Game Over: Quantifying and Simulating Unsustainable Fiscal Policy *

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Abstract

This paper studies the effects of a fiscal transfer program on the probability and timing of the insolvency of that program. We use an overlapping generations model with aggregate uncertainty in which households have rational expectations to show how the size of the fiscal transfer program affects the expected time in which the economy will reach its fiscal limit. We look at two results of insolvency. The first is a catastrophic government shut down in which the current economy ceases to function. The second is a less-severe permanent regime shift to a high proportional tax rate regime. In our example calibrated to the U.S. economy we show that the expected time until the fiscal limit is about 100 years. But we also find that there is a roughly 35 percent chance that the economy could hit its limit in 30 years. We also calculate measures of the fiscal gap and the equity premium. Our model with potential fiscal insolvency generates equity premia that are close to those observed in the data.

keywords: Social Security, intergenerational transfers, fiscal limits, regime switching.

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1 Introduction

The global recession that began in 2008 highlighted two key weaknesses in developed economies: excessive debt and unsustainable fiscal policies. This paper studies the effects of a fiscal transfer program on the probability and timing of the insolvency of that program. Although the fiscal issues of most developed countries also include healthcare spending and unemployment policy, the focus of this paper is pay-as-you-go transfer programs like the Social Security system in the United States.

Because age heterogeneity is central to questions about intergenerational transfers, we use an overlapping generations model with aggregate uncertainty in which households have rational expectations to show how the size of the fiscal transfer program affects the expected time in which the economy will reach its fiscal limit. We treat any type of restructuring of the transfer program as a default, and we look at two results of insolvency. The first is a catastrophic government shut down in which the current economy ceases to function. The second is a less-severe permanent regime shift to a high-proportional tax rate regime. Calibrating our model to the U.S. economy, we find that the expected time until the fiscal limit is about 100 years. But we also find that there is a roughly 35 percent chance that the economy could hit its limit in 30 years.

Using our model, we calculate measures of the fiscal gap as well as the equity premium as indicators of fiscal imbalance. Popular measures of a country's degree of indebtedness or fiscal insolvency, such as the deficit or the debt-to-GDP ratio, are not well defined or offer an incomplete picture. Our definition of the fiscal gap applies the generational accounting concept from Auerbach, Gokhale, and Kotlikoff (1991) and measures the long-run fiscal burden of a country as the net present value of expected future government revenues minus expenses as a percent of either current GDP or the net present value of GDP. Our measures of the fiscal gap as a percent of current GDP from our baseline model are between 1 and 4 percent and increase monotonically with the size of the transfer program.

The equity premia generated by our model with potential fiscal insolvency are about 6 percent in the initial period in the model in which insolvency triggers an economic shut down and about 2 percent in the model in which insolvency triggers a permanent switch to a high tax rate regime. The two specifications generate Sharpe ratios of 0.32 and 0.28, respectively. However, when tracking the equity premia and Sharpe ratios across time as the economy gets closer to its shut down or regime switch, the equity premium and Sharpe ratios rise to 7 percent and 0.33 in the shut down model and 6 percent and 0.50 in the regime switching model. The latter case is particularly interesting given that it predicts high equity premia and Sharpe ratios in economies that are close to their fiscal limit.

In treating fiscal insolvency as a type of default, the literature on sovereign default becomes informative.[1] However, Leeper and Walker (2011) argue that large restructuring of fiscal programs is probably more relevant to developed economies than is sovereign default. They define a fiscal limit as "the point beyond which taxes and government expenditures can no longer adjust to stabilize the value of government debt." A large literature focuses of fiscal stress and fiscal limits.[2]

Our paper abstracts from money, so it does not have the monetary and fiscal interaction described in Sargent and Wallace (1981) and highlighted in the recent fiscal limits research.[3] In this study, we treat the transfer program as having some fixed component that can be thought of as nonproportional to income and as hard to change quickly. The foundations of this idea of fiscal stickiness come from Alesina and Drazen (1991) and has been incorporated into stochastic OLG models by Auerbach and Hassett (1992, 2001, 2002, 2007) and Hassett and Metcalf (1999). We assume that a fiscal transfer program represents a promise to pay, and any restructuring of that program due to insolvency is defined as a default.

Further, our concept of what the government does after a fiscal restructuring is similar to the ideas in the regime switching literature, of which Hamilton (2008) is a

[1] See Yue (2010), Reinhart and Rogoff (2009), Arellano (2008), and Aguiar and Gopinath (2006).

[2] See Auerbach and Kotlikoff (1987), Kotlikoff, Smetters, and Walliser (1998a,b, 2007), İmrohoroğlu, İmrohoroğlu, and Joines (1995, 1999), Huggett and Ventura (1999), Cooley and Soares (1999), De Nardi, İmrohoroğlu, and Sargent (1999), Altig, Auerbach, Kotlikoff, Smetters, and Walliser (2001), Smetters and Walliser (2004), and Nishiyama and Smetters (2007).

[3] See also Cochrane (2011), Leeper and Walker (2011), Davig, Leeper, and Walker (2010, 2011), Davig and Leeper (2011a,b), and Trabandt and Uhlig (2009).

good survey. This study focuses on two potential regime switches—one more severe and one less severe. In each case, the first fiscal regime one in which a nonproportional transfer is collected from the young and given to the old. Our first specification is that the government completely shuts down if it cannot collect the transfer from the young. This could be interpreted as the young starving to death or as the economy degenerating to autarky. Our other specification is a permanent switch to a high proportional labor income tax rate regime when the fiscal transfer cannot be collected.

The paper proceeds as follows. Section 2 presents the model in which the economy shuts down if the fiscal transfer system becomes insolvent as well as the simulation of the economy, the measures of the fiscal gap, and the equity premium. Section 3 presents the model in which the fiscal transfer switches permanently to a high proportional tax regime if the transfer system becomes insolvent with its accompanying simulations, measures of the fiscal gap, and equity premium. Section 4 concludes.

2 Model with Shut Down

We study a simple 2-period-lived agent model in which the government promises to make a lump sum transfer $\bar{H} \geq 0$ from the young to the old each period. Ricardian equivalence holds because households have rational expectations. However, the constraints of the model generate states of the world in which the government can only make a transfer that is less that the promised amount $0 \leq H_t \leq \bar{H}$.

Our characterization of government shut down relies on the assumption that when the state of the world is such that $\bar{H}$ generates negative consumption for the young, the agents in the economy resort to autarky rather than starvation (negative consumption). This shut-down result would not hold if the government merely reduced the size of the transfer program in the face of a shut down. Rational agents would expect this and incorporate that risk on the payment $\bar{H}$ in the second period of their lives. We explore these types of less catastrophic regime shifts in Section 3.[4]

[4] A proportional transfer program will never shut down a government. However, if the government is locked in to some degree of nonproportional transfer program, then there are states of the world in which the government must either shut down or default on that debt. See Alesina and Drazen (1991)

2.1 Household problem

A unit measure of identical consumer-workers is born each period. They supply a unit of labor inelastically in the first period of life to identical perfectly competitive firms, and do not work in the second period of life,

$$l_{1,t} = \bar{l} = 1 \quad \forall t$$

where $l_{1,t}$ is the amount of labor inelastically supplied by age-1 workers at time t.

Consumer-workers live for $I = 2$ periods and choose each period how to divide their initial wealth and wages among consumption $c_{i,t}$ and capital investment in the firms $k_{i+1,t+1}$. The objective of a consumer-worker is maximize utility subject to a period budget constraint and two nonnegativity constraints,

$$\max_{c_{1,t},k_{2,t+1},c_{2,t+1}} \quad u(c_{1,t}) + \beta E_t\left[u(c_{2,t+1})\right]$$
$$\text{where} \quad c_{1,t} + k_{2,t+1} \leq w_t - H_t$$
$$\text{and} \quad c_{2,t+1} \leq (1 + r_{t+1} - \delta)k_{2,t+1} + H_{t+1}$$
$$\text{and} \quad c_{1,t}, c_{2,t+1}, k_{2,t+1} \geq 0$$
$$\text{and where} \quad u(c_{i,t}) = \frac{(c_{i,t})^{1-\gamma} - 1}{1-\gamma}$$

Let new consumer-workers have no initial capital $k_{1,t} = 0$. Note that the nonnegativity constraints on capital $k_{2,t+1}$ and consumption $c_{1,t}$ and $c_{2,t+1}$ are not strict inequalities. This is because we are allowing the government transfer program to zero out the consumption and savings of the young.

Consumption in the second period of life is characterized by the second period budget constraint.

$$c_{2,t+1} = (1 + r_{t+1} - \delta)k_{2,t+1} + H_{t+1} \tag{1}$$

Note that the nonnegativity constraint on consumption will never bind because ev-

for an early description of this type of policy inertia. If the government defaults in a way that the consumption of the young does not go to zero, then the government has changed its nonproportional transfer program to look like a proportional transfer program.

erything on the right-hand-side of (1) is weakly positive. Consumption in the first period of life $c_{1,t}$ and savings in the first period of life $k_{2,t+1}$ are jointly determined by the first period budget constraint and by the Euler equation.

$$c_{1,t} + k_{2,t+1} = w_t - H_t \tag{2}$$

$$u'(c_{1,t}) = \beta E_t\Big[\big(1 + r_{t+1} - \delta\big)u'(c_{2,t+1})\Big] \tag{3}$$

Note from the right-hand-side of (2) that the nonnegativity constraints on $c_{1,t}$ and $k_{2,t+1}$ bind when $w_t \leq \bar{H}$. It is in these cases that the government is only able to collect $H_t = w_t$ by forcing the consumption and savings of the young to zero. Economic shut down is characterized by this condition.

2.2 Firm problem

A unit measure of identical perfectly competitive firms exist in this economy that hire aggregate labor L_t at real wage w_t and rent aggregate capital K_t at real rental rate r_t every period in order to produce consumption good Y_t according to a Cobb-Douglas production function,

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \quad \forall t \tag{4}$$

where $A_t = e^{z_t}$ is distributed log normally, and z_t follows an $AR(1)$ process.

$$\begin{aligned} &z_t = \rho z_{t-1} + (1-\rho)\mu + \varepsilon_t \\ &\quad \text{where} \quad \rho \in [0,1), \quad \mu \geq 0, \quad \text{and} \quad \varepsilon_t \sim N(0,\sigma^2) \end{aligned} \tag{5}$$

Profit maximization implies that the real wage and real rental rate are determined by the standard first order conditions for the firm.

$$r_t = \alpha e^{z_t} K_t^{\alpha-1} L_t^{1-\alpha} \quad \forall t \tag{6}$$

$$w_t = (1-\alpha) e^{z_t} K_t^{\alpha} L_t^{-\alpha} \quad \forall t \tag{7}$$

2.3 Market clearing

Market clearing implies that the aggregate labor demand equals aggregate labor supply, aggregate capital demand equals aggregate capital supply, and output equals consumption minus investment in each period,

$$L_t = l_1 = \bar{l} = 1 \quad \forall t \tag{8}$$

$$K_t = k_{2,t} \quad \forall t \tag{9}$$

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t \quad \forall t \tag{10}$$

where C_t in (10) is aggregate consumption and is given by $C_t \equiv \sum_{i=1}^{2} c_{i,t}$.

2.4 Solution and calibration

A competitive equilibrium for a given $\bar{H}$ is defined in the following way.

Definition 1 (Competitive equilibrium). A competitive equilibrium in the overlapping generations model with 2-period lived agents and promised government transfer of $\bar{H}$ is defined as consumption $c_{1,t}$ and $c_{2,t}$ and savings $k_{2,t+1}$ allocations and a real wage w_t and real net interest rate r_t each period such that:

i. households optimize according to (1), (2) and (3),

ii. firms optimize according to (6) and (7),

iii. markets clear according to (8), (9), and (10).

The equilibrium can be solved in terms of either age-1 consumption $c_{1,t}$ or age-1 savings $k_{2,t+1}$. The following exposition and our numerical method obtains the solution by solving for the optimal $c_{1,t}$. We first write all the endogenous variables in terms of age-1 consumption $c_{1,t}$. The age-1 budget constraint (2) becomes

$$k_{2,t+1} = w_t - H_t - c_{1,t}. \tag{11}$$

Age-2 consumption in the next period is simply a function of $k_{2,t+1}$ as in (1). The real wage w_t and interest rate r_t from (6) and (7) are simply functions of the savings $k_{2,t}$

in equilibrium. So the Euler equation (3) can be written all in terms of parameters, period-t state variables, and $c_{1,t}$

$$u'\big(c_{1,t}\big) = \beta E_{z_{t+1}|z_t}\left[\left(1+\alpha e^{z_{t+1}}\left[(1-\alpha)e^{z_t}k_{2,t}^{\alpha}-\bar{H}-c_{1,t}\right]^{\alpha-1}-\delta\right)\times ... \right.$$
$$u'\left(\left[1+\alpha e^{z_{t+1}}\big([1-\alpha]e^{z_t}k_{2,t}^{\alpha}-\bar{H}-c_{1,t}\big)^{\alpha-1}-\delta\right]\big([1-\alpha]e^{z_t}k_{2,t}^{\alpha}-\bar{H}-c_{1,t}\big)+ ... \right.$$
$$\left.\left.\min\left\{(1-\alpha)e^{z_{t+1}}\big([1-\alpha]e^{z_t}k_{2,t}^{\alpha}-\bar{H}-c_{1,t}\big)^{\alpha},\bar{H}\right\}\right)\right] \tag{12}$$

Note that equation (12) characterizes $c_{1,t}$ for all t in which the nonnegativity constraint does not bind $w_t > \bar{H}$. In the other case when the wage is too low to be able to collect the transfer from the young $w_t \leq \bar{H}$, the government collects all that it can from the young $H_t = w_t$, transfers that amount to the old, and the young are left with zero consumption and savings $c_{1,t} = k_{2,t+1} = 0$. For this reason the amount of the transfer in equilibrium, in general, is

$$H_t = \min\{w_t, \bar{H}\} \quad \forall t \tag{13}$$

This expression implies the possibility that, in equilibrium, the government will not be able to collect the full promised transfer $\bar{H}$ in all states of the world. Because (12) characterizes $c_{1,t}$ in the cases in which the nonnegativity constraint does not bind, $H_t = \bar{H}$ in the equation. However, the last term in (12) shows that the integral over all shocks next period must include cases in which the nonnegativity constraint is not satisfied.

We calibrate the parameters of the model so that one period is equivalent to 30 years and then solve for the endogenous objects for a grid of points in the state space $(k_{2,t}, z_t)$.[5] The policy functions for $c_{1,t}$, $c_{2,t}$, $k_{2,t+1}$, Y_t, w_t, and r_t in terms of the state $(k_{2,t}, z_t)$ are all monotonically increasing in the productivity shock z_t, and all except for the interest rate r_t are monotonically increasing in the capital stock $k_{2,t}$.

[5]MatLab code for the computation is available upon request.

Table 1: Calibration of 2-period lived agent OLG model with promised transfer $\bar{H}$

Parameter	Source to match	Value
β	annual discount factor of 0.96	0.29
γ	coefficient of relative risk aversion between 1.5 and 4.0	2
α	capital share of income	0.35
δ	annual capital depreciation of 0.05	0.79
ρ	AR(1) persistence of normally distributed shock to match annual persistence of 0.95	0.21
μ	AR(1) long-run average shock level	0
σ	standard deviation of normally distributed shock to match the annual standard deviation of real GDP of 0.49	1.55
$\bar{H}$	set to be 32% of the median real wage	0.11

The Technical Appendix gives a detailed description of the calibration of all parameters.

2.5 Simulation

One way to measure the effect of fiscal policy on the probability of forcing an economy into a shut down scenario is to simulate the economy and observe when it is most likely to shut down relative to different sized transfer programs. In this section, we simulate the time series of the economy until it shuts down 3,000 times. And we do this for three different values of transfer program size $\bar{H} = \{0.05, 0.11, 0.17\}$ and for three different values of the initial value of the capital stock $k_{2,0} = \{0.11, 0.14, 0, 17\}$.[6] In each simulation we use an initial value of the productivity shock of its median value $z_0 = \mu$.

Table 2 shows how each parameterization for $\bar{H}$ and $k_{2,0}$ changes the median wage w_{med}, the median capital stock k_{med}, and the size of $\bar{H}$ and $k_{2,0}$ relative to the median wage w_{med} and the median capital stock k_{med}, respectively.

Using the calibrated parameters from Table 1 and the various values for $\bar{H}$ and $k_{2,0}$ from Table 2, we simulate the model 3,000 times. Table 3 presents the descriptive statistics of how many periods the simulations take to hit the economic shutdown point of $w_t \leq \bar{H}$. The middle row of Table 3 corresponding to $\bar{H} = 0.11$ shows that this model economy has a greater than 50 percent chance of shutting down in 60 years

[6]The three values for each roughly correspond to low, middle and high values. That is, $\bar{H} = 0.11$ is the value that is roughly equal to 32 percent of the median wage, and $k_{2,0} = 0.14$ is roughly equal to the median capital stock across simulations.

Table 2: Initial values relative to median values

	$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
	w_{med} $\bar{H}/w_{med}$	k_{med} $k_{2,0}/k_{med}$	w_{med} $\bar{H}/w_{med}$	k_{med} $k_{2,0}/k_{med}$	w_{med} $\bar{H}/w_{med}$	k_{med} $k_{2,0}/k_{med}$
$\bar{H} = 0.05$	0.3030	0.0992	0.3026	0.0996	0.3008	0.0991
	0.1650	1.1093	0.1652	1.4062	0.1662	1.7148
$\bar{H} = 0.11$	0.3445	0.1344	0.3433	0.1358	0.3474	0.1365
	0.3193	0.8187	0.3204	1.0311	0.3166	1.2457
$\bar{H} = 0.17$	0.2562	0.1043	0.2709	0.1090	0.2825	0.1134
	0.6635	1.0550	0.6275	1.2846	0.6018	1.4988

w_{med} is the median wage and k_{med} is the median capital stock across all 3,000 simulations before economic shut down.

