Financial reporting and market efficiency with extrapolative investors*

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Abstract

We consider a competitive financial market in which companies engage in strategic financial reporting knowing that investors only pay attention to finitely many aspects of firms' reports and extrapolate from their sample. We investigate the extent to which stock prices differ from the fundamental values, assuming that companies must report all their activities but are otherwise free to disaggregate their reports as they wish. We show that no matter how many aspects investors are able to consider, a monopolist can induce a price of its stock bounded away from the fundamental. Besides, competition between companies may exacerbate stock mispricing.

Keywords: Extrapolation, efficient market hypothesis, competition, sophistication, financial reporting.

JEL codes: C72, D53, G14.

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1 Introduction

Financial reporting is a highly strategic business for firms. Being strategic on the financial reporting may take different forms such as falsifying current statistics to meet the expected targets, engaging in earnings managements (aimed, for example, at smoothing earnings performances over time), or issuing more or less disaggregated earnings forecasts by type of activities (thereby making it more or less difficult to assess the overall profitability of the firm).

If investors were fully rational, such strategic considerations would presumably not cause major problems, as investors would be able to make the correct inference from what is being disclosed to them. But, many investors are not fully sophisticated, some of them taking at face value the financial reports they get exposed (or pay attention) to. In face of this, in the early days of accounting scandals, various forms of investor protection have been advocated so as to avoid that financial reports be too disconnected from the true performance of the firm.[1] Yet, no matter how powerful the regulatory body is, there seem to be some fundamental limits to investor protection and transparency requests. Firms are often (and for legitimate reasons) left with significant discretion in evaluating their activities and in communicating their evaluations to investors.

In this paper, we wish to explore the impact of strategic financial reporting on whether the prices of stocks correctly reflect the fundamental value of the firms in a world in which investors are not fully sophisticated. We address this question in a context in which firms are constrained in the kind of financial reporting they can consider, reflecting some form of investor protection. More precisely, using the metaphor of the issuance of earnings forecast, we allow each firm to choose freely how disaggregated it wishes to report its forecasts, but we require that every activity of the firm be included. As a result, what the firm reports to investors must be in aggregate correct.[2] We also impose a lower bound on the profitability on a single activity that can be reported by

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[1]See for example SEC Chairman Arthur Levitt's famous speech, Levitt (1998).

[2]This probably represents an idealization of how much investor protection can be expected to impose on firms. Our findings that prices of stocks can be far away from the fundamentals would *a fortiori* hold under weaker investor protection environments.

a firm, and we normalize it to zero.[3]

Clearly, if investors could process the entire information they get exposed to in a perfectly rational way, the frame of the financial report (how disaggregated it is) would make no difference. But, what if investors are not able to process the information fully? Specifically, we assume that investors pay only attention to a finite number of dimensions of the various financial reports (each investor considers a different sample drawn at random), and investors extrapolate from their sample so as to form an estimate of how profitable each firm is. Based on this, if the price of the stock is below the estimate they are willing to buy, and if it is above they are willing to (short) sell. In order to highlight the effect of extrapolation in the simplest setup, we assume that every investor can only trade one stock (either buy or short sell).

Assuming firms seek to maximize the price of their stocks,[4] the above setup defines a game between firms in which the strategy of a firm bears on how disaggregated it wishes to make its financial report. To make our investigation more focused, we assume there is no fundamental uncertainty in the economy. That is, the various firms have a deterministic fundamental value $\varphi > 0$ and this is the same for all firms (yet it is unknown to investors who form estimates about it, as explained above).

We ask ourselves: Are prices different from the fundamental? How is the answer affected by the sophistication of investors (as measured by the number of dimensions investors pay attention to)? How is the answer affected by the competitiveness of the financial market (as measured by the number of firms competing for investors' capital)?

Superficial reasoning would suggest that given that on average the estimates of investors have to be correct (since the dimensions looked at are independently drawn across investors), the possible mistakes made at the individual level would cancel out, and thus no significant distortion should be expected. Yet, this intuition is incorrect. To illustrate this most simply, consider the case of a monopolist facing investors who would just consider one aspect of the financial report. The market clearing price corresponds to the median of the distribution chosen by the monopolist (that is the price level at

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[3]One way of thinking of such an assumption is that all investors have an idea of what a lower bound on the profitability of a single activity in the firm can be.

[4]Various rationales can be proposed for this, in particular considering the common compensation schemes of managers in terms of stock options would naturally tilt the manager's objective in that direction. We briefly consider alternative objectives in Section 6.

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which there are as many investors willing to buy as there are investors willing to sell). Hence, the monopolist wishes to choose a distribution of non-negative signals whose mean coincides with the fundamental and whose median is the largest. Solving this problem reveals that the monopolist can achieve a price of its stock as high as 2φ by using a two signal distribution concentrated on 0 and 2φ with a weight slightly larger than $1/2$ on 2φ. So, in our model, a well designed financial reporting strategy would induce a significant distortion away from the fundamental.

The next question is: How is the distortion affected by the degree of sophistication of investors and by market competition?

Regarding sophistication, it would seem that as investors consider more and more dimensions in the financial report, they would approach the correct assessment of the profitability of the firm. If they could consider all dimensions, this would be true. But, suppose instead investors pay attention to a finite number of dimensions. No matter how large this number is, we show that a monopolist can guarantee that the price of its stock remains bounded away from the fundamental.[5]

Regarding competition, we consider the case in which investors only consider one dimension of the financial report, and we focus on the symmetric equilibrium which induces the highest stock prices (which we motivate based on tacit collusion considerations). Our main result is that more firms make the price of stocks further away from the fundamental. So, competition may magnify the mispricing of stocks. While the logic of this result is somewhat involved (it will be explained later on in detail),[6] such a result is strongly suggestive that it would be unwise to rely on the observation that there are many firms around to dismiss the potential effect of strategic financial reporting onto the mispricing of stocks.

Related literature

The sampling heuristic we consider for investors has been first proposed by

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[5] The key observation here is that the optimal reporting strategy endogenously depends on the sophistication of the investors. Because of this dependence, the law of large numbers does not apply and the estimates of investors are not necessarily close to the fundamental.

[6] Note that it was not a priori clear in which direction competition would tilt the bias. In fact, inducing a higher market clearing price would require attracting more demand and so tilt the financial reporting distribution toward higher signals. Yet, since the mean of the distribution has to coincide with the fundamental, that would have to be counter-balanced by having more weight on low signals. This makes it hard to identify how the most relevant deviations would look like and so what effect competition may have on stock prices.

Osborne and Rubinstein (1998) in a game-theoretic context. It captures the idea that agents may extrapolate too much from small samples, a cognitive bias referred to as the "law of small numbers" by Tversky and Kahneman (1971).[7] Several studies have documented the tendency of investors to extrapolate too much from small samples (see e.g. Benartzi (2001); Baquero and Verbeek (2008); Greenwood and Nagel (2009)),[8] but this paper seems to be the first one to address how such a bias may affect the choice of financial reporting strategy and the resulting pricing of stocks.

Since Osborne and Rubinstein (1998), the sampling heuristic has been considered in IO settings by Spiegler (2006a) and Spiegler (2006b) in which firms compete on prices and consumers choose their firms by a sampling procedure (the sampling bears on the quality in Spiegler (2006b) whereas it bears on the price draw as well in Spiegler (2006a)).[9] Our model follows the spirit of Spiegler in the modeling of investors' heuristics and in the questions that are being addressed (effect of sophistication, effect of competition). But our application is different, leading to different formulations of the game and different conclusions. In particular, in our setting, prices are determined through market clearing conditions, and the mean of the financial reporting distribution used by firms has to coincide with the fundamental, a constraint that has no analog in Spiegler' settings. In terms of results, our finding that more firms may induce larger distortions is somewhat different from that in Spiegler who obtains the weaker conclusion that distortions do not disappear with competition. Similarly, our finding that some significant distortion persists, even if investors consider arbitrarily large sample sizes has no counterpart in Spiegler.

From a more general perspective, alternative models of investors' overextrapolation have been considered in the context of financial markets. De Long, Shleifer, Summers and Waldmann (1990b) study whether arbitrageurs have a stabilizing role in the presence of extrapolative investors, while Barberis, Shleifer and Vishny (1998), Rabin (2002) and Rabin and Vayanos (2010) focus on how extrapolative investors react to news. None of these papers studies the issue of strategic financial reporting, which is the main focus of

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[7]See the recent book by Kahneman (2011) for several colorful accounts of this bias.

[8]More generally, several recent studies document the role of limited attention in investment decisions and its effect on financial markets (Barber and Odean (2008), Dellavigna and Pollet (2009), Hirshleifer, Lim and Teoh (2009)).

[9]See also Rubinstein and Spiegler (2008), who consider a speculative market in which investors randomly sample one price in the history of posted prices and buy if the current price is below the sampled price.

our paper. In a different vein, a few recent papers analyze stock prices in competitive equilibria with non-fully rational agents (in Gul, Pesendorfer and Strzalecki (2011), agents can only distinguish a limited number of contingencies; in Eyster and Piccione (2011), agents have a limited understanding of the functioning of the market). An essential distinctive feature of our study is the focus on how investors' beliefs may be manipulated, which has no counterpart in these papers. Finally, firms' strategies in financial reporting are analyzed in a large literature in accounting (see e.g. Verrecchia (2001) for a survey, and Hirshleifer and Teoh (2003) for a model in which investors have limited attention). This literature, however, generally abstracts from the role of improved investors' sophistication and of market competition on firms' reporting strategies.[10]

The rest of the paper is organized as follows. In Section 2 we present the model. In Section 3 we analyze the monopoly case in the simplest sophistication scenario. In Section 4 we study the effect of sophistication. In Section 5 we study the effect of competition. Section 6 offers a general discussion, in particular, discussing the role of bounded rationality in the results as well as the introduction of alternative investment heuristics. Section 7 concludes.

2   Model

Consider a stock market consisting of F firms $j = 1, ...F$, each having fundamental value φ. There is a unitary mass of investors trading on the stock market. Investors are unaware of the fundamental values of the firms. They assess the profitability of the various firms by taking at face value the financial reports they pay attention to (see details below). Each investor can only trade one stock (either buy or short sell), and he trades the one for which he perceives the highest gain from trade. The prices of the various stocks are determined through market clearing conditions.

Firms are assumed to know the procedure followed by investors, and they seek to maximize the price of their stocks. They choose the best financial reporting that consists in a choice of distribution of non-negative signals whose

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[10]Of course, there is also a large literature building on Crawford and Sobel (1982) that studies how much information can be transmitted from an informed sender to an uninformed decision-maker when the latter is assumed to be perfectly rational. We will discuss some of this literature in relation to the financial reporting application in Section 6.

mean is constrained to coincide with the fundamental value φ. There is complete information among firms, and we consider the Nash equilibria of the financial reporting game played by the firms. In particular, our analysis will focus on whether the prices of the stocks differ from the fundamental values, and how the sophistication of investors (see below for a measure of sophistication) and/or the degree of competitiveness (as measured by F) affect the result.

Formally, let σ^j denote the distribution of signals chosen by firm j and X^j be the support of σ^j.[11] We require that

$$X^j \subset \mathbb{R}_+, \tag{1}$$

and

$$E(X^j) = \varphi, \tag{2}$$

for each $j = 1, ..., F$.[12] We denote by Σ the set of signal distributions satisfying conditions (1)-(2), and we allow firms to choose any distribution in Σ. As mentioned, the objective of each firm j is to maximize its trading price p^j.

