Fiscal Devaluations*

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Abstract

We show that even when the exchange rate cannot be devalued, a small set of conventional fiscal instruments can robustly replicate the real allocations attained under a nominal exchange rate devaluation in a standard New Keynesian open economy environment. We perform the analysis under alternative pricing assumptions—producer or local currency pricing, along with nominal wage stickiness; under alternative asset market structures, and for anticipated and unanticipated devaluations. There are two types of fiscal policies equivalent to an exchange rate devaluation—one, a uniform increase in import tariff and export subsidy, and two, an increase in value-added tax and a uniform reduction in payroll tax. When the devaluations are anticipated, these policies need to be supplemented with a reduction in consumption tax and an increase in income taxes. These policies have zero impact on fiscal revenues. In certain cases equivalence requires in addition a partial default on foreign bond holders. We discuss the issues of implementation of these policies, in particular, under the circumstances of a currency union.

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1 Introduction

Exchange rate devaluations have long been proposed as a desirable policy response to macroeconomic shocks that impair a country’s competitiveness in the presence of price and wage rigidities. Milton Friedman famously argued for flexible exchange rates on these grounds. Yet, almost by definition, countries that wish to or have to maintain a fixed exchange rate (for instance, because they belong to a currency union) cannot resort to exchange rate devaluations. The question then is: How can fiscal policy be used to mimic an exchange rate devaluation, that is, generate the same real outcomes as those following a nominal exchange rate devaluation, while keeping the nominal exchange rate fixed?

This question about fiscal devaluations dates back to the period of the gold standard when countries could not devalue their currencies. At that time, Keynes (1931) had conjectured that a uniform ad valorem tariff on all imports plus a uniform subsidy on all exports would have the same impact as an exchange rate devaluation. The current crisis in the Euro area has once again brought fiscal devaluations to the forefront of policy. The Euro has been blamed for the inability of countries like Greece, Portugal, Spain and Ireland to devalue their exchange rates and restore their competitiveness in international markets. These statements are sometimes accompanied with calls for countries to exit the Euro. At the same time suggestions have been made to raise competitiveness through fiscal devaluations using value-added taxes and payroll subsidies. Despite discussions in policy circles, there is little formal analysis of the equivalence between fiscal devaluations and exchange rate devaluations. This is an area where the policy debate is ahead of academic knowledge. This paper is intended to bridge this gap, by providing a complete analysis of fiscal devaluations in a workhorse dynamic stochastic general equilibrium New Keynesian open economy model.

We define a fiscal devaluation of size $\delta_t$ at date $t$ to be a set of fiscal policies that implements the same real (consumption, output, labor supply) allocation as under a nominal exchange rate devaluation of size $\delta_t$, but holding the nominal exchange rate fixed. We explore a general path of $\delta_t$, including both expected and unexpected devaluations. Since the nature of price rigidity—whether prices are set in the currency of the producers or in local currency—is central for the real effects of nominal devaluations (see, for example, Lane, 2001; Corsetti, 2008), we study separately the case of producer (PCP) and local currency pricing (LCP), allowing in both cases for nominal wage rigidity. Additionally, we allow for different international asset market structures, including balanced trade, complete markets, and incomplete markets with international trade in risk-free nominal bonds and equities.

A main finding is that, despite the differences in allocations that accompany these various specifications, there exists a small set of fiscal instruments that can robustly replicate the effects of nominal exchange rate devaluations across all specifications. The

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exact details of which instruments need to be used depend on the extent of completeness of asset markets, the currency denomination of bonds and the expected or unexpected nature of devaluations.

There are two types of fiscal policies that generate fiscal devaluations. The first policy involves a uniform increase in import tariffs and export subsidies. The second policy involves a uniform increase in value-added taxes and a reduction in payroll taxes (e.g., social security contributions). Both of these policies, in general, need to be accompanied by a uniform reduction in consumption taxes and an increase in income taxes. However, under some circumstances, changes in consumption and income taxes can be dispensed with. When this latter option is possible depends on the extent of completeness of asset markets and whether the exchange rate movements that are being mimicked are anticipated or unanticipated.

To provide intuition for the mechanisms at play behind fiscal devaluations consider the case of producer currency pricing (PCP). The main channel through which a nominal devaluation raises relative output at home is through a deterioration of home’s terms of trade that makes home goods less expensive relative to foreign goods. For given producer prices, this movement in the terms of trade can be mimicked either through an import tariff and an export subsidy, or through an increase in the value-added tax (which is reimbursed to exporters and levied on importers). Additionally, to ensure that producer prices at home are the same as under a nominal devaluation, an increase in the value-added tax needs to be offset with a reduction of the payroll tax.

When is an reduction in consumption taxes and increase in income taxes required? Without a reduction in consumption taxes, fiscal devaluations result in an appreciated real exchange rate relative to a nominal devaluation. This is because fiscal devaluations, despite having the same effect on international relative prices by making home exports cheaper relative to foreign exports, lead to an increase in the price of home’s consumption bundle relative to that of foreign due to a tax on imports—an effect absent under nominal devaluations. This difference is of no consequence for the real allocation when trade is balanced or when countries can trade only risk-free bonds and the devaluation is unexpected, as neither risk-sharing nor saving decisions are affected under these circumstances. As a result, precisely in these two cases, we can dispense with the adjustment in consumption and income taxes. By contrast, with expected devaluations, in the absence of an adjustment in consumption and income taxes, the different behavior of the real exchange rate under nominal and fiscal devaluations induces different savings and portfolio decisions. These effects then need to be undone with a reduction in consumption taxes and an offsetting increase in income taxes. This allows to fully mimic the behavior of the real exchange rate under a nominal devaluation.

In the case of incomplete markets we highlight the role of the currency denomination of debt. When bonds are denominated in the foreign currency, no additional instruments are required for a fiscal devaluation. However, an important case when the proposed set of tax instruments does not suffice, is when international bonds are denominated

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3Consumption tax is equivalent to a sales tax that is applied only to final goods, and not to intermediate goods. In our setup all goods are final, and hence consumption and sales taxes are always equivalent. Further, under tariff-based policy, an increase in income tax should extend to both wage income and dividend income, while under VAT-based policy dividend-income tax should be left unchanged.
in the home currency. In this case equivalence requires a partial default by the home country. Specifically, a nominal devaluation depletes the foreign-currency value of home’s external debt if it was denominated in home currency. The proposed limited set of fiscal instruments cannot replicate this effect on home’s foreign obligations. This is why a fiscal devaluation under these circumstances must be accompanied by a partial default on home-currency debt of the home country.

We emphasize that the proposed fiscal devaluation policies are robust across a number of environments, despite the fact that the actual allocations induced by devaluations are sensitive to the details of the environment. Specifically, for a given asset market structure, fiscal devaluations work robustly independently of the degree of wage and price stickiness, and of the type of pricing—whether local or producer currency. Importantly, they are also revenue neutral.

In the case where production involves the use of capital, in addition to labor, the fiscal intervention that uses VAT tax increases and payroll subsidies needs to be extended to include a capital subsidy to firms. The capital subsidy is required because without it firms will have an incentive to substitute labor for capital in production under a fiscal devaluation given the payroll subsidy—an effect absent in a nominal devaluation. In the case of a one time unexpected devaluation, where a consumption subsidy is not used, this is the only additional instrument required. If however, the devaluation is anticipated and there is some risk-sharing then the consumption subsidy is required. In this case, to offset any distortions to households investment decisions an investment tax credit is required and the income tax should be extended to cover capital income (besides labor income).4

The equivalence of a δt VAT plus payroll intervention to a δt nominal devaluation relies on symmetry of pass-through of VAT increases and payroll subsidies into producer prices. In the absence of symmetry, for instance if pass-through from VAT exceeds pass-through from payroll, a δt nominal devaluation will require a larger fiscal devaluation.

The outline of the paper is as follows. Section 2 evaluates equivalence in a stylized static environment that is useful to convey the intuition for what follows in the dynamic analysis. Section 3 extends the results to a fully-fledged dynamic environment that features endogenous saving and portfolio choice decisions under a variety of asset market structures, as well as an interest-elastic money demand. Section 4 provides a numerical illustration of the equilibrium dynamics under nominal and fiscal devaluations against that under fixed exchange rate and passive fiscal policy. For our numerical example, we consider a small open economy with sticky wages and flexible prices, hit by a negative productivity shock. Under these circumstances, devaluation is the optimal policy, replicating the flexible-price, flexible-wage allocation. Lastly, Section 5 discusses the results with the emphasis on the issues of implementation of fiscal devaluations, in particular in the context of a currency union.

**Related literature.** Our paper contributes to an old literature, both positive and normative, that analyzes how to replicate the effects of exchange rate devaluations with fiscal instruments. The tariff-cum-export subsidy and the VAT increase-cum-payroll tax reduction are intuitive fiscal policies to replicate the effects of a nominal devaluations on

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4In the case where intervention involves an import tariff and export subsidy a similar extension of tax instruments is required. The capital subsidy is however never required.
international relative prices, and accordingly have been discussed before in the policy and academic literature. Most recently, Staiger and Sykes (2010) explore the equivalence using import tariffs and export subsidies in a partial equilibrium static environment with sticky or flexible prices, and under balanced trade. While the equivalence between a uniform tariff-cum-subsidy and a devaluation has a long tradition in the literature (as surveyed in Staiger and Sykes, 2010), most of the earlier analyses was conducted in static endowment economies (or with fixed labor supply). Berglas (1974) provides an equivalence argument for nominal devaluations, using VAT and tariff-cum-export-subsidy policies. The analysis is performed using a reduced-form model without micro-foundations, no labor supply and without specifying the nature of asset markets. Our departure from this literature is to perform a dynamic general equilibrium analysis with varying degrees of price rigidity, alternative asset market assumptions and for expected and unexpected devaluations. In contrast to the earlier literature, we allow for dynamic price setting as in the New Keynesian literature, endogenous labor supply, savings and portfolio choice decisions, as well as interest-elastic money demand. In doing so, we learn that the tariff-cum-subsidy and VAT-cum-payroll fiscal interventions do not generally suffice to attain equivalence. In the general case, additional tax instruments such as consumption taxes, income taxes or partial default are required. Moreover, some of the conclusions regarding which tax instruments suffice (such as import tariff only for local currency pricing as discussed in Staiger and Sykes, 2010) do not carry through in our more general environment. Furthermore, and this is more surprising, despite the different allocations being mimicked under different circumstances and a rich set of endogenous margins of adjustment, the additional instruments required are few in number. In other words, we find that a small number of instruments is all that is required to robustly implement fiscal devaluation under the fairly rich set of specifications we explore.

This paper is complementary to Adao, Correia, and Teles (2009) who show that any equilibrium allocation in the flexible price, flexible exchange rate economy can be implemented with fiscal and monetary policies that induce stable producer prices and constant exchange rates. Since the optimal policy is sensitive to details of the environment the fiscal instruments used will vary across environments and in general will require flexibly time varying taxes. Our approach is different and closer in focus to the previously discussed papers. We analyze simple fiscal policies that robustly replicate the effects of nominal devaluations across a wide class of environments, regardless of whether or not nominal devaluations exactly replicate flexible price allocations. We perform the analysis in a more general environment, with different types of price and wage stickiness, under a rich array of asset market structures, and for expected and unanticipated devaluations. An attractive feature of our findings is that the fiscal adjustments that are necessary to replicate nominal devaluations are to a large extent not dependent on the details of the environment.

Another important difference with Adao, Correia, and Teles (2009) lies in the set of fiscal instruments that we consider. First, their implementation requires time varying

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5The VAT policy with border adjustment has been the focus of Grossman (1980) and Feldstein and Krugman (1990), however, in an environment with flexible exchange rates and prices. Calmfors (1998) provides a policy discussion of the potential role of VAT and payroll taxes in impacting allocations in a currency union.
taxes both at Home and in Foreign. By contrast, ours requires only adjusting taxes at Home. This is an important advantage since it can be implemented unilaterally. Second, their implementation relies on income taxes and differential consumption taxes for local vs. imported goods. These taxes are typically not available. By contrast our implementation uses payroll and VAT taxes—tax instruments that have been proposed as potential candidates in policy circles\textsuperscript{6}.

This paper is also related to Lipińska and von Thadden (2009) and Franco (2011) who quantitatively evaluate the effects of a tax swap from direct (payroll) taxes to indirect taxes (VAT) under a fixed exchange rate.\textsuperscript{7} Neither of these studies however explores theoretical equivalence with a nominal devaluation, as we do in this paper. Another recent paper by Jeanne (2011) focuses on the ability of a country to generate a long-run real depreciation via controls on capital inflows in an endowment model without money. Our paper instead focuses on replicating both short-run and long-run effects of a nominal devaluation in a sticky price production economy. Lastly, the style of this paper is similar to Correia, Farhi, Nicolini, and Teles (2011) who, building on the general implementation results of Correia, Nicolini, and Teles (2008), use fiscal instruments to replicate the effects of the optimal monetary policy when the zero-lower bound on nominal interest rate is binding.

2 Static Analysis

We start with a one-period (static) setup with an arbitrary degree of price and wage flexibility, and consider in turn the cases of producer and local currency price setting. The model features two countries—Home (H) and Foreign (F). Foreign follows a passive policy of fixed money supply \(M^*\), while Home in addition to the money supply \(M\) can potentially use six different fiscal instruments—import and export tariffs, a value-added tax (with border adjustment),\textsuperscript{8} a payroll tax paid by the producers, and consumption and income taxes paid by the consumers. We consider various degrees of capital account openness—financial autarky (balanced trade), complete risk sharing with Arrow-Debreu securities, and an arbitrary net foreign asset position of the countries, where the last case takes the first two as special cases and allows us to study the valuation effects associated with devaluations.

Our central result is that there are two types of fiscal policies that can attain the same effects as a nominal devaluation while at the same time maintaining a fixed nominal exchange rate. We call these policies \textit{fiscal devaluations}. The first policy involves an increase in import tariff coupled with an equivalent increase in export subsidy. The other policy involves an increase in the value-added tax coupled with an equivalent reduction in the payroll tax. Under balanced trade no other fiscal instrument is needed. Outside of the case of balanced trade, adjustments in consumption and income taxes are generally needed. An expansion in the home money supply may or may not be needed in addition

\textsuperscript{6}See IMF Staff (2011).
\textsuperscript{7}Other quantitative analysis includes Boscam, Diaz, Domenech, Ferri, Perez, and Puch (2011) for Spain.
\textsuperscript{8}A VAT with border adjustment is reimbursed to the exporters and levied on the importers.
The following is important for the reader to keep in mind. In our formal analysis, we start from a situation where taxes are zero. This assumption is made only for simplicity and ease of exposition. Our results generalize straightforwardly to a situation where initial taxes are not zero. Indeed, our results characterize the changes in taxes that are necessary to engineer a fiscal devaluation. For example, a payroll subsidy should be interpreted as a reduction in payroll taxes if the economy starts in a situation where payroll taxes are positive. Similarly, a VAT should be interpreted as an increase in the VAT if the economy starts in a situation with a positive VAT.

2.1 General setup

Our static model features two countries and two goods, one produced at home and the other produced at foreign. Goods are produced from labor using a linear technology with productivity $A$ and $A^*$ respectively.

Consumers derive utility from both goods and disutility from labor. For simplicity, we assume that consumption and labor enter separably in the utility function, and that utility has a constant risk aversion $\sigma$ and a constant inverse Frisch elasticity $\varphi$:

$$U = \frac{1}{1-\sigma}C^{1-\sigma} - \frac{\kappa}{1+\varphi}N^{1+\varphi},$$

where $\kappa$ is the disutility of labor parameter. Our results generalize straightforwardly to a general non-separable utility function $U(C, N)$. Again for simplicity we focus on a Cobb-Douglas aggregator, but this can be easily extended to a CES aggregator and multiple home and foreign varieties, as we do in the dynamic analysis. We allow for home bias in preferences and denote by $\gamma$ the share of domestic goods in consumption expenditure in each country.\(^9\) Specifically, we have

$$C = C_H^\gamma C_F^{1-\gamma} \quad \text{and} \quad C^* = C_H^{*1-\gamma} C_F^{*\gamma},$$

where $C$ and $C^*$ are home and foreign aggregate consumption respectively. The associated price indexes are

$$P = \left(\frac{P_H}{\gamma}\right)^\gamma \left(\frac{P_F}{1-\gamma}\right)^{1-\gamma} \quad \text{and} \quad P^* = \left(\frac{P_H^*}{1-\gamma}\right)^{1-\gamma} \left(\frac{P_F^*}{\gamma}\right)^\gamma.$$ 

Here $P_H$ and $P_F$ are home-currency prices of the two goods before the consumption tax, but inclusive of the value-added tax and tariffs. Similarly, starred prices are foreign-currency prices of the two goods. Since these price indexes do not incorporate the consumption tax, they must be adjusted for the consumption tax in order to obtain the consumer prices of the home and foreign consumption baskets.

