The Cyclical Behavior of Equilibrium Unemployment and Vacancies in the US and Europe∗

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Abstract
We set-up a real business cycle model with search and matching frictions driven by several shocks, which nests full Nash Bargaining and wage rigidity as special cases and includes other transmission mechanisms suggested by the literature for the propagation and amplification of disturbances. The model is estimated using full information methods for two Anglo-Saxon countries (the US and the UK), two Continental European countries (France and Germany) and two Scandinavian countries (Norway and Sweden). We conduct inference with mixed frequency data, combining quarterly series for unemployment, vacancies, GDP, consumption, and investment, with annual data on unemployment flows. Parameters and shocks are estimated separately for each country, which can then vary in terms of search and hiring costs, workers’ bargaining power, unemployment benefits levels, wage rigidity and the stochastic properties of disturbances. Overall, the structural model accounts reasonably well for differences in labor market dynamics observed between the two sides of the Atlantic and within Europe. Our estimates indicate that there is considerable cross-country variation in the contribution of technology shocks to the cyclical fluctuations of the labor market. Technology shocks alone replicate remarkably well the volatility in vacancies, unemployment and finding probabilities observed in US, with mixed success in Europe. In contrast, matching shocks and job destruction shocks play a larger role in most European countries relative to the US.

JEL classification: E00, J60, O33.
Key words: Search frictions, business cycle, wage rigidity, OECD

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1 Introduction

There is much debate on whether the standard labor market search model can replicate the cyclical properties of the labor market. In an influential paper Shimer (2005a) has argued that the textbook search model described in Pissarides (2000) can not replicate the high degree of volatility of key labor market variables observed in US data. In contrast, Hagedorn and Manovskii (2008) reach the opposite conclusion in a specification of the model where unemployment has little cost. Shimer (2004), Hall (2005), Hall and Milgrom (2008), and Gertler and Trigari (2009) have noticed that modeling wage rigidities can also help to account for the cyclical volatility of vacancies and unemployment. Caballero and Hammour (1994, 1996, 1998) and more recently Pissarides (2009) and Silva and Toledo (2011) have instead emphasized the importance of allowing for some hiring costs, unrelated to aggregate labor market conditions. These costs make the ex ante net value of creating a job small, which amplifies the sensitivity of job creation to aggregate shocks for the same reasons as in Hagedorn and Manovskii (2008). Eyigungor (2010) exploits a similar mechanism.

The debate in this literature has for the most part focused on textbook versions of the search model, usually driven by just neutral technology shocks. In addition, quantitative analysis has by and large relied on calibration, assessing the predictions of the model using a few moments typically measured only in US data. This paper departs from this strand of work in three important dimensions. First, we endow the search and matching model with a rich set of shocks and transmission mechanisms, including those above mentioned (low cost of unemployment, wage rigidity and hiring costs) which may considerably amplify and propagate the effects of shocks. Second, we estimate the model using full information methods and seven observable series from the labor market and national accounts. Third, we study the empirical performance of the model in several European countries as well as in the US.

The focus of our analysis is on the extent to which technology shocks—as usually modeled in the real business cycle literature—can explain the cyclical properties of the labor market not only in the US but also in different European countries. In tackling this issue, we highlight some important differences in the cyclical behavior of unemployment, vacancies and workers flows between the US and Europe and across European countries. These differences allow us to quantify the importance of disturbances other than technology shocks for the cyclical dynamics of labor markets across countries.

The model that we take to the data is a conventional real business cycle search model, extended to incorporate many ingredients regarded as important by the business
cycle literature to enhance the model’s empirical performance. We allow for non linear preferences, endogenous capital utilization and for adjustment costs to capital and to job creation. Job separation probabilities are endogenous and privatively efficient—i.e. they are set to maximize the private net surplus of jobs. Recruiting efforts require as inputs both final output and labor. Job creation involves incurring the traditional costs of posting vacancies as well as other hiring costs unrelated to aggregate labor market conditions. To model the importance of wage rigidity, the wage setting mechanism is such that in a fraction of jobs wages are rigid as in Hall (2005), while in the remaining fraction wages are set through Nash bargaining as in the conventional formulation of the search model described in Pissarides (2000). This formulation nest full wage rigidity and only Nash bargaining as special cases.

The model is driven by six possible shocks that different strands of the business cycle and labor matching literature have regarded as important for the analysis of cyclical fluctuations. Allowing for several shocks is important to match key features of the data and to estimate the model with full information methods. These disturbances consist of neutral technology shocks, as in the conventional real business model by Kydland and Prescott (1982) and Prescott (1986) and Shimer (2005); shocks to investment-specific technology following Greenwood, Hercowitz, and Krusell (2000), Fisher (2006), Michelacci and Lopez-Salido (2007) and Justiniano, Primiceri, and Tambalotti (2010); stochastic variations in the discount factor of households as in Primiceri, Schaumburg, and Tambalotti (2006); shocks to the search and matching technology as in Blanchard and Diamond (1989, 1990), Hosios (1994) and Cheremukhin and Echavarria (2009); job destruction shocks that lead to movements in job separation probabilities unrelated to the net private surplus of jobs as in Shimer (2005a); and, finally, shocks to aggregate demand which cause exogenous changes in households’ wealth.

The model is taken to the data using a cross-country comparable data set for unemployment, vacancies, unemployment flows, output, consumption and investment. Worker flows data are taken from Elsby, Hobijn, and Sahin (2010). Our data set includes two Anglo-Saxon countries (the US and the UK), two Continental European countries (France, and Germany) and two Nordic countries (Norway and Sweden) over the period 1982-2007 although sample dates vary by country.

Our countries differ in terms of the average unemployment rate and in the rate at which workers lose their job when employed or find new jobs when unemployed. The cyclical properties of the labor market are also different across countries in terms of volatility and co-movement, in unemployment, vacancies and workers flows. While there is a large literature contrasting the properties of real business cycles across coun-
tries, (see for instance Ambler, Cardia, and Zimmermann 2004, and references therein) similar cross-country comparisons for the labor market are scant in quantitative work with search and matching models.

We separately estimate the model for each country, allowing parameters to vary along several dimensions including levels of unemployment rate, worker flows, search costs, workers’ bargaining power, unemployment benefits, wage rigidity as well as in the properties of shocks driving cyclical fluctuations. We adopt a Bayesian approach to inference, using mixed frequency data (quarterly and annual). To this end we rely on methods for state space models with temporarily aggregated observables, following Harvey (1990). Using mixed frequency data is important for our analysis since it enables us to cast the model at a quarterly horizon—given that in some countries worker transitions occur at a high frequency—while in our sample workers flow data are available just at an annual frequency. In addition to allowing for longer time series, estimation in mixed frequency accommodates missing observations, so as to deal with an unbalanced panel of aggregate time series, within each country.

Our main findings can be summarized as follows:

1. Labor markets and national accounts data. When focusing on labor market variables, there are substantial cross-country differences in the importance of shocks for cyclical variations and in the elasticities to shocks, as evident from variance decompositions and impulse response functions. Cross-country discrepancies are considerably smaller for national accounts variables.

2. Technology shocks. Technology shocks are the key driving force of national accounts data in all countries, while their contribution to labor market fluctuations vary substantially across countries. These disturbances account for the bulk of the cyclical fluctuations in unemployment, vacancies and job finding probability observed in the US data. They are also an important driving force of the cyclical dynamics of the labor market in some European countries, such as Germany and particularly Sweden. In France, Norway and the UK the contribution of technology shocks to the business cycle in the labor market is instead substantially more muted.

3. Matching and job destruction shocks. In Europe matching and job destruction shocks explain a larger share of the business cycle in unemployment, compared to the US, especially in France and the UK. The contribution of matching shocks to the cyclical variance in the finding probability is also considerably larger in Europe. These two observations reflect some salient features of the data that are hard to replicate with technology shocks only. First, the Beveridge curve is generally less
stable in European countries than in the US, resulting in lower associations between unemployment and vacancies. Second, the correlation between the job separation rate and the unemployment rate is substantially larger in Europe than in the US. Third, in France, Norway and the UK the correlation between the job finding and the job separation probability is greater than in the US.

4. Differences in parameter estimates. The estimated size of hiring costs unrelated to labor market conditions is generally small, albeit comparatively larger in the US and in Germany than in any other country. Job separation probabilities in France, Germany, and the UK respond more to labor market conditions compared to the US. This suggests that in these countries the separation margin is particularly important to characterize the transmission mechanism of shocks.

Our analysis constitutes a first step in understanding cross-country differences in the cyclical fluctuations of the labor market through the lens of a search model. Space constrains generated by the scope of the paper prevent us from analyzing a number of interesting questions raised by our results. For instance, we do not dissect the key transmission mechanism(s) that allows technology shocks to generate considerable volatility in aggregate labor market variables across different countries. Our empirical analysis also abstracts from direct measures of wages to identify important structural parameters of the model. This is an important omission, given that one of the main lessons by Hagedorn and Manovskii (2008) is that these data are important to identify crucial properties of the transmission mechanism in search models. Nonetheless, we have reasons to believe that at least some of our main conclusions should be robust to further scrutiny. The correlation structure of unemployment, vacancies and workers flows probabilities is significantly different in the US and in some European countries. It is these differences that make unlikely that neutral technology shocks (at least as usually modeled) can account for the lion’s share of the cyclical fluctuations of the labor market in all European countries.

Regarding the structure of the paper, Section 2 characterizes differences in labor market dynamics across the OECD. Section 3 characterizes the economy while Section 4 presents the equilibrium conditions of the decentralized economy. Section 6 discusses our choice for priors. Section 7 reports on estimation results. A Supplementary Appendix that presents additional details on the data, solves the social planner

\[\text{For the case of the US, we pursue the identification of alternative transmission mechanisms using the labor share as observable in Justiniano, Lopez-Salido, and Michelacci (2010).}\]
problem and discusses computational details about the model solution and estimation is available upon request.

2 Data description

We first briefly mention the sources for our data-set and then highlight important differences in the cross-country properties of selected variables in terms of means, volatilities, and cross-correlations. It is these differences that we interpret through the lens of the estimated structural model later on. Therefore, in sections 6 and 7 we extensively refer back to properties of the data highlighted here.

2.1 Data sources

The countries included in the analysis are the US, France, Germany, Sweden, Norway, and the UK. We look at national accounts data (GDP, consumption and investment), as well as of labor market variables (unemployment, vacancies, finding and separation probabilities). Time series for national income accounts, unemployment and vacancies are available at quarterly frequency. Worker flow probabilities are only available at an annual frequency.

To increase data comparability, we rely as much as possible on data compiled by the OECD. Data for GDP, consumption, and investment are taken from the OECD national income accounts except for Norway. Consumption corresponds to real personal final consumption expenditures, investment to total investment expenditures in private fixed investment. National accounts data for Norway are obtained from Statistics Norway and for Mainland only, hence excluding the part of Norwegian economic activity directly related to oil extraction and exploration in the North Sea. For Sweden, only GDP is taken from Statistics Sweden, due to longer time coverage, since in the OECD database, this series starts in 1993. In all countries, GDP, consumption, and investment are expressed in per-capita terms, i.e. divided by population.

Data on vacancies, unemployment, and employment are obtained from the OECD Main Economic Indicator online database. The unemployment rate is simply total number of unemployed over total labor force, which is measured by the sum of employed and unemployed workers. Data on job vacancies (in thousands) are used to construct the vacancy rate as the ratio of job vacancies to the sum of job vacancies and employment, consistent with the definition of the job opening rate used in JOLTS for the US.\textsuperscript{2}

\textsuperscript{2}For the US we splice, in 2001, the log detrended help wanted index with the log JOLTS job
Worker flows data are taken from Elsby, Hobijn, and Sahin (2010). Consistent with earlier comments an important feature of the Elsby et al. (2010) worker flow measures is that original data sources also come from the OECD, making them comparable across countries. These authors use annual measures of the unemployment stock by duration and quarterly measures of the unemployment rate to infer the (yearly) average continuous time Poisson exit rate from unemployment and entry rate into unemployment from employment. Their methodology is intended to correct for the time aggregation bias emphasized in Shimer (2005b). The implied worker flows rates are available at an annual frequency only, and they correspond to the average monthly Poisson rates in each year. We convert these Poisson rates into quarterly probabilities by calculating the quarterly job finding and job separation probability that generate an expected duration of an unemployment spell and an expected duration of an ongoing job consistent with the continuous time Poisson arrival rate calculated by Elsby et al. (2010). In the conversion we assume that a worker who loses a job in a quarter, can find a new one in the same quarter which is consistent with the timing convention of the structural model introduced in Section 3; see the Supplementary Appendix for further details.\footnote{Let $f_{m}^{\tau}$ and $s_{m}^{\tau}$ denote the average Elsby et al. (2010) monthly Poisson arrival rate in year $\tau$ for job finding and job separation, respectively. The implied expected duration of a job in quarters is $1/(3s_{m}^{\tau})$ while the analogous expected duration of an unemployment spell (again in quarters) is $1/(3f_{m}^{\tau})$. We define the yearly average of the quarterly job separation probability as equal to $\Lambda_{r} = 3s_{m}^{\tau}$, which is consistent with the model assumption in Section 3 that a new job lasts at least one quarter. The analogous job finding probability is set equal to $f_{r} = 1/\left[1/(3f_{m}^{\tau}) + 1\right]$, which is again consistent with the model assumption that workers, who lose their job in a period, can find another one in the same period. So the minimum duration of an unemployment spell is zero.}

### 2.2 Descriptive statistics

The sample periods are 1982:I-2007:IV for the United States (USA), 1989:I-2007:IV for France (FRA), 1991:I-2007:IV for Germany (DEU), 1980:I-2004:IV for the United Kingdom (GBR), 1983:I-2007:IV for Norway (NOR), and 1983:I-2004:IV for Sweden (SWE). Sample dates are largely determined by the Elsby et al. (2010) coverage of worker flows, although additional considerations constrain us further in a few countries. For instance, the starting date for Germany is due to German reunification, while the availability of data on vacancies determines the start of the sample in France, as well as the end date in the United Kingdom.