Investors ignore the fundamental values of the firms, and they employ a simple procedure in order to assess them. For each firm, they consider K independent random draws from the firm's signal distribution, and they interpret the average of these K signals as the firm's fundamental value.[13] Hence, if investor i observes signals $x_{i,1}^j, x_{i,2}^j, ..., x_{i,K}^j$ from firm j, his assessment of firm j's value is

$$\hat{x}_i^j = \frac{1}{K}\sum_{n=1}^{K} x_{i,n}^j.$$

The draws are assumed to be independent across investors.[14] Firms know that

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[11] Alternatively, σ^j may be interpreted as a sequence of activity reports with non-negative returns that add up to φ. We chose the distribution formulation because it is mathematically simpler to express.

[12] As already mentioned, condition (1) can be interpreted by saying that all investors have in mind a lower bound on the profitability of a single activity (here, normalized to 0). An alternative interpretation is that if on one activity the earnings report were too low, it would attract too much attention, and it would in the end be detrimental to the firm. Condition (2) can be motivated by investor protection considerations requiring that every single activity be included in the financial reporting but allowing the firm to disaggregate its reporting as it wishes.

[13] In our analysis, the same signal can be drawn several times. Yet, given that the firm can choose as many signals as it wishes, nothing would change if there were no replacements in the draws.

[14] From a theoretical perspective, note that such an assumption is the most favorable to

investors employ this procedure; in particular, firms know K.[15] We further assume that each investor is allowed to trade one unit of stock and he can either buy or short sell it.[16] Hence, investor i is willing to trade stock r if stock r is perceived to offer the highest gains from trade. That is, if

$$r \in \arg\max_{j} \left| p^j - \hat{x}_i^j \right|. \tag{3}$$

Investor i will buy stock r if $p^r < \hat{x}_i^r$ and he will short sell stock r if $p^r > \hat{x}_i^r$. Note that $\arg\max_j \left| p^j - \hat{x}_i^j \right|$ may sometimes consist of several stocks r, in which case investor i is indifferent between several options. In case of indifferences, a tie-breaking rule specifies the probability assigned to the various possible trades. We let Ω denote the set of tie-breaking rules and ω denote an element of Ω.

Based on investors' orders, and on the tie-breaking rule ω, demand and supply for firm j are denoted by $D^j(\sigma^j, \sigma^{-j}, p^j, p^{-j}, \omega)$ and $S^j(\sigma^j, \sigma^{-j}, p^j, p^{-j}, \omega)$, where σ^{-j} and p^{-j} denote respectively distributions and prices for all firms except j. We denote the profile of demand and supply for all firms as $D(\sigma, p, \omega)$ and $S(\sigma, p, \omega)$, where $\sigma = \{\sigma^j\}$ and $p = \{p^j\}$, $j = 1, .., F$. An equilibrium is defined as follows.

Definition 1 *(Equilibrium) The profile (σ, p, ω) is an equilibrium if: for each j, $\sigma^j \in \Sigma$, and*
a) $D(\sigma, p, \omega) = S(\sigma, p, \omega)$.
b) There is no distribution $\tilde{\sigma}^j \in \Sigma$, prices $\tilde{p}^j, \tilde{p}^{-j}$, and tie-breaking rule $\tilde{\omega} \in \Omega$ such that $D(\tilde{\sigma}^j, \sigma^{-j}, \tilde{p}^j, \tilde{p}^{-j}, \tilde{\omega}) = S(\tilde{\sigma}^j, \sigma^{-j}, \tilde{p}^j, \tilde{p}^{-j}, \tilde{\omega})$ and $\tilde{p}^j > p^j$.

The first condition requires that the markets clear. The second condition requires that there should be no profitable deviation for each firm j, where a profitable deviation $\tilde{\sigma}^j$ of firm j means that for the profile of distributions $(\tilde{\sigma}^j, \sigma^{-j})$, there exists a tie-breaking rule $\tilde{\omega}$ and prices $\tilde{p}^j, \tilde{p}^{-j}$ that clear the markets and such that firm j achieves a strictly higher price $\tilde{p}^j > p^j$.[17]

In the following analysis, we will prove the existence of an equilibrium (in a constructive manner). Discrete distributions with a finite number of signals will play an important role. We will denote by $\sigma = \{x_1, \mu_1; x_2, \mu_2; ..\}$ the distribution in which x_1 occurs with probability μ_1, x_2 occurs with probability μ_2, and so on.

3 Monopoly

We first consider a monopolistic firm facing investors who just consider one dimension in the financial report. That is, we set $F = 1$ and $K = 1$.

Since each investor only trades one stock, the market clearing price is the median of the firm's signals. At this price, half of the investors wants to buy and half of them wants to sell (so that the market clears). Hence, the monopoly's problem is to choose a distribution with the maximal median that satisfies the constraints (1) and (2) that signals should be non-negative and that the mean of the distribution should coincide with the fundamental φ.

Such a maximization is achieved with a two-signal distribution that puts weight on 0 and h and such that the median is just h (requiring that the weight on h is just above that on 0).[18] Consider then $\sigma = \{0, 1 - \mu; h, \mu\}$ with $\mu \geq 1/2$. The aggregation condition (2) implies that $\mu h = \varphi$, and thus the maximum price that can be achieved by the monopolist is 2φ. The following Proposition summarizes this.

Proposition 1 *Suppose $F = 1$ and $K = 1$. The firm chooses the distribution $\sigma^M = \{0, 1/2; 2\varphi, 1/2\}$. The price is $p^M = 2\varphi$.*

4 Monopoly and Sophistication

We now turn to a setting in which investors are more sophisticated in the sense of considering larger samples. More precisely, we consider a monopolistic firm and we assume that investors sample several ($K > 1$) signals in order to

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pectations about the ensuing market clearing prices). We chose this one to simplify the exposition of our results. Note also that any equilibrium as defined here would *a fortiori* be an equilibrium under the alternative specifications of profitable deviations.

[18] To see this, observe that any signal strictly above the median is a waste for the firm as reducing such a signal to the median while increasing all signals slightly so as to meet the aggregation condition would be profitable. Similarly, any signal strictly between 0 and the median is a waste, establishing the result.

evaluate the fundamental value of the firm. The question is whether the price gets close to the fundamental if we let K be sufficiently large.

Based on the law of large number, one might have expected that, for K large enough, investors would end up with a correct assessment of the fundamental value, and thus the market clearing price would have to be close to φ. Such an intuition would be true if the financial reporting strategy of the firm were set independently of K. But, this is not the relevant consideration here, given that the firm can adjust its financial reporting strategy to the number of draws K made by investors. Thus, the distribution chosen by the firm will typically change with K, and the law of large number can no longer be applied.

As we show now, the firm can always guarantee a price bounded away from the fundamental by a suitable choice of reporting strategy (that must depend on K by the previous argument).

To see this, consider the following two-signal distribution:

$$\sigma(K) = \left\{ 0, (1/2)^{1/K}; h, 1 - (1/2)^{1/K} \right\}, \tag{4}$$

and the price $p(K) = h/K$, with $h = \varphi/[1 - (1/2)^{1/K}]$ so that the mean of the distribution is φ.

An investor who gets K draws from the distribution and samples z times the signal h is willing to buy if the price does not exceeds zh/K. As the price equals h/K, only those who sample K times signal 0 are willing to sell, which is a proportion $[(1/2)^{1/K}]^K$ of investors. That is, at this price half of investors sell and half of the investors buy, so the market clears.

So given K, the monopolist can achieve a price of its stock no smaller than

$$p(K) = \frac{\varphi}{K[1 - (1/2)^{1/K}]}.$$

Simple algebra reveals that $p(K)$ is decreasing with K and that $p(K)$ converges to $\varphi/\ln 2$, which is strictly bigger than φ, as K grows arbitrarily large. Hence, we have established:

Proposition 2 *Suppose $F = 1$. Irrespective of K, the firm can attain a price no smaller than $\varphi/\ln 2$, which is strictly larger than φ.*

As described in (4), the distribution used to establish Proposition 2 requires that there is no upper bound on the signals that can be sent by the

firm ($h(K) = \varphi/[1 - (1/2)^{1/K}]$ goes to infinity as K goes to infinity). If there were an upper bound, the variance would have to be bounded, and the firm would not be able to obtain a price of its stock much away from the fundamental when K is large.

5 Competition

We now turn to investigate the effect of competition, i.e. having more than one firm $F > 1$. To keep the analysis tractable, we consider the case in which investors only consider one dimension in the financial reporting, i.e. $K = 1$.

Our main question of interest is whether more competition brings the prices of stocks closer to the fundamental values, and whether as there are many firms, there is still some distortion away from the fundamental.

We divide the investigation of this question into various subsections.

5.1 A non-transparency result

A first observation is that no matter how many firms are competing on the stock market, it cannot be an equilibrium that (all) firms choose a transparent financial reporting saying what their fundamental value is with probability 1. Indeed, if all firms send $\sigma = \{\varphi, 1\}$, then obviously the market clearing price for all stocks is $p = \varphi$. But, if firm j chooses the distribution displayed in the monopoly case $\sigma^j = \{0, 1/2; 2\varphi, 1/2\}$ (see Section 3), then firm j can achieve a price of its stock as high as 2φ, thereby showing that the deviation is profitable. This observation is summarized in the following Proposition.

Proposition 3 *Irrespective of F, there is no equilibrium in which firms report their fundamental value with probability* 1.

A second observation is that, irrespective of the strategy used by others, a firm can always guarantee that the price of its stock is at least the fundamental value. Indeed if firm j chooses $\sigma^j = \{\varphi, 1\}$ then $p^j = \varphi$ is necessarily a market clearing price for j (and there is no other possible market clearing price for j if some of the stocks j are to be traded).[19] Given our definition of an

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[19]This insight establishes within our setup that prices are more likely to exceed than to fall short of fundamentals. Such an asymmetry results in our model from firms' incentives to distort signals whenever they can induce prices above fundamentals while turning to full transparency if prices were to fall below fundamentals.

equilibrium, this shows the following Proposition:

Proposition 4 *In all equilibria, the price of stocks is no smaller than the fundamental value.*

5.2 The highest price equilibrium

Characterizing all equilibria is somewhat difficult because it requires getting into comparative statics properties of the Walrasian equilibria of the stock market as induced by the various possible choices of reporting strategies of the firms (which in turn affect in a complex way the demand and supply of the various stocks through the sampling strategy).[20]

For tractability reasons, we restrict our attention to symmetric equilibria. That is, we require that in equilibrium firms choose the same distribution of signals, the price of the various stocks is the same, and the tie-breaking rule is symmetric.[21] Moreover, among symmetric equilibria, we focus on the equilibrium that induces the highest prices of stocks. There are two ways to think of such a focus: 1) It highlights how much the prices can be far from the fundamental. 2) It is a natural benchmark equilibrium if we have in mind that the firms in the stock market can coordinate on the equilibrium they like best (a form of selection based on tacit collusion). We will also in the next subsection discuss other (symmetric) equilibria.

In order to characterize the highest price symmetric equilibrium, we proceed in several steps. We first consider a candidate symmetric equilibrium $\{\sigma, p, \omega\}$, with the distribution $\sigma = \{x_1, \mu_1; x_2, \mu_2; ...\}$. Suppose to start with that firm j faced no constraint in its choice of distribution so that it could choose the distribution $\tilde{\sigma}^j = \{x_1 + \varepsilon, \mu_1; x_2 + \varepsilon, \mu_2; ..\}$, which is derived by increasing all signals by ε and keeping the weights of the various signals unchanged. The prices $p^j = p + \varepsilon$ and $p^{-j} = p$ (together with the same tie-breaking rule ω) would clear the market,[22] and thus $\tilde{\sigma}^j$ would be a profitable deviation.

