Dynamics of Trade and Heterogeneity in General Equilibrium

Robert Dekle
University of Southern California

Hyeok Jeong
National Graduate Institute for Policy Studies (GRIPS)

Nobuhiro Kiyotaki
Princeton University

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Abstract

This paper develops a dynamic general equilibrium model that tries to reconcile the observation that aggregate movements of exports and imports are "disconnected" from real exchange rate movements, while firm-level exports co-move significantly with the real exchange rate. Firms are heterogenous, facing recurrent aggregate and firm-specific productivity shocks, choose which goods to export, and decide to enter and exit the business endogenously. We calibrate and estimate the model with both aggregate and firm level data from Japan.

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1 Introduction

Figure 1 displays the series of aggregate exports and imports together with the real exchange rate in Japan during the period of 1980-2009 in logarithmic scale. The real exchange rate is defined as the relative price between the two countries.[1] As Japanese goods become relatively more expensive, we would expect that exports would decrease and imports would increase. However, such a relationship between trade and the real exchange rate is not evident in Figure 1. As the yen becomes weaker, exports decline, and imports increase, which is the opposite of what we expect. During the entire sample period, the elasticity of exports with respect to the real exchange rate is -0.17, and that of imports is 0.08, although these estimates of elasticities are statistically insignificant. This lack of correlation, or correlation contrary to what we expect is an example of the so called "exchange rate disconnect puzzle," a long standing puzzle in international macroeconomics.[2] This weak or opposite correlation between aggregate exports (or imports) and the exchange rate is observed in many other countries as well (see Hooper, Johnson, and Marquez (2000), and Dekle, Jeong, and Ryoo (2007)).[3] Obstfeld and Rogoff (2000) mention that the exchange rate disconnect puzzle is one of the major puzzles in the international macroeconomics.[4]

Interestingly, after the year 2000, Figure 1 shows that aggregate exports positively co-moved with the real exchange rate, but aggregate imports also positively co-moved with the real exchange rate. These co-movements during this period suggests that a general equilibrium linkage may be important in order to understand the dynamics of trade and exchange rates in

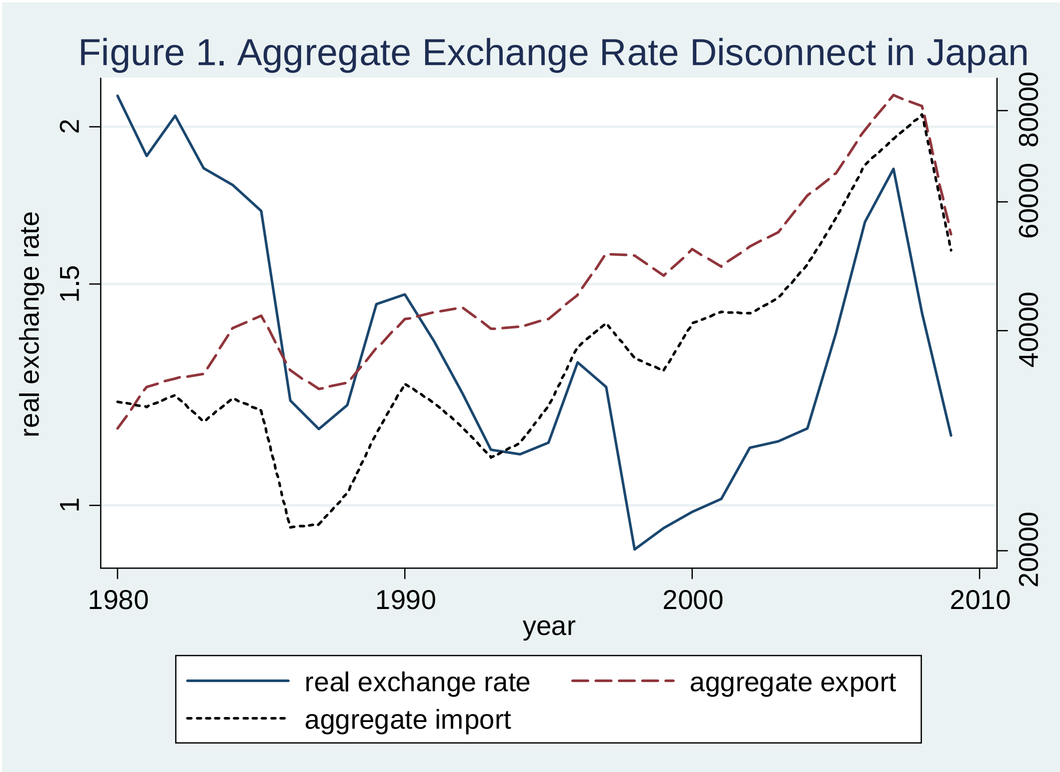
[1] The real exchange rate is measured as the ratio of Japanese prices to the weighted average of the prices of Japan's major trading partners, in yen terms, where the weights are the trading shares. The four major trading partner countries included here are the U.S., European Union, South Korea, and China and their trading shares are 0.49, 0.366, 0.095, and 0.044, respectively. (Sources: OECD Statistics)

Aggregate exports and imports are measured in billions of year 2000 yen. (Source: Ministry of Finance Trade Statistics: http://www.customs.go.jp/toukei/suii/html/time_e.htm)

[2] This empirical puzzle was first documented by Orcutt (1950).

[3] The list of other countries showing such weak correlation is Canada, France, Germany, Italy, the U.K., and the U.S.

[4] Note that this "exchange rate disconnect puzzle" is different from the so called "J-curve effect." The exchange rate disconnect puzzle is about the lack of association between the movements of exchange rates and *gross export quantities* while the J-curve effect is about the sluggish and J-shaped adjustment of trade balances (i.e., *net export sales*) in response to an improvement in the terms of trade. See Backus, Kehoe, and Kydland (1994) for the discussion of the J-curve effect.



Japan, where intermediate goods trade is dominant in imports, and increasingly more important in exports.

Recent empirical studies using firm-level data have found a more robust relationship between trade and the exchange rate. In contrast to the results using aggregate data, estimates using firm level tend to find a negative relationship between appreciating exchange rates and export quantities. Among other studies, Verhoogen (2008) finds that following the 1994 peso devaluation, Mexican firms increased their exports. Fitzgerald and Haller (2008), Dekle and Ryoo (2007), and Tybout and Roberts (1997) find a negative relationship between exports and exchange rate appreciation for Irish, Japanese and Colombian firms, respectively.

Some papers have tried to reconcile these aggregate and firm level results, but mostly in a partial equilibrium framework. Dekle, Jeong, and Ryoo (2007) show that in the aggregate export equation derived by consistently aggregating the firm level export equations, where industry level productivity and firm level (instrumented) export shares are controlled for, the disconnect puzzle disappears. Berman, Martin, and Mayer (2009) use a model with heterogeneous firms in the spirit of Melitz (2003) to show that high productivity firms (who are heavily involved in exports) will raise their prices–that is, increase their markups–instead of increasing their export quantities in response to an exchange rate depreciation. The authors show that this selection effect of low quantity response firms into the overall export market can explain the weak impact of exchange rate movements in aggregate data. There are some other recent papers that have tried to reconcile the discrepancy in a general equilibrium. Imbs and Majean (2009) and Feenstra, Russ, and Obstfeld (2010) show that the aggregation of heterogeneous industrial sectors can result in an aggregation bias in the elasticity of exports and imports with respect to exchange rates changes. Both of these papers examine only the steady-state.

In this paper, we develop a dynamic general equilibrium model with heterogeneous firms that attempts to reconcile the different responses of trade (exports and imports) to exchange rates at the aggregate- and at the firm-levels. Our model is a version of two-country real business cycle model with a rich production structure. Firms are heterogeneous, facing recurrent aggregate and firm-specific productivity shocks: they choose which varieties of goods to produce and export and decide to enter and exit endogenously. We calibrate and estimate our model with both aggregate and firm level data. We then carry out quantitative exercises regarding the impact of shocks to productivity and preferences on aggregate and firm-level exports and other variables of interest.[5]

We make a few choices to model heterogeneous firms to reflect our panel data of Japanese firms listed on the stock exchanges of Japan.[6] In a well-known paper, Melitz (2003) provides a

[5] One distinguishing feature of our work is the inclusion of heterogeneous firm dynamics that is actually estimated from firm level data. In the estimation of the firm-level responses, in addition to the firm level data, we rely on the aggregate variables and moments generated from the general equilibrium model. Thus, in a sense, we provide a general equilibrium model that is integrated with a structural model of heterogeneous firm dynamics that is estimated from actual firm level data.

[6] The raw data used here and in our paper are from almost all of the firms listed on the stock exchanges of Japan.The particular data set that we use were compiled by the Development Bank of Japan (or "Kaigin," in Japanese prior to the 2008 re-organization of government-owned enterprises, when the name of the bank was changed). Japanese listed firms cover a fairly respectable portion of the entire Japanese economy in terms of output (Fukao, et. al., 2008). In 2000, the gross sales of all the firms listed on the stock exchanges of Japan were 81 percent of Japanese nominal GDP, and 60 percent of total sales in the Japanese economy. However, listed firms are larger than the average firm in the economy. Thus, the number of listed firms account for less than 12 percent of the total number of Japanese firms, and the number of employees in listed firms are only 40 percent of all employees (Fukao, et. al., 2008).

framework where firms with different firm specific total factor productivity subject to fixed costs can generate heterogeneous exporting behavior. Das, Roberts, and Tybout (2007) provide an empirical study showing that this heterogeneity in total factor productivity among producers explains entry into and exit out of the export market, the so-called extensive margin of trade. In our Japanese panel data, there is a strong relationship between firm size and exporting status, as in Melitz (2003). The average total sales of the incumbent exporting firms is about double of the non-exporting firms. When firm level productivity is determined by a single factor of productivity, the Melitz type of trade model implies that the export share at the *intensive* margin (in addition to the extensive margin) be strongly correlated with firm size. Our Japanese firm level data show that this prediction is not true. The correlation between the export share and total sales is rather weak. The average correlation coefficient is only 0.08 among all firms. Among exporting firms, the correlation coefficient becomes even lower at 0.05. This weak correlation remains robust even after controlling for the industry and year effects.

Another interesting observation from Japanese firm level data is the significant presence of firms with negative profits staying in the market. About 8 percent of Japanese firms in our sample report negative profits. This fraction becomes even bigger at 11 percent among the always exporting firms, the biggest firms. Despite this presence of negative profits, Japanese listed firms do not easily exit from the business, although entry into and exit from the export market are rather frequent.

Given these empirical observations, we choose firms to produce multiple products and are heterogeneous in terms of the productivity of the best product as well as the span of the products. This two dimensional heterogeneity helps explain the weak relationships among size, the export share and profitability in our firm-level data. Our firms also face recurrent idiosyncratic productivity shocks, and thus they may not exit with temporary negative profits in order to enjoy the option value of continuing production.[7],[8] This explains our empirical

[7]Ghironi and Melitz (2005) analyze the dynamic effects of an aggregate productivity shock on the real exchange rate in a general equilibrium model with heterogeneous firms. But they concentrate on the extensive margin of products for export. Because there are no further idiosyncratic shocks after entry, there is no endogenous exit nor negative profits in their model.

[8]More broadly, our paper is related to the recent policy literature that examines how much of a real exchange

finding why Japanese firms with recurring negative profits resist to exit from their business.[9]

Section 2 presents the basic model of a small open economy and Section 3 presents the full model of two countries. In Section 4, we calibrate the parameters of the model either from structural estimation or from simply matching the moments. In Section 5, we quantitatively evaluate the model by simulating the aggregate dynamics.

2 A Basic Small Open Economy Model

Before examining the full model of the two countries, let us explain the mechanism by presenting a basic model of the small open economy. We ignore capital accumulation and the government sector in the basic model.

2.1 Set-up

There is a continuum of home firms $h \in \mathcal{H}_t$ each of which produces I_{ht} number of differentiated products for the home and export markets at date t. Firm h produces a differentiated product q^H_{hit} for the home market from labor l^H_{hit} and imported inputs m^{*H}_{hit}, according to a constant returns to scale technology

$$q^H_{hit} = a_{hit} Z_t \left(\frac{l^H_{hit}}{\gamma_L} \right)^{\gamma_L} \left(\frac{m^{*H}_{hit}}{1-\gamma_L} \right)^{1-\gamma_L}, \text{ for } i = 1, 2, .., I_{ht},$$

where a_{hit} is the productivity of firm h to produce the differentiated product (h, i) at date t and Z_t is the aggregate productivity shock and $\gamma_L \in (0, 1)$ is the labor share. Because no two firms produce the same product, each differentiated product is indexed by (h, i). Producing a differentiated product for the export market has the same marginal productivity as producing

rate depreciation is necessary to close a nation's current account imbalances. Obstfeld and Rogoff (2004) use a three-country model to calculate how much of a depreciation in the real exchange rate is needed to set the U.S. current account to zero. Dekle, Eaton, and Kortum (2008) fit their model to bilateral trade flows for 42 countries and solve for the new equilibrium in real exchange rates to eliminate all current account imbalances.

[9] Strictly speaking, in our sample of Japanese listed firms, firms that drop out of the sample are "delisted." Of the 2386 firms in our sample that we examine between 1985 and 1999, 104 firms became "delisted." We examined the circumstances surrounding the de-listing of all of these 104 firms and the vast majority were delisted because of bankruptcy or "ceasing to do business." A small number disappeared as independent firms because of mergers with stronger firms. Thus, we are on reasonably firm ground when we equate a firm that has been "delisted" as essentially "exiting" from production.

for the home market, but requires a fixed cost for each variety,[10]

$$q^F_{hit} = a_{hit} Z_t \left(\frac{l^F_{hit}}{\gamma_L} \right)^{\gamma_L} \left(\frac{m^{*F}_{hit}}{1-\gamma_L} \right)^{1-\gamma_L} - \phi, \text{ for } i = 1, 2, ..I_{ht}.$$

Home final goods are produced from a variety of differentiated products according to a constant returns to scale CES production function

$$Q^H_t = \left[\int_{h \in \mathcal{H}_t} \left(\sum_{i=1}^{I_{ht}} q^H_{hit}{}^{\frac{\theta-1}{\theta}} \right) dh \right]^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$ is the elasticity of substitution between among goods. Home output for export Q^F_t is produced from home differentiated goods

$$Q^F_t = \left[\int_{h \in \mathcal{H}_t} \left(\sum_{i=1}^{I_{ht}} q^F_{hit}{}^{\frac{\theta-1}{\theta}} \right) dh \right]^{\frac{\theta}{\theta-1}}.$$

Concerning the productivity of each differentiated product, a_{hit}, it is either high at $a_\alpha > 1$, low at 1 or zero. A new entrant firm who pays a sunk cost κ_E (in terms of home final goods) draws an opportunity of producing $b \in \{0, 1, 2\}$ number of new differentiated products

$$b = \begin{cases} 2, \text{ with probability } (wp.) \ \iota, \\ 1, \ wp. \ 1 - 2\iota, \\ 0, \ wp. \ \iota, \end{cases}$$

where $\iota \in (0, 1/2)$. The average number of new products drawn is normalized to be 1.[11] The productivity of each new product is independently distributed as

$$a_{hit} = \begin{cases} a_\alpha > 1, \ wp. \ \lambda\alpha \\ 1, \ wp. \ \lambda(1-\alpha), \\ 0, \ wp. \ 1 - \lambda. \end{cases}$$

Thus new entrants are heterogeneous in terms of number of differentiated products (width b) and the distribution of productivities of producible products (height a_{hit}). New firms can start production only from the next period.

[10] We assume that the fixed cost for exporting a differentiated product is constant in term of output - the first ϕ units of output exported are used to cover the fixed cost. Alternatively we can formulate the fixed cost in terms of input. We choose the current formulation because it is equally natural and slightly easier to aggregate.

[11] Because of this normalization, the dynamics of aggregate prices and quantities do not depend upon parameter ι, even though the dynamics of firm-level prices and quantities depend upon ι.

At the beginning of date t, the firm who has existing products must pay the fixed maintenance cost κ (in terms of home final goods) for each product in order to maintain its productivity. (The firm who wants to maintain I_{ht} number of products must pay $\kappa \cdot I_{ht}$). The product which the firm pays the fixed cost has the same productivity during this period ($a_{hit} = a_{hit-1}$) with probability $1-\delta$, and receives a new productivity draw according to the same distribution as a new entrant with probability δ. Thus the number of products each firm produces may increase or decrease depending upon the new draw of b, and the distribution of productivity of producible differentiated products changes depending upon the new draw of a_{hit}. Because firms are heterogeneous in terms of the width and the heights, we can show that there are only weak relationships among size, the export share and profitability across firms - a feature of our Japanese firm-level data.[12] If the firm does not pay the fixed cost κ for an existing product, it loses the technology for this product for sure and forever.

Because there is no capital accumulation nor the government sector, home final goods are either consumed or used for new entry and maintenance of the existing technology as

$$Q_t^H = C_t + \kappa_E N_{Et} + \kappa N_t',$$

where N_{Et} is the measure of entering firms and N_t' is the measure of differentiated products which firms try to maintain. We can think the cost of drawing new technology and maintaining old technology as investment in intangible capital.

