The Economics of Wild Goose Chases

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March 2012

Abstract

Authority, where employees are told what to do by their superiors, is a dominant feature of firms. There is consistent evidence that firms that rely on supervisor authority are less likely to provide formal incentives to their employees than when employees control their activities. This paper addresses the provision of incentives when such authority is present, where authority derives from superiors knowing more about the worker's productivity than do workers themselves. There are two kinds of tasks that workers can carry out - those where output is contractible and those where it is not. I show that authority can eliminate incentives, and will always do so with enough task assignment options in the baseline model. Furthermore, I show that when effort is feasible, authority is complementary with incentives when incentive provision is inexpensive, but harms incentive provision when incentives are difficult to provide. I also show how these authority issues affect intrinsic motivation and bureaucratic allocation in ways that share the theme of authority only being beneficial when contracting on performance is relatively easy.

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Most people follow orders in work. This is typically because others - usually their bosses - are better informed about opportunities and tradeoffs than they are, and hierarchical authority is a defining feature of organizational life. Yet firms also need to motivate workers to exert effort on their assigned tasks. This paper addresses how authority and motivation interact by considering incentive provision when workers are told what to do.

Ideally, workers should be assigned where they have the highest returns. It is then easy to see how authority can help incentive provision by orienting workers' efforts to their most productive ends. Given the simplicity of this logic, it is striking that empirical evidence consistently shows otherwise: workers are less likely to have formal incentives when supervisors have authority over their actions. This is shown in Table 1 in a wide variety of settings.

Table 1: The Relationship between Supervisor Authority and Worker Pay-For-Performance.

Authors	Occupation/Sample	Result
McLeod and Parent (1999)	National Survey (U.S.)	< 0
Nagar (2002)	Bank Branch Managers (U.S.)	< 0
Colombo and Delmastro (2004)	Manufacturing Workers (Italy)	< 0
Foss and Lauren (2005)	Managers (Denmark)	< 0
Wulf (2007)	Division Managers (U.S.)	< 0
De Varo and Kurtulus (2010)	National Sample (Britain)	< 0
Ghosh, Lafontaine, and Lo (2011)	Sales Force Workers (U.S.)	< 0

This finding motivates much of the paper's analysis. One possible explanation for it is that agents are better informed than their superiors (Prendergast, 2002), or need to be given incentives to become better informed (Aghion and Tirole, 1997). However, workers knowing more than their bosses seems far from ubiquitous, and here I argue that authority may impede incentives because workers do not trust how that authority is used. Specifically, workers fear being assigned to a task where their *observed* productivity is low, even when a better option is available - hence the "wild goose chases" of the paper's title.

The model below relies on one key assumption: that a supervisor may assign a worker to a task from which the worker gets little benefit from exerting effort, even if offered incentive pay. The shorthand for such activities is "non-contractible tasks". There are three natural cases that fit the spirit of this idea:

- *Non-Monetary Benefits:* One source of potential conflict between the interests of workers and employers is where the employer's benefit is non-monetary.[1] Any task that generates non-monetary benefits would satisfy the assumptions required below.

- *Timing:* In many settings, effort exerted today may take time to pay off, such as asking a manager to explore a possible new market. If those payoffs are far in the future, the agent may be long gone from the organization when the returns arise, and the agent receives no benefits.[2]

- *Risk:* Risk preferences may also generate conflict if some activities are unlikely to succeed, but have high payoffs conditional on success. These may be activities that the principal would like to explore, but a risk averse agent would prefer to avoid in favor of something more certain.

What matters below is not only that such tasks exist, but that a worker exerts effort that affects their productivity. There are two cases where she may do so. First, the worker may not know the "contractibility" of tasks - in the examples above, she knows little about the timing or riskiness of revenues or whether the principal attains private or non-monetary benefits from an activity. For example, a manager working on a new product introduction may know little about its likely profitability, whether it provides other benefits to the organization, and when any profits are likely to accrue. Second, it could be that efforts have returns that are not task-specific. For example, a worker can acquire human capital about how the business operates, can prepare presentations, can reorganize departments etc, that have value across many tasks.[3] In this setting, she could collect the skills while on some task, but could be subsequently reassigned to a non-contractible task. I deal with both cases below.

In the model below, a principal assigns a single agent to a task. There are two kinds of tasks - those that have returns that are measured (contractible tasks), and those that have returns which cannot be measured (non-contractible tasks). I begin with a baseline model, where the principal privately observes the marginal return to effort for n contractible tasks, and for n non-contractible tasks. In the baseline model, all tasks have marginal return to effort drawn from a similar (uniform) distribution.

[1] These benefits could be the kind of thing that affects the image or brand capital of the firm, from which the agent is unlikely to directly benefit. For example, is it hard to identify the monetary return to quickly and efficiently dealing with customer complaints. Or they could be private benefits to the principal, such as where certain outcomes could make the principal look good to the labor market. Finally, the principal could simply gain utility from the activity per se such as a "glamor project".

[2] As an example, fund raisers for organizations are often assigned "targets", yet they often have no idea whether their "target" is likely to donate in the near future.

[3] For example, much human capital in a sales setting is not client-specific.

The metric for the importance of authority is n. This is because in the first best, where the agent is always assigned to the highest productivity task, more authority (higher n) increases the expected marginal return to effort and so is complementary with incentive provision. Without incentives to worry about, this would occur. However, now suppose that the principal has to offer an incentive contract to the agent to induce her to exert effort, where she gets a share β of verifiable (or "observed") output. As the principal only has to pay the agent for contractible outcomes, he compares $1 - \beta$ of the productivity of the contractible task to the entire productivity of the non-contractible task when assigning her. This distortion lies behind all the paper's results.

The baseline model is symmetric, in the sense that the distributions of the non-contractible and contractible tasks are identical. I begin by showing that the exercise of authority can cause incentives to fail, in the sense that no contract can induce effort exertion. This result can occur even if the principal has few assignment options. More strikingly, incentives always fail if the principal has a sufficiently large set of assignment choices even though all tasks are ex ante identical. The reason for this is that with a wide enough span of authority, the probability of being assigned to a contractible task gets too small to make effort exertion worthwhile.[4] This incentive failure arises when the marginal product of effort is sufficiently *high*, so the beneficial effect of the principal's information is always outweighed by the agent's unwillingness to exert effort.

This stark benchmark is meant as a starting point for understanding the effect of authority on incentives - it is not intended to claim here that exercising authority must make incentives harder to provide. I address this by considering cases where incentives can still be provided despite the incentive to send the agent on a "wild goose chase". In this scenario, I show that the effect of increasing the span of the principal's authority on incentive provision depends on what I call an Incentive Multiplier. Based on this:

- When incentives are inexpensive to provide (in a welfare sense), increases in the principal's information make incentives even cheaper to provide, but
- When incentives are expensive to provide, increases in the principal's information make incentives yet more expensive.

The non-monotonicity of increased span of authority on incentive provision - where more

[4]As the agent is paid β of the contractible task's output, the best contractible task will only be assigned if it has productivity at least $\frac{1}{1-\beta}$ times that of the best non-contractible task. This can never happen with enough options for the principal.

assignment options enhances incentive provision only if incentives are inexpensive to provide - is a recurring theme in the paper.

Much of the recent agency literature has addressed instruments other than pay that could aid effort exertion. This model also offers some insight into these. I begin by addressing intrinsic motivation, where agents inherently value their outputs. An issue in this literature is how pay for performance affects intrinsic motivation (Ariely et al, 2009, Deci et al, 1999). This paper offers a novel reason why pay for performance adversely affects intrinsic motivation - the agent begins to distrust the instructions of the principal when incentive pay is used, and (correctly) infers that she is being sent on "wild goose chases". Furthermore, I show that the total effect of pay for performance on incentives depends on how well output is monitored - when monitoring is easy, total effort rises with incentive pay, while it declines when monitoring is hard.

The model also allows a role for bureaucracy, through the use of rules that restrict the principal's assignment options. Such restrictions on activities never occur when incentive provision is relatively low cost. However, as incentives become more costly to provide, there is an upper bound to the number of allowed tasks, and so bureaucracy is a natural implication of difficult agency settings.

These results are derived in a stark setting, where a single worker is assigned to one of n tasks, and n is allowed to vary. (For example, n could be the number of potential clients that a sales agent could be assigned to.) A realistic extension would be to allow the number of agents to increase with number of available tasks. This allows us to distinguish between simply increasing the "scale" of the principal's authority (where there is "more of everything") and the "scope" of his authority (where the number of options per worker rises). I show that what matters for more authority causing problems for incentives is not the scale of the manager's discretion but rather its scope.

The central point of the paper is that the exercise of authority by a better informed superior can make incentive provision difficult. The effect of increased authority is ambiguous because of two conflicting effects: (i) a better informed superior can make better choices, but (ii) workers don't trust those choices. In the baseline model - where the distributions are identical with a finite upper bound - the second effect always dominates the former with enough choices, and incentive provision ultimately becomes impossible. That incentives fail with enough choice need not arise with different distributional assumptions. I show this in two ways. First, I consider the case where the two kinds of tasks differ in their productivity. The only significant conceptual change is where the contractible tasks are on average better, in which greater scope of authority can relax the ability to provide incentives even for a principal with many options. However, this is only true if performance can be monitored sufficiently well. Second, I consider unbounded productivity distributions, where again incentives need not fail even with many assignment options.

