A Darwinian Perspective on the Chinese Exchange Rate

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June 9, 2011

Abstract

The conventional view is that a large departure from purchasing power parity and the existence of a large current account surplus make it indisputable that the Chinese real exchange rate is substantially undervalued. We argue that the conventional view misses something important (although not standard in the existing theory of exchange rate). A rise in the sex ratio (increasing relative surplus of men in the marriage market) in China, may have simultaneously generated a decline in the real exchange rate (RER) and a rise in the current account surplus. We demonstrate this logic through both a savings channel and an effective labor supply channel. In this model, a low RER is not a cause of the current account surplus, nor is it a consequence of currency manipulations. Empirically, those economies with a high sex ratio tend to have a low real exchange rate, beyond what can be explained by the Balassa-Samuelson effect, financial underdevelopment, dependence ratio, and exchange rate regime classifications. Once these factors are accounted for, the Chinese real exchange rate is estimated to be undervalued by only a relatively trivial amount.

Key words: surplus men, equilibrium real exchange rate, currency manipulation

JEL code: F3, F4, J1 and J7

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1 Introduction

The Chinese real exchange rate (RER) is widely believed to be substantially undervalued. This in turn has created enormous tension in the global monetary system. The standard narrative goes as follows. The Chinese exchange rate is undervalued largely through deliberate and massive government interventions in the currency market. The rapid accumulation of the country's foreign exchange reserve is prima facie evidence that the authorities have engaged in a massive currency market intervention. The undervalued currency has in turn created both a growing currency account surplus and an increasing departure from purchasing power parity.

However, this narrative is not an inevitable way to piece together the value of the real exchange rate, the current account and the foreign exchange reserve. In this paper, we explore an alternative narrative. It starts from some technology and policy shocks, unrelated to currency market interventions, that cause simultaneously a rise in the country's savings rate and an expansion in the country's effective labor supply. These developments in turn lead to a simultaneous decline in the value of the real exchange rate and a rise in the current account balance (even though the exchange rate decline is not the cause of the current account surplus). Once the current account is put into a surplus gear, the foreign exchange reserve accumulation can happen largely passively as a result of the country's capital control regime - put in place long before the exchange rate became an issue - which, as capital control regimes in many other countries, requires mandatory surrender of foreign exchange earnings by firms and households (i.e., selling foreign currencies to the central bank in return for RMB).

The initial technology shock in the new narrative was the spread of ultrasound B machines in China in the 1980s that allowed expectant parents to easily detect the gender of the fetus. 1985 was the first year in which half of the county level hospitals acquired at least one such machine (Li and Zheng, 2009). The initial policy shock was the implementation of a strict version of the family planning policy (popularly known as the "one-child policy") that severely restricts many couples' legally permissible number of children to a level below their desire. By interacting with a long-existing parental preference for sons, the combination of the two shocks started to produce an unnaturally high ratio of boys to girls at birth from the early 1980s, and the sex ratio at birth became worse progressively as the use of ultrasound machines became more widespread, and the enforcement of the family planning policy tightened over time. Around 2003, the first cohort born with an excess number of males was entering the marriage market. The competition for a marriage partner by young men became progressively more fierce. In 2007, the sex ratio for the pre-marital age cohort (5-20) was about 115 young men per 100 young women. This implies that about one out of every nine young men cannot get married, mathematically speaking.

How would a rise in the sex ratio imbalance trigger a significant increase in the savings rate? The key is that family wealth is a key status variable in the marriage market (other things equal). As the competition for brides intensifies, young men and their parents raise their savings rate in order to improve their relative standing in the marriage market. If the biological desire to have a female partner is strong, the response of the savings rate to a rise in the sex ratio can also be quantitatively large. Of course, any complete story has to investigate why the behavior by women or their parents does not undo the competitive savings story.

The empirical motivation for the savings channel comes from Wei and Zhang (2009). They provide evidence from China at both the household level and regional level. First, across rural households with a son, they document that the savings rate tends to be higher in regions with a higher sex ratio imbalance (holding constant family income, age, gender, and educational level of the household head and other household characteristics). In comparison, for rural households with a daughter, their savings rate appears to be uncorrelated with the local sex ratio. Across cities, both households with a son and households with a daughter tend to have a higher savings rate in regions with a more skewed sex ratio, although the elasticity of the savings rate with respect to the sex ratio tends to be bigger for son families. Second, across Chinese provinces, they find a strong positive correlation between the local savings rate and the local sex ratio, after controlling for the age structure of the local population, income level, inequality, recent growth rate, local birth rate, local enrollment rate in the social safety net, and other factors. Third, to go from correlation to causality, they explore regional variations in the enforcement of the family planning policy as instruments for the local sex ratio, and confirm the findings in the OLS regressions. The sex ratio effect is both economically and statistically significant. While the Chinese household savings rate approximately doubled from 16% (of disposable income) in 1990 to 31% in 2007, Wei and Zhang (2011) estimate that the rise in the sex ratio could explain about half the increase in the household savings rate.

When the economy-wide savings rate rises, the real exchange rate often falls. To see this, we recognize that a rise in the savings rate implies a reduction in the demand for both tradable and non-tradable goods. Since the price of the tradable good is tied down by the world market, this translates into a reduction in the relative price of the nontradable good, and hence a decline in the value of the real exchange rate (a departure from the PPP). The effect would be persistent if there are frictions that impede the reallocation of factors between the tradable and nontradable sectors.

The second channel for the sex ratio imbalance to affect the real exchange rate works through effective labor supply. A rise in the sex ratio can also motivate men to cut down leisure and increase labor supply. This leads to an increase in the economy-wide effective labor supply. If the nontradable sector is more labor intensive than the tradable sector, this generates a Rybzinsky-like effect, leading to an expansion of the nontradable sector at the expense of the tradable sector. The increase in the supply of nontradable good leads to an additional decline in the relative price of nontradable and a further decline in the value of the RER.

Putting the two channels together, a rise in the sex ratio generates a real exchange rate that appears too low relative to the purchasing power parity (or relative to the standard approach used by the IMF to assess equilibrium exchange rates that includes additional terms beyond a departure from PPP but does not include the sex ratio, savings rate, and effective labor supply). Because the effect of a skewed sex ratio on the real exchange rate comes from competition for sex partners, this is fundamentally a Darwinian perspective on the exchange rate.

Of course, other structural factors may also have contributed to an increase in the aggregate savings rate (e.g., an increase in government savings or an increase in private-sector precautionary savings) or an increase in the effective labor supply (e.g., gradual relaxation of restrictions on rural-urban migration). These other factors would reinforce the Darwinian mechanism discussed in this paper, causing the real exchange rate to fall further.

A desire to enhance one's prospect in the marriage market through a higher level of wealth could be a motive for savings even in countries with a balanced sex ratio. But such a motive is not as easy to detect when the competition is modest. When the sex ratio gets out of balance, obtaining a marriage partner becomes much less assured. A host of behaviors that are motivated by a desire to succeed in the marriage market may become magnified. But sex ratio imbalances so far have not been investigated by macroeconomists. This may be a serious omission.

A sex ratio imbalance is a common demographic feature in many economies, especially in East, South, and Southeast Asia, such as Korea, India, Vietnam, Singapore, Taiwan and Hong Kong, in addition to China. It is quite possible that the sex ratio effect plays an important role in the real exchange rate of these economies. To be clear, most countries in the world do not have a severe sex ratio imbalance. Correspondingly, it cannot be a significant determinant of the real exchange rate for them. However, if one only considers the standard determinants of the real exchange rate and ignores the sex ratio effect, one could mistakenly conclude that countries with a severe sex ratio imbalance to have a severely undervalued currency. This set of countries happens to include China - the world's second largest economy and the largest exporter. Given the enormous effort by international financial institutions and many national governments to pass judgment on its exchange rate, getting it right has global importance.

There are four bodies of work that are related to the current paper. First, the theoretical and empirical literature on the real exchange rate is too voluminous to summarize comprehensively here. Sarno and Taylor (2002) and Chinn (2011) provide recent surveys. Second, the literature on status goods, positional goods, and social norms (e.g., Cole, Mailath and Postlewaite, 1992, Corneo and Jeanne, 1999, Hopkins and Kornienko, 2004 and 2009) has offered many useful insights. One key point is that when wealth can improve one's social status (including improving one's standing in the marriage market), in addition to affording a greater amount of consumption goods, there is an extra incentive to save. This element is in our model as well. However, all existing theories on status goods feature a balanced sex ratio. Yet, an unbalanced sex ratio presents some non-trivial challenges. In particular, while a rise in the sex ratio is an unfavorable shock to men, it is a favorable shock to women. Could the women strategically reduce their savings so as to completely offset whatever increments in savings men may have? In other words, the impact on aggregate savings from a rise in the sex ratio appears ambiguous. Our model will address this question. In any case, the literature on status goods has no discernible impact in macroeconomic policy circles. For example, while there are voluminous documents produced by the International Monetary Fund or speeches by US officials on China's high savings rate and large current account surplus, no single paper or speech thus far has pointed to a possible connection with its high sex ratio imbalance.

A third related literature is the economics of family, which is also too vast to be summarized here comprehensively. One interesting insight from this literature is that a married couple's consumption has a partial public goods feature (Browning, Bourguignon and Chiappori, 1994; Donni, 2006). We make use of this feature in our model as well. An insightful paper by Bhaskar and Hopkins (2011) studies parental investment in their children before they go to the marriage market. When there is a surplus of boys, parents overinvest in boys and underinvest in girls but the total investment in children is excessive. Du and Wei (2010) examine the effect of higher sex ratios for aggregate savings and current account balances. None of the papers in this literature explores the general equilibrium implications for exchange rates from a change in the sex ratio.

The fourth literature examines empirically the causes of a rise in the sex ratio. The key insight is that the proximate cause for the recent rise in the sex ratio imbalance is sex-selective abortions, which have been made increasingly possible by the spread of Ultrasound B machines. There are two deeper causes for the parental willingness to disproportionately abort female fetuses. The first is the parental preference for sons, which in part has to do with the relatively inferior economic status of women. When the economic status of women improves, sex-selective abortions appear to decline (Qian, 2008). The second is either something that leads parents to voluntarily have a lower fertility rate than earlier generations, or a government policy that limits the number of children a couple can have. In regions of China where the family planning policy is less strictly enforced, there is also less sex ratio imbalance (Wei and Zhang, 2009). Bhaskar (2011) examines parental sex selections and their welfare consequences.

The rest of the paper is organized as follows. In Section 2, we construct a simple overlapping generations (OLG) model with only one gender, and show that structural shocks, by raising the savings rate, can simultaneously produce a real exchange rate depreciation and a current account surplus. In Section 3, we present an OLG model with two genders, and demonstrate that a rise in the sex ratio could lead to a rise in both the aggregate savings rate and the current account, and a fall in the value of the real exchange rate. In Section 4, we calibrate the model to see if the sex ratio imbalance can produce changes in the real exchange rate and current account whose magnitudes are economically significant. In section 5, we provide some empirical evidence on the connection between the sex ratio and the real exchange rate. Section 6 offers concluding remarks and discusses possible future research.

2 A benchmark model with one gender

We start with a simple benchmark model with one gender. This allows us to see the savings channel in a transparent way. The setup is standard, and the discussion will pave the way for a model in the next section that features two genders and an unbalanced sex ratio.

There are two types of agents: consumers and producers. Consumers consume and make the saving decisions to maximize their intertemporal utilities. Producers choose capital and labor input to maximize the profits.

2.1 Consumers

Consumers live for two periods: young and old. They receive labor income in the first period and nothing in the second period after retiring. In the first period, consumers consume a part of the labor income in the first period and save the rest for the second period.

The final good C_t consumed by consumers consists of two parts: a tradable good C_{Tt} and a nontradable good C_{Nt}.

$$C_t = \frac{C_{Nt}^{\gamma} C_{Tt}^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$$

We normalize the price of the tradable good to be one, and let P_{Nt} denote the relative price of the nontradable good. The consumer price index is $P_t = P_{Nt}^{\gamma}$.

Consumers earn labor income when they are young and retire when they are old. The optimization problem for a representative consumer is

$$\max \; u(C_{1t}) + \beta u(C_{2,t+1})$$

with the intertemporal budget constraint

$$P_t C_{1t} = (1 - s_t) y_t \text{ and } P_{t+1} C_{2,t+1} = R s_t y_t$$

where y_t is the disposable income and s_t is the savings rate of the young cohort. R is the gross interest rate in terms of the tradable good.

The optimal condition for the representative consumer's problem is

$$\frac{u'_{1t}}{P_t} = \beta R \frac{u'_{2,t+1}}{P_{t+1}} \tag{2.1}$$

We start with the case of a small open economy, and assume that the law of one price for the tradable good holds. The price of the tradable good is determined by the world market, and is set to be one in each period. The interest rate R in units of the tradable good is also a constant.

2.2 Producers

There are two sectors in the economy: a tradable good sector and a non-tradable good sector. Both markets are perfectly competitive. For simplicity, we make the same assumption as in Obstfeld and Rogoff (1996) that only the tradable good can be transformed into capital used in production.[1]

2.2.1 Tradable good producers

For simplicity, we assume a complete depreciation of capital at the end of every period. Tradable producers maximize

$$\max \; E_t \sum_{\tau=0}^{\infty} (R)^{-\tau} \left[Q_{T,t+\tau} - w_{t+\tau} L_{T,t+\tau} - K_{T,t+\tau+1} \right]$$

where the production function is

$$Q_{Tt} = \frac{A_{Tt} K_{Tt}^{\alpha_T} L_{Tt}^{1-\alpha_T}}{\alpha_T^{\alpha_T} (1-\alpha_T)^{1-\alpha_T}}$$

Without any unanticipated shocks, the factor demand functions are, respectively,

$$R = \frac{1}{\alpha_T^{\alpha_T} (1-\alpha_T)^{1-\alpha_T}} \alpha_T A_{Tt} \left(\frac{L_{Tt}}{K_{Tt}} \right)^{1-\alpha_T} \qquad (2.2)$$

$$w_t = \frac{1}{\alpha_T^{\alpha_T} (1-\alpha_T)^{1-\alpha_T}} (1-\alpha_T) A_{Tt} \left(\frac{K_{Tt}}{L_{Tt}} \right)^{\alpha_T} \qquad (2.3)$$

It is useful to note that when there is an unanticipated shock in period t, (2.2) does not hold since K_{Tt} is a predetermined variable.

2.2.2 Nontradable good producers

Nontradable good producers maximize the following objective function:

$$\max \; E_t \sum_{\tau=0}^{\infty} (R)^{-\tau} \left[P_{N,t+\tau} Q_{N,t+\tau} - w_{t+\tau} L_{N,t+\tau} - K_{N,t+\tau+1} \right]$$

with the production function given by

$$Q_{Nt} = \frac{A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}}$$

Without unanticipated shocks, we have

$$R = \frac{1}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}} P_{Nt} \alpha_N A_{Nt} \left(\frac{L_{Nt}}{K_{Nt}} \right)^{1-\alpha_N} \qquad (2.4)$$

$$w_t = \frac{1}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}} P_{Nt} (1-\alpha_N) A_{Nt} \left(\frac{K_{Nt}}{L_{Nt}} \right)^{\alpha_N} \qquad (2.5)$$

[1] Relaxing this assumption will not change any of our results qualitatively.

If there is an unanticipated shock in period t, (2.4) does not hold.

In equilibrium, the market clearing condition for the nontradable good pins down the price of the nontradable good,

$$Q_{Nt} = \frac{\gamma P_t \left(C_{2t} + C_{1t}\right)}{P_{Nt}} \tag{2.6}$$

The labor market clearing condition is given by

$$L_{Tt} + L_{Nt} = 1 \tag{2.7}$$

Assuming no labor income tax (for simplicity), $y_t = w_t$.

Definition 1 *An equilibrium in the small open economy is a set* $\{s_t, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *that satisfies the following conditions:*

(i) The households' savings rates, $s_t = \{s_{it}, s_{-i,t}\}$*, maximize the household's welfare*

$$s_t = \arg\max \{ V_t |\, s_{-i,t}, K_{Tt+1}, K_{Nt+t}, L_{Tt}, L_{Nt}, P_{Nt}\}$$

(ii) The allocation of capital stock and labor, and the output of the non-tradable good clear the factor and the output markets, and maximize the firms' profit. In other words, $\{K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *solves (2.2), (2.3), (2.4), (2.5), (2.6) and (2.7).*

2.3 A shock to the savings rate and the effect on the exchange rate

To illustrate the idea that a shock that raises the savings rate could lower the value of the real exchange rate, we now consider an unanticipated increase in the discount factor β that makes the young cohort more patient. In period t, (2.3) and (2.5) hold, but (2.2) and (2.4) fail.

The market clearing condition for the nontradable good can be re-written as

$$\frac{P_{Nt} A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}} = \gamma \left(R s_{t-1}^{young} w_{t-1} + (1 - s_t^{young}) w_t \right)$$

We can solve (2.1), (2.6), (2.3) and (2.5) to obtain the equilibrium in period t. Let $R = \frac{RP_t}{P_{t+1}}$ denote the real interest rate. We assume that the utility function is of the CRRA form, i.e., $u(C) = \frac{C^{1-\sigma}-1}{1-\sigma}$. Following Obstfeld and Rogoff (1996) and assuming that the nontradable good sector is relatively more labor-intensive, i.e., $\alpha_N < \alpha_T$, we can obtain the following proposition.

Proposition 1 *With an increase in the discount factor* β *of the young cohort, the aggregate savings rate rises, and the price of the nontradable good falls. As a result, the real exchange rate depreciates and the current account increases.*

Proof. See Appendix A. ■

In the period in which the shock occurs, as a representative consumer becomes more patient, he would save more and consume less. The reduction in aggregate consumption leads to a decrease in the relative price of nontradable good (and a depreciation of the real exchange rate). As the rise in savings is not accompanied by a corresponding rise in investment, the country's current account increases. In summary, without currency manipulations, real factors that lead to a rise in a country's savings rate can simultaneously produce a fall in the real exchange rate and a rise in the current account. The low value of the real exchange rate is not the cause of the current account surplus.

Note that the effect on the RER and the current account last for one period. In period $t+1$, since the shock has been observed and taken into account by consumers and firms, (2.2) and (2.4) hold in equilibrium. By solving (2.2), (2.3), (2.4) and (2.5), we have

$$P_{Nt} = R^{\frac{\alpha_N - \alpha_T}{1-\alpha_T}} \text{ and } P_{t+1} = R^{\frac{\gamma(\alpha_N - \alpha_T)}{1-\alpha_T}}$$

In other words, the price of the nontradable good and the consumer price index go back to their initial levels. Later in the paper, we will demonstrate how frictions in the factor market can produce longer-lasting effects on the real exchange rate and the current account.

3 Unbalanced sex ratios and real exchange rates

In this section, we extend our benchmark model to a two-sex overlapping generations model. Within each cohort, there are women and men. A marriage can take place at the beginning of a cohort's second period, but only between a man and a women in the same cohort. Once married, the husband and wife pool their first-period savings together and consume an identical amount in the second period. The second period consumption within a marriage has a partial public good feature. In other words, the husband and wife can each consume more than half of their combined second period income. Everyone is endowed with an ability to give his/her spouse some additional emotional utility (or "love"). This emotional utility is a random variable in the first period with a common and known distribution across all members of the same sex, and its value is realized and becomes public information when an individual enters the marriage market. There are no divorces.

Each generation is characterized by an exogenous ratio of men to women $\phi(\geq 1)$. All men are identical *ex ante*, and all women are identical *ex ante*. Men and women are symmetric in all aspects - in particular, men do not have an intrinsic tendency to save more - except that the sex ratio may be unbalanced.

Throughout the model, we maintain the assumption of an exogenous sex ratio. While it is surely endogenous in the long run as parental preference should evolve, the assumption of an exogenous sex ratio can be defended on two grounds. First, the technology that enables the rapid rise in the sex ratio has only become inexpensive and widely accessible in developing countries within the last 25 years or so. As a result, it is reasonable to think that the rising sex ratio affects only the relatively young cohort's savings decisions, but not those who have passed half of their working careers. Second, in terms of cross country experience, most countries with a skewed sex ratio have not shown a sign of reversal. This suggests that, if the sex ratio follows a mean reversion process, the speed of reversion is very low.

3.1 A small open economy

We start from a small open economy. As in the benchmark model, the price of the tradable good is always one and the interest rate in units of the tradable good is a constant R. As in Obstfeld and Rogoff (1995), we assume that only tradable goods can be converted into capital used in production.

A Representative Woman's Optimization Problem

A representative woman makes her consumption/saving decisions in her first period, taking into account the choices by men and all other women, and the likelihood that she will be married. If she fails to get married, her second-period consumption is $P_{t+1}C_{2,t+1}^{w,n} = Rs_t^w y_t^w$, where R, y_t^w and s_t^w are the gross interest rate of an international bond, her endowment, and her savings rate, respectively, all in units of the tradable good. If she is married, her second-period consumption is $P_{t+1}C_{2,t+1}^{w} = \kappa\left(Rs_t^w y_t^w + Rs_t^m y_t^m\right)$, where y_t^m and s_t^m are her husband's first period endowment and savings rate, respectively. κ ($\frac{1}{2} \leq \kappa \leq 1$) represents the notion that consumption within a marriage is a public good with congestion. As an example, if two spouses buy a car, both can use it. In contrast, were they single, they would have to buy two cars. When $\kappa = \frac{1}{2}$, the husband and the wife only consume private goods. When $\kappa = 1$, then all the consumption is a public good with no congestion[2].

