# The Market for OTC Derivatives*

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**Abstract**

Over-the-counter (OTC) derivatives markets are very large relative to banks' trading assets, and gross notionals are highly concentrated on the balance sheets of just a few large dealer banks. Moreover, the large volume of varied bilateral trades creates an intricate system of liability linkages between participating banks. These stylized observations have drawn the attention of policy makers and the public alike. They also clearly illustrate that trading patterns in OTC derivatives markets greatly differ from the Walrasian benchmark of centralized trading at a common price. In this paper, we develop a model of equilibrium entry, trade, and prices, in order to formally analyze positive and normative issues surrounding OTC derivatives markets. Our model illustrates the extent to which standard trading frictions, along with the standard risk sharing motive for trade, lead to a realistic market structure.

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# 1 Introduction

Over-the-counter (OTC) derivatives markets are very large relative to banks' trading assets, and the large volume of varied bilateral trades creates an intricate system of liability linkages between participating banks. Several stylized observations regarding trade in these markets have drawn the attention of policy makers and the public alike. Collectively, for the banks participating in OTC derivatives markets, the gross volume of trade greatly exceeds the net volume. This difference between gross and net volume is particularly striking for a few large "dealer" banks, which typically hold more than 95% of the aggregate gross positions in the market, but have long and short positions of nearly equal size. In contrast, medium sized "customer" banks typically have substantial differences between the size of their long and short positions. These markets are also segmented; most smaller banks do not participate at all. These trading patterns in OTC derivatives markets greatly differ from the Walrasian benchmark of centralized trading at a common price. In this paper, we develop a model of equilibrium entry, trade, and price determination, in order to formally analyze positive and normative issues surrounding OTC derivatives markets.

In our model, banks trade OTC derivatives to share an aggregate risk. This trade is subject to two key trading frictions. First, a fixed entry cost must be paid by participating banks, since trade in OTC derivaties markets requires specialized capital and expertise. Second, considerations about risk management limit the size of trades made by any one trader in a bank with his or her counterparty. We find that with these two trading frictions, this model leads to a realistic market structure. Our OTC market features in the aggregate a large gross volume of bilateral trades relative to the net volume of trade. This gross volume of trade is concentrated in large dealer banks which have low net volume. In contrast, medium size banks have larger net, and smaller gross, positions, and smaller banks choose not to participate. Although all banks in our model are endowed with access to the same trading technology, some large banks endogenously arise as "dealers", trading mainly to provide intermediation services, while medium sized banks endogenously participate as "customers" mainly to share risks. We show that this market structure arises from a combination of economies of scale, hedging needs, and incentives to provide intermediation services in the OTC market.

We next consider the link between the degree of trading frictions, and market size and concentration. We show that in an OTC market such as ours, conditional on entry patterns, total gross derivatives notionals are non-monotonic in trading frictions. When frictions are substantial, volume is accordingly low. As frictions decrease, volume and

gross exposures grow, volume starts exceeding the Walrasian volume, and concentration of gross notionals in dealer banks increases. Thus, a large aggregate notional concentrated in a few dealer banks can be seen as a side product of a better risk sharing resulting from increased market liquidity. When frictions are low enough, the Walrasian outcome of perfect risk sharing can be achieved with nearly zero excess volume, and hence very low gross notional exposures.

Finally, we also ask how a planner or policy maker might improve the OTC market structure. In our model, large banks choose to enter as dealers, going long and short simultaneously, to capture the trading profits that arise from intermediation. One policy question is whether private incentives to provide intermediation that arise endogenously in response to the trading frictions in the OTC market lead to a socially efficient market outcome. From a social welfare perspective, we show that the liability structure is indeed too concentrated in large banks: A social planner could improve welfare by removing some larger dealer banks from the market, and encouraging smaller banks to enter.

In our model, we consider a theoretical financial system composed of a continuum of financial institutions we call banks. A bank is viewed as a coalition of many risk-averse agents, called traders. Banks' coalitions have heterogenous sizes and heterogenous endowments of a non-tradable risky loan portfolio. The size of each bank's loan portfolio determines their initial exposure to an aggregate risk factor. Since banks start with different per capita exposures to the aggregate risk factor and and are risk averse, they would find it optimal to equalize these exposures. While, in our model, banks' initial risk exposures are non-tradable, we assume that banks can buy and sell insurance contracts resembling swaps to synthetically alter their exposure to the aggregate risk factor. Specifically, conditional on their size and initial exposure to aggregate risk, banks first choose whether to pay a fixed cost in order to enter into an OTC market for swaps. Next, participating banks trade swaps to share aggregate risk. Finally, banks consolidate their positions internally, and loans and swap contracts pay off. We first characterize the equilibrium aggregate and bilateral volume, as well as pricing conditional on entry patterns. Then, we consider how the joint distribution of participating banks' sizes and risk exposures is determined by their equilibrium entry decisions.

Conditional on entry patterns, the traders in each participating bank establish bilateral trading relationships almost surely with every other bank. Risk sharing among banks through these bilateral relationships is constrained by risk management practices. Specifically, each trader has a fixed trading capacity: she cannot trade more than a given amount of swap contracts, either long or short. This trading capacity constraint proxies for risk-management limits on individual trading desk positions in practice. We assume

that these trading limits are allocated to the traders in a bank before information about the risk exposure of each trader's counterparty is revealed. The restriction that banks cannot reallocate trading capacity across traders once trade has begun is what effectively limits risk sharing. We argue that these trading limits are realistic, since in practice banks typically limit the risk that any one trader can take with line limits. The magnitude and direction of bilateral exposures in our model financial system then depends on banks' sizes and pre-trade exposures to the aggregate risk factor, as well as on traders' capacity limits.

When the traders from two banks trade, they take their position limits as given bargain over the terms of that trade. Gains from trade, and hence swap spreads, are determined by the post-trade risk exposures of the traders' respective institutions. In particular, the "sign" of the contract which each trader executes depends on whether their counterparty's bank expects a larger or smaller post-trade exposure to the aggregate risk factor than their own bank. Thus, within a bank, some traders execute long contracts, and some enter short contracts. At the end of the period, the swap portfolio of a participating bank is made up of the swap contracts of all its traders. In equilibrium, traders thus share risk within their banks, and banks share aggregate risk amongst each other through the OTC derivatives market.

Banks' decisions to enter the market are driven by two motivations, the strength of which are determined by their sizes and initial risk exposures. One motivation to enter is to share aggregate risk. This motivation is strong when a bank's initial risk exposure is significantly higher or lower than the market-wide average. A second motivation for a bank to enter is to capture the trading profits due to the equilibrium price dispersion across different bilateral trades. These motivations result in the following entry patterns: Small-sized banks cannot spread the fixed entry cost over many traders, and choose not to enter. Medium-sized banks only find it optimal to enter the market if their gains from trading in the OTC market are large enough, which we show occurs when their initial risk exposure is significantly higher or lower than the market-wide average. They use the OTC market to take a large net position, either short or long, and in this sense act as customers. Finally, large-sized banks are willing to enter the swap market irrespective of their initial risk exposure. If their initial risk exposure is significantly higher or lower than the market-wide average, they enter as customers for the same reason as the medium-sized banks do. If their initial exposure is near the market-wide average, they do not desire much change in their risk exposure, and therefore do not have incentives to enter as customers. Nevertheless, they enter for a different reason: their size allows them to conduct sufficiently many offsetting trades for their intermediation profits to cover

their fixed cost of entry. In this sense, large banks with average risk exposures emerge endogenously as dealers providing intermediation services in the OTC market.

To fix ideas, we use terminology from the market for Credit Default Swaps (CDS) in the presentation of our model. In this case, the underlying aggregate risk factor is an aggregate default risk factor. Banks' exposures to this factor are determined by the size of their illiquid loan portfolios. In practice the vast majority of trade in credit is executed via CDS contracts due to the superior liquidity of the derivative contract. Other underlying risks might include interest rate risk, or foreign exchange risk, and the model can also be applied to consider the OTC markets for interest rate and currency swaps.

To provide context for the market structure which arises in our model, we develop the following stylized facts which describe the OTC market for CDS: First, the market is large. Second, there appear to be increasing returns to scale: gross CDS notionals increase more than proportionally with bank size. As a result, the market is highly concentrated amongst the largest participating banks, and moreover the vast majority of small banks choose not to even participate in the OTC market. Third, trading behavior differs amongst banks: larger banks appear to act as dealers, and medium sized banks as customers. Large banks perform significantly more netting of their long and short contracts within their CDS portfolios. As a result, these banks have gross notionals which greatly exceed their net notional. In other words, larger banks tend to have a great amount of intermediation volume. Larger banks are also less likely to be able to record their purchases of credit derivatives as guarantees for regulatory purposes. The large netting benefits and small hedging benefits garnered by large banks are consistent with these banks acting as intermediaries for medium-sized, customer banks who use credit derivatives to synthetically alter their net credit exposure. Fourth, consistent with trade resulting from search and bargaining in an OTC market, prices vary with counterparty characteristics. Fifth, and finally, all banks which participate in the OTC derivatives market become interconnected by a complex liability structure.

The paper proceeds as follows. Section 2 surveys the literature, Section 3 presents stylized facts characterizing the OTC market for CDS, Section 4 presents the economic environment, Section 5 solves for the equilibrium conditional on entry patterns, and Section 6 studies entry decisions. Finally, Section 7 analyzes efficiency and Section 8 concludes.

# 2 Related Literature

It is common to model the bilateral trade in OTC markets as being constrained by a friction. Our paper attempts to describe how certain banks may arise as key intermedi-

aries, and more generally how the patterns of bilateral trade are determined. By allowing the system of bilateral trades to arise endogenously as a result of market entry, and by studying the costs and benefits of a structure in which certain banks play a more important role in intermediating trade, we are able to study the costs and benefits of a more concentrated market structure.[1]

Several recent papers also consider ideas related to the role of the market structure in determining trading outcomes in OTC markets. Duffie and Zhu (2010) use a framework similar to that in Eisenberg and Noe (2001) to show that a central clearing party for CDS only may not reduce counterparty risk because such a narrow clearinghouse could reduce cross contract class netting benefits. Babus (2009) studies how the formation of long-term lending relationships allows agents to economize on costly collateral, and demonstrates the manner in which star-shaped networks arise endogenously in the corresponding network formation game. Gofman (2011) emphasizes the role of the bargaining friction in determining whether trading outcomes are efficient in an exogenously specified OTC trading system represented by a graph.

The effects of the trading structure on trading outcomes has also been studied in the literature on systemic risk.[2] Allen and Gale (2000) develop a theory of contagion in a circular system, which they use to consider systemic risk in interbank lending markets. This framework has been employed by Zawadowski (2011) to consider counterparty risk in OTC markets. Eisenberg and Noe (2001) also study systemic risk, but use lattice theory to consider the fragility of a financial system in which liabilities are taken as given. Our paper differs from the papers discussed above by considering entry.

One of the most commonly employed frictions used to study OTC markets, following Duffie, Gârleanu, and Pedersen (2005), is the search friction. Kiefer (2010) offers an early analysis of CDS pricing within this framework. Our paper is unique in that, in contrast to earlier models, we explicitly consider financial institutions comprised of many traders who sign derivatives contracts amongst each others, thereby creating liability linkages across banks. In that sense, our paper is most closely related to Afonso and Lagos (2011), who develop a different search model in order to explain trading dynamics in the Federal Funds Market. Their focus is on the dynamics of reserve balances, however they also consider the importance of intermediation by banks in the reallocation of reserves over the course of the

[1]The costs of concentration have been a key concern to regulators of OTC derivatives markets. See, for example, the quarterly reports from the Office of the Comptroller of the currency at http://www.occ.treas.gov/topics/capital-markets/financial-markets/trading/derivatives/derivatives-quarterly-report.html, as well as ECB (2009), and Terzi and Ulucay (2011).

[2]See Stulz (2011) for a discussion of the potential for systemic risk in CDS markets.

day. By collapsing all trade dynamics into a single multilateral trading session, our model becomes sufficiently tractable to analyze endogenous entry, explain empirical patterns of participation across banks of different sizes, and address normative issues regarding the size and composition of the market. Li, Rocheteau, and Weill (2011) develop a model illustrating the role of scarce collateral in OTC markets. One can interpret our trading capacity limit as a limit on per trader collateral.

# 3 CDS Stylized Facts

We collect data from the Office of the Comptroller of the currency for the top 25 bank holding companies in derivatives, and from these Bank Holding Companies' FR Y-9C filings, and document the stylized facts which characterize the market for CDS in the US. The market for CDS is large. In the third quarter of 2011, the top twenty-five US bank holding companies participating in over-the-counter (OTC) derivatives markets had $13.58 trillion in assets, and held almost twice as much, or $22.58 trillion, in credit derivatives notional.

There appear to be increasing returns to scale. That is, if one sorts banks according to trading asset size, gross notional relative to trading assets is increasing in trading assets. This can be seen in Figure 1, which graphs gross notional to trading assets, across banked ranked by trading assets, for the top 25 bank holding companies in derivatives.

The market is also highly concentrated. Figure 2 shows the gross notionals for credit derivatives for all of the top 25 bank holding companies in derivatives, from 2007 to 2011. Ninety-five percent of the gross notional in credit derivatives is consistently held by only five bank holding companies. Clearly there are few dominant bank holding companies, and many banks which participate to a lesser extent. Considering participation makes the apparent concentration more extreme. The Federal Reserve Bank of Chicago lists about 14,000 US bank holding companies, while Chen, Fleming, Jackson, Li, and Sarkar (2011) report that only about 900 bank holding companies worldwide trade in CDS.[3]

There is significant netting between long and short contracts for the largest banks and less netting for middle-sized banks. We report statistics for netting of long and short positions multilaterally, and across contracts. While ISDA master agreements account only for bilateral netting, the aggregate data we can access does not allow us to disentangle bilateral relationships. We can verify from the 10-Q's of individual firms that large banks

[3]See http://www.chicagofed.org/webpages/banking/financial_institution_reports/bhc_data.cfm Chen, Fleming, Jackson, Li, and Sarkar (2011) use a detailed data set on three months of CDS transactions to document the importance of dealer banks, and the relatively higher activity in index products relative to single name contracts.

indeed enjoy large netting benefits even when one adheres to the strict definition of netting according to ISDA.[4] Figure 3 plots net to gross notionals for the top 25 bank holding companies in derivatives and shows that this fraction is on average close to zero for the largest dealer banks, which appear on the right-hand side. For example, JP Morgan's net to gross notional ratio is -0.1%. Middle-sized banks, in the center of the graph, have much larger net to gross notional ratios. For instance, Bank of New York Mellon has a ratio close to 100%, meaning that nearly all its CDS positions are going in the same direction. Most small banks, on the left-hand side of the figure, have zero gross notional in credit derivatives, which we display using empty bars.

Smaller banks are more likely to be able to report purchased credit derivatives as guarantees for regulatory purposes than larger banks are. Starting in the first quarter of 2009, and implemented to a greater extent in the second quarter of 2009, bank holding companies' FR Y-9C filings report the notional of purchased credit derivatives that are recognized as a guarantee for regulatory capital purposes. Figure 4 compares the fraction of purchased credit derivatives from Q2 2009 to Q4 2011 that could be counted as a guarantee for regulatory purposes for the largest 12 vs. the rest of the top 25 bank holding companies in derivatives. For the largest 12 bank holding companies in derivatives (the top half in terms of trading asset size), less than 0.5% of purchased credit derivatives could be recognized as a guarantee. By contrast, for the smaller holding companies amongst the top 25, almost 40% of purchased credit derivatives were recognized as a guarantee. This is consistent with smaller banks on average being more likely to use purchased CDS to change their credit exposure and to hedge while larger banks simply trade CDS to earn spreads on intermediation volume.

Prices vary by counterparty. This is apparently true since pricing data from Markit are composite quotes from multiple sources. Arora, Gandhi, and Longstaff (2012) use heterogeneity in quotes from multiple dealers to a single customer in order to assess to what extent counterparty risk is priced. Interestingly, they find that little of the price dispersion is explained by counterparty risk. This is consistent with spreads being driven by post trade credit exposures, and by banks' outside trading options as in our model with no credit risk. Shachar (2012) also uses data on individual trades, but studies the impact of dealer exposures on their ability to provide liquidity. This evidence is consistent with the banks' preferences and pricing in our model; banks price each contract based on their pre-trade risk exposure combined with any additional default risk arising from the

[4]See, for example, the excerpts from the 10-Q's for Bank of America or Goldman Sachs in the Financial Times Alphaville at ftalphaville.ft.com/blog/2011/12/21/808181/do-you-believe-in-netting-part-1/

rest of their portfolio.

Finally, banks are connected by a complex liability structure. This is why regulators and the public are concerned about systemic risk. The OTC market for CDS is an opaque market in which the liability linkages are unknown. In our model, we construct a predicted liability structure based on banks' initial size and credit exposures. In this way, one might use the model to assess likely empirical CDS linkages given observed bank characteristics.

# 4 The economic environment

We develop our model in three steps. First, we describe the economic environment. Then, we describe the post-entry equilibrium in section 5. Finally, we consider the joint distribution for banks' sizes and pre-trade exposures in the CDS market which results from equilibrium entry in section 6. Then, after developing the model, and describing its positive features, we turn to a normative analysis in section 7. Proofs not given in the text are gathered in the appendix.

## 4.1 Preferences and endowments

The economy is populated by a unit continuum of risk-averse agents, called traders. Traders have identical constant absolute risk aversion and are endowed with a technology to make payment by producing storable consumption good at unit marginal cost.[5] To model the financial system, we assume that traders are organized into a continuum of large coalitions called banks. Banks are heterogenous along two dimensions: their size, which we identify with the number of traders in the coalition, and their per capita endowment of some non-tradeable risky loan portfolio.

Banks' sizes, denoted by $S$, are cross-sectionally distributed according to the continuous density $\varphi(S)$ over the support $[\underline{S}, \infty)$, $\underline{S} \geq 0$. The density has thin enough tails, in that $\lim_{S\to\infty} S^3\varphi(S)$ exists and is finite. Because the economy-wide number of traders is one, we must have $\int_{\underline{S}}^{\infty} S\varphi(S)\, dS = 1$.

Banks' per capita loan portfolio endowments, denoted by $\omega$, are cross-sectionally distributed according to a uniform distribution over $[0, 1]$, independently of bank sizes.[6] This

[5] Precisely, if an agent consumes $C$ and produces $H$, his utility is $U\{C-H\} = -\frac{1}{\alpha}e^{-\alpha(C-H)}$. Given no wealth effect, an equivalent interpretation is that the agent has a large endowment of storable consumption good that she uses to make payments.

[6] The independence assumption clarifies the economic forces at play. Indeed, while there is no relationship between sizes and per-capita endowment in the overall population of banks, entry decisions endogenously create a correlation between the two in the OTC market. That being said, our model is flexible enough to handle more general joint distributions of size and per capita endowments. For exam-

implies in particular that the economy-wide measure of per capita default risk exposure is equal to one half. The per capita payoff for the bank from its illiquid loan portfolio is the size of the portfolio, $\omega$, times each loan's payoff $1 - D$, where 1 represents the face value of the loans and default risk $D \in [0, 1]$ is a (non-trivial) random variable with a twice continuously differentiable moment generating function. Thus, $\omega$ represents a bank's pre-trade, per-trader's capita, exposure to the aggregate default risk factor.

Since different banks start with different exposures to the aggregate default risk factor, $D$, and have identical risk aversion, they would benefit from equalizing their exposures. While, in our model, loans are non tradable, we assume that banks can enter an OTC market to buy and sell derivatives contracts, resembling CDS, against default risk. Thus, in our model the CDS market allows banks to take synthetic long and short positions in the aggregate default risk factor.[7]

## 4.2 Entry, trading, and payoffs

The economy lasts for three periods. In the first period, each bank chooses whether or not to pay a fixed cost $c > 0$ to be active in the OTC market. In the second period, traders from active banks meet in the OTC market. Finally, in the third period, banks consolidate the positions of their traders and all payoffs realize.

### 4.2.1 Inactive banks

Traders in inactive banks consume the per-capita payoff of their loan portfolio endowment, $\omega(1 - D)$, with expected utility:[8]

$$\mathbb{E}\left[U\{\omega(1-D)\}\right] \equiv -\frac{1}{\alpha}\mathbb{E}\left[e^{-\alpha\omega(1-D)}\right].$$

The corresponding certainty-equivalent payoff is:

$$\mathrm{CE}_i(\omega) = \omega - \Gamma\left[\omega\right], \quad \text{where } \Gamma\left[\omega\right] \equiv \frac{1}{\alpha}\log\left(\mathbb{E}\left[e^{\alpha\omega D}\right]\right). \tag{1}$$

ple, in an earlier version of the paper, we provided a characterization of the post-entry equilibrium when larger banks have more neutral pre-trade exposures than smaller banks, for example through greater internal diversification.

[7] Our analysis applies more generally to OTC trading of credit derivatives contracts, in which counterparties make a fixed-for-floating exchange of cash flow streams, and in which the floating stream is exposed to aggregate risk. This includes, for examples, interest rate swaps, CDS on sovereign entities, CDS indices and, to the extent that default risk is correlated across firms, CDS on single firms.

[8] Given identical concave utility, this is indeed the *ex-ante* optimal allocation of risk amongst traders in the bank.

That is, $\text{CE}_i(\omega)$ is equal to the face value of the loan portfolio endowment, $\omega$, net of the certainty equivalent cost of bearing its default risk, $\Gamma[\omega]$.

**Lemma 1.** *The certainty equivalent cost bearing default risk, $\Gamma[\omega]$, is twice continuously differentiable, strictly increasing, $\Gamma'[\omega] > 0$, and strictly convex, $\Gamma''[\omega] > 0$.*

These intuitive properties follow by taking derivatives. Note that, when $D$ is normally distributed with mean $\mathbb{E}[D]$ and variance $\mathbb{V}[D]$, $\Gamma[\omega]$ is the familiar quadratic function:

$$\Gamma[\omega] = \omega\mathbb{E}[D] + \omega^2 \frac{\alpha\mathbb{V}[D]}{2}. \tag{2}$$

The first term is the expected loss $\omega\mathbb{E}[D]$ upon default. The second term is an additional cost arising because banks are risk averse and the loss is stochastic.[9]

### 4.2.2 Active banks

We now turn to banks who choose to be active in the OTC market. We let $N(\omega)$ denote the measure of traders in active banks with per capita endowment less than $\omega$. We assume that $N(\omega)$ admits a continuous density $n(\omega)$, positive almost everywhere. As will become clear shortly, our model has a natural homogeneity property: two banks with identical per capita loan portfolio endowments, $\omega$, have identical per capita trading behavior. As discussed formally after Proposition 1, this implies that after entry, size is no longer a state variable for the bank, and that the OTC market equilibrium will only depend on the distribution $n(\omega)$.

