# Optimal Multistage Adjudication

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*Abstract*

In many settings, there are preliminary or interim decision points at which legal cases may be terminated: e.g., motions to dismiss and for summary judgment in U.S. civil litigation, grand jury decisions in criminal cases, and agencies' screening and other exercises of discretion in pursuing investigations. This article analyzes how the decision whether to continue versus terminate should optimally be made when (A) proceeding to the next stage generates further information but at a cost to both the defendant and the government and (B) the prospect of going forward, and ultimately imposing sanctions, deters harmful acts and also chills desirable behavior. This subject involves a mechanism design analogue to the standard value of information problem, one that proves to be qualitatively different and notably more complex. Numerous factors enter into the optimal decision rule – some expected, some subtle, and some counterintuitive. The optimal rule for initial or intermediate stages is also compared to that for assigning liability at the final stage of adjudication.

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## 1. Introduction

Existing economics literature on decision-making in adjudication focuses on the final decision at the conclusion of a trial, i.e., on how stringent the burden of proof should be.[1] However, many cases are formally terminated earlier. In U.S. civil litigation, a court can grant a motion to dismiss after a case is filed or a motion for summary judgment after discovery but before trial. As an indication of the importance of these rules, the Supreme Court cases most cited by federal courts are a 1986 trilogy on summary judgment and one on motions to dismiss that was recently reversed in important respects.[2] In U.S. criminal cases, grand juries decide whether to issue indictments. In continental legal systems, adjudication has a sequential character, and some arbitration systems have the flexibility to operate in such a manner. More broadly and often less formally, investigators, prosecutors, and agencies screen cases at various points and make ongoing decisions whether to pursue cases or to terminate them.[3]

This article analyzes how optimally to make continuation decisions at each point in a multistage process. At first glance, this problem seems like familiar ones in decision analysis concerning the value of information: at any stage, one can make an ultimate decision or else choose to expend resources to obtain further information upon which a later decision can be based. In contrast to these standard settings, the present context involves decisions that centrally influence ex ante behavior.[4] Hence, the present investigation may be understood as developing a solution to the value of information problem in a mechanism design setting.

A central justification for the legal system is to deter harmful acts, and an important side effect is that the prospect of false positives and of incurring adjudication costs will chill desirable behavior (for example, mistaken liability for IPOs may discourage them and raise the cost of capital). When outcomes matter because of ex ante behavioral effects, the optimal decision problem – and, implicitly, determination of the value of information – is qualitatively different and considerably more complex. For example, Bayesian prior probabilities, instead of playing a central role, are irrelevant to the core behavioral components and enter additively (rather than by a difference or ratio) in a system cost component. In addition, even the cost of information is endogenous because the frequency with which the costs must be incurred depends on behavior, which in turn is influenced by the decisions that are anticipated to be made, including those

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[1]See, for example, Kaplow (2011), Lando (2002), and Rubinfeld and Sappington (1987). Some literature considers other aspects of adjudication not relevant for present purposes, such as regarding the resources devoted to the presentation of evidence or bringing suit (usually with exogenous underlying behavior). See, for example, Bernardo, Talley, and Welch (2000) and Hay and Spier (1997).

[2]On summary judgment, see Matsushita Elec. Indus. Co. v. Zenith Radio Corp., 475 U.S. 574 (1986), Anderson v. Liberty lobby, Inc., 477 U.S. 242 (1986), and Celotex Corp. v. Catrett, 477 U.S. 317 (1986). On motions to dismiss, see Bell Atlantic Corp. v. Twombly, 550 U.S. 544 (2007) (in the Court's language, "retir[ing]" the previously dominant version of the legal test from Conley v. Gibson, 355 U.S. 41 (1957)), and Ashcroft v. Iqbal, 556 U.S. 662 (2009).

[3]Some literature, notably in antitrust, has addressed information that may be used in screening but not how screening decisions should optimally made in light of available information. See, for example, Abrantes-Metz et al. (2006) and Harrington (2007).

[4]The more typical and straightforward version of the value of information problem would, however, be applicable in legal contexts that, unlike those examined here, have as their the main consequence the regulation of future conduct (licensing, zoning decisions, drug authorization, merger approval) or determination of eligibility for future government transfers.

about when to obtain additional information.

The analysis begins in section 2, which examines the optimal first stage decision in a model with two stages: a preliminary decision and, conditional on continuation to trial, a final judgment on liability. This initial modeling choice is made for tractability and also for clarity; a number of insights that prove to be robust can be gleaned from this simpler formulation. In the model, there are both harmful acts, which the legal system aims to deter, and benign acts that, unfortunately, the legal system may mistake for harmful ones and, as a consequence, may cause to be chilled. Individuals' decisions for each type of act reflect its associated expected sanctions as well as the adjudication costs expected to be borne. A fraction of committed acts of each type enters the legal system (determined by a regime of enforcement, taken here to be given). Based on a preliminary signal, the tribunal decides whether to let a case continue to final adjudication or to terminate it.[5] If the case proceeds, costs are incurred both by the individual and the government, and an additional signal is observed. Final adjudication uses all the information to determine whether to find liability and thus impose a sanction. Taking the functioning of this final stage as given in this section, the focus is on how optimally to make the first-stage continuation/termination decision.

Continuation relative to termination in a given state – i.e., for a given realization of the preliminary signal – has two types of effects. Most directly, there is an inframarginal cost regarding both harmful and benign acts that are committed: individuals and the government bear adjudication costs that otherwise would be spared. Additionally, there are marginal effects on behavior: greater deterrence and more chilling, as a consequence of keeping alive the possibility of imposing sanctions in an additional situation and also the certainty that, in the state under consideration, defendants in the system will bear additional adjudication costs. These inframarginal and marginal effects are qualitatively different. The former depend (among other factors) on the undeterred and unchilled masses for the two types of acts, which in turn depend on the cumulative distribution functions for the benefits of the two types of acts. The latter depend instead on the densities of these distributions for the marginal actor of each type. Accordingly, some factors affect just one or the other and some factors affect both but in different ways.

Some of the results for the optimal rule are expected while others are more counterintuitive. When the degree of underdeterrence is greater and when the cost of chilling benign acts is lower, continuation is more favorable. An implication is that decisions in other states affect the optimal decision in the situation at hand in multiple, subtle ways. Continuation is also favored by a relatively higher rate of harmful rather than benign acts entering the legal system and by higher diagnosticities of the signals received when entering the first stage and when moving to final adjudication. Higher adjudication costs have surprisingly complex effects on the optimality of continuation. The direct cost of continuation, of course, favors termination. However, the greater are adjudication costs, the greater is the social benefit of deterring a

[5] It is obvious that an unconstrained optimal system would also allow adjudication to conclude at the first stage or any intermediate stage with a finding of liability. Because this either is not permitted or tends to be relatively unimportant for most of the legal system features being modeled, this possibility is omitted. It will be clear how one could extend the model to incorporate it.

marginal harmful act and the less is the social cost of chilling a marginal benign act, so continuation's discouragement of both types of acts makes it more favorable when system costs are larger. Last, defendants' adjudication costs contribute to deterrence as well as to chilling, so higher defendants' costs can favor or oppose continuation on this behavioral dimension.

Section 2 also extends the analysis to allow the government to choose the level of enforcement effort and sanctions along with first-stage continuation/termination decisions. Many determinants of optimal effort differ qualitatively from those in prior literature. Optimal sanctions are maximal, a common feature of law enforcement models, but much of the basis here differs from the familiar argument, relating instead to the relative targeting precision of different enforcement instruments.

Section 3 presents a more general model for adjudication that allows any number of stages. This model is analyzed using recursion. The optimality condition for decisions in any given stage, except for the final stage, is similar to that for the first stage in section 2's two-stage model. Nevertheless, the more general formulation allows one to see how optimal decisions can differ at different stages and how they are interdependent (in both directions) across stages. An important application of this sort of analysis relates to the fact that actual legal systems may be subject to institutional constraints on decisions at some stages, which importantly influence how decisions at other stages, which may be less constrained, should optimally be made. For example, one can determine how an agency would optimally take into account subsequent rules governing adjudication in courts, or how courts would optimally adjust their rules if the inflow of cases (reflecting, for example, prosecutors' and agencies' decision-making) may deviate systematically from what is socially optimal.

Section 3 also analyzes the optimal decision rule for the final stage of adjudication. This rule differs from that applicable to all earlier stages as a consequence of there being no meaningful future continuation. As a consequence, the optimal rule can be represented as a standard likelihood ratio test, unlike at prior stages. More broadly, optimal decision rules at nonfinal stages cannot readily be compared in terms of stringency with each other or with that at the final stage. Conventional legal wisdom favoring increased stringency at later stages is difficult to rationalize as a general matter, among other reasons because more adjudication costs are sunk as cases proceed. Section 4 offers concluding remarks.

## 2. First Stage of Adjudication in a 2-Stage Model

### *A. Model*

There are two types of acts that may be committed, a harmful one, $H$, and a benign one, $B$. The harmful type of act imposes an external social cost of $h$; the benign type of act involves no externality.[6] A mass of individuals normalized to 1 may commit the harmful type of act. Those who may commit the benign type of act have a mass of $\gamma$; an interpretation is that $\gamma$

[6]One could readily allow a smaller negative externality or a positive externality.

indicates the relative quantity of benign acts that may be undertaken in situations in which they might initially be confused with harmful acts, for other benign acts do not face the risk of entering the legal system and being subject to sanctions.[7]

Individuals' benefits from committing an act are $b$, with density functions $f^i(b)$ (which are positive for positive values of $b$) and cumulative distribution functions $F^i(b)$, where $i = H, B$. Individuals know what type of act they are able to commit and its benefits to them, but the government initially has no knowledge of an act's type and never learns individuals' benefits from acts.

Individuals who commit these acts enter the legal system with probabilities $\pi^i$.[8] The government's policy choice is whether to allow cases to proceed ($\delta = 1$) or instead to terminate them ($\delta = 0$). Terminating a case may be thought of as a court granting a defendant's motion to dismiss or for summary judgment, a grand jury's decision not to issue an indictment, or an agency's or prosecutor's decision not to continue an investigation (screening out a case). For cases that continue to adjudication, two costs are incurred: $c$ is borne by the defendant and $k$ by the state.[9] These cases are ultimately tried and result in findings of liability with probabilities $p^i$, conditional on reaching the final stage of adjudication. Individuals found liable are subject to the sanction $s$, taken here to be a socially costless fine.[10] At this point, all legal system attributes except $\delta$ are taken as given; one may suppose that they are optimized or simply are fixed for other reasons. (Of particular interest, the $p^i$ can be understood to depend, in a manner suppressed in this section, on the information available in adjudication and the use of an optimal decision rule at the final stage, as analyzed explicitly in section 3.C.)

Cases entering the system are observationally distinguishable to an extent. In particular, there is a signal $\sigma$, distributed by $z^i(\sigma)$, positive on the real line, that is informative about the type of case.[11] The $p^i$ and the continuation costs, $c$ and $k$, each depend on $\sigma$.[12] (It is natural to think of

[7]One could instead imagine that the same individuals may commit both types of acts and that $\gamma$ indicates the relative frequency of opportunities to commit the benign type of act. Alternatively, one could allow individuals to choose one of the two acts or inaction, which would complicate the exposition but have only a modest effect on the qualitative results: deterrence of harmful acts would induce some individuals to switch to benign acts rather than inaction, and chilling of benign acts would cause some individuals to switch to harmful acts rather than inaction.

[8]The model takes as given some enforcement technology and (until section 2.C) the level of enforcement effort that generates these probabilities. The present formulation is most akin to monitoring (the posting of agents on the lookout for what appear to be harmful acts) or auditing (including inspections and the like). For further discussion of different enforcement technologies, see Shavell (1991), Mookherjee and Png (1992), and Kaplow (2011).