The representative household supplies labor L_t to earn wage income, consumes final goods C_t and holds home and foreign real bonds D_t^H and D_t^{*H} to maximize its expected utility,[13]

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \psi_0 \frac{L_t^{1+1/\psi}}{1+1/\psi} + \xi_t^{*H} \ln D_t^{*H} \right),$$

subject to the budget constraint

$$C_t + \kappa_E N_{Et} + \kappa N_t' + D_t^H + \epsilon_t D_t^{*H} = w_{Lt} L_t + \Pi_t + R_{t-1} D_t^H + \epsilon_t R_{t-1}^* D_{t-1}^{*H}. \quad (1)$$

[12] In the full model, we allow for more general distribution of width and heights. The evolution of the heights is more general too, even though we abstract from the evolution of the width.

[13] We index goods by the origin and user countries. For the origin, we label goods by naught for home and by * for foreign. For the user, we label goods by H for home and by F for foreign. For example, D_t^{*H} is foreign bond that are held in the home country.

w_{Lt} is real wage rate, ϵ_t is the real exchange rate (the relative price of foreign and home final goods), and R_t and R_t^* are home and foreign real gross interest rates. Π_t is the real gross profits distribution from businesses, and the net profit is $\Pi_t - \kappa_E N_{Et} - \kappa N_t'$. Home and foreign bonds are used as means of saving. In addition we assume that the holding of foreign bonds facilitates transactions and provides utility. The utility from holding foreign bonds is subject to the "liquidity" shock, ξ_t^{*H}.[14] We assume that foreigners do not hold home bonds in the basic model. Then because there is no government sector, the net supply of home bonds to home agents is zero,

$$D_t^H = 0.$$

We assume that all home imports are inputs to production, and that the imported input price is normalized to be one in terms of foreign final goods. We assume that foreign aggregate demand for home exports are given by

$$Q_t^F = \left(p_t^F\right)^{-\varphi} Y_t^*, \tag{2}$$

where Y_t^* is an exogenous foreign demand parameter, p_t^F is an endogenous export price in terms of foreign final goods, and $\varphi > 0$ is the constant elasticity of demand for home exports. Foreign bond holdings D_t^{*H} of the home representative household evolves along with exports and imports as

$$D_t^{*H} = R_{t-1}^* D_{t-1}^{*H} + p_t^F Q_t^F - M_t^{*H},$$

where $M_t^{*H} = \int_{h \in \mathcal{H}_t} \left[\sum_{i=1}^{I_{ht}} (m_{hit}^{*H} + m_{hit}^{*F})\right] dh$ are total imported input of the home country.

2.2 Competitive Equilibrium

All markets for the factors of production and outputs are perfectly competitive, except that the market for differentiated products are monopolistically competitive.

Consistent with the usual CES production function of final goods manufactured from differentiated products, each firm faces a downward sloping demand curve for its product in home

[14] The idea is similar to money in utility function. Section 5.3.8 of Obstsfeld and Rogoff (1998) presents a model with both home and foreign money in the utility function to analyze the phenomenon of dollarization. We have both home and foreign bonds in utility for the full model. We ignore the utility of home bonds in the basic model because the net supply of home bonds are zero.

and foreign markets as a function of prices p^H_{hit} and p^F_{hit}, such that

$$q^H_{hit} = \left(\frac{p^H_{hit}}{p^H_t}\right)^{-\theta} Q^H_t, \tag{3}$$
$$q^F_{hit} = \left(\frac{p^F_{hit}}{p^F_t}\right)^{-\theta} Q^F_t,$$

where p^H_t and p^F_t are the price indices of home final output for the home and export markets

$$1 = p^H_t = \left[\int_{h\in\mathcal{H}_t}\left(\sum_{j=1}^{I_{ht}}\left(p^H_{hit}\right)^{1-\theta}\right)dh\right]^{1-\theta}, \tag{4}$$
$$p^F_t = \left[\int_{h\in\mathcal{H}_t}\left(\sum_{j=1}^{I_{ht}}\left(p^F_{hit}\right)^{1-\theta}\right)dh\right]^{1-\theta}.$$

We use home final goods as the numeraire in the home market, and foreign final goods as the numeraire in the foreign market.

Note that the production functions of differentiated products all have a common component: the constant returns to scale Cobb-Douglas function. Moreover the ratio of labor to imported inputs is equal across firms when firms minimize the costs under perfectly competitive factor market. Using our notation of composite input use for producing a given differentiated product for the home x^H_{hit} and export x^F_{hit} markets , the production functions can be simplified as

$$q^H_{hit} = a_{hit}\cdot x^H_{hit},$$
$$q^F_{hit} = a_{hit}\cdot x^F_{hit} - \phi.$$

Aggregate production of the input composite should be equal to the sum of input composite use

$$X_t = Z_t\left(\frac{L_t}{\gamma_L}\right)^{\gamma_L}\left(\frac{M^{*H}_t}{1-\gamma_L}\right)^{1-\gamma_L} = \int_{h\in\mathcal{H}_t}\left(\sum_{i=1}^{I_{ht}}(x^H_{hit}+x^F_{hit})\right)dh. \tag{5}$$

Because the price of imported inputs at home is equal to the real exchange rate (due to our choice of numeraire), cost minimization implies that the unit cost of the composite input w_t, and the demand for imported inputs satisfy

$$w_t = (w_{Lt})^{\gamma_L}\epsilon_t^{1-\gamma_L}/Z_t,$$
$$M^{*H}_t = (1-\gamma_L)\frac{w_tX_t}{\epsilon_t}. \tag{6}$$

After maximizing current profits, each firm sets price as a mark-up over their unit production cost

$$p_{hit}^{H} = \frac{\theta}{\theta - 1}\frac{w_t}{a_{hit}}, \qquad (7)$$
$$p_{hit}^{F} = \frac{\theta}{\theta - 1}\frac{w_t/\epsilon_t}{a_{hit}},$$

for all products produced. Because there are only two productivity levels, either high at a_α or low at 1, there are only two price levels and there are only two output levels of differentiated products for the home $\left(q_{\alpha t}^{H}, q_{1t}^{H}\right)$ and export $\left(q_{\alpha t}^{F}, q_{1t}^{F}\right)$ markets. Although each firm may produce multiple differentiated products, the firm can decide how to produce and whether to continue producing each product independently from their choice of the other products.[15]

We conjecture that in equilibrium, all firms choose to pay the fixed maintenance cost with positive productivity $(a_{hit} = a_\alpha \text{ or } 1)$. Let N_{Et} be the measure of new entrants, and let $N_{\alpha t}$ and N_{1t} be the measures of products with high and low productivity. Then the measure of products that firms try to maintain at the beginning of date t+1 is equal to the sum of the measure of products with positive productivity at date t $(N_t = N_{\alpha t} + N_{1t})$, and the measure of new entrants that draw positive productivity λN_{Et}

$$N_{t+1}' = N_t + \lambda N_{Et},$$

When firms try to maintain existing products, only a fraction $1-\delta+\delta\lambda$ remain to have positive productivity.

$$N_{t+1} = (1 - \delta + \delta\lambda)\, N_{t+1}' = (1 - \delta + \delta\lambda)\,(N_t + \lambda N_{Et})\,. \qquad (8)$$

From the specific feature of our idiosyncratic productivity evolution, the ratio of high- and low-productivity products (that are produced) remains constant:

$$N_{1t} = (1 - \alpha)N_t, \;\; N_{\alpha t} = \alpha N_t.$$

[15] The founder of Kyocera, Mr Kazuo Inamori, proposes an "amoeba" management style, in which each production unit makes relatively independent production decisions, while the number of production units multiply like "amoebas." Our technology can be seen as a justification for the "amoeba" management sytle. See also Bernard, Redding and Schott. (2010, 2011).

Then from $(4,7)$, the price index for home final goods for the home market becomes

$$\begin{aligned} 1 &= p_t^H = \frac{\theta}{\theta-1}\frac{w_t}{\Theta_t^H}, \text{ where} \qquad (9) \\ \Theta_{at}^H &= [N_{1t} + N_{\alpha t}(a_\alpha)^{\theta-1}]^{\frac{1}{\theta-1}} = \overline{a} N_t^{\frac{1}{\theta-1}} \text{ and } \overline{a} = [1-\alpha+\alpha(a_\alpha)^{\theta-1}]^{\frac{1}{\theta-1}}. \end{aligned}$$

Θ_{at}^H is aggregate productivity of home firms for the home market, and $\overline{a}$ is the average productivity of products that are produced.

We also conjecture that low productivity products make zero profits for exports, and thus only a fraction - $N_{1t}^F \in (0, N_{1t})$ measure of products with low productivity - are produced for exports, while all high productivity products are produced for exports. The zero profit condition for a differentiated product with low productivity is

$$0 = \epsilon_t p_{1t}^F q_{1t}^F - w_t\left(q_{1t}^F + \phi\right) = w_t\left(\frac{1}{\theta-1}q_{1t}^F - \phi\right).$$

Using $(3,4)$, we have the zero profit condition and the price index of home exports

$$\begin{aligned} \phi &= \frac{1}{\theta-1}q_{1t}^F = \frac{1}{\theta-1}\frac{Q_t^F}{(\Theta_{at}^F)^\theta}, \\ p_t^F &= \frac{\theta}{\theta-1}\frac{w_t/\epsilon_t}{\Theta_{at}^F} = \frac{1}{\epsilon_t}\frac{\overline{a}}{[n_{1t}^F + \alpha(a_\alpha)^{\theta-1}]^{\frac{1}{\theta-1}}}, \text{ where} \\ \Theta_{at}^F &= [n_{1t}^F + \alpha(a_\alpha)^{\theta-1}]^{\frac{1}{\theta-1}} N_t^{\frac{1}{\theta-1}}, \text{ and } n_{1t}^F = N_{1t}^F/N_t \in [0, 1-\alpha]. \end{aligned}$$

Θ_{at}^F is aggregate productivity of home firms for the export market. This aggregate productivity is an increasing function of an extensive margin, n_{1t}^F - the fraction of low productivity products that are exported. Exports of low productivity $\left(q_{1t}^F\right)$ and high productivity $\left(q_{\alpha t}^F\right)$ products are fixed as

$$\begin{aligned} q_{1t}^F &= (\theta-1)\,\phi, \\ q_{\alpha t}^F &= \left(\frac{p_{\alpha t}^F}{p_{1t}^F}\right)^{-\theta} q_{1t}^F = (a_\alpha)^\theta\,(\theta-1)\,\phi. \end{aligned}$$

Then from the export demand function (2), the export market clearing condition implies

$$\begin{aligned} Q_t^F &= \left(\frac{1}{\epsilon_t}\frac{\overline{a}}{[n_{1t}^F + \alpha(a_\alpha)^{\theta-1}]^{\frac{1}{\theta-1}}}\right)^{-\varphi} Y_t^* \\ &= (\theta-1)\,\phi[N_{1t}^F + N_{\alpha t}(a_\alpha)^{\theta-1}]^{\frac{\theta}{\theta-1}} = (\theta-1)\,\phi[n_{1t}^F + \alpha(a_\alpha)^{\theta-1}]^{\frac{\theta}{\theta-1}} N_t^{\frac{\theta}{\theta-1}}, \end{aligned}$$

or

$$
\begin{aligned}
n_{1t}^F &= \left[\left(\frac{\epsilon_t}{\overline{a}}\right)^{\varphi}\frac{Y_t^*}{(\theta-1)\,\phi}\right]^{\frac{\theta-1}{\theta-\varphi}}\frac{1}{N_t^{\frac{\theta}{\theta-\varphi}}}-\alpha(a_\alpha)^{\theta-1}, \qquad (10)\\
Q_t^F &= \left[\left(\frac{\epsilon_t}{\overline{a}N_t^{\frac{1}{\theta-1}}}\right)^{\varphi\theta}\frac{(Y_t^*)^{\theta}}{[(\theta-1)\,\phi]^{\varphi}}\right]^{\frac{1}{\theta-\varphi}},\\
\epsilon_t p_t^F Q_t^F &= \left\{\left[(\theta-1)\,\phi\left(\overline{a}N_t^{\frac{1}{\theta-1}}\right)^{\theta}\right]^{1-\varphi}\epsilon_t^{(\theta-1)\varphi}(Y_t^*)^{\theta-1}\right\}^{\frac{1}{\theta-\varphi}}.
\end{aligned}
$$

2.2.1 The "Disconnect" Between the Aggregate and Firm Level Responses to the Real Exchange Rate

Because the number of products produced N_t is a state variable, the extensive margin (the number of low productivity products which are exported $N_{1t}^F = n_{1t}^F N_t$) reacts to an exogenous shift in foreign demand (Y_t^*) and an endogenous shift in the real exchange rate. When the elasticity of foreign demand for home products is relatively inelastic compared to the elasticity of substitution among differentiated products (φ is small relative to θ), then the export quantity Q_t^F and export value (in terms of home final goods) are relatively insensitive to the real exchange rate shift. In contrast, because all the adjustment is through the number of low productivity products which are exported, the average export of low productivity products ($N_{1t}^F q_{1t}^F / N_{1t}$) is sensitive to the real exchange rate shift. This is because as in Green (2009), low productivity products are sensitive to aggregate shocks - their exports drop like "flies" when there is an adverse shock such as a real exchange rate appreciation. However, within a period, total exports of high productivity products $N_{\alpha t} q_{\alpha t}$ are insensitive to aggregate shocks; thus aggregate exports are less sensitive contemporaneously to aggregate shocks.

Our Japanese firm-level data (Kaigin data) are mostly of relatively large firms, which typically produce multiple products - possibly after a number of good new draws of $b = 2$. If a firm h has I_{ht} number of active products, some are high productivity products and some are low productivity products, and some of the low productivity products are exported and others are not:

$$
\begin{aligned}
I_{ht} &= I_{ht}^{\alpha} + I_{ht}^{1} \\
&= I_{ht}^{\alpha} + I_{ht}^{1F} + I_{ht}^{1N},
\end{aligned}
$$

where I_{ht}^{α} is the number of high productivity products, I_{ht}^{1} is number of low productivity products, I_{ht}^{1F} is number of low productivity products exported, and I_{ht}^{1N} is that which is not exported. The export sales of firm h in terms of home final goods are

$$
\epsilon_t (I_{ht}^{\alpha} p_{\alpha t}^{F} q_{\alpha t}^{F} + I_{ht}^{1F} p_{1t}^{F} q_{1t}^{F}).
$$

If we randomly sample firms which has I_{ht} number of products, the probability that we observe a given export sales at date t would be equal to

$$
\frac{I_{ht}!}{I_{ht}^{\alpha}! \cdot I_{ht}^{1F}! \cdot I_{ht}^{1N}!} \cdot \alpha^{I_{ht}^{\alpha}} \cdot \left(n_{1t}^{F}\right)^{I_{ht}^{1F}} \cdot (1 - \alpha - n_{1t}^{F})^{I_{ht}^{1N}}.
$$

Note that this probability is a function of the fraction of exported low productivity products, n_{1t}^{F}. Therefore, even if the output of each differentiated product $\left(q_{\alpha t}^{F}, q_{1t}^{F}\right)$ is constant, we should observe firms with a large fraction of low productivity differentiated products react strongly to the real exchange rate shift, by through the adjustment of the extensive margin at product level n_{1t}^{F}.

This heterogeneous reaction of exports to the real exchange rate shift across different products explains why firm level exports co-move significantly with the real exchange rate, while aggregate exports appear "disconnected" from the real exchange rate.[16]

2.2.2 The Equilibrium Dynamics of Our Economy

For the extensive margin to be the relevant margin, we need $N_{1t}^{F} \in (0, N_{1t})$, or

$$
0 < n_{1t}^{F} = \frac{N_{1t}^{F}}{N_t} < 1 - \alpha. \tag{11}
$$

[16] Our explanation of the extensive margin adjustment at product level is consistent with Dekle, Jeong and Ryoo (2007), which find that the apparent lack of relationship between the exchange rate and aggregate exports occur through the intensive margin of export sales within firms, rather than through the extensive margin of entry and exit of firms in the export market.

The aggregate composite input used for the production for the home and export markets are given by

$$\begin{aligned} X_t^F &= N_{1t}^F q_{1t}^F + N_{\alpha t}\frac{1}{a_\alpha}(q_{\alpha t}^F+\phi) = N_{1t}^F\theta\phi + N_{\alpha t}\frac{1}{a_\alpha}\phi[\theta-1)(a_\alpha)^\theta+1] \\ &= \phi N_t\left\{\theta[n_{1t}^F+\alpha(a_\alpha)^{\theta-1}]-\alpha[(a_\alpha)^{\theta-1}-(a_\alpha)^{-1}]\right\}, \\ X_t &= X_t^H + X_t^F. \end{aligned} \tag{12}$$

The labor supply condition of the household is given by

$$w_{Lt} = \psi_0 L_t^{\frac{1}{\psi}} C_t.$$

Together with $(6,9)$, we have

$$L_t = \frac{1}{(\psi_0 C_t)^\psi}\left(\frac{Z_t}{\epsilon_t^{1-\gamma_L}}\frac{\theta-1}{\theta}\overline{a}N_t^{\frac{1}{\theta-1}}\right)^{\frac{\psi}{\gamma_L}},$$

$$\begin{aligned} X_t &= \left[Z_t\left(\frac{\theta-1}{\theta}\frac{\overline{a}N_t^{\frac{1}{\theta-1}}}{\epsilon_t}\right)^{1-\gamma_L}\right]^{\frac{1}{\gamma_L}}\frac{L_t}{\gamma_L} \\ &= \frac{1}{\gamma_L(\psi_0 C_t)^\psi}\left(\frac{Z_t}{\epsilon_t^{1-\gamma_L}}\right)^{\frac{1+\psi}{\gamma_L}}\left(\frac{\theta-1}{\theta}\overline{a}N_t^{\frac{1}{\theta-1}}\right)^{\frac{1-\gamma_L+\psi}{\gamma_L}}. \end{aligned} \tag{13}$$

Here employment and the supply of the composite input depend upon $Z_t/\epsilon_t^{1-\gamma_L}$ (the productivity of the composite input production adjusted by the use of imported intermediate goods) and $\frac{\theta-1}{\theta}\overline{a}N_t^{\frac{1}{\theta-1}}$ (the aggregate productivity measure of home firms in the home market adjusted by the mark-up distortion).