**CDS contracts in the OTC market.** In the OTC market, each trader is matched with probability one with a trader from some other bank to bargain over a CDS contract. All traders in the population are equally likely to be matched. The probability that a trader from one bank is matched with a trader from a bank whose per capita endowment is less than $\tilde{\omega} \in [0,1]$ is $N(\tilde{\omega})$. When a trader from a bank of type $\omega$ (an "$\omega$-trader") meets a trader from a bank of type $\tilde{\omega}$ (an "$\tilde{\omega}$-trader"), they bargain over the terms of a fixed-for-floating derivative contract resembling a CDS. The $\omega$-trader sells $\gamma(\omega,\tilde{\omega})$ contracts to the $\tilde{\omega}$-trader, whereby she promises to make the random payment $\gamma(\omega,\tilde{\omega})D$ at the end of the period, in exchange for the fixed payment $\gamma(\omega,\tilde{\omega})R(\omega,\tilde{\omega})$. If $\gamma(\omega,\tilde{\omega}) > 0$ then the $\omega$-trader sells insurance, and if $\gamma(\omega,\tilde{\omega}) < 0$ she buys insurance. Importantly for our

[9] Clearly, a normal distribution does not satisfy our assumption that $D \in [0,1]$. It also implies that $\Gamma[\omega]$ is decreasing for $\omega$ negative enough. However, and as will become clear as we progress, our results only rely on strict convexity and so they continue to hold with a normally distributed $D$.

results, we assume that in any bilateral meeting, a trader cannot sign more than a fixed amount of contracts, $k$, either long or short.

Taken together, the collection of CDS contracts signed by all banks $(\omega, \tilde{\omega}) \in [0,1]^2$ must therefore satisfy:

$$\gamma(\omega, \tilde{\omega}) + \gamma(\tilde{\omega}, \omega) = 0 \tag{3}$$

$$-k \leq \gamma(\omega, \tilde{\omega}) \leq k. \tag{4}$$

The level of frictions in the CDS market is thus determined by the size of the position limit, $k$. We do not model the microfoundations of the trading limit, however we note that, in practice, traders typically do face line limits.[10] For example, Saita (2007) states that the traditional way to prevent excessive risk taking in a bank "has always been (apart from direct supervision...) to set notional limits, i.e., limits to the size of the positions which each desk may take." Theoretically, one might motivate such limits as stemming from moral hazard problems , concerns about counterparty risk and allocation of scarce collateral, or from capital requirement considerations.

**Bank's per capita consumption.** A trader in this economy faces two kinds of risk. The first is idiosyncratic risk over the type of counterparty they will trade with, namely the size and risk exposure of their counterparty's bank. The second is aggregate default risk. But since there is a large number of traders in each bank, traders can diversify their individual counterparty-type risk so that they are left only with the per capita exposure to default risk. Specifically, we assume that, at the end of the period, traders of bank $\omega$ get together and consolidate all of their long and short CDS positions. By the law of large numbers, the per capita consumption in an active bank with per capita endowment $\omega$ and size $S$ is:[11]

$$-\frac{c}{S} + \omega(1-D) + \mu \int_0^1 \gamma(\omega, \tilde{\omega}) \Big( R(\omega, \tilde{\omega}) - D \Big) n(\tilde{\omega}) d\tilde{\omega}. \tag{5}$$

[10] In addition, measures such as DV01 or CS1% which measure positions' sensitivities to yield and credit spread changes, as well as risk weighted asset charges, are used to gauge and limit the positions of a particular desk's traders.

[11] As is well known from other models with "large coalition" or "large families," we could equivalently assume that traders can buy and sell CDS in two ways: i) with traders from other banks, in a bilateral OTC market and ii) with traders from the same bank, in an internal competitive market. The internal competitive market leads to full risk sharing within the bank, just as with the large coalition.

The first term is the per capita entry cost. The second term is the per capita payout of the loan portfolio endowment, after default. The third term is the per capita consolidated amount of fixed payments, $\gamma(\omega, \tilde{\omega})R(\omega, \tilde{\omega})$, and floating payments, $\gamma(\omega, \tilde{\omega})D$, on the portfolio of contracts signed by all $\omega$-traders. Note in particular that, given random matching, $n(\tilde{\omega})$ represents the fraction of $\omega$-traders who met $\tilde{\omega}$-traders. One can see that the position limit $k$ is indeed crucial. Banks in our model almost surely trade with every other bank and thus would want to allocate capacity to the "better trades", thereby achieving full risk-sharing. Again, we argue that in reality risk management practices aimed to alleviate standard moral problems prevent such reallocation of trading capacity.

Our assumption that traders consolidate their CDS positions captures some realistic features of banks in practice. Within a bank, some traders will go long, and some short, depending on whom they meet and trade with. Because of this, our model is able to distinguish between gross and net exposure to credit risk resulting from trades in the CDS market. Furthermore, as will become clear later, despite all banks being endowed with access to the same trading technology, some banks endogenously become intermediaries in this market, in the sense that their trades generate a gross exposure that greatly exceeds their net exposure.

**Certainty equivalent payoff.** To calculate the certainty equivalent payoff, it is useful to break down the bank's per capita consumption in equation (5) into a fixed and a floating component. Namely, in bank $\omega$, the per capita fixed payment is:

$$-\frac{c}{S} + \omega + \int_0^1 \gamma(\omega, \tilde{\omega})R(\omega, \tilde{\omega})n(\tilde{\omega})d\tilde{\omega}. \tag{6}$$

Similarly, the per capita floating payment is $-g(\omega)D$, where

$$g(\omega) \equiv \omega + \int_0^1 \gamma(\omega, \tilde{\omega})n(\tilde{\omega})d\tilde{\omega}, \tag{7}$$

is the sum of the initial exposure, $\omega$, and of the exposure acquired in bilateral matches. The function $g(\omega)$ thus represents the bank's *post-trade* exposure to default risk.

Just as with inactive banks in equation (1), we find that the per capita certainty equivalent payoff of an active bank is

$$\mathrm{CE}_a(\omega, S) = -\frac{c}{S} + \omega + \int_0^1 \gamma(\omega, \tilde{\omega})R(\omega, \tilde{\omega})n(\tilde{\omega})d\tilde{\omega} - \Gamma\left[g(\omega)\right], \tag{8}$$

the per-capita fixed payment, net of the certainty equivalent cost of bearing the floating

payment risk.

**Bargaining in the OTC market.** To determine the terms of trade in a bilateral meeting, we need to specify the objective function of a trader. To that end, we follow the literature which allows risk sharing within families, such as in Lucas (1990), Andolfatto (1996), Shi (1997), Shimer (2010), and others, and assume that a trader's objective is to maximize the marginal impact of her decision on her bank's utility. This assumption means that a trader is small relative to her institution and that she does not coordinate her strategy with other traders in the same institution. One could think, for instance, about a trading desk in which all traders work independently knowing that all risks will be pooled at the end of the day.

Precisely, when a trader signs $\gamma(\omega,\tilde{\omega})$ contracts at a price $R(\omega,\tilde{\omega})$ per contract, her marginal impact on her bank's utility is defined as:

$$\mathbb{E}\left[\Lambda(\omega,D)\gamma(\omega,\tilde{\omega})\Big(R(\omega,\tilde{\omega})-D\Big)\right],\ \text{where}\ \Lambda(\omega,D)\equiv\frac{U'\left\{y(S,\omega,D)\right\}}{\mathbb{E}\left[U'\left\{y(S,\omega,D)\right\}\right]},$$

and $y(S,\omega,D)$ is the bank's per capita consumption derived in equation (5).

The first term in the expectation, $\Lambda(\omega,D)$, is bank $\omega$'s stochastic discount factor. Since utility is exponential, there are no wealth effects and so $\Lambda(\omega,D)$ is invariant to deterministic changes in the level of consumption. In particular, it does not depend on the entry cost, $c/S$, and therefore does not depend on size.

The second term is the trader's contribution to her bank's consumption: the number of contracts signed, $\gamma(\omega,\tilde{\omega})$, multiplied by the net payment per contract, $R(\omega,\tilde{\omega})-D$. Using the formula for the cost of risk bearing, $\Gamma\left[g(\omega)\right]$, the $\omega$-trader's objective function can be simplified to:

$$\gamma(\omega,\tilde{\omega})\Big(R(\omega,\tilde{\omega})-\Gamma'\left[g(\omega)\right]\Big). \tag{9}$$

Note that this can be viewed as the trader's marginal contribution to the certainty equivalent payoff (8). The expression is intuitive. If the trader sells $\gamma(\omega,\tilde{\omega})$ CDS contracts, she receives the fixed payment $R(\omega,\tilde{\omega})$ per contract but, at the same time, she increases her bank's cost of risk bearing. Since the trader is small relative to her bank, she only has a marginal impact on the cost of risk bearing, equal to $\gamma(\omega,\tilde{\omega})\Gamma'\left[g(\omega)\right]$.

The objective of the other trader in the match, the $\tilde{\omega}$-trader, is similarly given by:

$$\gamma(\omega,\tilde{\omega})\Big(\Gamma'\left[g(\tilde{\omega})\right]-R(\omega,\tilde{\omega})\Big), \tag{10}$$

where we used the bilateral feasibility constraint of equation (3), stating that $\gamma(\tilde{\omega}, \omega) = -\gamma(\omega, \tilde{\omega})$. The trading surplus is therefore equal to the sum of (9) and (10):

$$\gamma(\omega, \tilde{\omega}) \Big( \Gamma' \left[ g(\tilde{\omega}) \right] - \Gamma' \left[ g(\omega) \right] \Big).$$

We assume that the terms of trade in a bilateral match between an $\omega$-trader and an $\tilde{\omega}$-trader are determined via Nash bargaining, with both traders having equal bargaining power. The first implication of Nash bargaining is that the terms of trade are (bilaterally) Pareto optimal, i.e, they must maximize the surplus shown above. Since the marginal cost of risk bearing, $\Gamma' [x]$, is incresasing, this immediately implies that:

$$\gamma(\omega, \tilde{\omega}) = \begin{cases} k & \text{if } g(\tilde{\omega}) > g(\omega) \\ [-k, k] & \text{if } g(\tilde{\omega}) = g(\omega) \\ -k & \text{if } g(\tilde{\omega}) < g(\omega). \end{cases} \tag{11}$$

This is intuitive: if the $\tilde{\omega}$-trader expects a larger post-trade exposure than the $\omega$-trader, $g(\tilde{\omega}) > g(\omega)$, then the $\omega$-trader sells insurance to the $\tilde{\omega}$-trader. And vice versa if $g(\tilde{\omega}) < g(\omega)$. When the post-trade exposures are the same, then any trade in $[-k, k]$ is optimal.

The second implication of Nash bargaining is that the unit price of a CDS, $R(\omega, \tilde{\omega})$, is set so that each trader receives exactly one half of the surplus. This implies that:

$$R(\omega, \tilde{\omega}) = \frac{1}{2} \Big( \Gamma' \left[ g(\omega) \right] + \Gamma' \left[ g(\tilde{\omega}) \right] \Big). \tag{12}$$

That is, the price is half-way between the two traders' marginal cost of risk bearing. As is standard in OTC markets models, prices depend on banks' infra-marginal characteristics. In particular, prices are dispersed in the cross-section of matches, and are increasing functions of traders' post-trade exposures.

It is important to note that a trader's reservation value in a match is determined by her *post-trade* exposure, which results from the simultaneous trades of all traders in her institution. This means that, although our model is static, outside options play a key role in determining prices: if a trader chooses not to trade in a bilateral match, she still enjoys the benefits created by the trades of all other traders in her institution. This is similar to the familiar outside option of re-trading later arising in a dynamic models.[12]

[12] As mentioned in footnote 11, allowing traders of the same bank to pool their CDS contracts is essentially equivalent to assuming that, after the OTC market, traders can exchange CDS in a competitive "intra-bank" market. In that market, the price of a CDS contract is $\Gamma' [g(\omega)]$. Thus, the outside option of a trader in a bilateral match can be viewed as the outside option of re-trading later in the intra-bank

# 5 Equilibrium in the OTC market

Conditional on the distribution of traders, $n(\omega)$, generated by entry decisions, an *equilibrium* in the OTC market is made up of measurable functions $\gamma(\omega, \tilde{\omega})$, $R(\omega, \tilde{\omega})$, and $g(\omega)$ describing, respectively, CDS contracts, CDS prices, and post-trade exposures, such that:

(i) CDS contracts are feasible: $\gamma(\omega, \tilde{\omega})$ satisfies (3) and (4);

(ii) CDS contracts are optimal: $\gamma(\omega, \tilde{\omega})$ and $R(\omega, \tilde{\omega})$ satisfies (11) and (12) given $g(\omega)$;

(iii) post-trade exposures are consistent: $g(\omega)$ satisfies (7) given $\gamma(\omega, \tilde{\omega})$.

## 5.1 Constrained efficiency

In order to show existence and uniqueness of an equilibrium, it is useful to first analyze its efficiency properties. To that end, we consider the planning problem of choosing a collection of CDS contracts, $\gamma(\omega, \tilde{\omega})$, in order to minimize the average cost of risk bearing across banks,

$$\inf \int_0^1 \Gamma\left[g(\omega)\right] n(\omega)\, d\omega, \tag{13}$$

with respect to some bounded measurable $\gamma(\omega, \tilde{\omega})$, subject to (3), (4), and (7). Given that certainty equivalents are quasi-linear, an allocation of risk solves the planning problem if and only if it is Pareto optimal, in that it cannot be Pareto improved by choosing another feasible collection of CDS contracts and making consumption transfers. We then establish:

**Proposition 1.** *The planning problem has at least one solution. All solutions share the same post-trade risk exposure, $g(\omega)$, almost everywhere. Moreover, a collection of CDS contracts, $\gamma(\omega, \tilde{\omega})$, solves the planning problem if and only if it is the basis of an OTC market equilibrium.*

It follows from this proposition that an equilibrium exists. Moreover, the equilibrium post-trade exposures, $g(\omega)$, and bilateral prices, $R(\omega, \tilde{\omega})$, are uniquely determined. Note that the Proposition shows that our restriction that CDS contracts only depend on $\omega$ is without much loss of generality. Indeed, if CDS contracts were allowed to depend on any other bank characteristics, such as size, then the same efficiency result would hold: equilibrium post-trade exposures would solve a generalized planning problem in which

market.

CDS contracts are allowed to depend on these characteristics. It is then easy to show that this generalized planning problem has the same solution as (13), i.e., the planner would find it strictly optimal to choose post–trade exposures that only depend on $\omega$.[13] Therefore, in any equilibrium, post-trade exposures coincide with the unique solution of (13).

## 5.2 Equilibrium post-trade exposures: some general results

We now establish elementary properties of the post-trade exposure function. First, we show that:

**Proposition 2.** *Post-trade exposures are non-decreasing and closer together than pre-trade exposures:*

$$0 \leq g(\tilde{\omega}) - g(\omega) \leq \tilde{\omega} - \omega, \text{ for all } \omega \leq \tilde{\omega}. \tag{14}$$

The left-hand inequality means that $g(\omega)$ is non-decreasing, i.e., banks starting with low pre-trade exposure end with low post-trade exposures, and vice versa. The right-hand side inequality is a manifestation of risk-sharing. For example, in the special case of full risk-sharing, then $g(\tilde{\omega}) - g(\omega) = 0$ and the inequality is trivially satisfied. With imperfect risk sharing, we obtain a weaker result: $g(\tilde{\omega}) - g(\omega)$ is smaller than $\tilde{\omega} - \omega$, but in general remains larger than zero.

**Proposition 3.** *If $g(\omega)$ is increasing at $\omega$, then:*

$$g(\omega) = \omega + k\left[1 - 2N(\omega)\right]. \tag{15}$$

*If $g(\omega)$ is flat at $\omega$ then:*

$$g(\omega) = \mathbb{E}\Big[\omega \,\Big|\, \omega \in [\underline{\omega}, \overline{\omega}]\Big] + k\left[1 - N(\overline{\omega})\right] - kN(\underline{\omega}), \tag{16}$$

*where the expectation is taken with respect to $n(\omega)$, conditional on $\omega \in [\underline{\omega}, \overline{\omega}]$, and where $\underline{\omega} \equiv \inf\{\tilde{\omega} : g(\tilde{\omega}) = g(\omega)\}$ and $\overline{\omega} \equiv \sup\{\tilde{\omega} : g(\tilde{\omega}) = g(\omega)\}$ are the boundary points of the flat spot surrounding $\omega$.*

[13] Precisely, suppose that CDS contracts depend on the pre-trade exposure, $\omega$, and on some other vector of characteristics denoted by $x$. Then, the CDS contracts $\hat{\gamma}(\omega, \tilde{\omega}) \equiv \int \gamma(\omega, x, \tilde{\omega}, \tilde{x}) n(dx \,|\, \omega) n(d\tilde{x} \,|\, \tilde{\omega})$ are feasible and generate post-trade exposures $\hat{g}(\omega) = \int g(\omega, x) n(dx \,|\, \omega)$. Because the cost of risk-bearing is convex, the planner prefers $\hat{\gamma}(\omega, \tilde{\omega})$ over $\gamma(\omega, x, \tilde{\omega}, \tilde{x})$, and strictly so if $g(\omega, x)$ varies with $x$.

The intuition for this result is the following. If $g(\omega)$ is strictly increasing at $\omega$, then it must be that a $\omega$-trader sells $k$ contracts to any trader $\tilde{\omega} > \omega$, and purchases $k$ contracts from any traders $\tilde{\omega} < \omega$. Aggregating across all traders in banks $\omega$, the total number of contracts sold by bank $\omega$ is $k[1 - N(\omega)]$ per-trader. Likewise, the total number of contracts purchased by bank $\omega$ is $kN(\omega)$ per capita. Adding all contracts sold and subtracting all contracts purchased, we obtain (15).

Now consider the possibility that $g(\omega)$ is flat at $\omega$ and define $\underline{\omega}$ and $\overline{\omega}$ as in the proposition. By construction, all banks in $[\underline{\omega}, \overline{\omega}]$ have the same post-trade exposure. Therefore, $g(\omega)$ must be equal to the average post-trade exposure across all banks in $[\underline{\omega}, \overline{\omega}]$ which is given in equation (16): the average pre-trade exposure across all banks in $[\underline{\omega}, \overline{\omega}]$, plus all the contracts sold to $\tilde{\omega} > \overline{\omega}$-traders, minus all the contracts purchased from $\tilde{\omega} < \underline{\omega}$-traders. The contracts bought and sold among traders in $[\underline{\omega}, \overline{\omega}]$ do not appear since, by (3), they must net out to zero.

To derive a sufficient condition for a flat spot, differentiate equation (15): $g'(\omega) = 1 - 2kn(\omega)$. Clearly, if this derivatives turns out negative, then (15) cannot hold, i.e., $g(\omega)$ cannot be increasing at $\omega$.

**Corollary 4.** *If $2kn(\omega) > 1$, then $g(\omega)$ is flat at $\omega$.*

This corollary means that, when $n(\omega)$ is large, then the post-trade exposure function is flat at $\omega$. Intuitively, when there's a large density of traders in the OTC market with similar endowments, these traders can find each other so easily that they manage to pool their risks fully in spite of the frictions they face.

A reasoning by contradiction offers a perhaps more precise intuition. Assume that $n(\omega)$ is large in some interval $[\omega_1, \omega_2]$, but that $g(\omega)$ is strictly increasing. Then, when two traders from this interval meet, it is always the case that the low-$\omega$ trader sells $k$ CDS to the high-$\omega$ trader. In particular, $\omega_1$ sells insurance to all traders in $(\omega_1, \omega_2]$, and $\omega_2$ buys insurance from all traders in $[\omega_1, \omega_2)$. If there are sufficiently many traders to be met in $[\omega_1, \omega_2]$, then this can imply that $g(\omega_1) > g(\omega_2)$, contradicting the property that $g(\omega)$ be non-decreasing.

The above results also provide a heuristic method for constructing the post-trade exposure function, $g(\omega)$, induced by some particular distribution of traders, $n(\omega)$. One starts from the guess that $g(\omega)$ is equal to $\omega + k[1 - 2N(\omega)]$, as in equation (15). If this function turns out to be non-decreasing, then it must be the equal to $g(\omega)$. Otherwise, one needs to "iron" its decreasing spots into flat spots. The levels of the flat spots are given by (16). The boundaries of the flats spots are pinned down by the continuity conditions that, at a each boundary point, post-trade exposures must satisfy both (15) and (16).

## 5.3 Example: $U$-shaped and symmetric distributions

To build more intuition, we solve for the equilibrium under the assumption that $n(\omega)$ is $U$-shaped and symmetric around $\frac{1}{2}$. That is, we assume that $n(\omega)$ is decreasing over $[0, \frac{1}{2}]$, increasing over $[\frac{1}{2}, 1]$ and satisfies $n(\omega) = n(1-\omega)$. Aside from the fact that it leads to a closed form solution, this type of distribution is of special interest because, under natural conditions, it will hold in the entry equilibrium of Section 6.

An example $U$-shaped and symmetric $n(\omega)$ is shown in Figure 5. In interpreting the figure, one should bear in mind that $n(\omega)$ is the product of the *number* of $\omega$-banks and of their *average size*. In particular, a large $n(\omega)$ does not imply that $\omega$-banks are large. In fact, we will show in Section 6 that extreme-$\omega$ banks are smaller, on average, while middle-$\omega$ banks are larger. That is, in the entry equilibrium to be described, the shape of the $n(\omega)$ distribution is ultimately driven by the number of banks entering at various point of the $\omega$ spectrum, and not by their sizes.