The optimal savings rate is chosen to maximize the following objective function:

$$V_t^w = \max_{s_t^w} u(C_{1t}^w) + \beta E_t\left[u(C_{2,t+1}^w) + \eta^m\right]$$

subject to the budget constraints that

$$P_t C_{1t}^w = (1 - s_t^w) y_t^w \tag{3.1}$$

$$P_{t+1}C_{2,t+1}^w = \begin{cases} \kappa\left(Rs_t^w y_t^w + Rs_t^m y_t^m\right) & \text{if married} \\ Rs_t^w y_t^w & \text{otherwise} \end{cases} \tag{3.2}$$

where E_t is the conditional expectation operator. η^m is the emotional utility (or "love") she obtains from her husband, which is a random variable with a distribution function F^m. Bhaskar (2011) also

[2] By assuming the same κ for the wife and the husband, we abstract from a discussion of bargaining within a household. In an extension later in the paper, we allow κ to be gender specifc, and to be a function of both the sex ratio and the relative wealth levels of the two spouses, along the lines of Chiappori (1988 and 1992) and Browning and Chiappori (1998). This tends to make the response of the aggregate savings stronger to a given rise in the sex ratio.

introduces a similar "love" variable.

A Representative Man's Optimization Problem

A representative man's problem is symmetric to a women's problem. In particular, if he fails to get married, his second period consumption is $P_{t+1}C_{2,t+1}^{m,n} = Rs_t^m y^m$. If he is married, his second period consumption is $P_{t+1}C_{2,t+1}^{m} = \kappa\left(Rs_t^w y_t^w + Rs_t^m y_t^m\right)$. He will choose his savings rate to maximize the following value function

$$V_t^m = \max_{s_t^m} u(C_{1t}^m) + \beta E_t\left[u(C_{2,t+1}^m) + \eta^w\right]$$

subject to the budget constraints that

$$P_t C_{1t}^m = (1 - s_t^m) y_t^m \tag{3.3}$$

$$P_{t+1}C_{2,t+1}^m = \begin{cases} \kappa\left(Rs_t^w y_t^w + Rs_t^m y_t^m\right) & \text{if married} \\ Rs_t^m y_t^m & \text{otherwise} \end{cases} \tag{3.4}$$

where V^m is his value function. η^w is the emotional utility he obtains from his wife, which is drawn from a distribution function F^w.

The Marriage Market[3]

In the marriage market, every woman (or man) ranks all members of the opposite sex by a combination of two criteria: (1) the level of wealth (which is determined solely by the first-period savings), and (2) the size of "love" she/he can obtain from her/his spouse. The weights on the two criteria are implied by the utility functions specified earlier. More precisely, woman i prefers a higher ranked man to a lower ranked one, where the rank on man j is given by $u(c_{2w,i,j}) + \eta_j^m$. Symmetrically, man j assigns a rank to woman i based on the utility he can obtain from her $u(c_{2m,j,i}) + \eta_i^w$. To ensure that the preference is strict for both men and women, whenever there is a tie in terms of the above criteria, we break the tie by assuming that a woman prefers j if $j < j'$ and a man does the same. Note that "love" is not in the eyes of a beholder in the sense that every woman (man) has the same ranking over men (women).

The marriage market is assumed to follow the Gale-Shapley algorithm, which produces a unique and stable equilibrium of matching (Gale and Shapley, 1962; and Roth and Sotomayor, 1990). The algorithm specifies the following: (1) Each man proposes in the first round to his most preferred choice of woman. Each woman holds the proposal from her most preferred suitor and rejects the rest. (2) Any man who is rejected in round k-1 makes a new proposal in round k to his most preferred woman among those who have not have rejected him. Each available woman in round k holds the proposal from her most preferred man and rejects the rest. (3) The procedure repeats itself until no further proposals are made, and the women accept the most attractive proposals.[4]

[3] We use the word "market" informally here. The pairing of husbands and wives is not done through prices.

[4] If only women can propose and men respond with deferred acceptance, the same matching outcomes will emerge.

With many women and men in the marriage market, all women (and all men) approximately form a continuum and each individual has a measure close to zero. Let I^w and I^m denote the continuum formed by women and men respectively. We normalize I^w and let $I^w = (0, 1)$. Since the sex ratio is ϕ, the set of men $I^m = (0, \phi)$. Men and women are ordered in such a way that a higher value in the set means a higher ranking by members of the opposite sex.

In equilibrium, there exists a unique mapping (π^w) for women in the marriage market, $\pi^w : I^w \rightarrow I^m$. That is, woman i ($i \in I^w$) is mapped to man j ($j \in I^m$), given all the savings rates and emotional utility draws. This implies a mapping from a combination (s_i^w, η_i^w) to another combination (s_j^m, η_j^m). Before she enters the marriage market, she knows only the distribution of her own type but not the exact value. As a result, the type of her future husband (s_j^m, η_j^m) is also a random variable. Woman i's second period expected utility is

$$\begin{aligned} & \int \max \left[\; u\left(c_{2w,i,j}\right) + \eta^m_{\pi^w\left(\,i|s_i^w,\eta_i^w,s_{-i}^w,\eta_{-i}^w,s^m,\eta^m\right)}, \quad u\left(R s_i^w y_i^w\right) \; \right] dF^w\left(\eta_i^w\right) \\ = \; & \int_{\bar{\pi}_i^w} \left[u\left(c_{2w,i,j}\right) + \eta^m_{\pi^w\left(\,i|s_i^w,\eta_i^w,s_{-i}^w,\eta_{-i}^w,s^m,\eta^m\right)} \right] dF^w\left(\eta_i^w\right) + \int^{\bar{\pi}_i^w} u\left(R s_i^w y_i^w\right) dF^w\left(\eta_i^w\right) \end{aligned}$$

where $\bar{\pi}_i^w$ is her threshold ranking on men such that she is indifferent between marriage or not. Any lower-ranked man, or any man with $\pi_i^w < \bar{\pi}_i^w$, won't be chosen by her.

Since we assume there are (weakly) fewer women than men, we expand the set I^w to $\widetilde{I}^w$ so that $\widetilde{I}^w = (0, \phi)$. In the expanded set, women in the marriage market start from value $\phi - 1$ to ϕ. The measure for women in the marriage market remains one. In equilibrium, there exists a unique mapping for men in the marriage market: $\pi^m : I^m \rightarrow \widetilde{I}^w$, where π^m maps man j ($j \in I^m$) to woman i ($i \in I^w$). Those men with a low value $i < \phi - 1$ in set $\widetilde{I}^w$ will not be married. In that case, $\eta^w_{\pi^m(j)} = 0$ and $c_{2m,j,i} = R s_j^m y_j^m$. In general, man j's second period expected utility is

$$\begin{aligned} & \int \max \left[\; u\left(c_{2m,j,i}\right) + \eta^w_{\pi^m\left(\,j|s_j^m,\eta_j^m,s_{-j}^m,\eta_{-j}^m,s^w,\eta^w\right)}, \quad u\left(R s_j^m y_j^m\right) \; \right] dF^m\left(\eta_j^m\right) \\ = \; & \int_{\bar{\pi}_j^m} \left[u\left(c_{2m,j,i}\right) + \eta^w_{\pi^m\left(\,j|s_j^m,\eta_j^m,s_{-j}^m,\eta_{-j}^m,s^w,\eta^w\right)} \right] dF^m\left(\eta_j^m\right) + \int^{\bar{\pi}_j^m} u\left(R s_j^m y_j^m\right) dF^m\left(\eta_j^m\right) \end{aligned}$$

where $\bar{\pi}_j^m$ is his threshold ranking on all women. Any woman with a poorer rank, $\pi_j^m < \bar{\pi}_j^m$, will not be chosen by him.

We assume that the density functions of η^m and η^w are continuously differentiable. Since all men (women) in the marriage market have identical problems, they make the same savings decisions. In equilibrium, a *positive assortative matching* emerges for those men and women who are married. In

What we have to rule out is that both men and women can propose, in which case, one cannot prove that the matching is unique.

other words, there exists a mapping M from η^w to η^m such that

$$\begin{aligned} 1-F^w(\eta^w) &= \phi\left(1-F^m\left(M(\eta^w)\right)\right) \\ &\Leftrightarrow \\ M(\eta^w) &= (F^m)^{-1}\left(\frac{F^w(\eta^w)}{\phi}+\frac{\phi-1}{\phi}\right) \end{aligned}$$

For simplicity, we assume that η^w and η^m are drawn from the same distribution, $F^w = F^m = F$. The lowest possible value of emotional utility $\eta^{\min}$ is sufficiently small (which can be negative) so that some women and some men may not get married. Let $\bar{\eta}^w$ and $\bar{\eta}^m$ denote the threshold values for women's and men's emotional utilities in equilibrium, respectively. Only women (men) with emotional utilities higher than the threshold value $\bar{\eta}^w$ ($\bar{\eta}^m$) will get married. In other words,

$$\bar{\eta}^w = \max\left\{u_{2m,n}-u_{2m}, M^{-1}\left(\bar{\eta}^m\right)\right\} \text{ and } \bar{\eta}^m = \max\left\{u_{2w,n}-u_{2w}, M(\bar{\eta}^w)\right\} \tag{3.5}$$

For woman i, given all her rivals' and men's savings decisions and η^w, her second period utility is

$$\delta_i^w u\left(\frac{\kappa\left(Rs_i^w y^w + Rs^m y^m\right)}{P_{t+1}}\right) + \left(1-\delta_i^w\right) u\left(\frac{Rs^w y^w}{P_{t+1}}\right) + \int_{\tilde{\eta}_i^w \geq \bar{\eta}^w} M(\tilde{\eta}_i^w) dF(\eta_i^w)$$

where $\tilde{\eta}_i^w = u\left(\frac{\kappa(Rs_i^w y^w + Rs^m y^m)}{P_{t+1}}\right) - u\left(\frac{\kappa(Rs^w y^w + Rs^m y^m)}{P_{t+1}}\right) + \eta_i^w$. δ_i^w is the probability that woman i will get married,

$$\begin{aligned} \delta_i^w &= \Pr\left(u\left(\frac{\kappa\left(Rs_i^w y^w + Rs^m y^m\right)}{P_{t+1}}\right) - u\left(\frac{\kappa\left(Rs^w y^w + Rs^m y^m\right)}{P_{t+1}}\right) + \eta_i^w \geq \bar{\eta}^w \middle| Rs^w y^w, Rs^m y^m\right) \\ &= 1-F\left(\bar{\eta}^w - u\left(\frac{\kappa\left(Rs_i^w y^w + Rs^m y^m\right)}{P_{t+1}}\right) + u\left(\frac{\kappa\left(Rs^w y^w + Rs^m y^m\right)}{P_{t+1}}\right)\right) \end{aligned} \tag{3.6}$$

Due to symmetry, we drop the sub-index i for women. Given men's savings decisions, the first order condition for her optimization problem is

$$-u'_{1w} y^w + \beta\left[\begin{array}{c} \delta^w u'_{2w} \frac{\partial c_{2w}}{\partial s^w} + \left(1-\delta^w\right) u'_{2w,n} \frac{RP_t}{P_{t+1}} y^w + \frac{\partial \int_{\tilde{\eta}^w \geq \bar{\eta}^w} M(\tilde{\eta}^w) dF(\eta^w)}{\partial s^w} \\ + \frac{\partial \delta^w}{\partial s^w}\left(u_{2w}-u_{2w,n}\right) \end{array}\right] = 0 \tag{3.7}$$

where

$$\begin{aligned} \frac{\partial \int_{\tilde{\eta}^w \geq \bar{\eta}^w} M(\tilde{\eta}^w) dF(\eta^w)}{\partial s^w} &= \kappa u'_{2w} \frac{RP_t}{P_{t+1}} y^w \left[\int_{\bar{\eta}^w} M'(\eta^w) dF(\eta^w) + M(\bar{\eta}^w) f(\bar{\eta}^w)\right] \\ \frac{\partial \delta^w}{\partial s^w} &= f\left(\bar{\eta}^w\right) \kappa u'_{2w} \frac{RP_t}{P_{t+1}} y^w \end{aligned}$$

Similarly, a representative man's second-period utility, given his rivals' and all women's savings decisions, is

$$\delta_j^m u\left(\frac{\kappa\left(Rs^w y^w + Rs_j^m y^m\right)}{P_{t+1}}\right) + (1-\delta_j^m) u\left(\frac{Rs_j^m y^m}{P_{t+1}}\right) + \int_{\tilde{\eta}_j^m \geq \bar{\eta}^m} M^{-1}\left(\tilde{\eta}_j^m\right) dF(\eta_j^m)$$

where $\tilde{\eta}_j^m = u\left(\frac{\kappa\left(Rs^w y^w + Rs_j^m y^m\right)}{P_{t+1}}\right) - u\left(\frac{\kappa(Rs^w y^w + Rs^m y^m)}{P_{t+1}}\right) + \eta_j^m$ and δ_j^m is the probability he gets married

$$\begin{aligned}
\delta_j^m &= \Pr\left(u\left(\frac{\kappa\left(Rs^w y^w + Rs_j^m y^m\right)}{P_{t+1}}\right) - u\left(\frac{\kappa\left(Rs^w y^w + Rs^m y^m\right)}{P_{t+1}}\right) + \eta_j^m \geq \bar{\eta}^m \middle| Rs^w y^w, Rs^m y^m\right) \\
&= 1 - F\left(\bar{\eta}^m - u\left(\frac{\kappa\left(Rs^w y^w + Rs_j^m y^m\right)}{P_{t+1}}\right) + u\left(\frac{\kappa\left(Rs^w y^w + Rs^m y^m\right)}{P_{t+1}}\right)\right) \quad (3.8)
\end{aligned}$$

The first order condition for a representative man's optimization problem is

$$-u'_{1m} y^m + \beta\left[\begin{array}{c} \delta^m u'_{2m}\frac{\partial c_{2m}}{\partial s^m} + \frac{\partial \int_{\tilde{\eta}^m \geq \bar{\eta}^m} M^{-1}(\tilde{\eta}^m) dF(\eta^m)}{\partial s^m} + (1-\delta^m)\, u'_{2m,n}\frac{RP_t}{P_{t+1}} y^m \\ +\frac{\partial \delta^m}{\partial s^m}\left(u_{2m} - u_{2m,n}\right) \end{array}\right] = 0 \quad (3.9)$$

where

$$\begin{aligned}
\frac{\partial \int_{\tilde{\eta}^m \geq \bar{\eta}^w} M^{-1}(\tilde{\eta}^m) dF(\eta^m)}{\partial s^m} &= \kappa u'_{2m}\frac{RP_t}{P_{t+1}} y^m \left[\int_{\bar{\eta}^m} \frac{\partial M^{-1}(\eta^m)}{\partial \eta^m} dF(\eta^m) + M^{-1}(\bar{\eta}^m) f(\bar{\eta}^m)\right] \\
\frac{\partial \delta^m}{\partial s^m} &= f\left(\bar{\eta}^m\right) \kappa u'_{2m}\frac{RP_t}{P_{t+1}} y^m
\end{aligned}$$

For simplicity, we assume that women and men will earn the same first period labor income and that there is no tax, i.e., $y_t^w = y_t^m = w_t$. We now define an equilibrium in this economy.

Definition 2 *An equilibrium is a set of savings rates, capital and labor allocation by sector, and the relative price of nontradable good* $\{s_t^w, s_t^m, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *that satisfies the following conditions:*

(i) The savings rates by the representative woman and the representative man, conditional on other women and men's savings rates, $s_t^w = \left\{s_{it}^w, s_{-i,t}^w\right\}$ *and* $s_t^m = \left\{s_{jt}^w, s_{-j,t}^m\right\}$*, maximize their respective utilities*

$$\begin{aligned}
s_{it}^w &= \arg\max\left\{V_t^w \,|\, s_{-i,t}^w, s_t^m, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\right\} \\
s_{jt}^w &= \arg\max\left\{V_t^m \,|\, s_t^w, s_{-j,t}^m, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\right\}
\end{aligned}$$

(ii) The markets for capital, labor, and tradable and nontradable goods clear, and firms maximize their profits. In other words, $\{K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *solves (2.2), (2.3), (2.4), (2.5), (2.6)*

and (2.7).

Shocks to the sex ratio We now consider an unanticipated shock to the young cohort's sex ratio, i.e., the sex ratio rises from one to $\phi(>1)$ from period t onwards. The nature of the shock is motivated by the facts about the sex ratio imbalance in China. Since a severe sex ratio imbalance for the pre-marital age cohort is a relatively recent phenomenon, the older generations' savings decisions were largely made when there was no severe sex ratio imbalance. As the shock is unanticipated, (2.2) and (2.4) do not hold in period t.

As in the benchmark model, the market clearing condition for the nontradable good can be re-written as

$$\frac{P_{Nt}A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}=\gamma\left(Rs_{t-1}w_{t-1}+(1-s_t)w_t\right) \tag{3.10}$$

where $s_t=\frac{\phi}{1+\phi}s_t^m+\frac{1}{1+\phi}s_t^w$ is the aggregate savings rate by the young cohort in period t.

By (2.3) and (2.5), we have

$$w_t=\frac{1}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}(1-\alpha_T)A_{Tt}\left(\frac{K_{Tt}}{1-L_{Nt}}\right)^{\alpha_T}=\frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}P_{Nt}(1-\alpha_N)A_{Nt}\left(\frac{K_{Nt}}{L_{Nt}}\right)^{\alpha_N} \tag{3.11}$$

We can solve (3.7), (3.9), (3.10) and (3.11) to obtain the equilibrium in period t. With some restrictions on the utility function and the distribution of emotional utility, we have the following proposition.

Proposition 2 *Assume that the utility function is of log form, $u(C)=\ln C$, for all men and women, and that η is drawn from a uniform distribution, then, as the sex ratio in the young cohort rises, a representative man in the cohort increases his savings rate while the savings response by a representative woman is ambiguous. However, the economy-wide savings rate increases unambiguously. The real exchange rate depreciates and the current account rises.*

Proof. See Appendix B. ■

A few remarks are in order. First, it is perhaps not surprising that the representative man raises his savings rate in response to a rise in the sex ratio because the need to compete in the marriage market becomes greater. Why does the representative woman reduce her savings rate? Because she anticipates a higher savings rate from her future husband, she does not need to sacrifice her first-period consumption as much as she otherwise would have to.

Second, why does the aggregate savings rate rise in response to a rise in the sex ratio? In other words, why is the increment in men's savings greater than the decline in women's savings? Intuitively, a representative man raises his savings rate for two reasons: in addition to improving his relative standing in the marriage market, he raises his savings rate to make up for the lower savings rate by his future wife. The more his future wife is expected to cut down her savings, the more he would have to raise his own savings to compensate. This ensures that his incremental savings is more than enough to offset any reduction in his future wife's savings. In addition, since men save more, the rising share of men in the population would also raise the aggregate savings rate. While both channels contribute to a rise in the aggregate savings rate, it is easy to verify that the first channel (the incremental competitive savings by any given man) is more important than the second effect (a change in the composition of the population with different saving propensities).

Third, once we obtain an increase in the aggregate savings rate, the logic from the previous one-gender benchmark model applies. In particular, the relative price of the non-tradable good declines (and hence the real exchange rate depreciates), and the current account rises.

Similar to the benchmark model with a single gender, once the shock is observed and taken into account in period $t+1$, (2.2) and (2.4) hold in equilibrium. By solving (2.2), (2.3), (2.4) and (2.5), we have

$$P_{Nt} = R^{\frac{\alpha_N - \alpha_T}{1-\alpha_T}} \text{ and } P_{t+1} = R^{\frac{\gamma(\alpha_N - \alpha_T)}{1-\alpha_T}}$$

This means that the real exchange rate and the current account will return to their previous values after one period.

3.2 Mixed-strategy equilibrium

In this section, we extend our benchmark model by considering an endogenous choice of entering/exiting the marriage market. Formally, we consider a mixed-strategy game in which (a) a representative woman will choose the probability of entering the marriage market ρ^w, a savings rate if she decides to enter, and a separate savings rate if she decides to abstain from the marriage market; and (b) a representative man has similar choices.

The representative woman will have the same optimization problem as in the previous section if she enters the marriage market. She can also choose to be single, and if she does so, her life-time utility is

$$V_n^w = \max_{s_n^w} u(c_{1w,n}) + \beta u(c_{2w,n})$$

where V_n^w denotes the value function of a woman who is single throughout her life.

Her overall optimization problem in the mixed-strategy game is

$$\max_{\rho^w, s^w, s_n^w} \rho^w V^w + (1-\rho^w) V_n^w$$

Obviously, she will choose $\rho^w = 1$ if and only if $V^w > V_n^w$.

Similarly, a representative man's overall optimization problem is

$$\max_{\rho^m, s^m, s_n^m} \rho^m V^m + (1-\rho^m) V_n^m$$

where V_n^m denotes the value function of a representative man who is single throughout his life, and ρ^m is his probability of entering the marriage market. He would decide to enter the marriage market with probability one if and only if the expected utility of doing so is greater than otherwise, or $V^m > V_n^m$.