(2 periods) under a fiscal transfer system that is calibrated to be close to that of the United States. Table 3 also shows that the probability of a shutdown increases or decreases drastically with the size of the fiscal transfer system.

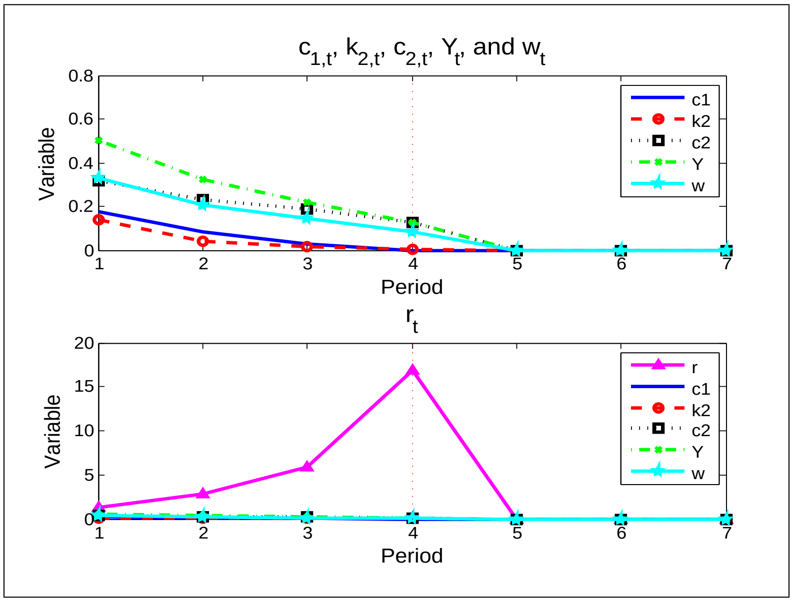
Table 3: Periods to shut down simulation statistics

		$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
		Periods	CDF	Periods	CDF	Periods	CDF
$\bar{H} = 0.05$	min	1	0.1620	1	0.1543	1	0.1477
	med	4	0.5370	4	0.5320	4	0.5283
	mean	5.95	0.6704	6.00	0.6703	6.04	0.6694
	max	45	1.0000	45	1.0000	45	1.0000
$\bar{H} = 0.11$	min	1	0.3623	1	0.3480	1	0.3357
	med	2	0.5653	2	0.5543	2	0.5433
	mean	3.29	0.7060	3.35	0.7029	3.41	0.7022
	max	24	1.0000	24	1.0000	25	1.0000
$\bar{H} = 0.17$	min	1	0.5203	1	0.4987	1	0.4807
	med	1	0.5203	2	0.6833	2	0.6707
	mean	2.42	0.7373	2.48	0.7336	2.54	0.7295
	max	18	1.0000	18	1.0000	18	1.0000

The "min", "med", "mean", and "max" rows in the "Periods" column represent the minimum, median, mean, and maximum number of periods, respectively, in which the simulated time series hit the economic shut down. The "CDF" column represents the percent of simulations that shut down in t periods or less, where t is the value in the "Periods" column. For the CDF value of the "mean" row, we used linear interpolation.

To illustrate the dynamics of the model, we show impulse response functions of the endogenous variables. Figure 1 shows the time series of the endogenous variables starting from the baseline values of $\bar{H} = 0.11$ and $k_{2,0} = 0.14$. With no shocks the economy hits its shut-down point in four periods. The lower pane in Figure 1 shows the interest rate separately because its magnitudes become so much larger than the other variables.[7]

Figure 1: Zero-shock time series of endogenous variables for $\bar{H} = 0.11$ and $k_{2,0} = 0.14$



Figures 2 and 3 show the impulse response functions for the model with shutdown and $\bar{H} = 0.11$ and $k_{2,0} = 0.14$ for a positive standard deviation productivity shock in period 2 and a negative standard deviation productivity shock in period 2, respectively. With the positive shock in period 2, the economy lasts until period 7 before shutting down. With the negative productivity shock in period 2, the economy shuts down immediately in period 2.

[7]Impulse response functions for the other starting values are included in the Technical Appendix.

Figure 2: Impulse response function for positive standard deviation productivity shock in period 2: $\bar{H} = 0.11$ and $k_{2,0} = 0.14$

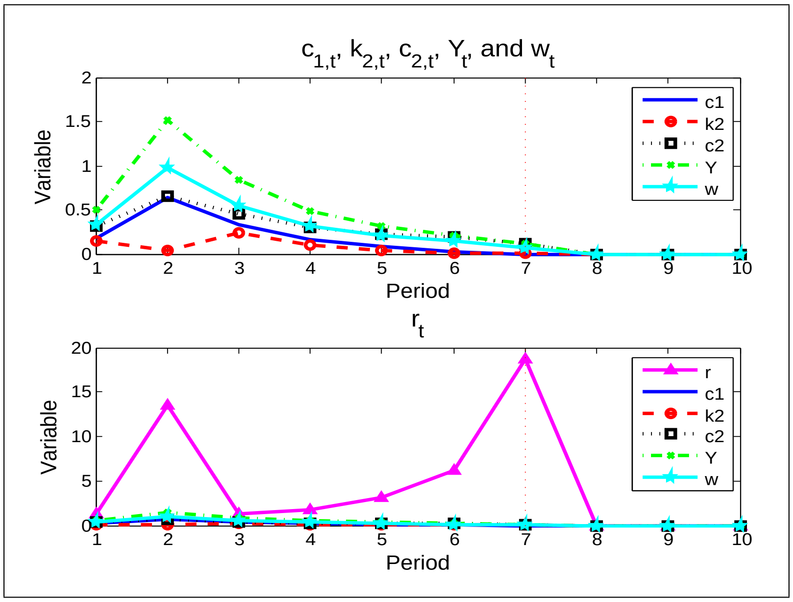
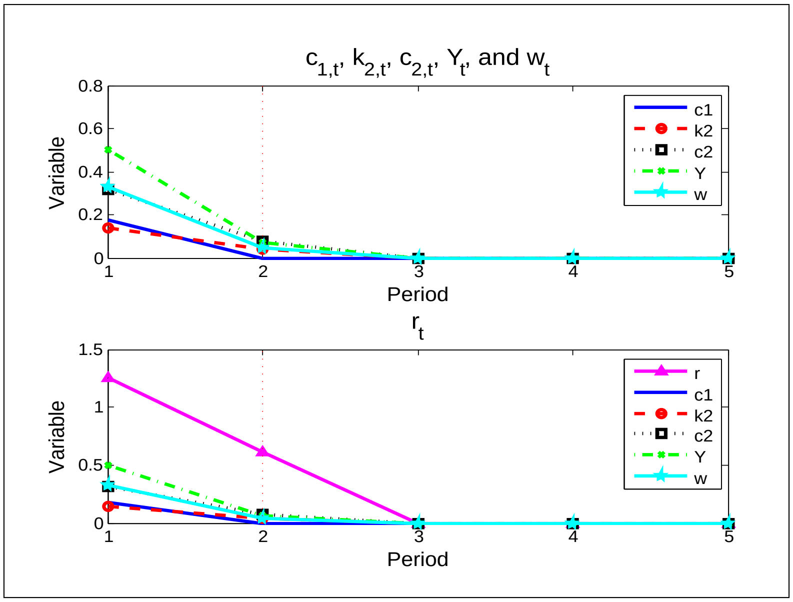


Figure 3: Impulse response function for negative standard deviation productivity shock in period 2: $\bar{H} = 0.11$ and $k_{2,0} = 0.14$



2.6 Fiscal gap and equity premium

Because the actual transfer is not always equal to the promised transfer $H_t \leq \bar{H}$, we define the fiscal gap as the deviation of the net present value of promised transfers from the net present value of actual transfers as a percent of the net present value of output.

$$\text{fiscal gap}_t = x_t \equiv \frac{NPV(\bar{H}) - NPV(H_t)}{NPV(Y_t)} \tag{14}$$

This measure does not suffer from the fungibility of short-run definitions of debts and deficits.

A difficulty with this long-run measure of the fiscal gap is that the lives of households are shorter lived than the horizon over which the net present values must be calculated. For the net present values in (14), we must use discount factors derived separately from the households' discount factor. If we define the discount factor in s periods from the current period as d_{t+s}, then we can rewrite the net present values in the measure of the fiscal gap from (14) in terms of the discount factors and expected streams of transfers and income.

$$x_t = \frac{\sum_{s=0}^{\infty} d_{t+s}\bar{H} - \sum_{s=0}^{\infty} d_{t+s}E\left[H_s\right]}{\sum_{s=0}^{\infty} d_{t+s}E\left[Y_s\right]} \tag{15}$$

We will compute four different measures of the fiscal gap using four different assumed sequences of discount factors d_{t+s}—two from our model and two from the literature. The first measure of the fiscal gap (fgap1) uses the prices of sure-return bonds that mature s periods from the current period t as the discount factors. Define $p_{t,j}$ as the price of an asset $B_{t,j}$ with a sure-return payment of one unit j periods in the future. If these assets can be bought and sold each period, then a household could purchase an asset that pays off after the household is dead and sell it before they die. Because each of these assets must be held in zero net supply, they do not change the equilibrium policy functions described in Section 2.4. The equations that characterized the prices $p_{t,j}$ for all t and j follow standard asset pricing theory.[8]

$$p_{t,j} = \begin{cases} 1 & \text{if} \quad j = 0 \\ \beta \frac{E_t[u'(c_{2,t+1})p_{t+1,j-1}]}{u'(c_{1,t})} & \text{if} \quad j \geq 1 \end{cases} \quad \forall t \tag{16}$$

With the starting value of the sure-return price $p_{t,0}$ pinned down, the prices of the assets that mature in future periods can be solved for recursively using equation (16).

Table 4 shows the calculated sure-return prices at each maturity—which we use as our discount factors—and their corresponding net discount rates shown at an annual rate. Each cell represents the computed prices and interest rates that correspond to a particular promised transfer value $\bar{H}$ and initial capital stock $k_{2,0}$. The first column in each cell displays the prices of the different maturity s of sure return bond $p_{t,t+s}$ computed using recursive equation (16). The second column in each cell represents the annualized version of the net return $r_{t,t+s}$APR or net interest rate.[9]

$$r_{t,t+s} = \left(\frac{1}{p_{t,t+s}}\right)^{\frac{1}{s30}} - 1 \quad \text{for} \quad s \geq 1 \tag{17}$$

The second fiscal gap measure (fgap 2) is calculated using a constant discount rate which is the current period risky return on capital R_t taken from the model. For example, the risky return on capital in period t is $R_t = 1.4971$ in the middle cell in which $\bar{H} = 0.11$ and $k_{2,0} = 0.14$. So the discount factors are $d_{t+s} = (1.4971)^{-s}$. Our third fiscal gap measure (fgap 3) uses a constant discount rate taken from International Monetary Fund (2009, Table 6.4). This study uses an annual discount factor of the growth rate in real GDP plus 1 percent to calculate the net present value of aging-related expeditures. This averages out among G-20 countries to be a discount rate of around 4 percent and for the U.S. is about 3.8 percent ($R_t \approx 3.1$). So the discount rates for fgap3 are $d_{t+s} = (3.05)^{-s}$. For the last measure of the fiscal gap

[8]We derive equation (16), as well as some other assets of interest, in detail in the Technical Appendix.

[9]The return or yield of a sure-return bond should increase with its maturity in an economy that never shuts down. However, the increasing probability of the economy shutting down in each future period counteracts the increasing value of the sure return in the future. This is why the interest rates in the second column of each cell in Table 4 seem to go toward an asymptote in the limit.

Table 4: Term structure of prices and interest rates

	s	$k_{2,0} = 0.11$ $p_{t,t+s}$	$k_{2,0} = 0.11$ $r_{t,t+s}$ APR	$k_{2,0} = 0.14$ $p_{t,t+s}$	$k_{2,0} = 0.14$ $r_{t,t+s}$ APR	$k_{2,0} = 0.17$ $p_{t,t+s}$	$k_{2,0} = 0.17$ $r_{t,t+s}$ APR
$\bar{H} = 0.05$	0	1	0	1	0	1	0
	1	1.5556	-0.0146	1.5897	-0.0153	1.6190	-0.0159
	2	0.3115	0.0196	0.3466	0.0178	0.3782	0.0163
	3	0.0385	0.0369	0.0441	0.0353	0.0493	0.0340
	4	0.0088	0.0403	0.0096	0.0395	0.0099	0.0392
	5	0.0049	0.0360	0.0063	0.0344	0.0063	0.0344
	6	0.0014	0.0372	0.0025	0.0338	0.0024	0.0342
$\bar{H} = 0.11$	0	1	0	1	0	1	0
	1	1.6771	-0.0171	1.7186	-0.0179	1.7673	-0.0188
	2	0.1543	0.0316	0.1793	0.0291	0.2137	0.0261
	3	0.0074	0.0560	0.0092	0.0535	0.0118	0.0506
	4	0.0072	0.0420	0.0077	0.0414	0.0085	0.0405
	5	0.0029	0.0397	0.0032	0.0390	0.0038	0.0379
	6	4.3×10^{-4}	0.0440	5.0×10^{-4}	0.0431	5.9×10^{-4}	0.0421
$\bar{H} = 0.17$	0	1	0	1	0	1	0
	1	1.5848	-0.0152	1.6811	-0.0172	1.7308	-0.0181
	2	0.0092	0.0812	0.0156	0.0718	0.0359	0.0570
	3	0.0010	0.0794	0.0031	0.0663	0.0038	0.0639
	4	9.0×10^{-5}	0.0808	0.0046	0.0459	0.0049	0.0453
	5	1.3×10^{-5}	0.0780	0.0010	0.0470	0.0011	0.0463
	6	1.7×10^{-5}	0.0630	5.6×10^{-5}	0.0558	6.1×10^{-5}	0.0554

The first column in each cell is the price of the sure-return bond $p_{t,t+s}$ at different maturities s as characterized by equation (16). The second column in each cell is the net interest rate $r_{t,t+s}$APR implied by the sure-return rate and given in annual percentage rate terms according to equation (17). Full descriptions of the term structure of prices and interest rates for all calibrations and for up to $s = 12$ is provided in the Technical Appendix.

(fgap4), we use the constant discount rate from Gohkhale and Smetters (2007) who use an annual discount rate of 3.65 percent for their discount factors in their NPV calculation. This is equivalent to a 30-year gross discount rate of $R_t \approx 2.9$. So the discount rates for fgap4 are $d_{t+s} = (2.93)^{-s}$. The expectations for H_t and Y_t are simply the average values from the 3,000 simulations described in Section 2.5.

Table 5 gives the computed fiscal gaps for the nine different combinations of promised transfer $\bar{H}$ and initial capital stock $k_{2,0}$ as a percent of the net present value of output.

Table 5: Measures of the fiscal gap as percent of NPV(GDP)

	$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
	fgap 1 fgap 3	fgap 2 fgap 4	fgap 1 fgap 3	fgap 2 fgap 4	fgap 1 fgap 3	fgap 2 fgap 4
$\bar{H} = 0.05$	0.0037 0.0033	0.0078 0.0035	0.0034 0.0030	0.0096 0.0032	0.0033 0.0028	0.0118 0.0029
$\bar{H} = 0.11$	0.0192 0.0168	0.0373 0.0176	0.0175 0.0152	0.0427 0.0159	0.0164 0.0140	0.555 0.0147
$\bar{H} = 0.17$	0.0474 0.0408	0.0876 0.0426	0.0421 0.0361	0.1041 0.0378	0.0385 0.0328	0.1171 0.0344

Fiscal gap 1 uses the gross sure return rates $R_{t,t+s}$ from Table 4 as the discount rates for NPV calculation. Fiscal gap 2 uses the current period gross return on capital R_t from the model as the constant discount rate. Fiscal gap 3 uses the International Monetary Fund (2009) method of an annual discount rate equal to 1 plus the average percent change in GDP plus 0.01 (≈ 2.05). And fiscal gap 4 uses the Gohkhale and Smetters (2007) method of an annual discount rate equal to 1 plus 0.0365 (≈ 1.93).

In similar fashion to how the fiscal gap is a measure of risk in the economy, we can use the difference in the expected risky return on capital one period from now $E[R_{t+1}]$ and the riskless return on the sure-return bond maturing one period from now $R_{t,t+1}$ to calculate an equity premium. A large literature has tried to explain why the equity premium observed in the real world is so large.[10] More recently, Barro (2009) has shown that incorporating rare disasters into an economic model produces risk premia and risk free rates that are similar to those observed in the data. In our model, we incorporate the rare disaster of an economic shutdown. As shown in

[10]See Shiller (1982), Mehra and Prescott (1985), Kocherlakota (1996), Campbell (2000), and Cochrane (2005, Ch. 21) for surveys of the equity premium puzzle.

Table 6, our model produces equity premia ranging from 4.7 percent to as high as 7.3 percent using only a coefficient of relative risk aversion of $\gamma = 2$.

Table 6: Components of the equity premium in period 1

		$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
		30-year	annual	30-year	annual	30-year	annual
$\bar{H} = 0.05$	$E[R_{t+1}]$	8.2070	1.0361	7.5150	1.0334	7.0113	1.0313
	$\sigma(R_{t+1})$	23.3433	n.a.	21.3222	n.a.	19.8511	n.a.
	$R_{t,t+1}$	0.6428	0.9854	0.6291	0.9847	0.6177	0.9841
	Equity premium $E[R_{t+1}] - R_{t,t+1}$	7.5641	0.0507	6.8859	0.0487	6.3936	0.0473
	Sharpe ratio $\frac{E[R_{t+1}]-R_{t,t+1}}{\sigma(R_{t+1})}$	0.3240	n.a.	0.3229	n.a.	0.3221	n.a.
$\bar{H} = 0.11$	$E[R_{t+1}]$	11.3042	1.0459	10.0769	1.0423	9.2241	1.0396
	$\sigma(R_{t+1})$	32.3859	n.a.	28.8049	n.a.	26.3140	n.a.
	$R_{t,t+1}$	0.5963	0.9829	0.5819	0.9821	0.5658	0.9812
	Equity premium $E[R_{t+1}] - R_{t,t+1}$	10.7080	0.0630	9.4950	0.0602	8.6582	0.0584
	Sharpe ratio $\frac{E[R_{t+1}]-R_{t,t+1}}{\sigma(R_{t+1})}$	0.3306	n.a.	0.3296	n.a.	0.3290	n.a.
$\bar{H} = 0.17$	$E[R_{t+1}]$	16.2082	1.0574	13.7520	1.0521	12.1889	1.0483
	$\sigma(R_{t+1})$	46.7126	n.a.	39.5389	n.a.	34.9735	n.a.
	$R_{t,t+1}$	0.6310	0.9848	0.5948	0.9828	0.5778	0.9819
	Equity premium $E[R_{t+1}] - R_{t,t+1}$	15.5772	0.0727	13.1572	0.0693	11.6112	0.0664
	Sharpe ratio $\frac{E[R_{t+1}]-R_{t,t+1}}{\sigma(R_{t+1})}$	0.3335	n.a.	0.3328	n.a.	0.3320	n.a.

