Trade and Uncertainty[*]

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Preliminary work in progress – comments welcome

Abstract

We offer a new explanation as to why international trade is so volatile in response to economic shocks. Our approach combines the uncertainty shock idea of Bloom (2009) with a model of international trade, extending the idea to the open economy. Firms import intermediate inputs from home or foreign suppliers, but with higher costs in the latter case. Due to fixed costs of ordering firms hold an inventory of intermediates. We show that in response to an uncertainty shock firms optimally adjust their inventory policy by cutting their orders of foreign intermediates disproportionately strongly. In the aggregate, this response leads to a bigger contraction in international trade flows than in domestic economic activity. We confront the model with newly-compiled monthly aggregate U.S. import data and industrial production data going back to 1962, and also with disaggregated data back to 1989. Our results suggest a tight link between uncertainty and the cyclical behavior of international trade.

Keywords: Uncertainty shock, trade collapse, inventory, real options, imports, intermediates

JEL Codes: E3, F1

1 Introduction

The recent global economic crisis was characterized by a sharp decline in economic output. However, the accompanying decline in international trade was even sharper, in some cases up to 50 percent. Standard models of international macroeconomics and international trade fail to account for the severity of the trade collapse.

In this paper, we attempt to explain why international trade is so volatile in response to economic shocks. On the theoretical side, we combine the uncertainty shock concept due to Bloom (2009) with a model of international trade. Bloom's (2009) real options approach is motivated by high-profile events that trigger an increase in uncertainty about the future path of the economy, for example the 9/11 terrorist attacks or the collapse of Lehman Brothers. In the wake of such events, firms adopt a 'wait-and-see' approach, slowing down their hiring and investment activities. Bloom (2009) shows that bouts of large uncertainty can be modeled as *second moment* shocks to demand or productivity and typically lead to sharp recessions. Once the degree of uncertainty subsides, firms revert to their normal hiring and investment patterns, and the economy recovers.

We extend the uncertainty shock approach to the open economy. In contrast to Bloom's (2009) closed-economy set-up, we develop a theoretical framework in which firms import intermediate inputs from foreign or domestic suppliers. This production structure is motivated by the empirical observation that a large fraction of international trade consists of capital-intensive intermediate goods such as car parts or electronic components.

Due to a fixed cost of ordering associated with transportation, firms hold an inventory of intermediate inputs. Following the inventory model with variable uncertainty by Hassler (1996), we show that in response to a large uncertainty shock to the demand for final products, firms optimally adjust their inventory policy by cutting their orders of foreign intermediates more strongly than orders for domestic intermediates. In the aggregate, this differential response leads to a bigger contraction and subsequently a stronger recovery in international trade flows than in domestic economic activity. Thus, international trade exhibits a larger variance and is more volatile than domestic economic activity. In a nutshell, uncertainty shocks magnify the response of international trade.

Our model generates a set of additional predictions. First, the magnification effect is increased by larger fixed costs of ordering. Intuitively, the larger the fixed costs of ordering, the more reluctant firms are to order intermediate inputs from abroad if uncertainty rises. This is a testable hypothesis to the extent that fixed ordering costs vary across domestic and foreign trading partners.

Second, the magnification effect is muted for industries characterized by high depreci-

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ation rates. Perishable goods are a case in point for extremely high depreciation rates. The fact that such goods have to be ordered frequently means that importers have little choice but to keep ordering them frequently even if uncertainty rises. Conversely, durable goods can be considered as the opposite case of very low depreciation rates. Our model predicts that for those goods we should expect the largest degree of magnification in response to uncertainty shocks. Intuitively, the option value of waiting can be most easily realized by delaying orders for durable goods. We find strong evidence of this pattern in the data when we examine the cross-industry response of imports to elevated uncertainty.

In sum, our model leads to various predictions in a unified framework. In contrast to conventional static trade models such as the gravity equation, we focus on the dynamic response of international trade. In addition, we focus on second moment shocks and thus move beyond the first moment shocks traditionally employed in the literature. Our approach is relevant for researchers and policymakers alike who seek to understand the recovery process in response to the Great Recession, and may also be relevant for understanding historical events like the Great Depression. It could also help predict the trajectory of international trade in future economic crises.

On the empirical side, we confront the model with high-frequency monthly U.S. import and industrial production data going back to 1962. Our results suggest a tight link between uncertainty shocks as identified by Bloom (2009) and the cyclical behavior of international trade. That is, the behavior of trade can be well explained with standard uncertainty measures such as a VXO stock market volatility index. Bloom (2009) identifies 17 high-volatility episodes since the early 1960s such as the assassination of JFK, the 1970s oil price shocks, the Black Monday of October 1987, the 1998 bailout of Long-Term Capital Management, 9/11 and the collapse of Lehman Brothers in September 2008. As Bloom (2009) shows, these high-volatility episodes are strongly correlated with alternative indicators of uncertainty.

In particular, we argue that the Great Trade Collapse of 2008/09 can to a large extent be explained by the large degree of uncertainty triggered by subprime lending and rising further up to, and especially after, the collapse of Lehman Brothers. According to our empirical results, the unusually large trade collapse of 2008/09 is thus a response to the unusually large increase in uncertainty at the time.[1] Although it stands out quantitatively, qualitatively the Great Trade Collapse is comparable to previous post-World War II slowdowns or contractions in international trade. In fact, our aim is to empirically account for trade recessions more generally, not only for the Great Trade Collapse. In addition, we

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[1]Similarly, Bloom, Bond and Van Reenen (2007) provide empirical evidence that fluctuations in uncertainty can lead to quantitatively large adjustments of firms' investment behavior.

confirm the cross-industry predictions coming from our theoretical model.

We are certainly not the first authors to consider general uncertainty and real option values in the context of international trade, but so far the literature has not examined the role of *uncertainty shocks* in an open-economy model of inventory investment. For example, Baldwin and Krugman (1989) adopt a real options approach to explain the hysteresis of trade in the face of large exchange rate swings but their model only features standard first moment shocks. More recently, the role of uncertainty has attracted new interest in the context of trade policy and trade agreements (Handley 2012, Handley and Limão 2012 and Limão and Maggi 2013). Similar to our approach, Taglioni and Zavacka (2012) empirically investigate the relationship between uncertainty and trade for a panel of countries with a focus on aggregate trade flows. But as they do not provide a theoretical mechanism, they do not speak to variation across industries.

The trade collapse of 2008/09 has been documented by various authors (see Baldwin 2009 for a collection of approaches and Bems, Johnson and Yi 2012 for a survey). Eaton, Kortum, Neiman and Romalis (2011) develop a structural model of international trade in which the decline in trade can be related to a collapse in demand for tradable goods and an increase in trade frictions. They find that a collapse in demand explains the vast majority of declining trade. Our approach is different in that we explicitly model the collapse in demand by considering second moment uncertainty shocks. Buyers react to the uncertainty by adopting a 'wait-and-see' approach. Thus, we do not require an increase in trade frictions to account for the excess volatility of trade. This approach is consistent with reports by Evenett (2010) and Bown (2011) who find that protectionism was contained during the Great Recession. This view is underlined by Bems, Johnson and Yi (2012).

Examining Belgian firm-level data during the 2008/09 recession, Behrens, Corcos and Mion (2013) find that most changes in international trade across trading partners and products occurred at the intensive margin, while trade fell most for consumer durables and capital goods. Similarly, Bricongne, Fontagné, Gaulier, Taglioni and Vicard (2012) confirm the overarching importance of the intensive margin for French firm-level export data. Levchenko, Lewis and Tesar (2010) stress that sectors used as intermediate inputs experienced substantially bigger drops in international trade. Likewise, Bems, Johnson and Yi (2011) confirm the important role of trade in intermediate goods. These findings are consistent with our modeling approach.

Our model is cast in terms of real variables, and we do not model monetary effects and prices. This modeling strategy is supported by the empirical regularity documented by Gopinath, Itskhoki and Neiman (2012) showing that prices of differentiated manufac-

tured goods (both durables and nondurables) hardly changed during the trade collapse of 2008/09. They conclude that the collapse in the value of international trade in differentiated goods was "almost entirely a quantity phenomenon." We therefore focus on modeling real variables.[2]

Amiti and Weinstein (2011) and Chor and Manova (2012) highlight the role of financial frictions and the drying up of trade credit. However, based on evidence from Italian manufacturing firms Guiso and Parigi (1999) show that the negative effect of uncertainty on investment cannot be explained by liquidity constraints. Paravisini, Rappoport, Schnabl, and Wolfenzon (2011) find that while Peruvian firms were affected by credit shocks, there was no significant difference between the effects on exports and domestic sales.

Engel and Wang (2011) point out the fact that the composition of international trade is tilted towards durable goods. Building a two-sector model in which only durable goods are traded, they can replicate the higher volatility of trade relative to general economic activity. Instead, we relate the excess volatility of trade to inventory adjustment in response to uncertainty shocks. As this mechanism in principle applies to any industry, compositional effects do not drive the volatility of international trade in our model.

Finally, our paper is related to Alessandria, Kaboski and Midrigan (2010a and 2011) who rationalize the decline in international trade by changes in firms' inventory behavior driven by a first moment shock to the cost of labor and a shock to the interest rate. In contrast, we focus on the role of increased uncertainty, modeled as a second-moment shock. Heightened uncertainty was arguably a defining feature of the Great Recession, and we employ an observable measure of it.

The paper is organized as follows. In section 2 we show how impulse response functions for aggregate U.S. imports and U.S. industrial production react to uncertainty shocks. To further motivate our approach, we also highlight how conventional static trade models, especially in the form of the gravity equation, fail to explain the extent of the Great Trade Collapse of 2008/09. In section 3, 4 and 5 we outline our theoretical model, conduct comparative statics and provide theoretical simulation results. Section 6 presents the main part of our empirical evidence. In section 7 we provide more specific details on aggregate inventory responses and the role of downstream intermediate use. In section 8 we ask to what extent uncertainty shocks can empirically account for the recent Great Trade Collapse. Section 9 concludes.

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[2]However, prices of non-differentiated manufactures declined sharply. But in the empirical section we most heavily rely on differentiated products.

2 Motivation: Uncertainty Shocks and International Trade

The world witnessed an unusually steep decline in international trade during the Great Recession of 2008/09, generally the steepest since the Great Depression. International trade plummeted by 30 percent or more in many cases. Some countries suffered particularly badly. For example, Japanese trade declined by about 50 percent from September 2008 to February 2009.

In addition, the decline was remarkably synchronized across countries. Baldwin (2009, introductory chapter) notes that "all 104 nations on which the WTO reports data experienced a drop in both imports and exports during the second half of 2008 and the first half of 2009." The synchronization hints at a common cause.

To motivate our approach, we first showcase the simplest possible evidence on the importance of uncertainty shocks for trade using aggregate data on real imports and industrial production (IP). We estimate a simple vector autoregression (VAR) with monthly data through 2012, following Bloom (2009) exactly (see the empirical section for details).

Figure 1 presents the VAR results for both imports and IP side by side. The impulse response functions (IRFs) are based on a one-period uncertainty shock where the Bloom uncertainty indicator increases by one unit (again, we describe the details in the empirical section). The bottom line is very clear from this figure. In response to the uncertainty shock, both industrial production and imports decline. But the response of imports is considerably stronger, about 5 to 10 times as strong in its period of peak impact during year one. The response of imports is also highly statistically significant. At its peak the IRF is 3 or 4 standard errors below zero, whereas the IRF for IP is only just about 2 standard errors below zero, and only just surmounts the 95% confidence threshold.

2.1 The Great Trade Collapse and the Failure of Gravity

While we will argue throughout the paper that uncertainty shocks can go a long way in explaining the behavior of international trade in recessions in general and during the Great Trade Collapse in particular, standard gravity approaches fall short of this objective. As a motivation, consider the workhorse gravity equation by Anderson and van Wincoop (2003):[3]

$$x_{ij} = \frac{y_i y_j}{y^W} \left(\frac{t_{ij}}{\Pi_i P_j} \right)^{1-\sigma},$$

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[3]Similar gravity equations can be derived from the models by Eaton and Kortum (2002) and Chaney (2008).

Figure 1: IRFs at Aggregate Level for Uncertainty Shocks
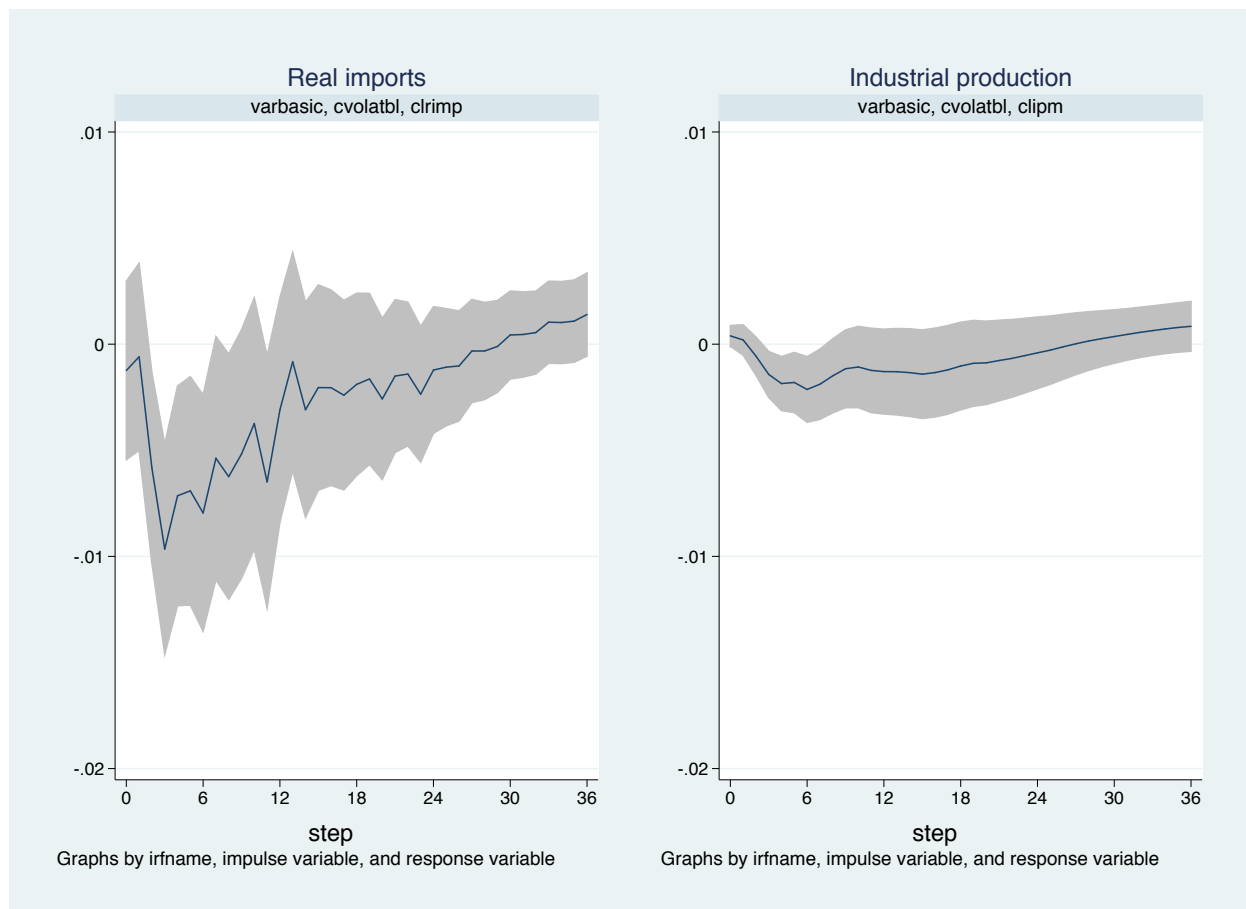
Source: Sample is 1962:1–2012:2. Imports from various sources, deflated by CPI; all other data as in Bloom (2009), updated. Uncertainty shocks for quadvariate VARs as in Bloom (2009). Ordering is stock market, volatility, log employment, followed lastly by either log real imports or log IP. Data updated through February 2012. No rescaling of shocks. 95% confidence intervals shown. See text.

where x_{ij} denotes imports to country i from country j, y_i and y_j denote the incomes of the two countries, y^W denotes world income, t_{ij} are bilateral trade costs, and Π_i and P_j are outward and inward multilateral resistance, respectively, which can be interpreted as average outward and inward bilateral trade costs. Taking logs and first differences yields

$$\Delta \ln \left(x_{ij}\right) = \Delta \ln \left(y_i\right) + \Delta \ln \left(y_j\right) - \Delta \ln \left(y^W\right) - (\sigma+1)\Delta \ln \left(t_{ij}\right) + (\sigma+1)\Delta \ln \left(\Pi_i P_j\right).$$

As most evidence indicates that trade costs and policy barriers moved very little during the recession (Evenett 2010), let us assume for simplicity that bilateral and average trade

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costs were unchanged:[4]

$$\Delta \ln \left(t_{ij} \right) = \Delta \ln \left(\Pi_i P_j \right) \approx 0.$$

To carry out a simple back-of-the-envelope calculation we suppose that income dropped by about 6 percent on average for OECD countries and around 3 percent for the global economy. It follows that bilateral trade amongst OECD countries should have dropped by only 9 percent on average:

$$
\begin{aligned}
\Delta \ln \left(x_{ij} \right) & \approx \Delta \ln \left(y_i \right) + \Delta \ln \left(y_j \right) - \Delta \ln \left(y^W \right) \\
& = -6 \text{ percent} - 6 \text{ percent} + 3 \text{ percent} = -9 \text{ percent}.
\end{aligned}
$$

However, in the data we observe a drop of around 30 percent. Clearly, the traditional gravity framework fails to explain the Great Trade Collapse.