I begin by describing the model in Section 1. I then consider the symmetric baseline case in Section 2 and show how incentives disappear with a sufficiently well informed principal, and also how the effect of greater span of authority on incentives depends on the ease of providing incentives. Section 3 shows how intrinsic motivation is affected by pay for performance and how the use of bureaucratic rules can relax the agency problem. Section 4 offers different interpretations of the span of authority to distinguish between scale and scope of authority. I follow this by considering the case where efforts are task specific in Section 5. Section 6 addresses possible institutional solutions to the potential abuse of authority. Section 7 deals with robustness and I conclude in Section 8.

1 The Model

A profit maximizing principal hires an agent to work on a single task. Productivity on that task depends on unobservable effort e, where e takes on a value of 1 or 0, where the marginal cost of effort of 1 is γ. Output is only produced in the event that effort of 1 is exerted.

There are a range of tasks to which the agent can be assigned. There are $2n$ such activities. The activities are of two types:

1. n of these activities produce an output that is contractible, and the return to effort on task i is given by d_i. The true value of d_i is privately observed by the principal, while the agent knows only that it is drawn from a Uniform distribution with support $[0, D]$.

2. The other n activities produce an output that is not contractible, and the return to effort on task i is given by b_i. The true value of b_i is privately observed by the principal, while the agent knows only that it is drawn from a Uniform distribution with support $[0, B]$.

As a concrete example, one can think of each draw as a potential client for the agent.[5] All

[5] It is not qualitatively important that some tasks are perfectly observed and others not at all. An alternative could be where the d type tasks have returns observed with probability $\overline{p}$ while the b type tasks are observed with probability $\underline{p} < \overline{p}$. The qualitative results extend to this case where the agent fears only receiving a return with probability $\underline{p}$ rather than $\overline{p}$.

draws are assumed to be independent. For the moment, assume that the principal assigns an agent to a task - formally assume that only the principal can distinguish between tasks.

Contracts The agent's pay can be conditioned on observed output.[6] The principal offers the agent a pay for performance contract where the agent is offered a share $\beta \geq 0$ of observed output and a fixed fee (or salary) β_0.

Monitoring One of the central themes of the agency literature is that output is not perfectly measured. I assume that the contractible task is measured with possible error. Consider the case where the agent is assigned to a task with return d_i. If she exerts effort of 1, I assume that output d_i is always observed, but if effort of 0 is exerted, output of d_i is observed with probability σ.

In the introduction, I described two reasons why an agent may exert effort on a non-contractible task - she may not know whether it is contractible, or effort could be exerted prior to task assignment. Both offer similar outcomes, but the latter case is somewhat simpler, so I being by considering timing where efforts are not task-specific, and address the task-specific case in Section 5.

Timing First, the principal offers the agent a contract with sharing rule β and fixed fee β_0. If the agent rejects, the game ends. If she accepts, the agent then exerts effort or not.[7] Following this, the principal observes the realization of the $2n$ random variables, and assigns the agent to one of the activities. The principal does so to maximize profits at that point. Output is then realized, and the agent is paid according to observed output. At that point, the game ends.

I characterize the surplus maximizing Bayesian Nash equilibrium[8] of the game. Because

[6] Meaning output that can be observed by a third party.

[7] That the effort is taken in the absence of a task assignment if, of course, purely an abstraction - it could be that the agent is always initially assigned to some base task, but future reassignments affect how likely she is to be rewarded for doing so. So in the example above, an agent incurs costs collecting human capital on the firm before he is assigned to a client. Or alternatively, she is reassigned to another client after collecting the human capital.

[8] The effort choice of the agent depends on what she expects the principal to do. There is always one trivial equilibrium here, where the agent assumes that the principal will assign her to a non-contractible task, and in response exerts no effort. Given no effort exerted, the principal is indifferent over a task to assign the agent, and indeed it is a Bayesian Nash equilibrium to assign her to such a task. I ignore this issue here by considering the surplus maximizing Bayesian Nash equilibrium of the game. An alternative

there are no restrictions on β_0, and contracts are chosen before private information is realized, the principal maximizes ex ante surplus. Specifically, he chooses β to maximize surplus produced before the realizations of the b and d distributions are known, subject to his own incentives to allocate tasks after observing the productivities.[9]

2 The Symmetric Case: $B = D = 1$

Begin by considering the symmetric case where the contractible and non-contractible tasks are drawn from the same distribution, whose support is normalized to 1.

The First best The principal receives $2n$ draws from the unit uniform, and chooses the task with highest realization. The first order statistic of the $2n$ independent unit uniforms has expected value $\frac{2n}{2n+1}$. Hence the expected surplus in the first best is

$$e[\frac{2n}{2n+1} - \gamma], \tag{1}$$

so that if $\gamma \geq \frac{2n}{2n+1}$, the agent should exert effort.[10] Let $\gamma^* = \frac{2n}{2n+1}$. Trivially, γ^* is increasing in n. In this sense, the importance of authority and incentive provision seem complementary.

Agent Incentives The agent is only rewarded if assigned to a contractible task, in which case the expected benefit to exerting effort is $\beta(1-\sigma)$ times its marginal product. She does not know whether the task is contractible, nor its marginal return, when choosing effort and so she will exert effort if and only if

$$\beta(1-\sigma)prob(task = d)E\{d_i|task = d\} \geq \gamma, \tag{2}$$

where the agent's beliefs are determined by the surplus maximizing equilbrium.

Lemma 1 *The agent's incentive compatibility constraint is given by*

$$\beta(1-\beta)^n \frac{n}{2n+1} \geq \frac{\gamma}{1-\sigma}. \tag{3}$$

solution to this issue would be to assume that there is some small probability of the b and d outputs being attained without effort exertion.

[9]Following Maskin and Tirole, 1990, there is research on the "informed principal" problem, where a privately informed principal contracts with an uninformed agent. In that setting, the emphasis is on screening contracts to reveal the principal's information at the contracting stage. By contrast, here the central concern is outcomes with non-contractible returns without adverse selection at the contracting stage.

[10]In Section 5 I deal with the case where effort is exerted after the principal knows the realization of the random variables, and the first best depends on these realizations.

The results of the paper largely derive from this condition. There are two conflicting pieces to it. First, the expected productivity of the contractible task, if assigned to such a task, is $\frac{2n}{2n+1}$, which is increasing in n. Second, the probability of being assigned to a contractible task is $\frac{(1-\beta)^n}{2}$ which is *decreasing* both in n and β. In words, both incentive pay and more options for the principal make assignment to a contractible task less likely. The product of these two effects yields (3) and the effect of authority on incentives is a horserace between these two effects.

2.1 The Failure of Incentives

Incentives can then only be provided if there exists some β such that (3) holds. As I consider the surplus maximizing Bayesian Nash equilibrium, the principal chooses his preferred β subject to this constraint. There is an allocative inefficiency from increasing β as the principal increasingly misallocates the agent to the best non-contractible task when the best contractible one has higher productivity. As a result, the principal will choose the lowest value of $\beta = \beta^{**}$ where

$$\beta^{**}(1-\beta^{**})^n \frac{n}{2n+1} = \frac{\gamma}{1-\sigma} \tag{4}$$

Does such a value of β^{**} exist? The maximized value of $\beta^{**}(1-\beta^{**})^n$ has a value of $\beta^{**} = \frac{1}{1+n}$. Proposition 1 immediately follows.

Proposition 1 *Incentives can be provided if and only if*

$$\frac{1}{2n+1}\left(\frac{n}{n+1}\right)^{n+1} \geq \frac{\gamma}{1-\sigma} \tag{5}$$

If this condition does not hold, there exists no incentive contract that can induce effort exertion. It is clear that this condition can fail if the costs of effort are high, or monitoring is poor. For example, with $n = 1$, incentives fails if $\frac{\gamma}{1-\sigma} > \frac{1}{12}$ even though the first best requires effort exertion for $\frac{\gamma}{1-\sigma} \leq \frac{2}{3}$. More interesting is that "enough" authority crowds out incentives in Proposition 2.

Proposition 2 *Incentives always fail if n is sufficiently large for any $\gamma > 0$ and $\sigma \geq 0$.*

In words, a sufficiently large set of options for the principal always eliminates incentives. Yet this is precisely when the return to the agent's effort is highest.

The reason that incentives always fail here is because the probability that the agent is assigned to a contractible task $\frac{(1-\beta)^n}{2}$ is decreasing in n and converges to 0 for any positive β. Said another way, when the principal has few options, the "competition" that best contractible task faces from the best non-contractible one is not so strong, in the sense that the marginal productivity of the two can be quite different. Such differences are necessary to overcome the incentive to assign the agent to a non-contractible task. When the principal has more options, the expected difference between the productivities of the best task of each type becomes small, and so she is unlikely to be assigned to the contractible task if incentive pay is used. But if this is the case, the agent never recovers her effort cost.

Small n So far, I have shown that with a sufficiently large number of options, authority crowds out incentives. What about when the number of options is smaller? To give incentive provision its best chance, consider the case where $\sigma = 0$. Let $\gamma^{**} = \frac{1}{2n+1}(\frac{n}{n+1})^{n+1}$ be the feasible effort cost below which the firm can induce effort. In the Appendix, I show that $\frac{d\gamma^{**}}{dn} < 0$, so that the feasibility of incentives is monotonically declining with the principal's options. The outcome of this section is described in Figure 1, where effort should be exerted everywhere below the first best line, but can only be induced if it lies below the feasible effort cost line.