The gross risky one-period return on capital is $R_{t+1} = 1 + r_{t+1} - \delta$. The annualized gross risky one-period return is $(R_{t+1})^{1/30}$. The expected value and standard deviation of the gross risky one-period return R_{t+1} are calculated as the average and standard deviation, respectively, across simulations. The annual equity premium is the expected value of the annualized risky return in the next period minus the annualized return on the one-period riskless bond.

We report the Sharpe ratio in Table 6 as well as all of the components of the equity premium and the Sharpe ratio. For the expected risky return $E[R_{t+1}]$, the one-period sure return $R_{t,t+1}$, and the equity premium (the difference between the two), we report results for both one period from the model (30 years) as well as the annualized (one-year) version. Our Sharpe ratios between 0.32 and 0.33 are in line with common estimates from the data.

Because the equity premium and the Sharpe ratio fluctuate from period-to-period, we report in Table 7 the average equity premium and Sharpe ratio across simulations in the period immediately before the economic shutdown as compared to their respective values in the first period. The average Sharpe ratio is above its initial value in the period immediately before shutdown in every case. This is evidence that Sharpe ratios increase as an economic approaches its critical value.

Table 7: Equity premium and Sharpe ratio in period immediately before shutdown

		$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
		Eq. prem.	Sharpe ratio	Eq. prem.	Sharpe ratio	Eq. prem.	Sharpe ratio
$\bar{H} = 0.05$	period 1	0.0507	0.3240	0.0487	0.3229	0.0473	0.3221
	before shutdown	0.0710	0.3356	0.0707	0.3337	0.0706	0.3370
	percent bigger	0.6617	0.5410	0.6843	0.5570	0.6960	0.5690
	percent smaller	0.1763	0.2970	0.1613	0.2887	0.1563	0.2833
$\bar{H} = 0.11$	period 1	0.0630	0.3306	0.0602	0.3296	0.0584	0.3290
	before shutdown	0.0679	0.3339	0.0667	0.3333	0.0664	0.3343
	percent bigger	0.3740	0.3760	0.4023	0.3970	0.4227	0.4153
	percent smaller	0.2637	0.2617	0.2497	0.2550	0.2417	0.2490
$\bar{H} = 0.17$	period 1	0.0727	0.3335	0.0693	0.3328	0.0664	0.3320
	before shutdown	0.0709	0.3353	0.0686	0.3354	0.0673	0.3348
	percent bigger	0.2027	0.2740	0.2253	0.2937	0.2543	0.3070
	percent smaller	0.2770	0.2057	0.2760	0.2077	0.2650	0.2123

The "period 1" row represents the equity premium and Sharpe ratio in the initial period for each specification. The "before shutdown" row represents the average equity premium and Sharpe ratio across simulations in the period immediately before shutdown for each specification. The "percent bigger" and "percent smaller" rows tell how many of the simulated ending values of the equity premium and Sharpe ratio were bigger than or less than, respectively, their initial period values. These percentages do not sum to one because the equity premium and Sharpe ratio do not change in the cases in which the economy shuts down in the second period.

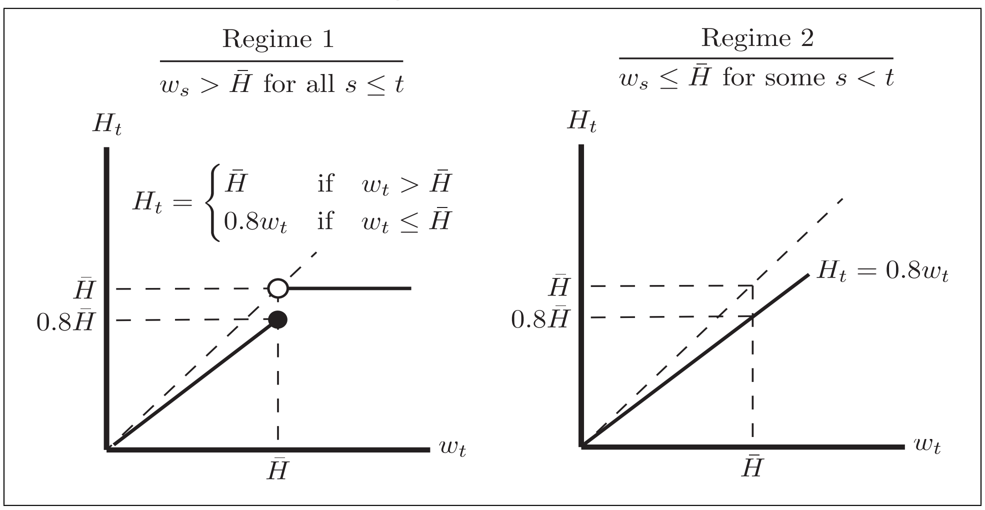
3 Model with Regime Change

In this section, we make the consequence of a default on the promised transfer $\bar{H}$ to be a regime switch to a proportional transfer system rather than the autarky shut down described in Section 2. We assume that when the government defaults on its promised transfer $w_t \leq \bar{H}$, the regime switches permanently to one in which the transfer is simply τ percent of the wage each period $H_t = \tau w_t$. We solve the model for the 80-percent tax rule $\tau = 0.8$ and for the 30-percent tax rule $\tau = 0.3$ in the case of a regime switch.

3.1 Regime change to 80-percent wage tax

Figure 4 illustrates the rule for the transfer H_t under regime 1 in which the transfer is $\bar{H}$ unless wages w_t are less than $\bar{H}$ and under regime 2 in which the transfer is permanently switched to the proportional transfer system $H_t = 0.8w_t$.

Figure 4: Transfer program H_t under regime 1 and regime 2: 80 percent wage tax



3.1.1 Household problem, firm problem, and market clearing

The characterization of the household problem remains the same as in equations (1), (2), and (3) from Section 2.1. The only difference is in the definition of H_t in those equations. With the new regime switching assumption, the transfer each period from the young to the old H_t is defined as follows.

$$H_t = \begin{cases} \bar{H} & \text{if} \quad w_s > \bar{H} \quad \text{for all} \quad s \leq t \\ 0.8w_t & \text{if} \quad w_s \leq \bar{H} \quad \text{for any} \quad s \leq t \end{cases} \tag{18}$$

The change is reflected in the expectations of the young of consumption when old $c_{2,t+1}$ in the savings decision (3).

The firm's problem and the characterization of output, aggregate productivity shock, and optimal net real return on capital and real wage are the same as equations (4) through (7) in Section 2.2. The market clearing conditions that must hold in each period are the same as (8), (9), and (10) from Section 2.3

3.1.2 Solution and calibration

A competitive equilibrium with a transfer program regime switch characterized in (18) is defined in the following way.

Definition 2 (Competitive equilibrium: 80-percent tax). A competitive equilibrium in the overlapping generations model with 2-period lived agents and promised government transfer of $\bar{H}$ that permanently switches to a proportional transfer of $0.8w_t$ if the government cannot collect $\bar{H}$ as in (18) is defined as consumption $c_{1,t}$ and $c_{2,t}$ and savings $k_{2,t+1}$ allocations and a real wage w_t and real net interest rate r_t each period such that:

i. households optimize according to (1), (2) and (3),

ii. firms optimize according to (6) and (7),

iii. markets clear according to (8), (9), and (10).

Characterizing the equilibrium from Definition 2 is simple because households in this model live for only two periods. If a period begins in the constant transfer regime $w_t > \bar{H}$ and $H_t = \bar{H}$, then the only difference from the model in Section 2 is that the young household's consumption and savings decision reflects the new possibility in expectation that next period's transfer could be $0.8w_{t+1}$ rather than $\bar{H}$.

This regime switch actually decreases the expected value of next period's transfer H_{t+1} for the current period's young—$0.8w_t$ instead of w_t. Thus, the current period young will have more precautionary savings $k_{2,t+1}$ than their Section 2 economic shutdown predecessors. However, this implicit tax increase on the period $t+1$ old allows the economy to persist and accumulate utility for generations in the future rather than die.

Once the regime has permanently switched to the high tax rate proportional transfer program of $H_t = 0.8w_t$, allocations each period are determined by the following two equations,

$$c_{2,t} = (1 + \alpha e^{z_t} k_{2,t}^{\alpha-1} - \delta)k_{2,t} + 0.8(1-\alpha)e^{z_t}k_{2,t}^{\alpha} \tag{19}$$

$$\begin{aligned} u'\left(c_{1,t}\right) = \beta E_{z_{t+1}|z_t}\Bigg[&\Big(1 + \alpha e^{z_{t+1}} k_{2,t+1}^{\alpha-1} - \delta\Big) \times ... \\ & u'\Big(\big[1 + \alpha e^{z_{t+1}} k_{2,t+1}^{\alpha-1} - \delta\big] k_{2,t+1} + 0.8(1-\alpha)e^{z_{t+1}}k_{2,t+1}^{\alpha}\Big)\Bigg] \end{aligned} \tag{20}$$

where,

$$k_{2,t+1} = 0.2(1-\alpha)e^{z_t}k_{2,t}^{\alpha} - c_{1,t} \tag{21}$$

and in which we have substituted in the expressions for r_t and w_t from (6) and (7), respectively, and $H_t = 0.8w_t$.

We calibrate the parameters of the model in the same way as in Table 1 for the economic shut down model with the exception of $\bar{H}$. We again calibrate $\bar{H}$ to be 32 percent of the median wage. However, we calculate the median wage from the time periods in the simulations before the regime switches (regime 1). Because the economy does not shut down any more, it is less risky in the long run. But the economy is actually more risky to the current period young in that the expected value of their transfer in the next period is decreased by a potential regime switch. Higher precautionary savings induces a higher median wage and a higher promised transfer $\bar{H} = 0.09$ in order to equal 32 percent of the regime 1 median wage. The policy functions for $c_{1,t}$, $c_{2,t}$, $k_{2,t+1}$, Y_t, w_t, and r_t in terms of the state $(k_{2,t}, z_t)$ are all monotonically increasing in the productivity shock z_t, and all except for the interest rate r_t are monotonically increasing in the capital stock $k_{2,t}$.

Table 8: Calibration of 2-period lived agent OLG model with promised transfer $\bar{H}$ and regime switching: 80-percent tax

Parameter	Source to match	Value
β	annual discount factor of 0.96	0.29
γ	coefficient of relative risk aversion between 1.5 and 4.0	2
α	capital share of income	0.35
δ	annual capital depreciation of 0.05	0.79
ρ	AR(1) persistence of normally distributed shock to match annual persistence of 0.95	0.21
μ	AR(1) long-run average shock level	0
σ	standard deviation of normally distributed shock to match the annual standard deviation of real GDP of 0.49	1.55
$\bar{H}$	set to be 32% of the median real wage	0.09

The Technical Appendix gives a detailed description of the calibration of all parameters.

3.1.3 Simulation

Analogous to the simulation of the model with economic shut down from Section 2.5, we simulate the regime switching model 3,000 times with various combinations of values for the promised transfer $\bar{H} \in \{0.09, 0.11\}$ and the initial capital stock $k_{2,0} \in \{0.0875, 0.14\}$. As shown in Table 9, our calibrated values of $\bar{H} = 0.09$ and $k_{2,0} = 0.0875$ correspond to 32 percent of the median real wage in regime 1 and the median capital stock in regime 1, respectively. In each simulation we use an initial value of the productivity shock of its median value $z_0 = \mu$.

Table 9: Initial values relative to median values from regime 1: 80-percent tax

	$k_{2,0} = 0.0875$		$k_{2,0} = 0.14$	
	w_{med} $\bar{H}/w_{med}$	k_{med} $k_{2,0}/k_{med}$	w_{med} $\bar{H}/w_{med}$	k_{med} $k_{2,0}/k_{med}$
$\bar{H} = 0.09$	0.2827 0.3184	0.0878 0.9967	0.2883 0.3121	0.0895 1.5642
$\bar{H} = 0.11$	0.2944 0.3736	0.0886 0.9873	0.3021 0.3641	0.0899 1.5567

w_{med} is the median wage and k_{med} is the median capital stock across all 3,000 simulations before the regime switch (in regime 1).

The upper left cell of Table 9 is analogous to the middle cell of Table 2 in that $\bar{H}$ is calibrated to be 32 percent of the regime 1 real wage and $k_{2,0}$ to equal the regime 1 median capital stock. However, the lower right cell of Table 9 has the same $\bar{H}$ and $k_{2,0}$ as the middle cell of Table 2. Notice that the median capital stock is higher in the regime switching economy ($k_{med} = 0.1.5567$ for $\bar{H} = 0.11$ and $k_{2,0} = 0.14$ in regime switching economy as compared to $k_{med} = 0.1.0311$ in the shutdown economy with the same $\bar{H}$ and $k_{2,0}$). This is because young households have an increased risk in the second period of life under the possibility of a regime switch because their transfer will be lower in the case of a default on $\bar{H}$.

Using the calibrated parameters from Table 8, we simulate the regime switching model 3,000 times for the four different combinations of $\bar{H}$ and $k_{2,0}$. Table 10 presents the descriptive statistics of how many periods the simulations take to hit the regime switch point of $w_t \leq \bar{H}$. Notice that the distribution of time until regime switch across simulations from the upper left cell of Table 10 is very similar to the middle cell in Table 3 from the shut down economy. Higher precautionary savings extends the time until a regime switch, but increased promised transfers reduce that time.

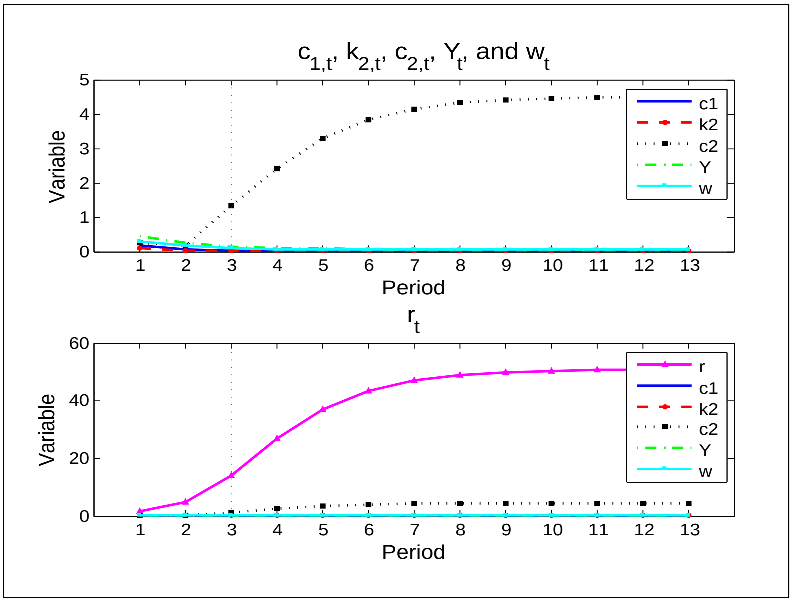
Table 10: Periods to regime switch simulation statistics: 80-percent tax

		$k_{2,0} = 0.0875$		$k_{2,0} = 0.14$	
		Periods	CDF	Periods	CDF
$\bar{H} = 0.09$	min	1	0.3677	1	0.3340
	med	2	0.5727	2	0.5470
	mean	3.25	0.7124	3.40	0.7066
	max	24	1.0000	25	1.0000
$\bar{H} = 0.11$	min	1	0.4517	1	0.4060
	med	2	0.6430	2	0.6127
	mean	2.78	0.7314	2.94	0.7244
	max	24	1.0000	24	1.0000

The "min", "med", "mean", and "max" rows in the "Periods" column represent the minimum, median, mean, and maximum number of periods, respectively, in which the simulated time series hit the regime switch condition. The "CDF" column represents the percent of simulations that switch regimes in t periods or less, where t is the value in the "Periods" column. For the CDF value of the "mean" row, we used linear interpolation.

As with the shut down model from Section 2.5, we show impulse response functions of the endogenous variables. Figure 5 shows the time series of the endogenous variables starting from the baseline values of $\bar{H} = 0.09$ and $k_{2,0} = 0.0875$. With no shocks the economy hits its shut-down point in 3 periods. The lower pane in Figure 5 shows the interest rate separately because its magnitudes become so much larger than the other variables.[11]

Figure 5: Zero-shock time series of endogenous variables for $\bar{H} = 0.09$ and $k_{2,0} = 0.0875$: 80-percent tax



Figures 6 and 7 show the impulse response functions for the model with shutdown and $\bar{H} = 0.09$ and $k_{2,0} = 0.0875$ for a positive standard deviation productivity shock in period 2 and a negative standard deviation productivity shock in period 2, respectively. With the positive shock in period 2, the economy lasts until period 6 before shutting down. With the negative productivity shock in period 2, the economy shuts down immediately in period 2.

[11]Impulse response functions for the other starting values are included in the Technical Appendix.