\(^9\)Our analysis immediately generalizes to the setup with a non-tradable good, as long as the same taxes apply to the non-tradable-good producers as to the producers of the home tradable good. Empirically, a relevant case may be when VAT does not apply to certain non-tradable industries, such as construction and housing. In this case, fiscal devaluations would simply require reducing the payroll tax only for the industries which face an increase in VAT.
With Cobb-Douglas preference aggregators, we can write the market clearing conditions for the two goods in the following way:

\[ Y = \gamma \frac{PC}{P_H} + (1 - \gamma) \frac{P^*C^*}{P^*_H} \quad \text{and} \quad Y^* = (1 - \gamma) \frac{PC}{P_F} + \gamma \frac{P^*C^*}{P^*_F}, \]  

where \( Y \) is production of the home good and \( PC \) is the before-consumption-tax expenditure of home consumers, and similarly for foreign. Here \( \gamma PC/P_H \), for example, is home demand for the home-produced good. Note that a consumption tax enters both the numerator and the denominator in this expression and hence cancels out.

We introduce money into the model by means of cash-in-advance constraints:

\[ \frac{PC}{1 + \varsigma^c} \leq M \quad \text{and} \quad P^*C^* \leq M^*, \]  

where \( \varsigma^c \) is a consumption subsidy at home. In the next section when we consider a dynamic environment we study a more general case of interest-elastic money demand.

In this static economy, home households face the following budget constraint:

\[ \frac{PC}{1 + \varsigma^c} + M + T \leq \frac{WN}{1 + \tau^n} + \frac{\Pi}{1 + \tau^d} + B^p, \]  

where \( \tau^n \) is the labor-income tax, \( \tau^d \) is the dividend-income tax, \( T \) is a lump-sum tax, and \( B^p \) are home household net foreign assets, possibly state-contingent, converted into home currency, \( WN \) is labor income and \( \Pi \) is firm profits introduced below. In this static section, \( \tau^d \) plays almost no role. For example, we could either set it to \( \tau^d = 0 \) or to \( \tau^d = \tau^n \). The only results that would be affected are those on the revenue impact of fiscal devaluations (Proposition 5). For this reason, we do not specify dividend tax adjustment until our discussion of this proposition. This irrelevance of dividend tax will not carry over to our dynamic setup in Section 3, where we specify dividend tax from the outset.

The home government budget constraint is given by

\[ M + T + TR + B^g \geq 0, \]  

where \( B^g \) is home government net foreign assets converted into home currency and \( TR \) stands for all non-lump-sum fiscal revenue of the home government.\(^{10}\) The two budget constraints together define the country-wide budget constraint, where \( B = B^p + B^g \) are the total home-country net foreign assets. The foreign household and government budget constraints are symmetric with the exception that Foreign does not use fiscal instruments. This assumption is made only for ease of exposition and has no consequence

\(^{10}\) Specifically, we have

\[ TR = \left( \frac{\tau^n}{1 + \tau^n} WN + \frac{\tau^d}{1 + \tau^d} \Pi - \frac{\varsigma^c}{1 + \varsigma^c} PC \right) + \left( \tau^n P_H C_H - \varsigma^p WN \right) + \left( \frac{\tau^n + \tau^m}{1 + \tau^m} P_F C_F - \varsigma^x E P^*_H C^*_H \right), \]

where the first two terms are the income taxes levied on and the consumption subsidy paid to home households; the next two terms are the value-added tax paid by and the payroll subsidy received by home firms; the last two terms are the import tariff and the VAT border adjustment paid by foreign exporters and the export subsidies to domestic firms, as we discuss below.
for our results, as long as foreign responds symmetrically to a fiscal as well as nominal devaluation. International asset market clearing requires $B + B^*E = 0$ state by state, where $B^*$ is foreign-country net foreign assets converted into foreign currency and $E$ is the nominal exchange rate. In this static setting, we take the asset positions $B$ and $B^*$ as exogenous, and we endogenize savings and portfolio choice decisions in the dynamic analysis of Section 3.

We analyze first the case of producer currency pricing and then the case of local currency pricing, allowing for an arbitrary degree of price stickiness. In both cases we also allow for an arbitrary degree of wage stickiness. For each case, we consider in turn various assumptions about international capital flows starting form the case of financial autarky and balanced trade. In all these cases, we characterize combinations of tax changes and money supplies in the home country that perfectly replicate the effects of a devaluation of the home currency but leave the nominal exchange rate unchanged, given that foreign follows a passive policy of constant money supply and no changes in taxes.

### 2.2 Producer currency pricing

We assume that prices and wages are partially (or fully) sticky in the beginning of the period, before productivity shocks and government policies are realized.

**Wage setting** We adopt the following specification for the equilibrium wage rate:

$$W = \bar{W}^{\theta_w} \left[ \frac{\mu_w 1 + \tau^n}{1 + \varsigma^c} \kappa PC^\sigma \left( \frac{YA}{A} \right)^{\varphi_n} \right]^{1-\theta_w},$$

(5)

where $\theta_w \in [0, 1]$ is the degree of wage stickiness, with $\theta_w = 1$ corresponding to fixed wages and $\theta_w = 0$ corresponding to fully flexible wages. Accordingly, $\bar{W}$ is the preset wage, while the term in the square bracket is the flexible wage. We denote by $\mu_w \geq 1$ the wage markup which may arise under imperfectly competitive labor market. The remaining terms in the square brackets define the consumer’s marginal rate of substitution between labor and consumption, where $\tau^n$ is an income tax and $\varsigma^c$ is a consumption subsidy.\(^{11}\) A symmetric equation (without taxes) characterizes the wage in the foreign country. This wage setting specification is motivated by the Calvo wage-setting model with monopsonistic labor supply of multiple types and a fraction $1 - \theta_w$ of types adjusting wages after the realization of the shocks. We spell out this model in greater detail when we carry out our dynamic analysis.

**Price setting** Under producer currency pricing, a home producer sets the same producer price, inclusive of the value-added tax, in home currency for both markets according to:

$$P_H = \bar{P}_H^{\theta_p} \left[ \frac{\mu_p 1 - \varsigma^p}{1 - \tau^v} \frac{W}{A} \right]^{1-\theta_p},$$

(6)

where $\theta_p \in [0, 1]$ is the measure of price stickiness, $\bar{P}_H$ is the preset price and the term in the square bracket is the flexible price, by analogy with wage setting (5). We denote

\(^{11}\)With a linear production technology, labor supply $N$ equals $YA$. 

by $\mu_p \geq 1$ the price markup, while the remaining terms in the square bracket are the firm’s marginal cost, where $\varsigma^p$ is a payroll subsidy and $\tau^v$ is a value-added tax. The foreign good price, $P^*_F$, is set symmetrically in the foreign currency, but with no payroll or value-added taxes. Note that by choosing $\theta_w$ and $\theta_p$ we can consider arbitrary degrees of wage and price stickiness. Furthermore, our results do not depend on whether foreign has the same or different price and wage stickiness parameters.

**International prices** Finally, we discuss international price setting. Under our assumption of PCP, home producers receives the same price from sales at home and abroad. Exports entails a subsidy, $\varsigma^x$, and also the value-added tax is reimbursed at the border. Therefore, the foreign-currency price of the home good is given by

$$P^*_H = P_H \frac{1 - \tau^v}{\mathcal{E}} \frac{1}{1 + \varsigma^x},$$

where $\mathcal{E}$ is the nominal exchange rate measured as units of home currency per unit of foreign currency so that higher values of $\mathcal{E}$ imply a depreciation of home currency. Expression (7) is a variant of the *law of one price* in our economy with tariffs and taxes. Similarly, the home-currency price of the foreign good is given by

$$P_F = P_F^* \mathcal{E} \frac{1 + \tau^m}{1 - \tau^v},$$

where $\tau^m$ is the import tariff, and the value-added tax is levied on imports at the border.

**Capital account openness** We now spell out various assumptions regarding capital account openness. Consider first the case of financial autarky, or balanced trade, which we model by imposing $B = B^p + B^g \equiv 0$ in (3)-(4), and consequently $B^* \equiv 0$. A constant zero net foreign asset position implies balanced trade, $P^*_{HC} = P^*_{HC}$. This can be alternatively stated as:

$$P^*_{YC} = P^*_{YC},$$

that is, the equality of total consumption expenditure and total production revenues in foreign. When trade is balanced and the import tariff equals the export subsidy ($\tau^m = \varsigma^x$), the home government makes no revenues from trade policy, and as a result $PC = PH Y$ also holds in equilibrium (to verify this, combine (3) and (4) and impose $\tau^m = \varsigma^x$).

$^{12}$Under PCP, the profits of a home firm can be written as $\Pi = (1 - \tau^v)PH Y - (1 - \varsigma^p)WN$, where $Y = AN$ is total output of the firm.

$^{13}$$P_F$ denotes the price to consumers before the consumption tax. The way we defined taxes, foreign firms receive $(1 - \tau^v)P_F/(1 + \tau^m)$ in home currency per unit exported, while the home government’s revenue is $(\tau^v + \tau^m)P_F/(1 + \tau^m)$ per unit imported.

$^{14}$Consider the foreign budget constraints in this case:

$$P^*_{YC} + M^* + T^* = W^* N^* + \Pi^* = P^*_F Y^*,$$

$$M^* + T^* = 0.$$

Subtracting one from the other, we immediately obtain (9). We can write $P^*_{YC} = P^*_H C_H^* + P^*_F C_F^*$ and $P^*_F Y = P^*_F (C_F + C_F^*)$, which together with (9) implies trade balance $P^*_F C_F = P^*_H C_H^*$, that is the equality of foreign export revenues and foreign import expenditure.
Next consider the case of perfect risk sharing. In this case, at the beginning of the period, before the realization of productivity and policy, private agents can trade Arrow-Debreu securities. The optimal risk sharing condition is the so-called Backus-Smith condition
\[
\frac{1}{\lambda} \left( \frac{C}{C^*} \right)^\sigma = \frac{P^* \mathcal{E}}{P} (1 + \varsigma^c) \equiv Q,
\]
where $Q$ is the consumer-price real exchange rate and $\lambda$ is the constant of proportionality. Without consequences for our results, we normalize $\lambda = 1$. Net foreign asset positions of the countries must be such that consumption satisfies (10) state by state.

Finally, consider the case where home’s net foreign assets $B$ is composed of an arbitrary portfolio of home-currency non-contingent assets $B^h$ and foreign-currency non-contingent assets $B^{f*}$:
\[
B = B^h + B^{f*} \mathcal{E},
\]
Combining the foreign country budget constraints with asset market clearing, we obtain the equilibrium condition in this case which generalizes (9):\footnote{The same condition can be derived from the home budget constraints (3)–(4), a consequence of Walras Law.}
\[
P^* C^* = P^* Y^* - \frac{B^h}{\mathcal{E}} - B^{f*}.
\]

**Nominal exchange rate** We have fully described the equilibrium structure of the PCP economy under various asset market structures. We only need to to characterize the equilibrium nominal exchange rate. We have: ¹⁶

Lemma 1 In a static PCP economy, the equilibrium nominal exchange rate is given by:

(i) under balanced trade,
\[
\mathcal{E} = \frac{1 - \tau^v}{1 + \tau^m} \frac{M}{M^*} (1 + \varsigma^c).
\]

(ii) with complete international risk-sharing,
\[
\mathcal{E} = \frac{M}{M^*} Q^{\sigma - 1}, \quad \text{where} \quad Q = \frac{P^* \mathcal{E}}{P} (1 + \varsigma^c).
\]

(iii) when the foreign asset position is a portfolio of home- and foreign-currency assets,
\[
\mathcal{E} = \frac{\frac{1 - \tau^v}{1 + \tau^m} M (1 + \varsigma^c) - \frac{1}{1 - \gamma} B^h}{M^* + \frac{1}{1 - \gamma} B^{f*}},
\]
where $B^h$ and $B^{f*}$ are respectively home- and foreign-currency assets of home.

¹⁵Proof of Lemma 1: (i) follows from trade balance (9), cash-in-advance (2) and market clearing (1). Combining these three and the law of one price (8) yields (12). (ii) follows from complete risk-sharing condition (10) and cash-in-advance (2) after rearranging terms and using the definition of the real exchange rate. (iii) follows from (11), together with (2), (1) and (8), just like in (i).
In all cases, the nominal exchange rate depends on money supply and fiscal policy. However, this relationship is different across the three asset market setups. In the case of balanced trade, this relationship is most direct, while in the other two cases it is partially mediated by the adjustment to shocks of prices or net foreign liabilities. Naturally, (12) is a special case of (14) with $B^h \equiv B^{f*} \equiv 0$.

**Fiscal devaluations** We can now formulate our main result. A nominal devaluation of size $\delta$ is the outcome of an increase in the home money supply $M$ so that $\Delta E / E = \delta$, without any change in taxes. We define a fiscal devaluation of size $\delta$ to be a set of fiscal policies, together with an adjustment in money supply, which implements the same consumption, labor, and output allocation as a nominal devaluation of size $\delta$, but holding the nominal exchange rate fixed.

We introduce two propositions that describe fiscal devaluations under various asset market structures:

**Proposition 1** In a static PCP economy under balanced trade or foreign-currency non-contingent assets ($B^{f*} \equiv \text{const}, B^h \equiv 0$ state-by-state), a fiscal devaluation of size $\delta$ can be attained by the following set of fiscal policies:

$$\tau^m = \zeta^x = \delta, \quad \zeta^c = \tau^n = \varepsilon, \quad \text{and} \quad \frac{\Delta M}{M} = \frac{\delta - \varepsilon}{1 + \varepsilon}.$$  

$$(\text{FD}')$$

or

$$\tau^v = \zeta^p = \frac{\delta}{1 + \delta}, \quad \zeta^c = \tau^n = \varepsilon, \quad \text{and} \quad \frac{\Delta M}{M} = \frac{\delta - \varepsilon}{1 + \varepsilon}.$$  

$$(\text{FD}'')$$

where $\varepsilon$ can be chosen arbitrarily.

**Proof:** Note that both (FD') and (FD'') have the same effect on international prices in (7) and (8) as a nominal devaluation $\Delta E / E = \delta$. Furthermore, for given $P_H$ and $P^*_F$, from (2) and (1) we see that a nominal and a fiscal devaluation will have the same effect on consumption and output in the two countries as long as the change in $M(1 + \zeta^c)$ is the same for all devaluation policies. Given prices, consumption and output, wage setting in (5) is the same across all devaluations. Given wages, price setting in (6) is the same across all devaluations. We went full circle, and now only need to check that fiscal devaluations keep the nominal exchange rate unchanged. In the case of balanced trade and foreign-currency debt, a nominal devaluation requires $\Delta M / M = \Delta E / E = \delta$, while fiscal devaluations hold $E$ constant and set, according to (12) and (14), $(M'(1 + \zeta^c) - M) / M = \delta$, where $M' = M + \Delta M$. Given $\zeta^e = \varepsilon$, we obtain the expression for $\Delta M / M$. $\blacksquare$

**Proposition 2** In a static PCP economy under complete international risk-sharing, a fiscal devaluation of size $\delta$ can be attained by the following set of fiscal policies:

$$\tau^m = \zeta^x = \delta, \quad \zeta^c = \tau^n = \delta, \quad \text{and} \quad \frac{\Delta M}{M} = \frac{\sigma - 1}{\sigma} \frac{\Delta Q}{Q},$$  

$$(\text{FD}')$$

or

$$\tau^v = \zeta^p = \frac{\delta}{1 + \delta}, \quad \zeta^c = \tau^n = \delta, \quad \text{and} \quad \frac{\Delta M}{M} = \frac{\sigma - 1}{\sigma} \frac{\Delta Q}{Q},$$  

$$(\text{FD}'')$$

where $\Delta Q / Q$ is the change in the real exchange rate following a nominal devaluation of the exchange rate of size $\delta$. 

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Proof: The proof follows along the exact same lines as that of Proposition 1. The difference is the following. Under complete international risk sharing, nominal and fiscal devaluations must have the same effect on the real exchange rate $Q$ in order to keep the relative consumption of the two countries unchanged, as follows from the risk-sharing condition (10). From (13), under nominal devaluation the change in $M$ equals the change in $\mathcal{E}/Q^{\frac{\sigma - 1}{\sigma}}$, while under fiscal devaluation the change in $M$ must equal the change in $Q^{\frac{\sigma - 1}{\sigma}}$. In all case, $\mathcal{E}(1 + \varsigma c)$ and $M(1 + \varsigma c)$ are unchanged, and therefore indeed consumption and output allocations must be the same. ■

The first type of fiscal devaluation (FD′) relies on an import tariff $\tau m$ combined with a uniform export subsidy $\varsigma x$, a policy advocated already by Keynes and recently studied in Staiger and Sykes (2010). The second fiscal devaluation policy (FD″) is driven by a value-added tax $\tau v$ with border adjustment,\textsuperscript{17} combined with a payroll subsidy $\varsigma p$ to neutralize the effects on price setting (see (6)). Of course, an appropriate combination of these two fiscal devaluation policies would also attain the same result.

The key to understanding the mechanism behind these fiscal devaluations is their effect on the terms of trade. For concreteness, we define the terms of trade as

$$S \equiv \frac{P^*_F}{P^*_H} = \frac{P^*_F}{P^*_H} \frac{1 + \varsigma x}{1 - \tau v},$$

(15)

where the second equality comes from the law of one price (7). Therefore, the terms of trade can be equivalently affected by a nominal or a fiscal devaluation. The remainder of the fiscal policies in (FD′) and (FD″) is needed to offset the additional consequences of fiscal devaluations. An increase in the export subsidy must be accompanied by an increase in the import tariff in order to ensure the same movement in international prices as under a nominal devaluation (see (8)). In sum, fiscal devaluations must ensure the same international relative price movements as a nominal devaluations.