[9]One could readily introduce defendant and government costs of entering the initial stage of adjudication, a possibility allowed in the more general formulation in section 3.

[10]One could introduce costly sanctions, which would have two competing effects on the optimal rules. On one hand, continuation (and, in the final stage, liability) would be more favorable because deterring and chilling marginal acts would have the added benefit of reducing expected sanction costs. On the other hand, in inframarginal cases (those that remain undeterred or unchilled, as the case may be), continuation (or ultimate liability) would be more costly.

[11]No particular requirement, such as satisfaction of the monotone likelihood ratio property, is imposed on the $z^i(\sigma)$; as will emerge, especially in section 3, little would be gained in the present setting. All that is assumed is that the functions are such that the integrals below are well defined.

[12]Regarding the $p^i$, the intended interpretation is that, for different $\sigma$, there are different expected distributions of evidence that will be used in adjudication and, for the given decision rule, this difference will imply differing probabilities of liability. See section 3.C. That the $\pi^i$ do not depend on $\sigma$ is without loss of generality. (If one did allow

σ as a vector, some elements of which may bear on the type of act and others on costs; a scalar representation is employed to simplify notation.) The government's problem is to choose the function $\delta(\sigma)$: that is, for each σ, to decide whether to allow the case to proceed or to terminate it.

The model can be summarized by reference to its timing:

1. The government sets all policy instruments, notably, the function $\delta(\sigma)$.
2. Individuals learn their type of act (*H* or *B*) and their private benefit *b*.
3. Individuals decide whether to act.
4. A portion of those who commit each type of act, $\pi^H$ and $\pi^B$, are identified and brought before a tribunal.
5. The signal σ is realized.
6. The tribunal either allows a type of case to continue ($\delta(\sigma) = 1$) or instead terminates it ($\delta(\sigma) = 0$).
7. If the case continues to final adjudication, the costs $c(\sigma)$ and $k(\sigma)$ are incurred by the defendant and the government, respectively.
8. If the case continues to final adjudication, individuals pay the sanction *s* with probabilities $p^H(\sigma)$ and $p^B(\sigma)$, as appropriate for their type of act.

Individuals are assumed to be risk neutral.[13] Accordingly, those whose type of act is *i* commit their act if and only if:

$$(1)\ b > \int_{-\infty}^{\infty} \pi^i \delta(\sigma)\Big(c(\sigma) + p^i(\sigma)s\Big)z^i(\sigma)d\sigma \equiv b^i .$$

That is, individuals act when their benefit exceeds their expected cost for their type of act, where their expected cost is (the integral, for each realization of σ, of) the product of (A) the likelihood that they will enter the initial stage of adjudication, (B) the likelihood that their case will continue rather than be terminated (which likelihood is one or zero, depending on the decision rule), and (C) the sum of: (i) the defendant's adjudication cost of going forward, and (ii) the likelihood (conditional on continuation) of being found liable times the sanction. As indicated on the right side of expression (1), it is convenient to define $b^i$ as the value of this expected cost for each type of act, that is, the benefit level of an individual who is just indifferent about whether to act for each type of act, *H* and *B*.

Social welfare, *W*, is taken to be the aggregate of individuals' benefits from acting minus the harm from the commission of acts of type *H* and the costs of defendants' and the

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this dependence, one could simply redefine variables so that what is here called $\pi^i$ would be the mean of that variable, with the variation absorbed in the corresponding density functions; below, $\pi^i$ and $z^i(\sigma)$ always appear as a product.) One could also extend the model to allow the $f^i$ and *h* (as well as the policy instrument *s*) to depend on σ, the implications of which would largely be straightforward.

[13]Introducing risk aversion would have two main effects. First, deterrence and chilling effects would rise nonlinearly with *s* (which is of little interest here since *s* is taken as given). Second, sanctions would then be socially costly, the effect of which is discussed in note 10.

government's expenditures on adjudication.

$$(2)\ W = \int_{b^H}^{\infty} \int_{-\infty}^{\infty} \Big(b - h - \pi^H \delta(\sigma)\big(c(\sigma) + k(\sigma)\big)\Big) z^H(\sigma) d\sigma\, f^H(b) db$$

$$+ \gamma \int_{b^B}^{\infty} \int_{-\infty}^{\infty} \Big(b - \pi^B \delta(\sigma)\big(c(\sigma) + k(\sigma)\big)\Big) z^B(\sigma) d\sigma\, f^B(b) db.$$

The first term indicates the benefits and harm attributable to harmful acts, where in addition to the direct external harm there is also the expected adjudication cost of defendants and the government. Similarly, the second term indicates the benefits from benign acts and the expected adjudication cost for them. In each instance, the lower limit of integration on the outer integral is the benefit of the individual just at the margin, with all individuals having greater benefits committing the act, as per expression (1).

*B. Analysis*

The government's problem is, for each possible realization of σ, say σ°, to decide whether to allow the case to continue (δ(σ°) = 1) or instead to terminate it (δ(σ°) = 0). It is optimal to allow the case to continue if and only if the value of *W* in expression (2) is greater when δ(σ°) = 1 than when δ(σ°) = 0. (At the optimum, this relationship will hold for all values of σ where δ(σ) = 1 and for none for which δ(σ) = 0, except on a set of measure zero, which qualification will be omitted in the exposition that follows).[14] Examination of expression (2) indicates that the difference in the pertinent values of *W* will involve two types of effects. First, the values of the $b^i$ will differ: if the case is allowed to continue (δ(σ°) = 1), these magnitudes will be higher, which is to say, the deterrence of harmful acts and the chilling of benign acts will be greater, in a manner determined by expression (1). Second, the values of the adjudication cost terms in both integrands will be larger since, in state σ°, defendant and government adjudication costs are incurred if δ(σ°) = 1 but not if δ(σ°) = 0.

Setting δ(σ°) = 1 is optimal if and only if:

[14]One can also contemplate a discrete version of the problem, where the variable σ designates partitions. For that problem, one might be concerned about whether other system variables would be optimized (in general, differently) for different choices of a particular δ(σ). For the continuous version examined here, such would not matter (and, in any case, for present purposes, other policy variables are taken to be fixed, although in this formulation we can imagine them to be at their optimal values, which are invariant when characterizing the optimal δ for a given σ°). Also, in the discrete version, the optimum may involve randomization, there being an optimal fraction of cases in some partition that would be continued rather than terminated.

$$(3)\ \pi^H\Big(c(\sigma^\circ)+p^H(\sigma^\circ)s\Big)z^H(\sigma^\circ)f^H(b^H)\left(h+\int_{-\infty}^{\infty}\pi^H\,\delta(\sigma)\big(c(\sigma)+k(\sigma)\big)z^H(\sigma)d\sigma-b^H\right)$$

$$-\int_{b^H}^{\infty}\pi^H\big(c(\sigma^\circ)+k(\sigma^\circ)\big)z^H(\sigma^\circ)f^H(b)db$$

$$+\gamma\pi^B\Big(c(\sigma^\circ)+p^B(\sigma^\circ)s\Big)z^B(\sigma^\circ)f^B(b^B)\left(\int_{-\infty}^{\infty}\pi^B\,\delta(\sigma)\big(c(\sigma)+k(\sigma)\big)z^B(\sigma)d\sigma-b^B\right)$$

$$-\gamma\int_{b^B}^{\infty}\pi^B\big(c(\sigma^\circ)+k(\sigma^\circ)\big)z^B(\sigma^\circ)f^B(b)db\ >0.$$

The first term (first row) in expression (3) is the deterrence effect of allowing cases with the signal σ° to continue. First, we have the deterrence punch: if cases in the legal system involving a harmful act are allowed to proceed (rather than being terminated), the pertinent fraction of harmful acts that enter the system are subject to both the defendant's adjudication costs (which will be incurred because such cases will continue) and an expected explicit sanction (the probability, conditional on being in the system and proceeding, of being found liable, times the sanction). These are weighted by the two densities, reflecting how many cases there are in this state σ° and, for a unit rise in individuals' expected cost for the harmful act, how many harmful acts are deterred, which is given by the density of the benefit distribution for harmful acts, evaluated at the marginal act. Second, this deterrence punch is multiplied by the effect on social welfare per marginal act that is deterred: the sum of the external harm that is avoided and the expected adjudication cost savings, minus the benefit of the marginal act that is forgone. To elaborate on the middle term in the large parentheses (which is an integral over σ rather than being evaluated at σ°): for acts that are deterred, they will not enter the system for any realization of σ, and thus the adjudication costs will be avoided for all such σ (not just for σ°).

The second term (second row) in expression (3) indicates the cost incurred, for (undeterred) harmful acts that end up in the system in state σ°, due to the fact that cases in this state will continue rather than terminate. The integral (over *b*, for harmful acts) covers all undeterred harmful acts. The internal weighting by $\pi^H$ reflects that costs are only incurred for cases that enter the system, and the weighting by $z^H(\sigma^\circ)$ that they are only incurred, as a consequence of this policy decision, for state σ°.

The third and fourth terms (third and fourth rows) in expression (3), for benign acts, are almost the same qualitatively as the first two terms. Each is weighted by γ, indicating the relative mass of benign acts. And the right component of the chilling term (third row) does not contain *h* because there is no external harm associated with benign acts.

It is helpful to rewrite expression (3) in the following, somewhat simplified form:

$$(4)\ \pi^H\left(c(\sigma^\circ)+p^H(\sigma^\circ)s\right)z^H(\sigma^\circ)f^H(b^H)\left(h+\kappa^H-b^H\right)\ >$$

$$\gamma\pi^B\left(c(\sigma^\circ)+p^B(\sigma^\circ)s\right)z^B(\sigma^\circ)f^B(b^B)\left(b^B-\kappa^B\right)$$

$$+\left(\left(1-F^H(b^H)\right)\pi^H z^H(\sigma^\circ)+\gamma\left(1-F^B(b^B)\right)\pi^B z^B(\sigma^\circ)\right)\left(c(\sigma^\circ)+k(\sigma^\circ)\right),$$

where

$$(5)\ \kappa^i\ =\int_{-\infty}^{\infty}\pi^i\delta(\sigma)\left(c(\sigma)+k(\sigma)\right)z^i(\sigma)d\sigma.$$

The variable $\kappa^i$ defined in expression (5) is the expected adjudication cost associated with each of the two types of acts. The $1-F^i(b^i)$ terms in the third row of expression (4) indicate the undeterred and unchilled fractions of those who have opportunities to commit harmful and benign acts, respectively.

On the left side of the inequality in expression (4) is the deterrence gain from allowing the case with the associated signal $\sigma^\circ$ to continue. We have the deterrence effect multiplied by the net benefit per act deterred. The first term on the right side of (4) is the corresponding chilling cost, reflecting that there is now a greater likelihood that benign acts will be subject to defendants' costs of adjudication and the possibility of being formally sanctioned. The second term on the right side (row three) is the rise in expected adjudication costs associated with state $\sigma^\circ$: for all acts (of both types) that flow into the system and are in that state, the decision to continue rather than terminate means that both defendants' and the government's adjudication costs associated with that state are incurred.

If the deterrence gain exceeds the sum of the chilling cost and supplemental adjudication cost, it is optimal to allow the case to continue. Observe that the chilling cost (row two) can readily be a net benefit: specifically, if the marginal benign act that is chilled has a benefit, $b^B$, lower than the expected adjudication cost associated with the act being committed, $\kappa^B$, social welfare would be higher as a consequence of chilling. (Note importantly that the benefit of the pertinent act chilled depends on the marginal benign act, that for which individuals are just indifferent, which in turn depends on the legal system, including on how continuation/ termination decisions are made.)