Now we can show a more general proposition in the following:

Proposition 3 *Assume that the utility function is of log form and that emotional utility is drawn from an independent and identical uniform distribution, then there exists a threshold value $\phi_1 > 1$ that satisfies $V^m = V_n^m$.*

(i) For $\phi < \phi_1$, both women and men choose to enter the marriage market with probability one. In addition, as the sex ratio rises, the savings rate of a representative man increases while the change in the savings rate of a representative woman is ambiguous. However, the economy-wide savings rate increases unambiguously, and the real exchange rate declines.

(ii) For $\phi \geq \phi_1$, as the sex ratio rises, a representative man chooses a positive probability of being single while a representative woman still chooses to enter the marriage market with probability one. The changes in the aggregate savings rate and the real exchange rate are ambiguous.

Proof. See Appendix C. ■

Two remarks are in order. First, the proposition states that as the sex ratio rises, up to a threshold ϕ_1, a representative man always chooses to enter the marriage market and raises his savings rate in response to a higher sex ratio. A representative woman also always chooses to enter the marriage market but reduces her savings rate in response to a higher sex ratio. This part is similar to Proposition 2. However, once the sex ratio exceeds the threshold ϕ_1, the representative man would respond to an additional increase in the sex ratio by choosing a progressively greater probability of not entering the marriage market. He does so because his savings rate is already high enough such that sharing his savings with a low-type spouse could yield him a lower utility. For the representative woman, entering the marriage market is still a dominant strategy even after the threshold.

Second, as the sex ratio rises, the representative man suffers a welfare loss from two sources. A higher sex ratio reduces his chance of marriage. In addition, while he has to increase his savings in order not to lose out to his competitors in the marriage market, the increased savings in the end does not alter his probability of marriage. Interestingly, the effect of a higher sex ratio on a presentative woman is ambiguous. On the one hand, she could gain both from her future husband's higher savings rates and from the improved probability to be matched with a man with a higher level of emotional utility. On the other hand, precisely because men have raised their savings rate, they become more reluctant to share their high savings rate with a low-type woman. As a result, a representative woman's chance of getting married declines. These two opposing forces produce an ambiguous net effect on the representative woman. It is useful to note that, while a representative woman could lose from a higher sex ratio, her utility level is always higher than that of the representative man.

3.3 Capital adjustment costs

Without additional frictions, a shock to the sex ratio can only affect the real exchange rate for one period. If there are capital adjustment costs in each sector, the effect on the real exchange rate can be prolonged. We assume that the capital accumulation in each sector is as following:

$$K_{t+1} = (1-\delta)K_t + I_t - \frac{b}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$$

where δ is the depreciation rate and I_t is investment. $\frac{b}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$ represents the adjustment cost as in Chari, Kehoe and McGrattan (2002).

Then (2.2) and (2.4) become, respectively,

$$\begin{aligned} R &= 1-\delta + \frac{1}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}\alpha_T A_{Tt}\left(\frac{L_{Tt}}{K_{Tt}}\right)^{1-\alpha_T} \\ &\quad -bR\left(\frac{I_{Tt}}{K_{Tt}} - \delta\right) - \frac{b}{2}\left(\left(\frac{I_{Tt}}{K_{Tt}}\right)^2 - \delta^2\right) \qquad (3.12) \\ R &= 1-\delta + \frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}P_{Nt}\alpha_N A_{Nt}\left(\frac{L_{Nt}}{K_{Nt}}\right)^{1-\alpha_T} \\ &\quad -bR\left(\frac{I_{Nt}}{K_{Nt}} - \delta\right) - \frac{b}{2}\left(\left(\frac{I_{Nt}}{K_{Nt}}\right)^2 - \delta^2\right) \qquad (3.13) \end{aligned}$$

Without capital adjustment cost, i.e., $b = 0$, the price of the nontradable good will go back to its equilibrium level in period $t+1$. If $b > 0$, then

$$P_{Nt} = \frac{\frac{1}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}\alpha_T A_{Tt+1}\left(\frac{L_{Tt+1}}{K_{Tt+1}}\right)^{1-\alpha_T} - bR\left(\frac{I_{Tt+1}}{K_{Tt+1}} - \frac{I_{Nt+1}}{K_{Nt+1}}\right) - \frac{b}{2}\left(\left(\frac{I_{Tt+1}}{K_{Tt+1}}\right)^2 - \left(\frac{I_{Nt+1}}{K_{Nt+1}}\right)^2\right)}{\frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}\alpha_N A_{Nt+1}\left(\frac{L_{Nt+1}}{K_{Nt+1}}\right)^{1-\alpha_T}}$$

P_{Nt} is now a function of $\frac{I_{Tt+1}}{K_{Tt+1}}$ and $\frac{I_{Nt+1}}{K_{Nt+1}}$. If $\frac{I_{Tt+1}}{K_{Tt+1}} \neq \frac{I_{Nt+1}}{K_{Nt+1}}$, P_{Nt} is not a constant. This means that, with capital adjustment costs, the price of the nontradable good does not return immediately to its long-run equilibrium level. As a result, a rise in the sex ratio can have a long-lasting and depressing effect on the real exchange rate.

3.4 Two large countries

We now turn to a world with two large countries: Home and Foreign. Assume that they are identical in every respect except for their sex ratios. Specifically, in period t, the sex ratio of the young cohort in Home rises from one to ϕ ($\phi > 1$), while Foreign always has a balanced sex ratio. Households in each country consume a tradable good and a nontradable good.

$$C_t = \frac{C_{Nt}^{\gamma} C_{Tt}^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \text{ and } C_t^* = \frac{(C_{Nt}^*)^{\gamma} (C_{Tt}^*)^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$$

where C_t and C_t^* represent home and foreign consumption indexes, respectively. Since we choose the tradable good as the numeraire, the consumer price index is $P_t = P_{Nt}^{\gamma}$, where P_{Nt} is the price of the home produced nontradable good. Similarly, the consumer price index in Foreign is $P_t^* = (P_{Nt}^*)^{\gamma}$.

The rise in Home's sex ratio in period t is assumed to be unanticipated. As a result, (2.2) and (2.4) fail in both Home and Foreign. By the same reasoning, Home experiences a real exchange rate depreciation in period t, but a real appreciation in period $t+1$. We can write the current account in Home and Foreign as follows:

$$CA_t = s_t w_t - s_{t-1} w_{t-1} + K_t - K_{t+1} \text{ and } CA_t^* = s_t^* w_t^* - s_{t-1}^* w_{t-1}^* + K_t^* - K_{t+1}^*$$

Before the shock, we had

$$s_{t-1} = s_{t-1}^*,\ w_{t-1} = w_{t-1}^* \text{ and } K_t = K_t^*$$

In period $t+1$, we have

$$P_{Nt} = P_{Nt},\ w_{t+1} = w_{t+1}^*, \text{ and } P_{t+1} = P_{t+1}^*$$

and the demand for the nontradable good is

$$Q_{N,t+1} = \frac{\gamma w_{t+1} \left((R-1)\, s_t + 1 \right)}{P_{Nt}} \text{ and } Q_{N,t+1}^* = \frac{\gamma w_{t+1}^* \left((R-1)\, s_t^* + 1 \right)}{P_{Nt}}$$

Since Home now has a higher sex ratio than Foreign, we have $s_t > s_t^*$, and therefore

$$Q_{N,t+1} > Q_{N,t+1}^*$$

We assume that the nontradable sector is more labor-intensive, i.e., $\alpha_N < \alpha_T$. Given the same technologies and the same labor endowments in the two countries, we have

$$K_{t+1} < K_{t+1}^*$$

In period t, since nothing changes in Foreign, it must be the case that $s_t^* w_t^* = s_{t-1} w_{t-1}$. Following the same steps as in the case of a small open economy, we can show that $s_t w_t > s_{t-1} w_{t-1} = s_t^* w_t^*$. Then it is easy to show that $CA_t > 0 > CA_t^*$. In other words, Home exhibits a current account surplus while Foreign experiences a current account deficit.

3.5 Endogenous labor supply

We turn to the case of endogenous labor supply. Just as a male raises his savings rate to gain a competitive advantage in the marriage market, he may choose to increase his supply of labor for the same reason in response to a rise in the sex ratio. This can translate into an increase in the effective aggregate labor supply if women do not decrease their labor supply too much. If the production of the nontradable good is more labor-intensive, the increase in the effective labor supply can reduce the relative price of the non-tradable good (and the value of the real exchange rate). Therefore, endogenous labor supply could reinforce the savings channel from the sex ratio shock, leading to an additional reduction in the real exchange rate.

We allow each person to endogenously choose the first period labor supply and the utility function of the first period is $u(C) + v(1-L)$, where L is the labor supply and $v(1-L)$ is the utility function of leisure. As in the standard literature, we assume that $v' > 0$ and $v'' < 0$. Again, for simplicity, we assume no tax on the labor income. The utility function governing the leisure-labor choice is the same for men and women. In other words, by assumption, men and women are intrinsically symmetric except for their ratio in the society.

We can rewrite the optimization problem for a representative woman as following:

$$\max \; u(C_{1t}^w) + v(1 - L_t^w) + \beta E_t \left[u(C_{2,t+1}^w) + \eta^m \right]$$

with the budget constraint

$$\begin{aligned} P_t C_{1t}^w &= (1 - s_t^w) w_t L_t^w \\ P_{t+1} C_{2,t+1}^w &= \begin{cases} \kappa \left(R s_t^w L_t^w + R s_t^m L_t^m \right) w_t & \text{if married} \\ R s_t^w w_t L_t^w & \text{otherwise} \end{cases} \end{aligned}$$

The first order condition with respect to her labor supply is

$$u'_{1w} \frac{(1 - s_t^w) w_t}{P_t} + \beta \left[\begin{array}{c} \delta^w u'_{2w} \frac{\partial c_{2w}}{\partial s^w} + (1 - \delta^w) \, u'_{2w,n} \frac{R P_t}{P_{t+1}} y^w + \frac{\partial \int_{\tilde{\eta}^w \geq \bar{\eta}^w} M(\tilde{\eta}^w) dF(\eta^w)}{\partial s^w} \\ + \frac{\partial \delta^w}{\partial s^w} \left(u_{2w} - u_{2w,n} \right) \end{array} \right] - v'_w = 0$$

Notice that $\frac{\partial C_{2,t+1}^w}{\partial L_t^w} = \frac{\partial C_{2,t+1}^w}{\partial s_t^w} \frac{s_t^w}{L_t^w}$ and $\frac{\partial \int M(\tilde{\eta}^w) d\tilde{F}^w(\tilde{\eta}^w)}{\partial L_t^w} = \frac{\partial \int M(\tilde{\eta}^w) d\tilde{F}^w(\tilde{\eta}^w)}{\partial s_t^w} \frac{s_t^w}{L_t^w}$. Combining the equation above with (3.7), we have

$$\frac{w_t}{P_t} = \frac{v'_w}{u'_{1w}} \tag{3.14}$$

The optimization problem for a representative man is similar:

$$\max\ u(C_{1t}^m) + v(1 - L_t^m) + \beta E_t\left[u(C_{2,t+1}^m) + \eta^w\right]$$

with the budget constraint

$$\begin{aligned} P_t C_{1t}^m &= (1 - s_t^m) w_t L_t^m \\ P_{t+1} C_{2,t+1}^m &= \begin{cases} \kappa\left(R s_t^m L_t^m + R s_t^m L_t^m\right) w_t & \text{if married} \\ R s_t^m w_t L_t^m & \text{otherwise} \end{cases} \end{aligned}$$

The optimization condition for his labor supply is

$$\frac{w_t}{P_t} = \frac{v'_m}{u'_{1m}} \tag{3.15}$$

On the supply side, all equilibrium conditions remain the same except for the labor market clearing condition, which now becomes

$$L_{Tt} + L_{Nt} = \frac{1}{1+\phi} L_t^w + \frac{\phi}{1+\phi} L_t^m \tag{3.16}$$

We now define an equilibrium for such an economy.

Definition 3 *An equilibrium is a set* $\{(s_t^w, L_t^w), (s_t^m, L_t^m), K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *that satisfies the following conditions:*

(i) The savings and labor supply decisions by women and men, $(s_t^w, L_t^w) = \left\{s_{it}^w, s_{-i,t}^w, L_{it}^w, L_{-i,t}^w\right\}$ *and* $(s_t^m, L_t^m) = \left\{s_{it}^m, s_{-i,t}^m, L_{it}^m, L_{-i,t}^m\right\}$*, maximize their utilities, respectively,*

$$\begin{aligned} (s_{it}^w, L_{it}^w) &= \arg\max\left\{V_t^w \middle| \left(s_{-i,t}^w, L_{-i,t}^w\right), (s_t^m, L_t^m), K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\right\} \\ \left(s_{jt}^m, L_{jt}^m\right) &= \arg\max\left\{V_t^m \middle| (s_t^w, L_t^w), \left(s_{-j,t}^m, L_{-j,t}^m\right), K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\right\} \end{aligned}$$

(ii) The markets for both goods and factors clear, and firms' profits are maximized. In other words, $\{K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *solves (2.2), (2.3), (2.4), (2.5), (2.6) and (3.16).*

As before, we assume that $u(C) = \ln C$. We let L_t denote the aggregate labor supply in period t, and assume that $\frac{v''L}{v'}$ is non-decreasing in L.

Proposition 4 *Under the same assumptions as in Proposition 2, as the sex ratio (in the young cohort) rises, a representative man increases both his labor supply and his savings rate, while the responses in a representative woman's savings rate and labor supply are ambiguous. However, the economy-wide labor supply and savings rate both increase unambiguously. The real exchange rate depreciates, and the current account rises.*

Proof. See Appendix C. ■

In response to a rise in the sex ratio, for the same reason that men may cut their consumption and increase their savings rate, they may cut down their leisure and increase their labor supply. Similarly, for women, for the same reason that induces them to reduce their savings, they may reduce their labor supply (and increase leisure). In the aggregate, for the same reason that the increase in savings by men is more than enough to offset the decrease in savings by women, the increase in labor supply by men is also larger than the decrease in labor supply by women. Therefore, the aggregate labor supply rises in response to a rise in the sex ratio.

With a fixed labor supply, it is worth remembering that the nontradable sector shrinks after a rise in the sex ratio. The reason is that a decline in the relative price of the nontradable good (due to the savings channel) makes it less attractive for labor and capital to stay in the nontradable sector. Now, with an endogenous labor supply, the total effective labor supply increases after a rise in the sex ratio according to Proposition 3. By a logic similar to the Rybzinksy theorem, this by itself has a tendency to induce an expansion of the nontradable sector if the production of the nontradable good is more labor intensive. Relative to the case of a fixed labor supply, adding the effect of endogenous labor supply leads to either an expansion of the nontradable sector, or at least a smaller reduction in the size of the nontradable sector. The exact scenario depends on parameter values. However, regardless of what happens to the size of the nontradable sector, the price of the nontradable good (and the value of the real exchange rate) must fall by a greater amount when the endogenous labor supply effect is added to the savings effect.

4 Numerical Examples

We start from a simple OLG model allowing mixed strategies in which every cohort lives two periods and there are no capital adjustment costs. We then add some more realisms by (1) assuming a 50-period life and (2) introducing capital adjustment costs.

4.1 Parameters

We take the annual interest rate $R = 1.04$ and $\beta = R^{-1}$. We assume the tradable sector has a higher capital intensity, $\alpha_T = 0.6$ and the nontradable sector has a lower capital intensity $\alpha_N = 0.3$. The share of the nontradable good consumption in the aggregate consumption is set to be 0.7, $\gamma = 0.7$. Within a family, the congestion for family consumption, $\kappa = 0.8$.

The emotional utility η needs to follow a continuously differential distribution. We assume a normal distribution which might be more realistic than the uniform distribution used in the analytical model. To choose the mean value of the emotional utility, we perform the following thought experiment. Holding all other factors constant, we can compute the income compensation needed to a life-time bachelor that can makes him indifferent between being married and being single.

$$u\left(\frac{1}{1+\beta}(1+m)y\right) = u\left(\frac{1}{1+\beta}y\right) + E(\eta)$$

where xy is the compensation paid to a life-time bachelor for being single and $\frac{1}{1+\beta}(1+x)y$ is his second period consumption. We calculate the value of x based on Blanchflower and Oswald (2004). Regressing self-reported well-being scores on income, marriage status, and other determinants, they estimate that a lasting marriage is, on average, worth $100,000 (in 1990 dollars) per year (every year) in the United States (compared to being widowed or separated) during 1972-1998. Since GDP per person employed is about $48,000 during the same period, this implies that a marriage is worth more than twice the average life-time income for employed people in the U.S. We take the ratio $m = 2$ as the benchmark and then the mean value of the emotional utility/love is:

$$E(\eta) = u\left(\frac{3y}{1+\beta}\right) - u\left(\frac{y}{1+\beta}\right)$$

As a robustness check, we will also consider $m = 0.5$.

To pin down the value of the standard deviation of emotional utility, we use the standard error associated with the estimate for the value of marriage in Blanchflower and Oswald (2004). More precisely, since the t statistic for the coefficient estimate on the dummy variable "never married" is about -20, the standard deviation of emotional utility, σ, is computed by

$$\frac{E(\eta)}{\sigma} = 20 \Longrightarrow \sigma = \frac{E(\eta)}{20} = \frac{\ln\left(\frac{3y}{1+\beta}\right) - \ln\left(\frac{y}{1+\beta}\right)}{20} \simeq 0.05$$

We also consider $\sigma = 0.1$ as a robustness check.

Choice of Parameter Values

Parameters	Benchmark	Source and robustness checks
Discount factor	$\beta = 0.45$	Prescott (1986), annual discount factor takes value 0.96 based on annual frequency. We take 20 years as one period, then $\beta = 0.96^{20} \simeq 0.45$
Share of nontradable good in the consumption basket	$\gamma = 0.7$	Burstein et al. (2001)
Nontradable sector capital-intensity	$\alpha_N = 0.3$	Burstein et al. (2001)
Tradable sector capital-intensity	$\alpha_T = 0.6$	Burstein et al. (2001)
Share of capital input	$\alpha = 0.35$	Bernanke, Gertler and Gilchrist (1999)
Congestion index	$\kappa = 0.8$	$\kappa = 0.7, 0.9$ in the robustness checks.
Love, standard deviation	$\sigma = 0.05$	$\sigma = 0.1$ in the robustness checks
Love, mean	$m = 2$	$m = 0.5$ in the robustness checks

4.2 Results for the 2-period OLG model

In Figure 1, we set parameter κ equal to 0.8. We set $m = 2$ and $\sigma = 0.05$ as a benchmark case. With an unbalanced sex ratio ($\phi > 1$), the real exchange rate depreciates. As the sex ratio rises from 1 to 1.15, the real exchange rate depreciates by 6.4%, while aggregate savings rate and current account both rise by around 2.9%. As the sex ratio continues to rise, the real exchange rate begins to appreciate. The turning point for the real exchange rate corresponds to when the sex ratio crosses the threshold ϕ_1 in Proposition 2.

As a first set of robustness checks, we experiment with different combinations of m and σ by setting $m = 0.5$ or 2, $\sigma = 0.05$ or 0.1. The results are also reported in Figure 1, and generally do not deviate from the benchmark by much.

We also set κ to be 0.7 or 0.9, respectively, and experiment with different combinations of other parameters. The results are reported in Figures 2 and 3. Generally speaking, the real exchange rate always depreciates more with a higher sex ratio. Both the savings rate and the current account (as a share of GDP) rise in response to a rise in the sex ratio.

We now consider endogenous labor supply in Figure 4. With $\kappa = 0.8$, $m = 2$ and $\sigma = 0.05$, we obtain a much stronger exchange rate depreciation. As the sex ratio rises from 1 to 1.15, the extent of the real exchange rate depreciation also rises from 0% to about 25%. The aggregate savings rate rises from 17% to 25%, while the current account surplus rises from 0% first to close to 9% of GDP. As the sex ratio continues to rise, it would cross the threshold value ϕ_1, at which point, the real exchange rate begins to appreciate. Correspondingly, the aggregate savings rate and current account both decline.

Robustness checks with other combinations of the parameters are reported in Figures 5 and 6. The results are broadly in line with the benchmark calibration. In particular, with an endogenous labor supply, a given rise in the sex ratio leads to a greater response in both the real exchange rate and the current account.

4.3 An OLG model in which a cohort lives 50 periods

We now extend our benchmark model by assuming that every cohort lives 50 periods. Everyone works in the first 30 periods, and retires in the remaining 20 periods. If one gets married, the marriage takes place in the τth period. While differences in the savings rates by parents with a son versus parents with a daughter are an important feature of the data (Wei and Zhang, 2009), we are not able to solve the problem that features simultaneously parental savings for children and a nontradable sector. Instead, we study a case in which men and women save for themselves. However, as we recognize the quantitative importance of parental savings in the data, we choose $\tau = 20$ as our benchmark case so the timing of the marriage is somewhere between the typical number of working years by parents when their child gets married and the typical number of working years by a young person when he/she gets married. Generally speaking, the greater the value of τ, the stronger is the aggregate savings response to a given rise in the sex ratio.

