k_F and r_L in addition to k_B and r_D to maximize their expected profits.

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8. Firms make zero expected profits in equilibrium.

9. The loan market clears.

As before, the equity and the deposit markets have to clear in equilibrium. Given the presence now of two sectors, the conditions for this to occur are slightly different. In particular, market clearing requires that

$$N_F k_F + N_B k_B \leq K \tag{10}$$

and

$$N_B(1 - k_B) \leq D, \tag{11}$$

where N_F and N_B are the number of firms and banks respectively. Conditions (10) and (11) require that the total capital used in the productive and the banking sectors does not exceed the available capital K, and that the total deposits in the banking sector do not exceed the total supply D in the economy. As in the case with the banking sector only, (10) and (11) hold with equality if $\rho > \frac{R}{2}$ and $u > 1$.

The loan market must clear so that

$$N_F(1 - k_F) = N_B. \tag{12}$$

This states that the total lending $N_F(1-k_F)$ needed by the firms equals the total resources available for lending at the N_B banks.

4.2 Bankruptcy costs in the banking and productive sectors ($0 \leq h_B, h_F < 1$)

We start by considering the case where there are bankruptcy costs in both sectors. In this case, each individual bank's maximization problem is now given by:

$$\max_{k_F, r_F, k_B, r_D} E\Pi_B = \int_{\overline{r}_B}^{\overline{r}_F} \left(\frac{h_F r}{1 - k_F} - r_D(1 - k_D) \right) \frac{1}{R} dr + \int_{\overline{r}_F}^{R} (r_L - r_D(1 - k_D)) \frac{1}{R} dr - \rho k_B \tag{13}$$

subject to

$$E\Pi_F = \int_{\overline{r}_F}^{R} (r - r_L(1 - k_F)) \frac{1}{R} dr - \rho k_F \geq 0 \tag{14}$$

$$EU_D = \int_0^{\overline{r}_B} \frac{h_B h_F r}{(1 - k_B)(1 - k_F)} \frac{1}{R} dr + \int_{\overline{r}_B}^{R} r_D \frac{1}{R} dr \geq u \geq 1 \tag{15}$$

$$0 \leq k_F \leq 1, \tag{16}$$

together with (5) and (6), where $\overline{r}_F$ is from (8) and $\overline{r}_B$ is from (9).

The first term in (13) represents the expected payoff to the bank when firms go bankrupt but the bank remains solvent for $\overline{r}_B < r < \overline{r}_F$. In this case, the bank obtains the firms' liquidation proceeds $\frac{h_F r}{1 - k_F}$ after repaying depositors. By contrast, when $\overline{r}_B = \overline{r}_F$, the bank goes bankrupt whenever the firm does so, and the first term in (13) becomes zero. The second term is the expected payoff to the bank from lending one unit to firms at the rate r_L after paying $r_D(1 - k_B)$ to its depositors. The last term ρk_B is the opportunity cost for bank shareholders. Constraint (14) requires the expected profit of the firm to be non-negative. The first term is the expected payoff to the firm from the investment in the risky technology after paying $r_L(1 - k_F)$ to the bank for $r > \overline{r}_F$. The last term ρk_F is the opportunity cost for firm shareholders. Constraint (15) requires that the depositors make at least their opportunity cost u in expectation. The first term is the payoff when the bank goes bankrupt for $r < \overline{r}_B$ and each depositor obtains a share $\frac{h_B}{(1 - k_B)}$ of the $\frac{h_F r}{(1 - k_F)}$

resources available at the bank. The second term is depositors' payoff for $r \geq \overline{r}_B$, when the bank remains solvent and each depositor obtains the promised repayment r_D.

We obtain the following result.

Proposition 5 *The unique equilibrium with $0 \leq h_B, h_F < 1$ is as follows:*

i) Banks hold $k_B = 0$ and set $r_D = r_L$.

ii) If $h_B = 0 \leq h_F < 1$, the equilibrium is as in Proposition 1 with the difference that firms hold the same capital as banks there, i.e., $k_F = \frac{4}{R} - 1$ for $R < \overline{R}$ and $k_F = \frac{\eta}{1+\eta}$ for $R \geq \overline{R}$.

iii) If $0 < h_B, h_F < 1$, the equilibrium is as in Proposition 2 with the difference that firms hold the same capital as banks there except that h_B is replaced by $h_B h_F$, i.e., $k_F = \frac{(2 - h_B h_F R)(2(2 - h_B h_F) - R)}{2(1 - h_B h_F)^2 R}$ for $R < \overline{R}$ and $k_F = \frac{\eta}{1+\eta}$ for $R \geq \overline{R}$.

The proposition states that in equilibrium banks are simply a conduit between depositors and firms and hold no capital. This allows a reduction in the deadweight costs associated with the bankruptcy of firms and banks. The result is illustrated in Figure 3, which shows the output of a single firm as a function of the return r, and how this is split among shareholders and depositors.

Consider first the case where both the bank and the firm hold positive capital and the firm goes bankrupt at a higher level of r than the bank, i.e., $\overline{r}_F' = r_L'(1 - k_F') > \overline{r}_B' = \frac{r_D'(1 - k_B')(1 - k_F')}{h_F}$. Region A represents the payoff to firm shareholders for $r \in (\overline{r}_F', R]$, when the firm remains solvent and repays $r_L'(1 - k_F')$ to the bank. Region $B + C$ represents the payoff to the bank shareholders. For $r \in [\overline{r}_F', R]$, the bank receives the promised repayment $r_L'(1 - k_F')$. For $r \in [\overline{r}_B', \overline{r}_F')$, the firm goes bankrupt and the bank receives $\frac{h_F r}{1 - k_F'}$. Region $D1$ represents the deadweight loss deriving from the bankruptcy of the firm. Region $E1 + F$ represents the payoff to bank depositors. For $r \in [\overline{r}_B', R]$, the bank is solvent, and each depositor receives the promised repayment r_D'. Since there are $(1 - k_B')(1 - k_F')$ depositors per firm, they obtain $r_D'(1 - k_B')(1 - k_F')$ in total. For $r \in [0, \overline{r}_B')$ the bank goes bankrupt. Each of the $(1 - k_B')$ depositors in the bank receives a share $\frac{h_B}{1 - k_B'}$ of the resources $\frac{h_F r}{1 - k_F'}$

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that the bank has. Thus, the $(1 - k'_B)(1 - k'_F)$ depositors per firm obtain $h_B h_F r$ in total.

Consider now transferring all capital from the bank to the firm and aligning the bankruptcy points of the bank and the firm. This entails setting $k^*_B = 0$ and $k^*_F = k'_B(1 - k'_F) + k'_F$. The firm then has a transfer of $k'_B(1 - k'_F)$, which is the amount of capital that the bank has per firm, in addition to its original amount k'_F. Since the bank has zero capital, it is possible to set $r^*_D = r^*_L = r'_D$ so that the bank becomes a conduit with zero profit. This aligns the firm and bank bankruptcy points and changes them to $\overline{r}^*_F = r^*_L(1 - k^*_F) = \overline{r}^*_B = r'_D(1 - k'_B)(1 - k'_F) < \overline{r}'_B = \frac{r'_D(1 - k'_B)(1 - k'_F)}{h_F}$. It is immediate to see that this allows the deadweight losses in Region $D1 + D2$ and $E2$ to be eliminated and improves the allocation. Both shareholders and depositors are better off than before the deviation.

This argument shows that in any equilibrium it must be the case that $k_B = 0$ and $r_L = r_D$. The optimal choice of k_F and r_L is then the same as the bank's choice of k_B and r_D when the bank invests directly in the risky asset except that the liquidation proceeds $h_B r$ are replaced by $h_B h_F r$. The equilibrium is then as described in Proposition 5.

4.3 Bankruptcy costs in the banking sector only $\left(h_B \geq 0 \text{ and } h_F = 1\right)$

We next consider the case where there are no bankruptcy costs in the productive sector $(h_F = 1)$, while keeping some bankruptcy costs in the banking sector $(h_B \geq 0)$. We have the following result.

Proposition 6 *With $h_F = 1$ and $h_B \geq 0$ there are multiple equilibria:*

i) If $h_B = 0$, the equilibrium values of r_D, ρ and u are as in Proposition 1.

ii) If $h_B > 0$, the equilibrium values of r_D, ρ and u are as in Proposition 2.

Banks and firms choose any level of k_B and k_F provided $\overline{r}_F = r_L(1 - k_F) \in [\overline{r}_B, R]$ such that

$$\overline{r}_B = \widehat{r}_D(1 - \widehat{k}_B) = \widehat{r}_D(1 - k_B)(1 - k_F), \tag{17}$$

where $\widehat{r}_D$ and $\widehat{k}_B$ come from Proposition 1 in i) and from Proposition 2 in ii). The value

of r_L does vary in the range $[r_D, R]$ with the split of capital so that $E\Pi_B = E\Pi_F = 0$. For any $k_F > 0$, the number of firms is $N_F = \widehat{N}_B = \frac{K}{\hat{k}_B}$ and the number of banks is $N_B = \frac{(1-\hat{k}_B)}{(1-k_B)}\widehat{N}_B < \widehat{N}_B$. The amount of deposits used in the banking sector remain as with one non-publicly traded productive sector.

The proposition shows that as in the case with a single non-traded productive sector, a Modigliani-Miller type of result holds when there are no bankruptcy costs in the traded productive sector. Depositors and shareholders make the same expected return in any of them as in Propositions 1 and 2 with a single non-traded productive sector, depending on the value of h_B. Bank and firm capital adjust in aggregate so as to guarantee those returns, but the specific capital structure of either banks or firms is irrelevant as long as (17) is satisfied. The loan rate adjusts to ensure that both bank and firm shareholders earn ρ. As in the case of Proposition 3, the multiplicity of equilibria and the irrelevance of capital structure composition do not translate, however, to a multiplicity of allocations. Given that there are bankruptcy costs in the banking sector, the allocation is unique and it is the same as in Propositions 1 or 2 depending on whether h_B is equal to zero or positive. It is only the way the system reaches that allocation, that is how capital is split among banks and firms, that is irrelevant. The other variable in addition to the loan rate r_L that changes depending on the distribution of capital between banks and firms is the number of banks. If $k_F > 0$, the number of banks is reduced. The number of firms and the amount of deposits used in the banking sector remain the same.

5 Two non-publicly traded productive sectors

So far we have considered the case where there is a single productive sector where all firms' payoffs are perfectly correlated. In this section, we analyze the effect of diversification on banks' capital structure as it is an alternative to equity capital for reducing bankruptcy costs. We assume that there are two sectors with independent returns r_1 and r_2, each uniformly distributed on the support $[0, R]$. We start with the simple case where banks

invest in the two sectors directly similarly to Section 2, and we then analyze the case of two publicly traded sectors in the next section.

As before, each bank raises k_B in capital and $1 - k_B$ in deposits, and invests 1 in total. Clearly, a diversified bank with two sectors will lend equally to each sector to maximize the benefit of diversification. The rest of the economy is as before. For simplicity, we focus on the case where $h_B = 0$ throughout so that there are full bankruptcy costs from the bank failing.

As the bank invests in equal shares in two independent projects, the return of the bank's portfolio is equal to the weighted sum of the returns of each sector, that is $x = \frac{r_1}{2} + \frac{r_2}{2}$. If $x \geq r_D(1 - k_B)$, that is $r_1 + r_2 \geq \overline{x}_B$, where

$$\overline{x}_B = 2r_D(1 - k_B), \tag{18}$$

then the bank remains solvent and repays depositors in full. Otherwise, it goes bankrupt, and depositors obtain nothing given that $h_B = 0$. To see when this occurs we consider the distribution of the sum of the returns r_1 and r_2 in Figure 4. The figure shows several regions depending on the values of r_1 and r_2. The bank goes bankrupt in Region A and remains solvent everywhere else. Region B captures the case where $r_1 + r_2 \geq \overline{x}_B$ for $r_1 \in [\, \overline{x}_B - r_2, R]$ and $r_2 \in [0, \overline{x}_B]$. Region C represents the case where $r_1 + r_2 \geq \overline{x}_B$ for $r_1 \in [0, R]$ and $r_2 \in [\overline{x}_B, R]$.

Given the return x and the areas of bank solvency as described above, the bank's maximization problem is now given by

$$\max_{k_B, r_D} E\Pi_B = \int_0^{\overline{x}_B} \int_{\overline{x}_B - r_2}^R \left(\frac{r_1}{2} + \frac{r_2}{2} - r_D(1 - k_B) \right) \frac{1}{R^2} dr_1 dr_2 \tag{19}$$
$$+ \int_{\overline{x}_B}^R \int_0^R \left(\frac{r_1}{2} + \frac{r_2}{2} - r_D(1 - k_B) \right) \frac{1}{R^2} dr_1 dr_2 - \rho k_B$$

subject to

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$$EU_D = \int_0^{\overline{x}_B} \int_{\overline{x}_B - r_2}^R r_D \frac{1}{R^2} dr_1 dr_2 + \int_{\overline{x}_B}^R \int_0^R r_D \frac{1}{R^2} dr_1 dr_2 \geq u, \qquad (20)$$

together with (5) and (6).

