

PRELIMINARY AND INCOMPLETE:  
NOT FOR QUOTATION

PERFORMANCE, CAREER DYNAMICS, AND SPAN OF CONTROL

by

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## ABSTRACT

There is an extensive theoretical literature based on what is called the scale-of-operations effect, i.e., the idea that the return to managerial ability is higher the more resources the manager influences with his or her decisions. This idea leads to various testable predictions including that higher ability managers should supervise more subordinates or equivalently have a larger span of control. And although some of this theory's predictions have been empirically investigated, there has been little systematic investigation of the theory's predictions concerning span of control. In this paper we first extend the theoretical literature on the scale-of-operations effect to allow firms' beliefs concerning a manager's ability to evolve over the manager's career, where much of our focus is the determinants of span of control. We then empirically investigate testable predictions from this theoretical analysis using a unique single firm dataset that contains detailed information concerning the reporting relationships at the firm. Our investigation provides strong support both for the model's predictions concerning wages, wage changes, and probability of promotion, and also for the model's predictions concerning span of control including predictions derived from the learning component of the model. Overall, our investigation supports the notion that the scale-of-operations effect and additionally learning are important determinants of the internal organization of firms including span of control.

## I. INTRODUCTION

A standard building block of hierarchical models of the firm is the scale-of-operations effect. The basic idea is that a manager's ability or productivity affects the productivity of workers below the manager in the firm's hierarchy, so there are large returns to ability at higher levels of the hierarchy. This standard building block which can be found in various theoretical models of the firm, such as Lucas (1978), Rosen (1982), and Garicano (2000), leads to a number of testable predictions concerning various important aspects of internal organization such as assignment, promotion, and span of control.<sup>1</sup> And although some of these predictions have been investigated previously, there has been little systematic empirical study of the theory's predictions concerning span of control. This paper is a theoretical and empirical investigation concerning the scale-of-operations effect and, in particular, what the scale-of-operations effect tells us about span of control.

The most basic prediction that the scale-of-operations effect makes concerning span of control is that higher ability or more productive managers should have larger spans of control. In the standard model that makes this prediction a subordinate's productivity is positively related to the ability of the subordinate's manager, but there is also a cost of having a manager that supervises a large number of workers such as having the extra productivity due to a higher ability manager be lower the more workers the manager supervises, i.e., the larger is the manager's span of control. In deciding upon a manager's span of control, therefore, a firm must trade-off the benefit of having this extra productivity apply to a larger number of workers with the costs associated with a larger span of control. In the standard set-up, in turn, this trade-off yields that a higher ability manager should have a larger span of control.

This argument which can be found in early papers on the topic such as Lucas (1978) and Rosen (1982) ignores the idea that during a manager's career firms' beliefs concerning the manager's ability will evolve as the manager accumulates human capital and firms observe

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<sup>1</sup> The scale-of-operations effect was first discussed in Mayer (1960). See Section II for a discussion.

managerial performance in each period. Gibbons and Waldman (1999a, 2006) consider models with this type of learning and show that they match well with various empirical findings in the literature concerning wage and promotion dynamics, including findings in the well known study of Baker, Gibbs, and Holmstrom (1994a,b). But Gibbons and Waldman's models do not incorporate the scale-of-operations effect and also do not consider span of control (in the Gibbons and Waldman models there is a job ladder but there is no sense in which a manager supervises a particular set of workers). In this paper we incorporate the human capital and learning assumptions from the Gibbons and Waldman analyses into a model of the scale-of-operations effect and span of control.

In our basic theoretical analysis workers vary in terms of their innate ability levels, while effective ability is a function of innate ability and general human capital which workers accumulate as they gain labor market experience. When a worker enters the labor market firms initially do not know a worker's innate and effective ability levels but firms update their beliefs concerning these abilities when output is produced, where we assume output realizations are publicly observable. We assume identical competitive firms described by a two-level job ladder where each division in a firm has a single worker on an upper level who supervises employees at a lower level. Consistent with the above discussion, we further assume that the return to managerial ability increases with the number of workers the manager supervises. We also consider a number of extensions including what happens when workers vary in terms of publicly observable schooling levels and what happens when there are three-level job ladders.

We first show that this model is consistent with the basic scale-of-operations result that higher ability managers have larger spans of control, where to be precise in this model this positive correlation is between a manager's expected effective ability and the manager's span of control. We also find a number of other theoretical results that have been found elsewhere such as a positive correlation between wage increases and performance, a positive correlation between worker education and probability of subsequent promotion holding worker performance fixed,

and a positive correlation between worker performance and probability of subsequent promotion holding the education level fixed.

We then derive a number of results concerning span of control that are not found in previous theoretical analyses. These new results are based on the idea that span of control in our framework is positively correlated with a manager's expected effective ability, so factors that are correlated with expected effective ability will also have the same correlation with span of control. For example, we find that, holding education fixed, a manager's span of control should be positively correlated with previous performance. The logic is that higher performance causes firms to positively update their beliefs concerning the manager's expected effective ability, so the positive correlation between expected effective ability and span of control translates into a positive correlation between prior performance and span of control. Similarly, holding performance fixed, a manager's span of control should be positively correlated with the manager's education. The logic here is similar. Because performance ratings are noisy measures of expected effective ability, higher education even controlling for performance ratings is positively correlated with expected effective ability. So the positive correlation between expected effective ability and span of control now translates into a positive correlation between the manager's education and span of control.

In the second part of the paper we test these predictions using confidential performance and personnel data from a large EU "high tech" manufacturing firm that has production facilities in various counties around the world and whose products are sold globally. What distinguishes this dataset from the data set investigated in Baker, Gibbs, and Holmstrom (1994a,b) and data sets used in other similar investigations is that, in addition to performance ratings like those found in the Baker, Gibbs, and Holmstrom study, this dataset includes information about the firm's chain of command, i.e., we know who each individual worker and manager reports to.<sup>2</sup>

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<sup>2</sup> Other studies of single firm data sets that find results similar to those found by Baker, Gibbs, and Holmstrom include Lazear (1992), Seltzer and Merrett (2000), Treble et al. (2001), and Dohmen, Kriechel, and Pfann (2004). A similar dataset is also investigated in the well known studies of Medoff and Abraham (1980, 1981), although those studies mostly focus on different issues.

And we are able to use this information to calculate the span of control for each manager and how that span of control varies over time.

In our empirical analysis we test various predictions of our theoretical analysis. We start with predictions that are consistent with earlier related theoretical studies and for which there is already empirical evidence. For example, as predicted by the theory, we find that wage growth is positively related to a worker's performance evaluation. We also find that the probability of promotion is positively related to the worker's education level and to the most recent performance rating. Note that, consistent with our discussion, related empirical results can be found in various empirical studies such as Baker, Gibbs, and Holmstrom (1994a,b), Seltzer and Merrett (2000), and DeVaro and Waldman (2012).

We then turn our attention to predictions concerning span of control many of which are new and for all of which there is little or no previous empirical evidence in the literature. As discussed, our theoretical model predicts that span of control should be positively related to factors positively correlated with expected effective ability – education and performance ratings. We find evidence consistent with both predictions, although span of control does not seem to be higher for workers with advanced degrees relative to workers with a bachelors degree. We then conduct an additional test which we believe is a better test of the theory because it is less likely to be explained by alternative theories. If span of control is positively related to expected effective ability as our theory predicts, then changes in span of control should be positively correlated with factors that are positively correlated with changes in expected effective ability. Specifically, performance ratings should be positively correlated with how a manager's span of control varies over time. And indeed we find clear evidence that changes in span of control are positively correlated with performance ratings.

Overall, our theoretical and empirical analysis strongly supports the idea that the scale-of-operations effect and learning are important determinants of the design of job hierarchies and, in particular, play an important role in span of control and how a manager's span of control varies over the manager's career. The paper thus contributes to the literature referred to above

that investigates the role of job assignment, learning, and human capital acquisition in the operation of internal labor markets. The earlier literature has mostly focused on showing that these factors can explain a wide variety of empirical findings concerning wage and promotion dynamics in internal labor markets, while here we show that a related approach is also consistent with empirical evidence concerning span of control.

The outline for the paper is as follows. Section II reviews the related literature. Section III presents and analyzes a theoretical model that combines the scale-of-operations effect with human capital acquisition and learning about worker ability as workers gain labor market experience. Section IV describes the data and presents some basic facts about the firm. Section V focuses on tests of predictions of our theoretical model, where much of the focus is on tests concerning span of control because many of these predictions are new. Section VI discusses what we learn from our empirical findings. Section VII provides concluding remarks.

## II. RELATED LITERATURE

As indicated in the Introduction, there is an extensive theoretical literature concerning the operation of hierarchies and much of this literature is consistent with the scale-of-operations effect which was first discussed in Mayer (1960). For example, numerous models that focus on assignment such as Lucas (1978), Rosen (1982), and Waldman (1984a) are characterized by the scale-of-operations effect, while the knowledge-based hierarchy models of Garicano (2000) and Garicano and Rossi-Hansberg (2006) are also consistent with this idea. In addition, there are other models of hierarchical production such as the supervision models of Williamson (1967), Calvo and Wellisz (1978, 1979), and Qian (1994) and tournament models such as Lazear and Rosen (1981) and Rosen (1986) where there is no clear scale-of-operations effect either because workers do not vary in terms of ability (the supervision models) or the managerial production

function is not modeled (the tournament models).<sup>3</sup> But we believe that it is possible to build extensions of such models that would be consistent with the scale-of-operations effect.<sup>4</sup> In our theory and empirical work we do not try to distinguish between different approaches that might yield the scale-of-operations effect, but rather we focus on identifying the empirical implications of the scale-of-operations effect and the extent to which our data is consistent with those implications.