A representative woman's optimization problem is

$$\max \sum_{t=1}^{\tau-1} \beta^{t-1} u(c_t^w) + E_1 \left[\sum_{t=\tau}^{50} \beta^{t-1} \left(u(c_t^w) + \eta^m \right) \right]$$

For $t < \tau$, when the woman is still single, the intertemporal budget constraint is

$$A_{t+1} = R \left(A_t + y_t^w - P_t c_t^w \right)$$

where A_t is the wealth held by the woman at the beginning of period t. $y_t^w = w_t L_t^w$ is her labor income at the age t. After marriage ($t \geq \tau$), her family budget constraint becomes

$$A_{t+1}^H = \begin{cases} R\left(A_t^H + w_t L_t^w - P_t c_t\right) & \text{if } t \leq 30 \\ R\left(A_t^H - c_t^w\right) & \text{if } t > 30 \end{cases}$$

where A_t^H is the level of family wealth (held by wife and husband) at the beginning of period t. c_t is the public good consumption by wife and husband, which takes the same form as in the two period OLG model. The optimization problem for a representative man is similar. To simplify the calculation and generate interesting results, we assume that there is a lower bound of labor supply $\bar{L}$, $L_t^i \geq \bar{L}$ $(i = w, m)$. [One justification: In the real world labor market, it may be difficult to find a job that allows for downward adjustment of working hours in a flexible manner. Part-time jobs such as babysitting are not generally available in all industries at all skill levels. On the other hand, one can often do overtime or moonlight for a second job.]

As before, we take $R = 1.04$ as the annual gross interest rate. The subjective discount factor now takes the value of $\beta = 1/R$. We assume capital accumulation evolves in the following way:

$$K_{t+1} = (1-\delta)K_t + I_t - \frac{b}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$$

where $\frac{b}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$ represents the quadratic capital adjustment cost. Following Chari, Kehoe and McGrattan (2002), we assume $\delta = 0.1$ and $b = 3$.

To think of the sex ratio shock, we use demographic changes in China over the last two decades as a guide. As the data exhibits a steady increase in the sex ratio in the pre-marital age cohort, we let the sex ratio at birth in the model rise continuously and smoothly until it reaches 1.2 in period 20. The sex ratio then stays at that level in all subsequent periods. For technical reasons, we set $\sigma = 0.1$. Under such a standard deviation, ϕ_1 does not appear in this experiment.

The simulation results are shown in Figure 7. As the sex ratio rises from 1 in period 0 to 1.2 in period 20, the real exchange rate depreciates by around 9 percent. The economy-wide savings rate and the current account rise by around 3.5 percent of GDP. As a robustness check, if capital adjusts more slowly, i.e., with a higher cost of capital adjustment, the real exchange rate depreciates by almost 10 percent. The converse is true when the adjustment cost is lower.

5 Some empirics

Since the sex ratio effect is novel, it is useful to present and discuss some empirical evidence. We recall first the evidence in Wei and Zhang (2009) that a higher sex ratio has led to a rise in the household savings rate in China. Chinese households with a son in both rural and urban areas tend to save more in regions with a more skewed sex ratio. The savings rate by urban households with a daughter also tends to rise with the local sex ratio, although the savings rate by rural households with a daughter appears to be insensitive to the local sex ratio. The savings behavior by daughter-households is consistent with the notion that intra-household bargaining is sufficiently important that they do not cut down savings rate in response to a higher sex ratio (the models of Du and Wei, 2010, and Bhaskar and Hopkins, 2011, formalize this intuition). Using regional variations in the enforcement of the family planning policy as instruments for the local sex ratio, Wei and Zhang (2009) suggest that the positive correlation reflects a causal effect from a higher sex ratio to a higher savings rate. Based on the IV regressions, they estimate that the rise in the sex ratio may explain about half of the observed rise in the household savings rate in the last two decades.

Some evidence that a higher sex ratio has increased effective labor supply is provided in Wei and Zhang (2011). In particular, the number of days a rural migrant worker chooses to work away from home tends to rise with the local sex ratio, especially if the migrant worker has a son at home. Similarly, migrant workers with a son from a region with a more skewed sex ratio are also more willing to work in jobs that are more dangerous and less pleasant, such as in mining or construction, or with exposure to extreme heat, cold or hazardous material, presumably for a better wage.

We now provide some suggestive cross-country evidence on how the sex ratio imbalance may affect the real exchange rate. We first run regressions based on the following specification:

$$\ln RER_i = \alpha + \beta \cdot \text{sex ratio} + \gamma \cdot Z + \varepsilon_i$$

where RER_i is the real exchange rate for country i. Z is the set of control variables. We consider a sequentially expanding list of control variables including log GDP per capita, financial development index, government fiscal deficit, dependence ratio, and *de facto* exchange rate regime classifications.

The data for the real exchange rate and real GDP per capita are obtained from Penn World Table 6.3. The "price level of GDP" in the Penn World Table is equivalent to the inverse of the real exchange rate in the model: A higher value of the "price level of GDP" means a lower value of the real exchange rate. The sex ratio data is obtained from the World Factbook. As we are not able to find the sex ratio for the age cohort 10-25 for a large number of countries, we use age group 0-15 instead to maximize the country coverage.

We use two proxies for financial development. The first is the ratio of private credit to GDP, from the World Bank's WDI dataset. This is perhaps the most commonly used proxy in the standard literature. There is a clear outlier with this proxy: China has a very high level of bank credit, exceeding 100% of GDP. However, 80% of the bank loans go to state-owned firms, which are potentially less efficient than private firms (see Allen, Qian, and Qian, 2004). To deal with this problem, we modify the index by multiplying the credit to GDP ratio for China by 0.2. Because this measure is far from being perfect, we also use a second measure, which is the level of financial system sophistication as perceived by a survey of business executives reported in the Global Competitiveness Report (GCR).

For exchange rate regimes, we use two de facto classifications. The first comes from Reinhart and Rogoff (2004), who classify all regimes into four groups: peg, crawling peg, managed floating and free floating. The second classification comes from Levy-Yeyati and Sturzenegger (2005), who use three groups: fix, intermediate and free float.

For the dependent variable, log RER, and most regressors where appropriate, we use their average values over the period 2004-2008. The averaging process is meant to smooth out business cycle fluctuations and other noises. The period 2004-2008 is chosen because it is relatively recent, and the data are available for a large number of countries. (We have also examined a single year, 2006, and obtained similar results).

Table 1 provides summary statistics for the key variables. The log RER ranges from -2.22 to 0.41 in the sample, with a mean of -0.74 and a standard deviation of 0.59. The value of log RER for China indicates a substantial undervaluation on the order of 45% when compared to the simple criterion of purchasing power parity.

For the sex ratio for the age cohort 0-15, both the mean and the median across countries are 1.04, and the standard deviation is 0.02. For this age cohort, all countries in the sample have a sex ratio that is at least 1. The sex ratio for most of the countries is between 1 and 1.07. The following economies have a sex ratio that is 1.07 or higher: China (1.13), Macao (1.11), Korea (1.11), Singapore (1.09), Switzerland (1.08), Hong Kong (1.08), Vietnam (1.08), Jordan (1.07), Portugal (1.07) and India (1.07). They represent the most skewed sex ratios in the sample. China, by far, has the most unbalanced sex ratio in the world. If the same sex ratio persists into the marriage market, then at least one out of every eight young men cannot get married. As wives are typically a few years younger than their husbands, the actual probability of not being able to marry is likely to be modestly better in a country with a growing population (for which later cohorts are slightly larger). Nonetheless, the relative tightness of the marriage market for men across countries should still be highly correlated with this sex ratio measure. In addition, unlike most other countries, China exhibits a progressively smaller age cohort over time as a result of its strict family planning policy. As a result, the relative tightness of the marriage market for Chinese men when compared to their counterparts in other countries is likely to be worse than what is represented by this sex ratio. Furthermore, the Chinese sex ratios at birth in 1990 and 2005 are estimated to be 1.15 and 1.20, respectively (see Wei and Zhang, 2009). This implies that the sex ratio for the pre-marital age cohort will likely worsen in the foreseeable future.

We present a series of regressions in Table 2. The first column shows that the real exchange rate tends to be lower in poorer countries. This is commonly interpreted as confirmation of the Balassa-Samuelson effect. In Column 2, we add a proxy for financial development by the ratio of private sector credit to GDP. The positive coefficient on the new regressor indicates that countries with a poorer financial system tend to have a lower RER. In Column 3, we add the sex ratio. The coefficient on the sex ratio is negative and statistically significant, indicating that countries with a higher sex ratio tend to have a lower RER. Since oil exporting countries have a current income that is likely to be substantially higher than their permanent income (until they run out of the oil reserve), their current account and RER patterns may be different from other economies. In Column 4, we exclude major oil exporters and re-do the regression. This turns out to have little effect on the result. In particuar, countries with a higher sex ratio continue to exhibit a lower RER.

In Column 5 of Table 2, we add several additional control variables: government fiscal deficit, terms of trade, capital account openness, and dependency ratio. Due to missing values for some of these variables, the sample size is dramatically smaller (a decline from 123 in Column 4 to 75 in Column 5). Of these variables, the dependence ratio is the only significant variable. The positive coefficient on the dependence ratio (0.0093) means that countries with a low dependency ratio (fewer children and retirees as a share of the population) tend to have a low RER. By the logic of the life-cycle hypothesis, a lower dependency ratio produces a higher savings rate. By the model in Section 2, this could lead to a reduction in the value of the real exchange rate. It is noteworthy, however, that even with these additional controls and in a smaller sample, the sex ratio effect is still statistically significant, although its point estimate is slightly smaller.

In Column 6 of Table 2, we take into account exchange rate regimes using the Reinhart-Rogoff (2004) de facto regime classifications. Relative to the countries on a fixed exchange rate regime (the omitted group), those on a crawling peg appear to have a lower RER. Countries on other currency regimes do not appear to have a systematically different RER. With these controls, the negative effect of the sex ratio on the RER is still robust. In Column 7, we measure exchange rate regimes by the de facto classifications proposed by Levy Yeyati and Sturzeneger (2003). It turns out this does not affect the relationship between the sex ratio and the real exchange rate.

In Table 3, we re-do the regressions in Table 2 except that we now measure a country's financial development by the financial system sophistication index from the Global Competitiveness Report. The results are broadly similar to Table 2. In particular, the coefficients on the sex ratio are negative in all five cases, and are significant in four of the five cases. The sex ratio coefficient is (marginally) not significant in Column 6 of Table 3, where the Reinhart-Rogoff exchange rate classifications are used as controls. We note, however, that this regression also has far fewer observations (35 only), which also reduces the power of the test. In any case, when the LYS exchange rate classifications are used instead (reported in Column 7), the sex ratio coefficient becomes significant again.

In Tables 4 and 5, we examine the relationship between the sex ratio and the (private-sector) current account. Because our theory does not discuss government savings behavior, we choose to define the dependent variable as a country's current account (as a share of GDP) minus the government savings (as a share of GDP). Otherwise, the regression specifications are similar to those in Tables 2 and 3. The sex ratio has a positive coefficient which is statistically significant in almost all cases except when the sample size becomes very small.

In sum, we find that the sex ratio has a significant impact on the real exchange rate and current account in a way consistent with our theory: as the sex ratio rises, a country tends to have a real exchange rate depreciation and a current account surplus. (An important caveat is that we do not have a clever idea to instrument for the sex ratio in the cross country context; future research will have to investigate the causality more thoroughly.)

To be clear, as the sex ratio imbalance is a severe problem only in a subset of countries, it is not a key fundamental for the real exchange rate in most countries. Nonetheless, for those countries with a severe sex ratio imbalance, including China, one might not have an accurate view on the equilibrium exchange rate unless one takes it into account. To illustrate the quantitative significance of the empirical relations, we compute the extent of the Chinese real exchange rate undervaluation (or the value of the RER relative to what can be predicted based on the fundamentals) by taking the point estimates in Columns 1-2 and 5 of Tables 2-5, respectively, at their face value. The results are tabulated in Table 6. As noted earlier, relative to the simple-minded PPP, the Chinese exchange rate is undervalued by about 45%. Once we adjust for the Balassa-Samuelson effect, the extent of the undervaluation becomes 55% (column 1 of Table 6) - apparently the Chinese RER is even lower than other countries at the comparable income level. If we additionally consider financial underdevelopment (proxied by the ratio of private sector loans to GDP), the Chinese RER undervaluation is reduced to 43% (column 2, row 1 of Table 6), which is still economically significant. If we also take into account government deficit, terms of trade, and capital account openness, the extent of the RER undervaluation is 35% (column 3, row 1). If we further take into account the dependency ratio, the extent of undervaluation drops to 18% (column 4, row 1). Finally, if we add the sex ratio effect, the extent of undervaluation becomes 8% (column 5, row 1 of Table 6). The last number represents a relatively trivial amount of undervaluation since major exchange rates (e.g., the euro/dollar rate or the yen/dollar rate) could easily fluctuate by more than 8% in a year. If we proxy financial development by the rating of financial system sophistication, and also take into account the sex ratio effect and other structural variables, the extent of the Chinese RER undervaluation becomes 2% (column 5, row 2 of Table 6), an even smaller amount.

We can do similar calculations for the Chinese (private sector) current account (as a share of GDP) in excess of the fundamentals. If we only take into consideration the regularity that poorer countries tend to have a lower current account balance, the Chinese excess current account is on the order of 14%. If we take into account the sex ratio effect as well as financial underdevelopment, the dependency ratio and other variables in the regressions, the excess amount of current account becomes somewhere between 0.3% and 2.0%, depending on which proxy for financial development is used. These numbers illustrate that the sex ratio is a quantitatively important structural factor, though it is not the only one. In particular, the dependency ratio is also a very important factor. In any case, if these structural factors are not taken into account, one might mistakenly exaggerate the role of currency manipulation in affecting both the RER and the current account.

6 Conclusion

A low value of the real exchange rate (i.e., deviations from the PPP from below), a large current account surplus, and accumulation of foreign exchange reserves are the commonly used criteria for judging currency undervaluation or manipulation. We argue that none of them is a logically sound criterion. Instead, a dramatic rise in the sex ratio for the premarital age cohort in China since 2003, could generate both a depreciation of the real exchange rate and a rise in the current account surplus. With capital controls (including mandatory surrender of foreign exchange earnings), a persistent current account surplus can mechanically be converted into a rise in a country's foreign exchange reserve.

If other factors, in addition to a rise in the sex ratio, have also contributed to a rise in the Chinese savings rate, such as a reduction in the dependency ratio, or a rise in the corporate and government savings rates, they can complement the sex ratio effect and reinforce an appearance of an undervalued currency even when there is no manipulation. To be clear, this paper is not saying that no manipulations have occurred in any particular country. Instead, it illustrates potential pitfalls in assessing the equilibrium exchange rate when important structural factors are not accounted for.

Empirically, countries with a high sex ratio do appear to have a low value of the real exchange rate and a current account surplus. If we take the econometric point estimates at face value, it appears that the Chinese real exchange rate has only a relatively small amount of undervaluation (2-8%) once we take into account the sex ratio effect and other structural factors.

In future research, the model could be extended to allow for endogenous adjustment of the sex ratio. This will help us to assess the speed of the reversal of the sex ratio and the unwinding of the current account surplus and currency "undervaluation."

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A Proof of Proposition 1

Proof. We totally differentiate the system and have

$$\Omega \cdot \begin{pmatrix} ds_t \\ dw_t \\ dP_{Nt} \\ dL_{Nt} \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}$$

where

$$\begin{aligned}
\Omega_{11} &= (u_1'' + \beta R u_2'') \frac{w_t}{P_t},\ \Omega_{12} = \Omega_{13} = \Omega_{14} = 0 \\
\Omega_{21} &= \gamma w_t, \Omega_{22} = -\gamma(1-s_t),\ \Omega_{23} = \frac{A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}},\ \Omega_{24} = \frac{P_{Nt}(1-\alpha_N) A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}} \\
\Omega_{31} &= \Omega_{33} = 0,\ \Omega_{32} = -1,\ \Omega_{34} = \left(\frac{\alpha_T}{1-\alpha_T}\right)^{1-\alpha_T} (1-\alpha_T) A_{Tt} K_{Tt}^{\alpha_T} (1-L_{Nt})^{-\alpha_T - 1} \\
\Omega_{41} &= 0,\ \Omega_{42} = -1,\ \Omega_{43} = \frac{w_t}{P_{Nt}},\ \Omega_{44} = -\left(\frac{\alpha_T}{1-\alpha_T}\right)^{1-\alpha_T} (1-\alpha_T) A_{Tt} K_{Tt}^{\alpha_T} (1-L_{Nt})^{-\alpha_N - 1}
\end{aligned}$$

and

$$z_1 = -R u_2',\ z_2 = z_3 = z_4 = 0$$

The determinant of matrix Ω is

$$\det(\Omega) = \Omega_{11} \cdot \det \begin{pmatrix} \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix}$$

and

$$\begin{aligned}
\det \begin{pmatrix} \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix} &= \text{negative}\,.terms + \gamma(1-s_t) \left(\frac{w_t}{P_{Nt}}\right)^2 \frac{1-\alpha_T}{L_T} \\
&\quad - \left(\frac{w_t}{P_{Nt}}\right) \left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right) C_{Nt}
\end{aligned}$$

Since the consumption on the nontradable goods by the young cohort must be less than the aggregate nontradable good consumption, it follows that $\gamma(1-s_t)w_t < P_{Nt} C_{Nt}$. Therefore,

$$\det \begin{pmatrix} \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix} < 0$$

and $\det(\Omega) > 0$

Then it is easy to show that

$$\frac{ds_t}{d\beta} = \frac{\det\begin{pmatrix} z_1 & \Omega_{12} & \Omega_{13} & \Omega_{14} \\ z_2 & \Omega_{22} & \Omega_{23} & \Omega_{24} \\ z_3 & \Omega_{32} & \Omega_{33} & \Omega_{34} \\ z_4 & \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix}}{\det(\Omega)} = \frac{z_1}{\Omega_{11}} > 0$$

and the price of the nontradable good

$$\frac{dP_{Nt}}{d\beta} = \frac{\det\begin{pmatrix} \Omega_{11} & \Omega_{12} & z_1 & \Omega_{14} \\ \Omega_{21} & \Omega_{22} & z_2 & \Omega_{24} \\ \Omega_{31} & \Omega_{32} & z_3 & \Omega_{34} \\ \Omega_{41} & \Omega_{42} & z_4 & \Omega_{44} \end{pmatrix}}{\det(\Omega)} = \frac{z_1\Omega_{21}\Omega_{32}(\Omega_{44} - \Omega_{34})}{\det(\Omega)} < 0$$

The labor input in the nontradable sector

$$\frac{dL_{Nt}}{d\beta} = \frac{\det\begin{pmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & z_1 \\ \Omega_{21} & \Omega_{22} & \Omega_{23} & z_2 \\ \Omega_{31} & \Omega_{32} & \Omega_{33} & z_3 \\ \Omega_{41} & \Omega_{42} & \Omega_{43} & z_4 \end{pmatrix}}{\det(\Omega)} = -\frac{z_1\Omega_{21}\Omega_{32}\Omega_{34}}{\det(\Omega)} < 0$$

In period $t+1$, the shock has been observed, (2.2) and (2.4) hold in equilibrium. By solving (2.2), (2.3), (2.4) and (2.5), we have

$$P_{Nt} = R^{\frac{\alpha_N - \alpha_T}{1-\alpha_T}} \quad \text{and} \quad P_{t+1} = R^{\frac{\gamma(\alpha_N - \alpha_T)}{1-\alpha_T}}$$

which means that one period after the shock occurs, the price of the nontradable good and the consumer price index will go back to their initial levels. As for the current account,

$$CA_t = P_{Nt}Q_{Nt} + Q_{Tt} + (R-1) \cdot NFA_{t-1} - P_tC_t - K_{t+1}$$

where NFA_{t-1} is the net foreign asset holdings in period $t-1$ and K_{t+1} is the sum of capital input in both the nontradable sector and the tradable sector in period $t+1$. Since

$$s_{t-1}w_{t-1} = NFA_{t-1} + K_t$$

Then

$$CA_t = s_t w_t - s_{t-1} w_{t-1} - \Delta K_{t+1}$$

where $\Delta K_{t+1} = K_{t+1} - K_t$. The demand for the nontradable good is now

$$Q_{N,t+1} = \frac{\gamma w \left((R-1) s_t + 1 \right)}{P_N}$$

where we drop the time subindex because wage rate and the relative price of the nontradable good will go back to their initial levels. It is easy to see that since $s_t > s_{t-1}$, $Q_{N,t+1} > Q_{N,t-1}$.

As $\alpha_N < \alpha_T$, the nontradable sector has a lower capital-intensity than the tradable sector. Then, in period $t+1$, $K_{t+1} < K_{t-1}$.