Data are expressed in logs, multiplied by 100 and detrended using the Hodrick-Prescott filter with smoothing parameter equal to 1600 for observables available at a quarterly frequency and equal to 100 for observables at the annual frequency. Due to
the mixed frequency of the data, in statistics reported below involving finding and separation probabilities the time unit is one year, while it is one quarter for all remaining series.

To begin highlighting the diversity in labor markets across the countries in our sample, Panel A) in Table 1 reports sample means for the unemployment rate $u$, and the job finding and job separation probabilities, denoted by $f$ and $\Lambda$, respectively. As expected the unemployment rate in France, Germany and the UK is higher than in the US. In contrast, the unemployment rate of the two Scandinavian countries is below that in the US. Finding and separation probabilities in France, Germany, and the UK are significantly lower than in the US, while Nordic countries lie in between the levels in the US and these other European countries. The level of the Unemployment Insurance replacement rate measured by the percentage of net earning in work over five years of unemployment, comes from the OECD database on Benefits and Wages. As it is well known, benefit levels are substantially lower in the US than in any of these other countries. Panel A also reports a cursory look at the value of the matching elasticity to unemployment implied by a simple OLS regression of the log job finding probability $f$ on the logged vacancy-unemployment ratio $v/u$. The inferred elasticity of the matching function to unemployment is in the range 0.7-0.8. At least with this simple regression, this is slightly higher than the conventional estimates discussed in Petrongolo and Pissarides (2001) but not too far away from the range of values of 0.7-0.75 obtained by Shimer (2005a) using a similar methodology on US data.

The standard deviation of our variables, using each country’s specific sample are reported in Panel B. The analogous value relative to the same statistic in the US when restricting the analysis to a common sample is shown in Panel C. To save space we just omit the relative volatilities for consumption and investment. The overall picture emerging from these comparisons is that while GDP, consumption and investment are in general more volatile in Europe than in the US, differences in volatilities are larger for labor market variables, both across the Atlantic and within Europe. Observe for instance that France, Germany and the UK have lower standard deviations in unemployment and vacancies than the US, although the reverse is true for GDP, consumption and investment. Scandinavian countries have instead larger standard deviations in unemployment and vacancies than the US, although this higher volatility is in line with the larger standard deviations observed for GDP, consumption, and investment. A salient feature of the data is that finding and separation rates are considerably more volatile in all five European countries relative to the US. This last observation may be puzzling. However, we have found similar results when using alternative measures
Table 1: Descriptive statistics.

<table>
<thead>
<tr>
<th>Country:</th>
<th>US</th>
<th>France</th>
<th>Germany</th>
<th>Norway</th>
<th>Sweden</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A) Mean (%)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment $u$</td>
<td>6.0</td>
<td>9.1</td>
<td>8.7</td>
<td>4.0</td>
<td>4.4</td>
<td>8.8</td>
</tr>
<tr>
<td>Finding $f$</td>
<td>62</td>
<td>18</td>
<td>15</td>
<td>52</td>
<td>43</td>
<td>25</td>
</tr>
<tr>
<td>Separation $\Lambda$</td>
<td>10</td>
<td>2.5</td>
<td>1.6</td>
<td>4.7</td>
<td>3.9</td>
<td>3.1</td>
</tr>
<tr>
<td>UI-replacement rate</td>
<td>36</td>
<td>57</td>
<td>66</td>
<td>58</td>
<td>63</td>
<td>53</td>
</tr>
<tr>
<td>Matching elasticity to $u$</td>
<td>78</td>
<td>73</td>
<td>70</td>
<td>74</td>
<td>80</td>
<td>70</td>
</tr>
<tr>
<td><strong>B) Standard Deviation (%)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD(Unemployment $u$)</td>
<td>8.2</td>
<td>5.3</td>
<td>7.2</td>
<td>13.6</td>
<td>16.3</td>
<td>8.1</td>
</tr>
<tr>
<td>SD(Vacancies $v$)</td>
<td>13.5</td>
<td>6.2</td>
<td>13.6</td>
<td>16.0</td>
<td>22.0</td>
<td>11.5</td>
</tr>
<tr>
<td>SD(Finding $f$)</td>
<td>4.8</td>
<td>5.9</td>
<td>9.6</td>
<td>10.4</td>
<td>9.5</td>
<td>9.1</td>
</tr>
<tr>
<td>SD(Separation $\Lambda$)</td>
<td>2.5</td>
<td>8.0</td>
<td>10.3</td>
<td>14.7</td>
<td>21.5</td>
<td>9.0</td>
</tr>
<tr>
<td>SD(Consumption $c$)</td>
<td>0.7</td>
<td>0.7</td>
<td>1.4</td>
<td>1.7</td>
<td>1.6</td>
<td>1.5</td>
</tr>
<tr>
<td>SD(Investment $i$)</td>
<td>3.7</td>
<td>2.6</td>
<td>4.4</td>
<td>6.5</td>
<td>5.2</td>
<td>4.2</td>
</tr>
<tr>
<td>SD(GDP)</td>
<td>0.9</td>
<td>0.9</td>
<td>1.4</td>
<td>1.7</td>
<td>1.5</td>
<td>1.2</td>
</tr>
<tr>
<td><strong>C) Relative SD over common sample, US series=1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD(Unemployment $u$)</td>
<td>1</td>
<td>0.61</td>
<td>0.86</td>
<td>1.7</td>
<td>1.9</td>
<td>1.0</td>
</tr>
<tr>
<td>SD(Vacancies $v$)</td>
<td>1</td>
<td>0.49</td>
<td>1.1</td>
<td>1.3</td>
<td>1.5</td>
<td>1.1</td>
</tr>
<tr>
<td>SD(Finding $f$)</td>
<td>1</td>
<td>1.1</td>
<td>1.9</td>
<td>2.2</td>
<td>2.0</td>
<td>1.8</td>
</tr>
<tr>
<td>SD(Separation $\Lambda$)</td>
<td>1</td>
<td>3.2</td>
<td>3.9</td>
<td>5.7</td>
<td>8.9</td>
<td>3.8</td>
</tr>
<tr>
<td>SD(GDP)</td>
<td>1</td>
<td>0.94</td>
<td>1.6</td>
<td>1.9</td>
<td>1.5</td>
<td>1.2</td>
</tr>
<tr>
<td><strong>D) First order autocorrelation</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment $u$</td>
<td>0.89</td>
<td>0.89</td>
<td>0.87</td>
<td>0.82</td>
<td>0.92</td>
<td>0.94</td>
</tr>
<tr>
<td>Vacancies $v$</td>
<td>0.93</td>
<td>0.79</td>
<td>0.90</td>
<td>0.76</td>
<td>0.89</td>
<td>0.90</td>
</tr>
<tr>
<td>Finding $f$</td>
<td>0.65</td>
<td>0.18</td>
<td>0.19</td>
<td>0.61</td>
<td>0.66</td>
<td>0.42</td>
</tr>
<tr>
<td>Separation $\Lambda$</td>
<td>0.13</td>
<td>0.30</td>
<td>0.31</td>
<td>0.11</td>
<td>0.74</td>
<td>0.12</td>
</tr>
<tr>
<td>GDP</td>
<td>0.84</td>
<td>0.89</td>
<td>0.84</td>
<td>0.19</td>
<td>0.62</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>07:IV</td>
<td>07:IV</td>
<td>07:IV</td>
<td>00:IV</td>
<td>07:IV</td>
<td>04:IV</td>
</tr>
</tbody>
</table>

Notes: The UI-replacement rate is the percentage of net earnings in work received as unemployment benefits as reported from the OECD. The matching elasticity to unemployment is obtained from an OLS regression of the log job finding probability $f$ on the logged vacancy-unemployment ratio $v/u$. Standard deviations and correlations are calculated for the variable in logs. For finding and separation probabilities the serial correlation is at the annual level.
for unemployment flows available for France and the UK constructed by Petrongolo and Pissarides (2008), once these are aggregated and detrended at an annual frequency. The Supplementary Appendix compares the properties of the Petrongolo and Pissarides (2008) and Elsby et al. (2010) datasets.

Panel D) of Table 1 reports the first order autocorrelation of variables—recall differences in time units across series. In general, there are no major differences across countries in the serial correlation of unemployment and vacancies. In contrast, finding probabilities are more persistent in the US than in the UK, and, in particular, France and Germany. But generally cross-country differences in persistence are small. Overall, separation rates exhibit little serial correlation in the US and Europe, with the exception of Sweden where they are more persistent than finding rates. Finally, the serial correlation of GDP in Norway is remarkably smaller than in any other country, likely due to oil production, which indirectly affects the value of mainland GDP.

Figure 1 plots the Beveridge curve for the six OECD countries in our sample. The vacancy rate and the unemployment rate are both in logs, and the scale of axes is maintained unchanged for the different countries. The figure highlights some important cross-country differences in the cyclical properties of the labor market. In the US, Norway and to a lesser extent Sweden vacancies and unemployment line up along a well behaved negatively sloped relation. In contrast, the Beveridge curve would not seem fairly stable in France, Germany and the UK.

Table 2 reports contemporaneous correlations focusing on labor market variables listed by row. The correlation between unemployment and vacancies is substantially higher in the US than in Germany, UK, and in particular France, where it is close to zero. In contrast, the same statistic is quite similar across the Scandinavian countries in our sample and the US. These two observations are in line with the evidence in Figure 1. The degree of comovement between unemployment and the finding rate is highest in the US, with the converse being true for separations. This suggests that the separation margin may be relatively more important in explaining unemployment dynamics in Europe than in the US, consistent with the conclusions by Elsby et al. (2010). Finding and separation probabilities are negatively correlated in all countries except Norway where the correlation is positive and in the UK, where it is close to zero. The positive association of the separation rate with the finding rate in Norway is noteworthy, although visual inspection of the series suggests that might have partially to do with a couple of outliers.

To summarize, the properties of labor market data differ, at times considerably, both between the US and Europe, as well as within European countries. Broadly
Figure 1: The Beveridge curve in different OECD countries

Notes: The Beveridge curve in the US and Europe. All series are detrended. Vacancy rate is total vacancy over the sum of vacancy and employment, unemployment rate is unemployment over total labor force, both in logs.
<table>
<thead>
<tr>
<th>Data:</th>
<th>Unemp.</th>
<th>GDP</th>
<th>Cons.</th>
<th>Inv.</th>
<th>Vacancy</th>
<th>Finding</th>
<th>Separation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>US</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemp.</td>
<td>1.00</td>
<td>-0.80</td>
<td>-0.63</td>
<td>-0.80</td>
<td>-0.79</td>
<td>-0.90</td>
<td>0.28</td>
</tr>
<tr>
<td>Vacancy</td>
<td>-0.79</td>
<td>0.74</td>
<td>0.66</td>
<td>0.81</td>
<td>1.00</td>
<td>0.80</td>
<td>-0.39</td>
</tr>
<tr>
<td>Finding</td>
<td>-0.90</td>
<td>0.76</td>
<td>0.67</td>
<td>0.77</td>
<td>0.80</td>
<td>1.00</td>
<td>-0.13</td>
</tr>
<tr>
<td>Separation</td>
<td>0.28</td>
<td>-0.21</td>
<td>0.03</td>
<td>-0.17</td>
<td>-0.39</td>
<td>-0.13</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>France</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemp.</td>
<td>1.00</td>
<td>-0.66</td>
<td>-0.61</td>
<td>-0.65</td>
<td>-0.05</td>
<td>-0.61</td>
<td>0.64</td>
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</table>

Notes: See notes at Table 1.
speaking, differences with the US seem more marked with Germany, UK and particularly France, rather than with Norway and Sweden. Nonetheless, the last two countries stand out in the relatively high volatility of the separation rate. As discussed in Section 7, these empirical regularities play a crucial role in understanding the identification of shocks as well as cross-country differences in the importance of different disturbances for fluctuations within our estimated structural model, to which we now turn.

3 Description of the decentralized equilibrium

We describe the assumptions that characterize the decentralized equilibrium.

Job output and technologies There is one consumption good, the numeraire, which is produced according to

$$Y = F_t(jK, N) = A_t(jK)^\alpha N^{1-\alpha},$$

with $0 < \alpha < 1$. Here $j$ denotes capital utilization, $K$ the capital stock, $N$ the amount of labor intensive intermediate goods used in production, while $A_t$ is the standard source of temporary cyclical fluctuations considered in general equilibrium versions of the standard search model, see Andolfatto (1996), Merz (1995), and Den-Haan, Ramey, and Watson (2000). We assume that

$$a_t = \ln A_t = \rho a_{t-1} + \epsilon_{at}.$$  

Labor intensive intermediate goods are produced in jobs which consist of firm-worker pairs. A worker can be employed in at most one job. A job produces a unit of intermediate goods.