Another literature our paper builds on is the literature on symmetric learning in labor markets studied initially in Harris and Holmstrom (1982) and Holmstrom (1982). The basic idea in this literature is that firms are imperfectly informed about a worker's ability when the worker enters the labor market and learn about ability gradually as worker outputs are publicly observed.<sup>5</sup> Specifically, we build on the more recent contributions found in Gibbons and Waldman (1999, 2006) which consider this type of symmetric learning in a setting characterized by workers who accumulate human capital with labor market experience and move up a job ladder as they age. Gibbons and Waldman show that such a model can explain various empirical findings concerning wage and promotion dynamics found in Baker, Gibbs, and Holmstrom (1994a,b) and elsewhere.<sup>6</sup> But, as mentioned earlier, the Gibbons and Waldman models do not capture the scale-of-operations effect because there is no span of control in their model. We

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<sup>3</sup> Calvo and Wellisz (1979) do allow for workers who vary in terms of ability, but they purposely do not incorporate the scale-of-operations effect because they want to show that in a model of supervision higher ability workers wind up at higher levels of job hierarchies even in the absence of the scale-of-operations effect.

<sup>4</sup> See Sattinger (1993) and Gibbons and Waldman (1999b) for surveys that discuss many of these literatures.

<sup>5</sup> See Farber and Gibbons (1996) and Altonji and Pierret (2001) for empirical studies that find evidence consistent with the symmetric learning approach. The alternative assumption is that learning is asymmetric which means that only a worker's current employer directly gathers information concerning the worker's ability and other firms observe the actions of the current employer in updating beliefs about the worker. This approach was initially modeled in Greenwald (1979, 1986) and Waldman (1984b), while Gibbons and Katz (1991), Pinkston (2009), and DeVaro and Waldman (2012) find empirical evidence consistent with this assumption. See the Conclusion for a related discussion.

<sup>6</sup> Other important early papers in this literature include Lazear (1992), McLaughlin (1994), McCue (1996), and Podolny and Baron (1997). Also, see Kauhanen and Napari (2011) for empirical results similar to those found by Baker, Gibbs, and Holmstrom but in a study that employs a large linked employer-employee panel dataset, while a number of recent studies including Lima and Pereira (2003), Lluís (2005), Dias da Silva and van der Klaauw (2011), and Hunnes (2011) empirically estimate the Gibbons and Waldman (1999) model using large datasets from other countries and in general these studies find evidence that supports the framework. See Waldman (2011) for a recent discussion of this literature.

incorporate span of control into a Gibbons and Waldman type set-up, where much of our focus is the resulting testable implications concerning span of control and whether the data is consistent with those implications.

This paper is closely related to Smeets and Warzynski (2008). That paper considers personnel and performance data from the same firm considered here. Smeets and Warzynski start by analyzing how the firm's hierarchy has evolved over time and show that the hierarchy has become flatter and that span of control has increased consistent with earlier results in Rajan and Wulf (2006). The conclusion of that analysis is that results are best explained by the knowledge-based theory of hierarchical production and, in particular, by the specific model put forth in Garicano and Rossi-Hansberg (2006) under the assumption that communication costs have fallen over time. They then consider the dynamics of managerial careers including various factors including promotions, compensation, and span of control. Here they find a number of results they suggest might be explained by combining elements of Rosen's (1982) static analysis of assignment and hierarchies with the learning and human capital acquisition elements of Gibbons and Waldman (1999), and then suggest that theoretically and empirically investigating such a model would be worthwhile. Our paper follows up this suggestion.

Our paper also contributes to a small but important empirical literature concerning hierarchical production, where most of this literature is based on survey data. Ortin-Angel and Salas-Fumas (2002) use survey data from a large number of Spanish firms for the period 1990 to 1992. Their main findings are that the elasticity of managerial compensation to number of subordinates seems to be less than one and that differences in measurable human capital explain a large fraction of wage differences within the firm. Another important paper is Rajan and Wulf (2006) referred to above. They focus on 300 large US firms over the period 1986 to 1999 and, as indicated, document a flattening of firms' hierarchies over time, i.e., span of control has increased while number of layers has decreased. They also find growth in pay inequality. Guadalupe and Wulf (2010) is a related study that shows that one of the forces that can lead to this type of flattening is trade liberalization and the increases in competition that typically

follow. Garicano and Hubbard (2007, 2009) employ the 1992 Census of Services to look at hierarchical production in law firms where much of their focus is on specialization. They find a number of results including that, as predicted by the theory they develop, span of control depends on the extent of the market. Finally, Fox (2009) uses a Swedish linked employer-employee dataset and US data from the 1996 Survey of Income and Program Participation to show that the wage gap between large and small firms increases with “job responsibility.” He argues that his results are consistent with predictions of hierarchical models characterized by the scale-of-operations effect. Note that none of these papers consider our main focus which is how managerial ability is related to span of control and how factors related to the evolution of beliefs concerning managerial ability are related to span of control.

### III. MODEL AND THEORETICAL ANALYSIS

In this section we first present and analyze a model with identical competitive firms described by a two-level job ladder and multiple divisions, where a division consists of a single manager at an upper level who supervises workers on a lower level. There are overlapping generations of workers, where workers are in the labor market for exactly two periods but enter the labor market with heterogeneous schooling levels. The model captures both the scale-of-operations effect as in, for example, Lucas (1978) and Rosen (1982), and symmetric learning about worker abilities as in, for example, Gibbons and Waldman (1999, 2006). We then discuss what happens when the analysis is extended to workers with longer labor market lives and to job ladders with more levels.

#### A) The Model

There is free entry into production, where firms are identical and the only input is labor. A worker’s career lasts two periods and labor supply in each period is fixed at one unit for each worker. A worker is referred to as young in the worker’s first period in the labor market and old in the second. Worker  $i$  enters the labor market with schooling level  $s_i$ , where  $s_i$  can take on any

integer value between 1 and  $S$ . We also assume that each cohort of workers has exactly  $z_s$  workers of schooling level  $s$ , where  $z_s > 0$  for all  $s, s=1, \dots, S$ .

Let  $\eta_{it}$  denote worker  $i$ 's expected effective ability in period  $t$ , where

$$(1) \quad \eta_{it} = \theta_i f(x_{it}).$$

In equation (1),  $\theta_i$  is the worker's innate ability and  $x_{it}$  is the worker's labor market experience prior to period  $t$ , i.e.,  $x_{it}=0$  for young workers and  $x_{it}=1$  for old workers. Also, we assume  $f(1) > f(0) > 0$  which captures that workers acquire general human capital as they age. We assume that innate ability can be either high or low:  $\theta_i \in [\theta_L, \theta_H]$ . The ex ante probability that a worker with schooling level  $s$  has high innate ability equals  $p(s)$ , where  $p(s) > p(s-1)$  for all  $s, s=2, \dots, S$ . That is, a higher schooling level translates into a higher probability that the worker has high innate ability. This can be the case either because innate ability and schooling are positively correlated which could occur because of education signaling like in Spence (1973), or because schooling enhances human capital. In the latter case it might be more appropriate to refer to  $\theta_i$  as worker  $i$ 's starting ability rather than worker  $i$ 's innate ability.

A firm consists of two job levels and  $m$  divisions, where in each division a single worker is assigned to job level 2 while the number of workers assigned to job level 1 in a division is a choice variable for the firm. Note that a richer and more realistic specification would make the number of divisions in each firm a choice variable, but that enrichment would not change any of the testable predictions we derive in the next subsection. So for tractability reasons the number of divisions is exogenously determined.

If worker  $i$  is assigned to job 1 in period  $t$ , then the worker produces

$$(2) \quad y_{it} = (1 + v_{it}) [c_1 + c_2 (\eta_{it} + \varepsilon_{it})],$$

where  $c_1$  and  $c_2$  are constants known to all labor market participants and  $\varepsilon_{it}$  is a noise term drawn from a normal distribution with mean 0 and variance  $\sigma_1^2$ . The term  $v_{it}$  equals  $v, v > 0$ , if the worker was employed at the firm in the previous period and zero otherwise (so all young workers assigned to job 1 in period  $t$  are characterized by  $v_{it}=0$ ). The term  $v_{it}$  thus captures that workers

acquire firm specific human capital as they gain experience at a firm and this human capital increases productivity at job level 1.

It is assumed that firm specific human capital is required to produce at the high level job, so only an old worker with previous experience at the firm is ever assigned to job 2. If old worker  $i$  with previous experience at firm  $k$  is assigned to job 2 at division  $j$  in firm  $k$  in period  $t$ , then the worker produces

$$(3) \quad y_{ijkt} = g(n_{jkt})(\eta_{it} + \varepsilon_{i2t}),$$

where  $n_{jkt}$  is the number of level 1 workers employed in division  $j$  in firm  $k$  in period  $t$  and  $\varepsilon_{i2t}$  is a noise term drawn from a normal distribution with mean 0 and variance  $\sigma_2^2$ . We assume  $g(0) = G$ ,  $g' > 0$ , and  $g'' < 0$ . The assumption  $g' > 0$  captures the scale-of-operations effect – formally, the partial derivative of  $y_{ijkt}$  with respect to  $\eta_{it}$  is increasing in  $n_{jkt}$ . We also assume that  $g'' < 0$  and that the concavity of the  $g(\cdot)$  function is such that firms are “small” relative to the population of workers, or equivalently “many” firms operate in equilibrium.<sup>7</sup> We further assume that  $G > (1 + \nu)c_2$  which ensures that each firm finds it profitable to assign the old workers with the highest expected effective abilities to the job 2 positions (see Waldman (1984a) for a related discussion).<sup>8</sup>

At the beginning of a worker’s career, a worker with schooling level  $s$  is known to be of innate ability  $\theta_H$  with probability  $p(s)$  and of innate ability  $\theta_L$  with probability  $1 - p(s)$ . Learning takes place at the end of the worker’s first period in the labor market when the realization of the worker’s output for that period becomes common knowledge. The presence of the noise term in equation (2) means that learning at the end of this first period is incomplete, i.e., there is

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<sup>7</sup> We also assume that the  $g(\cdot)$  function is such that in each period a firm hires at least  $m$  total young workers so that it is able to fully staff its managerial positions in the following period.