In period $t+1$,

$$A_{Nt} K_{N,t+1}^{\alpha_N} L_{N,t+1}^{1-\alpha_N} = \frac{\gamma w \left((R-1) s_t + 1 \right)}{P_{N,t+1}}$$

In the equilibrium, all markets clear and we can obtain

$$K_{t+1} = \frac{\alpha_T - \gamma(\alpha_T - \alpha_N) \left[(R-1) s_t + 1 \right]}{(1-\alpha_T) R} w$$

and then

$$CA_t = s_t w_t - s_{t-1} w + \frac{(\alpha_T - \alpha_N)(R-1) \left(s_t - s_{t-1} \right)}{(1-\alpha_T) R} w$$

To show $\frac{dCA_t}{d\beta} > 0$, we only need to show $\frac{d(s_t w_t - s_{t-1} w_{t-1})}{d\beta} > 0$. One sufficient condition for the inequality is

$$s_t P_{Nt} > s_{t-1} P_{Nt}$$

To show this inequality, we just need to show

$$s_t \frac{dP_{Nt}}{d\beta} + P_{Nt} \frac{ds_t}{d\beta} > 0$$

which means

$$\frac{dP_{Nt}/d\beta}{ds_t/d\beta} + \frac{P_{Nt}}{s_t} > 0$$

Plugging the expressions of $\frac{dP_{Nt}}{d\beta}$ and $\frac{ds_t}{d\beta}$, we have

$$\begin{aligned} \frac{dP_{Nt}}{ds_t} + \frac{P_{Nt}}{s_t} &= \frac{-\gamma(1-s_t) w_t C_{Nt} \left(\frac{w_t}{P_{Nt}} \right) \left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}} \right) + P_{Nt} C_{Nt} \left(\frac{w_t}{P_{Nt}} \right) \left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}} \right)}{s_t \cdot \text{positive}\, .terms} + \text{positive}\, .term \\ &= \frac{\left(P_{Nt} C_{Nt} - \gamma(1-s_t) w_t \right) \left(\frac{w_t}{P_{Nt}} \right) \left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}} \right)}{s_t \cdot \text{positive}\, .terms} + \text{positive}\, .term \end{aligned}$$

As shown above, $P_{Nt}C_{Nt} - \gamma(1-s_t)w_t > 0$, then $\frac{dCA_t}{d\beta} > 0$, in period t, the country will experience a current account surplus. ■

B Proof of Proposition 2

Proof. At $\phi = 1$, all women and men are symmetric and they make the same savings decisions. Since $\frac{1}{2} \leq \kappa \leq 1$,

$$\kappa(Rs_t^m w_t + Rs_t^w w_t) \geq \max\left(Rs_t^w w_t,\, Rs_t^m w_t\right) \tag{B.1}$$

Then, in the neighbourhood of $\phi = 1$, we have $\kappa u'_{2m} < u'_{2m,n}$.[5]

We proceed in two steps. In the first step, we assume that inequality $\kappa u'_{2m} < u'_{2m,n}$ holds for all values of ϕ, and prove that a higher sex ratio leads to a higher savings rate. In the second step, we prove by contradiction that the inequality indeed holds for all values of ϕ.

Assuming that inequality $\kappa u'_{2m} < u'_{2m,n}$ holds for all values of $\phi \geq 1$, the first order conditions for a woman and a man, respectively, are:

$$-u'_{1w} + \beta R \frac{P_t}{P_{t+1}} \left[\begin{array}{c} \kappa u'_{2w}\left(\delta^w + \left[\frac{1}{\phi}\left(1 - F\left(\bar{\eta}^w\right)\right) + M(\bar{\eta}^w) f(\bar{\eta}^w)\right]\right) + \left(1-\delta^w\right) u'_{2w,n} \\ + f\left(\bar{\eta}^w\right) \kappa u'_{2w}\left(u_{2w} - u_{2w,n}\right) \end{array} \right] = 0 \tag{B.2}$$

$$-u'_{1m} + \beta R \frac{P_t}{P_{t+1}} \left[\begin{array}{c} \kappa u'_{2m}\left(\delta^m + \left[\phi\left(1 - F\left(\bar{\eta}^m\right)\right) + M^{-1}(\bar{\eta}^m) f(\bar{\eta}^m)\right]\right) + \left(1-\delta^m\right) u'_{2m,n} \\ + f\left(\bar{\eta}^m\right) \kappa u'_{2m}\left(u_{2m} - u_{2m,n}\right) \end{array} \right] = 0 \tag{B.3}$$

We show by contradiction that $\bar{\eta}^w = u_{2m,n} - u_{2m}$ and $\bar{\eta}^m = M(\bar{\eta}^w)$ hold for $\phi \geq 1$. Suppose not, then

$$\bar{\eta}^m > M(\bar{\eta}^w) \geq \bar{\eta}^w$$

where the second inequality holds because $\phi \geq 1$. Then we have

$$u\left(\frac{Rs_t^w w_t}{P_{t+1}}\right) - u\left(\frac{\kappa(Rs_t^w w_t + Rs_t^m w_t)}{P_{t+1}}\right) > u\left(\frac{Rs_t^m w_t}{P_{t+1}}\right) - u\left(\frac{\kappa(Rs_t^w w_t + Rs_t^m w_t)}{P_{t+1}}\right)$$

and hence, $s_t^w > s_t^m$.

Then

$$\begin{aligned} u'_{1w} &= \delta^w \kappa u'_{2w}\left(1 + \left[\frac{1}{\phi}\left(1 - F\left(\bar{\eta}^w\right)\right) + M(\bar{\eta}^w) f(\bar{\eta}^w)\right]\right) + \left(1-\delta^w\right) u'_{2w,n} + f\left(\bar{\eta}^w\right) \kappa u'_{2w}\left(u_{2w} - u_{2w,n}\right) \\ &< \delta^m \kappa u'_{2w}\left(1 + \left[\frac{1}{\phi}\left(1 - F\left(\bar{\eta}^w\right)\right) + M(\bar{\eta}^w) f(\bar{\eta}^w)\right]\right) + \left(1-\delta^m\right) u'_{2w,n} + f\left(\bar{\eta}^w\right) \kappa u'_{2w}\left(u_{2w} - u_{2w,n}\right) \\ &< \delta^m \kappa u'_{2m}\left(1 + \left[\phi\left(1 - F\left(\bar{\eta}^m\right)\right) + M^{-1}(\bar{\eta}^m) f(\bar{\eta}^m)\right]\right) + \left(1-\delta^m\right) u'_{2m,n} + f\left(\bar{\eta}^m\right) \kappa u'_{2m}\left(u_{2m} - u_{2m,n}\right) = u'_{1m} \end{aligned}$$

[5] The condition for the equality $\kappa u'_{2m} = u'_{2m,n}$ is $\kappa(Rs^m y + Rs^w y) \geq \max\left(Rs^w y,\, Rs^m y\right)$ and $\kappa = 1$, which is not possible in the model.

[6]which means that

$$s_t^m > s_t^w$$

Contradiction! Therefore, we have $\bar{\eta}^m = M(\bar{\eta}^w)$ and $s^m \geq s^w$ for $\phi \geq 1$.

We totally differentiate the system and have

$$\Omega \cdot \begin{pmatrix} ds_t^w \\ ds_t^m \\ dw_t \\ dP_{Nt} \\ dL_{Nt} \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{pmatrix}$$

where Ω is a 5×5 matrix with elements

$$\begin{aligned}
\Omega_{11} &= u''_{1w} w_t + \beta \left(R \frac{P_t}{P_{t+1}} \right)^2 w_t \left[\begin{array}{c} \kappa^2 u''_{2w} \left(\left(1 + \frac{1}{\phi} \right) (1 - F(\bar{\eta}^w)) \right) + (1 - \delta^w) u''_{2w,n} \\ + f(\bar{\eta}^w) \kappa^2 u''_{2w} (u_{2w} + M(\bar{\eta}^w) - u_{2w,n}) + 2 f(\bar{\eta}^w) \kappa u'_{2w} \left(\kappa u'_{2w} - u'_{2w,n} \right) \end{array} \right] \\
\Omega_{12} &= \beta \left(R \frac{P_t}{P_{t+1}} \right)^2 w_t \left[\begin{array}{c} \kappa^2 u''_{2w} \left(\left(1 + \frac{1}{\phi} \right) (1 - F(\bar{\eta}^w)) \right) + f(\bar{\eta}^w) \kappa^2 u''_{2w} (u_{2w} + M(\bar{\eta}^w) - u_{2w,n}) \\ + f(\bar{\eta}^w) \kappa^2 u'^2_{2w} + f(\bar{\eta}^w) \left(u'_{2m,n} - \kappa u'_{2m} \right) \left(u'_{2w,n} - \kappa u'_{2w} \right) \end{array} \right] \\
\Omega_{13} &= \Omega_{14} = \Omega_{15} = 0 \\
\Omega_{21} &= \beta \left(R \frac{P_t}{P_{t+1}} \right)^2 w_t \left[\kappa^2 u''_{2m} ((1 + \phi)(1 - F(M(\bar{\eta}^w)))) + f(\bar{\eta}^w) \kappa u'_{2m} \left(\left(1 + \frac{1}{\phi} \right) \kappa u'_{2m} - \frac{1}{\phi} u'_{2m,n} \right) \right] \\
\Omega_{22} &= u''_{1m} w_t + \beta \left(R \frac{P_t}{P_{t+1}} \right)^2 w_t \left[\begin{array}{c} \kappa^2 u''_{2m} ((1 + \phi)(1 - F(M(\bar{\eta}^w)))) + (1 - \delta^m) u''_{2m,n} \\ + f(\bar{\eta}^w) \left(\kappa u'_{2m} - u'_{2m,n} \right) \left(\left(1 + \frac{1}{\phi} \right) \kappa u'_{2m} - \frac{1}{\phi} u'_{2m,n} \right) \end{array} \right] \\
\Omega_{23} &= \Omega_{24} = \Omega_{25} = 0
\end{aligned}$$

$$\begin{aligned}
\Omega_{31} &= \frac{\gamma w_t}{1 + \phi}, \ \Omega_{32} = \frac{\gamma \phi w_t}{1 + \phi}, \ \Omega_{33} = -\gamma(1 - s_t), \ \Omega_{34} = \frac{A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N} (1 - \alpha_N)^{1-\alpha_N}}, \Omega_{35} = \frac{P_{Nt}(1 - \alpha_N) A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{-\alpha_N}}{\alpha_N^{\alpha_N} (1 - \alpha_N)^{1-\alpha_N}} \\
\Omega_{41} &= \Omega_{42} = 0, \ \Omega_{43} = -1, \ \Omega_{44} = 0, \ \Omega_{45} = \left(\frac{\alpha_T}{1 - \alpha_T} \right)^{1-\alpha_T} (1 - \alpha_T) A_{Tt} K_{Tt}^{\alpha_T} (1 - L_{Nt})^{-\alpha_T - 1} \\
\Omega_{51} &= \Omega_{52} = 0, \ \Omega_{53} = -1, \ \Omega_{54} = \frac{w_t}{P_{Nt}}, \ \Omega_{55} = -\left(\frac{\alpha_N}{1 - \alpha_N} \right)^{1-\alpha_T} (1 - \alpha_N) A_{Nt} K_{Nt}^{\alpha_N} {L_{Nt}}^{-\alpha_N - 1}
\end{aligned}$$

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[6]**The second inequality holds because (i)**

$$\frac{1}{\phi} (1 - F(\bar{\eta}^w)) + M(\bar{\eta}^w) f(\bar{\eta}^w) = \phi (1 - F(\bar{\eta}^m)) + M^{-1}(\bar{\eta}^m) f(\bar{\eta}^m)$$

by using the uniform distribution assumption; and (ii),

$$u_{2m} - u_{2m,n} > u_{2w} - u_{2w,n}$$

and

$$z_1 = 0,\ z_2 = \frac{1}{\phi^2}\left[1 - F(\bar{\eta}^w)\right]\left(\kappa u'_{2m} - u'_{2m,n}\right),\ z_3 = -\frac{\gamma w_t\left(s_t^m - s_t^w\right)}{1+\phi},\ z_4 = z_5 = 0$$

The determinant of matrix Ω is

$$\det(\Omega) = \det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \cdot \det\begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix}$$

It is easy to show that

$$\det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} > 0$$

and

$$\begin{aligned} \det\begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix} &= \text{negative}\,.terms + \gamma(1-s_t)\left(\frac{w_t}{P_{Nt}}\right)^2 \frac{1-\alpha_T}{L_T} \\ &\quad - \left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right) C_{Nt} \end{aligned}$$

Notice that the consumption of the nontradable good by the young cohort must be less than the aggregate nontradable good consumption, then $\gamma(1-s_t)w_t < P_{Nt}C_{Nt}$. Therefore,

$$\det\begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix} < 0$$

and $\det(\Omega) < 0$.

Then
$$\frac{ds_t^m}{d\phi} = \frac{z_2\Omega_{11}}{\det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}} > 0 \text{ and } \frac{ds_t^w}{d\phi} = \frac{z_2\Omega_{12}}{\det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}}$$

The sign of $\frac{ds_t^w}{d\phi}$ is ambiguous. However, the aggregate savings rate by the young cohort, $s_t^{young} =$

$\frac{\phi}{1+\phi}s_t^m + \frac{1}{1+\phi}s_t^w$, rises as the sex ratio becoms more unbalanced.

$$\begin{aligned}
\frac{ds_t^{young}}{d\phi} &= \frac{s_t^m - s_t^w}{(1+\phi)^2} + \frac{\phi-1}{1+\phi}\frac{ds_t^m}{d\phi} + \frac{1}{1+\phi}\left(\frac{ds_t^m}{d\phi} + \frac{ds_t^w}{d\phi}\right) \\
&= \frac{s_t^m - s_t^w}{(1+\phi)^2} + \frac{\phi-1}{1+\phi}\frac{ds_t^m}{d\phi} \\
&\quad + \frac{1}{1+\phi}\frac{z_2\left(u''_{1w}y + Ry\left[(1-\delta^w)\,u''_{2w,n} - f(\bar{\eta}^w)\left(\kappa u'_2\left(u'_{2w,n} - u'_{2m,n}\right) + u'_{2w,n}u'_{2m,n}\right)\right]\right)}{\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}}
\end{aligned}$$

where all the terms on the right hand side are positive and hence, $\frac{ds_t^{young}}{d\phi} > 0$.

As for the price of the nontradable good,

$$\frac{dP_{Nt}}{d\phi} = -\frac{z_3\frac{w_t^2}{P_{Nt}}\frac{\alpha_T}{L_{Tt}}}{\det\begin{pmatrix}\Omega_{33} & \Omega_{34} & \Omega_{35}\\ \Omega_{43} & \Omega_{44} & \Omega_{45}\\ \Omega_{53} & \Omega_{54} & \Omega_{55}\end{pmatrix}} + \frac{z_2\left(\Omega_{11}\Omega_{32} - \Omega_{12}\Omega_{31}\right)\frac{w_t}{P_{Nt}}\frac{\alpha_T}{L_{Tt}}}{\det(\Omega)}$$

It is easy to show that $\Omega_{11}\Omega_{32} - \Omega_{12}\Omega_{31} < 0$, and since $z_2 < 0$, $\frac{dP_{Nt}}{d\phi} < 0$, which results in a fall in the consumption price index and therefore a real exchange rate depreciation in period t.

As for the current account,

$$CA_t = P_{Nt}Q_{Nt} + Q_{Tt} + (R-1)\cdot NFA_{t-1} - P_tC_t - K_{t+1}$$

where NFA_{t-1} is the net foreign asset holdings in period $t-1$ and K_{t+1} is the sum of capital input in both the nontradable sector and the tradable sector in period $t+1$.

Notice that

$$s_{t-1}w_{t-1} = NFA_{t-1} + K_t$$

Then

$$CA_t = s_tw_t - s_{t-1}w_{t-1} - \Delta K_{t+1}$$

where $\Delta K_{t+1} = K_{t+1} - K_t$. By Obstfeld and Rogoff (1995), if the sex ratio remains constant ϕ after period t, the price of the nontradable good will go back to its initial level, which means that the real exchange rate will appreciate in period $t+1$. In this perfect foresight setup, when firms make their optimal decisions, equations (2.2) and (2.4) hold. If we assume log utility function, the aggregate savings rate by the young cohort will remain the same after period t.

The demand for the nontradable good is now

$$Q_{N,t+1} = \frac{\gamma w \left((R-1) s_t + 1 \right)}{P_{N,t+1}}$$

where we drop the time subscript because both the wage rate and the relative price of the nontradable good would go back to their initial levels. It is easy to see that since $s_t > s_{t-1}$, $Q_{N,t+1} > Q_{N,t-1}$.

As in Obstfeld and Rogoff (1995), we assume that $\alpha_N < \alpha_T$, the nontradable sector has a lower capital-intensity than the tradable sector. Then, in period $t+1$, $K_{t+1} < K_{t-1}$.

In period $t+1$,

$$A_{Nt} K_{N,t+1}^{\alpha_N} L_{N,t+1}^{1-\alpha_N} = \frac{\gamma w \left((R-1) s_t + 1 \right)}{P_{N,t+1}}$$

In the equilibrium, all markets clear and we can obtain

$$K_{t+1} = \left[\frac{\alpha_T}{(1-\alpha_T)} - \left(\frac{\alpha_T}{1-\alpha_T} - \frac{\alpha_N}{1-\alpha_N} \right) \gamma \left((R-1)s + 1 \right) \right] \frac{w}{R}$$

and then

$$\begin{aligned} \Delta K_{t+1} &= \gamma \left(\frac{\alpha_T}{1-\alpha_T} - \frac{\alpha_N}{1-\alpha_N} \right) (R-1) \left(s_t - s_{t-1} \right) \frac{w}{R} \\ CA_t &= s_t w_t - s_{t-1} w + \gamma \left(\frac{\alpha_T}{1-\alpha_T} - \frac{\alpha_N}{1-\alpha_N} \right) (R-1) \left(s_t - s_{t-1} \right) \frac{w}{R} \end{aligned}$$

To show $\frac{dCA_t}{d\phi} > 0$, we only need to show $\frac{d(s_t w_t - s_{t-1} w_{t-1})}{d\phi} > 0$. By (3.9), one sufficient condition for the inequality is

$$s_t P_{Nt} > s_{t-1} P_{Nt}$$

To show this inequality, we just need to show

$$s_t \frac{dP_{Nt}}{d\phi} + P_{Nt} \frac{ds_t}{d\phi} > 0$$

which means

$$\frac{dP_{Nt}/d\phi}{ds_t/d\phi} + \frac{P_{Nt}}{s_t} > 0$$

Plugging the expressions of $\frac{dP_{Nt}}{d\phi}$ and $\frac{ds_t}{d\phi}$, we have

$$\begin{aligned} \frac{dP_{Nt}}{ds_t} + \frac{P_{Nt}}{s_t} &= \frac{-\gamma(1-s_t) w_t C_{Nt} \left(\frac{w_t}{P_{Nt}} \right) \left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}} \right) + P_{Nt} C_{Nt} \left(\frac{w_t}{P_{Nt}} \right) \left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}} \right)}{s_t \cdot \text{positive}.terms} + \text{positive}.term \\ &= \frac{\left(P_{Nt} C_{Nt} - \gamma(1-s_t) w_t \right) \left(\frac{w_t}{P_{Nt}} \right) \left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}} \right)}{s_t \cdot \text{positive}.terms} + \text{positive}.term \end{aligned}$$

As shown above, $P_{Nt}C_{Nt} - \gamma(1-s_t)w_t > 0$, then $\frac{dCA_t}{d\phi} > 0$, in period t, the country will experience a current account surplus.

We now show by contradiction that $\kappa u'_{2m} < u'_{2m,n}$ must hold for all ϕs. Suppose not, then $\kappa u'_{2m} < u'_{2m,n}$ may fail sometime. Due to continuity of z_2, there exists a level of sex ratio ϕ_0 at which $\kappa u'_{2m} = u'_{2m,n}$, which implies that $z_2 = 0$.

As in Du and Wei (2010), we can show that

$$z_2|_{\phi=\phi_0} = 0 \text{ and } \left.\frac{d^k z_2}{d\phi^k}\right|_{\phi=\phi_0} = 0 \text{ for any } k > 0$$

which means that $z_2 = 0$ for all ϕs. This contradicts the assumption that $z_2 < 0$ when $\phi = 1$. Therefore, the inequality $\kappa u'_{2m} < u'_{2m,n}$ holds for all ϕs.

[QY: In the last paragraph, can you eliminate "as in Du and Wei (2010), and fill in necessary details? The goal is to make the paper as self-contained as possible.] ■

C Proof of Proposition 3

Proof. If $u(c) = \ln c$, solving the first order condition under a balanced sex ratio for both men and women in the marriage market, we can obtain

$$-\frac{1}{1-s_t} + \beta R \frac{P_t}{P_{t+1}} \frac{1}{s_t} = 0$$

which is the same optimal condition when a man or a woman chooses to be single. For a representative woman, at the balanced sex ratio, if she chooses to enter the marriage market, with probability $F(\bar{\eta})$ she can get married and receive welfare

$$\begin{aligned} V_t^w &= \ln\left(\frac{(1-s_t)w_t}{P_t}\right) + \beta F(\bar{\eta}) \ln\left(\frac{\kappa R(2s_t) w_t}{P_{t+1}}\right) + \beta(1 - F(\bar{\eta})) \ln\left(\frac{Rs_t w_t}{P_{t+1}}\right) + E[\eta | \eta^w \geq \bar{\eta}] \\ &\geq \ln\left(\frac{(1-s_t)w_t}{P_t}\right) + \beta \ln\left(\frac{Rs_t w_t}{P_{t+1}}\right) = V_{n,t}^w \end{aligned}$$

where the inequality holds because $\kappa > 1/2$ and $E[\eta | \eta^w \geq \bar{\eta}] \geq 0$. Therefore, entering the marriage market is a dominant strategy for all women. Since men and women are symmetric when $\phi = 1$, all men and all women will enter the marriage market with probability one at the balanced sex ratio.