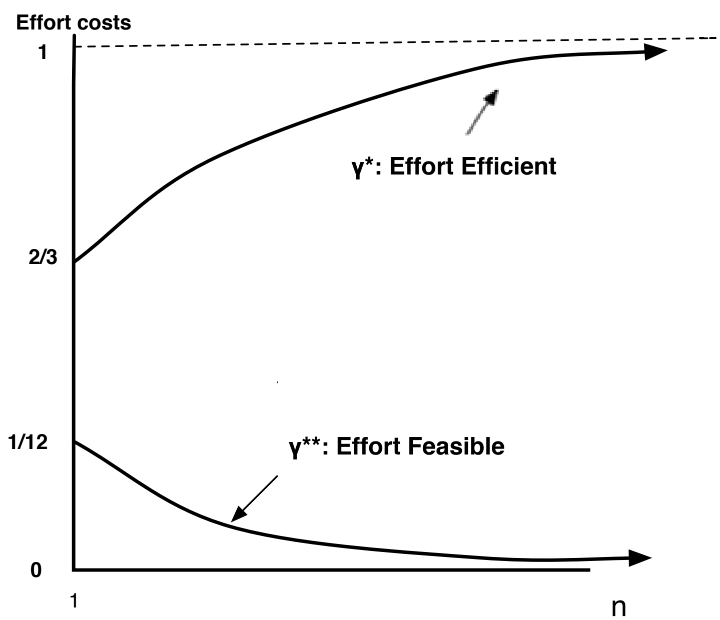

Figure 1: **Feasible and First Best Effort**

Unbalanced tasks The baseline model assumes an equal number of each type of task. Consider a simple alternative parameterization where the ratio of contractible to non-contractible tasks is $k > 1$, so there are n non-contractible tasks and kn contractible tasks. Then the incentive compatibility constraint is $\beta(1-\beta)^n \frac{kn}{(k+1)n+1} \geq \frac{\gamma}{1-\sigma}$, and the feasibility condition is given by $\frac{k}{(k+1)n+1}(\frac{n}{n+1})^{n+1} \geq \frac{\gamma}{1-\sigma}$. Not surprisingly, these conditions are easier to satisfy than in the balanced case, as the expected value of the best contractible task is higher. However, the left hand side continues to declines in n, so it remains the case more options for the principal both make effort more desirable yet less feasible. Finally, incentives continue to always fail for n large enough, as in the baseline model.

Correlation in Returns: The returns to the two tasks are independent. Correlation in returns make incentive provision harder, because the best contractible task is only assigned if its return is sufficiently better than the best non-contractible task. To see this, consider the simplest case where $n = 1$ but with probability ϕ the returns of the two tasks are independent, and with probability $1 - \phi$ the returns are identical. Then the feasibility condition becomes $\frac{\phi}{12} \geq \frac{\gamma}{1-\sigma}$, which is harder to satisfy than (5) with $n = 1$.

One final point is worth making about the comparative statics of agency problems. A seemingly reasonable implication of agency theory is that incentives should be more likely when the marginal return to effort rises. This is not true here when marginal returns are generated by increased n - while the marginal benefits of effort rise in n the agent trusts the principal so much less in these settings that she becomes less likely to incur the costs of doing so.

2.2 When Incentives Can Be Provided

So far, one issue has been addressed - where the agent trusts the principal so little that incentives fail. Now consider the case when the agent can be induced to exert effort: some β exists to satisfy (4). It is useful to begin by considering surplus with effort exerted. This is computed in the Appendix to be

$$S(\beta^{**}) = \frac{n}{2n+1}(1-\beta^{**})^{n+1} + \frac{n}{2n+1}(1-\beta^{**})^n + \frac{n}{n+1}[1-(1-\beta^{**})^{n+1}] - \gamma. \quad (6)$$

This is decreasing in β^{**} as the agent is sent on more "wild goose chases". As a result, if β^{**} declines in n, incentives can be provided more cheaply with a larger span of authority. I call the relationship between β^{**} and n the Incentive Multiplier, which is positive if $\frac{d\beta^{**}}{dn} < 0$.

The Incentive Multiplier is positive[11] if and only if

$$log(1-\beta^{**}) + \frac{1}{n(2n+1)} > 0. \tag{7}$$

This has an indeterminate sign because there are two conflicting effects of increasing the principal's options. First, the productivity of the best contractible task increases - this is the $\frac{1}{n(2n+1)}$ term. Second, the agent is more likely to be "cheated" by being given a non-contractible task. This effect is proportional to $log(1-\beta^{**})$, and the total effect is determined by the sum of these two factors.

The Incentive Multiplier depends on β^{**} and n. For low enough β^{**} the Incentive Multiplier is always positive. Said another way, when incentives can be provided inexpensively (in a surplus sense) a greater span of authority further reduces the cost. Incentives are inexpensive to provide when γ and σ are low, so a greater span of authority is complementary with incentive provision when monitoring is easy and costs low. By contrast, when β or n are high, the Incentive Multiplier always reduces surplus.[12]

In effect, exercising a greater span of authority imposes an externality on incentive provision. When incentives are inexpensive to provide, this externality is positive. By contrast, when incentives are hard to provide, a greater span of authority imposes a negative externality on incentive provision.

The outcome of this section is illustrated in Figure 2, where optimal incentives (β^{**}) are plotted against n. The downward sloping hashed line gives the feasibility constraint, $\beta^{**} = \frac{1}{1+n}$, and only outcomes that lie below this line are feasible. I then distinguish between two cases - where incentive costs rise in n at $n = 1$, and where they do not. First consider the case where $log(1-\beta^{**}) + \frac{1}{n(2n+1)} < 0$ at $n = 1$ so $log(1-\beta^{**}) < -\frac{1}{3}$ and the Incentive Multiplier is always negative. Second, consider the case where $log(1-\beta^{**}) \geq -\frac{1}{3}$: here more options for the principal initially reduces required incentive payments, and greater span of authority is complementary with incentive provision. However, even in that case, at some point n rises by enough to make $log(1-\beta^{**}) + \frac{1}{n(2n+1)} < 0$ and so eventually the two variables become positively related - incentives are harmed by more authority - and eventually hit the

[11] $\frac{d\beta^{**}}{dn} = \frac{-[log(1-\beta^{**})+\frac{1}{n(2n+1)}]}{\frac{1}{\beta^{**}} - \frac{n}{1-\beta^{**}}}$. The denominator is positive by the second order condition as we are considering the case where $\beta^{**} < \frac{1}{n+1}$, and reflects simply that the incentives of the agent must be weakly increasing when β^{**} rises.

[12] Note however that β cannot be too high, or else the feasibility constraint is violated, so is it the case that for feasible levels of β, the required β can be increasing in n? The answer is yes for any n. For $\beta = \frac{1}{1+n}$, $log(1-\beta) + \frac{1}{n(2n+1)} < 0$ for any finite n so there is always a range of feasible levels of β where the incentive multiplier is negative.

feasibility condition, but at a higher n than for the poor monitoring case.

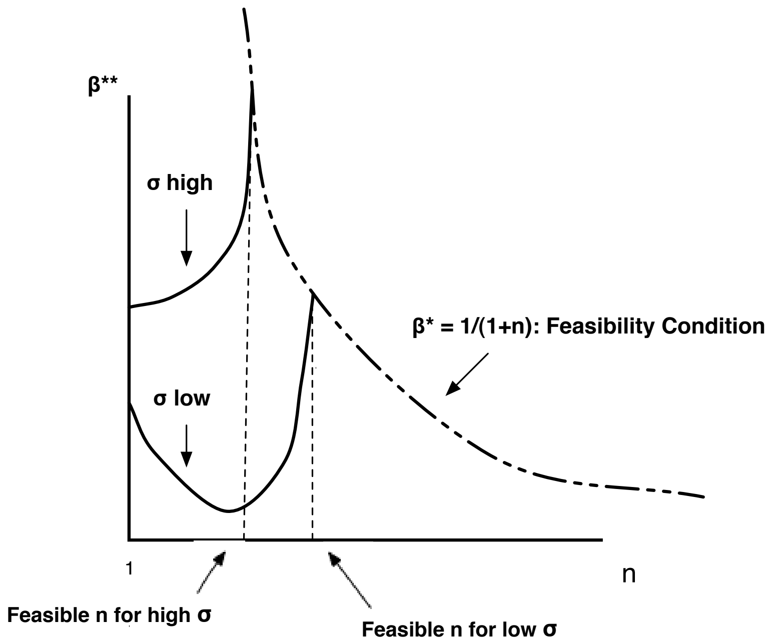

Figure 2: **The Incentive Multiplier**

Effect on Surplus Changing the number of task assignment options has implications for surplus in (6) beyond the Incentive Multiplier. First, the principal finds better tasks on average - the $\frac{n}{2n+1}$ and $\frac{n}{n+1}$ terms reflect the value of more information - which increases surplus. Second, there is the "competition effect" - as n increases the principal misallocates the agent more to the non-contractible task, which will reduce surplus. This arises through varying n for the $(1-\beta^{**})^{n+1}$ and $(1-\beta^{**})^{n}$ terms. Finally, there is the Incentive Multiplier.