As before, the bank chooses k_B and r_D to maximize its expected profit subject to the depositors obtaining their opportunity cost $u \geq 1$ in expectation, but the problem is algebraically more complicated now. Expression (19) represents the bank's expected profit net of shareholders' opportunity cost ρk_B. The first term is what the bank obtains after paying $r_D(1 - k_B)$ to the depositors when the bank remains solvent in Region B for $r_1 \in [\overline{x}_B - r_2, R]$ and $r_2 \in [0, \overline{x}_B]$. The second term is what it obtains in Region C where $r_1 \in [0, R]$ and $r_2 \in [\overline{x}_B, R]$. Similarly, expression (20) represents depositors' expected utility to depositors when the bank remains solvent in Regions B and C. The last constraints (5) and (6) are the usual non-negative profit condition for the bank and the feasibility constraint on the amount of capital.

We obtain the following result.

Proposition 7 *When $h_B = 0$, two-sector banks dominate one-sector banks and the unique equilibrium is as follows:*

i) For $R < \overline{R}, k_B = 1 - \frac{R\rho\sqrt{2(3\rho^2 - 4\rho + 1)}}{(3\rho - 1)^2} \in (0, 1)$, $r_D = \frac{(3\rho - 1)}{2\rho}$, $\rho = \rho_1(R), E\Pi = 0$, $EU = u = 1$.

ii) For $R \geq \overline{R}, k_B = \frac{\eta}{1+\eta} \in (0, 1), r_D = \frac{u(3\rho - u)}{2\rho}, \rho = \rho_2(R), u = u(R), E\Pi = 0, EU = u > 1$.

The expressions for $\overline{R}, \rho_1(R)$ and $\rho_2(R)$ are defined in the appendix.

The proposition states that banks always choose to diversify their business when they can invest in two non-publicly traded productive sectors. The reason is that diversification reduces the probability for the bank to go bankrupt, as can be understood from Figure 4. If a bank invests in one sector only, it will go bankrupt in Region $A2 + B2 + C2$ where $r < \overline{r}_B = r_D(1 - k_B)$. By splitting investment between two sectors instead, the bank goes bankrupt in Region $A1 + A2$ where $r_1 + r_2 \leq \overline{x}_B$. For given r_D and k_B, Regions $A1$ and

$B2$ are of the same size since $\overline{x}_B = 2\overline{r}_B$ and the line $r_1 = \overline{x}_B - r_2$ has slope equal to -1. Thus, diversification allows the bank to remain solvent in Region $C2$ where it would go bankrupt if it invested in one sector only. In this region, r_1 is low and the bank would have gone bankrupt if it only invested in one sector, but r_2 is sufficiently high to ensure that the bank remains solvent if it diversifies.

6 Two publicly traded sectors

In this section we turn to the case where there are two publicly traded sectors with independent returns r_1 and r_2, each uniformly distributed on the support $[0, R]$. As before, each firm requires 1 unit of funds and finances this with equity k_F and loans from banks of $1 - k_F$. The former has an opportunity cost $\rho \geq \frac{R}{2}$, while the latter has a promised per unit rate of r_L. If $r_i \geq \overline{r}_F = r_L(1 - k_F)$, firms in sector i are solvent and repay $\frac{r_L}{2}$ to the banks. If $r_i < \overline{r}_F$, firms in sector i go bankrupt and the liquidation proceeds $h_F r_i$ with $h_F \in [0, 1]$ are distributed pro-rata to the banks providing the $1 - k_F$ in loans. A bank diversifying across the two sectors raises k_B in capital and $1 - k_B$ in deposits, and lends to each sector equally. Two-sector banks lend equally to both sectors so that the firms in the two sectors are symmetric, and it is assumed for simplicity $h_B = 0$ as in the previous section. The rest of the economy is as before. We start with the simplest case with $h_B = h_F = 0$.

6.1 Full bankruptcy costs in the banking and in the two publicly traded sectors $(h_B = h_F = 0)$

As before, the equilibrium requires banks choose firm capital k_F, the loan rate r_L, the amount of bank capital k_B and the deposit rate r_D to maximize their expected profits, while guaranteeing depositors their opportunity cost u and firm shareholders their opportunity cost ρ. To derive the bank's maximization problem we need to distinguish different cases depending on how firm and bank bankruptcies are related to each other.

When $h_F = 0$, a two-sector bank only receives a positive payoff when firms do not go bankrupt, that is for $r_i > \overline{r}_F = r_L(1 - k_F)$. If firms in only one sector are solvent, then the bank receives $\frac{r_L}{2}$. The first possibility is that the solvency of firms in one sector is sufficient to guarantee bank solvency so

$$\frac{r_L}{2} \geq r_D(1 - k_B). \tag{21}$$

This implies that the bank goes bankrupt only when firms in both sectors go bankrupt as the bank obtains nothing then since $h_F = 0$. The case is illustrated in Figure 5. The bank remains solvent in all regions except Region A. Regions C and D correspond to where firms in only one sector go bankrupt and the bank obtains $\frac{r_L}{2}$. Finally, Region E is where all firms remain solvent and the bank receives r_L in total. The bank's maximization problem is then given by

$$
\begin{aligned}
\underset{k_F, r_L, k_B, r_D}{Max} E\Pi_B \ &= \ \int_0^{\overline{r}_F} \int_{\overline{r}_F}^R \left(\frac{r_L}{2} - r_D(1 - k_B)\right) \frac{1}{R^2} dr_1 dr_2 \\
&+ \int_{\overline{r}_F}^R \int_0^{\overline{r}_F} \left(\frac{r_L}{2} - r_D(1 - k_B)\right) \frac{1}{R^2} dr_1 dr_2 \\
&+ \int_{\overline{r}_F}^R \int_{\overline{r}_F}^R \left(r_L - r_D(1 - k_B)\right) \frac{1}{R^2} dr_1 dr_2 - \rho k_B
\end{aligned}
\tag{22}
$$

subject to

$$E\Pi_{Fi} = \int_{\overline{r}_F}^R (r_i - r_D(1 - k_B)) \frac{1}{R} dr_i - \rho k_F \geq 0 \tag{23}$$

$$EU_D = \int_0^{\overline{r}_F} \int_{\overline{r}_F}^R r_D \frac{1}{R^2} dr_1 dr_2 + \int_{\overline{r}_F}^R \int_0^{\overline{r}_F} r_D \frac{1}{R^2} dr_1 dr_2 + \int_{\overline{r}_F}^R \int_{\overline{r}_F}^R r_D \frac{1}{R^2} dr_1 dr_2 \geq u, \tag{24}$$

where the subscript Fi in (23) with $i = 1, 2$ represents firms operating in either of the two sectors, together with the the non-negativity constraint (5) for banks' expected profit, and the feasibility conditions (16) and (6).

Expression (22) is the bank expected profit net of the shareholders' opportunity cost

ρk_B. The first and second terms correspond to the expected payoff to the bank in Regions C and D where firms only in one sector remain solvent, the bank receives $\frac{r_L}{2}$ and repays $r_D(1-k_B)$ to the depositors. The last term corresponds to the bank net payoff in Region E when firms in both sectors remain solvent and obtain r_L in total. Constraint (23) requires the expected profit of the firm to be non-negative. Constraint (24) is the usual depositor participation constraint. Depositors receive r_D in Regions C, D and E when the bank is solvent.

The second case to be considered is where the solvency of firms in one sector is not sufficient to guarantee bank solvency so

$$\frac{r_L}{2} < r_D(1 - k_B). \tag{25}$$

In this case the bank remains solvent only in Region E where firms in both sectors are solvent and repay the promised loan rate r_L. This implies that only the last integral term in (22) and (24) remains positive.

We next show that when $h_B = h_F = 0$ two-sector banks will not survive in equilibrium.

Proposition 8 *When $h_B = h_F = 0$, one-sector banks strictly dominate two-sector banks. The equilibrium is then the same as in part i) and ii) of Proposition 5.*

The equilibrium involves specialized banks lending to one sector only. The proposition can be understood with the use of Figure 5. Consider a candidate equilibrium with two-sector banks. We can show that a one-sector bank lending the same amount $1 - k_F$ at loan rate r_L to firms is always more profitable so the candidate equilibrium is not viable. Only a one-sector bank equilibrium exists.

Firms in sector i go bankrupt for $r_i < \overline{r}_F = r_L(1 - k_F)$. In the case where $\frac{r_L}{2} < r_D(1 - k_B)$ the two-sector bank remains solvent only in Region E while the one-sector bank is also solvent in Regions C and D. Thus, the latter does better since it goes bankrupt less.

When $\frac{r_L}{2} \geq r_D(1 - k_B)$, both the two-sector bank and the one-sector bank remain solvent in Regions C, D and E. Since Regions C and D are the same and in the case of two sectors the bank invests equally in both sectors, the two-sector bank and the one-sector bank generate the same expected output. However, the one-sector bank can always do better by choosing a capital structure that has a lower funding cost. In Region E the two-sector bank receives $r_L > r_D(1 - k_B)$ and would make positive expected profits if it had $k_B = 0$ but this is inconsistent with equilibrium. Thus, in any equilibrium with $\frac{r_L}{2} \geq r_D(1 - k_B)$, the two-sector bank must use capital $k_B > 0$. For a given k_F and r_L, the one-sector bank can choose $k_B = 0$ and fund more cheaply. This breaks the equilibrium with the two-sector banks. The only possible equilibrium is with one-sector banks choosing $k_B = 0$ and setting $r_L = r_D$.

The advantage of one-sector banks is that they can align the firm and bank bankruptcy thresholds by being a conduit. The two-sector bank cannot do this. One possibility is to choose to go bankrupt when only firms in one sector go bankrupt ($\frac{r_L}{2} < r_D(1 - k_B)$) but in this case the output of the firms in the solvent sector is wasted since $h_B = 0$. The other possibility is to choose to go bankrupt only when the firms in both sectors go bankrupt ($\frac{r_L}{2} \geq r_D(1 - k_B)$). This requires that the two-sector bank can pay its depositors even if it receives the promised payment on half of its loans. This generates the same expected output as a one-sector bank when $h_F = 0$. However, in a competitive environment the two-sector bank must use costly capital to achieve this, whereas the one-sector bank does not need it.

6.2 No bankruptcy costs in the productive sectors ($h_B = 0$ and $h_F = 1$)

When $h_F = 1$, the bank receives the promised r_L when firms do not go bankrupt, that is for $r > \overline{r}_F = r_L(1 - k_F)$, and a share $\frac{r_i}{2(1 - k_F)}$ when firms in sector i go bankrupt. In contrast to the previous subsection where $h_F = 0$, diversification is optimal since there are no bankruptcy costs in the productive sectors. As in Section 4.3, this lack of bankruptcy costs also leads to a multiplicity of equilibria in banks' and firms' capital structure, but

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not in terms of allocation of resources.

Proposition 9 *With $h_B = 0$ and $h_F = 1$, two-sector banks strictly dominate one-sector banks and there are multiple equilibria. The equilibrium values of r_D, ρ and u are as in Proposition 7. Banks and firms choose any level of k_B and $k_F \in [0, \overline{k}_F]$ such that*

$$\overline{x}_B = 2\widehat{r}_D(1 - \widehat{k}_B) = 2\widehat{r}_D(1 - k_B)(1 - k_F), \tag{26}$$

where $\overline{k}_F$ is such that $\overline{x}_B = \overline{r}_F$, and $\widehat{r}_D$ and $\widehat{k}_B$ come from Proposition 7. The value of r_L does vary in the range $[r_D, R]$ with the split of capital so that $E\Pi_B = E\Pi_F = 0$. For any $k_F > 0$, the number of firms is $N_F = \widehat{N}_B = \frac{K}{\widehat{k}_B}$ and the number of banks is $N_B = \frac{(1 - \widehat{k}_B)}{(1 - k_B)}\widehat{N}_B < \widehat{N}_B$. The amount of deposits is as in the case of two non-publicly traded sectors.

The intuition behind this result can be understood by considering the case with $r_L = R$ and $k_F = 0$. The two-sector bank is then just like the bank in Section 5 that invests directly in two non-productive sectors. We know from the discussion there that a two-sector bank has the advantage that diversification allows bank bankruptcy to be reduced and increases expected output. As a result the two-sector bank strictly dominates the one-sector bank. The alignment of the firm and bank bankruptcy with the one-sector bank does not bring any benefit since there are no bankruptcy costs for firms.