### 5.3.1 Post-trade exposures

We focus attention on $\omega \in [0, \frac{1}{2}]$ because the construction over $[\frac{1}{2}, 1]$ is symmetric. First, since $n(\omega)$ is decreasing over $[0, \frac{1}{2}]$ it follows that $\omega + k\,[1 - 2N(\omega)]$ is increasing over $[0, \frac{1}{2}]$ if and only if it is increasing for $\omega = 0$, that is if and only if $2kn(0) \leq 1$. If that condition is satisfied then clearly $g(\omega)$ is non-dereasing and is given by equation (15). Otherwise, we guess that $g(\omega)$ is first flat over some interval $[0, \underline{\omega}]$, and then increasing over the subsequent interval $[\underline{\omega}, \frac{1}{2}]$. The boundary $\underline{\omega}$ of the flat spot must satisfy two conditions. First, the post-trade exposure must be equal to

$$g(\underline{\omega}) = \underline{\omega} + k\,[1 - 2N(\underline{\omega})]\,.$$

That is, a trader just to the right of $\underline{\omega}$ must buy $k$ contracts from all $\tilde{\omega} < \underline{\omega}$ and sell $k$ contracts to all $\tilde{\omega} > \underline{\omega}$. The second condition is given by Proposition 3, which states that post-trade exposures in the flat spot must be equal to

$$g(\underline{\omega}) = \mathbb{E}\Big[\omega \,\Big|\, \omega \in [0, \underline{\omega}]\Big] + k\,[1 - N(\underline{\omega})]$$

Taking the difference between the two we obtain:

$$H(\underline{\omega}) = 0, \qquad \text{where } H(\omega) \equiv \int_0^{\omega} (\omega - \tilde{\omega})\, n(\tilde{\omega})\, d\tilde{\omega} - kN(\omega)^2.$$

If there is some $\underline{\omega} \in (0, \frac{1}{2})$ such that $H(\underline{\omega}) = 0$, then we have found the upper boundary of the flat spot. Otherwise, the post-trade exposures must be flat over the entire interval $[0, \frac{1}{2}]$. The construction is illustrated in Figure 6, and summarized below:

**Proposition 5.** *Suppose that the distribution of traders, $n(\omega)$, is U-shaped and symmetric around $\omega = \frac{1}{2}$. Then, there are $\underline{\omega} \in [0, \frac{1}{2}]$ and $\overline{\omega} = 1 - \underline{\omega}$ such that, for $\omega \in [0, \underline{\omega}]$ and $\omega \in [\overline{\omega}, 1]$, $g(\omega)$ is flat, and for $\omega \in [\underline{\omega}, \overline{\omega}]$, $g(\omega)$ is increasing and equal to $g(\omega) = \omega + k\,[1 - 2N(\omega)]$. Moreover:*

- *if $k \leq \frac{1}{2}\,[n\,(0)]^{-1}$, then $g(\omega)$ has no flat spot.*
- *if $\frac{1}{2}\,[n\,(0)]^{-1} < k < 1 - 2\mathbb{E}\left[\omega \,|\, \omega \leq \frac{1}{2}\right]$, then $g(\omega)$ has flat and increasing spots.*
- *if $k > 1 - 2\mathbb{E}\left[\omega \,|\, \omega \leq \frac{1}{2}\right]$, then $g(\omega)$ is flat everywhere and equal to $\frac{1}{2}$.*

### 5.3.2 CDS contracts

The post-trade exposures of Proposition 5 are implemented with the following collection of CDS contracts. For all $\omega \in [\underline{\omega}, 1 - \underline{\omega}]$, the implementation is straightforward: since $g(\omega)$ is increasing, it must be the case that a $\omega$ trader buys $k$ contracts from all $\tilde{\omega} < \omega$, and sells $k$ contracts to all $\tilde{\omega} > \omega$. Matters are more subtle within the flat spots: indeed, when two traders $(\omega, \tilde{\omega})$ in $[0, \underline{\omega}]^2$ or $[1 - \underline{\omega}, 1]^2$ meet, all trades in $[-k, k]$ leave them indifferent. Yet, they must trade in such a way that their respective institutions wind up with identical post-trade exposures, $g(\underline{\omega})$. To find bilaterally feasible contracts delivering identical post-trade exposures, we guess that, when two traders $\omega < \tilde{\omega}$ meet, the $\omega$-trader sells to the $\tilde{\omega}$ trader a number of contracts, which we denote by $z(\tilde{\omega})$, that only depends on $\tilde{\omega}$. When the $\omega$ trader meets a trader $\tilde{\omega} > \underline{\omega}$, he must sell $k$ contracts since in this case $g(\tilde{\omega}) > g(\omega)$. This guess is illustrated in Figure 7 and means that:

$$g(\underline{\omega}) = \omega - z(\omega)N(\underline{\omega}) + \int_{\omega}^{\underline{\omega}} z(\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega} + k\,[1 - N(\underline{\omega})]$$

The first term is the initial exposure. The second term adds up all the contracts purchased from $\tilde{\omega} < \omega$; the third term adds up all the contracts sold to $\tilde{\omega} \in (\omega, \underline{\omega}]$; and the fourth term adds up all the contracts sold to $\tilde{\omega} \in (\underline{\omega}, 1]$. Taking derivatives delivers an ordinary differential equation for $z(\omega)$, which we can solve explicitly with the terminal condition $z(\underline{\omega}) = k$.

**Proposition 6.** *The post-trade exposures of Proposition 5 are implemented by the following CDS contracts. For all* $\omega \in \left[0, \frac{1}{2}\right]$ *and* $\tilde{\omega} > \omega$*:*

$$\text{if } \tilde{\omega} \leq \tfrac{1}{2} \,:\, \gamma(\omega, \tilde{\omega}) = \min\left\{k, z(\tilde{\omega})\right\};$$
$$\text{if } \tilde{\omega} > \tfrac{1}{2} \,:\, \gamma(\omega, \tilde{\omega}) = \min\left\{k, z\left(\tfrac{1}{2}\right)\right\}, \text{ where } z(\omega) \equiv \frac{\int_0^{\omega} (\omega - \tilde{\omega})\, n(\tilde{\omega})\, d\tilde{\omega}}{N(\omega)^2},$$

*All other* $\gamma(\omega, \tilde{\omega})$ *are then uniquely determined by symmetry,* $\gamma(1-\omega, 1-\tilde{\omega}) = -\gamma(\omega, \tilde{\omega})$*, and bilateral feasibility,* $\gamma(\omega, \tilde{\omega}) + \gamma(\tilde{\omega}, \omega) = 0$*.*

While bilateral feasibility puts non-trivial restrictions on CDS contracts, there can be multiple collections of CDS contracts implementing the same equilibrium. The one proposed above has, however, two appealing features: it is consistent with the intuitive notion that banks with low exposure sell to banks with high exposures, and it is continuous at the boundary of the flat spot.

### 5.3.3 Notionals

We now study banks' trading behavior across the $\omega$ spectrum. We show in particular that, in our OTC market, traders employed by the same bank execute both long and short contracts, and as a result a bank's gross notional can greatly exceed its net notional. For brevity we only offers a graphical analysis but a precise analytical characterization can be found in Appendix A.10.1.

**Contracts sold and bought.** Following some of the measurements performed by the US Office of the Comptroller of the Currency (OCC), we let the (per-capita) number of contracts sold and bought by bank $\omega$:

$$G^+(\omega) = \int_{\omega}^{1} \gamma(\omega, \tilde{\omega}) n(\tilde{\omega}), \quad \text{and} \quad G^-(\omega) = -\int_{0}^{\omega} \gamma(\omega, \tilde{\omega}) n(\tilde{\omega}),$$

keeping in mind that, with the network of CDS contracts of Proposition 6, $\gamma(\omega, \tilde{\omega}) > 0$ for $\tilde{\omega} > \omega$, and $\gamma(\omega, \tilde{\omega}) < 0$ for $\tilde{\omega} < \omega$. The number of contracts sold, $G^+(\omega)$ in the left panel, is decreasing over $[0, 1]$ and equal to zero for $\omega = 1$: this reflects the fact that low-$\omega$ banks, with low pre-trade exposure to the aggregate default risk factor, have more risk bearing capacity, and hence tend to supply more insurance to others than high-$\omega$ banks, with high pre-trade exposures. Note that $G^+(\omega)$ is positive even for banks $\omega > \frac{1}{2}$: that is, even though banks with $\omega > \frac{1}{2}$ are net buyers of insurance, $g(\omega) - \omega < 0$, their traders sell insurance when they meet traders of banks $\tilde{\omega} > \omega$. Symmetrically, the number of

contracts sold, $G^-(\omega)$ in the right panel, is zero for $\omega = 0$, and then is increasing over $[0, 1]$. Because banks with high pre-trade exposures to credit risk have less risk bearing capacity, they demand more insurance from others.

**Intermediation.** As shown in Figure 10, all banks $\omega \in (0, 1)$ provide some intermediation: they simultaneously buy and sell CDS contracts since both $G^+(\omega)$ and $G^-(\omega)$ are positive. A natural measure of intermediation volume is $\min\{G^+(\omega), G^-(\omega)\}$. This measure provides, for each bank $\omega$, the per-capita volume of fully offsetting CDS contracts. One sees that this volume is smallest for extreme-$\omega$ banks, and largest for middle-$\omega$ banks. Extreme-$\omega$ banks play the role of "customer banks" and do not provide much intermediation. In order to bring their exposures closer to $\frac{1}{2}$ most of their traders sign contracts in the same direction. Middle-$\omega$ banks, on the other hand, play the role of "dealer banks", who provide more intermediation: they do not need to change their exposure much, and so they can use their trading capacity to take large offsetting long and short positions.

**Gross notional.** The gross notional is $G^+(\omega)+G^-(\omega)$, the total number of CDS signed. One sees that the gross notional is largest and the net notional is smallest for middle-$\omega$ dealer banks. Indeed, middle-$\omega$ banks lie in an increasing spot of $g(\omega)$, so their traders always use all of their capacity limit, either selling $k$ or purchasing $-k$. Extreme-$\omega$ customer banks, on the other hand, lie in a flat spot of $g(\omega)$, and their traders do not use all of their capacity when they meet other traders from the same flat spot. This feature of the equilibrium will be a key driver of some of the cross sectional variation across banks' trading behavior.

**Net notional.** The net notional is the difference between contracts sold and purchased, $\left|G^+(\omega) - G^-(\omega)\right| = \left|g(\omega) - \omega\right|$. Clearly, it is lowest for middle-$\omega$ dealer banks, which enter the OTC market with $\omega \simeq \frac{1}{2}$ and thus do not need to change their risk exposures much.

### 5.3.4 Bilateral prices

As shown in equation (12), in the equilibrium of our CDS market, prices are dispersed. In particular, the average price faced by a $\omega$ bank is:

$$\frac{1}{2}\left(\Gamma'\left[g(\omega)\right] + \int_0^1 \Gamma'\left[g(\tilde{\omega})\right] n(\tilde{\omega})\, d\tilde{\omega}\right)$$

Thus, the average price faced by a $\omega$-bank is increasing in its post-trade exposure, $g(\omega)$. In particular, banks with high post-trade exposures find it very beneficial to buy insurance, and so they face higher prices. Conversely, banks with low post-trade exposures do not find it very costly to provide insurance, and so they trade at lower prices. Note also that customers with the most risky post trade positions face the highest prices. Since post trade risk exposure is likely to be related to variables that measure default risk, this is consistent with the evidence in Arora, Gandhi, and Longstaff (2012), which shows that counterparty risk is to some extent priced in CDS markets. In our model, the adverse pricing offered to banks with large post trade risk exposures reflects their large gains from trade. However, empirically, such variation in the gains from trade may appear to be related to counterparty credit risk. Thus, what appears to be variation in counterparty risk may actually be measuring variation in gains from trade.

### 5.3.5 Market outcomes with vanishing frictions

What is the impact of reducing trading frictions on market outcomes? From Proposition 5 one can easily show that, when $k$ increases, the two flat spots become larger and closer to $\frac{1}{2}$. This phenomenon reflects better risk sharing and higher welfare. It is intuitive that welfare improves since the equilibrium allocation is socially efficient conditional on entry, and since any collection of CDS contracts that is feasible with a lower $k$ is obviously feasible with a higher $k$.

According to Proposition 6, the function $z(\omega)$, which determines trades in the flat spot, does not change as $k$ increases. This implies that an extreme-$\omega$ bank in the flat spot $[0, \underline{\omega}]$ or $[1-\underline{\omega}, 1]$, will not change its trades with other banks in the same flat spot. It will only use its larger capacity to increase its trades with banks outside the flat spot. Precisely, for such a customer bank, we show in Appendix A.6 that the derivative of gross notional with respect to $k$ is equal to $\left[1 - N(\underline{\omega})\right] < 1$. Middle-$\omega$ banks in the increasing spot, however, will increase their trades with all other banks. For such a dealer bank, the derivative of gross notional with respect to $k$ is larger, and equal to 1. Therefore, our model has the prediction that, as $k$ grows and the market becomes more efficient, gross notionals increase for all banks but become more concentrated in dealer banks.

The model also implies that trading volume is non-monotonic in $k$, in the following sense. When $k$ is small then mechanically trading volume is small because traders have little capacity. When $k$ is greater than $1 - 2\mathbb{E}\left[\omega \,|\, \omega \leq \frac{1}{2}\right]$ but less than 1, we show in Appendix A.6 that, for *any* equilibrium set of CDS contracts, not only that of Proposition 6, the OTC market optimally circumvents frictions by creating excess trading volume, relative to its Walrasian counterpart. Note that this is in spite of the fact that the post-

trade exposures are the same as in the Walrasian equilibrium. Finally, when $k$ is large enough, one can find CDS contracts generating a volume that is arbitrarily close to the Walrasian volume.

# 6 Equilibrium entry

In this section we study the entry of banks into the OTC market, thus endogenizing $n(\omega)$. We first characterize entry incentives and offer a general existence result. Next, we provide conditions under which the equilibrium $n(\omega)$ turns out to be $U$-shaped and symmetric. Lastly, we show that, with entry, our model explains qualitative empirical relationships between bank size, net notionals, gross notionals, and intermediation activity. In the model, entrants to the OTC market must pay a fixed cost. In practice, mass processing of OTC derivatives order tickets requires a large infrastructure, and trading in derivatives requires specialized expertise which is costly to acquire. In the CDS market, it is those banks with a broad client base, which includes smaller banks with loan portfolios as well as hedge funds with little initial exposure to credit risk, that became dealers. Because the market depends largely on bilateral relationships, perceived credit risk also plays an important role in succeeding as a dealer. In our model, the fixed cost implies that only relatively large banks will enter and trade. Furthermore, large banks which have neutral exposure essentially have a broad client base of both extreme high and low $\omega$ customers. These banks become the key CDS market dealers in the sense that their balance sheets have the largest gross positions, and they net the largest amount of long short trades.

## 6.1 Entry incentives

If the distribution of active banks in the market is $n(\omega)$, then the per-capita and before entry cost certainty equivalent payoff of entering the market is, using (1) and (8),

$$\Delta(\omega) \equiv \Gamma\left[\omega\right] - \Gamma\left[g(\omega)\right] + \int_0^1 \gamma(\omega, \tilde{\omega}) R(\omega, \tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega}.$$

The first-term is the change in exposure: it is negative if the bank is a net seller of insurance, and positive if it is a net buyer. The second term is the sum of all CDS premia collected and paid by the bank. It is positive if the bank is a net seller, and is typically negative if it is a net buyer. To gain further insights into $\Delta(\omega)$, we use the decomposition:

$$\Delta(\omega) = K(\omega) + \frac{1}{2} F(\omega)$$

where the function $K(\omega)$ is:

$$\begin{aligned} K(\omega) &\equiv \Gamma\left[\omega\right] - \Gamma\left[g(\omega)\right] + \int_0^1 \Gamma'\left[g(\omega)\right]\gamma(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega} \\ &= \Gamma\left[\omega\right] - \Gamma\left[g(\omega)\right] + \Gamma'\left[g(\omega)\right]\big(g(\omega)-\omega\big), \end{aligned}$$

because $\int_0^1 \gamma(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega} = g(\omega)-\omega$. The function $F(\omega)$ is:

$$\begin{aligned} F(\omega) &\equiv 2\bigg(\int_0^1 \gamma(\omega,\tilde{\omega})R(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega} - \int_0^1 \Gamma'\left[g(\omega)\right]\gamma(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}\bigg) \\ &= \int_0^1 \gamma(\omega,\tilde{\omega})\big(\Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\omega)\right]\big)n(\tilde{\omega})\,d\tilde{\omega} \\ &= k\int_0^1 \big|\Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\omega)\right]\big|n(\tilde{\omega})\,d\tilde{\omega}, \end{aligned}$$

where the first equality follows from using the formula (12) for $R(\omega,\tilde{\omega})$, and the second equality follows from the optimality condition (11).

The function $K(\omega) \geq 0$ represents the per-capita *competitive surplus* of bank $\omega$. For a net seller of insurance, $g(\omega) > \omega$, $K(\omega)$ is simply a producer surplus: the first two terms represent the utility cost of producing $g(\omega)-\omega$ units of insurance by changing exposure, and the last term is the marginal cost of producing this insurance. Vice versa, for a net buyer of insurance, $K(\omega)$ is a consumer surplus. In this sense, the function $K(\omega)$ measures the benchmark entry incentives provided by a frictionless competitive market. Notice that it is positive if $g(\omega) \neq \omega$ and zero if $g(\omega) = \omega$. In other words, $K(\omega)$ only measures incentives to change exposure and does not account for incentives to provide intermediation.

The function $F(\omega)$ is also positive and measures the *frictional surplus* of bank $\omega$. When $\Gamma'\left[g(\tilde{\omega})\right]$ is not equalized across banks, a $\omega$-bank uses its bargaining power to sell insurance at a price higher than its marginal cost, and to buy insurance at price lower than its marginal value. The profits thus generated add up to $\frac{1}{2}F(\omega)$, where the frictional surplus is multiplied by $\frac{1}{2}$ because a trader only has half of the bargaining power. Notice in particular that, unlike $K(\omega)$, the function $F(\omega)$ is positive even when $g(\omega) = \omega$, and so it accounts for incentives to provide intermediation.

When $D$ is normally distributed and $n(\omega)$ is $U$-shaped and symmetric, we obtain a sharper characterization of both functions:

**Proposition 7** (Entry Incentives)**.** *Suppose that the distribution of active traders, $n(\omega)$, is U-shaped and symmetric, and assume that $D$ is normally distributed. Then both the competitive surplus, $K(\omega)$, and the frictional surplus, $F(\omega)$, are U-shaped and symmetric*

*around* $\frac{1}{2}$.

The proposition means that extreme-$\omega$ "customer" banks, who have incentives to share risk in the market and acquire large net positions, have the greatest incentives to enter. At the same time, middle-$\omega$ "dealer" banks, who acquire small net positions but large gross positions, have the smallest incentives to enter. Thus, intermediation activity endogenously has a small profit margin in our model.

Moreover, the $U$-shape pattern of incentives holds both for the competitive surplus and for the frictional surplus. The competitive surplus, $K(\omega)$, is $U$-shaped because a bank has greatest incentives to enter if it either has a large risk bearing capacity (small $\omega$), or if it has a large need to unwind its risk (large $\omega$). The frictional surplus, $F(\omega)$, measures the average absolute distance between bank $\omega$'s marginal cost of risk bearing and that of other banks, and so it is minimized for the median bank. With a symmetric $n(\omega)$ the median bank is at $\omega = \frac{1}{2}$.

## 6.2 Equilibrium

Keeping in mind that a bank has to pay a fixed cost $c$ to enter the market, it will enter if and only if:

$$\Delta(\omega) - \frac{c}{S} \geq 0 \quad \Longleftrightarrow \quad S \geq \Sigma(\omega) \text{ where } \Sigma(\omega) \equiv \frac{c}{\Delta(\omega)}.$$

Let $\Psi(S)$ be the measure of traders in banks with sizes greater than $S$. For $S \geq \underline{S}$, $\Psi(S) = \int_S^\infty x\varphi(x)\,dx$, and for $S \leq \underline{S}$, $\Psi(S) = 1$. Given that sizes and per-capita endowments are independent in the banks' cross section, the *measure* of active traders with endowment $\omega$ is $\Psi\left[\Sigma(\omega)\right]$. The *distribution* of active traders is, then:

$$T\left[n\right](\omega) = \frac{\Psi\left[\Sigma(\omega)\right]}{\int_0^1 \Psi\left[\Sigma(\tilde{\omega})\right]\,d\tilde{\omega}},$$

where the denominator normalizes the measure of active traders by the total measure of active traders in the OTC market, keeping in mind that $\omega$ is uniformly distributed in the banks' cross-section. Note that this is a fixed point equation, since $\Delta(\omega)$ and thus $\Sigma(\omega)$ implicitly depend on $n(\omega)$.

**Proposition 8** (Existence)**.** *There exists a continuous function, $n$, with $\int_0^1 n(\tilde{\omega})\,d\tilde{\omega} = 1$, solving the functional equation $T[n] = n$.*

Because our proof applies the Schauder fixed point Theorem, it establishes existence

but not uniqueness.[14]

Proposition 7 showed that a $U$-shaped and symmetric $n(\omega)$ maps into a $U$-shaped and symmetric $\Delta(\omega)$ and, therefore, into a hump-shaped and symmetric threshold, $\Sigma(\omega)$. Since the function $\Psi(x)$ is decreasing, it follows that $T[n](\omega)$ is $U$-shaped and symmetric. Therefore, the operator $T$ preserves the $U$-shaped and symmetric property. This immediately implies that:

**Corollary 9.** *If $D$ is normally distributed, there exists an equilibrium $n(\omega)$ that is $U$-shaped and symmetric.*

In the entry equilibrium of Corollary 9, small-sized banks do not enter the OTC market because their entry cost per capita is too large. Middle-sized banks enter, but only at the extremes of the $\omega$ spectrum, where $\Delta(\omega)$ is largest. In other words, middle-sized banks tend to be customers. Large banks enter at all points of the $\omega$ spectrum: at the extreme, as customers, and in the middle, as dealers. This pattern of participation thus implies that middle-$\omega$ dealer banks are, on average, larger than extreme-$\omega$ customer banks.

## 6.3 Empirical implications

In this section we study the implications of our model for a cross-section of banks sorted by size, and we study the economic forces determining the concentration of gross notional in the OTC market for CDS's.

### 6.3.1 Conditional moments

In section 3, we provided stylized facts about gross and net CDS notional in a cross-section sorted by trading assets, a natural empirical measure of bank size. To derive model counterparts of these facts, we calculate population moments conditional on size, for several variables of interest. Precisely, let $n(\omega \mid S)$ be the distribution of traders conditional on size,[15] and let $x(\omega)$ be some cross-sectional variable of interest (price, notional, etc...). The conditional moment of $x(\omega)$ is defined as:

$$\mathbb{E}_S\left[x(\omega)\right] \equiv \int_0^1 x(\omega) n(\omega \mid S)\, d\omega,$$

[14] A crucial step in applying the Schauder fixed point Theorem is to prove that the set of distributions generated by the operator $T[n]$ is equicontinuous. What delivers this property in our model is the observation, from Proposition 2, that the post-trade exposure function $g(\omega)$ is Liptchitz with a coefficient that does not depend on $n(\omega)$.

