Costs and Benefits of Dynamic Trading in a Lemons Market

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Abstract

We study a dynamic market with asymmetric information that induces the lemons problem. We compare efficiency of the market under different assumptions about the timing of trade. We show that when efficiency can be improved by temporarily closing the market as compared to continuous trading opportunities.

1 Introduction

Consider an owner of an asset who is facing liquidity needs and would like to sell the asset in a market where buyers compete. The seller is privately informed about the value of the asset. Although there is common knowledge of gains from trade, the buyers would not be willing to pay the average value if at that value the highest seller type would not be willing to trade. Hence, the competitive equilibrium price must be lower. As pointed out in the seminal paper by Akerlof (1970) this logic leads to an inefficiently low amount of trade.

The implicit assumption in Akerlof's model is that the seller has a unique opportunity to interact with the potential buyers. If the offered price is rejected there are no further opportunities to trade. It is natural to think that in many instances buyers will get additional opportunities to trade with the seller. The possibility of being able to access the market again reduces the incentives of the seller to sell for two reasons: (1) by rejecting the first offer the

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buyers update upwards their posterior about the seller's type and are willing to make higher offers in the future (2) the fact that there is another opportunity to sell the good reduces the costs of not reaching an agreement. Both of these forces lead to less trade in the first period. Although this would decrease efficiency, allowing for an extra period might allow some higher types that would not have traded in the one shot model to trade in the second period. Depending on how many of these types are around and how the surplus from trade is distributed across types opening the market for a second round of trade might be either good or bad for efficiency.

In this paper we study the optimality of allowing for more opportunities to trade than the single initial offer. In addition, we also allow for the possibility that the adverse selection problem be only short lived. That is, there is some date T at which the seller's type is revealed and trade can take place efficiently. In the Akerlof model $T = \infty$ but there might be situations were a finite T is a more natural assumption.[1]

We start our analysis with an example with linear valuations and uniform distribution of types and show that the market with restricted trading opportunities (allowing trades only at zero and at T) generates higher expected gains from trade than a market where continuous trading is possible. In general, there is a tradeoff: under dynamic (continuous) trading the lemons market problem gets worse and hence even fewer types than in the classic static model trade initially. On the other hand, the buyers can use time to screen the seller types and eventually more types can trade in the continuous case. Indeed we show by construction that one can find combinations of distributions and valuations for which continuous trading dominates restricted trading if the informational asymmetry is long lived.

Intuitively, the restricted market design is useful because it gives a lot of incentives to trade at time zero (there is a big surplus loss -from delay- if trade does not take place). On the other hand, because all the types that trade at zero must receive the same price this is a very blunt tool to separate the different types. With continuous trading we have the extreme opposite, there are little instantaneous incentives to trade but we can smoothly screen all types and hence eventually generate more trade. For continuous trading to dominate we need two things (1) that the equilibrium with restricted trading leave a lot of valuable trading opportunities unconsummated and (2) that frequent trading would lead to these trades taking place not too far away in the future.

One could naturally ask: Why restrict the analysis to the extreme cases of a market that is always open or one that closes after the initial offer and does not re-open until the information

[1] In Section 5.1 we consider information arriving at random time.

is revealed? Indeed, one could imagine a market that opens and closes several times might be better than either extreme. To address this possibility we use a mechanism design approach which allows for any of these possibilities. We characterize a sufficient condition (related to monotone marginal revenue condition in optimal auctions literature) under which infrequent trading dominates all other possible trading designs not just continuous trading.

What if this condition is not met? We show in Proposition 5 that a market design in which the initial trading opportunity at $t = 0$ is followed by a "market closure" (or a lockup period) of size Δ generates higher welfare than continuous trading. We establish this by showing that for small Δ the total volume of trade is larger under the "market closure" design. Using the same techniques as in the proof of Proposition 5 we also show that if the private information is short-lived (i.e. T is small) then, again in general, a market with continuous trading is strictly less efficient than the restricted-trading market.

When a market is designed to restrict trade not at time 0 but as some intermediate interval (for example, no trade in the time interval (t^*, T) before the information arrives at T) then new effects appear. As with the restriction at time zero, closing the market at $t^* > 0$ helps reduce the lemons problem at that time and generates a mass of trade at closure. In turn, this mass creates an endogenous "quiet period" before t^*. The intuition is as follows. Competition drives the price at t^* to the expected value of the good conditional on the range of types that trade at this time. This expectation is strictly higher than the value corresponding to the lowest type in that range. If the market were screening types continuously before t^*, prices would jump up at t^*. That cannot happen in equilibrium since some seller types would have an incentive to wait for the jump. As a result, we show that the equilibrium must have an additional endogenous period of no trade before t^* even though the market design allows for trade. This endogenous quiet period and the associated delay in trade can erode all the benefits from the acceleration of trade at t^*. Indeed, we can construct examples for which trading pauses after time zero can either increase or decrease welfare. In either case the gains and losses are very small. In practice there are several cases of restrictions to trading just before information is revealed. Unlike the case of the lockup periods after initial trade, our model does not provide a first-order justification for them.

Beyond regulating the frequency with which the market is open, the government might want to intervene directly in the market. We have seen several of these interventions during the recent financial crisis. The government could offer guarantees to put a lower bound on the return of certain assets -this was done with the debt issues by several companies and as part of some of the takeover deals of financially distressed banks. The direct purchase of real estate loan portfolios from banks by the government has been done in Ireland and is being discussed as a remedy for the unfolding Spanish banking crisis. Our setting allows us to study the impact of these policies. As we know from Akerlof (1970) there could be complete unraveling and no trade even when there is only one opportunity to trade. This would arise for example if the seller's value/cost is proportional to the buyer's value and the support includes goods for which there are no gains from trade. By offering to provide insurance or by buying low quality assets directly, the government can kick-start the market. The post-intervention market would then have strictly positive gains from trade even for the lowest types and hence there would be some trade in equilibrium. If the market is open continuously and T is large most types would eventually trade. This highlights another interesting effect that seems to be absent of most of the public discussion about the government bailouts. It is not just the banks that participate in the asset buyback or debt guarantee programs that benefit from the government's intervention. The whole financial sector benefits because liquidity is restored to markets and even that helps even non-lemons that manage to realize gains from trade thanks to the intervention.

2 The Model

As in the classic market for lemons, a potential seller owns one unit of an indivisible asset. When the seller holds the asset it generates for him a revenue stream $c \in [0,1]$ that is private information of the seller. c is drawn from a distribution $F(c)$, which is common knowledge, atomless and has a continuous, strictly positive density $f(c)$.

There is a competitive market of potential buyers. Each buyer values the asset at $v(c)$ which is strictly increasing, thrice differentiable, $v(c) > c$ for all $c < 1$ (i.e. common knowledge of gains from trade) and $v(1) = 1$ (i.e. no gap on the top). These assumptions imply that in the static Akerlof (1970) problem some types will trade, but that the lemons problem is present and not all the types trade in equilibrium.[2]

Time is $t \in [0,T]$ and we consider different market designs in which the market is opened in different moments in that interval. We start the analysis with two extreme market designs: "infrequent trading" (or "restricted trading") in which the market is opened only twice at $t \in \{0,T\}$ and "continuous trading" in which the market is opened in all $t \in [0,T]$. Let $\Omega \subset [0,T]$ denote the set of times that the market is open (we assume that at the very minimum $\{0,T\} \subset \Omega$).

Every time the market is opened buyers make public price offer to the seller and the seller

[2] This allows us not to have to worry about out of equilibrium beliefs after a history where all types where supposed to accept but trade did not take place. We discuss these assumptions further in Section 5.3.

either accepts one of them (which ends the game) or rejects and the game moves to the next time the market is opened. If no trade takes place by time T the type of the seller is revealed and the price in the market is $v(c)$, at which all seller types trade.