We note that in the absence rising trade costs, 'back-and-forth trade' or 'vertical specialization' generally cannot explain the excessive responsiveness of trade to the decline in income. For example, if demand for the final good drops by 10 percent, then demand for intermediates in all stages is also expected to drop by 10 percent. If trade cost elasticities are sufficiently high, only an increase in bilateral trade frictions such as a tariff hike or sudden protectionism could explain a disproportionately strong decline in trade. But there is little evidence of increased protectionism during the Great Recession, while freight rates declined for most modes of shipping.

3 A Model of Trade with Uncertainty Shocks

We build on Hassler's (1996) setting of investment under uncertainty to construct a model of trade in intermediate goods. Following the seminal contribution by Bloom (2009) we then introduce second moment uncertainty shocks.

Hassler's (1996) model starts from the well-established premise that uncertainty has an adverse effect on investment. In our set-up we model 'investment' as firms' investing in intermediate goods. Due to fixed costs of ordering firms build up an inventory of intermediate goods that they run down over time and replenish at regular intervals. The intermediate goods can be either ordered domestically or imported from abroad. Thus, we turn the model into an open economy set-up.

In addition, firms face uncertainty over 'business conditions,' which means they ex-

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[4]The assumption of constant multilateral resistance also entails that world income shares did not change, e.g. the U.S. share of world income was constant. We maintain this assumption here as a rough approximation.

perience unexpected fluctuations in productivity and demand. What's more, the degree of uncertainty varies over time. Firms might therefore enjoy periods of calm when business conditions are relatively stable, or they might have to weather 'uncertainty shocks' that lead to a volatile business environment characterized by large fluctuations. Overall, this formulation allows us to model the link between production, international trade and shifting degrees of uncertainty. Hassler's (1996) key innovation is to formally model how *changes* in uncertainty influence investment. His model therefore serves as a natural starting point for our analysis of uncertainty shocks.

3.1 Production and Demand

As in Bloom (2009), each firm has a Cobb-Douglas production function

$$F(A, K, L) = AK^\alpha L^{1-\alpha}, \tag{1}$$

where A is productivity, L is domestic labor and K is an intermediate input that depreciates at rate δ. Each firm faces isoelastic demand Q with elasticity ε

$$Q = BP^{-\varepsilon}, \tag{2}$$

where B is a demand shifter. As we focus on the firm's short-run behavior, we assume that the firm takes the wage rate and the price of the intermediate input as given.[5] We thus adopt a partial equilibrium approach to keep the model tractable.

3.2 Inventory and Trade

The input K is an intermediate good (or a composite of such goods). As the firm has to pay fixed costs of ordering per shipment f, it stores the intermediate as inventory and follow an s, S inventory policy. Scarf (1959) shows that in the presence of such fixed costs of ordering, an s, S policy is an optimal solution to the dynamic inventory problem. We assume that the intermediate good can be either ordered from abroad or sourced domestically, leading to imports or domestic trade flows, respectively. The corresponding fixed costs are f_F and f_D with $f_F \geq f_D > 0$, where F stands for foreign and D for domestic.

Given the intermediate input price and the wage rate, it follows that the firm employs

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[5]We will for most part think of the intermediate input as imported from abroad. The prices of differentiated manufactured goods in international trade were essentially unchanged during the trade collapse of 2008/09, as documented by Gopinath, Itskhoki and Neiman (2012). Their evidence further motivates our assumption of a given input price.

a constant ratio of intermediates and labor regardless of productivity fluctuations. That is, the Cobb-Douglas production function (1) implies that the firm's use of intermediates K is proportional to output Q. Similar to Hassler (1996) we assume that the firm has a target level of intermediates to be held as inventory, denoted by K^*, which is proportional to output Q. Thus, we can write

$$k^* = c + q, \tag{3}$$

where c is a constant, $k^* \equiv \ln(K^*)$ denotes the logarithmic target and $q \equiv \ln(Q)$ denotes logarithmic output. Grossman and Laroque (1990) show that such a target level can be rationalized as the optimal solution to a consumption problem in the presence of adjustment costs. In our context the target can be similarly motivated if it is costly for the firm to adjust production up or down.

We follow Hassler (1996) in modeling the dynamic inventory problem. In particular, we assume a quadratic loss function that penalizes deviations z from the target k^* as $\frac{1}{2}z^2$ with $z \equiv k - k^*$.[6] Clearly, in the absence of ordering costs the firm would continuously set k equal to the target k^*. However, we assume positive ordering costs ($f > 0$) so that the firm faces a non-trivial trade-off as it needs to balance the fixed costs on the one hand and the costs of deviating from the target on the other.

We solve for the optimal solution to this inventory problem subject to a stochastic process for output q. The optimal control solution can be characterized as follows: when deviation of inventory z reaches a lower trigger point s, the firm orders the amount ϕ so that the inventory rises to a return point of deviation $S = s + \phi$. Formally, we can state the problem as

$$\min_{\{I_t, z_t\}_0^\infty} \left\{ \mathrm{E}_0 \int_0^\infty e^{-rt} \left(\frac{1}{2}z_t^2 + I_t f \right) \mathrm{d}t \right\} \tag{4}$$

subject to

$$z_0 = \bar{z},$$

$$z_{t+\mathrm{d}t} = \begin{cases} \text{free} & \text{if } k_t \text{ is adjusted,} \\ z_t - \delta\,\mathrm{d}t - \mathrm{d}q & \text{otherwise,} \end{cases}$$

$$I_t\,\mathrm{d}t = \begin{cases} 1 & \text{if } k_t \text{ is adjusted,} \\ 0 & \text{otherwise,} \end{cases}$$

I_t is a dummy variable that takes on the value 1 whenever k_t is adjusted, $r > 0$ is a

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[6]The loss associated with a negative deviation could be seen as the firm's desire to avoid a stockout, while the loss associated with a positive deviation could be interpreted as the firm's desire to avoid excessive storage costs.

constant discount factor, and $\delta > 0$ is the depreciation rate for the intermediate so that $\mathrm{d}\, K_t / K = \delta\, \mathrm{d}\, t$.[7]

3.3 Business Conditions with Time-Varying Uncertainty

Due to market clearing output can move because of shifts in productivity A in equation (1) or demand shifts B in equation (2). Like Bloom (2009), we refer to the combination of supply and demand shifters as *business conditions*. Specifically, we assume that output q follows a stochastic marked point process that is known to the firm. With an instantaneous probability $\lambda/2$ per unit of time and $\lambda > 0$, q shifts up or down by the amount ε:

$$
q_{t+\mathrm{d}\, t} = \begin{cases} q_t + \varepsilon & \text{with probability } (\lambda/2)\, \mathrm{d}\, t, \\ q_t & \text{with probability } 1 - \lambda\, \mathrm{d}\, t, \\ q_t - \varepsilon & \text{with probability } (\lambda/2)\, \mathrm{d}\, t. \end{cases} \tag{5}
$$

The shock ε can be interpreted as a sudden change in business conditions. Through the proportionality between output and the target level of inventory embedded in equation (3) a shift in q leads to an updated target inventory level k^*. Following Hassler (1996) we assume that ε is sufficiently large such that it becomes optimal for the firm to adjust k.[8] That is, a positive shock to output increases k^* sufficiently to lead to a negative deviation z that reaches below the lower trigger point s. As a result the firm restocks k. Vice versa, a negative shock reduces k^* sufficiently such that z reaches above the upper trigger point and the firm destocks k.[9] Thus, to keep our model tractable we allow the firm to both restock and destock depending on the direction of the shock. The firm knows the stochastic process (5) and takes it into account when solving its optimization problem (4).[10]

The arrival rate of shocks λ is the measure of uncertainty and thus a key parameter of interest. We interpret changes in λ as changes in the degree of uncertainty. Note that λ determines the frequency of shocks, not the size of shocks. This feature is consistent with λ determining the *second moment* of shocks, not their first moment. More specifically,

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[7]The intermediate factor K is a stock variable in our model. In our trade and production data at the 4-digit industry level, examples of such factors include 'electrical equipment', 'engines, turbines, and power transmission equipment', 'communications equipment' and 'railroad rolling stock.' We can think of the firm described in our model as ordering a mix of such products.

[8]Hassler (1996, section 4) reports that relaxing the large shock assumption, while rendering the model more difficult to solve, appears to yield no qualitatively different results.

[9]To keep the exposition concise we do not explicitly describe the upper trigger here and focus on the lower trigger point s and the return point S. But it is straightforward to characterize the upper trigger point. See Hassler (1996) for details.

[10]When we simulate the model in Section 5, we consider a large number of firms that receive idiosyncratic shocks to business conditions but that are otherwise identical. Those firms do not behave strategically, and there are no self-fulfilling bouts of uncertainty.

as the simplest possible set-up we follow Hassler (1996) by allowing uncertainty λ_ω to switch stochastically between two states $\omega \in \{0,1\}$: a state of low uncertainty λ_0 and a state of high uncertainty λ_1 with $\lambda_0 < \lambda_1$. The transition of the uncertainty states follows a Markov process

$$\omega_{t+dt} = \begin{cases} \omega_t & \text{with probability } 1 - \gamma_\omega \, dt, \\ \overline{\omega}_t & \text{with probability } \gamma_\omega \, dt, \end{cases} \tag{6}$$

where $\overline{\omega}_t = 1$ if $\omega_t = 0$, and vice versa. The probability of switching the uncertainty state in the next instant dt is therefore $\gamma_\omega \, dt$, with the expected duration until the next switch given by γ_ω^{-1}. Below we will choose parameter values for λ_0, λ_1, γ_0 and γ_1 that are consistent with uncertainty fluctuations as observed over the past few decades.[11]

3.4 Solving the Inventory Problem

The Bellman equation for the inventory problem is

$$V(z_t, \omega_t) = \frac{1}{2} z_t^2 \, dt + (1 - r \, dt) \, \mathrm{E}_t V(z_{t+dt}, \omega_{t+dt}). \tag{7}$$

The cost function $V(z_t, \omega_t)$ at time t in state ω_t thus depends on the instantaneous loss element from the minimand (4), $z_t^2 \, dt/2$, as well as the discounted expected cost at time $t + dt$. The second term can be further broken down as follows:

$$\mathrm{E}_t V(z_{t+dt}, \omega_{t+dt}) = \begin{aligned} &V_z(z_t, \omega_t) - \delta \, dt V_z(z_t, \omega_t) \\ &+\lambda_\omega \, dt \, \{V(S_\omega, \omega_t) + f - V(z_t, \omega_t)\} \\ &+\gamma_\omega \, dt \, \{V(z_t, \overline{\omega}_t) - V(z_t, \omega_t)\}, \end{aligned} \tag{8}$$

where V_z denotes the derivative of V with respect to z. The expected cost at time $t + dt$ thus takes into account the cost of depreciation over time through the term involving δ. It also captures the probability $\lambda_\omega \, dt$ of a shock hitting the firm's business conditions (in which case the firm would pay the ordering costs f to restock to its return point S_ω), as well as the probability $\gamma_\omega \, dt$ that the uncertainty state switches from ω_t to $\overline{\omega}_t$.

Equations (7) and (8) form a system of two differential equations for the two possible states ω_t and $\overline{\omega}_t$. Following Hassler (1996) we show in the technical appendix how standard stochastic calculus techniques lead to a solution for the system. We have to use numerical methods to obtain values for the four main endogenous variables of interest:

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[11]Overall, the stochastic process for uncertainty is consistent with Bloom's (2009). In his setting uncertainty also switches between two states (low and high uncertainty) with given transition probabilities.

the bounds s_0 and S_0 for the state of low uncertainty λ_0, and the bounds s_1 and S_1 for the state of high uncertainty λ_1. It turns out that in either state, the cost function V reaches its lowest level at the respective return point S. This point represents the level of inventory the firm ideally wants to hold given the expected outlook for business conditions and given it has just paid the fixed costs f for restocking.[12]

Following Hassler (1996) it can be shown that the following condition can be derived from the Bellman equation:[13]

$$\frac{1}{2}\left(s_\omega^2 - S_\omega^2\right) = (r + \lambda_\omega)f + \gamma_\omega\left\{f - (V(s_\omega, \overline{\omega}_t) - V(S_\omega, \overline{\omega}_t))\right\} > 0. \qquad (9)$$

This expression is strictly positive as $(r + \lambda_\omega)f > 0$ and $\gamma_\omega\left\{f - (V(s_\omega, \overline{\omega}_t) - V(S_\omega, \overline{\omega}_t))\right\} \geq 0$. This last non-negativity result holds because the smallest value of V can always be reached by paying the fixed costs f and stocking up to S_ω. That is, for any z_t, the cost value $V(z_t, \overline{\omega}_t)$ can never exceed the minimum value $V(S_\omega, \overline{\omega}_t)$ plus f. It therefore also follows that $V(s_\omega, \overline{\omega}_t)$ can never exceed $V(S_\omega, \overline{\omega}_t) + f$, i.e., $V(s_\omega, \overline{\omega}_t) \leq V(S_\omega, \overline{\omega}_t) + f$.