It should not be surprising that the aggregate effect of more information on surplus is ambiguous. There is one positive effect (the principal gets better draws), one that is negative (the contractible tasks face stiffer "competition"), and one which is ambiguous, the Incentive Multiplier. In general, this cannot be signed. However, there is a little more that we can say here.

Proposition 3 *Surplus is increasing in n when incentives are inexpensive to provide (β close to 0) but decreasing in n when incentives are sufficiently expensive to provide (close to $\beta = \frac{1}{n+1}$).*

2.3 The Empirical Evidence

Part of the paper's motivation is that formal incentives are less frequent when supervisors retain authority, yet no alternative to either supervisor authority or pay for performance has been offered.

2.3.1 Monitoring Inputs

An alternative to rewarding outputs is to directly monitor agent effort (*input* monitoring). Assume now that monitoring effort directly with no error has a (welfare) cost of $\kappa > 0$, independent of the task to which the agent is assigned.

The downside of input monitoring is this additional cost. Yet it has the upside that agents are always assigned correctly to tasks, as the cost of incentive provision is independent of task assigned. By monitoring inputs, the principal creates surplus of $\frac{2n}{2n+1} - (\gamma + \kappa)$. Then input monitoring will occur in either of two cases: (i) incentives are not feasible and $\frac{2n}{2n+1} - (\gamma + \kappa) \geq 0$, or (ii) incentives are feasible but the required incentive pay β^{**} is sufficiently high such that $S(\beta^{**}) < \frac{2n}{2n+1} - (\gamma + \kappa)$. It should be clear that a sufficient condition for input monitoring is if either n or σ is large enough.

2.3.2 Delegation

An alternative to supervisor authority is to allow the agent to choose, where she randomly chooses a task.[13] Call this delegation.[14] Up to now, little has been said about whether the agent knows the contractibility of tasks. If she does not know, she has a 50% chance of choosing a contractible task and her return to effort is $[(1-\sigma)\frac{1}{4} - \gamma]$, while if she knows whether a task is contractible, she will choose a contractible task and receive marginal return of $[(1-\sigma)\frac{1}{2} - \gamma]$. To give delegation its best chance, consider the case where the agent knows if tasks are contractible and $\frac{1-\sigma}{2} > \gamma$, in which case the agent will exert effort if she chooses the task. Then delegation is optimal[15] only if $n > n^*$ where $\frac{1}{2n^*+1}(\frac{n^*}{n^*+1})^{n^*+1} = \frac{\gamma}{1-\sigma}$, and κ is

[13]There is a large body of work following Aghion and Tirole, 1997, and Dessein, 2002, on how delegation of control to agents affects incentive provision.

[14]Note that delegation is identical to the principal simply randomizing over tasks. This tie is easily broken by imagining that the agent has some private benefit from carrying out actions, where that private benefit is small enough not to overturn the optimal allocation rule, and where the private benefit is privately known to the agent. Then the agent should be delegated control over the decision as the expected value of that benefit can be extracted in the up front payment.

[15]There is a better outcome than pure delegation, namely probabilistic delegation. Let M^d be the marginal return to exerting effort when he has control over the task carried out, and let M^p be her marginal return

sufficiently large.[16]

Two final observations yield a negative correlation between supervisor authority and pay for performance. First, if the choice is delegated to the worker, input monitoring is never used, because $\kappa > 0$ and there is no distortion from pay for performance. Second, input monitoring may be the preferred choice when the supervisor retains authority. This will arise for small enough κ and large enough n. These two additions yield a negative empirical relationship between authority and incentive pay consistent with Table 1.

3 Other Instruments for Providing Incentives

In this section, I address other avenues by which the agent could be provided incentives, namely intrinsic motivation, bureaucracy, and more general output-based contracts.

3.1 Intrinsic Motivation

There has been considerable interest recently in the issue of intrinsic motivation, and the effect that it has on optimal pay for performance (Delfgaauw and Gur, 2003, Benabou and Tirole, 2003, Besley and Ghatak, 2007, and Prendergast, 2007). A central question in this literature is whether using pay for performance demotivates workers through reducing their intrinsic motivation. This paper offers an alternative and very simple reason why pay for performance can demotivate, namely, that it causes workers to trust their bosses less, and as a result doubt that the instructions they are being given are truly the right ones. If this effect is large enough relative to the usual motivating effects of pay for performance, workers have less incentive to exert effort. Furthermore, the total effect of pay for performance depends on the ability to monitor - specifically, when monitoring is poor, the incentive to exert effort always falls, while if monitoring is good, incentives rise.

To see this, consider a scenario where the worker intrinsically values output Y at vY, where $v < 1$. Note here that the agent has the same objective as the principal (if muted), and values both the contractible and non-contractible outcomes. Expected output is $\frac{n}{2n+1}(1 -$

when the principal allocates tasks. Then delegation is only relevant if $M^p \leq \gamma$, but $M^d > \gamma$. But as the principal makes better allocative decisions than random choice of the agent, there is no reason to keep the agent's incentive compatibility constraint slack. Instead, the optimal probability of delegation is given by ρ^*, where $\rho^* M^d + (1 - \rho^*)M^p = \gamma$.

[16] There is one other conceivable reason why the agent may be delegated control, namely, that holding effort constant, the marginal surplus from random choice exceeds that of the principal choosing. This cannot occur because the principal never induces marginal returns worse than random choice.

$\beta)^{n+1} + \frac{n}{2n+1}(1-\beta)^n + \frac{n}{n+1}[1-(1-\beta)^{n+1}]$ and so the incentive compatibility constraint becomes

$$v\left[\frac{n}{2n+1}(1-\beta)^{n+1} + \frac{n}{2n+1}(1-\beta)^n + \frac{n}{n+1}[1-(1-\beta)^{n+1}]\right] + (1-\sigma)\beta(1-\beta)^n \frac{n}{2n+1} \geq \gamma. \tag{8}$$

The piece that is new here is that β affects the agent's perception of output. Expected output is declining in β (holding effort constant), and so intrinsic incentives are harmed. Specifically, if U is the utility of the agent, then holding effort constant,

$$\frac{dU}{d\beta} = e(1-\beta)^n \left[(1-\sigma)(1-\frac{n\beta}{1-\beta}) - \frac{vn^2\beta}{(2n+1)(1-\beta)}\right]. \tag{9}$$

This term is negative for large σ and positive for small σ. In words, the marginal return to exerting effort falls in β if monitoring is poor but increases if monitoring is good.

3.2 Bureaucracy

Bureaucracy generally refers to the use of rules over allowing discretion in firms (Milgrom, 1988). Consider a scenario where the principal can commit to only choose from a predetermined random set $2m$ of tasks, where $m \leq n$.[17] It should be obvious how restricting authority can improve incentives, given the results of the last section. First consider the case where the feasibility constraint is violated. One way to allow effort exertion is to restrict tasks to some m no higher than the largest m where

$$\frac{1}{2m+1}(\frac{m}{m+1})^{m+1} \geq \frac{\gamma}{1-\sigma}. \tag{10}$$

Now consider the case where incentives are feasible. Remember from above the effect of n on surplus is indeterminate. As a result, it is hard to make concrete statements about the extent of bureaucratic restrictions when effort is feasible. However, from Proposition 3 surplus is decreasing in n close to the maximum feasible level $\beta = \frac{1}{1+n}$. As a result, the principal will also optimally restrict his options in this case so that bureaucracy will arise in some settings even when effort is feasible.

[17] This occurs at the same time as the contract choice in the timing above. Whether such restrictions are possible depends very much on the environment. In a setting where it is clear what the possible tasks are, it is feasible to imagine how these restrictions could be implemented, whereas in others, it may be difficult to delineate what a task means.

3.3 Optimal Output-Based Contracts

So far, I have assumed linear contracts, where the share of output obtained by the agent is independent of output produced. Another way to induce effort more efficiently may be through non-linear contracts, so a relevant question here is the extent to which the results depend on the assumption of linearity.

Proposition 4 *Consider a more general contract where the agent receives a transfer $\beta(y)$ when output of y is observed. The unique optimal contract is linear in y for any n.*

Linearity in optimal contracts is unusual, except in the well known Holmstrom and Milgrom (1992) setting. It arises here because if F is the CDF and f the density function of the first order statistic for the unit uniform with n draws, $\frac{F(y-\beta^*(y))}{f(y-\beta^*(y))} = \frac{y-\beta^*(y)}{n}$ is linear in $\beta(y)$ and so the optimal contract is linear.

4 The Span of Authority

The model offers one view of the principal's span of authority - namely, he assigns a single worker to one task among n. The purpose of this section is address whether the idea that more authority can harm incentive provision depends on this stark assumption. To do so, I consider two other cases involving a return to assignment.

4.1 Matching Ability to Tasks

So far, there has been some redundancy of tasks, in the sense that tasks remain unstaffed in equilibrium. However, this is not necessary for the problem to arise. Consider the case where there is one task of each type ($n = 1$) and where there are two workers.