All players discount payoffs at a rate r and we let $\delta = e^{-rT}$. The values c and $v(c)$ are normalized to be in total discounted terms. Therefore, if trade happens at time t at a price p_t then the seller payoff is

$$\left(1 - e^{-rt}\right) c + e^{-rt} p_t$$

and the buyer's payoff is

$$e^{-rt} \left(v(c) - p_t\right)$$

A competitive equilibrium of this market is a pair of functions $\{p_t, k_t\}$ for $t \in \Omega$ where p_t is the market price at time t and k_t is the highest type of the seller that trades at time t.[3] These functions satisfy:

(1) Zero profit condition: $p_t = E\left[v(c) | c \in [k_{t-}, k_t]\right]$ where k_{t-} is the cutoff type at the previous time the market is open before t (with $k_{t-} = 0$ for the first time the market is opened)[4]

(2) Seller optimality: given the process of prices each seller type maximizes profits by trading according to the rule k_t.

(3) No (Unrealized) Deals: in any period the market is open, the price is at least $p_t \geq v(k_{t-})$ since it is common knowledge that the value of the seller asset is at least that much (this condition removes some trivial multiplicity of equilibria, for example an equilibrium in which $p_t = k_t = 0$ for all periods).

Condition (3) is a weaker notion of the No Unrealized Deals Condition in Daley and Green (2011) (see Definition 2.1 there).[5]

We assume that the players publicly observe all the trades and hence after a buyer obtains the object, if he tries to put it back on the market the market can infer something about c based on the history. Since all buyers value the good at the same amount, there will not be any profitable trade between buyers after the first transaction with the seller and hence we ignore that possibility in our model.

[3] Since we know that the skimming property holds in this environment it is simpler to directly define the competitive equilibrium in terms of cutoffs.

[4] In continuous time we use a convention $k_{t-} = \lim_{s \uparrow t} k_s$, and $E\left[v(c) | c \in [k_{t-}, k_t]\right] = \lim_{s \uparrow t} E\left[v(c) | c \in [k_s, k_t]\right]$ and $v(k_{t-}) = \lim_{s \uparrow t} v(k_s)$.

[5] We could use their condition but this weaker and simpler to verify condition is sufficient in our setting.

3 Motivating Example

Before we present the general analysis of the problem, consider the following example. c is distributed uniformly over $[0,1]$ and $v(c) = \frac{1+c}{2}$.

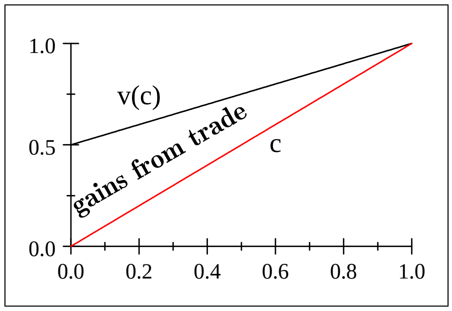

Figure 1

We compare two possible market organizations. First, infrequent trading, that is $\Omega_I = \{0, T\}$. Second, continuous trading, $\Omega_C = [0, T]$.

Infrequent Trading The "infrequent trading" market design corresponds to the classic market for lemons as in Akerlof (1970). The equilibrium in this case is described by a price p_0 and a cutoff k_0 that satisfy that the cutoff type is indifferent between trading at $t = 0$ and waiting till T:

$$p_0 = (1-\delta) k_0 + \delta \frac{1+k_0}{2}$$

and that the buyers break even in expectations:

$$p_0 = E[v(c) | c \leq k_0]$$

The solution is $k_0 = \frac{2-2\delta}{3-2\delta}$ and $p_0 = \frac{4-3\delta}{6-4\delta}$. The expected gains from trade are

$$S_0 = \int_0^{k_0} (v(c) - c)\, dc + \delta \int_{k_0}^{1} (v(c) - c)\, dc = \frac{4\delta^2 - 11\delta + 8}{4(2\delta - 3)^2}$$

Continuous Trading The above outcome cannot be sustained in equilibrium if there are multiple occasions to trade before T. If at $t = 0$ types below k_0 trade, the next time the market opens we require the price to be at least $v(k_0)$, but that means that types close to k_0 would be strictly better off delaying trade. As a result for any richer set Ω than in the infrequent case, there will be less trade in period 0.

If we look at the case of continuous trading, $\Omega_C = [0, T]$, then the equilibrium with continuous trade is a pair of two processes $\{p_t, k_t\}$ that satisfy:

$$\begin{aligned} p_t &= v(k_t) \\ r(p_t - k_t) &= \dot{p}_t \end{aligned}$$

Since the process k_t is continuous, the zero profit condition is that the price is equal to the value of the current cutoff type. The second condition is the indifference of the current cutoff type between trading now and waiting for a dt and trading at a higher price. These conditions yield a differential equation for the cutoff type

$$r(v(k_t) - k_t) = v'(k_t)\,\dot{k}_t$$

with the boundary condition $k_0 = 0$. In our example this process has a simple solution:

$$k_t = 1 - e^{-rt}.$$

The total surplus from continuous trading is

$$\begin{aligned} S_C &= \int_0^T e^{-rt}(v(k_t) - k_t)\,\dot{k}_t dt + e^{-rT}\int_{k_T}^1 (v(c) - c)\,dc \\ &= \int_0^T e^{-rt}\left(\frac{1}{2}e^{-rt}\right)\left(re^{-rt}\right)dt + e^{-rT}\int_{1-e^{-rT}}^1 \left(\frac{1-c}{2}\right)dc \\ &= \frac{1}{12}\left(2 + \delta^3\right). \end{aligned}$$

Remark 1 *While we look at competitive equilibria, it is also possible to write a game-theoretic version of the model allowing two buyers to make public offers every time the market is open. If we write the model having $\Omega = \{0, \Delta, 2\Delta, ..., T\}$ then we can show that there is a unique Perfect Bayesian Equilibrium in our example for every T and $\Delta > 0$. When $\Delta = T$ then the equilibrium coincides with the equilibrium in the infrequent trading market we identified above. As we take the sequence of equilibria as $\Delta \to 0$, the equilibrium path converges to the competitive equilibrium we identified for our "continuous trading" design. In other words, the equilibria we described above have a solid game-theoretic foundation.*

Comparing Infrequent and Continuous trading The graph below (left) compares the dynamics of trade in these two settings for the case $T = \infty$. The dashed line at 2/3 is the equilibrium price and cutoff when there is only one opportunity to trade. With continuous trading the cutoff starts at zero and gradually rises towards one. In terms of efficiency there is a trade-off. With restricted trading opportunities more trade takes place at time zero but some types never trade.

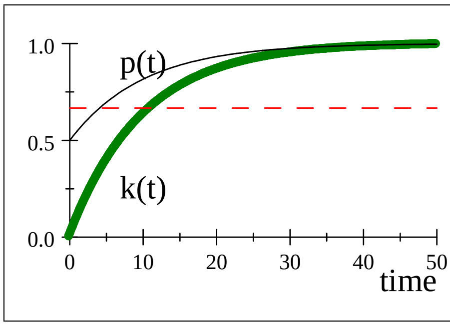

Figure 2: Trade Dynamics

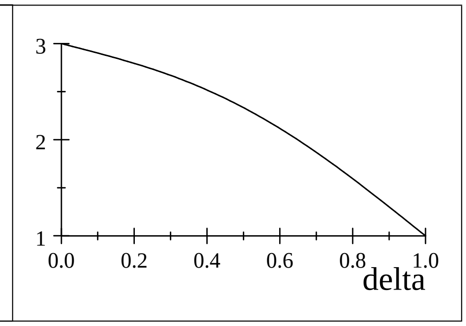

Figure 3: Efficiency

So how do gains from trade compare in these two cases? Figure 3 shows the ratio $\frac{S_{FB}-S_C}{S_{FB}-S_I}$ where S_{FB} is the trade surplus if trade was efficient, S_I and S_C the trade surpluses computed above. The ratio represents the relative efficiency loss from adverse selection in the two markets:

- When $\delta \to 0$ (i.e. as $rT \to \infty$, the private information is long-lived) we get $\frac{S_{FB}-S_C}{S_{FB}-S_I} \to 3$ so the efficiency loss with continuous trading design is three times higher than with infrequent trading.