Figure 6: Impulse response function for positive standard deviation productivity shock in period 2: $\bar{H} = 0.09$, $k_{2,0} = 0.0875$, 80-percent tax

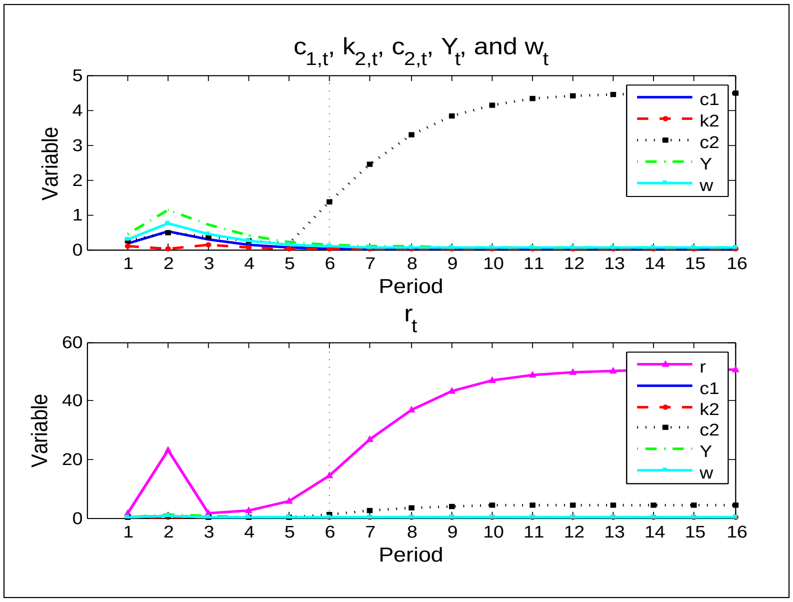
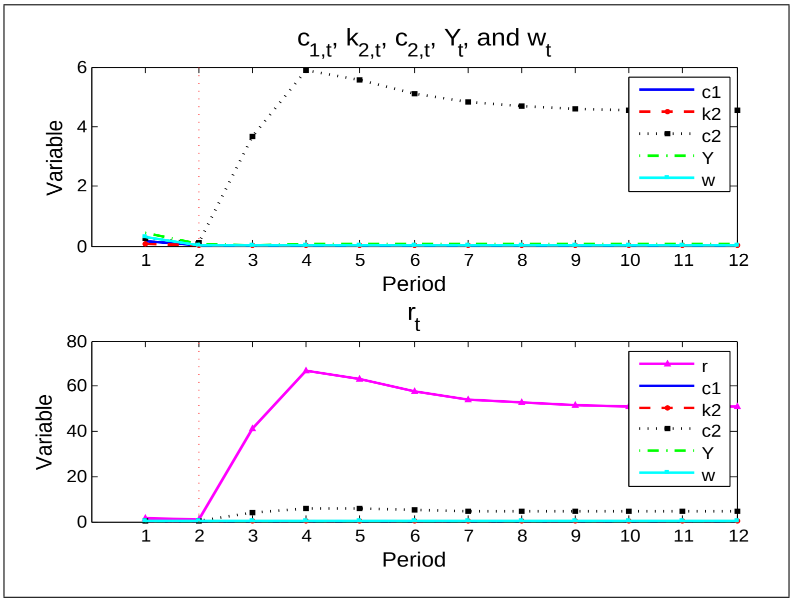


Figure 7: Impulse response function for negative standard deviation productivity shock in period 2: $\bar{H} = 0.09$, $k_{2,0} = 0.0875$, 80-percent tax



3.1.4 Fiscal gap and equity premium

For the model with regime switching to an 80-percent wage tax, we define the fiscal gap in the same way as in equation (14) from Section 2.6. The discount factors used to calculate the net present values in the fiscal gap measures from the regime switching model are calculated in the same way as described in Section 2.6. Table 11 shows the calculated sure-return prices and their corresponding annualized discount rates for this regime switching economy. Each cell represents the computed prices and interest rates that correspond to a particular promised transfer value $\bar{H}$ and initial capital stock $k_{2,0}$.

Table 11: Term structure of prices and interest rates in regime switching economy: 80-percent tax

		$k_{2,0} = 0.0875$		$k_{2,0} = 0.14$	
	s	$p_{t,t+s}$	$r_{t,t+s}$ APR	$p_{t,t+s}$	$r_{t,t+s}$ APR
$\bar{H} = 0.09$	0	1	0	1	0
	1	0.3269	0.0380	0.4645	0.0259
	2	1.1607	-0.0025	2.5547	-0.0155
	3	0.3534	0.0116	0.4138	0.0099
	4	0.6753	0.0033	1.2121	-0.0016
	5	0.4117	0.0059	0.2982	0.0081
	6	0.1304	0.0114	0.4420	0.0045
$\bar{H} = 0.11$	0	1	0	1	0
	1	0.2328	0.0498	0.3227	0.0384
	2	1.3063	-0.0044	1.5334	-0.0071
	3	2.5521	-0.0104	1.5811	-0.0051
	4	0.2606	0.0113	0.8424	0.0014
	5	1.7532	-0.0037	1.8832	-0.0042
	6	0.3762	0.0054	0.4895	0.0040

The first column in each cell is the price of the sure-return bond $p_{t,t+s}$ at different maturities s as characterized by equation (16). The second column in each cell is the net interest rate $r_{t,t+s}$APR implied by the sure-return rate and given in annual percentage rate terms according to equation (17). Full descriptions of the term structure of prices and interest rates for all calibrations and for up to $s = 12$ is provided in the Technical Appendix.

Table 12 shows our four measures of the fiscal gap as a percent of the net present value of GDP for each of our four combinations of $\bar{H}$ and $k_{2,0}$. One difference in the regime switching model is that the economy never shuts down. Also striking is that some of the fiscal gap measures are negative. This occurs because some of the discount rates decay more slowly than others (fgap 1 is the slowest) and because expected H_t is higher than $\bar{H}$ after the regime switch. Even though the impulse response of w_t decays to a lower level after the regime switch (see Figure 5), the expected H_t can be high because of the high variance in productivity shocks. A median value would be lower. We therefore can get negative fiscal gap measures, even though $\bar{H}$ is big enough to trigger a regime switch in relatively few periods. Table 12 gives the computed fiscal gaps as a percent of the net present value of output as in equation (14) for the four combinations of values for the promised transfer $\bar{H}$ and the initial capital stock $k_{2,0}$.

Table 12: Measures of the fiscal gap with regime switching as percent of NPV(GDP): 80-percent tax

	$k_{2,0} = 0.0875$		$k_{2,0} = 0.14$	
	fgap 1	fgap 2	fgap 1	fgap 2
	fgap 3	fgap 4	fgap 3	fgap 4
$\bar{H} = 0.09$	-0.0519	0.0003	-0.0343	-0.0157
	0.0067	0.0066	0.0052	0.0051
$\bar{H} = 0.11$	-0.0861	0.0057	-0.0749	-0.0075
	0.0130	0.0129	0.0103	0.0102

Fiscal gap 1 uses the gross sure return rates $R_{t,t+s}$ from Table 4 as the discount rates for NPV calculation. Fiscal gap 2 uses the current period gross return on capital R_t from the model as the constant discount rate. Fiscal gap 3 uses the International Monetary Fund (2009) method of an annual discount rate equal to 1 plus the average percent change in GDP plus 0.01 ($\approx$ 2.05). And fiscal gap 4 uses the Gohkhale and Smetters (2007) method of an annual discount rate equal to 1 plus 0.0365 ($\approx$ 1.93).

Note also in Table 12 that the fiscal gap measure fgap1 becomes even more negative as $\bar{H}$ increases. This is caused by the higher $\bar{H}$ shortening the periods until the regime switch or higher H_t values. In other words, the positive effect on the fiscal gap from a higher $\bar{H}$ in the pre-switch periods is dominated by the negative effect on the fiscal gap from the more periods of high regime 2 H_t. For the other measures of the fiscal gap, the second effect dominates so the fiscal gap increases with the size of the promised transfer $\bar{H}$.

Lastly, we also caclulate the equity premium and Sharpe ratio for this regime switching model using the difference in the expected risky return on capital one period from now $E[R_{t+1}]$ and the riskless return on the sure-return bond maturing one period from now $R_{t,t+1}$. In reference to the Barro (2009) result, our model with regime switching delivers equity premia that are significantly lower than the riskier model with shut down from Section 2.6 and do not match as closely estimated equity premia and Sharpe ratios. As shown in Table 13, our regime switching model produces equity premia around 2 percent and Sharpe ratios around 0.28.

Table 13: Components of the equity premium with regime switching: 80-percent tax

		$k_{2,0} = 0.0875$		$k_{2,0} = 0.14$	
		30-year	annual	30-year	annual
$\bar{H} = 0.09$	$E[R_{t+1}]$	17.1319	1.0592	12.9708	1.0503
	$\sigma(R_{t+1})$	49.4105	n.a.	37.2570	n.a.
	$R_{t,t+1}$	3.0589	1.0380	2.1526	1.0259
	Equity premium $E[R_{t+1}] - R_{t,t+1}$	14.0731	0.0213	10.8182	0.0244
	Sharpe ratio $\frac{E[R_{t+1}]-R_{t,t+1}}{\sigma(R_{t+1})}$	0.2848	n.a.	0.2904	n.a.
$\bar{H} = 0.11$	$E[R_{t+1}]$	22.1773	1.0678	16.0801	1.0572
	$\sigma(R_{t+1})$	64.1466	n.a.	46.3385	n.a.
	$R_{t,t+1}$	4.2960	1.0498	3.0985	1.0384
	Equity premium $E[R_{t+1}] - R_{t,t+1}$	17.8813	0.0180	12.9816	0.0188
	Sharpe ratio $\frac{E[R_{t+1}]-R_{t,t+1}}{\sigma(R_{t+1})}$	0.2788	n.a.	0.2801	n.a.

The gross risky one-period return on capital is $R_{t+1} = 1 + r_{t+1} - \delta$. The annualized gross risky one-period return is $(R_{t+1})^{1/30}$. The expected value and standard deviation of the gross risky one-period return R_{t+1} are calculated as the average and standard deviation, respectively, across simulations. The annual equity premium is the expected value of the annualized risky return in the next period minus the annualized return on the one-period riskless bond.

However, the real equity premium story in the model with the 80-percent tax rate regime switch is what happens to the equity premium as the economy approaches its critical value. Table 14 reports the average equity premium and Sharpe ratio across simulations in the period immediately before the regime switch as compared to their respective values in the first period. The average equity premium and Sharpe ratio increase significantly from the initial period to the period right before the regime switch in every case.

Table 14: Equity premium and Sharpe ratio in period immediately before regime switch: 80-percent tax

		$k_{2,0} = 0.0875$		$k_{2,0} = 0.14$	
		Eq. prem.	Sharpe ratio	Eq. prem.	Sharpe ratio
$\bar{H} = 0.09$	period 1	0.0213	0.2848	0.0244	0.2904
	before shutdown	0.0737	0.3231	0.0773	0.3272
	percent bigger	0.6287	0.5353	0.6600	0.5523
	percent smaller	0.0037	0.0970	0.0060	0.1137
$\bar{H} = 0.11$	period 1	0.0180	0.2788	0.0188	0.2801
	before shutdown	0.0637	0.3152	0.0675	0.3201
	percent bigger	0.5457	0.4770	0.5910	0.5180
	percent smaller	0.0027	0.0713	0.0030	0.0760

The "period 1" row represents the equity premium and Sharpe ratio in the initial period for each specification. The "before shutdown" row represents the average equity premium and Sharpe ratio across simulations in the period immediately before shutdown for each specification. The "percent bigger" and "percent smaller" rows tell how many of the simulated ending values of the equity premium and Sharpe ratio were bigger than or less than, respectively, their initial period values. These percentages do not sum to one because the equity premium and Sharpe ratio do not change in the cases in which the economy shuts down in the second period.

3.2 Regime change to 30-percent wage tax

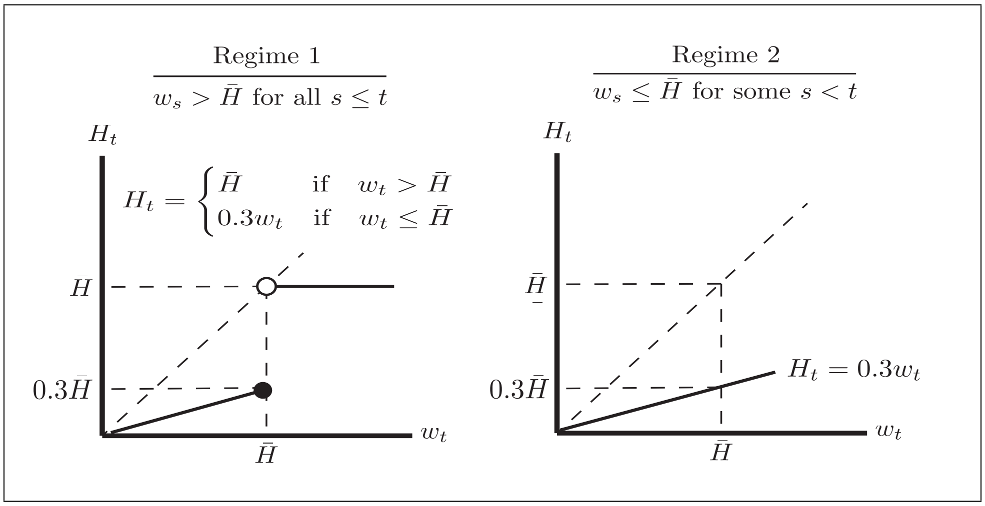
Figure 8 illustrates the rule for the transfer H_t under regime 1 in which the transfer is $\bar{H}$ unless wages w_t are less than $\bar{H}$ and under regime 2 in which the transfer is permanently switched to the proportional transfer system $H_t = 0.3w_t$.

3.2.1 Household problem, firm problem, and market clearing

The characterization of the household problem remains the same as in equations (1), (2), and (3) from Section 2.1. The transfer each period from the young to the old H_t is defined as follows.

$$H_t = \begin{cases} \bar{H} & \text{if} \quad w_s > \bar{H} \quad \text{for all} \quad s \leq t \\ 0.3w_t & \text{if} \quad w_s \leq \bar{H} \quad \text{for any} \quad s \leq t \end{cases} \tag{22}$$

Figure 8: Transfer program H_t under regime 1 and regime 2: 30 percent wage tax



The change is reflected in the expectations of the young of consumption when old $c_{2,t+1}$ in the savings decision (3).

The firm's problem and the characterization of output, aggregate productivity shock, and optimal net real return on capital and real wage are the same as equations (4) through (7) in Section 2.2. The market clearing conditions that must hold in each period are the same as (8), (9), and (10) from Section 2.3

3.2.2 Solution and calibration

A competitive equilibrium with a transfer program regime switch characterized in (22) is defined in the following way.

Definition 3 (**Competitive equilibrium: 30-percent tax**)**.** A competitive equilibrium in the overlapping generations model with 2-period lived agents and promised government transfer of $\bar{H}$ that permanently switches to a proportional transfer of $0.3w_t$ if the government cannot collect $\bar{H}$ as in (22) is defined as consumption $c_{1,t}$ and $c_{2,t}$ and savings $k_{2,t+1}$ allocations and a real wage w_t and real net interest rate r_t each period such that:

i. households optimize according to (1), (2) and (3),

ii. firms optimize according to (6) and (7),

iii. markets clear according to (8), (9), and (10).

Once the regime has permanently switched to the high tax rate proportional transfer program of $H_t = 0.3w_t$, allocations each period are determined by the following two equations,

$$c_{2,t} = (1 + \alpha e^{z_t} k_{2,t}^{\alpha-1} - \delta)k_{2,t} + 0.3(1-\alpha)e^{z_t}k_{2,t}^{\alpha} \tag{23}$$

$$\begin{aligned} u'\big(c_{1,t}\big) = \beta E_{z_{t+1}|z_t}\Bigg[\Big(1 + \alpha e^{z_{t+1}} k_{2,t+1}^{\alpha-1} - \delta\Big) \times ... \\ u'\Big(\big[1 + \alpha e^{z_{t+1}} k_{2,t+1}^{\alpha-1} - \delta\big] k_{2,t+1} + 0.3(1-\alpha)e^{z_{t+1}}k_{2,t+1}^{\alpha}\Big)\Bigg] \end{aligned} \tag{24}$$

where,

$$k_{2,t+1} = 0.7(1-\alpha)e^{z_t}k_{2,t}^{\alpha} - c_{1,t} \tag{25}$$

and in which we have substituted in the expressions for r_t and w_t from (6) and (7), respectively, and $H_t = 0.3w_t$. We use the same calibration as in Table 8 with the 80-percent tax regime shift.

3.2.3 Simulation

We simulate the regime switching model 3,000 times with the same combinations of values for the promised transfer $\bar{H} \in \{0.09, 0.11\}$ and the initial capital stock $k_{2,0} \in \{0.0875, 0.14\}$ as in Section 3.1. As shown in Table 15, our calibrated values of $\bar{H} = 0.09$ and $k_{2,0} = 0.0875$ correspond to about 32 percent of the median real wage in regime 1 and close to the median capital stock in regime 1, respectively. Note that none of these regime 1 values change much from Table 9 even though the regime 2 tax plan is significantly different. In each simulation we use an initial value of the productivity shock of its median value $z_0 = \mu$.

The upper left cell of Table 15 is analogous to the middle cell of Table 2 in that $\bar{H}$ is calibrated to be 32 percent of the regime 1 real wage and $k_{2,0}$ to equal the regime 1 median capital stock. However, the lower right cell of Table 15 has the same $\bar{H}$ and $k_{2,0}$ as the middle cell of Table 2. Notice that the median capital stock is higher in the

Table 15: **Initial values relative to median values from regime 1: 30-percent tax**

	$k_{2,0} = 0.0875$		$k_{2,0} = 0.14$	
	w_{med}	k_{med}	w_{med}	k_{med}
	$\bar{H}/w_{med}$	$k_{2,0}/k_{med}$	$\bar{H}/w_{med}$	$k_{2,0}/k_{med}$
$\bar{H} = 0.09$	0.2828	0.0864	0.2880	0.0885
	0.3183	1.0130	0.3125	1.5819
$\bar{H} = 0.11$	0.2963	0.0868	0.3051	0.0877
	0.3712	1.0082	0.3605	1.5970

w_{med} is the median wage and k_{med} is the median capital stock across all 3,000 simulations before the regime switch (in regime 1).

regime switching economy ($k_{med} = 0.1.5970$ for $\bar{H} = 0.11$ and $k_{2,0} = 0.14$ in regime switching economy as compared to $k_{med} = 0.1.0311$ in the shutdown economy with the same $\bar{H}$ and $k_{2,0}$). This is because young households have an increased risk in the second period of life under the possibility of a regime switch because their transfer will be lower in the case of a default on $\bar{H}$.