With no other additions to the model, there is no relevant sense of authority as it does not matter who does which one. However, now assume that the agents vary in their "ability", meaning the marginal return to their effort. Specifically, assume that one agent has marginal return to effort $a > 1$ times that of the other agent, where the other's is normalized to 1 as in the baseline model. Then the first best involves assigning the more able agent (the a one) to the task with the highest marginal productivity. If the highest productivity task is non-contractible, then this will always occur. However, if the highest productivity task is contractible, then at the point of task assignment, the principal receives $a(1-\beta)d - b$ by assigning workers efficiently, and $ab - (1-\beta)d$ by inefficient assignment. Then the principal assigns efficiently only if $(1-\beta)d-b>0$, which is identical to the baseline model. Hence the logic of the distortion in the previous section carries over to this assignment problem.

4.2 More Tasks and Agents

One concern with the exercise of increasing n above is that the number of possible tasks becomes large without allowing the number of agents to change. Here I also allow the number of agents to simultaneously change to address the effect of increased n on incentive provision.

It is difficult to attain closed form solutions for small sample order statistics other than the best and worst elements so here I compare two cases where a closed form is easily attained. First, I consider the case of $n=1$ above where there are two tasks and one worker. Here the incentive compatibility constraint is $\frac{\beta(1-\beta)}{3} \geq \frac{\gamma}{1-\sigma}$ if incentives can be provided. I compare that to the case where there are a large number of tasks *and* workers, by considering the limiting case where $n \to \infty$ but now a fraction t of those tasks is carried out. A useful benchmark here is constant returns to scale: $t=\frac{1}{2}$, where half of all tasks are carried out as in the $n=1$ case. This is a useful benchmark as this is the case where the scale of the principal's authority has been increased, without changing its scope.

Proposition 5 *Let β_1 be the smallest value to solve $\frac{\beta_1(1-\beta_1)}{3}=\frac{\gamma}{1-\sigma}$ and assume incentives can be provided with $n=1$. Define t^* by*

$$\frac{2(1-\beta_1)}{3}=1-\left(\frac{2(1-t^*)}{2-\beta_1}\right)^2, \tag{11}$$

where $t^*<\frac{1}{2}$. For all $t \geq t^*$ incentives per worker are cheaper to provide than with $n=1$ while if $t<t^*$, they are more expensive.

In words, incentives become harder to provide only if the number of possible assignments per worker increases - with constant returns to scale ($t=\frac{1}{2}$), it always becomes easier to provide incentives. This result illustrates that what matters for the potentially harmful effect of authority on incentives is not "scale", but rather "scope".

5 When Efforts are Task Specific

In this section, I consider the symmetric case where effort is exerted after the agent has been assigned to a task, in an environment where the agent does not know the contractibility of the task to which she is assigned. Relative to the previous sections, there is one conceptual novelty - the principal knows the realization of the worker's marginal product on her assigned task, which she could reveal to the agent. In this section, I consider such options for information revelation. For simplicity, restrict attention to pure strategy outcomes and where $\sigma = 0$.

Cheap talk will not suffice as a means of persuading the agent - instead, as is usual in signaling settings, credible information must involve a cost to the principal. This cost is in the form of a discretionary transfer made to the agent before effort exertion. As effort is binary, and we are considering pure strategies, there is never more than one such credible transfer or "gift" offered in equilibrium.[18] Accordingly, consider the case where after observing the realizations of the marginal productivities, the principal can, at his discretion, offer g to the agent.

Modified Timing: First, the principal offers the agent a contract with sharing rule β, a fixed fee β_0, and a discretionary transfer g. If the agent rejects, the game ends. If she accepts, the principal privately observes the realization of the $2n$ random variables and the contractibility of the task. He then assigns the agent to one of the activities and chooses whether to offer g. The agent then exerts effort or not. Output is then realized, and the agent is paid according to observed output. At that point, the game ends.

Consider an equilibrium of the form where receipt of g results in effort exertion, but a failure to receive g results in no effort. (This is the only relevant case: g could be 0 of course.) In the usual logic of signaling, the principal offers g only if his profit from the agent exerting effort from its receipt is at least g. As there are two kinds of tasks, the principal offering g implies that either $max\{b_i\}$ or $(1-\beta)max\{d_i\}$ exceeds g. By change of variables this is equivalent to the principal revealing to the agent that either the best contractible task has productivity above y^* or the best non-contractible task has productivity above $(1-\beta)y^*$.

The baseline model is equivalent to $y^* = 0$ and the agent computes the returns to effort conditional on the productivity being in the support $[0, 1]$ for both tasks. With a gift, her productivity lies in the support $[y^*, 1]$ for the contractible task and $[(1-\beta)y^*, 1]$ for the non-contractible task. The firm chooses y^* to maximize ex ante surplus. Deriving the incentive compatibility constraint involves computing Bayes Rule over these supports.

[18] The identity of the task cannot be used to signal here as all tasks are identical to the agent.

Lemma 2 *The agent's incentive compatibility constraint when the principal can choose y^* is given by*

$$R(y^*(\beta))\beta(1-\beta)^n \frac{n}{2n+1} \geq \gamma, \tag{12}$$

where $R(y^*(\beta)) = \frac{1-y^{*(2n+1)}}{1-(1-\beta)^n y^{*2n}}$. Furthermore, $R(0)=1$, $R(1)=0$, and $\frac{dR}{dy^*} > (<)0$ at $y^* = 0(1)$.

This is similar to the initial incentive constraint - it adds only the term $R(y^*(\beta))$ to agent's incentives. Incentives can always be relaxed by offering a small gift, as R is declining in y^* at $y^* = 0$. Hence gifts can be used to overcome some of the issues in the previous sections.[19] However, R is non-monotonic in y^*. In words, offering a small gift relaxes the incentive constraint but, as $R(1) = 0$, a large enough gift makes incentives impossible - hence signaling can only relax the incentive provision problem to a degree.[20] Note also that R is decreasing in β so this model has the same qualitative features as the baseline case but where the principal can choose y^* as desired.

6 Potential Solutions

The principal's preferences have largely been taken as given here, where he has an incentive to abuse his authority to reduce the agent's pay. Yet there are conceivably ways to limit the principal's interest in doing so.

Fixed Wage Bills In settings where there is more than one agent, a possible solution is to use some form of relative performance evaluation with a fixed wage bill, through something like a tournament. In this way, the only discretion that the principal holds is over who gets which rewards, rather than the total allocation. To the extent that fixed wage bills do not result in collusion by agents, these can alleviate the problem.

Payments to Third Parties In the baseline model, the principal gains when the agent is not paid. But the agent not being paid is easily contractible, so another solution may be

[19] The optimal choice of y^* is then the usual monopoly tradeoff: higher y^* may relax the incentives for the group that receives the gift, but fewer agents are offered it and hence exert effort.

[20] A large gift make incentives hard because the agent is very unlikely to be assigned to a contractible task. To see this, consider the limiting case where the principal offers a fixed fee of $g = 1-\beta$. The agent then knows that upon being offered $g = 1-\beta$ that either $max\{d_i\} = 1$ or $max\{b_i\} \geq 1-\beta$. Bayes Rule implies that the likelihood that the agent is type $max\{d_i\} = 1$ is close to zero here, and so exerts no effort.

to penalize the principal whenever that happens, where the principal makes a transfer t to a third party when incentive pay is 0. (Third parties are necessary here in order to retain incentives to agents.) Or said another way, the principal is only rewarded on profits excluding wage costs, as in Zabojnik, 1998. In this way, his incentive to reduce wage payments can be reduced.

More Complex Mechanisms So far it has been assumed that only the principal can assign tasks, through the assumption that only he can distinguish between tasks. If this assumption is dropped, it may be possible to design mechanisms in ways that can improve efficiency. In the Appendix, I show that if a mechanism designer can identify tasks and more complex mechanisms are allowed, the first best can be approximated. This also requires deep pockets for the principal. In this mechanism, the principal reports the realizations of the d and b vectors to a mechanism designer, and both the implemented task and payments are contingent on the reports. By using a Becker-like mechanism - investigate all states with small probability to get truth-telling (this is where both the ability to identify tasks and deep pockets matter) - the principal can be induced to tell the truth over the productivities of all contractible tasks. Given this information, the designer then taxes the principal for implementing a *non-contractible* task by exactly the wage savings he would have received by assigning the worker to the best *contractible* task. In this way, the first best can be approximated.

The plausibility of this mechanism is debatable. First, it requires that the mechanism designer choose the task, which implies identifying whether the appropriate "task" has been implemented. While the outcomes of tasks may be sometimes easily identifiable, the tasks themselves are often so amorphous and fluid that their ex ante identification may be simply too difficult for a third party. Second, the mechanism requires large transfers from the principal to the agent, requiring deep pockets for the principal. Finally, in reality the mechanism may be too complex for the agent to compute both its value to her and how it solves the principal's problems. The spirit of the paper is one where agents are poorly informed about the technology used by the firm, and this may go well beyond knowing the realization of the $2n$ random variable. For these reasons, the use of this kind of mechanism is likely to be limited.

7 Robustness

The central point of the paper is that a better informed supervisor can make incentive provision more difficult, because his assignment choices are not trusted. So far, I have addressed cases where one stark result arises, where a principal with sufficient options always causes incentives to fail. Though the setting for this is far from pathological - identical distributions with a finite upper bound - this result does not generalize to other distributions. I show this here in two ways.

7.1 Non-Symmetric Cases

Up to now, I have only considered the case where the returns to the contractible and non-contractible tasks were drawn from the same distribution. Here I allow one of the distributions to have higher expected returns than the other. There are two cases - (i) where the non-contractible activities are ex ante more productive, and (ii) where the contractible returns are more productive. I consider each in turn.