The profits arising from exporting for the low- and high productivity products are

$$\begin{aligned} \pi_{1t}^F &= 0, \\ \pi_{\alpha t}^F &= \epsilon_t p_{\alpha t}^F q_{\alpha t}^F - w_t\frac{1}{a_\alpha}(q_{\alpha t}^F+\phi) \\ &= w_t\phi[(a_\alpha)^{\theta-1}-(a_\alpha)^{-1}] = \phi[(a_\alpha)^{\theta-1}-(a_\alpha)^{-1}]\frac{\theta-1}{\theta}\overline{a}N_t^{\frac{1}{\theta-1}}, \text{ (by (9))}. \end{aligned}$$

The profit arising from selling in the home market are

$$\begin{aligned} \pi_{1t}^H &= p_{1t}^H q_{1t}^H - w_t x_{1t}^H = \frac{1}{\theta}\overline{a}N_t^{\frac{1}{\theta-1}}\frac{X_t^H}{\overline{a}^{\theta-1}N_t} \\ \pi_{\alpha t}^H &= \frac{1}{\theta}\overline{a}N_t^{\frac{1}{\theta-1}}(a_\alpha)^{\theta-1}\frac{X_t^H}{\overline{a}^{\theta-1}N_t}. \end{aligned}$$

Let $V_{\alpha t}$ and V_{1t} be the values of the firms with high- and low productivities, respectively, in the previous period (who decided to pay the fixed cost of maintenance at the beginning of this period). The Bellman equations are

$$\begin{aligned} V_{\alpha t} &= -\kappa + (1-\delta+\delta\lambda\alpha)[\pi_{\alpha t}^{H} + \pi_{\alpha t}^{F} + E_t(\Lambda_{t,t+1}V_{\alpha t+1})] + \delta\lambda(1-\alpha)[\pi_{1t}^{H} + E_t(\Lambda_{t,t+1}V_{1t+1})], \\ V_{1t} &= -\kappa + \delta\lambda\alpha[\pi_{\alpha t}^{H} + \pi_{\alpha t}^{F} + E_t(\Lambda_{t,t+1}V_{\alpha t+1})] + [1-\delta+\delta\lambda(1-\alpha)]\,[\pi_{1t}^{H} + E_t(\Lambda_{t,t+1}V_{1t+1})], \end{aligned}$$

where $\Lambda_{t,t+1} = \beta C_t/C_{t+1}$. The necessary and sufficient condition that the firm strictly prefers to maintain a low productivity product by paying the fixed cost is

$$V_{1t} > 0. \tag{14}$$

Notice that this condition is satisfied even if realized current profits of each product (net of fixed cost) is negative, because the firm does not know its productivity this period when it chooses whether to pay the fixed cost of maintaining its existing products at the beginning of this period. (Recall that products that were productive earlier lose their productivity at the rate $\delta(1-\lambda)$). Net profits of low productivity products may also be negative $\left(-\kappa + \pi_{1t}^{H} < 0\right)$ because there is an option value for the low productivity product to become a high productivity product.[17] This helps explain why firms often record negative profits after paying their fixed costs of maintaining the business. In addition, because firms may have a large number of low productivity products, there is only a weak correlation between size and profitability across firms - another curious aspect of Japanese firms.

The value function of the average product produced is

$$\begin{aligned} \overline{V}_t &= \alpha V_{\alpha t} + (1-\alpha)V_{1t} \qquad (15) \\ &= -\kappa + (1-\delta+\delta\lambda)[\overline{\pi}_t + E_t\left(\Lambda_{t,t+1}\overline{V}_{t+1}\right)], \end{aligned}$$

[17]The option value due to the idiosyncratic productivity shock cannot be too large, because we conjecture that the firm will not maintain the product with zero productivity. Firms with zero productivity will not pay the fixed cost if

$$0 > -\kappa + \delta\lambda\alpha[\pi_{\alpha t}^{H} + \pi_{\alpha t}^{F} + E_t(\Lambda_{t,t+1}V_{\alpha t+1})] + \delta\lambda(1-\alpha)[\pi_{1t}^{H} + E_t(\Lambda_{t,t+1}V_{1t+1})].$$

where $\overline{\pi}_t$ is the average profit per product: $\alpha\left(\pi^F_{\alpha t}+\pi^H_{\alpha t}\right)+(1-\alpha)\left(\pi^F_{1t}+\pi^H_{1t}\right)$. Then we have

$$\overline{\pi}_t=\frac{1}{\theta}\overline{a}N_t^{\frac{1}{\theta-1}}\left\{\frac{X_t^H}{N_t}+\alpha\phi(\theta-1)[(a_\alpha)^{\theta-1}-(a_\alpha)^{-1}]\right\}. \tag{16}$$

The free entry condition for a new entrant is

$$\kappa_{Et}=\lambda E_t\left(\Lambda_{t,t+1}\overline{V}_{t+1}\right). \tag{17}$$

The final goods market clearing implies

$$C_t+\kappa_E N_{Et}+\kappa N_t'=\overline{a}N_t{}^{\frac{1}{\theta-1}}X_t^H. \tag{18}$$

Net foreign assets evolve as

$$D_t^{*H}=R_{t-1}^*D_{t-1}^{*H}+\left\{\left[(\theta-1)\,\phi\left(\frac{\overline{a}}{\epsilon_t}N_t{}^{\frac{1}{\theta-1}}\right)^{\theta}\right]^{1-\varphi}(Y_t^*)^{\theta-1}\right\}^{\frac{1}{\theta-\varphi}}-(1-\gamma_L)\frac{\theta-1}{\theta}\overline{a}N_t{}^{\frac{1}{\theta-1}}\frac{X_t}{\epsilon_t}. \tag{19}$$

From the utility maximization of the household, we have the demand for net foreign asset as

$$\epsilon_t-R_t^*E_t\left(\Lambda_{t,t+1}\epsilon_{t+1}\right)=\xi_t^{*H}\frac{C_t}{D_t^{*H}},$$

The left hand side (LHS) is the opportunity cost of holding one unit of the foreign bond. The right hand side (RHS) is the ratio of the marginal utility of foreign bond holdings and consumption. Because foreigners do not exchange foreign bonds for home bonds, the market clearing condition for net foreign assets is given as

$$D_t^{*H}=\frac{\xi_t^{*H}C_t}{\epsilon_t-R_t^*E_t\left(\Lambda_{t,t+1}\epsilon_{t+1}\right)}. \tag{20}$$

The aggregate state of our small open economy is described by the state variables $\mathcal{M}_t=\left(N_t,D_{t-1}^{*H},Z_t,\xi_t^{*H},Y_t^*,R_t^*\right)$, where the first two are endogenous and the last four are exogenous. The market equilibrium condition of exports, labor, composite input, home final goods and the net foreign assets together with the average value function, average profit function, free entry condition, the evolution of number of differentiated goods produced, and the net foreign assets $(8,10,12,13,15,16,17,18,19,20)$ determine $\left(n_{1t}^F,X_t,X_t^H,C_t,\epsilon_t,\overline{V}_t,\overline{\pi}_t,N_{Et},N_{t+1},D_t^{*H}\right)$ as a function of the state variables. The budget constraint (1) is automatically satisfied once all the market clearing conditions are satisfied, noting that aggregate profit is equal to the average profit multiplied by the number of products produced ($\Pi_t=\overline{\pi}_tN_t$).

2.2.3 Solving for the Model Equilibrium

We first examine the market clearing condition for net foreign assets (20) to fix intuition. Suppose as it is likely, that a liquidity shock to foreign bonds ξ_t^{*H} is very volatile in the short-run. The supply of net foreign assets changes sluggishly over time through the current account. Consumption is relatively smooth by permanent income theory if labor supply is relatively inelastic and the investment on technology $(\kappa_E N_{Et} + \kappa N_t')$ serves as a buffer to absorb shocks (which we have to verify later). Then since the liquidity shock to foreign bonds appears only in the market clearing condition for net foreign assets, the real exchange rate has to adjust quickly to the volatile liquidity shock, that is, adjust at high frequency - even though at low frequency, the adjustment of the current account and consumption are as important as the real exchange rate adjustment.

Thus in our economy, the high frequency movement of the real exchange rate is dominated by the liquidity shock. Empirically, we can treat the short-run movement of the real exchange rate as almost "exogenous", because we can always find a liquidity shock to justify the observed movement of real exchange rates as long as our boundary conditions $(11, 14)$ are satisfied and the evolution of net foreign assets is stable in the long-run. In addition, net foreign assets appear only in the equation for the evolution of net foreign assets (19).

Therefore, below we consider the following abbreviated or "shrunk" model. We take $\mathcal{M}_t' = (N_t, Z_t, Y_t^*, \epsilon_t^*)$ as the state variables, and solve for the eight endogenous variables $\left(n_{1t}^F, X_t, X_t^H, C_t, \overline{V}_t, \overline{\pi}_t, N_{Et}, N_{t+1}\right)$ as functions of the state variables which satisfy the eight equilibrium conditions $(8, 10, 12, 13, 15, 16, 17, 18)$. Here there is only one endogenous state variable, N_t, which greatly simplifies our analysis.

Equation (10) provides the expression for the quantity and the value of total exports as explicit functions of the state variables. Our discussion of the adjustment of exports through the extensive margin (number of low productive products) can then be made clearer, because we can now take the real exchange rate as exogenous. Moreover from $(15, 17)$, we have

$$\overline{V}_t = -\kappa + (1 - \delta + \delta\lambda)\left(\overline{\pi}_t + \frac{\kappa_E}{\lambda}\right).$$

Multiplying both sides of this expression at date t+1 by $\Lambda_{t,t+1}$, we have

$$\frac{\kappa_E}{\lambda} = E_t \Lambda_{t,t+1} \overline{V}_{t+1} = E_t \left\{ \Lambda_{t,t+1} \left[-\kappa + (1 - \delta + \delta\lambda) \left(\overline{\pi}_{t+1} + \frac{\kappa_E}{\lambda} \right) \right] \right\},$$

or

$$\kappa_E \left[1 - (1 - \delta + \delta\lambda) E_t \left(\Lambda_{t,t+1} \right) \right] = \lambda E_t \left\{ \Lambda_{t,t+1} \left[\left[1 - \delta + \delta\lambda \right) \overline{\pi}_{t+1} - \kappa \right] \right] \right\}.$$

The LHS is the cost of entering now instead of the next period, and the RHS is the expected net profits of the next period which the firm would lose by delaying entry. (Remember the entering firm can only produce from the next period). Using $(10, 12, 13, 16)$, this can be written as

$$\begin{aligned} & \kappa_E \left[1 - (1 - \delta + \delta\lambda) E_t \left(\beta \frac{C_t}{C_{t+1}} \right) \right] \qquad (21) \\ = \; & \lambda E_t \left[\beta \frac{C_t}{C_{t+1}} \left(-\kappa + \frac{1 - \delta + \delta\lambda}{N_{t+1}} \left\{ \begin{array}{c} \frac{\left(\frac{\theta - 1}{\theta} \right)^{\frac{1 - \gamma_L + \psi}{\gamma_L}}}{\theta \gamma_L (\psi_0 C_{t+1})^{\psi}} \left(\frac{Z_t}{\epsilon_{t+1}^{1 - \gamma_L}} \overline{a} N_{t+1}{}^{\frac{1}{\theta - 1}} \right)^{\frac{1 + \psi}{\gamma_L}} \\ + \alpha \phi \left(a_\alpha{}^{\theta - 1} - a_\alpha \right) \overline{a} N_{t+1}{}^{\frac{\theta}{\theta - 1}} \\ - \left[\left(\frac{\epsilon_{t+1}^{\varphi} Y_{t+1}^*}{\theta - 1} \right)^{\theta - 1} \phi^{1 - \varphi} \left(\overline{a} N_{t+1}{}^{\frac{1}{\theta - 1}} \right)^{\theta (1 - \varphi)} \right]^{\frac{1}{\theta - \varphi}} \end{array} \right\} \right) \right] \end{aligned}$$

Using $(8, 10, 12, 13)$, the goods market equilibrium (18) can be written as

$$\begin{aligned} & C_t + \kappa \frac{N_t}{1 - \delta + \delta\lambda} + \frac{\kappa_E}{\lambda} \left(\frac{N_{t+1}}{1 - \delta + \delta\lambda} - N_t \right) \qquad (22) \\ = \; & \frac{\left(\frac{\theta - 1}{\theta} \right)^{\frac{1 - \gamma_L + \psi}{\gamma_L}}}{\gamma_L (\psi_0 C_t)^{\psi}} \left(\frac{Z_t}{\epsilon_t^{1 - \gamma_L}} \overline{a} N_t{}^{\frac{1}{\theta - 1}} \right)^{\frac{1 + \psi}{\gamma_L}} + \alpha \phi \left(a_\alpha{}^{\theta - 1} - a_\alpha \right) \overline{a} N_t{}^{\frac{\theta}{\theta - 1}} \\ & - \theta \left[\left(\frac{\epsilon_t^{\varphi} Y_t^*}{\theta - 1} \right)^{\theta - 1} \phi^{1 - \varphi} \left(\overline{a} N_t{}^{\frac{1}{\theta - 1}} \right)^{\theta (1 - \varphi)} \right]^{\frac{1}{\theta - \varphi}} \end{aligned}$$

We can find the equilibrium function $C_t = C(\mathcal{M}_t')$ and $N_{t+1} = N(\mathcal{M}_t')$ which satisfy the two equilibrium conditions (21, 22).

Using (6) (10) and (12), home real GDP is given by

$$\begin{aligned} Y_t &= Q_t^H + \epsilon_t p_t^F Q_t^F - \epsilon_t M_t^{*H} \\ &= \overline{a} N_t^{\frac{1}{\theta - 1}} \left\{ X_t^H + \frac{\theta - 1}{\theta} \left[X_t^F + \alpha \phi (a_\alpha^{\theta - 1} - a_\alpha^{-1}) N_t \right] - (1 - \gamma_L) \frac{\theta - 1}{\theta} X_t \right\} \\ &= \overline{a} N_t^{\frac{1}{\theta - 1}} \left\{ \gamma_L \frac{\theta - 1}{\theta} X_t + \frac{1}{\theta} X_t - \phi (n_{1t} + \alpha a_\alpha^{-1}) N_t \right\}. \end{aligned}$$

The first term of RHS is wage, the second is gross profit and the last is the fixed cost for export.

Appendix A examines the steady state in order to derive condition for the extensive margin of products to adjust to the shocks in the export market.

3 The Full Two-Country Model

In this section, we extend the basic model into two countries in order to analyze fully the general equilibrium interactions. The full model also serves as the basis for our calibration and empirical study.

3.1 Physical Setup

There are two countries, home and foreign. Home final goods Y_t are produced at date t from home produced goods Y_t^H and foreign produced goods Y_t^{*H} such that

$$Y_t = \left[(\eta_t)^{\frac{1}{\varphi}} (Y_t^H)^{\frac{\varphi-1}{\varphi}} + (Y_t^{*H})^{\frac{\varphi-1}{\varphi}} \right]^{\frac{\varphi}{\varphi-1}}, \tag{23}$$

and foreign final goods Y_t^* are produced from home produced goods Y_t^F and foreign produced goods Y_t^{*F} such that

$$Y_t^* = \left[(Y_t^F)^{\frac{\varphi-1}{\varphi}} + (\eta_t^*)^{\frac{1}{\varphi}} (Y_t^{*F})^{\frac{\varphi-1}{\varphi}} \right]^{\frac{\varphi}{\varphi-1}}, \tag{24}$$

where η_t and η_t^* are taste shocks of home and foreign countries, respectively.