Capital Accumulation Households accumulate capital and rent it out to firms. Following Christiano, Eichenbaum, and Evans (2005), we allow for the presence of investment adjustment cost. Adjustment costs to capital are important to reproduce the high serial correlation of investment in the data. As a result, the law of accumulation of the capital in the hand of the representative household (see below) can be described as follows:

$$K_{t+1} = [1 - \delta(j_t)] K_t + e^{\phi_t} \left[ 1 - T \left( \frac{I_t}{I_{t-1}} \right) \right] I_t,$$

where $\delta(j_t)$ is the depreciation rate while the function $T$ satisfies $T = T' = 0$ in deterministic steady state and $T'' > 0$. Out of the deterministic steady state, $T, T'$
and $T''$ are all strictly positive. These assumptions imply that at the steady state the relative price of installed capital in terms of new capital goods equals unity. In the above expression $I_t$ is the amount of investment expenditures measured in final output. The variable $\varphi_t$ represents the investment specific technology, as in Solow (1960) and Greenwood, Hercowitz, and Krusell (1997). It evolves according to the following AR(1) process:

$$\varphi_t = \rho \varphi_{t-1} + \varepsilon_{\varphi_t}. \quad (4)$$

At a more general level, these shocks characterize the effects of arbitrary shocks to the demand for capital. Following this logic, Shimer (2010) interprets a shock to $\varphi$ as characterizing the effects of a change in financial frictions on the demand for capital.

As in Greenwood, Hercowitz, and Krusell (2000) and Schmitt-Grohe and Uribe (2008, 2010), owners of physical capital can control the intensity with which the capital stock is utilized. Formally, we let $j_t$ measure capacity utilization in period $t$. The effective amount of capital services that households supply to firms in period $t$ is given by $j_tK_t$. We assume that increasing the intensity of capital utilization $j$ implies a faster rate of capital depreciation $\delta(j)$ so that

$$\delta(j) = \delta_0 + \delta_1(j - 1) + \frac{\delta_2}{2}(j - 1)^2 \quad (5)$$

To guarantee that the depreciation rate is an increasing and convex function of the rate of capacity utilization we assume that $\delta_0, \delta_1, \delta_2$ are non negative. We normalize capital utilization to one in steady state. This modeling of capital utilization guarantees the existence of a steady state equilibrium even in the presence of a trend in the investment-specific technology.

**Search frictions** The labor market for workers is subject to search frictions. The matching process within a period takes place before production in the period. So workers and firms that are matched in period $t$ begin active relationships in the same period. Unmatched workers remain jobless. Workers and firms whose matches are severed can enter their respective matching pools and be re-matched within the same period. We modify standard timing conventions in the discrete time version of the search model (see for example Shimer, 2010) to guarantee that unemployment duration spells can be arbitrarily close to zero. This is important to match the empirical

---

4In practice in the second order expansion below we assume that the function $T$ takes the form

$$T(x) = \frac{T''}{2}(x - 1)^2,$$

where $T'' > 0$. 

---
evidence of some countries such as the US where unemployment duration is remarkably small.\footnote{For example, with this formulation the expected duration of unemployment in steady state is equal to $\frac{1}{f} - 1$, where $0 < f < 1$ denotes the job finding probability.} Following Pissarides (2000), we model the flow of viable matches using a matching function

\[ n_t(s, v) = (M_t s)^\eta v^{1-\eta}, \]

whose arguments $s$ and $v$ denote the masses of workers searching for a new job and of vacancies, respectively. This function is homogeneous of degree one, increasing in each of its arguments, concave, and continuously differentiable.\footnote{We will check that, over the relevant range, it always satisfies $n_t(s, v) \leq \min(s, v)$.} Under (6), the probability that a vacancy gets filled is given by:

\[ q(f) = \frac{n_t(s, v)}{v} = \left(\frac{n}{s}\right)^{-\frac{\eta}{1-\eta}} = f^{-\frac{n}{1-\eta}}, \]

which is decreasing in the rate at which an unemployed worker finds a job given by $f \equiv n/s$. We assume that

\[ m_t \equiv \ln M_t = (1 - \rho_m)m + \rho_m m_{t-1} + \varepsilon_{mt}, \]

which characterizes a shock to the matching technology, i.e. a skill mismatch shock. These shocks do not affect the productivity of a job, but have a direct effect on the outside options of workers. These shocks tend to induce a positive co-movement in the job finding and job separation rate and shift the Beveridge curve.\footnote{Shocks with this property are usually dubbed reallocative, see Blanchard and Diamond (1989, 1990), Davis and Haltiwanger (1999) and Balakrishnan and Michelacci (2001) for evidence about the relevance of these shocks. Cheremukhin and Echavarria (2009) also argue that these shocks are important to explain the cyclical volatility of labor market variables and of the labor wedge in the US.}

**Job creation** Free entry by firms determines the size of the vacancy pools. Processing the applications for a vacancy requires some recruiting services that are exchanged in a perfectly competitive market. The amount of recruiting services required for training $n$ workers and processing applications for $v$ vacancies is given by:

\[ \bar{R}(n, v) = \gamma_n n + \gamma_v v, \quad \gamma_n, \gamma_v \geq 0, \]

Notice that after using (6) to substitute for $v = (M_t s)^{-\frac{1}{1-\eta}} n^{\frac{1}{1-\eta}}$, the amount of recruiting services can be written as

\[ R_t(s, n) = \left[ \gamma_n + \gamma_v \left( \frac{n}{M_t s} \right)^{\frac{n}{1-\eta}} \right] n, \]
which is an extended formulation of the conventional search model. The parameter \( \gamma_n \) matters for the costs of search inefficiencies. This is because training costs are paid before wage bargaining takes place, which leads to a natural hold-up problems. As emphasized by Pissarides (2009) this inefficiency matters little for results. In the standard formulation of the search model (see for example Pissarides, 2000) creation costs are linear in vacancies which corresponds to the case \( \gamma_n = 0 \). Pissarides (2009) emphasizes the importance of job creation costs unrelated to aggregate labor market conditions, \( \gamma_n > 0 \), for reproducing the volatility of key cyclical variables in the US, see also Rotemberg (2006) and Silva and Toledo (2011). The term in \( \frac{n}{M_{ts}} \) in (9) are due to search frictions in the labor market and they imply that job creation costs fall when more workers are searching for a job. This represents the search component of the total costs of job creation.

Recruiting services are produced by combining labour intensive intermediate goods and some final output services whose unitary cost is normalized to one. We also allow for the presence of adjustment costs in the supply of recruiting services. The supply of recruiting services at time \( t \) is denoted by \( R_t \) and is equal to:

\[
R_t = S_t \left[ 1 - G \left( \frac{S_t}{S_{t-1}} \right) \right],
\]

(10)

where \( S_t \) is the input in the production of recruiting services. These input services are obtained by using \( X \) units of labour intensive intermediate goods and \( O \) units of output services according to the following Cobb-Douglas production function:

\[
S = \left( \frac{X}{\kappa} \right)^\kappa \left( \frac{0}{1 - \kappa} \right)^{1 - \kappa}.
\]

The function \( G \) characterizes adjustment costs in the production of recruiting services. It satisfies the condition \( G = G' = 0 \) in deterministic steady state and \( G'' > 0 \). Out of the deterministic steady state, \( G, G' \) and \( G'' \) are all strictly positive.\(^8\) These assumptions imply that in steady state, adjustment costs are irrelevant and the relative price of \( S \) and \( R \) are equal. Adjustment costs in the supply of recruiting services slow down the adjustment of vacancies. This helps in reproducing the strong serial correlation of vacancies observed in the data and the fact that the response of vacancies to shocks is typically hump-shaped (vacancies are sluggish to respond). Fujita and

\(^8\)In practice in the second order expansion below we assume that the function \( G \) takes the form

\[
G(x) = \frac{G''}{2} (x - 1)^2,
\]

where \( G'' > 0 \).
Ramey (2007) and Ravn and Simonelli (2008) have emphasized that the conventional search model has problems in reproducing this feature of the data.

Cost minimization implies that the unitary cost of producing an input service \( S \) is

\[
    r_t = p_t^\kappa, \quad (11)
\]

where \( 0 < \kappa < 1 \), and \( p_t \) denotes the equilibrium price of a labor intensive intermediate good. This is achieved by using

\[
    x_t = \kappa \left( \frac{1}{p_t} \right)^{1-\kappa}, \quad (12)
\]

units of labour intensive intermediate goods and by spending

\[
    o_t = (1 - \kappa) r_t \quad (13)
\]

units of final output. The expression for the cost of recruiting services in (11) allows for differences in the factor content of recruiting costs which Shimer (2010) has shown to matter for the response of the economy to shocks. When recruiting services are obtained by just using labor—which in our formulation corresponds to the case \( \kappa = 1 \)—Shimer (2010) derives a neutrality proposition whereby shocks to labor productivity have no effects on unemployment.\(^9\) Absence of adjustment costs requires setting \( G'' = 0 \). Notice that with this formulation there are no rents generated by the sector producing recruiting services.

**Job destruction** The worker in the job needs to invest to maintain the job productive. Greater effort in maintenance involves a greater survival probability of the job. But greater investment in maintenance also comes at cost to the worker because it reduces the amount of leisure he enjoys. We assume that, when the job destruction probability is \( \Lambda_t \), the worker enjoys utility from leisure equal to \( e^{-\Lambda_t} O(\Lambda_t) \) which is increasing and concave in \( \Lambda_t \), \( O' > 0 \), \( O'' < 0 \). We also assume that in steady state \( O = 0 \), which is just a normalization. The stochastic disturbance \( \lambda_t \) evolves as

\[
    \lambda_t = \rho_{\lambda} \lambda_{t-1} + \varepsilon_{\lambda_t}, \quad (14)
\]

so it has mean zero in steady state. The shock \( \varepsilon_{\lambda_t} \) will induce a specific shock to the job separation rate. Greater \( \lambda_t \) reduces the value of leisure enjoyed by the worker and thereby reduces the job separation probability. This modeling of the job separation rate

\(^9\)In addition to \( \kappa = 1 \), the analysis in Shimer (2010) regarding the neutrality proposition would require that capital is absent \( \alpha = 0 \) and that \( \gamma_n = 0 \) so that \( \bar{R}(n,v) = \gamma_v v \).
is convenient because it allows to solve the model using linear methods, still preserving key properties of models with endogenous job separation. In particular the equilibrium job separation rate will fall when the job net surplus increases, which is the key insight of any model where job separation is set optimally to maximize the job net surplus, see Mortensen and Pissarides (1994). Moreover, shocks to the endogenously determined separation rate will have nil first order effects on the expected net value of a newly created job, due to a conventional envelope condition. As emphasized by Pissarides (2009), this is another important property of models with endogenous separation.

We assume that job destruction decisions are privately efficient and are always set so as to maximize the private net surplus of a job. This is coherent with Barro (1977) who argue that two parties in direct contact with one another can always arrange the terms of their relationship so to achieve private bilateral efficiency. This is the natural equilibrium outcome if investment in job maintenance is observable and verifiable and we assume that firms and workers sign long-term contracts that specify fully contingents plans for workers’ investment in job maintenance. The same outcome would also be obtained in equilibrium if we were to follow Hall (2009) in assuming that the worker at the start of the relationship buys out the firm by paying to the firm the full value of the job.

Unemployment benefits A worker searching for a job who remains jobless at the end of period receives unemployment insurance benefits equal to $z$. Benefits are financed through lump sum taxes and the government budget is balanced in each period.

Splitting of surplus If a firm and a worker who have met separated, both would loose the opportunity of producing and each would have to go through a time-consuming process of search before meeting a new suitable partner. Hence, there is a surplus from a job. We allow for different ways of splitting such surplus. The surplus splitting mechanism is determined at the time when the match is formed. We assume that with probability $1 - \theta$ the wage determination process in a job is governed by Nash bargaining (Pissarides, 2000). In this case the worker and the firm split the net surplus of a job by using a generalized Nash bargaining solution in which the bargaining powers of the worker and the firm are $\beta$ and $1 - \beta$, respectively. Division of the surplus is accomplished via wage payments.\textsuperscript{10} Bargaining takes place after the searching process.