We now discuss the role of consumption subsidies. From (2) and (1), we see the effect of a consumption subsidy on consumption and output is the same as the effect of an expansion in money supply. Indeed, under balanced trade and foreign-currency risk-free debt, a consumption subsidy is not essential and it can be replaced by an increase in money supply which does not affect the nominal exchange rate as long as $M(1 + \tau c)$ increases in proportion with the import tariff or value-added tax (see (12) and (14)). A fiscal devaluation without consumption subsidy requires an increase in the money supply in order to keep trade balanced (otherwise there would be a trade surplus).

With complete international risk-sharing, a consumption subsidy is needed even if we allow the home money supply to adjust. This is because the proposed tariff and VAT changes, although they affect the terms of trade in the same way as a nominal devaluation, have the opposite effect on the real exchange rate. Using the definition of the real exchange rate in (10) together with (7)–(8), we can write

$$Q = S^{2(\gamma - 1)} \frac{(1 - \tau v)(1 + \varsigma c)}{(1 + \varsigma c)^{\gamma}(1 + \tau m)^{1-\gamma}}.$$

\textsuperscript{17}In contrast to the results in Grossman (1980) and Feldstein and Krugman (1990) derived under flexible exchange rate and prices, border adjustment is indispensable for our results.
Therefore, given the movement in terms of trade $S$, fiscal devaluations, in the absence of consumption subsidies, lead to an extra appreciation of the real exchange rate $Q$. Both fiscal and nominal devaluations make home exports cheaper relative to foreign exports. However, nominal devaluations achieve this outcome by making all home-produced goods relatively cheaper, while fiscal devaluations makes home consumption relatively more expensive by taxing imports. This leads to a differential movement in the real exchange rate, which under complete markets results in real transfers through asset markets, between the two countries. A consumption subsidy is needed exactly to mimic these effects by depreciating the real exchange rate under a fiscal devaluation. In turn, this consumption subsidy limits the need for a monetary expansion since it has the same effect on consumption through the cash-in-advance constraint (2). Finally, an income tax $\tau^n$ is only needed to offset the labor wedge created by a consumption subsidy, as can be seen from (5).

To summarize, the consumption subsidy (accompanied by an income tax) is needed in order to keep international risk sharing and relative consumption allocation across countries unchanged. Under balanced trade the relative consumption allocation does not depend on the real exchange rate since there are no state-contingent transfers across countries. However, under complete markets these transfers depend on the value of the real exchange rate, and hence the consumption subsidy (and the offsetting income tax) is indispensable.

Further observe that under the circumstances of Proposition 1 (e.g., balanced trade), fiscal devaluations can be attained with no change in the home money supply, by using instead a consumption subsidy and an income tax. Under complete markets (Proposition 2) however, both the use of a consumption subsidy and a change in the home money supply are needed. This raises certain issues about implementation of this policy in a monetary union with no independent money supply, which we discuss in Section 5.

Also note that both proposed fiscal devaluations are neutral, in the sense that they have no effect on consumption and output allocation when prices and wages are fully flexible. Propositions 1 and 2 apply to arbitrary degrees of wage and price stickiness.

When prices or wages are sticky, fiscal devaluations have the same real effects on the economy as those brought about by a nominal devaluation driven by an expansion in the money supply. It is important to note however that the effects of a $\delta$-devaluation (nominal or fiscal) are different for different asset market structures. Under different asset market structures, a given devaluation is attained by a different expansion in the money supply. Specifically, under balanced trade a $\delta$-devaluation requires $\Delta M/M = \delta$, or alternatively $\varsigma^c = \delta$. However, under complete international risk sharing, as long as there is home bias ($\gamma > 1/2$) and utility is not log in consumption ($\sigma \neq 1$), a nominal devaluation will be associated with a depreciation of the real exchange rate, which in turn limits (amplifies) the required money supply expansion when $\sigma > 1$ ($\sigma < 1$), as can be seen from (13).

Valuation Effects For discussion of valuation effects see, for example, Gourinchas and Rey (2007).

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18 For discussion of valuation effects see, for example, Gourinchas and Rey (2007).
(\(B^h \neq 0\)), the fiscal instruments specified in Proposition 1 no longer suffice. Instead they must be supplemented with a partial default \(d = \delta/(1 + \delta)\), or a tax, on the home-currency-denominated debt of the home country held by foreign, in order to replicate the effects of a devaluation. That is, the post-devaluation debt position of home becomes \((1 - d)B^h = B^h/(1 + \delta)\). The difference in the equivalence proposition between foreign- and home-currency denominated debt can be understood by studying the foreign budget constraint (11). When \(B^h = 0\) then a nominal devaluation has no effect on the foreign-currency value of the debt. If instead \(B^h < 0\) a nominal devaluation reduces the foreign-currency value of the debt owed by home to foreign to \(B^h/(1 + \delta)\). The partial default \(d\) is then needed to exactly mimic this reduction in the foreign-currency value of debt in a fiscal devaluation when the exchange rate is held fixed.

An alternative approach to understanding the difference is to study home’s consolidated budget constraint which is given by,

\[-\mathcal{E}B_f^* - B^h = \frac{1 - \tau^v}{1 + \tau^m}(P_HY - PC).\]

If Home has positive debt, repayment requires \((P_HY - PC) > 0\). If this debt is denominated in foreign currency, then a devaluation has the direct effect of raising the local currency value of the debt \(-\mathcal{E}B_f^*\), and increases the payments in local currency to the foreign country. This same effect follows an increase in a uniform import tariff-cum-export subsidy or an increase in value-added tax-cum-payroll tax reduction. Now if the debt is denominated in home currency, a devaluation has no direct effect on the value of debt in home currency, but the increase in fiscal taxes will raise the transfers to the foreign country. To undo this requires a partial default/tax on foreign holders of home debt.

We summarize this discussion in:

**Proposition 3** With home-currency debt \((-B^h \neq 0\)), a fiscal devaluation of size \(\delta\) can be attained by the same set of fiscal policies as in Proposition 1, combined with a partial default on the home-currency denominated debt of the home country, \(d = \delta/(1 + \delta)\), and a suitable adjustment in the money supply.\(^{19}\)

Note that this partial default is a direct transfer of wealth from foreign to home households without involving taxation by the government. Symmetrically, if home has home-currency assets \((B^h > 0)\), then equivalence requires debt forgiveness to foreign that reduces homes assets to the level \((1 - d)B^h\). An implication of this analysis is that in the case when there are heterogenous agents in the economy with different portfolios of foreign- and home-currency assets, exchange rate devaluations will effect the cross-sectional distribution of wealth differently from a fiscal devaluation, unless all agents with home-currency liabilities partially default on them with the haircut given by \(d = \delta/(1 + \delta)\).

### 2.3 Local currency pricing

We now consider briefly the alternative case of local currency pricing. We show that the results are exactly the same under local currency pricing. This is surprising because the

\(^{19}\)The required adjustment in the money supply can be inferred from equation (14), given the desired size of the devaluation, fiscal policies used, and the amount of home-currency debt.
mechanism of a nominal devaluation under LCP is quite different from that under PCP. While under PCP a nominal devaluation affects international relative consumer prices, under LCP it affects the profit margins of the firms. In both cases, these are exactly the effects attained by a fiscal devaluation.

Formally, the international law of one price (7)–(8) no longer holds and firms set prices separately for domestic and foreign consumers. We assume that prices are preset inclusive of all taxes and subsidies, apart from the consumption subsidy given directly to the consumers. This assumption, while consistent with the logic of local currency pricing, is crucial for our equivalence results, since otherwise fiscal policies will not be able to attain the same effects on international relative prices as a nominal devaluation.

All equilibrium conditions remain unchanged under LCP, apart from (7)–(8) that are replaced with the following price-setting equations:

\[
P_H^* = \tilde{P}_H^* \left[ \frac{1 - \xi^p}{1 + \xi^p} \frac{1}{W} \right]^{1-\theta_\omega}, \tag{16}
\]

\[
P_F = \tilde{P}_F^* \left[ \frac{1 + \tau_m}{1 - \xi^v} \frac{W}{A^*} \right]^{1-\theta_\omega}, \tag{17}
\]

that parallel (6). From (16)–(17) we see that as international prices adjust, they are affected in the same way by fiscal and nominal devaluations. However when prices are fixed, neither devaluation has an effect on international prices.

With fixed prices, however, profits must adjust. For example, the profits of a representative foreign firm are given by

\[
\Pi^* = P_F^* C_F^* + P_F C_F^* \frac{1}{1 + \tau_m} - W^* N^*.
\]

From this expression it is clear that a fiscal devaluation affects profits \(\Pi^*\) in the same way as a nominal devaluation.

Finally, we characterize equilibrium nominal exchange rate under LCP:

**Lemma 2** Lemma 1 applies to the case of LCP as well, and the nominal exchange rate is given by (12), (13) or (14) depending on the structure of the asset market.

**Proof:** The proof for the case of complete markets does not rely on the type of price setting, PCP or LCP. The case of trade balance and non-zero net foreign liabilities is more involved. Consider the household and government budget constraints in foreign:

\[
P^* C^* + M^* + T^* = W^* N^* + \Pi^* + B^{ps},
\]

\[
M^* + T^* + B^{gs} = 0.
\]

Combining this with the expression for \(\Pi^*\) above, we obtain

\[
P^* C^* = P_F C_F^* + P_F C_F^* \frac{1}{1 + \tau_m} - B^h - B^{fs},
\]
where we have used the fact that \( B^* = B^p + B^g = -B/E = -B^h/E - B^f^* \) in equilibrium. Now use cash-in-advance (2) and Cobb-Douglas demand for home and foreign goods to obtain

\[
M^* = \gamma M^* + (1 - \gamma) M \left( 1 + \varsigma c^e \right) \frac{1}{E} \frac{1 - \tau v}{1 + \tau m} - \frac{B^h}{E} - B^f^*,
\]

which immediately implies (14), and hence (12) as a special case when \( B^h \equiv B^f^* \equiv 0 \) and trade is balanced. ■

With Lemma 2, we can immediately generalize the results in Proposition 1, 2 and 3 to the case of LCP (the proof follows exactly the same steps as above):

**Proposition 4** With LCP, fiscal policies \((FD')\) and \((FD'')\) constitute fiscal devaluations under balanced trade, complete international risk sharing, and foreign-currency risk-free debt, just like under PCP: Propositions 1, 2 and 3 apply.

We have identified a robust set of fiscal policies—fiscal devaluations—which achieve the same allocations as nominal devaluations, but keep the exchange rate unchanged. It is important to note that the allocations themselves are very different under LCP and PCP. As surveyed in Lane (2001), a monetary expansion under PCP has a positive spillover for the foreign country through a depreciation of the home terms of trade. Under PCP, nominal devaluation generates a production boom at home and a consumption boom worldwide. By contrast, a monetary expansion under LCP is beggar-thy-neighbor due to a terms of trade depreciation of foreign and a reduction in foreign firms’ profit margins.\(^{20}\) Under LCP, a nominal devaluation generates a consumption boom at home and a production boom worldwide. It is immediate to extend our results to environments with a mix of producer and local currency pricing, as for example in Devereux and Engel (2007).

### 2.4 Revenue Neutrality

The last question we address before moving on to the dynamic analysis, is whether the proposed fiscal policies lead to government budget surplus or deficit. We have:

**Proposition 5** Under both PCP and LCP, as regards non-lump-sum tax revenue \( TR \): \((FD')\) is revenue-neutral if all taxes are adjusted by the same amount \((\varepsilon = \delta)\) or if trade is balanced, and in both cases, if the dividend tax is set to \( \tau^d = \varepsilon \); \((FD'')\) is revenue-neutral if all taxes are adjusted by the same amount \((\varepsilon = \delta)\), but dividend tax is set to \( \tau^d = 0 \).

\(^{20}\)Home terms of trade under LCP are given by

\[
S = \frac{P_F}{P_H} \frac{1 - \tau v}{E} \frac{1}{1 + \tau m},
\]

and nominal (as well as fiscal) devaluation leads to its appreciation, in contrast to the PCP case where the terms of trade depreciates (see (15)), as emphasized in Obstfeld and Rogoff (2000).
Proof: In general, non-lump-sum revenues $TR$ are given by

$$TR = (\tau^n WN + \tau^d \Pi - \frac{\zeta^c PC}{1+\zeta^c}) + (\tau^n P_H C_H - \zeta^p WN) + \left(\frac{\tau^n + \tau^m}{1+\tau^m} P_F C_F - \zeta^c \mathcal{E} P_H^* C_H^*\right),$$

Under (FD'), profits are given by

$$\Pi = P_H C_H + (1+\delta) \mathcal{E} P_H^* C_H^* - WN,$$

both under PCP and LCP, where $\mathcal{E}$ is the constant value of the nominal exchange rate. Under LCP, $P_H^*$ is the sticky consumer price, while under PCP it is given by the law of one price $P_H^* = P_H/[\mathcal{E}(1+\delta)]$, where $P_H$ is the sticky producer price. Furthermore, tax revenues in this case are given by

$$TR = \frac{\tau^d}{1+\tau^d} \Pi + \frac{\varepsilon}{1+\varepsilon} (WN - PC) + \frac{\delta}{1+\delta} P_F C_F - \delta \mathcal{E} P_H^* C_H^*$$

$$= \left[\frac{\tau^d}{1+\tau^d} - \frac{\varepsilon}{1+\varepsilon}\right] \Pi + \left[\frac{\delta}{1+\delta} - \frac{\varepsilon}{1+\varepsilon}\right] (P_F C_F - (1+\delta) \mathcal{E} P_H^* C_H^*),$$

where the second equality substitutes in the expression for profits and rearranges terms using $PC = P_H C_H + P_F C_F$. We also used $\zeta^c = \tau^n = \varepsilon$ with either $\varepsilon = \delta$ as in Proposition 2 or as a free parameter as in Proposition 1. Hence, we can always set $\varepsilon = \delta$ and $\tau^d = \varepsilon$ and have $TR = 0$. If we choose $\varepsilon = 0$, $TR$ has the same sign as the trade balance of foreign.

Similarly, in the case of (FD'')

$$TR = \frac{\tau^d}{1+\tau^d} \Pi + \frac{\varepsilon}{1+\varepsilon} (WN - PC) + \frac{\delta}{1+\delta} (P_H C_H - WN) + \frac{\delta}{1+\delta} P_F C_F$$

$$= \left[\frac{\delta}{1+\delta} - \frac{\varepsilon}{1+\varepsilon}\right] (PC - WN) + \frac{\tau^d}{1+\tau^d} \Pi.$$

With $\varepsilon = \delta$ and $\tau^d = 0$, $TR = 0$. When $\varepsilon < \delta$ and $\tau^d = 0$, $TR \geq 0$ whenever $PC > WN$.

The key conclusion here is that our proposed fiscal devaluations can always be implemented with a balanced budget, an important property for a viable devaluation policy under most circumstances. Note that by focusing on non-lump-sum government revenues $TR$, we have effectively excluded seigniorage from our analysis of revenue neutrality. However, apart from seigniorage, $TR$ defines the primary fiscal surplus of the home country.

Finally, we emphasize the robustness of our revenue-neutrality result. First, it applies equally to both PCP and LCP environments. Second, this argument directly extends to a dynamic environment, as the one considered in the next section, since a dynamic fiscal devaluation can be implemented with $TR = 0$ period-by-period.\(^{21}\)

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\(^{21}\)Period-by-period neutrality is no longer true when a fiscal devaluation does not involve consumption subsidies and income taxes. As shown in the proof, when $\zeta^c = \tau^n = \tau^d = 0$ under (FD’), government revenues from a fiscal devaluation are proportional to home trade deficit. Therefore, the net present value of revenues from fiscal devaluation, by the intertemporal budget constraint, must be proportional to the initial net foreign assets of home. Under (FD''), the same is true when $\zeta^c = \tau^n = 0$ and $\tau^d = -\delta/(1+\delta)$, that is, when a dividend income subsidy is in place. With $\tau^d = 0$, revenues from the fiscal devaluation are greater (more positive or less negative) in proportion to the aggregate profits of the economy.
3 Dynamic Analysis

In this section we extend our results from the static model to a fully-fledged New Keynesian dynamic stochastic model, which incorporates standard Calvo sticky prices and wages. Again we allow for price setting both in producer and local currency. We start with a model without capital and later analyze how the introduction of capital changes our conclusions. In contrast to a static model, within a dynamic framework households face endogenous savings and portfolio choice decisions. We consider different asset market structures, including complete markets, home or foreign-currency risk-free bond economies, and international trade in equities. Additionally, we generalize our setup to allow for interest-elastic dynamic money demand. Finally, a dynamic framework allows us to discuss separately both one-time unexpected devaluations, as well as expected dynamic devaluations. Overall, the proposed fiscal devaluations work robustly in our significantly more general dynamic environment, with certain modifications and caveats that we emphasize below.

3.1 Baseline setup

For concreteness, we begin by laying out the general features of our model economy for the case of complete international asset markets and Producer Currency Pricing (PCP). We later consider in turn alternative international asset market structures, and then show the robustness of our results under Local Currency Pricing (LCP), and to the introduction of capital.