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[20] The theory of general equilibrium has essentially produced existence and efficiency results but very few instances in which Walrasian prices can be explicitly derived from the demand and supply structure. For our purpose, it is the latter that is required though.

[21] That is, if a mass μ of investors ends up with the same assessment about a set of N firms, each of these firms receives a fraction μ/N of the trades.

[22] This follows from the observation that all demand and supply decisions are based on the profiles of $p^j - \hat{x}_i^j$ and thus the increase of $\hat{x}_i^j$ by ε would be exactly offset by the increase of p^j by ε.

Of course, such a deviation is not feasible, as it implies $E(X^j) = \varphi + \varepsilon$. However, as we show next, the firm can implement a nearby deviation whenever there exists a signal $x > 0$ which is sent with positive probability and such that signal $2p - x$ is not in the support of the distribution. The proposed deviation is such that all signals but x are shifted by ε with the same weight and signal x is split between an upward shift of ε and a downward shift of δ, where δ and the weights of $x - \delta$ and $x + \varepsilon$ are chosen to satisfy the aggregation condition (2). One can then define a tie-breaking rule such that $p^j = p + \varepsilon$ and $p^{-j} = p$ clear the markets.[23] The following Lemma proven in the Appendix makes use of this observation:

Lemma 1 *Consider a symmetric equilibrium profile* $\{\sigma, p, \omega\}$. *If signal x is part of σ and $x > 0$, then also signal $\tilde{x}$ where $x + \tilde{x} = 2p$ is part of σ.*

Lemma 1 defines a class of candidate distributions for symmetric equilibrium profiles. In these distributions, positive signals need to be paired around the price. Specifically, we say that

$$\widehat{\sigma} \in \hat{\Sigma},$$

if $\widehat{\sigma} \in \Sigma$ (as defined by conditions (1) and (2)), and if there is no signal $x > 0$ which is part of $\widehat{\sigma}$ and such that signal $\tilde{x} = 2p - x$ is not part of $\widehat{\sigma}$. Given Lemma 1, in order to find out symmetric equilibria, we can focus on distributions in which there exists a $\widehat{\sigma} \in \hat{\Sigma}$ such that $\sigma^j = \widehat{\sigma}$ for all firms j.

The second step in our argument is to characterize the distribution in $\hat{\Sigma}$ that (together with the symmetric tie-breaking rule) induces the largest common market clearing price. We start by observing that to achieve the largest price, the distribution $\widehat{\sigma}$ in $\hat{\Sigma}$ should assign positive weight to at most three signals. To see this, suppose that $\widehat{\sigma}$ assigns positive weight to n signals and $n > 3$. Denote by p the corresponding price of stocks. Then one can

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[23]To get a sense of this, assume that $x - \delta > p$. As a result of the deviation, firm j loses the demand in case a signal $x - \delta$ is drawn together with a signal x from another firm (because now at the proposed prices the gains of trade are smaller with firm j than with the other firms). Such a lower demand for firm j can however be compensated by assigning more demand to firm j when signal $x + \varepsilon$ is drawn from firm j together with signal x from the non-deviating firms. For ε and δ sufficiently small, and since signal $2p - x$ is not part of the distribution, these are the only draws which are affected by the proposed deviation, and thus the prices $p^j = p + \varepsilon$ and $p^{-j} = p$ do clear the markets with the corresponding tie-breaking rule.

define another distribution $\tilde{\sigma} \in \hat{\Sigma}$ which involves at most $n-1$ signals and that induces a price $\tilde{p} \geq p$ (assuming again a symmetric tie-breaking rule and that $\sigma^j = \tilde{\sigma}$ for all j). The idea is to remove the two signals closest to the price and move their mass either to the price (if the weight of the higher of the two signals is no smaller than the weight of the smaller one) or to the adjacent signals further away from the price (if the weight of the smaller signal is bigger than the weight of the higher signal).[24]

Iterating the argument, one gets a distribution with at most three signals, $0, p, 2p$. Then, one can move equal mass from p to 0 and $2p$ or vice-versa and so end up with a two-signal distribution which takes one of the following forms: $\sigma_a = \{0, 1 - \mu_a; 2p_a, \mu_a\}$ or $\sigma_b = \{0, 1 - \mu_b; p_b, \mu_b\}$.[25]

Consider σ_a. Investors are indifferent between trading stock j and stock r whenever they sample signal $2p_a$ from firm j and signal 0 from firm r. The highest aggregate demand is obtained by letting investors buy j whenever indifferent between buying j and selling another stock. In that case, the aggregate supply includes only those who sample signal 0 from all firms, which has probability $(1 - \mu_a)^F$. Hence, market clearing requires $(1 - \mu_a)^F \leq 1/2$. If $(1 - \mu_a)^F < 1/2$, one can decrease slightly μ_a and increase all signals by ε and obtain a price which is ε higher. Hence, among distributions σ_a, the price is maximized by setting $(1 - \mu_a)^F = 1/2$. The highest market clearing price from distributions σ_a is thus obtained with

$$\sigma^* = \left\{ 0, (1/2)^{1/F}; \varphi/[1 - (1/2)^{1/F}], 1 - (1/2)^{1/F} \right\}, \qquad (5)$$

and the resulting market clearing price is

$$p^* = \frac{\varphi}{2[1 - (\frac{1}{2})^{1/F}]}. \qquad (6)$$

Consider $\sigma_b = \{0, 1 - \mu_b; p_b, \mu_b\}$. The aggregate demand equals at most those who sample signal p_b from all firms, so market clearing requires $(\mu_b)^F \geq 1/2$. Due to the aggregation condition (2), $p_b \leq \varphi(2)^{1/F}$ which is lower than p^*. This in turn leads to the next Lemma, whose detailed proof appears in the

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[24]Intuitively, such a move can be done while respecting the market clearing conditions. The direction of the move is then dictated so that the aggregation condition can be satisfied by moving all signals (except possibly 0) as well as the price upwards.

[25]The distribution $\sigma_c = \{p_c, 1 - \mu_c; 2p_c, \mu_c\}$ is easily ruled out given that by (2) it would imply that $p_c < \varphi$, which is clearly not the highest achievable price (see Proposition 4).

Appendix.

Lemma 2 *Suppose that there exists $\widehat{\sigma} \in \hat{\Sigma}$ such that $\sigma^j = \widehat{\sigma}$ for all j and consider a symmetric tie-breaking rule. The resulting market clearing price $\widehat{p}$ is no larger than p^*, as defined in (6). Moreover, p^* is obtained with the distribution σ^*, as defined in (5).*

Our last step is to show that $\sigma^j = \sigma^*$ together with $p^j = p^*$ for all j and the symmetric tie-breaking rule defines an equilibrium. The complete proof is a bit tedious as it requires checking all sorts of multi-signal deviations. Yet, given the above use of the deviation obtained by shifting all signals by ε, let us explain here why this is not profitable whenever firms choose the distribution σ^*. Suppose firm j deviates to a two signal-distribution concentrated on ε and $h + \varepsilon$ where $h = \varphi/[1 - (1/2)^{1/F}]$. By the aggregation condition, it should be that the weight on $h + \varepsilon$ is strictly smaller than $1 - (1/2)^{1/F}$. But, this would then imply that it is not possible to sustain $p^j = p^* + \varepsilon$ and $p^{-j} = p^*$ as market clearing prices given that the aggregate supply of all stocks would be strictly larger than $1/2$.[26] As it turns out, with such a deviation the market clearing prices would be $p^j = \varepsilon$ and $p^{-j} = 0$, and thus the deviation would not be profitable. Extending that kind of arguments, we can show:

Lemma 3 *There is a symmetric equilibrium in which firms choose the distribution σ^* and the price is p^*, as defined respectively in (5) and (6).*

Combining Lemmas 1, 2 and 3, we get:

Proposition 5 *Suppose $K = 1$ and $F > 1$. The maximal price achieved in a symmetric equilibrium is $p^* = \frac{\varphi}{2[1-(\frac{1}{2})^{1/F}]}$. This price increases in F, and $p^* \to \infty$ as $F \to \infty$.*

Proposition 5 shows that p^* increases with the number of competing firms. The reason is that in this equilibrium investors are induced to sell stock j only when they sample F low signals. For a given probability μ of high signal, the more firms, the lower the chance that the signals drawn from all firms are low. To clear the market, one should thus decrease μ, which by the aggregation condition requires pushing the high signal (and thus the price which is half the

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[26]There is surely supply whenever the draw from each distribution is the smaller signal, and this would have probability strictly larger than 1/2.

high signal) upwards. Thus, when considering the highest price equilibrium, more competition induces more distortion away from the fundamental. In the limit, the market clearing price can grow arbitrarily large as F goes to infinity.

A question arises as to how the market clearing price in the competitive case compares with the monopoly price (see Proposition 1). Simple calculations reveal that the price in the duopoly case is smaller than in the monopoly case, but the price for any other market structure configuration ($F > 2$) is larger than in the monopoly case.[27]

5.3   Other Equilibria

As mentioned before, we think of the highest price symmetric equilibrium as the most meaningful one based on tacit collusion considerations. Yet, abstracting from such considerations, we wish here to highlight that there may be other (symmetric) equilibria. To illustrate this, we exhibit a symmetric equilibrium that induces a market clearing price as low as the fundamental (which combined with Propositions 4 and 5 allows us to show the range of market clearing prices that can be sustained in symmetric equilibria). More precisely, we have:

Proposition 6 *Suppose $K = 1$. For every $F > 1$, there is a symmetric equilibrium with market clearing prices $p = \varphi$. The common distribution of signals has support $(0, 2\varphi)$. It is centered around φ, and it is such that the probability of sampling $F - 1$ signals within distance z from φ is linear in z. When $F = 2$, it is the uniform distribution on $(0, 2\varphi)$.*

To get some intuition for the Proposition, consider the duopoly case $F = 2$. If firm 1 chooses a uniform distribution of signals between 0 and 2φ, it is not hard to see that irrespective of the choice of distribution of firm 2, the market clearing price for firm 1 must be $p_1 = \varphi$. Indeed at this price, and given the symmetry of the distribution of firm 1 around φ, there is as much demand as there is supply for firm 1. More important for our purpose though is the observation that when firm 1 chooses such a distribution, the market clearing price of firm 2 cannot be larger than φ. If the support of the distribution of firm 2 coincides with $(0, 2\varphi)$, one can show that the market clearing price

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[27] These considerations imply that if a monopolistic firm could split its activity into several companies with different stocks, it would benefit from it given the heuristic of the investors.

of the two firms will have to be φ, and any positive measure of signal above 2φ would lead to a strictly lower price for firm 2. This in turn establishes Proposition 6 for the duopoly case and the argument can be generalized for an arbitrary number of firms (see the Appendix).

Two further comments about the equilibrium displayed in Proposition 6 are worth mentioning. First, as F increases, the corresponding distribution of signals becomes more concentrated around φ (so for this equilibrium, more competition eventually induces financial reports that get close to reporting the fundamental value with probability 1). Second, the equilibrium shown in Proposition 6 suffers from the following fragility. While the equilibrium requires that firms choose a distribution with continuous density, an obvious alternative (and simpler) best-response would be for the firms to choose a distribution putting mass 1 on the fundamental value. Yet, if firms were to choose such a financial reporting strategy, this would not be an equilibrium (see Proposition 3).