\textsuperscript{10}As emphasized by Haefke, Sonntag, and Van-Rens (2007) and Pissarides (2009), the allocation of resources in the decentralized equilibrium is unaffected by whether bargaining occurs continuously over time or just at the start of the employment relationship—and then wages are set through long-
has concluded, so unemployment is the relevant outside option for the worker. When unemployed, the worker cashes unemployment benefits and has to wait for next period to search for a job. This timing assumption is particularly appropriate because it will imply that the Hosios (1990) condition is satisfied.\footnote{An alternative assumption would be to assume that matching and bargaining occurs simultaneously. This would be a fiction given that newly created jobs are created after a match. Under this alternative assumption the Hosios condition would not be satisfied.}

With probability $\theta$ the job is instead characterized by rigid wages as in Shimer (2004) and Hall (2005). If these wages are inside the bargaining set, then the firm pays to the worker the deterministic quantity $\omega$. If the wage is outside the bargaining set the outside option of either the worker or the firm binds.\footnote{In the presence of a unique deterministic rigid wage, the wage can jump discontinuously in response to a shock that makes the outside option of one party binding. To solve this problem we follow Hall (2005), Hall and Milgrom (2008) and Gertler and Trigari (2009) in assuming that wages are always strictly within the bargaining set. Although this assumption is reasonable, this approach might create problems in estimating the model. In Justiniano, Lopez-Salido, and Michelacci (2010) we allow $\omega$ to be an job specific time varying idiosyncratic shock. This makes solution methods based on linearizing the equilibrium conditions of the model more appropriate. With stochastic rigid wages outside options always bind for at least some workers and some firms and shocks affect the fraction of workers and firms for which outside options are binding. Since fractions move continuously in response to shocks, solution methods based on linearization are appropriate. To simplify exposition, here we avoided pursuing this line of reasoning.}

**Aggregate resource constraint** The aggregate resource constraint is:

$$Y_t = I_t + C_t + D_t + L_t,$$

where $D_t$ is an aggregate demand exogenous component (say due to government expenditures or net trade balances) that we assume evolves as

$$d_t \equiv \ln D_t = (1 - \rho_d)d + \rho_d d_{t-1} + \varepsilon_{dt}. \quad (15)$$

Moreover we have that

$$L_t = (1 - \kappa) p_t^\kappa S_t \quad (16)$$

represent the total amount of output units spent for job creation purposes, which is obtained combining (13) and (8). Notice that

$$L^x_t = \kappa p_t^\kappa S_t, \quad (17)$$

denotes instead the total recruiting costs due to the purchase of labour intensive intermediate goods.
Representative household  The economy is populated by a continuum of identical infinitely-lived households of measure one. Each household is thought of as a large extended family which contains a continuum of workers. The population of workers in the economy is normalized to one and there are no movements in and out of the labor force. We follow, among others, Andolfatto (1996) and Den-Haan, Ramey, and Watson (2000) in assuming that workers pool their income at the end of the period and choose consumption and effort costs to maximize the sum of the expected utility of the household’s members; thus a representative household exists. Workers can be either employed or non-employed. The utility obtained by the representative household in a period is given by:

\[ U(C_t) - e_t \Psi + e_{t-1} e^{-\lambda t} O(\Lambda_t) \]

where \( e_t \) denotes the number of employed worker at the end of period \( t \), \( C_t \) denotes aggregate consumption, \( \Psi \) is a leisure cost of working, the last term accounts for job maintenance costs which are incurred by all jobs producing in the previous period. We assume that consumption utility is a standard constant relative risk aversion utility function so that

\[ U(C_t) = \frac{C_t^{1-\chi} - 1}{1 - \chi} \]

When \( \chi = 1 \) preferences are logarithmic, when \( \chi = 0 \) preferences are linear as in the textbook presentation of the search model in Pissarides (2000). The household’s discount factor is \( B_t \) where

\[ b_t \equiv \ln B_t = (1 - \rho_b) b + \rho_b b_{t-1} + \epsilon_{bt}, \]

so that as in Primiceri, Schaumburg, and Tambalotti (2006) and Justiniano, Primiceri, and Tambalotti (2010) we allow for shocks to the discount factor. Generally speaking, these shocks characterize the effects of changes in the supply of capital—due for example to changes in financial market conditions that affects households’ ability to save and invest. For example, notice that, when preferences are linear \( (\chi = 0), \) \( b_t \) represents an exogenous shocks to the rental price of capital services.

We assume that the claims on the profit streams of firms are traded. In equilibrium the household owns a diversified portfolio of all such claims, implying that the discount factor used by firms to discount future profits from time \( t+i \) to \( t \) is consistent with the household’s intertemporal decisions and so they share the same discount factor. The representative household maximizes the expected present value of its instantaneous utility (18), subject to the per period budget constraint:

\[ C_t + K_{t+1} = [1 - \delta (j_t)] K_t + [\theta \omega + (1 - \theta) u_l^b] e_t + r_t j_t K_t + (1 - e_t) z - \Upsilon_t \]
where $w^b_t$ is the average wage paid in a bargained wage job while $\Upsilon_t$ represents lump sump taxes, which satisfies

$$\Upsilon_t = D_t + (1 - e_t)z.$$  

This last follows from the fact that government budget is balanced. Notice that (20) incorporates the assumption that households accumulate capital, they decide capital utilization and thereby the capital services $j_tK_t$ to supply to firms.

**Timing** We adopt the following convention about the timing of events within a period $t$:

i. Aggregate shocks $\varepsilon_{at}$, $\varepsilon_{ct}$, $\varepsilon_{mt}$, $\varepsilon_{lt}$, $\varepsilon_{bt}$, and $\varepsilon_{dt}$ are realized;

ii. Investment in job maintenance;

iii. Old jobs realize whether they are destroyed (which occurs with probability $\Lambda_t$) and workers can search for a new job in the same period;

iv. Decisions about job creation are taken;

v. Old jobs and new jobs (resulting from matches at time $t$) produce output. Then income is pooled, invested and consumed. Next period begins.

4 Equilibrium conditions

We now characterize the equilibrium conditions of the decentralized economy.

**Consumption, investment and capital utilization** At every point in time the marginal value of wealth $\pi_t$ (i.e. the Lagrange multiplier of the representative household’s budget constraint) is equal to the marginal utility of consumption so that

$$\pi_t = \frac{1}{C_t^X}$$  

(21)

The Euler condition for the optimal choice of investment is

$$\pi_t = e^{\bar{r}t} \left[ 1 - T \left( \frac{I_t}{I_{t-1}} \right) - T' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] \Omega_t + B_tE_t \left[ e^{\bar{r}t+1} T' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \Omega_{t+1} \right],$$  

(22)
which establishes a marginal indifference condition between increasing consumption $C_t$ or increasing investment $I_t$. In the expression $\Omega_t$ is the time-$t$ expected shadow value.
Table 3: Legend (continued)

**Values**

<table>
<thead>
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<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$H$</td>
<td>value of searching</td>
</tr>
<tr>
<td>$U$</td>
<td>value of being unemployed at end of period</td>
</tr>
<tr>
<td>$V$</td>
<td>net (private) surplus of a job</td>
</tr>
<tr>
<td>$J$</td>
<td>value of a job net of the value of searching</td>
</tr>
<tr>
<td>$W^b$</td>
<td>value to the worker of a bargained wage job</td>
</tr>
<tr>
<td>$W^r$</td>
<td>value to the worker of a rigid wage job</td>
</tr>
<tr>
<td>$P^b$</td>
<td>value to the firm of a bargained wage job</td>
</tr>
<tr>
<td>$P^r$</td>
<td>value to the firm of a sticky wage job</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>marginal value of capital</td>
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<tr>
<td>$\pi$</td>
<td>marginal value of wealth</td>
</tr>
</tbody>
</table>

**Variables**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_t$</td>
<td>output</td>
</tr>
<tr>
<td>$K_t$</td>
<td>capital</td>
</tr>
<tr>
<td>$j_t$</td>
<td>capital utilization rate</td>
</tr>
<tr>
<td>$N_t$</td>
<td>labor intensive intermediate goods used to produce output</td>
</tr>
<tr>
<td>$I_t$</td>
<td>investment expenditures</td>
</tr>
<tr>
<td>$C_t$</td>
<td>consumption</td>
</tr>
<tr>
<td>$D_t$</td>
<td>exogenous aggregate demand component</td>
</tr>
<tr>
<td>$L_t$</td>
<td>total output units cost of job creation</td>
</tr>
<tr>
<td>$S_t$</td>
<td>input in the production of recruiting services</td>
</tr>
<tr>
<td>$n_t$</td>
<td>new jobs created</td>
</tr>
<tr>
<td>$s_t$</td>
<td>number workers searching</td>
</tr>
<tr>
<td>$u_t$</td>
<td>unemployment rate</td>
</tr>
<tr>
<td>$e_t$</td>
<td>number of workers producing</td>
</tr>
<tr>
<td>$f_t$</td>
<td>finding rate</td>
</tr>
<tr>
<td>$p_t$</td>
<td>marginal value of one labor intensive intermediate good</td>
</tr>
<tr>
<td>$w^b_t$</td>
<td>wages paid to worker in a bargained wage job</td>
</tr>
<tr>
<td>$r_t$</td>
<td>cost of one unit of recruiting services</td>
</tr>
<tr>
<td>$r_{vt}$</td>
<td>cost of processing applications for one vacancy</td>
</tr>
<tr>
<td>$r_{nt}$</td>
<td>cost of training one worker</td>
</tr>
<tr>
<td>$r_{kt}$</td>
<td>price of one unit of capital services</td>
</tr>
</tbody>
</table>
of capital at time $t+1$, which satisfies the following arbitrage condition:

$$
\Omega_t = B_t E_t \left\{ \left[ 1 - \delta(j_{t+1}) \right] \Omega_{t+1} + \pi_{t+1} r_{kt+1} j_{t+1} \right\},
$$

(23)

where

$$
r_{kt+1} \equiv \alpha A_{t+1} \left( \frac{N_{t+1}}{j_{t+1} K_{t+1}} \right)^{1-\alpha}
$$

is the equilibrium price of one unit of capital services at time $t+1$ equal to the marginal productivity of a capital service in the period.

The optimal choice of the intensity of capacity utilization will satisfy

$$
\pi_t r_{kt} = \delta'(j_t) \Omega_t
$$

(24)

where $\delta'(j_t)$ denotes the derivative of the function $\delta$ in (5) with respect to capital utilization. Equation (24) equates the marginal gains of increasing capital utilization to its marginal cost. The gain is the value of the increase in income. The cost is the value of the fall in capital of $\delta'(j_t)$ at time $t+1$.

**Value of a job** The value of searching for a job in period $t$ measured in utils is denoted by $H_t$ which solves the asset type equation

$$
H_t = f_t \left[ \theta W_t^b + (1 - \theta) W_t^r \right] + (1 - f_t) U_t
$$

(25)

where $W_t^i$, $i = b, r$ denotes the value to the worker of being employed in a job where wages are set through Nash bargaining $i = b$, or where wages are rigid $i = r$. The right hand side of (25) takes into account that with probability $1 - f_t$ the worker remains unemployed whose value is denoted by $U_t$ that solves

$$
U_t = \pi_t z + B_t E_t (H_{t+1}).
$$

(26)

This incorporates the fact that the unemployed worker cashes unemployment insurance benefits $z$ and he wait for next period before searching for a job. The time-$t$ (private) net value in utils of a job is defined as equal to $V_t \equiv P_t^i + W_t^i - U_t$. This is the net surplus that workers and firms have to split when bargaining over wages. This incorporates the assumption that bargaining takes place after the searching process has concluded, so unemployment is the relevant outside option for the worker. We prove below that $V_t$ satisfies

$$
V_t \equiv \pi_t p_t - \Psi - \pi_t z + B_t E_t \left[ (1 - \Lambda_{t+1}) J_{t+1} + e^{-\lambda_{t+1}} O(\Lambda_{t+1}) \right]
$$

(27)

where

$$
p_t = (1 - \alpha) A_t \left( \frac{j_t K_t}{N_t} \right)^{\alpha}
$$

(28)
is the equilibrium price of one unit of labor intensive intermediate goods while

\[ J_t \equiv P^i_t + W^i_t - H_t \equiv V_t + U_t - H_t \]  

(29)

is the value of a job net of the expected value of searching for a new job, which is equal to \( H_t \). The first three terms in (27) measure the net instantaneous gains of the job, equal to the difference between the value produced in the job and the sum of the effort cost of working and the benefits that the worker would obtain if unemployed. The last term in (27) is the future gains from producing today. These gains are net of the future investment in job maintenance and they are obtained only if the job is not destroyed.\(^{13}\)

Notice \( O \) is utility from leisure and so it enters positively in the expression. This is convenient to simplify notation.