Consumer problem The model features two countries, each populated by a continuum of symmetric households. Households are indexed by $h \in [0, 1]$, but we often omit the index $h$ when it does not lead to confusion. Household $h$ maximizes expected utility

$$
E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, m_t, N_t),
$$

where $C_t$ denotes consumption, $N_t$ is labor, and $m_t = M_t(1 + \varsigma^c_t)/P_t$ denotes real money balances; $P_t$ is the consumer price before consumption subsidy $\varsigma^c_t$, which is why $P_t/(1 + \varsigma^c_t)$ deflates nominal money balances in the utility function.

The utility function is given by

$$
U(C_t, m_t, N_t) = \frac{1}{1 - \sigma} C_t^{1-\sigma} + \frac{\chi}{1 - \nu} m_t^{1-\nu} - \frac{\kappa}{1 + \phi} N_t^{1+\phi}.
$$

Consumption $C_t$ is an aggregator of home and foreign goods that features a home bias:

$$
C_t = C^H_t C^F_t^{1-\gamma}, \quad \gamma > 1/2,
$$

The analysis of the case of financial autarky (balanced trade) is essentially identical to that of our static model, and is omitted for brevity.

For simplicity, we again start from a situation where taxes are zero. As discussed in the static section, our results generalize immediately to a situation where initial taxes are not zero. Indeed, our results characterize the changes in taxes that are necessary to engineer a fiscal devaluation.

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22The analysis of the case of financial autarky (balanced trade) is essentially identical to that of our static model, and is omitted for brevity.

23For simplicity, we again start from a situation where taxes are zero. As discussed in the static section, our results generalize immediately to a situation where initial taxes are not zero. Indeed, our results characterize the changes in taxes that are necessary to engineer a fiscal devaluation.
and where consumption of both home and foreign goods is given by CES aggregators of individual varieties:

\[
C_{Ht} = \left[ \int_0^1 C_{Ht}(i)^{\frac{\rho}{\rho-1}} \, di \right]^{\frac{\rho-1}{\rho}} \quad \text{and} \quad C_{Ft} = \left[ \int_0^1 C_{Ft}(i)^{\frac{\rho}{\rho-1}} \, di \right]^{\frac{\rho-1}{\rho}},
\]

with \( \rho > 1 \). Our assumptions of a Cobb-Douglas upper-tier consumption aggregator and separable utility in consumption and labor can be immediately generalized without affecting the results, and we adopt them exclusively for convenience of exposition.

Note that we have introduced money into the utility function. Our results require money to enter separably in the utility function except in the case of a one-time unanticipated devaluation as we discuss below. For anticipated devaluations, non-separability of money in the utility function requires an additional tax instrument such as a tax on cash holdings. The cash-in-advance economy can be thought of as a special limiting case of money-in-the-utility function with money entering non-separably, but our baseline results generalize to this limiting case without requiring additional tax instruments. Note that these cases—money in the utility function entering separably or cash-in-advance constraints—are the focus of the New-Keynesian literature.

In each period, each household chooses consumption \( C_t \), money \( M_t \) and state-contingent one-period bonds \( B_{t+1} \). It also sets a wage rate \( W_t \), and supplies labor in order to satisfy demand at this wage rate. We describe the wage setting process below. Households face the following flow budget constraint:

\[
\frac{P_t C_t}{1 + \zeta_t} + M_t + \mathbb{E}_t \{ \Theta_{t+1} B_{t+1} \} \leq B_t + M_{t-1} + \frac{W_t N_t}{1 + \tau_t^a} + \frac{\Pi_t}{1 + \tau_t^d} - T_t.
\]

In this equation, \( \Theta_{t+1} \) is the home-currency nominal stochastic discount factor and \( \Pi_t = \int \Pi_t^i \, di \) is aggregate profits of the home firms. As before, \( \tau_t^a \) is the labor-income tax, \( \tau_t^d \) is the dividend-income tax, and \( T_t \) is the lump-sum tax.

Solving the consumers problem yields the following expressions for the stochastic discount factor

\[
\Theta_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \frac{1 + \zeta_{t+1}^c}{1 + \zeta_t^c},
\]

the nominal interest rate

\[
\frac{1}{1 + i_{t+1}} = \mathbb{E}_t \left\{ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \frac{1 + \zeta_{t+1}^c}{1 + \zeta_t^c} \right\} = \mathbb{E}_t \Theta_{t+1},
\]

and money demand

\[
\chi C_t^\sigma \left( \frac{M_t(1 + \zeta_t^c)}{P_t} \right)^{-\nu} = \frac{i_{t+1}}{1 + i_{t+1}}.
\]

\[24\] Specifically, cash-in-advance corresponds to a constant-proportions Leontief utility function over consumption and real money balances, which is the limiting case of a CES utility as the elasticity of substitution goes to zero.

\[25\] Under these two assumptions, the nominal interest rate is the only money market relevant variable for the rest of the allocation, justifying the focus on the corresponding cashless limit in the New Keynesian literature.
Given the choice of consumption $C_t$, each household purchases home and foreign goods according to
\[
C_{Ht} = \gamma \frac{P_tC_t}{P_{Ht}} \quad \text{and} \quad C_{Ft} = (1 - \gamma) \frac{P_tC_t}{P_{Ft}},
\]  
where $P_{Ht}$ and $P_{Ft}$ are the price indexes of home and foreign goods. Finally, household demand for a given variety $i$ of home and foreign goods is given by:
\[
C_{Hi}(i) = \left( \frac{P_{Hi}(i)}{P_{Ht}} \right)^{-\rho} C_{Ht} \quad \text{and} \quad C_{Fi}(i) = \left( \frac{P_{Fi}(i)}{P_{Ft}} \right)^{-\rho} C_{Ft}.
\]  

Foreign households face a symmetric problem with the exception that the foreign government imposes no taxes or subsidies and foreign consumers have a home bias towards foreign-produced goods. The optimal choices of foreign consumers are characterized by similar conditions which we omit for brevity. The foreign-currency nominal stochastic discount factor is given by $\Theta_{t+1}^* = \Theta_{t+1}E_{t+1}/E_t$, where $E_t$ is the nominal exchange rate defined as in the static analysis. Complete international asset markets allow for perfect international risk-sharing, resulting in the well-known Backus-Smith condition:
\[
\left( \frac{C_t}{C_t^*} \right)^{\sigma} = \lambda \frac{P_{t}^*E_t}{P_t/(1 + \zeta_t)},
\]  
where $\lambda$ is a constant of proportionality pinned down by the initial net foreign asset positions of the two countries.

**Producer problem** In each country there is a continuum $i \in [0, 1]$ of firms producing different varieties of goods using a linear technology in labor. Specifically, firm $i$ produces according to
\[
Y_t(i) = A_t N_t(i),
\]  
where $A_t$ is the country-specific level of productivity and $N_t(i)$ is the firm’s labor input.

Under PCP and with CES demand functions, firm $i$ charges the same producer price $P_{Hi}(i)$ on both its domestic and foreign sales. Therefore, the profits of the firm are given by:\[\tag{28}\]
\[
\Pi_t^i = (1 - \tau_t^v)P_{Hi}(i)Y_t(i) - (1 - \zeta_t^p)W_tN_t(i),
\]
where \( \tau^v_t \) is the value-added tax (VAT) and \( \varsigma^p_t \) is the payroll subsidy. Note that we define the producer price to be inclusive of the VAT.

Firm \( i \) faces the following demand

\[
Y_t(i) = C_{Ht}(i) + C^*_H(i) = \left( \frac{P_{Ht}(i)}{P_{Ht}} \right)^{-\rho} (C_{Ht} + C^*_H),
\]

where the second equality uses (20) and the fact that the law of one price holds under PCP (see below).\(^{29}\) At the aggregate, this condition implies market clearing

\[
C_{Ht} + C^*_H = Y_t \equiv \left[ \int_0^1 Y_t(i) \frac{\tau^v_t}{\rho} \, di \right]^\frac{\rho}{\rho - 1}
\]

where \( Y_t \) is the aggregate real output of the home country.

Price setting is subject to a Calvo friction: in any given period, a firm can adjust its price with probability \( 1 - \theta_p \), and must maintain its previous-period price with probability \( \theta_p \). The price \( \bar{P}_{Ht}(i) \) set by a firm that gets to change its price in period \( t \) maximizes the expected net present value of profits conditional on no price change

\[
\mathbb{E}_t \sum_{s=t}^{\infty} \theta_p^{s-t} \Theta_{t,s} \frac{\Pi^i_s}{1 + \tau^d_s} \left[ (1 - \tau^v_s) \bar{P}_{Ht}(i) - \frac{\rho}{\rho - 1} (1 - \varsigma^p_s) \frac{W_s}{A_s} \right] = 0. \tag{22}
\]

This implies that the preset price \( \bar{P}_{Ht}(i) \) is a constant markup over the weighted-average expected future marginal costs over the period of price duration. All firms adjusting prices at \( t \) are symmetric (there are no idiosyncratic shocks), and hence choose the same reset price, \( \bar{P}_{Ht}(i) \equiv \bar{P}_{Ht} \).\(^{30}\) As a result, the aggregate price level \( P_t \) satisfies

\[
P_{Ht} = \left[ \theta_p P_{H,t-1}^{1-\rho} + (1 - \theta_p) \bar{P}_{Ht}^{1-\rho} \right]^{\frac{1}{1-\rho}}. \tag{23}
\]

The price-setting problem of foreign firms is identical, with the exception that there are no taxes or subsidies in the foreign country. The foreign price index \( P_{Ft} \) follows a symmetric law of motion. International prices satisfy the law of one price

\[
P_{Ht}(i) = P_{Ht}(i) \frac{1}{\mathcal{E}_t} \frac{1 - \tau^v_t}{1 + \varsigma^v_t}, \tag{24}
\]

\[
P_{Ft}(i) = P_{Ft}(i) \mathcal{E}_t \frac{1 + \tau^m_t}{1 - \tau^v_t}. \tag{25}
\]

\(^{29}\)We also implicitly aggregate across households. Formally, \( C_{Ht}(i) = \int_0^1 C_{Ht}(h,i) \, dh \), but since households are ex ante symmetric and have access to complete markets, we have \( C_{Ht}(h,i) = C_{Ht}(i) \) for all \( h \).

\(^{30}\)We adopt this assumption exclusively for convenience of exposition, as our results generalize immediately to the environment with heterogenous firms and workers charging respectively different prices and wages upon adjustment.
Foreign prices $P_{Ht}(i)$ are obtained from home prices by a conversion into foreign currency and an adjustment for import tariffs, export subsidies and value-added tax (that is reimbursed for exports and levied on imports). Since both countries face the same aggregators for home and foreign varieties, the law of one price also holds at the aggregate level for home and foreign goods (but not for consumption goods because of home bias).

**Wage setting** Labor input $N_t$ is a CES aggregator of the individual varieties supplied by each household:

$$N_t = \left[ \int_0^1 N_t(h) \frac{q-1}{\eta} \, dh \right]^{\eta-1}, \quad \eta > 1.$$  

Therefore, aggregate demand for each variety of labor is given by

$$N_t(h) = \left( \frac{W_t(h)}{W_t} \right)^{-\eta} N_t,$$

where $N_t = \int_0^1 N_t(i) \, di$ is aggregate labor demand in the economy, $W_t(h)$ is wage rate charged by household $h$ for its variety of labor services and $W_t$ is the wage for a unit of aggregate labor input in the home economy.\(^{31}\)

Households are subject to a Calvo friction when setting wages: in any given period, with probability $1 - \theta_w$ they can adjust their wage, but with probability $\theta_w$ they have to keep their wage unchanged. The optimality condition for wage setting is given by (see the Appendix):

$$E_t \sum_{s=t}^{\infty} \theta_w^{s-t} \Theta_{t,s} N_s W_s^{\eta(1+\varphi)} \left[ \frac{\eta}{\eta - 1} \left( 1 + \frac{\eta}{1 + \tau_s} \right) - \frac{1}{1 + \tau_s} \frac{\bar{W}_t(h)^{1+\eta\varphi}}{W_s^{\eta\varphi}} \right] = 0. \quad (26)$$

This implies that the wage $\bar{W}_t(h)$ is preset as a constant markup over the expected weighted-average between future marginal rates of substitution between labor and consumption and aggregate wage rates, over the duration of the wage. This is a standard result in the New Keynesian literature, as derived, for example, in Galí (2008).

All adjusting households at time $t$ set the same wage $\bar{W}_t(h) \equiv \bar{W}_t$, so that $W_t$ satisfies

$$W_t = \left[ \theta_w W_{t-1}^{1-\eta} + (1 - \theta_w) \bar{W}_t^{1-\eta} \right]^{1/\eta}. \quad (27)$$

Foreign wages are set in a symmetric way with the exception that foreign households are not subject to consumption subsidies or income taxes.

Finally, the aggregate production function (or aggregate labor demand) can be written as

$$A_t N_t = \int_0^1 Y_t(i) \, di = Y_t \left[ \int_0^1 \left( \frac{P_{Ht}(i)}{P_{Ht}} \right)^{-\rho} \, di \right]. \quad (28)$$

\(^{31}\)Formally, home aggregate wage is given by a standard CES price index $W_t = \left[ \int_0^1 W_t(h)^{1-\eta} \, dh \right]^{1/(1-\eta)}$.  

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**Government** We assume that the government must balance its budget each period. This is without loss of generality since Ricardian equivalence holds in this model. The government budget constraint in period $t$ is

$$M_t - M_{t-1} + T_t + TR_t = 0,$$

where tax revenues from distortionary taxes $TR_t$ are given by

$$TR_t = \left( \frac{\tau_t^m}{1 + \tau_t^d} W_t N_t + \frac{\tau_t^d}{1 + \tau_t^d} \Pi_t - \frac{\tau_t^c}{1 + \tau_t^c} P_t C_t \right)$$

$$+ \left( \tau_t^v P_{Ht} C_{Ht} - \zeta_t^p W_t N_t \right) + \left( \frac{\tau_t^v + \tau_t^m}{1 + \tau_t^m} P_{Ft} C_{Ft} - \zeta_t^x E_t P_{Ht}^* C_{Ht}^* \right),$$

just as in the static economy.\(^{32}\)

Combining this together with the household budget constraint and aggregate profits, we arrive at the aggregate country budget constraint

$$P_t C_t + E_t \{ \Theta_{t+1} B_{t+1} \} = B_t + P_{Ht} Y_t + E_t \left( \frac{\tau_t^v + \tau_t^m}{1 - \tau_t^m} P_{Ft}^* C_{Ft} - \zeta_t^x + \frac{\tau_t^v}{1 - \tau_t^v} P_{Ht}^* C_{Ht}^* \right), \quad (29)$$

which using the law of one price can be rewritten as:\(^{33}\)

$$E_t \{ \Theta_{t+1} B_{t+1} - B_t \} = E_t \{ P_{Ht}^* C_{Ht}^* - P_{Ft}^* C_{Ft} \}.$$  

The foreign country aggregate budget constraint is symmetric with $B_t + B_t^* = 0$ (by market clearing). It is redundant and can be dropped by Walras law.

To finish the description of the model setup, we provide the aggregate goods market clearing conditions, equalizing output of each country with world demand for the respective good:

$$Y_t = \gamma \frac{P_t C_t}{P_{Ht}} + (1 - \gamma) \frac{P_t^* C_t^*}{P_{Ht}^*} \quad \text{and} \quad Y_t^* = (1 - \gamma) \frac{P_t C_t}{P_{Ft}} + \gamma \frac{P_t^* C_t^*}{P_{Ft}^*}. \quad (30)$$

### 3.2 Fiscal devaluations under complete markets

Consider an equilibrium path of our model economy along which the nominal exchange rate follows

$$E_t = E_0 (1 + \delta_t) \quad \text{for} \quad t \geq 0.$$  

Here $\delta_t$ denotes the percent nominal devaluation relative to period 0. We refer to such an equilibrium path as a *nominal $\{\delta_t\}$-devaluation.* Denote by $\{M_t\}$ the path of home money supply that is associated with the nominal devaluation. A *fiscal $\{\delta_t\}$-devaluation* is a sequence $\{M_t^{'}, \tau_t^m, \tau_t^v, \tau_t^d, \zeta_t^p, \zeta_t^c, \zeta_t^x, \Theta_{t+1} \}_{t \geq 0}$ of money supply and taxes that achieves the

---

\(^{32}\)Note that $W_t N_t = \int_0^1 W_t(h) N_t(h) dh$ and $P_t C_t = \int_0^1 P_{Ht}(i) C_{Ht}(i) i + \int_0^1 P_{Ft}(i) C_{Ft}(i) i$.  

\(^{33}\)Note that the home country trade balance can be written as $TB_t = E_t \{ P_{Ht}^* C_{Ht}^* - P_{Ft}^* C_{Ft} \}$. Furthermore, we have $P_{Ht}^* C_{Ht}^* - P_{Ft}^* C_{Ft} = P_{Ht}^* C_{Ht} - P_{Ft}^* Y_t^*$. Hence the intertemporal budget constraint can be written as $B_0 + \sum_{t=0}^{\infty} \Theta_{0,t} E_t [P_t^* C_t^* - P_{Ft}^* Y_t^*] = 0$.  

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same equilibrium allocation of consumption, output and labor supply, but for which the
equilibrium exchange rate is fixed $E_t' \equiv \varepsilon_0$ for all $t \geq 0$.