Because of this point, it is not necessarily true, as in standard enforcement models (see

Polinsky and Shavell 2007), that at the optimum there will be underdeterrence (relative to the first best, wherein acts are committed if and only if the private benefit exceeds $h$). Suppose, however, that the right side of (4) is positive – a possibility that may well hold even if chilling did happen to be desirable, as a consequence of the other term, reflecting total adjudication costs in state $\sigma°$ if the decision is to continue. Because the deterrence benefit on the left side includes not only avoiding $h$ but also avoiding $\kappa^H$, the left side could be positive even if there was overdeterrence (that is, if $h < b^H$).[15]

Reflection on expression (4) suggests a number of features regarding how the optimal first-stage decision should be made in a given state (for a given signal $\sigma°$), some of which might readily have been anticipated and others that are not as obvious and may initially seem counterintuitive. Regarding the Propositions to follow, two important observations should be noted. First, as just explained, it is possible that the chilling effect, the first term on the right side of (4), involves a net benefit rather than a cost. Some of the statements are qualified accordingly (by the further stipulation that $b^B > \kappa^B$).

Second, each of the propositions stipulates that "all else is equal." The meaning is straightforward when the claim involves an exogenous parameter that, moreover, has no influence on any of the endogenous variables, notably, the $b^i$ and the $\kappa^i$ (the former of which, in turn, affects the $f^i$ and the $F^i$). When an exogenous parameter does influence an endogenous variable, the meaning is that the values of other exogenous variables in other states, which do not directly appear in expression (4), are taken to be adjusted so as to keep those endogenous variables constant. Accordingly, the claim will involve a comparison of two different worlds (sets of exogenous variables) in which, in the state in question, only the posited variable differs. Similarly, when a claim concerns an endogenous variable, it is assumed that exogenous variables in other states differ so that, in the state under consideration, the only resulting difference regards the endogenous variable that is the subject of the claim. In these latter instances, it will be explained (mostly in notes) how such situations in which "all else is equal" can be constructed. Formal proofs of the actual propositions are omitted because, given this proviso, the derivation of each result is straightforward.

The first set of results pertains to the benefits and costs of deterrence and chilling.

*Proposition 1: When all else is equal,*

*a: a higher $h$ favors $\delta(\sigma°) = 1$,*

*b: a higher $\gamma$ favors $\delta(\sigma°) = 0$ (assuming that $b^B > \kappa^B$)*

*c: a higher achieved degree of deterrence ($b^H$ being larger) and a higher achieved degree of chilling ($b^B$ being larger) each favor $\delta(\sigma°) = 0$, and*

*d: a higher $\kappa^H$ and a higher $\kappa^B$ each favor $\delta(\sigma°) = 1$.*

Proposition *1.a* is obvious: raising $h$ increases the deterrence benefit on the left side of expression (4) and has no other effect. Likewise for Proposition *1.b*: raising $\gamma$ (the mass of

[15] It is also possible that, considering all three elements, the gain per act deterred can be negative, which case will not be considered explicitly (and will not be optimal in typical settings).

opportunities for benign acts) increases two of the costs (the chilling cost and a component of adjudication costs) on the right side of expression (4). (The reason for the additional proviso, a sufficient condition for the result, is given above.)

Proposition *1.c* reflects that both deterrence and chilling involve the cost of forgoing individuals' benefits from their acts, and the greater the forgone benefit per act deterred or chilled, the larger is that cost. (For elaboration on the "all else equal" proviso regarding this result and the next, see the note.[16]) An important lesson is that the optimal decision in a given state depends on the characteristics of other states. For example, anticipating Propositions *2.a* and *2.c*, if the signal in many other states strongly indicates a harmful act, so as to lead to continuation, the achieved degree of deterrence may be great, making continuation in the present state less advantageous. Similarly, if continuation in many other states involves substantial chilling, termination in the present state is favored. This result casts in a different light one of the central understandings of courts and legal scholars: that continuation (to allow discovery and the like) is necessarily favored in cases in which information is primarily in the possession of the defendant. On one hand, if this is generally true and it leads to termination in most states, then achieved deterrence and chilling may tend to be low; on the other hand, if it is generally false but idiosyncratically true in the case (state) at hand, then the opposite situation may prevail, favoring termination. (And, as will be discussed in a moment, the result in Proposition *1.d* also tends to cut against the standard view.)

Note further that the levels of achieved deterrence and chilling described in Proposition *1.c* may in practice depend on considerations outside this model. If adjudication involves private suits but there is also strong public enforcement that operates independently or there are strong market forces that contribute to deterrence (perhaps through reputation), then achieved deterrence may be high, favoring termination. The overall lesson is that, because ex ante behavioral effects are central, the aggregation of all forces – these external factors and, as emphasized just above, decisions in other states – importantly influences the deterrence benefit and the chilling cost of continuation in a given state. This conclusion contrasts with that in conventional, purely forward-looking decision problems involving the value of information, where hypothetical decisions in other states tend to be irrelevant to the decision at hand.

The result in Proposition *1.d*, that greater expected costs of adjudication favor continuation over termination, may at first seem surprising. The explanation is that, the larger are these costs, the more is saved as a consequence of both greater deterrence and greater chilling. Note that these $\kappa^i$ are the aggregate expected costs associated with each type of act, of which the costs in a given state $\sigma^\circ$ are infinitesimal. The effects of the state-specific costs are

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[16]For *1.c*, the variables $b^i$, from expression (1), are endogenous, determined by other parameters. Note that the values of the $p^i(\sigma)$ only appear directly in (4) for the specific signal $\sigma^\circ$, so it is possible to have different settings in which only the $b^i$ differ by supposing different values of $p^i(\sigma)$ solely in other states. There is another complication with the "all else equal" proviso mentioned earlier in the text: the $b^i$ also enter (4) through the values of the corresponding density and distribution functions. Here, one could postulate different benefit distributions such that, comparing two situations with different values of the $b^i$, these two values would be the same. For Proposition *1.d*, where the $\kappa^i$ differ, one could allow the value of $k$ to differ in states other than $\sigma^\circ$. (In this instance, note that differing $k$ are not borne by actors and hence do not feed back on the $b^i$.)

considered below. Of course, higher expected (average) costs are associated with higher costs in at least some states. The point of this Proposition, juxtaposed with the others, is to distinguish average adjudication costs from those in the particular situation at hand – which costs may be atypically high or low. Propositions *1.d*, *3.a*, and *3.b* together indicate that the relationship among components of adjudication costs – average versus state-specific and, for the latter, defendants' versus the government's – matters a good deal and that some of the effects are contrary to what one might intuitively have supposed.

Further elaborating the "all else equal" stipulation of Proposition 1, observe that many of the factors that influence the $b^i$ also influence the $\kappa^i$. Indeed, expressions (1) and (5) are quite similar. In (4), these factors appear as a difference: for example, $b^B - \kappa^B$ in the second row. Subtracting the terms in larger parentheses in the two integrands in (1) and (5), we can see that the net effect depends on the difference between the government's adjudication cost in each state and the pertinent expected sanction in each state. (The actor's adjudication costs cancel reflecting that, for each act discouraged, this cost is saved to society, but it also equals a subcomponent of the forgone benefit from the marginal act that is discouraged – each weighted by the same requisite probability.) Relatedly, it is this difference (along with the avoided external harm, in the case of acts of type $H$) that determines the net effect of discouraging acts. Note that, in the preceding discussion of cross-state dependence of optimal decision rules, it was noted that lower preexisting deterrence – say, due to terminations in most other states – made deterrence more valuable at the margin. We can now see that, when we do not hold all else equal, it also means that expected adjudication costs per undeterred act will be less, which makes deterrence less valuable at the margin. Whether the net effect of, say, more terminations in other states, makes deterrence more or less valuable will depend on which of these forces is greater. (In this regard, note that $h$ is a constant, independent of the level of achieved deterrence.) And, of course, the same logic applies to chilling.

The next set of results pertains to the relative magnitude of the deterrence punch and the chilling punch that are generated when cases are allowed to continue. (Note that the proviso that chilling is net desirable is included for all of these claims; in each instance, it is a sufficient condition.)

*Proposition 2: When all else is equal and $b^B > \kappa^B$,*

*a: a higher $z^H(\sigma°)$ relative to $z^B(\sigma°)$ favors $\delta(\sigma°) = 1$,*

*b: a higher $\pi^H$ relative to $\pi^B$ favors $\delta(\sigma°) = 1$,*

*c: a higher $p^H(\sigma°)$ and a lower $p^B(\sigma°)$ each favor $\delta(\sigma°) = 1$, and*

*d: a higher $f^H(b^H)$ and a lower $f^B(b^B)$ each favor $\delta(\sigma°) = 1$.*

Proposition *2.a* asserts that, the greater the relative likelihood that the signal in question, $\sigma°$, is associated with harmful rather than benign acts, the more valuable it is to allow the case to proceed because the relative magnitude of deterrence versus chilling is greater. Raising $z^H(\sigma°)$ increases the left side of expression (4) and (only) part of the second term on the right side by the same proportion, leaving the other terms on the right side of (4), both of which are costs, unaffected. Lowering $z^B(\sigma°)$ reduces these other cost terms on the right side of expression (4), leaving the rest unaffected. Both changes favor continuation. (Another way to see this result is to divide both sides of expression (4) by $z^B(\sigma°)$. Likewise with $\pi^B$ for the next claim.)

Proposition *2.b* asserts that, the more likely it is that cases flowing into the legal system, being considered at the stage-one continue/terminate phase, involve harmful rather than benign acts, the more valuable it is to allow cases to proceed because the greater will be the deterrence punch relative to the chilling punch. This Proposition may seem essentially the same as *2.a* because, after all, the $\pi^i$'s and the corresponding $z^i(\sigma°)$'s always appear as a product in expression (4). The result is nevertheless stated separately in part because the "all else is equal" proviso is more involved in this instance since both the $b^i$'s and the $\kappa^i$'s depend on the $\pi^i$'s.[17] It is, however, possible to construct comparisons in which all else is indeed equal by adjusting other exogenous parameters in other states in a manner that keeps them constant.[18]

Proposition *2.c* indicates that higher diagnosticity of stage-two adjudication – the more (less) likely it is that sanctions will be applied to harmful (benign) acts – favorably influences the magnitudes of the deterrence punch and the chilling punch and thus favors continuation.

Regarding Proposition *2.d*, in determining the deterrence and chilling punches, what matters are not only the changes in individuals' expected costs for the two types of acts but also how many harmful and benign acts will be deterred for given increases in their expected costs. This feature, in turn, is indicated by the magnitudes of the densities for individuals' benefits from acts, each evaluated for the pertinent marginal act. When near the peaks of the distributions, these effects are large, whereas at other points the effects can be small. In general, either of these densities might be higher. Which it is – and how large any difference might be – will depend on the nature of the two distributions as well as on all the features of the legal system. Regarding the latter, even if the two distributions were the same, since individuals' expected costs for the two types of acts will differ, perhaps substantially, so may the values of these density functions.

The final set of results concerns adjudication costs from continuation, the last term in expression (4).

*Proposition 3: When all else is equal,*

a. *a higher $c(\sigma°)$ has an ambiguous effect on the optimal $\delta(\sigma°)$,*
b. *a higher $k(\sigma°)$ favors $\delta(\sigma°) = 0$, and*
c. *a higher $1-F^H(b^H)$ and a higher $1-F^B(b^B)$ each favor $\delta(\sigma°) = 0$.*

Proposition *3.a* may seem surprising, especially when juxtaposed with Proposition *3.b*, which is obvious. Regarding the former, it is true that, just as with $k(\sigma°)$, a higher $c(\sigma°)$ raises the last term in expression (4), indicating that it is more expensive to go forward. However, a higher $c(\sigma°)$ also raises the deterrence and chilling punches (reflected on the left side of

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[17] In Proposition *2.a*, we were considering changes in particular $z^i(\sigma°)$'s, so these added effects would be infinitesimal.