**A derivation for the expression of the net surplus in (27)** We now derive from first principle the expression for the net surplus of a job in (27). Let denote by \( P_t \) the value of a job to the firm and by \( W_t \) the value of the job to the worker. Both expressed in utility terms. With this notation we have that

\[ V_t \equiv P_t + W_t - U_t \]

Let \( w_t \) denote a worker’s wage. Then

\[ P_t = \pi_t (p_t - w_t) + B_t E_t \left[ (1 - \Lambda_{t+1}) P_{t+1} \right] \]

The value of a job to the worker is equal to

\[ W_t = \pi_t w_t - \Psi + B_t E_t \left[ (1 - \Lambda_{t+1}) W_{t+1} + \Lambda_{t+1} H_{t+1} + e^{-\lambda_{t+1}/\mu} \cdot O(\Lambda_{t+1}) \right] \]

which incorporates the fact that the worker loses the job in the next period with probability \( \Lambda_{t+1} \). In that case the worker can search for another job in the same period, whose value is \( H_{t+1} \). By summing \( P_t \) to \( W_t \) and then subtracting the expression for \( U_t \) in (26) from the resulting expression we obtain that

\[ V_t \equiv \pi_t p_t - \Psi - \pi_t z - B_t E_t (H_{t+1}) \]

\[ + B_t E_t \left[ (1 - \Lambda_{t+1}) (J_{t+1} + W_{t+1}) + \Lambda_{t+1} H_{t+1} + e^{-\lambda_{t+1}/\mu} \cdot O(\Lambda_{t+1}) \right] \]

which is equal to

\[ V_t \equiv \pi_t p_t - \Psi - \pi_t z + B_t E_t \left[ (1 - \Lambda_{t+1}) (V_{t+1} + U_{t+1} - H_{t+1}) + e^{-\lambda_{t+1}/\mu} \cdot O(\Lambda_{t+1}) \right] \]

which is analogous to (27). \( \blacksquare \)

\(^{13}\)An alternative assumption would be to assume that \( V_t \equiv J_t + W_t - H_t \). This is equivalent to assuming that matching and bargaining occurs simultaneously. This would be a fiction given that newly created jobs are created after a match. Under this alternative assumption the Hosios condition would not be satisfied.
**Job destruction** The optimal level for job maintenance is set to equate the marginal cost of the investment in maintenance to its return—equal to the expected increase in value due to the marginal fall in the job separation probability, so

\[ e^{-\frac{\Lambda t}{\mu}} O'(\Lambda_t) = J_t. \] (30)

By inverting this expression we can obtain that

\[ \Lambda_t \equiv \Lambda \exp \left[ -\mu (\ln J_t - \ln J) - \lambda_t \right] \] (31)

where \( J_t \) is given in (29), \( J \) is its steady state value while \( \lambda_t \) characterizes a job specific shock to the separation rate. This last specification is parsimonious and captures key properties of model with endogenous job separation. When \( \mu = 0 \), the job separation rate is just driven by the exogenous shock \( \lambda_t \). When \( \mu > 0 \), investment in job maintenance is more valuable when the job net surplus is greater. As a result the job destruction rate falls. This negative relation between job destruction and the job net surplus is the key insight of any model where job separation rate is endogenously determined so as to maximize the job net surplus (see for example Mortensen and Pissarides, 1994). In the Supplementary Appendix we provide a functional form that leads exactly to (31) as an equilibrium outcome, but equation (31) generally hold up to a first order approximation where \( \mu \equiv \left| \frac{O'}{O''} \right| \). Notice that (30) implies that the relation between the net surplus and the separation rate is characterized by an envelope condition. As emphasized by Pissarides (2009) this implies that changes in the separation rate has nil first order effects on the expected net value of a newly created job. The model is parameterized just in terms of \( \Lambda \) and \( \mu \). Intuitively \( \Lambda \) determines the average steady state separation rate while \( \mu \) determines the elasticity of the job separation rate to the job net surplus.

**Splitting of the job net surplus** Let \( P^i_t, i = b, r \) denote the value to the firm of a job where wages are set through Nash bargaining \( i = b \), or where wages are rigid \( i = r \). Recall that \( W^i_t, i = b, r \) denotes the analogous value to the worker. Notice that for any \( i \) we have \( V_t = P^i_t + W^i_t - U_t \). We assume that Nash bargaining implies that

\[ W^b_t = \beta V_t + U_t \quad \text{and} \quad P^b_t = (1 - \beta) V_t \]

The value to the worker of a job with rigid wages satisfies

\[ W^r_t = V_t - P^r_t + U_t \]

while the value to a firm of a job with rigid wages satisfies

\[ P^r_t = \pi_t (p_t - \omega) + B_t E_t [(1 - \Lambda_{t+1}) P^r_{t+1}] \] (32)
To understand the expression notice that the first two terms calculate the value of the job net profits, the last term is simply the expected future value of the job to the firm. The job in the next period is destroyed with probability $\Lambda_{t+1}$.

**Free entry** Since vacancies are posted till the exhaustion of any rents, in equilibrium their value would be equal to zero so the following free entry condition will hold

$$\pi_t r_{vt} + q(f_t) \pi_t r_{nt} = q(f_t) \left[ \theta P^r_t + (1 - \theta) P^b_t \right]$$

where

$$f_t = \frac{n_t}{s_t}$$

is the job finding rate while $r_{vt}$ and $r_{nt}$ denote the cost of processing the applications for a vacancy and training a worker, respectively. This expression equates the expected cost of filling the vacancy (equal to the sum of the cost of processing application for the vacancies plus the cost for the worker training) to the expected net capital gains (the term in the right hand side). In equilibrium the cost of processing the applications is equal to its marginal cost, that given the functional form for $\tilde{R}$ is equal to

$$r_{vt} = r_t \frac{\partial \tilde{R}(n_t, v_t)}{\partial v_t} = r_t \gamma_v$$

while the marginal cost of training a worker is given by

$$r_{nt} = r_t \frac{\partial \tilde{R}(n_t, v_t)}{\partial n_t} = r_t \gamma_n$$

Since (6) implies that

$$v_t = (M_t s_t)^{-\frac{n_t}{n}} \frac{1}{n_t^{-\frac{n_t}{n}}}$$

we have that

$$q_t = \frac{n_t}{v_t} = \left( \frac{n_t}{M_t s_t} \right)^{-\frac{n_t}{n}}$$

which can be used to rewrite (33) as follows:

$$\pi_t r_{vt} \left[ \gamma_n + \gamma_v \left( \frac{n_t}{M_t s_t} \right)^{-\frac{n_t}{n}} \right] = \theta P^r_t + (1 - \theta) P^b_t.$$

This conditions simply says that jobs are created up to the point where the average cost of creating a new job is equal to its expected net value.
The market for recruiting services  The price of one recruiting services, denoted by \( r_t \), will satisfy the following intertemporal conditions:

\[
p^\kappa_t = r_t \left[ 1 - G \left( \frac{S_t}{S_{t-1}} \right) - G' \left( \frac{S_t}{S_{t-1}} \right) \frac{S_t}{S_{t-1}} \right] + B_t E_t \left[ G' \left( \frac{S_{t+1}}{S_t} \right) \left( \frac{S_{t+1}}{S_t} \right)^2 r_{t+1} \right]
\] (35)

Notice that when \( G = G' = 0 \), we obtain the familiar condition \( p^\kappa_t = r_t \). This condition equates the marginal costs of producing one unit of \( S \) to the marginal gains.\(^{14}\) The first term in the right hand side of (35) is the net increase in income due to the increase in the current period supply of recruiting services. The second term is simply the increase in income due to the increase in the supply of recruiting services of next period, which is due to the reduction of next period adjustment costs. The condition (35) is derived below.

In equilibrium the total demand of recruiting services has to be equal to its supply. So the following condition will hold:

\[
\left[ \gamma_n + \gamma_v \left( \frac{f_t}{M_t} \right)^{\frac{\alpha}{\alpha+n}} \right] n_t = S_t \left[ 1 - G \left( \frac{S_t}{S_{t-1}} \right) \right]
\] (36)

\(^{14}\)From using (13) and the definition of \( S_t \)—which is the input in the production of recruiting services—we obtain the amount of output expenditures in recruiting services can be expressed as follows:

\[
L_t = (1 - \kappa) p^\kappa_t S_t
\] (37)

The total amount of labor intensive intermediate goods produced is equal to \( e_t \). Given (12) and (13), the amount of labor intensive intermediate goods used for producing final output is then given by

\[
N_t = (1 - u_t) - \frac{\kappa L_t}{(1 - \kappa) p_t}
\] (38)

This again follows from using (12) and the definition of \( S_t \)

Unemployment and pool of searchers  There is an inflow to the stock of workers searching equal to \( \Lambda_{\ell} e_{\ell-1} \), and an outflow equal to \( f_{\ell-1} s_{\ell-1} \). Due to this the

---

\(^{14}\)Equation (35) can be easily obtained by solving the problem of the producers of recruiting services. They maximize profits given by

\[
\max_{S_t} \sum_{t=0}^{\infty} \left\{ B_t r_t S_t \left[ 1 - G \left( \frac{S_t}{S_{t-1}} \right) \right] - p^\kappa_t S_t \right\}
\]

where \( p^\kappa_t \) is the cost of producing one unit of \( S_t \). The sector producing recruiting services takes this price as given. By writing the first order condition with respect to \( S_t \), we immediately obtain (35).
stock of workers searching for a job evolves as

\[ s_t = (1 - f_{t-1}) s_{t-1} + \Lambda_t e_{t-1} \]  

(39)

Since the labor force is normalized to one, the number of employed workers who produce output at the end of period satisfies

\[ e_t + u_t = 1 \]  

(40)

where \( u_t \) denotes the unemployment rate, i.e. the number of worker who do not produce in the period. This is equal to

\[ u_t = (1 - f_t) s_t \]  

(41)

which evolves as

\[ u_t = u_{t-1} + \Lambda_t (1 - u_{t-1}) - f_t s_t \]  

(42)

This says that unemployment changes are equal to the difference between the inflow into and outflow from unemployment.\(^{15}\) Notice that this formulation implies that unemployment at time \( t \) is influenced by both the separation rate and the finding rate at time \( t \). The outflow rate from unemployment is equal to

\[ out_t = f_t \]  

(43)

the inflow rate to unemployment is

\[ int_t = \Lambda_t \]  

(44)

which is consistent with the definition in Elsby et al. (2010).

The full characterization of the equilibrium of the model, its steady state properties are in the Supplementary Appendix.

\(^{15}\)Notice that if we define the separation rate as equal to

\[ \tilde{\Lambda}_t = (1 - f_t) \Lambda_t, \]

we obtain a more canonical expression for the law of motion of unemployment Lagging the second and replacing in the first and then into the second again

\[
\begin{align*}
   u_t &= (1 - f_t)[u_{t-1} + \Lambda_t(1 - u_{t-1})] \\
&= (1 - f_t)u_{t-1} + (1 - f_t)\Lambda_t - (1 - f_t)\Lambda_t u_{t-1} \\
\end{align*}
\]

which simplifies to

\[ u_t = u_{t-1} - f_t u_{t-1} + \tilde{\Lambda}_t (1 - u_{t-1}) \]

which is a more conventional expression for the law of motion of unemployment. Under this definition steady state unemployment is equal to

\[ u = \frac{(1 - f)\tilde{\Lambda}}{(1 - f)\tilde{\Lambda} + \tilde{f}} = \frac{\tilde{\Lambda}}{\tilde{\Lambda} + \tilde{f}}. \]
4.1 Welfare

It is always useful to analyze the welfare properties of the model relative to the Hosios (1990) benchmark—often labeled as the $\beta = \eta$ condition. We prove in the Supplementary Appendix that

**Proposition 1** The decentralized equilibrium is socially efficient if the Hosios condition holds ($\beta = \eta$), there are no appropriability problems ($\gamma_n = 0$), wages are flexible ($\theta = 0$) and there are no unemployment benefits ($z = 0$).

The Proposition highlights that there are three possible sources of inefficiencies in the model. All them stem from frictions in the labour market and they have already been emphasized before. Inefficiencies arise because of either failure of the Hosios (1990) condition ($\beta$ is different from $\eta$); or because of appropriability problems in creation costs as in Caballero and Hammour (1996, 1998) ($\gamma_n$ or $\gamma_v$ is different from zero); or because of wage rigidity as in Shimer (2004) and Hall (2005) ($\theta$ is different from zero). Notice that these frictions could have different welfare costs depending of the type of shocks and their sign. When the Hosios condition is satisfied it is optimal to set benefits to zero, $z = 0$. Notice that if either $\gamma_n$ is positive or $\theta$ is positive it never exists a value of $\beta$ that makes the decentralized equilibrium efficient.

5 Model solution and state-space representation

This section briefly describes the solution of the model and its state space representation. Let

$$E_t [f (\zeta_{t+1}, \zeta_t, \zeta_{t-1}, \varepsilon_t, \Gamma)] = 0,$$  \hspace{1cm} (45)

denote the collection of equilibrium conditions from Section 4, where $\zeta_t$, $\varepsilon_t$ and $\Gamma$ are vectors of endogenous and exogenous variables, exogenous i.i.d. disturbances, and, unknown parameters, respectively.

For a given $\Gamma$, the first step is to find the non-stochastic steady state, which requires solving for the root of a function in $f$, the finding probability (see Supplementary Appendix for details). Having obtained the non-stochastic steady state we then log-linearize (45) around it and solve the resulting linear system of rational expectation equations using the Anderson and Moore algorithm (see Anderson (2008)). This procedure yields the following system of transition equations

$$\hat{\xi}_t = G (\Gamma) \hat{\xi}_{t-1} + M (\Gamma) \varepsilon_t,$$  \hspace{1cm} (46)
where the $\hat{\cdot}$ denotes log deviations from the steady state, while $G(\Gamma)$ and $M(\Gamma)$ are conformable matrices whose elements are functions of $\Gamma$.

The state space representation of the model solution has (46) as transition equation. The associated observation equation

$$y_t = Z\hat{\zeta}_t + R\eta_t$$

maps some of the elements in $\hat{\zeta}_t$ into a vector of observables, $y_t$, through the selection matrix $Z$. The vector $\eta_t$ represents idiosyncratic disturbances that do not enter the equilibrium conditions, and the matrix $R$ maps each of the i.i.d. elements in $\eta_t$ to a single observable series, with $R$ having more columns than rows. Idiosyncratic disturbances capture deviations between some observables and the corresponding variable in the model and are discussed at length in the next Section.

Given the data, $Y = [y_1, y_2, ..., y_T]$, the likelihood function associated with each parameter $\Gamma$ is obtained through the Kalman filter using the transition equation (46) and observation equation (47). This allows us not only to estimate model parameters but also to infer (through the Kalman smoother) the shocks, $[\varepsilon'_t, \eta'_t]$, buffeting our economy at each point in time. We later exploit this feature to decompose cyclical fluctuations across shocks and to conduct counterfactual experiments.