6.3   Intermediate bankruptcy costs in the productive sectors ($h_B = 0$ and $h_F < 1$)

In Subsection 6.1 we showed that one-sector banks dominate two-sector banks when $h_F = 0$. Diversification is not beneficial as it does not allow the two-sector bank to achieve higher expected output. In one-sector banks the alignment firm and bank bankruptcy minimizes total bankruptcy costs. In Subsection 6.2 we showed that two-sector banks

dominate when $h_F = 1$ so there are no firm bankruptcy costs. The reason is that now diversification allows the two-sector bank to minimize its bankruptcy costs.

We now turn to the intermediate case where $h_F < 1$. We demonstrate in the context of an example that when $u = 1$ there is a critical value $\overline{h}_F$ above which two-sector banks are optimal and below which one-sector banks are optimal. When $u > 1$ there is a range of values of h_F between $\underline{h}_F$ and $\overline{h}_F$ such that both one-sector banks and two-sector banks are optimal and coexist. Between $\underline{h}_F$ and $\overline{h}_F$ the proportion of one-sector banks goes from one to zero, while the proportion of two-sector banks goes from zero to one.

As above, we have to distinguish different cases depending on how firm and bank bankruptcies are related. We start with the case where $\frac{r_L}{2} > r_D(1 - k_B)$. This implies that the bank goes bankrupt only when firms in both sectors go bankrupt and $\frac{h_F(r_1 + r_2)}{2(1 - k_F)} < r_D(1 - k_B)$, or equivalently

$$r_1 + r_2 < \overline{x}_B = \frac{2r_D(1 - k_B)(1 - k_F)}{h_F}. \tag{27}$$

The case is illustrated in Figure 6, where $\overline{x}_B < \overline{r}_F = r_L(1 - k_F)$. The bank remains solvent in all regions except Region A. Region B captures the case where both firms go bankrupt and the bank receives $\frac{h_F(r_1 + r_2)}{2(1 - k_F)} > r_D(1 - k_B)$. Region C and D correspond to where firms in only one sector go bankrupt and the bank obtains $\frac{h_F r_i}{2(1 - k_F)} + \frac{r_L}{2}$. Finally, Region E is where all firms remain solvent and the bank receives r_L in total.

The next case to be considered is $\frac{r_L}{2} < r_D(1 - k_B)$. This case is illustrated in Figure 7. The first difference is that now $\overline{x}_B > \overline{r}_F$. This means that the bank can now go bankrupt when one sector is solvent and returns in the bankrupt sector are sufficiently low, i.e., when $\frac{r_L}{2} + \frac{h_F r_i}{2(1 - k_F)} < r_D(1 - k_B)$ or, equivalently when

$$r_i < \widehat{r}_i = \frac{2r_D(1 - k_B)(1 - k_F)}{h_F} - \frac{r_L(1 - k_F)}{h_F} = \overline{x}_B - \frac{\overline{r}_F}{h_F}. \tag{28}$$

This changes the shape of Region A where the bank goes bankrupt relative to Figure 6. The other main difference is that Regions C_1 and D_1 are new. These are bounded

below by $\widehat{r}_i$ and above by r_i^*. The latter is the value of r_i where $\frac{h_F(r_i+r_{-i})}{2(1-k_F)} = r_D(1-k_B)$ and $r_{-i} = r_L(1-k_F)$, i.e.,

$$r_i^* = \frac{2r_D(1-k_B)(1-k_F)}{h_F} - r_L(1-k_F) = \overline{x}_B - \overline{r}_F. \tag{29}$$

The final case is when

$$\frac{r_L}{2} = r_D(1-k_B), \tag{30}$$

which, from (28), implies $\widehat{r}_i = 0$. This corresponds to the special case in Figure 7 where the boundaries below Regions C_1 and D_1 become the axes.

Solving the bank's problem when $h_F < 1$ is analytically intractable. We therefore solve two numerical examples with $R = 2.5$ or $R = 4$ and $\eta = 0.1$. In both examples the solution involves the case where (30) holds so that $\widehat{r}_i = 0$. We describe the full bank maximization problems as well as the solution to the examples in detail in Appendix B.

We start with $R = 2.5$ and $\eta = 0.1$. The results are shown in Table 1. The two-sector bank equilibrium is calculated as described in Appendix B. The one-sector bank equilibrium comes from parts i) and ii) of Proposition 5 and does not depend on the size of h_F because $h_B = 0$ so any proceeds from the firm bankruptcy are lost in the bank bankruptcy. The table shows that the two-sector bank equilibrium Pareto dominates the one-sector bank equilibrium for $h_F = 0.9 > \overline{h}_F = 0.825$ because $u = 1$ in both cases but ρ is higher in the two-sector bank equilibrium. Each firm holds $k_F = 0.341$ while each bank holds $k_B = 0.327$. The opportunity cost of capital is $\rho = 1.361$ with two-sector banks while it is only 1.333 with one-sector banks. The total number of banks N_B is 11.841, and the total number of firms is 17.963 divided equally between the two sectors.

As h_F reaches $\overline{h}_F = 0.825$, the two equilibria are equivalent in terms of the remuneration to shareholders, which is now reduced to $\rho = 1.333$, and depositor expected utility $u = 1$. Since bankruptcy costs are increased as h_F is lowered and u remains constant at 1, ρ falls. The amount of capital used by the firm in the two-sector bank equilibrium is

now substantially higher at 0.423 compared to 0.341 when $h_F = 0.9$. As a consequence, r_L also drops to 1.422 from 1.482 when $h_F = 0.9$. The amount of capital used by the bank is increased from 0.327 to 0.341 and r_D goes from 1.102 to 1.080. In the one-sector bank equilibrium capital is much higher in the firm and much less in the bank, where it is 0. In the two-sector bank equilibrium capital in the bank is useful because it maximizes the benefit from diversification. In the one-sector bank equilibrium, bank and firm bankruptcies are aligned by setting capital in the bank to be zero. The number of firms and banks is different in the two equilibria. In the two-sector bank equilibrium there are fewer firms and more banks ($N_F = 16.124$ and $N_B = 9.302$) than in the one-sector bank equilibrium where $N_F = 16.667$ and $N_B = 6.667$. The size of the productive sector is larger in the one-sector bank equilibrium and more capital is used in this sector. The size of the financial system is larger in the two-sector equilibrium. However, financial inclusion, as represented by the total amount of deposits used by banks, is smaller in the two-sector equilibrium because each bank uses less deposits (6.124 instead of 6.667). This implies that the amount invested in depositors' alternative opportunity is higher in the two-sector bank equilibrium.

For $h_F = 0.8 < \overline{h}_F$ shareholders are worse off with two-sector banks and so the equilibrium will involve one-sector banks only. The cost of firm bankruptcy when there are two-sector banks outweighs the benefit from diversification net of the higher cost of bank capital and one-sector banks dominate.

We now increase the return of the risky technology to $R = 4$ and in this case $u > 1$. The results are shown in Table 2. As with $u = 1$, for high values of h_F the equilibrium involves two-sector banks and for low values of h_F it involves one-sector banks. The main difference is that there is a range of values $[\underline{h}_F, \overline{h}_F]$ there is a mixed equilibrium in which both one-sector banks and two-sector banks are optimal and coexist. Another difference is that now the one-sector bank values depend on h_F in the interval $[\underline{h}_F, \overline{h}_F]$, while they are still independent from h_F in the range of values $[0, \underline{h}_F]$ where the one-sector banks dominate. The reason is that in this range the one-sector banks and the two-sector banks

are competing and the latter are affected by the size of h_F.

For h_F in the range $(0.9092, 1]$ the two-sector bank equilibrium Pareto dominates the one-sector bank equilibrium because it leads to higher opportunity costs for both shareholders and depositors. For h_F in the range $(0.8994, 0.9092)$ the equilibrium still involves only two-sector banks. Depositors are worse off than in the one-sector bank equilibrium but shareholders are better off. The equilibrium involves two-sector banks because the one-sector banks cannot cover the opportunity cost of the shareholders. However, the one-sector bank equilibrium does not exist because two-sector banks can successfully enter and make strictly positive profits. For values of h_F above $\overline{h}_F$, the fact that with two-sector banks firm and bank bankruptcies are not aligned is not very costly and the diversification effect more than makes up for this. In this range the return on capital ρ increases as h_F decreases. The reason is that capital becomes more valuable in terms of avoiding bankruptcy costs. Since bankruptcy costs are going up, u must fall if ρ goes up.

As h_F falls towards $\overline{h}_F$, the lack of alignment of bank and firm bankruptcies is more and more costly. One-sector banks where bank and firm bankruptcies are aligned become increasingly desirable. At $h_F = \overline{h}_F = 0.8994$ it is just worthwhile for one-sector banks to enter because they can match the return on capital $\rho = 7.191$ using funds that in the one-sector bank equilibrium go to depositors, i.e. 1.091 versus 1.081. For h_F in the range $[\underline{h}_F, \overline{h}_F]$ the equilibrium involves a mixture of both types of banks. At $\overline{h}_F$ the proportion of firms borrowing from one-sector banks, as represented by υ in the table, is 0 and the proportion of firms borrowing from two-sector banks is 1. As h_F falls to $\underline{h}_F$, the proportion of firms borrowing from two-sector banks falls to 0 and the proportion of firms borrowing from one-sector banks increases to 1. With two-sector banks capital is more valuable because it allows bankruptcy costs to be reduced in both firms and banks, while in one-sector banks firm and bank bankruptcies are aligned. This is why differently from Table 1, the return ρ to shareholders falls as the proportion of two-sector banks falls. By a similar argument, u increases as the proportion of one-sector banks rises because deposits are more valuable for them.

For values of h_F below $\underline{h}_F$ one-sector banks dominate. Then the value of h_F does not matter since firm and bank bankruptcies are aligned and all proceeds from firm liquidation are lost in the bank bankruptcy since $h_B = 0$.

7   Concluding remarks

We have developed a general equilibrium model of banks and firms to endogenize the equity cost of capital in the economy. The two key assumptions of our model are that deposit and equity markets are segmented and there are bankruptcy costs for banks and firms. We have shown that in equilibrium equity capital has a higher expected return than investing directly in the risky asset. Deposits are a cheaper form of finance as their return is below the return on the risky asset. This implies that equity capital is costly relative to deposits. When banks directly finance risky investments, they hold a positive amount of equity capital as a way to reduce bankruptcy costs and always prefer to diversify if possible. In contrast, when banks provide loans to non-financial firms that invest in risky assets, diversification is not always optimal. Diversification is only relevant when firms are bankrupt otherwise the bank simply receives a fixed return on its loans. There is then a trade-off. In order for the bank to reap the benefits of diversification, it must remain solvent when firms are bankrupt and this requires it to hold positive capital even though this is costly relative to deposits. When bankruptcy costs are significant, banks finance themselves with deposits only and specialize in lending to one sector. Diversification is not worthwhile because very little is received by the banks when bankruptcy costs are high. It is more efficient for firms to hold all the equity capital and minimize their probability of bankruptcy. When bankruptcy costs are low, diversification across different sectors is optimal because banks receive high returns in this case. They also hold positive bank capital to lower their probability of bankruptcy. For intermediate values of bankruptcy costs, both undiversified and diversified banks coexist.

We have excluded bond finance in our analysis. An important extension is to take

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account of this possibility. Since both banks and firms use bond finance presumably there is less or no segmentation between bond markets and equity markets.

Much of the recent literature on bank capital structure has been concerned with issues of regulation (e.g., Hellmann et al. (2000), Van den Heuvel (2008), Admati et al. (2010), Acharya, Mehran and Thakor (2012)). In our model there are no benefits from regulating bank capital. The market solution is efficient since there are no pecuniary or other kinds of externalities. There are many ways to introduce reasons for regulation as is done in the regulation literature. Our purpose in this paper is to consider the effect of bankruptcy costs and market segmentation without complicating the model with other factors. Another important extensions is to consider the interaction between standard rationales for capital regulation with our approach.

Finally, there is a growing, mostly empirical, literature on financial inclusion (see, e.g., Demirgüç-Kunt, Beck and Honohan (2008)). Our paper provides a framework to think about financial inclusion and its effects on real economic activity. We hope to pursue this in future work.