[18] Starting with the $b^i$'s, one could adjust the $c$'s or $p^i$'s in other states. Then, for the $\kappa^i$'s, one could adjust the $k$'s. (If the former is done by adjusting the $c$'s, that also influences the $\kappa^i$'s, so the needed adjustment to the $k$'s would need to reflect that as well.)

expression (4) and the first term on the right, respectively) because individuals thereby anticipate paying greater costs when their cases will proceed to final adjudication. In an optimal system, this deterrence gain will tend to exceed the chilling cost (because if it did not, it would probably be true that too few cases are being terminated, in light of adjudication costs). Accordingly, we can imagine situations under which a higher defendant's adjudication cost would tip the balance in favor of continuation. Specifically, suppose that the factors in Propositions *1* and *2* would favor continuation in a wide range of states were it not for the fact that, in most of those states, $k(\sigma)$ was so high that termination was best. Now, in the state $\sigma°$ under consideration, suppose that $k(\sigma°)$ is barely high enough that termination is optimal. In that setting, a higher $c(\sigma°)$ might produce a deterrence gain that is relatively large compared to both the chilling cost and the added adjudication cost, swinging the balance in favor of continuation.

Proposition *3.c* is straightforward from examination of the final term in expression (4). Note that these factors involve inframarginal effects; specifically, they refer to the undeterred and unchilled portions of the population. By contrast, both the deterrence and chilling effects depend on the densities (see Proposition *2.d*), reflecting that they involve *changes* in behavior due to switching the decision from terminate to continue, rather than the existing *stock* of behavior, in aggregate.[19] (Note that it is only here – really, in the last line of expression (4), showing the direct costs of continuation – that inframarginal effects matter, whereas they are central in the more familiar, forward-looking, value of information problem. Moreover, here the two inframarginal measures enter additively, which is qualitatively different from how they affect the standard problem.)

*C. Additional Enforcement Instruments*

The present analysis can be extended and thereby more closely compared with prior literature on the economics of law enforcement by allowing the government also to choose the level of sanctions and enforcement effort. The discussion in this section will be brief and informal, with derivations relegated to an appendix.

Begin with the sanction *s*. It does not appear directly in *W* (see expression 2), its influence being through deterrence and chilling, as indicated by expression (1). The first-order condition for an optimal interior sanction equates the deterrence gain to the chilling cost. (See expression A2.) Obviously, if both involve a net benefit (recall the above discussion), the optimal sanction is maximal. More broadly, if the government can adjust all policy instruments – for the moment, continuation/termination decisions and the sanction – the optimal sanction will be maximal regardless, a result reminiscent of that suggested by Becker (1968), although the analysis differs in the present setting.

*Proposition 4: If $p^H(\sigma) \geq p^B(\sigma)$ for all $\sigma$ and the government may choose the $\delta(\sigma)$, then the optimal sanction is maximal.*

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[19]Observe further that it is easy to construct changes in the $F^i$ that do not affect the $f^i$ by moving some of the mass from one end of the support to the other (that is, not at the $b^i$ in question).

To see why this result arises, suppose that $s$ is not maximal. Consider the experiment of marginally raising $s$ and also switching from continuation to termination in states with the worst effective diagnosticity, that is, the highest $[c(\sigma)+p^B(\sigma)s]z^B(\sigma) / [c(\sigma)+p^H(\sigma)s]z^H(\sigma)$. (See the discussion of expression A5.) Furthermore, make these adjustments to an extent that keeps deterrence, that is, $b^H$, constant. It can be shown that there are three effects, all of which are favorable.

First, terminations save adjudication costs. Second, chilling falls due to the better targeting of the sanction $s$ across states. The heightened $s$ applies (probabilistically) in all states with continuation – an average effect – whereas the now-terminated states had the worst (and hence necessarily a below-average) targeting of expected system costs. Third, chilling also falls because (assuming that $p^B(\sigma)/p^H(\sigma) < 1$) sanctions are better targeted on harmful versus benign acts than are defendants' continuation costs, $c(\sigma)$. (See the discussion of expression A4.) Recall from expression (1) that a key component of both deterrence and chilling punches, for each state, is the sum of the defendant's continuation costs and the probability of the sanction times its magnitude. Continuation costs are the same for harmful and benign acts, whereas the likelihood of being sanctioned at the final stage is taken to be higher for the former. Hence, switching from continuations to a higher $s$ as the source of a given level of deterrence relies more on the sanction and less on continuation costs, which is relatively favorable for benign acts. Interestingly, the latter two effects indicate that a concern for chilling, which arises from the legal system's incidental inclusion of innocent behavior, bolsters the case of a maximal sanction in an optimized enforcement regime.

Now consider enforcement effort. This may be introduced by supposing that the $\pi^i$ are each a function of expenditures $e$. Assume that $\pi^{i\prime}(e) > 0$, $\pi^{i\prime\prime}(e) \leq 0$, for $i = H, B$. (It may be natural for some interpretations that there be a maximum level of $e$, such as when an audit rate reaches one hundred percent.) In expression (2) for social welfare, one would insert an additional term "$-e$" at the end. The first-order condition for an interior optimum includes deterrence and chilling effects (through the influence of $e$ on the $\pi^i$ in expression 1 for the $b^i$), inframarginal costs due to the fact that all undeterred and unchilled acts are now more likely to enter the legal system (see the integrands in expression 2), and a "$-1$" reflecting the cost of enforcement effort itself. (See expression A6.)

In achieving, say, a given level of deterrence, it is interesting to compare the level of $e$ with the choice of the set of states in which cases are continued rather than terminated, in a manner analogous to the foregoing discussion. As with raising $s$ and terminating in more states, raising $e$ and terminating in more states again saves continuation costs and can also be beneficial with regard to chilling effects by terminating in those states with the weakest diagnosticity (because greater enforcement effort increases the flow in all states, a sort of average effect). However, the benefit with regard to chilling that $s$ operates only through the $p^i$ is inapplicable because, like continuation, raising $e$, which increases the $\pi^i$, achieves both deterrence and chilling also through defendants' adjudication costs $c$ (see expression 1).

By comparison to raising $s$, raising $e$ has additional disadvantages. Most obvious is the cost of the expenditure itself. In addition, by raising the $\pi^i$, more undeterred and unchilled acts enter the legal system in all states, as mentioned, which thereby raises total system costs in all states where continuation is still employed. A final point concerns the targeting of increases in $e$. If the enforcement technology involves purely random audits, raising $e$ would raise the $\pi^i$ in proportion. However, for other means of enforcement – or for audits that are prioritized, say, by some preliminary signal – it will be optimal to target the most promising cases first, so there will be diminishing returns in terms of targeting precision. Put another way, raising $e$ would tend to raise $\pi^B$ relatively more than $\pi^H$, and this will be disadvantageous. Taken together, some factors favor raising $e$ and terminating in more states, and others are opposed. Accordingly, the optimal system will ordinarily involve an intermediate level of enforcement effort and continuation in some but not all states.

Anticipating section 3, note further that the enforcement effort decision can in many respects be interpreted as an additional (earlier) stage of adjudication. When enforcement actions are guided by some signal, the analogy is fairly precise. In that event, the choice between enforcement effort and continuation/termination decisions at stage 1 can instead be understood as one about optimal continuation/termination decisions at different stages.

Finally, compare the choice between enforcement effort and sanctions, a central topic in the literature on the economics of law enforcement (Polinsky and Shavell 2007) and the locus of Becker's (1968) original argument favoring raising $s$ and lowering $e$, keeping deterrence constant but saving enforcement resources. (For the analogue here, see expression A7.) The present model adds benign activity that may be chilled and multiple stages of adjudication. Drawing on the preceding discussion, we can see that Becker's experiment plausibly remains favorable, and for a number of additional reasons. The direct cost point (reducing $e$ saves resources but raising $s$ does not cost any) continues to hold. There is the additional inframarginal savings because fewer undeterred and unchilled acts enter the legal system and thus (probabilistically, depending on the state) result in continuation costs. In addition, because of chilling effects, there may well be additional advantages: $e$ influences behavior through defendants' continuation costs $c$, which are less diagnostic than $s$, and for some enforcement technologies, $e$ is subject to diminishing returns in targeting efficiency. As shown in the appendix, however, the point that continuation costs more poorly target harmful versus benign acts is more complex and, without further assumptions, is ambiguous, in contrast to the situation in the earlier construction involving an increase in $s$ accompanied by termination in additional states. (See the discussion of expression A8.)

## 3. Multistage Adjudication

### *A. Model*

Let $t$ denote the stage of adjudication, with the final stage designated by $T$, where $T > 2$. The analysis in section 2 can be understood as applicable to $t = 1$, where the stage was suppressed and all variables involving continuation or the final stage can now be understood as implicitly referring to pertinent expected values, as will be made explicit here.

Consider first the decision whether to allow a case that enters stage $t < T$ to continue to stage $t$+1 ($\delta_t = 1$) or instead to terminate it at that point ($\delta_t = 0$). For a case that continues, the two adjudication costs in moving to the next stage are now designated as $c_t$ borne by the defendant and $k_t$ by the government. It is also useful to introduce the notation $E_t^i(c)$ and $E_t^i(k)$, where $E_t^i(\cdot)$, the expected value operator as of stage $t$, refers to the expected sum of the pertinent costs borne in all future stages, conditioning on the fact that one has reached stage $t$ and decides to continue (see expressions (7) and (9) below). Note that the costs $c_t$ and $k_t$, in moving from stage $t$ to $t$+1, are certain when $\delta_t = 1$, whereas for all subsequent stages (if any), they are conditional on the subsequent signals and on the decision rules contingent on those signals being such that the case is continued yet again rather than terminated. (All costs incurred to reach stage $t$ are sunk and thus are excluded from this expression for expected continuation costs.) Also, the superscript $i$ appears on the expectation operator because, although the costs incurred in moving to the next stage are taken to be the same for the two types of acts, expected costs for subsequent stages depend on subsequent decisions, which in turn depend on the subsequent signals, the likelihoods of which generally differ for the two types of acts.

A case that reaches stage $T$ is tried and results in a finding of liability ($\delta_T = 1$) or instead is terminated ($\delta_T = 0$). As before, individuals found liable are subject to the socially costless sanction $s$. At nonfinal stages $t$, the pertinent probability of ultimately being sanctioned is designated $E_t(p^i)$, which here refers to the probability that the final stage $T$ is ultimately reached and that liability is also found at that stage, all conditional on a case already having arrived at stage $t$ (see expression (11) below).

As before, individuals who commit their acts enter the legal system (at the first stage) with probabilities $\pi^i$. For this multistage version of the problem, it is supposed that, at the outset of each stage $t$, there is a signal $\theta_t$. Accordingly, in any stage $t$, the realizations of signals up to and including that stage are known. It is convenient to adjust the previous notation to use, for each stage $t$, the symbol $\sigma_t$ to denote the vector of these signals: i.e., $\sigma_t = (\theta_1, ..., \theta_t)$. At each stage, the signal $\theta_t$ has a conditional density – i.e., it is conditioned on $\sigma_{t-1} = (\theta_1, ..., \theta_{t-1})$ – given by the function $z_t^i(\sigma_t)$, positive on the real line, where the superscript $i$ again indicates that these densities depend on the type of act. (For elaboration, see the footnote.[20])

The $E_t^i(c)$, $E_t^i(k)$, and $E_t(p^i)$ each depend on $\sigma_t$. The government's problem at stage $t$ is to choose the function $\delta_t(\sigma_t)$: that is, for each $\sigma_t$, to decide whether to allow the case to continue to stage $t$+1 (if $t = T$, to assign liability) or instead to terminate it.