7.1.1 When Non-Contractible Tasks Have Higher Expected Return

First consider the case where the expected returns from the non-contractible activity are higher, where $B > 1$ while D remains equal to 1. Here the incentive compatibility constraint becomes $(\frac{D}{B})^n \beta^{**}(1-\beta^{**})^n \frac{n}{2n+1} \geq \frac{\gamma}{1-\sigma}$. The maximized value of $\beta^{**}(1-\beta^{**})^n$ remains $\beta^{**} = \frac{1}{1+n}$, and hence incentives can only be provided if and only if

$$\left(\frac{D}{B}\right)^n \frac{1}{2n+1}\left(\frac{n}{n+1}\right)^{n+1} \geq \frac{\gamma}{1-\sigma}. \tag{13}$$

This is harder to satisfy than before as $D < B$. Hence the problems that plague effort exertion in the baseline hold with greater force in this situation.

7.1.2 When Contractible Tasks Have Higher Expected Return

Now consider the case where $B < 1$ and $D = 1$. This case is more conceptually distinct, as there is now "daylight" between the best contractible task's productivity and that of the non-contractible task, and right tail events become more important as the principal has more options. The key issue for whether the qualitative nature of the comparative statics on n change is not whether D exceeds B, but rather whether $D(1-\beta)$ exceeds B. To see this, note that if $D(1-\beta) \geq B$, the incentive constraint is given by

$$\beta\left[\frac{n}{n+1}D\left[1-\left(\frac{B}{D(1-\beta)}\right)^{n+1}\frac{n}{2n+1}\right]\right] \geq \frac{\gamma}{1-\sigma}. \tag{14}$$

Unlike the previous sections, the incentive constraint now becomes easier to satisfy as n increases.

By contrast, if $D(1-\beta) < B$, the incentive constraint is $\left(\frac{D}{B}\right)^n \beta^{**}(1-\beta^{**})^n \frac{n}{2n+1} \geq \frac{\gamma}{1-\sigma}$. Conceptually this is no different from above - incentives may be somewhat easier to achieve than in the symmetric case (as $\frac{D}{B} > 1$), but it remains the case that incentives fail with enough options for the principal. Hence what matters is whether $D(1-\beta) < B$ or not. However, this condition does not help as β is endogenous, so to make more progress I identify a lower bound on β.

Proposition 6 *The lowest possible value of β is given by $\underline{\beta} = \frac{\gamma}{D(1-\sigma)}$. A necessary condition for increases in n to relax the feasibility condition is*

$$D(1-\underline{\beta}) \geq B. \tag{15}$$

Proposition 6 illustrates the possibility of greater scope of authority helping incentive provision, even for a principal with many options. The reason is that if (15) holds, then for sufficiently large n, it will be the case that $D(1-\beta) > B$, and so more scope of authority increases the likelihood of being assigned to a contractible task, even though the principal has to give a piece of its returns to the agent. Note however, that there are three caveats to this more optimistic view of managerial authority on the provision of incentives. First, if monitoring is poor (σ sufficiently large), it can never be the case that (15) holds. Second, even if monitoring is perfect, it is not enough that the upper bound of the contractible distribution exceeds the non-contractible one, instead it must exceed it by the cost of effort for (15) to hold. Finally, the condition above implies that when the principal's information set becomes sufficiently rich, further increases in n make incentives easier to provide. It does not imply that for lower levels of n.[21]

[21] The reason is very simple. The first best calculation is based on the agent always being assigned to a contractible task and hence being rewarded. But for smaller n the probability is less than 1, which implies that β must increase to compensate the agent. But increases in β make it less likely that $D(1-\beta) \geq B$ and so the results of the previous section continue to hold.

7.2 Other Distributions

Consider a more general distribution, where the two kinds of tasks have identical distributions, but where the CDF of the distribution of the most productive task is $F_n(z)$ defined from 0 to ∞, with density $f_n(z)$. Then with a linear contract, the worker's incentive compatibility constraint is given by $\int_0^\infty \beta z F_n((1-\beta)z) f_n(z) dz \geq \frac{\gamma}{1-\sigma}$. Increasing β continues to have the effect of making an assignment to a contractible task (weakly) less likely, so the first implication above continues to hold. However, it is not necessarily the case that increases in the scope of the principal's authority makes "wild goose chases" more likely.

To see this, it is worth considering two distributions, the Freschet and Gumbel.[22] Proposition 7 shows that greater scope of authority does not leads to more misallocation in these cases.

Proposition 7 *The probability of being assigned to a contractible tasks, holding β fixed, is independent of n for both the Freschet and Gumbel distributions.*

Consequently, greater scope of authority is always beneficial to the provision of incentives, for the reason that the principal is getting better and better draws - so β can fall - with no reduced likelihood of being assigned to a "wild goose chase" for a given β.

8 Conclusion

Providing incentives and exercising authority are two of the most important roles played by managers. The central empirical point of this work is that authority and worker incentive pay may be difficult bedfellows, as suggested by the empirical evidence described in the introduction. While Aghion and Tirole, 1997, and Prendergast, 2002, address this issue when workers are better informed, the emphasis here is where principals know more about firm objectives, payoff, or requirements to coordinate,[23] but workers simply don't trust them to use that information appropriately.

[22] The reason is that for order statistics that have a limiting non-degenerate distribution, the distribution of the first order statistic must converge in n to one of three distributions - Weibull, Freschet, or Gumbel. The commonly used Normal, Log-Normal, Exponential, Gamma, Log, and Weibull distributions converge to Gumbel, while the fatter tailed Cauchy, Pareto, and Freschet converge to Freschet. I focus on the latter two, as these retain their shape as n increases, in the sense that the first order statistic of a Gumbel is Gumbel, and similarly for the Freschet. As a result, by considering these two distributions I can make statements about both small n and the limiting case.

[23] As in say Dessein, Garicano, and Gertner, 2008.

There is a large literature that addresses the issue of agents not trusting the motives of their principals. Mostly this literature is in the context of repeated interaction, where the constraining factor to efficiency is the value of the future relationship. For details, see Bull, 1987, Baker, Gibbons and Murphy, 1993, Levin, 2003, and Powell, 2011.[24] One purpose of this paper is partly to address the issue of untrustworthy principals in a static setting where the role of authority on incentives becomes a horserace between the possibility of better choices by the principal, but choices that the agent does not always trust.

A further motivation comes from recent work on incentives offered to Chicago schoolchildren by Fryer, 2010. Incentives offered to these children to improve test scores failed, not for the usual reasons, but because the students did not know how to produce good test scores. For incentive contracts to have been effective, students would have needed help from others (teachers in this case) to identify the return to various kinds of studying. The key issue in such settings is not simply that agents don't know their marginal product but rather why don't principals simply tell them what to do? While this solution may be the appropriate solution in some settings,[25] one of the objectives of this paper is to show that generally there are reasons not to trust the motives of principals who also use incentive pay.

[24] There is some work in a static setting, such as Kahn and Huberman, 1988, and Tirole, 1992, but the emphasis is very different.

[25] One suspects it would be a potentially successful avenue in the context of schoolteachers telling their students how to improve test scores as they have no countervailing incentive not to see them succeed.

References

Aghion, Philippe and Jean Tirole (1997), "Formal and Real Authority in Organizations", *Journal of Political Economy*, 105, 1-29.

Ariely, D., U. Gneezy, G. Loewenstein, and N. Mazar (2009), "Large Mistakes and Big Mistakes," *Review of Economic Studies*, 2009.

Baker, George, Robert Gibbons and Kevin J. Murphy (1994), "Subjective Performance Measures in Optimal Incentive Contracts", *Quarterly Journal of Economics*, 109, 1125-56.

Benabou, Roland, and Jean Tirole (2003), "Intrinsic and Extrinsic Motivation," *Review of Economic Studies*, 70(3), 488-520.

Besley, Tim, and Maitreesh Ghatak (2005), "Competition and Incentives with Motivated Agents," *American Economic Review*, 95(3), 616-636.

Bull, Clive (1987), "The Existence of Self-Enforcing Wage Contracts", *Quarterly Journal of Economics*, 102, 147-59.

Colombo, Massimo G., and Marco Delmastro (2004) "Delegation of Authority in Business Organizations: An Empirical Test", *Journal of Industrial Economics*, Vol. 52, No. 1, pp. 53-80.

Deci, E.L., R. Koestner, and R. Ryan (1999), "A Meta-Analytic Review of Experiments Examining the Effects of Extrinsic Rewards on Intrinsic Motivation," *Psychological Bulletin*, vol. 125, 627-668.

Delfgaauw, Josse, and Robert Dur (2003), "Signaling and Screening of Workers' Motivation," mimeo, Erasmus University.

Dessein, Wouter (2002), "Authority and Communucation in Organizations", *Review of Economics Studies*, 811-38.

Dessein, Wouter, Luis Garicano and Robert Gertner (2008), "Organizing for Synergies: A Theory of Hybrids", mimeo, Chicago GSB.

DeVaro, Jed, and Fidan Ana Kurtulus (2010), "An Empirical Analysis of Risk, Incentives, and the Delegation of Worker Authority", *Industrial and Labor Relations Review*, 63(4), 641-61.