Recall that the lower trigger point s_ω is expressed as a deviation from the target level k^*. We therefore have $s_\omega < 0$. Conversely, the return point S_ω is always positive, $S_\omega > 0$. The fact that expression (9) is positive implies $-s_\omega > S_\omega$, or $|s_\omega| > S_\omega$. This means that the inventory level is on average below the target if the firm always runs down its stock until it hits s_ω. This reflects the uncertainty and the option value of waiting. Intuitively, in the absence of uncertainty the firm would stock as much inventory as to be at the target value on average. That is, its inventory would be above and below the target exactly half of the time. However, in the presence of uncertainty it becomes optimal for the firm to adopt a more cautious stance. Given that output q and thus the target level k^* are stochastic according to equation (5), the firm is better off holding less inventory on average and placing a large order only once the need arises.[14] This logic follows immediately from the literature on uncertainty and the option value of waiting (McDonald and Siegel 1986, Dixit 1989, Pindyck 1991).

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[12]It would not be optimal for the firm to return to a point at which the cost function is above its minimum. The intuition is that in that case, the firm would on average spend less time in the lowest range of possible cost values.

[13]For details of the derivation see Hassler (1996, Appendix 2).

[14]Although we will fill in more details in section 4, we can refer interested readers to Figure 5 where we illustrate the difference between the absence of uncertainty (cases 1 and 2) and positive uncertainty (case 3).

4 Comparative Statics

The main purpose of this section is to explore how the firm endogenously changes its s, S bounds in response to increased uncertainty. Our key result is that the firm *lowers* the bounds in response to increased uncertainty. In addition, we are interested in comparative statics for the depreciation rate δ and the fixed cost of ordering f. As explained by Hassler (1996), the model cannot be solved analytically. Numerical methods must be used but they uniquely define $V(z)$ in the Bellman equation (7).

4.1 Parameterizing the Model

We choose the same parameter values for the interest rate and rate of depreciation as in Bloom (2009), i.e., $r = 0.065$ and $\delta = 0.1$ per year. For the stochastic uncertainty process described by equations (5) and (6) we choose parameter values that are consistent with Bloom's (2009) data on stock market volatility. In his Table II he reports that an uncertainty shock has an average half-life of two months. This information can be expressed in terms of the transition probabilities in equation (6) with the help of a process of exponential decay for a quantity D_t:

$$D_t = D_0 \exp(-gt).$$

Setting t equal to 2 months yields a rate of decay $g = 0.3466$ for D_t to halve. The decaying quantity D_t in that process can be thought of as the number of discrete elements in a certain set. We can then compute the average length of time that an element remains in the set, which corresponds to the expected duration of the high-uncertainty state, γ_1^{-1}. This is the mean lifetime of the decaying quantity, and it is given by $1/g$. Thus, the average duration of the high-uncertainty period corresponds to 1/0.3466=2.8854 months or 0.2404 years in this case. It follows $\gamma_1 = 1/0.2404 = 4.1588$ when the duration is expressed in years.

Bloom (2009) furthermore reports 17 uncertainty shocks in 46 years. Hence, an uncertainty shock arrives on average every 46/17=2.71 years, or 32.5 months. Ignoring the relatively short duration of high-uncertainty periods, this means an average duration of the low-uncertainty period of 2.71 years. It follows $\gamma_0 = 1/2.7059 = 0.3696$.

The uncertainty term $\lambda \, \mathrm{d} t$ in the marked point process (5) indicates the probability that output is hit by a supply or demand shock in the next instant that is sufficiently large to shift the target level of inventory. Thus, the expected length of time until the next shock is $1/\lambda$. It is difficult to come up with an empirical counterpart of the frequency of such shocks since they are unobserved. For the baseline level of uncertainty we set $\lambda_0 = 2$,

which implies that shocks hit on average every 1/2=0.5 years. However, we note that the results are not very sensitive to the λ_0 value. In our baseline specification we follow Bloom (2009, Table II) in setting the value for the high-uncertainty state twice as large as in the low-uncertainty state, i.e., $\lambda_1 = 2\lambda_0 = 4$. But in the comparative statics below we experiment with a range of values for λ_1.

Finally, we need to find an appropriate value for the fixed costs of ordering f. Based on data for a US steel manufacturer Alessandria, Kaboski and Midrigan (2010b) report that "domestic goods are purchased every 85 days, while foreign goods are purchased every 150 days." To match the behavior of foreign import flows we set f to ensure that the interval between orders is on average 150 days in the low-uncertainty state. This implies $f_F = 0.0000588$ as our baseline value. Matching the interval of 85 days for domestic flows would imply $f_D = 0.00001066$. These fixed costs differ by a large amount (a factor of about 5.5) and that might seem implausibly large. However, we show in the simulation section that quantitatively that we can obtain large declines in trade flows for fixed costs that are not as high as f_F in our baseline specification. That is, we are able to obtain a large decline in trade flows for a lower ratio of f_F/f_D as implied by the above values.

4.2   A Rise in Uncertainty

Given the above parameter values we solve the model numerically. Figure 2 illustrates the change in s, S bounds in response to a rising level of uncertainty λ. Note that there are two sets of s, S bounds: one set for the low-uncertainty state 0, and the other for the high-uncertainty state 1. The level of low uncertainty is fixed at $\lambda_0 = 2$ but the level of high uncertainty λ_1 varies on the horizontal axis. As the s, S bounds are endogenous, all of them in principle shift in response to a change in one of the model's parameters. But clearly, the bounds for the low-uncertainty state are hardly affected by changes in λ_1.

The vertical scale indicates the percentage deviation from the target k^*. Two observations stand out. First, the bounds for the high-uncertainty state are lower, i.e., $S_1 < S_0$ and $s_1 < s_0$. Second, the negative deviations for the lower trigger points s_0 and s_1 are higher in absolute value than positive deviations for the return points S_0 and S_1, i.e., $|s_1| > S_1$ and $|s_0| > S_0$. On average the firm's inventory is thus below target. This reflects the option value of waiting.

Figure 2 illustrates that the firm optimally responds to increased uncertainty by lowering both s, S bounds for the high-uncertainty state. Figure 3 shows that the decline in the lower trigger point s_1 compared to s_0 can be quite substantial for high degrees of uncertainty. Intuitively, when uncertainty rises, the firm becomes more cautious and adopts

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Figure 2: Change in s, S bounds due to higher uncertainty. The low-uncertainty state is in grey, the high-uncertainty state is in black.
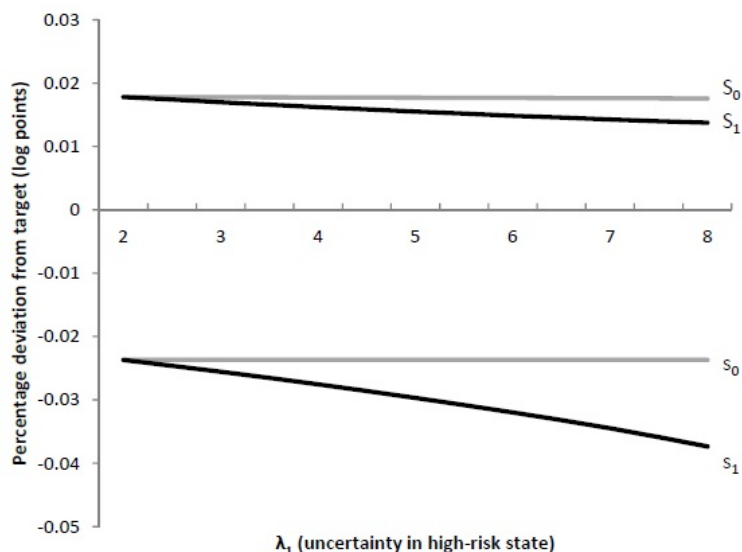
a wait-and-see attitude. It runs down its inventory further than in the low-uncertainty state, and it does not stock up to as high a level. It turns out that the firm lowers s_1 more than S_1 so that the bandwidth $(S_1 - s_1)$ rises in response to higher uncertainty. Figure 4 plots this increase in bandwidth.

Figure 5 summarizes the main results in a compact way. Case 1 depicts the situation where both fixed costs f and uncertainty λ are essentially 0. The s_1 and S_1 bounds are symmetric around the target level k^*, and the bandwidth (i.e., the height of the box) is small. In case 2 the fixed costs are positive, which pushes both s_1 and S_1 further away from the target but in a symmetric way. Case 3 corresponds to the circumstances implied by Figures 2–4. Uncertainty has increased, which induces two effects. First, both s1 and S1 shift down so that they are no longer symmetric around the target. Second, the bandwidth increases further (see Figure 4).

4.3   Varying the Depreciation Rate

Some types of imports are inherently difficult to store as inventory, for instance food products and other perishable goods. We model this inherence difference in storability with a higher rate of depreciation. Figure 6 illustrates the effect of a higher depreciation rate that doubles to $\delta = 0.2$ compared to the baseline scenario of $\delta = 0.1$ in Figure 3. In general,

Figure 3: How uncertainty decreases the lower trigger point.(compared to the low-uncertainty state).
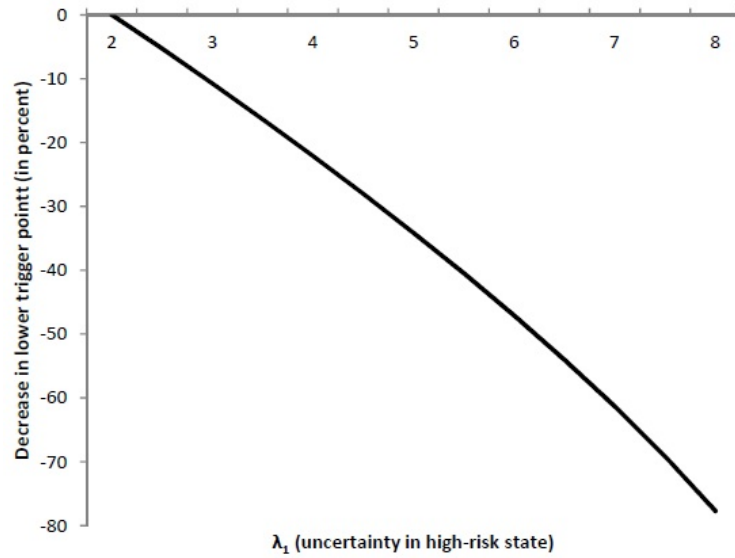

Figure 4: How uncertainty increases the s, S bandwidth (compared to the low-uncertainty state).
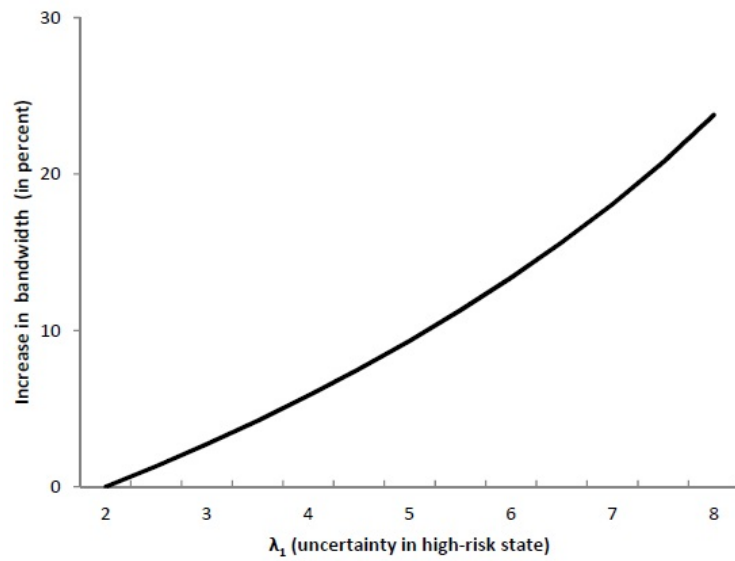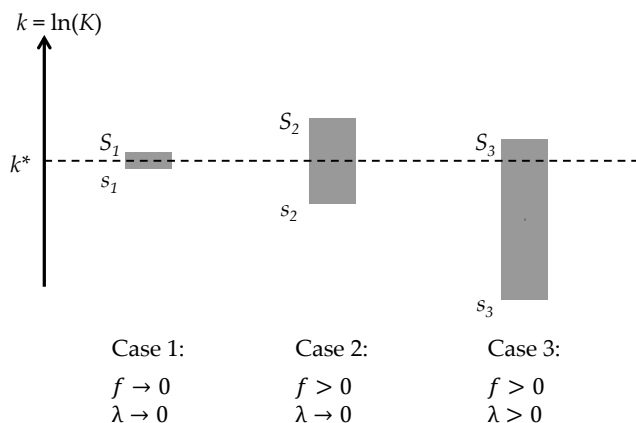
Figure 5: Summary: How uncertainty pushes down the s, S bounds and increases the bandwidth.

the larger the depreciation rate, the smaller the decreases in s_1 and S_1. Intuitively, with a larger depreciation rate the firm orders more frequently. The value of waiting is therefore not as important and does not respond as strongly to changes in uncertainty. Figure 6 graphs the decrease in the lower trigger point s_1 relative to s_0 for both the baseline depreciation rate and the higher value.

4.4   Varying the Fixed Costs of Ordering

We expect fixed costs of ordering to be lower when the intermediate input is ordered domestically, i.e., $f_D \leq f_F$. Figure 7 shows the effect of using the value f_D from above that corresponds to an average interval of 85 days between domestic orders compared to the baseline value f_F in Figure 3. Not surprisingly, the lower the fixed costs of ordering, the bigger the incentive for the firm to keep inventory close to the target level.