<sup>8</sup> Frequently the scale-of-operations effect, as in the Introduction, is described in terms of managerial ability positively affecting the output of workers below the manager in the hierarchy. An alternative specification more similar to this description would be to assume that a manager directly produces zero, but each worker in division  $j$  in firm  $k$  in period  $t$  has an extra term in his or her production function equal to  $g(n_{jkt})(\eta_{jkt} + \varepsilon_{j2t})/n_{jkt}$ , where  $\eta_{jkt}$  is the effective ability of the manager in division  $j$  in firm  $k$  in period  $t$ . We employ the specification described in equations (2) and (3) above rather than this alternative because it simplifies the learning process.

updating after the worker's first period output is realized but after this updating there is still uncertainty concerning the worker's true innate ability.

Workers and firms are assumed to be risk neutral and there is no discounting, while workers face no mobility costs and firms face no hiring or firing costs. To make the model consistent with standard wage determination at most firms we assume spot-market contracting. Also, we focus on wages paid in advance of production rather than piece-rate contracts.

At the beginning of each period, all firms simultaneously offer each old worker a wage for that period and the worker then chooses to work for the firm that offers the highest wage. Further, if multiple firms are tied for this highest wage, the worker chooses randomly among the firms unless one is the worker's employer from the previous period in which case the worker remains with that firm. After this stage, firms hire young workers. Because within a schooling group all young workers look ex ante identical, in any period the young worker wage will vary with the schooling level but within a schooling group all young workers receive the same wage where this wage equates supply and demand for young workers with that schooling level.<sup>9</sup>

Finally, by prior assumption we know that no old worker switches employers to work at job 2 in a new firm. We also assume that the magnitude of firm specific human capital in the production function for job level 1,  $v$ , is sufficiently large that no old worker switches firms to work at job level 1 at a new firm. See DeVaro and Morita (2012) for a related analysis which allows for turnover.

## B) Equilibrium and Testable Implications

In this subsection we describe equilibrium behavior in this model and derive testable implications. In the next subsection we discuss extensions. Note that our focus here is on

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<sup>9</sup> This specification for the young worker wage determination process is consistent with young workers not being able to observe prior wage and promotion practices of each firm so firms are not able to establish reputations concerning these practices.

properties that an equilibrium must satisfy rather than on proving an equilibrium exists. Also, we focus on behavior in equilibria in which there is no entry or exit after the first period.<sup>10</sup>

We start by describing the nature of equilibrium when  $S=1$ , i.e., workers do not vary in terms of schooling levels. This case is a bit simpler and analyzing this case first helps build intuition. Consider firm  $k$  in period  $t$ . The firm starts the period with some number of old workers it employed in job 1 in period  $t-1$  when the workers were young, where these workers vary in terms of their period  $t-1$  outputs. Let  $\eta_{it}^e$  be the market's expectation of worker  $i$ 's expected effective ability in period  $t$ . Given there is a single schooling level so all workers are ex ante identical, for every young worker  $i$  the firm employed in period  $t-1$  we have  $\eta_{it-1}^e = [p(1)\theta_H + (1-p(1))\theta_L]f(0)$ . In turn, after observing period  $t-1$  outputs the market updates its beliefs based on these outputs. Specifically, let  $\theta_{it}^e$  denote the expected innate ability of old worker  $i$  in period  $t$ , i.e.,  $\theta_{it}^e = E(\theta_{it}|y_{i,t-1})$ . Then the expected effective ability of old worker  $i$  in period  $t$  is given by  $\eta_{it}^e = \theta_{it}^e f(1)$ , where  $\eta_{it}^e$  is an increasing function of  $y_{i,t-1}$ . That is, because of Bayesian updating, the expectation concerning expected effective ability when a worker is old is positively related to the worker's output when the worker was young.

Now consider how firm  $k$  assigns these old workers to jobs in period  $t$  (remember we are focused on equilibria in which there is no turnover). Firm  $k$  has  $m$  divisions and a single job level 2 or managerial job in each division and effective ability is more valuable in job level 2 than in job level 1. So the firm promotes the  $m$  old workers with the highest values for  $\eta_{it}^e$ , i.e., the firm promotes the  $m$  old workers with the highest period  $t-1$  outputs and the remaining old workers are assigned to job level 1. In turn, taking the market clearing wage for young workers as given, the firm hires the number of young workers such that each division has the efficient number of level-1 workers. The result is that, given the scale-of-operations effect captured by

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<sup>10</sup> If firm specific human capital is sufficiently high there will never be exit. And if the managerial job is sufficiently important in terms of a division's total production then there will never be entry after the beginning of the game.

the  $g(n)$  term in equation (3), the number of workers in a division is positively related to the division manager's expected effective ability.

Proposition 1 formalizes results from the above discussion. Note that all proofs are in the Appendix.

*Proposition 1:* Consider any firm  $k$  and period  $t$ . If  $S=1$ , then the following describe firm  $k$ 's behavior in period  $t$ .

- i) The firm promotes the  $m$  old workers it employed in the previous period who produced the highest outputs, while the other workers are assigned to job 1.
- ii) The wage increase from period  $t-1$  to  $t$  for old workers at the firm is a strictly positive function of the period  $t-1$  output (this results holds in the economy generally).
- iii) Promoted workers at firm  $k$  receive larger pay increases from  $t-1$  to  $t$  than the workers who were not promoted.
- iv) All young workers hired by the firm are assigned to job 1 and are paid the same wage.
- v) Managerial span of control (weakly) increases with the manager's  $t-1$  output.

We now consider how the nature of equilibrium changes when  $S>1$ , i.e., there are multiple education groups. Consider firm  $k$  in period  $t$  that starts the period with some number of workers it employed in job 1 in period  $t-1$ , where these workers vary in terms of their education levels and their period  $t-1$  outputs.<sup>11</sup> Because the schooling level determines the ex ante probability a worker has high ability, for every young worker  $i$  with schooling level  $s_i$  the firm employed in period  $t-1$  we now have  $\eta_{it-1}^e = [p(s_i)\theta_H + (1-p(s_i))\theta_L]f(0)$ . Further, we also again have that after observing period  $t-1$  outputs the market updates its beliefs concerning the

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<sup>11</sup> Although we do not show it formally, in our model the return to hiring a young worker with a high education level falls with the number of other young workers with high education levels employed. The reason is that the probability the worker will be assigned to a managerial job in the following period is lower when the firm has a large number of other young workers with high education levels. The result is that, rather than a firm hiring a homogeneous group of young workers, in our model the typical case is that a firm hires a heterogenous group of young workers.

workers' innate ability levels. So expected innate ability of old worker  $i$  in period  $t$  is given by  $\theta_{it}^e = E(\theta_{it} | s_i, y_{it-1})$ . In turn, expected effective ability of old worker  $i$  in period  $t$  is still given by  $\eta_{it}^e = \theta_{it}^e f(1)$ . The difference is that now the expectation concerning expected effective ability when a worker is old increases with  $y_{it-1}$  holding  $s_i$  fixed and also increases with  $s_i$  holding  $y_{it-1}$  fixed.

Now consider firm  $k$ 's assignment decisions in period  $t$ . As in the single education group case, the firm promotes the  $m$  old workers with the highest values for expected effective ability to the level-2 or managerial jobs and the remaining old workers are assigned to level-1 jobs. There is a difference, however, which is that this decision rule does not translate into promoting the  $m$  old workers who produced the highest outputs in period  $t-1$ . Because expected effective ability is a function of both the period  $t-1$  output and the schooling level, there can be a pair of workers only one of whom is promoted where the promoted worker had a lower  $t-1$  output but a higher education level. Further, because of the scale-of-operations effect, a manager's span of control is again a positive function of the manager's expected effective ability. In this case this means that span of control will be positively related to output in  $t-1$  holding the schooling level fixed, and will also be positively related to the schooling level holding output in period  $t-1$  fixed.

Proposition 2 formalizes results from the above discussion.

*Proposition 2:* Consider any firm  $k$  and period  $t$ . If  $S > 1$ , then the following describe firm  $k$ 's behavior in period  $t$ .

- i) The firm promotes the  $m$  old workers it employed in the previous period with the highest values for expected effective ability, while the other workers are assigned to job 1.
- ii) Within any schooling group the old workers promoted are the ones who produced the highest outputs, but there can be pairs of old workers where only one is promoted and this worker produced less in  $t-1$  but has a higher education level.
- iii) The period- $t$  wage for old workers at the firm and the wage increase from  $t-1$  to  $t$  for old workers at the firm both increase with the  $t-1$  output holding education fixed, while the

wage for old workers increases with education holding the t-1 output fixed (these results hold in the economy generally).

- iv) Within a schooling group, promoted workers at firm  $j$  receive larger pay increases from  $t-1$  to  $t$  than the workers who were not promoted.
- v) All young workers hired by the firm are assigned to job 1, where the young worker wage increases with the worker's education level.
- vi) Managerial span of control (weakly) increases with the manager's t-1 output holding education fixed, and also (weakly) increases with the manager's education level holding the t-1 output fixed.