Using the calibrated parameters from Table 8, we simulate the regime switching model 3,000 times for the four different combinations of $\bar{H}$ and $k_{2,0}$. Table 16 presents the descriptive statistics of how many periods the simulations take to hit the regime switch point of $w_t \leq \bar{H}$. Notice that the distributions of time until regime switch across simulations in all the cells of Table 16 are very similar to the distributions from the 80-percent tax economy in Table 10. Higher precautionary savings extends the time until a regime switch, but increased promised transfers reduce that time.

The impulse response functions of the endogenous variables are given in Figures 9, 10, and 11.Figure 9 shows the time series of the endogenous variables starting from the baseline values of $\bar{H} = 0.09$ and $k_{2,0} = 0.0875$. With no shocks the economy hits its shut-down point in 3 periods. The lower pane in Figure 9 shows the interest rate separately because its magnitudes become so much larger than the other variables.[12]

Figures 10 and 11 show the impulse response functions for the model with shutdown and $\bar{H} = 0.09$ and $k_{2,0} = 0.0875$ for a positive standard deviation productivity shock in period 2 and a negative standard deviation productivity shock in period 2,

[12]Impulse response functions for the other starting values are included in the Technical Appendix.

Table 16: Periods to regime switch simulation statistics: 30-percent tax

		$k_{2,0} = 0.0875$		$k_{2,0} = 0.14$	
		Periods	CDF	Periods	CDF
$\bar{H} = 0.09$	min	1	0.3677	1	0.3340
	med	2	0.5697	2	0.5440
	mean	3.28	0.7116	3.42	0.7054
	max	24	1.0000	25	1.0000
$\bar{H} = 0.11$	min	1	0.4517	1	0.4060
	med	2	0.6390	2	0.6080
	mean	2.80	0.7302	2.96	0.7228
	max	24	1.0000	24	1.0000

The "min", "med", "mean", and "max" rows in the "Periods" column represent the minimum, median, mean, and maximum number of periods, respectively, in which the simulated time series hit the regime switch condition. The "CDF" column represents the percent of simulations that switch regimes in t periods or less, where t is the value in the "Periods" column. For the CDF value of the "mean" row, we used linear interpolation.

respectively. With the positive shock in period 2, the economy lasts until period 6 before shutting down. With the negative productivity shock in period 2, the economy shuts down immediately in period 2.

Figure 9: Zero-shock time series of endogenous variables for $\bar{H} = 0.09$ and $k_{2,0} = 0.0875$: 30-percent tax

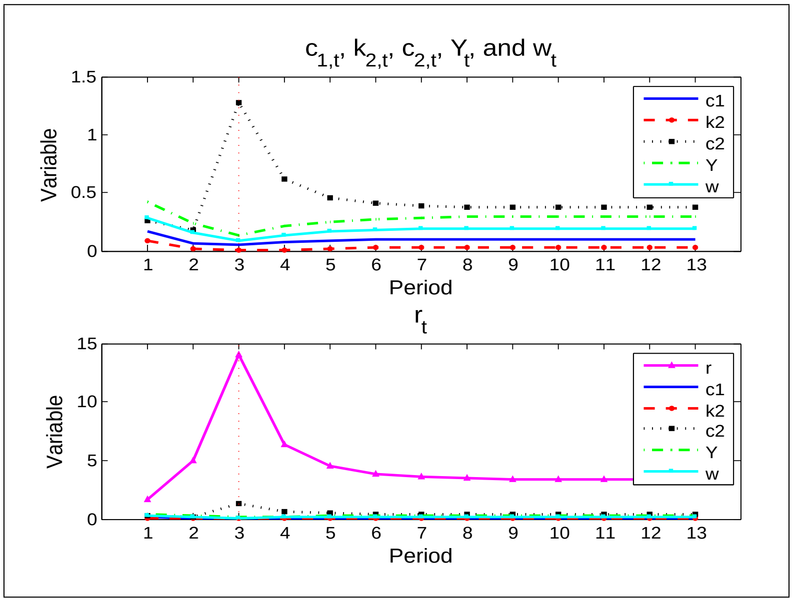


Figure 10: Impulse response function for positive standard deviation productivity shock in period 2: $\bar{H} = 0.09$, $k_{2,0} = 0.0875$, 30-percent tax

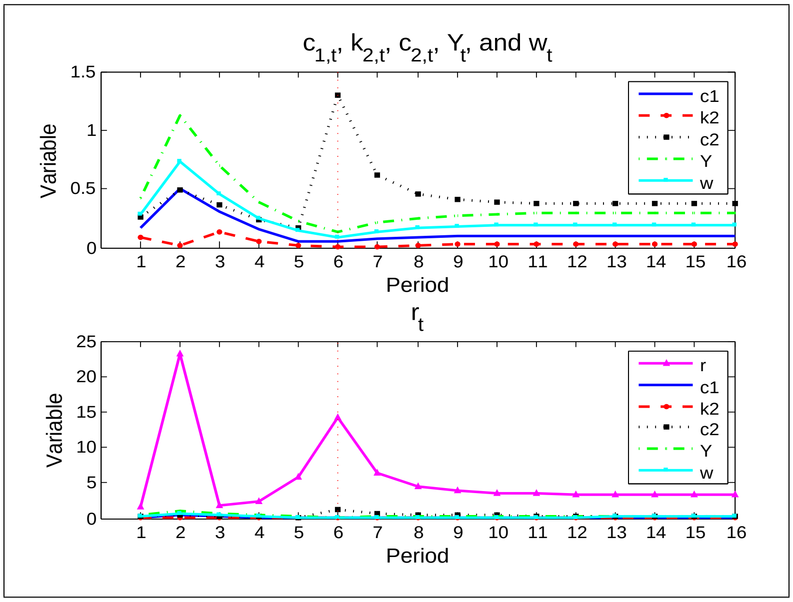
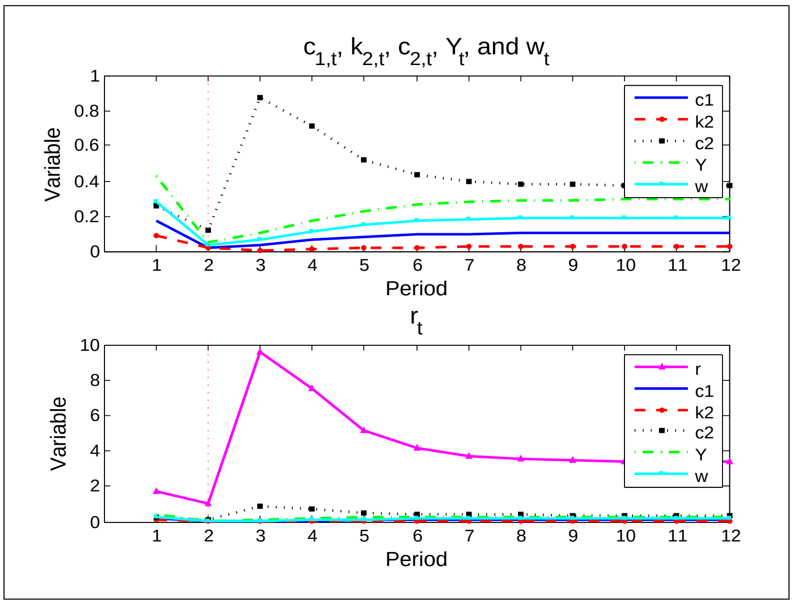


Figure 11: Impulse response function for negative standard deviation productivity shock in period 2: $\bar{H} = 0.09$, $k_{2,0} = 0.0875$, 30-percent tax



3.2.4 Fiscal gap and equity premium

For the model with regime switching to a 30-percent wage tax, we define the fiscal gap in the same way as in equation (14) from Section 2.6. The discount factors used to calculate the net present values in the fiscal gap measures from the regime switching model are calculated in the same way as described in Section 2.6. Table 17 shows the calculated sure-return prices and their corresponding annualized discount rates for this regime switching economy. Each cell represents the computed prices and interest rates that correspond to a particular promised transfer value $\bar{H}$ and initial capital stock $k_{2,0}$.

Table 17: Term structure of prices and interest rates in regime switching economy: 30-percent tax

		$k_{2,0} = 0.0875$		$k_{2,0} = 0.14$	
	s	$p_{t,t+s}$	$r_{t,t+s}$ APR	$p_{t,t+s}$	$r_{t,t+s}$ APR
$\bar{H} = 0.09$	0	1	0	1	0
	1	0.3367	0.0370	0.4453	0.0273
	2	6.0523	-0.0296	8.0476	-0.0342
	3	2.0412	-0.0079	6.7823	-0.0210
	4	8.5075	-0.0177	16.8480	-0.0233
	5	15.9863	-0.0183	25.3856	-0.0213
	6	7.5427	-0.0112	6.1479	-0.0100
$\bar{H} = 0.11$	0	1	0	1	0
	1	0.2326	0.0498	0.3225	0.0384
	2	7.3132	-0.0326	7.1394	-0.0322
	3	11.5166	-0.0268	5.8534	-0.0194
	4	16.4777	-0.0231	12.1299	-0.0206
	5	9.2992	-0.0148	15.5375	-0.0181
	6	23.4145	-0.0174	31.7886	-0.0190

The first column in each cell is the price of the sure-return bond $p_{t,t+s}$ at different maturities s as characterized by equation (16). The second column in each cell is the net interest rate $r_{t,t+s}$APR implied by the sure-return rate and given in annual percentage rate terms according to equation (17). Full descriptions of the term structure of prices and interest rates for all calibrations and for up to $s = 12$ is provided in the Technical Appendix.

Table 18 shows our four measures of the fiscal gap as a percent of the net present value of GDP for each of our four combinations of $\bar{H}$ and $k_{2,0}$. Similar to the 80-

percent tax regime switch model, all the measures for the first measure of the fiscal gap (fgap1) are negative. These negative fiscal gaps—and relatively low measures of the fiscal gap for the other measures—occur because the expected H_t after the regime swith is significantly higher than $\bar{H}$. But in all cases, increased $\bar{H}$ increases the fiscal gap.

Table 18: Measures of the fiscal gap with regime switching as percent of NPV(GDP): 30-percent tax

	$k_{2,0} = 0.0875$		$k_{2,0} = 0.14$	
	fgap 1 fgap 3	fgap 2 fgap 4	fgap 1 fgap 3	fgap 2 fgap 4
$\bar{H} = 0.09$	-0.1241 0.0099	0.0002 0.0096	-0.1214 0.0079	-0.0148 0.0078
$\bar{H} = 0.11$	-0.1194 0.0172	0.0064 0.0171	-0.1190 0.0139	-0.0108 0.0138

Fiscal gap 1 uses the gross sure return rates $R_{t,t+s}$ from Table 4 as the discount rates for NPV calculation. Fiscal gap 2 uses the current period gross return on capital R_t from the model as the constant discount rate. Fiscal gap 3 uses the International Monetary Fund (2009) method of an annual discount rate equal to 1 plus the average percent change in GDP plus 0.01 ($\approx$ 2.05). And fiscal gap 4 uses the Gohkhale and Smetters (2007) method of an annual discount rate equal to 1 plus 0.0365 ($\approx$ 1.93).

Lastly, we also caclulate the equity premium and Sharpe ratio for this regime switching model using the difference in the expected risky return on capital one period from now $E[R_{t+1}]$ and the riskless return on the sure-return bond maturing one period from now $R_{t,t+1}$. The equity premium results from Table 19 show very little change from the Table 13 results from the 80-percent tax regime model. This means that the form of the regime change has little effect on the initial period equity premium. The equity premia here around 2 percent with Sharpe ratios around 0.28.

And as with the other regime switching model, the real equity premium story in the model with the 30-percent tax rate regime switch is what happens to the equity premium as the economy approaches its critical value. Table 20 reports the average equity premium and Sharpe ratio across simulations in the period immediately before the regime switch as compared to their respective values in the first period.

Table 19: Components of the equity premium with regime switching: 30-percent tax

		$k_{2,0} = 0.0875$		$k_{2,0} = 0.14$	
		30-year	annual	30-year	annual
$\bar{H} = 0.09$	$E[R_{t+1}]$	17.1319	1.0592	12.9708	1.0503
	$\sigma(R_{t+1})$	49.4105	n.a.	37.2570	n.a.
	$R_{t,t+1}$	2.9703	1.0370	2.2457	1.0273
	Equity premium $E[R_{t+1}] - R_{t,t+1}$	14.1616	0.0223	10.7251	0.0229
	Sharpe ratio $\frac{E[R_{t+1}]-R_{t,t+1}}{\sigma(R_{t+1})}$	0.2866	n.a.	0.2879	n.a.
$\bar{H} = 0.11$	$E[R_{t+1}]$	22.1773	1.0678	16.0801	1.0572
	$\sigma(R_{t+1})$	64.1466	n.a.	46.3385	n.a.
	$R_{t,t+1}$	4.2986	1.0498	3.1006	1.0384
	Equity premium $E[R_{t+1}] - R_{t,t+1}$	17.8787	0.0180	12.9795	0.0187
	Sharpe ratio $\frac{E[R_{t+1}]-R_{t,t+1}}{\sigma(R_{t+1})}$	0.2787	n.a.	0.2801	n.a.

The gross risky one-period return on capital is $R_{t+1} = 1 + r_{t+1} - \delta$. The annualized gross risky one-period return is $(R_{t+1})^{1/30}$. The expected value and standard deviation of the gross risky one-period return R_{t+1} are calculated as the average and standard deviation, respectively, across simulations. The annual equity premium is the expected value of the annualized risky return in the next period minus the annualized return on the one-period riskless bond.

The average equity premium and Sharpe ratio increase significantly from the initial period to the period right before the regime switch in every case. In the case of both the 80-percent wage tax regime switch and the 30-percent wage tax regime switch, the equity premia in the period before the shift are much closer to those observed in the data, notwithstanding the initial period equity premia are smaller.

Table 20: Equity premium and Sharpe ratio in period immediately before regime switch: 30-percent tax

		$k_{2,0} = 0.0875$		$k_{2,0} = 0.14$	
		Eq. prem.	Sharpe ratio	Eq. prem.	Sharpe ratio
$\bar{H} = 0.09$	period 1	0.0223	0.2866	0.0229	0.2879
	before shutdown	0.0819	0.3266	0.0848	0.3276
	percent bigger	0.6290	0.5367	0.6617	0.5660
	percent smaller	0.0033	0.0957	0.0043	0.1000
$\bar{H} = 0.11$	period 1	0.0180	0.2787	0.0187	0.2801
	before shutdown	0.0701	0.3173	0.0739	0.3199
	percent bigger	0.5460	0.4807	0.5913	0.5153
	percent smaller	0.0023	0.0677	0.0027	0.0787

The "period 1" row represents the equity premium and Sharpe ratio in the initial period for each specification. The "before shutdown" row represents the average equity premium and Sharpe ratio across simulations in the period immediately before shutdown for each specification. The "percent bigger" and "percent smaller" rows tell how many of the simulated ending values of the equity premium and Sharpe ratio were bigger than or less than, respectively, their initial period values. These percentages do not sum to one because the equity premium and Sharpe ratio do not change in the cases in which the economy shuts down in the second period.

4 Conclusion

In this paper, we have used a stylized model to quantify the degree to which a fiscal transfer program might impose systemic risk on an economy. We estimate the time until a fiscal limit is reached by simulating the model under various assumptions about the severity of regime switch after the fiscal limit. We also calculate various measures of the fiscal gap for each set of assumptions.

Our results show that the expected time until the fiscal limit is reached for our example calibrated to the United States is about 35 years. However, the chance of hitting the fiscal limit within 30 years is more than 30 percent under every assumption.

We also calculated equity premia and Sharpe ratios for every period in every simulation in our models. For the model in which the economy shuts down when the fiscal limit is reached, the equity premium in the first period of the baseline case is about 6 percent and rises slightly as the economy approaches the fiscal limit. As in the channel proposed by Barro (2009), the rare disaster of an economic shutdown produces equity premia that are closer to what is observed in the data without resorting to unreasonably high risk aversion.

In the models in which the fiscal system permanently switches to a proportional tax rate upon reaching the fiscal limit, the equity premia in the first period are much lower at around 2 percent. But they rise dramatically to an average of between 7 and 8 percent in the period immediately before the fiscal limit is reached. This result suggests that the equity premium might be a good indicator of proximity to the fiscal limit.

An obvious extension of this work is to augment the model with agents that live for more than two periods and to perhaps model some other dimensions of heterogeneity within each age cohort such as ability. A more detailed model will give better predictions about the risk of hitting the fiscal limit. However, the computational burden of obtaining a solution will increase exponentially.

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TECHNICAL APPENDIX

T-1 Description of calibration

This section details how we arrived at the calibrated parameter values listed in Table 1. The 30-year discount factor β is set to match the annual discount factor common in the RBC literature of 0.96.

$$\beta = (0.96)^{30}$$

We set the coefficient of relative risk aversion at a midrange value of $\gamma = 2$. This value lies in the midrange of values that have been used in the literature.[13] The capital share of income parameter is set to match the U.S. average $\alpha = 0.35$, and the 30-year depreciation rate δ is set to match an annual depreciation rate of 5 percent.

$$\delta = 1 - (1 - 0.05)^{30}$$

The equilibrium production process in our 2-period model is the following,

$$Y_t = e^{z_t} K_t^{\alpha} \quad \forall t$$

where labor is supplied inelastically and z_t is the aggregate total factor productivity shock. We assume the shock z_t is an AR(1) process with normally distributed errors.

$$\begin{aligned} &z_t = \rho z_{t-1} + (1-\rho)\mu + \varepsilon_t \\ &\quad \text{where} \quad \rho \in [0,1), \quad \mu \geq 0, \quad \text{and} \quad \varepsilon_t \sim N(0,\sigma^2) \end{aligned} \tag{5}$$

This implies that the shock process e^{z_t} is lognormally distributed $LN(0,\sigma^2)$. The RBC literature calibrates the parameters on the shock process (5) to $\rho = 0.95$ and $\sigma = 0.4946$ for annual data.