[15] Note that, since we're conditioning on a population of banks who have identical size, this distribution of traders must coincide with the distribution of establishments conditional on size.

the expectation of $x(\omega)$ with respect to the distribution of traders conditional on $S$. We obtain:

**Proposition 10** (Cross-sectional facts)**.** *In an entry equilibrium with a U-shaped and symmetric $n(\omega)$:*

- *conditional gross notional, $\mathbb{E}_S\big[G^+(\omega) + G^-(\omega)\big]$, is non-decreasing in $S$;*
- *conditional net notional, $\mathbb{E}_S\big[|G^+(\omega) - G^-(\omega)|\big]$, is non-increasing in $S$;*
- *conditional intermediation, $\mathbb{E}_S\big[\min\{G^+(\omega), G^-(\omega)\}\big]$, is non-decreasing in $S$;*
- *conditional price dispersion, $\mathbb{E}_S\big[R(\omega,\tilde{\omega})^2\big] - \mathbb{E}_S\big[R(\omega,\tilde{\omega})\big]^2$, is non-increasing in $S$.*

These results, illustrated in Figure 11, are the model counterparts of the stylized facts we document using US data from the OCC and bank holding companies' financial reports. Just as we normalize by trading assets in the data to control for the mechanical effect of size on various volume measures, the proposition focuses on "per capita" quantities. To build intuition, recall that middle-$\omega$ banks have the largest gross notional, the smallest net notional, and the largest intermediation volume. This is because, respectively, middle-$\omega$ banks lie in an increasing post-trade exposure region and thus use all of their trading capacity, they do not desire a change in their credit exposure since their pre-trade exposure is near the aggregate of one-half, and for the same reason they conduct a large amount of offsetting long and short trades in order to earn trading profits on CDS spreads. Because only large banks can recoup their entry cost by conducting a large enough volume of intermediation, in the entry equilibrium, these middle-$\omega$ banks are predominantly large. As a result, gross notional increases with size, net notional decreases with size, and intermediation volume increases with size.

The last result of the proposition is that price dispersion decreases with size. Because middle-sized banks tend to have extreme $\omega$'s, a large number of middle-sized matches involve either two low-$\omega$ banks, who trade at low prices, or two high-$\omega$ banks, who trade at a high prices. The prevalence of either low-price or high-price matches results in a large degree of price dispersion. On the other hand, there are more large-sized banks with middle $\omega$, and therefore more matches at average prices. This effect implies that price dispersion decreases with size.

It is intuitive large middle-$\omega$ banks, which act as dealer banks, should trade with each other at common prices since they all have similar outside options. Then, these dealer banks trade with typically smaller banks with more extreme $\omega$'s, i.e., end users or customers, at heterogeneous prices that depend on the customers' post trade positions. In

the earlier work of Duffie, Gârleanu, and Pedersen (2005) and Lagos and Rocheteau (2009), it is assumed that market makers or dealers trade in a frictionless market at common prices. In our model, such an inter-dealer market arises endogenously amongst large banks that are central to the market. One advantage of not assuming a frictionless inter-dealer market is that, empirically, Arora, Gandhi, and Longstaff (2012) show that there is substantial heterogeneity across dealers in CDS spreads offered to the same customer. This seems consistent with the fact that, as in our model, dealer characteristics are also key determinants of the spread.

### 6.3.2 Concentration

Taking stock of the above results, our model shows that the concentration of gross exposures in large banks is the result of several forces working in the same direction. First, on the extensive margin, small banks participate less in the OTC market than large banks: small-sized banks do not participate, middle-sized banks participate only if their per capita endowment, $\omega$, is extreme enough. Second, on the intensive margin, middle-sized banks sign less CDS contracts than larger banks. Some of this intensive margin effect is purely mechanical: middle-sized banks have less traders, so they sign proportionally less CDS contracts. But some of the effect arises because middle-sized banks tend to have extreme $\omega$'s, to lie in a flat spot of $g(\omega)$, and to thus find it optimal to use less than their full trading capacity, $k$. Large-sized banks tend to have average $\omega$, to lie in an increasing spot, and to thus use all their trading capacity. This is another channel driving notional concentration.

Figure 12 shows a heat map of bilateral gross notionals, per capita, across size percentiles. Variation along the diagonal of the heat map illustrates that gross notionals are larger amongst larger banks, as we already knew from Proposition 10. The off-diagonal vectors of the heat map offer new information: smaller banks trade more with larger banks than amongst each other. This further illustrates the manner in which, in spite of our assumption that all matching is random, large banks endogenously emerge as central counterparties in the OTC market.

# 7 Welfare

This section shows that, at the margin of the entry equilibrium, a planner can increase welfare by reducing the entry of middle-$\omega$ banks and increasing the entry of extreme-$\omega$ banks.

## 7.1 The welfare impact of marginal changes in entry

Consider any pattern of entry characterized by a threshold $\Sigma(\omega) > \underline{S}$ such that $\omega$-banks enter if and only if $S \geq \Sigma(\omega)$. Let $M$ denote the corresponding measure of traders in the OTC market, and let $n(\omega) > 0$ denote the corresponding distribution of traders. We now study the impact of changing the measure of traders entering at $\omega$ by $\varepsilon\delta(\omega)$, for some small $\varepsilon > 0$ and some continuous function $\delta(\omega)$. If $\delta(\omega) < 0$, then the measure of traders at $\omega$ decreases, and if $\delta(\omega) > 0$ it increases. The changes $\varepsilon\delta(\omega)$ are engineered by changing the entry size threshold from $\Sigma(\omega)$ to $\Sigma_\varepsilon(\omega)$ such that:

$$\Psi\left[\Sigma_\varepsilon(\omega)\right] = Mn(\omega) + \varepsilon\delta(\omega).$$

That is, the total measure of traders in banks of size greater than $\Sigma_\varepsilon(\omega)$ is equal to $Mn(\omega) + \varepsilon\delta(\omega)$.

Without loss of generality, we assume that CDS contracts are efficient conditional on entry, i.e., they solve the planning problem of Proposition 1. If the planner can transfer goods (but not assets) across banks at time zero, then $\delta(\omega)$ increases welfare if and only if it increases:

$$\begin{aligned} W(\varepsilon,\delta) \equiv &- \int_0^1 \big(1 - Mn(\omega) - \varepsilon\delta(\omega)\big)\Gamma\left[\omega\right]\, d\omega - \int_0^1 \big(Mn(\omega) + \varepsilon\delta(\omega)\big)\Gamma\left[g_\varepsilon(\omega)\right]\, d\omega \\ &- c\int_0^1 \Phi\left[\Sigma_\varepsilon(\omega)\right]\, d\omega, \end{aligned}$$

where $g_\varepsilon(\omega)$ is the post-trade exposure solving the planning problem conditional on the distribution of traders induced by $Mn(\omega) + \varepsilon\delta(\omega)$. The first term is the cost of risk bearing for traders who stay out of the OTC market, and the second term is the cost of risk bearing for traders who enter. The last term, on the second line, is the sum of all fixed entry costs incurred by bank establishments, with $\Phi\left[S\right] \equiv \int_S^\infty \varphi(x)\, dx$ denoting the measure of banks with sizes greater than $S$.

To study marginal changes in entry we evaluate the directional derivative $W'(0,\delta)$. Because post-trade exposures conditional on entry are efficient, an envelope theorem of Milgrom and Segal (2002) implies that the marginal impact of entry is found by differentiating $W(\varepsilon,\delta)$ holding the collection of CDS contracts, $\gamma(\omega,\tilde{\omega})$, constant.

**Proposition 11** (Directional Derivative)**.** *The derivative of* $W(\varepsilon,\delta)$ *at* $\varepsilon = 0$ *is:*

$$W'(0,\delta) = \int_0^1 \delta(\omega)\left( -\frac{c}{\Sigma(\omega)} + K(\omega) + F(\omega) - \frac{1}{2}\int_0^1 F(\tilde{\omega})n(\tilde{\omega})\, d\tilde{\omega}\right) d\omega, \qquad (17)$$

*where $K(\omega)$ and $F(\omega)$ are the competitive and frictional surpluses defined in Section 6.1.*

The intuition for each term is the following. First $\delta(\omega)$ creates additional entry costs: at $\omega$, the marginal bank is of size $\Sigma(\omega)$, so each new $\omega$-trader in this bank must pay a per-capita entry cost $\frac{c}{\Sigma(\omega)}$. Second, entering banks change the exposure of their traders, from $\omega$ to $g(\omega)$, and provide risk-sharing services to incumbents. These two effects combined are measured by $K(\omega) + F(\omega)$, the sum of the competitive surplus and of the frictional surplus. Finally, the last term arises because the entry of new banks creates congestion in the OTC market. Indeed, since the matching function has constant returns to scale, the total number of matches received by an incumbent stays equal to 1, regardless of the size of the market. Thus, mechanically, any match creation between incumbents and entrants results in a corresponding amount of match destruction amongst incumbents. In particular, and to a first order, there are $\varepsilon\delta(\omega)$ new matches between $\omega$-entrants and incumbents,[16] and therefore a corresponding destruction of matches amongst incumbents. The last term in equation (17) demonstrates that the associated surplus destruction is equal to half of the average frictional surplus. It is intuitive that this surplus destruction is proportional to the average frictional surplus because matches amongst incumbents are, by virtue of random matching, effectively destroyed at random. The multiplication by $\frac{1}{2}$ corrects for double counting. Indeed, in the average frictional surplus, $k \int_0^1 \int_0^1 \left|\Gamma'\left[g(\omega)\right] - \Gamma'\left[g(\tilde{\omega})\right]\right| n(\tilde{\omega}) n(\omega)\, d\tilde{\omega}\, d\omega$, the surplus of each incumbents' match is counted twice: once at $(\omega, \tilde{\omega})$ and once at $(\tilde{\omega}, \omega)$.

## 7.2 Improving upon the entry equilibrium

To see that the distribution of traders resulting from equilibrium entry is inefficient, recall that the free entry condition can be written:

$$\Delta(\omega) = K(\omega) + \frac{1}{2}F(\omega) = \frac{c}{\Sigma(\omega)}.$$

Plugging this back into (17), we obtain

$$W'(0,\delta) = \frac{1}{2}\int_0^1 \delta(\omega)\left(F(\omega) - \int_0^1 F(\tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega}\right) d\omega,$$

a formula with several implications. First, a necessary condition for first-order welfare improvement is that $F(\omega) \neq 0$ for some positive measure of $\omega$, i.e., some frictional surplus

[16] To a first order, the probability of matching with an entrant of type $\omega$ is $\varepsilon\delta(\omega)/M$, and the measure of incumbents is $M$. Multiplying the two, we find that the total number of new matches between type-$\omega$ entrants and incumbents is $\varepsilon\delta(\omega)$.

is still up for grabs in the OTC market. Second, if $\delta(\omega)$ is proportional to $n(\omega)$, then clearly $W'(0,\delta) = 0$. In words, the average frictional surplus in matches between entrants and incumbents exactly offsets the average frictional surplus destroyed in matches amongst incumbents. This means that, to a first order, there are no welfare gains from changing the size of the market while keeping its composition, $n(\omega)$, the same. In Appendix A.12, we show that this is, in fact, a global result: an entry equilibrium has a socially optimal size given $n(\omega)$. The intuition is that, when the composition of the market does not change, entry is equivalent to creating two separate markets with identical $n(\omega)$, one for the incumbents and one for the entrants. Therefore, entry has no externality on incumbents. Finally, one sees from the formula that $W'(0,\delta)$ increases when banks with larger-than-average frictional surplus enter, while banks with lower-than-average surplus exit. When $D$ is normally distributed, we obtain a sharper characterization:

**Proposition 12.** *Assume that $D$ is normally distributed, consider an entry equilibrium in which $n(\omega)$ is U-shaped and symmetric, and assume that $\Sigma(\omega) > \underline{S}$ for all $\omega$, and that $F(\omega) \neq 0$ for some positive measure of $\omega$. Then, welfare increases if middle-$\omega$ banks exit and extreme-$\omega$ banks enter. Formally, if $\int_0^1 \delta(\tilde{\omega})\, d\tilde{\omega} = 0$, then $W'(0,\delta) > 0$ when $\delta(\omega)$ is U-shaped and symmetric, and $W'(0,\delta) < 0$ when $\delta(\omega)$ is hump-shaped and symmetric.*

The assumption that $\int_0^1 \delta(\tilde{\omega})\, d\tilde{\omega} = 0$ is without loss of generality because we already know that changing size without changing the composition does not create a welfare improvement. The result then follows directly from Proposition 7 in which we showed that $F(\omega)$ is $U$-shaped and symmetric, i.e., middle-$\omega$ banks have lowest frictional surplus. Now recall that the marginal middle-$\omega$ bank is large-sized, while the marginal extreme-$\omega$ bank is middle-sized. Our result thus implies that the planner should subsidize the entry of small banks, in order to foster entry in the extremes of the $\omega$ spectrum. At the same time, the planner should discourage entry of large banks, in order to foster exit in the middle of the $\omega$ spectrum.

It is important to note that this result does not mean that middle-$\omega$ banks should not enter in the OTC market at all. In our model, any bank, and in particular a middle-$\omega$ bank, always has a positive social value before accounting for the entry cost. To see this, note that

**Lemma 2.** *For for any $n(\omega)$, the frictional surplus created by an entering bank of type $\omega$ is always larger than the congestions it imposes on incumbents:*

$$F(\omega) - \frac{1}{2}\int_0^1 F(\tilde{\omega})\, n(\tilde{\omega})\, d\tilde{\omega} \geq 0$$

*with a strict inequality if $F(\omega) \neq 0$ for a positive measure of $\omega$.*

The result follows by an application of the first triangle inequality. It implies that, even for a bank that creates no competitive surplus, $K(\omega) = 0$, the social value of entry is positive before entry costs. This means that, as long as there is not full risk sharing, it is socially optimal to have some large enough $\omega = \frac{1}{2}$ banks entering in the OTC market. In the entry equilibrium, however, these banks enter too much. Their large size means that even though they do not gain in utility from altering their exposure to the default risk factor, their scale allows them to defease the entry cost with a small intermediation profit on every trade they make. On the other hand, a smaller bank with a high pre-trade exposure would gain a lot in utility from insuring against default risk, but finds the entry cost too high given their small scale.

# 8 Conclusion

The OTC market for CDS's is very large relative to banks' trading assets, and gross notionals are highly concentrated on the balance sheets of just a few large dealer banks. Moreover, the varied bilateral trades executed by banks' many traders create an intricate system of liability linkages. In this paper we have sought to uncover the economic forces which determine this empirical trading structure in the OTC credit derivatives market. To this end, we have developed a model in which banks trade credit default swaps in an OTC market in order to share aggregate credit default risk. The equilibrium distribution of trades in our model has many realistic features. Gross notionals greatly exceed net notionals. The market is highly concentrated, and features increasing returns to scale. Small-sized banks do not participate, middle-sized banks arise as customers which trade at dispersed prices, whereas a small measure of large banks arise as key dealers and trade at near common prices. Finally, all banks are connected through the bilateral trades of participating banks' many traders. We argue that capturing these positive features gives credence to our model as a laboratory in which to study the normative features OTC derivatives markets, as well as the policy questions surrounding them.

We study the key normative features of our theoretical market setup, namely the size and concentration of the market. We find that the market is not too large in the sense that a planner could not improve welfare by adding or subtracting banks while leaving the composition of traders' pre-trade risk endowment the same. On the other hand, we also show that in our OTC market, even if frictions decrease enough to enable full risk sharing, volume and gross notional balances still exceed their Walrasian counterparts. In this sense, the OTC market is too large, however without trading costs excess volume does

not reduce welfare. We also consider whether the model OTC market is too concentrated. Concentration arises in our model market due to optimal bank entry with fixed costs of participation. We find that the market is indeed too concentrated, in the sense that a planner would want to remove some large dealer banks and replace them with smaller customer banks. However, we also show that in our model, as frictions decrease and the risk allocation improves, not only does volume increase, but concentration increases as well. Thus, drawing normative conclusions about market concentration involves substantial subtlety.

Finally, we argue that it is important to understand how the characteristic market features arise in order to answer the current regulatory questions surrounding OTC derivatives markets. For example, the proposed "Volcker rule" aims to restrict derivatives trading which is not directly tied to underlying exposures. This may reduce risk taking by banks. However, one potential side effect is a decline in market intermediation and liquidity. Another proposed provision in the Dodd Frank Act aims to restrict exposure to any one counterparty. Are such limits welfare improving? If so, how should each bilateral limit be determined? Current bankruptcy law and capital requirement regulation seem to favor banks with large offsetting long and short positions. Moreover, policies such as too-big-to-fail and FDIC insurance may provide forces for concentrating derivatives trading in explicitly or implicitly insured institutions. How much of the observed concentration of gross CDS notionals is due to traditional considerations of economies of scale that we study, and how much is driven by regulation favoring large dealer banks? In future work, we plan to address questions such as these using the framework we develop in this paper.

**Figure 1:** Increasing Returns to Scale in CDS Markets

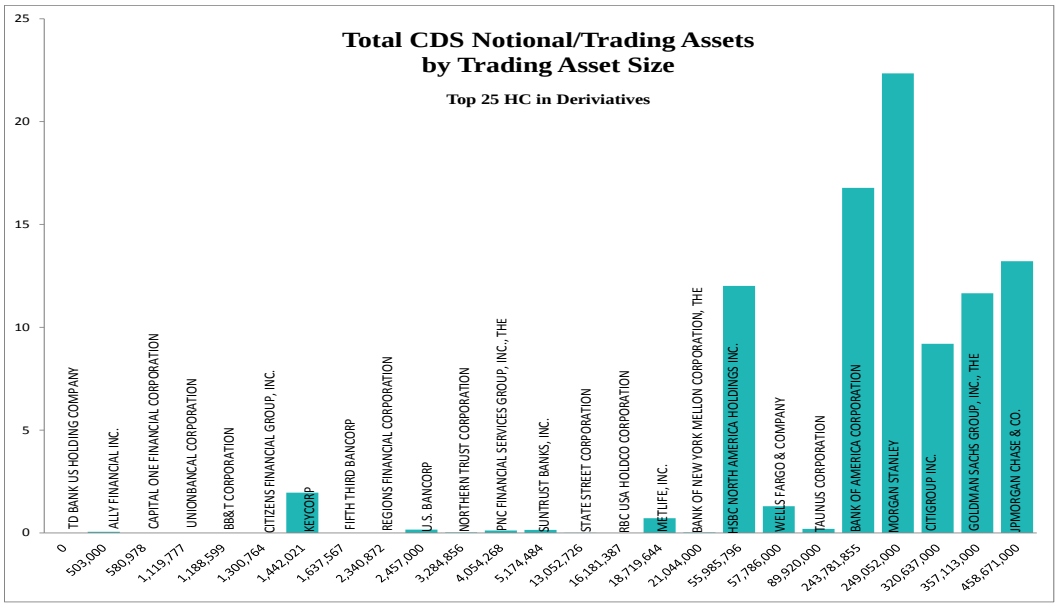


Figure 1 plots gross notional to trading assets by trading assets by trading assets for the top 25 bank holding companies in derivatives according to the OCC quarterly report on bank trading and derivatives activities third quarter 2011. Data are from bank holding companies' FR Y-9C filing from Q3 2011. Trading assets in thousands.

**Figure 2:** CDS Market Concentration

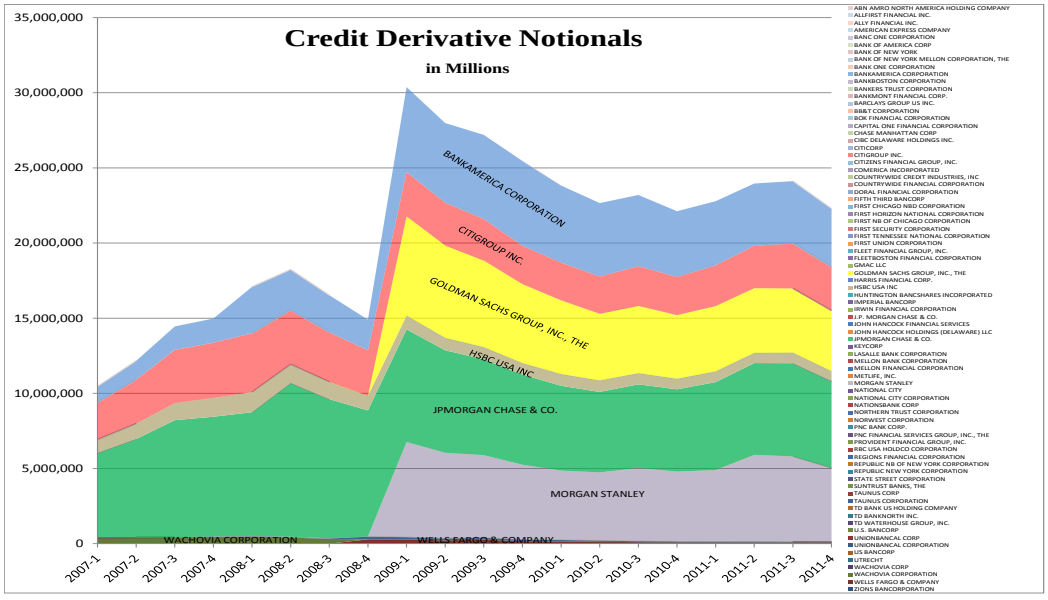


Figure 2 plots gross notionals from 2007-2011 for banks that are among the top 25 bank holding companies in OTC derivatives during that time period. Data are from the OCC quarterly report on bank trading and derivatives activities third quarter 2011. Derivatives notionals in millions.