5   Simulating an Uncertainty Shock

So far we have described the behavior of a single firm. We now simulate identical 50,000 firms that receive shocks according to the stochastic uncertainty process in equations (5)

Figure 6: The effect of a higher depreciation rate on the decrease in the lower trigger point.
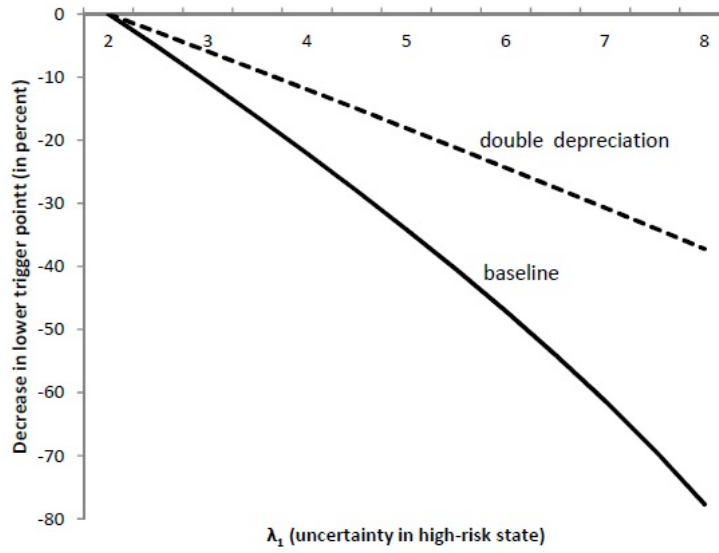

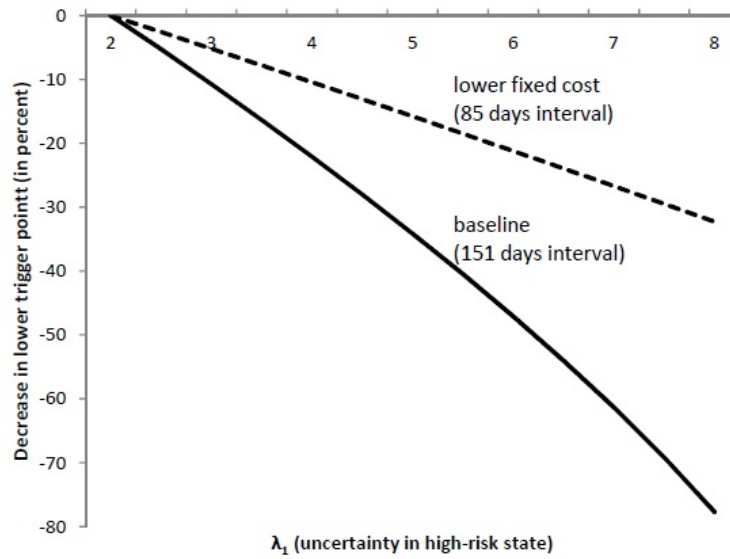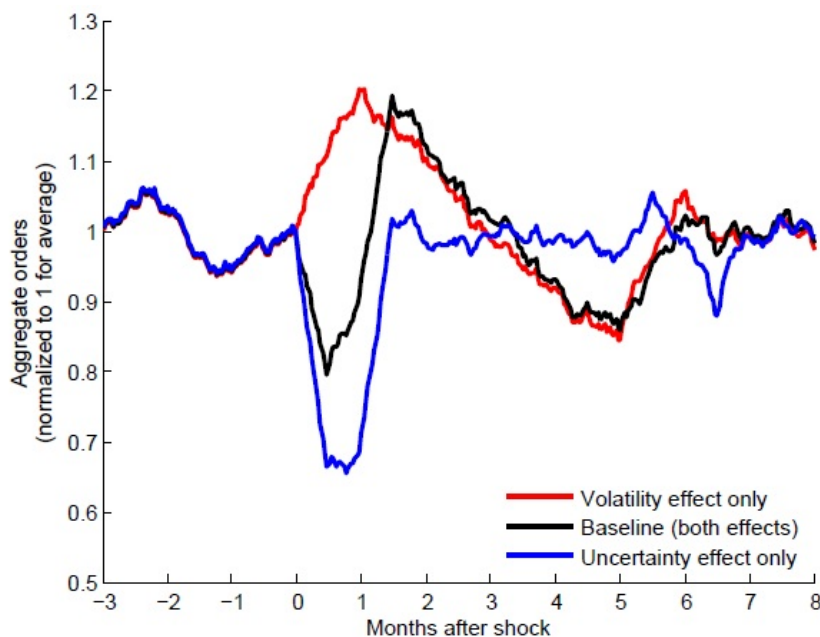Figure 7: The effect of a lower fixed costs of ordering on the decrease in the lower trigger point.

Figure 8: Simulating the response of aggregate orders to an uncertainty shock: the total effect decomposed into an uncertainty effect and a volatility effect.

and (6). These shocks are idiosyncratic for each firm but drawn from the same distribution. We use the same parameter values as described in section 4.1. We should add that firms neither enter nor exit over the simulation period. This assumption seems reasonable given that most of the changes in the value of international trade during the trade collapse of 2008/09 happened at the intensive margin (see Behrens, Corcos and Mion 2013 and Bricongne et al. 2012). Allowing for extensive margin responses would be an important avenue for future research. We conjecture that the extensive margin would amplify the effect of uncertainty shocks. Firms would be likely to exit in the face of higher uncertainty and likely to enter once the recovery takes hold.

A key result from section 4 is that firms lower their s, S bounds in response to increased uncertainty. This shift leads to sharp downward adjustments of intermediate input inventories and thus a decline in imports. In contrast, output is not characterized by systematic fluctuations in our model because as a result of the stochastic process (5), output is driven by idiosyncratic mean-zero shocks that wash out in the aggregate. With regard to the data our framework can therefore best be interpreted as explaining the excess volatility of trade flows that arises in addition to movements of output, or as explaining the magnified response of trade flows.

Figure 8 plots the aggregate orders of imported intermediate goods in the economy (normalized to 1 for the long-run average value). We simulate a permanent shift from low uncertainty to high uncertainty. Again, we stress that this is purely a second moment shock, not a first moment shock. Average aggregate orders in the long run are therefore the same before and after the uncertainty shock hits. The reason is that the extent of orders is ultimately determined by the depreciation rate δ since intermediates depreciate over time.[15]

The black graph in Figure 8 represents the reaction of aggregate orders. Given our parameterization they decrease by about 20 percent in the short run following the shock. Note that any first moment shifts such as, say, a ten percent decline in demand, would accrue in addition. The total effect can be decomposed into a 'pure' *uncertainty effect* (in blue) and a *volatility effect* (in red). The uncertainty effect is related to the downshifting of the s, S bounds (holding the degree of volatility fixed as implied by λ_0). Once the uncertainty shock hits, firms decrease their lower trigger point such that they initially take longer to run down their inventory. This leads to a sharp drop in orders of imported intermediate inputs. Once firms approach the new lower trigger point, they start restocking. This leads to a sharp recovery in orders. As in Bloom (2009), this pattern of sharp contraction and strong recovery is typical for uncertainty shocks.

The volatility effect is an overshoot caused by the increased probability of firms receiving a shock (holding the s, S bounds fixed). Recall that a shock ε moves output symmetrically in either direction with equal probability. If a lot of firms are close to the return point, then negative orders (induced by z being pushed above the upper trigger point) and positive orders (induced by z being pushed below the lower trigger point) should be of the same size such that in the aggregate the two cancel (i.e., aggregate orders are close to their average level). However, if a lot of firms have not stocked up in a while (which happens once s falls) so that the average firm has moved closer to the lower trigger point due to depreciation, then the uncertainty shock tends to push up aggregate orders. This effect is analogous to Bloom's (2009) 'volatility overshoot.' The volatility effect only arises in the medium term, whereas the uncertainty effect happens immediately.[16]

We illustrate the inventory position of the average firm in Figure 9. Specifically, we

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[15]As firms are equally likely to receive positive or negative shocks, the effects of restocking and destocking cancel on average.

[16]One implication is that the upper trigger point does not matter for the size of orders. The reason is that the shock size is such that once a shock hits, there is always adjustment (i.e., the upper trigger point is always breached given a shock in that direction). Therefore, only the return point and the lower trigger point matter for the size of orders as they mark the range of inventory that the average firm holds. Since depreciation can only decrease but never increase inventory, the average firm's inventory can never be above the return point.

Figure 9: Simulating the inventory position of the average firm.
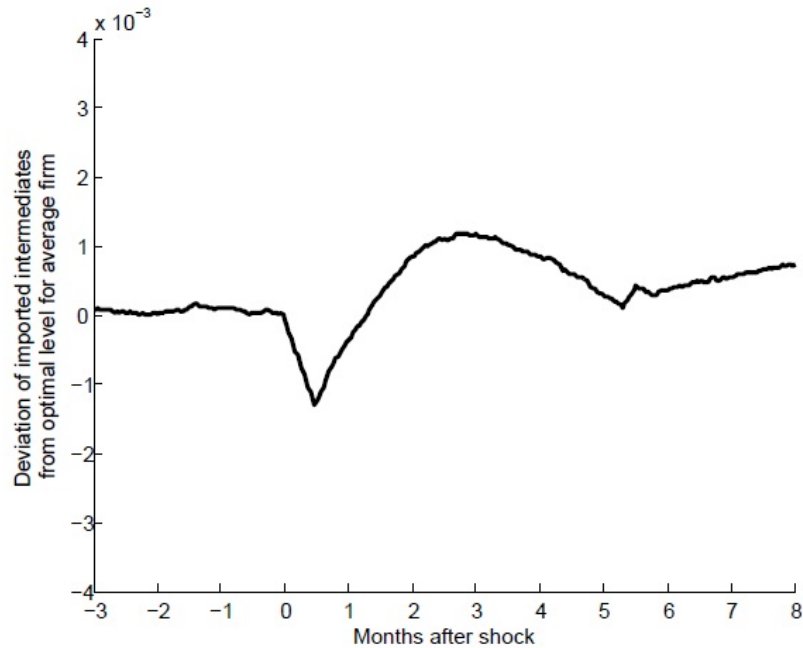
plot the average deviation of imported intermediates from the optimal level. Before the uncertainty shock, this deviation is close to zero as firms on average hold precisely the amount of inventory that minimizes their loss function. But once the shock hits, their average inventories decline sharply since they decrease their lower trigger point. This is driven by the uncertainty effect described above. But at the same time, the higher volatility implied by the uncertainty shock means that firms are more likely to restock so that on average their inventories are closer to the return point. This phenomenon is the volatility effect described above. It phases in slowly over time since although volatility increases immediately once the uncertainty shock hits, it takes time until more and more firms get actually hit by idiosyncratic shocks. In Figure 9 the volatility effect starts to dominate the uncertainty after about half a month, which is when average inventories start increasing again. Subsequently it takes another two to three months until the volatility effect peaks. Note that since the uncertainty shock in this simulation is permanent, average inventories in the new steady state are higher than in the old steady state because the higher level of volatility pushes firms closer to the return point and thus above the target level.

In the previous simulation the trade collapse and recovery happen quickly within two or three months. However, in the data, for instance during the Great Recession, this

Figure 10: Simulating aggregate orders with different values of fixed costs of ordering f.
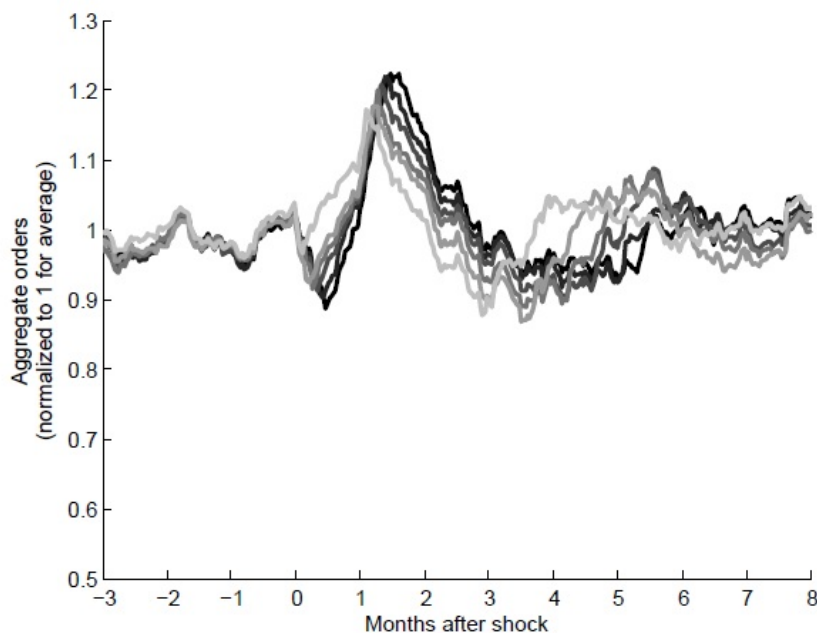
process takes longer, typically at least a year. Such persistence could be introduced into our model by staggering firms' responses. Currently, all firms in the simulation perceive the rise and fall of uncertainty in exactly the same way and thus synchronize their reactions. It might be more realistic to introduce some degree heterogeneity by allowing firms to react at slightly different times. In particular, firms might have different assessments as to the time when uncertainty has faded and business conditions have normalized. This would tend to stretch out the recovery of trade. Moreover, delivery lags could be introduced that vary across industries. We abstracted from such extensions here in order to keep the model tractable.

In Figure 10 we plot the total effect of an uncertainty shock for six different values of fixed costs f. The black line corresponds to the baseline value of $f_F = 0.0000588$. The remaining five lines in grey correspond to declining values of f (with the light grey line corresponding to the smallest value). Their values in declining order are 0.00005066, 0.00004066, 0.00003066, 0.00002066 and 0.00001066. The last value corresponds to the domestic fixed cost value above, f_D. Given the domestic fixed cost value of $f_D = 0.00001066$ (see the light grey line), domestic trade would decline to only about 97 percent of its average level.

The main insight is that although the trade collapse is less severe with smaller fixed

cost values (as predicted by the theory), quantitatively the collapse is not as sensitive to fixed costs above a certain threshold. For instance, for the baseline value trade declines to about 88 percent of its average level upon impact of the uncertainty shock. But even with a substantially smaller fixed cost value of $f_F = 0.00002066$, trade still declines to about 92 percent of its average level upon impact. Thus, we may not need f_F to be substantially larger than f_D to generate a strong decline in international trade. In this case, the foreign fixed cost value is only about twice as large as the domestic fixed cost value, $f_F/f_D = 1.94$. In contrast, Alessandria, Kaboski and Midrigan (2010a) use a ratio of $f_F/f_D = 6.54$.[17]

6   Empirical Evidence

To explore the relationship between uncertainty, production, and international trade we run vector autoregressions (VARs) with U.S. data. In particular, we follow the seminal work of Bloom (2009) in running a VAR to generate an impulse response function (IRF) relating the reactions of key model quantities, in this case not only industrial production but also imports, to the underlying impulses which take the form of shocks to uncertainty.

   We contend that, as with the application to production volatility, the payoffs to an uncertainty- based approach will turn out to be substantial again in the new setting we propose for modeling trade volatility. Recall that in the view of Bloom (2009, p. 627):

> More generally, the framework in this paper also provides one response to the "where are the negative productivity shocks?" critique of real business cycle theories. In particular, since second-moment shocks generate large falls in output, employment, and productivity growth, it provides an alternative mechanism to first-moment shocks for generating recessions.

   The same might then be said of theories of trade collapse that rely on negative productivity shocks, or other first moment shocks. So by the same token, the framework in our paper provides one response to the "where are the increases in trade frictions?" objection that is often cited when standard static models are unable to otherwise explain the amplified nature of trade collapses in recessions, relative to declines in output. Our theoretical model, and empirical evidence, can thus be seamlessly integrated with the Bloom (2009) view of uncertainty-driven recessions, whilst matching one other crucial and recurrent feature of international economic experience: the highly magnified volatility of

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[17]Alessandria, Kaboski and Midrigan (2010a, Table 4) choose values for the fixed costs of ordering that correspond to 23.88 percent of mean revenues for foreign orders and 3.65 percent of mean revenues for domestic orders.

trade, which has been a focus of inquiry since at least the 1930s, and which, since the on-set of the Great Recession has flared again as an object of curiosity and worry to scholars and policymakers alike.

6.1 Four Testable Hypotheses

To look ahead and quickly sum up the bottom line, our empirical results expose several new and important stylized facts, all of which are consistent with, and thus can motivate our previously described theoretical framework. Specifically we focus on testing four empirical propositions that would be implied by our theory.

- First, trade volumes do respond to uncertainty shocks, the effects are quantitatively and statistically significant, and are robust to different samples and methods. In addition, trade volumes respond much more to uncertainty shocks than does the volume of industrial production; that is to say, there is something fundamentally different about the dynamics of traded goods supplied via the import channel, as compared to supply originating from domestic industrial production.

- Second, we confirm that these findings are true not just at the aggregate level, but also at the disaggregated level, indicating that the amplified dynamic response of traded goods is not just a sectoral composition effect.

- Third, we find that the dynamic response of traded goods to uncertainty shocks is greatest in durable goods sectors as compared to nondurable goods sectors, con-sistent with the theoretical model where a decrease in the depreciation parameter (interpreted as an increase in perishability) leads to a larger response.

- Fourth, we find corroborative evidence in the response of aggregate input invento-ries to uncertainty shocks, and in the stronger response of imports to uncertainty shocks in in those sectors where a larger share of the product is used downstream as an intermediate input.