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[20]Specifically, for any stage $t$, let $Z_t^i(\theta_1, ..., \theta_t)$ be the joint density function, positive on the domain $\mathbb{R}^t$. Then, the conditional density function specified in the text is defined by this joint density function divided by the pertinent marginal density (which is just the joint density on the conditioned variables from the prior stage):

$$z_t^i(\sigma_t) = z_t^i(\theta_1,...,\theta_t) \equiv Z_t^i(\theta_t|\theta_1,...,\theta_{t-1}) = \frac{Z_t^i(\theta_1,...,\theta_t)}{Z_{t-1}^i(\theta_1,...,\theta_{t-1})}.$$

(For $t = 1$, the denominator on the right side is taken to equal 1.) For convenience, this further notation is suppressed throughout.

The model for adjudication with $T$ stages can be summarized by reference to its timing:

1. The government sets all policy instruments, notably, the functions $\delta_t(\sigma_t)$, for all $1 \le t \le T$.
2. Individuals learn their type of act ($H$ or $B$) and their private benefit $b$.
3. Individuals decide whether to act.
4. A portion of those who commit each type of act, $\pi^H$ and $\pi^B$, are identified and brought before a tribunal, in which event costs $c_0$ and $k_0$ are incurred by the defendant and the government, respectively.
5. The stage is $t = 1$, and the signal $\theta_1$ is realized.
6. The tribunal either allows the case to continue ($\delta_t(\sigma_t) = 1$) or instead terminates it ($\delta_t(\sigma_t) = 0$); in the latter case, the game ends.
7. If the case continues:
    a. Costs $c_t(\sigma_t)$ and $k_t(\sigma_t)$ are incurred by the defendant and the government, respectively.
    b. The case enters the next stage ($t$ is incremented by 1).
    c. The new signal $\theta_t$ is realized.
    d. If $t < T$, the case reenters step 6; else, it goes to step 8.
8. In final adjudication, stage $T$, the tribunal decides whether to find liability ($\delta_T(\sigma_T) = 1$) and thus apply the sanction $s$ or instead to find no liability ($\delta_T(\sigma_T) = 0$).

Those whose type of act is $i$ commit their act if and only if:

$$(6)\ b > E_0^{\,i}(c) + E_0(p^i)s \equiv b^i .$$

Expression (6) is the multistage analogue to expression (1), when the process was taken to involve only a single preliminary stage ($t = 1$) and a final stage ($T = 2$). The pertinent expectation operators in expression (6) are assessed at what is here referred to as stage 0. Recall that stage 1 is the stage at which a case enters the legal system. Stage 0 is to be interpreted as the preceding point in time at which individuals decide whether to act (step 3 in the above time line).

Let us now derive the expected values in expression (6) and some others that will be needed below, using backward induction and recursion. Begin with prospective defendants' expected adjudication costs. For $t \ge 1$, recall that $E_t^i(c)(\sigma_t)$ is the expected sum of the defendant's adjudication costs borne in all future stages, conditioning on the fact that the case has reached stage $t$ and the decision is to continue. In stage $T$, such expected costs are 0 because continuation in stage $T$ means assignment of liability and application of the sanction, with no further adjudication. In stage $T-1$, continuation means that the cost $c_{T-1}(\sigma_{T-1}^{\circ})$ is incurred, where the history of the signals to (and including) that stage is $\sigma_{T-1}^{\circ}$. For all prior stages, generically $t$, the expected cost if there is continuation is the sum of (A) the (certain) cost of moving from stage $t$ to stage $t$+1, which is $c_t(\sigma_t^{\circ})$, and (B) the expected value of all subsequent costs. The latter, for each possible realization of the signal $\theta_{t+1}$, either is zero if the case is terminated in stage $t$+1 or, otherwise, it is the pertinent probability of being in that situation times the expected continuation costs apropos for stage $t$+1 (in that case, in moving to stage $t$+2). Accordingly, for

$1 \le t < T$, we can write

$$(7)\ E_t{}^i(c)(\sigma_t{}^\circ) = c_t(\sigma_t{}^\circ) + \int_{-\infty}^{\infty} \delta_{t+1}(\sigma_{t+1}) E_{t+1}{}^i(c)(\sigma_{t+1}) z_{t+1}{}^i(\sigma_{t+1}) d\theta_{t+1}.$$

(When $t = T-1$, the right side of expression (7) just equals $c_{T-1}(\sigma_{T-1}{}^\circ)$ since $E_T{}^i(c)(\sigma_T) = 0$, consistent with the prior explanation.) Note that the variable of integration is $\theta_{t+1}$: although this variable may not be apparent in the integrand, recall that $\sigma_{t+1}$ is a vector, the last element of which is $\theta_{t+1}$. (Regarding the conditional density $z_{t+1}{}^i(\sigma_{t+1})$, see note 20.)

For use in expression (6), we need a parallel formulation for what is being called stage 0, when individuals contemplate whether to act, which is:

$$(8)\ E_0{}^i(c) = \pi^i\left(c_0 + \int_{-\infty}^{\infty} \delta_1(\sigma_1) E_1{}^i(c)(\sigma_1) z_1{}^i(\sigma_1) d\theta_1\right).$$

Here, we add a preceding factor $\pi^i$, the probability that an individual who commits an act of a given type will enter the legal system in the first place. Also note that it is allowed that there be a cost, denoted $c_0$, of initially entering the system. (For convenience, this latter possibility was omitted in section 2.)

The corresponding expressions for the government's cost $k$ are analytically the same:

$$(9)\ E_t{}^i(k)(\sigma_t{}^\circ) = k_t(\sigma_t{}^\circ) + \int_{-\infty}^{\infty} \delta_{t+1}(\sigma_{t+1}) E_{t+1}{}^i(k)(\sigma_{t+1}) z_{t+1}{}^i(\sigma_{t+1}) d\theta_{t+1}.$$

$$(10)\ E_0{}^i(k) = \pi^i\left(k_0 + \int_{-\infty}^{\infty} \delta_1(\sigma_1) E_1{}^i(k)(\sigma_1) z_1{}^i(\sigma_1) d\theta_1\right).$$

We can now use the same technique to derive the expected probability of liability. For $t \ge 1$, $E_t(p^i)(\sigma_t)$ is the conditional expected probability of liability. In stage $T$, this probability is 1 if there is continuation and 0 otherwise. In earlier stages, continuation carries no immediate consequence. The only effect is that, once in the next stage, there is the possibility of further continuation, ultimately reaching stage $T$. For $1 \le t < T$, we can write

$$(11)\ E_t(p^i)(\sigma_t{}^\circ) = \int_{-\infty}^{\infty} \delta_{t+1}(\sigma_{t+1}) E_{t+1}(p^i)(\sigma_{t+1}) z_{t+1}{}^i(\sigma_{t+1}) d\theta_{t+1}.$$

(As just indicated, when $t = T-1$, we have $E_T(p^i)(\sigma_T) = 1$, with the interpretation that, when the decision in period *T* is to continue, this means that liability is assigned with certainty.)

Next, as with adjudication costs, we can state the appropriate expression for what is referred to as stage 0, when individuals contemplate whether to act:

$$(12)\ E_0(p^i) = \pi^i \int_{-\infty}^{\infty} \delta_1(\sigma_1) E_1(p^i)(\sigma_1) z_1{}^i(\sigma_1) d\theta_1.$$

Once again, we have the preceding factor $\pi^i$, the probability that an individual who commits an act of a given type will initially enter the legal system.

Finally, as in section 2, social welfare is taken to be the aggregate of individuals' benefits from acting minus the harm from the commission of acts of type *H* and the costs of defendants' and the government's expenditures on adjudication.

$$(13)\ W = \int_{b^H}^{\infty} \left(b - h - E_0{}^H(c) - E_0{}^H(k)\right) f^H(b) db + \gamma \int_{b^B}^{\infty} \left(b - E_0{}^B(c) - E_0{}^B(k)\right) f^B(b) db.$$

As mentioned, the government's problem is, for each possible realization $\sigma_t°$ at each stage *t*, to decide whether to allow the case to continue or instead to terminate it. Just as when analyzing the first stage ($t = 1$) in the two-stage setting, examination of expression (13) indicates that the difference in the pertinent values of *W* will involve two types of effects. First, the values of the $b^i$ will differ: if the case is allowed to continue, these magnitudes will be higher, which is to say, the deterrence of harmful acts and the chilling of benign acts will be greater. Second, for any nonfinal stage ($1 \le t < T$), the values of the adjudication cost terms in both integrands will be larger since, if the case continues rather than terminates, defendants' and the government's adjudication costs are incurred both in moving to the next period, $t$+1, and, with a probability, to subsequent periods as well.

Before proceeding, it is useful to introduce one last bit of notation. Because we will contemplate decision choices at various stages after stage 1, it is convenient to have an expression for the probability that a given stage *t* (including final stage *T*) is entered along the particular path of signals embodied in $\sigma_t°$. (Note that we are interested in this probability for the particular path and not aggregated along all paths because the stage *t* continuation/termination decision in a given state, $\delta(\sigma_t°)$, is made with full knowledge of the prior signals and, in general, will differ depending on the realizations of those signals.) For entering the first stage, these probabilities, the $\pi^i$, were already postulated. More generally, define

$$(14)\ \pi_t{}^i(\sigma_t°) = \pi^i \prod_{\tau=1}^{t-1} \delta_\tau(\sigma_\tau°) z_\tau{}^i(\sigma_\tau°).$$

(And, consistent with the previous statement, when $t = 1$, the expression involving the product operator on the right side of (14) should be taken to equal 1.) Expression (14) states that the probability of entering stage $t$ along a particular signal path is the probability of entering the first stage, multiplied by the product of the indicator variable and of the density of the signal at each subsequent stage up to (but not including) stage $t$.[21] Note that if any decision before stage $t$ involves termination, the probability of entering stage $t$ along that path is zero (making the decision at that stage or any subsequent stage moot).

*B. Intermediate Stages*

The condition at stage $t$ in state $\sigma_t°$ for continuation ($\delta_t(\sigma_t°) = 1$) to be optimal is:

$$(15)\ \pi_t^{\ H}(\sigma_t°)\Big(E_t^{\ H}(c)(\sigma_t°) + E_t(p^H)(\sigma_t°)s)\Big)z_t^{\ H}(\sigma_t°)f^{\,H}(b^H)\Big(h + E_0^{\ H}(c) + E_0^{\ H}(k) - b^H\Big)$$

$$-\int_{b^H}^{\infty}\pi_t^{\ H}(\sigma_t°)\Big(E_t^{\ H}(c)(\sigma_t°) + E_t^{\ H}(k)(\sigma_t°)\Big)z_t^{\ H}(\sigma_t°)f^{\,H}(b)db$$

$$+\gamma\pi_t^{\ B}(\sigma_t°)\Big(E_t^{\ B}(c)(\sigma_t°) + E_t(p^B)(\sigma_t°)s)\Big)z_t^{\ B}(\sigma_t°)f^{\,B}(b^B)\Big(E_0^{\ B}(c) + E_0^{\ B}(k) - b^B\Big)$$

$$-\gamma\int_{b^B}^{\infty}\pi_t^{\ B}(\sigma_t°)\Big(E_t^{\ B}(c)(\sigma_t°) + E_t^{\ B}(k)(\sigma_t°)\Big)z_t^{\ B}(\sigma_t°)f^{\,B}(b)db > 0.$$

All legal system attributes except the $\delta_t(\sigma_t°)$ under consideration are taken as given. The other $\delta_t$ can be taken to be optimal or simply as stipulated. To determine the optimal sequence of the $\delta_t$ for each possible sequence of signals, one would use backward induction.[22]

The reader will note that expression (15) is quite similar to expression (3) for the first-stage decision ($t = 1$) in the two-stage model. This connection can be seen even more readily if we rewrite expression (15) analogously to what we did before, as follows:[23]

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[21] Because what happens in stage $t$ is excluded, the probability in expression (14) does not actually depend on the signal in stage $t$; however, as will be apparent below, it is appealing to use the notation for the signal vector through stage $t$ since this is the pertinent vector for all the other variables in the optimality condition.