**Figure 3:** CDS Net to Gross Notional

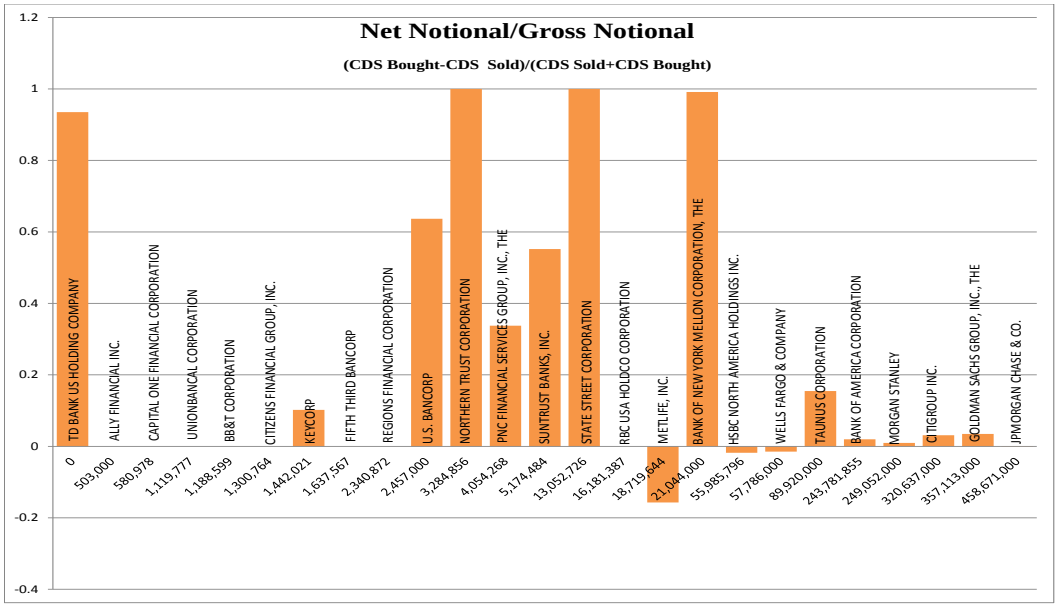


Figure 3 plots net to gross notional for the top 25 bank holding companies in derivatives according to the OCC quarterly report on bank trading and derivatives activities third quarter 2011. Data are from bank holding companies' FR Y-9C filing from Q3 2011. Trading assets in thousands. Empty bars denote zero gross CDS notional.

**Figure 4:** Fraction of purchased credit derivatives recorded as guarantee

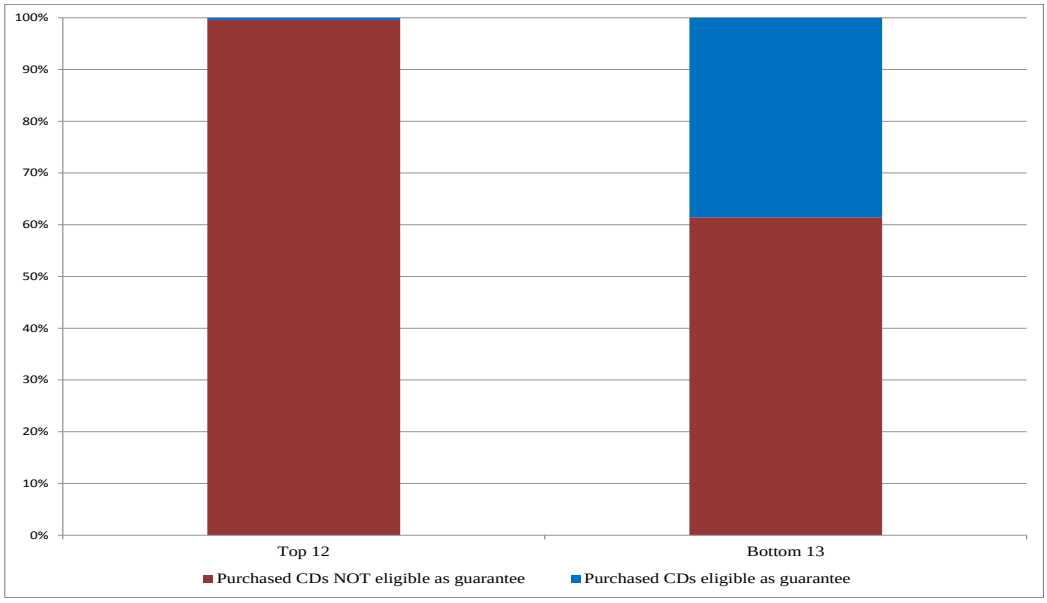


Figure 4 plots the fraction of purchased credit derivatives from Q2 2009 to Q4 2011 that could be counted as a guarantee for regulatory purposes for the larger vs. the smaller top 25 bank holding companies in derivatives. Data are from bank holding companies' FR Y-9C filings. Size is measured by trading assets.

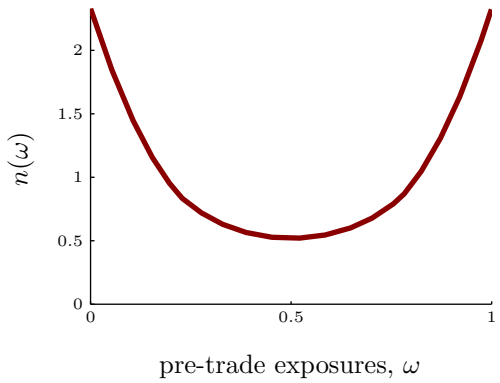


**Figure 5:** A $U$-shaped and symmetric distribution.

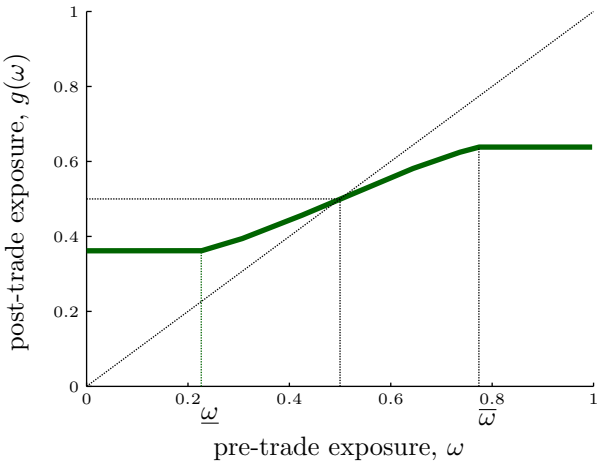


**Figure 6:** The post-trade exposure function when the traders' distribution is $U$-shaped and symmetric.

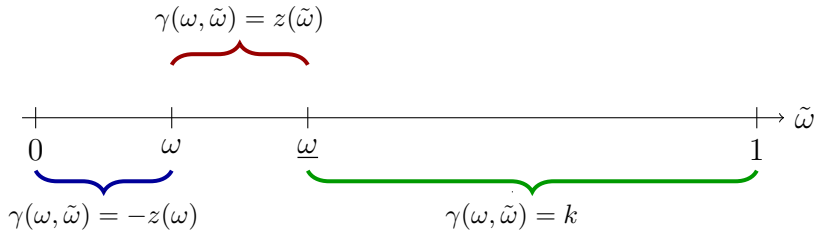


**Figure 7:** Portfolio of CDS contracts of a bank with pre-trade exposure $\omega \in [0, \frac{1}{2}]$.

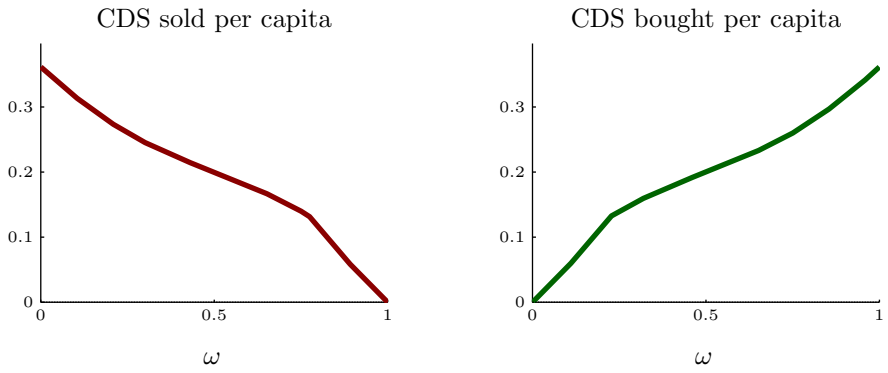


**Figure 8:** The number of contract sold, $G^+(\omega)$, and purchased, $G^-(\omega)$.

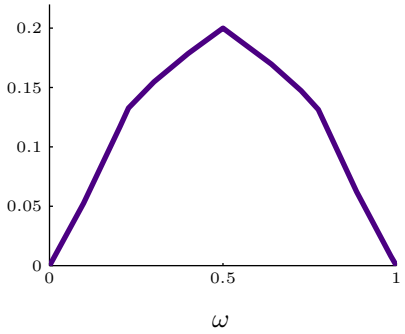


**Figure 9:** The volume of fully offsetting CDS contracts, $\min\{G^+(\omega), G^-(\omega)\}$.

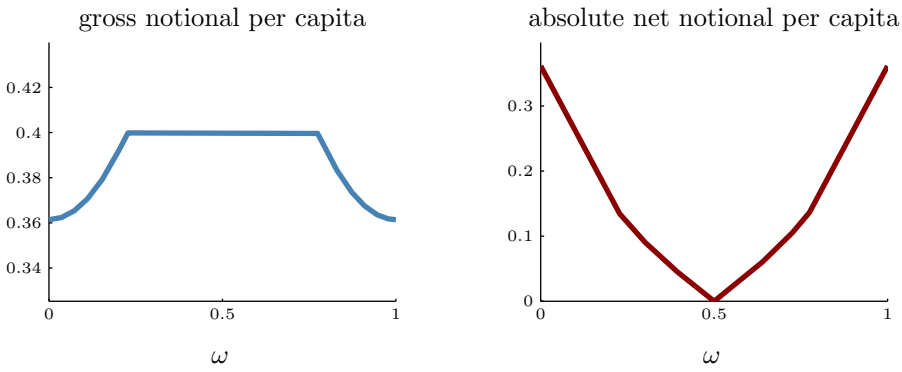


**Figure 10:** Gross and net notional.

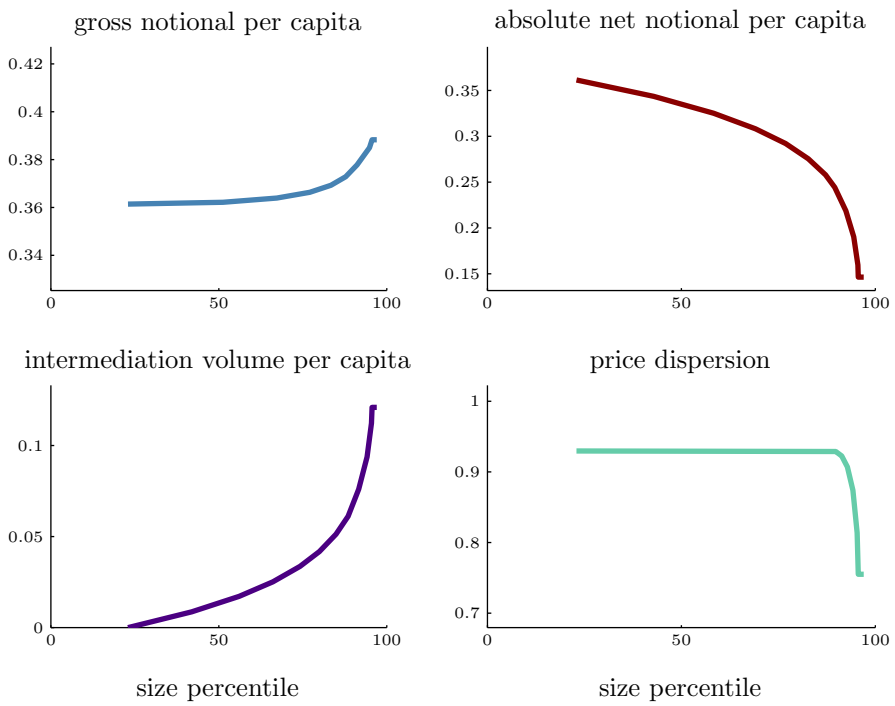


**Figure 11:** Conditional moments. On all panels, the $x$-axis is the size percentile of a bank in the overall cross-section. The dotted vertical line indicates the entry threshold, $\Sigma(0)$.

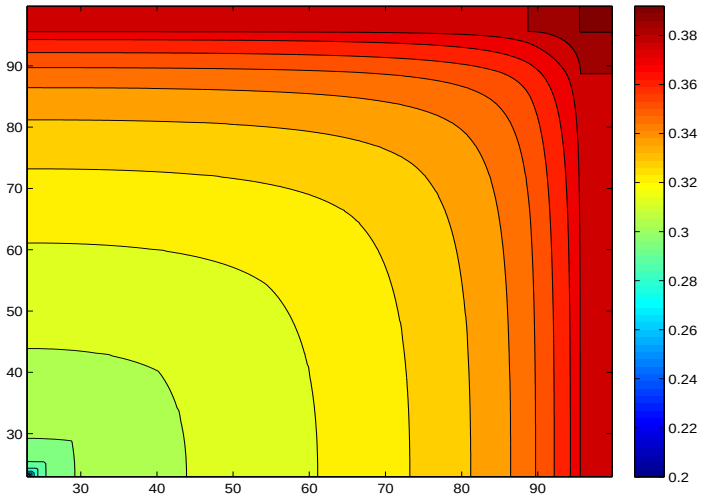


**Figure 12:** Bilateral gross notionals between size percentiles, per-capita.

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# A Proofs

## A.1 Proof of Lemma 1

The first derivative of $\Gamma(g)$ is:

$$\Gamma'[g] = \frac{\mathbb{E}\left[De^{\alpha g D}\right]}{\mathbb{E}\left[e^{\alpha g D}\right]},$$

which is evidently positive since $D$ is positive. The second derivative is:

$$\Gamma''[g] = \alpha \frac{\mathbb{E}\left[D^2 e^{\alpha g D}\right]\mathbb{E}\left[e^{\alpha g D}\right] - \mathbb{E}\left[De^{\alpha g D}\right]^2}{\mathbb{E}\left[e^{\alpha g D}\right]^2}.$$

The numerator is positive since, by the Cauchy-Schwarz inequality, we have

$$\mathbb{E}\left[De^{\alpha g D}\right]^2 = \mathbb{E}\left[De^{\alpha g/2 D}e^{\alpha g/2}x\right]^2 < \mathbb{E}\left[D^2 e^{\alpha g D}\right]\mathbb{E}\left[e^{\alpha g D}\right].$$

The denominator is, evidently, positive as well. We thus obtain that $\Gamma''(g) > 0$ as claimed. □

## A.2 Proof of Proposition 1

The planner's objective is convex, and it constraint set is convex and bounded. Given that $\gamma(\omega, \tilde{\omega})$ is measurable and bounded by $k$, it belongs to the Hilbert space of square integrable functions. Existence then follows from an application of Proposition 1.2, Chapter II in Eckland and Téman (1987). That all solutions of the planning problem share the same post-trade exposure almost everywhere follows from the strict convexity of the function $\Gamma[g]$.

To show that any solution of the planning problem is the basis of an equilibrium, consider the following variational experiment. Starting from a solution, $\gamma(\omega, \tilde{\omega})$ and $g(\omega)$, consider the alternative feasible allocation $\gamma(\omega, \tilde{\omega}) + \varepsilon\Delta(\omega, \tilde{\omega})$ for some small $\varepsilon > 0$ and some bounded $\Delta(\omega, \tilde{\omega})$ such that

$$\Delta(\omega, \tilde{\omega}) + \Delta(\tilde{\omega}, \omega) = 0 \tag{18}$$
$$\gamma(\omega, \tilde{\omega}) = k \Rightarrow \Delta(\omega, \tilde{\omega}) \leq 0 \tag{19}$$
$$\gamma(\omega, \tilde{\omega}) = -k \Rightarrow \Delta(\omega, \tilde{\omega}) \geq 0. \tag{20}$$

The corresponding post-trade exposure is $g(\omega) + \varepsilon \int_0^1 \Delta(\omega, \tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega}$. The change in the objective is, up to second-order terms:

$$\begin{aligned}
\delta J =& \varepsilon \int_0^1 n(\omega)\Gamma'[g(\omega)] \int_0^1 \Delta(\omega, \tilde{\omega})\, n(\tilde{\omega})\, d\tilde{\omega}\, d\omega \\
=& \frac{\varepsilon}{2}\int_0^1 \int_0^1 \Gamma'[g(\omega)]\, \Delta(\omega, \tilde{\omega}) n(\tilde{\omega}) n(\omega)\, d\tilde{\omega}\, d\omega + \frac{\varepsilon}{2}\int_0^1 \int_0^1 \Gamma'[g(\omega)]\, \Delta(\omega, \tilde{\omega}) n(\tilde{\omega}) n(\omega) d\tilde{\omega}\, d\omega \\
=& \frac{\varepsilon}{2}\int_0^1 \int_0^1 \Gamma'[g(\omega)]\, \Delta(\omega, \tilde{\omega}) n(\tilde{\omega}) n(\omega)\, d\tilde{\omega}\, d\omega + \frac{\varepsilon}{2}\int_0^1 \int_0^1 \Gamma'[g(\tilde{\omega})]\, \Delta(\tilde{\omega}, \omega) n(\omega) n(\tilde{\omega}) d\omega\, d\tilde{\omega}
\end{aligned}$$

$$
\begin{aligned}
=&\frac{\varepsilon}{2}\int_0^1\int_0^1 \Gamma'\left[g(\omega)\right]\Delta(\omega,\tilde{\omega})n(\tilde{\omega})n(\omega)\,d\tilde{\omega}\,d\omega - \frac{\varepsilon}{2}\int_0^1\int_0^1 \Gamma'\left[g(\tilde{\omega})\right]\Delta(\omega,\tilde{\omega})n(\omega)n(\tilde{\omega})d\omega\,d\tilde{\omega}\\
=&\frac{\varepsilon}{2}\int_0^1\int_0^1 \Big(\Gamma'\left[g(\omega)\right]-\Gamma'\left[g(\tilde{\omega})\right]\Big)\Delta(\omega,\tilde{\omega})n(\omega)n(\tilde{\omega})\,d\tilde{\omega}\,d\omega.
\end{aligned}
$$

where: the first equality follows trivially; the second equality from relabeling $\omega$ by $\tilde{\omega}$ and vice versa; the third equality from the fact that $\Delta(\omega,\tilde{\omega})+\Delta(\tilde{\omega},\omega)=0$; the fourth equality by collecting terms. Since we started from a solution to the planning problem, it must be that $\delta J \geq 0$. For this inequality to hold for any $\Delta(\omega,\tilde{\omega})$ satisfying (18)-(20), it must be the case that $\gamma(\omega,\tilde{\omega})$ and $g(\omega)$ satisfy (11) almost everywhere. Clearly, $\gamma(\omega,\tilde{\omega})$ is basis of an equilibrium, after perhaps modifying it on a measure zero set so that it satisfies (11) everywhere. For brevity we omit the precise construction of such a modification. The details are available from the authors upon request.

Conversely, consider any equilibrium $\gamma(\omega,\tilde{\omega})$ and $g(\omega)$. Given that $\Gamma[g]$ is convex, the difference between the planner's cost for the equilibrium allocation and the planner's cost for any other allocation $\hat{\gamma}(\omega,\tilde{\omega})$ and $\hat{g}(\omega)$ is smaller than:

$$
J-\hat{J}\leq\int_0^1 n(\omega)\Gamma'\left[g(\omega)\right]\int_0^1\Big(\gamma(\omega,\tilde{\omega})-\hat{\gamma}(\omega,\tilde{\omega})\Big)n(\tilde{\omega})d\tilde{\omega}d\omega.
$$

Note that, given $\gamma(\omega,\tilde{\omega})+\gamma(\tilde{\omega},\omega)=0$, we have using the same manipulations as before:

$$
\int_0^1\int_0^1 \Gamma'\left[g(\omega)\right]\gamma(\omega,\tilde{\omega})n(\tilde{\omega})n(\omega)d\tilde{\omega}\,d\omega=\frac{1}{2}\int_0^1\int_0^1\Big(\Gamma'\left[g(\omega)\right]-\Gamma'\left[g(\tilde{\omega})\right]\Big)\gamma(\omega,\tilde{\omega})n(\omega)n(\tilde{\omega})d\omega\,d\tilde{\omega}.
$$

Plugging this back into the expression for $J-\hat{J}$, we obtain:

$$
J-\hat{J}\leq\int_0^1\int_\omega^1\Big(\Gamma'\left[g(\omega)\right]-\Gamma'\left[g(\tilde{\omega})\right]\Big)\Big(\gamma(\omega,\tilde{\omega})-\hat{\gamma}(\omega,\tilde{\omega})\Big)n(\tilde{\omega})n(\omega)d\tilde{\omega}d\omega.
$$

Because of (11) the integrand is negative, implying that $J\leq\hat{J}$ and establishing the claim that $\gamma(\omega,\tilde{\omega})$ solves the planner's problem. □

## A.3 Proof of Proposition 2

To prove that $g(\omega)$ is non-decreasing, suppose, constructing a contradiction, that there are $\omega\leq\tilde{\omega}$ such that $g(\tilde{\omega})<g(\omega)$. Then, by (11), it must be the case that for any counterparty $x$, $\gamma(\omega,x)\leq\gamma(\tilde{\omega},x)$, i.e., bank $\tilde{\omega}$ sells more insurance than bank $\omega$. But since bank $\tilde{\omega}$ starts with weakly larger exposure, $\omega\leq\tilde{\omega}$, this implies that $g(\tilde{\omega})\geq g(\omega)$, which would be a contradiction.