Home output for domestic use Q_t^H, for final goods Y_t^H and intermediate input M_t^H, is produced from a variety of home differentiated goods $Q_{ht}^H(\omega)$ such that

$$Y_t^H + \frac{M_t^H}{Z_{Mt}^H} = Q_t^H = \left[\int_{h \in \mathcal{H}_t} \int_{\omega \in \Omega} Q_{ht}^H(\omega)^{\frac{\theta-1}{\theta}} d\omega dh \right]^{\frac{\theta}{\theta-1}} \equiv \Theta_Q \left\{ \left[Q_{ht}^H(\omega) \right] \right\}, \tag{25}$$

where $\theta > 1$ and Θ_Q is the quantity aggregator over the differentiated products. There is a continuum of home firms indexed by $h \in \mathcal{H}_t$, each of which produces a continuum of differentiated goods indexed by $\omega \in \Omega = [0, \infty)$, and none of the differentiated product is produced by two firms. Hence each differentiated product is indexed by (h, ω). Home output for foreign use Q_t^F is produced from home differentiated goods such that

$$Y_t^F + \frac{M_t^F}{Z_{Mt}^F} = Q_t^F = \Theta_Q \left\{ \left[Q_{ht}^F(\omega) \right] \right\}. \tag{26}$$

Similarly, foreign output for its own use Q_t^{*F} and home country's use Q_t^{*H} are produced from foreign differentiated goods $Q_{ft}^{*F}(\omega)$ as

$$Y_t^{*F} + \frac{M_t^{*F}}{Z_{Mt}^{*F}} = Q_t^{*F} = \Theta_Q \left\{ \left[Q_{ft}^{*F}(\omega) \right] \right\}, \tag{27}$$

$$Y_t^{*H} + \frac{M_t^{*H}}{Z_{Mt}^{*H}} = Q_t^{*H} = \Theta_Q \left\{ \left[Q_{ft}^{*H}(\omega) \right] \right\}. \tag{28}$$

$\left(Z_{Mt}^{H}, Z_{Mt}^{F}, Z_{Mt}^{*H}, Z_{Mt}^{*F}\right)$ are intermediate input specific technology shocks.[18]

Each home firm h produces a variety of differentiated goods for home and foreign use from input composites $X_{ht}^{H}(\omega)$ and $X_{ht}^{F}(\omega)$, respectively. Exports require the fixed cost ϕ of producing each variety, and output shrinks by factor $1/\tau < 1$ during the export transportation so that outputs of home variety ω for home and foreign countries are

$$Q_{ht}^{H}(\omega) = a_{ht}(\omega) X_{ht}^{H}(\omega), \tag{29}$$

$$Q_{ht}^{F}(\omega) = \frac{1}{\tau} \left[a_{ht}(\omega) X_{ht}^{F}(\omega) - \phi \right], \tag{30}$$

where $a_{ht}(\omega)$ is the productivity level for variety ω specified as

$$a_{ht}(\omega) = A_{ht} \exp\left(-\frac{\omega}{B_h} \right). \tag{31}$$

Each firm's productivity is characterized by its productivity *profile* over the differentiated varieties $[a_{ht}(\omega)]_{\omega \in [0,\infty)}$ which is determined by the A_{ht} measuring the "height" and B_h measuring the "width" of firm h's productivity. A_{ht} corresponds to the productivity level of its best product. The number of products whose productivity is within a certain percentage deviation from the highest level is proportional to B_h. The height can vary over time, but the width is assumed to be time-invariant.

A new firm that pays a sunk cost κ_E at date t can independently draw a height A_{ht} and a width B_h from lognormal distributions of $\digamma_A^o(A_{ht})$ and $\digamma_B^o(B_h)$ such that

$$\ln A_{ht} - \mu_E = \nu_{E,ht} \sim \mathcal{N}(0, \sigma_{\nu_E}^2), \tag{32}$$

$$\ln B_h - \mu_b \sim \mathcal{N}(0, \sigma_b^2). \tag{33}$$

[18] These shocks include exogenous shock to the relative price of intermediate inputs such as an oil shock.

The new firm can start producing from the next period.

Each firm (including new entrants) faces an idiosyncratic shock to lose the technology and exits exogenously with probability $\delta_N \in (0,1)$. The firm whose height and width were A_{ht} and B_h in period t must pay a fixed cost κB_h in the beginning of period $t+1$ to draw a height A_{ht+1} from a distribution $F(A_{ht+1}|A_{ht})$ following an AR(1) process such that

$$\ln A_{ht+1} = (1-\rho_a)\mu_a + \rho_a \ln A_{ht} + \nu_{a,ht+1}, \text{ where } \nu_{a,ht+1} \sim \mathcal{N}(0, \sigma^2_{\nu_a}). \tag{34}$$

Without paying κB_h, the firm loses the technology (i.e., $A_{ht+1} = 0$ with probability 1) and exits. The distribution $\widetilde{\Phi}_t$ of the productivity of home firms (A_{ht}, B_h) evolves through entry, exit and the evolution of A_{ht} such as

$$N_{t+1}\widetilde{\Phi}_{t+1}(A_{ht+1}, B_h^{'}) = \begin{array}{l} \int_0^{B_h^{'}} \int_0^{\infty} F(A_{ht+1}|A_{ht}) 1_{t+1}(A_{ht}, B_h) \times \\ (1-\delta_N)\left[N_{Et} \digamma^o_A(dA_{ht}) \digamma^o_B(dB_h) + N_t \widetilde{\Phi}_t(dA_{ht}, dB_h)\right], \end{array} \tag{35}$$

where N_t is the number of firms that maintain the technology in the beginning of period t, and N_{Et} is the number of new entrants. $1_{t+1}(A_{ht}, B_h) = 1$ if the firm of productivity (A_{ht}, B_h) chooses to stay at the beginning of period t+1, and $1_{t+1}(A_{ht}, B_h) = 0$ otherwise.[19]

Similarly, each foreign firm f produces a variety of goods for home and foreign uses as

$$Q^{*H}_{ft}(\omega) = \frac{1}{\tau^*}\left[a^*_{ft}(\omega) X^{*H}_{ft}(\omega) - \phi^*\right], \tag{36}$$

$$Q^{*F}_{ft}(\omega) = a^*_{ft}(\omega) X^{*F}_{ft}(\omega), \tag{37}$$

where the productivity for a foreign firm f to produce a variety ω is

$$a^*_{ft}(\omega) = A^*_{ft} \exp\left(-\frac{\omega}{B^*_f}\right). \tag{38}$$

A foreign entrant pays a sunk cost κ^*_E to draw a new technology from lognormal distributions of $\digamma^{o*}_A(A^*_{ft})$ and $\digamma^{o*}_B(B^*_f)$ such that

$$\ln A^*_{ft} - \mu^*_E = \nu^*_{E,ft} \sim \mathcal{N}(0, \sigma^{*2}_{\nu_E}), \tag{39}$$

$$\ln B^*_f - \mu^*_b \sim \mathcal{N}(0, \sigma^{*2}_b). \tag{40}$$

[19] Note N_t denotes the measure of active firms here, while in the basic model, N_t denoted the measure of products produced.

Each foreign firm exits exogenously with probability $\delta_N^* \in (0,1)$ each period, and needs to pay $\kappa^* B_f^*$ in order to draw a height A_{ft+1}^* from a distribution $F(A_{ft+1}^*|A_{ft}^*)$ following an AR(1) process such that

$$\ln A_{ft+1}^* = (1-\rho_a^*)\mu_a^* + \rho_a^* \ln A_{ft}^* + \nu_{a,ft+1}^*, \text{ where } \nu_{a,ft+1}^* \sim \mathcal{N}(0, \sigma_{\nu_a^*}^2). \tag{41}$$

The productivity distribution of foreign firms $\widetilde{\Phi}_t^*(A_f^*, B_f^*)$ evolves through entry, exit, and the evolution of A_{ft}^* similarly as in (35).

The input composite is produced from labour L_t, capital K_t, home intermediate goods M_t^H, and foreign intermediate goods M_t^{*H} according to a constant returns to scale Cobb-Douglas production function. The resource constraint of the input composite for home country is

$$\begin{aligned} X_t &= Z_t \left(\frac{L_t}{\gamma_L}\right)^{\gamma_L} \left(\frac{K_t}{\gamma_K}\right)^{\gamma_K} \left(\frac{M_t^H}{\gamma_M^H}\right)^{\gamma_M^H} \left(\frac{M_t^{*H}}{\gamma_M^{*H}}\right)^{\gamma_M^{*H}}, \\ &= \int_{h\in\mathcal{H}_t}\int_{\omega\in\Omega} \left[X_{ht}^H(\omega) + X_{ht}^F(\omega)\right] d\omega dh, \end{aligned} \tag{42}$$

where Z_t is the home TFP and $\gamma_L + \gamma_K + \gamma_M^H + \gamma_M^{*H} = 1$. The resource constraint of the input composite for foreign country is similarly given as

$$\begin{aligned} X_t^* &= Z_t^* \left(\frac{L_t^*}{\gamma_L^*}\right)^{\gamma_L^*} \left(\frac{K_t^*}{\gamma_K^*}\right)^{\gamma_K^*} \left(\frac{M_t^F}{\gamma_M^F}\right)^{\gamma_M^F} \left(\frac{M_t^{*F}}{\gamma_M^{*F}}\right)^{\gamma_M^{*F}}, \\ &= \int_{f\in\mathcal{F}_t}\int_{\omega\in\Omega} \left[X_{ft}^{*H}(\omega) + X_{ft}^{*F}(\omega)\right] d\omega df, \end{aligned} \tag{43}$$

where Z_t^* is the foreign TFP and $\gamma_L^* + \gamma_K^* + \gamma_M^F + \gamma_M^{*F} = 1$.

The resource constraint of final goods for the home country is

$$C_t + G_t + \frac{1}{Z_{It}}\left[K_{t+1} - (1-\delta_K)K_t\right] + \kappa_E N_{Et} + \kappa E_t(B_h) N_t = Y_t. \tag{44}$$

C_t and G_t are consumption expenditures of households and the government. $\frac{1}{Z_{It}}\left[K_{t+1} - (1-\delta_K)K_t\right]$ is the tangible capital investment subject to investment specific technology shock Z_{It}. $\kappa_E N_{Et}$ is the cost of drawing new technology. $\kappa E_t(B_h)N_t$ is the cost of maintaining the existing technology, where $E_t(B_h)$ is the average width of the staying firms. The last two terms can be considered as gross investment in intangible capital. Similarly, the resource constraint of final goods for the foreign country is

$$C_t^* + G_t^* + \frac{1}{Z_{It}^*}\left[K_{t+1}^* - (1-\delta_K)K_t^*\right] + \kappa_E^* N_{Et}^* + \kappa^* E_t(B_f^*)N_t^* = Y_t^*. \tag{45}$$

3.2 Decentralized Economy

All the markets for factors of production and output are perfectly competitive, except that the markets for differentiated products are monopolistically competitive.

3.2.1 Government and Representative Household

The home government consumes G_t, taxes home households lump sum by T_t and issues short-term government bonds, D_t at the market gross real interest rate R_t subject to the budget constraint

$$G_t + R_{t-1}D_{t-1} = T_t + D_t. \tag{46}$$

The representative household owns the entire home capital stock and firms.[20] The household consumes, invests and purchases home and foreign government bonds to maximize their expected discounted utility

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \psi_0 \frac{L_t^{1+1/\psi}}{1+1/\psi} + \xi_t^H \ln D_t^H + \xi_t^{*H} \ln D_t^{*H} \right), \tag{47}$$

subject to the budget constraint as

$$\begin{aligned} & C_t + \frac{1}{Z_{It}}\left[K_{t+1} - (1-\delta_K)K_t\right] + \kappa_E N_{Et} + \kappa \overline{B} N_t + D_t^H + \epsilon_t D_t^{*H} \\ = \;& w_{Lt}L_t + w_{Kt}K_t + \Pi_t - T_t + R_{t-1}D_{t-1}^H + \epsilon_t R_{t-1}^* D_{t-1}^{*H}. \end{aligned} \tag{48}$$

The aggregate profit Π_t is defined as

$$\Pi_t = \overline{B} N_t \int_0^\infty \pi(A_{ht}, \mathcal{M}_t) d\Phi_t\left(A_{ht}\right),$$

[20] This is an important limitation of our analysis, because there are extensive holdings of foreign equities and capital. But because the home bias of equity holdings is still strong especially for our data up to 1999, this assumption may not be a bad approximation. This assumption greatly simplifies our analysis, because we can analyze the behavior of the representative household as if there were no stock markets, and can use the resulting consumption rule to compute the stochastic discount factor $\Lambda_{t,t+1}$.

Perhaps a more serious limitation of our analysis is the absence of overseas production of home firms. Perhaps in future, we can extend our analysis by allowing home firm h to spend a sunk cost κ_E^F to draw a new technology (A_h^*, B_h^*) for foreign production which depends on (A_h, B_h).

where $\pi(A_{ht}, \mathcal{M}_t)$ is given in (61). In the utility function, we include the utility of home and foreign bond holdings to capture the transaction frictions in home and foreign markets. Their first order conditions imply

$$\begin{aligned} w_{Lt} &= \psi_0 L_t^{1/\psi} C_t, & (49) \\ \frac{1}{Z_{It}} &= E_t \left[\Lambda_{t,t+1} \left(w_{Kt+1} + \frac{1-\delta_K}{Z_{It+1}} \right) \right], & (50) \\ D_t^H &= \xi_t^H \left[1 - R_t E_t \left(\Lambda_{t,t+1} \right) \right]^{-1} C_t, \\ D_t^{*H} &= \xi_t^{*H} \left[1 - R_t^* E_t \left(\Lambda_{t,t+1} \epsilon_{t+1} / \epsilon_t \right) \right]^{-1} C_t / \epsilon_t. \end{aligned}$$

The gap between unity and $R_t E_t (\Lambda_{t,t+1})$ is the opportunity cost of holding home bonds, which reflects the liquidity service of home bonds that facilitate transactions.

Similarly, the foreign representative household owns the entire foreign capital stock and firms and maximizes the utility

$$U_0^* = E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t^* - \psi_0^* \frac{L_t^{*1+1/\psi^*}}{1+1/\psi^*} + \xi_t^F \ln D_t^F + \xi_t^{*F} \ln D_t^{*F} \right), \quad (51)$$

subject to the budget constraint as

$$\begin{aligned} & C_t^* + \frac{1}{Z_{It}^*} \left[K_{t+1}^* - (1-\delta_K) K_t^* \right] + \kappa_E^* N_{Et}^* + \frac{D_t^F}{\epsilon_t} + D_t^{*F} & (52) \\ = \; & w_{Lt}^* L_t^* + w_{Kt}^* K_t^* + \Pi_t^* - T_t^* + R_{t-1} \frac{D_{t-1}^H}{\epsilon_t} + R_{t-1}^* D_{t-1}^{*F}. \end{aligned}$$

The bond market clearing conditions are the demands by foreign households

$$\begin{aligned} D_t &= D_t^H + D_t^F \\ &= \frac{\xi_t^H C_t}{1 - R_t E_t (\Lambda_{t,t+1})} + \frac{\xi_t^F C_t^* \epsilon_t}{1 - R_t E_t \left(\Lambda_{t,t+1}^* \epsilon_t / \epsilon_{t+1} \right)}, & (53) \\ D_t^* &= D_t^{*H} + D_t^{*F} \\ &= \frac{\xi_t^{*H} C_t / \epsilon_t}{1 - R_t^* E_t (\Lambda_{t,t+1} \epsilon_{t+1} / \epsilon_t)} + \frac{\xi_t^{*F} C_t^*}{1 - R_t^* E_t \left(\Lambda_{t,t+1}^* \right)}. & (54) \end{aligned}$$

These shocks to the utility of bond holdings $\left(\xi_t^H, \xi_t^{*H}, \xi_t^F, \xi_t^{*F} \right)$ follow an AR(1) process, and help explain the volatile movement of the real exchange rate, even though we will not use all the shocks in the calibration.

3.2.2 Production of firms

A home firm h owns the same constant returns to scale technology as (42), to produce the composite input and use this composite input to produce a variety of differentiated goods for home and foreign uses according to the menu of technologies described by equations (29) to (34). Consistent with the CES quantity indices in (25) and (26), each firm faces a downward sloping demand curve for its products in home and foreign markets, as a function of prices $p_{ht}^{H}(\omega)$ and $p_{ht}^{F}(\omega)$ such that

$$
\begin{aligned}
Q_{ht}^{H}(\omega) &= \left[\frac{p_{ht}^{H}(\omega)}{p_t^{H}}\right]^{-\theta} Q_t^{H}, \\
Q_{ht}^{F}(\omega) &= \left[\frac{p_{ht}^{F}(\omega)}{p_t^{F}}\right]^{-\theta} Q_t^{F},
\end{aligned}
$$

where p_t^H is the price index of home output for home use corresponding to the quantity index of Q_t^H in (25) and p_t^F is that of home output for foreign use corresponding to Q_t^F in (26), given by

$$
\begin{aligned}
p_t^H &= \left[\int_{h\in\mathcal{H}_t}\int_{\omega\in\Omega} p_{ht}^{H}(\omega)^{1-\theta} d\omega dh\right]^{\frac{1}{1-\theta}} \equiv \Theta_P\left\{\left[p_{ht}^{H}(\omega)\right]\right\}, \qquad (55)\\
p_t^F &= \Theta_P\left\{\left[p_{ht}^{F}(\omega)\right]\right\}.
\end{aligned}
$$

We use home final goods as the numeraire in the home market, and foreign final goods as the numeraire in the foreign market.