[22] Note that in general there can exist multiple local optima. To illustrate, suppose that adjudication costs in continuing from stage 1 are a relatively large factor. There might exist a local optimum in which most cases are terminated at stage 1 (which, note, implies that expected system costs per undiscouraged act, the $\kappa^i$'s in expression (17), will be low, which makes deterrence and chilling less valuable, suggesting the desirability of termination). And there may exist a local optimum in which most cases are continued at stage 1 (which reverses the foregoing).

[23] It would be possible to rewrite all the optimality conditions in this section by: noting that $\pi_t^{\ i}(\sigma_t°)$ and $z_t^{\ i}(\sigma_t°)$ always appear as a product, substituting for the former using expression (14), and then substituting for the latter (and its appearances in (14)) using the explicit notation for the joint density functions in note 20. Specifically, one can show that

$$(16)\ \pi_t^{\,H}(\sigma_t{}^\circ)\Big(E_t^{\,H}(c)(\sigma_t{}^\circ)+E_t(p^H)(\sigma_t{}^\circ)s\Big)z_t^{\,H}(\sigma_t{}^\circ)f^{\,H}(b^H)\Big(h+\kappa^H-b^H\Big)\ >$$

$$\gamma\pi_t^{\,B}(\sigma_t{}^\circ)\Big(E_t^{\,B}(c)(\sigma_t{}^\circ)+E_t(p^B)(\sigma_t{}^\circ)s\Big)z_t^{\,B}(\sigma_t{}^\circ)f^{\,B}(b^B)\Big(b^B-\kappa^B\Big)$$

$$+\Big(1-F^H(b^H)\Big)\pi_t^{\,H}(\sigma_t{}^\circ)z_t^{\,H}(\sigma_t{}^\circ)\Big(E_t^{\,H}(c)(\sigma_t{}^\circ)+E_t^{\,H}(k)(\sigma_t{}^\circ)\Big)$$

$$+\gamma\Big(1-F^B(b^B)\Big)\pi_t^{\,B}(\sigma_t{}^\circ)z_t^{\,B}(\sigma_t{}^\circ)\Big(E_t^{\,B}(c)(\sigma_t{}^\circ)+E_t^{\,B}(k)(\sigma_t{}^\circ)\Big),$$

where

$$(17)\ \kappa^i = E_0^{\,i}(c)+E_0^{\,i}(k).$$

To reinforce the similarity and to solidify understanding of the decision at intermediate stages, it is useful to review afresh the terms in expression (16). On the left side of the inequality (the first row) is the deterrence benefit of allowing a case to proceed at stage *t* in state $\sigma_t{}^\circ$, rather than to terminate it at that point. The deterrence effect is the product of: (A) the probability of having entered that stage with respect to a harmful act for the pertinent signal path; (B) the increment to the expected cost imposed on harmful acts if the case is allowed to continue, which is the sum of (i) the additional expected defendants' adjudication costs incurred, going forward from stage *t*, and (ii) the expected sanction if the current stage *t* is survived, which is the product of the expected probability of a sanction when the stage is survived and the sanction; (C) the magnitude of the density for being in state $\sigma_t{}^\circ$ when the act is of the harmful type; and (D) the magnitude of the density for marginal harmful acts (indicating, as before, the quantity of acts deterred per unit increase in the deterrence punch). This deterrence effect is, again, multiplied by the deterrence benefit per act deterred for the marginal act. The avoided expected adjudication cost now reflects, as expressed in (17), the expected costs through all stages for a harmful act.

The first term on the right side of the inequality (the second row) is the chilling cost. It is determined in a fashion precisely analogous to that for the deterrence benefit. The final two

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$$\pi_t^{\,i}(\sigma_t{}^\circ)z_t^{\,i}(\sigma_t{}^\circ)=\pi^i\Delta_{t-1}(\sigma_{t-1}{}^\circ)Z_t^{\,i}(\sigma_t{}^\circ),$$

where $\Delta_{t-1}(\sigma_{t-1}{}^\circ)$ is the product of the indicator variables from all stages before *t* (i.e., it equals 1 rather than 0 if and only if all prior decisions were to continue). This $\Delta_{t-1}(\sigma_{t-1}{}^\circ)$ term cancels, for example, in expression (16) because it appears identically in all four rows (unlike the other components, which depend on the type of act *i*). Although more explicit, this substitution is not made because the resulting condition is less intuitive: having $\pi^i$ in lieu of $\pi_t^{\,i}(\sigma_t)$ no longer refers to the probability of entering the stage under consideration, and the use of $Z_t^{\,i}(\sigma_t)$ rather than $z_t^{\,i}(\sigma_t)$ refers to a joint density function of all the signals through stage *t* rather than the density of the signal at stage *t*, conditional on the prior signals.

terms (the third and fourth rows) are, as in section 2's model, the increment to the expected adjudication costs incurred with respect to inframarginal acts of both types. For each type of act, we have the product of: (A) the mass of acts of that type that are committed; (B) the probability that the type of act enters stage *t* and also is associated with the pertinent signal path; and (C) the sum of expected defendants' and government costs, going forward. Note that this latter component excludes the costs incurred in reaching stage *t*: for inframarginal acts, these prior-stage costs are sunk; specifically, they would not be avoided if a case was terminated at stage *t* rather than continued, which is the decision under consideration.

Although notationally more complex, the decision rule represented in expression (16) is qualitatively the same at nonfinal stages in the *T*-stage model as it is at the first stage in the two-stage framework (expression 4). Hence, the elaboration on the analytics appearing in section 2.B does not need to be repeated.

Nevertheless, this framework with multiple stages prior to the final stage of adjudication suggests a number of interpretations that go beyond those in the simpler version of the problem. Begin with the deterrence and chilling effects, the first two rows in expression (16). Consider first the factors $\pi_t^i(\sigma_t^\circ)$, the probability that an act of the pertinent type enters stage *t* along the posited path of signal realizations. When entering the first stage (and in the analysis of section 2 where there was only a single nonfinal stage, see expression 4), we simply had $\pi^i$. From expression (14), we can see that in subsequent stages, the magnitude of $\pi_t^H(\sigma_t^\circ)$ relative to $\pi_t^B(\sigma_t^\circ)$ will depend on the discriminating power of the signals in prior stages. Accordingly, the more it is true on a signal path that prior decisions to continue were not close calls, the more likely it is that it would be optimal to continue in stage *t* because the deterrence effect will be larger relative to the chilling effect, ceteris paribus.

The next terms in the first two rows of expression (16) represent the increment to individuals' expected costs from the decision to proceed in stage *t*. On one hand, as a case proceeds to later stages, subsequent adjudication costs are lower since additional stages' costs are sunk. See expression (7). On the other hand, the expected probability of liability with continuation tends to be higher as the case gets closer to the final stage and thus a possible finding of liability. See expression (11). Taken together, the aggregate expected cost of continuation decisions to an actor could be rising or falling as cases progress to later stages. The two densities are interpreted as before: the relative likelihood of the signal in the current period (now conditional on past realizations) and the relative magnitude of the density functions for the distributions of individuals' benefits from the two types of acts, evaluated for the marginal act (a factor independent of what stage a case is in). Note also that both the gain per harmful act deterred (avoided harm and adjudication costs minus the benefit from the act forgone) and the cost per benign act chilled (forgone benefit minus avoided adjudication costs) do not depend on the stage a case is in. As expression (17) depicts, the avoided adjudication costs in question with regard to the first two rows in expression (16) are from discouraged acts, so they are the expected adjudication costs of the entire process, not just that for particular stages.

Continuation also increases inframarginal adjudication costs for undeterred harmful acts and for unchilled benign acts, as indicated in the third and fourth rows of expression (16). The masses of activity for the two types of acts do not depend on the stage. For the $\pi_t^i(\sigma_t^\circ)$ and $z_t^i(\sigma_t^\circ)$, what matters here are the absolute magnitudes rather than the relative magnitudes since both involve costs (whereas for deterrence and chilling, we had a benefit and a cost that we needed to trade off). But this was similarly true in the first stage. What differs are the final terms, the expected defendants' and government adjudication costs of going forward. In later stages, more of both components of total adjudication costs are sunk, so it takes less of a deterrence gain relative to the chilling cost to warrant continuation. That is, this factor favors an increasingly lenient approach toward continuation as cases progress through the legal system, contrary to conventional wisdom in legal scholarship. (Indeed, this section identifies a number of factors bearing on the standard view; although some are supportive, others – and not just this point about more costs being sunk – are opposed.)

Stepping back from these particulars, it is interesting to reflect more broadly on the manner in which greater generosity or stringency in making continuation decisions at some stages influences how decisions are optimally made at others. In particular, we now consider decisions as a whole rather than just on a particular path of signal realizations because a number of key variables depend on the overall operation of the system, while the contribution of a specific decision at a particular stage on a particular path is negligible.

There are two sets of such variables, as explained in the discussion of Propositions *1.c* and *1.d*. First, the values of the $b^i$ – i.e., the benefits of the acts of each type that are just at the margin – depend on the extent of deterrence and chilling: when they are larger, the benefit of the marginal forgone private benefit is greater, so deterrence is less valuable and chilling more costly. Second, the $\kappa^i$ depend on decisions not only in other states at the same stage (as discussed in section 2) but also on decisions at other stages. As before, these two sets of considerations (marginal forgone benefits and expected adjudication costs) often cut in the opposite direction. For simplicity, the following discussion abstracts from this second factor, which would be correct in terms of the sign if the change in its magnitude was smaller.

Suppose now that, in the first stage, a large portion of cases are terminated. As a consequence, the value of deterring a marginal act will be high and the cost of chilling a marginal act will be low, favoring continuation at subsequent stages, all else equal.[24] In addition, the stringent test at stage 1 implies that the mix of cases remaining at later stages involves a heavier concentration of harmful acts, which also favors subsequent continuation. Conversely, if in the first stage a large portion of cases are continued, then termination will be relatively more favorable at subsequent stages on account of these considerations.

In interpreting such statements, however, an important distinction must be made concerning the basis for the stringency or generosity of treatment at various stages. Factors favoring, say, termination in many states at stage one – e.g., low $h$ or high $\gamma$ – will tend to favor

[24]Obviously, all else is not equal. In addition to the immediately preceding discussion (if a large portion of cases is terminated, total adjudication costs per undeterred and unchilled act will be lower, making deterrence and chilling less valuable, favoring termination), it is also true that the densities of the distributions of individuals' benefits for the two types of acts will, in general, have different values, which could cut either way with respect to both deterrence and chilling.

similar, not opposing, decisions at later stages. Even so, three important points remain true. First, as mentioned, decisions at prior stages will affect the mix of cases that remain and hence what is optimal going forward. Second, some factors are stage specific: for example, costs of continuing to stage 2 may be distinctively large. Third, it is of practical relevance to entertain the possibility that decisions at some stages are not made optimally because legal systems may impose institutional constraints on how cases can be handled at particular points in adjudication. For example, it may be impermissible to terminate cases at some stages, or decisions may be made by other agents with different objectives. In such circumstances, if there is flexibility at other stages, counterbalancing action may tend to be optimal in the sort of situation described here. In this regard, keep in mind that expression (16) indicates when continuation at a given stage, for a given signal path, is optimal taking as given decisions at other stages and on other paths, i.e., not supposing them to be made optimally.