As we have shown in Proposition 2,

$$\frac{ds_t^m}{d\phi} > 0 \text{ and } \frac{ds_t^m}{d\phi} + \frac{ds_t^w}{d\phi} > 0$$

we can show that

$$\begin{aligned}\frac{\partial V_t^m}{\partial \phi} &= y\left(-u'_{1m}+\beta R\frac{P_t}{P_{t+1}}\left(\kappa\delta^m u'_{2m}+(1-\delta^m)u'_{2m,n}\right)\right)\frac{ds_t^m}{d\phi} \\ &\quad +\beta R\frac{P_t}{P_{t+1}}\delta^m y\kappa u'_{2w}\frac{ds_t^w}{d\phi}-\beta\int_{M(\bar\eta^w)}[1-F(\eta)]\,d\eta \\ &< -\beta\int_{M(\bar\eta^w)}[1-F(\eta)]\,d\eta-\beta R\frac{P_t}{P_{t+1}}(\phi-1)\,\delta^m y\kappa u'_{2w}\frac{ds_t^m}{d\phi}<0\end{aligned} \tag{C.1}$$

where the first equality in (C.1) holds because

$$\begin{aligned}\frac{\partial\delta^m}{\partial\phi} &= -\frac{1-F(\bar\eta^w)}{\phi^2}(u_{2m}-u_{2m,n}) \\ &\quad -\frac{Ryf(\bar\eta^w)}{\phi}\left[u'_{2m,n}\frac{ds_t^m}{d\phi}-\kappa u'_{2m}\left(\frac{ds_t^m}{d\phi}+\frac{ds_t^w}{d\phi}\right)\right](u_{2m}-u_{2m,n}) \\ \frac{\partial\left(\int_{M(\bar\eta^w)}M^{-1}(\eta^m)dF(\eta^m)\right)}{\partial\phi} &= -\int_{M(\bar\eta^w)}[1-F(\eta)]\,d\eta-\frac{\bar\eta^w(1-F(\bar\eta^w))}{\phi^2} \\ &\quad -\frac{\bar\eta^w f(\bar\eta^w)}{\phi}\left[u'_{2m,n}\frac{ds_t^m}{d\phi}-\kappa u'_{2m}\left(\frac{ds_t^m}{d\phi}+\frac{ds_t^w}{d\phi}\right)\right]\end{aligned}$$

and the first inequality in (C.1) holds because

$$\delta^m<1<\phi(1-F(\bar\eta^m))+\bar\eta^m f(\bar\eta^m)\text{ and }\frac{ds_t^w}{d\phi}\le\frac{ds_t^m}{d\phi}$$

Men lose as the sex ratio rises while the effect on women's welfare is ambiguous.

Now consider women's welfare. Given the equilibrium s_t^m and s_t^w under a sex ratio ϕ, if one woman deviates from the equilibrium choice s_t^w, for instance, by choosing a savings rate $s_t^{w\prime}=s_t^m$, she would receive a lower life-time utility $V_t^{w\prime}$ $(\le V_t^w)$. Since $s_t^{w\prime}=s_t^m\ge s_t^w$, this woman will have a better situation than all other women in the marriage market, i.e., she is more likely to get married and also more likely to marry a better man. Then

$$\begin{aligned}V_t^{w\prime} &= u_{1w'}+\beta\left[\delta' u_{2w'}+\left(1-\delta'\right)u_{2w',n}+\int_{\bar\eta^w}M\left(\eta^w+u_{2w'}-u_{2w}\right)dF\left(\eta^w\right)\right] \\ &\ge u_{1w'}+\beta\left[(1-F(\bar\eta^w))\,u_{2w'}+F\left(\bar\eta^w\right)u_{2w',n}+\int_{\bar\eta^w}M\left(\eta^w\right)dF\left(\eta^w\right)\right] \\ &= u_{1m}+\beta\left[(1-F(\bar\eta^w))\,u_{2m}+F\left(\bar\eta^w\right)u_{2m,n}+\int_{\bar\eta^w}M\left(\eta^w\right)dF\left(\eta^w\right)\right] \\ &\ge u_{1m}+\beta\left[(1-F(M\left(\bar\eta^w\right)))\,u_{2m}+F\left(M\left(\bar\eta^w\right)\right)u_{2m,n}+\int_{M(\bar\eta^w)}M^{-1}\left(\eta^m\right)dF\left(\eta^m\right)\right]=V_t^m\end{aligned}$$

where $u_{1w'}$, $u_{2w'}$ and $u_{2w',n}$ denote the first period consumption-led utility, the second period consumption-led utility when she gets married, and the second period utility when she fails to get matched with any man, respectively. u_{2w} is the second period consumption-led utility for all other women who get married. The first inequality holds because the woman faces a greater possibility of getting married and also she will receive a higher expected emotional utility from her husband. The second inequality holds both because she is more likely than the representative man to get married and because she is expecting to receive higher emotional utility from her spouse than the representative man.

Therefore, for $\phi \geq 1$, we can show that $V_t^w \geq V_t^{w\prime} \geq V_t^m$, the representative woman always achieves higher welfare than the representative man.

For the representative man in the marriage market, given his rivals' choices, if he choose to stay in the marriage market, he will follow the first order condition (B.3) and achieves an approximate life time utility $u_{1m} + \beta u_{2m,n}$. If he chooses to be single, he maximizes the life time utility $u_1 + \beta u_2$. The first order condition in this case is

$$-u'_{1m} + u'_{2m} = 0 \tag{C.2}$$

The two savings decisions, in the marriage market and being single, will be different since the man will follow different first order conditions. Then

$$V_n^m = \max u_1 + \beta u_2 > u_{1m} + \beta u_{2m,n} \to V^m$$

when $\phi \to \infty$. The representative man will then choose to be single which violates the assumption that, for all ϕs, entering the marriage market is the dominant strategy for all men. Therefore, a threshold as ϕ_1 exists and at $\phi \geq \phi_1$, $V_n^m = V^m$.

For $\phi \geq \phi_1$, with probability $\frac{\phi_1}{\phi}$, the representative man will choose to enter the marriage market, and with probability $1 - \frac{\phi_1}{\phi}$, he remains single. For the representative woman, since she earns the same first period income as he, we can show that

$$V_n^w = V_n^m = V^m < V^w$$

the representative woman will enter the marriage market with probability one.

As for the aggregate savings rate in the young cohort, we have shown in Proposition 1 that for $\phi < \phi_1$, as the sex ratio rises, the aggregate savings rate in the young cohort will rise. For $\phi \geq \phi_1$, as the sex ratio rises, some men begin quitting the marriage market and choose a different savings rate according to (C.2). Compare (B.3) with (C.2), it is ambiguous whether $s^m > s_n^m$ or not, then the effect on the aggregate savings rate is ambiguous. ■

D Proof of Proposition 4

Proof. If $u(C) = \ln C$, for $\phi < \phi_1$, by the optimal labor supply condition, we have

$$0 < \frac{dL_t^i}{ds_t^i} = \frac{1}{1-s_t^i}\frac{v_i' L_t^i}{v_i' - v_i'' L_t^i} \tag{D.1}$$

where $i = w, m$.

Similar to the proof of Proposition 2, we can show that $\bar{\eta}^m = M(\bar{\eta}^w)$ and $s^m L^m \geq s^w L^w$ for $\phi \geq 1$. Since at $\phi = 1$, women and men are symmetric, and hence $s^m = s^w$ and $L^m = L^w$. For $\phi \geq 1$, by (D.1), $s^m L^m \geq s^w L^w$ means $s^m \geq s^w$ and $L^m \geq L^w$.

$$\Omega \cdot \begin{pmatrix} ds_t^w \\ ds_t^m \\ dw_t \\ dP_{Nt} \\ dL_{Nt} \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{pmatrix}$$

where

$$\begin{aligned}
\Omega_{11} &= u_{1w}''\left(1-(1-s_t^w)\frac{dL_t^w}{ds_t^w}\right)\frac{w_t}{P_t} \\
&\quad +\beta\left(R\frac{P_t}{P_{t+1}}\right)^2\left[\begin{array}{c} \kappa^2 u_{2w}''\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right)+\left(1-\delta^w\right)u_{2w,n}'' \\ +f\left(\bar{\eta}^w\right)\kappa^2 u_{2w}''\left(u_{2w}+M(\bar{\eta}^w)-u_{2w,n}\right)+2f\left(\bar{\eta}^w\right)\kappa u_{2w}'\left(\kappa u_{2w}'-u_{2w,n}'\right)\end{array}\right]\left(L_t^w+s_t^w\frac{dL_t^w}{ds_t^w}\right)\frac{w_t}{P_t} \\
\Omega_{12} &= \beta\left(R\frac{P_t}{P_{t+1}}\right)^2\left[\begin{array}{c} \kappa^2 u_{2w}''\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right)+\left(1-\delta^w\right)u_{2w,n}'' \\ +f\left(\bar{\eta}^w\right)\kappa^2 u_{2w}''\left(u_{2w}+M(\bar{\eta}^w)-u_{2w,n}\right)+2f\left(\bar{\eta}^w\right)\kappa u_{2w}'\left(\kappa u_{2w}'-u_{2w,n}'\right)\end{array}\right]\left(L_t^m+s_t^m\frac{dL_t^m}{ds_t^m}\right)\frac{w_t}{P_t} \\
\Omega_{13} &= \Omega_{14}=\Omega_{15}=0 \\
\Omega_{21} &= \beta\left(R\frac{P_t}{P_{t+1}}\right)^2\left[\begin{array}{c} \kappa^2 u_{2m}''\left(\left(1+\phi\right)\left(1-F\left(M\left(\bar{\eta}^w\right)\right)\right)\right) \\ +f\left(\bar{\eta}^w\right)\kappa u_{2m}'\left(\left(1+\frac{1}{\phi}\right)\kappa u_{2m}'-\frac{1}{\phi}u_{2m,n}'\right)\end{array}\right]\left(L_t^w+s_t^w\frac{dL_t^w}{ds_t^w}\right)\frac{w_t}{P_t} \\
\Omega_{22} &= u_{1m}''\left(1-(1-s_t^w)\frac{dL_t^m}{ds_t^m}\right)\frac{w_t}{P_t} \\
&\quad +\beta\left(R\frac{P_t}{P_{t+1}}\right)^2\left[\begin{array}{c} \kappa^2 u_{2m}''\left(\left(1+\phi\right)\left(1-F\left(M\left(\bar{\eta}^w\right)\right)\right)\right)+\left(1-\delta^m\right)u_{2m,n}'' \\ +f\left(\bar{\eta}^w\right)\left(\kappa u_{2m}'-u_{2m,n}'\right)\left(\left(1+\frac{1}{\phi}\right)\kappa u_{2m}'-\frac{1}{\phi}u_{2m,n}'\right)\end{array}\right]\left(L_t^m+s_t^m\frac{dL_t^m}{ds_t^m}\right)\frac{w_t}{P_t} \\
\Omega_{23} &= \Omega_{24}=\Omega_{25}=0
\end{aligned}$$

$$
\begin{aligned}
\Omega_{31} &= \frac{\gamma w_t}{1+\phi}\left(L_t^w + s_t^w \frac{dL_t^w}{ds_t^w}\right),\ \Omega_{32} = \frac{\gamma \phi w_t}{1+\phi}\left(L_t^m + s_t^m \frac{dL_t^m}{ds_t^m}\right) \\
\Omega_{33} &= -\gamma\left[\frac{(1-s_t^w)L_t^w}{1+\phi} + \frac{\phi(1-s_t^m)L_t^m}{1+\phi}\right],\ \Omega_{34} = \frac{A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}},\ \Omega_{35} = \frac{P_{Nt}(1-\alpha_N)A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}} \\
\Omega_{41} &= -\frac{\alpha_T w_t}{\frac{1}{1+\phi}L_t^w + \frac{\phi}{1+\phi}L_t^m - L_{Nt}} \frac{1}{1+\phi}\frac{dL_t^w}{ds_t^w},\ \Omega_{42} = -\frac{\alpha_T w_t}{\frac{1}{1+\phi}L_t^w + \frac{\phi}{1+\phi}L_t^m - L_{Nt}} \frac{\phi}{1+\phi}\frac{dL_t^m}{ds_t^m} \\
\Omega_{43} &= -1,\ \Omega_{44} = 0,\ \Omega_{45} = \left(\frac{\alpha_T}{1-\alpha_T}\right)^{1-\alpha_T}(1-\alpha_T)A_{Tt}K_{Tt}^{\alpha_T}\left(\frac{1}{1+\phi}L_t^w + \frac{\phi}{1+\phi}L_t^m - L_{Nt}\right)^{-\alpha_T-1} \\
\Omega_{51} &= \Omega_{52} = 0,\ \Omega_{53} = -1,\ \Omega_{54} = \frac{w_t}{P_{Nt}},\ \Omega_{55} = -\left(\frac{\alpha_N}{1-\alpha_N}\right)^{1-\alpha_N}(1-\alpha_N)A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}{}^{-\alpha_N-1}
\end{aligned}
$$

and

$$
\begin{aligned}
z_1 &= 0,\ z_2 = \frac{1}{\phi^2}\left[1-F(\bar{\eta}^w)\right]\left(\kappa u'_{2m} - u'_{2m,n}\right),\ z_3 = -\frac{\gamma w_t\left(s_t^m L_t^m - s_t^w L_t^w\right)}{1+\phi} \\
z_4 &= \frac{\alpha_T w_t}{\frac{1}{1+\phi}L_t^w + \frac{\phi}{1+\phi}L_t^m - L_{Nt}} \frac{L_t^m - L_t^w}{(1+\phi)^2},\ z_5 = 0
\end{aligned}
$$

The determinant of matrix Ω is

$$
\det(\Omega) = \det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \cdot \det\begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix}
$$

Under the assumption that $E\eta$ is sufficiently large, it is easy to show that

$$
\det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} > 0
$$

and

$$
\begin{aligned}
\det\begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix} &= \text{negative}\,.terms + \gamma\left[\frac{(1-s_t^w)L_t^w}{1+\phi} + \frac{\phi(1-s_t^m)L_t^m}{1+\phi}\right]\left(\frac{w_t}{P_{Nt}}\right)^2 \frac{1-\alpha_T}{L_T} \\
&\quad -\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right)C_{Nt}
\end{aligned}
$$

Notice that the consumption of the nontradable good by the young cohort must be less than the aggregate nontradable good consumption, then $\gamma\left[\frac{(1-s_t^w)L_t^w}{1+\phi} + \frac{\phi(1-s_t^m)L_t^m}{1+\phi}\right]w_t < P_{Nt}C_{Nt}$. Therefore,

$$
\det\begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix} < 0
$$

and $\det(\Omega) < 0$

Then

$$\frac{ds_t^m}{d\phi} = -\frac{z_2\Omega_{11}}{\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}} > 0 \text{ and } \frac{ds_t^w}{d\phi} = \frac{z_2\Omega_{12}}{\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}}$$

The sign of $\frac{ds_t^w}{d\phi}$ is ambiguous. By (D.1), we have

$$\frac{dL_t^m}{d\phi} > 0$$

and the sign of $\frac{dL_t^w}{d\phi}$ is ambiguous.

The aggregate savings rate by the young cohort $s_t^{young} = \frac{\phi}{1+\phi}s_t^m + \frac{1}{1+\phi}s_t^w$,

$$\frac{ds_t^{young}}{d\phi} = \frac{\phi}{1+\phi}\frac{ds_t^m}{d\phi} + \frac{1}{1+\phi}\frac{ds_t^w}{d\phi} + \frac{s_t^m - s_t^w}{(1+\phi)^2} > 0$$

The aggregate labor supply in period t

$$\frac{dL_t}{d\phi} = \frac{\phi}{1+\phi}\frac{dL_t^m}{ds_t^m}\frac{ds_t^m}{d\phi} + \frac{1}{1+\phi}\frac{dL_t^w}{ds_t^w}\frac{ds_t^w}{d\phi} + \frac{L_t^m - L_t^w}{(1+\phi)^2}$$

Under the assumption $\frac{v''L}{v'}$ is non-decreasing in L, by (D.1), $\frac{dL_t^m}{ds_t^m} > \frac{dL_t^w}{ds_t^w}$, then we have $\frac{dL_t}{d\phi} > 0$, which means the aggregate labor supply is increasing in the sex ratio.

As for the price of the nontradable good,

$$\frac{dP_{Nt}}{d\phi} = -\frac{z_3\left(\frac{w_t^2}{P_{Nt}}\right)\frac{\alpha_T}{L_{Tt}} + z_4\left(\Omega_{34}\Omega_{55} - \Omega_{35}\Omega_{54}\right)}{\det\begin{pmatrix}\Omega_{33} & \Omega_{34} & \Omega_{35}\\ \Omega_{43} & \Omega_{44} & \Omega_{45}\\ \Omega_{53} & \Omega_{54} & \Omega_{55}\end{pmatrix}} + \frac{z_2\left(\Omega_{11}\Omega_{32} - \Omega_{12}\Omega_{31}\right)\frac{w_t^2}{P_{Nt}}\frac{\alpha_T}{L_{Tt}}}{\det(\Omega)}$$

It is easy to show that $\Omega_{34}\Omega_{55} - \Omega_{35}\Omega_{54} < 0$, then $\frac{dP_{Nt}}{d\phi} < 0$, which results in a fall in the consumption price index and therefore a real exchange rate depreciation in period t.

As for the current account,

$$CA_t = P_{Nt}Q_{Nt} + Q_{Tt} + (R-1)\cdot NFA_{t-1} - P_tC_t - K_{t+1}$$

where NFA_{t-1} is the net foreign asset holdings in period $t-1$ and K_{t+1} is the sum of capital input in both the nontradable sector and the tradable sector in period $t+1$.

Notice that

$$s_{t-1}w_{t-1}L_{t-1} = NFA_{t-1} + K_t$$

Then

$$CA_t = \left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right) w_t - s_{t-1} w_{t-1} L_{t-1} - \Delta K_{t+1}$$

where $\Delta K_{t+1} = K_{t+1} - K_t$. Following Obstfeld and Rogoff (1995), if the sex ratio remains constant at ϕ after period t, the price of the nontradable good will go back to its initial level, which means that the real exchange rate will appreciate in period $t+1$. In this perfect foresight setup, when firms make their optimal decisions, equations (2.2) and (2.4) hold. If we take the log utility function, the aggregate savings rate by the young cohort will remain the same after period t.

The demand for the nontradable good is now

$$Q_{N,t+1} = \frac{\gamma w \left((R-1)\left(\frac{s_{t+1}^w}{1+\phi} + \frac{\phi s_{t+1}^m}{1+\phi}\right) + 1\right)}{P_{N,t+1}}$$

where we drop the time subindex because wage rate and the relative price of the nontradable good will go back to their initial levels. It is easy to see that since $s_t > s_{t-1}$, $Q_{N,t+1} > Q_{N,t-1}$.

In period $t+1$,

$$A_{Nt} K_{N,t+1}^{\alpha_N} L_{N,t+1}^{1-\alpha_N} = \frac{\gamma w \left((R-1)\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right) + 1\right)}{P_{N,t+1}}$$

In equilibrium, all markets clear and we can obtain

$$K_{t+1} = \frac{\alpha_T - \gamma(\alpha_T - \alpha_N)\left[(R-1)\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right) + 1\right]}{(1-\alpha_T)R} w$$

and then

$$CA_t = \left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right) w_t - s_{t-1} w L_{t-1} + \frac{(\alpha_T - \alpha_N)(R-1)\left(\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right) - s_{t-1} L_{t-1}\right)}{(1-\alpha_T)R} w$$

To show $\frac{dCA_t}{d\phi} > 0$, we only need to show $\frac{d\left(\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right) w_t - s_{t-1} w L_{t-1}\right)}{d\phi} > 0$. By (3.9), one sufficient condition for the inequality is

$$\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right) P_{Nt} > s_{t-1} L_{t-1} P_{Nt}$$

To show this inequality, we just need to show

$$\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right) \frac{dP_{Nt}}{d\phi} + P_{Nt} \frac{d\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right)}{d\phi} > 0$$

which means

$$\frac{dP_{Nt}/d\phi}{d\left(\frac{s_t^w L_t^w}{1+\phi}+\frac{\phi s_t^m L_t^m}{1+\phi}\right)/d\phi}+\frac{P_{Nt}}{s_t}>0$$

Plugging the expressions of $\frac{dP_{Nt}}{d\phi}$ and $\frac{ds_t}{d\phi}$, we have

$$\begin{aligned}
\frac{dP_{Nt}}{ds_t}+\frac{P_{Nt}}{s_t} &= \frac{-\gamma(1-s_t)w_t C_{Nt}\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}}+\frac{1-\alpha_N}{L_{Nt}}\right)+P_{Nt}C_{Nt}\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}}+\frac{1-\alpha_N}{L_{Nt}}\right)}{s_t\cdot \text{positive}.terms}+\text{positive}.term \\
&= \frac{\left(P_{Nt}C_{Nt}-\gamma(1-s_t)w_t\right)\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}}+\frac{1-\alpha_N}{L_{Nt}}\right)}{s_t\cdot \text{positive}.terms}+\text{positive}.term
\end{aligned}$$

As shown above, $P_{Nt}C_{Nt}-\gamma(1-s_t)w_t>0$, then $\frac{dCA_t}{d\phi}>0$, in period t, the country will experience a current account surplus. ■

E Welfare analysis and discussions of policy interventions

We conduct a simple welfare analysis and use it as a basis for evaluating policy interventions aimed at reducing current account imbalances. Consider a benevolent central planner who cares about the overall welfare of men and women when utility is transferable. The central planner can do anything, including cutting down the sex ratio. We first compute the welfare loss of a rise in the sex ratio. Then we compare the welfare consequences of two different ways to reduce the current account surplus: (i) taxing the tradable good and (ii), reducing the sex ratio.