Because of the common constant returns to scale production function in (42), the unit cost of the composite input w_t is

$$
w_t = w_{Lt}{}^{\gamma_L} w_{Kt}{}^{\gamma_K}\left(w_{Mt}^{H}\right)^{\gamma_M^H}\left(w_{Mt}^{*H}\right)^{\gamma_M^{*H}}/Z_t,
$$

where w_{Lt} is real wage rate, w_{Kt} is cost of capital, and w_{Mt}^H and w_{Mt}^{*H} are the costs of home and foreign produced intermediate inputs. w_{Mt}^H is equal to $\frac{p_t^H}{Z_{Mt}^H}$ because the intermediate goods specific technology shock Z_{Mt}^H in (25) captures the movement of relative price between home final goods and home intermediate goods. Similarly, $w_{Mt}^{*H}=\frac{p_t^{*H}}{Z_{Mt}^{*H}}$, where p_t^{*H} is the price index of foreign output for home use. Thus, the unit cost w_t can also be written as

$$w_t = w_{Lt}{}^{\gamma_L} w_{Kt}{}^{\gamma_K} \left(\frac{p_t^H}{Z_{Mt}^H} \right)^{\gamma_M^H} \left(\frac{p_t^{*H}}{Z_{Mt}^{*H}} \right)^{\gamma_M^{*H}} / Z_t. \tag{56}$$

Maximizing current profits subject to the constraints of the present technology and aggregate market conditions, each firm sets the price as a mark-up over their unit production costs as

$$\begin{aligned} p_{ht}^H(\omega) &= \frac{\theta}{\theta - 1} \frac{w_t}{a_{ht}(\omega)}, \\ p_{ht}^F(\omega) &= \frac{\theta}{\theta - 1} \frac{\tau w_t / \epsilon_t}{a_{ht}(\omega)}, \end{aligned}$$

for all ω produced, where ϵ_t is the real exchange rates (the relative price of foreign to home final goods). Because all firms produce all the available products for the home market, the price index for home output for home use becomes

$$p_t^H = \frac{\theta}{\theta - 1} \frac{w_t}{\Theta_{at}^H}, \tag{57}$$

where

$$\begin{aligned} \Theta_{at}^H &= \Theta_a^H \{[a_{ht}(\omega)]\} \\ &\equiv \left[\int_{h \in \mathcal{H}_t} \int_0^\infty a_{ht}(\omega)^{\theta - 1} d\omega dh \right]^{\frac{1}{\theta - 1}}, \\ &= \left[\int_{h \in \mathcal{H}_t} \frac{B_h}{\theta - 1} A_{ht}{}^{\theta - 1} dh \right]^{\frac{1}{\theta - 1}}. \end{aligned}$$

using a one-to-one mapping between product variety ω and the productivity level a_{ht} as $\omega = B_h (\ln A_{ht} - \ln a_{ht})$ from (31). Θ_{at}^H is an aggregate productivity measure of home firms for the home market.

Firm h produces goods for export, only if the profit is non-negative:

$$0 \leq \epsilon_t p_{ht}^F(\omega) Q_{ht}^F(\omega) - w_t X_{ht}^F(\omega) = \frac{w_t}{a_{ht}(\omega)} \left[\frac{\tau}{\theta - 1} Q_{ht}^F(\omega) - \phi \right].$$

Then the home h firm exports a good ω if and only if productivity $a_{ht}(\omega)$ is higher than a common lower bound $\underline{a}_t$ that is characterized by

$$\phi = \frac{\tau}{\theta-1}\left(\frac{\underline{a}_t}{\Theta^F_{at}}\right)^{\theta}\left(Y^F_t + \frac{M^F_t}{Z^F_{M,t}}\right), \tag{58}$$

$$p^F_t = \frac{\theta}{\theta-1}\frac{\tau w_t/\epsilon_t}{\Theta^F_{at}}, \tag{59}$$

where

$$\Theta^F_{at} = \Theta^F_a\left\{[a_{ht}(\omega)]\right\} = \left[\int_{h\in\mathcal{H}_t|A_{ht}\geq\underline{a}_t}\frac{B_h}{\theta-1}({A_{ht}}^{\theta-1} - \underline{a}_t^{\theta-1})dh\right]^{\frac{1}{\theta-1}}.$$

The home firm h exports if and only if its highest productivity satisfies $A_{ht} \geq \underline{a}_t$. This is the feature that Melitz (2003) and Eaton and Kortum (2002) emphasized. Here, however, the productivity of the firm has two dimensions, height A_{ht} and the width B_h. Thus, even if a firm has large width, the firm exports only if the height is high. On the other hand, there are many smaller and medium size firms with small width who are heavily involved in exports because their height is high. This helps to explain the weak cross-sectional correlation between exports and size which is observed in our Japanese firm level data. The aggregate productivity measure of home firms for exports Θ^F_{at} is smaller than the aggregate productivity measure for the home market Θ^H_{at}, because a smaller number of varieties are produced for exports.

Using the above production choice, total sales S_{ht} and the gross profit Π_{ht} of firm h before subtracting the fixed cost of maintaining the technology are now summarized as:

$$S_{ht} = \frac{B_h w_t \theta}{\theta-1}\left\{\begin{array}{l}\frac{Q^H_t}{\left(\Theta^H_{at}\right)^{\theta}}\frac{(A_{ht})^{\theta-1}}{\theta-1}+ \\ 1(A_{ht}\geq\underline{a}_t)\frac{\tau Q^F_t}{\left(\Theta^F_{at}\right)^{\theta}}\left[\frac{(A_{ht})^{\theta-1}}{\theta-1} - \frac{(\underline{a}_t)^{\theta-1}}{\theta-1}\right]\end{array}\right\} \tag{60}$$

$$\begin{aligned}\Pi_{ht} &= \frac{B_h w_t}{\theta-1}\left\{\begin{array}{l}\frac{Q^H_t}{\left(\Theta^H_{at}\right)^{\theta}}\frac{(A_{ht})^{\theta-1}}{\theta-1}+ \\ 1(A_{ht}\geq\underline{a}_t)\left[\frac{\tau Q^F_t}{\left(\Theta^F_{at}\right)^{\theta}}\left(\frac{(A_{ht})^{\theta-1}}{\theta-1} - \frac{(\underline{a}_t)^{\theta-1}}{\theta-1}\right) - \phi\left(\theta-1\right)\left(\frac{1}{\underline{a}_t} - \frac{1}{A_{ht}}\right)\right]\end{array}\right\} \\ &\equiv B_h\pi(A_{ht},\mathcal{M}_t),\end{aligned} \tag{61}$$

where $1(A_{ht}\geq\underline{a}_t) = 1$ if and only if $A_{ht}\geq\underline{a}_t$ (indicating export status), and $\frac{Q^H_t}{\left(\Theta^H_{at}\right)^{\theta}}$ and $\frac{\tau Q^F_t}{\left(\Theta^F_{at}\right)^{\theta}}$ summarize the market conditions for home output in home and foreign countries. Observe that profit is proportional to the width of the firm, because both demand for its product and cost are proportional to the width, and profit per width $\pi(A_{ht},\mathcal{M}_t)$ only depends upon the height

A_{ht} and the market condition $\mathcal{M}_t$ which we will specify more explicitly later. Observe also that the exporting firm's profitability is more sensitive to foreign market conditions and height of productivity, because of the presence of the fixed cost ϕ of production for exports.

3.2.3 Exit, Entry and Distribution of Firms

Each firm must pay a fixed cost κB_h to receive a technology draw according to (34) at the beginning of period t before idiosyncratic technology shock realizes. Thus, the firm makes an exit decision based on the information available at the beginning of period t. Let $V(A_{ht-1}, \mathcal{M}_t)$ be the value per width of the active firm with date t-1 height A_{ht-1} before drawing a date t productivity. Then, the Bellman equation for this firm is

$$\begin{aligned} V(A_{ht-1}, \mathcal{M}_t) &= -\kappa + \\ & (1-\delta_N) E\{\pi(A_{ht}, \mathcal{M}_t) + \Lambda_{t,t+1} Max[0, V(A_{ht}, \mathcal{M}_{t+1})] | A_{ht-1}, \mathcal{M}_t\}, \end{aligned} \tag{62}$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor. Because we assume home firms are all owned by home households, the stochastic discount factor is equal to the marginal rate of substitution between date $t+1$ and date t consumption of the home representative household, i.e., $\Lambda_{t,t+1} = \beta \frac{C_t}{C_{t+1}}$. Because the RHS is monotonically increasing in the height of the previous period A_{ht-1}, firm's exit decision follows a reservation rule, i.e., firm h exits if and only if $A_{ht-1} \leq \underline{A}(\mathcal{M}_t)$, where

$$V(\underline{A}(\mathcal{M}_t), \mathcal{M}_t) = 0. \tag{63}$$

Because $\pi(A_{ht}, \mathcal{M}_t)$ is stochastic when the firm chooses whether to stay or exit, the net profit $\pi(A_{ht}, \mathcal{M}_t) - \kappa$ can be negative for low realization of A_{ht} for some firms that decided to stay based on their information $(A_{ht-1}, \mathcal{M}_t)$. In addition, because there is uncertainty about future value of the firm $V(A_{ht}, \mathcal{M}_{t+1})$, there is an option value to stay in business at the beginning of date t. This helps explain why negative profits are often observed among staying firms.

Firms are also free to enter to draw a new technology according to (32) and (33). Because the new entrant can only start producing in the next period, it also chooses whether to exit or stay. Then we have the free entry condition as

$$\kappa_E = \overline{B} \int_{A_{ht} \geq \underline{A}(\mathcal{M}_{t+1})} (1-\delta_N) E\{\Lambda_{t,t+1} Max[0, V(A_{ht}, \mathcal{M}_{t+1})] | A_{ht}, \mathcal{M}_t\} F_A^o(dA_{ht}), \tag{64}$$

where $\overline{B} = E_t(B_h) = \exp(\mu_b + \frac{\sigma_b^2}{2})$.

The distribution $\widetilde{\Phi}_t$ of the productivity of home firms (A_{ht}, B_h) evolves through the entry, exit and the evolution of A_{ht}. Because the evolution of height is independent from the time invariant width, we have

$$\begin{aligned} \widetilde{\Phi}_t(A_{ht}, B_h) &= \Phi_t(A_{ht}) F_B^o(B_h) \\ N_{t+1}\Phi_{t+1}(A_{ht+1}) &= \int_{A_{ht} \geq \underline{A}(\mathcal{M}_{t+1})}^{\infty} F(A_{ht+1}|A_{ht})(1-\delta_N)[N_{Et} F_A^o(dA_{ht}) + N_t \Phi_t(dA_{ht})]. \end{aligned} \tag{65}$$

The distribution function of the height $\Phi_t(A_{ht})$ summarizes the productivity distribution of heterogeneous home firms.

The aggregate productivity measures of home firms for home and foreign markets are now simplified to

$$\Theta_{at}^H = \left(\overline{B} N_t\right)^{\frac{1}{\theta-1}} \Theta(0, \Phi_t), \tag{66}$$

$$\Theta_{at}^F = \left(\overline{B} N_t\right)^{\frac{1}{\theta-1}} \Theta(\underline{a}_t, \Phi_t), \tag{67}$$

where

$$\Theta(\underline{a}_t, \Phi_t) = \left\{ \int_{A_{ht} \geq \underline{a}_t} \frac{1}{\theta - 1} \left[(A_{ht})^{\theta-1} - (\underline{a}_t)^{\theta-1} \right] d\Phi_t(A_{ht}) \right\}^{\frac{1}{\theta-1}}. \tag{68}$$

The lower bound $\underline{a}_t$ in (58) and price indices p_t^H and p_t^F in (57) and (59) are determined by (66) and (67).

The behavior of foreign firms and price indices are described similarly to home firms such that

$$
\begin{aligned}
w_t^* &= {w_{Lt}^*}^{\gamma_L^*} {w_{Kt}^*}^{\gamma_K^*} \left(\frac{p_t^F}{Z_{Mt}^F}\right)^{\gamma_M^F} \left(\frac{p_t^{*F}}{Z_{Mt}^{*F}}\right)^{\gamma_M^{*F}} / Z_t^*, && (69)\\
p_t^{*H} &= \frac{\theta}{\theta-1}\frac{\tau^*\epsilon_t w_t^*}{\left(\overline{B}^* N_t^*\right)^{\frac{1}{\theta-1}}\Theta\left(\underline{a}_t^*, \Phi_t^*\right)}, && (70)\\
p_t^{*F} &= \frac{\theta}{\theta-1}\frac{w_t^*}{\left(\overline{B}^* N_t^*\right)^{\frac{1}{\theta-1}}\Theta\left(0, \Phi_t^*\right)}, && (71)\\
\phi^* &= \frac{\tau^*}{\theta-1}\left[\frac{\underline{a}_t^*}{\left(\overline{B}^* N_t^*\right)^{\frac{1}{\theta-1}}\Theta\left(\underline{a}_t^*, \Phi_t^*\right)}\right]^\theta (Y_t^{*H} + \frac{M_t^{*H}}{Z_{Mt}^{*H}}), && (72)
\end{aligned}
$$

where $\overline{B}^* = E(B_f^*) = \exp\left(\mu_b^* + \frac{\sigma_b^{*2}}{2}\right)$, N_t^* is number of foreign firms, and $\Theta\left(\underline{a}_t^*, \Phi_t^*\right) = \left\{\int_{A_{ft}^* \geq \underline{a}_t^*} \frac{1}{\theta-1}\left[\left(A_{ft}^*\right)^{\theta-1} - \left(\underline{a}_t^*\right)^{\theta-1}\right] d\Phi_t^*\left(A_{ft}^*\right)\right\}^{\frac{1}{\theta-1}}$.

3.2.4 Market Equilibrium

From the optimal choice of home and foreign produced goods for production of final goods in (23) and (24), we have

$$
\begin{aligned}
\frac{Y_t^H}{Y_t^{*H}} &= \eta_t\left(\frac{p_t^H}{p_t^{*H}}\right)^{-\varphi}, && (73)\\
\frac{Y_t^{*F}}{Y_t^F} &= \eta_t^*\left(\frac{p_t^{*F}}{p_t^F}\right)^{-\varphi}.
\end{aligned}
$$

Then as we chose final goods as the numeraire in home and foreign countries, we have

$$
\begin{aligned}
1 &= \left[\eta_t\left(p_t^H\right)^{1-\varphi} + \left(p_t^{*H}\right)^{1-\varphi}\right]^{1/(1-\varphi)}, && (74)\\
1 &= \left[\left(p_t^F\right)^{1-\varphi} + \eta_t^*\left(p_t^{*F}\right)^{1-\varphi}\right]^{1/(1-\varphi)}. && (75)
\end{aligned}
$$

Thus we have

$$
\begin{aligned}
Y_t^H &= \eta_t\left(p_t^H\right)^{-\varphi} Y_t, \quad Y_t^{*H} = \left(p_t^{*H}\right)^{-\varphi} Y_t,\\
Y_t^F &= \left(p_t^F\right)^{-\varphi} Y_t^*, \quad Y_t^{*F} = \eta_t^*\left(p_t^{*F}\right)^{-\varphi} Y_t^*. && (76)
\end{aligned}
$$

Concerning the composite input, from the cost minimization of the input composite in (42), we have

$$M_t^H = \gamma_M^H \frac{X_t}{p_t^H / Z_{Mt}^H},\ \ M_t^{*H} = \gamma_M^{*H} \frac{X_t}{p_t^{*H} / Z_{Mt}^{*H}}. \tag{77}$$

Together with production function in (42), the home supply of the input composite is

$$X_t = \left[Z_t \left(\frac{Z_{Mt}^H}{p_t^H} \right)^{\gamma_M^H} \left(\frac{Z_{Mt}^{*H}}{p_t^{*H}} \right)^{\gamma_M^{*H}} \left(\frac{L_t}{\gamma_L} \right)^{\gamma_L} \left(\frac{K_t}{\gamma_K} \right)^{\gamma_K} \right]^{1/(\gamma_L + \gamma_K)}. \tag{78}$$

The demand for the home composite input is

$$\begin{aligned} X_t &= \frac{\left(Y_t^H + \frac{M_t^H}{Z_{Mt}^H} \right)}{\left(\overline{B} N_t \right)^{\frac{1}{\theta - 1}} \Theta \left(0, \Phi_t \right)} + \frac{\tau \left(Y_t^F + \frac{M_t^F}{Z_{Mt}^F} \right)}{\left(\overline{B} N_t \right)^{\frac{1}{\theta - 1}} \Theta \left(\underline{a}_t, \Phi_t \right)} \\ &\quad + \phi \overline{B} N_t \int_{A_{ht} \geq \underline{a}_t} \left(\frac{1}{\underline{a}_t} - \frac{1}{A_{ht}} \right) d\Phi_t \left(A_{ht} \right). \end{aligned} \tag{79}$$

Similarly for the foreign country, the intermediate goods uses are

$$M_t^F = \gamma_M^F \frac{X_t^*}{p_t^F / Z_{Mt}^F},\ \ M_t^{*F} = \gamma_M^{*F} \frac{X_t^*}{p_t^{*F} / Z_{Mt}^{*F}}, \tag{80}$$

supply of foreign input composite is

$$X_t^* = \left[Z_t^* \left(\frac{Z_{Mt}^F}{p_t^F} \right)^{\gamma_M^F} \left(\frac{Z_{Mt}^{*F}}{p_t^{*F}} \right)^{\gamma_M^{*F}} \left(\frac{L_t^*}{\gamma_L^*} \right)^{\gamma_L^*} \left(\frac{K_t^*}{\gamma_K^*} \right)^{\gamma_K^*} \right]^{1/(\gamma_L^* + \gamma_K^*)}. \tag{81}$$

demand for the foreign composite input is

$$\begin{aligned} X_t^* &= \frac{\left(Y_t^{*H} + \frac{M_t^{*H}}{Z_{Mt}^{*H}} \right)}{\left(\overline{B}^* N_t^* \right)^{\frac{1}{\theta - 1}} \Theta \left(\underline{a}_t^*, \Phi_t^* \right)} + \frac{\tau \left(Y_t^{*F} + \frac{M_t^{*F}}{Z_{Mt}^{*F}} \right)}{\left(\overline{B}^* N_t^* \right)^{\frac{1}{\theta - 1}} \Theta \left(0, \Phi_t^* \right)} \\ &\quad + \phi^* \overline{B}^* N_t^* \int_{A_{ft}^* \geq \underline{a}_t^*} \frac{1}{\theta - 1} \left(\frac{1}{\underline{a}_t^*} - \frac{1}{A_{ft}^*} \right) d\Phi_t^* \left(A_{ft}^* \right). \end{aligned} \tag{82}$$

Concerning aggregate values of export and import, we have

$$\begin{aligned} \text{home export} &= \epsilon_t \text{foreign import} = \epsilon_t p_t^F \left(Y_t^F + \frac{M_t^F}{Z_{Mt}^F} \right), \\ \text{home import} &= \epsilon_t \text{foreign export} = p_t^{*H} \left(Y_t^{*H} + \frac{M_t^{*H}}{Z_{Mt}^{*H}} \right). \end{aligned}$$

Thus, home trade balance is $TB_t = \epsilon_t p_t^F \left(Y_t^F + \frac{M_t^F}{Z_{Mt}^F} \right) - p_t^{*H} \left(Y_t^{*H} + \frac{M_t^{*H}}{Z_{Mt}^{*H}} \right)$, home capital account balance is $KB_t = \epsilon_t \left(R_{t-1}^* - 1 \right) D_{t-1}^{*H} - \left(R_{t-1} - 1 \right) D_{t-1}^F$, and the home current account balance is $CB_t = TB_t + KB_t$. The international account balances for foreign country can be similarly defined, and the foreign current account balance is $CB_t^* = -\frac{1}{\epsilon_t} CB_t$. Note that we allow non-zero international account balances. In Japan, intermediate goods have dominated the share of imports, and recently the share of intermediate goods in exports have risen significantly. We will see if this is one of the reasons why aggregate imports move together with aggregate production and aggregate exports, especially after the year 2000.