### 6 Bayesian Inference

We use Bayesian methods to characterize the posterior distribution of the structural parameters (see An and Schorfheide 2007 for a survey). The posterior distribution combines the prior with the likelihood function obtained with the Kalman filter. The likelihood is based on the following vector of observable variables:

$$[\ln u_t, \ln v_t, \ln GDP_t, \ln C_t, \ln I_t, \ln f^A_t, \ln \Lambda^A_t],$$

(48)
corresponding to quarterly ($t$) unemployment rate, vacancy rate, GDP, consumption and investment, as well as annual ($\tau$) averages of the quarterly finding and separation probabilities, respectively. GDP is defined as output net of recruiting costs, $GDP = Y - L$. The data are discussed in Section 2, which also details the country specific samples. Our guiding choices in eliciting priors warrant a thorough discussion, especially given the relatively short samples used in the estimation.

**Calibrated parameters** A few parameters that are difficult to pin down without level information are calibrated. In some cases we set the same value for all countries.
For instance, the quarterly depreciation rate of capital $\delta$ is set to 10 percent per year, 2.5 percent per quarter which is similar to Greenwood, Hercowitz, and Krusell (1997). The discount factor $B$ is fixed to .9901, delivering a steady state real interest rate of 4 percent, while the elasticity of output to capital, $\alpha$, is calibrated to 0.33. Finally, we impose a coefficient of relative risk aversion, $\chi$, equal to one, corresponding to log preferences. Other parameters are calibrated using country specific information. The replacement rate of unemployment benefits, is set to the value reported in Table 1 for each country. The steady state value of $\frac{D_{GDP}}{C+Y_{GDP}}$, which measures the share of GDP not accounted for by the sum of consumption and investment, is chosen using nominal national accounts data. This delivers percentage shares of 17% for the US, 24% in France, 21% Germany, 19% the United Kingdom, 27% Norway, and, 34% Sweden. Notice that this share is smallest in the US, although close to that in the UK.16. Finally, the steady state value of the separation probability, $\Lambda$, is calibrated to the sample means reported in Table 1.

For the remaining parameters, the prior is identical across countries and described in Table 4. The second column corresponds to the type of density, with "N" denoting Normal, "B" Beta, "G" Gamma, "U" Uniform, and "I" Inverse Gamma1. Beta distributions are reserved for variables defined over the [0,1] intervals, while G and I are only defined for the positive real line. For each distribution the third and fourth column report the prior mean and standard deviation, respectively. To help gauge the range of values entailed by each density, the last column reports 98 percent prior probability intervals.

**Matching elasticity, factor content of recruiting service, and effort cost**

The elasticity of the matching function with respect to unemployment, $\eta$, is centered around 0.6. This value accords well with the evidence summarized by Petrongolo and Pissarides (2001) who conclude that “a plausible range for the empirical elasticity of unemployment is 0.5 to 0.8 ”. Shimer (2005a) argues in favor of a higher value of $\eta = 0.72$, in line with the simple regression results reported in Table 1. There is scant evidence about the value of $\kappa$ that measures the weight of labor goods in creation costs, though Shimer (2010) argues that this parameter might matter for cyclical fluctuations. For this reason we specify a fairly flat Beta prior, with prior probability intervals that cover the unit interval. Our prior for the disutility of labor $\Psi$ is informed by the estimation of the model with higher frequency data for the US. In Justiniano, Lopez-

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16More specifically $\frac{D_{GDP}}{C+Y_{GDP}} = 1 - \frac{C}{C+Y_{GDP}}$, encompassing therefore government expenditures and net exports
### Table 4: Parameters Prior

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Mean</th>
<th>SD</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>B</td>
<td>0.5</td>
<td>0.25</td>
<td>0.03–0.97</td>
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<td>$\Psi$</td>
<td>G</td>
<td>0.4</td>
<td>0.1</td>
<td>0.21–0.56</td>
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<tr>
<td>$\kappa$</td>
<td>B</td>
<td>0.5</td>
<td>0.25</td>
<td>0.03–0.97</td>
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<td>$\gamma_n$</td>
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<td>144</td>
<td>5–495</td>
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<td>$\gamma_v$</td>
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<td>144</td>
<td>5–495</td>
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<td>1</td>
<td>2.05–6.69</td>
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<tr>
<td>$G''$</td>
<td>G</td>
<td>4</td>
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<td>2.05–6.69</td>
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<td>$\delta_2$</td>
<td>G</td>
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<td>$\infty$</td>
<td>0.53–11.59</td>
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<tr>
<td>$\sigma_b$</td>
<td>I</td>
<td>1</td>
<td>$\infty$</td>
<td>0.26–5.62</td>
</tr>
<tr>
<td>$\sigma_d$</td>
<td>I</td>
<td>1</td>
<td>$\infty$</td>
<td>0.26–5.62</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>I</td>
<td>1</td>
<td>$\infty$</td>
<td>0.26–5.62</td>
</tr>
<tr>
<td>$\sigma_\varphi$</td>
<td>I</td>
<td>2</td>
<td>$\infty$</td>
<td>1.05–22.5</td>
</tr>
<tr>
<td>SD idiosyn. shock in $f$, $\Lambda$ and $v$</td>
<td>B</td>
<td>0.3</td>
<td>0.2</td>
<td>0.01–0.82</td>
</tr>
</tbody>
</table>

**Steady states priors**

- $u \times 100$ | B | sample mean | .02 | -0.467–0.467 |
- $r \left( \frac{\gamma_n + \gamma_v \sqrt{2\pi}}{\pi} \right)$ | B | 0.15 | 0.1 | 0.01–0.47 |

**Notes:** Priors in estimation. First column, corresponds to the parameters names. Second column is the type of density. “N” is for Normal, “B” for Beta, “G” for Gamma, “U” for Uniform, and “I” for Inverse Gamma. Third column is the mean while the fourth is the standard deviation. The last column reports the prior percentage band with 98 percent probability coverage. For unemployment the band is reported as a difference with the sample mean. The replacement rate of unemployment benefits, the steady state value of the $D$ over $Y - L$ ratio and the separation rate $\Lambda$ are calibrated to their value in Table 1.

Salido, and Michelacci (2010), we estimate the model for the US using monthly and quarterly data which, given the larger sample, delivers more precise estimates.\(^{17}\) The estimate for $\Psi$ obtained in that sample under a uniform [0,50] prior, once converted \(^{17}\)We checked that the US results with the higher frequency data-set used in Justiniano, Lopez-Salido, and Michelacci (2010) and the low-frequency data set used here yield consistent results.
to a quarterly frequency, suggests a value for this parameter considerably smaller than 0.4. However, in order not to penalize specifications with low net (private) surplus for a job, we center our prior around this higher value. Furthermore, prior beliefs regarding $\Psi$ for other countries are grounded on the assumption that workers preferences are similar across countries, so that working effort costs also vary little across countries.

**Adjustment costs and job separation elasticity** For the parameter $T''$ that governs adjustment costs in capital, we follow Justiniano, Primiceri, and Tambalotti (2010) (and references therein) in centering the Gamma density at 4, albeit with a fairly large standard error. Absent any evidence to guide an alternative choice, we specify the same distribution for adjustment costs in recruiting services, $G''$. The elasticity of depreciation with respect to utilization is determined by the ratio $\frac{\delta_2}{\delta_1}$, where the denominator is pinned down by the steady state and it is equal across countries to 0.035. We elicit a prior for $\delta_2$ which implies an apriori median for $\frac{\delta_2}{\delta_1}$ of roughly 5.5, but with a standard deviation of 5.5 as well, thereby encompassing a wide range of values suggested in the literature, see Rios-Rull, Schorfheide, Fuentes-Albero, Santaeulalia-Llopis, and Kryshko (2011) for a discussion. Finally given the large debate on the importance of fluctuations in job separation probabilities over the business cycle we set a pretty loose prior for the parameter $\mu$ that governs the sensitivity of job separation to the job net surplus.

**Wage rigidity and bargaining power** We consider fairly uninformative priors for the importance of wage stickiness $\theta$, and workers’ bargaining power $\beta$. There is much debate in the literature about the value of these parameters. Hall (2005) argues in favor of $\theta = 1$, while Hagedorn and Manovskii (2008) are inclined toward $\theta = 0$. Similar considerations apply for the value of $\beta$ that characterize workers’ bargaining power. For this reason, we use a Beta-distribution with mean 0.5 and standard deviation 0.25, which is very close to a uniform distribution and, as mentioned above, covers the unit interval. We impose that in steady state wages are the the same in a rigid and a bargaining wage job. This implies that the two types of jobs differ *just* in their wage response to aggregate labor market fluctuations and formalizes the idea that wage rigidity does not matter for steady state allocations. From the estimation point of view, this parametrization reduces by one the set of parameters to be estimated.

**Steady state unemployment rate** Following the methodology outlined in DelNegro and Schorfheide (2008) we also impose priors on two relevant steady state quantities. First, we set a tight prior on (100 times) the steady state unemployment rate,
choosing a Normal with a mean informed by the sample average reported in Table 1 and standard deviation of 0.2. This approach helps shield against implausibly large steady state unemployment rates which might spuriously help the model to fit the data for reasons discussed in Cole and Rogerson (1999).

**Job creation costs** The second steady state prior concerns the costs of job creation, and requires a detailed explanation. As emphasized by Pissarides (2009), job creation costs, particularly if unrelated to labor market conditions, can significantly influence the transmission mechanisms of neutral shocks. We discipline this mechanism through a prior on the steady state costs of job creation, while adopting an agnostic Uniform $[0,500]$ for the parameters governing the contribution of vacancy costs, $\gamma_v$, and training costs, $\gamma_n$. More specifically, to restrict the value of recruiting costs we focus on the ratio

$$\vartheta = \frac{r \left( \gamma_n + \gamma_v f^{\frac{\eta}{1-\eta}} \right)}{p}$$

Roughly $\vartheta$ measures the cost of hiring a worker relative to the productivity of the worker in the job. Silva and Toledo (2011) calculate that hiring a worker requires 4.3 percent of the quarterly wage of a newly hired worker and that hiring also requires some training costs that amounts to 55 percent of quarterly wages. Under these values

$$\vartheta = \frac{(0.043 + 0.55)w}{p}$$

where $\frac{w}{p}$ is roughly the labor share multiplied by $1 - \alpha$. So we obtain that, when the labor share is equal to $1 - \alpha$,

$$\vartheta \sim (0.043 + 0.55)^2 \frac{2}{3} (1 - \alpha) \sim 0.593.$$  

We take this value of 0.593 as an upper bound for the set of reasonable values of $\vartheta$. For this reason, the prior for this parameter is centered at 0.15, with a standard deviation of 0.1. This has a 98 percent prior probability interval roughly covering $[0.01,0.47]$, making evident that this density favors values considerably smaller than those suggested by Silva and Toledo (2011). Hagedorn and Manovskii (2008) also add some costs from keeping capital idle which might justify increasing the variance for the prior.\(^{18}\)

**Shocks** All shocks are normalized to zero in logs (such that means are one in levels), except $B$ (discount factor), and $D$ (exogenous demand component), whose calibrated

\(^{18}\)The median is 0.12 while the 90 percent prior probability band covers instead $[0.03,0.34]$, making evident that this density favors values considerably smaller than those suggested by Silva and Toledo (2011). Hagedorn and Manovskii (2008) also add some costs from keeping capital idle which might justify increasing the variance for the prior.
levels have been already discussed. Regarding persistence, our prior Beta density for $\rho_a$ is centered at 0.85, suggesting that neutral technology shocks are highly autocorrelated, following the RBC literature. For all remaining shocks, $\rho_i$, $i = \varphi, \lambda, b, d, m, \ldots$, the prior mean is 0.6 with a standard deviation of 0.2, which allows a fairly broad degree of autocorrelation.

As it is customary in the empirical DSGE literature, Inverse Gamma 1 densities are preferred for the prior standard deviation of the innovation to the shocks. While we allow the means to differ somewhat across shocks, we select the degrees of freedom that parametrize this density to be equal to 2, which results in an infinite variance.

The estimation of model parameters through likelihood based methods is unfeasible when the number of shocks is smaller than the number of series in the dataset. In this case the model is stochastically singular, counterfactually predicting that some linear combination of the observables must hold exactly in the data. To break stochastic singularity we introduce idiosyncratic errors in vacancies, finding and separation probabilities. These shocks, given by $\eta_t$ in section 5, enter only the observation equation of each corresponding series, and are assumed serially uncorrelated and orthogonal to any other disturbance in the model. Therefore, idiosyncratic disturbances are unable to pick up the comovement in the data.

Aside from the above technical considerations, there are good reasons to allow for some idiosyncratic error in the measure of vacancies, job finding and job separation probabilities. For the case of vacancies, this idiosyncratic error may capture some well-known measurement issues with the Help Wanted Index. With regards to labor flows, idiosyncratic shocks may account for time aggregation issues. Furthermore, alternative measures of these probabilities tend to accord well in their cyclical components but can display notable differences in high frequency behavior. This is evident for France and the UK in comparing our measures of worker flows probabilities with those based on the data by Petrongolo and Pissarides (2008) and for the US when contrasting our time series with those in Shimer (2005b).\textsuperscript{19} Idiosyncratic shocks can also pick up model misspecification, due to movements in and out of the labor force that the model ignores.