Note that, in general, we do not restrict the path of the exchange rate under a nominal
devolution. For example, one can look at the case of a probabilistic one-time devaluation
where $\delta_t$ is a Markov process with two states $\{0, \delta\}$ where $\delta$ is an absorbing state, or the
case of a deterministic devaluation where $\delta_t = 0$ for $t < T$ and $\delta_t = \delta$ for $t \geq T$. We will also consider an interesting variant, a one-time unexpected devaluation under which $\delta_t = 0$
for $t < T$ and $\delta_t = \delta > 0$ with probability one for $t \geq T$; in addition $\Pr_t\{\delta_{t+j} = 0\} = 1$
for $t < T$ and $j \geq 0$.

We can now state the main result of this section:

**Proposition 6** In a dynamic PCP economy with interest-elastic money demand and com-
plete international asset markets, a fiscal $\{\delta_t\}$-devaluation can be achieved either by

$$\tau^{m}_t = \zeta^{x}_t = \zeta^{c}_t = \tau^{d}_t = \delta_t \quad \text{for} \quad t \geq 0 \quad \text{(FDD')}$$

or by

$$\tau^{v}_t = \zeta^{p}_t = \frac{\delta_t}{1 + \delta_t}, \quad \zeta^{c}_t = \tau^{n}_t = \delta_t \quad \text{and} \quad \tau^{d}_t = 0 \quad \text{for} \quad t \geq 0, \quad \text{(FDD'')}$$

as well as a suitable choice of $M'_{t}/M_t$ for $t \geq 0$. Under a one-time unexpected fiscal deval-
uation, $M'_{t}/M_t = 1/(1 + \delta_t)$ and for (FDD') we can also use $\tau^{d}_t = 0$ for all $t \geq 0$.

**Proof:** The idea of the proof is to show that all the equilibrium conditions hold for the
allocation associated with the nominal devaluation, with the same prices and wages, but
with a fixed exchange rate $E_t' = \varepsilon_0$ under either (FDD') or (FDD'').

First, note from the law of one price equations (24)–(25) that a fiscal devaluation
requires the suggested movement in $\tau^{m}_t = \zeta^{x}_t$ or in $\tau^{v}_t$. Otherwise international relative
prices are different under the proposed fiscal devaluation and under the nominal deval-
uation. But then the terms of trade are not the same, leading to a violation of market
clearing (30) for the original allocation.

The international risk sharing condition (21) implies that the consumption subsidy
$\zeta^{c}_t$ has to follow the suggested path so that the real exchange rate is the same under
the proposed fiscal devaluation and the nominal devaluation, otherwise original relative
consumption across countries would not satisfy optimal risk sharing.

The money demand condition (18) can be written as

$$\chi C_t^\sigma P_t^\nu = \left(M_t(1 + \zeta^{c}_t)\right)^\nu \left[1 - \beta E_t \left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}} \frac{1 + \zeta^{c+1}_t}{1 + \zeta^{c}_t}\right\}\right].$$

Given $P_t$, both sides of this equation must remain identical under the proposed fiscal
devolution and the nominal devaluation. This requires that

$$\frac{M'_{t}}{M_t} = \frac{1}{1 + \delta_t} \left(\frac{1 - \beta E_t \left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}} \frac{1 + \zeta^{c+1}_t}{1 + \zeta^{c}_t}\right\}}{1 - \beta E_t \left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}} \frac{1 + \delta_t + \zeta^{c+1}_t}{1 + \delta_t + \zeta^{c}_t}\right\}\right)^{1/\nu}.$$
Note that under a one-time unanticipated devaluation, the bracket on the right-hand side is simply 1.

Finally, we verify that wages and prices are the same under the proposed fiscal devaluation and under the nominal devaluation. From the wage setting equation (26) it is clear that as long as \( \tau^*_n = \varsigma^c_n \), and given the path \( \{P_t, C_t, N_t, W_t\} \), the reset wage \( \bar{W}_t \) stays the same. This in turn implies the same path for \( W_t \) in (27). Turning to prices, note that we can rewrite price-setting condition (22) as

\[
\bar{P}_{Ht} = \frac{\rho}{\rho - 1} \frac{\mathbb{E}_t \sum_{s \geq t} (\beta \theta_p)^{s-t} C_s^{-\sigma} P_s^{-1} P^\rho_{Hs} Y_s \frac{(1+\varsigma^c_s)(1-\varsigma^d_s)}{1+\tau^d_s} W_s}{\mathbb{E}_t \sum_{s \geq t} (\beta \theta_p)^{s-t} C_s^{-\sigma} P_s^{-1} P^\rho_{Hs} Y_s \frac{(1+\varsigma^c_s)(1-\tau^d_s)}{1+\tau^d_s}},
\]

where we have substituted in the expression for the nominal stochastic discount factor

\[
\Theta_{t,s} = \beta^{s-t} \left( \frac{C_s}{C_t} \right)^{-\sigma} \frac{P_t}{P_s} \frac{1 + \varsigma^c_s}{1 + \varsigma^c_t} \quad \text{for} \quad s \geq t.
\]

Note that under \( \text{FDD}'' \), \( (1 + \varsigma^c_s)(1 - \tau^d_s) = (1 + \varsigma^c_s)(1 - \varsigma^d_s) = 1 \) for all \( s \geq 0 \), and therefore given the path \( \{C_t, P_s, Y_s, P_{Hs}, W_s, A_s\} \), and with \( \tau^d_s = 0 \), the reset price \( \bar{P}_{Ht} \) stays the same, and hence so does \( P_{Ht} \). Under \( \text{FDD}' \), however, we need to choose \( \tau^d_s = \delta_s \) in order to achieve this outcome. Under a one-time unexpected devaluation, we can choose \( \tau^d_t = 0 \) for all \( t \geq 0 \).

Thus, the proposed set of policies ensures an equilibrium with \( \mathcal{E}'_t = \mathcal{E}_0 \) for all \( t \geq 0 \), and unchanged consumption, output, and labor supply allocations, as well as wages and prices, as under a nominal \( \{\delta_t\} \)-devaluation. ■

Just like in the static model, under complete markets, a fiscal devaluation requires the use of a consumption subsidy. This instrument is necessary in order to match the path of the real exchange rate obtained under a nominal devaluation, which in turn affects state-contingent transfers and relative consumption allocation across countries.

The fact that money demand is interest-elastic does not require additional tax instruments. If the nominal devaluation is expected, its fiscal equivalent will involve a different path for the nominal interest rate. This requires adjustments in the money supply. For example an expected stochastic or deterministic one-time nominal devaluation is associated with an increase in the interest rate differential \( (i_{t+1}^* - i_{t+1}^*) \) before the devaluation, which does not occur under an equivalent fiscal devaluation. This naturally leads to a higher money demand at home which needs to be accommodated with a higher money supply.\(^{34}\) Additionally, a consumption subsidy expands effective real balances for a given money supply, and hence money supply can be reduced under a fiscal devaluation relative to a nominal devaluation, when a consumption subsidy is in place. Both these effects can be observed in the expression for \( M'_t/M_t \) in the proof. When money enters non-separably into the utility function, our conclusions still hold true for an unexpected devaluation, but implementing an expected devaluation requires an additional fiscal instrument affecting

\(^{34}\)Indeed, the difference in the interest rate differential comes from \( i_{t+1} - i_{t+1}^* \), since \( i_{t+1}^* \) remains unchanged across fiscal and nominal devaluations.
money demand.\footnote{Suppose money enters non-separably into the utility function. Now the marginal utility of consumption depends on both consumption $C_t$ and the level of real money balances $M_t(1+\varsigma^c_t)/P_t$. Therefore, risk-sharing and other conditions that involve the SDF (price setting, budget constraints) now depend on the path of real money balances. Previously, a differential movement in the nominal interest rate could be offset simply by adjusting $M_t(1+\varsigma^c_t)$ appropriately (see the proof). However, with non-separability $M_t(1+\varsigma^c_t)$ enters not only demand for money as previously, but also the marginal utility of consumption. In particular, the risk sharing condition now requires $M_t(1+\varsigma^c_t)$ to be the same across nominal and fiscal devaluations. This generates a conflict unless it is a one time unexpected devaluation and consequently the interest rate does not change. More generally, with anticipated devaluations, an equivalent fiscal devaluation is possible only with an additional instrument to offset the effects of movements in interest rates on money demand, for instance a tax on money holdings.}

Interestingly, a fiscal devaluation using tariff policy (FDD') without an adjustment in dividend tax has second order effects on price setting. This is because time-varying consumption taxes change the stochastic discount factor of consumers, which is used to discount firm profits. These effects are exactly offset by the adjustment in the VAT and the payroll tax under (FDD''), but under (FDD') they must be offset by an adjustment in dividend tax, except in the case of a one-time unexpected devaluation.

Importantly, an unexpected one-time nominal devaluation can be mimicked only by an equally unexpected fiscal policy change. An unexpected devaluation implies a zero-probability one-time event. It is important in our complete markets analysis that agents share risk optimally even across these zero-probability events, which requires the use of international bonds contingent on these zero-probability events. Deviating from this assumption requires taking a stand on a particular asset market structure. Indeed, we consider below the case of incomplete markets with trade in risk-free bonds and equities.

### 3.3 Incomplete asset markets

We now consider a number of cases with incomplete international asset markets. We start with the case where the only internationally-traded asset is a foreign-currency risk-free bond. Our results generalize to this case most straightforwardly. We then look at the case of home-currency risk-free bonds, and international trade in equities. In all these cases we assume that markets are complete within countries.\footnote{This assumption emphasizes greater risk-sharing within countries as compared to imperfect international risk-sharing. It also greatly simplifies wage setting, leaving it unchanged from our baseline framework above.}

**Foreign-currency bond economy** The home representative household’s budget constraint is given by

$$
\frac{P_tC_t}{1+\varsigma^f_t} + M_t + Q^*_t \mathcal{E}_t B^f_{t+1} \leq B^f_{t+1} \mathcal{E}_t + M_{t-1} + \frac{W_t N_t}{1+\tau^u_t} + \frac{\Pi_t}{1+\tau^d_t} - T_t,
$$

where $Q^*_t$ is the foreign-currency price of a risk-free bond paying one unit of foreign currency in all states next period, and $B^f_{t+1}$ is the quantity of these bonds held by the home household.
Most equilibrium condition are the same as with complete markets, with the exception of the international risk sharing condition (21) and of the country budget constraints (29). The optimal risk sharing condition is now replaced by

\[
Q_t^* = \beta \mathbb{E}_t \left\{ \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \cdot \frac{P_t^*}{P_{t+1}^*} \right\} = \beta \mathbb{E}_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \cdot \frac{P_t}{P_{t+1}} \cdot \frac{(1 + \epsilon_i t) \mathcal{E}_{t+1}}{(1 + \epsilon_f t) \mathcal{E}_t} \right\},
\]

which guarantees that consumption growth is equalized across countries in expectation, but no longer state-by-state as under complete markets.

The country budget constraints become

\[
Q_t^* \mathcal{E}_t B^f_{t+1} - \mathcal{E}_t B^f_t = P_{Ht} Y_t - P_tC_t + \mathcal{E}_t \left( \frac{\tau^m v + \tau_t m}{1 - \tau_t v} \cdot P_t^* C_{Ft} - \frac{\epsilon_t + \tau_t v}{1 - \tau_t v} \cdot P_{Ht}^* C_{Ht}^* \right),
\]

\[
Q_t^* B^f_{t+1} - B^f_t = P^*_t Y_t^* - P^*_t C^*_t,
\]

and the bonds-market clearing condition is \(B^f_t + B^f_{t+1} = 0\) for all \(t\). One of the two budget constraints is redundant by Walras Law. We can combine either one of the budget constraints with market clearing (30) and the law of one price (25) to obtain

\[
Q_t^* B^f_{t+1} - B^f_t = (1 - \gamma) \left[ P^*_t C^*_t - P_tC_t \cdot \frac{1 - \tau_t v}{1 + \tau_t m} \right].
\]

This leads us to:

**Proposition 7** In a dynamic PCP economy with foreign-currency bonds only, (i) a fiscal \(\{\delta_t\}\)-devaluation can be achieved either by (FDD) or by (FDDa), exactly as in Proposition 6; (ii) a one-time unexpected fiscal devaluation can be implemented without the use of consumption subsidy and income tax \((\epsilon_t^c = \tau_t^m = \tau_t^d = 0\) for all \(t \geq 0\)), with the other fiscal instruments still given by (FDD) or (FDDa), and with the same money supply as under a nominal devaluation \((M'_t = M_t\) for all \(t \geq 0\)).

**Proof:** (i) The proof follows the same steps as the proof of Proposition 6. The consumption subsidy \(\epsilon_t^c = \delta_t\) ensures that the new risk sharing condition (31) is met under the proposed fiscal devaluation with \(\mathcal{E}_t = \mathcal{E}_0\). The use of the VAT \(\tau_t^v = \delta_t/(1 + \delta_t)\) or import tariff \(\tau_t^m = \delta_t\) ensures that the right-hand side of (32) is unchanged, so that the choice of \(B^f_{t+1}\) is unaltered.

(ii) Note that under the unexpected devaluation, consumption risk-sharing in (31) is unaltered if \(\epsilon_t^c = 0\) because \(\Pr_t\{\mathcal{E}_s = 0\} = 1\) for all \(t < T, s \geq t\) and \(\Pr_t\{\mathcal{E}_s = \delta\} = 1\) for all \(t \geq T, s \geq t\), so that for all \(t \geq 0\)

\[
\mathbb{E}_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \cdot \frac{P_t}{P_{t+1}} \cdot \mathcal{E}_{t+1}/\mathcal{E}_t \right\} = \mathbb{E}_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \cdot \frac{P_t}{P_{t+1}} \right\}.
\]

With the rest of the fiscal instruments as in (FDD) or (FDDa), the choice of \(B^f_{t+1}\) is again unaltered in (32). Finally, with \(\epsilon_t^c = 0\), we need \(M'_t = M_t\) in order for (18) to hold. This is because under an unexpected nominal devaluation \(1 + i_{t+1} = 1 + i_{t+1}^*/Q^*_t\), and the same is true under an unexpected fiscal devaluation.
In contrast to the static case, we cannot generally carry out a fiscal devaluation in a dynamic foreign-currency bond economy without using consumption subsidies. This is because dynamic savings decisions depend on the dynamics of the real exchange rate. Consumption subsidies are needed in order to replicate the path of the real exchange rate that would obtain under a nominal devaluation. However, unlike the complete markets case, only the expected path of the real exchange rate matters, since risk is only shared in expectation. As a result, we can dispense with consumption subsidies when the devaluation is one-time unexpected.

What is the intuition behind the different conclusions for expected and unexpected devaluations? When devaluations are unexpected, the availability of a foreign-currency risk-free bond does not affect risk sharing relative to financial autarky, from the perspective of sharing risk of a devaluation. As a result, when the devaluations are unexpected, fiscal devaluations work very much like in a balanced-trade economy. Foreign-currency risk-free bonds help only to smooth consumption ahead of expected devaluations. When devaluations are expected, fiscal devaluations must replicate the effect of expected real exchange rate movements on optimal saving decisions, requiring the use of a consumption subsidy.

**Home-currency bond** We know from the static analysis that partial defaults on home-currency debt are a necessary ingredient of fiscal devaluations. We therefore introduce the possibility of partial defaults on home-currency bonds from the outset, in the form of a contingent sequence \( \{d_t\} \).

The international risk sharing condition becomes

\[
Q_t = \beta \mathbb{E}_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \frac{1 + \zeta_{t+1}^c (1 - d_{t+1})}{1 + \zeta_t^c} \right\} = \beta \mathbb{E}_t \left\{ \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{P_{t+1}^* \mathbb{E}_t}{P_{t+1}^* \mathbb{E}_{t+1}} (1 - d_{t+1}) \right\},
\]

where \( Q_t \) is the home-currency price of a home-currency risk-free bond; the bond promises to pay one unit of home currency at \( t + 1 \), and \( d_{t+1} \) is the haircut (partial default) on the holders of the bond. The country budget constraint can now be written as

\[
Q_t B_{t+1}^h - (1 - d_t) B_t^h = (1 - \gamma) \left[ \mathbb{E}_t P_{t+1}^* C_t^* - P_t C_t \frac{1 - \tau_t^v}{1 + \tau_t^m} \right],
\]

where \( B_t^h \) denotes home’s holdings of home-currency bonds.

A fiscal \( \{\delta_t\} \)-devaluation now requires:

\[
\zeta_t^c = \delta_t \quad \text{and} \quad 1 - d_t = \frac{1 + \delta_{t-1}}{1 + \delta_t}.
\]

This ensures that the foreign-currency value of the home bonds \( b_t^h = B_t^h / \mathbb{E}_{t-1} \) is the same in every period under both policies.\(^{37}\) This in turn guarantees that both the risk-sharing

\(^{37}\)To see this, rewrite the home country budget constraint (34) as

\[
Q_t b_{t+1}^h - (1 - d_t) \frac{\mathbb{E}_{t-1}}{\mathbb{E}_t} b_t^h = (1 - \gamma) \left[ P_{t+1}^* C_t^* - P_t C_t \frac{1 - \tau_t^v}{\mathbb{E}_t (1 + \tau_t^m)} \right].
\]
condition (33) and the country budget constraint (34) still hold under a fiscal devaluation for the allocation and prices associated with the nominal devaluation. Importantly, the partial default required in period \( t \) under a fiscal devaluation is equal to the devaluation rate that would occur under the nominal devaluation. Hence, a one-time devaluation requires a one-time partial default.