6 Discussion

6.1 On investors' rationality

The above analysis has assumed a specific form of bounded rationality on investors' investment strategy. In this Subsection, we review three different sets of issues. 1) Do we need bounded rationality to produce our results? 2) What if investors use alternative heuristic procedures to make their investment decisions? 3) What if investors use heterogeneous procedures to make their investment decisions?

6.1.1 Is bounded rationality needed?

This is a standard (and legitimate) question to be asked about models which introduce some form of bounded rationality. In our economy, there is no fundamental uncertainty, as firms have a uniquely defined fundamental value. Thus, if investors were fully rational, there would be no point for firms to be strategic on their financial reporting and trades would necessarily occur at the fundamental value.

One might consider enriching the setup by introducing some asymmetric information with only the firms knowing their fundamental values, and the

17

financial reporting playing the role of how much information firms are willing to release about their value. Even maintaining the assumption of full rationality of investors, there are different ways of modeling this, either assuming that firms can commit ex ante to whatever disclosure sounds best (as in Kamenica and Gentzkow (2011), Rayo and Segal (2010) or Jehiel (2011)) or alternatively assuming that the disclosure strategy is chosen at the time the firms know the realization of their private information (as in models of cheap talk, à la Crawford and Sobel (1982)). While a complete analysis of such models goes beyond the scope of this paper, it sounds unlikely (based on previous works) that they would deliver insights similar to ours: 1) If trades are purely speculative as in our model, one could not generate trades based on the classic no trade argument; 2) If noise traders are added (as in Grossman and Stiglitz (1980)), it sounds plausible that such models would deliver the classic insight that for asymptotically large economies prices aggregate well the dispersed information and thus correctly represent the fundamental values of firms.

To sum up, while there are obviously rational approaches to the modeling of strategic information transmission, it is dubious that such approaches can deliver insights similar to ours.

6.1.2 On alternative heuristic procedures

Our main reason for studying the above specific form of bounded rationality is that we believe it reflects a general tendency agents have about extrapolating from small samples, and our aim was to investigate the impact this could have in financial markets. Yet, there are several additional ingredients that could be added to the considerations of investors. For example, investors could consider that the price itself is indicative of the fundamental value. Alternatively, investors could base their estimate of the fundamental value not only on the part of the financial reporting they pay attention to but also on the market sentiment (De Long, Shleifer, Summers and Waldmann (1990a)). Finally, investors could take into account that their estimate is noisy and adjust their investment decision accordingly.

There are several possible ways to incorporate such ideas into the heuristic of investors. We propose some that allow us to preserve our qualitative analysis, thereby suggesting some form of robustness of our results with respect to richer specifications of investors' procedures.

In the main model, investor i made an estimate of the fundamental value of firm j based on the average sample signal $\hat{x}_i^j$ from j. Suppose instead that investor i assesses the fundamental value of firm j according to

$$v_i^j = \lambda_i p^j + (1 - \lambda_i)\hat{x}_i^j,$$

where $\lambda_i \in [0, 1)$ reflects the subjective weight attached by investor i to the informativeness of the price relative to the informativeness of the private signal $\hat{x}_i^j$. Trading j would be assessed to give gains of $\left|p^j - v_i^j\right| = (1 - \lambda_i)\left|p^j - \hat{x}_i^j\right|$ and thus, our previous analysis would apply equally to this new specification.

Regarding the idea of "market sentiment", we may model the latter as the average belief of the various investors about the profitability of the firm. Given that as already noted, the mean of the financial reporting distribution has to coincide with the fundamental, the average belief about firm j corresponds to its fundamental value. Thus, an investor i receiving an average sample $\hat{x}_i^j$ from j would assess firm j according to

$$v_i^j = \lambda_i \varphi + (1 - \lambda_i)\hat{x}_i^j$$

where $\lambda_i \in [0, 1)$ represents the weight given by investor i to the market sentiment. The gains from trade attached to asset j would be perceived to be $\left|p^j - \lambda_i \varphi - (1 - \lambda_i)\hat{x}_i^j\right|$, and the main messages of our previous analysis would remain qualitatively the same.[28]

Finally, we could incorporate the idea that investors would take into account that their estimate of the fundamental value is noisy. For example, when investors draw several signals $K > 1$, instead of simply considering the mean of the signal and reason as if it were the fundamental value, investors could also consider the empirical variance in the sample and reason as if the fundamental value was a random variable normally distributed with mean and variance coinciding with the corresponding empirical values in the sample. With risk neutral investors (as we assumed) this would have no consequence. With risk-averse investors, it is not clear a priori in which way our main analysis would change given that both buying and short selling would be

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[28] As K grows large in the monopoly case, the price would be bounded by $\lambda_i \varphi + (1 - \lambda_i)\varphi / \ln 2$, which exceeds φ. In the oligopoly case with $K = 1$, as F grows large, the maximal price sustainable in equilibrium would be $\lambda_i \varphi + (1 - \lambda_i)p^*$, where p^* denotes the price characterized in Section 5.

perceived as risky. A more systematic investigation of such heuristics should be the subject of future work.

6.1.3 Playing on investors' cognitive types

In the case of multiple firms, we assumed that $K = 1$. Analyzing fully a general setting with arbitrary K is a bit cumbersome. Yet, we conjecture that similar insights obtain, and, in particular, more competition may still drive the price further away from the fundamental in the $K > 1$ case. Following the logic of our previous analysis, one can show that, for any F, there is a market clearing price which is bounded away from the fundamental no matter how large is K. Moreover, irrespective of K, the market clearing price can grow arbitrarily large as F grows large.[29]

In the main analysis, all investors were assumed to have the same K. What if investors can have different K? Dealing in general with the case of heterogeneous populations is quite involved. We consider here the special case in which investors are either fully rational ($K = \infty$) with probability α or they are assumed to follow the K^*- sampling procedure with probability $1 - \alpha$. We still assume that investors whether fully rational or boundedly rational can trade only one stock. As in our setting the fundamental value of each firm is deterministic, rational investors know it with certainty. Given that the price is typically above the fundamental value, rational investors would all go for short selling. Our previous equilibrium constructions should then be modified by adding a fraction α to the aggregate supply. Yet, it is not difficult to show that, provided α is not too large, our previous insights carry through.[30]

6.2 Alternative objectives for firms

In our main model, firms were assumed to maximize the price of their stocks. We think this is a natural objective for managers whose managerial compensation is to a non-negligible extent indexed on the value of the stock. Yet, there may be other motives driving the choice of financial reporting strategy. For example, reputation considerations may lead firms to look as transparent

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[29] This can be shown by applying a very similar argument as in Sections 4 and 5 to the distribution $\sigma = \{0, 1 - \mu; h, \mu\}$, where μ is defined by market clearing as $(1 - \mu)^{FK} = 1/2$.

[30] To see this, one can apply the same analysis as above and just modify the market clearing conditions. In Section 4, market clearing would require $(1 - \mu)^{K^*}(1 - \alpha) + \alpha = 1/2$. In Section 5, it would require $(1 - \mu)^F (1 - \alpha) + \alpha = 1/2$.

as possible, which would clearly alleviate the distortions highlighted in this paper. Alternatively, managers may try to look better than their competitors. We analyze briefly this case assuming that there are two firms, $F = 2$ and that the sample size of investors is $K = 1$.

Firm j is now assumed to maximize $p^j - p^{-j}$.[31] This modified objective function leads to a different game between firms, and in the $F = 2$ case, the resulting game is a symmetric zero-sum game whose value is 0 (since picking the same strategy as the competitor gives in expectation 0).

Yet, knowing the value of the game does not tell us what the resulting price of stocks is. Exploiting the analysis of Section 5, we now observe that there is an equilibrium in which the price coincides with the fundamental $p = \varphi$, which is sustained by having the two firms using a uniform distribution between 0 and 2φ, that is, $\sigma^R \sim U(0, 2\varphi)$. This follows from the proof of Proposition 6, in which we show that if firm j chooses σ^R, then we have $p^j = \varphi$ and $p^{-j} \leq \varphi$ for any distribution chosen by firm $-j$. Hence, no firm can get a price higher than the competitor when the competitor chooses σ^R.[32]

Proposition 7 *Suppose $F = 2$ and firms seek to maximize relative price. There is a symmetric equilibrium in which firms choose the uniform distribution on $(0, 2\varphi)$ and the price of stocks is φ.*

We suspect that the equilibrium shown in Proposition 7 is the only equilibrium.[33] If so, the change of objective of firms from absolute to relative price would affect dramatically the conclusion. Now competition brings the prices of stocks down to the fundamental (even if again firms do not use a transparent financial reporting strategy).

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[31] Condition (b) in the definition of equilibrium should be modified accordingly by requiring that there is no distribution $\tilde{\sigma}^j \in \Sigma$, prices $\tilde{p}^j, \tilde{p}^{-j}$, and tie-breaking rule $\tilde{\omega} \in \Omega$ such that $D(\tilde{\sigma}^j, \sigma^{-j}, \tilde{p}^j, \tilde{p}^{-j}, \tilde{\omega}) = S(\tilde{\sigma}^j, \sigma^{-j}, \tilde{p}^j, \tilde{p}^{-j}, \tilde{\omega})$ and $\tilde{p}^j - \tilde{p}^{-j} > p^j - p^{-j}$.

[32] Surprisingly, Myerson (1993) obtains the same uniform distribution as an equilibrium in the Blotto game (with a continuum of battle fields). While the aggregation condition is clearly the same in the two problems, the market clearing condition seems to have no analog in the Blotto game, making the connection unexpected.

[33] We were able to show that there is no equilibrium with discrete distributions (as moving all signals by ε and changing slightly the weights would induce a profitable deviation). We suspect there is no other equilibrium with continuous density, thereby explaining the conjecture.

7　Conclusion

This paper has considered a stylized financial market in which firms may strategically frame their financial reports so as to induce higher stock prices. We have illustrated how the introduction of less sophisticated (though not stupid) investors in such a setting could alter dramatically the analysis of market efficiency. Capital market competition has been shown to be ineffective in ensuring that prices are close to fundamentals. We note that it is unclear what kind of regulation could improve the working of the financial reporting business. A form of investor protection requesting that overall there should be no lie in the financial reporting was shown in this paper not to restore market efficiency. Of course, distilling the idea to investors and managers that transparency and simplicity of financial reporting are desirable could alleviate the problems we have highlighted, but it is unclear whether just relying on such vague recommendations could be enough.