To limit the influence of idiosyncratic disturbances, the volatility of idiosyncratic shock $i = v, \text{find, sep}$ in country $j$ is confined to the interval $[0, q_j^i]$, with $q_j^i$ equal to half of the standard deviation of series $i$ in that country. The estimated standard deviation of each idiosyncratic shock is equal to $\xi q_j^i$, where $\xi$ has a Beta prior centered at 0.3 with standard deviation 0.2. Despite this approach, the extent to which idiosyncratic

\textsuperscript{19}See the discussion in Supplementary Appendix.
disturbances may undermine our ability to learn about the cyclical behavior of the labor market should be judged by how important are these shocks over the business cycle, an issue analyzed extensively in the next section.

7 Estimation results

The presentation of our results begins with the estimated parameters in Section 7.1. Section 7.2 comments on the model’s ability to fit cross-correlations and volatilities. The role of technology shocks in generating fluctuation is taken up in Section 7.3. To this end, we use a historical decomposition of the observables as driven by only technology shocks (both neutral and investment specific) to show that their contribution to labor market fluctuations varies considerably across countries. Before remarking on other shocks, Section 7.4 briefly digresses on the role of wage rigidity in increasing the response of labor market variables to neutral technology shocks. The contribution of all shocks to business cycle variations across countries and series is presented in section 7.5. We conclude in section 7.6 with a cross-country comparison of transmission mechanisms, which helps probe why the importance of shocks differs across countries and series.

7.1 Parameters estimates

The posterior mode for the parameters in each country is reported in Table 5, other steady state quantities are reported in Table 6, while measures of uncertainty are omitted due to space considerations.\textsuperscript{20}

The model estimate for the degree of wage rigidity, $\theta$, is highest in the US at 0.57, suggesting that wages are more rigid than in Europe. This might be because European trade unions target their demand for wages in new jobs to aggregate labor market conditions. Conversely, $\beta$ in the US is estimated at 0.51, hinting that workers have greater bargaining power in Europe than in the US.

The elasticity of the matching function to unemployment $\eta$ is similar across countries and close to the value reported in table 1. In contrast, estimates for the elasticity of separations $\mu$ vary considerably across countries, from 0.44 in Sweden to almost 2 in Norway, with the US on the low side at 0.83. This is in line with the conclusions

\textsuperscript{20}When estimating structural models it is not uncommon to find alternative parameter configurations that provide similar characterizations of the data. To gauge whether identification issues manifest in multiple local modes, we maximize the posterior density using alternative optimization algorithms initialized with at least 50 random draws generated from a uniform grid, overdispersed relative to our prior. While this procedure is silent on the role of priors in achieving identification, it does reveal a unique parameter mode for each country.
## Table 5: Cross-country estimates, parameters values

<table>
<thead>
<tr>
<th>Country:</th>
<th>US</th>
<th>France</th>
<th>Germany</th>
<th>Norway</th>
<th>Sweden</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>β</td>
<td>0.51</td>
<td>0.76</td>
<td>0.57</td>
<td>0.63</td>
<td>0.59</td>
<td>0.80</td>
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<td>Ψ</td>
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<td>0.36</td>
<td>0.24</td>
<td>0.34</td>
<td>0.38</td>
</tr>
<tr>
<td>δ₀</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>δ₁</td>
<td>0.035</td>
<td>0.035</td>
<td>0.035</td>
<td>0.035</td>
<td>0.035</td>
<td>0.035</td>
</tr>
<tr>
<td>δ₂ × 100</td>
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<td>0.18</td>
<td>0.17</td>
<td>0.18</td>
<td>0.12</td>
<td>0.27</td>
</tr>
<tr>
<td>κ</td>
<td>0.90</td>
<td>0.78</td>
<td>0.78</td>
<td>0.61</td>
<td>0.71</td>
<td>0.69</td>
</tr>
<tr>
<td>γₙ</td>
<td>0.36</td>
<td>0.00</td>
<td>0.14</td>
<td>0.18</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>γᵥ</td>
<td>1.25</td>
<td>499.65</td>
<td>13.67</td>
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<td>10.21</td>
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<td>0.79</td>
<td>0.82</td>
<td>0.69</td>
<td>0.75</td>
<td>0.81</td>
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</tr>
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<td>0.58</td>
<td>0.63</td>
<td>0.53</td>
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<tr>
<td>θ</td>
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</tr>
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</tr>
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<td>μ</td>
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<tr>
<td>T''</td>
<td>3.62</td>
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<td>3.11</td>
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<tr>
<td>G''</td>
<td>3.28</td>
<td>3.51</td>
<td>3.22</td>
<td>3.76</td>
<td>3.20</td>
<td>3.49</td>
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<tr>
<td>ρₐ</td>
<td>0.95</td>
<td>0.96</td>
<td>0.94</td>
<td>0.93</td>
<td>0.96</td>
<td>0.95</td>
</tr>
<tr>
<td>ρ₇</td>
<td>0.64</td>
<td>0.70</td>
<td>0.60</td>
<td>0.11</td>
<td>0.52</td>
<td>0.71</td>
</tr>
<tr>
<td>ρ₈</td>
<td>0.64</td>
<td>0.69</td>
<td>0.60</td>
<td>0.11</td>
<td>0.52</td>
<td>0.70</td>
</tr>
<tr>
<td>ρ₉</td>
<td>0.68</td>
<td>0.68</td>
<td>0.79</td>
<td>0.32</td>
<td>0.38</td>
<td>0.73</td>
</tr>
<tr>
<td>ρₐ</td>
<td>0.78</td>
<td>0.52</td>
<td>0.61</td>
<td>0.62</td>
<td>0.72</td>
<td>0.70</td>
</tr>
<tr>
<td>ρₜ</td>
<td>0.48</td>
<td>0.60</td>
<td>0.25</td>
<td>0.13</td>
<td>0.22</td>
<td>0.25</td>
</tr>
<tr>
<td>σₐ</td>
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<td>0.23</td>
<td>0.31</td>
<td>0.42</td>
<td>0.34</td>
<td>0.27</td>
</tr>
<tr>
<td>σ₇</td>
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<td>6.15</td>
<td>14.64</td>
<td>12.89</td>
<td>9.90</td>
<td>4.31</td>
</tr>
<tr>
<td>σ₈</td>
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<td>0.31</td>
<td>0.64</td>
<td>3.16</td>
<td>1.22</td>
<td>0.37</td>
</tr>
<tr>
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<td>2.04</td>
<td>2.58</td>
<td>5.71</td>
<td>2.48</td>
<td>3.45</td>
</tr>
<tr>
<td>σₐ</td>
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<td>7.00</td>
<td>9.84</td>
<td>12.54</td>
<td>2.79</td>
<td>8.92</td>
</tr>
<tr>
<td>σ₉</td>
<td>3.69</td>
<td>2.65</td>
<td>6.90</td>
<td>13.17</td>
<td>9.53</td>
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<tr>
<td>SD idiosyn. shock, f</td>
<td>0.98</td>
<td>0.95</td>
<td>0.96</td>
<td>0.97</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>SD idiosyn. shock, λ</td>
<td>0.72</td>
<td>0.95</td>
<td>0.96</td>
<td>0.95</td>
<td>0.99</td>
<td>0.96</td>
</tr>
<tr>
<td>SD idiosyn. shock, v</td>
<td>0.08</td>
<td>0.90</td>
<td>0.08</td>
<td>0.51</td>
<td>0.57</td>
<td>0.59</td>
</tr>
</tbody>
</table>

Notes: Estimates. In all estimates χ is set to one. The replacement rate of unemployment in benefits is set to the value in Table 1. The parameter α and B are set equal to the 0.33 and 0.99, respectively. The value of the wage in bargained jobs in steady state is equal to ω.

by Elsby et al. (2010) that the separation margin matters more in Europe, see also Section 2. Recruiting relies mostly on labor, especially in the US where κ is highest at 0.9, consistent with the model specification in Shimer (2010).
According to our estimates there is a fair degree of adjustment costs in both capital and recruiting efforts in most countries. While the former friction is standard in empirical RBC models, we find that a similar mechanism is crucial to account for the high persistence of vacancies. As emphasized by Fujita and Ramey (2007) and Ravn and Simonelli (2008), this is a salient empirical feature of vacancies and labor market tightness. Posterior estimates for $\delta_2$ imply relatively large elasticities of capital utilization to variations in the return to capital in all countries. As explained later, this accords well with the importance of variable capital utilization for the propagation of intertemporal shocks, such as variations in the households discount factor and the investment specific technology (Greenwood, Hercowitz, and Huffman 1988).

Table 6 reports the steady state implied by these estimates. In each country the finding probability and unemployment rate are well in line with the sample means in Table 1.

The estimates of $\gamma_n$ in Table 5 tends to be higher in Germany than in other countries but training costs are in general small. Vacancies posting costs $\gamma_v$ vary substantially across countries. However, this masks significant differences in the magnitude of job creation costs as a share of GDP. For this reasons, Table 6 also reports the ratio between total job creation costs equal to

$$L^T = r \left( \gamma_n + \gamma_v f^{1-\eta} \right) n,$$

and aggregate GDP equal to $Y - L$. This ratio tends to be lower in Europe due to lower worker turnover.\(^{21}\)

To characterize the value of rents in existing jobs, Table 6 reports two statistics. The first is the flow value of \textit{ex-post} surplus in output units relative to productivity given by

$$PFS = \frac{p - \frac{\psi}{x} - z}{p} \quad (49)$$

The second is an analogous measure for the value of \textit{ex-ante} surplus in output units given by

$$AFS = \frac{p - \frac{\psi}{x} - z - \frac{r+\Lambda}{1+r} \gamma_n}{p} \quad (50)$$

\(^{21}\)To see this result more formally notice that $Y = \frac{p}{1-\alpha} N$ and $L = (1 - \kappa) L^T$. So

$$\frac{L^T}{GDP} \sim \frac{r \left( \gamma_n + \gamma_v f^{1-\eta} \right) \Lambda}{\frac{p}{1-\alpha} - r \left( \gamma_n + \gamma_v f^{1-\eta} \right) \Lambda} = \left[ \frac{p}{(1 - \alpha) r \left( \gamma_n + \gamma_v f^{1-\eta} \right) \Lambda} - 1 \right]^{-1}$$

which is increasing in $f$ and $\Lambda$ which are both larger in Europe. This explain why $\frac{L^T}{GDP}$ tends to be low in Europe even if $\gamma_n$ and $\gamma_v$ are higher.
The latter expression takes into account that training is a cost paid at the start of the employment relationship. This expense affects the incentive to create new jobs. The term \(\frac{r+\Lambda}{1+r}\gamma_n\) in (50) is simply the flow value equivalent of the ex ante cost in training.\(^{22}\)

The numbers for AFS and PFS reported in Table 6 indicate that ex-ante and ex-post surpluses are similar. This implies that training costs matter little for the economic responses to neutral technology shocks for the reasons discussed in Pissarides (2009). Our estimates for the values of the job net surplus seem to be in line with the values used by Shimer (2005a) in his calibration—although one has to be careful here because the choice for the time length of a period matters. As a point of reference the flow value of surplus is equal to 0.05 in the calibration of the search model in Hagedorn and Manovskii (2008) and equal to 0.6 in the calibration by Shimer (2005a).

\begin{table}[h]
\centering
\begin{tabular}{lcccccc}
\hline
\textbf{Country:} & US & France & Germany & Norway & Sweden & UK \\
\hline
Finding, \(f\) & 0.609 & 0.203 & 0.15 & 0.52 & 0.436 & 0.238 \\
Unemployment, \(u\) & 0.06 & 0.089 & 0.083 & 0.042 & 0.047 & 0.088 \\
Separation, \(\Lambda\) & 0.1 & 0.025 & 0.016 & 0.047 & 0.038 & 0.03 \\
Job creation, \(n\) & 0.094 & 0.023 & 0.015 & 0.045 & 0.036 & 0.027 \\
Labor for cons., \(N\) & 0.897 & 0.906 & 0.914 & 0.951 & 0.947 & 0.909 \\
\(C/GDP\) & 0.593 & 0.524 & 0.554 & 0.493 & 0.424 & 0.574 \\
\(I/GDP\) & 0.237 & 0.236 & 0.236 & 0.237 & 0.236 & 0.236 \\
\(L^T/GDP\) (\(\times 100\)) & 3.566 & 0.457 & 0.298 & 0.849 & 0.611 & 0.323 \\
Labor share & 0.66 & 0.67 & 0.67 & 0.67 & 0.67 & 0.67 \\
\(\vartheta = r(\gamma_n + \gamma_v f^{\frac{\alpha}{\pi}})/p\) & 0.50 & 0.27 & 0.28 & 0.27 & 0.24 & 0.16 \\
PFS & 0.63 & 0.38 & 0.29 & 0.40 & 0.33 & 0.42 \\
AFS & 0.61 & 0.38 & 0.28 & 0.38 & 0.33 & 0.42 \\
\hline
\end{tabular}
\caption{Cross-country estimates, means}
\end{table}

Notes: Steady state means corresponding to the parameter estimates in Table 5. \(L^T\) denotes the total costs for job creation (both for labor and output). GDP is equal to \((Y - L)\). PFS is equal to \((p - \frac{\pi}{\pi} - z)/p\) and corresponds to the ex-post flow value of the net surplus of a job. AFS is equal to \((p - \frac{\pi}{\pi} - z - \frac{r+\Lambda}{1+r}\gamma_n)/p\) and corresponds to the ex-ante flow value of the net surplus of a job.