As long as we supplement our fiscal devaluations with an appropriate partial default policy, the results of our foreign-currency bond economy in Propositions 7 extend straightforwardly to the economy with home-currency bonds. The intuition for this result follows similar lines to that in the static section. We have:

**Proposition 8** In a dynamic PCP economy with home-currency bonds, (i) a fiscal \( \{\delta_t\}\)-devaluation can be achieved either by (FDD') or by (FDD''), exactly as in Proposition 6, supplemented with a partial default policy \( 1 - d_t = (1 + \delta_{t-1}) / (1 + \delta_t) \); (ii) a one-time unexpected fiscal devaluation of size \( \delta \) at \( t = T \) can be implemented without the use of consumption subsidy and income tax \( (\zeta_t = \tau^n_t = \tau^d_t = 0) \), with the other fiscal instruments still given by (FDD') or (FDD''), supplemented with an unexpected one-time partial default on outstanding home-currency debt, \( d_T = \delta / (1 + \delta) \), and \( d_t = 0 \) for all \( t \neq T \), while maintaining the same money supply as under a nominal devaluation (\( M_t' = M_t \)).

It is important to clarify our terminology. Home country holds debt when \( B_t^h < 0 \), and in this case a fiscal devaluation is indeed associated with a partial default of home on its debt obligations. In the alternative case when home holds net foreign assets, \( B_t^h > 0 \), instead of partial default a fiscal devaluation requires a partial debt forgiveness to the foreign country. Both partial default and partial debt forgiveness are direct transfers between home and foreign households and do not involve government taxation.

**Domestic and Foreign Equities** We can additionally consider economies with international trade in equities, that is shares in home and foreign firms. The home budget constraint is in this case:

\[
\frac{P_t C_t}{1 + \xi_t^c} + M_t + (\omega_{t+1} - \omega_t) \mathbb{E}_t \{ \Theta_{t+1} V_{t+1} \} - (\omega_t^* - \omega_t^*) \mathbb{E}_t \{ \Theta_{t+1} E_{t+1} V_{t+1}^* \} \leq \frac{W_t N_t}{1 + \tau_t^n} + \omega_t \frac{\Pi_t}{1 + \tau_t^d} + (1 - \omega_t^*) \mathcal{E}_t \Pi_t^* + M_{t-1} - T_t, \tag{35}
\]

where \( \omega_t \) and \( 1 - \omega_t^* \) are the shares of the home country in home and foreign firms respectively, and we have used the fact that the sum of shares held by home and foreign residents have to add up to one. \( V_t \) and \( V_t^* \) are date-\( t \) cum-dividend home-currency value of home stock market (i.e., aggregate of all home firms) and foreign-currency value of foreign stock market respectively. The dividend tax \( \tau_t^d \) on profits of home firms applies equally to home and foreign shareholders.\(^{38}\) All the other variables are as defined in

\(^{38}\)Foreign budget constraint is given respectively by

\[
\frac{P_t^* C_t^*}{1 + \xi_t^c} + M_t^* - (\omega_{t+1} - \omega_t) \mathbb{E}_t \left\{ \Theta_{t+1}^* \frac{V_{t+1}}{E_{t+1}^*} \right\} + (\omega_t^* - \omega_t^*) \mathbb{E}_t \left\{ \Theta_{t+1}^* V_{t+1}^* \right\} \leq \frac{W_t^* N_t^*}{1 + \tau_t^d E_t} + \omega_t^* \frac{\Pi_t^*}{1 + \tau_t^d E_t} + M_{t-1}^* - T_t^*,
\]
Section 3.1. The international risk-sharing conditions with trade in equity become:

\[
\mathbb{E}_t \sum_{s=t}^{\infty} \left( \Theta_{t,s} - \Theta_{t,s}^* \right) \frac{\Pi_s}{1 + \tau_s^d} = 0 \quad \text{and} \quad \mathbb{E}_t \sum_{s=t}^{\infty} \left( \Theta_{t,s}^* \frac{\xi_s}{\xi_t} - \Theta_{t,s}^* \right) \Pi_s^* = 0. \quad (36)
\]

With this, we can now prove:

**Proposition 9** *In a dynamic PCP economy with international trade in equities, (FDD′) and (FDD″), exactly as in Proposition 6, constitute a fiscal \{\delta_t\}-devaluation.*

The proof of this proposition is somewhat tedious and we provide it in the Appendix. The idea behind the proof is the following. A nominal devaluation leads to a negative valuation effect for foreigners holding home equity, and a positive valuation effect for home shareholders in foreign firms. This must be mimicked under a fiscal devaluation. Under (FDD″) the VAT-plus-payroll subsidy is an effective tax on profits of home firms, which reduces the dividends for its shareholders, home and foreign alike. A consumption subsidy for home households undoes the effects of this ‘profit tax’ for home households, and as such creates an effective subsidy on holding foreign equity, just like the valuation effects under nominal devaluation. This also ensures that under (FDD″), the risk-sharing conditions continue to hold for the same portfolio choice as under the nominal devaluation. As a result, both the risk-sharing conditions and the country budget constraints continue to hold for the original allocations; the same is true for the rest of the equilibrium system just like in the proof of Proposition 6. (FDD′) requires the tax on dividends of home firms to generate the same negative valuation effects for the foreign shareholders in the home firms.

Naturally, the policies proposed in Proposition 9 still constitute a fiscal devaluation when both foreign-currency bonds and equities are traded internationally. Additionally introducing home-currency debt, a fiscal devaluation would require a partial default characterized in Proposition 8. In a similar way we can extend the analysis to other asset classes without having to solve explicitly for the international portfolio allocation of countries—a problem with no general solution (see e.g., Devereux and Sutherland, 2008).

### 3.4 Local currency pricing

We now offer a brief treatment of the LCP case, leaving most formal details to the Appendix. The main conclusion here is that our results generalize to the case of local currency pricing. The LCP case is notationally somewhat more tedious, this is why we choose to separate it out from PCP, but conceptually there are no differences between the two analysis. We start by summarizing the changes in the equilibrium conditions in the LCP case relative to the PCP case. We then explain how the results of the PCP case extend to the LCP case.

and can be omitted by Walras Law. Value functions are given by

\[ V_t = \mathbb{E}_t \sum_{s=t}^{\infty} \Theta_{t,s} [\Pi_s / (1 + \tau_s^d)] \quad \text{and} \quad V^*_t = \mathbb{E}_t \sum_{s=t}^{\infty} \Theta^*_{t,s} \Pi^*_s, \]

and optimal risk-sharing conditions (36) below imply that households in both countries equally assess the valuation of the firms (this must be true in any equilibrium, in which both countries hold interior equity positions in both home and foreign firms, which for concreteness is the focus of our analysis).
Equilibrium conditions  The only fundamental change in the equilibrium conditions concerns price setting and international prices. The law of one price equations (24) and (25) no longer hold, and instead home firms set \( \bar{P}^*_Ht(i) \) and foreign firms set \( \bar{P}_Ft(i) \), when they get to adjust prices respectively. Our fiscal devaluations does not rely on whether firms adjust their domestic and international prices on the same dates or with the same frequency. Firms choose prices to maximize the net present value of profits in the domestic and international markets during the duration of the respective price. As a result, we have two new first order price-setting conditions for international prices, and we need to adjust the price-setting equation (22) for the domestic market (and its counterpart for foreign firms). The profits of home firms from home and foreign sales can be written as:

\[
\Pi_i^{Ht} = (1 - \tau^t_i)P_{Ht}(i)C_{Ht}(i) - (1 - \zeta^t_i)W_tN_{Ht}(i),
\]

and

\[
\Pi^*_i^{Ht} = (1 + \zeta^t_i)E_tP^*_Ht(i)C^*_Ht(i) - (1 - \zeta^t_i)W_tN^*_Ht(i),
\]

where \( N_t(i) = N_{Ht}(i) + N^*_Ht(i) \) is total labor use by home firm \( i \) which is split between production for home and foreign markets. Given the expression for profits, the optimality conditions for price setting can be characterized in the same way as in Section 3.1, the description of which we leave for the Appendix. The dynamics of prices satisfies similar laws of motion to (23), but we now need to follow the dynamics of four price indexes, \( \{P_{Ht}, P^*_Ht, P^*_Ft, P_{Ft}\} \).

The other changes to the equilibrium system are largely notational. Demand for good \( i \) is now

\[
Y_{Ht} = C_{Ht}(i) + C^*_Ht(i) = \left( \frac{P_{Ht}(i)}{P^*_Ht} \right)^{-\rho} C_{Ht} + \left( \frac{P^*_Ht(i)}{P_{Ht}} \right)^{-\rho} C^*_Ht,
\]

and we cannot in general take the price terms outside the brackets since the law of one price does not hold under LCP. As a result, we now need to treat \( C_{Ht} \) and \( C^*_Ht \) as two separate goods that have a non-unit relative price in general. The aggregate market clearing conditions (30) are now replaced by

\[
Y_{Ht} = C_{Ht} = \gamma \frac{P_tC_t}{P_{Ht}} \quad \text{and} \quad Y^*_Ht = C^*_Ht = (1 - \gamma) \frac{P^*_tC^*_t}{P^*_Ht},
\]

and the same for foreign good. Labor market clearing (28) must be adjusted in a similar way.

Importantly, the expressions for budget constraints of the countries under LCP are exactly the same as those under PCP (for example, see equations (32) and (34)).

\[\text{Notation under LCP case is more general, since it does not require that the law of one price holds, and equally applies under PCP, where a simplification is possible.}\]

\[\text{The only difference in the budget constraint (29) under complete markets is that we need to write home revenues more generally as } P_{Ht}C_{Ht} + (1 + \zeta^t_i)E_tP^*_HtC^*_Ht/(1 - \tau^t_i), \text{ instead of } P_{Ht}Y_t \text{ which is a simplification under the law of one price. The same is true in the case of international trade in equities. Note that there the risk-sharing conditions apply to total firm profits: } \Pi_t \equiv \int \Pi_t[i, di], \text{ where } \Pi_t = \Pi_{Ht} + \Pi^*_Ht, \text{ and similarly for } \Pi^*_t. \text{ When domestic dividends are taxed, the tax must apply equally to domestic and foreign operations.}\]
Generalization of the results The results of the PCP case extend directly: Propositions 6, 7, 8 and 9 hold with no change. This is because under the proposed fiscal devaluations, \( (FDD') \) or \( (FDD'') \), all four reset prices \( \{\bar{P}_{Ht}, \bar{P}_{Ht}^*, \bar{P}_{Ft}, \bar{P}_{Ft}^*\} \) are left unchanged relative to the nominal devaluation under consideration, for the same reasons we discussed following the proof of Proposition 6. Given prices, the other equilibrium relationships remain the same across LCP and PCP setups, and hence the results extend immediately. The formal details are in the Appendix.

These conclusions equally apply under all the international asset market structures we have considered—perfect risk sharing, trade in risk-free bonds denominated in either currency, and trade in equities. This underscores the robustness of the proposed fiscal devaluation policies, which can replicate the allocations under a nominal devaluation in different environments, even though the allocations per se are different from one environment to the other.

3.5 Capital

In this section, we explain how our characterization of fiscal devaluations change when we introduce capital in the model. We have streamlined the exposition to emphasize the main changes compared to the no-capital case. We start by summarizing the main changes in the model and equilibrium conditions. We then explain how the results of the no-capital case extend when capital is introduced in the model.

As we shall see shortly, when capital is added to the model, additional instruments are required. Specifically, we introduce an investment subsidy (an investment tax credit) \( \varsigma_{t}^{I} \), a tax on capital income \( \tau_{t}^{K} \), and a capital subsidy (a subsidy on the rental rate of capital) \( \varsigma_{t}^{K} \). The main conclusion is that once those instruments are introduced, our results generalize.

3.5.1 Equilibrium conditions

We first describe the main changes in the model and its equilibrium conditions. We adopt a formalization where firms rent the services from labor and capital on centralized markets, at prices \( W_{t} \) and \( i_{t}^{K} \), and capital is accumulate by households according to

\[
K_{t+1} = K_{t} (1 - \delta) + I_{t},
\]

where gross investment \( I_{t} \) combines the different goods in the exact same way as the consumption bundle \( C_{t} \).

Households face the following sequence of budget constraints

\[
\frac{P_{t}C_{t}}{1 + \varsigma_{t}^{c}} + M_{t} + \mathbb{E}_{t}\{\Theta_{t+1}B_{t+1}\} + \frac{P_{t}I_{t}}{1 + \varsigma_{t}^{I}} \leq B_{t} + M_{t-1} + \frac{W_{t}N_{t}}{1 + \tau_{t}^{n}} + \frac{i_{t}^{K}K_{t}}{1 + \tau_{t}^{K}} + \frac{\Pi_{t}}{1 + \tau_{t}^{d}} - T_{t}.
\]

where \( \varsigma_{t}^{I} \) is an investment tax credit. \( \tau_{t}^{K} \) is a tax on capital income.

The household first-order conditions are the same as in the model without capital. But there is now one additional first-order condition for capital accumulation

\[
\frac{U_{c,t} (1 + \varsigma_{t}^{c})}{(1 + \varsigma_{t}^{I})} = \beta\mathbb{E}_{t}U_{c,t+1} \left[ \frac{i_{t+1}^{K} (1 + \varsigma_{t+1}^{c})}{P_{t+1} (1 + \tau_{t+1}^{K})} + (1 - \delta) \frac{(1 + \varsigma_{t+1}^{c})}{(1 + \varsigma_{t+1}^{I})} \right].
\]
We now turn to producers. We assume that each firm operates a neoclassical production function

\[ Y_t(i) = A_t F (K_t(i), N_t(i)), \]

where \( K_t(i) \) is the firm’s capital input. Profits are given by

\[ \Pi_t^i = (1 - \tau_t^i)P_{ht}(i)Y_t(i) - (1 - \varsigma_t^p)W_tN_t(i) - (1 - \varsigma_t^k)i_t^kK_t(i), \]

where \( \varsigma_t^k \) is the capital subsidy. We can then solve the firm’s problem. The only non-trivial part concerns the reset price, which is given by

\[
\bar{P}_{ht}(i) = \frac{\rho}{\rho - 1} \frac{\mathbb{E}_t \sum_{s=t}^{\infty} (\beta \theta_p)^{s-t} C_s^{-\sigma} P_s^{-1} (1 + \varsigma_h^s) P_H^{\rho H_s} Y_s}{\mathbb{E}_t \sum_{s=t}^{\infty} (\beta \theta_p)^{s-t} C_s^{-\sigma} P_s^{-1} P_H^{\rho H_s} Y_s (1 + \varsigma_h^s)^{\frac{1 - \tau_t^c}{1 + \tau_t^c}}}. \]

In addition, the firm’s optimal mix of labor and capital use is given by

\[
\frac{L}{K} = \frac{\alpha}{1 - \alpha} \frac{1 - \varsigma_t^k}{1 - \varsigma_t^p}. \]

### 3.5.2 Generalization of the results

Consider first the complete markets case. Then a fiscal \( \{\delta_t\} \)-devaluation can be engineered exactly as in Proposition 6 supplemented with the following tax adjustments. For (FDD') an investment subsidy \( \varsigma_t^i = \delta_t \) and a tax on capital income \( \tau_t^k = \delta_t \) are needed. For (FDD''), a subsidy on the rental rate of capital \( \varsigma_t^k = \delta_t/(1 + \delta_t) \) is also needed.

Consider now the incomplete markets case. Then depending on whether only foreign-currency bonds or only domestic-currency bonds are traded, a fiscal \( \{\delta_t\} \)-devaluation can be engineered exactly as in Proposition 7 or Proposition 8, supplemented with the same tax adjustments on \( \varsigma_t^i, \tau_t^c \) and \( \varsigma_t^k \) described in the previous paragraph. Exactly as in Propositions 7 and 8, in the case where the fiscal devaluation is one-time unexpected, one can dispense with the use of the consumption subsidy, income tax and dividend tax (\( \varsigma_t^c = \tau_t^a = \tau_t^d = 0 \) for all \( t \geq 0 \)). Moreover, we can then dispense with the use of the investment subsidy and the tax on capital income (\( \varsigma_t^i = \tau_t^k = 0 \) for all \( t \geq 0 \)).

It is easiest to grasp the intuition for these results in the incomplete markets case, when the devaluation is one-time unexpected. In this case, under (FDD'), no further tax adjustments are required when capital is introduced into the model. This is not so for (FDD'') where a subsidy on the rental rate of capital is needed: exactly for the same reason that a payroll subsidy \( \varsigma_t^p = \delta_t/(1 + \delta_t) \) is needed to offset the effect of the VAT \( \tau_t^v = \delta_t/(1 + \delta_t) \), an equivalent subsidy on the rental rate of capital income \( \varsigma_t^k = \delta_t/(1 + \delta_t) \) is needed. Without this adjustment in rental rate subsidies, firms would have incentives to substitute labor for capital in production under a fiscal devaluation—an effect absent in a nominal devaluation.