To prove that post-trade exposures are closer together than pre-trade exposures, consider again two banks $\omega\leq\tilde{\omega}$ and bear in mind that we have just shown that $g(\omega)\leq g(\tilde{\omega})$. If $g(\omega)=g(\tilde{\omega})$, then the result is trivially true. Otherwise if $g(\omega)<g(\tilde{\omega})$, then it must be the case that, for any counterparty $x$, bank $\tilde{\omega}$ sells less insurance than bank $\omega$, i.e. $\gamma(\tilde{\omega},x)\leq\gamma(\omega,x)$. Therefore $\int_0^1\gamma(\tilde{\omega},x)n(x)\,dx\leq\int_0^1\gamma(\omega,x)n(x)\,dx$ which is, by definition of $g(\omega)$, equivalent to $g(\tilde{\omega})-\tilde{\omega}\leq g(\omega)-\omega$. □

## A.4 Proof of Proposition 3

All $\omega$-traders in $[\underline{\omega}, \overline{\omega}]$ sell $k$ CDS contracts to any trader $\tilde{\omega} > \overline{\omega}$ and buy $k$ CDS contracts from any trader $\tilde{\omega} < \underline{\omega}$. With traders $\tilde{\omega} \in [\underline{\omega}, \overline{\omega}]$, the number of CDS contracts bought and sold is indeterminate. For any $\omega \in [\underline{\omega}, \overline{\omega}]$ we thus have:

$$g(\underline{\omega}) = g(\overline{\omega}) = g(\omega) = \omega - kN(\underline{\omega}) + \int_{\underline{\omega}}^{\overline{\omega}} \gamma(\omega, \tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega} + k\left[1 - N(\overline{\omega})\right].$$

Now multiply by $n(\omega)$ and integrate from $\underline{\omega}$ to $\overline{\omega}$ and note that, by (3) it must be the case that:

$$\int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\omega}}^{\overline{\omega}} \gamma(\omega, \tilde{\omega}) n(\omega) n(\tilde{\omega}) d\omega d\tilde{\omega} = 0,$$

i.e. the net trade across all matches $(\omega, \tilde{\omega}) \in [\underline{\omega}, \overline{\omega}]^2$ must be equal to zero. Collecting terms we obtain expression (16). □

## A.5 Proof of Proposition 5 and 6

### A.5.1 Two preliminary results

First we establish that, when $n(\omega)$ is symmetric around $\frac{1}{2}$, the equilibrium is symmetric as well:

**Lemma 3.** *Suppose that the distribution of traders satisfies $n(\omega) = n(1-\omega)$. Then equilibrium post-trade exposures are symmetric, i.e. they satisfy $g(1-\omega) = 1 - g(\omega)$, and can be implemented by a symmetric collection of CDS contract, i.e. such that $\gamma(\omega, \tilde{\omega}) = -\gamma(1-\omega, 1-\tilde{\omega})$.*

To see this consider some equilibrium collection of CDS contract, $\gamma(\omega, \tilde{\omega})$ and its associated post-trade exposures, $g(\omega)$. Now, the alternative collection of CDS $\hat{\gamma}(\omega, \tilde{\omega}) = -\gamma(1-\omega, 1-\tilde{\omega})$ is feasible and generates post-trade exposures:

$$\begin{aligned}\hat{g}(\omega) &= \omega - \int_0^1 \gamma(1-\omega, 1-\tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega} = \omega - \int_0^1 \gamma(1-\omega, 1-\tilde{\omega}) n(1-\tilde{\omega})\, d\tilde{\omega} \\ &= 1 - \left(1 - \omega + \int_0^1 \gamma(1-\omega, \hat{\omega}) n(\hat{\omega})\, d\hat{\omega}\right) = 1 - g(1-\omega),\end{aligned}$$

where the first equality holds by definition of $\hat{\gamma}(\omega, \tilde{\omega})$, the second equality because $n(\tilde{\omega}) = n(1-\tilde{\omega})$, and the third equality by change of variables $\hat{\omega} = 1 - \tilde{\omega}$. Now it is easy to see that $\hat{\gamma}(\omega, \tilde{\omega})$ satisfies (11). Indeed, $\hat{g}(\omega) < \hat{g}(\tilde{\omega})$ is equivalent to $g(1-\tilde{\omega}) < g(1-\omega)$, which implies from (11) that $\gamma(1-\tilde{\omega}, 1-\omega) = -\gamma(1-\omega, 1-\tilde{\omega}) = \hat{\gamma}(\omega, \tilde{\omega}) = k$. Since equilibrium post-trade exposures are uniquely determined, we conclude from this that $\hat{g}(\omega) = 1 - g(1-\omega) = g(\omega)$.

To see that $g(\omega)$ can be implemented using a symmetric collection of CDS contract, consider $\gamma^*(\omega, \tilde{\omega}) = \frac{1}{2}\big(\gamma(\omega, \tilde{\omega}) + \hat{\gamma}(\omega, \tilde{\omega})\big)$, for the same $\hat{\gamma}(\omega, \tilde{\omega})$ defined above. By construction, we have that $\gamma^*(1-\omega, 1-\tilde{\omega}) = -\gamma^*(\omega, \tilde{\omega})$, and $g^*(\omega) = \frac{1}{2}\big(g(\omega) + \hat{g}(\omega)\big) = g(\omega)$, given that we have just shown that $g(\omega)$ is symmetric. □

The second preliminary result concerns the function $H(\omega)$:

**Lemma 4.** *Let, for $\omega \in [0, \frac{1}{2}]$, $H(\omega) \equiv \int_0^\omega (\omega - \tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega} - kN(\omega)^2$.*

- *if $k \leq \frac{1}{2}\left[n\left(0\right)\right]^{-1}$, then $H(\omega) \geq 0$.*

- *if* $\frac{1}{2}[n(0)]^{-1} < k < 1-2\mathbb{E}\left[\omega \,|\, \omega \leq \frac{1}{2}\right]$, *then there is a unique* $\underline{\omega} \in (0, \frac{1}{2})$ *such that* $H(\omega) < 0$ *for* $\omega \in (0, \underline{\omega})$ *and* $H(\omega) > 0$ *for* $\omega \in (\underline{\omega}, \frac{1}{2})$.
- *if* $k \geq 1 - 2\mathbb{E}\left[\omega \,|\, \omega \leq \frac{1}{2}\right]$, *then* $H(\omega) \leq 0$.

To prove this result note that $H'(\omega) = N(\omega)[1 - 2kn(\omega)]$ and keep in mind that $n(\omega)$ is non-increasing over $[0, \frac{1}{2}]$. If $2kn(0) \leq 1$, then $H'(\omega) \geq 0$ over $[0, \frac{1}{2}]$. Given that $H(0) = 0$, this establishes the first point of the Lemma. If $2kn(0) > 1$, then it follows that $H(\omega)$ is initially decreasing and may subsequently become increasing: therefore, the equation $H(\omega) = 0$ has at most one solution in $(0, \frac{1}{2}]$. Such a solution exists if and only if:

$$\begin{aligned} H(\tfrac{1}{2}) \geq 0 &\iff \tfrac{1}{2}N\left(\tfrac{1}{2}\right) - \int_0^{\frac{1}{2}} \tilde{\omega} n(\tilde{\omega})\, d\tilde{\omega} - kN\left(\tfrac{1}{2}\right)^2 \geq 0 \\ &\iff k \leq 1 - 2\mathbb{E}\left[\omega \,|\, \omega \leq \tfrac{1}{2}\right], \end{aligned}$$

since $n(\omega)$ symmetric implies $N(\frac{1}{2}) = \frac{1}{2}$.

Lastly, we verify that $1 - 2\mathbb{E}\left[\omega \,|\, \omega \leq \frac{1}{2}\right] \geq \frac{1}{2}[n(0)]^{-1}$, with an equality if and only if $n(\omega)$ is uniform. This follows from two implications of the fact that $n(\omega)$ is non-increasing over $[0, \frac{1}{2}]$. First,

$$N(\tfrac{1}{2}) = \tfrac{1}{2} = \int_0^{\frac{1}{2}} n(\omega)\, d\omega \leq \tfrac{n(0)}{2} \Longrightarrow n(0) \geq 1.$$

Second, a uniform distribution over $[0, \frac{1}{2}]$ first-order stochastically dominates $n(\omega \,|\, \omega \leq \frac{1}{2})$, implying that:

$$\mathbb{E}\left[\omega \,|\, \omega \leq \tfrac{1}{2}\right] \leq \tfrac{1}{2} \iff 1 - \mathbb{E}\left[\omega \,|\, \omega \leq \tfrac{1}{2}\right] \geq \tfrac{1}{2},$$

with a strict inequality if and only if $n(\omega \,|\, \omega \leq \frac{1}{2})$, and thus $n(\omega)$, is uniform. Combining the two we obtain the desired inequality. □

It follows directly from the Lemma that:

**Corollary 13.** *For* $\omega \in (0, \frac{1}{2})$, $z(\omega) \leq k$ *if and only if* $H(\omega) \leq 0$.

### A.5.2 Proof of the two propositions

Given Lemma 3 it is enough to focus on $\omega \in [0, \frac{1}{2}]$.

**When there is no flat spot.** Consider first the case when $k \leq \frac{1}{2}[n(0)]^{-1}$. Then, by Lemma 4, it follows that $z(\omega) \geq k$ for all $\omega \in (0, \frac{1}{2}]$. Therefore, the candidate CDS contracts of Proposition 6 are such that $\gamma(\omega, \tilde{\omega}) = k$ if $\omega < \tilde{\omega}$. The corresponding post trade exposure is thus equal to $g(\omega) = \omega + k[1 - 2N(\omega)]$. Note that, since $n(\omega)$ is $U$ shaped and symmetric, the maximum of $n(\omega)$ is $n(0)$ and so $2kn(\omega) \leq 1$ for all $\omega$. Therefore the function $g(\omega)$ is non-decreasing over $[0, 1]$. This establishes that the CDS contracts of Proposition 6 are indeed the basis of an equilibrium.

**When there are flat and increasing spots.** Now assume that $[n(0)]^{-1} < k < 1 - 2\mathbb{E}\left[\omega \,|\, \omega \leq \frac{1}{2}\right]$ and consider some $\omega \in [0, \underline{\omega}]$. By Lemma 4, we have that $\gamma(\omega, \tilde{\omega}) = -z(\omega)$ for

$\tilde{\omega} < \omega$, $\gamma(\omega, \tilde{\omega}) = z(\tilde{\omega})$ for $\tilde{\omega} \in (\omega, \underline{\omega})$, and $\gamma(\omega, \tilde{\omega}) = k$ for $\tilde{\omega} \geq \underline{\omega}$. Therefore, the corresponding post-trade exposure is

$$g(\omega) = \omega - \int_0^{\omega} z(\omega)\, n(\tilde{\omega})\, d\tilde{\omega} + \int_{\omega}^{\underline{\omega}} z(\tilde{\omega})\, n(\tilde{\omega})\, d\tilde{\omega} + k\,[1 - N(\underline{\omega})]\,.$$

Differentiating with respect to $\omega$ one directly verifies that $g'(\omega) = 0$ so that $g(\omega)$ is flat over $[0, \frac{1}{2}]$. Next consider $\omega \in (\underline{\omega}, \frac{1}{2}]$. By Lemma 4, we have that $\gamma(\omega, \tilde{\omega}) = -k$ for $\tilde{\omega} < \omega$, and $\gamma(\omega, \tilde{\omega}) = k$ for $\tilde{\omega} > \omega$. Therefore the corresponding post-trade exposure is $g(\omega) = \omega + k\,[1 - 2N(\omega)]$. Differentiating we obtain that $g'(\omega) = 1 - 2kn(\omega)$, which is positive since, by Lemma 4, it must be the case that $H'(\omega) > 0$ for $\omega \in (\underline{\omega}, \frac{1}{2}]$. Taken together, this implies that the CDS contracts of Proposition 6 are indeed the basis of an equilibrium.

**When there is full risk sharing.** Lastly assume that $k \geq 1 - 2\mathbb{E}\left[\omega \,|\, \omega \leq \frac{1}{2}\right]$. Then $z(\omega) \leq k$ for all $\omega \in [0, \frac{1}{2}]$. For all $\omega \in [0, \frac{1}{2}]$, $g(\omega)$ becomes

$$g(\omega) = \omega - \int_0^{\omega} z(\omega)\, n(\tilde{\omega})\, d\tilde{\omega} + \int_{\omega}^{\frac{1}{2}} z(\tilde{\omega})\, n(\tilde{\omega})\, d\tilde{\omega} + z(\tfrac{1}{2})\left[1 - N(\tfrac{1}{2})\right],$$

i.e., the same as above with $\underline{\omega}$ replaced by $\frac{1}{2}$ and $k$ replaced by $z(\frac{1}{2})$. Taking derivatives show that $g'(\omega) = 0$ and so $g(\omega) = g(\frac{1}{2})$. Now given that $N(\frac{1}{2}) = \frac{1}{2}$, it follows that $g(\frac{1}{2}) = \frac{1}{2}$ and so the CDS contracts of Proposition 6 are the basis of an equilibrium. □

## A.6 Proof of the results discussed in Section 5.3.5

The result that $\underline{\omega}$ increases with $k$ follows immediately because $\underline{\omega}$ solves the implicit equation $H(\underline{\omega}) = 0$ and because, at $\underline{\omega}$, the function $H(\omega)$ is increasing in $\omega$ and decreasing in $k$. The level of the flat spot goes up with $\underline{\omega}$ because $g(\omega)$ is increasing in $[\underline{\omega}, 1 - \underline{\omega}]$.

To study the derivatives of gross notional with respect to $k$, note that the gross notional of a bank in the increasing spot is equal to $k$, since traders go to corner in all meetings. For a bank in the flat spot, gross notional is equal to:

$$z(\omega)N(\omega) + \int_{\omega}^{\underline{\omega}} z(\tilde{\omega})n(\tilde{\omega})\, d\tilde{\omega} + k\big[1 - N(\underline{\omega})\big].$$

Taking derivatives with respect to $k$ we obtain:

$$\frac{\partial \underline{\omega}}{\partial k} z(\underline{\omega})n(\underline{\omega}) + \big[1 - N(\underline{\omega})\big] - kn(\underline{\omega})\frac{\partial \underline{\omega}}{\partial k}$$

and the result follows because $z(\underline{\omega}) = k$ by construction.

Finally, let us turn to the trading volume. Trivially, when $k \to 0$, volume must also goes to zero. Now we show that, when $k$ is greater than $1 - 2\mathbb{E}\left[\omega \,|\, \omega \leq \frac{1}{2}\right]$ but less than one, then the volume must be greater than its Walrasian counterpart. Suppose towards a contradiction that volume is exactly equal to the Walrasian counterpart. That is, for any $\omega$, the gross exposure is equal to the absolute net exposure, $|\frac{1}{2} - \omega|$. Since $g(\omega) = \frac{1}{2} = \omega + \int_0^1 \gamma(\omega, \tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega}$, this implies that for $\omega < \frac{1}{2}$, we must have $\gamma(\omega, \tilde{\omega}) \geq 0$ and for $\omega > \frac{1}{2}$, $\gamma(\omega, \tilde{\omega}) \leq 0$. In other words, the only way there is no excess trading volume is that any given bank $\omega$ only sign CDS contracts in one direction. But this also means that there cannot be trades amongst $(\omega, \tilde{\omega}) \in [0, \frac{1}{2})^2$, and

amongst $(\omega, \tilde{\omega}) \in (\frac{1}{2}, 1]^2$. Therefore, for a $\omega < \frac{1}{2}$ bank:

$$\tfrac{1}{2} - \omega = \int_{\frac{1}{2}}^{1} \gamma(\omega, \tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega} \leq \frac{k}{2}.$$

But this inequality can't hold if $\omega \simeq 0$ and $k$ is less than one, a contradiction.

Finally we show how to find CDS contracts whose volume is arbitrarily close to the Walrasian volume, for $k$ large enough. The construction is based on the following intuition: in the absence of any trading limits, the Walrasian allocation obtains by trading the quantity $\frac{1}{2} - \omega$ of CDS's with the symmetric bank, $1 - \omega$, and nothing with banks $\tilde{\omega} \neq 1 - \omega$. When trading capacity is large, then approximate Walrasian volume obtains by trading with a small measure of banks surrounding the symmetric $1 - \omega$ bank. Formally let us assume for this argument that $n(\omega) > 0$ eveywhere so that it is bounded below by $\underline{n} > 0$. Given some integer $I$ consider the sequence $\omega_i = \frac{1}{2} + \frac{i}{2I}$, for $i \in \{1, \ldots, I\}$ and the symmetric sequence $\omega_{-i} = 1 - \omega_i$. Let $\Omega_i \equiv [\omega_{i-1}, \omega_i]$ and its symmetric $\Omega_{-i} \equiv [\omega_{-i}, \omega_{-i+1}]$. The mean endowment in $\Omega_i$ is $\omega_i^\star = \mathbb{E}\big[\omega \,|\, \omega \in \Omega_i\big]$, and by symmetry the mean endowment in $\Omega_{-i}$ is $\omega_{-i}^\star = 1 - \omega_i^\star$. Now consider the collection of CDS contracts $\gamma_A(\omega, \tilde{\omega}) + \gamma_B(\omega, \tilde{\omega})$, defined as follows. For $\omega \in \Omega_i$ and $\tilde{\omega} \in \Omega_{-i}$:

$$\gamma_A(\omega, \tilde{\omega}) = \frac{\frac{1}{2} - \omega_i^\star}{\big(N(\omega_i) - N(\omega_{i-1})\big)},$$

and for $\omega \in \Omega_i$ and $\tilde{\omega} \notin \Omega_{-i}$, $\gamma_A(\omega, \tilde{\omega}) = 0$. For $\omega \in \Omega_i$ and $\tilde{\omega} \in \Omega_j$,

$$\gamma_B(\omega, \tilde{\omega}) = \big(\omega_j^\star - \tilde{\omega}\big) - \big(\omega_i^\star - \omega\big).$$

The contracts $\gamma_A(\omega, \tilde{\omega})$ prescribe trades amongst $(\omega, \tilde{\omega})$ belonging to symmetric intervals $\Omega_i$ and $\Omega_{-i}$. They are designed to bring a $\omega$-trader's exposure close to $\frac{1}{2}$. Namely after conducting all the trades in $\gamma_A(\omega, \tilde{\omega})$ the exposure of a $\omega$-trader is $\frac{1}{2} - (\omega_i^\star - \omega)$. When $I$ is large enough, $\omega \simeq \omega_i^\star$ and so this exposure is very close to $\frac{1}{2}$. Note that, because the aggregate exposure to default risk is equal to $\frac{1}{2}$, the average residual exposure $\omega_i^\star - \omega$ must be equal to zero.

The contracts $\gamma_B(\omega, \tilde{\omega})$ prescribe a $\omega$-trader to swap his residual endowment, $\omega_i^\star - \omega$, with the residual endowment of everyone else. Therefore, he ends up with the average residual endowment, which is equal to zero as noted above. Taken together, the combined contracts, $\gamma_A(\omega, \tilde{\omega}) + \gamma_B(\omega, \tilde{\omega})$, lead to post trade exposures $g(\omega) = \frac{1}{2}$.

Clearly, gross notionals created by $\gamma_A(\omega, \tilde{\omega})$ can be made arbitrarily close to their Walrasian counterpart for $I$ large enough. Likewise, the gross notionals created by $\gamma_B(\omega, \tilde{\omega})$ can be made arbitrarily close to zero. Given that $N(\omega_i) - N(\omega_{i-1}) > \underline{n}\,(\omega_i - \omega_{i-1}) = \underline{n}/(2I) > 0$, choosing $k$ large enough makes these contracts feasible, establishing the claim. □

## A.7 Proof of Proposition 7

When $D$ is normally distributed, $\Gamma[x]$ is quadratic, and so we have the identity:

$$\Gamma[y] = \Gamma[x] + \Gamma'[x]\,(y - x) + \frac{\Gamma''}{2}\,(y - x)^2,$$

where $\Gamma'' = \alpha \mathbb{V}[D]$ is the constant second derivative of $\Gamma[x]$. It thus follows that competitive surplus must be equal to:

$$K(\omega) = \Gamma[\omega] - \Gamma[g(\omega)] + \Gamma'[g(\omega)]\big(g(\omega) - \omega\big) = \frac{\alpha \mathbb{V}[D]}{2}\big(g(\omega) - \omega\big)^2.$$

The property that $K(\omega)$ is U-shaped obtains from the fact that $g'(\omega) \leq 1$ and $g(\frac{1}{2}) = \frac{1}{2}$.

The formula from the frictional surplus is:

$$\begin{aligned} F(\omega) &= \int_0^1 \big|\Gamma'[g(\tilde{\omega})] - \Gamma'[g(\omega)]\big| n(\tilde{\omega})\, d\tilde{\omega} \\ &= \int_0^{\omega} \big(\Gamma'[g(\omega)] - \Gamma'[g(\tilde{\omega})]\big) n(\tilde{\omega})\, d\tilde{\omega} + \int_{\omega}^1 \big(\Gamma'[g(\tilde{\omega})] - \Gamma'[g(\omega)]\big) n(\tilde{\omega})\, d\tilde{\omega}. \end{aligned}$$

Now take derivatives

$$F'(\omega) = \Gamma''[g(\omega)]\, g'(\omega)\big(2N(\omega) - 1\big),$$

which is negative for $\omega$ below the median, and positive for $\omega$ above the median, which in our symmetric case is $\omega = \frac{1}{2}$.