Interdependence of decisions across stages also applies in reverse: optimal decisions at stage $t$ depend on the conditional expectations regarding future decisions, a point reflected by the expectation operators that appear throughout expression (16). However, the implications need not be the same. For example, if at a later (or final) stage, it were known that institutional constraints required termination in some subset of cases, it may not be wise to be more generous in allowing continuation at early stages if the continued cases will probably be terminated later in any event, with adjudication costs being incurred along the way. (Proposition *3.a* offers a caveat, reflecting that defendants' adjudication costs serve a deterrence function that is independent of whether liability is ultimately imposed.) On the other hand, if later stages will be too lenient due to institutionally constrained decisionmaking, then earlier decisions may well best tilt toward termination under the conditions postulated here.

*C. Final Stage*

Consider now the final stage $T$. To do so, we can simply interpret expression (16) for $t = T$. This expression indicates when it is optimal to continue ($\delta_T(\sigma_T) = 1$) rather than terminate ($\delta_T(\sigma_T) = 0$), but in this instance continuation means assigning liability, i.e., imposing the sanction $s$. Therefore, $E_T(p^i) = 1$, for $i = H, B$, as indicated in section 3.A. It was also explained that, because there is no subsequent stage, $E_T{}^i(c) = 0$ and $E_T{}^i(k) = 0$. Accordingly, for stage $T$, expression (16) reduces to

$$(18)\ \pi_T{}^H(\sigma_T{}^\circ)z_T{}^H(\sigma_T{}^\circ)sf^H(b^H)\left(h+\kappa^H-b^H\right) \ >$$

$$\gamma\pi_T{}^B(\sigma_T{}^\circ)z_T{}^B(\sigma_T{}^\circ)sf^B(b^B)\left(b^B-\kappa^B\right).$$

This expression simply compares the deterrence benefit to the chilling cost (there being no additional adjudication costs of continuation associated with inframarginal acts, as there were in expression (16), for $t < T$). The deterrence benefit is the rise in individuals' expected cost for harmful acts (the probability of harmful acts entering this stage, times the likelihood of the pertinent signal when there is a harmful act before the tribunal, times the sanction) times the density of harmful acts (together giving the incremental quantity of harmful acts deterred) times the benefit per harmful act that is deterred (which is the same as before). Likewise for the chilling cost.

It is also informative to rewrite expression (18) as follows:

$$(19)\ \frac{\pi_T{}^H(\sigma_T{}^\circ)z_T{}^H(\sigma_T{}^\circ)}{\pi_T{}^B(\sigma_T{}^\circ)z_T{}^B(\sigma_T{}^\circ)} > \frac{\gamma f^B(b^B)\left(b^B-\kappa^B\right)}{f^H(b^H)\left(h+\kappa^H-b^H\right)}.$$

We can see that the rule for optimal final-stage decisionmaking is a likelihood ratio test.[25] The left side of expression (19) gives the likelihood ratio for state $\sigma_T{}^\circ$. The right side is the critical likelihood ratio (it does not depend on $\sigma_T{}^\circ$). This critical ratio is the chilling cost per unit increase in individuals' expected cost for benign acts over the deterrence benefit per unit increase in individuals' expected cost for harmful acts.[26]

It is useful to compare expressions (18) and (19) to expression (16), the latter for cases in which $t < T$. Expression (16) also could have been put in the form of a likelihood ratio test. However, the resulting critical likelihood ratio, corresponding to the right side of expression (19), would depend on $\sigma_t{}^\circ$, so we would not have a conventional likelihood ratio test. Relatedly, we can see that the determinants of the optimal decision rule for intermediate stages (including the first stage) are a good deal more complex than those for final adjudication. Reviewing the results in section 2.B, it is apparent that Propositions *2c*, *3.a*, *3.b*, and *3.c* have no analogue for the final stage here.

To pursue this comparison a bit further, one could divide both sides of expression (16) by $\pi_t{}^B(\sigma_t{}^\circ)z_t{}^B(\sigma_t{}^\circ)$. The likelihood ratio (corresponding to the left side of expression 19) would then appear in two places: multiplying the deterrence term (the first row of modified expression 16) and the first of the two inframarginal adjudication cost terms (the third row of modified expression 16). Ceteris paribus, a higher likelihood ratio at a given stage in a given state would indeed favor continuation: it raises the benefit from continuation, but only one of the three costs, by a common factor, leaving unchanged the other two costs (the chilling cost and the other inframarginal adjudication cost). Hence, fixing all the other values, one could state a critical likelihood for the given stage and path of signal realizations. Note, however, that this statement implies that the critical likelihood would, in general, differ, even for a given stage *t*, depending on the signal path. This point should not be surprising because, among other reasons, different prior paths may imply different adjudication costs in going to the next stage. A further

[25]See, for example, Neyman and Pearson (1933), Karlin and Rubin (1956), and Milgrom (1981).

[26]The discussion in the text supposes that both the numerator and denominator on the right side are positive. If (only) the numerator is negative – which indicates that it is net desirable to chill benign acts because the marginal benign act has a benefit less than the expected adjudication costs it generates – then expression (18) indicates that liability would optimally be assigned in any state. If (only) the denominator is negative (which indicates overdeterrence), then, in moving from (18) to (19), the inequality would reverse, and the interpretation is that liability would not optimally be assigned in any state of the world. Of course, in such instances, one would optimally have terminated at stage 1.

difficulty, already suggested, with a likelihood ratio interpretation is that other terms cannot be regarded as fixed. In particular, all of the expectation operators depend on the signal distributions in subsequent periods, which in turn depend on the prior realizations, just as do the $\pi_t^i(\sigma_t^\circ)$ (see expression 14) and the $z_t^i(\sigma_t^\circ)$. See expressions (7), (9), and (11). Note that, when there is no fixed critical likelihood ratio, it is possible that it is optimal to continue in some state $\sigma_t^\circ$ that has a lower likelihood ratio than that in some other state $\sigma_t'$ in which it is optimal to terminate (perhaps state-specific continuation costs are atypically low in the former state and high in the latter). Put another way, the likelihood ratio is not a sufficient statistic for making the continuation/termination decision.

In sum, it is inapt to think of the optimal decision rules for nonfinal stages in terms of a standard likelihood ratio test even though it is natural to do so for the final stage, as per expression (19). A related point is that it is not very meaningful to say whether optimal decision rules at the initial stage or intermediate stages are more or less stringent than each other or than the optimal rule at the final stage. By contrast, it is ordinarily supposed in legal writing on the subject that continuation rules should and do become increasingly stringent as a case moves through the system. Specifically, in U.S. civil litigation, it is considered to be relatively easy for a case to survive a motion to dismiss (at the outset), harder to survive a defendant's motion for summary judgment (after discovery), and harder still to win at trial. On reflection, however, the conventional wisdom (which both purports to describe existing law and to be prescriptive) can readily be seen as dubious regarding optimal multistage decision-making because so many factors change (and in different directions) at subsequent stages – including, notably, that more adjudication costs are sunk as cases proceed through the system.

## 4. Conclusion

Most systems of adjudication have multiple stages. More broadly, investigators, prosecutors, and regulatory agencies have internal processes that screen out cases at various points and make interim decisions whether to expend resources to gather additional information. This article formally models multistage adjudication in order to determine how decisions are optimally made at initial and interim stages as well as at the final stage. As mentioned in the introduction, the present analysis can be understood as addressing the value of information problem in a mechanism design context. Most of the complexity is due to two factors: first, decisions at all stages influence ex ante behavior – both the deterrence of harmful acts and the chilling of benign acts – making the flow of cases endogenous; and second, decisions in other states and at other stages influence what decision is optimal in a given state at a given stage.

Optimal decision-making at nonfinal stages reflects a large number of factors, some pertaining to marginal effects on behavior (deterrence and chilling), some to inframarginal effects on adjudication costs (continuation raises these costs for undeterred and unchilled acts), and some to both. As an example of the latter, the diagnosticity of signals at each stage enters through relative effects for deterrence and chilling (which are traded off) but in terms of levels for adjudication costs (since they are summed, as part of overall costs, rather than compared for harmful versus benign acts). And some factors can have subtle and counterintuitive effects: e.g., individual actors' prospective adjudication costs are a social cost that is ideally minimized but also are part of the total cost of committing acts, including harmful acts, and thus contribute to deterrence (as well as to chilling); moreover, more generous continuation, while raising total costs per case, also reduces the total number of cases through deterrence and chilling. Additionally, in the extension that allows the level of the sanction to be adjusted, it was observed that the possibility of mistakes that burden innocent behavior actually favors higher sanctions because, combined with more stringent termination decisions, deterrence can be maintained at a lower chilling cost due to improved targeting on two dimensions.

Another implication for system design concerns how decisions at one stage influence optimal decisions at other stages – both at subsequent stages, by influencing the mix of cases that remain in the system and also the levels of deterrence and chilling, and at prior stages, since optimal decisions there depend on the consequences of continuation. Furthermore, because many actual legal systems impose institutional constraints that may render decisions at some stages suboptimal, the present analysis allows one to assess how decisions at other stages should be adjusted to compensate, to the extent such flexibility exists. In addition, it is explained how optimal final stage decisions involve a conventional likelihood ratio test whereas those at nonfinal stages do not. Relatedly, there is no simple way to rank decisions at different stages by their stringency, and conventional legal wisdom and practice that seems to favor increasing stringency as one proceeds from the initial stage to the final stage is difficult to rationalize. This latter point reflects that many factors change (and in different directions) as cases proceed through the system, including that more costs are sunk.

The models analyzed here, although general on many dimensions, are oversimplified on others, suggesting potential extensions. The method of enforcement embedded here is akin to the posting of monitors or auditing (including inspections and the like), whereas in some settings, notably crime, investigation (information gathering triggered by the observation of particular harmful acts) is more commonly employed.[27] Another important variant concerns adjudication aimed primarily at regulating future conduct rather than at influencing ex ante behavior (deterrence), a simpler problem discussed briefly in note 4. In addition, for some legal systems, the structuring of the stages is itself a decision variable, so one could analyze, for example, whether consecutive stages should be combined (balancing cost reduction due to economies of scope in information collection against the forgone option value) and how stages should optimally be ordered (information with a high ratio of diagnosticity to cost would optimally be gathered first). Finally, although atypical in most legal settings mentioned previously, it is natural to consider as well the possibility of assigning liability (as a third alternative to termination and continuation) at nonfinal stages, the analysis of which would closely mirror that presented here.

A different sort of extension would be to model the behavior of those who initiate and

[27]In preliminary notes, I have analyzed enforcement by investigation. The results are more complicated, although most of the qualitative effects derived here are preserved. Many of the differences concern the fact that greater deterrence has the added effect that fewer investigations are triggered (for a given enforcement probability), which in turn reduces the number of benign acts that enter the legal system.

pursue cases. For the government, this would include police, prosecutors, and agency officials. For private litigation, the focus would be on the incentives of plaintiffs and their lawyers.[28] These actors influence which cases enter the legal system and also incentives to make expenditures to gather information at each stage. The present analysis is complementary in this regard because it analyzes how optimally to make decisions at every stage – from the initial one to final determinations of liability – taking all other decisions and information as given, that is, without supposing that other parts of the system operate optimally.

[28]For example, a literature (surveyed in Spier 2007) considers the credibility of suits – usually in models with exogenous behavior and with only one, final stage of adjudication and in which the decision rule there is taken as given. Allowing multiple stages, with termination possible at each, affects litigants' filing decisions and their willingness to continue. Relatedly, the present analysis also abstracts from settlement (including plea bargaining), which of course will reflect expectations of how cases will otherwise be resolved. Hence, the present analysis can be employed to determine what settlements litigants would find attractive. Also, the greater the fraction of cases that settle, the lower will be ex ante expected adjudication costs (the $\kappa^i$) and the inframarginal expected adjudication costs from continuation (the third and fourth terms in expressions 4 and 16), both of which influence the optimal decision rules.