The above discussion and Proposition 2 tell us that our model characterized by the scale-of-operations effect and learning yields a number of testable implications. First, the wage for young workers and wage increases for old workers should be increasing functions of worker schooling levels. Second, within a schooling group, promoted workers should be those who performed better prior to promotion. Third, within a schooling group, performance should be positively related to subsequent wage increases and there is a corresponding prediction that promoted workers receive larger wage increases than those that are not promoted. Fourth, a manager's span of control should be positively related to prior performance holding the schooling level constant. Fifth, span of control should also be positively related to the schooling level holding performance constant. Note that the first three of these testable implications are also implications of the Gibbons and Waldman (1999, 2006) analyses.<sup>12</sup>

In summary, in this subsection we have incorporated the scale-of-operations effect into a Gibbons and Waldman (1999, 2006) type model of job assignment, human capital acquisition, and symmetric learning. The model captures results concerning wage and promotion dynamics

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<sup>12</sup> All of these implications would also hold in the absence of learning, i.e., if each worker's innate ability was known with certainty when the worker entered the labor market. We come back to this issue in the next subsection where we discuss extensions.

similar to those found in the earlier Gibbons and Waldman analyses and also captures new results concerning the determinants of span of control. The basic idea is that wages, wage changes, probability of promotion, and span of control are all related to a worker's expected effective ability, so the determinants of expected effective ability – in this model schooling and performance – are also determinants of wages, wage changes, promotions, and span of control.

### C) Extensions

In the previous subsection we formally analyzed a model with two job levels and workers whose labor market lives are two periods. In this subsection we provide informal discussions of two extensions of the model. We begin with a discussion of what happens when workers are in the labor market for more than two periods. We then consider what happens when there are more than two job levels in addition to workers being in the labor market for more than two periods.

Suppose that everything is the same as in the model considered in the previous subsection except that labor market careers last  $X$  periods rather than two and equations for expected effective ability and production are changed to accommodate the longer labor market lives. Specifically, we now have the following. First, equation (1) still determines expected effective ability, where now we assume  $f(X-1) > f(X-2) > \dots > f(1) > f(0) > 0$ . Second, worker  $i$ 's output in job 1 in period  $t$  is now given by

$$(4) \quad y_{i1t} = (1 + v_1(r_{ikt})) [c_1 + c_2(\eta_{it} + \varepsilon_{i1t})],$$

where  $r_{ikt}$  is worker  $i$ 's tenure at the firm,  $v_1' > 0$ , and  $v_1'' < 0$ . This equation captures that firm specific human capital continues to accumulate as the worker's tenure at the firm grows. Third, as previously, at each firm  $k$  only workers with at least one period of prior experience at the firm are assigned to job 2. Further, if worker  $i$  with at least one period of prior experience at firm  $k$  is assigned to job 2 at division  $j$  of firm  $k$  in period  $t$ , then the worker produces

$$(5) \quad y_{ijkt} = v_2(r_{ikt}) g(\eta_{jkt})(\eta_{it} + \varepsilon_{i2t}),$$

where  $v_2' > 0$  and  $v_2'' < 0$ . Equation (5) captures that, in addition to firm specific human capital being sufficiently important for job level 2 that workers without previous experience at the firm produce zero if assigned to job-level 2, it is also the case that firm tenure beyond a single period increases expected output in job-level 2.

This model yields results similar to those found in the previous subsection. For example, focusing on workers of a given schooling level, labor market experience, and performance history assigned to job 1 in period  $t-1$ , the workers promoted to job 2 in period  $t$  will be those with higher period  $t-1$  outputs. Further, for those promoted span of control will also be positively related to performance in  $t-1$ . Similarly, education will also affect probability of promotion and span of control in basically the same way it did in the previous subsection.

One interesting aspect of this extension is that it allows us to distinguish between our approach with symmetric learning and a model where firms have full information about each worker's effective ability. As mentioned in footnote 12, a model characterized by full information can capture most or all of the testable implications derived in the previous subsection. For example, in such a model wages and probability of promotion would vary positively with effective ability. So if effective ability positively depends on schooling, then in this type of alternative model wages and probability of promotion will depend positively on schooling just like they did in our model with symmetric learning.

But now consider the model just analyzed in which careers last  $X$  periods rather than two. In this model wage changes and changes in span of control are determined by changes in expected effective ability. So wage changes and changes in span of control are correlated with the most recent performance even after controlling for the prior wage, education, labor market experience, and firm tenure, because the most recent performance is correlated with changes in expected effective ability. But in a world characterized by full information this is not the case because firms know each worker's innate ability, so effective ability changes with labor market experience and firm tenure but after controlling for these factors, education, and the prior wage, changes in expected effective ability should be uncorrelated with the most recent performance.

Now suppose that we extend this model further so that there is a third job level in each firm's hierarchy. Specifically, each firm continues to have  $m$  divisions with a single manager at the top of the division who is now defined to be in the division's single level-3 job. What is new is that each division now has an endogenously determined number of subdivisions, where each subdivision is characterized by a single manager defined to be in the subdivision's single level-2 job and an endogenously determined number of workers reporting to the subdivision's manager. Further, assume that the production functions are analogous to those in the two-level model above so that the scale-of-operations effect applies both to a level-2 manager in terms of the number of workers who report to that manager and to a level-3 manager in terms of the number of level-2 managers who report to this level-3 manager.

Analysis of this model would yield results similar to those discussed above for firms with two job levels, except many of the results concerning wages, wage changes, promotions, and span of control would now apply to multiple levels of the hierarchy. For example, holding job level, schooling level, labor market experience, and previous performance fixed, it will be the workers with high current performance who will be promoted in the next period for workers both in job level 1 and job level 2 (since there are only three job levels in this model workers in job level 3 cannot be promoted). Similarly, holding the same set of factors fixed and focusing on individuals who are on levels 2 or 3 both in this period and the next period, we get the prediction that current performance will be positively related to the change from this period to the next period in the managerial span of control.

In summary, in this subsection we have argued that extending the model by allowing for longer work lives and more jobs does not change the basic predictions of the model where workers have two-period careers and there are two jobs. But it does result in the new prediction that changes in span of control should be positively correlated with performance.

#### IV. DATA

In this section we describe our data and present some basic facts about the firm. In the next section we provide tests of our model.

We received confidential performance data from one large European Union “high tech” manufacturing firm that produces in various countries around the globe and sells its products in almost every country. The data covers the calendar years 2006 to 2010, where each year’s data were collected in the spring of the following year. The data quality is especially high for the home country, Denmark, and the US and is also high for most countries (especially China and Japan) in 2008, 2009, and 2010. While the firm initially focused its attention on the top managers of the firm, in later years the ratings were extended to include almost all of the firm’s white-collar workers. The total number of ratings increased from 1,867 for 2006 (mostly Danish and US workers) to 10,882 performance ratings for 2008 (out of 27,241 employees in December 2008) and then to 15,916 for 2009 (out of 29,959 employees in March 2010), with wider global coverage in later years (in particular China and Japan). Our focus will be on Danish workers since that is the location where the data is most complete and the distribution of performance ratings varied significantly from country to country (see below).

The firm employs a 1 to 5 scale for its evaluation ratings, where 1 denotes the lowest performance and 5 the highest.<sup>13</sup> Figure 1 shows the distribution of performance ratings in Denmark for our sample period, while Table 1 shows in detail how the distribution of performance ratings evolved over the sample period. The figure and table show that the distribution of the performance ratings was relatively constant over the sample period, where in each year approximately ninety percent of the workers received a 3 or 4, approximately six percent received a 5, two to three percent received a 2, and very few workers (less than one-half percent) received a 1.

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<sup>13</sup> To be precise, 1 means the worker does not meet expectations, 2 means the worker’s performance approaches expectations and goals, 3 means performance meets expectations, 4 is for performance that exceeds expectations and goals, and 5 is for outstanding performance.

Figure 2 shows how the distribution of performance ratings varies across the four main countries in the dataset (Denmark, China, Japan, and the US) in 2009. The figure shows that the distribution varies substantially across countries. For example, the percentage of workers who received a 4 or 5 was roughly twice as high in Denmark than in Japan, while this percentage for each of China and the US is between the values for Denmark and Japan. Also, the percentage of workers who received a 1 or 2 was much higher in Japan than in any of the other countries. There are various possible explanations for these differences. They could be due to differences in the quality of the firm's labor force across the countries, differences in organizational structures, or cultural differences that result in differences in how workers are evaluated across the countries.

We combine the data on performance ratings with data from confidential monthly personnel records we received for all workers from January 2003 (January 1997 for Danish workers) to January 2011. In this way we create a panel dataset for the years 2006 to 2010 that includes for each observation the worker's firm tenure, age, salary, bonus, job category, job level, nationality, gender, schooling level, a promotion indicator variable (1 if the worker was promoted that year and 0 if not), and performance evaluation.<sup>14</sup>

What is distinctive about this dataset which sets it apart from other similar datasets based on firms' personnel records is that we were also provided information about the firm's chain of command. Before describing this part of the dataset and how we used it we need to describe the hierarchical structure of the firm. Until relatively recently, the firm was organized into five hierarchical layers. Workers at job level 0 are the non-managerial employees. Workers at job level 1 are lower management and these workers are referred to as managers. Job level 2 refers to middle management and these workers are referred to as VPs. Job level 3 refers to upper management and workers at this level are referred to as SVPs. And workers at job level 4 are the top management and these workers are referred to as EVPs. Also, to cope with significant

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<sup>14</sup> Job category refers to the firm's cost center categories, i.e., the functional divisions of the firm. These are administration, administration in production, production, sales and marketing, and research and development.

growth during the last decade the firm recently introduced two new levels. Team leaders or assistant managers are between the old job levels 0 and 1 (above non-managerial employees but below managers), while corporate vice presidents, CVPs, are between the old job levels 2 and 3 (above VPs but below SVPs).