## A.8 Proof of Proposition 8

### A.8.1 Preliminaries

To apply the fixed point Theorem, we first need to establish properties of the operators mapping the distribution of traders, $n$, to the post-trade exposure function, $g$, and two the entry payoff, $\Delta$. Consider the set $\mathcal{C}^0([0,1])$ of continuous functions over $[0,1]$, equipped with the sup norm. Let $\mathcal{N}$ be the set of $n \in \mathcal{C}^0([0,1])$ such that $n(\omega) > 0$ almost everywhere and $\int_0^1 n(\omega)\, d\omega = 1$. Let $\mathcal{G}$ be the set of post-trade exposures functions, $g$, and let $\mathcal{D}$ be the set of entry payoff functions, $\Delta$, generated by distributions in $\mathcal{N}$. Clearly, by Proposition 2, $\mathcal{G}$ is a subset of $\mathcal{C}^0([0,1])$, and so is $\mathcal{D}$. Moreover:

**Lemma 5.** *The sets $\mathcal{G}$ and $\mathcal{D}$ are equibounded and equicontinuous, and so their closures, $\bar{\mathcal{G}}$ and $\bar{\mathcal{D}}$, are compact.*

For $\mathcal{G}$, boundedness follows because $g(\omega) \in [0,1]$ and equicontinuity because, as shown in Proposition 2, post-trade exposure are Lipchitz with coefficient 1. For $\mathcal{D}$, boundedness follows because $g$ is bounded. For equicontinuity, it is enough to show that all $\Delta \in \mathcal{D}$ are Lipchitz with a coefficient that is independent of $n \in \mathcal{N}$. For this note first that the competitive surplus $K(\omega) = \Gamma[\omega] - \Gamma[g(\omega)] + \Gamma'[g(\omega)]\big(g(\omega) - \omega\big)$ is Lipchitz with a coefficient independent of $n$. This is because $g$ is Lipchitz with coefficient one, because both $\Gamma[x]$ and $\Gamma'[x]$ are continuously differentiable, and because the Lipchitz property over a compact is preserved by sum, product, and composition. So all we need to show is that the frictional surplus is Lipchitz with a coefficient

that is independent from $n$. To that end consider $\omega_2 > \omega_1$ and note that:

$$\begin{aligned}
\left|F(\omega_1) - F(\omega_2)\right| &= \left| \int_0^1 \left( \left|\Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\omega_2)\right]\right| - \left|\Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\omega_1)\right]\right| \right) n(\tilde{\omega})\, d\tilde{\omega} \right| \\
&\leq \int_0^1 \left| \left|\Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\omega_2)\right]\right| - \left|\Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\omega_1)\right]\right| \right| n(\tilde{\omega})\, d\tilde{\omega} \\
&\leq \int_0^1 \left|\Gamma'\left[g(\omega_2)\right] - \Gamma'\left[g(\omega_1)\right]\right| n(\tilde{\omega})\, d\tilde{\omega} = \left|\Gamma'\left[g(\omega_2)\right] - \Gamma'\left[g(\omega_1)\right]\right|,
\end{aligned}$$

where the inequality on the third line follows by an application of the reverse triangle inequality. Since $\Gamma'[g]$ is Lipchitz, and since $g$ is Lipchitz with coefficient one, the result follows. The compactness of $\bar{\mathcal{G}}$ and $\bar{\mathcal{D}}$ then follows from Arzela-Ascoli Theorem (see Theorem 11.28 in Rudin, 1974). □

Next, we establish a first continuity property:

**Lemma 6.** *The operator mapping distributions, $n \in \mathcal{N}$, into post-trade exposures, $g \in \mathcal{G}$, is continuous.*

To show this result consider some $n \in \mathcal{N}$, and let $g$ and $\gamma$ be the associated equilibrium post-trade exposures and CDS contracts. Let $n^{(p)}$ be a sequence of distributions converging to $n$, and let $\gamma^{(p)}$ and $g^{(p)}$ be the associated sequences of CDS contracts and post-trade exposures. Keep in mind that we equipped $C^0([0,1])$ with the sup norm, so convergence is uniform. Given that $\bar{\mathcal{G}}$ is compact, in order to show continuity it is sufficient to show that all convergent subsequences of $g^{(p)}$ share the same limit, and that this limit is equal to $g$. Without loss of generality, we thus assume that $g^{(p)}$ is convergent, and denote its limit by $g^\star$. Note that, since all $g^{(p)}$ are continuous and convergence is uniform, $g^\star$ must be continuous. By construction, $g^{(p)}$, $\gamma^{(p)}$ and $n^{(p)}$ satisfy:

$$g^{(p)}(\omega) = \omega + \int_0^1 \gamma^{(p)}(\omega, \tilde{\omega}) n^{(p)}(\tilde{\omega})\, d\tilde{\omega}.$$

Consider the auxiliary post-trade exposures:

$$\begin{aligned}
\hat{g}^{(p)}(\omega) &\equiv \omega + \int_0^1 \gamma^{(p)}(\omega, \tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega} \\
\tilde{g}^{(p)}(\omega) &\equiv \omega + \int_0^1 \gamma(\omega, \tilde{\omega}) n^{(p)}(\tilde{\omega})\, d\tilde{\omega}.
\end{aligned}$$

In words, $\hat{g}^{(p)}$ is the post-trade exposure generated by $\gamma^{(p)}$ if the underlying distribution of traders is $n$, and $\tilde{g}^{(p)}$ is the post-trade exposure generated by $\gamma$ if the underlying distribution is $n^{(p)}$. Note that, since $\gamma^{(p)}$ is bounded by $k$ and since $n^{(p)} \to n$, we have $||\hat{g}^{(p)} - g^{(p)}|| \to 0$ and so $\hat{g}^{(p)} \to g^\star$. Likewise, $\tilde{g}^{(p)} \to g$.

Let $J(g,n) \equiv \int_0^1 \Gamma\left[g(\omega)\right] n(\omega)\, d\omega$ be the average cost of risk bearing generated by post-trade exposures $g$ under distribution $n$. Recall that $(\gamma, g)$ solves the planning problem of Proposition 1 given $n$, i.e., it minimizes $J$ amongst feasible risk allocations. Since $(\gamma^{(p)}, \hat{g}^{(p)})$ is feasible for this planning problem, we must have $J(g,n) \leq J(\hat{g}^{(p)}, n)$. Going to the limit $p \to \infty$, we obtain by dominated convergence that $J(g,n) \leq J(g^\star, n)$. Likewise, $g^{(p)}$ solves the planning problem given $n^{(p)}$. Since $(\gamma, \tilde{g}^{(p)})$ is feasible for this planning problem, we must have $J(g^{(p)}, n^{(p)}) \leq J(\tilde{g}^{(p)}, n^{(p)})$. Going to the limit $p \to \infty$, we obtain by dominated convergence that $J(g^\star, n) \leq$

$J(g,n)$. Taken together, we thus have that $J(g^\star,n)=J(g,n)$. Now suppose that $g^\star\neq g$. Then by convexity of $\Gamma[g]$ we have that for all $\lambda\in(0,1)$:

$$J(\lambda g+(1-\lambda)g^\star,n)<J(g,n).$$

But $\lambda\gamma+(1-\lambda)\gamma^{(p)}$ is feasible, generates post trade exposure $\lambda g+(1-\lambda)\hat{g}^{(p)}$ given $n$, and converges to $\lambda g+(1-\lambda)g^\star$ as $p\to\infty$. Therefore, for $p$ large enough, $J(\lambda g+(1-\lambda)\hat{g}^{(p)},n)<J(g,n)$, which is impossible given that $(\gamma,g)$ solves the planning problem. Therefore, $g=g^\star$ almost everywhere, since we restricted attention to $n$ such that $n(\omega)>0$ almost everywhere. Since $g$ and $g^\star$ are continuous, we obtain that $g=g^*$ everywhere. □

Next we have the corollary:

**Corollary 14.** *The operator mapping distributions, $n\in\mathcal{N}$, into entry-utility functions, $\Delta\in\mathcal{D}$, is continuous.*

Note that $\Delta(\omega)=A\big(\omega,g(\omega),N(\omega)\big)+B\big(\omega,g,n\big)$, where

$$A(\omega,g,N)=\Gamma\left[\omega\right]-\Gamma\left[g\right]-\Gamma'\left[g\right](\omega-g)+\frac{k}{2}\left(2N-1\right)\Gamma'\left[g\right]$$
$$B\big(\omega,g(\cdot),n(\cdot)\big)=-\frac{k}{2}\int_0^\omega\Gamma'\left[g(\tilde{\omega})\right]n(\tilde{\omega})\,d\tilde{\omega}+\frac{k}{2}\int_\omega^1\Gamma'\left[g(\tilde{\omega})\right]n(\tilde{\omega})\,d\tilde{\omega}.$$

Now given any $n^{(p)}\to n$, we already know that the associated $g^{(p)}\to g$. Note also, that $N^{(p)}\to N$ (uniformly as well). Since $A(\omega,g,N)$ is continuous and $(\omega,g,N)\in[0,1]^3$, it follows that $A(\omega,g^{(p)}(\omega),N^{(p)}(\omega))\to A(\omega,g(\omega),N(\omega))$ uniformly. Likewise, by dominated convergence, $B\big(\omega,g^{(p)},n^{(p)}\big)\to B\big(\omega,g,n\big)$ uniformly. □

Lastly an important property for what follows is:

**Lemma 7** ($0\neq\bar{\mathcal{D}}$)**.** *The function $\Delta(\omega)=0$ does not belong to the closure of $\mathcal{D}$.*

Towards a contradiction assume it does: there is a sequence $n^{(p)}$ such that the associated $\Delta^{(p)}\to 0$. Note that, by strict convexity of $\Gamma\left[g(\omega)\right]$, $\Gamma\left[\omega\right]-\Gamma\left[g(\omega)\right]+\Gamma'\left[g(\omega)\right]\big(g(\omega)-\omega\big)\geq 0$, with an equality if and only if $g(\omega)=\omega$. Given that the frictional surplus is non-negative, this implies that:

$$\Gamma\left[\omega\right]-\Gamma\left[g^{(p)}(\omega)\right]-\Gamma'\left[g^{(p)}(\omega)\right]\left(\omega-g^{(p)}(\omega)\right)\to 0,$$

and that all convergent subsequence of $g^{(p)}$ are such that $g^{(p)}(\omega)\to\omega$. Given that $\bar{\mathcal{G}}$ is compact, we thus have that $g^{(p)}(\omega)\to\omega$ uniformly over $\omega\in[0,1]$. Now turning to the last term of $\Delta(\omega)$ evaluated at $\omega=0$, we have:

$$\int_0^1\left(\Gamma'\left[g^{(p)}(\tilde{\omega})\right]-\Gamma'\left[g^{(p)}(0)\right]\right)n^{(p)}(\tilde{\omega})\,d\tilde{\omega}\to 0,$$

where we can remove the absolute value since $g^{(p)}(\omega)\geq g^{(p)}(0)$. In particular, for any $\omega>0$, we have that $\int_\omega^1\left(\Gamma'\left[g^{(p)}(\tilde{\omega})\right]-\Gamma'\left[g^{(p)}(0)\right]\right)n^{(p)}(\tilde{\omega})\,d\tilde{\omega}\to 0$. Given that $g^{(p)}(\tilde{\omega})\geq g^{(p)}(\omega)$ for $\tilde{\omega}\in[\omega,1]$, this implies in turn that $\left(\Gamma'\left[g^{(p)}(\omega)\right]-\Gamma'\left[g^{(p)}(0)\right]\right)\left(1-N^{(p)}(\omega)\right)\to 0$. Since $g^{(p)}(\omega)\to\omega$, we thus have that $N^{(p)}(\omega)\to 1$, i.e., the distribution $n^{(p)}$ converges to a Dirac at $\omega=0$. The intuition is simple: the only way $\omega=0$ has no gain from entering, i.e. $\Delta(\omega)=0$, is if it only meets traders of her kind, i.e., $\tilde{\omega}=0$ with probability one. But this means that $\omega\neq 0$

must have strictly positive gains from entering. Formally, given that $g^{(p)}(\omega)$ converges towards $\omega$, uniformly over $\omega \in [0,1]$, this implies that, for every $\omega \in (0,1]$, the last term of $\Delta^{(p)}(\omega)$ converges to $\frac{k}{2}\left|\Gamma'(0) - \Gamma'(\omega)\right| > 0$, which is a contradiction.

### A.8.2 Properties of the fixed-point equation

For this section it is convenient to rewrite the fixed-point equation as:

$$T[n](\omega) = \frac{\Upsilon[\Delta(\omega)]}{\int_0^1 \Upsilon[\Delta(\tilde{\omega})]\, d\tilde{\omega}}, \text{ where } \Upsilon(x) \equiv \Psi\left(\frac{c}{x}\right). \tag{21}$$

We start by establishing basic properties of the function $\Upsilon(x)$:

**Lemma 8.** *Let, for $x > 0$, $\Upsilon(x) \equiv \Psi\left(\frac{c}{x}\right)$ and let $\Upsilon(0) = 0$. Then the function $\Upsilon(x)$ is bounded, non-decreasing, continuous, piecewise continuously differentiable with bounded derivative.* □

The function $\Upsilon(x)$ is bounded and non-deacreasing since $\Psi(x) \in [0,1]$ and non-increasing. It is obviously continuous for $x > 0$, and it is also continuous at zero since $\lim_{S\to\infty} \Psi(S) = 0$. It is differentiable for all $x > 0$ and $x \neq \frac{c}{\underline{S}}$. For $x \in (0, \frac{c}{\underline{S}})$, $\Upsilon'(x) = -c^2/x^3\varphi(c/x)$. Note that $\lim_{x\to 0} \Upsilon'(x) = \frac{1}{c} \times \lim_{S\to\infty} S^3\varphi(S)$, which we assumed exists. Given that $\varphi(S)$ is continuous, this implies in turns that $\Psi'(x)$ is bounded over $\left[0, \frac{c}{\underline{S}}\right)$. For $x > \frac{c}{\underline{S}}$, $\Upsilon'(x) = 0$ and is obviously continuously differentiable and bounded. Moreover,

$$\frac{1}{x}\Psi\left(\frac{c}{x}\right) = \frac{1}{c} \times \frac{c}{x}\int_{\frac{c}{x}}^{\infty} z\varphi(z)\, dz = \frac{1}{c} \times \frac{c}{x}\int_0^{\frac{x}{c}} \frac{1}{y^3}\varphi\left(\frac{1}{y}\right) dy \to \frac{1}{c} \times \lim_{S\to\infty} S^3\varphi(S),$$

where the first equality follows from the definition of $\Psi(S)$, the second equality from the change of variable $y = 1/z$, and the third equality follows because of our assumption that $\lim_{S\to\infty} S^3\varphi(S)$ exists. Therefore, $\Upsilon(x)$ is continuously differentiable at zero, implying that its derivative is bounded over $[0, \frac{c}{\underline{S}})$. □

**Lemma 9** (Properties of $T$)**.** *The operator $T$ is continuous and uniformly bounded. The set $T[\mathcal{N}]$ is included in $\mathcal{N}$ and is equicontinuous.*

Continuity follows because $\Upsilon(x)$ is continuous, and because, by Corollary 14, the operator mapping $n$ to $\Delta$ is continuous. For boundedness, note first that the numerator of (21) is positive and less than one. So all we need to show is that denominator is bounded away from zero. For this it suffices to show that:

$$\inf_{\Delta\in\bar{\mathcal{D}}} \int_0^1 \Upsilon\big[\Delta(\omega)\big]\, d\omega > 0.$$

Since $\bar{\mathcal{D}}$ is compact by Lemma 5, and since the functional that is minimized is continuous in $\Delta$, it follows that the infimum is achieved for some continuous function $\Delta^* \in \mathcal{D}$. By Lemma 7, $\Delta^* \neq 0$. Since $\Psi(x) \geq 0$ with an equality if and only if $x = 0$, it follows that the infimum is strictly positive.

Next we show that $T[\mathcal{N}] \subseteq \mathcal{N}$, i.e., that $T[n](\omega)$ is continuous, satisfies $\int_0^1 n(\omega)\, d\omega = 1$, and $n(\omega) > 0$ almost everywhere. Continuity follows because $\Delta(\omega)$ and $\Upsilon(x)$ are continuous; $\int_0^1 n(\omega)\, d\omega = 1$ follows by construction. To show that $T[n](\omega) > 0$ almost everywhere, we show that there is at most one $\omega^\star$ such that $T[n](\omega^\star) = 0$. Indeed, since $\Upsilon(x) = 0$ if and only if

$x = 0$, $T[n](\omega^\star) = 0$ implies $\Upsilon\left[\Delta(\omega^\star)\right] = \Delta(\omega^\star) = K(\omega^\star) = F(\omega^\star) = 0$. Because $F(\omega^\star) = 0$, and keeping in mind that $n(\omega) > 0$ almost everywhere, it follows that $\Gamma'\left[g(\omega)\right] = \Gamma'\left[g(\omega^\star)\right]$ and thus $g(\omega) = g(\omega^\star)$ almost everywhere. Since $g(\tilde{\omega})$ is continuous, it follows that $g(\omega)$ is constant and equal to $\int_0^1 \omega n(\omega)\, d\omega$. But then $K(\omega^*) = 0$ implies that $\omega^\star = \int_0^1 \omega n(\omega)\, d\omega$, which has a unique solution.

Turning to equicontinuity, note that since $\Psi\left[x\right]$ is piecewise differentiable with bounded derivatives, it is Lipchitz. Recall from the proof of Lemma 5 that all $\Delta \in \mathcal{D}$ are Lipchitz with a coefficient that is independent of $n \in \mathcal{N}$. Given that the Lipchitz property is preserved by composition, this shows that $\omega \mapsto \Psi\left[\Delta(\omega)\right]$ is Lipchitz with a coefficient that is independent of $n \in \mathcal{N}$. Lastly, from argument above, the denominator of $T[n](\omega)$ is bounded away from zero. Taken together, this shows that the function $T[n]$ is Lipchitz with a coefficient that is independent of $n \in \mathcal{N}$. □

### A.8.3 Applying the Schauder fixed point Theorem

Let $M$ be the uniform upper bound of the operator $T$ we obtained in the proof of Lemma 9. Consider the closed and bounded set $\mathcal{N}_M$ of all $n \in \mathcal{N}$ such that $||n|| \leq M$. Note that $\mathcal{N}$ is convex and that a convex combination of functions bounded by $M$ remains bounded by $M$. Therefore, $\mathcal{N}_M$ is convex as well. Clearly, the operator $T$ maps $\mathcal{N}_M$ into itself. Moreover, by Lemma 9, the family $T\left[\mathcal{N}_M\right]$ is equicontinuous. Taken together all these properties allow to apply Theorem 17.4 in Stokey and Lucas (1989), establishing the existence of a fixed point. □

## A.9 Proof of Corollary 9

The result follows by applying the Schauder fixed point Theorem in the set $\mathcal{N}_S$ of $n \in \mathcal{N}_M$ which are U-Shaped and symmetric. Clearly, this set is closed and convex. Moreover, when $\Gamma\left[x\right]$ the operator $T$ maps $\mathcal{N}_S$ into itself. The results follows. □

## A.10 Proof of Proposition 10

### A.10.1 Preliminary: notional analytics

In this section we establish:

**Proposition 15.** *Suppose that $n(\omega)$ is U-shaped, symmetric, positive, and consider the equilibrium CDS contracts of Proposition 6. Then:*

*(i)* $G^+(\omega) = G^-(1-\omega)$;

*(ii)* $G^+(\omega)$ *is decreasing and* $G^-(\omega)$ *is increasing;*

*(iii)* $G^+(\omega) = G^-(\omega)$ *if and only if* $\omega = \frac{1}{2}$;

*(iv)* $G^+(\omega) + G^-(\omega)$ *is weakly hump-shaped and symmetric around* $\frac{1}{2}$*: it is increasing over* $[0, \underline{\omega})$*, constant over* $(\underline{\omega}, 1-\underline{\omega})$ *and decreasing over* $(1-\underline{\omega}, 1]$;

*(v)* $\left|G^+(\omega) - G^+(\omega)\right|$ *is U-shaped and symmetric around* $\frac{1}{2}$;

*(vi)* $\min\{G^+(\omega), G^-(\omega)\}$ *is hump-shaped and symmetric around* $\frac{1}{2}$.

**Point (i).** The first point of the Proposition follows from the fact that $\gamma(\omega, \tilde{\omega}) = -\gamma(1-\omega, 1-\tilde{\omega})$.

**Point (ii).** Given the first point, to prove the second point we only need to show that $G^+(\omega)$ is decreasing over $[0, \frac{1}{2}]$, and that $G^-(\omega)$ is increasing over $[0, \frac{1}{2}]$. For this weconsider various subcases. If $g(\omega)$ is increasing, then $G^+(\omega) = k\left[1 - N(\omega)\right]$ and so is clearly decreasing. Likewise, $G^-(\omega) = kN(\omega)$ and is clearly increasing. If $g(\omega)$ has flat and possibly increasing spot, i.e. $\underline{\omega} \in (0, \frac{1}{2}]$, then for $\omega \in (\underline{\omega}, \frac{1}{2}]$, $G^+(\omega) = k\left[1 - N(\omega)\right]$ and $G^-(\omega) = kN(\omega)$, which are respectively decreasing and increasing. For $\omega \in [0, \underline{\omega}]$

$$G^+(\omega) = \int_0^{\underline{\omega}} z(\tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega} + k\left[1 - N(\underline{\omega})\right],$$

which is decreasing. Likewise, when $\omega \in [0, \underline{\omega}]$,

$$G^-(\omega) = N(\omega) z(\omega) = \frac{\int_0^{\omega} (\omega - \tilde{\omega})\, n(\tilde{\omega})\, d\tilde{\omega}}{N(\omega)}$$

Taking derivatives we obtain

$$\begin{aligned}
\frac{dG^-}{d\omega}(\omega) &= \frac{N(\omega)^2 - n(\omega)\int_0^{\omega} \big(\omega - \tilde{\omega}\big) n(\tilde{\omega})\, d\tilde{\omega}}{N(\omega)^2} \\
&\geq \frac{\omega n(\omega) N(\omega) - n(\omega)\int_0^{\omega} \big(\omega - \tilde{\omega}\big) n(\tilde{\omega})\, d\tilde{\omega}}{N(\omega)^2} = \frac{n(\omega)\int_0^{\omega} \tilde{\omega} n(\tilde{\omega})\, d\tilde{\omega}}{N(\omega)^2} \geq 0
\end{aligned}$$

where the second line follows from the fact that $n(\omega)$ is decreasing over $[0, \frac{1}{2}]$, implying that $N(\omega) \geq \omega n(\omega)$.

**Point (iii).** The first point implies that $G^+(\frac{1}{2}) = G^-(\frac{1}{2})$. This is the unique solution of $G^+(\omega) = G^-(\omega)$ given that $G^+(\omega)$ is decreasing and $G^-(\omega)$ is increasing.

**Point (iv).** Given the first point it is enough to show that $G^+(\omega) + G^-(\omega)$ is non-decreasing over $[0, \frac{1}{2}]$. If $g(\omega)$ is increasing then $G^+(\omega) + G^-(\omega) = k$. Now suppose that $g(\omega)$ has a flat and possibly an increasing spot. For all $\omega \in [\underline{\omega}, \frac{1}{2}]$, $g(\omega)$ is increasing and so $G^+(\omega) + G^-(\omega) = k$. For $\omega \in [0, \underline{\omega}]$, the gross exposure is:

$$G^+(\omega) + G^-(\omega) = z(\omega) N(\omega) + \int_{\omega}^{\underline{\omega}} z(\tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega} + z(\underline{\omega})\left[1 - N(\underline{\omega})\right].$$

Now recall:

$$g(\underline{\omega}) = \omega - z(\omega) N(\omega) + \int_{\omega}^{\underline{\omega}} z(\tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega} + z(\underline{\omega})\left[1 - N(\underline{\omega})\right].$$

This implies:

$$G^+(\omega) + G^-(\omega) = g(\underline{\omega}) - \omega + 2z(\omega)N(\omega) = g(\underline{\omega}) - \omega + 2\frac{\int_0^\omega (\omega - \tilde{\omega})\, n(\tilde{\omega})\, d\tilde{\omega}}{N(\omega)}$$
$$= g(\underline{\omega}) + \frac{\int_0^\omega (\omega - 2\tilde{\omega})\, n(\tilde{\omega})}{N(\omega)},$$

where the first equality follows from plugging in the expression for $z(\omega)$ from Proposition 6. Taking derivatives with respect to $\omega$ we find:

$$\frac{N(\omega)\left(-\omega n(\omega) + N(\omega)\right) - n(\omega)\int_0^\omega (\omega - 2\tilde{\omega})\, n(\tilde{\omega})\, d\tilde{\omega}}{N(\omega)^2}$$
$$= \frac{N(\omega) - 2n(\omega)\int_0^\omega (\omega - \tilde{\omega})\, n(\tilde{\omega})\, d\tilde{\omega}}{N(\omega)} \geq \frac{n(\omega)}{N(\omega)}\left(\omega - 2\int_0^\omega (\omega - \tilde{\omega})\ n(\tilde{\omega})\, d\tilde{\omega}\right),$$

where the last inequality follows because $n(\omega)$ is non-increasing over $[0, \frac{1}{2}]$, and so $N(\omega) \geq \omega n(\omega)$. The result follows because the term in parenthesis is equal to zero when $\omega = 0$, and its derivative is $1 - 2N(\omega) > 0$ over $[0, \frac{1}{2})$.