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A Proofs

Proof of Proposition 1: Solving (4) with equality for k_B after setting $h_B = 0$, we find

$$k_B = 1 - \frac{(r_D - u)}{r_D^2}R. \tag{31}$$

Substituting this into (3), differentiating with respect to r_D, and solving for r_D gives

$$r_D = \frac{u(2\rho - u)}{\rho}. \tag{32}$$

Substituing this into (31) gives

$$k_B = 1 - \frac{\rho R(\rho - u)}{u(2\rho - u)^2}. \tag{33}$$

Using (32) and (33) in (3), we obtain

$$E\Pi_B = \frac{\rho^2 R}{2u(2\rho - u)} - \rho. \tag{34}$$

Equating this to zero since $E\Pi_B = 0$ in equilibrium, and solving for ρ gives

$$\rho = \frac{2u^2}{4u - R}. \tag{35}$$

Substituting (35) into (32) leads to

$$r_D = \frac{R}{2},$$

and together with (31), this implies

$$k_B = \frac{4u}{R} - 1. \tag{36}$$

If depositors use their alternative opportunity then $u = 1$. This will occur if

$$\frac{k_B}{1 - k_B} > \eta.$$

In this case, banks will be formed until all the capital is used up. However, there will still be deposits left over, which depositors will put in their alternative opportunity, so $u = 1$. Using (36) in this gives

$$R < \overline{R} = \frac{4(1+\eta)}{1+2\eta}.$$

Putting $u = 1$ in (35) and (36) gives $\rho = \frac{2}{4-R}$ and $k_B = \frac{4}{R} - 1$. It can easily be checked that $\rho > \frac{R}{2}$ and $k_B \in (0,1)$. Substituting the expressions for r_D and k_B into (2) gives $\overline{r}_B = R - 2$. Given $\rho > \frac{R}{2}$ and $u = 1$, we have $N_B k_B = K$ and $N_B(1 - k_B) < D$. This gives the first part of the proposition.

For $R \geq \overline{R}$, deposits are in short supply and in this case $u > 1$. The equilibrium level of u is found from

$$\frac{k_B}{1 - k_B} = \eta,$$

where k_B is given by (36). Solving gives

$$u = \frac{1 + 2\eta}{2 + 2\eta} \frac{R}{2}.$$

Using this in (35) and (36) gives $\rho = \frac{1+4\eta(1+\eta)}{4\eta(1+\eta)} \frac{R}{2}$ and $k_B = \frac{\eta}{1+\eta}$. It can be checked that $\rho > \frac{R}{2}$, $k_B \in (0,1)$ and $u \in [1, \frac{R}{2})$. Substituting the expressions for r_D and k_B into (2) gives $\overline{r}_B = \overline{r}_B = \frac{1}{(1+\eta)} \frac{R}{2}$. Given $\rho > \frac{R}{2}$ and $u > 1$, we have $N_B k_B = K$ and $N_B(1 - k_B) = D$. This gives the second part of the proposition. $\square$

Proof of Proposition 2: Solving (4) with equality for k_B and $h_B > 0$, we find

$$k_B = 1 - \frac{2(r_D - u)}{(2 - h_B)r_D^2} R. \tag{37}$$

Substituting this into (3), differentiating with respect to r_D, and solving for r_D gives

$$r_D = \frac{2u\left((2 - h_B)\rho - u\right)}{(2 - h_B)\rho - uh_B}. \tag{38}$$

Using (38) in (3), equating to zero since $E\Pi_B = 0$ in equilibrium, and solving for ρ gives

$$\rho = \frac{u\left(2u - h_B R\right)}{2u(2 - h_B) - R}. \tag{39}$$

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Substituting (39) into (38) leads to

$$r_D = \frac{2u(1 - h_B)R}{2u(2 - h_B) - h_B R},\tag{40}$$

and together with (37), this implies

$$k_B = \frac{(2u - h_B R)(2u(2 - h_B) - R)}{2u(1 - h_B)^2 R}.\tag{41}$$

Similarly to the case with $h_B = 0$, depositors use their alternative opportunity and thus $u = 1$ when

$$\frac{k_B}{1 - k_B} > \eta.$$

Using (41) in this gives

$$R < \overline{R} = \frac{2(1 + \eta) - (1 - h_B)\left(1 - h_B + \sqrt{4\eta(1 + \eta) + (1 - h_B)^2}\right)}{(1 + \eta)h_B}.\tag{42}$$

Putting $u = 1$ in (41), (40) and (39) gives $k_B = \frac{(2 - h_B R)(2(2 - h_B) - R)}{2(1 - h_B)^2 R}$, $r_D = \frac{2(1 - h_B)R}{2(2 - h_B) - h_B R}$ and $\rho = \frac{2 - h_B R}{2(2 - h_B) - R}$. To show that r_D, ρ and k_B are positive, we start by showing that $2 - h_B R > 0$ and $2(2 - h_B) - R > 0$ for any $R < \overline{R}$. Substituting (42) into $2 - h_B R$, we obtain

$$
\begin{aligned}
2 - h_B \overline{R} &= \frac{2(1 + \eta)h_B - h_B\left(2(1 + \eta) - (1 - h_B)^2 - (1 - h_B)\sqrt{4\eta(1 + \eta) + (1 - h_B)^2}\right)}{(1 + \eta)h_B} \\
&= \frac{(1 - h_B)\left(1 - h_B + \sqrt{4\eta(1 + \eta) + (1 - h_B)^2}\right)}{(1 + \eta)} > 0.
\end{aligned}\tag{43}
$$

Then, substituting (42) into $2(2 - h_B) - \overline{R}$, we obtain

$$
\begin{aligned}
2(2 - h_B) - \overline{R} &= \frac{(4 - 2h_B)(1 + \eta)h_B - \left(2(1 + \eta) - (1 - h_B)\left(1 - h_B + \sqrt{4\eta(1 + \eta) + (1 - h_B)^2}\right)\right)}{(1 + \eta)h_B} \\
&= (1 - h_B)\frac{\sqrt{4\eta(1 + \eta) + (1 - h_B)^2} - (1 + 2\eta)(1 - h_B)}{(1 + \eta)h_B}.
\end{aligned}
$$

The sign of the numerator is the same as the sign of

$$4\eta(1+\eta) + (1-h_B)^2 - (1+2\eta)^2(1-h_B)^2.$$

This simplifies to

$$4\eta(1+\eta)\left(1 - (1-h_B)^2\right) > 0,$$

so that

$$2(2-h_B) - \overline{R} > 0. \tag{44}$$

This implies that r_D is positive and less than R as $h_B < 1$ and

$$R - r_D = \frac{(2-h_BR)R}{(2(2-h_B) - h_BR))} > 0 \text{ for } R \leq \overline{R}.$$

Finally, it can be seen that $\rho > \frac{R}{2}$, as

$$\rho - \frac{R}{2} = \frac{(R-2)^2}{2(2(2-h_B) - R)} > 0 \text{ for } R \leq \overline{R}.$$

It follows from (43) and (44) that $k_B > 0$. Also, $k_B < 1$ since, using the expression for k_B in the proposition, we have

$$2(1-h_B)^2R - (2-h_BR)(2(2-h_B) - R) = (R-2)(2(2-h_B) - h_BR) > 0 \text{ for } R < \overline{R}.$$

Substituting then the expressions for k_B and r_D into $\overline{r}_B = r_D(1-k_B)$ gives $\overline{r}_B = \frac{R-2}{1-h_B}$. This completes the first part of the proposition.

For $R > \overline{R}$, deposits are in short supply and in this case $u > 1$. The equilibrium level of u is found from

$$\frac{k_B}{1-k_B} = \eta, \tag{45}$$

where k_B is given by (41). Solving gives

$$u = \frac{2(1+\eta) - (1-h_B)\left(1 - h_B + \sqrt{4\eta(1+\eta) + (1-h_B)^2}\right)}{2(1+\eta)(2-h_B)} \frac{R}{2}. \tag{46}$$

We know from the definition of $\overline{R}$ that $u = 1$ at $R = \overline{R}$. It follows from (46) that $u > 1$ for $R > \overline{R}$. Also, from (44) it follows that $u < \frac{R}{2}$ since $(2(2-h_B) - h_BR) > 0$ and thus $\frac{h_B\overline{R}}{2(2-h_B)} < 1$.

From (45) we have $k_B = \frac{\eta}{1+\eta}$. Closed form solutions for r_D and ρ can be found by using (46) in the expressions (40) and (39). To check that $r_D < R$, we calculate

$$R - r_D = \frac{(2u - h_B R)R}{(2u(2 - h_B) - R)}.$$

Substituting for u from (46), the numerator becomes

$$2u - h_B R = \left(\frac{2(1 + \eta) - (1 - h_B)\left(1 - h_B - \sqrt{4\eta(1 + \eta) + (1 - h_B)^2}\right)}{(1 + \eta)(2 - h_B)} - h_B \right) R$$

$$= \left(\frac{1 + \eta h^2 + (1 - h_B)\left(2\eta + \sqrt{4\eta(1 + \eta) + (1 - h_B)^2}\right)}{(1 + \eta)(2 - h_B)} \right) R > 0.$$

Substituting the expression for u from (46), the denominator becomes

$$2u(2 - h_B) - R = \left(\frac{2(1 + \eta) - (1 - h_B)\left(1 - h_B - \sqrt{4\eta(1 + \eta) + (1 - h_B)^2}\right)}{(1 + \eta)} - 1 \right) R$$

$$= \left(\frac{1 + \eta - (1 - h_B)\left(1 - h_B - \sqrt{4\eta(1 + \eta) + (1 - h_B)^2}\right)}{(1 + \eta)} \right) R > 0.$$

This implies that $r_D < R$ for $R > \overline{R}$. Moreover, it is easy to see that $\rho > \frac{R}{2}$, since

$$\rho - \frac{R}{2} = \frac{(R - 2u)^2}{2\left(2u(2 - h_B) - R\right)} > 0.$$

Substituting then the expressions for k_B and r_D into $\overline{r}_B = r_D(1 - k_B)$ gives $\overline{r}_B = \frac{2u(1 - h_B)R}{(1 + \eta)(2u(2 - h_B) - h_B R)}$. This completes the second part of the proposition. $\square$

Proof of Proposition 3: Since there are no bankruptcy costs, there are no efficiency gains from having capital in the banks. This means it is always possible to set up a bank with $r_D = R$ and $k_B = 0$ such that

$$EU_D = \int_0^R r \frac{1}{R} dr = \frac{R}{2}.$$

Thus, in equilibrium depositors must always receive $EU = \frac{R}{2}$. Since capital providers can always invest directly in the risky technology, they receive at least $\frac{R}{2}$ as well. Since total

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output with no bankruptcy costs is $\frac{R}{2}$ for each unit invested, the capital providers will earn exactly $\frac{R}{2}$. So one equilibrium involves all depositors using banks with no capital and all capital providers investing in their alternative opportunity. However, there exist many other equilibria. In these, banks choose a level of capital k_B and r_D such that $E\Pi_B = 0$ and $EU_D = \frac{R}{2}$. Substituting $\rho = \frac{R}{2}$ in (3) and solving $E\Pi_B = 0$ with respect to r_D gives r_D as in the proposition. $\square$

Proof of Proposition 4: Using the expression for $\overline{R}$ from Proposition 2, it can be seen that $\underset{h\to 1}{Lim}\overline{R} \to 2$. For $R \leq \overline{R}$, substituting $\overline{R}$ into the expression for k_B in part i) of Proposition 2 and taking the limit for $h_B \to 1$ gives $k_B \to \frac{\eta}{1+\eta}$. Similarly, it can be shown that at $R = \overline{R}$, $\underset{h_B\to 1}{Lim} r_D \to \frac{1+\eta}{1+\eta+\sqrt{\eta(1+\eta)}}$, $\underset{h_B\to 1}{Lim}\rho \to \frac{R}{2} \to \frac{\overline{R}}{2}$ and $\underset{h_B\to 1}{Lim} EU_D \to \frac{\overline{R}}{2}$. The case for $R > \overline{R}$ can be shown similarly. The limit equilibrium with $h_B \to 1$ is one of the multiple equilibria in Proposition 3. It is the one with $k_B = \frac{\eta}{1+\eta}$ and $r_D = \frac{(1-\sqrt{k_B})}{1-k_B}R = \frac{1+\eta}{1+\eta+\sqrt{\eta(1+\eta)}}R$. $\square$

Proof of Proposition 5: It was argued above that equilibrium is inconsistent with $\overline{r}_F < \overline{r}_B$ so that any equilibrium must involve $\overline{r}_F \geq \overline{r}_B$. We show that $\overline{r}_F > \overline{r}_B$ cannot hold in equilibrium and that equilibrium entails $k_B = 0$ and $r_L = r_D$.

Suppose there exists a candidate equilibrium, defined as X, with

$$k_B' > 0, k_F' > 0, r_L' > r_D', \rho' \geq \frac{R}{2}, u' \geq 1, \ \overline{r}_F' = r_L'(1 - k_F') > \overline{r}_B' = \frac{r_D'(1 - k_B')(1 - k_F')}{h_F}. \tag{47}$$

This cannot be an equilibrium because, by transferring the capital of the bank to the firm and aligning the bankruptcy thresholds of the bank and the firm, it is possible to reduce overall bankruptcy costs. To see this, consider the following deviation, which we denote Z, where

$$k_B^* = 0, k_F^* = k_B'(1 - k_F') + k_F', r_D^* = r_L^* = r_D', \overline{r}_F^* = \overline{r}_B^* = r_L^*(1 - k_F^*) < \overline{r}_B'. \tag{48}$$

It can be seen from Figure 3 that this deviation eliminates the firm bankruptcy costs represented by Region $D1 + D2$, and the bank bankruptcy costs represented by $E2$. The shareholders are better off by the amount $D1 + D2$ and the depositors are better off by the amount $E2$. This implies that the deviation Z represents a Pareto improvement.