- When $\delta \to 1$, which means that $T \to 0$, the private information is very short-lived and the organization of the market does not matter since even by waiting till T players can achieve close to full efficiency in either case.

Why is it that in our example restricting trading opportunities is advantageous? Committing to only one opportunity to trade generates a big loss of surplus if players do not reach an agreement in the current period. This clearly leaves a lot of unrealized gains from trade. But it is this inefficiency upon disagreement that helps overcome the adverse selection problem and increases the amount of trade in the initial period. Continuous trading on the other hand does not provide many incentives to trade in the current period since there a negligible loss of surplus from waiting an extra instant to trade. This leads to an equilibrium with smooth trading over time. Although this leads to a slow screening of types and delay of trade the advantage is that eventually (in particular for large T) higher types will receive attractive offers (since they are no longer pooled with lower types) and there will be more trade. In determining which trading environment is better one has to weight the cost of delaying trade with low types that would trade immediately in the batch case with the advantage of eventually trading with more types.

Since in our example types are uniformly distributed and there are higher gains from trade with the low types the advantage of getting more low types to trade without delay overcomes the benefit of getting to eventually trade with higher types. In the next Sections we will formalize these ideas.

3.1 Can Continuous Trading be better?

Our example above demonstrates a case of $v(c)$ and $F(c)$ such that for every T the infrequent trading market is more efficient than the continuous trading market. Furthermore, the greater T, the greater the efficiency gains from using infrequent trading. Is it a general phenomenon? The answer is no:

Proposition 1 *There exist $v(c)$ and $F(c)$ such that for T large enough the continuous trading market generates more gains from trade than the infrequent trading market*

Proof. Consider a distribution that approximates the following: with probability ε c is drawn uniformly on $[0,1]$; with probability $\alpha(1-\varepsilon)$ it is uniform on $[0,\varepsilon]$; and with probability $(1-\alpha)(1-\varepsilon)$ it is uniform on $[c_1, c_1+\varepsilon]$ for some $c_1 > v(0)$. In other words, the mass is concentrated around 0 and c_1. Let $v(c) = \frac{1+c}{2}$ as in our example.

For small ε there exists $\alpha < 1$ such that

$$E[v(c) | c \leq c_1 + \varepsilon] < c_1$$

so that in the infrequent trading market trade will happen only with the low types. In particular, if α is such that

$$\alpha v(0) + (1-\alpha) v(c_1) < c_1$$

then as $\varepsilon \to 0$ and $T \to \infty$, the infrequent trading equilibrium price converges to $v(0)$ and the surplus converges to

$$\lim_{\varepsilon \to 0, T \to \infty} S_I = \alpha v(0) + (1-\alpha) c_1$$

The equilibrium path for the continuous trading market is independent of the distribution and hence

$$\begin{aligned}\lim_{\varepsilon\to 0, T\to\infty} S_C &= \alpha v(0) + (1-\alpha)\left[e^{-r\tau(c_1)} v(c_1) + \left(1 - e^{-r\tau(c_1)}\right) c_1\right] \\ &= \lim_{\varepsilon\to 0, T\to\infty} S_I + (1-\alpha)\left(e^{-r\tau(c_1)}\left(v(c_1) - c_1\right)\right)\end{aligned}$$

where $\tau(k)$ is the inverse of the function k_t. The last term is strictly positive for any $c_1 < v(c_1)$. In particular, with $v(c) = \frac{1+c}{2}$, $e^{-r\tau(c)} = (1-c)$ and $v(c_1) - c_1 = \frac{1}{2}(1-c_1)$, so

$$\lim_{\varepsilon\to 0, T\to\infty} S_C = \lim_{\varepsilon\to 0, T\to\infty} S_I + \frac{1}{2}(1-\alpha)(1-c_1)^2 .$$

■

The example used in this proof illustrates what is needed for the continuous trading market to dominate the infrequent one: we need a large mass at the bottom of the distribution, so that the infrequent trading market gets "stuck" with these types while under continuous trading these types trade quickly. Additionally, we need some mass on higher types that would be reached in the continuous market after some time, generating additional surplus.

In the rest of the paper we offer general results that allow us to compare the continuous trading market design to several other designs, including the infrequent trading one.

4 Optimality of Restricting Trading Opportunities

We now return to the general model. We first describe the equilibrium in the continuous time trading

Proposition 2 (Continuous trading) *For $\Omega_C = [0, T]$ the competitive equilibrium (unique up to measure zero of times) is the unique solution to:*

$$\begin{aligned} p_t &= v(k_t) \\ k_0 &= 0 \\ r(v(k_t) - k_t) &= v'(k_t)\dot{k}_t \end{aligned}$$

Proof. First note that our requirement $p_t \geq v(k_{t-})$ implies that there cannot be any atoms of trade, i.e. that k_t has to be continuous. Suppose not, that at time s types $[k_{s-}, k_s]$ trade with $k_{s-} < k_s$. Then at time $s + \varepsilon$ the price would be at least $v(k_s)$ while at s the price would be strictly smaller to satisfy the zero-profit condition. But then for small ε types close to k_s would be better off not trading at s, a contradiction. Therefore we are left with processes such that k_t is continuous and $p_t = v(k_t)$. For k_t to be strictly increasing over time we need that $r(p_t - k_t) = \dot{p}_t$ for almost all t: if price was rising faster, current cutoffs would like to wait, a contradiction. If prices were rising slower over any time interval starting at s, there would be an atom of types trading at s, another contradiction. So the only remaining possibility is that $\{p_t, k_t\}$ are constant over some interval $[s_1, s_2]$. Since the price at s_1 is $v\left(k_{s_{1-}}\right)$ and the price at s_2 is $v(k_{s_2})$, we would obtain a contradiction that there is no atom of trade in equilibrium. In particular, if $p_{s_1} = p_{s_2}$ (which holds if and only if $k_{s_{1-}} = k_{s_1} = k_{s_2}$) then there exist types $k > k_{s_1}$ such that

$$v(k_{s_1}) > \left(1 - e^{r(s_2 - s_1)}\right) k + e^{r(s_2 - s_1)} v(k_{s_1})$$

and these types would strictly prefer to trade at $t = s_1$ than to wait till s_2, a contradiction again. ■

On the other extreme, with infrequent trading, Ω_I, the equilibrium is:[6]

Proposition 3 (Infrequent/Restricted Trading) *For $\Omega_I = \{0, T\}$ there exists a competitive equilibrium $\{p_0, k_0\}$. Equilibria are a solution to:*

$$p_0 = E[v(c) \,|c \in [0, k_0]] \tag{1}$$
$$p_0 = \left(1 - e^{-rT}\right) k_0 + e^{-rT} v(k_0) \tag{2}$$

If $\frac{f(c)}{F(c)}(v(c) - c) - \frac{\delta}{1-\delta} v'(c)$ is strictly decreasing then the equilibrium is unique.

Proof. 1) **Existence**. The equilibrium conditions follow from the definition of equilibrium. To see that there exists at least one solution to (1) and (2) note that if we write the condition for the cutoff as:

$$E[v(c) \,|c \leq k_0] - \left(\left(1 - e^{-rT}\right) k_0 + e^{-rT} v(k_0)\right) = 0 \tag{3}$$

then the LHS is continuous in k_0, it is positive at $k_0 = 0$ and negative at $k_0 = 1$. So there exists at least one solution. [7]

[6] The infrequent trading model is the same as the model in Akerlof (1970) if $T = \Delta = \infty$. Even with $T < \infty$ the proof of existence and inefficiency of the equilibrium is standard. The somewhat novel part of the proof is the sufficient condition for uniqueness in our environment.