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8 Appendix

Proof of Proposition 1

As shown in the text, the price $p^M = 2\varphi$ clears the market when the firm sends the distribution $\sigma^M = \{0, 1/2; 2\varphi, 1/2\}$. We now show that no distribution induces a higher price. Suppose that the firm sends the distribution $\sigma = \{x_0, \mu_0; x_1, \mu_1; x_2, \mu_2; ..; x_N, \mu_N\}$ with $0 = x_0 < x_1 < x_2 < ... < x_N$ and $\mu_n \geq 0$ for $n = 0, ..., N$. (We consider a discrete distribution for simplicity of notation, the argument is the same if we consider continuous distributions.) Market clearing requires that the price is the median of the distribution. If there are several medians (because of the discreteness of the distribution),

then considering the largest median is enough to characterize the largest market clearing price. Thus, we let $p = x_N$ if $\mu_N \geq 1/2$; $p = x_{N-1}$ if $\mu_N < 1/2$ and $\mu_N + \mu_{N-1} \geq 1/2$; and more generally for $n \in [1, N-1]$

$$p = x_n, \text{ if } \sum_{w=0}^{N-n-1} \mu_{N-w} < 1/2 \text{ and } \sum_{w=0}^{N-n} \mu_{N-w} \geq 1/2.$$

Maximizing x_n while satisfying the above constraints and the constraint in (2) requires setting $\mu_w = 0$ for all $w \in [1, n-1]$. Moreover, by setting $x_w = x_n$ for all $w \in [n+1, N]$, x_n can be increased, and so p can be increased, while still satisfying condition (2). Hence, we are left with a distribution $\sigma = \{0, 1 - \mu; x_n, \mu\}$ with $\mu \geq 1/2$. Condition (2) requires $x_n \leq \varphi/\mu$. As we need $\mu \geq 1/2$ to have $p > 0$, it follows that $x_n \leq 2\varphi$. Thus, no alternative distribution can induce a price higher than p^M. **Q. E. D.**

Proof of Lemma 1

Consider the equilibrium profile $\{\sigma, p, \omega\}$ and suppose that there exists a signal $x > 0$ which is part of σ (that is, to which the distribution σ assigns positive mass) and such that signal $\tilde{x} = 2p - x$ is not part of σ. Let μ_x denote its weight. We show that one firm (say firm j) has a profitable deviation. In particular, suppose firm j deviates to the distribution σ^ε, in which all signals are increased by ε and all weights are kept constant except the one on signal x. The weight on signal x is split into two signals, $x + \varepsilon$ with mass μ_ε and $x - \delta$ with mass $\mu_x - \mu_\varepsilon$, where $\varepsilon > 0$ is small and δ is defined by condition (2) so that $\mu_x x = \mu_\varepsilon(x + \varepsilon) + (\mu_x - \mu_\varepsilon)(x - \delta) + (1 - \mu_x)\varepsilon$. We then show that there exists a tie-breaking rule $\tilde{\omega}$ and a μ_ε such that the market clears with $p^j = p + \varepsilon$ and $p^{-j} = p$.

To establish this, suppose that $x > p$ (the argument is symmetric with $x < p$). Denote by μ_z the total mass of signals that are at a distance to the price smaller than x is. That is, $\mu_z = \sum_{\{n \text{ s.t. } 2p-x < x_n < x\}} \mu_n$. The tie-breaking rule $\tilde{\omega}$ after the deviation is the same as that before the deviation except when the draw for firm j is $x + \varepsilon$ and (some) draws from firms $-j$ are x in which case we assume the entire demand (if any) goes to firm j (remember we are postulating that the prices after the deviation are $p^j = p + \varepsilon$ and $p^{-j} = p$, hence the ties for the corresponding draws).

The demand for each firm before the deviation can be written as

$$D = \mu_x \sum_{y=0}^{F-1} \binom{F-1}{y} \frac{1}{F-y} (\mu_x)^{F-1-y} (\mu_z)^y + W,$$

where W is unaffected by the proposed deviation (provided that ε and δ are sufficiently small).

The new demand for the deviating firm j is

$$D^j = \mu_\varepsilon \sum_{y=0}^{F-1} \binom{F-1}{y} (\mu_x)^{F-1-y} (\mu_z)^y + (\mu_x - \mu_\varepsilon)(\mu_z)^{F-1} + W,$$

assuming that ε and δ are sufficiently small.

The assumption that there is no signal $2p - x$ guarantees that the total demand (of all firms) is the same after and before the deviation and so is the total supply, provided that ε and δ are small enough (which will be checked ex post). Thus, it is enough to check that at prices $p^j = p + \varepsilon$ and $p^{-j} = p$, we can have $D = D^j$ for ε and δ small enough. The condition $D = D^j$ can be written as

$$\mu_x \sum_{y=0}^{F-2} \binom{F-1}{y} \frac{1}{F-y} (\mu_x)^{F-1-y} (\mu_z)^y = \mu_\varepsilon \sum_{y=0}^{F-2} \binom{F-1}{y} (\mu_x)^{F-1-y} (\mu_z)^y.$$

That is,

$$\mu_\varepsilon = \frac{\sum_{y=0}^{F-2} \binom{F-1}{y} \frac{1}{F-y} (\mu_x)^{F-1-y} (\mu_z)^y}{\sum_{y=0}^{F-2} \binom{F-1}{y} (\mu_x)^{F-1-y} (\mu_z)^y} \mu_x. \tag{7}$$

It is readily verified from (7) that $\frac{\mu_x}{F} < \mu_\varepsilon < \frac{\mu_x}{2}$, which implies that $\delta \to 0$ as $\varepsilon \to 0$, and thus, δ and ε can be chosen arbitrarily small while satisfying (7) and condition (2).

Hence, if firm j deviates to $x + \varepsilon$ with mass μ_ε and $x - \delta$ with mass $\mu_x - \mu_\varepsilon$, where μ_ε is defined in (7), firm j can get $p^j = p + \varepsilon$ (while $p^{-j} = p$), which shows that we cannot have a $x > 0$ which belongs to the equilibrium strategy σ and such that signal $\hat{s} = 2p - x$ is not part of σ. **Q. E. D.**

Proof of Lemma 2

Suppose all firms send the same distribution $\sigma \in \hat{\Sigma}$ and consider a symmetric tie-breaking rule. Denote by p the market clearing price. Suppose that σ assigns positive mass to $2n + 1$ signals, 0, x_1^-, x_2^-, .., x_n^-, x_n^+, .., x_2^+, x_1^+ with $0 \le x_1^- < x_2^- < .. < x_n^- < p < x_n^+ < .. < x_2^+ < x_1^+$ and $x_t^+ + x_t^- = 2p$ for all $t = 1, .., n$. Suppose there are also atomless parts of the distribution over the intervals $[a_1^-, b_1^-]$ and $[b_1^+, a_1^+]; ...; [a_n^-, b_n^-]$ and $[b_n^+, a_n^+]$, where $0 \le a_1^- < b_1^- < .. < a_n^- < b_n^- < p < b_n^+ < a_n^+ < .. < b_1^+ < a_1^+$ and $a_t^+ + a_t^- = b_t^+ + b_t^- = 2p$ for all $t = 1, .., n$. In steps 1-4, we show that one can induce a price $\hat{p} \ge p$ by sending at most two signals. For this purpose, it is convenient to define $\hat{X}$ as the set of signals x in the support of the distribution such that there exists a signal $2p - x$ in the support of the distribution, that is

$$\hat{X} = \left\{ x \in \sigma : x \ge \min \left\{ x_1^-, a_1^- \right\} \right\}.$$

Let μ_0 denote the weight attached by σ on signal 0. The set $\hat{X}$ is then simply the set of strictly positive signals in σ if $\min \left\{ x_1^-, a_1^- \right\} > 0$ and $\mu_0 > 0$ or the set of all signals in σ if $\min \left\{ x_1^-, a_1^- \right\} = 0$. In step 5, we show that no distribution with at most two signals induces a price higher than p^*, as defined in (6). We conclude that p^* is the maximal market clearing price when firms choose a distribution $\sigma \in \hat{\Sigma}$.

Step 1. Consider signal x_n^-, x_n^+. Suppose $\mu_{x_n^+} \ge \mu_{x_n^-}$ and $b_n^- < x_n^-$; that is, there is no atomless part of the distribution at a lower distance from the price (we consider the atomless parts of the distribution in step 3 below). Then one can induce a price $p + \Delta_1$, where $\Delta_1 \ge 0$ will be defined below, by first moving x_n^+ and x_n^- to p and then moving all signals $x \in \hat{X}$ up by Δ_1. To show this, we first show that by moving x_n^+ and x_n^- to p one can induce the same market clearing price p and employ a signal distribution whose average is lower than φ. Then, we can move all signals $x \in \hat{X}$ up by Δ_1 to obtain a price $p + \Delta_1$ with a signal distribution in $\hat{\Sigma}$ whose average is φ. To see this, suppose firms send signal p with weight $\mu_{x_n^+} + \mu_{x_n^-}$ instead of sending signals x_n^+ and x_n^-. Those who sample signal p for all firms are indifferent between buying and selling. Denote as τ_1 the fraction of them who buy. Suppose first that, before the change in the distribution, whenever an investor sampled signal x_n^+ from firm j and signal x_n^- from firm $\hat{j}$ he bought stock j. The old aggregate demand

is

$$D_1 = \sum_{y=1}^{F} \binom{F}{y} (\mu_{x_n^+})^y (\mu_{x_n^-})^{F-y} + Z_1,$$

where Z_1 depends on the signals further away from p and is unaffected with the proposed change. The new aggregate demand (after the change) is

$$\hat{D}_1 = \tau_1 (\mu_{x_n^+} + \mu_{x_n^-})^F + Z_1.$$

Market clearing at p requires $\hat{D}_1 = D_1$, that is

$$\tau_1 = \frac{\sum_{y=1}^{F} \binom{F}{y} (\mu_{x_n^+})^y (\mu_{x_n^-})^{F-y}}{(\mu_{x_n^+} + \mu_{x_n^-})^F}.$$

Notice that $\tau_1 < 1$ since $(\mu_{x_n^+} + \mu_{x_n^-})^F = \sum_{y=0}^{F} \binom{F}{y} (\mu_{x_n^+})^y (\mu_{x_n^-})^{F-y}$ and that exceeds $\sum_{y=1}^{F} \binom{F}{y} (\mu_{x_n^+})^y (\mu_{x_n^-})^{F-y}$. Suppose instead that whenever an investor sampled signal x_n^+ from firm j and signal x_n^- from firm $\hat{j}$ he sold stock $\hat{j}$. The old demand is $\tilde{D}_1 = (\mu_{x_n^+})^F + Z_1$ and so market clearing at p requires $\tilde{D}_1 = \hat{D}_1$, that is $\tau_1 = (\mu_{x_n^+})^F / (\mu_{x_n^+} + \mu_{x_n^-})^F$, where again $\tau_1 < 1$.

Notice now that $\mu_{x_n^+} x_n^+ + \mu_{x_n^-} x_n^- \geq (\mu_{x_n^+} + \mu_{x_n^-}) p$ since by definition $x_n^- = 2p - x_n^+$ and so that requires $x_n^+ (\mu_{x_n^+} - \mu_{x_n^-}) \geq p(\mu_{x_n^+} - \mu_{x_n^-})$, that is $\mu_{x_n^+} \geq \mu_{x_n^-}$. Hence, we can define

$$\Delta_1 = \frac{1}{1 - \mu_0} [\mu_{x_n^+} x_n^+ + \mu_{x_n^-} x_n^- - (\mu_{x_n^+} + \mu_{x_n^-}) p],$$

and move all signals $x \in \hat{X}$ up by Δ_1 so as to satisfy condition (2) and have a price $p + \Delta_1$. Note that the resulting distribution still belongs to $\hat{\Sigma}$. The same will be true in the next steps.