The following parts of this section are structured as follows. The first part briefly spells out the empirical VAR methods we employ based on Bloom (2009). The second part spells out the data we have at our disposal, some of it newly collected, to examine the differences between trade and industrial production in this framework. The subsequent parts discuss our findings on the first three testable hypotheses noted above, and we discuss the corroborative evidence in the next section, before concluding.

6.2 Computing the Responses to an Uncertainty Shock

In typical business cycle empirical work, researchers are often interested in the response of key variables, most of all output, to various shocks, most often a shock to the level of technology or productivity. The analysis of such first moment shocks has long been a centerpiece of the macroeconomic VAR literature. The innovation of Bloom (2009) was to construct, simulate and empirically estimate a model where the key shock of interest is a second moment shock, which is conceived of as an "uncertainty shock" of a specific form. This shock amounts to an increase in the variance, but not the mean, of a composite "business condition" disturbance in the model, which can be flexibly interpreted as a demand or supply shock. For empirical purposes when the model is estimated using data on the postwar U.S., Bloom proposes that changes in the market price of the VXO index, a daily options-based implied stock market volatility for a 30-day horizon, be used as a proxy for the uncertainty shock, with realized volatility used when the VXO is not available. A plot of this series, scaled to an annualized form, and extended to 2012, is shown in Figure 11.[18]

Following Bloom (2009) we evaluate the impact of uncertainty shocks using VARs on monthly data from June 1962 (the same as in Bloom) to February 2012 (going beyond Bloom's end date of June 2008). Bloom's full set of variables, in VAR estimation order are as follows: log(S&P500 stock market index), stock-market volatility indicator, Federal Funds Rate, log(average hourly earnings), log(consumer price index), hours, log(employment), and log(industrial production).[19]

For simplicity, for the main results of this paper presented in this section, all VARs of this form are estimated using a quad-variate VAR (log stock-market levels, the volatility indicator, log employment, and lastly the industrial production or trade indicator).

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[18]As Bloom (2009, Figure 1) notes: "Pre-1986 the VXO index is unavailable, so actual monthly returns volatilities are calculated as the monthly standard deviation of the daily S&P500 index normalized to the same mean and variance as the VXO index when they overlap from 1986 onward. Actual and VXO are correlated at 0.874 over this period. A brief description of the nature and exact timing of every shock is contained in the empirical appendix [in progress]. The asterisks indicate that for scaling purposes the monthly VXO was capped at 50. Uncapped values for the Black Monday peak are 58.2 and for the credit crunch peak are 64.4. LTCM is Long Term Capital Management." For comparability, we follow exactly the same definitions here and we thank Nicholas Bloom for providing us with an updated series extended to 2012.

[19]In terms of VAR variable ordering and variable definitions we follow Bloom (2009) exactly for comparability. As Bloom notes: "This ordering is based on the assumptions that shocks instantaneously influence the stock market (levels and volatility), then prices (wages, the consumer price index (CPI), and interest rates), and finally quantities (hours, employment, and output). Including the stock-market levels as the first variable in the VAR ensures the impact of stock-market levels is already controlled for when looking at the impact of volatility shocks."

Figure 11: The Bloom (2009) Index: Monthly U.S. stock market volatility, 1962–2012
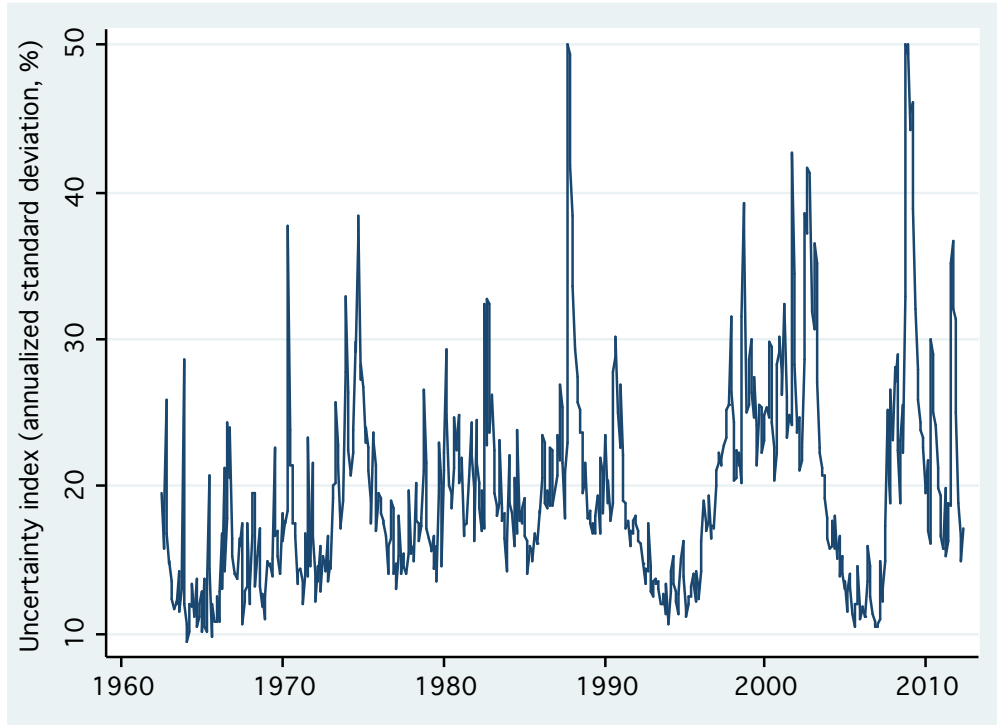
Source: Bloom (2009) and personal correspondence. See footnote 18.

6.3 Data

Many of our key variables are taken from the exact same sources as Bloom (2009). As it is noted: "The full set of VAR variables in the estimation are log industrial production in manufacturing (Federal Reserve Board of Governors, seasonally adjusted), employment in manufacturing (BLS, seasonally adjusted), average hours in manufacturing (BLS, seasonally adjusted), log consumer price index (all urban consumers, seasonally adjusted), log average hourly earnings for production workers (manufacturing), Federal Funds rate (effective rate, Federal Reserve Board of Governors), a monthly stock-market volatility indicator (described below)[above here], and the log of the S&P500 stock-market index. All variables are HP detrended using a filter value of $\lambda = 129,600$." We follow these definitions exactly.

However, in some key respects, our data requirement are much larger than this. For starters, we are interested in assessing the response of trade, so we needed to collect monthly import volume data. In addition, we are interested in computing disaggregated responses of trade and industrial production (IP) in different sectors, in the aftermath of uncertainty shocks, in an attempt to gauge whether some of the key predictions of

our theory are sustained. Thus, we needed to assemble new monthly trade data (aggregate and disaggregate) as well as new disaggregated monthly IP data to complement the Bloom data.

We briefly explain the provenance of these newly collected data, all of which will also be HP filtered for use in the VARs as above.

- U.S. aggregated monthly real import volume. These data run from January 1962 to February 2012. After 1989, total imports for general consumption were obtained from the USITC dataweb, where the data can be downloaded online. From 1968 to 1988 data were collected by hand from FT900 reports, where the imports series are only available from 1968 as F.A.S. at foreign port of export, general imports, seasonally unadjusted; the series then change to C.I.F. value available beginning in 1974, and the definition changes to customs value in 1982. Prior to 1968 we use NBER series 07028, a series that is called "total imports, free and dutiable" or else "imports for consumption and other"; for the 1962 to 1967 window this NBER series is a good match, as it is sourced from the same FT900 reports as our hand compiled series. The entire series was then deflated by the monthly CPI.

- U.S. disaggregated monthly real imports. These data only run from January 1989 to February 2012. In each month total imports for general consumption disaggregated at the 4-digit NAICS level were obtained from the USITC dataweb, where the data can be downloaded online. All series were then deflated by the monthly CPI. In this way 108 sector-level monthly real import series were compiled.

- U.S. disaggregated monthly industrial production. These data only run from January 1972 to February 2012 at a useful level of granularity. Although aggregate IP data are provided by the Fed going back to February 1919, the sectorally disaggregated IP data only start in 1939 for 7 large sectors, with ever finer data becoming available in 1947 (24 sectors), 1954 (39 sectors) and 1967 (58 sectors). However, it is in 1972 that IP data are available using the 4-digit NAICS classification which will permit sector-by-sector compatibility with the import data above. Starting in 1972 we use the Fed G.17 reports to compile sector-level IP indices, which affords data on 98 sectors at the start, expanding to 99 in 1986.

6.4   Results 1: IRFs at Aggregate Level for Trade versus IP

We begin with the simplest possible evidence on the importance of uncertainty shocks for trade, using aggregate data on real imports and industrial production.

Following Bloom (2009) exactly, a baseline quad-variate VAR is estimated for both series, which are place last in the ordering. Ordering is stock market, volatility, log employment, followed lastly by either log real imports or log IP. Data differ from Bloom in that we have updated all series through February 2012, so as to include the response to the 2008 financial crisis. However our results are not sensitive to this extension of the sample [empirical appendix – in progress]. The presentation also differs from Bloom in that we do not rescale the IRFs at this stage, since we are only interested in the comparative responses of internationally traded and domestically produced goods.

In Figure 1 we already presented the VAR results for both imports and IP side by side. The impulse response functions (IRFs) are based on a one-period uncertainty shock where the Bloom uncertainty indicator (that is, VXO or its proxy) increases by one unit. The bottom line is very clear from this figure. In response to the uncertainty shock, both industrial production and imports decline. But the response of imports is considerably stronger, about 5 to 10 times as strong in its period of peak impact during year one. The response of imports is also highly statistically significant. At its peak the IRF is 3 or 4 standard errors below zero, whereas the IRF for IP is only just about 2 standard errors below zero, and only just surmounts the 95% confidence threshold.

These results offer prima facie confirmation of the mechanisms suggested in our theoretical model. Indeed to the extent that the Bloom (2009) result for IP has proven novel, robust, and influential, one might argue that our finding of a import response to uncertainty that is almost an order of magnitude larger is also notable, especially since it opens an obvious route towards finding an explanation for the amplification effects seen during the recent trade collapse, a puzzle where, as we have seen, no fully convincing theoretical explanation has yet been given.

However, to make that claim more solid, we must convince the reader that the theoretical mechanisms we propose are indeed at work. To do that, we delve more deeply into the dynamics of disaggregated trade and IP in the wake of uncertainty shocks. The following empirical sections demonstrate that, taking into account cross-sectoral variations in perishability/durability and also in the intensity of downstream intermediate use, the empirical evidence closely matched our model's predictions. We find that imports of any good are, in general, more responsive to uncertainty shocks than domestic IP, whether in broad sectors (like End Use categories), or at a much more disaggregated level (e.g. NAICS 4-digit sectors). However, the aggregate results seen above will be shown to mask substantial sectoral heterogeneity in responses to uncertainty shock. With that taken into account at the end we will be able to weight the responses according, for both imports and IP, and compute a simulated response to the 2008 uncertainty shock aggre-

gated across sectors. We will show that this response closely matches the observed data, with import volume falling about twice as much as a basket of industrial production.

6.5   Results 2: IRFs with Coarse Disaggregation

Proceeding to a coarse level of disaggregation we now investigate IRFs for uncertainty shock when the trade and IP data are divided into either End Use categories (a BEA classification) or into Market Groups (a Fed classification). The purpose is to see whether the aggregate result holds up at the sectoral level, and, to the extent that there is any departure, to see if there is any systematic variation that is yet consistent with our model's more detailed predictions for heterogeneous goods.

Figure 12 shows IRF for real imports disaggregated into 6 BEA 1-digit End Use categories. The response to an uncertainty shock varies considerably across these sectors, but in a manner consistent with some predictions from theory.

There is essentially no response for the most perishable, or least durable, types of goods found in End Use category 0. These goods include foods, feeds and beverages. This response matches up with cases in our model when the depreciation parameter is set very high. In this case the response to uncertainty shocks diminishes towards zero. Responses are also weak for other nondurable consumer goods (End Use 4) and the residual category of imports not elsewhere specified (End Use 5).

In contrast, some sectors show a very large response to an uncertainty shocks, notably End Use categories 1, 2, and 3, which include industrial inputs, capital goods, and autos. These are all sectors characterized by either high durability and/or high downstream intermediate use. Again, our theory predicts that it is precisely these sectors that will experience the largest amplitude response to an uncertainty shock.

It is not possible to compare these IRFs to the corresponding response of domestic IP using the same End Use classification, since we cannot obtain IP disaggregated by End Use code. However, we can obtain both imports and IP disaggregated in a matched way at a coarse level by using the Fed's Market Group categories. IP is available directly in this format on a monthly basis and we were able to allocate imports to this classification by constructing a concordance (with some weighting using 2002 data on weights) mapping from 4-digit NAICS imports to Fed Market Group.

Figure 13 shows IRF for real imports (upper panel) and IP (lower panel) disaggregated into Fed Market Group categories. Again, the response to an uncertainty shock varies considerably across these sectors, and we can compare the import and IP responses directly. To facilitate this, all responses are shown on the same scale.

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Figure 12: Import IRFs by End Use category for Uncertainty Shocks
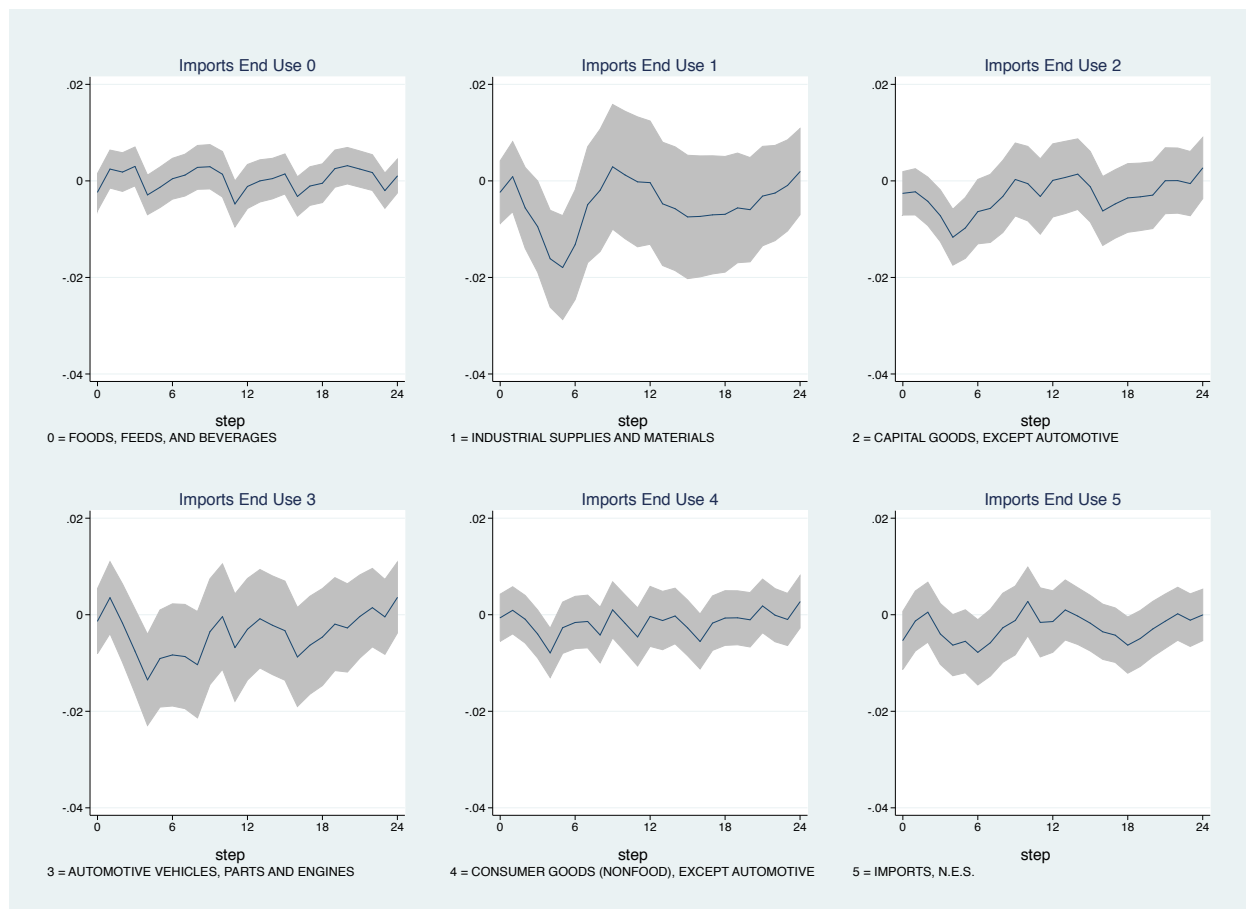
Source: Sample is 1989:1–2012:2. Imports by End Use 1-digit from USITC dataweb, deflated by CPI; all other data as in Bloom (2009), updated. Uncertainty shocks for quadvariate VARs as in Bloom (2009). Ordering is stock market, volatility, log employment, followed lastly by either log real imports or log IP. Data updated through February 2012. No rescaling of shocks. 95% confidence intervals shown. See text.