When $k_B' = 0$, it must be the case that $r_L' = r_D'$ for bank expected profits to be zero. In this case, $\overline{r}_F' = \overline{r}_B'$ and this is the equilibrium since no profitable deviation is possible. The choice of the optimal value of k_F and r_L are then identical to the choice of k_B and

r_D in the case when the bank invests directly in the risky asset except the liquidation proceeds are $h_F h_B r$ instead of $h_B r$. $\square$

Proof of Proposition 6: It can be seen that an equilibrium exists for part i) (part ii)) as in Proposition 1 for $h_B = 0$ (2 for $h_B > 0$) with $k_B = \widehat{k}_B$ and $k_F = 0$. Choose any non-negative k_B and k_F such that (17) is satisfied and $\overline{r}_F$ lies in the range $[\overline{r}_B, R]$. Then it follows from the proof of Proposition 5 that, since $h_F = 1$, $Y_Z = Y_X$, the shareholders of the banks and firms will have the same expected return as in Proposition 1(2). Although r_D, ρ and u remain constant, r_L adjusts to ensure banks' and firms' shareholders earn ρ.

To find N_F and N_B, recall that from Proposition 1 it holds $\widehat{N}_B = \frac{K}{\widehat{k}_B}$ and from (17) it is

$$\widehat{k}_B = 1 - (1 - k_B)(1 - k_F)$$

or

$$k_F = 1 - \frac{1 - \widehat{k}_B}{1 - k_B}. \tag{49}$$

Then, substituting (12) and (49) into (10), we obtain

$$N_F \left(1 - \frac{1 - \widehat{k}_B}{1 - k_B} \right) (1 - k_B) + N_F k_B = K,$$

which simplifies to $N_F \widehat{k}_B = K$ and thus

$$N_F = \widehat{N}_B = \frac{K}{\widehat{k}_B}.$$

Substituting this and (49) into (12) gives

$$N_B = \frac{1 - \widehat{k}_B}{1 - k_B} \frac{K}{\widehat{k}_B} < \widehat{N}_B$$

since $\widehat{k}_B > k_B$. This completes the proposition. $\square$

Proof of Proposition 7: Consider the two-sector banks first. Solving (20) equal to u, we find

$$k_B = 1 - \frac{\sqrt{r_D(r_D - u)}}{\sqrt{2}r_D^2} R.$$

Substituting this into (18), differentiating with respect to r_D and solving for r_D gives

$$r_D = \frac{u(3\rho - u)}{2\rho}. \tag{50}$$

Putting this back in the expression for k_B gives

$$k_B = 1 - \frac{\rho R \sqrt{2(3\rho^2 - 4\rho u + u^2)}}{u(3\rho - u)^2}. \tag{51}$$

Substituting (50) and (51) in (18), we obtain

$$E\Pi_B = \frac{R(9\rho u - 3u^2 + 2(\rho - u)\sqrt{2(3\rho^2 - 4\rho u + u^2)})}{6u(3\rho - u)} - \rho. \tag{52}$$

To see that two-sector banks always dominate one-sector banks, we consider the difference in the expected profits between the two. Substracting (34) from (52) and simplifying gives

$$-\frac{R(\rho - 1)\sqrt{3\rho^2 - 4\rho u + u^2}}{6u(6\rho^2 - 5\rho u + u^2)} \left[3\sqrt{(3\rho^2 - 4\rho u + u^2)} - 2\sqrt{2}(2\rho - u)\right].$$

Since $\rho \geq u$, the denominator is positive. Also, for $\rho = u = 1$, $3\rho^2 - 4\rho u + u^2 = 0$. Differentiating with respect to ρ gives $6\rho - 4u > 0$. So $3\rho^2 - 4\rho u + u^2 > 0$ for $\rho > u \geq 1$. This implies that two-sector banks dominate if

$$2\sqrt{2}(2\rho - u) > 3\sqrt{(3\rho^2 - 4\rho u + u^2)}.$$

Squaring both sides and simplifying gives

$$5\rho^2 + 4\rho u - u^2 > 0.$$

This implies that two-sector banks dominate.

To characterize the equilibrium with two-sector banks, we now solve (52) equal to zero with respect to ρ after setting $u = 1$ gives one real positive solution as follows

$$\rho_1(R) = \frac{3 + 9R - 3R^2}{(27 - 2R^2)} + \frac{\sqrt{3(12 - 36R + 83R^2 - 40R^3 + 6R^4)}\left[Cos(\frac{t}{3}) + \sqrt{3}\,Sin(\frac{t}{3})\right]}{(27 - 2R^2)},$$

where

$$t = Arctan\left(\frac{R(R-2)(27-2R^2)\sqrt{6(R-2)(-8+36R-6R^2+3R^3}}{(-72+324R+954R^2-1557R^3+960R^4-258R^5+24R^6)}\right).$$

The derivation of this solution makes use of de Moivre's Theorem and it is available from the authors upon request. Depositors make use of their alternative investment opportunity and $u = 1$ if

$$\frac{k_B}{1-k_B} > \eta. \tag{53}$$

Substituting the solution $\rho = \rho_1(R)$ into (51), and solving (53) with equality with respect to R gives

$$\overline{R} = \frac{(3\overline{\rho}-1)^2}{\overline{\rho}(1+\eta)\sqrt{2(3\overline{\rho}^2-4\overline{\rho}+1)}},$$

where $\overline{\rho} = \rho_1(\overline{R})$. Putting $u = 1$ in (50) and (51) completes part i) of the proposition.

For $R > \overline{R}$, $u > 1$ and all deposits are used in the banking sector. The equilibrium level of u is found again from

$$\frac{k_B}{1-k_B} = \eta,$$

where k_B is given by (51). Rearranging the expression gives

$$\frac{u(3\rho-u)^2}{\rho R\sqrt{2(3\rho^2-4\rho u+u^2)}} = 1+\eta \tag{54}$$

Substituting then r_D as in (50) and k_B as in (51) into (18) gives

$$\Pi(\rho,u) = \frac{6\rho u(u-3\rho)+R(9\rho u-3u^2+2(\rho-u)\sqrt{2(3\rho^2-4\rho u+u^2)}}{6u(3\rho-u)}. \tag{55}$$

Solving (54) and (55) equal to zero with respect to u and ρ gives the solutions $u = u(R)$ and $\rho = \rho_2(R)$. This completes part ii) of the proposition. $\square$

Proof of Proposition 8: Suppose first that there exists a two-sector bank with $\frac{r'_L}{2} \geq r'_D(1-k'_B)$ that pays $\rho' \geq \frac{R}{2}$ per unit of capital to its shareholders and gives $EU = u' < \frac{R}{2}$ per unit of funds to its depositors. Given that in any equilibrium banks must make zero expected profits, $k'_B > 0$ must hold. Otherwise, $k'_B = 0$, banks would have positive expected profits since they obtain $\frac{r_L}{2}$ from both sectors with positive probability. Consider then a conduit one-sector bank with $r^*_D = r^*_L = r'_L, k^*_B = 0$ and $k^*_F = k'_F$. Firms are indifferent between borrowing from two-sector and one-sector banks because

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their shareholders earn ρ' per unit of capital in both cases. The total expected payment from firms to two-sector banks is

$$\int_0^{\overline{r}'_F} \int_{\overline{r}'_F}^R \frac{r'_L}{2} \frac{1}{R^2} dr_1 dr_2 + \int_{\overline{r}'_F}^R \int_0^{\overline{r}'_F} \frac{r'_L}{2} \frac{1}{R^2} dr_1 dr_2 + \int_{\overline{r}'_F}^R \int_{\overline{r}'_F}^R r'_L \frac{1}{R^2} dr_1 dr_2 = \frac{r'_L \left(R - r'_L (1 - k'_F) \right)}{R}.$$

The total expected payment from firms to one-sector banks is

$$\int_{\overline{r}^*_F}^R r^*_L \frac{1}{R} dr_1 = \frac{r^*_L \left(R - r^*_L (1 - k^*_F) \right)}{R} = \frac{r'_L \left(R - r'_L (1 - k'_F) \right)}{R}.$$

With one-sector banks the total expected payment is received by depositors in return for the one unit of funds that they supply. With two-sector banks the one unit of funds is supplied partially by depositors and partially by shareholders because $k'_B > 0$. Since the shareholders in the two-sector banks receive a return $\rho' \geq \frac{R}{2}$, the depositors in the one-sector bank must be strictly better off than the depositors in the two-sector bank. Therefore it is possible for the one-sector bank to reduce r^*_D so that positive profits are made. Hence, there cannot be two-sector banks in equilibrium.

Consider now the case where $\frac{r'_L}{2} < r'_D (1 - k'_B)$ and the bank remains solvent only when firms in both sectors repay r_L. Here the total expected payment received by depositors and shareholders in the two-sector banks is less than the total expected payments to the depositors in one-sector banks,

$$\int_{\overline{r}'_F}^R \int_{\overline{r}'_F}^R r'_L \frac{1}{R^2} dr_1 dr_2 = \frac{r'_L \left(R - r'_L (1 - k'_F) \right)^2}{R^2} < \frac{r^*_L \left(R - r^*_L (1 - k^*_F) \right)}{R},$$

since $r^*_L = r'_L$ and $k^*_F = k'_F$. This implies that one-sector banks again dominate.

Consider next the one-sector bank equilibrium as in Proposition 5. A similar argument to above can be used to show that two-sector banks cannot compete. Two-sector banks must offer contracts to firms that are at least as attractive as those offered by the one-sector banks. Suppose they offer the same contract. Then the output of two-sector banks is either equal or less depending on whether $\frac{r'_L}{2}$ is greater or smaller than $r'_D (1 - k'_B)$. In the former case, it is not possible to make depositors and shareholders as well off since output is the same and depositors in one-sector banks receive it all while they do not in two-sector banks. In the latter case, when output is strictly less, it is again clear that depositors cannot be made as well off in two-sector banks. $\square$

Proof of Proposition 9: Each firm's expected profit remains as in (23). Banks'

expected profits and depositors' utility are as in (56) and (57) when $\frac{r_L}{2} > r_D(1 - k_B)$ and as in (58) and (59) when $\frac{r_L}{2} < r_D(1 - k_B)$ after substituting $h_F = 1$.

The two-sector banks dominate one-sector banks just as in Proposition 7 with $r_L = R$ and $k_F = 0$. The multiplicity of equilibria with respect to k_B and k_F that satisfy (26) and r_L such that $E\Pi_B = E\Pi_F = 0$ follows similarly to the proof of Proposition 6. $\square$

B Derivation of the equilibrium in the case of two publicly traded sectors with $h_B = 0$ and $h_F < 1$

In the case where $\frac{r_L}{2} > r_D(1 - k_B)$ and Figure 6 is relevant, a two-sector bank's maximization problem is given by:

$$
\begin{aligned}
\underset{r_L, r_D, k_F, k_B}{Max} E\Pi_B \;=\; & \int_0^{\overline{x}_B} \int_{\overline{x}_B - r_2}^{\overline{r}_F} \left(\frac{h_F(r_1 + r_2)}{2(1 - k_F)} - r_D(1 - k_B) \right) \frac{1}{R^2} dr_1 dr_2 \qquad (56) \\
& + \int_{\overline{x}_B}^{\overline{r}_F} \int_0^{\overline{r}_F} \left(\frac{h_F(r_1 + r_2)}{2(1 - k_F)} - r_D(1 - k_B) \right) \frac{1}{R^2} dr_1 dr_2 \\
& + \int_0^{\overline{r}_F} \int_{\overline{r}_F}^{R} \left(\frac{r_L}{2} + \frac{h_F r_2}{2(1 - k_F)} - r_D(1 - k_B) \right) \frac{1}{R^2} dr_1 dr_2 \\
& + \int_{\overline{r}_F}^{R} \int_0^{\overline{r}_F} \left(\frac{h_F r_1}{2(1 - k_F)} + \frac{r_L}{2} - r_D(1 - k_B) \right) \frac{1}{R^2} dr_1 dr_2 \\
& + \int_{\overline{r}_F}^{R} \int_{\overline{r}_F}^{R} (r_L - r_D(1 - k_B)) \frac{1}{R^2} dr_1 dr_2 - \rho k_B,
\end{aligned}
$$

subject to

$$
EU_D = \int_0^{\overline{x}_B} \int_{\overline{x}_B - r_2}^{R} r_D \frac{1}{R^2} dr_1 dr_2 + \int_{\overline{x}_B}^{R} \int_0^{R} r_D \frac{1}{R^2} dr_1 dr_2 = u \geq 1 \qquad (57)
$$

together with (23) and the usual non-negativity constraints.