Outside of this case, the consumption subsidy \( \varsigma_t^c \), the (labor) income tax \( \tau_t^a \) and the dividend tax \( \tau_t^d \) cannot be set to zero (even without capital). Instead one must set \( \varsigma_t^c = \tau_t^a = \tau_t^d = \delta_t \). This requires also adjusting investment subsidy \( \varsigma_t^i = \delta_t \) and a tax on capital income \( \tau_t^k = \delta_t \) so as not to distort investment decisions of households. Note that
the required adjustment on the labor income and capital income taxes are the same. This is consistent with adjusting a single tax on total income (from both labor and capital).

4 Optimal Devaluation: Numerical Illustration

So far we have not focused on whether a devaluation is optimal or desirable; we have simply asked whether it is possible to robustly replicate the real allocations that would follow a nominal devaluation, but keeping the nominal exchange rate fixed. This is because while the optimality of a devaluation is model dependent, equivalence, which is the focus of this paper is robust across many environments.

There are cases when a devaluation is optimal going back to the argument Milton Friedman made in favor of flexible exchange rates in an environment where prices are rigid in the producer’s currency (for a recent formal analysis of this argument see Devereux and Engel, 2007).41 In this section we examine another case where wages are rigid but prices are flexible. In this environment the optimal policy response to a negative productivity shock is a devaluation: nominal or fiscal.

We provide a simple numerical illustration of this case. For simplicity, we consider a small-open economy. The only international asset is risk-free foreign-currency bond traded at a constant rate $r^*$ such that $\beta (1 + r^*) = 1$. We introduce money into the model by way of a cash-in-advance constraint. The relevant parameters are chosen as follows: $\beta = 0.99$, $\theta_w = 0.75$, $\gamma = 2/3$, $\sigma = 4$, $\varphi = 1$, $\kappa = 1$, $\eta = 3$. Hence, a period corresponds to a quarter and the average wage duration is one year. The choice of the utility parameters does not affect qualitative properties of the dynamics of the small open economy, as long as the relative risk aversion is greater than one ($\sigma > 1$).

We consider the following experiment. The economy starts initially in a non-stochastic steady state with productivity $A_0 = 1$. At $t = 1$, home productivity permanently and unexpectedly drops by 10%.43 Because home is a small open economy, all the foreign variables remain unchanged. We consider equilibrium dynamic response to this shock under two regimes. First, the economy implements the optimal nominal or fiscal devaluation, and second, the economy maintains a fixed exchange rate and no change in the fiscal policy.

Figure 1 describes the dynamic path for the economy under the two regimes. First, consider the regime under which the exchange rate is devalued by 5%. Exactly the same outcome could be achieved through a fiscal devaluation, either by increasing import tariffs and export subsidies by 5 percentage points, or by lowering the payroll tax and increasing the VAT by 5 percentage points, with no need to use additional fiscal instruments. Recall that this equivalence result follows from Proposition 7 (part (ii)) of our dynamic analysis.

41 Hevia and Nicolini (2011) propose a New Keynesian small-open-economy model with trade in commodities as intermediate inputs. In this environment, a nominal devaluation can be the constrained optimal response to an exogenous terms-of-trade shock.

42 When $\sigma = 1$, productivity does not affect equilibrium nominal wage under fixed exchange rate, and therefore wage stickiness is not a binding constraint in the experiment we consider below. For $\sigma < 1$, under fixed exchange rate nominal wages increase in response to a negative productivity shock.

43 In our model, this drop in productivity given the nominal wage rate is equivalent to starting the economy at an initial nominal wage which is too high given productivity and price level.
Figure 1: Dynamic path of the economy under optimal devaluation and fixed exchange rate, following a one-time unanticipate 10% fall in productivity.

*RER is real exchange rate. In this economy, changes in RER are proportional to changes in the terms of trade, $\hat{Q} = \gamma \hat{S}$, therefore the dynamics of RER is qualitatively the same as that for the terms of trade, with RER being less volatile since $\gamma < 1$. 

---

*[72x693] Figure 1: Dynamic path of the economy under optimal devaluation and fixed exchange rate, following a one-time unanticipate 10% fall in productivity

[0x0] 0 10 20 30
[0x0] −0.1
[0x0] 0 Productivity

[0x0] 0 10 20 30
[0x0] 0.05
[0x0] Money

[0x0] 0 10 20 30
[0x0] 0
[0x0] Nominal Exchange Rate

[0x0] 0 10 20 30
[0x0] 0
[0x0] Terms of Trade (and RER*)

[0x0] 0 10 20 30
[0x0] 0
[0x0] Nominal Wage

[0x0] 0 10 20 30
[0x0] 0
[0x0] Real Wage

[0x0] 0 10 20 30
[0x0] 0.1
[0x0] Price of Home Good

[0x0] 0 10 20 30
[0x0] 0.1
[0x0] CPI

[0x0] 0 10 20 30
[0x0] 0.05
[0x0] Hours

[0x0] 0 10 20 30
[0x0] 0
[0x0] Output

[0x0] 0 10 20 30
[0x0] −5 $ \times 10^{-3}$
[0x0] Trade Balance

[0x0] 0 10 20 30
[0x0] −0.04
[0x0] Net Foreign Assets

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*xOptimal Devaluation

---

*xFixed Exchange Rate
in Section 3.

This devaluation replicates the flexible-price, flexible-wage allocation with no wage inflation. This allocation is perfectly constant: consumption drops, hours increase, output drops, the real wage drops, and the terms of trade appreciate. Note that the net foreign asset position remains at zero. Because the shock is permanent and the allocation constant, there are no additional opportunities for consumption smoothing through international borrowing and lending.

One way to understand why a devaluation achieves the flexible-price, flexible-wage outcome is as follows. With the productivity shock, two relative prices need to adjust: the real wage and the terms of trade. The combination of a nominal devaluation and a jump in the home price level is enough to perfectly and instantly hit both targets. Alternatively, one can think of the devaluation as a way to achieve the desired real wage adjustment without any nominal wage adjustment. All in all, a devaluation circumvents the sticky wage constraint.\(^{44}\)

Figure 1 also describes the dynamic path for the economy under fixed exchange rate—that is, an economy with neither nominal, nor fiscal devaluation following the productivity shock. Just like the flexible-price, flexible-wage economy, the sticky wage economy eventually achieves a lower real wage and an appreciated terms of trade. However, the initial adjustment in the home price level cannot alone (without a simultaneous adjustment in the nominal exchange rate) hit the two relative price targets that are the real wage and the terms of trade of the flexible-price, flexible-wage economy. Instead, part of the adjustment now comes in the form of a protracted wage deflation. The initial increase in the home price level results in a decrease in the real wage and appreciation of the terms of trade. But the initial appreciation in the terms of trade overshoots its long run level—the terms of trade appreciates more in the short run—while the real wage undershoots its long term value—the real wage decreases less the short run. In other words, the resulting short-run wage markup is too high, explaining why wage deflation takes place. This in turn leads to depressed hours and a negative output gap. Finally, that the terms of trade initially appreciate more than in the long run results in trade deficits, followed by trade surpluses. The trade deficits that occur early on can be seen as symptoms of a competitiveness problem.\(^{45}\)

5 Discussion

In this paper we propose two types of fiscal policies that can robustly implement allocations stemming from a nominal devaluation, but in an economy with a fixed exchange rate. One way to interpret these results is that when fiscal instruments are available, exchange rate stability need not imply that flexible-exchange-rate allocations are no longer feasible. The impossible trinity, or trilemma, in international macroeconomics refers to the impossibility of having an independent monetary policy in an open economy with fixed exchange rate.\(^{46}\)

\(^{44}\)In our economy, flexible-price, flexible-wage allocation is the first best if monopolistic markups in price and wage-setting are offset with appropriate subsidies.

\(^{45}\)It is also possible to understand these developments from the perspective of the capital account. While the shock is permanent, the transitional dynamics due to wage stickiness generates a recession in the short run, the effects of which on consumption can be smoothed through international borrowing.
exchange rates and free capital mobility (for a recent reference, see Obstfeld, Shambaugh, and Taylor, 2010). While it is indeed the case that fixed exchange rates and free capital mobility restrict monetary independence since nominal interest rates are tied down by a parity condition even in our framework, what we show is that when fiscal policies are added to the mix of instruments the allocations that are attainable are the same as those with a flexible exchange rate, or independent monetary policy. Therefore the restrictiveness on monetary policy is without cost in this model. This result is similar to that of Adao, Correia, and Teles (2009), with the difference that we consider allocations in a sticky price environment when nominal devaluations have real effects.

Next we discuss a number of important implementation issues.

**Revenue Neutrality** The proposed fiscal devaluations are robustly revenue-neutral in the sense that they do not lead to an increase in the primary deficit of the government. This is crucial for the viability of fiscal devaluations for many countries who consider implementing a competitive devaluation given the already high level of indebtedness of their governments.

**Non-uniform VAT** The implementability of fiscal devaluations may be limited by the fact that VAT often does not apply to certain non-tradable services such as construction and housing and that labor constitutes not the only variable factor of production for the firms. As we explained in footnote 9, an exact VAT-based fiscal devaluation requires *either* that an increase in VAT applies uniformly to all home-produced goods, including all non-tradables, or *or* that reduction in payroll tax extends only to the industries that face an increase in the VAT. Therefore, non-uniform application of the VAT does not limit considerably the implementability of a VAT-based devaluation.

**Multiple variable inputs** Our baseline assumption that labor is the only variable factor (apart from intermediates) is probably good in the short run. Hence our baseline analysis should still apply in the short-run when labor is likely to be the only variable factor, and when the most real effects of a devaluations are realized. The plausibility of the assumption is the only variable input decreases over time: at longer horizons there are other variable inputs in production (apart from intermediates). Then the government needs to subsidize these other factors, alongside with labor (again apart from intermediates). We illustrated this logic in the case of capital in Section 3.5 but this principle is more general.

**Tax Pass-through** We discuss our assumptions on the sensitivity of prices to exchange rate and tax shocks and relate it to existing empirical evidence. We restrict attention to the VAT+ payroll policy given its greater implementability. The propositions on equivalence rely on two sets of assumptions that would be normal to impose in a standard new Keynesian environment: One, foreign firms pass-through of exchange rate and VAT tax changes into prices at which they sell to the domestic market is the same, all else equal, that is conditional on the foreign wage. Two, domestic firms pass-through of VAT and payroll tax to domestic prices is the same, conditional on the domestic wage.
In the long-run, when prices are fully flexible these assumptions are natural. When the exchange rate and tax changes are large the long-run can be attained very quickly since firms will choose to adjust prices immediately. The question then is about the short-run, when as a large body of evidence suggests, prices adjust infrequently and respond sluggishly to shocks.

We now survey what empirical evidence exists on the short-run response of prices to exchange rate and tax policy changes. The first assumption requires symmetry of pass-through of exchange rate shocks and VAT shocks into foreign firms prices to the domestic market. Since existing papers in the literature do not directly address this question one is necessarily comparing evidence across different data sets and more importantly comparing cases where the tax shocks and exchange rate shocks are not necessarily similarly unanticipated or anticipated. Nevertheless, what evidence exists appears to support the assumption of similar pass-through rates. For instance, Campa, Golberg and Gonzalez-Minguez (2005) estimate that short-run (1 month) pass-through into import prices in the Euro Area is 66% (and 81% in 4 months (long-run)). Andrade, Carre, Benassy-Quere (2010) examine data on French exports to the Euro zone over the 1996-2005 period and document that median pass-through of VAT shocks that occurred in eleven EMU12 partner countries over this period is 70-82% at a one year horizon. While they lack higher frequency data they conclude that the evidence is consistent will similar pass-through behavior for exchange rate and VAT shocks over a year. The evidence also appears consistent with producer currency pricing.

Evidence on the second assumption on responses of domestic prices to VAT and payroll is even harder to come by. First, while there exist some studies on VAT pass-through at various horizons there are very few equivalent studies for payroll taxes. Carbonnier (J Pub E, 2007, 91) studies two French reforms that involved steep decreases in VAT in 1987 and then in 1999 and finds that domestic prices decreased by 57% in the new car sales market and 77% in the household repair services market very quickly. The extent of pass-through therefore varies by market. There is however no similar evidence for payroll tax changes in these markets. Further, the tax changes were of a very large magnitude and consequently more revealing of long-run pass-through. The one case study that involved both a VAT tax increase and a payroll tax cut is the German VAT increase of 3 percentage points and cut in employer and employee payroll contributions by 2.3 percentage points in 2007. Carare and Damminger (2010 IMF working paper) examine the effect of these policy changes on core inflation. They find evidence of staggered price adjustment to tax shocks. The tax policies were announced 13 months ahead of actual implementation and consistent with infrequent price adjustment they find that prices adjusted upward prior to implementation. They conclude that overall pass-through from VAT was 73% with about half of this occurring in the run-up to implementation and the other half at the time of implementation. This evidence however cannot be directly used to shed light on the symmetry assumption. Firstly, they focus on core inflation and do not distinguish between domestic and foreign price pass-through. Secondly, there is no evidence on pass-through of the payroll tax. Given that their identification

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46In September 1987, the VAT rate on car sales went down from the luxury-rate of 33.33% to the full-rate of 18.6%. In September 1999, the VAT rate on housing repair services went down from the full-rate of 20.6% to the reduced-rate of 5.5%
relies on comparing VAT effected goods with non-VAT goods they isolate only the VAT pass-through component. This evidence also does not shed light or unanticipated tax changes.

The existing evidence therefore does not shed much light on the second assumption. Consequently we briefly discuss how the equivalence proposition is impacted in the case of short-run asymmetry in pass-through rates between VAT and payroll. Most of the intuition can be conveyed using the static model of Section 2. We further restrict the environment to a small open economy so foreign prices are taken as exogenous. Specifically suppose that prices in the producer currency, while sticky in the short-run, can be indexed to VAT and payroll tax changes and by different degrees. That is,

\[ \hat{P}_H = \left[ \hat{P}_H \cdot \frac{(1 - \varsigma^p)^{\xi_p}}{(1 - \tau^v)^{\xi_r}} \right]^{\theta_p} \left[ \frac{1 - \varsigma^p W}{1 - \tau^v A} \right]^{1 - \theta_p} \]

where \( \xi_p \) and \( \xi_r \) measures the extent of indexation to the payroll subsidy and VAT respectively. Now Proposition 1 is altered as:

**Proposition 10** In a static PCP economy under balanced trade or foreign-currency non-contingent assets \((B^f \equiv \text{const}, B^h \equiv 0 \text{ state-by-state})\), a fiscal devaluation of size \( \delta \) can be attained by the following set of fiscal policies:

\[
\tau^v = \varsigma^p = \tau = 1 - \frac{1}{1 + \delta} \left( \frac{\theta_p (\xi_r - \xi_p)}{1 - 2 \gamma (1 - \theta_w) (1 - \theta_p)} \right),
\]

with \( M \) adjusted appropriately.

When \( \xi_r - \xi_p = 0 \), that is there is symmetry in pass-through of VAT and payroll tax or when prices are fully flexible, \( \theta_p = 0 \) we obtain our previous result of \( \tau = \frac{\delta}{1 + \delta} \). Otherwise the tax rate \( \tau \) is an increasing function of \( (\xi_r - \xi_p) \). The extreme case when equivalence breakdown is when \( \xi_r - \xi_p = 1 \) and \( \theta_p = 1 \).

**Implementation in a monetary union** We turn to the most pressing issue—how to implement a fiscal devaluation in a monetary union, where the member-countries give up their monetary policy independence and adopt a common currency hence abandoning the possibility of a nominal devaluation. In fact, current circumstances in the Euro Zone are the principal source of motivation for our analysis.

Generally, our proposed fiscal devaluations require a change in money supply at home. Note, however, that with cash-in-advance and under certain asset market structures (see Proposition 1 in the static analysis section), a change in money supply is not needed, and a fiscal devaluation can be implemented by means of a consumption subsidy only. Scenarios where changes in the home money supply are required raise interesting implementation questions in the context of a currency union, where money supply is controlled by a union central bank. In these cases implementation may call for an increase in money supply by the union central bank with the seignorage income from this policy transferred to the
home country.\textsuperscript{47} Equivalently, the union central bank can let the national central bank of the country under consideration print the required money. Such implementations cannot be thought of as a unilateral policy change by the home country. However, when home is small relative to the overall size of the currency union, no coordinated policy by the union central bank is needed, and a fiscal devaluation can be achieved by the home’s unilateral change in fiscal policy. The formal analysis of implementation in a monetary union is provided in the Appendix.\textsuperscript{48}

\textsuperscript{47}Formally, the monetary union’s central bank controls total money supply to all union members, as well as transfers of seigniorage income to individual countries, while money supply to individual countries is determined endogenously in order to maintain fixed exchange rate. In the (empirically relevant) limiting case as seigniorage income constitutes negligible share of country’s GDP, the transfer of seigniorage income becomes inessential.