There are two sources of market failures that the central planner would avoid: (a) men save competitively to improve their relative standing in the marriage market; and (b) both men and women may under-save as they do not take into account the benefits of their own savings for the well-being of their future spouses. The central planner assigns the marriage market matching outcome and optimally chooses women's and men's savings rates to maximize the social welfare function,

$$\max\ U=\frac{1}{1+\phi}U^w+\frac{\phi}{1+\phi}U^m$$

The first order conditions are

$$-u'_{1w}+\beta R\frac{P_t}{P_{t+1}}\left[2(1-F\left(\bar{\eta}^w\right))\kappa u'_{2w}+F(\bar{\eta}^w)u'_{2w,n}\right] = 0 \tag{E.1}$$

$$-u'_{1m}+\beta R\frac{P_t}{P_{t+1}}\left[2(1-F\left(M\left(\bar{\eta}^w\right)\right))\kappa u'_{2m}+F(M\left(\bar{\eta}^w\right))u'_{2m,n}\right] = 0 \tag{E.2}$$

Comparing (E.1), (E.2) to (3.7) and (3.9), in general, it is not obvious whether women or men will save at a higher rate in a decentralized equilibrium than that under central planning. However, when $\phi=1$, we can show that the two sets of first order conditions are identical, and therefore, women and men will save at the same rates under a central planning economy as in a decentralized economy.

There are two opposing effects. On one hand, a part of the savings in the competitive equilibrium is motivated by a desire to out-save one's competitors in the marriage market. The increment in the savings, while individually rational, is not useful in the aggregate, since when everyone raises the savings rate by the same amount, the ultimate marriage market outcome is not affected by the increase in the savings. In this sense, the competitive equilibrium produces too much savings. On the other hand, because the savings contribute to a pubic good in a marriage (an individual's savings raises the utility of his/her partner), but an individual in the first period does not take this into account, he/she may under-save relative to the social optimum. These two effects offset each other. Therefore, when $\phi = 1$, the final savings rate in the decentralized equilibrium could be the same as the social optimum.

In calibrations with a log utility function, we show that men's welfare under a decentralized equilibrium relative to the central planner's economy declines as the sex ratio increases. In comparison, women's relative welfare increases as the sex ratio goes up. The social welfare (a weighted average of men's and women's welfare) goes down as the sex ratio rises.

As a thought experiment, one may also consider what the central planner would do if she can choose the sex ratio (in addition to the savings rates) to maximize the social welfare. The new first order condition with respect to ϕ is

$$\frac{U^m - U^w}{(1+\phi)^2} = 0 \tag{E.3}$$

The only sex ratio that satisfies (E.3) is $\phi = 1$. In other words, the central planner would have chosen a balanced sex ratio. Deviations from a balanced sex ratio represent welfare losses.

We now consider the welfare effect of two policy interventions aimed at reducing the current account imbalance: i) taxing the tradable good and ii), reducing the sex ratio.

We first consider the case of taxing the tradable good. Suppose Home will impose a tax τ on the tradable good in period t and fully rebate this tax revenue to consumers, then the price taken by the tradable good producers will be $1-\tau$. In period $t+1$, when the current account goes back to zero, Home will reduce the tax to zero. During the period in which the shock occurs, (3.11) becomes

$$w_t^\tau = \frac{1-\tau}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}(1-\alpha_T)A_{Tt}\left(\frac{K_{Tt}^\tau}{1-L_{Nt}^\tau}\right)^{\alpha_T} = \frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}P_{Nt}(1-\alpha_N)A_{Nt}\left(\frac{K_{Nt}^\tau}{L_{Nt}^\tau}\right)^{\alpha_N}$$

where variable Z^τ denotes the variable when there is a tax on the tradable good.

As we have shown in the proof of Proposition 2,

$$CA_t = s_t y_t - s_{t-1} w + \gamma\left(\frac{\alpha_T}{1-\alpha_T} - \frac{\alpha_N}{1-\alpha_N}\right)(R-1)\left(s_t - s_{t-1}\right)\frac{w}{R} \tag{E.4}$$

where y_t is the first period income of the young cohort. We assume that a fraction a ($0 \le a \le 1$) of the tax revenue will be distributed to the young cohort in period t while the rest will be refunded to the old cohort. Then the nontradable good market clearing condition can be re-written as

$$\frac{P_{Nt}A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}} = \gamma\left(Rs_{t-1}w_{t-1} + (1-a)\tau Q_{Tt} + (1-s_t)\left(w_t + aQ_{Tt}\right)\right) \tag{E.5}$$

and the wage parity is

$$w_t = \frac{1-\tau}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}(1-\alpha_T)A_{Tt}\left(\frac{K_{Tt}}{1-L_{Nt}}\right)^{\alpha_T} = \frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}P_{Nt}(1-\alpha_N)A_{Nt}\left(\frac{K_{Nt}}{L_{Nt}}\right)^{\alpha_N} \tag{E.6}$$

Given K_{Tt} and K_{Nt} are predetermined, we can show the following proposition:

Proposition 5 *If the tax revenue from the tradable good will only be refunded to the working people,*

(i) If

$$\alpha_T\left(Rs_{t-1}\frac{w_{t-1}}{w_t} + 1 - s_t\right) + (1-s_t)\left(w_t - P_{Nt}\right)\left(\alpha_T + L_{Nt} - 1\right) \geq 0$$

taxing the tradable good cannot reduce the current account surplus.

(ii) If

$$\alpha_T\left(Rs_{t-1}\frac{w_{t-1}}{w_t} + 1 - s_t\right) + (1-s_t)\left(w_t - P_{Nt}\right)\left(\alpha_T + L_{Nt} - 1\right) < 0$$

taxing the tradable good can reduce the current account surplus. However, everyone in Home will experience a welfare loss (on top of the welfare loss associated with an unbalanced sex ratio).

Proof. As we have shown in Proposition 2, if the utility function is of log form, then savings rates will not depend on the first period income. We then can take the savings rates as given. We totally differentiate the system which consists of (E.5) and (E.6) and obtain

$$\Omega \cdot \begin{pmatrix} dP_{Nt} \\ dw_t \\ dL_{Nt} \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} d\tau$$

where

$$\begin{aligned}
\Omega_{11} &= C_{Nt} \\
\Omega_{12} &= -\gamma(1-s_t) \\
\Omega_{13} &= \gamma\left(1-a+a\left(1-s_t\right)\right)\frac{(1-\alpha_T)Q_{Tt}}{1-L_{Nt}} \\
\Omega_{21} &= 0 \\
\Omega_{22} &= 1 \\
\Omega_{23} &= -\alpha_T\frac{w_t}{1-L_{Nt}} \\
\Omega_{31} &= \frac{w_t}{P_{Nt}} \\
\Omega_{32} &= 1 \\
\Omega_{33} &= \alpha_N\frac{w_t}{L_{Nt}}
\end{aligned}$$

and

$$\begin{aligned}
z_1 &= \gamma\left(1-a+a\left(1-s_t\right)\right)Q_{Tt} \\
z_2 &= -\frac{w_t}{1-\tau} \\
z_3 &= 0
\end{aligned}$$

The determinant of matrix Ω is

$$\begin{aligned}
\det(\Omega) &= wC_N\left(\frac{\alpha_N}{L_N}+\frac{\alpha_T}{1-L_N}\right)+\frac{w}{P_N}\left(\frac{\gamma(1-s_t)\alpha_T w}{1-L_N}-\frac{\gamma(1-a+a(1-s_t))(1-\alpha_T)Q_{Tt}}{1-L_N}\right) \\
&= \text{positive}\,.terms+\frac{w}{P_N}\left(P_N C_N\left(\frac{\alpha_N}{L_N}+\frac{\alpha_T}{1-L_N}\right)-\gamma(1-a+a(1-s_t))w\right) \\
&> \frac{w}{P_N}\left(P_N C_N-\gamma(1-a+a(1-s_t))w\right)
\end{aligned}$$

The last inequality holds because we use the fact that

$$\frac{\alpha_N}{L_N}+\frac{\alpha_T}{1-L_N}\geq\left(\sqrt{\alpha_N}+\sqrt{\alpha_T}\right)^2$$

where the equality holds when $L_N=\left(1+\sqrt{\frac{\alpha_T}{\alpha_N}}\right)^{-1}$. In the standard literature, both α_N and α_T take value greater than 0.25, then $\frac{\alpha_N}{L_N}+\frac{\alpha_T}{1-L_N}>1$.

Notice that $\gamma(1-a+a(1-s_t))w$ is only part of the demand for the nontradable good, which must be smaller than $P_N C_N$, therefore, $\det(\Omega)>0$.

Then we can calculate

$$\begin{aligned}\left.\frac{dy_t}{d\tau}\right|_{\tau=0} &= \frac{dw_t}{d\tau}+Q_{Tt}\\ &= \frac{w_t}{\det(\Omega)}\left[\frac{\alpha_T C_N}{1-\alpha_T}+\gamma(1-s_t)\left(\frac{\alpha_T}{1-L_N}-1\right)\left(\frac{w_t}{P_{Nt}}-1\right)Q_{Tt}\right]\\ &= \frac{\gamma w_t^2}{P_{N,t+1}(1-\alpha_T)\det(\Omega)}\left[\alpha_T\left(Rs_{t-1}\frac{w_{t-1}}{w_t}+1-s_t\right)+(1-s_t)(w_t-P_{Nt})(\alpha_T+L_{Nt}-1)\right]\end{aligned}$$

In period $t-1$, $w_{t-1}=R^{-\frac{\alpha_T}{1-\alpha_T}}<R^{\frac{\alpha_N-\alpha_T}{\alpha_T}}=P_{Nt}$. In period t, when shock occurs, as we have shown in Proposition 2, $\frac{w_t}{P_{Nt}}$ increases. However, it is unclear whether it exceeds one. Therefore, the sign of $\frac{dy_t}{d\tau}$ at $\tau=0$ is ambiguous.

If $\alpha_T\left(Rs_{t-1}\frac{w_{t-1}}{w_t}+1-s_t\right)+(1-s_t)(w_t-P_{Nt})(\alpha_T+L_{Nt}-1)\geq 0$, then $\left.\frac{dy_t}{d\tau}\right|_{\tau=0}\geq 0$. By (E.4), taxing the tradable good cannot reduce the current account surplus caused by the unbalanced sex ratio. On the other hand, if $\alpha_T\left(Rs_{t-1}\frac{w_{t-1}}{w_t}+1-s_t\right)+(1-s_t)(w_t-P_{Nt})(\alpha_T+L_{Nt}-1)<0$, then $\left.\frac{dy_t}{d\tau}\right|_{\tau=0}<0$. Taxing the tradable good can achieve the goal of cutting down the current account surplus. However, this also reduces the first period income by the young cohort. The welfare of young women and young men will be worse off.

And

$$\begin{aligned}\left.\frac{dC_{2t}}{d\tau}\right|_{\tau=0} &= -\frac{Rs_{t-1}w}{P_{Nt}}\frac{dP_{Nt}}{d\tau}\\ &= -\frac{\gamma Rs_{t-1}w}{P_{Nt}}(1-s_t)\left(\frac{\alpha_N}{L_N}+\frac{2\alpha_T}{1-L_N}-1\right)<0\end{aligned}$$

then the old cohort in period t also suffers from the tax on the tradable good sector.

Therefore, if $\frac{\alpha_T\left(Rs_{t-1}\frac{w_{t-1}}{w_t}+1-s_t\right)}{1-\alpha_T}+(1-s_t)(w_t-P_{Nt})\frac{\alpha_T+L_{Nt}-1}{1-\alpha_T}<0$, taxing the tradable good will cut down the current account surplus; however, at the same time, it will reduce the economy-wide welfare. ■

When Home taxes the tradable good sector, the wage rate in that sector decreases immediately, which induces a migration of labor from the tradable sector to the nontradable good sector. The tradable good sector shrinks. Since the young people also get the tax refund, whether this tax refund can offset the decrease in the wage rate is ambiguous. Since the total tax refund equals the tax on per unit tradable good multiplied by the quantity of tradable output, a shrinkage of the tradable good sector implies less tax revenue from the tradable sector and a smaller transfer to consumers. However, consumers only bear a part of the tax burden through a lower wage. Firms bear the other part of tax burden by receiving a lower return to capital. Since the entire tax revenue is transferred to consumers, there is an indirect transfer from firms in the tradable sector to consumers. The net effect on the first period income of the young cohort is ambiguous.

If the central planner can reduce the sex ratio, then as shown Proposition 2, a reduction in the sex ratio will yield a fall in the current account. Correspondingly, there will be a welfare gain for young men but a welfare loss for young women. The aggregate social welfare will improve.

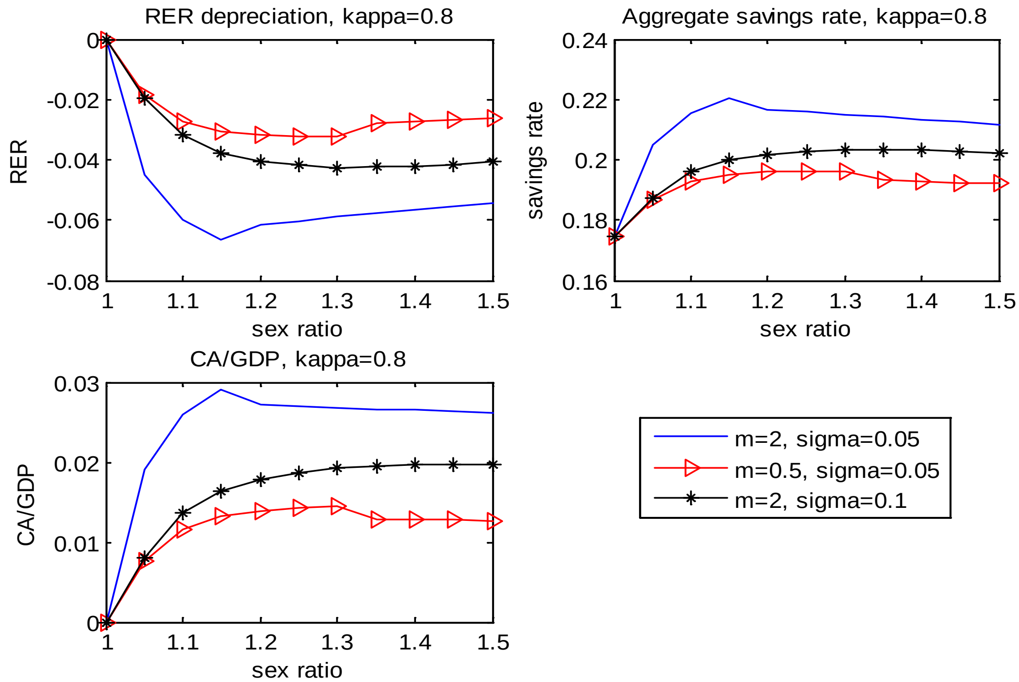

Figure 1: RER, aggregate savings rate, CA/GDP vs sex ratio, no labor supply effect, kappa=0.8

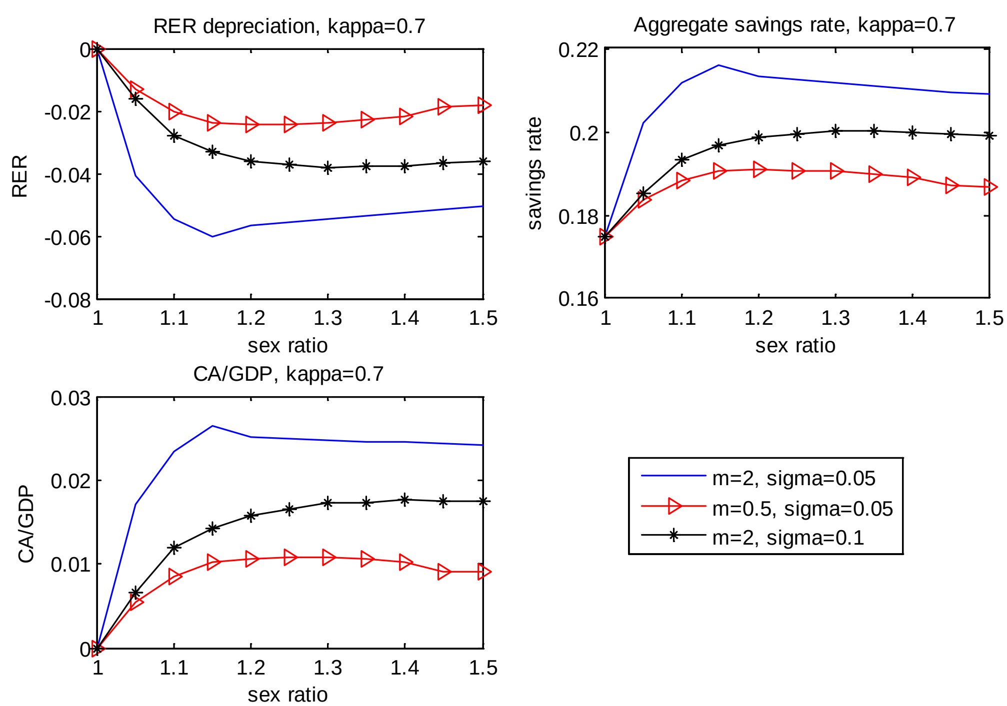

Figure 2: RER, aggregate savings rate, CA/GDP vs sex ratio, no labor supply effect, kappa=0.7

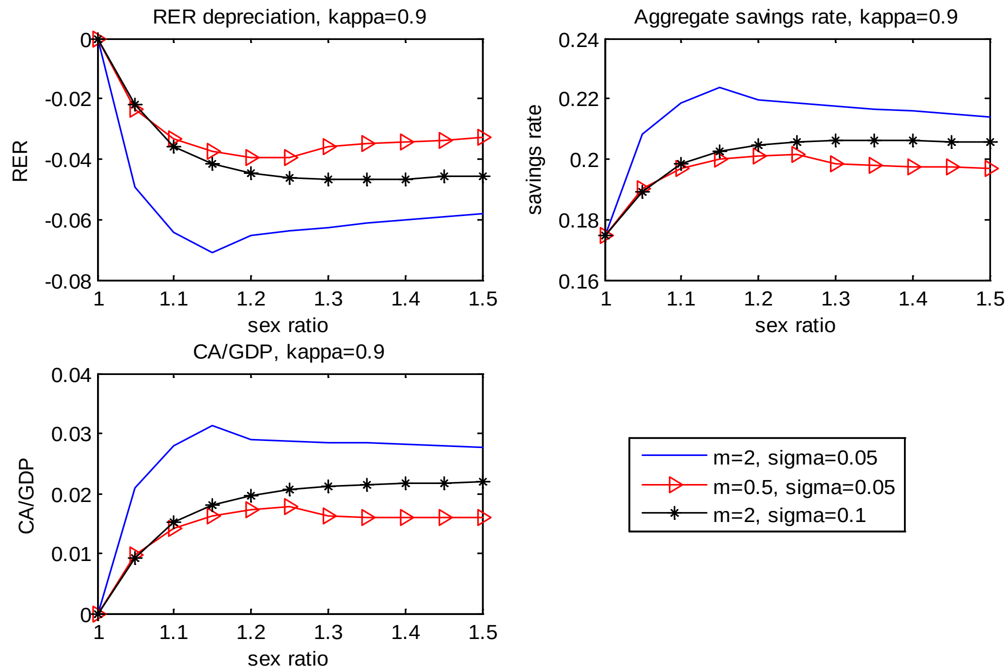

Figure 3: RER, aggregate savings rate, CA/GDP vs sex ratio, no labor supply effect, kappa=0.9