[7] If there are multiple solutions, a game theoretic-model would refine some of them, see section 13.B of Mas-Colell, Whinston and Green (1995) for a discussion.

2) **Uniqueness**. To see that there is a unique solution under the two assumptions, note that the derivative of the LHS of (3) at any k is

$$\frac{f(k)}{F(k)}\left(v(k)-E\left[v(c)\,|c\leq k\right]\right)-(1-\delta)-\delta v'(k)$$

When we evaluate it at points where (3) holds, the derivative is

$$\frac{f(k)}{F(k)}\left(v(k)-k\right)(1-\delta)-(1-\delta)-\delta v'(k)$$

and that is by assumption decreasing in k.

Suppose that there are at least two solutions and select two: the lowest k_L and second-lowest k_H. Since k_L is the lowest solution, at that point the curve on the LHS of (3) must have a weakly negative slope (since the curve crosses zero from above). However, our assumption implies that curve has even strictly more negative slope at k_H. That leads to a contradiction since by assumption between $[k_L, k_H]$ the LHS is negative, so with this ranking of derivatives it cannot become 0 at k_H ■

4.1 Other Market Designs

So far we have compared only the continuous trading market with the infrequent trading. But one can imagine many other ways to organize the market. For example, the market could clear every day, for some Δ which is smaller than T but larger than 0. Or the market could start with a positive Δ and then be opened continuously. Or, the market could start being opened continuously and close some Δ before T (i.e. at $t = T - \Delta$). In this section we consider some of these alternative timings.

4.1.1 When Infrequent Trading is Optimal

We start with providing a sufficient condition for the infrequent trading to dominate all these other possible designs:

Proposition 4 *If* $\frac{f(c)}{F(c)}\frac{v(c)-c}{1-\delta+\delta v'(c)}$ *and* $\frac{f(c)}{F(c)}(v(c)-c)$ *are decreasing*[8] *then infrequent trading,* $\Omega_I = \{0, T\}$ *, generates higher expected gains from trade than any other market design.*

Proof. We use mechanism design to establish the result. We expand the set of possible market designs to allow for any trading mechanism that is incentive compatible, does not

[8] A sufficient condition is that $v''(c) \geq 0$ and $\frac{f(c)}{F(c)}(v(c)-c)$ is decreasing.

require the buyers to lose money on average. For every market design, the equilibrium outcome can be replicated by such a mechanism (but not necessarily vice versa). We then show that under the conditions in the proposition, infrequent trading replicates the outcome of the best mechanism and hence any other market design generates lower expected gains from trade.

As usual, given a T, let $\delta = e^{-rT}$.

A general direct revelation mechanism can be described by 3 functions $x(c), y(c)$ and $P(c)$, where $y(c)$ is the probability that the seller will not trade before information is released, $x(c)$ is the discounted probability of trade over all possible trading times and $P(c)$ is the transfer received by the seller conditional on trading before information is released.[9] We restrict the mechanism to satisfy that the highest type does not trade before T (as needs to be the case in a market equilibrium for any Ω). Note that $y(c) \in [0,1]$ but $x(c) \in [\delta, 1]$. The seller's value function in the mechanism is then:

$$\begin{aligned} U(c) &= y(c)\left[(1-\delta)c + \delta v(c)\right] + (1-y(c))\left[P(c) + (1-x(c))c\right] && (4) \\ &= \max_{c'} y(c')\left[(1-\delta)c + \delta v(c)\right] + (1-y(c'))\left[P(c') + (1-x(c'))c\right] && (5) \end{aligned}$$

Using the envelope theorem

$$\begin{aligned} U'(c) &= y(c)\left[(1-\delta) + \delta v'(c)\right] + (1-y(c))(1-x(c)) \\ &= \delta y(c)(v'(c)-1) + 1 - x(c)(1-y(c)) \end{aligned}$$

Let $V(c) = \delta v(c) + (1-\delta)c$ be the no-trade surplus, so that:

$$\begin{aligned} U'(c) - V'(c) &= \delta y(c)(v'(c)-1) + 1 - x(c)(1-y(c)) - (\delta v'(c) + (1-\delta)) \\ &= (1-y(c))(-x(c) - \delta(v'(c)-1)) \end{aligned}$$

[9] Letting $G_t(c)$ denote for a given type the distribution function over the times of trade:

$$x(c) = \int_0^T e^{-rt} dG_t(c).$$

write the expected gains from trade as:

$$\begin{aligned}
S &= \int_0^1 (U(c) - V(c)) f(c)\, dc \\
&= (U(c) - V(c)) F(c) |_{c=0}^{c=1} - \int_0^1 (U'(c) - V'(c)) F(c)\, dc \\
&= \int_0^1 (1 - y(c)) [x(c) - \delta (1 - v'(c))] F(c)\, dc \qquad (6)
\end{aligned}$$

The no–losses-on-average constraint is in this problem:

$$\int_0^1 (1 - y(c)) (x(c) v(c) - P(c)) f(c)\, dc \geq 0$$

From the expression for $U(c)$ we have

$$\begin{aligned}
U(c) - y(c) [(1 - \delta) c + \delta v(c)] - (1 - y(c)) (1 - x(c)) c &= (1 - y(c)) P(c) \\
U(c) - V(c) + (1 - y(c)) (\delta (v(c) - c) + x(c) c) &= (1 - y(c)) P(c)
\end{aligned}$$

So the constraint can be re-written as a function of the allocations alone:

$$\int_0^1 (1 - y(c)) (x(c) - \delta) (v(c) - c) f(c)\, dc - \int_0^1 (U(c) - V(c)) f(c)\, dc \geq 0 \qquad (7)$$

We now optimize (6) subject to (7), ignoring necessary monotonicity constraints on $x(c)$ and $y(c)$ that assure that reporting c truthfully to the mechanism is incentive compatible (we will check later that they are satisfied in the solution). The bang-for-the-buck formula (i.e. looking at the ratio of the derivative of the objective function to the derivative of the constraint) with respect to $x(c)$ is

$$\frac{1}{Z_x - 1}$$

where

$$Z_x = \frac{f(c)}{F(c)} \frac{(1 - y(c)) (v(c) - c)}{(1 - y(c))} = \frac{f(c)}{F(c)} (v(c) - c)$$

Hence, if $\frac{f(c)}{F(c)} (v(c) - c)$ is decreasing, for every $y(c) < 1$ it is optimal to set $x(c) = 1$ for small c and $x(c) = \delta$ for all the high $c's$. It is optimal to set $y(c) < 1$ only if $x(c) = 1$.

The bang-for-the-buck condition with respect to $y(c)$ is $\frac{1}{Z-1}$where:[10]

$$Z = \frac{f(c)}{F(c)} \frac{(x(c) - \delta)(v(c) - c)}{x(c) - \delta(1 - v'(c))}$$

For any c such that optimal $y(c)$ is less than 1, we argued above that $x(c) = 1$. That changes the expression to:

$$Z = \frac{f(c)}{F(c)} \frac{(1 - \delta)(v(c) - c)}{1 - \delta + \delta v'(c)}$$

so the sufficient condition is that Z is decreasing in c. If $v''(c) \geq 0$ then a sufficient condition for all δ is that $\frac{f(c)}{F(c)}(v(c) - c)$ is decreasing.

Under these conditions, the solution to our relaxed problem is to find a c^* such that all types $c \leq c^*$ trade immediately and all types $c > c^*$ that trade at T. This solution implies that maximand (5) is supermodular in c and c' so under the appropriate choice of $P(c)$ (so it satisfies the envelope formula), truthtelling is a best response.