Step 2. The procedure in step 1 can be repeated until one considers signals x_m^-, x_m^+ where $m \equiv max_t \left\{ t : \mu_{x_t^+} < \mu_{x_t^-} \right\}$ (if $\mu_{x_n^+} < \mu_{x_n^-}$, then $m = n$), or until one encounters an atomless part of the distribution at a lower distance from the price. Suppose one ends up with weight μ_{p_2} on signal p_2 and market clearing requiring that a fraction τ_2 of those who sample signal p_2 for all firms buy. Consider first x_m^-, x_m^+. Following the same logic of step 1, one can move

28

x_m^- to x_{m-1}^- and x_m^+ to x_{m-1}^+ and then move all signals $x \in \hat{X}$ up by some $\Delta_2 \geq 0$ so as to induce a price $p_2 + \Delta_2$. To see this, consider the following weights: $\hat{\mu}_{x_{m-1}^-} = \mu_{x_m^-} + \mu_{x_{m-1}^-} - k_2$ and $\hat{\mu}_{x_{m-1}^+} = \mu_{x_m^+} + \mu_{x_{m-1}^+} - k_2$; and p_2 with probability $\hat{\mu}_{p_2} = \mu_{p_2} + 2k_2$. Suppose a fraction τ_m of those who sample signal $\tilde{p}$ for all firms buy. We wish to define a $k_2 \in (0, \mu_{x_m^+})$ and a $\tau_m \in (0,1)$ such that p_2 clears the market. Suppose first that whenever an investor samples signal x_m^+ from firm j and signal x_m^- from firm $\hat{j}$ he buys stock j and similarly for signals x_{m-1}.[34] The pre-change aggregate demand is

$$D_2 = \sum_{y=1}^{F} \binom{F}{y} (\mu_{x_{m-1}^+})^y (\mu_{x_{m-1}^-} + \mu_{x_m^-} + \mu_{x_m^+} + \mu_{p_2})^{F-y}$$
$$+ \sum_{y=1}^{F} \binom{F}{y} (\mu_{x_m^+})^y (\mu_{x_m^-} + \mu_{p_2})^{F-y} + \tau_2 (\mu_{p_2})^F + Z_2.$$

The new aggregate demand (considering the same symmetric tie-breaking rule after the change of distribution) is

$$\hat{D}_2 = \sum_{y=1}^{F} \binom{F}{y} (\mu_{x_{m-1}^+} + \mu_{x_m^+} - k_2)^y (\mu_{x_{m-1}^-} + \mu_{x_m^-} + \mu_{p_2} + k_2)^{F-y} + \tau_m (\mu_{p_2} + 2k_2)^F + Z_2.$$

Suppose $k_2 = 0$ and $\tau_m = 1$. Notice that

$$\hat{D}_2 - D_2 = [(\mu_{x_{m-1}^-} + \mu_{x_m^-} + \mu_{x_m^+} + \mu_{p_2})^F - (\mu_{x_{m-1}^-} + \mu_{x_m^-} + \mu_{p_2})^F$$
$$- (\mu_{x_m^+} + \mu_{x_m^-} + \mu_{p_2})^F + (\mu_{x_m^-} + \mu_{p_2})^F] + [(1 - \tau_2)(\mu_{p_2})^F].$$

The first term in square brackets is positive (because of the convexity of $x \to x^F$ for $F \geq 2$); the second term is also positive. Hence, $\hat{D}_2 > D_2$ when $k_2 = 0$ and $\tau_m = 1$. Let instead $k_2 = \mu_{x_m^+}$ and $\tau_m = 0$. Then $\hat{D}_2 - D_2 = -\sum_{y=1}^{F} \binom{F}{y} (\mu_{x_m^+})^y (\mu_{x_m^-} + \mu_{p_2})^{F-y} - \tau_2 (\mu_{p_2})^F < 0$. Hence there exists a $k_2 \in (0, \mu_{x_m^+})$ and a $\tau_m \in (0,1)$ such that $\hat{D}_2 = D_2$.

Suppose instead that, before the change, whenever an investor sampled signal x_m^+ from firm j and signal x_m^- from firm $\hat{j}$ he sold stock $\hat{j}$ and similarly

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[34]We can wlog assume the indifferences are broken in the same way when x_m^- vs x_m^+ or x_{m-1}^- vs x_{m-1}^+ are drawn by satiating demand in one or the other.

for signals x_{m-1}. The old aggregate demand is

$$\tilde{D}_2 = \sum_{y=1}^{F} \binom{F}{y} (\mu_{x_{m-1}^+})^y (\mu_{x_m^-} + \mu_{x_m^+} + \mu_{p2})^{F-y} + \sum_{y=1}^{F} \binom{F}{y} (\mu_{x_m^+})^y (\mu_{p2})^{F-y} + \tau_2 (\mu_{p2})^F + Z_2,$$

and the new aggregate demand (considering the same symmetric tie-breaking rule after the change of distribution) is

$$\bar{D}_2 = \sum_{y=1}^{F} \binom{F}{y} (\mu_{x_{m-1}^+} + \mu_{x_m^+} - k_2)^y (\mu_{x_m^-} + \mu_{p2} + k_2)^{F-y} + \tau_m (\mu_{p2} + 2k_2)^F + Z_2.$$

Similarly to above, suppose $k_2 = 0$ and $\tau_m = 1$, then since $(\mu_{x_m^-} + \mu_{x_m^+} + \mu_{p2})^F - (\mu_{x_m^-} + \mu_{p2})^F - (\mu_{x_m^+} + \mu_{p2})^F + (\mu_{p2})^F$ is positive and $(\mu_{p2})^F > \tau_2(\mu_{p2})^F$, we have $\hat{D}_2 > D_2$. If instead $k_2 = \mu_{x_m^+}$ and $\tau_m = 0$ then $\bar{D}_2 < \tilde{D}_2$ and so there exists a $k_2 \in (0, \mu_{x_m^+})$ and a $\tau_m \in (0,1)$ such that $\bar{D}_2 = \tilde{D}_2$.

Notice now that, following the same procedure as in the step 1, $\mu_{x_m^+} x_m^+ + \mu_{x_{m-1}^+} x_{m-1}^+ + \mu_{x_m^-} x_m^- + \mu_{x_{m-1}^-} x_{m-1}^- \geq (\mu_{x_m^+} + \mu_{x_{m-1}^+} - k_2) x_{m-1}^+ + (\mu_{x_m^-} + \mu_{x_{m-1}^-} - k_2) x_{m-1}^- + 2p_2 k_2$ is equivalent to $\mu_{x_m^-} \geq \mu_{x_m^+}$. Hence, we can define

$$\Delta_2 = \frac{1}{1 - \mu_0} [\mu_{x_m^+} x_m^+ + \mu_{x_{m-1}^+} x_{m-1}^+ + \mu_{x_m^-} x_m^- + \mu_{x_{m-1}^-} x_{m-1}^-$$
$$- (\mu_{x_m^+} + \mu_{x_{m-1}^+} - k_2) x_{m-1}^+ - (\mu_{x_m^-} + \mu_{x_{m-1}^-} - k_2) x_{m-1}^- - 2p_2 k_2],$$

where $\Delta_2 \geq 0$, and move all signals $x \in \hat{X}$ up by Δ_2 so as to satisfy condition (2) and have a price $p_2 + \Delta_2$.

Step 3. Suppose one encounters an atomless part of the distribution and there is no other signal at a lower distance from the price. Suppose the price is p_3 and consider the distribution with density $g(x)$ over the interval $[a_n^-, b_n^-]$ and density $h(x)$ over $[b_n^+, a_n^+]$. The logic of the previous steps can be applied by dividing the intervals $[a_n^-, b_n^-]$ and $[b_n^+, a_n^+]$ into sufficiently small subintervals. Consider first the intervals $[b_n^- - \varepsilon, b_n^-]$ and $[b_n^+, b_n^+ + \varepsilon]$, where ε is small. Let $\mu^+ = \int_{b_n^+}^{b_n^+ + \varepsilon} h(x) dx$ and $\mu^- = \int_{b_n^- - \varepsilon}^{b_n^-} g(x) dx$ and let x^+ denote the average over $[b_n^+, b_n^+ + \varepsilon]$, that is $x^+ = \frac{1}{\mu^+} \int_{b_n^+}^{b_n^+ + \varepsilon} x h(x) dx$, and similarly let

30

$x^- = \frac{1}{\mu^-} \int\limits_{b_n^- - \varepsilon}^{b_n^-} xg(x)dx$. Suppose first that $\mu^+ > \mu^-$ and one moves all signals $x \in [b_n^- - \varepsilon, b_n^-]$ and all $x \in [b_n^+, b_n^+ + \varepsilon]$ to p_3. Applying the logic in Step 1, the moving is profitable if

$$\mu^+ x^+ + \mu^- x^- > (\mu^+ + \mu^-)p,$$

that is $\mu^+(x^+ - p) > \mu^-(p - x^-)$. For $\varepsilon \to 0$, $x^+ \to b_n^+$ and $x^- \to b_n^-$, where by definition $b_n^- = 2p - b_n^+$. Hence, the previous inequality writes as $\mu^+(b_n^+ - p) > \mu^-(b_n^+ - p)$, that is the case since $\mu^+ > \mu^-$. Similarly, suppose that $\mu^+ < \mu^-$ and one moves all signals $x \in [b_n^- - \varepsilon, b_n^-]$ to $b_n^- - \varepsilon$ and all $x \in [b_n^+, b_n^+ + \varepsilon]$ to $b_n^+ + \varepsilon$. Applying the logic in Step 2, the moving is profitable if

$$\mu^+ x^+ + \mu^- x^- > \mu^+(b_n^+ + \varepsilon) + \mu^-(b_n^- - \varepsilon),$$

that is $\mu^+(b_n^+ + \varepsilon - x^+) < \mu^-(x^- - b_n^- + \varepsilon)$. For $\varepsilon \to 0$, the previous inequality writes as $\mu^+(\varepsilon) < \mu^-(\varepsilon)$, that is the case since $\mu^+ < \mu^-$. Finally, suppose that $\mu^+ = \mu^-$ for all $\varepsilon \in [0, a_n^+ - b_n^+]$. Consider then $\varepsilon = a_n^+ - b_n^+$. We have $x^+ = 2p - x^-$ and so the same price p_3 can be obtained by moving all the mass μ^+ into x^+ and all the mass μ^- into x^-.

Step 4. The argument in steps 1-3 can be iterated until one obtains a distribution $0, x_1^-, p_4, x_1^+$, with $x_1^+ = 2p_4 - x_1^-$ and $x_1^- \geq 0$ with weights $\mu_0, \mu_{x_1^-}, \mu_{p_4}, \mu_{x_1^+}$. Suppose $\mu_{x_1^-} < \mu_{x_1^+}$. Then one can increase the price by repeating the argument in step 1 and moving x_1^- and x_1^+ to p_4. If $\mu_0 = 0$, we would end up with a one-signal distribution. If $\mu_0 > 0$, we would end up with a two-signals distribution with signals 0 and $\hat{p}_4$. Suppose instead $\mu_{x_1^-} \geq \mu_{x_1^+}$. Then one could increase the price by repeating the argument in step 2 and moving x_1^- to 0 and x_1^+ to $2p_4$. We would end up with a three-signals distribution with $0, \bar{p}, 2\bar{p}$. Now consider the distribution $0, \bar{p}, 2\bar{p}$, with weights respectively $\tilde{\mu}_0, \mu_p, \mu_{2p}$. The aggregate supply is at least

$$S_4 = \sum_{y=1}^{F} \binom{F}{y} (\mu_p)^{F-y} (\tilde{\mu}_0)^y = (\tilde{\mu}_0 + \mu_p)^F - (\mu_p)^F.$$

We show that there exists a two-signals distribution inducing a larger price. Suppose a mass k_4 is moved from $\bar{p}$ to 0 and a mass k_4 is moved from $\bar{p}$ to

$2\bar{p}$. Condition (2) holds and there exists a tie breaking rule so that the new aggregate supply is

$$\tilde{S}_4 = \sum_{y=1}^{F} \binom{F}{y}(\tilde{\mu}_0 + k_4)^y(\mu_p - 2k_4)^{F-y} = (\tilde{\mu}_0 + \mu_p - k_4)^F - (\mu_p - 2k_4)^F.$$