\textsuperscript{48}Implementation in a currency union is additionally constrained by the mobility of labor across countries, while we assume throughout international immobility of labor. However, since our proposed fiscal devaluations have the same real effects as those of a nominal devaluation, the consequences of both types of policies for international labor movements must also be the same.
A Derivations for Section 3

A.1 Consumer problem and wage setting

The problem of a home household $h$ can be described by the following pair of Bellman equations

$$J_t^h = \max_{C_t^h, M_t^h, N_t^h, W_t^h} \left\{ U \left( C_t^h, \frac{M_t^h(1 + \zeta_t^h)}{P_t}, N_t^h \right) + \beta \theta_w \mathbb{E}_t J_{t+1}^h, W_t^h \right\} + \beta(1 - \theta_w) \mathbb{E}_t J_{t+1}^h,$$

$$J_t^h(W_{t-1}^h) = \max_{C_t^h, M_t^h, N_t^h} \left\{ U \left( C_t^h, \frac{M_t^h(1 + \zeta_t^h)}{P_t}, N_t^h \right) + \beta \theta_w \mathbb{E}_t J_{t+1}^h, W_t^h \right\} + \beta(1 - \theta_w) \mathbb{E}_t J_{t+1}^h,$$

where $J_t^h$ denotes the value of the household at $t$ upon adjusting its wage, and $J_t^h(W_{t-1}^h)$ is the value of the household which does not adjust its wage at $t$. In this later case, $W_t^h = W_{t-1}^h$. In both cases, the household faces the flow budget constraint

$$\frac{P_t C_t^h}{1 + \zeta_t^h} + M_t^h + \mathbb{E}_t \Theta_{t+1} B_{t+1}^h \leq B_t^h + M_{t-1}^h + \frac{W_t^h N_t^h}{1 + \tau_t} + \frac{\Pi_t}{1 + \tau_t} - T_t,$$

and labor demand

$$N_t^h = \left( \frac{W_t^h}{W_t} \right)^{-\eta} N_t,$$

taking $N_t$, $W_t$ and other prices as given, and given individual state vector $(B_t^h, M_{t-1}^h)$. Substitute labor demand into the utility and the budget constraint, and denote by $\mu_t^h$ a Lagrange multiplier on the budget constraint. Note that there exists a separate budget constraint for each state of the world at each date. The description of the state of the world includes whether the household resets its wage rate. State-contingent bonds allow to share risk across states when the household adjusts and does not adjust its wage. Since wage-adjusting event is an idiosyncratic risk, $\Theta_{t+1}$, and hence $\mu_t^h$, do not depend on whether the household adjusts its wage. Using this fact, we can manipulate the first order and envelope conditions for the household problem, to obtain the expression for $\Theta_{t+1}$ in the text and money demand (18). Additionally, we have:

$$\mu_t^h P_t / (1 + \zeta_t) = U C_t = C_t^{-\sigma}.$$

Now consider the optimality condition for the choice of $\tilde{W}_t^h$. The first-order and envelope conditions are:

$$0 = -\eta \kappa (W_t^h)^{-\eta(1+\varphi)-1}(W_t^h N_t)^{1+\varphi} + \frac{\mu_t^h}{1 + \tau_t^h}(1 - \eta)(W_t^h)^{-\eta} W_t^h N_t + \beta \theta_w \mathbb{E}_t \partial J_{t+1}^h / \partial W_t^h,$$

$$\frac{\partial J_t^h}{\partial \tilde{W}_{t-1}^h} = -\eta \kappa (\tilde{W}_t^h)^{-\eta(1+\varphi)-1}(W_t^h N_t)^{1+\varphi} + \frac{\mu_t^h}{1 + \tau_t^h}(1 - \eta)(\tilde{W}_t^h)^{-\eta} W_t^h N_t + \beta \theta_w \mathbb{E}_t \partial J_{t+1}^h / \partial \tilde{W}_t^h.$$

Combining these two conditions and solving forward imposing a terminal condition, we obtain the optimality condition for wage setting:

$$\mathbb{E}_t \sum_{s \geq t} (\beta \theta_w)^{s-t} \left[ -\eta \kappa (\tilde{W}_s^h)^{-\eta(1+\varphi)-1}(W_s^h N_s)^{1+\varphi} + \frac{\mu_s^h}{1 + \tau_s^h}(1 - \eta)(\tilde{W}_s^h)^{-\eta} W_s^h N_s \right] = 0.$$

Substituting in $\mu_t^h$ and doing standard manipulations results in the wage-setting condition (26) provided in the text.
A.2 Proof of Proposition 9: International trade in equities

Money demand, market clearing, good demand, and wage and price-setting equations (including international prices) are exactly as in Section 3.1, and we do not reproduce them. We only consider the equilibrium conditions that are different—the risk-sharing equations (36) and the budget constraints (35).

First, consider the risk-sharing conditions (36). Recall that the stochastic discount factors are

\[ \Theta_{t,s} = \beta \left( \frac{C_s}{C_t} \right)^{-\sigma} \frac{P_t}{P_s} \left( 1 + \zeta_s^c \right) \quad \text{and} \quad \Theta_{t,s}^* = \beta \left( \frac{C_s^*}{C_t^*} \right)^{-\sigma} \frac{P_t^*}{P_s^*}, \]

and aggregate after-tax profits are

\[ \frac{\Pi_t}{1 + \tau_t^d} = \frac{1 - \tau_t^v}{1 + \tau_t^d} P_{Ht} Y_t - \frac{1 - \zeta_t^p}{1 + \tau_t^d} W_t N_t \quad \text{and} \quad \Pi_t^* = P_{Ft}^* Y_t^* - W_t^* N_t^*. \]

Substituting these expressions into (36), one can directly verify that the equilibrium allocation of a nominal devaluation continues to satisfy (36) under both proposed (FDD') and (FDD'') and a constant nominal exchange rate.

Now we need to combine the home household budget constraint (35) with that for the foreign government budget constraint by

\[ M_t - M_{t-1} + T_t + TR_t = 0, \]

where now

\[ TR_t = \left( \frac{\tau_t^v}{1 + \tau_t^d} W_t N_t - \frac{\zeta_t^c}{1 + \tau_t^d} P_t C_t \right) + \frac{\tau_t^d}{1 + \tau_t^d} \Pi_t \]

\[ + \left( \tau_t^v P_{Ht} C_{Ht} - \zeta_t^p W_t N_t \right) + \left( \tau_t^v + \tau_t^m \right) P_{Ft}^* C_{Ft} - \zeta_t^p E_t P_{Ht}^* C_{Ht}^*. \]

Combining this with (35) and manipulating the expression (in particular, making use of the expression for profits), we get:

\[ \left[ 1 - \omega_t \right] \frac{\Pi_t}{1 + \tau_t^d} - \left( 1 - \omega_t^* \right) E_t \Pi_t^* + E_t \left( P_{Ft}^* C_{Ft} - P_{Ht}^* C_{Ht}^* \right) \]

\[ = \left( \omega_{t+1} - \omega_t^* \right) E_t \left\{ \Theta_{t+1} E_{t+1} V_{t+1}^* \right\} \]

The LHS of this equality reflects home’s Current Account deficit—the sum of international movement of dividend income (from home to foreign) and home’s trade deficit. The RHS is the Capital Account—adjustments in international portfolio positions (relocation of portfolio shares towards foreign). We now do the final manipulation by dividing the country budget constraint by \( E_t \) and applying the fact that home’s and foreign’s valuation of equities coincide. We obtain:

\[ \left[ 1 - \omega_t \right] \frac{\Pi_t}{E_t(1 + \tau_t^d)} - \left( 1 - \omega_t^* \right) \Pi_t^* + \left( P_{Ft}^* C_{Ft} - P_{Ht}^* C_{Ht}^* \right) \]

\[ = \left( \omega_{t+1} - \omega_t^* \right) E_t \left\{ \Theta_{t+1} V_{t+1}^* \right\} \]
By substituting in the expression for profits, one can directly verify that this budget constraint holds for the same allocation (including portfolio share \( \{\omega_t, \omega_t^s\} \)) under nominal devaluation and under both fiscal devaluations of Proposition 9.

This completes the proof as we demonstrated that the same allocation (real allocations, including portfolio shares, wages, producer prices) satisfy all the equilibrium conditions under both nominal and fiscal devaluations. ■

A.3 Dynamic analysis with LCP

Under LCP, a home firm \( i \) sets its domestic and export prices to maximize

$$\max_{P_t(i)} \mathbb{E}_t \sum_{s \geq t} \theta_p^{s-t} \Theta_{t,s} \frac{\Pi_t^{i_s} \Pi_t^{i_s}}{1 + \tau_d^s}$$

respectively, and where the expressions for profits from domestic and foreign operations are provided in the text. Note that labor demand of the firm for the two operation satisfies respectively \( N_{Ht}(i) = C_{Ht}(i)/A_t \) and \( N_{Ft}(i) = C_{Ft}(i)/A_t \), while the demand equations for \( C_{Ht}(i) \) and \( C_{Ft}(i) \) are the same as under PCP (see Section 3.1). Taking the first-order conditions for price setting and manipulating them in the same way as in the case of PCP, we obtain the expression for optimal price setting:

$$P_{Ht} = \left[ \rho_p P_{Ht}^{1 - \rho} + (1 - \rho_p) \bar{P}_{Ht}^{1 - \rho} \right]^{1/\rho}$$

$$P_{Ft}^{*} = \left[ \rho_p P_{Ft}^{1 - \rho} + (1 - \rho_p) \bar{P}_{Ft}^{1 - \rho} \right]^{1/\rho}$$

where we dropped firm-identifier \( i \) since all firms adjusting prices at \( t \) are symmetric and set the same prices. Note that the law of one price no longer holds in general. Finally, as under PCP, price index dynamics can be written as

$$A_t N_t = \int_0^1 Y_t(i) \, di = Y_{Ht} \int_0^1 \left( \frac{P_{Ht}(i)}{P_t} \right)^{-\rho} \, di + Y_{Ft}^{*} \int_0^1 \left( \frac{P_{Ft}^{*}(i)}{P_t} \right)^{-\rho} \, di.$$

\(^{49}\)Explicitly, we have \( \Pi_{Ft} = (1 - \tau_t^p) \rho_{Ft} \rho_t \), \( C_{Ft}(i) = W_t \), \( N_{Ft}(i) = C_{Ft}(i)/A_t^s \), and therefore

$$P_{Ft} = \frac{\rho}{1 - \rho} \mathbb{E}_t \sum_{s \geq t} (\beta \theta_p)^{s-t} C_s^{1 - \sigma} P_s^{\sigma} \frac{P_{Ft}^{*} \Pi_t^{i_s} \Pi_t^{i_s}}{1 + \tau_d^s}.$$

\( P_{Ft}^{*} \) satisfies a similar expression, but without \( (1 - \tau_t^p)/(1 + \tau_s^m) \) in the denominator.
All other equations, including country budget constraints and risk-sharing conditions under all asset-market structures, remain the same (see remarks in footnote 40).

We can now extend the PCP results to the case of LCP:

**Complete markets (extension of Proposition 6)** We only need to verify that \( \{\bar{P}_{Ht}, \bar{P}_{Ht}^*, \bar{P}_{Ft}, \bar{P}_{Ft}\} \) stay unchanged under (FDD') and (FDD'') relative to nominal devaluation. This is indeed the case. This also implies that \( \{P_{Ht}, P_{Ht}^*, P_{Ft}, P_{Ft}, P_t, P_t^*\} \) also stay unchanged. The rest of the proof is exactly the same as that of Proposition 6 in the text, since all remaining blocks of the equilibrium system are unchanged under LCP relative to PCP (in particular the perfect risk-sharing condition (21)). This is intuitive since the difference between LCP and PCP is only in the price-setting block of the equilibrium system.

**Incomplete markets (extension of Propositions 7-9)** As above, under LCP all price setting and price dynamics remain unchanged across fiscal and nominal devaluations. As we argued in the main text, country budget constraints under LCP are the same as those under PCP. The same is true for risk-sharing conditions under all asset-market structures. Therefore, again, the remainder of the proofs directly extend from the PCP case. As an example, consider the risk-sharing condition when home-firm equities are traded:

\[
E_t \sum_{s=t}^{\infty} \left( \Theta_{t,s} - \Theta_{t,s}^* \frac{\varepsilon_t}{\bar{E}_s} \right) \frac{\Pi_s}{1 + \tau^d_s} = 0,
\]

where now

\[
\frac{\Pi_t}{1 + \tau^d_t} = \frac{1 - \tau^w_t}{1 + \tau^d_t} P_{Ht} C_{Ht} + \frac{(1 + \sigma^z_t) \varepsilon_t}{1 + \tau^d_t} P_{Ht}^* C_{Ht}^* - \frac{1 - \tau^p_t}{1 + \tau^d_t} W_t N_t.
\]

Clearly, a nominal-devaluation allocation still satisfies this risk-sharing condition under both (FDD') and (FDD'').

**B Implementation in a Monetary Union**

Consider the dynamic model. In a monetary union, countries give up monetary independence, and delegate monetary policy to union central bank. As a result, the home’s national government budget constraint becomes

\[
T_t + \Omega_t + TR_t = 0,
\]

where \( \Omega_t \) denotes transfers from the union central bank, and again assuming (without loss of generality due to Ricardian equivalence) that the government runs a balanced budget. The foreign’s national government budget constraint is now \( T_t^* + \Omega_t^* = 0 \).

Naturally, since members of a currency union adopt the same currency, we have

\[
\varepsilon_t \equiv \bar{\varepsilon} = 1.
\]
The union central bank supplies money $\bar{M}_t$ and transfers the proceeds of seigniorage to the national governments:

$$\bar{M}_t - \bar{M}_{t-1} + \Omega_t + \Omega^*_t = 0.$$  

Total money supply is equal to $\bar{M}_t$, while we denote money demand by $M_t$ and $M^*_t$ in home and foreign respectively. Money market clearing requires

$$\bar{M}_t = M_t + M^*_t.$$  

Finally, the policy variables now are union money supply and transfers of seigniorage revenues, $\{\bar{M}_t, \Omega_t, \Omega^*_t\}$, along with fiscal policies of the national governments. Currency (money) supply to the member-states is now endogenous (effectively ensuring a constant nominal exchange rate), with union-wide money supply given by $\bar{M}_t$.

The remainder of the equilibrium system stays unchanged, including household budget constraints in home and foreign. Denote with primes ' the counterfactual values of variables under a fiscal devaluation when countries are not part of a monetary union. Section 3 described how to implement a fiscal devaluation under these circumstances using fiscal policies and national money supply. It is immediately obvious that implementing a fiscal devaluation in a monetary union requires, apart from adopting the same fiscal policies, that union-wide money supply follows:

$$\bar{M}_t = M_t' + M^*_t',$$

that is, exactly mimics the aggregate money supply in the two economies under a fiscal devaluation when they are not part of a monetary union. If this is not followed, money demands would not be satisfied at the nominal-devaluation allocations.

When international asset markets are complete, this is the only required policy of the union central bank, since state-contingent-bond positions in the decentralized asset markets will take care of the transfers of seigniorage across countries. Outside the case of complete markets, the union central bank must distribute seigniorage according to

$$\Omega_t = \Delta M'_t \quad \text{and} \quad \Omega^*_t = \Delta M'^*_t = 0.$$  

That is, since we considered the case of passive monetary policy by the foreign under a nominal devaluation, foreign earns no seigniorage revenues in that case. All seigniorage revenues from the monetary slackening by home remains with the home central bank under a nominal or fiscal devaluation without a currency union. This has to be mimicked under the monetary union as well, by having the union central bank make according transfers of union-wide seigniorage to national governments.

This fully characterizes implementation under a monetary union. $M_t$ and $M^*_t$ endogenously adjust to satisfy money demands at the nominal-devaluation allocations. To summarize, in general implementation in a monetary union requires active monetary policy by the union central bank, as well as a particular transfer policy of the corresponding seigniorage to the national governments.  

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50 Clearly, this implicitly assumes $E_t \equiv 1$ for all $t \geq 0$ under a fiscal devaluation without currency union, otherwise we need to normalize all home-currency variables by $E_0$.  

51 Naturally, the union-wide monetary policy ($\bar{M}_t$ money supply) can be implemented by means of a union-wide interest rate rule (by targeting $i^*_t + 1$, that is interest rate in foreign under a fiscal devaluation without monetary union).
There are two important special cases however (apart from the trivial case in which $\Delta M'_t = \Delta M''_t = 0$), when the required actions by the union central bank are reduced. First consider the limiting case of $\chi \to 0$, in which money demand shrinks holding constant consumption, prices and interest rates. For concreteness consider $\sigma = \nu = 1$, so we can rewrite (18) as

$$M_t = \chi \frac{1 + i_{t+1}}{(1 + \zeta_t)_{t+1}} P_t C_t,$$

that is, money demand shrinks relative to the nominal expenditure of the economy. In this case, $\Omega_t = \Delta M'_t$ becomes increasingly a more trivial component of home-economy revenues and expenditures, and in the limit it is zero. This means that when $\chi \approx 0$, transfers of seigniorage from the union central bank are (nearly) inconsequential for allocations in the economy.

The other special case is that of a small home relative to the size of the monetary union (in particular, in terms of money demand). In this case, $M_t / \bar{M}_t \to 0$, and hence $\Delta \bar{M}_t / \bar{M}_t = \Delta M'_t / \bar{M}_t \to 0$. Therefore, there is no need for the union central bank to emit additional money under these circumstances. Money supply endogenously relocate from foreign to home in response to a fiscal devaluation, without causing consequences for foreign due to the small (negligible) economic size of home. When $\chi \gg 0$, replicating a fiscal devaluation under a currency union additionally requires a (negligibly small from the point of view of foreign) transfer to home in the size of $\Delta M'_t$ (which might, however, not be negligible for home). Taking the limit $\chi \to 0$, again makes the size of the transfer negligible, and under these circumstances a fiscal devaluation can be implemented in the monetary union without any actions by the union central banks or exogenous transfers between the union members.
References