In panel (a) the results for imports are compatible with those above based on the End Use categories. Here, under the Fed Market Groups the largest amplitude responses to an uncertainty shock are seen for materials, business equipment and consumer durables. The responses here are between a 1 and 2 percent drop at peak. The weakest response is for consumer nondurables, which shows about a 0.5 percent drop at the peak, although this is barely statistically significant at the 95% level.

By contrast, in panel (b) the results for IP are very muted indeed. Confidence intervals are tighter, so these responses do in all cases breach the 95% confidence interval within a range of steps. However, the magnitude of the response is qualitatively different from imports. The consumer durables response is just below 1 percent at peak for IP, whereas it had been about twice as large, near 1.5 percent for imports. Materials and business

30

equipment fall at peak by about 0.25 percent for IP, but fell about four times as much in the case of imports. Consumer nondurables in IP are barely perturbed at all.

6.6 Results 3: IRFs with Finer Disaggregation

Our final set of results aims to study dynamic responses to uncertainty shocks at an even finer level of disaggregation, whilst still allowing for comparability between import and IP responses. For these purposes we move to the 3- or 4-digit NAICS level of classification, again sourcing the date from USITC dataweb and the Fed G.17 releases at a monthly frequency starting in 1989. The overlap between these two sources allows us to work with 51 individual sectors, as seen in Figure 14. A list of NAICS codes is provided in the empirical appendix.

A similar pattern emerges here, consistent with previous results, whereby the responsiveness in any sector is higher for real imports (CPI deflated) than for industrial production. There are some exceptions but these are generally to be found in only a few sectors. The bars in Figure 14 is ordered from top to bottom starting with largest negative real import response measured by the average sectoral IRF over months 1–12.

Some of the sectors are also obviously quite peculiar. One is basically a nonmanufacturing sector, and not very tradable — namely logging (NAICS 1133, which is resource intensive and not highly traded outside imports from Canada). This does fit the general pattern of imports being more volatile than domestic output, but this may reflect downstream use in the heavily procyclical construction industry (we discuss downstream use in the next section). Another oddity is tobacco manufacturing (NAICS 3122), where the response goes heavily against the prevailing pattern, with tobacco imports rising sharply after an uncertainty shock, and domestic supply basically flat. Still, this response is consistent with clinical studies which show that the use of tobacco may rise, and the ability of people to quit smoking may fall, in stressful periods of economic hard times.

Less unusual cases where the negative response of IP exceeds real imports are: Audio and video equipment manufacturing (NAICS 3343); Household and institutional furniture and kitchen cabinet manufacturing (3371); Industrial machinery manufacturing (3332); Ventilation, heating, air-conditioning, and commercial refrigeration equipment manufacturing (3334); Leather and allied product manufacturing (316); Apparel manufacturing (315); Nonmetallic mineral mining and quarrying (2123); Metalworking machinery manufacturing (3335). Still, out of 51 sectors, these are a minority.
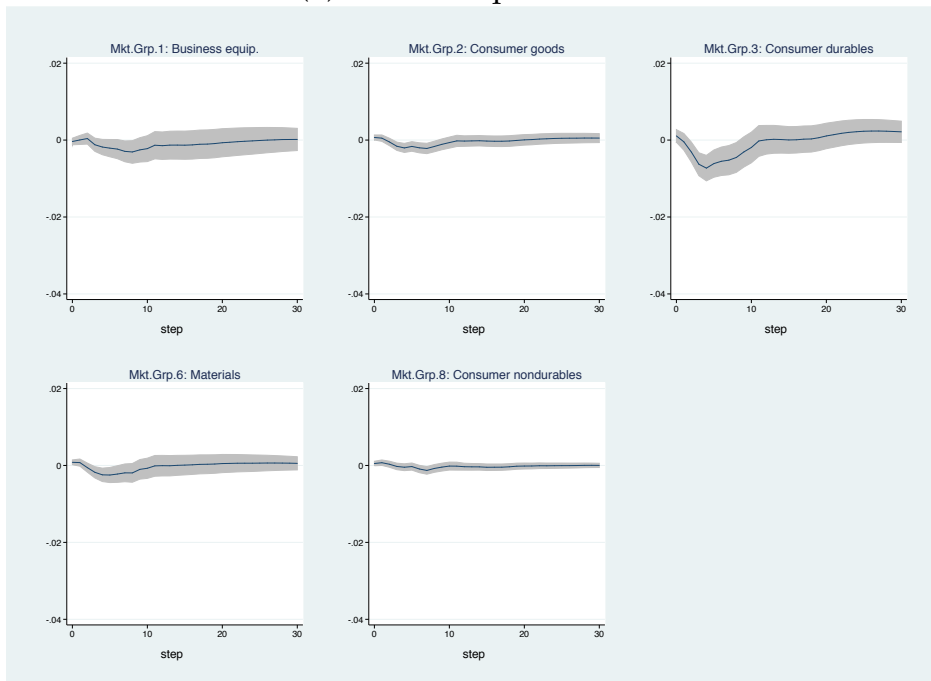
But generally, and especially for the high response sectors where responses are significantly different from zero, the real import bar is larger and more negative than the IP bar.

Figure 13: Import and IP IRFs by Fed Market Group for Uncertainty Shocks
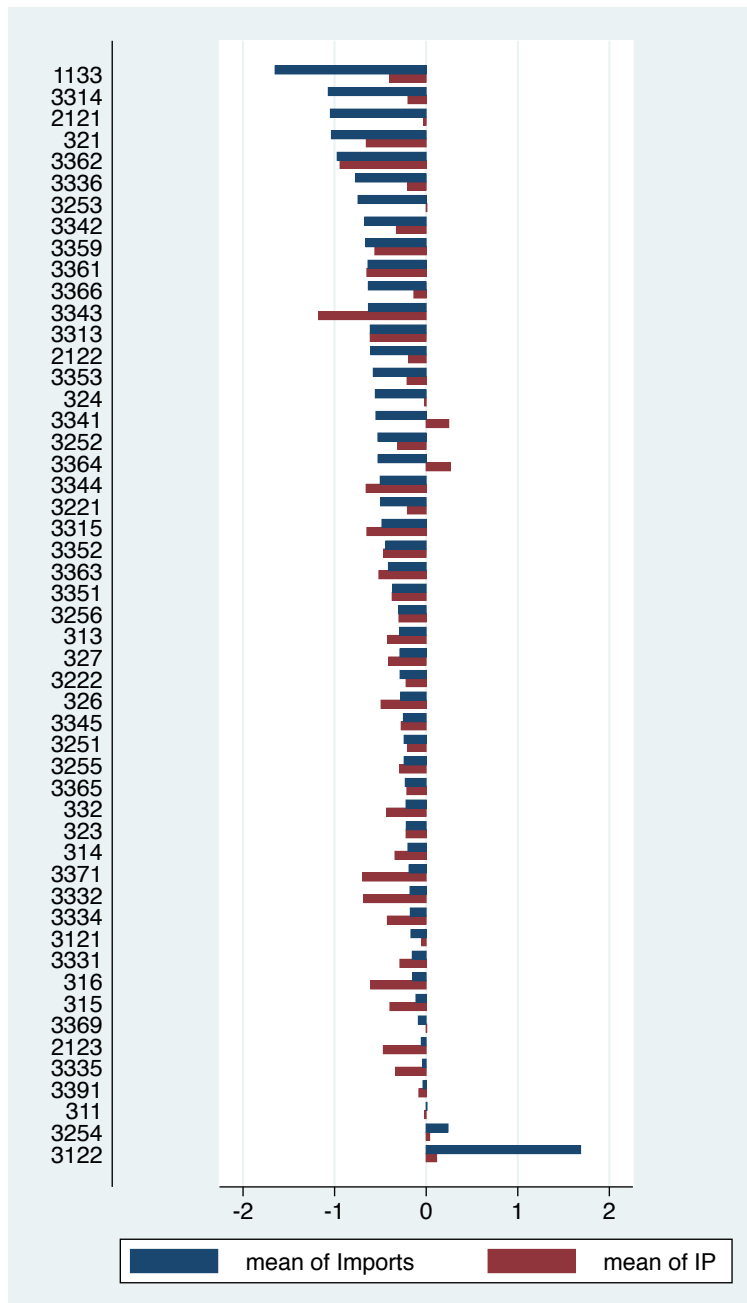
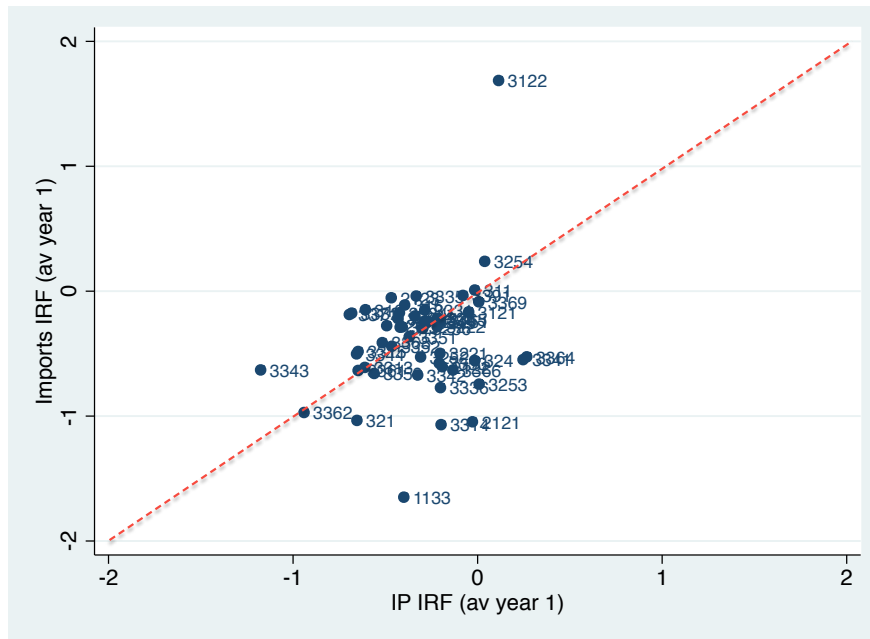(a) Real imports
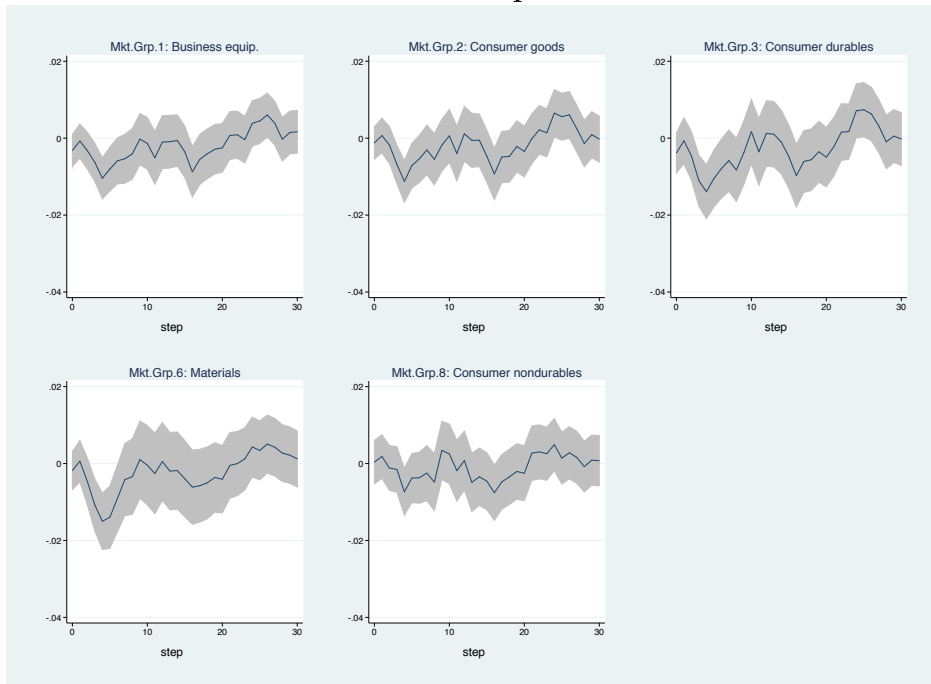
(b) Industrial production

Source: Sample is 1989:1–2012:2. Imports via concordance from USITC dataweb, deflated by CPI; IP from Fed G.17; all other data as in Bloom (2009), updated. Uncertainty shocks for quadvariate VARs as in Bloom (2009). Ordering is stock market, volatility, log employment, followed lastly by either log real imports or log IP. Data updated through February 2012. No rescaling of shocks. 95% confidence intervals shown. See text.

Figure 14: Import and IP IRFs Compared in Months 1–12

Source: Sample is 1989:1–2012:2. Imports from USITC dataweb, deflated by CPI; IP from Fed G.17; all other data as in Bloom (2009), updated. Uncertainty shocks for quadvariate VARs as in Bloom (2009). Ordering is stock market, volatility, log employment, followed lastly by either log real imports or log IP. Data updated through February 2012. No rescaling of shocks. Average IRF for months 1–12. See text.

Figure 15: Import and IP IRFs Compared in Months 1–12

The essence of this pattern is revealed in Figure 15 which presents a scatter of the average real import one-year IRF on the vertical axis against the average IP one-year IRF on the horizontal axis. There is only a very weak correlation between these responses (0.25) and when the outlier tobacco sector is excluded the correlation essentially vanishes (it falls to 0.15, but is not statistically significant).

What is more striking in the figure, however, is the general asymmetry relative to the 45 degree line. Most points lie in the lower-left quadrant where both real imports and IP react negatively to an uncertainty. In that quadrant, we do find points above the diagonal, where the IP response is more negative than the real import response – but generally these deviations from the diagonal are small. In contrast several sectors fall well below the diagonal by a significant margin, indicating a much sharper negative response of real imports compared to IP.

7 Inside the Black Box: Aggregate Inventory Responses and the Role of Downstream Intermediate Use

So far our empirical analysis has taken our model seriously but looked only for validation at the point where uncertainty shocks make their presence felt in the patterns of aggregate or sectoral supply responses, whether from IP at home or imports from abroad. However, our model makes other predictions about the mechanisms whereby these uncertainty shocks feed into the supply responses, and we would like to check these intermediate linkages to see whether the entire story fits together at each step; that is, to get beyond the "black box" of the impulse responses seen so far and look for further corroboration in other dynamics. In this section we do that by looking at two other series and their response to the uncertainty shocks.