That induces a higher price if $\tilde{S}_4 < S_4$, that is the case if $\tilde{S}_4$ decreases in k_4 at $k_4 = 0$. Taking the derivative of $\tilde{S}_4$ with respect to k_4, we need that

$$\frac{d\tilde{S}_4}{dk_4} = (2)^{1/(F-1)}(\mu_p - 2k_4) - (\tilde{\mu}_0 + \mu_p - k_4) < 0.$$

Notice that $d\tilde{S}_4/dk_4$ is decreasing in k_4 $(1 - 2^{\frac{F}{F-1}} < 0$ for all $F \geq 2)$, hence if $d\tilde{S}_4/dk_4 < 0$ at $k_4 = 0$ then it is negative everywhere. Hence, we need that

$$(2)^{1/(F-1)}(\mu_p) \leq (\tilde{\mu}_0 + \mu_p). \tag{8}$$

If condition (8) holds, setting $k_4 = \mu_p/2$ we obtain a two-signals distribution which induces a higher price. Suppose instead

$$(2)^{1/(F-1)}(\mu_p) > (\tilde{\mu}_0 + \mu_p), \tag{9}$$

and suppose a mass k_4 is moved from 0 to $\bar{p}$ and a mass k_4 is moved from $2\bar{p}$ to $\bar{p}$. Condition (2) holds and there exists a tie breaking rule so that the new aggregate supply is

$$\hat{S}_4 = \sum_{y=1}^{F} \binom{F}{y}(\tilde{\mu}_0 - k_4)^y(\mu_p + 2k_4)^{F-y} = (\tilde{\mu}_0 + \mu_p + k_4)^F - (\mu_p + 2k_4)^F.$$

Similarly to above, that induces a higher price if $\hat{S}_4 < S_4$, that is the case if $\hat{S}_4$ decreases in k_4 at $k_4 = 0$. Taking the derivative of $\hat{S}_4$ with respect to k_4, we need that

$$\frac{d\hat{S}_4}{dk_4} = (2)^{1/(F-1)}(\mu_p + 2k_4) - (\tilde{\mu}_0 + \mu_p + k_4) > 0.$$

Notice that $d\hat{S}_4/dk_4$ is increasing in k_4, hence if $d\hat{S}_4/dk_4 > 0$ at $k_4 = 0$ then it is positive everywhere. Hence, we need that $(2)^{1/(F-1)}(\mu_p) > (\tilde{\mu}_0 + \mu_p)$, that is indeed condition (9). If condition (9) holds, setting $k_4 = \min\{\tilde{\mu}_0, \mu_{2p}\}$

32

we obtain a two-signals distribution which induces a higher price. Hence, the highest market clearing price is obtained with a two-signals distribution.

Step 5. We are then left with a two-signals distribution which takes one of the following forms: $\sigma_a = \{0, 1 - \mu_a; 2p_a, \mu_a\}$ or $\sigma_b = \{0, 1 - \mu_b; p_b, \mu_b\}$ or $\sigma_c = \{p_c, 1 - \mu_c; 2p_c, \mu_c\}$. For the argument developed in the main text, among those distributions, the highest price is p^*, as defined in (6), and it is achieved by σ^*, as defined in (5). **Q. E. D.**

Proof of Lemma 3

We show that σ^* are p^*, as defined respectively in (5) and (6), are part of an equilibrium. To simplify the exposition, denote with h the positive signal which is part of σ^* and with μ its weight, that is

$$h = \varphi/[1 - (1/2)^{1/F}], \text{ and } \mu = 1 - (1/2)^{1/F}.$$

First, we show that there exists a tie-breaking rule such that (σ^*, p^*) clears the market. Suppose that whenever indifferent between buying firm r and selling another firm j the investor buys r. Then since $p^* = h/2$ only those who sample a signal 0 for all firms sell. The aggregate supply is $(1 - \mu)^F$ that equals $1/2$. Hence the aggregate demand is also $1/2$, i.e. aggregate demand equals aggregate supply. As the equilibrium is symmetric, that implies that the market clears for each firm.

Consider now the possibility of deviations. Suppose firm j deviated by sending n signals $\sigma^j = \{x_n, \mu_n\}$, we then show that there is no combination of weights μ_n such that the market clears and $p^j > h/2$. We first consider the case in which, after the deviation, all non deviating firms (we denote them as $-j$) are traded at the same price, which we denote as p (we later consider the possibility of assigning different prices to firms $-j$). It is first useful to distinguish those signals, which we call marginal, that make investors indifferent between trading j and another firm, that is the set of $x_n : |p^j - x_n| = p$ or $|p^j - x_n| = h - p$. Since firms $-j$ send two signals, there can be at most four marginal signals in σ^j. We denote them as a, b, c, d with $a < b < c < d$. Non-marginal signals will be denoted as $x_0, x_1, ..., x_n$ with $0 \leq x_0 < x_1 < ... < x_n$. Denote with $\mu_0 = \sum_{x_n < a} \mu_n$, $\mu_1 = \sum_{x_n \in (a,b)} \mu_n$, $\mu_2 = \sum_{x_n \in (b,c)} \mu_n$, $\mu_3 = \sum_{x_n \in (c,d)} \mu_n$ and $\mu_4 = \sum_{x_n > d} \mu_n$. As we do not impose that any of these weights is strictly positive, this formulation includes the case in which firm j sends any number of signals. (As it will be clear, this

formulation also includes the case in which σ^j has atomless parts.)

Suppose first that $p > h/2$. The aggregate demand for $-j$ is at most $D^{-j} = \mu^{F-1}(\mu_b + \mu_c + \mu_2)$, while the aggregate supply for $-j$ is at least $S^{-j} = (1 - \mu^{F-1})(\mu_b + \mu_c + \mu_2 + \mu_1 + \mu_3)$. Since $\mu^{F-1} < 1 - \mu^{F-1}$ (for all $F \geq 2$), there is always excess supply in $-j$. So we cannot have $p > h/2$.

Suppose now $p \leq h/2$. Suppose there exists a market clearing price $p^j = h/2 + \varepsilon$, with $\varepsilon > 0$. Due to the definition of marginal signal, the following relations hold: $p^j - a = h - p$ and so $a = \varepsilon + p - h/2$; $p^j - b = p$ and so $b = h/2 + \varepsilon - p$; $c - p^j = p$ and so $c = h/2 + \varepsilon + p$ and $d - p^j = h - p$ and so $d = 3h/2 + \varepsilon - p$.

Because of requirement (2), it must be that

$$\Psi = (\frac{h}{2} + \varepsilon - p)(\mu_b + \mu_2)$$
$$+ (\frac{h}{2} + \varepsilon + p)(\mu_c + \mu_3) + (\frac{3h}{2} + \varepsilon - p)(\mu_d + \mu_4) \leq \varphi, \quad (10)$$

where Ψ is derived by letting $x_n \to 0$ for all $x_n < b$, $x_n \to b$ for all $x_n \in (b, c)$, $x_n \to c$ for all $x_n \in (c, d)$, $x_n \to d$ for all $x_n > d$. Since signals cannot be negative, their average cannot fall short of Ψ. Hence, if $\Psi > \varphi$, the distribution σ^j would violate condition (2).

The highest p^j is achieved when all investors buy stock j whenever indifferent between buying j and trading $-j$ and they instead trade a stock in $-j$ whenever indifferent between selling j and trade $-j$ since in this way aggregate demand for j would be maximized. In this case, market clearing in j requires

$$\mu_4 + \mu_d + (\mu_3 + \mu_c)(1 - \mu)^{F-1} \geq \mu_0 + (\mu_1 + \mu_a)(1 - \mu)^{F-1}. \quad (11)$$

We also need that aggregate demand equals aggregate supply, which requires

$$\mu_4 + \mu_d + (\mu_3 + \mu_c)(1 - \mu)^{F-1} + (1 - (1 - \mu)^{F-1})(\mu_b + \mu_2 + \mu_1 + \mu_a + \mu_3 + \mu_c) \geq$$
$$\mu_0 + (\mu_1 + \mu_a)(1 - \mu)^{F-1} + (\mu_b + \mu_2)(1 - \mu)^{F-1}. \quad (12)$$

In (12), the left hand side is the maximal aggregate demand while the right hand side is the minimal aggregate supply, as derived by letting investors buy whenever indifferent between buying and selling. Equation (12) can be

rewritten as

$$\mu_4 + \mu_d + \mu_3 + \mu_c \geq \mu_0 + (\mu_1 + \mu_a + \mu_b + \mu_2)(2(1-\mu)^{F-1} - 1), \qquad (13)$$

which implies $\mu_4 + \mu_d + \mu_3 + \mu_c \geq (\mu_0 + \mu_1 + \mu_a + \mu_b + \mu_2)(2(1-\mu)^{F-1} - 1)$ since $2(1-\mu)^{F-1} - 1 \leq 1$. Substituting $\mu_0 = 1 - \mu_1 - \mu_a - \mu_b - \mu_2 - \mu_4 - \mu_d - \mu_3 - \mu_c$, we have

$$(\mu_4 + \mu_d + \mu_3 + \mu_c)2(1-\mu)^{F-1} \geq 2(1-\mu)^{F-1} - 1, \qquad (14)$$

that is

$$\mu_4 + \mu_d + \mu_3 + \mu_c \geq \mu. \qquad (15)$$

Suppose first $d\Psi/dp < 0$. Then it is necessary that condition (10) holds when $p \to h/2$ and $\varepsilon \to 0$, that is we need $h(\mu_c + \mu_3 + \mu_d + \mu_4) < \varphi$, that is

$$\mu_c + \mu_3 + \mu_d + \mu_4 < \mu, \qquad (16)$$

which is inconsistent with (15).

Suppose instead $d\Psi/dp \geq 0$. Then it is necessary that condition (10) holds when $p \to 0$ and $\varepsilon \to 0$. That writes

$$(1/2)(\mu_b + \mu_2 + \mu_c + \mu_3) + (3/2)(\mu_d + \mu_4) \leq \mu. \qquad (17)$$

That is $(1/2)(1 - \mu_1 - \mu_a - \mu_0) + (\mu_d + \mu_4) \leq \mu$, or $1 - 2\mu + 2(\mu_d + \mu_4) \leq \mu_1 + \mu_a + \mu_0$. Since $(1-\mu)^{F-1} \leq 1$, it follows from (11) that $(\mu_0 + \mu_1 + \mu_a)(1-\mu)^{F-1} \leq \mu_4 + \mu_d + (\mu_3 + \mu_c)(1-\mu)^{F-1}$. That is

$$\mu_0 + \mu_1 + \mu_a \leq \frac{\mu_4 + \mu_d}{(1-\mu)^{F-1}} + (\mu_3 + \mu_c).$$

Combining the two, $1 - 2\mu + 2(\mu_d + \mu_4) \leq \frac{\mu_4 + \mu_d}{(1-\mu)^{F-1}} + (\mu_3 + \mu_c)$, that is

$$1 - 2\mu \leq \left(\frac{1}{(1-\mu)^{F-1}} - 2\right)(\mu_d + \mu_4) + (\mu_3 + \mu_c). \qquad (18)$$

Since $\frac{1}{(1-\mu)^{F-1}} < 2$ for all F, that implies $\mu_3 + \mu_c \geq 1 - 2\mu$. But $c > h/2$ and $(1 - 2\mu)h/2 = h/2 - \varphi > \varphi$ for $F > 2$ so the requirement $\mu_3 + \mu_c \geq 1 - 2\mu$ violates condition (2) when $F > 2$.