Foss, Nicolai J., and Keld Laursen (2005), "Performance Pay, Delegation and Multitasking Under Uncertainty and Innovativeness: An Empirical Investigation," *Journal of Economic Behavior and Organization*, Vol. 58, No. 2, pp. 246-76.

Freyer, Roland (2010) ,"Financial Incentives and Student Achievement: Evidence from Randomized Trials", mimeo, Harvard University.

Ghosh, M,. F. Lafontaine, and D. Lo (2011), "Delegation and Pay-for-Performance: Evidence from Industrial Sales Force ", mimeo.

Holmstrom, Bengt and Paul Milgrom (1992), "Multi-Task Principal Agent Analyses: Linear Contracts, Asset Ownership and Job Design", *Journal of Law, Economics, and Organization*, 7, 24-52.

Kahn, Charles and Gur Huberman (1988), "Two-Sided Uncertainty and Up-or-Out Contracts", *Journal of Labor Economics*, 6, 423-44.

Jonathan Levin (2003), "Relational Incentive Contracts", *American Economic Review*, 93(3), 835-857.

MacLeod, W. Bentley and Daniel Parent (1999), "Job Characteristics and the Form of Compensation", in Solomon Polacheck, ed., *Research in Labor Economics*, Vol. 18, pp. 177242. Stamford, Conn.: JAI.

Maskin, Eric, and Jean Tirole (1990), "The Principal-Agent Relationship with an Informed Principal: The Case of Private Values", *Econometrica*, 58(2), 379-409.

Milgrom, Paul (1988), "Employment Contracts, Influence Activities, and Efficient Organizational Design", *Journal of Political Economy*, 96(1), 42-60.

Nagar, Venky (2002), "Delegation and Incentive Compensation", *The Accounting Review*, Vol. 77, No. 2, pp. 379-95.

Powell, Michael (2011), "Productivity and Credibility in Industry Equilibrium", mimeo, MIT.

Prendergast, Canice (2002), "The Tenuous Tradeoff of Risk and Incentives", *Journal of Political Economy*, 110(5), 1071-1102.

Prendergast, Canice (2007), "The Motivation and Bias of Bureaucrats," *American Economic Review*, 97(1), 180-96.

Tirole, Jean (1992), “Collusion and the Theory of Organizations”, in *Advances in Economic Theory: Sixth World Congress, Vol. 2.* J. Laffont, ed. Cambridge: Cambridge University Press.

Wulf, Julie (2007), “Authority, Risk, and Performance Incentives: Evidence from Division Manager Positions Inside Firms”, *Journal of Industrial Economics*, Vol. 55, No. 1, pp. 169-96.

Zabojnik, Jan (1998) “Sales Maximization and Specific Human Capital”, *RAND Journal of Economics*, 29, 790-802.

9 Appendix

Proof of Lemma 1 Consider the task assignment decision of the principal. The principal will either assign the agent to the highest d_i or the highest b_i, so what matters for incentives is the distribution of the first order statistics of the two types of tasks. The cdf of the first order statistic of a unit uniform with n draws is $F(y) = y^n$ with density $f(y) = ny^{n-1}$. The principal's objective at the point of assigning a task is to maximize

$$max\{max\{b_i\}, max\{d_i(1-\beta)\}\}Ee \tag{16}$$

where Ee is the principal's expectation of the agent's effort. The agent will be assigned to a contractible task if $(1-\beta)$ of its productivity exceeds the productivity of the best non-contractible task, so that if the agent draws a value z for the best contractible task, the probability that it will be assigned to the agent is given by $F((1-\beta)z) = ((1-\beta)z)^n$. But as the agent does not know z when choosing effort, she does to so maximize $e[\beta \int_0^1 zf(z)F((1-\beta)z)dz - \gamma] = e[\beta \int_0^1 znz^{n-1}((1-\beta)z)^n dz - \gamma]$ which is

$$e[(1-\sigma)\beta(1-\beta)^n \frac{n}{2n+1} - \gamma]. \tag{17}$$

Incentives can then only be provided if there exists some β such that

$$\beta(1-\beta)^n \frac{n}{2n+1} \geq \frac{\gamma}{1-\sigma}. \tag{18}$$

Proof of Proposition 1 The expected return to exerting effort is given by (17). But as the maximized value of $\beta(1-\beta)^n$ arises at $\beta = \frac{1}{n+1}$, simple substitution yields Proposition 1.

Proof of Proposition 2 Let γ^{**} be the feasible effort cost, namely the effort cost below which the firm can induce effort given the constraint that (5) must hold so $\frac{1}{2n+1}(\frac{n}{n+1})^{n+1} = \gamma^{**}$. Note that by contrast to the first best, $\frac{d\gamma^{**}}{dn} < 0$, as $\frac{d\gamma^{**}}{dn} = \frac{n}{(2n+1)(n+1)^n}[-\frac{2n}{(2n+1)(n+1)} + \frac{1}{(n+1)^2} + \frac{n}{n+1}log(\frac{n}{n+1})] < 0$ because $\frac{n}{n+1} < 1$ and $\frac{2n}{2n+1} > \frac{1}{n+1}$. Furthermore, $\gamma^{**} \to 0$ as $n \to \infty$.

Computation of Surplus If the agent exerts effort of $e = 1$, there are three possible outcomes.

- The maximum b_i exceeds $1-\beta$. The (unconditional) surplus created then given by

$$\int_{1-\beta}^{1} yny^{n-1}dy = \frac{n}{n+1}(1-(1-\beta)^{n+1}). \tag{19}$$

- The maximum b_i is below $1-\beta$ and the maximum b_i exceeds the maximum d_i. The (unconditional) surplus created then given by

$$\int_0^{1-\beta} yny^{n-1}(\frac{y}{1-\beta})^n dy = \frac{n}{2n+1}(1-\beta)^{n+1}. \tag{20}$$

- The maximum b_i is below $1-\beta$ and the maximum d_i exeeds the maximum b_i. The (unconditional) surplus created then given by

$$\int_0^{1} yny^{n-1}(y(1-\beta))^n dy = \frac{n}{2n+1}(1-\beta)^{n}. \tag{21}$$

Surplus is then the sum of these three terms.

Proof of Proposition 3 First consider surplus. There are three cases to consider - (i) where the maximum b_i exceeds $1-\beta$ and hence there is no value of d_i that can beat it, (ii) where where the maximum b_i is below $1-\beta$ and wins, and (iii) where where the maximum b_i is below $1-\beta$ and loses to the maximum d_i. Surplus is given by the sum of these three states and is given by

$$S = \int_{1-\beta}^{1} nx^{n-1}xdx + \int_0^{1-\beta} xnx^{n-1}(\frac{x}{1-\beta})^n dx + \int_0^1 xnx^{n-1}(x(1-\beta))^n dx. \tag{22}$$

Integration yields

$$S(\beta^{**}) = \frac{n}{2n+1}(1-\beta^{**})^{n+1} + \frac{n}{2n+1}(1-\beta^{**})^{n} + \frac{n}{n+1}[1-(1-\beta^{**})^{n+1}] - \gamma. \tag{23}$$

The effect of n on surplus is given by

$$\frac{dS}{dn} = \frac{n}{(n+1)(2n+1)}\frac{d[(1-\beta)^{n+1}]}{dn} - \frac{d[\beta(1-\beta)^n\frac{n}{2n+1}]}{dn} + \frac{2(1-\beta)^{n+1}}{(2n+1)^2} + \frac{1-(1-\beta)^{n+1}}{(n+1)^2} \tag{24}$$

where $\frac{d[(1-\beta)^{n+1}]}{dn} = (1-\beta)^{n+1}log(1-\beta) - [(n+1)(1-\beta)^n]\frac{d\beta}{dn}$, $\frac{d[\beta(1-\beta)^n\frac{n}{2n+1}]}{dn} = 0$ from (4), and $\frac{d\beta}{dn} = \frac{-[log(1-\beta^{**})+\frac{1}{n(2n+1)}]}{\frac{1}{\beta^{**}}-\frac{n}{1-\beta^{**}}}$. At β close to 0,

$$\frac{dS}{dn} = \frac{2}{(2n+1)^2} > 0 \tag{25}$$

and surplus is enhanced by the principal being better informed. By contrast, remember that the maximum value of β is $\frac{1}{n+1}$. Evaluating (24) at this point, the sign of the effect on total surplus is the sign of $-[log(1-\beta^{**})+\frac{1}{n(2n+1)}]$ at $\beta^{**} = \frac{1}{n+1}$, which is always negative.

Proof of Proposition 4: Consider a more general contract where the agent receives a transfer $\beta(y)$ when output of y is realized. Then as the density of the first order statistic is given by $f(y) = ny^{n-1}$, and the probability of contractible output y "winning" is $(y-\beta(y))^n$, the relevant incentive compatibility constraint is given by

$$\int_0^1 ny^{n-1}\beta(y)(y-\beta(y))^n dy \geq \frac{\gamma}{1-\sigma} \tag{26}$$

Then consider the principal's objective. When the principal increases $\beta(y)$ for realized output y, the marginal loss to him is as follows - rather than create surplus of y, instead $y - \beta(y)$ is produced, which occurs whenever *both* the best contractible outcome is y and the best non-contractible outcome is $y - \beta(y)$. Hence the marginal loss is given by $L = -\beta(y)ny^{n-1}n(y-\beta(y))^{n-1}$. Then if λ is the Lagrange multiplier on the incentive constraint, the optimal choice, $\beta^*(y)$ is given by $L = -\lambda\frac{d[\int_0^1 ny^{n-1}\beta(y)(y-\beta(y))^n dy - \frac{\gamma}{1-\sigma}]}{d\beta(y)}$ or after a small amount of manipulation,

$$\beta^*(y) = \frac{\lambda y}{\lambda(n+1)+n}. \tag{27}$$

Hence, the unique optimal contract is linear in output, with slope $\frac{\lambda}{\lambda(n+1)+n}$ and so there is no loss from the assumption of linearity above. This formulation also makes intuitive sense where when the incentive constraint is weak, and $\lambda \to 0$, then $\beta^* \to 0$, while when the incentive constraint becomes very binding, where $\lambda \to \infty$, maximum incentives converge to $\beta^* \to \frac{1}{n+1}$, as above in (5).