For data in which one period is 30 years, we have to recalculate the analogous $\tilde{\rho}$ and $\tilde{\sigma}$.

$$\begin{aligned} z_{t+1} &= \rho z_t + (1-\rho)\mu + \varepsilon_{t+1} \\ z_{t+2} &= \rho z_{t+1} + (1-\rho)\mu + \varepsilon_{t+2} \\ &= \rho^2 z_t + \rho(1-\rho)\mu + \rho\varepsilon_{t+1} + (1-\rho)\mu + \varepsilon_{t+2} \\ z_{t+3} &= \rho z_{t+2} + (1-\rho)\mu + \varepsilon_{t+3} \\ &= \rho^3 z_t + \rho^2(1-\rho)\mu + \rho^2\varepsilon_{t+1} + \rho(1-\rho)\mu + \rho\varepsilon_{t+2} + (1-\rho)\mu + \varepsilon_{t+3} \\ &\vdots \\ z_{t+j} &= \rho^j z_t + (1-\rho)\mu \sum_{s=1}^{j} \rho^{j-s} + \sum_{s=1}^{j} \rho^{j-s}\varepsilon_{t+s} \end{aligned}$$

[13] Estimates of the coefficient of relative risk aversion γ mostly lie between 1 and 10. See Mankiw and Zeldes (1991), Blake (1996), Campbell (1996), Kocherlakota (1996), Brav, Constantinides, and Geczy (2002), and Mehra and Prescott (1985).

With one period equal to thirty years $j = 30$, the shock process in our paper should be:

$$z_{t+30} = \rho^{30} z_t + (1-\rho)\mu \sum_{s=1}^{30} \rho^{30-s} + \sum_{s=1}^{30} \rho^{30-s} \varepsilon_{t+s} \tag{T.1.1}$$

Then the persistence parameters in our one-period-equals-thirty-years model should be $\tilde{\rho} = \rho^{30} = 0.2146$. Define $\tilde{\varepsilon}_{t+30} \equiv \sum_{s=1}^{30} \rho^{30-s} \varepsilon_{t+s}$ as the summation term on the right-hand-side of (T.1.1). Then $\tilde{\varepsilon}_{t+30}$ is distributed:

$$\tilde{\varepsilon}_{t+30} \sim N\left(0, \left[\sum_{s=1}^{30} \rho^{2(30-s)}\right] \sigma^2\right)$$

Using this formula, the annual persistence parameter $\rho = 0.95$, and the annual standard deviation parameter $\sigma = 0.4946$, the implied thirty-year standard deviation is $\tilde{\sigma} = 1.5471$. So our shock process should be,

$$z_t = \tilde{\rho} z_{t-1} + (1-\rho)\tilde{\mu} + \tilde{\varepsilon}_t \quad \forall t \quad \text{where} \quad \tilde{\varepsilon} \sim N(0, \tilde{\sigma}^2)$$

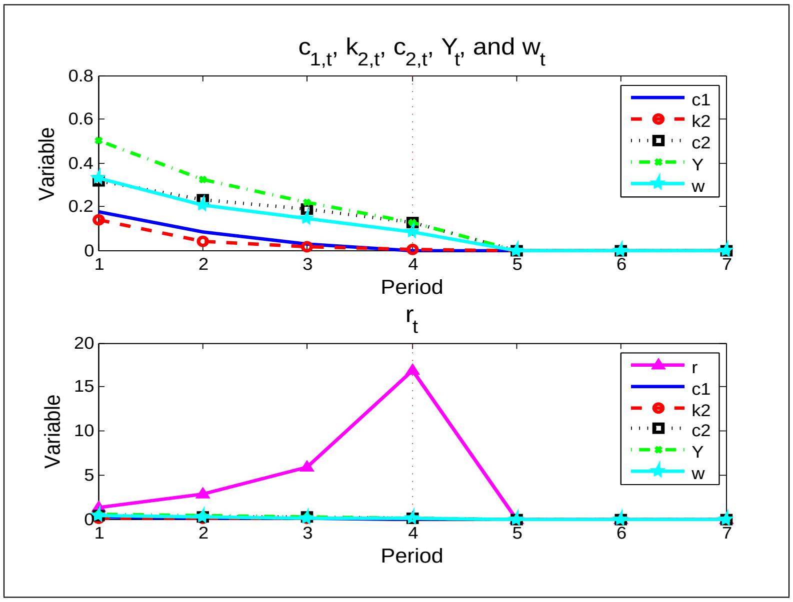
where $\tilde{\rho} = 0.2146$ and $\tilde{\sigma} = 1.5471$. We calibrate μ, and therefore $\tilde{\mu}$, so that the median wage is 50,000.

Lastly, we set the size of the promised transfer $\bar{H}$ to be 32 percent of the median real wage. This level of transfers is meant to approximately match the average per capita real transfers in the United States to the average real wage in recent years. We get the median real wage by simulating a time series of the economy until it hits the shut down point, and we do this for 3,000 simulated time series. We take the median wage from those simulations. In order to reduce the effect of the initial values on the median, we take the simulation that lasted the longest number of periods before shutting down and remove the first 10 percent of the longest simulation's periods from each simulation for the calculation of the median.

T-2 Impulse response functions for all specifications

T-2.1 Shut down model impulse response functions

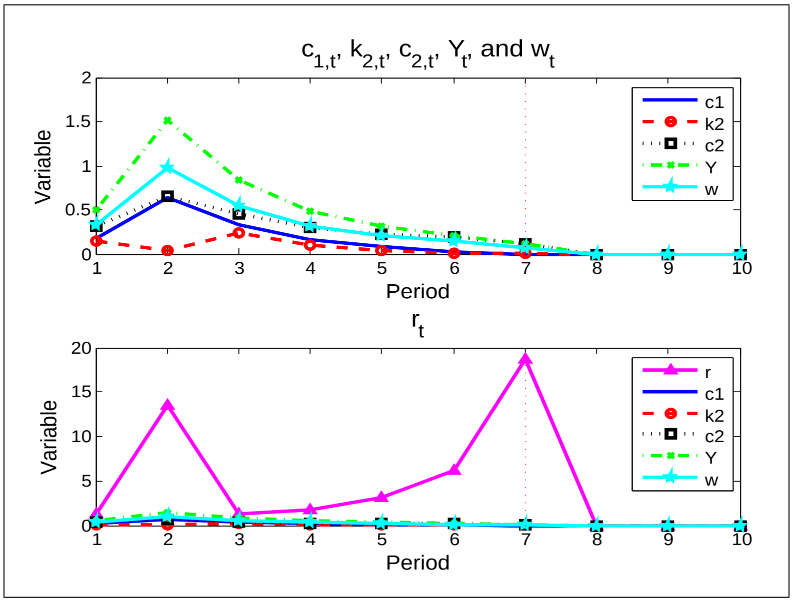
Figure 12: Zero-shock time series of endogenous variables for $\bar{H} = 0.11$ and $k_{2,0} = 0.14$



T-2.2 80-percent wage tax regime switch impulse response functions

T-2.3 30-percent wage tax regime switch impulse response functions

Figure 13: Impulse response function for positive standard deviation productivity shock in period 2: $\bar{H} = 0.11$ and $k_{2,0} = 0.14$



T-3 Derivation of government discount factors

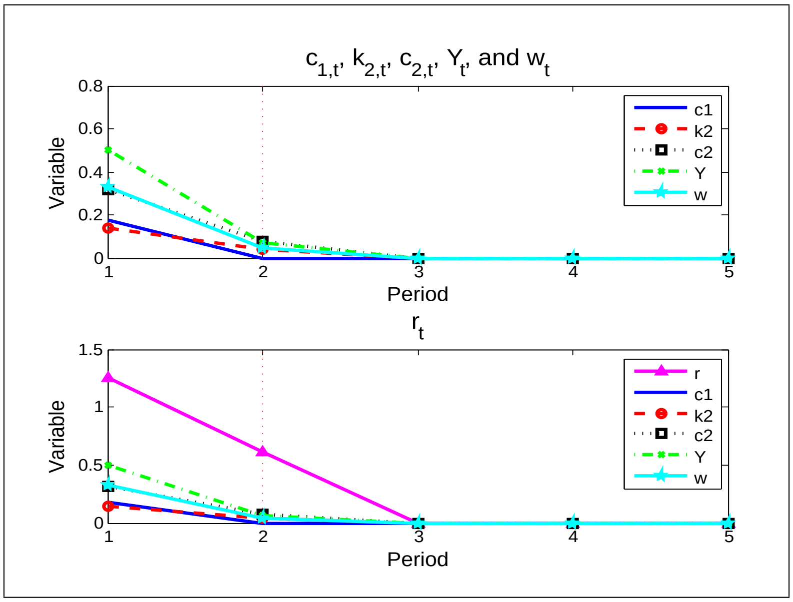
In this section, we derive the discount factors that the government uses in order to calculate the net present value of transfers H_t, promised transfers $\bar{H}$, and output Y_t into the infinite future. The discount factors must be computed separately from the households' discount factor because the life of the policy variables outlasts the life of each household.

Our primary method of computing the discount factors is to use the term structure of period-t prices of assets that give a sure return in the future. Because we are interested in discounting assets with conditional returns $H_t = \min\{w_t, \bar{H}\}$ and Y_t in addition to a sure return $\bar{H}$, we need to define three different assets and three different prices. Define $p_{t,j}$ as the price of an asset $B_{t,j}$ that guarantees a payment of one unit j periods in the future. Define $q_{t,j}$ as the price of an asset $D_{t,j}$ that guarantees a payment of $H_t = \min\{w_t, \bar{H}\}$ units j periods in the future. And define $s_{t,j}$ as the price of an asset $F_{t,j}$ that guarantees a payment of Y_t units j periods in the future. If these assets can be bought and sold each period, then a household could purchase an asset that pays off after the household is dead and sell it before they die.

Because each of these assets must be held in zero net supply, they do not change the equilibrium policy functions described in Section 2. The budget constraints in the households' problem become the following,

$$c_{1,t} + k_{2,t+1} \leq w_t - H_t - \sum_{j=0}^{\infty} p_{t,j} B_{t,j} - \sum_{j=0}^{\infty} q_{t,j} D_{t,j} - \sum_{j=0}^{\infty} s_{t,j} F_{t,j}$$

Figure 14: Impulse response function for negative standard deviation productivity shock in period 2: $\bar{H} = 0.11$ and $k_{2,0} = 0.14$



$$c_{2,t+1} \leq (1 + r_{t+1} - \delta)k_{2,t+1} + H_{t+1} + ...$$
$$\sum_{j=0}^{\infty} p_{t+1,j}B_{t,j+1} + \sum_{j=0}^{\infty} q_{t+1,j}D_{t,j+1} + \sum_{j=0}^{\infty} s_{t+1,j}F_{t,j+1}$$

The equilibrium solutions for the prices on the assets that pay off in the current period are,

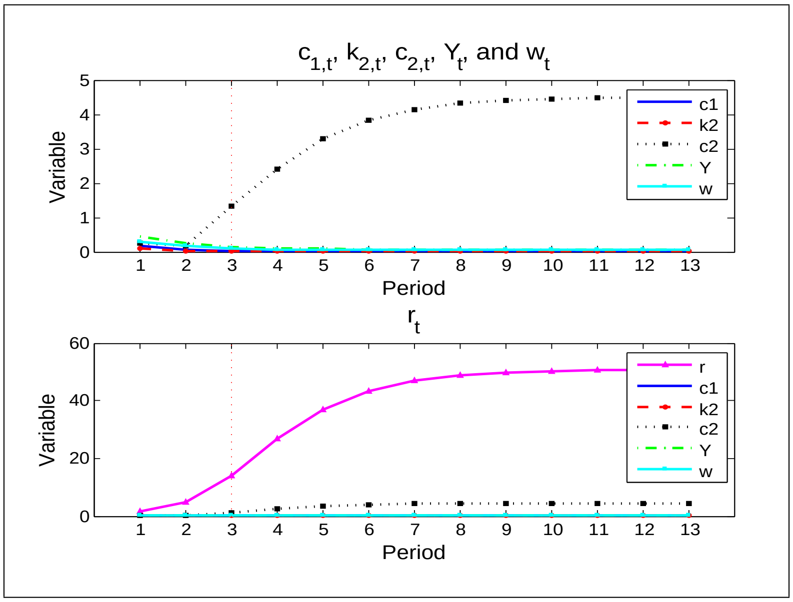
$$p_{t,0} = 1 \tag{T.3.1}$$
$$q_{t,0} = H_t = \min\{w_t, \bar{H}\} \tag{T.3.2}$$
$$s_{t,0} = Y_t \tag{T.3.3}$$

The first order conditions for the households' optimal choices of $B_{t,j}$, $D_{t,j}$, and $F_{t,j}$, for all $j \geq 1$, give the following standard asset pricing Euler equations that pin down the prices $p_{t,j}$, $q_{t,j}$, and $s_{t,j}$ in recursive fashion.

$$p_{t,j} = \beta \frac{E_t\left[u'\left(c_{2,t+1}\right)p_{t+1,j-1}\right]}{u'\left(c_{1,t}\right)} \quad \forall t \quad \text{and} \quad j \geq 1 \tag{T.3.4}$$
$$q_{t,j} = \beta \frac{E_t\left[u'\left(c_{2,t+1}\right)q_{t+1,j-1}\right]}{u'\left(c_{1,t}\right)} \quad \forall t \quad \text{and} \quad j \geq 1 \tag{T.3.5}$$
$$s_{t,j} = \beta \frac{E_t\left[u'\left(c_{2,t+1}\right)s_{t+1,j-1}\right]}{u'\left(c_{1,t}\right)} \quad \forall t \quad \text{and} \quad j \geq 1 \tag{T.3.6}$$

We compute the prices of the sure return assets $p_{t,j}$, $q_{t,j}$, and $s_{t,j}$ by discretizing the state space and then approximating the exact integrals from the right-hand side

Figure 15: Zero-shock time series of endogenous variables for $\bar{H} = 0.09$ and $k_{2,0} = 0.0875$: 80-percent tax



of (T.3.4), (T.3.5), and (T.3.6) for each point in the discretized state space using polynomial interpolation.[14]

i. We choose the nodes in the support of $k_{2,t}$ to be N_k equally spaced points on a log scale between the minimum value recorded in the 2,000 simulations from Section 2.5 and the value of the 95th percentile of the simulations for the maximum. The log scale increases the accuracy of the discretized approximation because most of the realizations of $k_{2,t}$ in the simulations are concentrated in the lower end of the range. We set the number of nodes in the discretized support of $k_{2,t}$ to $N_k = 151$.

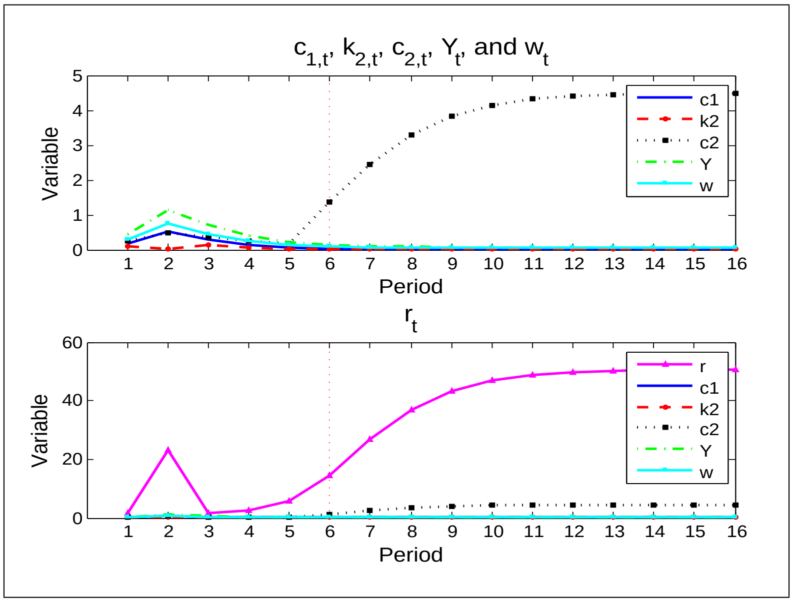
ii. We choose N_z nodes in the support of z_t and calculate a Markov transition matrix for the discretized approximation of z_t using Gaussian quadrature as described in Tauchen and Hussey (1991) and computed using the implementation from Flodén (2008). We set $N_z = 7$.

iii. The next step is to compute the exact solution for all the endogenous objects from Section 2 for all $N_k \times N_z = 1,057$ points in the state space: $c_{1,t}(k_{2,t}, z_t)$, $c_{2,t}(k_{2,t}, z_t)$, $k_{2,t+1}(k_{2,t}, z_t)$, $Y_t(k_{2,t}, z_t)$, $w_t(k_{2,t}, z_t)$, and $r_t(k_{2,t}, z_t)$.

iv. With the solutions for the endogenous objects from step (iii) we can solve for the prices of the assets that mature in the current period $p_{t,0}$, $q_{t,0}$, and $s_{t,0}$ for every value of the discretized state using equations (T.3.1), (T.3.2), and (T.3.3).

[14]The MatLab code for this computation is available upon request.

Figure 16: Impulse response function for positive standard deviation productivity shock in period 2: $\bar{H} = 0.09$, $k_{2,0} = 0.0875$, 80-percent tax



v. Because we can express the one-period-ahead prices $p_{t+1,0}$, $q_{t+1,0}$, and $s_{t+1,0}$ from step (iv) as closed form functions of $k_{2,t+1}$ and z_{t+1}, we can solve exactly for the current period prices of the assets that mature one period from now $p_{t,1}$, $q_{t,1}$, and $s_{t,1}$ using equations (T.3.4), (T.3.5), and (T.3.6).

vi. We solve for the rest of the j-period-ahead prices $p_{t,j}$, $q_{t,j}$, and $s_{t,j}$ recursively from equations (T.3.4), (T.3.5), and (T.3.6) using interpolation on the one-period-ahead version of the price function for the bond that matures in $j-1$ periods.