The mechanism is incentive compatible and has transfer $P(c) = E[v(c)|c \leq c^*]$ for all $c \leq c^*$ and $P(c) = 0$ for all higher types (to guarantee that the buyers make zero profit). Finally, since $U(c)$ is continuous, it must be that $\lim_{c \uparrow c^*} U(c) = \lim_{c \downarrow c^*} U(c)$ which implies that c^* solves $E[v(c)|c \leq c^*] = c^*$. This (plus the implied price) is exactly the equilibrium for the infrequent trading market, which completes the proof. ■

The condition in the proposition is similar to the standard condition in optimal auction theory/pricing theory that the virtual valuation/marginal revenue curve be monotone. In particular, think about a static problem of a monopolist buyer choosing a cutoff and probability to trade, $F(c)$, and making a take-it-or-leave-it offer $P(c)$, where $P(c) = (1 - \delta)c + \delta v(c)$. In that problem $\frac{f(c)}{F(c)} \frac{v(c) - c}{1 - \delta + \delta v'(c)}$ decreasing guarantees that the marginal profit crosses zero exactly once.[11]

Note as well that the proof was constructed requiring only that buyers break even on average. That is, we were considering a relaxed problem were buyers buying in a given period could potentially subsidize buyers buying in another period. This would of course be a problem if the solution had the optimal market timing include several trading opportunities. One would then need to verify that indeed at each time the market is open the buyers break even in expectation. Otherwise, the solution characterized could not be implemented simply by determining the times at which the market is open. Given that the solution calls for the

[10] Note that when $T = \infty$ and hence $\delta = 0$ this expression coincides with Z_x and hence the analysis is even simpler.

[11] The FOC of the monopolist problem choosing c is: $(1 - \delta) f(c)(v(c) - c) - F(c)((1 - \delta) + \delta v'(c)) = 0$.

market only being open once this turns out not be an issue we must contend with.

4.1.2 Closing the Market Briefly after Initial Trade.

Even if the condition in Proposition 4 does not hold, we can show that under very general conditions it is possible to improve upon the continuous trading market.

In particular, consider the design $\Omega^{EC} \equiv \{0\} \cup [\Delta, T]$: there is trade at $t = 0$, then the market is closed till $\Delta > 0$ and then it is opened continuously till T. We call this design "early closure". We show that there always exists a small delay that improves upon continuous trading:

Proposition 5 *There exists $\Delta > 0$ such that the early closure market design $\Omega^{EC} = \{0\} \cup [\Delta, T]$ yields higher gains from trade than the continuous trading design $\Omega_C = [0, T]$.*

Proof. To establish that early closure increases efficiency of trade we show an even stronger result: that for small Δ with Ω^{EC} there is more trade at $t = 0$ than with Ω_C by $t = \Delta$. Let k_Δ^{EC} be the highest type that trades at $t = 0$ when the design is Ω^{EC}. Let k_Δ^C the equilibrium cutoff at time Δ in design Ω_C. Then the stronger claim is that for small Δ, $k_\Delta^C < k_\Delta^{EC}$. Since $\lim_{\Delta \to 0} k_\Delta^{EC} = \lim_{\Delta \to 0} k_\Delta^C = 0$ (for k_Δ^{EC} see discussion in Step 1 below). So it is sufficient for us to rank:

$$\lim_{\Delta \to 0} \frac{\partial k_\Delta^{EC}}{\partial \Delta} \text{ vs. } \lim_{\Delta \to 0} \frac{\partial k_\Delta^C}{\partial \Delta}$$

Step 1: Characterizing $\lim_{\Delta \to 0} \frac{\partial k_\Delta^{EC}}{\partial \Delta}$.

Consider Ω^{EC}. When the market reopens at $t = \Delta$ the market is continuously open from then on. Hence, the equilibrium in the continuation game is the same as the equilibrium characterized in Proposition (2) albeit with a different starting lowest type. Namely, for $t \geq \Delta$

$$\begin{aligned} p_t &= v(k_t) \\ r(v(k_t) - k_t) &= v'(k_t)\dot{k}_t \end{aligned}$$

with a boundary condition:

$$k_\Delta = k_\Delta^{EC}.$$

The break even condition for buyers at $t = 0$ implies:

$$p_0 = E\left[v(k) \,|\, k \in \left[0, k_\Delta^{EC}\right]\right]$$

and type k_Δ^{EC} must be indifferent between trading at this price at $t=0$ or for $p_\Delta = v\left(k_\Delta^{EC}\right)$ at $t=\Delta$:

$$v\left(k_\Delta^{EC}\right) - p_0 = \left(1-e^{-r\Delta}\right)\left(v\left(k_\Delta^{EC}\right) - k_\Delta^{EC}\right)$$

For small Δ , $E\left[v\left(c\right)|c \leq k_\Delta^{EC}\right] \approx \frac{v\left(k_\Delta^{EC}\right)}{2}$ so the benefit of waiting is approximately $\frac{v\left(k_\Delta^{EC}\right)}{2}$ while the cost is approximately $rTv\left(0\right)$ so k_Δ^{EC} for small T solves approximately

$$\frac{v\left(k_\Delta^{EC}\right)}{2} \approx rTv\left(0\right)$$

and more precisely:

$$\lim_{\Delta\to 0}\frac{\partial k_\Delta^{EC}}{\partial \Delta} = \frac{2rv\left(0\right)}{v'\left(0\right)}$$

Step 2: Characterizing $\lim_{\Delta\to 0}\frac{\partial k_\Delta^{C}}{\partial \Delta}$.

Consider Ω^C. Since k_t is defined by the differential equation

$$r\left(v\left(k_t\right) - k_t\right) = v'\left(k_t\right)\dot{k}_t,$$

for small Δ :

$$k_\Delta^C \approx rT\frac{v\left(0\right)}{v'\left(0\right)},$$

and more precisely:

$$\lim_{\Delta\to 0}\frac{\partial k_\Delta^{C}}{\partial \Delta} = \frac{rv\left(0\right)}{v'\left(0\right)}.$$

Summing up steps 1 and 2, we have:

$$\lim_{\Delta\to 0}\frac{\partial k_\Delta^{EC}}{\partial \Delta} = 2\lim_{\Delta\to 0}\frac{\partial k_\Delta^{C}}{\partial \Delta}$$

which implies the claim. ■

A closely related result is that when the private information is short lived, closing the market after the initial trade and waiting until the information is reveal dominates having continuous trading.

Corollary 1 *Fix $v\left(c\right)$, $F\left(c\right)$ and r. There exists a $T^* > 0$ such that for all $T \leq T^*$ the infrequent trading market generates higher expected gains from trade than the continuous trading market.*

The proof is analogous to the proof of the previous Proposition by noting that in either situation: $\Omega^{EC} = \{0\}\cup[\Delta, T]$ or $\Omega_I = \{0, T=\Delta\}$ the cutoff type trading at time 0 chooses between p_0 and price $v(k_0)$. In case information is revealed at T this is by assumption that the market is competitive at T. In case the market is open continuously after the early closure it is by our observation that the continuation equilibrium has smooth screening of types so the first price after closure is $p_\Delta = v\left(k_\Delta^{EC}\right)$.

4.1.3 Closing the Market Briefly before Information Arrives

The final design we consider is the possibility of keeping the market opened continuously from $t = 0$ till $T - \Delta$ and then closing it till T. Such a design may be more realistic since in practice it may be easier to determine when some private information is likely to be revealed than when it is that the seller of the asset is hit by liquidity needs (i.e. when is $t = 0$).

The comparison of this "late closure" market with the continuous trading market is much more complicated than in the previous section for two related reasons. First, if the market is closed from $T - \Delta$ to T, there will be an atom of types trading at $T - \Delta$. As a result, there will be a "quiet period" before $T - \Delta$: there will be some time interval $[t^*, T - \Delta]$ such that despite the market being opened, there will be no types that trade on the equilibrium path in that time period. The equilibrium outcome until t^* is the same in the "late closure" and continuous trading designs, but diverges from that point on. That brings the second complication: starting at time t^*, the continuous trading market will benefit from some types trading earlier than in the "late closure" market. Therefore it is not sufficient to show that by T there are more types that trade in the late closure market, we actually have to compare directly the total surplus generated between t^* and T. These two complications are not present when we consider the "early closure" design since there is no t^* before $t = 0$ for the early closure to affect trade before it.