As mentioned above, we were provided information about the firm's chain of command. That is, for each individual and each year we were given the names of all the individuals directly above the worker in the firm's hierarchy. For example, in the latter part of the dataset, for each worker at level 0 we know the names of the team leader, manager, VP, CVP, SVP, and EVP, as well as the name of the department, the name of the subsidiary, and the geographic area where it operates. We use this information to construct the span of control for workers above level 0 (see Smeets and Warzynski (2008) for more detail). One interesting aspect of span of control at this firm is that, as captured in Figure 3, the average span of control has been falling over time.

## V. EMPIRICAL TESTS

In this section we present various empirical tests many of which are tests of the theory developed in Section III. The theory makes predictions about three sets of variables: i) wages and wage changes; ii) probability of promotion; and iii) span of control and changes in span of control. In the first subsection we consider wages, wage changes, and probability of promotion. In the second subsection we consider tests concerning span of control and changes in span of control.

### A) Wages, Wage Changes, and Probability of Promotion

We start with tests concerning wage determination. The theory predicts that wages should be related to job level (since wages rise with promotions), human capital variables, and the worker's performance history. In considering the determinants of the wage in our dataset we employ an ordinary least squares regression where the dependent variable is the log of the worker's wage in period  $t$ . Our independent variables include the worker's job level in period  $t$

which we capture with a set of indicator variables (the lowest job level is the omitted category), while our human capital variables are experience, experience squared, firm tenure, firm tenure squared, and education indicator variables. Specifically, one education indicator variable is for workers with bachelors degrees, a second is for workers with advanced degrees (masters degree or PhD), while other education outcomes are grouped into the omitted category. In our first wage regression we capture the worker's performance history by including the worker's performance rating six months prior to  $t$ . We also include cost center indicator variables.

The first column of Table 2 reports results not including the performance variable. The results concerning job level and education are consistent with the theory. The coefficients on the job level indicator variables increase with job level. Further, in many of the cases the differences in coefficients are statistically significant. We also find results concerning education that are consistent with the theory. Specifically, the coefficient on the indicator variable for a bachelors degree is positive and statistically significant, while the coefficient on the indicator variable for an advanced degree is positive, statistically significant, and also larger than the coefficient on the indicator for a bachelors degree (the difference between the coefficients is also statistically significant).

The coefficients on the four variables concerning experience and tenure are not as clearly supportive of the theory. The coefficient on experience is positive and statistically significant but the coefficient on experience squared is negative and statistically significant, so there is no clear result concerning how wages change as experience increases. Also, the coefficients on both the tenure and tenure squared variables are small and statistically insignificant. But this is really not that surprising. Holding job level fixed, higher values for experience and tenure could indicate a lower value for innate ability because higher ability workers are promoted to the next level, so there is not a clear theoretical prediction that increases in experience and tenure should be associated with a higher wage.

In column 2 we add the performance variable and the result is that the coefficients on the other variables are mostly qualitatively unchanged, while consistent with the theory the current

wage is positively related to the most recent performance evaluation.<sup>15</sup> Note that the theory predicts that it is not just the most recent performance that should matter, but rather the whole history of performance evaluations should matter. Including more than the most recent performance evaluation causes us to lose observations, but results are consistent with the theory. For example, as reported in column 3 of Table 2, when the most recent performance measure and the prior performance measure are both included each is positive and statistically significant.

Our next tests concern wage changes. The theory predicts that wage changes should be positively related to the most recent performance evaluation and whether the worker was promoted in the most recent period. To investigate the determinants of wage changes we employ an ordinary least squares regression where the dependent variable is the change in the log wage from  $t-1$  to  $t$ . Our independent variables are an indicator for whether the worker was promoted in period  $t$ , the worker's most recent performance evaluation, and the log wage in  $t-1$ . We include the log wage in  $t-1$  to capture the possibility that average percentage wage changes vary with the  $t-1$  wage. We also include year and cost center indicator variables.

The first column of Table 3 reports results not including the promotion variable, while the second column reports results when the promotion variable but not the performance variable is included. In both columns we find a statistically significant negative coefficient on the log wage in  $t-1$  which means that percentage wage increases are on average smaller for workers who are already being paid a high wage. In column 1 we also find that the most recent performance evaluation is positively related to the change in log wage, while column 2 shows that being promoted is positively related to the change in log wage. In the third column we include both the performance and promotion variables and find that when both are included each coefficient remains positive and statistically significant, while in column 4 we add the prior performance variable and find that the coefficients on both performance variables and the promotion variable

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<sup>15</sup> The only noticeable changes are that the coefficient on the tenure variable is now negative and statistically significant at the ten percent level, while the coefficient on the tenure squared variable is positive and statistically significant at the five percent level. That these coefficients change substantially between columns 1 and 2 is likely due to a correlation between firm tenure and performance which biases the firm tenure coefficients in column 1 where performance variables are not included.

are all positive and statistically significant. Note that one could argue that since in the theory promotions are associated with large wage increases because they are associated with high performance, in columns 3 and 4 there should be little or no correlation between the promotion variable and wage increases because we control for performance. But given that the performance measure has only five categories which likely is too coarse to fully capture true performance, promotions could be positively correlated with true performance even when measured performance is controlled for, i.e., the theory is in fact consistent with a positive coefficient on the promotion variable in columns 3 and 4.<sup>16</sup>

In our next set of tests we consider probability of promotion. In the theory promotion is determined by both the education level and performance. That is, holding performance fixed the probability of promotion rises with the education level, while holding education fixed the probability of promotion rises with performance. As a reminder, the reason that education matters even after controlling for performance is that the learning process in this model is gradual, i.e., observing a worker's most recent performance is informative about the worker's ability but not fully informative, so the worker's education level still provides valuable information.

To investigate probability of promotion we conduct probit tests where the dependent variable is an indicator that takes on a value of one if the worker was promoted that period and zero if not, where independent variables include performance measures, indicators for the worker's education, experience and tenure variables, and year and cost center indicator variables. We conduct these tests for promotions from level 0 to level 1 (promotion from worker to manager) and from level 1 to level 2 (promotion from manager to vice president), and we also

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<sup>16</sup> One might argue, however, that in moving from column 2 to columns 3 and 4 the theory predicts larger decreases for the coefficients on the promotion variable than we observe. This suggests that there may be reasons for why promotions are associated with large wage increases not captured in our theoretical model such as that promotion wage increases are prizes in a tournament such as in Lazear and Rosen (1981) or promotions serve as signals such as in Waldman (1984b). See DeVaro and Waldman (2012) for a related analysis and the Conclusion for further discussion.

include an indicator variable which takes on a value of one if the observation is of an individual who prior to the promotion decision is a manager rather than a worker.

Results are reported in Table 4. In column 1 we report marginal effects from a probit regression that includes only the most recent performance measure. One result is that the coefficient on the manager indicator variable is positive and strongly statistically significant. This is likely due to span of control being smaller for vice presidents than for managers. That is, in this firm the span of control is smaller on average for vice presidents than for managers, so in the competition for promotion a manager on average faces fewer competitors than does a worker with the possible result being a higher probability of promotion. We also find consistent with the theory that performance is positively related to the probability of promotion, while mixed evidence for the theory is that the coefficients on both education indicator variables are positive but not statistically significant.

In column 2 we add as an explanatory variable the performance measure from one period earlier. In this test the coefficient on the most recent performance variable remains positive and strongly significant although it is smaller in absolute value than the corresponding coefficient in column 1, while the coefficient on the lagged performance variable is positive but not statistically significant. In terms of the other coefficients we still find that being a manager rather than being a worker is positively related to the probability of promotion while now the coefficients the two education variables are both positive and statistically significant. However, inconsistent with the theory the coefficient on the advanced degree indicator variable is smaller than the coefficient on the bachelors degree indicator variable. One possibility here is that individuals with advanced degrees frequently compete for promotion against other workers with advanced degrees which limits the effect that having an advanced degree has on the promotion probability.

To further investigate the difference between columns 1 and 2 concerning the coefficients on the education variables, in column 3 we rerun the column 1 test but use only the 2347 observations from column 2. Here we find that the coefficient on the performance variable is

still positive and statistically significant, although the coefficient is smaller than the corresponding coefficient in column 1. What is new here is that, in contrast to column 1, the education coefficients are both positive and statistically significant, although like in column 2 the coefficient on the advanced degree indicator variable is smaller rather than larger than the coefficient on the bachelors degree indicator variable. This indicates that the lack of statistical significance for the education coefficients in column 1 is due to education not being important for promotion for the observations included in column 1 but not included in columns 2 and 3. These are observations for which there is a most recent performance measure but not a prior performance measure. These will frequently be workers who were recently hired by the firm. So the table suggests that for workers recently hired education is not important for the promotion decision. But for workers who have been at the firm longer higher levels of education do increase the promotion probability.<sup>17</sup>

So overall our results in this subsection are mostly consistent with the theory presented in Section III. First, wages are positively related to performance, education, and job level. Second, wage increases are positively related to performance and receiving a promotion. Third, performance and education are positively related to the probability of promotion, although the evidence concerning education is a bit mixed. Note, however, although the results are in general consistent with Section III's model, they can also be explained by the Gibbons and Waldman (1999, 2006) models. In the next subsection we focus on predictions from our model that are not found in the Gibbons and Waldman papers.