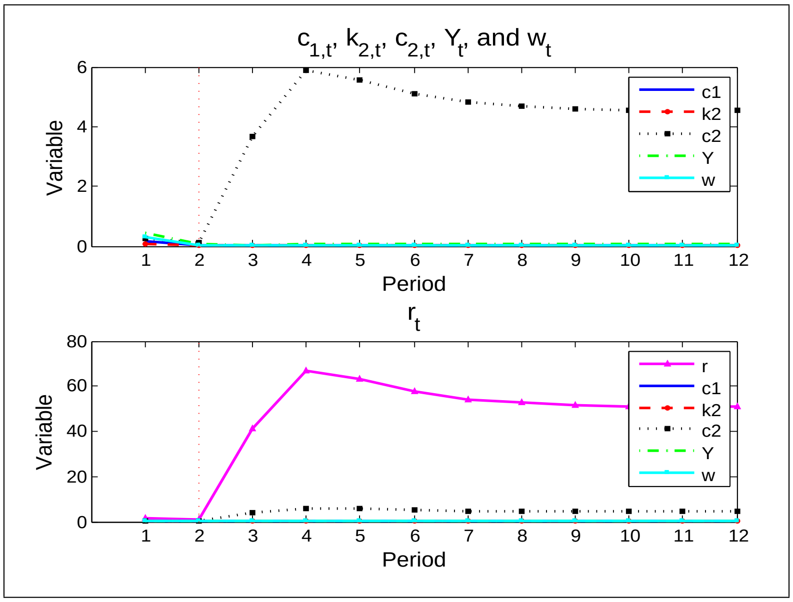
(a) Because we don't have a closed form function for $p_{t+1,j-1}$, $q_{t,j-1}$, and $s_{t,j-1}$ for $j \geq 2$, we cannot compute the exact integral in the numerator of the right-hand-side of equations (T.3.4), (T.3.5), and (T.3.6).

(b) We take a linear interpolation of the discretized functions $p_{t+1,j-1}$, $q_{t,j-1}$, and $s_{t,j-1}$ in the $k_{2,t+1}$ dimension from the policy function $k_{2,t+1}(k_{2,t}, z_t)$.

(c) We then fit a polynomial in z to the pricing functions $p_{t+1,j-1}$, $q_{t,j-1}$, and $s_{t,j-1}$ to match the nonzero nodes in the computed functions in the z_{t+1} dimension. We use a quadratic polynomial approximation for values of $k_{2,t+1}$ that have only three nonzero nodes in the z_{t+1} dimension, and we use a cubic polynomial to approximate all other price functions for a given $k_{2,t+1}$. The price functions are smooth enough that a cubic polynomial is sufficient to closely approximate them.

(d) We then integrate over the closed form solution for the marginal utility

Figure 17: Impulse response function for negative standard deviation productivity shock in period 2: $\bar{H} = 0.09$, $k_{2,0} = 0.0875$, 80-percent tax



of consumption tomorrow and the polynomial approximation for the pricing function $\int_{z_{min>0}}^{z_{max}} \text{Prob}(z_{t+1}|z_t) u'(c_{2,t+1}) p_{t+1.j-1} dz_{t+1}$. We set the upper bound of the support of z_{t+1} over which we integrate equal to the largest node in the discretized support of z because the probability of higher realizations of z is very close to zero. We set the lower bound of the support of z_{t+1} over which we integrate equal to the largest node in the price function that is equal to zero. This is approximately equivalent to integrating over all z for which prices are positive.

vii. We continue recursively computing prices $p_{t,j+1}$, $q_{t,j+1}$, and $s_{t,j+1}$, until they get close to zero. In our case, we compute prices for $j = 0, 1, 2, ...9$. Tables 21 through 29 list the values for the prices $p_{t,j}$ for each maturity of asset for our 9 different calibrations described in Section 2.5.

Table 21: Term structure of prices and interest rates: $\bar{H} = 0.05$, $k_{2,0} = 0.11$

s	$p_{t,t+s}$	$R_{t,t+s}$	$R_{t,t+s}$ APR	$r_{t,t+s}$ APR
0	1	1	1	0
1	1.5556	0.6428	0.9854	-0.0146
2	0.3115	3.2105	1.0196	0.0196
3	0.0385	25.9903	1.0369	0.0369
4	0.0088	113.9341	1.0403	0.0403
5	0.0049	202.6663	1.0360	0.0360
6	0.0014	722.2930	1.0372	0.0372
7	2.8695×10^{-4}	3.4849×10^{3}	1.0396	0.0396
8	1.3004×10^{-4}	7.6900×10^{3}	1.0380	0.0380
9	3.0166×10^{-5}	3.3150×10^{4}	1.0393	0.0393
10	7.6699×10^{-6}	1.3038×10^{5}	1.0400	0.0400
11	2.2726×10^{-6}	4.4003×10^{5}	1.0402	0.0402
12	8.3032×10^{-7}	1.2044×10^{6}	1.0397	0.0397

The gross sure return $R_{t,t+s} = (p_{t,t+s})^{-1}$ is the inverse of the sure return bond price. $R_{t,t+s}$ APR is the annualized gross sure return, where $R_{t,t+s}$ APR $= R_{t,t+s}^{1/s30}$. The net annualized sure return is simply $r_{t,t+s}$ APR $= R_{t,t+s}$ APR $- 1$.

Table 22: Term structure of prices and interest rates: $\bar{H} = 0.05$, $k_{2,0} = 0.14$

s	$p_{t,t+s}$	$R_{t,t+s}$	$R_{t,t+s}$ APR	$r_{t,t+s}$ APR
0	1	1	1	0
1	1.5897	0.6291	0.9847	-0.0153
2	0.3466	2.8853	1.0178	0.0178
3	0.0441	22.6875	1.0353	0.0353
4	0.0096	104.0359	1.0395	0.0395
5	0.0063	159.0087	1.0344	0.0344
6	0.0025	396.0301	1.0338	0.0338
7	6.8826×10^{-4}	1.4529×10^{3}	1.0353	0.0353
8	1.7310×10^{-4}	5.7770×10^{3}	1.0367	0.0367
9	5.1573×10^{-5}	1.9390×10^{4}	1.0372	0.0372
10	1.1606×10^{-5}	8.6162×10^{4}	1.0386	0.0386
11	3.4871×10^{-6}	2.8677×10^{5}	1.0388	0.0388
12	1.0859×10^{-6}	9.2093×10^{5}	1.0389	0.0389

The gross sure return $R_{t,t+s} = (p_{t,t+s})^{-1}$ is the inverse of the sure return bond price. $R_{t,t+s}$ APR is the annualized gross sure return, where $R_{t,t+s}$ APR $= R_{t,t+s}^{1/s30}$. The net annualized sure return is simply $r_{t,t+s}$ APR $= R_{t,t+s}$ APR $- 1$.

Table 23: Term structure of prices and interest rates: $\bar{H} = 0.05$, $k_{2,0} = 0.17$

s	$p_{t,t+s}$	$R_{t,t+s}$	$R_{t,t+s}$ APR	$r_{t,t+s}$ APR
0	1	1	1	0
1	1.6190	0.6177	0.9841	-0.0159
2	0.3782	2.6440	1.0163	0.0163
3	0.0493	20.2780	1.0340	0.0340
4	0.0099	100.0359	1.0392	0.0392
5	0.0063	159.6110	1.0344	0.0344
6	0.0024	423.1373	1.0342	0.0342
7	4.0991×10^{-4}	2.4395×10^{3}	1.0378	0.0378
8	1.7858×10^{-4}	5.5996×10^{3}	1.0366	0.0366
9	4.6981×10^{-5}	2.1285×10^{4}	1.0376	0.0376
10	8.6992×10^{-6}	1.1495×10^{5}	1.0396	0.0396
11	2.7552×10^{-6}	3.6295×10^{5}	1.0396	0.0396
12	1.1390×10^{-6}	8.7793×10^{5}	1.0387	0.0387

The gross sure return $R_{t,t+s} = (p_{t,t+s})^{-1}$ is the inverse of the sure return bond price. $R_{t,t+s}$ APR is the annualized gross sure return, where $R_{t,t+s}$ APR $= R_{t,t+s}^{1/s30}$. The net annualized sure return is simply $r_{t,t+s}$ APR $= R_{t,t+s}$ APR $- 1$.

Table 24: Term structure of prices and interest rates: $\bar{H} = 0.11$, $k_{2,0} = 0.11$

s	$p_{t,t+s}$	$R_{t,t+s}$	$R_{t,t+s}$ APR	$r_{t,t+s}$ APR
0	1	1	1	0
1	1.6771	0.5963	0.9829	-0.0171
2	0.1543	6.4811	1.0316	0.0316
3	0.0074	134.2966	1.0560	0.0560
4	0.0072	138.6856	1.0420	0.0420
5	0.0029	344.5899	1.0397	0.0397
6	4.3310×10^{-4}	2.3089×10^{3}	1.0440	0.0440
7	3.9482×10^{-5}	2.5328×10^{4}	1.0495	0.0495
8	2.7294×10^{-5}	3.6638×10^{4}	1.0448	0.0448
9	9.0193×10^{-6}	1.1087×10^{5}	1.0440	0.0440
10	1.1851×10^{-6}	8.4381×10^{5}	1.0465	0.0465
11	1.3306×10^{-7}	7.5152×10^{6}	1.0491	0.0491
12	9.5400×10^{-8}	1.0482×10^{7}	1.0459	0.0459

The gross sure return $R_{t,t+s} = (p_{t,t+s})^{-1}$ is the inverse of the sure return bond price. $R_{t,t+s}$ APR is the annualized gross sure return, where $R_{t,t+s}$ APR $= R_{t,t+s}^{1/s30}$. The net annualized sure return is simply $r_{t,t+s}$ APR $= R_{t,t+s}$ APR $- 1$.

Table 25: Term structure of prices and interest rates: $\bar{H} = 0.11$, $k_{2,0} = 0.14$

s	$p_{t,t+s}$	$R_{t,t+s}$	$R_{t,t+s}$ APR	$r_{t,t+s}$ APR
0	1	1	1	0
1	1.7186	0.5819	0.9821	-0.0179
2	0.1793	5.5768	1.0291	0.0291
3	0.0092	108.7856	1.0535	0.0535
4	0.0077	129.7630	1.0414	0.0414
5	0.0032	308.9255	1.0390	0.0390
6	5.0106×10^{-4}	1.9958×10^{3}	1.0431	0.0431
7	4.1821×10^{-5}	2.3911×10^{4}	1.0492	0.0492
8	2.8161×10^{-5}	3.5510×10^{4}	1.0446	0.0446
9	1.0005×10^{-5}	9.9946×10^{4}	1.0436	0.0436
10	1.3691×10^{-6}	7.3040×10^{5}	1.0460	0.0460
11	1.2989×10^{-7}	7.6990×10^{6}	1.0492	0.0492
12	1.0361×10^{-7}	9.6515×10^{6}	1.0457	0.0457

The gross sure return $R_{t,t+s} = (p_{t,t+s})^{-1}$ is the inverse of the sure return bond price. $R_{t,t+s}$ APR is the annualized gross sure return, where $R_{t,t+s}$ APR $= R_{t,t+s}^{1/s30}$. The net annualized sure return is simply $r_{t,t+s}$ APR $= R_{t,t+s}$ APR $- 1$.

Table 26: Term structure of prices and interest rates: $\bar{H} = 0.11$, $k_{2,0} = 0.17$

s	$p_{t,t+s}$	$R_{t,t+s}$	$R_{t,t+s}$ APR	$r_{t,t+s}$ APR
0	1	1	1	0
1	1.7673	0.5658	0.9812	-0.0188
2	0.2137	4.6801	1.0261	0.0261
3	0.0118	84.9122	1.0506	0.0506
4	0.0085	117.2211	1.0405	0.0405
5	0.0038	266.5164	1.0379	0.0379
6	5.9449×10^{-4}	1.6821×10^{3}	1.0421	0.0421
7	4.4991×10^{-5}	2.2227×10^{4}	1.0488	0.0488
8	3.3257×10^{-5}	3.0069×10^{4}	1.0439	0.0439
9	1.2022×10^{-5}	8.3183×10^{4}	1.0429	0.0429
10	1.6211×10^{-6}	6.1686×10^{5}	1.0454	0.0454
11	1.4999×10^{-7}	6.6671×10^{6}	1.0488	0.0488
12	1.1393×10^{-7}	8.7771×10^{6}	1.0454	0.0454

The gross sure return $R_{t,t+s} = (p_{t,t+s})^{-1}$ is the inverse of the sure return bond price. $R_{t,t+s}$ APR is the annualized gross sure return, where $R_{t,t+s}$ APR $= R_{t,t+s}^{1/s30}$. The net annualized sure return is simply $r_{t,t+s}$ APR $= R_{t,t+s}$ APR $- 1$.

Table 27: Term structure of prices and interest rates: $\bar{H} = 0.17,\ k_{2,0} = 0.11$

s	$p_{t,t+s}$	$R_{t,t+s}$	$R_{t,t+s}$ APR	$r_{t,t+s}$ APR
0	1	1	1	0
1	1.5848	0.6310	0.9848	-0.0152
2	0.0092	108.2899	1.0812	0.0812
3	0.0010	970.3013	1.0794	0.0794
4	8.9671×10^{-5}	1.1152×10^{4}	1.0808	0.0808
5	1.2850×10^{-5}	7.7820×10^{4}	1.0780	0.0780
6	1.6796×10^{-5}	5.9539×10^{4}	1.0630	0.0630
7	9.4392×10^{-7}	1.0594×10^{6}	1.0683	0.0683
8	1.1858×10^{-7}	8.4330×10^{6}	1.0687	0.0687
9	1.1900×10^{-7}	8.4034×10^{6}	1.0608	0.0608
10	1.1339×10^{-8}	8.8189×10^{7}	1.0629	0.0629
11	1.3094×10^{-9}	7.6368×10^{8}	1.0639	0.0639
12	5.7012×10^{-10}	1.7540×10^{9}	1.0609	0.0609

The gross sure return $R_{t,t+s} = (p_{t,t+s})^{-1}$ is the inverse of the sure return bond price. $R_{t,t+s}$ APR is the annualized gross sure return, where $R_{t,t+s}$ APR $= R_{t,t+s}^{1/s30}$. The net annualized sure return is simply $r_{t,t+s}$ APR $= R_{t,t+s}$ APR$-$ 1.

Table 28: Term structure of prices and interest rates: $\bar{H} = 0.17,\ k_{2,0} = 0.14$

s	$p_{t,t+s}$	$R_{t,t+s}$	$R_{t,t+s}$ APR	$r_{t,t+s}$ APR
0	1	1	1	0
1	1.6811	0.5948	0.9828	-0.0172
2	0.0156	64.0010	1.0718	0.0718
3	0.0031	322.3614	1.0663	0.0663
4	0.0046	217.5026	1.0459	0.0459
5	0.0010	981.4442	1.0470	0.0470
6	5.6471×10^{-5}	1.7708×10^{4}	1.0558	0.0558
7	2.4281×10^{-5}	4.1184×10^{4}	1.0519	0.0519
8	1.0641×10^{-5}	9.3977×10^{4}	1.0489	0.0489
9	8.4137×10^{-7}	1.1885×10^{6}	1.0532	0.0532
10	1.6832×10^{-7}	5.9411×10^{6}	1.0534	0.0534
11	1.0340×10^{-7}	9.6715×10^{6}	1.0499	0.0499
12	8.0409×10^{-9}	1.2436×10^{8}	1.0531	0.0531

The gross sure return $R_{t,t+s} = (p_{t,t+s})^{-1}$ is the inverse of the sure return bond price. $R_{t,t+s}$ APR is the annualized gross sure return, where $R_{t,t+s}$ APR $= R_{t,t+s}^{1/s30}$. The net annualized sure return is simply $r_{t,t+s}$ APR $= R_{t,t+s}$ APR $- 1$.