7.1 Aggregate IRFs for Input Inventories

The first thing we check is aggregate input inventories of domestic (U.S.) firms. If our argument is true then the response of these inventories, denoted "materials and supplies" in the NIPA data from BEA (real 2005 prices, seasonally adjusted), should also show a signature of the uncertainty shock. We are careful to focus only on this component of NIPA inventories, and exclude "work in process" and "finished goods" items as these do not correspond to quantities in our model, where the key concept is input inventories. However, one shortcoming is that the NIPA inventories data are classified by the sector of the good produced, and are not broken down according to the sector of origin of the input used. It is the latter we would ideally like to see, so as to match responses with, say, the NAICS disaggregated IP and import data which represent the supply of goods being demanded as input classified by input type. At present, we have yet to find a direct way around this data limitation.

Pressing on, Figure 16 shows the IRF for input inventories of materials and supplies at the aggregate level. Sample is 1989:1–2012:12, so that the start date matches the 1989 beginning year of our disaggregated data samples, and runs through 2012. The message is clear that, despite, possible aggregation biases, the level of input inventories does show a negative response to uncertainty shocks, although this is small and somewhat delayed. The scale of this effect is bound to be small, however, compared to our earlier results, given that NIPA input inventories are measured as a *stock* whereas all of our earlier results pertain to *flows*, of either IP or imports.

We also have top recognize another shortcoming here: we are looking at input on

Figure 16: Aggregate IRF for Input Inventories
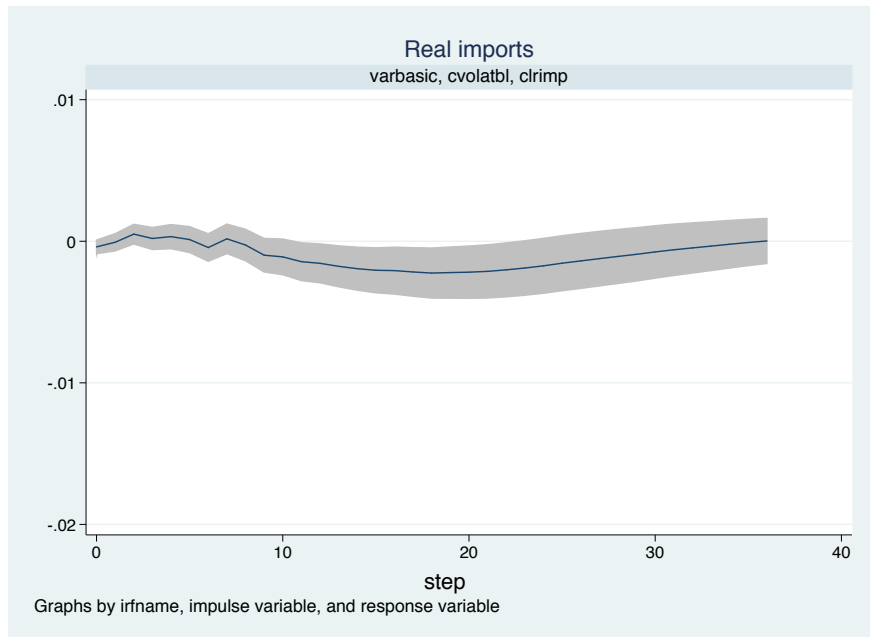
Source: Sample is 1989:1–2012:12. Response is for real inventories, materials and supplies, all manufacturing, real 2005 prices, seasonally adjusted, from BEA NIPA. IP from Fed G.17; all other data as in Bloom (2009), updated. Uncertainty shocks for quadvariate VARs as in Bloom (2009). Ordering is stock market, volatility, log employment, followed lastly by real inventories. Data updated through December 2012. No rescaling of shocks. Average IRF for months 1–12. See text.

hand as inventories and these may not decline as much as actual input orders themselves, because the decline in stocks will be to some extent mitigated by the decline in industrial activity itself in the using sector. [references to be added]

## 7.2   Disaggregated IRFs versus Downstream Intermediate Use

The second thing we look at is the relationship of the uncertainty IRFs to the intermediate input characteristics of the goods looking at the covariation of the IRFs with a measure of downstream intermediate use (see Levchenko, Lewis and Tesar 2010). Whilst not as clean as a direct input measure, we lack that on a flow or order basis for the using sector, or even in the aggregate, as noted above, and can only use aggregate input stocks. So at the moment the only sectoral disaggregation we can exploit that might proxy for sectoral differences in the uncertainty IRFs will be based on a measure of how intensively imports and IP in a given sector are involved in downstream intermediate use (versus going direct into final demand). This measure, which is not time-varying, can be obtained from the latest BEA use tables.

Figure 17 shows that for real imports at the sectoral level the average IRF in year one is significantly correlated with the extent to which the product is involved in downstream intermediate use (based on BEA 2002 MakeUse data for each of the 51 NAICS sectors). The figure also shows that this relationship is much weaker (and is not statistically significant) for IP at the sectoral level.

These findings can be related to predictions in our model, and are consistent with our model's predictions about downstream linkages and differential trade costs between home and foreign sourced goods.

[To be completed.]

Figure 17: Import and IP IRFs versus Downstream Intermediate Use
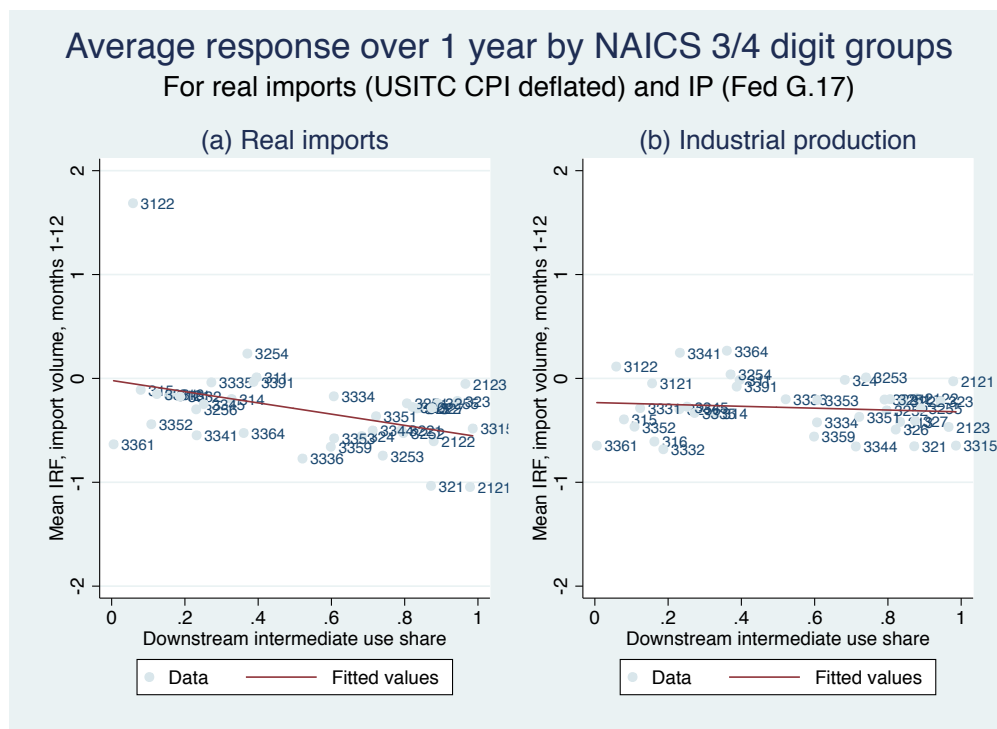
Source: Sample is 1989:1–2012:2. Imports from USITC dataweb, deflated by CPI; IP from Fed G.17; all other data as in Bloom (2009), updated. Uncertainty shocks for quadvariate VARs as in Bloom (2009). Ordering is stock market, volatility, log employment, followed lastly by either log real imports or log IP. Data updated through February 2012. No rescaling of shocks. Average IRF for months 1–12. See text.

Notes: The bivariate regressions of MEANIRF (dependent variable) on DOWNSTREAMUSE are as follows.

	(1) Imports all	(2) Imports ex autos 3361	(3) Imports ex tobacco 3122	(4) IP all	(5) IP ex autos 3361	(6) IP ex tobacco 3122
DOWNSTREAMUSE	-0.541*	-0.638**	-0.297*	-0.0898	-0.155	-0.0404
	(-2.64)	(-3.06)	(-2.09)	(-0.70)	(-1.19)	(-0.31)
Constant	-0.0195	0.0506	-0.201*	-0.233**	-0.185*	-0.269**
	(-0.15)	(0.38)	(-2.23)	(-2.89)	(-2.24)	(-3.25)
Observations	39	38	38	39	38	38

t-statistics in parentheses.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

The charts use fitted values from the full-sample regressions in columns (1) and (4), but the results are robust to the exclusion of autos (a key sector) or tobacco (an outlier, as seen in the imports chart), as seen in the other columns. In all cases downstream use is correlated with import response, but not IP response.

8 Can the Great Trade Collapse Be Explained?

The four months following the collapse of Lehman Brothers were characterized by particularly strong increases in uncertainty as measured by the volatility index (Sept.–Dec. 2008). To simulate this shock we observe that the own-response of volatility to itself in the orthogonalized impulse response is about 3. We perturbed the model with four successive +10-unit shocks to volatility, which, given the scaling corresponds to four +30-unit shocks to the VXO index. In actuality, the VXO rose from its pre-crisis mean of about 20 to almost 90 in the last quarter of 2008, and given the decay of the shocks, this set of impulses matches the actual path of VXO quite well, as shown in Figure 18.

Given these shocks he actual aggregate responses to IP and real imports are shown in Figure 19. As can be seen the IRF for IP is capable of explaining a large fraction of actual observed IP response. But it is wholly incapable of explaining the real import response. Our simulations can explain about 75-80 percent of the subsequent imports collapse. But especially in the first quarter of 2009, additional factors must be at work that are not captured by heightened uncertainty. Nonetheless, this shows that the evidence is consistent with a large fraction of the Trade Collapse being explicable in terms of second moment uncertainty shocks, rather than the conventional first moment explanations.

[Preliminary.]

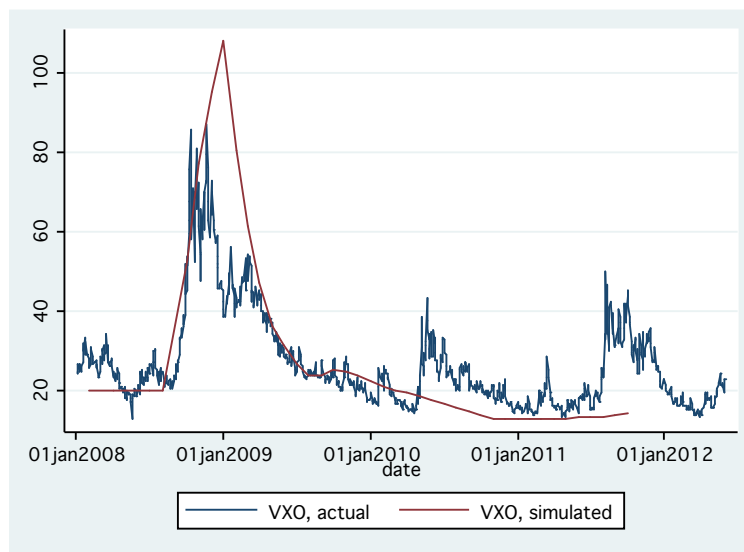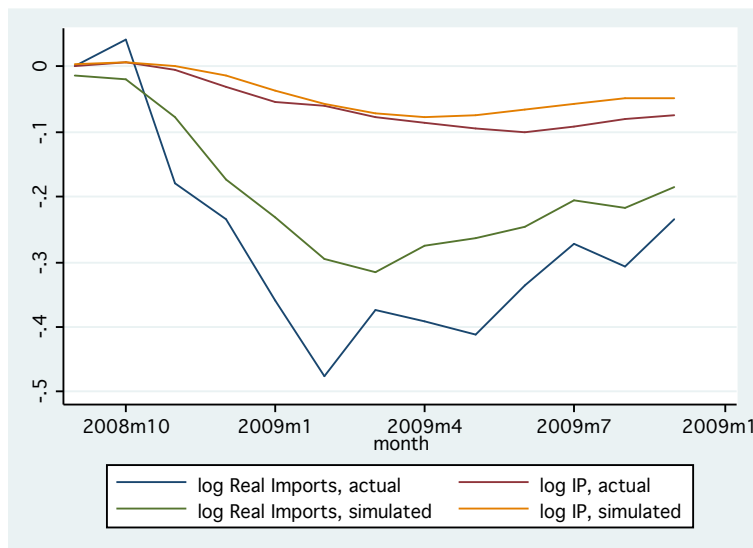Figure 18: Actual and simulated VXO, Aggregate Model

Figure 19: Actual and simulated Real Imports and IP, Aggregate Model

9 Conclusion

Following the seminal paper by Bloom (2009), we introduce second moment uncertainty shocks into a dynamic, open-economy model of international trade. Firms import intermediate inputs and due to fixed costs of ordering store them according to an optimal s, S inventory policy. We show that elevated uncertainty leads firms to shift down their s, S bands. This induces a sharp trade contraction of international trade flows followed by a swift recovery. However, in the absence of conventional first moment shocks output remains unaffected. Uncertainty shocks can therefore explain why trade is more volatile than domestic economic activity.

Our results offer an explanation for the Great Trade Collapse of 2008/09 and previous trade slowdowns in a way that differs from the conventional static trade models or dynamic inventory models seen before. We argue that imports and industrial production can be modeled as reacting to uncertainty shocks in theory and in practice. Such second moment shocks are needed since the required first moment shocks are either absent on the impulse side or insufficient on the propagation side (for plausible parameters) to explain the events witnessed. We also show that there is substantial heterogeneity in responses at the sectoral level, both for imports and industrial production, in a way consistent with the model.

References

[1] Alessandria, G., Kaboski, J., Midrigan, V., 2010a. The Great Trade Collapse of 2008–09: An Inventory Adjustment? *IMF Economic Review* 58, pp. 254–294.

[2] Alessandria, G., Kaboski, J., Midrigan, V., 2010b. Inventories, Lumpy Trade, and Large Devaluations. *American Economic Review* 100, pp. 2304–2339.

[3] Alessandria, G., Kaboski, J., Midrigan, V., 2011. US Trade and Inventory Dynamics. *American Economic Review* 101 Paper & Proceedings, pp. 303–307.

[4] Amiti, M., Weinstein, D., 2011. Exports and Financial Shocks. *Quarterly Journal of Economics* 126, pp. 1841–1877.

[5] Anderson, J., van Wincoop, E., 2003. Gravity with Gravitas: A Solution to the Border Puzzle. *American Economic Review* 93, pp. 170–192.

[6] Baldwin, R. (ed.), 2009. The Great Trade Collapse: Causes, Consequences and Prospects. Centre for Economic Policy Research, London.

[7] Baldwin, R., Krugman, P., 1989. Persistent Trade Effects of Large Exchange Rate Shocks. *Quarterly Journal of Economics* 104, pp. 635–654.

[8] Behrens, K., Corcos, G., Mion, G., 2013. Trade Crisis? What Trade Crisis? *Review of Economics and Statistics*, forthcoming.