Suppose $F = 2$, $p \leq h/2$ and $d\Psi/dp \geq 0$. The demand for firm j cannot exceed $\mu_4 + \mu\mu_d + (1-\mu)(\mu_d + \mu_3) + \tau_1(1-\mu)\mu_c$ which is derived by allocating

all the orders to firm j in case investors draw signal d from j and signal h from $-j$ and in which τ_1 is the fraction of orders allocated to firm j in case investors draw signal c from j and signal 0 from $-j$. The supply for firm j cannot fall short of $\mu_0 + (1-\mu)(\mu_a + \mu_1) + \tau_2\mu\mu_a$, which is derived by allocating all the orders to firm $-j$ in case investors draw signal b from j and signal 0 from $-j$ and in which τ_2 is the fraction of orders allocated to firm j in case investors draw signal a from j and signal h from $-j$. Hence, market clearing for firm j requires

$$\mu_4 + \mu\mu_d + (1-\mu)(\mu_d + \mu_3) + \tau_1(1-\mu)\mu_c \geq \mu_0 + (1-\mu)(\mu_a + \mu_1) + \tau_2\mu\mu_a. \quad (19)$$

Similarly, market clearing for firms $-j$ requires

$$\mu(\mu_c + \mu_b + \mu_3 + \mu_2 + \mu_1) + (1-\tau_2)\mu\mu_a \leq (1-\mu)\mu_2 + (1-\mu)\mu_b + (1-\tau_1)(1-\mu)\mu_c. \quad (20)$$

From these two equations, it follows that the left hand side of (19) plus the right hand side of (20) exceeds the the right hand side of (19) plus the left hand side of (20). Substituting $\mu_0 = 1 - \mu_1 - \mu_a - \mu_b - \mu_2 - \mu_4 - \mu_d - \mu_3 - \mu_c$ and simplifying, we obtain

$$(2 - 2\mu)(\mu_b + \mu_2) + (2 - 2\mu)(\mu_c + \mu_3) + 2(\mu_d + \mu_4) \geq 1. \quad (21)$$

In order to satisfy (17), it is then necessary that $2 - 2\mu \geq 1/2\mu$ or that $2 \geq 3/2\mu$. Both of these requirements are violated by μ.

Finally, suppose $F > 2$ and non-deviating firms are traded at a different price. Suppose two non-deviating firms, say firm 1 and firm 2, are traded respectively at prices p_1 and p_2. Assume wlog that $p_1 < p_2$. Let D_1 and D_2 denote the demand respectively for firm 1 and 2, and S_1 and S_2 the corresponding supply. Suppose first that $p_1 < h/2 < p_2$. Let κ denote the probability that, for all firms $r \neq 2$, no signal s^r from firm r is drawn such that $|s^r - p^r| > p_2$. Since $p_2 > h/2$, κ is not smaller than the probability that, for all firms $r \neq 2$, no signal s^r from firm r is drawn such that $|s^r - p^r| \geq h - p_2$. Hence, we have $S_2 \geq (1-\mu)^2\kappa$ and $D_2 \leq \mu(1-\mu)\kappa$, so

$$\frac{S_2}{D_2} \geq \frac{(1-\mu)^2}{\mu(1-\mu)},$$

but $(1-\mu)^2 > \mu(1-\mu)$ since $\mu < 1/2$ for all F. Hence, $S_2 > D_2$ and there is

no market clearing for firm 2 when $p_1 < h/2 < p_2$. Suppose instead $p_2 \leq h/2$. When signal 0 from firm 1 is drawn together with signal 0 from firm 2 an investor prefers trading stock 2 (since $p_1 < p_2$) and when signal 0 from firm 1 is drawn with signal h from firm 2 an investor prefers trading stock 2 (since $p_1 < h - p_2$). Hence, $S_1 = 0$ and firm 1 is not traded. Similarly, if $p_1 \geq h/2$ an investor always prefers trading stock 1 whenever signal h is drawn from firm 2. Hence, $D_2 = 0$ and firm 2 is not traded.

We are then left with a market in which $F - 1$ firms are traded. Suppose that, among them, the non-deviating firms are traded at the same price p. We can then repeat the above argument employing $F-1$ instead of F. As we show, the argument holds a fortiori since μ decreases in F. Suppose first that $p > h/2$. The aggregate demand for $-j$ is at most $D^{-j} = \mu^{F-2}(\mu_b + \mu_c + \mu_2)$, while the aggregate supply for $-j$ is at least $S^{-j} = (1 - \mu^{F-2})(\mu_b + \mu_c + \mu_2 + \mu_1 + \mu_3)$. Since $\mu^{F-2} < 1 - \mu^{F-2}$ (for all $F \geq 3$), there is always excess supply in $-j$. So we cannot have $p > h/2$.

Suppose now $p \leq h/2$ and $d\Psi/dp < 0$. Following the same logic behind equation (14) and substituting $F - 1$ instead of F, we need $(\mu_4 + \mu_d + \mu_3 + \mu_c)2(1 - \mu)^{F-2} \geq 2(1 - \mu)^{F-2} - 1$, that is

$$\mu_4 + \mu_d + \mu_3 + \mu_c \geq 1 - \frac{1}{2(1 - \mu)^{F-2}}.$$

Since $1 - \frac{1}{2(1-\mu)^{F-2}} > \mu$, it is necessary that condition (15) holds, which is however inconsistent with (16).

If instead $p \leq h/2$ and $d\Psi/dp \geq 0$, following the logic behind condition (18) substituting $F - 1$ instead of F, we obtain

$$1 - 2\mu \leq \left(\frac{1}{(1 - \mu)^{F-2}} - 2\right)(\mu_d + \mu_4) + (\mu_3 + \mu_c),$$

and since $\frac{1}{(1-\mu)^{F-2}} < 2$, that requires $1 - 2\mu \leq \mu_3 + \mu_c$. As shown above, however, the last condition violates condition (2).

Finally, suppose that among the $F - 1$ traded firms there exist two non-deviating firms which are traded at a different price. Then one can repeat the above argument and end up with $F - 2$ traded firms. Iterating, we would end up with 2 traded firms. Let p be the price of the non-deviating firm. The argument to rule out $p > h/2$ is unchanged. Also the argument to rule out $p \leq h/2$ is the same than the case with $F = 2$ above (conditions (16) and (21)

are violated for all F). We conclude that there is no profitable deviation and so the profile (σ^*, p^*) is part of an equilibrium. **Q. E. D.**

Proof of Proposition 6

Suppose firms send a distribution with support on $[0, 2\varphi]$, density g symmetric around φ and cdf G such that $[1 - 2G(x)]^{F-1} = 1 - x/\varphi$ for $x < \varphi$ and $[2G(x) - 1]^{F-1} = x/\varphi - 1$ for $x \geq \varphi$.

Step 1. Market clearing requires $p = \varphi$. In fact, at $p = \varphi$, aggregate demand is

$$D = F \int_{\varphi}^{2\varphi} g(x)[2G(x) - 1]^{F-1} dx = F \frac{1}{\varphi} \int_{\varphi}^{2\varphi} g(x)(x - \varphi) dx,$$

while aggregate supply is

$$S = F \int_{0}^{\varphi} g(x)[1 - 2G(x)]^{F-1} dx = F \frac{1}{\varphi} \int_{0}^{\varphi} g(x)(\varphi - x) dx.$$

Market clearing requires $D = S$, that is

$$\int_{0}^{\varphi} g(x)(\varphi - x) dx = \int_{\varphi}^{2\varphi} g(x)(x - \varphi) dx,$$

which is the case since by definition $\int_{0}^{2\varphi} g(x) dx = 1$ and $\int_{0}^{2\varphi} xg(x) dx = \varphi$ due to condition (2).

Step 2. There is no profitable deviation. To see this, suppose firm j deviates to a distribution H with density h. Denote with p^j the price of j and with p the price of non-deviating firms. Notice first that market clearing requires $p = \varphi$. In fact, if $p = \varphi$ aggregate demand for non-deviating firms is

$$D^{-j} = (F - 1) \int_{\varphi}^{2\varphi} g(x)[2G(x) - 1]^{F-2} [H(p^j + x - \varphi) - H(\varphi + p^j - x)] dx,$$

while aggregate supply is

$$S^{-j} = (F-1)\int\limits_{0}^{\varphi} g(x)[1 - 2G(x)]^{F-2}[H(p^j + \varphi - x) - H(p^j + x - \varphi)]dx.$$

Notice that by symmetry of g, for any $x \leq \varphi$ there exists a signal $v = 2\varphi - x$ such that $g(x) = g(v)$. Hence, $G(x) = 1 - G(v)$, $H(x - \varphi) = H(\varphi - v)$ and $H(\varphi - x) = H(v - \varphi)$. Hence,

$$\int\limits_{0}^{\varphi} g(x)[1 - 2G(x)]^{F-2}[H(p^j + \varphi - x) - H(p^j + x - \varphi)]dx =$$

$$\int\limits_{\varphi}^{2\varphi} g(v)[2G(v) - 1]^{F-2}[H(p^j + v - \varphi) - H(p^j + \varphi - v)]dv,$$

that is $D^{-j} = S^{-j}$ for $p = \varphi$. To see that $p = \varphi$ is the only market clearing price for non-deviating firms, suppose $p > \varphi$. Then the new aggregate demand is

$$\hat{D}^{-j} = (F-1)\int\limits_{p}^{2\varphi} g(x)[G(x) - G(2p - x)]^{F-2}[H(p^j + x - p) - H(p + p^j - x)]dx,$$

but notice that $p > \varphi$ implies $G(x) - G(2p - x) < G(x) - G(2\varphi - x)$ and $H(p^j + x - p) - H(p + p^j - x) > H(p^j + x - \varphi) - H(\varphi + p^j - x)$, so it must be that $\hat{D}^{-j} < D^{-j}$. Similarly, the new aggregate supply is

$$\hat{S}^{-j} = (F-1)\int\limits_{0}^{p} g(x)[G(2p - x) - G(x)]^{F-2}[H(p^j + p - x) - H(p^j + x - p)]dx,$$

where $\hat{S}^{-j} > S^{-j}$. Hence, there is excess supply and so $p > \varphi$ does not clear the market. The argument which rules out $p < \varphi$ is symmetric. Suppose then $p^j = \varphi$. The demand for j would be

$$D^j = \int\limits_{\varphi}^{\infty} h(x)[2G(x) - 1]^{F-1}dx = \frac{1}{\varphi}\int\limits_{\varphi}^{2\varphi} h(x)(x - \varphi)dx + \int\limits_{2\varphi}^{\infty} h(x)dx,$$

while the supply of j would be

$$S^j = \int_0^\varphi h(x)[1 - 2G(x)]^{F-1}dx = \frac{1}{\varphi}\int_0^\varphi h(x)(\varphi - x)dx.$$

We claim that, $D^j \leq S^j$ at $p^j = \varphi$. Since $\int_0^\infty xh(x)dx = \varphi$ and $\int_0^\infty h(x)dx = 1$, that writes as

$$2\varphi\int_{2\varphi}^\infty h(x)dx \leq \int_{2\varphi}^\infty xh(x)dx,$$

which is indeed the case. Since $D^j \leq S^j$ at $p^j = \varphi$, it must be that $p^j \leq \varphi$. Hence, there is no profitable deviation. **Q. E. D.**