#### B) Span of Control

In this subsection we consider span of control and changes in span of control, where we begin with tests concerning span of control. The scale-of-operations effect which is a driving force in our theoretical model predicts that a manager's span of control should be positively

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<sup>17</sup> The theory does not allow for workers who switch employers during their careers, so in a sense a test eliminating workers who were recently hired is a more direct test of the theory than a test that includes such workers.

related to the manager's ability level (expected ability level in the model). In turn, translating this into a testable prediction yields that an individual's span of control should be positively related to the manager's performance history and the manager's education level.

To investigate our predictions concerning span of control we conduct ordinary least squares regressions using a sample consisting of all observations of individuals at level 1 (managers) and level 2 (vice presidents). The dependent variable in our regressions is the individual's span of control, while independent variables include performance measures, an indicator variable that takes on a value of zero if the individual was a manager rather than a vice president and zero otherwise, education indicator variables, job category indicator variables, and year indicator variables.

Results are reported in Table 5. In column 1 we report results for an ordinary least squares regression where only the individual's most recent performance is included. One clear result here is that, as mentioned earlier, in our firm managers have on average a higher span of control than vice presidents. In terms of theoretical predictions, we find that the coefficient on the performance variable is positive and strongly statistically significant, while the coefficients on both education variables are positive. However, only the coefficient on the indicator for individuals with bachelors degrees is statistically significant where this coefficient is statistically significant at the one percent level.

In column 2 we add as an explanatory variable the lagged value for the individual's performance. The main finding here is that the coefficient on the lagged performance measure is positive and statistically significant at the one percent level while the coefficient on the most recent performance measures is close to zero and statistically insignificant. One possibility here is that, in contrast to what is true in the theoretical model, there are costs to changing span of control quickly with the result that an individual's actual span is more closely correlated with lagged performance than with the most recent performance. One additional result in Table 5 worth mentioning is that in each column the coefficient on the bachelors degree indicator variable is positive and statistically significant at the one percent level, while the coefficient on

the advanced degree indicator variable is smaller in magnitude and not statistically significant. One possibility for why the advanced degree indicator coefficients are insignificant and smaller than the bachelors degree indicator coefficients is that individuals with advanced degrees who are managers or higher are managing different types of workers where the work is more complex and managing more workers is harder with the result being a systematic reduction in span of control.<sup>18</sup>

In our last test we consider changes in span of control. To investigate changes in the span of control we consider an ordinary least squares regression where change in the log of span of control is the dependent variable, while independent variables are an indicator for whether the individual is a manager, the starting value for the log of span of control, performance measures, job category indicator variables, and year indicator variables. Results are reported in Table 6. In the first column we only include the most recent performance measure. One result here is that the coefficient on the starting value for the log of span of control is negative and statistically significant which means that the percentage increase in the log of span of control is on average smaller when the starting value for the span of control is larger. The other main result is that the coefficient on the performance variable is positive and statistically significant which is exactly what one would expect if changes in span of control are driven by changes in expected effective ability as predicted by the theory.

In column 2 we add the lagged performance measure to the regression. The main result here is that coefficients on the most recent performance measure and the lagged performance measure are both positive, similar in size, and both are statistically significant at the ten percent level. This is a bit different than what might have been expected given the results in column 2 of

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<sup>18</sup> In column 3 we investigate whether the difference between columns 1 and 2 concerning the coefficient on the most recent performance variable is due to different sample sizes or the inclusion of the lagged performance variable in column 2. Specifically, in column 3 we run the column 1 regression on the column 2 sample. Although somewhat inconclusive, the test mostly points to the inclusion of the lagged performance measure being the source of the difference between columns 1 and 2 concerning this coefficient.

Table 5 where the lagged performance measure was more important than the most recent performance measure as a determinant of span of control.<sup>19</sup>

In columns 3 and 4 of Table 6 we add the education indicator variables. In each column the coefficients on the education indicator variables are statistically insignificant, the size or sizes of the coefficients on the performance variables are basically unchanged, and the statistical significance of the coefficient on the most recent performance measure is also unchanged. Columns 3 and 4 tell us that the positive correlation between change in log span of control and performance is not driven by a positive correlation between performance and education.

## VI. DISCUSSION

As briefly mentioned at the end of Subsection V.A, the results in that subsection can be explained by the models investigated in Gibbons and Waldman (1999, 2006). Like the model investigated here, those models combine job assignment, human capital acquisition, and learning in an exploration of wage and promotion dynamics in internal labor markets. They are consistent with our findings that wages are positively related to performance, education, and job level, wage changes are positively related to performance and having received a promotion, and that the probability of promotion is positively correlated with performance and education.

But the Gibbons and Waldman (1999, 2006) models do not say anything about span of control. In those models there are two or three job levels and like the model in Section III the return to ability is higher at higher job levels. But in those models there is no sense in which a worker assigned to a higher level job supervises workers at a lower level, so those models do not capture the idea of span of control. So it is not the case that any of our empirical findings in Subsection V.B are inconsistent with testable predictions of the Gibbons and Waldman models,

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<sup>19</sup> That the coefficient on the most recent performance measure in column 2 is the same size as the lagged performance measure is consistent with the coefficient on the lagged performance measure in column 2 of Table 5 being larger than the coefficient on the most recent performance measure, i.e., this follows since the Table 6 column 2 results indicate that performance affects subsequent growth in span of control for multiple periods. But the results do not seem consistent with the near zero coefficient on the most recent performance measure in column 2 of Table 5.

but rather those models are simply silent concerning how span of control should work in real world firms. Thus, to the extent our findings in Subsection V.B concerning span of control are consistent with testable predictions of our model, this is evidence that our extension of the Gibbons and Waldman framework provides a more complete picture of the operation of internal labor markets than the Gibbons and Waldman models.

The first finding in Subsection V.B is that span of control is positively correlated with both the most recent performance measure and education, although the positive correlation between span of control and education is stronger for workers with bachelors degrees relative to workers with advanced degrees. This first finding is generally consistent with the scale-of-operations effect being important at this firm. That is, the scale-of-operations effect predicts that span of control should be higher for higher ability managers, so span of control being positively correlated with both performance and education is consistent with the scale-of-operations effect since performance and education should both be positively correlated with managerial ability. But this first finding tells us nothing about whether learning is an important determinant of span of control at our firm. Or another way to put this is that this first finding can be explained by static analyses such as in Lucas (1978) and Rosen (1982) and does not require the learning component of our theoretical approach.

Our second finding in Subsection V.B is that, when a lagged performance measure is added to the empirical specification, span of control is more closely positively correlated with lagged performance than with the most recent performance measure. This suggests that learning is important. Lagged performance and the most recent performance measure should be approximately equally correlated with ability, so if there was no learning the two should be approximately equally correlated with span of control. The fact that the correlation is much stronger for the lagged performance variable suggests that learning is important but that span of control only adjusts slowly to changes in expected effective ability.

Our last finding in Subsection V.B is that change in span of control is positively correlated with performance and this is the case even after controlling for education. This

finding also suggests that learning is an important determinant of span of control since, if learning was unimportant, then perceptions about ability would not change with performance so changes in span of control which should depend on changes in such perceptions would be uncorrelated with performance.

So overall we find results consistent with the Gibbons and Waldman (1999, 2006) approach that combines job assignment, human capital acquisition, and symmetric learning, results consistent with the scale-of-operations effect and span of control as in the early papers of Lucas (1978) and Rosen (1982), and results that can be explained by combining the two approaches like we do in Section III's model. We also do not know of any alternative theoretical approach capable of explaining our findings, although as we discuss in the Conclusion the pure symmetric learning assumption we employ in our theoretical model may not fully capture how learning works in our firm.

## VII. CONCLUSION

A standard idea in the theoretical literature concerning hierarchies is the scale-of-operations effect, i.e., the idea that the return to managerial ability is higher the more resources the manager influences with his or her decisions. This theoretical idea leads to various testable implications including that higher ability managers should have a higher span of control. And although the empirical literature on organizations finds evidence consistent with some predictions that follow from the scale-of-operations effect, predictions concerning span of control have not been investigated.

In this paper we first extended the theory by looking at the scale-of-operations effect in a model in which there is uncertainty concerning workers' innate abilities when they enter the labor market and firms learn about these innate abilities as careers progress. This model yields a large number of testable predictions some of which are consistent with models focused solely on the scale-of-operations effect, some of which are consistent with models focused solely on learning, and some of which follow from combining the scale-of-operations effect with learning.

These various testable predictions concern wages, wage changes, promotion probabilities, and span of control.

We then empirically investigated these predictions using a unique single firm dataset that contains detailed information concerning the reporting relationships at the firm. Our results support our theoretical predictions. We find results concerning wages, wage changes, promotion probabilities, and span of control all consistent with our theoretical model. Most interestingly, we find results that follow from combining the scale-of-operations effect with learning about worker ability. For example, in a world characterized by the scale-of-operations effect but no learning, performance should have little or no effect on changes to a manager's span of control because with no learning performance does not change beliefs concerning the manager's ability. But with learning there should be a positive correlation because performance is positively correlated with subsequent changes in expected effective ability and changes in expected effective ability are correlated with changes in span of control. Our empirical analysis finds a positive correlation between performance and subsequent changes in span of control as predicted by our analysis that combines the scale-of-operations effect with learning. Our conclusion, therefore, is that both the scale-of-operations effect and learning are important factors in the operation of real-world hierarchies and, further, that the interaction of these two factors which has not previously been explored is in its own right an important factor.