The equilibrium in the "late closure" design is as follows. Let $p^*_{T-\Delta}, k^*_{T-\Delta}$ and t^* be a solution to the following system of equations:

$$
\begin{aligned}
E\left[v(c)\,|c \in [k_{t^*}, k_{T-\Delta}]\right] &= p_{T-\Delta} &(8)\\
\left(1 - e^{-r\Delta}\right) k_{T-\Delta} + e^{-r\Delta} v\left(k_{T-\Delta}\right) &= p_{T-\Delta} &(9)\\
\left(1 - e^{-r(T-\Delta-t^*)}\right) k_{t^*} + e^{-r(T-\Delta-t^*)} p_{T-\Delta} &= v\left(k_{t^*}\right) &(10)
\end{aligned}
$$

where the first equation is the zero-profit condition at $t = T - \Delta$, the second equation is the indifference condition for the highest type trading at $T - \Delta$ and the last equation is the indifference condition of the lowest type that reaches $T - \Delta$, who chooses between trading at t^* and at $T - \Delta$. The equilibrium for the late closure market is then:

1) at times $t \in [0, t^*]$, (p_t, k_t) are the same as in the continuous trading market
2) at times $t \in (t^*, T - \Delta)$, $(p_t, k_t) = (v(k_{t^*}), k_{t^*})$
3) at $t = T - \Delta$, $(p_t, k_t) = \left(p^*_{T-\Delta}, k^*_{T-\Delta}\right)$

Condition (10) guarantees that given the constant price at times $t \in (t^*, T - \Delta)$ it is indeed optimal for the seller not to trade. There are other equilibria that differ from this equilibrium in terms of the prices in the "quiet period" time: any price process that satisfies in this time period

$$\left(1 - e^{-r(T-\Delta-t)}\right) k_{t^*} + e^{-r(T-\Delta-t)} p_{T-\Delta} \geq p_t \geq v(k_{t^*})$$

satisfies all our equilibrium conditions. Importantly, however, all these paths yield the same equilibrium outcome.

Despite this countervailing inefficiency, for our leading example:

Proposition 6 *Suppose $v(c) = \frac{1+c}{2}$ and $F(c) = c$. For every r, T there exists a $\Delta > 0$ such that the "late closure" market design $\Omega^{LC} = [0, T - \Delta] \cup \{T\}$ generates higher expected gains from trade than the continuous trading market, Ω_C. Yet, the gains from late closure are smaller than the gains from early closure.*

The proof is in the appendix. It shows third-order gains of welfare from the late closure (while the gains from early closure are first-order). Figure 4 below illustrates the reason the gains from closing the market are smaller relative to when the market is closed at time zero. The bottom two lines show the evolution of the cutoff type in Ω_C (continuous curve) and in Ω^{LC} (discontinuous at $t = T - \Delta = 0.9$). The top two lines show the corresponding path of prices. The gains from bringing forward trades that would have occurred when the market is exogenously closed in $t \in (9, 10)$ (i.e. the jump in types at $t = 0.9$) are partially offset by the delay of types in the endogenous quiet period $t \in (8.23, 9)$. If we close the market for $t \in (0, \Delta)$ instead, there is no loss from some types postponing trade because there is no time before 0.

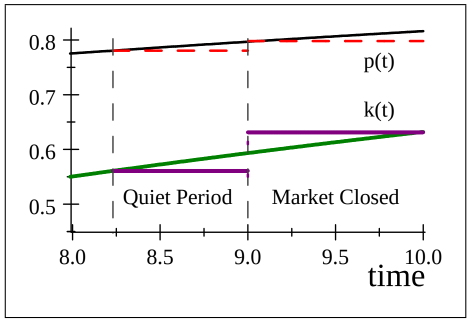

Figure 4: Late Closure

$$T = 10 \quad \Delta = 1 \qquad r = 0.1 \qquad v(c) = \frac{c+1}{2} \quad F(c) = c$$

Given our results so far showing the benefits of restricting opportunities to trade, one might speculate that the optimal Ω may not contain any continuous-trading intervals but be instead characterized by a discrete grid of trading times $\Omega = \{0, \Delta_1, \Delta_2, \Delta_2, ..., T\}$. We do not know how to prove or disprove this claim without putting any restrictions on $v(c)$ and $f(c)$.

What we can show is that there are cases when some restrictions of continuous trading, even small, can reduce welfare. An example of such a situation is $f(c) = 2 - 2c$ and $v(c) = c + 1$. In this case, by direct calculations we can show that "late closure" reduces expected gains from trade. The intuition is that even though the gains from trade are constant across all types, since $f(c)$ is decreasing, the distribution assigns a higher weight to the types that delay in the endogenous "quiet period" than to the types that speed up thanks to closure.

5 Discussion

In this section we will first explore relaxing some of the assumptions of the model. We will then explore the role of the government when the adverse selection problem is severe.

5.1 Stochastic Arrival of Information

So far we have assumed that it is known that the private information is revealed at T. However, in some markets even if the private information is short-lived, the market participants may be uncertain about the timing of its revelation. We now return to our motivating example to illustrate that trade-offs we have identified for the deterministic duration of private information apply also to the stochastic duration case.

Seller type c is distributed uniformly over $[0,1]$ and $v(c) = \frac{1+c}{2}$. Suppose that with a Poisson rate λ the type c gets publicly revealed and at that time the seller trades immediately at a price $v(c)$. Analogously to the previous definitions, let "infrequent trading" market be such that the seller can trade only either at $t = 0$ or upon arrival of information. Also let the continuous trading market be such that the seller can trade at any time.

In the infrequent trading market, the equilibrium (p_0, k_0) is determined by:

$$\begin{aligned} p_0 &= \frac{\lambda}{\lambda + r} v(k_0) + \frac{r}{\lambda + r} k_0 \\ p_0 &= E\left[v(c) \,|c \leq k_0\right] \end{aligned}$$

where the first equation is the indifference condition of the cutoff type and the second equation is the usual zero-profit condition. In our example we get

$$\begin{aligned} k_0 &= \frac{2r}{3r + \lambda} \\ p_0 &= \frac{4r + \lambda}{6r + 2\lambda} \end{aligned}$$

In the continuous trading market the equilibrium is described by the same differential equation:

$$\begin{aligned} r(p_t - k_t) &= \dot{p}_t \\ p_t &= v(k_t) \\ k_0 &= 0 \end{aligned}$$

Since it is the same differential equation as in deterministic duration case, the solution is again

$$k_t = 1 - e^{-rt}$$

We now can compare the gains from trade. The total surplus in the infrequent trading market is

$$\begin{aligned} S_0 &= \int_0^{k_0} (v(c) - c)\, dc + \frac{\lambda}{\lambda + r} \int_{k_0}^{1} (v(c) - c)\, dc \\ &= \frac{1}{4} \frac{5z + 8 + z^2}{(3 + z)^2} \end{aligned}$$

where $z \equiv \frac{\lambda}{r}$.

In the continuous trading market the surplus is

$$S_C = \int_0^{+\infty} \lambda e^{-\lambda t} \left(\int_0^{k_t} e^{-r\tau(c)} \left(v\left(c\right) - c \right) dc + e^{-rt} \int_{k_t}^{1} \left(v\left(c\right) - c \right) dc \right) dt$$

where $\tau(c)$ is the time type c trades if there is no arrival before his time of trade. In our example $\tau(c) = -\frac{\ln(1-c)}{r}$ and $e^{-r\tau(c)} = 1 - c$, so the expected surplus is:

$$S_C = \frac{1}{6} + \frac{z}{12\left(3+z\right)}$$

The difference is:

$$S_0\left(z\right) - S_C\left(z\right) = \frac{1}{2}\left(z+3\right)^{-2} > 0$$

So at least in our example, for every λ, the infrequent trading market is more efficient than the continuous trading market.