Figure 18: Zero-shock time series of endogenous variables for $\bar{H} = 0.09$ and $k_{2,0} = 0.0875$: 30-percent tax

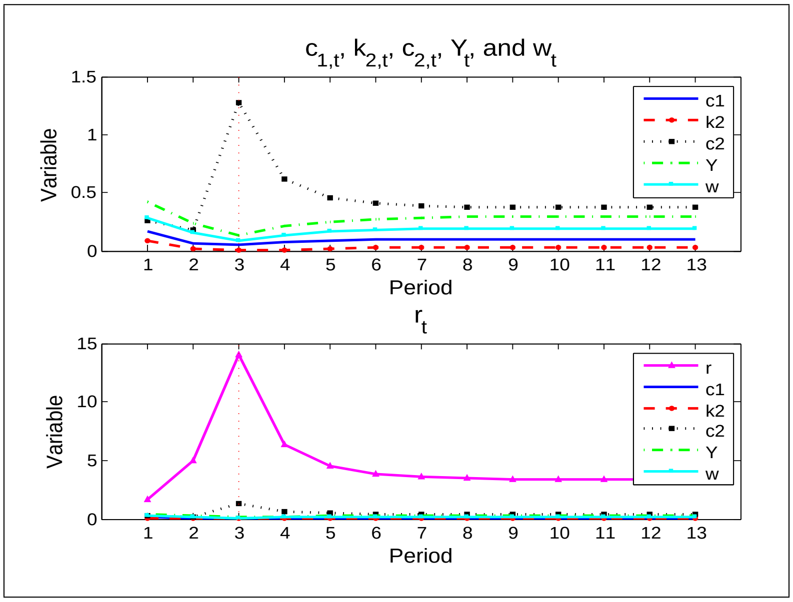


Figure 19: Impulse response function for positive standard deviation productivity shock in period 2: $\bar{H} = 0.09$, $k_{2,0} = 0.0875$, 30-percent tax

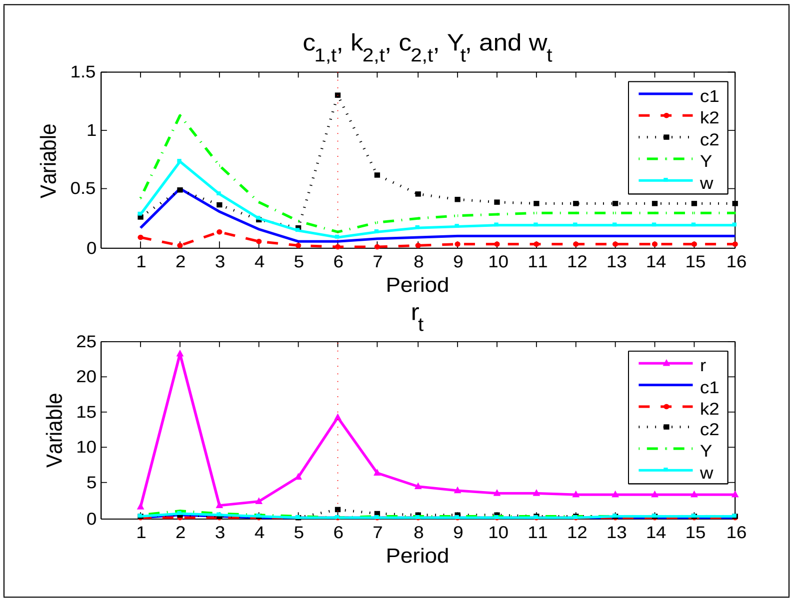


Figure 20: **Impulse response function for negative standard deviation productivity shock in period 2: $\bar{H} = 0.09$, $k_{2,0} = 0.0875$, 30-percent tax**

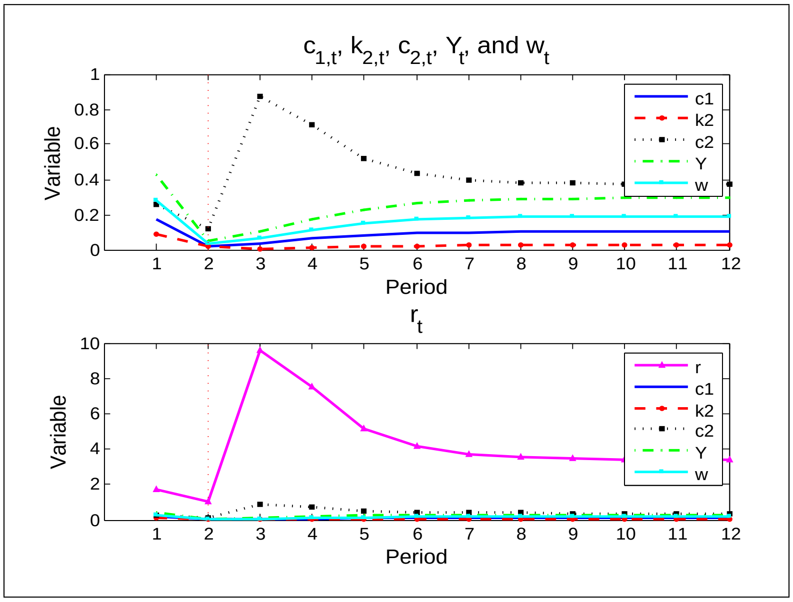


Table 29: **Term structure of prices and interest rates: $\bar{H} = 0.17$, $k_{2,0} = 0.17$**

s	$p_{t,t+s}$	$R_{t,t+s}$	$R_{t,t+s}$ APR	$r_{t,t+s}$ APR
0	1	1	1	0
1	1.7308	0.5778	0.9819	-0.0181
2	0.0359	27.8392	1.0570	0.0570
3	0.0038	263.0105	1.0639	0.0639
4	0.0049	203.9569	1.0453	0.0453
5	0.0011	890.2539	1.0463	0.0463
6	6.0795×10^{-5}	1.6449×10^{4}	1.0554	0.0554
7	2.5424×10^{-5}	3.9332×10^{4}	1.0517	0.0517
8	1.1716×10^{-5}	8.5355×10^{4}	1.0484	0.0484
9	9.5619×10^{-7}	1.0458×10^{6}	1.0527	0.0527
10	1.6125×10^{-7}	6.2016×10^{6}	1.0535	0.0535
11	1.1130×10^{-7}	8.9845×10^{6}	1.0497	0.0497
12	1.4073×10^{-8}	7.1056×10^{7}	1.0515	0.0515

The gross sure return $R_{t,t+s} = (p_{t,t+s})^{-1}$ is the inverse of the sure return bond price. $R_{t,t+s}$ APR is the annualized gross sure return, where $R_{t,t+s}$ APR $= R_{t,t+s}^{1/s30}$. The net annualized sure return is simply $r_{t,t+s}$ APR $= R_{t,t+s}$ APR $- 1$.

Tables 30 through 33 represent the term structure of prices and interest rates for the four combinations of $\bar{H}$ and $k_{2,0}$ from Section ??.

Table 30: Term structure of prices and interest rates with regime switching: $\bar{H} = 0.110$, $k_{2,0} = 0.14$

s	$p_{t,t+s}$	$R_{t,t+s}$	$R_{t,t+s}$ APR	$r_{t,t+s}$ APR
0	1	1	1	0
1	1.9094	0.5237	0.9787	-0.0213
2	0.2084	4.7994	1.0265	0.0265
3	0.1759	5.6849	1.0195	0.0195
4	0.0485	20.6398	1.0255	0.0255
5	0.0081	124.0635	1.0327	0.0327
6	0.0078	128.3248	1.0273	0.0273
7	0.0029	347.5132	1.0283	0.0283
8	2.1955×10^{-4}	4.5548×10^{3}	1.0357	0.0357
9	1.3069×10^{-4}	7.6517×10^{3}	1.0337	0.0337
10	1.0594×10^{-4}	9.4391×10^{3}	1.0310	0.0310
11	9.4124×10^{-6}	1.0624×10^{5}	1.0357	0.0357
12	5.6738×10^{-6}	1.7625×10^{5}	1.0341	0.0341

The gross sure return $R_{t,t+s} = (p_{t,t+s})^{-1}$ is the inverse of the sure return bond price. $R_{t,t+s}$ APR is the annualized gross sure return, where $R_{t,t+s}$ APR $= R_{t,t+s}^{1/s30}$. The net annualized sure return is simply $r_{t,t+s}$ APR $= R_{t,t+s}$ APR $- 1$.

Table 31: Term structure of prices and interest rates with regime switching: $\bar{H} = 0.110$, $k_{2,0} = 0.18$

s	$p_{t,t+s}$	$R_{t,t+s}$	$R_{t,t+s}$ APR	$r_{t,t+s}$ APR
0	1	1	1	0
1	1.9606	0.5100	0.9778	-0.0222
2	0.2387	4.1888	1.0242	0.0242
3	0.2781	3.5956	1.0143	0.0143
4	0.0426	23.4731	1.0266	0.0266
5	0.0084	118.6004	1.0324	0.0324
6	0.0074	135.8980	1.0277	0.0277
7	0.0029	340.8212	1.0282	0.0282
8	2.8337×10^{-4}	3.5290×10^{3}	1.0346	0.0346
9	2.6734×10^{-4}	3.7405×10^{3}	1.0309	0.0309
10	1.0882×10^{-4}	9.1897×10^{3}	1.0309	0.0309
11	9.4293×10^{-6}	1.0605×10^{5}	1.0357	0.0357
12	8.8712×10^{-6}	1.1272×10^{5}	1.0328	0.0328

The gross sure return $R_{t,t+s} = (p_{t,t+s})^{-1}$ is the inverse of the sure return bond price. $R_{t,t+s}$ APR is the annualized gross sure return, where $R_{t,t+s}$ APR $= R_{t,t+s}^{1/s30}$. The net annualized sure return is simply $r_{t,t+s}$ APR $= R_{t,t+s}$ APR $- 1$.

Table 32: Term structure of prices and interest rates with regime switching: $\bar{H} = 0.119$, $k_{2,0} = 0.14$

s	$p_{t,t+s}$	$R_{t,t+s}$	$R_{t,t+s}$ APR	$r_{t,t+s}$ APR
0	1	1	1	0
1	1.8979	0.5269	0.9789	-0.0211
2	0.1901	5.2608	1.0281	0.0281
3	0.0670	14.9250	1.0305	0.0305
4	0.0458	21.8314	1.0260	0.0260
5	0.0135	74.2269	1.0291	0.0291
6	0.0062	161.2355	1.0286	0.0286
7	9.7773×10^{-4}	1.0228×10^{3}	1.0336	0.0336
8	2.5035×10^{-4}	3.9943×10^{3}	1.0352	0.0352
9	1.9703×10^{-4}	5.0754×10^{3}	1.0321	0.0321
10	6.6588×10^{-5}	1.5018×10^{4}	1.0326	0.0326
11	6.1179×10^{-6}	1.6345×10^{5}	1.0370	0.0370
12	7.0348×10^{-6}	1.4215×10^{5}	1.0335	0.0335

The gross sure return $R_{t,t+s} = (p_{t,t+s})^{-1}$ is the inverse of the sure return bond price. $R_{t,t+s}$ APR is the annualized gross sure return, where $R_{t,t+s}$ APR $= R_{t,t+s}^{1/s30}$. The net annualized sure return is simply $r_{t,t+s}$ APR $= R_{t,t+s}$ APR $- 1$.

Table 33: Term structure of prices and interest rates with regime switching: $\bar{H} = 0.119$, $k_{2,0} = 0.18$

s	$p_{t,t+s}$	$R_{t,t+s}$	$R_{t,t+s}$ APR	$r_{t,t+s}$ APR
0	1	1	1	0
1	1.9561	0.5112	0.9779	-0.0221
2	0.2195	4.5568	1.0256	0.0256
3	0.0850	11.7710	1.0278	0.0278
4	0.0463	21.6143	1.0259	0.0259
5	0.0109	91.8687	1.0306	0.0306
6	0.0073	136.5303	1.0277	0.0277
7	0.0012	856.1382	1.0327	0.0327
8	3.6734×10^{-4}	2.7223×10^{3}	1.0335	0.0335
9	2.1539×10^{-4}	4.6428×10^{3}	1.0318	0.0318
10	5.6401×10^{-5}	1.7730×10^{4}	1.0331	0.0331
11	8.6027×10^{-6}	1.1624×10^{5}	1.0360	0.0360
12	7.4281×10^{-6}	1.3462×10^{5}	1.0334	0.0334

The gross sure return $R_{t,t+s} = (p_{t,t+s})^{-1}$ is the inverse of the sure return bond price. $R_{t,t+s}$ APR is the annualized gross sure return, where $R_{t,t+s}$ APR $= R_{t,t+s}^{1/s30}$. The net annualized sure return is simply $r_{t,t+s}$ APR $= R_{t,t+s}$ APR $- 1$.

T-4 Policy functions of equilibrium objects

Figure 21 shows the policy functions for the equilibrium objects $c_{1,t}$, $c_{2,t}$, $k_{2,t+1}$, Y_t, w_t, and r_t in terms of the state $(k_{2,t}, z_t)$ from Section 2.

Figure 21: Equilibrium policy functions

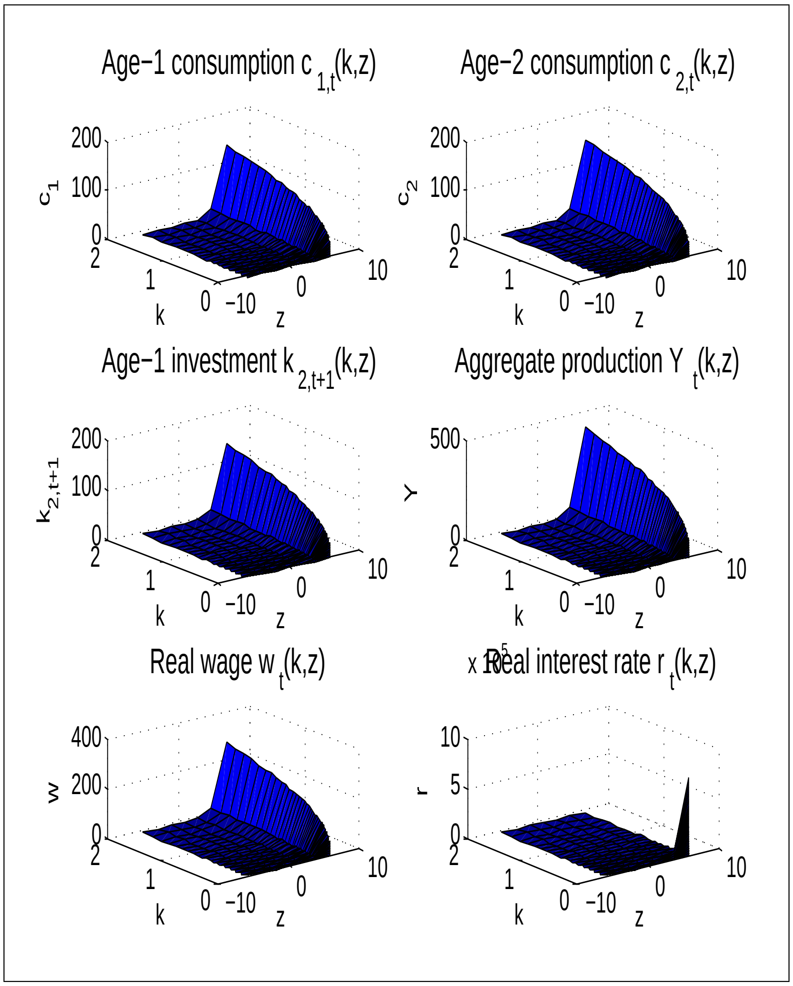
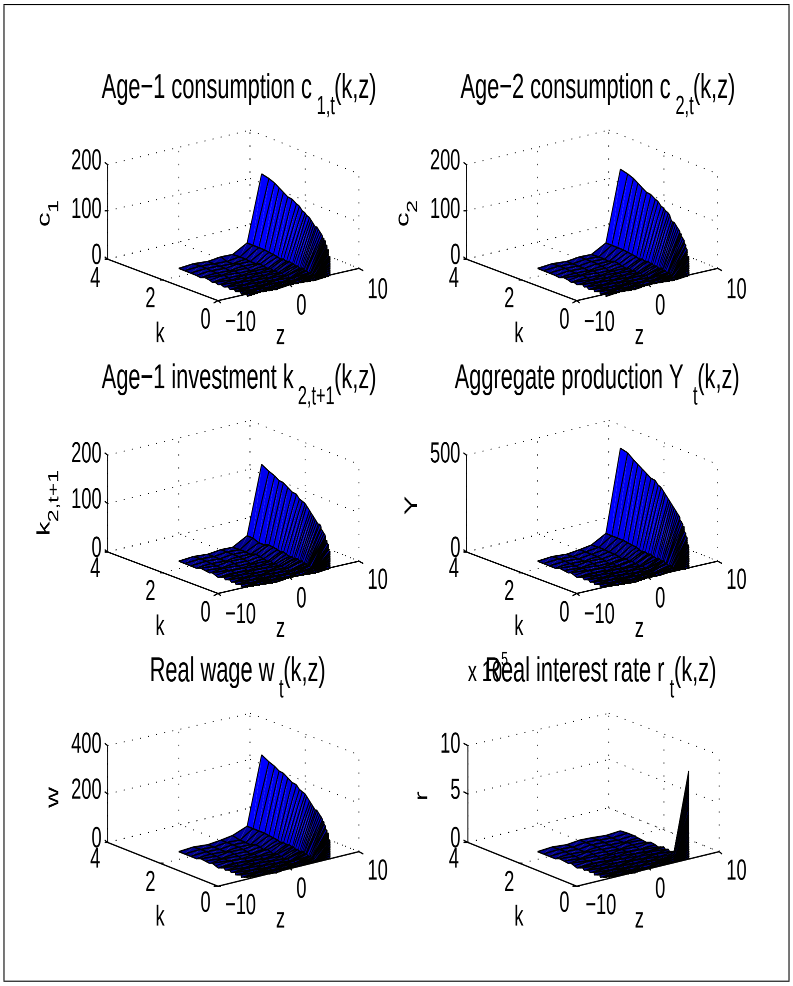


Figure 22 shows the policy functions for the equilibrium objects $c_{1,t}$, $c_{2,t}$, $k_{2,t+1}$, Y_t, w_t, and r_t in terms of the state $(k_{2,t}, z_t)$ from Section 3.

Figure 22: Equilibrium policy functions with regime switching



T-5 Equilibrium Euler equation for model with regime change

In this section we report the equilibrium equations for the model with regime change in which H_t is given by equation (18) from Section 3. If a period begins in the constant transfer regime $w_t > \bar{H}$ and $H_t = \bar{H}$, then the only difference from the model in Section 2 is that the young household's consumption and savings decision reflects the new possibility in expectation that next period's transfer could be $0.8w_{t+1}$ rather than $\bar{H}$,

$$u'\big(c_{1,t}\big) = \beta E_{z_{t+1}|z_t}\Bigg[\bigg(1 + \alpha e^{z_{t+1}}\big[(1-\alpha)e^{z_t}k_{2,t}^{\alpha} - \bar{H} - c_{1,t}\big]^{\alpha-1} - \delta\bigg) \times ...$$

$$u'\Bigg(\bigg[1 + \alpha e^{z_{t+1}}\big([1-\alpha]e^{z_t}k_{2,t}^{\alpha} - \bar{H} - c_{1,t}\big)^{\alpha-1} - \delta\bigg]\big([1-\alpha]e^{z_t}k_{2,t}^{\alpha} - \bar{H} - c_{1,t}\big) + H_{t+1}\Bigg)\Bigg] \tag{T.5.1}$$

where H_{t+1} is defined by (18). The only difference between equation (T.5.1) and equation (12) is the definition of the last term representing H_{t+1} and its implication on expectations.