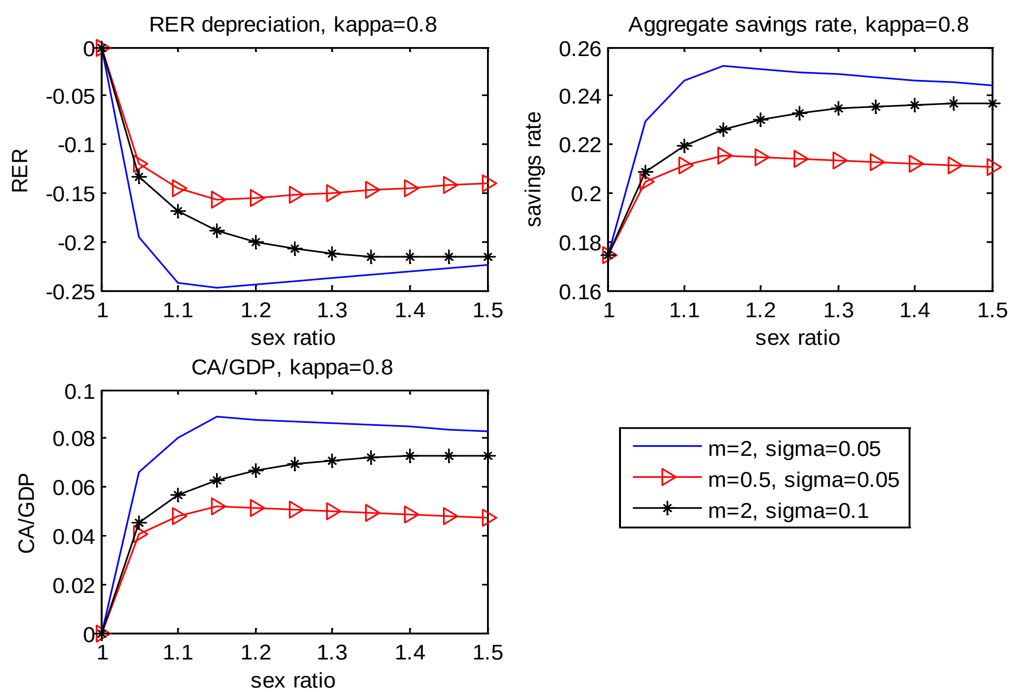

Figure 4: RER, aggregate savings rate, CA/GDP vs sex ratio, with labor supply effect, kappa=0.8

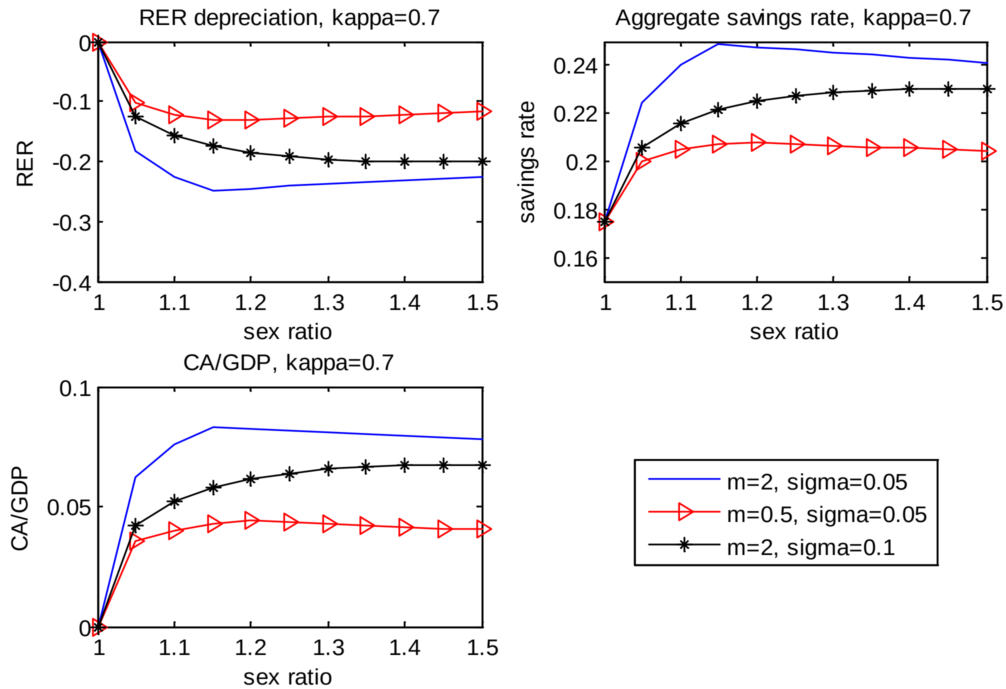

Figure 5: RER, aggregate savings rate, CA/GDP vs sex ratio, with labor supply effect, kappa=0.7

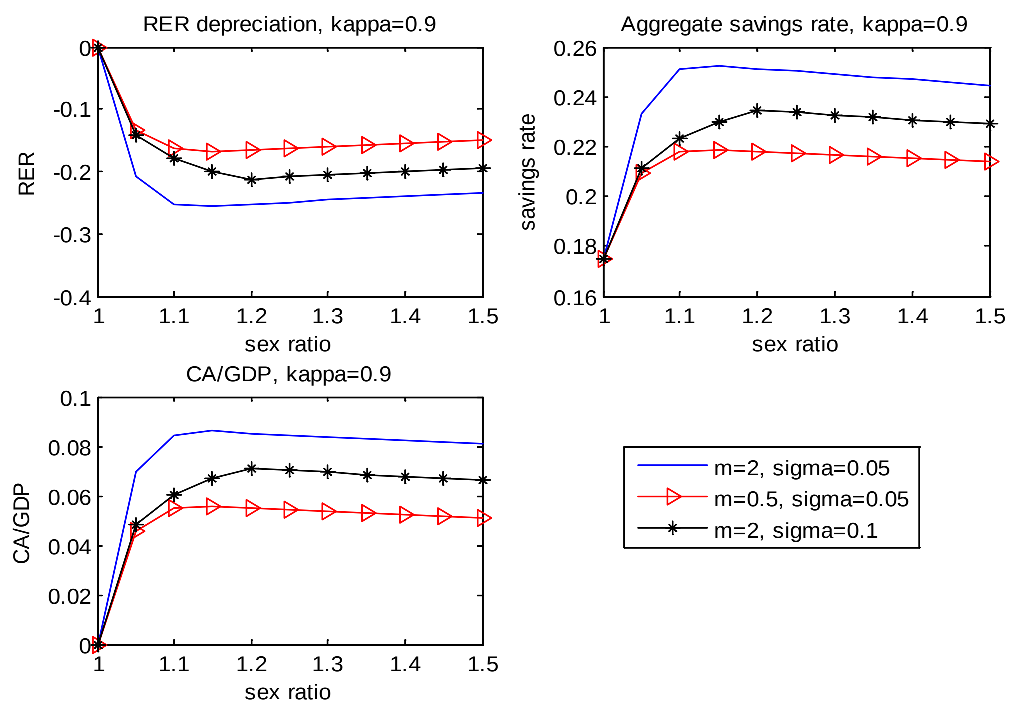

Figure 6: RER, aggregate savings rate, CA/GDP vs sex ratio, with labor supply effect, kappa=0.9

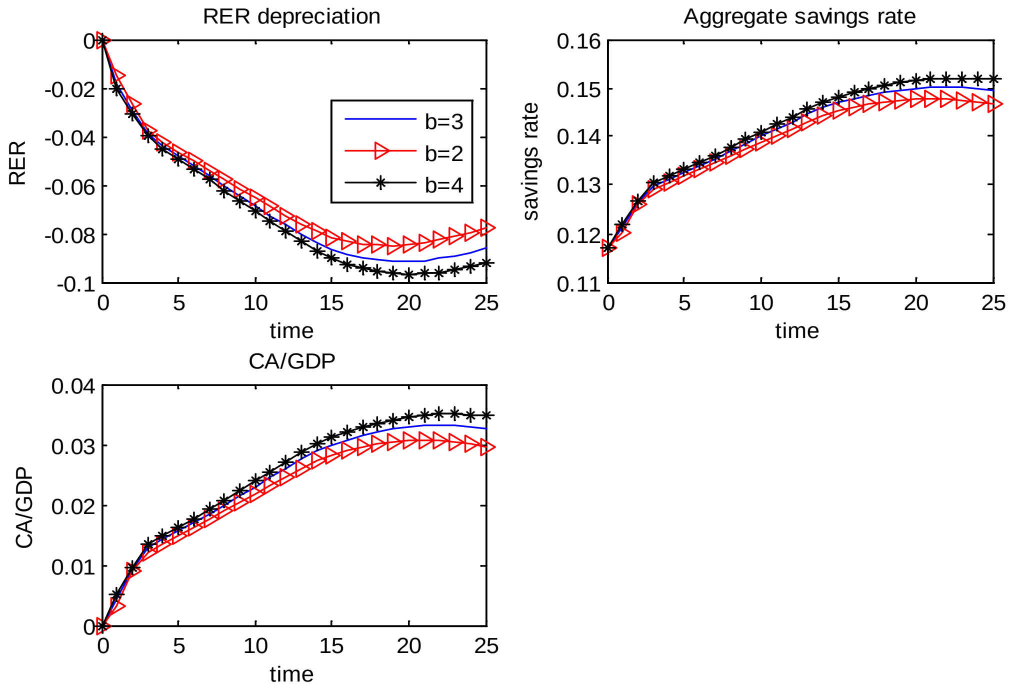

Figure 7: Impulse responses of RER, aggregate savings rate and CA/GDP, τ=20

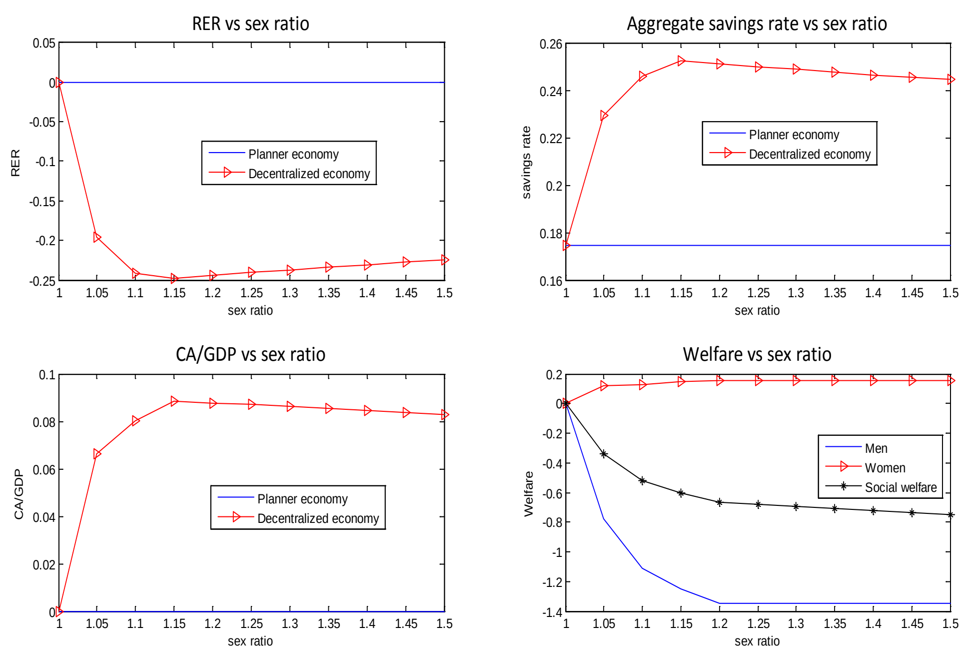

Figure 8: Planner's economy vs Decentralized economy, with labor supply effect, κ=0.8, m=2, σ=0.05

Table 1: Summary statistics, 2004-2008 average

Variable	Mean	Median	Standard deviation	Min value	Max value
Ln(RER)	-0.74	-0.80	0.59	-2.22	0.41
(Private Sector) Current account	-3.63	-2.93	9.32	-31.51	26.91
Real GDP per capita (US$)	12986	7747	13733	367	77057
Private credit (% of GDP)	56.63	38.70	52.26	2.08	319.72
Financial system sophistication	3.78	3.66	0.79	2.52	5.28
Sex ratio	1.04	1.04	0.02	1.00	1.13
Fiscal deficit (% of GDP)	-1.47	-0.37	5.98	-25.98	11.38
Terms of trade	113	102	33.8	70.0	205.8
Capital account openness	0.53	0.118	1.64	-1.83	2.50
Dependency ratio	60.75	54.84	17.64	28.47	107.60

- The real exchange rate data is obtained from Penn World Tables 6.3. The variable "*p*" (called "price level of GDP") in the Penn World Tables is equivalent to the real exchange rate relative to the US dollar: A lower value of *p* means a depreciation in the real exchange rate.
- Private Sector Current account = current account to GDP ratio minus the government savings to GDP ratio.
- For the ratio of private credit (% of GDP), we follow Allen, Qian and Qian (2004) and modify the measure for China by multiplying 0.2 to the credit to GDP ratio. This is to correct for the fact that only 20% of the bank loans go to private firms. Financial system sophistication from the Global Competitiveness Report is another measure for the financial development.
- Fiscal deficit data is obtained from IFS database. Terms of trade index is defined as the ratio of export price index to the import price index, which is from Worldbank database. We use the capital account openness index in Chinn and Ito (2008) to measure the degree of capital controls. A higher value means less capital control. Dependency ratio data can be obtained from Worldbank database.

Table 2: Ln(real exchange rate) and the sex ratio, using private credit to GDP ratio as the measure of financial development

	(1) All countries	(2) All countries	(3) All countries	(4) Excluding major oil exporters	(5) Excluding major oil exporters	(6) Excluding major oil exporters	(7) Excluding major oil exporters
Sex ratio			-4.290**	-4.012**	-3.193*	-3.408**	-3.500**
			(1.667)	(1.713)	(1.797)	(1.568)	(1.754)
Ln(GDP per capita)	0.318**	0.190**	0.236**	0.233**	0.360**	0.402**	0.359**
	(0.030)	(0.038)	(0.041)	(0.044)	(0.073)	(0.063)	(0.073)
Private credit (% of GDP)		0.004**	0.004**	0.004**	0.003**	0.002**	0.002**
		(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Fiscal deficit					-0.007	0.002	-0.005
					(0.009)	(0.008)	(0.009)
Terms of trade					0.0002	-0.001	0.0003
					(0.001)	(0.001)	(0.001)
Capital account openness					0.060**	0.029	0.058**
					(0.027)	(0.024)	(0.027)
Dependency ratio					0.009**	0.010**	0.008*
					(0.004)	(0.004)	(0.004)
Crawling peg (RR)						-0.397**	
						(0.075)	
Managed floating (RR)						-0.036	
						(0.077)	
Free floating (RR)						-0.081	
						(0.119)	
Intermediate (LYS)							-0.078
							(0.092)
Float (LYS)							-0.145*
							(0.085)
Observations	142	132	132	123	92	89	92
R-squared	0.444	0.542	0.564	0.579	0.706	0.801	0.716

Dependent variable = ln(RER). Standard errors are in parentheses, ** $p<0.05$, * $p<0.1$

Table 3: Ln(real exchange rate) and the sex ratio, using financial system sophistication as the measure of financial development

	(1) All countries	(2) All countries	(3) All countries	(4) Excluding major oil exporters	(5) Excluding major oil exporters	(6) Excluding major oil exporters	(7) Excluding major oil exporters
Sex ratio			-6.192**	-6.255**	-5.051*	-4.664*	-4.430
			(1.964)	(1.995)	(2.500)	(2.802)	(2.908)
Ln(GDP per capita)	0.318**	0.480**	0.443**	0.447**	0.529**	0.526**	0.531**
	(0.030)	(0.082)	(0.077)	(0.088)	(0.123)	(0.119)	(0.127)
Financial system sophistication		0.170*	0.252**	0.245**	0.099	0.034	0.086
		(0.089)	(0.086)	(0.099)	(0.110)	(0.121)	(0.116)
Fiscal deficit					-0.022	-0.014	-0.025
					(0.015)	(0.015)	(0.017)
Terms of trade					-0.004	-0.006**	-0.005
					(0.003)	(0.003)	(0.003)
Capital account openness					0.063	0.058	0.073
					(0.042)	(0.047)	(0.047)
Dependency ratio					0.014**	0.017**	0.017*
					(0.007)	(0.007)	(0.008)
Crawling peg (RR)						-0.285*	
						(0.147)	
Managed floating (RR)						0.045	
						(0.102)	
Free floating (RR)						0.053	
						(0.173)	
Intermediate (LYS)							-0.052
							(0.137)
Float (LYS)							0.044
							(0.125)
Observations	142	54	54	49	43	42	43
R-squared	0.444	0.748	0.791	0.797	0.844	0.866	0.845

- Dependent variable = log(RER). Standard errors are in parentheses, ** p<0.05, * p<0.1

Table 4: Non-governmental CA/GDP vs sex ratio, using private credit to GDP ratio as the measure of financial development

	(1) All countries	(2) All countries	(3) All countries	(4) Excluding major oil exporters	(5) Excluding major oil exporters	(6) Excluding major oil exporters	(7) Excluding major oil exporters
Sex ratio			66.43*	78.43**	134.7**	111.6**	94.24
			(37.09)	(36.65)	(37.52)	(56.43)	(56.51)
Ln(GDP per capita)	2.025**	3.683**	2.964**	2.050**	4.941**	4.035	3.834
	(0.639)	(0.876)	(0.957)	(0.975)	(1.529)	(3.415)	(3.115)
Private credit (% of GDP)		-0.048**	-0.046**	-0.030*	-0.054**	-0.053**	-0.051**
		(0.018)	(0.018)	(0.018)	(0.018)	(0.025)	(0.024)
Fiscal deficit					0.079	-0.031	0.101
					(0.187)	(0.379)	(0.345)
Terms of trade					0.021	0.127	0.131*
					(0.029)	(0.076)	(0.076)
Capital account openness					-0.315	-0.017	-0.081
					(0.563)	(1.508)	(1.353)
Dependency ratio					0.175*	0.209	0.439
					(0.089)	(0.745)	(0.720)
Share of working age people						0.163	0.797
						(1.884)	(1.885)
Social security expenditure (% of GDP)						0.137	0.115
						(0.250)	(0.233)
Crawling peg (RR)						3.413	
						(3.694)	
Managed floating (RR)						0.957	
						(2.840)	
Free floating (RR)						1.556	
						(5.730)	
Intermediate (LYS)							1.789
							(2.911)
Float (LYS)							0.117
							(2.469)
Continent dummies	N	N	N	N	N	Y	Y
Observations	130	127	127	120	91	47	48
R-squared	0.073	0.125	0.147	0.121	0.275	0.543	0.532

Table 5: Non-governmental CA/GDP vs sex ratio, using financial system sophistication as the measure of financial development

	(1) All countries	(2) All countries	(3) All countries	(4) Excluding major oil exporters	(5) Excluding major oil exporters	(6) Excluding major oil exporters	(7) Excluding major oil exporters
Sex ratio			103.3**	77.57**	129.6**	51.98	13.34
			(43.29)	(37.42)	(52.89)	(96.33)	(80.39)
Ln(GDP per capita)	2.025**	1.715	2.327	-0.470	2.009	3.752	-0.194
	(0.639)	(1.744)	(1.688)	(1.641)	(2.613)	(5.031)	(3.845)
Financial system sophistication		-0.925	-2.290	0.876	-0.477	4.029	3.480
		(1.889)	(1.896)	(1.861)	(2.337)	(3.336)	(2.951)
Fiscal deficit					-0.185	0.081	0.385
					(0.313)	(0.533)	(0.512)
Terms of trade					0.039	-0.070	0.0772
					(0.055)	(0.118)	(0.105)
Capital account openness					-0.103	-1.300	-0.569
					(0.879)	(2.619)	(1.765)
Dependency ratio					0.251	0.167	-1.807
					(0.150)	(5.527)	(2.375)
Share of working age people						0.125	-3.446
						(12.23)	(5.400)
Social security expenditure (% of GDP)						0.056	-0.092
						(0.302)	(0.257)
Crawling peg (RR)						7.405	
						(5.893)	
Managed floating (RR)						2.020	
						(3.055)	
Free floating (RR)						-8.510	
						(7.890)	
Intermediate (LYS)							7.626**
							(3.571)
Float (LYS)							-3.593
							(4.050)
Continent dummies	N	N	N	N	N	Y	Y
Observations	130	54	54	49	43	32	33
R-squared	0.073	0.023	0.123	0.118	0.200	0.478	0.505

Table 6: Real exchange rate undervaluation and excess current account: The case of China

	% of RER undervaluation					Excess (non-governmental) current account				
	(1) Only BS	(2) FD+BS	(3) Add GD +TT+KA	(4) Add DR	(5) Add SR	(1) Only BS	(2) FD+BS	(3) Add GD +TT+KA	(4) Add DR	(5) Add SR
Financial development index										
Private credit (% of GDP)	55.26	43.45	35.44	17.91	7.86	13.52	12.06	11.39	8.97	2.01
Financial system sophistication	55.26	46.38	31.31	16.78	2.24	13.52	10.26	10.11	7.97	0.37

Notes:

A. Excess RER undervaluation = model prediction – actual log RER. (A positive number describes % undervaluation).

B. Excess current account = private sector current account (i.e., current account net of government savings) – model prediction;

C. The five columns include progressively more regresors:

- (1) The only regressor (other than the intercept) is log income, a proxy for the Balassa-Samuelson (BS) effect;
- (2) Add financial development (FD) to the list of regressors;
- (3) Add government fiscal deficit (GD), terms of trade (TT), and capital account openness (KA);
- (4) Add the dependence ratio (DR);
- (5) Add the sex ratio (SR)

D. The last two rows correspond to estimates when two different proxies for financial development are used. The first row uses the ratio of credit to the private sector to GDP, and the second row uses an index of local financial system sophistication from the Global Competitiveness Report.