The aggregate state of the economy in period t is $\mathcal{M}_t = \left(K_t, K_t^*, N_t, N_t^*, D_{t-1}, D_{t-1}^*, \Phi_t, \Phi_t^*\right)$. The home and foreign governments choose (G_t, T_t) and (G_t^*, T_t^*) as functions of the aggregate state. The thirteen prices $\left(p_t^H, p_t^{*H}, p_t^F, p_t^{*F}, w_t, w_t^*, w_{Lt}, w_{Lt}^*, w_{Kt}, w_{Kt}^*, R_t, R_t^*, \epsilon_t\right)$ and twenty two quantities $\left(\underline{a}_t, \underline{a}_t^*, Y_t, Y_t^*, X_t, X_t^*, L_t, L_t^*, Y_t^H, Y_t^{*H}, Y_t^F, Y_t^{*F}, M_t^H, M_t^{*H}, M_t^F, M_t^{*F}, C_t, C_t^*, \underline{A}_t, \underline{A}_t^*, N_{Et}, N_{Et}^*\right)$ and the aggregate state of the next period $\mathcal{M}_{t+1}$ are determined as functions of the current aggregate state $\mathcal{M}_t$ in the recursive equilibrium of our economy.[21]

3.3 Steady State

3.4 Equilibrium Dynamics

4 Parameter Selection

4.1 Estimation of Firm Level Productivity

The domestic sales component of individual home firm in (60) together with (57) implies

$$\ln S_{ht}^H = \widetilde{\theta} + \ln Q_t^H + \theta \ln p_t^H - (\theta - 1)\ln(w_t) + \ln B_h + (\theta - 1)\ln A_{ht},$$

where $\widetilde{\theta}$ is constant which depends upon θ. Using this equation to infer the parameter θ involves two complications. The aggregate variables Q_t^H and p_t^H are functions of θ, and w_t would depend on Z_t in equilibrium. Because of these issues, we do not aim to estimate θ from this equation. Our goal of using this equation is to infer the distributions of B_h and A_{ht}'s by estimating this

[21] In addition to (65) and the corresponding foreign equation to determine $\left(N_{t+1}, N_{t+1}^*, \Phi_{t+1}, \Phi_{t+1}^*\right)$, we have *thirty nine* independent equations: (23), (24), (44), (45), (56), (57), (58), (59), (63), (64), (two more equations for foreign country corresponding to (63), (64)), (69), (70), (71), (72), (46), (48), (49), (50), (four more equations for foreign countries corresponding to (46), (48), (49), (50)), (53), (54), (74), (75), four equations in (76), two equations in (77), (78), (79), two equations in (80), (81), and (82). One of these forty equations is not independent because of Walras's Law.

equation by replacing all aggregate effects with time dummies D_t such that

$$\ln S_{ht}^H = \widehat{\theta} + D_t + b_h + a_{ht} \tag{83}$$

where

$$\begin{aligned} \widehat{\theta} &= \widetilde{\theta} + \mu_b + (\theta - 1)\,\mu_a, \\ b_h &= \ln B_h - \mu_b, \\ a_{ht} &= (\theta - 1)\left(\ln A_{ht} - \mu_a\right), \end{aligned}$$

and μ_b is the mean of $\ln B_h$ and μ_a is the mean of the stationary distribution of the AR(1) process of $\ln A_{ht} = (1 - \rho_a)\,\mu_a + \rho_a \ln A_{h,t-1} + \nu_{a,ht}$. That is, a_{ht} follows the AR(1) process

$$a_{ht} = \rho_a a_{h,t-1} + \widetilde{\nu}_{a,ht},$$

where $\widetilde{\nu}_{a,ht} = (\theta - 1)\,\nu_{a,ht}$.

We estimate the equation (83) by the fixed effect model with autoregressive disturbances. The estimates are reported in Table 1, with standard errors in parentheses. Figure 2 plots the kernel density estimate of b_h overlaid with the normal density, showing that the lognormal distribution is a reasonable approximation of the distribution of B_h. Figure 3.1 compares the shapes of the time-varying kernel density estimates of a_{ht} across three sample years of 1986, 1993 and 1999. Figure 3.2 shows that the average of a_{ht} monotonically converges to zero while the dispersion (measured by standard deviation) declines and then becomes stabilized as time passes.

Table 1. Estimates of Firm Level Productivity Parameters

Parameters	$\widehat{\theta}$	σ_b	ρ_a	$\sigma_{\widetilde{\nu}_a}$
Estimates	17.84 (.002)	1.413	.689	.183

4.2 Estimation of Aggregate Shocks Processes

4.2.1 Aggregate Productivity Shocks

The aggregate productivity variables Z_t and Z_t^* are measured by TFP from the industry level growth accounting rather than by national growth accounting, so that our measures of aggregate

Figure 2. Kernel Density of Log of Normalized Width

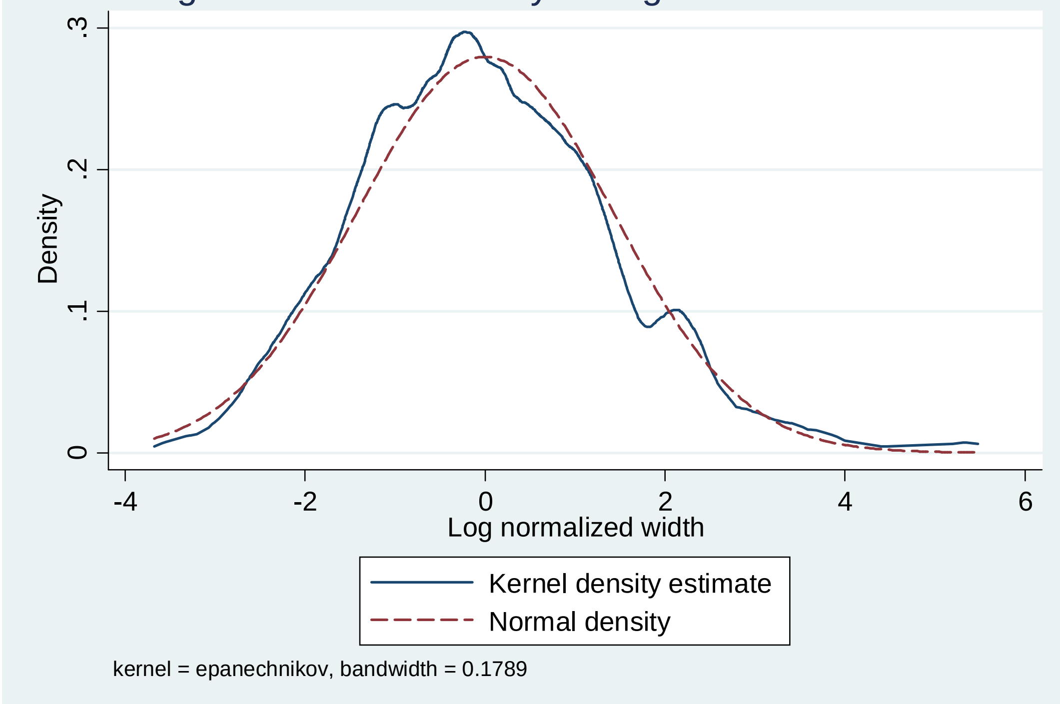
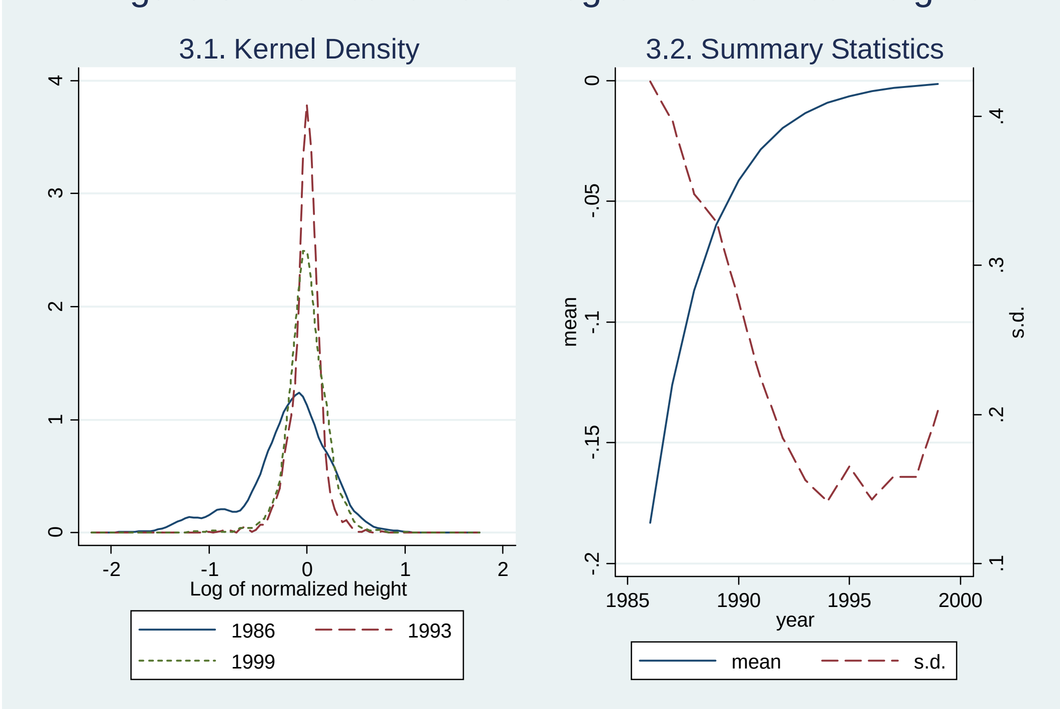


Figure 3. Distribution of of Log of Normalized Heights



productivity may reflect the productivity from technology rather than from various institutional factors.[22] Figure 4 plots the measured Z_t and Z_t^* in logarithmic scale. The existence of a trend is clear for foreign productivity while Japanese productivity grew faster than foreign productivity until 1991 and then flattened during the 1990's, starting to recover after 2002. Due to the presence of trends in productivity, we estimate the AR(1) process of the log difference of productivity. We initially allowed for the cross-country dependence in aggregate productivity growth. The estimated bivariate AR(1) process is given by

$$\begin{bmatrix} \Delta \ln Z_t \\ \Delta \ln Z_t^* \end{bmatrix} = \begin{bmatrix} .003\ (.003) \\ .006\ (.002) \end{bmatrix} + \begin{bmatrix} .345\ (.193) & .201\ (.357) \\ .133\ (.105) & \text{-}.304\ (.195) \end{bmatrix} \begin{bmatrix} \Delta \ln Z_{t-1} \\ \Delta \ln Z_{t-1}^* \end{bmatrix} + \begin{bmatrix} \widetilde{v}_{Zt} \\ \widetilde{v}_{Zt}^* \end{bmatrix}$$

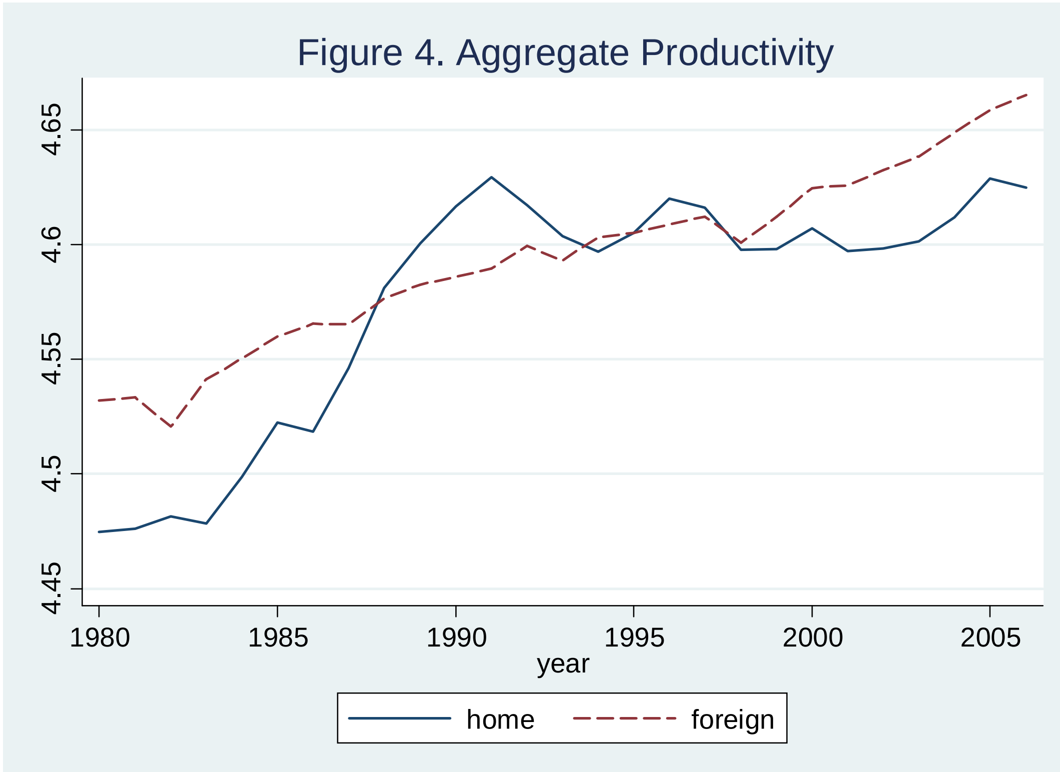
where the standard errors are in parentheses. Not surprisingly, due to the substantial differences in the movements between the two productivity measures, the estimates of the cross-country correlations 0.201 and 0.133 are insignificant. (Their p-values are 0.572 and 0.205, respectively.) Given this weak correlation, we estimate the process of productivity by the univariate AR(1) process for each country such that

$$\begin{aligned} \Delta \ln Z_t &= (1-\rho_Z)\,\mu_Z + \rho_Z \Delta \ln Z_{t-1} + v_{Zt}, \\ \Delta \ln Z_t^* &= (1-\rho_Z^*)\,\mu_Z^* + \rho_Z^* \Delta \ln Z_{t-1}^* + v_{Zt}^*, \end{aligned}$$

where μ_Z and μ_Z^* correspond to the steady state growth rate of Z_t and Z_t^*, respectively, while ρ_Z and ρ_Z^* measure the autocorrelation of the growth rates during transition. The estimates of these parameters are reported in Table 2, with standard errors in parentheses. Figure 5 compares the estimated process of productivity growth with actual productivity growth.

Table 2. Estimates of Aggregate Productivity Parameters

[22] The domestic aggregate productivity Z_t is measured by the Japanese industry level productivity data provided by EU KLEMS Release November 2009. The foreign aggregate productivity Z_t^* is measured by the trade-share weighted sum of those of U.S., E.U., and Korea. The aggregate productivity for U.S. and E.U. are measured from the same source, EU KLEMS Release November 2009. Korean aggregate productivity is obtained from the Korean Industrial Productivity (KIP) database rather than using the EU KLEMS data, because of the substantial and more precise revision of depreciation rate of physical capital in Korean growth accounting by the KIP. Reliable measures of aggregate productivity are not yet available for China; hence Chinese aggregate productivity is not included in Z_t^*. Given the small trade share of Japan with China during the sample period (4 percent), this omission may not have substantial influence on Z_t^*.