More generally, for any common distribution of first order statistics F for both contractible and non-contractible returns, the optimal choice of $\beta^*(y)$ is given by

$$f(y)f(y-\beta^*(y))\beta^*(y) = -\lambda f(y)[F(y-\beta^*(y)) - \beta^*(y)f(y-\beta^*(y))] \tag{28}$$

or

$$\beta^*(y) = -\lambda[\frac{F(y-\beta^*(y))}{f(y-\beta^*(y))} - \beta^*(y)]. \tag{29}$$

For the distribution of the first order statistic for the uniform with n draws, $\frac{F(y-\beta^*(y)}{f(y-\beta^*(y))} = \frac{y-\beta^*(y)}{n}$ is linear in $\beta(y)$ and so the optimal contract is linear.

Proof of Proposition 5 If a measure m of tasks are done, then if all contractible tasks above d^* are assigned and all non-contractible tasks above b^*, then as $(1-d^*)+(1-b^*) = m$, yet $b^* = (1-\beta)d^*$, so

$$d^* = \frac{2-m}{2-\beta}. \tag{30}$$

If the agent knows that the probability she is assigned a task for which she is paid is only $1 - \frac{2-m}{2-\beta}$, the agent's incentive compatibility constraint is given by

$$\beta \frac{2-m}{2(2-\beta)} \geq \frac{\gamma}{1-\sigma}. \tag{31}$$

In order to identify how increasing scale affects the ability to induce effort exertion, consider the case the value of m for which incentives are identical to the case of $n = 1$. This is given by

$$\frac{1-\beta_1}{3} = \frac{1}{2}(1 - (\frac{2-m^*}{2-\beta_1})^2). \tag{32}$$

Note that $m^* < 1$. But $t = \frac{m}{2}$, so the relevant condition in terms of fraction of tasks done is $\frac{1-\beta_1}{3} = \frac{1}{2}(1 - (\frac{2(1-t^*)}{2-\beta_1})^2)$.

Proof of Lemma 2 The conditional probability that $max\{d_i\} \geq y^*$ given the fixed fee being offered is given by

$$\frac{1-y^{*n}}{[1-y^{*n}] + y^{*n}[1-((1-\beta)y^*)^n]} = \frac{1-y^{*n}}{1-(1-\beta)^n y^{*2n}}, \tag{33}$$

and the incentive compatibility constraint is then given by

$$\beta(\frac{1-y^{*n}}{1-(1-\beta)^n y^{*2n}}) \int_{y^*}^{1} z \frac{nz^{n-1}}{1-y^{*n}}((1-\beta)z)^n dz \geq \gamma, \tag{34}$$

or

$$\frac{1-y^{*(2n+1)}}{1-(1-\beta)^n y^{*2n}} \beta(1-\beta)^n \frac{n}{2n+1} \geq \gamma. \tag{35}$$

Proof of Proposition 6: Initially consider the outcome as $n \to \infty$. In the limit, the principal receives a first order statistic of D from the contractible distribution and B from the non-contractible one. The first best arises here if $D(1-\beta) \geq B$, as the agent is always assigned a contractible task. If the agent believes that she is always assigned the contractible task, then the lowest possible value that β can take to satisfy incentive compatibility is given by $\beta = \frac{\gamma}{D(1-\sigma)}$. Then the equilibrium inference is indeed true if the agent is always allocated to the contractible task, which occurs if and only if $D - \frac{\gamma}{1-\sigma} \geq B$.

Why begin with the first best condition? In the first best outlined above, the share that the agent receives is at its lowest possible level because she is always rewarded if she exerts effort. If $D - \frac{\gamma}{1-\sigma} < B$, the principal always distorts job assignments and β must rise to compensate as the agent is not always rewarded for exerting effort. But this guarantees that $D(1-\beta) < B$ for any smaller n if $D - \frac{\gamma}{1-\sigma} < B$, in which case increases in n tighten the feasibility constraint. The fact that D exceeds B may relax the incentive constraint, but the logic of incentives ultimately failing in n continues to hold. As a result, the results of the previous section generalize to these settings.

An Approximate First Best Mechanism in Section 6: Here I show that with sufficient flexibility in the contracting environment, the first best can be achieved. The mechanism consists of the principal making a report of $\hat{b}$, the highest productivity non-contractible task, and $\{\hat{d}_1, \hat{d}_2, ...\hat{d}_n\}$, the entire vector of contractible outcomes, to a mechanism designer, where they are ordered such that $\hat{d}_1$ is the lowest and $\hat{d}_n$ is the highest. The mechanism is as follows:

- If $\hat{d}_n < \hat{b}$, then
  - With probability ϵ, where ϵ is small, the principal randomly implements one of the 1 to n contractible projects, even though they are lower than the best reported non-contractible project. Let y_i be the observed output if the ith highest element of the $\hat{d}$ vector is implemented. Then if $y_i = \hat{d}_i$, the agent is paid 0, but if $y_i \neq \hat{d}_i$ but $y_i > 0$, then the principal makes a transfer of $T > 0$ to the mechanism designer, where T is large.
  - With probability $1 - \epsilon$, $\hat{b}$ is implemented and the agent is paid a fixed payment of $\beta^{**}\hat{d}_n$, where $\beta^{**}\frac{n}{2n+1}(1-\epsilon) = \frac{\gamma}{1-\sigma}$.
- If $\hat{d}_n \geq \hat{b}$, implement $\hat{d}_n$ with probability 1 and the agent is offered β^{**}.

Why does this mechanism induce the first best? Begin by assuming that the principal truthfully reveals the realizations of all the contractible variables. Then in the mechanism the principal will choose a non-contractible task if $(1-\beta^{**})max\{d_i\} \geq max\{b_i\} - \beta^{**}\hat{d}_n = max\{b_i\} - \beta^{**}max\{d_i\}$or $max\{d_i\} \geq max\{b_i\}$, which yields the first best outcome. Hence, if the principal can be induced to tell the truth over the maximum d, the mechanism designer can impose a penalty (a fixed fee to the agent so has no effect on incentives) for choosing a non-contractible task such that efficient choices are made.

But the principal can be induced to tell the truth about the d vector by random monitoring, where large penalties are imposed if the outcome does not accord with the reported outcome. Then, as in the usual Becker logic, by increasing T and reducing ϵ such that the principal is indifferent about lying about the states, the principal can be induced to tell the truth with (almost) no distortion in task assignments. For any finite return to deviating from d_n to $\hat{d}_n$, T can be chosen large enough for any ϵ to deter deviation. T, of course needs to be large and so deep pockets are necessary. Finally, note that the agent has incentives to exert effort with incentive payments given by β^{**}. Hence the first best is attainable with these more complex mechanisms and deep pockets for the principal.

Proof of Proposition 7:

Freschet The Freschet has cdf given by $F(x) = exp\{-(\frac{x}{s})^{-\alpha}\}$, $\alpha > 1$ and $x \geq 0$. The mean of this distribution is given by $s\Gamma(1 - \frac{1}{\alpha})$, where Γ is the gamma function. Now consider the first order statistic of this Freschet with n draws. It is Freschet with the only change from the intial distribution being that s becomes $s_n = s(n^{\frac{1}{\alpha}})$. For simplicity consider the case where $s = 1$. The distribution of the first order statistic is then Freschet with mean $n^{\frac{1}{\alpha}}\Gamma(1 - \frac{1}{\alpha})$ and variance that is $(n^{\frac{1}{\alpha}})^2$ times the variance of the base distribution. The distribution of the first order statistic is distributed identically to $n^{\frac{1}{\alpha}}$ times the distribution of the initial distribution. What this implies is that the likelihood that the non-contractible task beats $(1-\beta)$ times the contractible task is independent of n because if $max\{d_i\}(1-\beta) \geq max\{b_i\}$, then $n^{\frac{1}{\alpha}}max\{d_i\}(1 - \beta) \geq n^{\frac{1}{\alpha}}max\{b_i\}$ and so n plays no role.

Gumbel The Gumbel distribution is given by $F(x) = exp^{-exp(x)}$ where $-\infty \leq x < \infty$. This distribution has mean η, where $\eta = 0.5772$. The first order statistic from n draws of this distribution is Gumbel with $F(x) = exp^{-exp(x-log(n))}$ with mean $0.5772 + log(n)$, but with unchanged other moments. As a result, the only change from adding more observations is to increase the mean - the shape of the distribution remains unchanged. As a result, the probability of being assigned to a contractible tasks for a given β is independent of n