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**Appendix: Additional Enforcement Instruments**

Following the text in section 2.C, begin with the problem in which the government chooses not only the δ(σ) but also the sanction *s*. The effect of the sanction on deterrence and chilling can be assessed by taking the derivative of expression (1) with respect to *s*:

$$(A1)\ \frac{db^i}{ds} = \int_{-\infty}^{\infty} \pi^i \delta(\sigma) p^i(\sigma) z^i(\sigma) d\sigma.$$

Using expression (2), we can determine that:

$$(A2)\ \frac{dW}{ds} = \left(h + \kappa^H - b^H\right) f^H(b^H)\frac{db^H}{ds} - \gamma\left(b^B - \kappa^B\right) f^B(b^B)\frac{db^B}{ds}.$$

Throughout, the net benefit per deterred act (the leading parenthetical expression in the first term on the right side of A2) is taken to be positive. As mention, it is obvious if the net welfare impact per chilled act (the leading parenthetical expression in the second term) is also favorable (i.e., $b^B < \kappa^B$), this derivative must be positive, implying that the optimal sanction is maximal if it holds throughout. Hence, the main case of interest in attempting to establish Proposition 4 is that involving an imagined optimum with a nonmaximal sanction, with the social impact of chilling the marginal act being negative.

The construction suggested in the text can be presented more precisely as follows. To begin, let the set Σ denote all σ such that, at the hypothesized optimum, we have δ(σ) = 1. (Note that it is not necessary that the δ(σ) have been set optimally.) In the first step of the experiment, raise *s* slightly and, for each σ ∈ Σ, reduce δ(σ) slightly such that the contribution to deterrence from the state is constant. That is, as one raises *s*, the δ(σ) change as follows:

$$(A3)\ \frac{d\delta(\sigma)}{ds} = -\frac{p^H(\sigma)}{c(\sigma) + p^H(\sigma)s}.$$

The derivation of (A3) is straightforward from expression (1): the numerator on the right side indicates how much, conditional on a case entering the legal system and being in state σ, the expected cost of a harmful act rises per unit increase in *s*, and the denominator indicates the change in the expected cost of committing a harmful act per unit change in δ(σ). Because this relationship holds for all σ ∈ Σ, it is obvious that, when we integrate over these states, the value of $b^H$ given by expression (1) remains constant.

Next, we can determine how the chilling impact, the level of $b^B$, changes. For this, we take $db^B/ds$, using expression (1), where the δ(σ) adjust according to expression (A3). For each state σ (i.e., conditional on a case entering the legal system and being in that state), the effect on $b^B$ is given by:

$$(A4)\ p^B(\sigma) - \left(\frac{p^H(\sigma)}{c(\sigma)+p^H(\sigma)s}\right)\left(c(\sigma)+p^B(\sigma)s\right) =$$

$$p^H(\sigma)\left(\frac{p^B(\sigma)s}{p^H(\sigma)s} - \frac{c(\sigma)+p^B(\sigma)s}{c(\sigma)+p^H(\sigma)s}\right) \le 0,$$

with strict inequality for any state in which $p^H(\sigma) > p^B(\sigma)$. Intuitively, when we increase $s$ and terminate rather than continue just often enough to keep the contribution to deterrence in each state constant, chilling is reduced to the extent that there is any diagnosticity in the application of the explicit sanction $s$. The reason is that the defendants' continuation cost, $c(\sigma)$, influences deterrence and chilling but lacks any diagnosticity (conditional on being in the state σ); hence, relying less on it and more on $s$ improves targeting. Finally, what is true in each state is true in aggregate, so if deterrence is held constant by this experiment, chilling must fall. (If $p^H(\sigma) = p^B(\sigma)$ for all $\sigma \in \Sigma$, chilling remains constant, which is sufficient in establishing the Proposition.)

In the second step of the construction, we will now raise these δ(σ) back to 1.0, and keep deterrence constant by terminating (with certainty) in those states that have the worst overall targeting. Specifically, as suggested in the main text, we can order all the $\sigma \in \Sigma$ from highest (worst) to lowest (best) in terms of the ratio:

$$(A5)\ r(\sigma) = \frac{\left(c(\sigma)+p^B(\sigma)s\right)z^B(\sigma)}{\left(c(\sigma)+p^H(\sigma)s\right)z^H(\sigma)}.$$

Now, having first restored all the δ(σ) back to 1.0, we can remove states from Σ, starting from those with the highest $r(\sigma)$, until we have reached the point at which deterrence is back to its initial level. It should be clear that this step reduces chilling (or keeps it constant if the $r(\sigma)$ are uniform across all σ). The reason is simply that $r(\sigma)$ indicates the contribution to chilling – to the level of $b^B$ – per unit contribution to deterrence – the level of $b^H$ – for changes in the level of δ(σ). See expression (1). Because (combining the components of this second step) we raise δ(σ) (to 1) in all $\sigma \in \Sigma$ with $r(\sigma) \le r^*(\sigma)$, for some critical value $r^*(\sigma)$, and reduce δ(σ) (to 0) in all $\sigma \in \Sigma$ with $r(\sigma) > r^*(\sigma)$, it follows that chilling falls further. (If all the $r(\sigma)$ are equal, this step has no effect on chilling, which is sufficient in establishing the Proposition.)

Finally, observe that, after this construction is fully implemented, we have terminations in some states in Σ (and no continuations in any states not in Σ). Hence, on this account, continuation costs fall. Taken together, we have deterrence constant, a (weak) decline in chilling (which was assumed to be costly in this part of the proof), and a strict decline in continuation costs. Hence, welfare must be higher. Therefore, a nonmaximal sanction $s$ cannot be optimal.■

Again paralleling section 2.C, we now consider enforcement effort, where we have $\pi^i(e)$ such that $\pi^{i\prime}(e) > 0$, $\pi^{i\prime\prime}(e) \le 0$, for $i = H, B$. Using expressions (1) and (2), modified accordingly, the first-order condition for (an intermediate) $e$ is given by:

$$(A6)\ \frac{dW}{de} = \left(h + \kappa^H - b^H\right) f^H(b^H) b^H \frac{\pi^{H\prime}(e)}{\pi^H(e)} - \gamma\left(b^B - \kappa^B\right) f^B(b^B) b^B \frac{\pi^{B\prime}(e)}{\pi^B(e)}$$

$$- \left(1 - F^H(b^H)\right) \frac{\pi^{H\prime}(e)}{\pi^H(e)} \kappa^H - \gamma\left(1 - F^B(b^B)\right) \frac{\pi^{B\prime}(e)}{\pi^B(e)} \kappa^B - 1 = 0.$$

The discussion in section 2.C of this first-order condition and what immediately follows (which considers raising $e$ and switching to termination in some states) can be related to expression (A6) in a straightforward manner.

Finally, consider the classic Becker (1968) experiment of raising $s$ and lowering $e$ so as to keep deterrence ($b^H$) constant. Using expression (1), the requisite adjustment in $e$ is as follows:

$$(A7)\ \frac{de}{ds} = -\frac{\int_{-\infty}^{\infty} \pi^H(e) \delta(\sigma) p^H(\sigma) z^H(\sigma) d\sigma}{\int_{-\infty}^{\infty} \pi^{H\prime}(e) \delta(\sigma) \left(c(\sigma) + p^H(\sigma) s\right) z^H(\sigma) d\sigma}.$$

The numerator indicates the rise in deterrence on account of a unit increase in $s$, and the denominator on account of a unit increase in $e$. As we can see, raising $e$, because it increases the flow of cases into the legal system, raises the expected costs of those who commit harmful acts both due to the increase in the expected (explicit) sanction and the increase in defendants' expected adjudication costs. We have already seen that the latter is, in an important sense, less well targeted, so it might seem that we could prove that chilling must fall.

To determine the effect on chilling ($b^B$), we can also use expression (1), taking the derivative with respect to $s$, wherein $e$ changes according to expression (A7). The sign of the net effect on chilling is given by the sign of:

$$(A8)\ \frac{\int_{-\infty}^{\infty}\delta(\sigma)p^B(\sigma)z^B(\sigma)d\sigma}{\int_{-\infty}^{\infty}\delta(\sigma)p^H(\sigma)z^H(\sigma)d\sigma} - \frac{\pi^{B'}(e)\Big/\pi^B(e)}{\pi^{H'}(e)\Big/\pi^H(e)}\ \frac{\int_{-\infty}^{\infty}\delta(\sigma)\big(c(\sigma)+p^B(\sigma)s\big)z^B(\sigma)d\sigma}{\int_{-\infty}^{\infty}\delta(\sigma)\big(c(\sigma)+p^H(\sigma)s\big)z^H(\sigma)d\sigma}.$$

(For convenience, the $\pi^i(e)$ elements in the numerator and denominator of the first term were divided out, so they could be included with the $\pi^{i\prime}(e)$ elements in the second term.) The first term (subject to the stated adjustment) gives the ratio of chilling to deterrence per unit increase in $s$ and the second term (subject to the stated adjustment) gives the ratio for the calibrated change in $e$. If the latter is larger, chilling falls.

The point in section 2.C about diminishing returns in terms of targeting precision as one increases $e$, for some enforcement technologies, is apparent from the leading component of the second term. It indicates the relative rise in the probability of benign acts entering the legal system, compared to that in the probability of harmful acts entering the legal system. If this ratio is constant, let us say, we are left, in terms of chilling, with the fact that defendants' continuation costs play a greater role when using $e$ rather than $s$ to achieve a given level of deterrence.

Now, in the earlier analysis of raising $s$ and terminating in the weakest states, so as to keep deterrence constant, this differential was favorable. While there may well be such a tendency as a practical matter in the current setting, however, it is not necessarily true. The reason is that, when we raise $s$ and reduce $e$, the latter reduces the extent to which defendants bear adjudication costs and (conditional) expected (explicit) sanctions in all $\sigma \in \Sigma$, not just in the worst states in terms of targeting. Recall, moreover, just how worst is defined in the present context, as indicated by the ratio $r(\sigma)$ given by expression (A5). We have not only $[c(\sigma)+p^B(\sigma)s] / [c(\sigma)+p^H(\sigma)s]$ – which was at the core of the argument based on the difference in expression (A4) – but also $z^B(\sigma)/z^H(\sigma)$, which can confound the preference for relying more on $s$ and less on the $c(\sigma)$.

To see this point, assume that the $p^i(\sigma)$ are barely diagnostic, so that the targeting advantage of greater reliance on $s$ rather than the $c(\sigma)$ is slight. Furthermore, suppose that in some states with substantial mass, we have very low $z^B(\sigma)/z^H(\sigma)$ and very large $c(\sigma)$. In that event, the $c(\sigma)$ in those states are very well targeted, more so than is *s on average*. Now, when $s$ is raised, $e$ is reduced, and perhaps substantially because the $p^i(\sigma)$ that are not very diagnostic may nevertheless be large in states with high $z^B(\sigma)/z^H(\sigma)$. This reduction in $e$, in turn, makes the powerful targeting from the states with very low $z^B(\sigma)/z^H(\sigma)$ but very large $c(\sigma)$ less important. Taken together, chilling could rise. And if it rose enough to outweigh the other advantages of the classic Becker experiment, the net welfare impact could be negative.

One could put forward some fairly inelegant, although perhaps practically plausible, sufficient conditions to rule this possibility out. (Note, for example, that if $z^B(\sigma)/z^H(\sigma)$ is very low, it seems unlikely that, in a well-operating system, the $p^i(\sigma)$ would fail to be highly diagnostic in such states.) In any event, since Proposition 4's claim that the optimal sanction is maximal can be established through the earlier construction, further exploration is of limited interest.