**Point (v).** Given the fist point it is enough to show that $G^+(\omega) - G^-(\omega)$ is decreasing over $[0, \frac{1}{2}]$. this note that $|G^+(\omega) - G^-(\omega)| = |g(\omega) - \omega|$. For $\omega \in [0, \frac{1}{2}]$, Proposition 2 implies that $g(\frac{1}{2}) - g(\omega) \leq \frac{1}{2} - \omega$. Given symmetry, $g(\frac{1}{2}) = \frac{1}{2}$, so $g(\omega) \geq 0$ and $|G^+(\omega) - G^-(\omega)| = g(\omega) - \omega$. When $g(\omega)$ is flat, then clearly this is a decreasing function. When $g(\omega)$ is increasing, then $g'(\omega) = 1 - 2kn(o)$, and so $g'(\omega) - 1 = -2kn(\omega) < 0$.

**Point (vi).** From Point (ii) and (iii), it follows that, for $\omega \in [0, \frac{1}{2}]$, $\min\{G^+(\omega), G^-(\omega)\} = G^+(\omega)$ is increasing. □

### A.10.2 Proof of the main proposition

The first step is to derive the distribution of traders conditional on size, $n(\omega \,|\, S)$. Recall that, the entry threshold at $\omega$, $\Sigma(\omega)$, is a hump-shaped function of $\omega$. This means that, if a bank of size $S$ is active in the market, then its endowment per capita, $\omega$, must be either small enough or large enough. Precisely, for $S \in [\Sigma(\frac{1}{2}), \infty)$, let $\bar{\omega}(S) \in [0, \frac{1}{2}]$ be the solution of $\Sigma(\omega) = S$. For $S \geq \Sigma(\frac{1}{2})$, let $\bar{\omega}(S) = \frac{1}{2}$. Then if a bank of size $S \in [\Sigma(\frac{1}{2}), \infty)$ is active in the OTC market, its endowment per-capita of a bank of size must either belong to $[0, \bar{\omega}(S)]$ or $[1 - \bar{\omega}(S), 1]$. Now recall that the endowment per trader is drawn uniformly conditional on $S$. This implies that the measure of traders conditional on $S$ must be uniform over its support, that is:

$$n(\omega \,|\, S) = \frac{1}{2\bar{\omega}(S)} \text{ if } \omega \in [0, \bar{\omega}(S)] \cup [1 - \bar{\omega}(S), 1]$$

and $n(\omega \,|\, S) = 0$ otherwise. Clearly, if $S' > S$, then $n(\omega \,|\, S')$ has puts more mass towards the middle of the $\omega$ spectrum than $n(\omega \,|\, S)$, because middle-$\omega$ banks are predominantly large. Mathematically, given symmetry, this property can be expressed as follows:

**Lemma 10.** *Consider $(S, S'$ in $[\Sigma(0), \Sigma(\frac{1}{2})]$. If $S' > S$, then $n(\omega \,|\, S', \omega \leq \frac{1}{2})$ first order stochastically dominates $n(\omega \,|\, S, \omega \leq \frac{1}{2})$.*

Now consider any function $x(\omega)$ that is U-shaped and symmetric and calculate:

$$\mathbb{E}_S\left[x(\omega)\right] = \int_0^1 x(\omega)n(\omega \mid S)\,d\omega = 2\int_0^{\frac{1}{2}} x(\omega)n(\omega \mid S, \omega \leq \tfrac{1}{2})\,d\omega,$$

where the second equality follows because $x(\omega)$ is symmetric. Now, given that $x(\omega)$ is decreasing over $[0, \frac{1}{2}]$, it follows from the above lemma that $\mathbb{E}_S\left[x(\omega)\right]$ is non-increasing. The opposite is true if $x(\omega)$ is hump-shaped and symmetric. Except for price dispersion, the results then follow from Proposition 15. For price dispersion, recall that, given that $\Gamma'\left[g\right] = \mathbb{E}\left[D\right] + \alpha\frac{\mathbb{V}[D]}{2}g$ and so:

$$R(\omega, \tilde{\omega}) = \mathbb{E}\left[D\right] + \frac{\alpha\mathbb{V}\left[D\right]}{2}\Big(g(\omega) + g(\tilde{\omega})\Big)$$

Given that $n(\omega \mid S)$ is symmetric around $\frac{1}{2}$, $\mathbb{E}_S\left[g(\omega)\right] = \frac{1}{2}$ and so the conditional mean of CDS price is:

$$\mathbb{E}_S\left[R(\omega, \tilde{\omega})\right] = \mathbb{E}\left[D\right] + \frac{\alpha\mathbb{V}\left[D\right]}{2}.$$

The conditional dispersion is, up to some multiplicative constant:

$$\mathbb{E}_S\left[\left(g(\omega) - \tfrac{1}{2} + g(\tilde{\omega}) - \tfrac{1}{2}\right)^2\right] = 2\mathbb{E}_S\left[\left(g(\omega) - \tfrac{1}{2}\right)^2\right] = 2\int_0^{\frac{1}{2}} \left(g(\omega) - \tfrac{1}{2}\right)^2 n(\omega \mid S, \omega \leq \tfrac{1}{2})\,d\omega,$$

where the first equality follows because $\omega$ and $\tilde{\omega}$ are drawn independently, and the second equality follows by symmetry. Now observe that, over $[0, \frac{1}{2}]$, $g(\omega) - \frac{1}{2}$ is non-decreasing and negative, so that $\left(g(\omega) - \frac{1}{2}\right)^2$ is non-increasing. The result then follows from the Lemma. □

## A.11 Proof of Proposition 11

We study each terms of $W(\varepsilon, \delta)$ in turns. To ease exposition, we start with the third term, then move on to the first term, and finally study the second term.

**Derivative of the third term.** Note that $\Sigma_\varepsilon(\omega) = \Psi^{-1}\left[Mn(\omega) + \varepsilon\delta(\omega)\right]$ and that the derivative of $\Phi \circ \Psi^{-1}[x]$ is $1/\Psi^{-1}[x]$. Therefore, the derivative of the third term is:

$$-c\int_0^1 \frac{\delta(\omega)}{\Sigma(\omega)}\,d\omega.$$

**Derivative of the first term.** The derivative of the first term is, clearly:

$$\int_0^1 \delta(\omega)\Gamma\left[\omega\right]\,d\omega.$$

**Derivative of the second term.** For this define $\bar{\delta} \equiv \int_0^1 \delta(\omega)\,d\omega$ and let

$$n_\varepsilon(\omega) \equiv \frac{Mn(\omega) + \varepsilon\delta(\omega)}{M + \varepsilon\bar{\delta}}.$$

Recall that, conditional on entry, the equilibrium allocation of risk is efficient. That is, the second term can be written:

$$-\left(M+\varepsilon\bar{\delta}\right)\times\inf_{\gamma(\omega,\tilde{\omega})}\int_0^1 n_\varepsilon(\omega)\Gamma\left[\omega+\int_0^1\gamma(\omega,\tilde{\omega})\,n_\varepsilon(\tilde{\omega})d\tilde{\omega}\right]d\omega,$$

By the envelope theorem (see paragraph below for the detailed formal argument), the derivative of "inf" is equal to the partial derivative of the objective with respect to $\varepsilon$ evaluated at the optimal $\gamma(\omega,\tilde{\omega})$. Given that $\frac{\partial n_\varepsilon(\omega)}{\partial\varepsilon}\big|_{\varepsilon=0}=\frac{\delta(\omega)-n(\omega)\bar{\delta}}{M}$, the derivative of the third term is:

$$\begin{aligned}&-\bar{\delta}\int_0^1 n(\omega)\Gamma\left[g(\omega)\right]\,d\omega-\int_0^1\left(\delta(\omega)-n(\omega)\bar{\delta}\right)\Gamma\left[g(\omega)\right]\,d\omega\\&-\int_0^1 n(\omega)\Gamma'\left[g(\omega)\right]\int_0^1\gamma(\omega,\tilde{\omega})\left(\delta(\tilde{\omega})-\bar{\delta}n(\tilde{\omega})\right)d\tilde{\omega}d\omega.\end{aligned}$$

Changing the order of integration and using $\gamma(\omega,\tilde{\omega})=-\gamma(\tilde{\omega},\omega)$ we obtain that the term on the second line above is:

$$\begin{aligned}&-\int_0^1 n(\omega)\Gamma'\left[g(\omega)\right]\int_0^1\gamma(\omega,\tilde{\omega})\left(\delta(\tilde{\omega})-\bar{\delta}n(\tilde{\omega})\right)d\tilde{\omega}\,d\omega\\&=\int_0^1\left(\delta(\omega)-\bar{\delta}n(\tilde{\omega})\right)\int_0^1\Gamma'\left[g(\tilde{\omega})\right]\gamma(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}\,d\omega\end{aligned}$$

**Some more algebra.** Collecting the derivatives of the first, second, and third term we obtain:

$$W'(0,\delta)=\int_0^1\delta(\omega)\left(-\frac{c}{\Sigma(\omega)}+\Gamma\left[\omega\right]-\Gamma\left[g(\omega)\right]+\int_0^1\Gamma'\left[g(\tilde{\omega})\right]\gamma(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}\right)d\omega \tag{22}$$

$$-\bar{\delta}\int_0^1\int_0^1\Gamma'\left[g(\tilde{\omega})\right]\gamma(\omega,\tilde{\omega})n(\omega)n(\tilde{\omega})\,d\omega d\tilde{\omega}. \tag{23}$$

Just as in the calculation of entry incentives, in Section 6.1, we add and subtract $\int_0^1\Gamma'\left[g(\omega)\right]\gamma(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}=\Gamma'\left[g(\omega)\right]\left(g(\omega)-\omega\right)$ to the integral in the first line, (22). We obtain that this integral is equal to:

$$\int_0^1\delta(\omega)\left(-\frac{c}{\Sigma(\omega)}+K(\omega)+F(\omega)\right)d\omega.$$

Now moving on to the second line, (23), we note that:

$$\begin{aligned}
&\int_0^1 \int_0^1 \Gamma'\left[g(\tilde{\omega})\right] \gamma(\omega,\tilde{\omega}) n(\tilde{\omega}) n(\omega)\, d\tilde{\omega} d\omega \\
=&\frac{1}{2}\int_0^1 \int_0^1 \Gamma'\left[g(\tilde{\omega})\right] \gamma(\omega,\tilde{\omega}) n(\tilde{\omega}) n(\omega)\, d\tilde{\omega} d\omega + \frac{1}{2}\int_0^1 \int_0^1 \Gamma'\left[g(\tilde{\omega})\right] \gamma(\omega,\tilde{\omega}) n(\tilde{\omega}) n(\omega)\, d\tilde{\omega} d\omega \\
=&\frac{1}{2}\int_0^1 \int_0^1 \Gamma'\left[g(\tilde{\omega})\right] \gamma(\omega,\tilde{\omega}) n(\tilde{\omega}) n(\omega)\, d\tilde{\omega} d\omega - \frac{1}{2}\int_0^1 \int_0^1 \Gamma'\left[g(\tilde{\omega})\right] \gamma(\tilde{\omega},\omega) n(\tilde{\omega}) n(\omega)\, d\tilde{\omega} d\omega \\
=&\frac{1}{2}\int_0^1 \int_0^1 \Gamma'\left[g(\tilde{\omega})\right] \gamma(\omega,\tilde{\omega}) n(\tilde{\omega}) n(\omega)\, d\tilde{\omega} d\omega - \frac{1}{2}\int_0^1 \int_0^1 \Gamma'\left[g(\omega)\right] \gamma(\omega,\tilde{\omega}) n(\omega) n(\tilde{\omega})\, d\omega d\tilde{\omega} \\
=&\frac{1}{2}\int_0^1 \int_0^1 \Big(\Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\omega)\right]\Big) \gamma(\omega,\tilde{\omega}) n(\tilde{\omega}) n(\omega)\, d\tilde{\omega} d\omega = \frac{1}{2}\int_0^1 F(\omega) n(\omega)\, d\omega.
\end{aligned}$$

where: the first equality follows trivially; the second equality follows from $\gamma(\tilde{\omega},\omega) = -\gamma(\omega,\tilde{\omega})$; the third equality from relabeling $\omega$ by $\tilde{\omega}$ and vice versa; and the last line by collecting terms, using (11), as well as the definition of the frictional surplus. Collecting all terms we arrive at the formula of the proposition.

**The formal application of the Envelope Theorem.** Consider the optimization problem

$$K(\varepsilon) = \min_{\gamma(\omega,\tilde{\omega})} \int_0^1 \phi(\omega,\gamma,\varepsilon)\, d\omega, \tag{24}$$

subject to $\gamma(\omega,\tilde{\omega}) + \gamma(\tilde{\omega},\omega) = 0$ and $\gamma(\omega,\tilde{\omega}) \in [-k,k]$ and where:

$$\begin{aligned}
\phi(\omega,\gamma,\varepsilon) &\equiv n_\varepsilon(\omega)\Gamma\left[\omega + \int_0^1 \gamma(\omega,\tilde{\omega})\, n_\varepsilon(\tilde{\omega}) d\tilde{\omega}\right] d\omega \\
&= \frac{M n(\omega) + \varepsilon\delta(\omega)}{M + \varepsilon\bar{\delta}}\Gamma\left[\omega + \frac{M}{M+\varepsilon\bar{\delta}}\int_0^1 \gamma(\omega,\tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega} + \frac{\varepsilon}{M+\varepsilon\bar{\delta}}\int_0^1 \gamma(\omega,\tilde{\omega})\delta(\tilde{\omega})\, d\tilde{\omega}\right].
\end{aligned}$$

Clearly, both $\frac{\partial\phi}{\partial\varepsilon}$ and $\frac{\partial^2\phi}{\partial\varepsilon^2}$ exist. Moreover, since because $\gamma(\omega,\tilde{\omega})$, $n(\tilde{\omega})$ and $\delta(\tilde{\omega})$ are bounded, these derivatives are bounded uniformly in $(\omega,\varepsilon) \in [0,1]^2$. Therefore, using Theorem 9.42 in Rudin (1953), we obtain $\frac{\partial f}{\partial\varepsilon}$ by differentiating $\int_0^1 \phi(\omega,\gamma,\varepsilon)\, d\omega$ under the integral sign. Moreover, since $\frac{\partial^2\phi}{\partial\varepsilon^2}$ is bounded uniformly in $(\omega,\varepsilon)$, it follows that $\frac{\partial\phi}{\partial\varepsilon}$ and thus $\frac{\partial\phi}{\partial\varepsilon}$ is Lipchitz with respect to $\varepsilon$, with a Lipchitz coefficient that is independent from $\omega$. This allows to apply Theorem 3 in Milgrom and Segal (2002) and assert that:

$$K'(0) = \lim_{\varepsilon\to 0}\int_0^1 \frac{\partial\phi}{\partial\varepsilon}(\omega,\gamma_\varepsilon,0)\, d\omega,$$

where $\gamma_\varepsilon$ is, for each $\varepsilon$, a solution of the minimization problem (24). All we need to show is, therefore, that

$$\lim_{\varepsilon\to 0}\int_0^1 \frac{\partial\phi}{\partial\varepsilon}(\omega,\gamma_\varepsilon,0)\, d\omega = \int_0^1 \frac{\partial\phi}{\partial\varepsilon}(\omega,\gamma,0)\, d\omega,$$

where $\gamma$ is a solution of the minimization problem when $\varepsilon = 0$. To that end we first take derivatives with respect to $\varepsilon$:

$$M \int_0^1 \frac{\partial \phi}{\partial \varepsilon}(\omega, \gamma_\varepsilon, \varepsilon)\, d\omega = \int_0^1 \left[\delta(\omega) - \delta n(\omega)\right] \Gamma\left[\tilde{g}_\varepsilon(\omega)\right]\, d\omega$$
$$+ \int_0^1 n(\omega) \Gamma'\left[\tilde{g}_\varepsilon(\omega)\right] \left( -\delta \int_0^1 \gamma_\varepsilon(\omega, \tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega} + \int_0^1 \gamma_\varepsilon(\omega, \tilde{\omega}) \delta(\tilde{\omega})\, d\tilde{\omega} \right) d\omega,$$

where $\tilde{g}_\varepsilon(\omega) \equiv \omega + \int_0^1 \gamma_\varepsilon(\omega, \tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega}$ is the post-exposure generated by $\gamma_\varepsilon(\omega, \tilde{\omega})$ if the underlying distribution of traders is $n(\omega)$. Given that $n_\varepsilon(\omega) \to n(\omega)$ uniformly, we know from the proof of Lemma 6 that $\tilde{g}_\varepsilon(\omega) \to g(\omega)$ uniformly. Given the definition of $\tilde{g}_\varepsilon(\omega)$ this also implies that $\int_0^1 \gamma_\varepsilon(\omega, \tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega} = \tilde{g}_\varepsilon(\omega) - \omega \to g(\omega) - \omega = \int_0^1 \gamma(\omega, \tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega}$ uniformly. This implies that all terms except perhaps the last one converge to their $\varepsilon = 0$ counterparts. To show that the last term converges as well, rewrite it as:

$$\int_0^1 \int_0^1 n(\omega) \Gamma'\left[\tilde{g}_\varepsilon(\omega)\right] \gamma_\varepsilon(\omega, \tilde{\omega}) \delta(\tilde{\omega})\, d\tilde{\omega}\, d\omega$$
$$= \int_0^1 \int_0^1 n(\omega) \Big( \Gamma'\left[\tilde{g}_\varepsilon(\omega)\right] - \Gamma'\left[\tilde{g}_\varepsilon(\tilde{\omega})\right] \Big) \gamma_\varepsilon(\omega, \tilde{\omega}) \delta(\tilde{\omega})\, d\tilde{\omega}\, d\omega + \int_0^1 \int_0^1 n(\omega) \Gamma'\left[\tilde{g}_\varepsilon(\tilde{\omega})\right] \gamma_\varepsilon(\omega, \tilde{\omega}) \delta(\tilde{\omega})\, d\tilde{\omega}\, d\omega$$
$$= -\int_0^1 \int_0^1 n(\omega) \Big| \Gamma'\left[\tilde{g}_\varepsilon(\omega)\right] - \Gamma'\left[\tilde{g}_\varepsilon(\tilde{\omega})\right] \Big| k \delta(\tilde{\omega})\, d\tilde{\omega}\, d\omega - \int_0^1 \Gamma'\left[\tilde{g}_\varepsilon(\omega)\right] \delta(\omega) \int_0^1 \gamma_\varepsilon(\omega, \tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega}\, d\omega,$$

where the second line follows from subtracting and adding $\Gamma'\left[g_\varepsilon(\tilde{\omega})\right]$, and the third line follows, for the first term, from the optimality condition (11) and, for the second term, from the fact $\gamma(\omega, \tilde{\omega}) = -\gamma(\tilde{\omega}, \omega)$ and by switching integrating variables. The result follows because, as noted before, both $\tilde{g}_\varepsilon(\tilde{\omega})$ and $\int_0^1 \gamma_\varepsilon(\omega, \tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega}$ converge uniformly to their $\varepsilon = 0$ counterparts. □

### A.11.1 Proof of Lemma 2

Note that

$$F(\omega) - \frac{1}{2} \int_0^1 F(\tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega} = F(\omega) - \frac{1}{2} \int_0^1 \int_0^1 \left| \Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\hat{\omega})\right] \right| n(\tilde{\omega}) n(\hat{\omega})\, d\tilde{\omega}\, d\hat{\omega}.$$

Now, by the first triangle inequality, it follows that:

$$\left| \Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\hat{\omega})\right] \right| \leq \left| \Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\omega)\right] \right| + \left| \Gamma'\left[g(\omega)\right] - \Gamma'\left[g(\hat{\omega})\right] \right|$$

which, after substituting in the above, gives:

$$F(\omega) - \frac{1}{2} \int_0^1 F(\tilde{\omega})\, n(\tilde{\omega})\, d\tilde{\omega} \geq 0$$

## A.12 Equilibrium market size is socially optimal given $n(\omega)$

Go back to the beginning of Section 7.1 and let the planner choose the number $M$ of traders in the OTC market, holding market composition $n(\omega)$ the same. The corresponding entry threshold is $\Sigma_M(\omega)$ and solves $\Sigma_M(\omega) = \Psi^{-1}\left[M n(\omega)\right]$. Moreover the threshold $\Sigma_M(\omega)$ is decreasing in $M$ because $\Psi\left[S\right]$ is decreasing in $M$. Note as well that there is a maximum market size, $\bar{M}$,

consistent with $n(\omega)$. The maximum size $\bar{M}$ is attained when the threshold reaches zero for some $\omega$. When $M$ rises above $\bar{M}$, it is no longer possible to increase entry at the extreme and so it is no longer possible to keep market composition the same. Given that the threshold is lowest at $\omega = 0$, $\bar{M}$ solves $\Psi[0] = \bar{M}n(0)$.

For $M \in (0, \bar{M}]$ the derivative of the welfare function with respect to $M$ is given by the directional derivative of Proposition 11, evaluated at $\Sigma_M(\omega)$ and $\delta(\omega) = n(\omega)$:

$$\int_0^1 n(\omega)\left\{\Gamma[g(\omega)] - \Gamma[\omega] - \frac{c}{\Sigma_M(\omega)}\right\} d\omega.$$

Note that $g(\omega)$ only depends on $n(\omega)$ and, therefore, do not depend on $M$. Therefore, the directional derivative only depends on $M$ through $\Sigma_M(\omega)$. Given that $\Sigma_M(\omega)$ is a decreasing function of $M$, the derivative is decreasing, the planning problem is strictly concave in $M$, and the first-order condition for optimality is sufficient.