There are a number of directions the analysis in this paper could be extended. One that we think has particular promise is to introduce an element of asymmetric learning into the analysis. In this paper we have focused on symmetric learning similar to the approach taken, for example, in Harris and Holmstrom (1982), Holmstrom (1982), and Gibbons and Waldman (1999, 2006). In a world of symmetric learning all firms are equally informed about each worker's ability at any point in time. An alternative approach first explored in Greenwald (1979, 1986) and Waldman (1984b) is that learning is asymmetric, i.e., only a worker's current employer directly receives information about the worker's ability, while other firms learn about the worker's ability by observing the actions of the current employer such as promotion

decisions. There are numerous empirical studies that test for asymmetric learning in labor markets such as Gibbons and Katz (1991), Schonberg (2007), Pinkston (2009), and DeVaro and Waldman (2012) and, on net, we believe the evidence supports asymmetric learning being important. We thus believe it would be interesting to incorporate an element of asymmetric learning into our theoretical analysis and then empirically investigate the new testable predictions that result.

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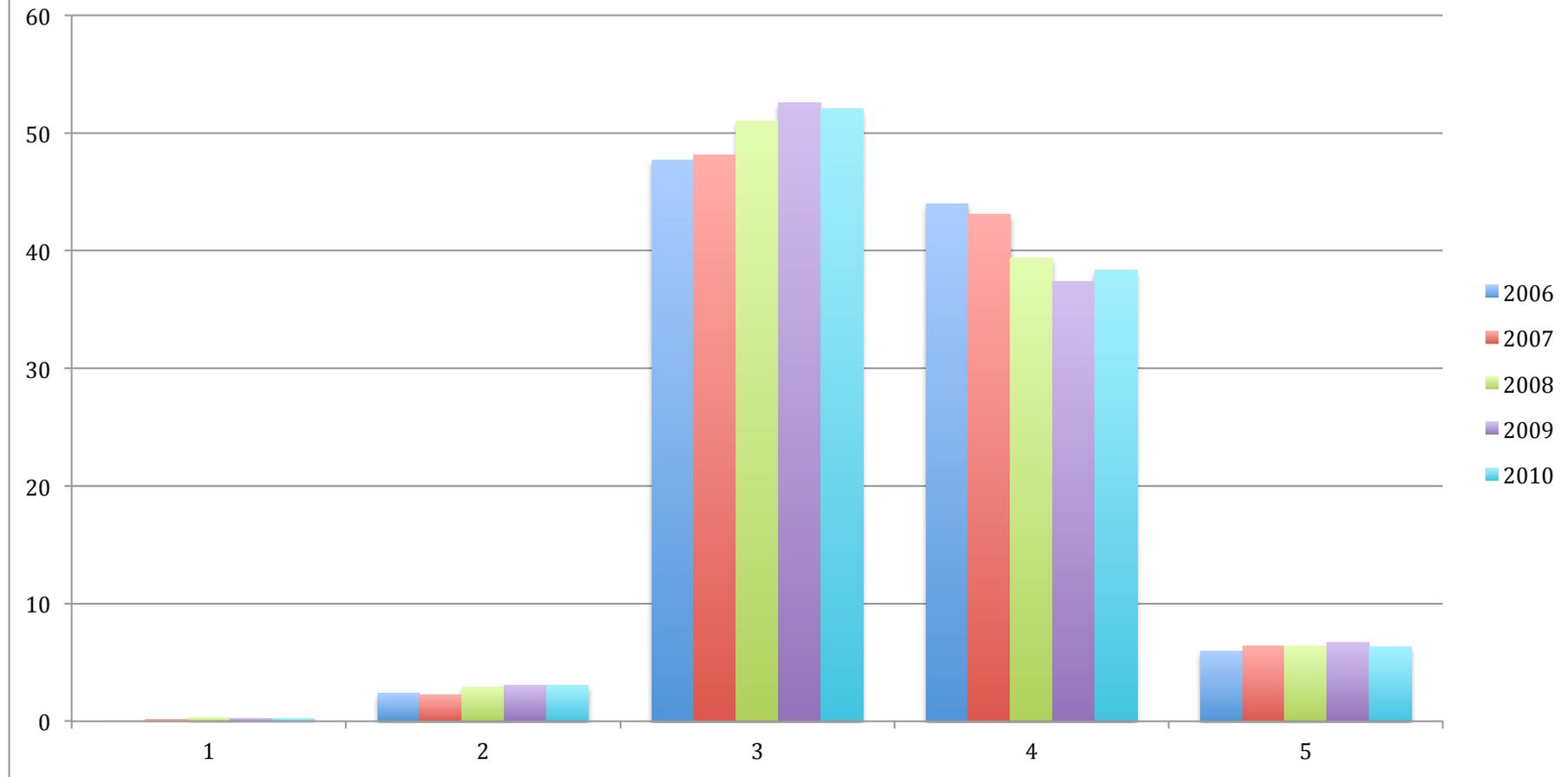
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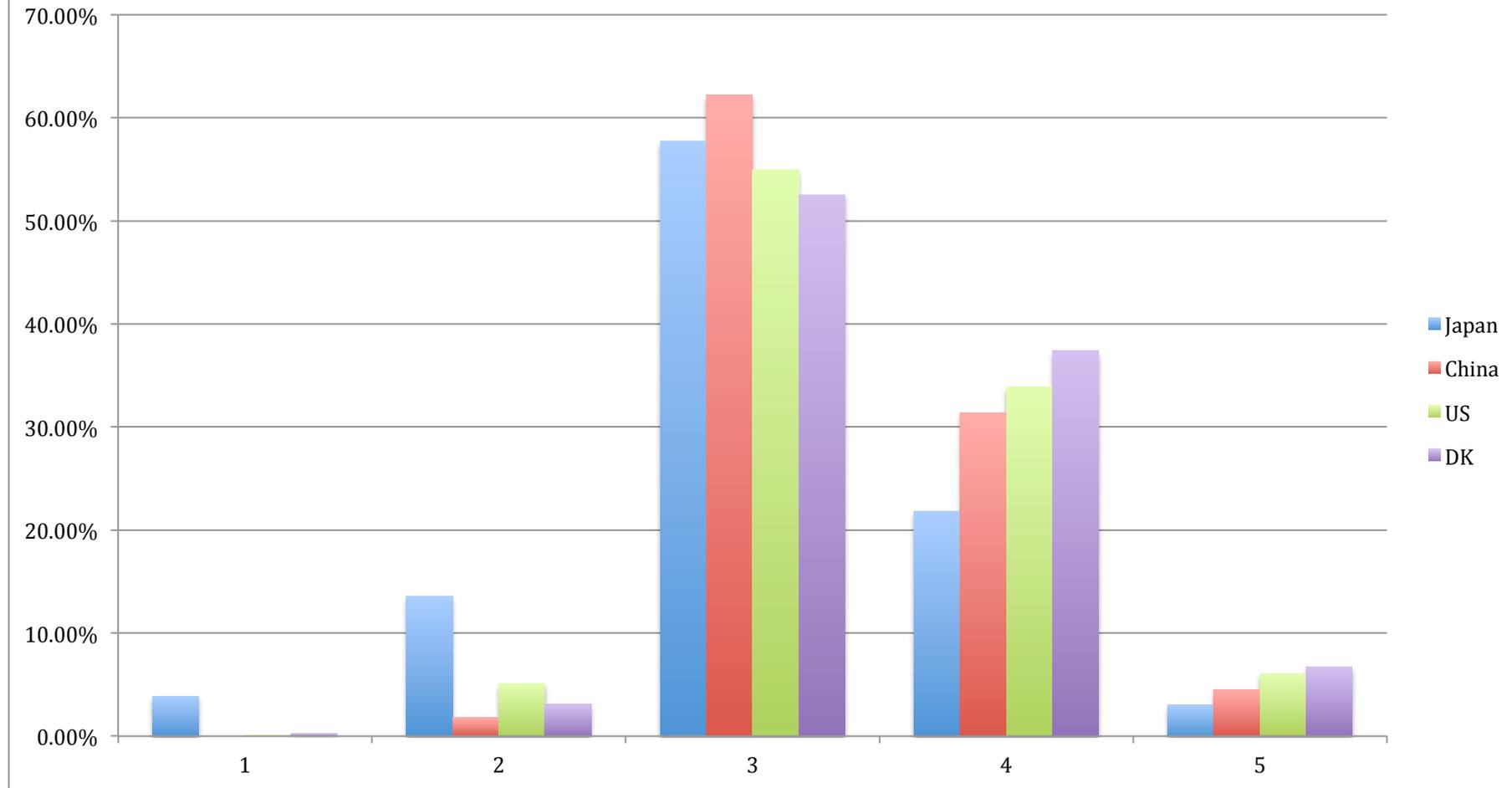
**Table 1: Evolution of the performance distribution in Denmark (in %)**

	1	2	3	4	5	# obs.
2006	0	2.39	47.71	43.97	5.93	962
2007	0.15	2.26	48.16	43.07	6.37	2,749
2008	0.29	2.94	51.04	39.36	6.38	4,561
2009	0.27	3.09	52.55	37.38	6.71	5,174
2010	0.23	3.05	52.06	38.34	6.33	7,077