The first two terms in (56) represent the net payoff to the bank in Region B where both firms go bankrupt and the bank receives $\frac{h_F(r_1 + r_2)}{2(1 - k_F)} > r_D(1 - k_B)$. The third and fourth terms are the expected payoff to the bank in Regions C and D where firms in only one sector remain solvent and the bank receives $\frac{r_L}{2} + \frac{h_F r_i}{2(1 - k_F)}$. The fifth term is the payoff in Region E where all firms remain solvent and the bank receives r_L. The last term ρk_B is the usual opportunity cost for bank shareholders. Constraint (57) requires depositors' expected utility to be at least u. Depositors receive r_D whenever the bank is solvent. The first term represents the payoff to depositors in Regions B_1 and D_1. The second term groups together all other regions, i.e., $B2 + C + D2 + E$, in the figure for $r_1 \in [0, R]$ and

$r_2 \in [\overline{x}_B, R]$. Constraint (23) requires that the firms' shareholders obtain at least their opportunity cost ρ.

In the case where $\frac{r_L}{2} < r_D(1 - k_B)$ and Figure 7 is relevant, a two-sector bank's maximization problem is given by:

$$\underset{r_L, r_D, k_F, k_B}{Max} E\Pi_B = \int_{r_2^*}^{\overline{\tau}_F} \int_{\overline{x}_B - r_2}^{\overline{\tau}_F} \left(\frac{h_F(r_1 + r_2)}{2(1 - k_F)} - r_D(1 - k_B) \right) \frac{1}{R^2} dr_1 dr_2 \qquad (58)$$
$$+ \int_{\overline{\tau}_F}^{R} \int_{\widehat{r}_1}^{\overline{\tau}_F} \left(\frac{h_F r_1}{2(1 - k_F)} + \frac{r_L}{2} - r_D(1 - k_B) \right) \frac{1}{R^2} dr_1 dr_2$$
$$+ \int_{\widehat{r}_2}^{\overline{\tau}_F} \int_{\overline{\tau}_F}^{R} \left(\frac{r_L}{2} + \frac{h_F r_2}{2(1 - k_F)} - r_D(1 - k_B) \right) \frac{1}{R^2} dr_1 dr_2$$
$$+ \int_{\overline{\tau}_F}^{R} \int_{\overline{\tau}_F}^{R} (r_L - r_D(1 - k_B)) \frac{1}{R^2} dr_1 dr_2 - \rho k_B,$$

subject to

$$EU_D = \int_{\widehat{r}_2}^{r_2^*} \int_{\overline{\tau}_F}^{R} r_D \frac{1}{R^2} dr_1 dr_2 + \int_{r_2^*}^{\overline{\tau}_F} \int_{\overline{x}_B - r_2}^{R} r_D \frac{1}{R^2} dr_1 dr_2 \qquad (59)$$
$$+ \int_{\overline{\tau}_F}^{R} \int_{\widehat{r}_1}^{R} r_D \frac{1}{R^2} dr_1 dr_2 = u \geq 1,$$

(23) and the usual non-negativity constraints.

The problem is similar to before. The first term in (58) represents the net payoff to the bank in Region B where both sectors go bankrupt. The second and third terms are the net payoff in Regions C and D, where either sector 1 or 2 goes bankrupt. Finally, the last term is the payoff in Region E where all firms remain solvent. Constraint (59) requires that depositors obtain at least u in expectation. The first term represents the payoff to depositors in Region D_1; the second term corresponds to Regions B and D_2, while the last one groups Regions C and E.

Finally, for the case where $\frac{r_L}{2} = r_D(1 - k_B)$ and $\widehat{r}_i = 0$, the bank still maximizes (58) subject to (59) and (23) once $\widehat{r}_i = 0$ is substituted in.

The main difference with two publicly traded sectors is the possibility of a mixed equilibrium with both one-sector and two-sector banks. When the equilibrium is not mixed, that is it involves only one type of banks, then the equilibrium conditions are similar to conditions 1 to 9 in Sections 3 and 4.1 with the only difference that in (10) and (12) N_F refers now to the total number of firms in the two sectors with half of this total in each sector.

When the equilibrium is mixed, however, the market clearing conditions change from those above. We use N_{Fj} with $j = 1, 2$ to denote the number of firms borrowing from

j-sector banks and N_{Bj} for the number of j-sector banks. Similarly, k_{Fj} is the capital used by firms borrowing from j-sector banks and k_{Bj} is the capital used by j-sector banks. The market clearing condition for capital is then

$$N_{F1}k_{F1} + N_{F2}k_{F2} + N_{B1}k_{B1} + N_{B2}k_{B2} \leq K. \tag{60}$$

The market clearing condition for deposits is

$$N_{B1}(1 - k_{B1}) + N_{B2}(1 - k_{B2}) \leq D. \tag{61}$$

The loan market condition is

$$N_{Fj}(1 - k_{Fj}) = N_{Bj}. \tag{62}$$

As before, one-sector banks optimally set $k_{B1} = 0$. Substituting (62) into the ratio of (60) and (61) gives

$$\eta = \frac{N_{F1}k_{F1} + N_{F2}k_{F2} + N_{F2}(1 - k_{F2})k_{B2}}{N_{F1}(1 - k_{F1}) + N_{F2}(1 - k_{F2})(1 - k_{B2})}. \tag{63}$$

Defining the proportion of firms borrowing from one-sector banks as $\upsilon = \frac{N_{F1}}{N_{F1}+N_{F2}}$ and the proportion of firms borrowing from two-sector banks as $1 - \upsilon = \frac{N_{F2}}{N_{F1}+N_{F2}}$, (63) becomes

$$\eta = \frac{\upsilon k_{F1} + (1 - \upsilon)\left[k_{F2} + (1 - k_{F2})k_{B2}\right]}{\upsilon(1 - k_{F1}) + (1 - \upsilon)(1 - k_{F2})(1 - k_{B2})}. \tag{64}$$

This now becomes the market clearing condition when we solve for the mixed equilibrium.

Solving for equilibrium in the case of two publicly traded sectors is complex. To do this, we start by finding $k_F = k_F(r_L)$ from (23) equal to zero and $k_B = k_B(r_L, k_F, r_D)$ from either (57) and (59) equal to $u \geq 1$. We can further substitute k_F into k_B and find $k_B(r_L, r_D)$. Substituting $k_F(r_L)$ and $k_B(r_L, r_D)$ into the expression for the bank's expected profits as in (56) or (58) as appropriate gives $E\Pi_B = E\Pi_B(r_L, r_D)$.

To define the three regions for the two-sector bank's maximization problem as a function of r_D and r_L where $\frac{r_L}{2} \gtreqqless r_D(1-k_B)$, we next consider the boundaries given by $\widehat{r}_i = 0$ and $r_i^* = 0$ as defined by (28) and (29), respectively. This allows us to obtain $\widehat{r}_D = \widehat{r}_D(r_L)$ and $r_D^* = r_D^*(r_L)$ and define the regions as illustrated in Figure 8. Region I is the case illustrated in Figure 6 where both $\widehat{r}_i < 0$ and $r_i^* < 0$. Region II represents the case where $\widehat{r}_i < 0$ and $r_i^* > 0$. This is the special case in Figure 7 where the boundaries below Regions C_1 and D_1 are the axes. Finally, Region III corresponds to the case illustrated in Figure 7 where $\widehat{r}_i > 0$ and $r_i^* > 0$.

We illustrate the form of equilibrium with two numerical examples where $R = 2.5$ or $R = 4$ and $\eta = 0.1$. We conjecture that the solution is where $\widehat{r}_i = 0$ and so $\widehat{r}_D = \widehat{r}_D(r_L)$.

Substituting this into $E\Pi_B(r_L, r_D)$ gives $E\Pi_B = E\Pi_B(r_L)$. We then optimize with respect to r_L and hence solve for the two-sector bank's maximization. We find the conjectured equilibrium ρ in the case where $R = 2.5$ so that $u = 1$ from $E\Pi_B(r_L) = 0$. Similarly, we find the conjectured equilibrium ρ and $u > 1$ in the case where $R = 4$ from $E\Pi_B(r_L) = 0$ and the market clearing conditions after substituting $k_F(r_L)$ and $k_B(r_L, \widehat{r}_D(r_L))$. Finally, we check that our conjectured solution with $\widehat{r}_i = 0$ is in fact the optimal solution through a grid search.

The solutions are shown in Tables 1 and 2 and discussed in Section 6.3.

h_F	type	r_L	k_F	r_D	k_B	ρ	u	N_F	N_B	Total deposits
0.9	$2S$	1.482	0.341	1.102	0.327	1.361	1	17.963	11.841	7.963
$\overline{h}_F = 0.825$	$2S$	1.422	0.423	1.080	0.341	1.333	1	16.124	9.302	6.124
0.8	$2S$	1.407	0.447	1.075	0.346	1.327	1	15.666	8.656	5.666
any	$1S$	1.25	0.6	1.25	0	1.333	1	16.667	6.667	6.667

Table 1: The allocation with one-sector ($1S$) and two-sector banks ($2S$) in the case of publicly traded productive sectors for $R = 2.5, K = 10$ and $D = 100$.

h_F	type	r_L	k_F	r_D	k_B	ρ	u	N_B	N_F	υ
1	$2S$	3.108	0.016	1.682	0.076	6.821	1.190	108.21	110	0
0.95	$2S$	2.970	0.021	1.599	0.071	6.989	1.133	107.65	110	0
0.9092	$2S$	2.870	0.025	1.539	0.067	7.150	1.091	107.22	110	0
$\overline{h}_F = 0.8994$	$2S$	2.848	0.026	1.525	0.066	7.191	1.081	107.12	110	0
$\overline{h}_F = 0.8994$	$1S$	2	0.081	2	0	7.191	1.081	0	0	0
0.899	$2S$	2.843	0.027	1.526	0.069	6.975	1.084	71.55	73.56	0.331
0.899	$1S$	2	0.084	2	0	6.975	1.084	33.37	36.44	0.331
$\underline{h}_F = 0.8981$	$2S$	2.831	0.030	1.528	0.074	6.545	1.091	0	0	1
$\underline{h}_F = 0.8981$	$1S$	2	0.091	2	0	6.545	1.091	100	110	1