5.2 Asset Purchases by the Government

Beyond its potential role in shaping Ω the government could intervene more directly in the market by purchasing some assets or subsidizing certain trades. During the recent financial crisis several markets effectively shut down or became extremely illiquid. One of the main reasons cited for this was the realization by market players that the portfolios of asset backed securities that banks held were not all investment grade as initially thought. Potential buyers of these securities which used to trade them without much concern suddenly became very apprehensive of purchasing these assets for the potential risk of buying a lemon. The Treasury and the Federal Reserve tried many different things to restore liquidity into the markets. Some of the measures were aimed at providing protection against downside risk via guarantees effectively decreasing the adverse selection problem or by removing the most toxic assets from the banks' balance sheets.

Our model provides a natural framework to study the potential role for government. To illustrate consider the case in which if $v(c) = \gamma c$ for $2 > \gamma > 1$ and $F(c) = c$.[12] Then for all Ω the unique equilibrium is for there never to be any trade before the information is revealed. So the market is completely illiquid and no gains from trade are realized. The government

[12] This model arises for example if the seller has a higher discount rate than the buyers.

could intervene in this market by making an offer $p_g > 0$ to buy any asset backed securities sellers are willing to sell at that price. The average quality of these securities will be $\frac{p_g}{2}$ and hence the government will lose money on them. On the bright side is that once the toxic assets have been removed from the market and the remaining distribution is truncated to $c \in \left[\frac{p_g}{\gamma}, 1\right]$ now even if $\Omega = [0_+, T]$ buyers would be willing to start making offers again. Note that this government intervention not only benefits the direct recipients of government funds but also all other sellers since by reducing the adverse selection problem in the market they will now have an opportunity trade with a private counterparty.

Somewhat similar potential roles for the government have been recently discussed in Tirole (2012) and in Philippon and Skreta (2010).

5.3 Common Knowledge of Gains from Trade

We assumed that there are strict gains from trade with the lowest type: $v(0) > 0$ and that there are no gains from trade at the top $v(1) = 1$. What is the role of these assumptions?

5.3.1 Role of $v(0) > 0$

If $v(0) = 0$ then Proposition (4) still applies. Moreover, the continuously open market has no trade in equilibrium before T. For example, if $v(c) = \sqrt{c}$ and $f(c) = 1$ then for all T the conditions in Proposition (4) are satisfied and $\Omega_I = \{0, T\}$ is welfare-maximizing, while $\Omega_C = [0, T]$ is welfare-minimizing over all Ω.

Assuming strict gains from trade at the bottom are useful since in its absence in equilibrium there may be no trade. For example, if $v(c) = \gamma c$ for $2 > \gamma > 1$ and $F(c) = c$ then for all Ω the unique equilibrium there is no trade before the information is revealed.[13] With $v(c) = \gamma c$ for $2 > \gamma > 1$ then for general $F(c)$ there will be trade in equilibrium if the opportunities for trade are sufficiently restricted but not if the market is opened continuously.

5.3.2 Role of $v(1) = 1$

By assuming that there are no gains from trade at the top we are sure that when we define this market in formal game theoretic terms we do not have to deal with off-equilibrium beliefs and the multiplicity of equilibria they can sustain.

To elaborate, when $v(1) = 1$, the highest type never trades in equilibrium no matter how large is T.. This in turn implies that on the equilibrium path there are no offers that all

[13]This example also has strict gains from trade at the top but that is irrelevant for the point we are making.

types should accept and so we do need to consider the buyer beliefs about remaining types after such a history. This facilitates our definition of competitive equilibrium: condition (3) "No (Unrealized) Deals" is simpler than Definition 2.1 in Daley and Green (2011) precisely because we do not have to consider such histories.

To illustrate how the freedom in selecting off-equilibrium-path beliefs can lead to a multiplicity of equilibria with radically different outcomes consider the following example:

$$F(c) = c \ ; \ \ v(c) = c + s$$

with $\Omega = \{0, \Delta, 2\Delta, ..., \infty\}$ for $\Delta > 0$. Let $s > \frac{1}{2}$ so that in a static problem trade would be efficient.

Case 1: Assume that when an offer that is supposed to be accepted by all types on the equilibrium path is rejected, buyers believe the seller has the highest type, $c = 1$. Then, taking a sequence of equilibria as $\Delta \to 0$, we can show that in the limit trade is smooth over time (no atoms) with:

$$\begin{aligned} p_t(k) &= v(k_t) \\ k_t &= rst \end{aligned}$$

On equilibrium path all types trade by:

$$\tau = \frac{1}{rs}$$

unless $\tau < T$. If the last offer, $p_\tau = 1 + s$ is rejected, the price stays constant after that, consistently with the believes and competition.

Case 2: Alternatively, assume that if an offer p_t that on the equilibrium path is accepted by all types is rejected, buyers do not update their believes. That is, after that history they believe the seller type is distributed uniformly over $[k_t, 1]$ (k_t is derived from the history of the game). In that case the following is an equilibrium for all $\Delta > 0$. At $t = 0$ there is an initial offer $p_0 = \frac{1}{2} + s$ and all types trade. If that initial offer is rejected, the buyers believe $c\tilde{\ }U[0, 1]$ and continue to offer $p_t = p_0$ for all $t > 0$ (and again all types trade). This is indeed and equilibrium since the buyers break even at time zero (and all future times given their beliefs) and no seller type is better off by rejecting the initial offer.

These equilibria are radically different in terms of efficiency: only the second one is efficient. It is beyond the scope of this paper to study in what situations or under what model extensions this multiplicity could be resolved.

6 Related Literature

We now relate our model to the theoretical literature. Our model is a dynamic version of the market for lemons in Akerlof (1970). Several papers have looked at dynamic models of trading with correlated values. We can classify this literature by the market structure it studies.

On the one hand Evans (1989), Vincent (1989) and Deneckere and Liang (2006) among others have studied bargaining models (bilateral monopoly) with correlated values.[14] They show that with correlated values and a strong enough adverse selection problem, the equilibrium outcome is inefficient even when offers can be made continuously. Even though offers are made frequently, trade takes place slowly over time.[15] This literature has not considered an optimal Ω design.

There are also models where one informed agent faces an uninformed competitive fringe. The most recent work that is closest to ours is Daley and Green (2011).[16] The focus of their very nice paper is on the role of exogenous information flowing into the market and not on the timing at which the market is open. When they explore the limit as the signal becomes uninformative (and for sufficiently low priors), they characterize an equilibrium in which only a fraction of the lemons trade at time zero, after that buyers mix between a non-offer and an offer acceptable to both types. There is strictly positive probability of trade in any time interval. When we look at the market that is continuously open we show that when there is a continuous distribution of seller types the equilibrium is unique with prices smoothly increasing over time and there is no mass of trade at time zero.

Janssen and Roy (2002) take a Walrasian approach in a discrete time model with a mass 1 of privately informed sellers and a larger mass of uninformed buyers with value proportional to the seller's value. In discrete time and $T = \infty$, they have shown that in equilibrium there is skimming over time: prices increase over time and eventually every type trades. They point out the outcome is still inefficient even as the per period discounting disappears (which is equivalent to taking a limit to continuous trading in our model) since some trades are delayed even in the limit. They do not explore the optimal period length (interval of time the market is closed) or other Ω designs. Yet, we share with their model the observation that dynamic trading with $T = \infty$ leads to more and more types trading over time.

[14] See also Fuchs and Skrzypacz (2010) & (2012) and Olsen (1992)

[15] In contrast, when values are not correlated then as offers can be made continuously the first best outcome is attained, a result known as the Coase conjecture.