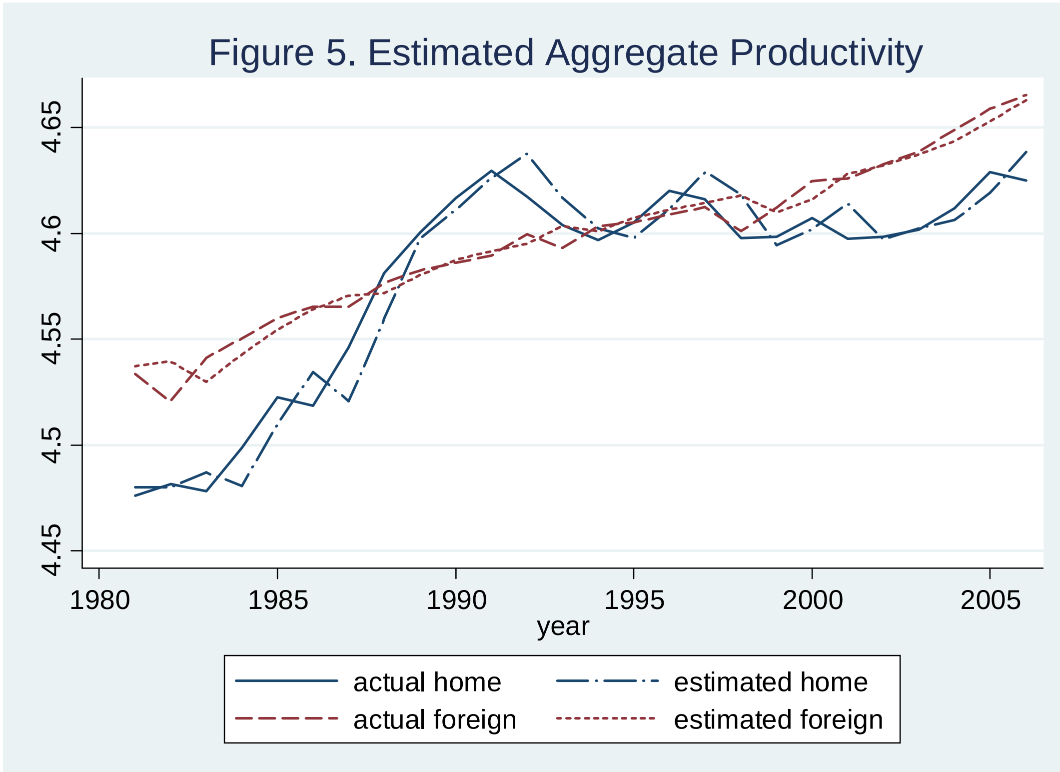
Parameters	μ_Z	ρ_Z	μ_Z^*	ρ_Z^*	σ_{v_Z}	$\sigma_{v_Z^*}$
Estimates	.005 (.004)	.363 (.171)	.005 (.001)	-.231 (.202)	.0123 (.0014)	.0069 (.0012)

4.2.2 Taste Shocks

The allocation between home produced final goods Y_t^H and imported final goods Y_t^{*H} is determined by (73), where η_t is the home bias taste shock in (23) such that $\ln \eta_t \sim N\left(\mu_\eta, \sigma_\eta^2\right)$. Taking logs, we have

$$\ln\left(\frac{p_t^H}{p_t^{*H}}\right) = \frac{\mu_\eta}{\varphi} - \frac{1}{\varphi}\ln\left(\frac{Y_t^H}{Y_t^{*H}}\right) + \widetilde{\eta}_t,$$

where $\widetilde{\eta}_t = \frac{1}{\varphi}\left(\ln \eta_t - \mu_\eta\right)$ and $\ln \eta_t \sim N(\mu_\eta, \sigma_\eta^2)$. We estimate the elasticity of substitution parameter φ from this equation. Given the parameter φ, the process of η_t can be identified by the constant term and the variance of the residuals of this regression. Here, the ratio of home produced goods to imported goods would depend on the taste shock, hence we instrument $\frac{Y_t^H}{Y_t^{*H}}$ by the government consumption, which we consider is exogenously determined but heavily ori-



ented to home produced goods in its composition. Table 3 reports the estimates with standard errors in parentheses. Figure 6 plots the estimated path of the taste shocks for Japan.

Table 3. Estimates of Taste Shock Parameters

Parameters	φ	μ_η	σ_η
Estimates	.99 (.15)	2.57 (.035)	.190 (.015)

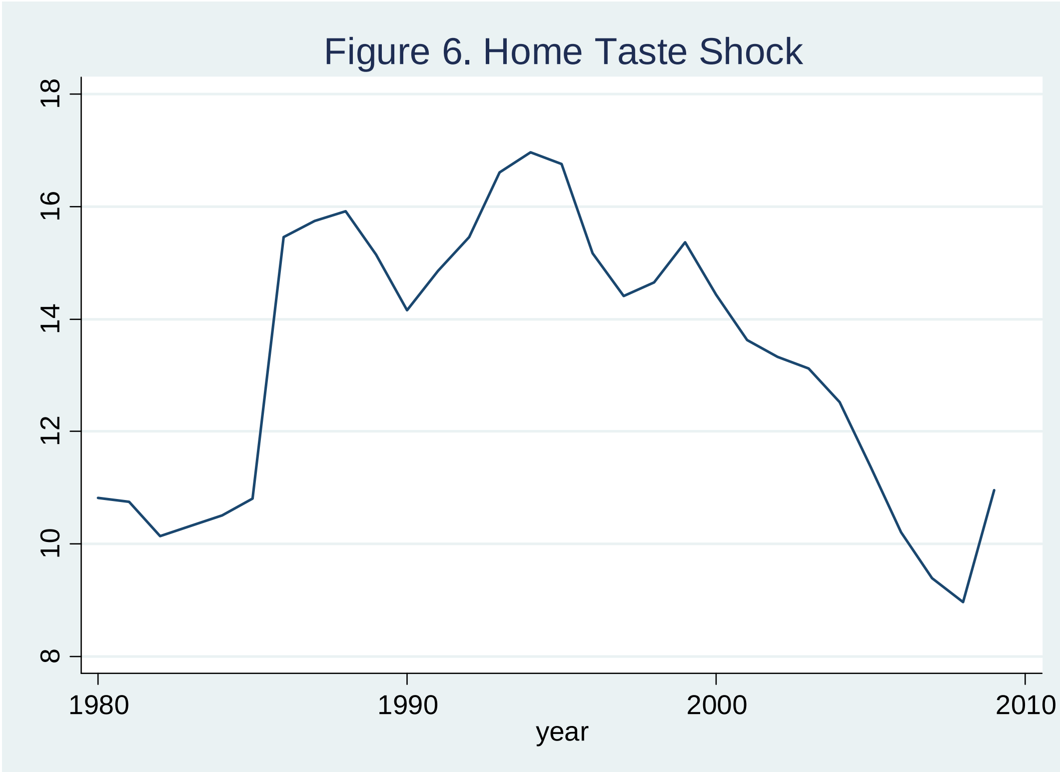
4.2.3 Exchange Rate Shocks

The reduced form real exchange rate process is estimated by the following AR(1) process

$$\ln \epsilon_t = (1 - \rho_\epsilon)\, \mu_\epsilon + \rho_\epsilon \ln \epsilon_{t-1} + \nu_{\epsilon t},$$

where $\nu_{\epsilon t} \sim N\left(0, \sigma^2_{v_\epsilon}\right)$. Table 4 reports the estimates with standard errors in parentheses. Figure 7 compares the estimated and the actual processes of the real exchange rates in logs.

Table 4. Estimates of Real Exchange Rate Parameters



Parameters	μ_ϵ	ρ_ϵ	σ_{v_ϵ}
Estimates	.329 (.192)	.853 (.103)	.133 (.019)

This reduced form estimates give us information about the stochastic process for the utility shocks for home and foreign bond holdings in utility functions $(47, 51)$.

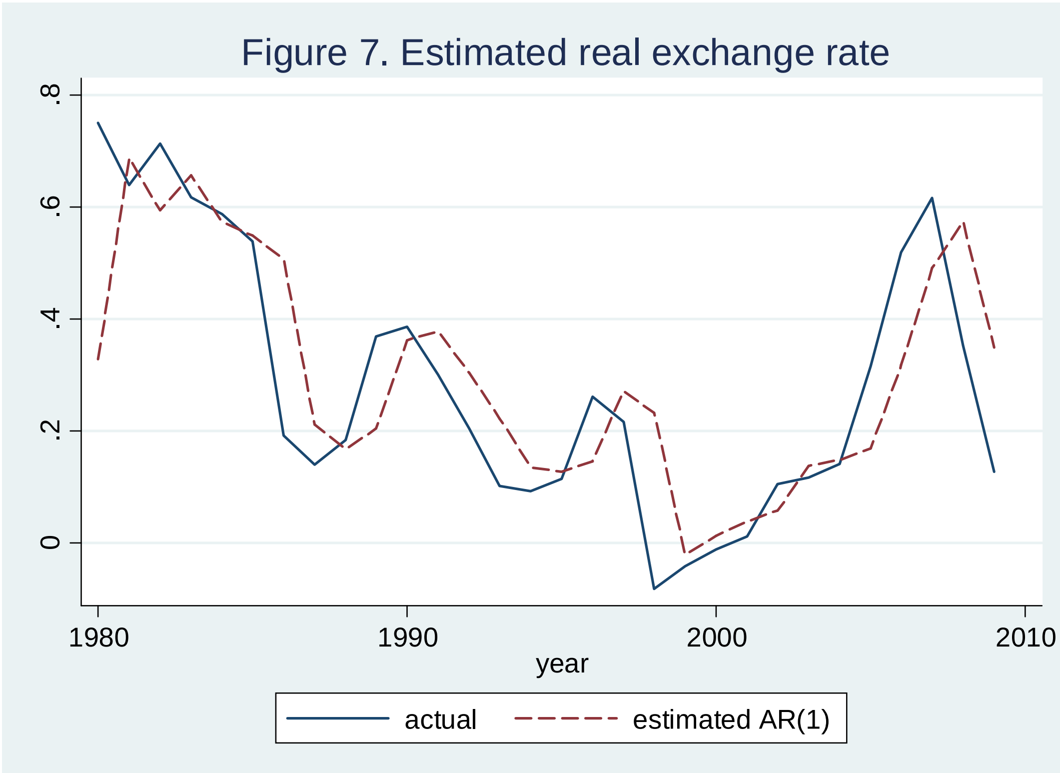
4.2.4 Government Fiscal Policy Shocks

4.2.5 Investment Specific Technology Shocks

The investment specific technology shocks (Z_{It}, Z_{It}^*) are measured by the relative prices

$$Z_{It} = \frac{p_t}{p_{It}},$$
$$Z_{It}^* = \frac{p_t^*}{p_{It}^*},$$

where p_{It} and p_{It}^* are the price levels of investment goods in home and foreign countries.



4.2.6 Intermediate Goods Specific Technology Shocks

The intermediate goods specific technology shocks $\left(Z^H_{Mt}, Z^F_{Mt}, Z^{*H}_{Mt}, Z^{*F}_{Mt}\right)$ are measured by the relative prices

$$\begin{aligned} Z^H_{M,t} &= \frac{p^H_t}{w^H_{M,t}} \\ Z^F_{Mt} &= \frac{p^F_t}{w^F_{M,t}} \\ Z^{*H}_{Mt} &= \frac{p^{*H}_t}{w^{*H}_{M,t}} \\ Z^{*F}_{Mt} &= \frac{p^{*F}_t}{w^{*F}_{M,t}}, \end{aligned}$$

where $w^H_{M,t}$ and $w^F_{M,t}$ are the price of home produced intermediate goods in the home and foreign markets. $w^{*H}_{M,t}$ and $w^{*F}_{M,t}$ are the prices of foreign produced intermediate goods in home and foreign markets.

4.3 Calibration of Other Parameters

5 Simulation

6 Conclusion

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A Steady State of the Basic Model

From equation above (15,17) with $\Lambda_{t,t+1} = \beta$ in the steady state, we have

$$\overline{\pi} = \frac{\kappa}{1-\delta+\delta\lambda} + \left[\frac{1}{\beta\left(1-\delta+\delta\lambda\right)} - 1\right]\frac{\kappa_E}{\lambda}. \tag{A1}$$

This equation implies $\overline{\pi}$ is a function of exogenous parameters in the steady state. The from (16), we have

$$\overline{\pi} = \frac{1}{\theta}\overline{a}N^{\frac{1}{\theta-1}}\left[\frac{X^H}{N} + \alpha\phi\left(\theta-1\right)\left(a_\alpha^{\theta-1} - a_\alpha^{-1}\right)\right],$$

or

$$\overline{a}N^{\frac{1}{\theta-1}}\frac{X^H}{N} = \theta\overline{\pi} - \alpha\phi\left(\theta-1\right)\left(a_\alpha^{\theta-1} - a_\alpha^{-1}\right)\overline{a}N^{\frac{1}{\theta-1}}. \tag{A2}$$

From (8),

$$N_E = \frac{\delta\left(1-\lambda\right)}{\lambda\left(1-\delta+\delta\lambda\right)}N.$$

Then, from (18) and (A1), we have

$$\frac{C}{N} = \theta\overline{\pi} - \alpha\phi\left(\theta-1\right)\left(a_\alpha^{\theta-1} - a_\alpha^{-1}\right)\overline{a}N^{\frac{1}{\theta-1}} - \frac{\kappa + \kappa_E\delta\left(1-\lambda\right)/\lambda}{1-\delta+\delta\lambda}. \tag{A3}$$

From (10) and (12),

$$\overline{a}N^{\frac{1}{\theta-1}}\frac{X^F}{N} = \phi\left[\theta\left(\frac{\epsilon^{\varphi}Y^*}{\overline{a}^{\varphi}\left(\theta-1\right)\phi}\right)^{\frac{\theta-1}{\theta-\varphi}}\frac{\overline{a}N^{\frac{1}{\theta-1}}}{N^{\frac{\theta}{\theta-\varphi}}} - \alpha(a_\alpha^{\theta-1} - a_\alpha^{-1})\overline{a}N^{\frac{1}{\theta-1}}\right].$$

Together with (13) and (A2), we have

$$\begin{aligned}
\overline{a}N^{\frac{1}{\theta-1}}\left(\frac{X^H}{N} + \frac{X^F}{N}\right) &= \theta\left[\overline{\pi} + \left(\frac{\epsilon^{\varphi}Y^*}{\overline{a}^{\varphi}\left(\theta-1\right)\phi}\right)^{\frac{\theta-1}{\theta-\varphi}}\frac{\overline{a}N^{\frac{1}{\theta-1}}}{N^{\frac{\theta}{\theta-\varphi}}} - \alpha\phi(a_\alpha^{\theta-1} - a_\alpha^{-1})\overline{a}N^{\frac{1}{\theta-1}}\right] \\
&= \overline{a}N^{\frac{1}{\theta-1}}\frac{X}{N} \\
&= \frac{\left(\frac{\theta-1}{\theta}\right)^{\frac{1-\gamma_L+\psi}{\gamma_L}}}{\gamma_L\left(\psi_0 C/N\right)^{\psi}}\left(\frac{\overline{a}Z}{\epsilon^{1-\gamma_L}}\right)^{\frac{1+\psi}{\gamma_L}}N^{\left[\frac{1}{(\theta-1)\gamma_L}-1\right](1+\psi)}.
\end{aligned}$$

Using (A3), equilibrium N solves

$$
\begin{aligned}
0 &= J(N,\epsilon,Y^*,Z) \qquad\qquad (A4)\\
&= -\left[\theta\overline{\pi}-\alpha\phi\left(\theta-1\right)\left(a_\alpha^{\theta-1}-a_\alpha^{-1}\right)\overline{a}N^{\frac{1}{\theta-1}}-\frac{\kappa+\kappa_E\delta\left(1-\lambda\right)/\lambda}{1-\delta+\delta\lambda}\right]^{\psi}\\
&\quad\cdot\left[\overline{\pi}+\left(\frac{\epsilon^{\varphi}Y^*}{\overline{a}^{\varphi}\left(\theta-1\right)\phi}\right)^{\frac{\theta-1}{\theta-\varphi}}\frac{\overline{a}N^{\frac{1}{\theta-1}}}{N^{\frac{\theta}{\theta-\varphi}}}-\alpha\phi(a_\alpha^{\theta-1}-a_\alpha^{-1})\overline{a}N^{\frac{1}{\theta-1}}\right]\\
&\quad+\frac{\left(\frac{\theta-1}{\theta}\right)^{\frac{1-\gamma_L+\psi}{\gamma_L}}}{\gamma_L\left(\psi_0C/N\right)^{\psi}}\left(\frac{\overline{a}Z}{\epsilon^{1-\gamma_L}}\right)^{\frac{1+\psi}{\gamma_L}}\frac{1}{N^{\left[1-\frac{1}{(\theta-1)\gamma_L}\right](1+\psi)}}.
\end{aligned}
$$

$\frac{\partial J}{\partial N}<0$ in the neighborhood of $J(N,\epsilon,Y^*,Z)=0$, iff

$$
\begin{aligned}
&\left[1-\frac{1}{\left(\theta-1\right)\gamma_L}\right]\left(1+\psi\right) \qquad\qquad (A5)\\
>\;&\frac{X^F+\alpha\phi(a_\alpha^{\theta-1}-a_\alpha^{-1})N}{X}\left(\frac{\theta}{\theta-\varphi}-\frac{1}{\theta-1}\right)+\frac{\alpha\phi(a_\alpha^{\theta-1}-a_\alpha^{-1})N}{X}\frac{1}{\theta-1}\\
&+\psi\frac{\alpha\phi(\theta-1)(a_\alpha^{\theta-1}-a_\alpha^{-1})\overline{a}N^{\frac{1}{\theta-1}}}{C/N}\frac{1}{\theta-1}\\
=\;&\frac{X^F}{X}\left(\frac{\theta}{\theta-\varphi}-\frac{1}{\theta-1}\right)+\alpha\phi(a_\alpha^{\theta-1}-a_\alpha^{-1})\left[\frac{N}{X}\frac{\theta}{\theta-\varphi}+\psi\frac{\overline{a}N^{\frac{1}{\theta-1}}}{C/N}\right].
\end{aligned}
$$

Assume that this inequality is satisfied because θ is relatively large. Because we know $\frac{\partial J}{\partial\epsilon}<0$ $\frac{\partial J}{\partial Y^*}<0$ $\frac{\partial J}{\partial Z}<0$, we learn

$$
\frac{\partial N}{\partial\epsilon}<0,\frac{\partial N}{\partial Y^*}<0,\frac{\partial N}{\partial Z}>0.
$$

Thus if TFP is higher, the number of differentiated products produced is higher. If the export market expands by the real exchange rate depreciation or increase of foreign demand, there are more low productive firms producing for export with zero profits, which crowds out the profits at home and reduces the number of differentiated products produced in the steady state.