Table 2: The allocation with one-sector ($1S$) and two-sector banks ($2S$) in the case of publicly traded productive sectors for $R = 4, K = 10$ and $D = 100$.
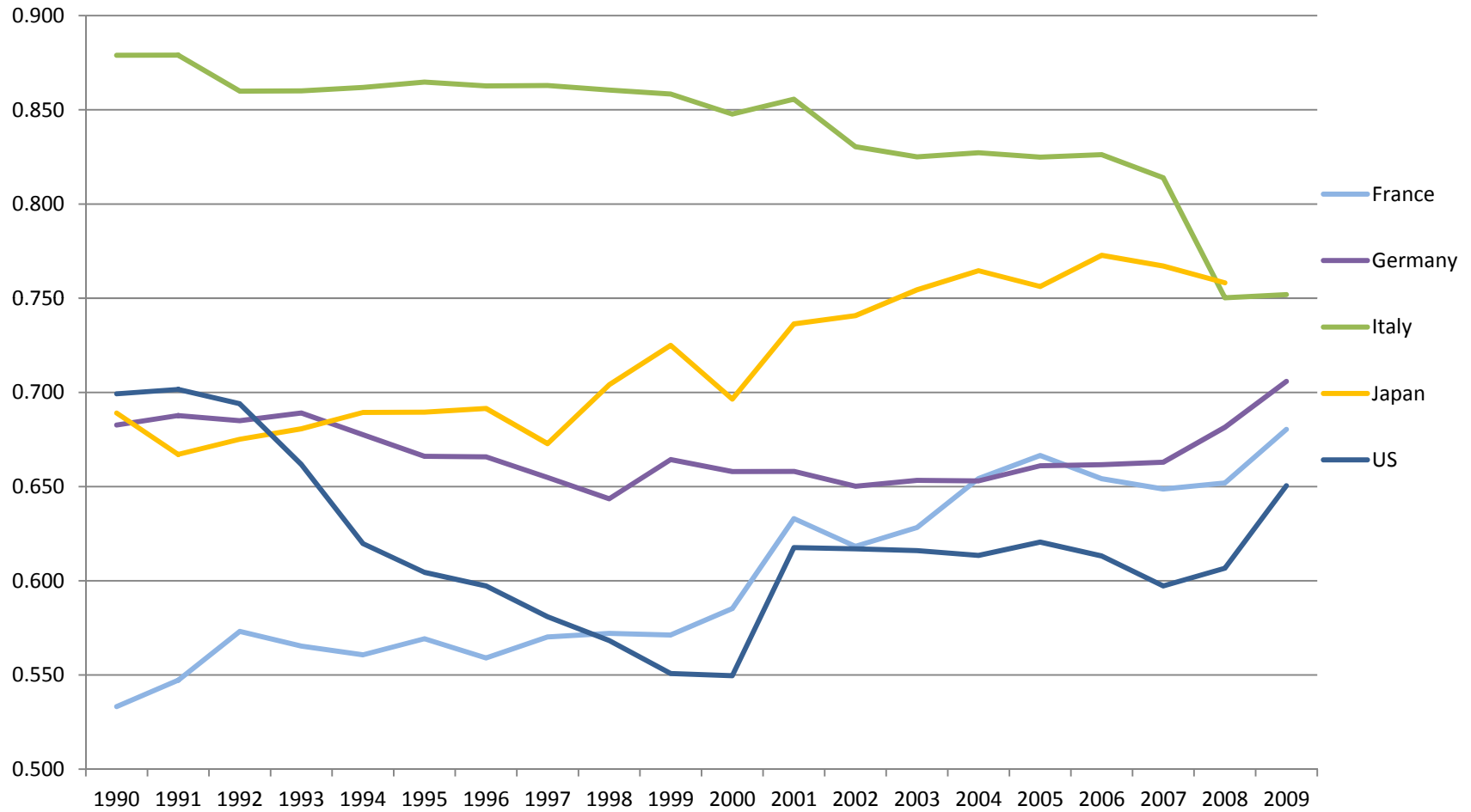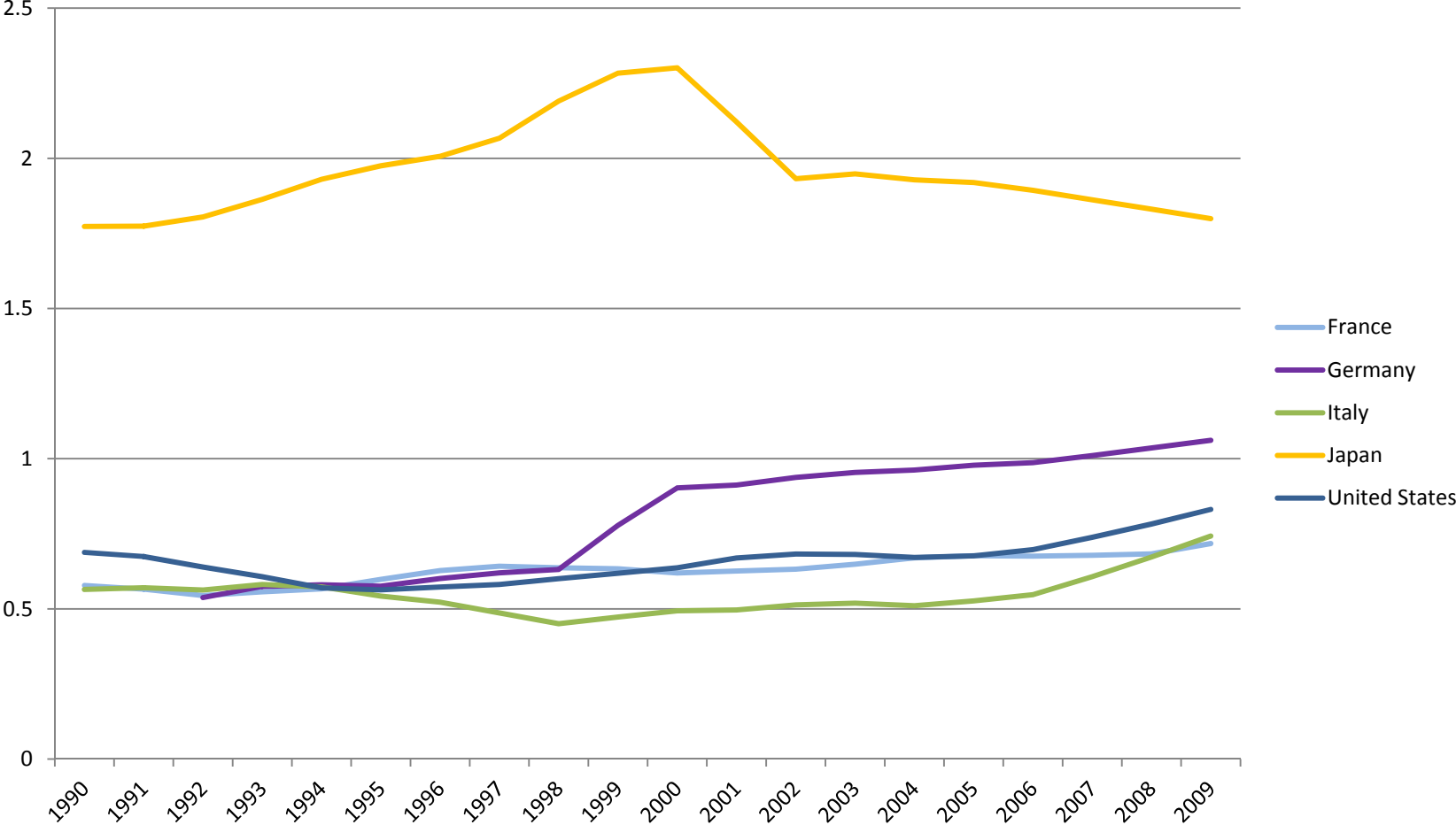
Figure 1: Deposits over bank liabilities

Figure 2: Bank deposits over GDP

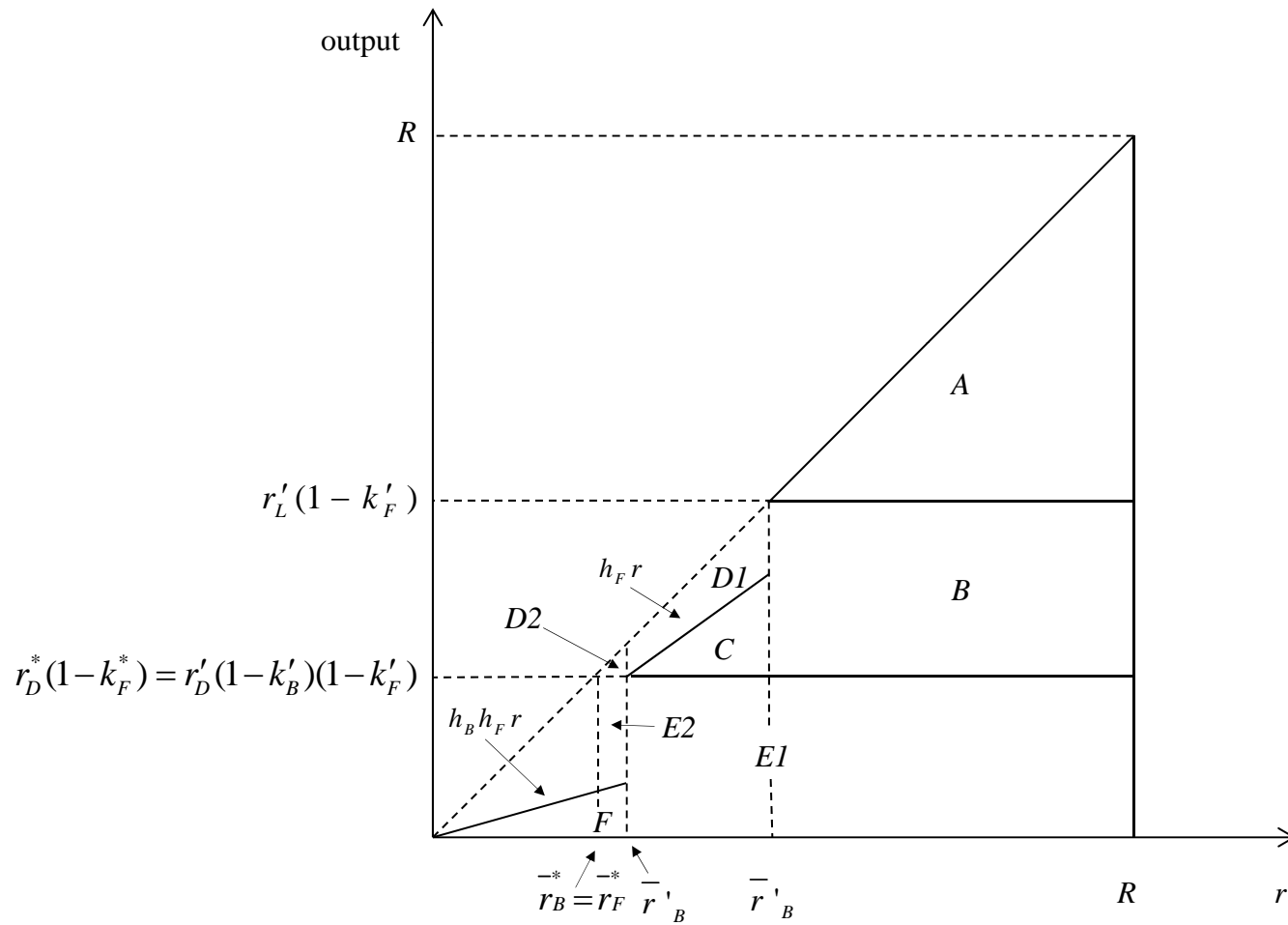Legend: France, Germany, Italy, Japan, United States