[16] See also the informative discussion of Swinkels (1999), Kremer and Skrzypacz (2007) and Noldeke and van Damme (1990) in Daley and Green (2011)

There is also a recent literature of adverse selection with correlated values in search models.[17] Rather than having just one market in which different quality sellers sell at different times, the separation of types in these models is achieved because each different market has a different market tightness with the property that in the market with low prices a seller can find a buyer very quickly and in the market with high prices it takes a long time to find a buyer. Low quality sellers which enjoy less holding their good then self-select into the low price market while high quality sellers which are less eager to let go of their good are happy to wait in the high price market for a sale to take place. The question posed in this paper translated to a search setting corresponds to studying the efficiency consequences of closing certain types of market. A way implement this for example would be by imposing a price ceiling. This would correspond to closing the market after some time in our setting.

Lastly, Hörner and Vieille (2009) analyzed a similar model to ours but with one buyer per period (i.e. two-sided monopoly in every period but a new buyer every period). They study $T = \infty$ with a constant duration of periods. Their focus is on comparing equilibrium outcomes in case rejected offers are public or private (we assume that all prices are public).[18] In the game with many trading opportunities, their equilibrium is quite different than ours. When offers are public, they comment about their main result: "Proposition 1 is paradoxical: there is a unique equilibrium outcome, according to which the first buyer's offer is rejected with positive probability, and all subsequent offers are rejected with probability 1." Indeed this surprising outcome is not sustained when there is a finite horizon or intra-period competition as considered in this paper.[19]

In summary, although there is an extensive literature which looks at dynamic trading with adverse selection, some of the papers study different market structures than we do, but more importantly none of the papers have considered the efficiency trade-offs in choosing among possible Ω designs.

7 Appendix

Proof of Proposition 6**.**

[17] See for example Guerrieri, Shimer and Wright (2010), Guerrieri and Shimer (2011) and Chang (2012).

[18] See Fuchs, Öry and Skrzypacz (2012) for an analysis of the public vs. private offers in our setup.

[19] The comparison of the two models is a bit indirect since we study a competitive equilibrium while they study a game-theoretic model. However, in our leading example and with a finite horizon, we can show that trade happens in every period even in a game theoretic model.

In this case the equilibrium conditions $(8), (9)$ and (10) simplify to

$$\begin{aligned}
\frac{1}{2}+\frac{k_{t^*}+k_{T-\Delta}}{4} &= p_{T-\Delta} &(6')\\
\left(1-e^{-r\Delta}\right)k_{T-\Delta}+\left(\frac{1}{2}+\frac{k_{T-\Delta}}{2}\right)e^{-r\Delta} &= p_{T-\Delta} &(7')\\
\left(1-e^{-r\Delta_2}\right)k_{t^*}+e^{-r\Delta_2}p_{T-\Delta} &= \frac{1}{2}+\frac{k_{t^*}}{2} &(8')
\end{aligned}$$

where $\Delta_2 = T-\Delta-t^*$.

Solution of the first two equations is:

$$\begin{aligned}
k_{T-\Delta} &= \frac{k_{t^*}+2-2e^{-r\Delta}}{3-2e^{-r\Delta}}\\
p_{T-\Delta} &= \frac{1}{2}\left(\frac{2-e^{-r\Delta}}{3-2e^{-r\Delta}}k_{t^*}+\frac{4-3e^{-r\Delta}}{3-2e^{-r\Delta}}\right)
\end{aligned}$$

Substituting the price to the last condition yields

$$\left(1-e^{-r\Delta_2}\right)k_{t^*}+e^{-r\Delta_2}\left(\frac{1}{2}\left(\frac{2-e^{-r\Delta}}{3-2e^{-r\Delta}}k_{t^*}+\frac{4-3e^{-r\Delta}}{3-2e^{-r\Delta}}\right)\right)=\frac{1}{2}+\frac{k_{t^*}}{2}$$

which can be solved for Δ_2 independently of k_{t^*} (given our assumptions about $v(c)$ and $F(c)$).

$$r\Delta_2 = -\ln\frac{3-2e^{-r\Delta}}{4-3e^{-r\Delta}}$$

Note that

$$\lim_{\Delta\to 0}\frac{\partial\Delta_2}{\partial\Delta}=\lim_{\Delta\to 0}\frac{\partial}{\partial\Delta}\frac{1}{r}\left(-\ln\frac{3-2e^{-r\Delta}}{4-3e^{-r\Delta}}\right)=1$$

so Δ_2 is approximately equal to Δ.

In the continuous trading cutoffs follow $k_t = 1-e^{-rt}$, $\dot{k}_t = re^{-rt}$. Normalize $T=1$ (and rescale r appropriately). Then

$$\begin{aligned}
k_{t^*} &= 1-e^{-r(1-\Delta-\Delta_2)} = 1-\frac{4-3e^{-r\Delta}}{3-2e^{-r\Delta}}e^{r\Delta}\delta\\
\text{where } \delta &= e^{-r} \text{ and}\\
t^* &= 1-\Delta-\Delta_2 = 1-\Delta+\frac{1}{r}\ln\frac{3-2e^{-r\Delta}}{4-3e^{-r\Delta}}
\end{aligned}$$

We can now compare gains from trade in the two cases. The surplus starting at time t^* is (including discounting):

$$\begin{aligned} S_c(\Delta) &= \int_{k_{t^*}}^{1-e^{-r}} e^{-r\tau(c)} \left(v(c)-c\right) dc + \delta \int_{1-e^{-r}}^{1} \left(v(c)-c\right) dc \\ &= \int_{k_{t^*}}^{1-e^{-r}} (1-c)\left(\frac{1-c}{2}\right) dc + \delta \int_{1-e^{-r}}^{1} \left(\frac{1-c}{2}\right) dc \end{aligned}$$

where we used $e^{-r\tau(c)} = 1-c$.

$$\frac{\partial S_c(\Delta)}{\partial \Delta} = -\frac{\partial k_{t^*}}{\partial \Delta}\frac{(1-k_{t^*})^2}{2}$$

and since $\lim_{\Delta\to 0} \frac{\partial k_{t^*}}{\partial \Delta} = -2r\delta$ we get that

$$\lim_{\Delta\to 0} \frac{\partial S_c(\Delta)}{\partial \Delta} = r\delta^3$$

For the "late closure" market the gains from trade are

$$S_{LC}(\Delta) = e^{-r(1-\Delta)} \int_{k_{t^*}}^{k_{T-\Delta}} \left(v(c)-c\right) dc + e^{-r} \int_{k_{T-\Delta}}^{1} \left(v(c)-c\right) dc$$

after substituting the computed values for k_{t^*} and $k_{T-\Delta}$ it can be verified that

$$\lim_{\Delta\to 0} \frac{\partial S_{LC}(\Delta)}{\partial \Delta} = r\delta^3$$

which is the same as in the case of continuous market, so to the first approximation even conditional on reaching t^* the gains from trade are approximately the same in the two market designs.

We can compare the second derivatives:

$$\begin{aligned} \lim_{\Delta\to 0} \frac{\partial S_{LC}^2(\Delta)}{\partial \Delta^2} &= 3\delta^3 r^2 \\ \lim_{\Delta\to 0} \frac{\partial S_c^2(\Delta)}{\partial \Delta^2} &= 3\delta^3 r^2 \end{aligned}$$

and even these are the same. Finally, comparing third derivatives:

$$
\begin{aligned}
\lim_{\Delta \to 0} \frac{\partial S_{LC}^{3}(\Delta)}{\partial \Delta^{3}} &= 13r^{3}\delta^{3} \\
\lim_{\Delta \to 0} \frac{\partial S_{c}^{3}(\Delta)}{\partial \Delta^{3}} &= 9r^{3}\delta^{3}
\end{aligned}
$$

so we get that for small Δ, the "late closure" market generates slightly higher expected surplus, but the effects are really small. ■

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