**Figure 1: distribution of subjective performance ratings in Denmark, 2006- 2010 (in %)**



**Figure 2: distribution of subjective performance evaluation ratings in 4 countries**



**Figure 3: Evolution of the average span of control of managers**

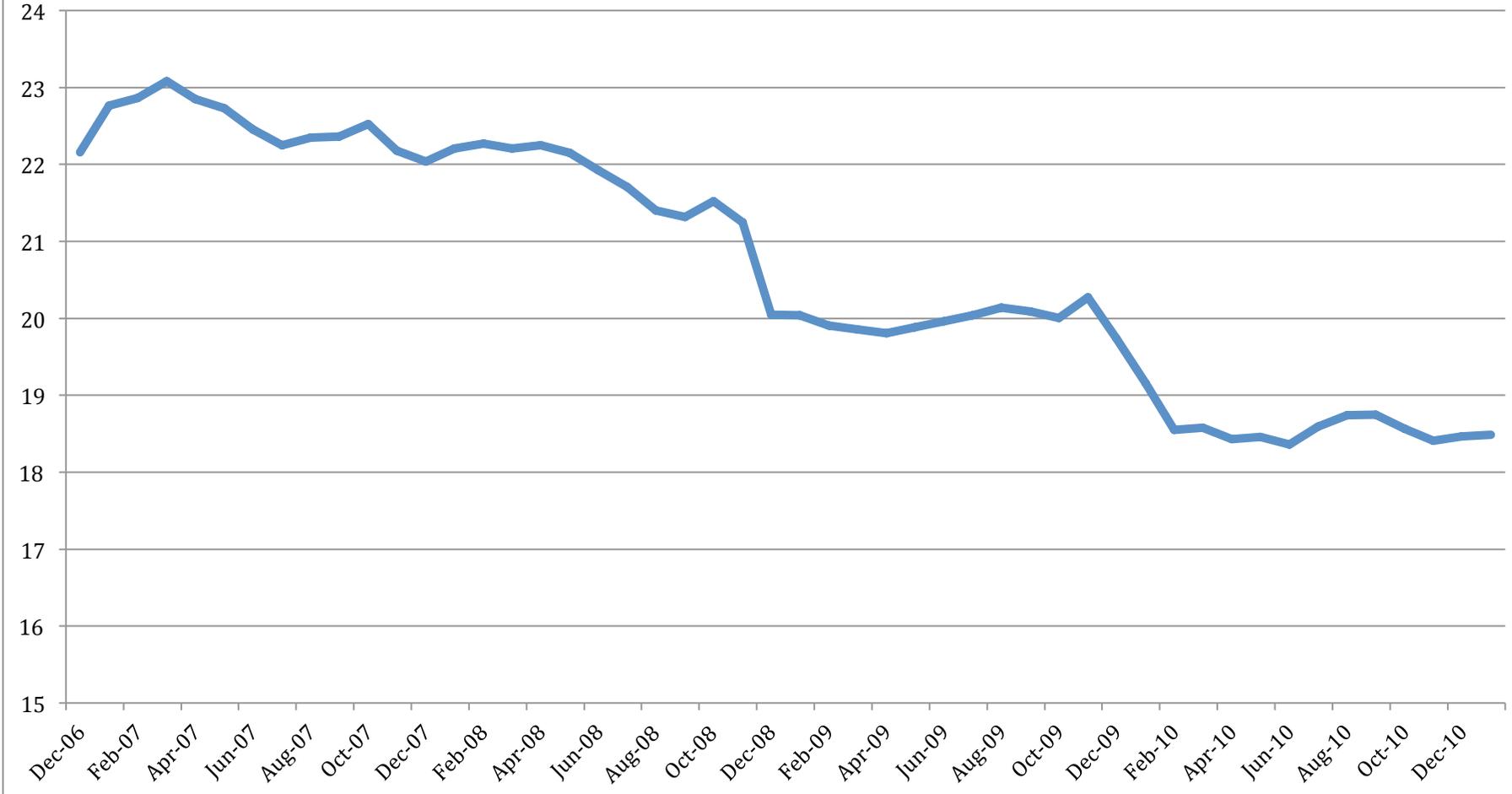


Table 2 : Wages, Human Capital and Performance

Log wage	(1)	(2)	(3)
Experience	0.018*** (0.001)	0.018*** (0.001)	0.014*** (0.001)
Experience <sup>2</sup> /100	-0.029*** (0.001)	-0.028*** (0.001)	-0.021*** (0.002)
Tenure	-0.0003 (0.0004)	-0.0007* (0.0005)	-0.001* (0.0006)
Tenure <sup>2</sup> /100	0.0016 (0.0014)	0.0027** (0.0013)	0.003 (0.002)
Bachelor	0.192*** (0.003)	0.190*** (0.003)	0.175*** (0.005)
Master and above	0.266*** (0.003)	0.266*** (0.003)	0.237*** (0.005)
Team Leader	0.195*** (0.004)	0.192*** (0.004)	0.185*** (0.005)
Specialist	0.298*** (0.003)	0.295*** (0.003)	0.299*** (0.004)
Manager	0.407*** (0.004)	0.399*** (0.004)	0.410*** (0.005)
Senior Specialist	0.531*** (0.009)	0.523*** (0.009)	0.535*** (0.010)
Vice President	0.653*** (0.008)	0.642*** (0.008)	0.610*** (0.010)
Corporate Vice President	0.828*** (0.009)	0.809*** (0.009)	0.815*** (0.010)
Performance in t-1	-	0.031*** (0.002)	0.023*** (0.002)
Performance in t-2	-	-	0.012*** (0.002)
Constant	10.19*** (0.007)	10.08*** (0.009)	10.13*** (0.015)
Year dummies	YES	YES	YES
Cost center dummies	YES	YES	YES
Adj. R2	0.84	0.84	0.86
N	10,710	10,710	5,272

**Table 3 : Wage Growth and Performance**

$\Delta$ Log wage	(1)	(2)	(3)	(4)
Log wage	-0.011*** (0.001)	-0.014*** (0.001)	-0.015*** (0.001)	-0.018*** (0.001)
Performance in t-1	-	0.010*** (0.0004)	0.010*** (0.0004)	0.008*** (0.0006)
Performance in t-2	-	-		0.003*** (0.0006)
Promotion	0.038*** (0.005)	-	0.032*** (0.004)	0.029*** (0.005)
Constant	0.150*** (0.013)	0.147*** (0.012)	0.155*** (0.012)	0.190*** (0.015)
Year dummies	YES	YES	YES	YES
Cost center dummies	YES	YES	YES	YES
Adj. R2	0.03	0.11	0.12	0.11
N	6,478	6,478	6,478	3,184

**Table 4 : Promotion and Performance (marginal effects reported)**

Promotion	(1)	(2)	(3)
Experience	0.0006 (0.0006)	0.001** (0.00056)	0.001* (0.00058)
Experience <sup>2</sup> /100	-0.0017 (0.0014)	-0.0026* (0.0014)	-0.0027* (0.0014)
Tenure	0.0002 (0.0004)	0.00004 (0.0002)	0.00005 (0.0002)
Tenure <sup>2</sup> /100	-0.0008 (0.004)	0.00008 (0.0008)	0.00006 (0.0008)
Bachelor	0.003 (0.008)	0.719*** (0.299)	0.689*** (0.299)
Master and above	0.004 (0.004)	0.083*** (0.053)	0.080*** (0.050)
Performance in t-1	0.008*** (0.001)	0.004*** (0.001)	0.005*** (0.001)
Performance in t-2	-	0.001 (0.0009)	-
Manager	0.012*** (0.004)	0.003** (0.0018)	0.004** (0.002)
Year dummies	YES	YES	YES
Cost center dummies	YES	YES	YES
Pseudo R2	0.20	0.23	0.23
Log Likelihood	-286.54	-155.96	-156.80
N	4,774	2,347	2,347

**Table 5 : Span of Control and Performance (managers and VPs pooled)**

Log span	(1)	(2)	(3)
Experience	0.026 (0.016)	0.026 (0.019)	0.025 (0.019)
Experience <sup>2</sup> /100	-0.069* (0.036)	-0.075* (0.042)	-0.075* (0.042)
Tenure	0.042*** (0.009)	0.047*** (0.010)	0.047*** (0.010)
Tenure <sup>2</sup> /100	-0.073*** (0.026)	-0.079*** (0.029)	-0.078*** (0.029)
Bachelor	0.315*** (0.099)	0.403*** (0.113)	0.427*** (0.113)
Master and above	0.098 (0.095)	0.177 (0.108)	0.198* (0.109)
Performance in t-1	0.091*** (0.031)	0.001 (0.039)	0.064* (0.038)
Performance in t-2	-	0.119*** (0.038)	-
Manager	1.477*** (0.078)	1.485*** (0.086)	1.454*** (0.088)
Constant	0.457*** (0.244)	0.345*** (0.313)	0.489*** (0.309)
Year dummies	YES	YES	YES
Cost center dummies	YES	YES	YES
Adj. R2	0.41	0.44	0.43
N	1,158	847	847

**Table 6 : Change in Span and Performance**

$\Delta$ Log span	(1)	(2)	(3)	(4)
log span	-0.008** (0.003)	-0.007* (0.004)	-0.008** (0.003)	-0.006 (0.004)
Performance in t-1	0.008** (0.004)	0.008* (0.005)	0.009** (0.004)	0.009* (0.005)
Performance in t-2	-	0.008* (0.005)	-	0.008* (0.005)
Manager	0.005 (0.009)	0.001 (0.011)	0.004 (0.010)	-0.0001 (0.011)
Bachelor	-	-	-0.009 (0.011)	-0.014 (0.013)
Master and above	-	-	-0.015 (0.011)	-0.016 (0.012)
Experience	0.002 (0.002)	0.002 (0.002)	0.001 (0.002)	0.002 (0.002)
Experience <sup>2</sup>	-0.004 (0.004)	-0.006 (0.005)	-0.004 (0.004)	-0.006 (0.005)
Tenure	0.001 (0.001)	0.002 (0.001)	0.001 (0.001)	0.002 (0.001)
Tenure <sup>2</sup>	-0.003 (0.003)	-0.005 (0.003)	-0.003 (0.003)	-0.005 (0.003)
Constant	-0.034 (0.027)	-0.075** (0.036)	-0.018 (0.025)	-0.057 (0.038)
Year dummies	YES	YES	YES	YES
Cost center dummies	YES	YES	YES	YES
Adj. R2	0.02	0.03	0.02	0.03
N	1,017	749	1,017	749