Figure 3: Output of a single firm and returns to shareholders and depositors as a function of the return r in the case of a single productive sector.
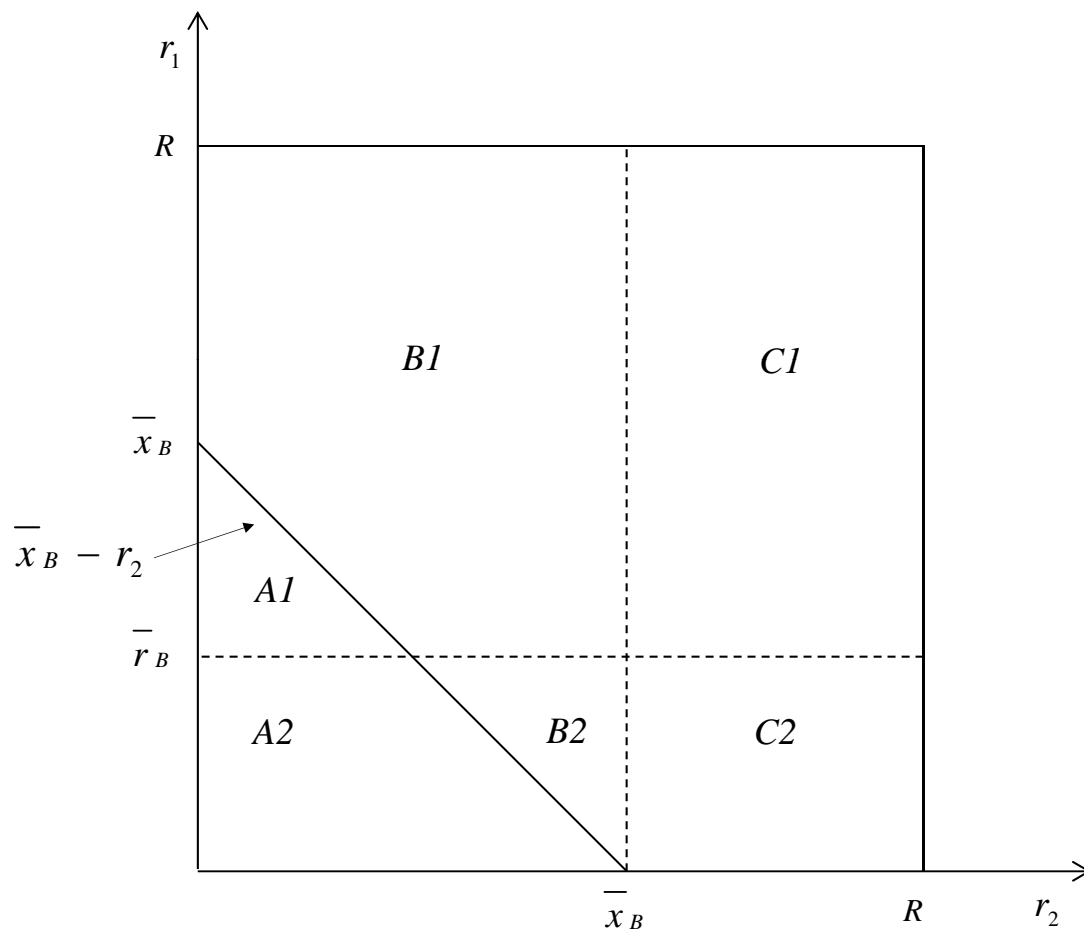
Figure 4: Regions where firms and banks go bankrupt as a function of r_1 and r_2 in the case of non-publicly traded productive sectors.
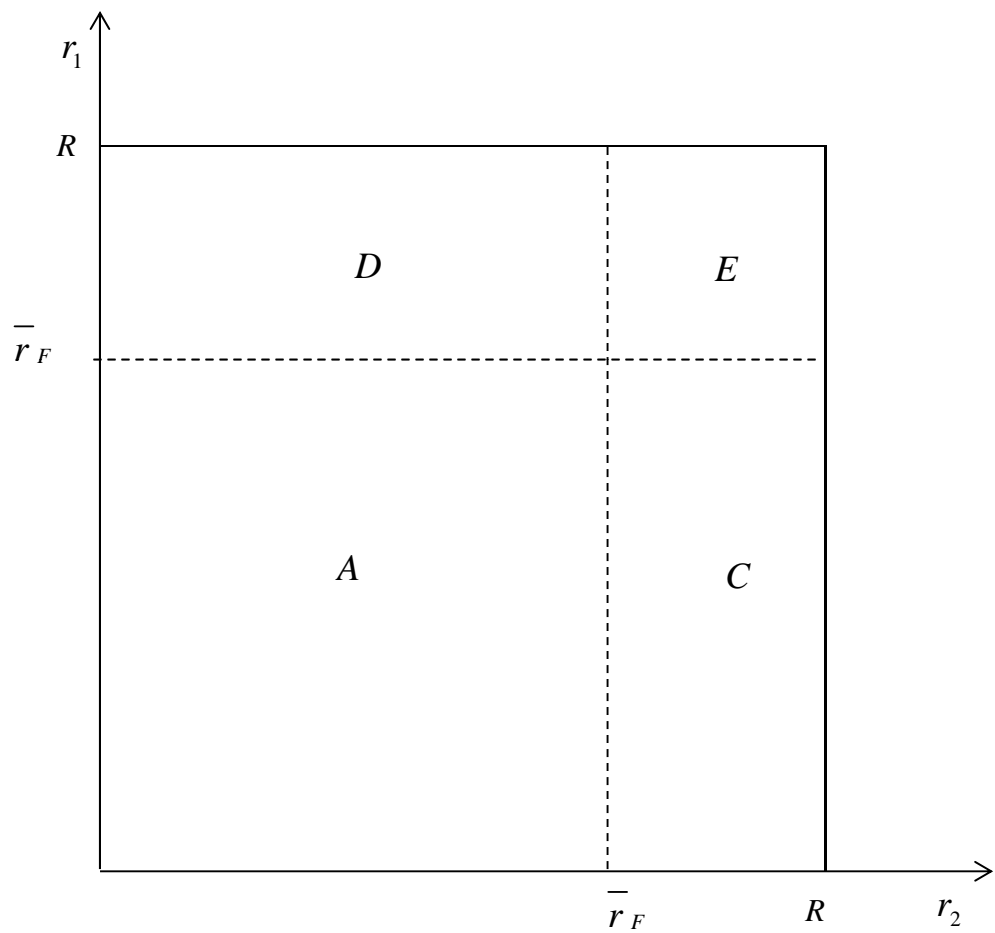
Figure 5: Regions where firms and banks go bankrupt as a function of r_1 and r_2 in the case of two publicly traded productive sectors when $h_B=h_F=0$.
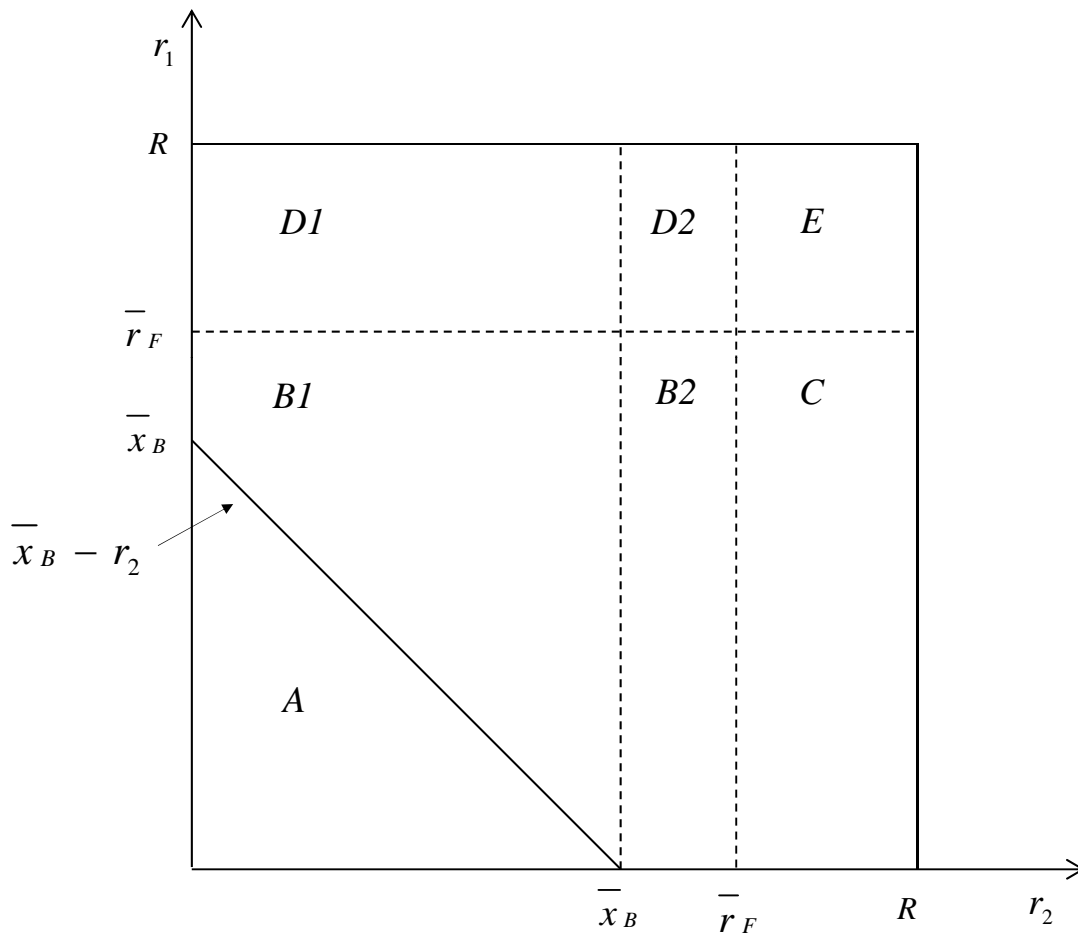
Figure 6: Regions where firms and banks go bankrupt as a function of r_1 and r_2 in the case of two publicly traded productive sectors for $r_L/2 > r_D(1 - k_B)$.
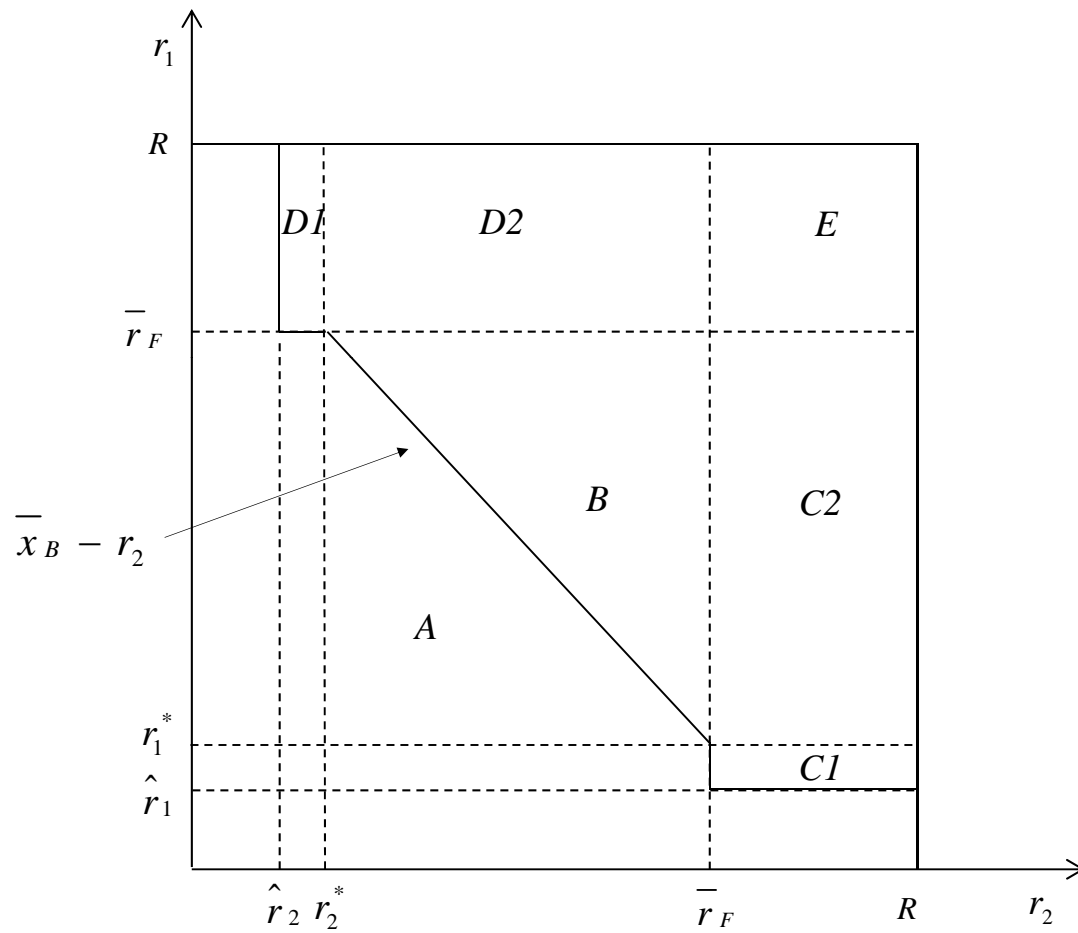
Figure 7: Regions where firms and banks go bankrupt as a function of r_1 and r_2 in the case of two publicly traded productive sectors for $r_L/2 < r_D(1 - k_B)$.
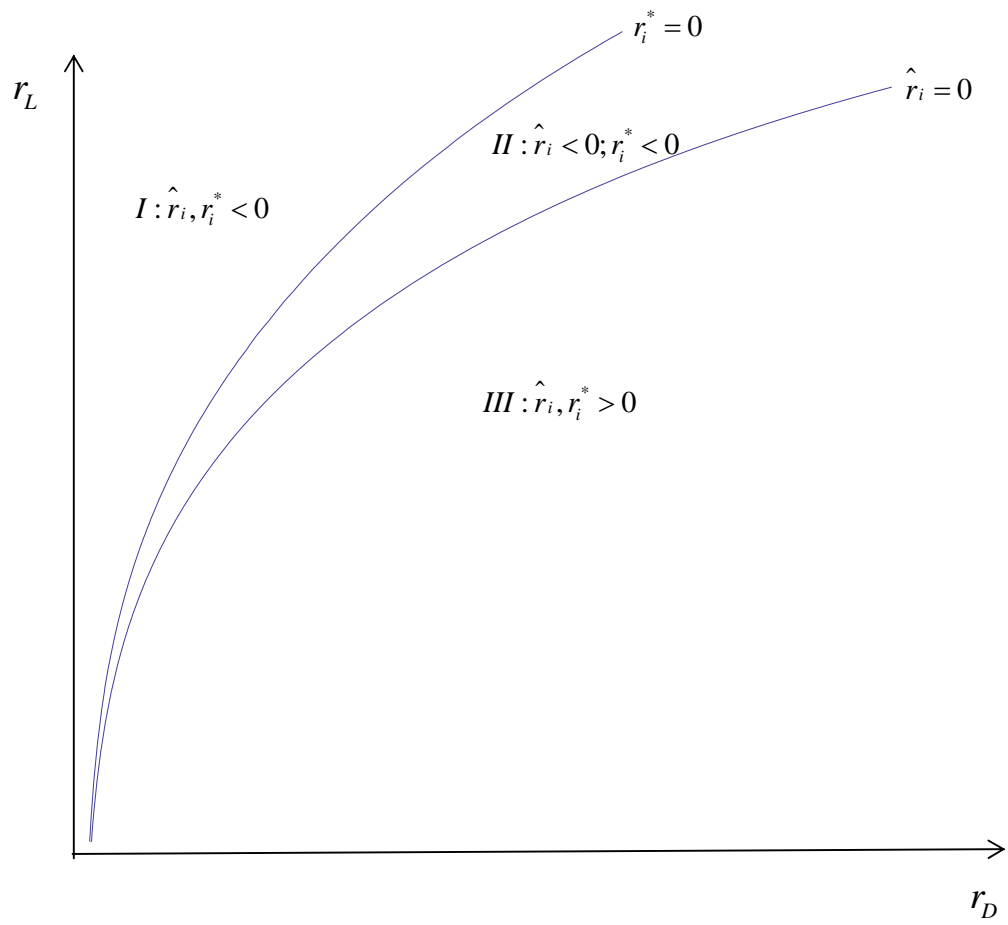
Figure 8: Regions defining the sign of the thresholds $\hat{r}_i$ and r_i^* as a function of r_D and r_L.