Note from (10),

$$
n_1^F+\alpha a_\alpha^{\theta-1}=\left(\frac{\epsilon^{\varphi}Y^*}{\overline{a}^{\varphi}\left(\theta-1\right)\phi}\right)^{\frac{\theta-1}{\theta-\varphi}}\frac{1}{N^{\frac{\theta}{\theta-\varphi}}}=f(N).
$$

Define $\underline{N}$ and $\overline{N}$ as

$$
\begin{aligned}
f(\underline{N}) &= \overline{a}^{\theta-1},\\
f(\overline{N}) &= \alpha a_\alpha^{\theta-1}.
\end{aligned}
$$

Then inequality (11) holds if and only if

$$J(\underline{N}, \epsilon, Y^*, Z) > 0 > J(\overline{N}, \epsilon, Y^*, Z) \tag{A6}$$

This is the necessary and sufficient condition for the extensive margin adjusts against shock in the export market.

With straight-forward and tedious algebra, the inequality (14) is satisfied in the steady state if and only if

$$(1-\beta+\beta\delta)\left[(1-\delta+\delta\lambda)\pi_1^H-\kappa\right]+\delta\lambda\alpha(\pi_\alpha^H+\pi_\alpha^F-\pi_1^H)>0,$$

or

$$\begin{aligned}
& \frac{\theta-1}{\theta}\alpha\phi(a_\alpha^{\theta-1}-a_\alpha^{-1})\overline{a}N^{\frac{1}{\theta-1}}(1-\delta)(1-\delta+\delta\lambda)\overline{a}N^{\frac{1}{\theta-1}} \\
< \; & \left[(1-\beta+\beta\delta)\left(1-\delta+\delta\lambda\right)+\delta\lambda\alpha(a_\alpha^{\theta-1}-1)\right]\frac{\kappa_E}{\lambda\beta}-(1-\delta)\alpha(a_\alpha^{\theta-1}-1)\kappa.
\end{aligned}$$

The inequality of Footnote 17 is satisfied if and only if

$$\kappa > \delta\lambda\left(\overline{\pi}+\beta\overline{V}\right),$$

or

$$\delta\kappa_E < (1-\delta)\,\kappa.$$

B Social Planner's Problem for the Full Model

Consider there is no friction in transaction so that there is no utility derived from holding home and foreign bond.

$$U_0 = E_0\sum_{t=0}^{\infty}\beta^t\left(\ln C_t-\psi_0\frac{L_t^{1+1/\psi}}{1+1/\psi}\right), \tag{84}$$

$$U_0^* = E_0\sum_{t=0}^{\infty}\beta^t\left(\ln C_t^*-\psi_0^*\frac{L_t^{*1+1/\psi^*}}{1+1/\psi^*}\right). \tag{85}$$

The efficient allocation in which the social planner chooses the resource allocation in order to maximize the weighted average of the utility of home and of the foreign representative households $U_0^S = \vartheta U_0 + (1-\vartheta)U_0^*$ subject to the above technology and resource constraints. This social planner's allocation serves as a benchmark comparison to the decentralized economy. We can define the Lagrangian of the planner as

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t (\vartheta \left[\ln C_t - \psi_0 \frac{L_t^{1+1/\psi}}{1+1/\psi} \right] + (1-\vartheta) \left[\ln C_t^* - \psi_0^* \frac{L_t^{*1+1/\psi^*}}{1+1/\psi^*} \right]$$

$$+\mathcal{P}_t \left\{ \left[(\eta_t)^{\frac{1}{\varphi}} (Y_t^H)^{\frac{\varphi-1}{\varphi}} + (Y_t^{*H})^{\frac{\varphi-1}{\varphi}} \right]^{\frac{\varphi}{\varphi-1}} \text{-} C_t \text{-} G_t \text{-} \frac{1}{Z_{It}} \left[K_{t+1} \text{-} (1\text{-}\delta_K) K_t \right] \text{-} \kappa_E N_{Et} \text{-} \kappa E_t(B_h) N_t \right\}$$
$$+\mathcal{P}_t^* \left\{ \left[(\eta_t)^{\frac{1}{\varphi}} (Y_t^F)^{\frac{\varphi-1}{\varphi}} + (Y_t^{*F})^{\frac{\varphi-1}{\varphi}} \right]^{\frac{\varphi}{\varphi-1}} \text{-} C_t^* \text{-} G_t^* \text{-} \frac{1}{Z_{It}^*} \left[K_{t+1}^* \text{-} (1\text{-}\delta_K) K_t^* \right] \text{-} \kappa_E^* N_{Et}^* \text{-} \kappa^* E_t(B_f^*) N_t^* \right\}$$

$$+p_t^H \mathcal{P}_t \left\{ \left[\int_{h\in\mathcal{H}_t} \int_{\omega\in\Omega} Q_{ht}^H(\omega)^{\frac{\theta-1}{\theta}} d\omega dh \right]^{\frac{\theta}{\theta-1}} - \frac{M_t^H}{Z_{Mt}^H} - Y_t^H \right\}$$
$$+p_t^F \mathcal{P}_t^* \left\{ \left[\int_{h\in\mathcal{H}_t} \int_{\omega\in\Omega} Q_{ht}^F(\omega)^{\frac{\theta-1}{\theta}} d\omega dh \right]^{\frac{\theta}{\theta-1}} - \frac{M_t^F}{Z_{Mt}^F} - Y_t^F \right\}$$
$$+p_t^{*H} \mathcal{P}_t \left\{ \left[\int_{f\in\mathcal{F}_t} \int_{\omega\in\Omega} Q_{ft}^{*H}(\omega)^{\frac{\theta-1}{\theta}} d\omega df \right]^{\frac{\theta}{\theta-1}} - \frac{M_t^{*H}}{Z_{Mt}^{*H}} - Y_t^{*H} \right\}$$
$$+p_t^{*F} \mathcal{P}_t^* \left\{ \left[\int_{f\in\mathcal{F}_t} \int_{\omega\in\Omega} Q_{ht}^{*F}(\omega)^{\frac{\theta-1}{\theta}} d\omega df \right]^{\frac{\theta}{\theta-1}} - \frac{M_t^{*F}}{Z_{Mt}^{*F}} - Y_t^{*F} \right\}$$

$$+ \int_{h\in\mathcal{H}_t} \int_{\omega\in\Omega} p_{ht}^H(\omega) \mathcal{P}_t \left[A_{ht} e^{-\frac{\omega}{B_h}}(\omega) X_{ht}^H(\omega) - Q_{ht}^H(\omega) \right] d\omega dh$$
$$+ \int_{h\in\mathcal{H}_t} \int_{\omega\in\Omega} p_{ht}^F(\omega) \mathcal{P}_t^* \left\{ \frac{1}{\tau} \left[A_{ht} e^{-\frac{\omega}{B_h}} X_{ht}^F(\omega) - \phi \right] - Q_{ht}^F(\omega) \right\} d\omega dh$$
$$+ \int_{f\in\mathcal{F}_t} \int_{\omega\in\Omega} p_{ft}^{*H}(\omega) \mathcal{P}_t \left\{ \frac{1}{\tau^*} \left[A_{ft}^* e^{-\frac{\omega}{B_f^*}} X_{ft}^{*H}(\omega) - \phi^* \right] - Q_{ft}^{*H}(\omega) \right\} d\omega df$$
$$+ \int_{f\in\mathcal{F}_t} \int_{\omega\in\Omega} p_{ft}^{*F}(\omega) \mathcal{P}_t^* \left[A_{ft}^* e^{-\frac{\omega}{B_f^*}} X_{ft}^{*F}(\omega) - Q_{ft}^{*F}(\omega) \right] d\omega df$$
$$+w_t \mathcal{P}_t \left\{ \begin{array}{c} Z_t \left(\frac{L_t}{\gamma_L}\right)^{\gamma_L} \left(\frac{K_t}{\gamma_K}\right)^{\gamma_K} \left(\frac{M_t^H}{\gamma_M^H}\right)^{\gamma_M^H} \left(\frac{M_t^{*H}}{\gamma_M^{*H}}\right)^{\gamma_M^{*H}} \\ - \int_{h\in\mathcal{H}_t} \int_{\omega\in\Omega} \left[X_{ht}^H(\omega) + X_{ht}^F(\omega) \right] d\omega dh \end{array} \right\}$$
$$+w_t^* \mathcal{P}_t^* \left\{ \begin{array}{c} Z_t^* \left(\frac{L_t^*}{\gamma_L^*}\right)^{\gamma_L^*} \left(\frac{K_t^*}{\gamma_K^*}\right)^{\gamma_K^*} \left(\frac{M_t^F}{\gamma_M^F}\right)^{\gamma_M^F} \left(\frac{M_t^{*F}}{\gamma_M^{*F}}\right)^{\gamma_M^{*F}} \\ - \int_{f\in\mathcal{F}_t} \int_{\omega\in\Omega} \left[X_{ft}^{*H}(\omega) + X_{ft}^{*F}(\omega) \right] d\omega df \end{array} \right\}$$

$$+\int_0^\infty V_{t+1}(A_{ht},B_h)\left\{\begin{array}{c}\int_0^\infty\int_0^\infty F(dA_{ht+1}|A_{ht})1_{t+1}(A_{ht},B_h)(1-\delta_N)\cdot\\ \left[N_{Et}F_A^o(dA_{ht})F_B^o(dB_h)+N_t\widetilde{\Phi}_t(dA_{ht},dB_h)\right]\\ -N_{t+1}\widetilde{\Phi}_{t+1}(dA_{ht+1},dB_h)\end{array}\right\}$$

$$+\int_0^\infty V_{t+1}^*(A_{ft}^*,B_f^*)\left\{\begin{array}{c}\int_0^\infty\int_0^\infty F^*(dA_{ft+1}^*|A_{ft}^*)1_{t+1}^*\left(A_{ft}^*,B_f^*\right)(1-\delta_N)\cdot\\ \left[N_{Et}^*F_A^{o*}(dA_{ft}^*)F_B^{o*}\left(dB_f^*\right)+N_t^*\widetilde{\Phi}_t^*\left(dA_{ft}^*,dB_f^*\right)\right]\\ -N_{t+1}^*\widetilde{\Phi}_{t+1}^*(dA_{ft+1}^*,dB_f^*)\end{array}\right\}). \tag{86}$$

From the first order condition of $Q_{ht}^H(\omega)$, we learn

$$Q_{ht}^H(\omega) = \left[\frac{p_{ht}^H(\omega)}{p_t^H}\right]^{-\theta} Q_t^H, \text{ where} \tag{87}$$

$$p_t^H = \left[\int_{h\in\mathcal{H}_t}\int_{\omega\in\Omega} p_{ht}^H(\omega)^{1-\theta}d\omega dh\right]^{\frac{1}{1-\theta}} \equiv \Theta_P\left\{\left[p_{ht}^H(\omega)\right]\right\}. \tag{88}$$

Also from the first order condition of $X_{ht}^H(\omega)$, we have

$$p_{ht}^H(\omega) = \frac{w_t}{A_{ht}e^{-(\omega/B_h)}} = \frac{w_t}{a_{ht}(\omega)} \tag{89}$$

This is familiar condition of the price being equal to the marginal cost. Then we have

$$\begin{aligned} p_t^H &= \frac{w_t}{\Theta_a^H\{[a_{ht}(\omega)]\}}, \text{ where} \\ \Theta_a^H\{[a_{ht}(\omega)]\} &\equiv \left[\int_{h\in\mathcal{H}_t}\int_{\omega\in\Omega} a_{ht}(\omega)^{\theta-1}d\omega dh\right]^{\frac{1}{\theta-1}} \\ &= \left[\int_{h\in\mathcal{H}_t}\frac{1}{\theta-1}{A_{ht}}^{\theta-1}B_h dh\right]^{\frac{1}{\theta-1}} \end{aligned}$$

using one-to-one mapping between product variety ω and productivity level a as $\omega = B_h(\ln A_{h,t} - \ln a)$ from (31). $\Theta_a\{[a_h(\omega)]\}$ can be thought of the aggregate productivity measure of home firms for home market.

Similarly from the first order condition of $Q_{ht}^F(\omega)$ and $X_{ht}^F(\omega)$, we learn

$$Q_{ht}^F(\omega) = \left[\frac{p_{ht}^F(\omega)}{p_t^F}\right]^{-\theta} Q_t^F, \text{ where} \tag{90}$$

$$p_t^F = \Theta_P\left\{\left[p_{ht}^F(\omega)\right]\right\}, \text{ and} \tag{91}$$

$$\epsilon_t p_{ht}^F(\omega) = \tau\frac{w_t}{a_{ht}(\omega)}, \text{ where} \tag{92}$$

$$\epsilon_t \equiv \mathcal{P}_t^*/\mathcal{P}_t. \tag{93}$$

where ϵ_t can be considered as the real exchange rate. But because of the fixed cost for producing each variety, firm h produce differentiated goods for export if and only if $\omega \leq \overline{\omega}_{ht}$ in which the first order condition for $\overline{\omega}_h$ is

$$p_t^F \mathcal{P}_t^* \frac{\theta}{\theta - 1} \left(Q_t^F\right)^{\frac{1}{\theta}} \left(Q_t^F \left(\overline{\omega}_{ht}\right)\right)^{\frac{\theta-1}{\theta}} = w_t \mathcal{P}_t X_{ht}^F \left(\overline{\omega}_{ht}\right). \tag{94}$$

The LHS is the marginal value of having a variety $\overline{\omega}_{ht}$ and the right hand side (RHS) is the cost of producing $\overline{\omega}_{ht}$. Then using (30), (90) and (92), we learn

$$a_{ht}\left(\overline{\omega}_{ht}\right) = \underline{a}_t, \text{ for all } h \in \mathcal{H}_t, \tag{95}$$

$$Q_{ht}^F(\omega) = \left(\frac{a_{ht}(\omega)}{\underline{a}_t}\right)^{\theta} \frac{\theta - 1}{\tau}\phi, \text{ for } a_{ht}(\omega) \geq \underline{a}_t, \tag{96}$$

$$p_t^F = \frac{\tau w_t / \epsilon_t}{\Theta_a^F \{[a_h(\omega)]\}}, \text{ where} \tag{97}$$

$$\Theta_a^F \{[a_h(\omega)]\} = \left[\int_{h \in \mathcal{H}_t | A_{ht} \geq \underline{a}_t} \frac{1}{\theta - 1} ({A_{ht}}^{\theta-1} - \underline{a}_t^{\theta-1}) B_h dh\right]^{\frac{1}{\theta-1}}. \tag{98}$$

Home firm h exports if and only if its highest productivity satisfies $A_{ht} \geq \underline{a}_t$.The aggregate productivity measure of home firms for export $\Theta_a^F \{[a_h(\omega)]\}$ is aggregate productivity measure for home market, because smaller number of varieties are produced.

From the first order condition of Y_t^H and Y_t^{*H}, we have

$$Y_t^H = \eta_t \left(p_t^H\right)^{-\varphi} Y_t$$

$$Y_t^{*H} = \left(p_t^{*H}\right)^{-\varphi} Y_t$$

$$1 = \left[\eta_t \left(p_t^H\right)^{1-\varphi} + \left(p_t^{*H}\right)^{1-\varphi}\right]^{\frac{1}{1-\varphi}}.$$

From the first order condition for C_t and C_t^*, we have

$$\mathcal{P}_t = \vartheta / C_t, \ \mathcal{P}_t^* = (1 - \vartheta)/C_t^*,$$

$$\epsilon_t \equiv \frac{\mathcal{P}_t^*}{\mathcal{P}_t} = \frac{1 - \vartheta}{\vartheta} \frac{C_t}{C_t^*}.$$

The marginal cost of labour supply and the cost of capital satisfies the conditions

$$w_{Lt} = \vartheta \psi_0 L_t^{1/\psi} / \mathcal{P}_t = \psi_0 L_t^{1/\psi} C_t,$$

$$\frac{1}{Z_{It}} = E_t \left[\left(\frac{\beta C_t}{C_{t+1}}\right)\left(w_{Kt+1} + \frac{1 - \delta_K}{Z_{It+1}}\right)\right],$$

where the cost of capital is the opportunity cost of suing capital for one period. Then the optimal choice of input leads to

$$
\begin{aligned}
L_t &= \gamma_L \frac{w_t X_{,t}}{w_{Lt}}, && (99)\\
K_t &= \gamma_K \frac{w_t X_t}{w_{Kt}}, && (100)\\
M_t^H &= \gamma_M^H \frac{w_t X_t}{p_t^H / Z_{Mt}^H}, && (101)\\
M_{h,t}^{*H} &= \gamma_M^H \frac{w_t X_t}{p_t^{*H} / Z_{Mt}^{*H}}, && (102)\\
w_t &= {w_{Lt}}^{\gamma_L} {w_{Kt}}^{\gamma_K} \left(p_t^H / Z_{Mt}^H\right)^{\gamma_M^H} \left(p_t^{*H} / Z_{Mt}^{*H}\right)^{\gamma_M^{*H}} / Z_t. && (103)
\end{aligned}
$$