[9] Bems, R., Johnson, R., Yi, K., 2011. Vertical Linkages and the Collapse of Global Trade. *American Economic Review* 101, pp. 308–312.

[10] Bems, R., Johnson, R., Yi, K., 2012. The Great Trade Collapse. NBER Working Paper 18632.

[11] Bloom, N., 2009. The Impact of Uncertainty Shocks. *Econometrica* 77, pp. 623–685.

[12] Bloom, N., Bond, S., Van Reenen, J., 2007. Uncertainty and Investment Dynamics. *Review of Economic Studies* 74, pp. 391–415.

[13] Bown, C. (ed.), 2011. The Great Recession and Import Protection: The Role of Temporary Trade Barriers. Centre for Economic Policy Research and World Bank, Washington, D.C.

[14] Bricongne, J., Fontagné, L., Gaulier, G., Taglioni, D., Vicard, V., (2012). Firms and the Global Crisis: French Exports in the Turmoil. *Journal of International Economics* 87, pp. 134–146.

[15] Chaney, T., 2008. Distorted Gravity: The Intensive and Extensive Margins of International Trade. *American Economic Review* 98, pp. 1707–1721.

[16] Chor, D., Manova, K., 2012. Off the Cliff and Back? Credit Conditions and International Trade during the Global Financial Crisis. *Journal of International Economics* 87, pp. 117–133.

[17] Dixit, A., 1989. Entry and Exit Decisions under Uncertainty. *Journal of Political Economy* 97, pp. 620–638.

[18] Eaton, J., Kortum, S., 2002. Technology, Geography and Trade. *Econometrica* 70, pp. 1741–1779.

[19] Eaton, J., Kortum, S., Neiman, B., Romalis, J., 2011. Trade and the Global Recession. NBER Working Paper 16666.

[20] Engel, C., Wang, J., 2011. International Trade in Durable Goods: Understanding Volatility, Cyclicality, and Elasticities. *Journal of International Economics* 83, pp. 37–52.

[21] Evenett, S. (ed.), 2010. Tensions Contained... For Now: The 8th GTA Report. Centre for Economic Policy Research, London.

[22] Gopinath, G., Itskhoki, O., Neiman, B., 2012. Trade Prices and the Global Trade Collapse of 2008–2009. *IMF Economic Review* 60, pp. 303–328.

[23] Grossman, S., Laroque, G., 1990. Asset Pricing and Optimal Portfolio Choice in the Presence of Illiquid Durable Consumption Goods. *Econometrica* 58, pp. 25–51.

[24] Guiso, L., Parigi, G., 1999. Investment and Demand Uncertainty. *Quarterly Journal of Economics* 114, pp. 185–227.

[25] Handley, K., 2012. Exporting under Trade Policy Uncertainty: Theory and Evidence. Working Paper, University of Michigan.

[26] Handley, K., Limão, N., 2012. Trade and Investment under Policy Uncertainty: Theory and Firm Evidence. NBER Working Paper 17790.

[27] Hassler, J., 1996. Variations in Risk and Fluctuations in Demand: A Theoretical Model. *Journal of Economic Dynamics and Control* 20, pp. 1115–1143.

[28] Levchenko, A., Lewis, L., Tesar, L., 2010. The Collapse of International Trade during the 2008–2009 Crisis: In Search of the Smoking Gun. *IMF Economic Review* 58, pp. 214–253.

[29] Limão, N., Maggi, G., 2013. Uncertainty and Trade Agreements. NBER Working Paper 18703.

[30] McDonald, R., Siegel, D., 1986. The Value of Waiting to Invest. *Quarterly Journal of Economics* 101, pp. 707–728.

[31] Paravisini, D., Rappoport, V., Schnabl, P., Wolfenzon, D. 2011. Dissecting the Effect of Credit Supply on Trade: Evidence from Matched Credit-Export Data. NBER Working Paper 16975.

[32] Pindyck, R., 1991. Irreversibility, Uncertainty, and Investment. *Journal of Economic Literature* 29, pp. 1110–1148.

[33] Scarf, H., 1959. The Optimality of (S,s) Policies in the Dynamic Inventory Problem. In: Arrow, K., Karlin, S., Suppes, P. (eds.), Mathematical Methods in the Social Sciences, Stanford University Press, pp. 196–202.

[34] Taglioni, D., Zavacka, V., 2012. Innocent Bystanders: How Foreign Uncertainty Shocks Harm Exporters. Working Paper, European Bank for Reconstruction and Development.

Technical Appendix

This appendix shows how the solution to the system of differential equations implied by equations (7) and (8) can be found. We closely follow Hassler (1996) and refer to his appendix for further details.

We plug the expression for $E_t V(z_{t+dt}, \omega_{t+dt})$ from equation (8) into equation (7). We then set $d t^2 = 0$ and divide by $d t$ to arrive at the following system of differential equations:

$$
rV(z_t, \omega_t) = \begin{array}{l} \frac{1}{2}z_t^2 - \delta V_z(z_t, \omega_t) \\ + \lambda_\omega \left\{ V(S_\omega, \omega_t) + f - V(z_t, \omega_t) \right\} + \gamma_\omega \left\{ V(z_t, \overline{\omega}_t) - V(z_t, \omega_t) \right\}. \end{array}
$$

The set of solutions to this system is given by

$$
V(z_t, 0) = \begin{array}{l} \frac{\alpha_0}{2}z_t^2 + \beta_0 z_t + c_1 \exp\left(\rho_1 z_t\right) + c_2 \exp\left(\rho_2 z_t\right) \\ + \phi_0 + \frac{1}{\Delta}\left\{ \lambda_1 \gamma_0 V(S_1, 1) + \lambda_0 \psi_1 V(S_0, 0) \right\} \end{array} \tag{10}
$$

for the state of low uncertainty, and

$$
V(z_t, 1) = \begin{array}{l} \frac{\alpha_1}{2}z_t^2 + \beta_1 z_t + v_1 c_1 \exp\left(\rho_1 z_t\right) + v_2 c_2 \exp\left(\rho_2 z_t\right) \\ + \phi_1 + \frac{1}{\Delta}\left\{ \lambda_1 \psi_0 V(S_1, 1) + \lambda_0 \gamma_1 V(S_0, 0) \right\} \end{array} \tag{11}
$$

for the state of high uncertainty, where c_1 and c_2 are the integration constants. The parameters ψ_0, ψ_1, Δ, α_0, α_1, β_0, β_1, ϕ_0 and ϕ_1 are given by

$$
\psi_\omega = r + \lambda_\omega + \gamma_\omega,
$$

$$
\Delta = \psi_0 \psi_1 - \gamma_0 \gamma_1,
$$

$$
\alpha_\omega = \frac{1}{\Delta}\left(r + \lambda_{\overline{\omega}} + \gamma_{\overline{\omega}} + \gamma_\omega\right),
$$

$$
\beta_\omega = -\frac{\delta}{\Delta}\left(\psi_{\overline{\omega}}\alpha_\omega + \gamma_\omega \alpha_{\overline{\omega}}\right),
$$

$$
\phi_\omega = \frac{1}{\Delta}\left(\psi_{\overline{\omega}}\left(\lambda_\omega f - \delta\beta_\omega\right) + \gamma_\omega\left(\lambda_{\overline{\omega}}f - \delta\beta_{\overline{\omega}}\right)\right),
$$

where $\overline{\omega} = 1$ if $\omega = 0$, and vice versa. $[v_i, 1]'$ is the eigenvector that corresponds to the eigenvalue ρ_i of the matrix

$$
\frac{1}{\delta}\begin{bmatrix} -(r + \lambda_1 + \gamma_1) & \gamma_1 \\ \gamma_0 & -(r + \lambda_0 + \gamma_0) \end{bmatrix}
$$

for $i = 1, 2$. Expressions for $V(S_0, 0)$ and $V(S_1, 1)$ can be obtained by setting $V(z_t, 0) = V(S_0, 0)$ and $V(z_t, 0) = V(S_1, 1)$ in equations (10) and (11), respectively, and then solving the two resulting equations.

Six key equations describe the solution. They are two value-matching conditions positing for each state of uncertainty that the value of the cost function at the return point must be equal to the value at the lower trigger point less the fixed ordering costs f:

$$
\begin{aligned}
V(S_0, 0) &= V(s_0, 0) - f, \\
V(S_1, 1) &= V(s_1, 1) - f.
\end{aligned}
$$

The remaining four equations are smooth-pasting conditions:

$$
\begin{aligned}
V_z(S_0, 0) &= 0, \\
V_z(s_0, 0) &= 0, \\
V_z(S_1, 1) &= 0, \\
V_z(s_1, 1) &= 0.
\end{aligned}
$$

These six conditions determine the six key parameters: the return points S_0 and S_1, the lower trigger points s_0 and s_1 as well as the two integration constants c_1 and c_2. Numerical methods have to be used to find them.

Empirical Appendix

[To be completed.]

List of NAICS 4-digit codes

1111   OILSEEDS AND GRAINS
1112   VEGETABLES AND MELONS 1113   FRUITS AND TREE NUTS
1114   MUSHROOMS, NURSERY AND RELATED PRODUCTS
1119   OTHER AGRICULTURAL PRODUCTS
1121   CATTLE
1122   SWINE
1123   POULTRY AND EGGS
1124   SHEEP, GOATS AND FINE ANIMAL HAIR
1125   FARMED FISH AND RELATED PRODUCTS
1129   OTHER ANIMALS
1132   FORESTRY PRODUCTS
1133   TIMBER AND LOGS
1141   FISH, FRESH, CHILLED OR FROZEN AND OTHER MARINE PRODUCTS
2111   OIL AND GAS
2121   COAL AND PETROLEUM GASES
2122   METAL ORES
2123   NONMETALLIC MINERALS
3111   ANIMAL FOODS
3112   GRAIN AND OILSEED MILLING PRODUCTS
3113   SUGAR AND CONFECTIONERY PRODUCTS
3114   FRUIT AND VEGETABLE PRESERVES AND SPECIALTY FOODS
3115   DAIRY PRODUCTS
3116   MEAT PRODUCTS AND MEAT PACKAGING PRODUCTS
3117   SEAFOOD PRODUCTS PREPARED, CANNED AND PACKAGED
3118   BAKERY AND TORTILLA PRODUCTS
3119   FOODS, NESOI
3121   BEVERAGES
3122   TOBACCO PRODUCTS
3131   FIBERS, YARNS, AND THREADS
3132   FABRICS
3133   FINISHED AND COATED TEXTILE FABRICS
3141   TEXTILE FURNISHINGS
3149   OTHER TEXTILE PRODUCTS
3151   KNIT APPAREL
3152   APPAREL
3159   APPAREL ACCESSORIES
3161   LEATHER AND HIDE TANNING
3162   FOOTWEAR
3169   OTHER LEATHER PRODUCTS
3211   SAWMILL AND WOOD PRODUCTS
3212   VENEER, PLYWOOD, AND ENGINEERED WOOD PRODUCTS
3219   OTHER WOOD PRODUCTS
3221   PULP, PAPER, AND PAPERBOARD MILL PRODUCTS
3222   CONVERTED PAPER PRODUCTS
3231   PRINTED MATTER AND RELATED PRODUCT, NESOI
3241   PETROLEUM AND COAL PRODUCTS
3251   BASIC CHEMICALS
3252   RESIN, SYNTHETIC RUBBER, & ARTIFICIAL & SYNTHETIC FIBERS & FILIMENT
3253   PESTICIDES, FERTILIZERS AND OTHER AGRICULTURAL CHEMICALS
3254   PHARMACEUTICALS AND MEDICINES
3255   PAINTS, COATINGS, AND ADHESIVES
3256   SOAPS, CLEANING COMPOUNDS, AND TOILET PREPARATIONS
3259   OTHER CHEMICAL PRODUCTS AND PREPARATIONS
3261   PLASTICS PRODUCTS
3262   RUBBER PRODUCTS
3271   CLAY AND REFRACTORY PRODUCTS
3272   GLASS AND GLASS PRODUCTS
3273   CEMENT AND CONCRETE PRODUCTS
3274   LIME AND GYPSUM PRODUCTS
3279   OTHER NONMETALLIC MINERAL PRODUCTS
3311   IRON AND STEEL AND FERROALLOY
3312   STEEL PRODUCTS FROM PURCHASED STEEL
3313   ALUMINA AND ALUMINUM AND PROCESSING
3314   NONFERROUS METAL (EXCEPT ALUMINUM) AND PROCESSING
3315   FOUNDRIES
3321   CROWNS, CLOSURES, SEALS AND OTHER PACKING ACCESSORIES
3322   CUTLERY AND HANDTOOLS
3323   ARCHITECTURAL AND STRUCTURAL METALS
3324   BOILERS, TANKS, AND SHIPPING CONTAINERS
3325   HARDWARE
3326   SPRINGS AND WIRE PRODUCTS
3327   BOLTS, NUTS, SCREWS, RIVETS, WASHERS AND OTHER TURNED PRODUCTS
3329   OTHER FABRICATED METAL PRODUCTS
3331   AGRICULTURE AND CONSTRUCTION MACHINERY
3332   INDUSTRIAL MACHINERY
3333   COMMERCIAL AND SERVICE INDUSTRY MACHINERY

3334   VENTILATION, HEATING, AIR-CONDITIONING, AND COMMERCIAL REFRIGERATION EQUIPMENT
3335   METALWORKING MACHINERY
3336   ENGINES, TURBINES, AND POWER TRANSMISSION EQUIPMENT
3339   OTHER GENERAL PURPOSE MACHINERY
3341   COMPUTER EQUIPMENT
3342   COMMUNICATIONS EQUIPMENT
3343   AUDIO AND VIDEO EQUIPMENT
3344   SEMICONDUCTORS AND OTHER ELECTRONIC COMPONENTS
3345   NAVIGATIONAL, MEASURING, ELECTROMEDICAL, AND CONTROL INSTRUMENTS
3346   MAGNETIC AND OPTICAL MEDIA
3351   ELECTRIC LIGHTING EQUIPMENT
3352   HOUSEHOLD APPLIANCES AND MISCELLANEOUS MACHINES, NESOI
3353   ELECTRICAL EQUIPMENT
3359   ELECTRICAL EQUIPMENT AND COMPONENTS, NESOI
3361   MOTOR VEHICLES
3362   MOTOR VEHICLE BODIES AND TRAILERS
3363   MOTOR VEHICLE PARTS
3364   AEROSPACE PRODUCTS AND PARTS
3365   RAILROAD ROLLING STOCK
3366   SHIPS AND BOATS
3369   TRANSPORTATION EQUIPMENT, NESOI
3371   HOUSEHOLD AND INSTITUTIONAL FURNITURE AND KITCHEN CABINETS
3372   OFFICE FURNITURE (INCLUDING FIXTURES)
3379   FURNITURE RELATED PRODUCTS, NESOI
3391   MEDICAL EQUIPMENT AND SUPPLIES
3399   MISCELLANEOUS MANUFACTURED COMMODITIES
5111   NEWSPAPERS, BOOKS & OTHER PUBLISHED MATTER, NESOI
5112   SOFTWARE, NESOI
5122   PUBLISHED PRINTED MUSIC AND MUSIC MANUSCRIPTS
9100   WASTE AND SCRAP
9200   USED OR SECOND-HAND MERCHANDISE
9800   GOODS RETURNED TO CANADA (EXPORTS ONLY); U.S. GOODS RETURNED AND REIMPORTED ITEMS (IMPORTS ONLY)
9900   SPECIAL CLASSIFICATION PROVISIONS, NESOI