\(^{22}\)The expression \(\frac{r+\Lambda}{1+r}\gamma_n\) comes from solving for the value of \(x\) that solves the equation

\[
\sum_{s=0}^{\infty} \left(\frac{1-A}{1+r}\right)^s x = \gamma_n
\]

This means that \(x\) is the flow value equivalent of the ex ante cost in training.

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Properties of shocks  Neutral technology shocks are highly persistent in line with the RBC literature. The volatility of its innovations is lowest in the US and highest in Norway and Sweden, consistent with the discrepancies in business cycle variability documented in Table 1. Nonetheless, as it will become evident shortly, cross-countries differences in the volatility of neutral shocks are rather small, at least relative to the variation in estimates for the standard deviation and persistence of other disturbances.

The properties of investment shocks, $\varphi$, vary considerably across countries. These shocks are most persistent in France, with $\rho_\varphi$ at 0.6, but much closer to white noise in the remaining European countries, particularly Norway where $\rho_\varphi$ is 0.13—in line with the very low correlation for GDP reported in Table 1. In general investment shocks are substantially more volatile in Europe than in the US, especially so in Norway. For Norway this last observation likely reflects the indirect effects of oil extraction, a notoriously capital intensive industry. As for the remaining European countries, the volatility of these shocks is somewhat out of line with the differences in standard deviations for investment documented in section 2.2.

Estimates of the volatility of job destruction shocks $\sigma_\lambda$ are also fairly different across countries, being five times larger in Norway than in the US, and even more so in Germany. The variability of innovations to matching shocks, $\sigma_m$ is lowest in the US and Sweden, and highest in Germany, Norway and the UK. These differences in estimates accord well with the high correlation between finding and separation probabilities observed for these countries, see Table 2. This is because, as emphasized by Hosios (1994) and further discussed in Section 7.6, disturbances to the matching technology have the distinctive feature of generating a positive comovement between finding and separation probabilities. Regarding persistence, the autocorrelation in separation and matching shocks are broadly in the 0.5-0.8 range, while Norway stands out again, with 0.11 estimates for $\rho_\lambda$ and $\rho_m$.

Idiosyncratic shocks are quite volatile, and more so in Europe, which largely reflects the larger standard deviations in labor market data compared to the US. The variance of idiosyncratic shocks is pretty large in all countries for both finding and separation probabilities and close to the upper bounds allowed, that is the estimated $\tau$ close to 1. The sole exception is the US, where the standard deviation of idiosyncratic disturbances to unemployment inflows is significantly smaller than the upper bound. This is an indirect indication that the model fits this series best in the US, as we latter corroborate. Idiosyncratic shocks to vacancies are generally small with the exception of France. This is mostly due to the low association between the vacancy rate and the unemployment rate and finding probability present in the French data (see Table 2),
which is a common theme throughout our discussion of results for France.

7.2 Model fit: correlogramm and volatilities

We now discuss the model’s ability to fit key moments of the data. We first focus on the comovement properties and then analyze how the model performs in matching the volatility of national accounts and labor market variables.

Figure 2 presents the actual and model-implied correlation pattern of the seven variables used to estimate the model for the US. Figure 2b-f in the Appendix present analogous graphs for the other five countries in the sample. The panel in row \(i\) and column \(j\) corresponds to the cross-correlation between series \(i\) and up to 3 lags, \(k = 0, 1, 2, 3\), of series \(j\). Given the mixed frequency of the data, one lag corresponds to one year for moments involving unemployment flow variables, and to one quarter for the remaining series. The moments in the data are given by the solid line. Dashed lines instead report 95 percent posterior probability bands obtained for each country using the estimated parameter mode reported in Table 5 to simulate 4000 artificial model replications of length equal to the data. The median of the implied posterior distribution of the model generated correlations is shown by the dotted line.

Since each figure reports a fairly large number of moments, we limit our discussion to fairly broad features of the model and the data, and we concentrate on the US. Overall, Figure 2a suggests that the model matches reasonably well the correlation pattern of all the seven observables in the US although, admittedly, some confidence intervals are large, which reflects both the relatively short sample period and the low sample frequency considered for some variables. When looking at labor market variables, the model is successful in reproducing the remarkably high serial correlation of vacancies, unemployment and job finding probabilities observed in the data. The cross-correlations between the unemployment rate, worker flows probabilities and vacancies generated by the model align reasonably well with their empirical counterparts. For instance, median model-based statistics for the separation probability are fairly close to the actual moments, particularly at lags of this variable. Similarly, the comovement properties between vacancies and unemployment generated by the model generally agree with the data, although for leads of vacancies the correlations are somewhat larger than those observed.
Figure 2: Cross-correlogram of the US model

Notes: Theoretical correlation of the US model in the data (solid line) and in the model (dotted line). Dashed line are 95 percent confidence interval.
Consistent with ample evidence from the RBC literature, the model captures the serial correlation properties of national accounts variables quite well. Overall, the empirical correlation between labor market and national accounts variables are also within the posterior probability bands, although the model predicts a somewhat stronger association between GDP and unemployment than in the data. Within the national accounts series, the model underpredicts the correlation between consumption and investment, mainly due to the substitution effect induced by investment specific technology shocks. The model fit of the correlation pattern for the other countries is somewhat similar and omitted to save space, see Figure 2b-f in the Appendix.

Table 7 characterizes the model’s ability to reproduce the standard deviation of the observables. Each column corresponds to a different country, each row to a different variable. As in the case of cross-correlations, we report the median and [5-95] percentiles of the model implied distribution of standard deviations, together with the same statistic in data—identical to the number reported in panel (B) of Table 1. Note that the cross-country variation in implied volatilities is consistent with the heterogeneity observed in the data. Model-based standard deviations of unemployment and vacancies also broadly agree with the data, being if, anything, slightly larger in the model than in the data, in Germany, and, for vacancies in Sweden.

Empirical and model generated standard deviations of finding and separation probabilities are in close agreement, except for the UK and Sweden where the model falls short of capturing the high volatility in separations. Focusing on national accounts data, model-based volatilities in GDP and investment are reasonably in line, albeit somewhat above, their empirical counterparts. In general the model overstates the volatility of consumption, particularly in the Scandinavian countries, owing in part to the comovement problem induced by investment specific technology shocks, alluded to earlier. In summary, with only a few exceptions, the model accounts reasonably well for the observed differences in volatility across countries.
Table 7: Cross-country estimates, volatilities

<table>
<thead>
<tr>
<th>Country:</th>
<th>US</th>
<th>France</th>
<th>Germany</th>
<th>Norway</th>
<th>Sweden</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unemployment rate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td>11.4</td>
<td>6.4</td>
<td>11.3</td>
<td>17.2</td>
<td>21.3</td>
<td>8.0</td>
</tr>
<tr>
<td>5-95 percentiles</td>
<td>7.5-17.6</td>
<td>4.0-10.1</td>
<td>6.2-19.8</td>
<td>12.5-24.3</td>
<td>13.4-35.0</td>
<td>5.0-12.7</td>
</tr>
<tr>
<td>Data</td>
<td>8.2</td>
<td>5.3</td>
<td>7.2</td>
<td>13.6</td>
<td>16.3</td>
<td>8.1</td>
</tr>
<tr>
<td><strong>Vacancy rate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td>15.0</td>
<td>9.7</td>
<td>18.1</td>
<td>17.6</td>
<td>31.7</td>
<td>12.5</td>
</tr>
<tr>
<td>5-95 percentiles</td>
<td>9.4-23.6</td>
<td>6.1-16.1</td>
<td>11.1-29.0</td>
<td>12.7-25.7</td>
<td>19.3-52.7</td>
<td>7.9-20.5</td>
</tr>
<tr>
<td>Data</td>
<td>13.5</td>
<td>6.2</td>
<td>13.6</td>
<td>16.0</td>
<td>22.0</td>
<td>11.5</td>
</tr>
<tr>
<td><strong>Finding probability</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td>4.4</td>
<td>5.7</td>
<td>10.3</td>
<td>9.6</td>
<td>9.1</td>
<td>9.2</td>
</tr>
<tr>
<td>5-95 percentiles</td>
<td>2.9-6.8</td>
<td>4.1-7.8</td>
<td>6.6-15.9</td>
<td>6.9-13.0</td>
<td>5.7-15.0</td>
<td>6.3-13.2</td>
</tr>
<tr>
<td>Data</td>
<td>4.8</td>
<td>5.9</td>
<td>9.6</td>
<td>10.4</td>
<td>9.5</td>
<td>9.1</td>
</tr>
<tr>
<td><strong>Separation probability</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median</td>
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<td>8.0</td>
<td>10.6</td>
<td>14.6</td>
<td>10.1</td>
<td>6.6</td>
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<tr>
<td>5-95 percentiles</td>
<td>2.9-3.9</td>
<td>7.0-9.3</td>
<td>7.5-14.2</td>
<td>11.0-18.8</td>
<td>8.0-19.0</td>
<td>4.7-8.9</td>
</tr>
<tr>
<td>Data</td>
<td>2.5</td>
<td>8.0</td>
<td>10.3</td>
<td>14.7</td>
<td>21.5</td>
<td>9.0</td>
</tr>
<tr>
<td><strong>GDP</strong></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Median</td>
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<td>1.2</td>
<td>1.7</td>
<td>2.1</td>
<td>2.0</td>
<td>1.4</td>
</tr>
<tr>
<td>5-95 percentiles</td>
<td>0.8-1.9</td>
<td>0.7-2.0</td>
<td>1.0-3.0</td>
<td>1.6-2.9</td>
<td>1.3-3.2</td>
<td>0.9-2.3</td>
</tr>
<tr>
<td>Data</td>
<td>0.9</td>
<td>0.9</td>
<td>1.4</td>
<td>1.7</td>
<td>1.5</td>
<td>1.2</td>
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<tr>
<td><strong>Consumption</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td>1.2</td>
<td>1.2</td>
<td>1.7</td>
<td>2.8</td>
<td>2.6</td>
<td>1.6</td>
</tr>
<tr>
<td>5-95 percentiles</td>
<td>1.0-1.6</td>
<td>0.9-1.7</td>
<td>1.3-2.3</td>
<td>2.4-3.4</td>
<td>2.1-3.4</td>
<td>1.2-2.1</td>
</tr>
<tr>
<td>Data</td>
<td>0.7</td>
<td>0.7</td>
<td>1.4</td>
<td>1.7</td>
<td>1.6</td>
<td>1.5</td>
</tr>
<tr>
<td><strong>Investment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td>5.0</td>
<td>4.3</td>
<td>6.5</td>
<td>8.1</td>
<td>7.2</td>
<td>6.0</td>
</tr>
<tr>
<td>5-95 percentiles</td>
<td>3.4-7.4</td>
<td>2.6-7.1</td>
<td>3.9-10.8</td>
<td>5.9-11.3</td>
<td>4.7-11.3</td>
<td>4.0-9.1</td>
</tr>
<tr>
<td>Data</td>
<td>3.7</td>
<td>2.6</td>
<td>4.4</td>
<td>6.5</td>
<td>5.2</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Notes: For each variable and country first row is standard deviation of variable in logs in the model, second row is the 5-95 percentiles of the posterior distribution in the model, and, third row is data. For finding and separation probability they correspond to annual data, for the other variable they are at the quarterly frequency.
7.3 Technology component in historical fluctuations

We now show that there are important cross-country differences in the ability of technology shocks to explain cyclical fluctuations in labor market variables. To isolate the role of technology shocks alone in inducing volatility we perform a historical decomposition of the observables. This is possible thanks to the linearity of the model’s state space representation, which allows decomposing the observed time series of each variable as the sum of the components due to each shock. In this way, the historical paths of the observables that would have been obtained in response only to technology shocks (either neutral or investment specific) is shown as the solid lines in Figure 3. The actual data are represented by the dashed line. Panels a-f correspond to a different country. In each panel we consider all observables with the exception of consumption, due to space considerations.

The bottom row in each panel makes evident that in all countries the technology component accounts for the bulk of the cyclical variation in GDP and investment. In contrast, important cross-country differences are visible when focusing on labor market series. In the US (see panel a), technology shocks capture remarkably well the historical evolution of unemployment, vacancies and finding probability—although they slightly under-predict the fall in the finding probability during the recession of the early 1990s. This success in the US is more modest for separation probabilities. In Europe, technology shocks explain virtually all of the cyclical fluctuations of unemployment and vacancies in Scandinavian countries (panels d and e) and to a lesser extent in Germany (panel c). But the technology component alone fails to match the magnitude of unemployment fluctuations in France (panel b) and the UK (panel f), particularly so during downturns. Technology shocks track the contours of finding probabilities in Germany and Sweden, with fluctuations being considerably smaller that in the data for Norway, the UK and particularly France. In Europe the technology component also fails to reproduce an important part of the fluctuations of the job separation rate, especially when considering the two Scandinavian countries. This last shortcoming seems substantially more severe in all European countries than in the US.

\[23\] More specifically, the Kalman smoother is used to infer the unique sequence of shocks, \([\xi_t, \eta_t]\)
(in the notation in section 5), which reproduce the observables. One can then feed subsets of these shocks, e.g. neutral and investment specific, through the model to obtain the components for each series driven by those disturbances only.
Figure 3: Technology component due to technology shocks, Cross-country comparison

Notes: Historical decomposition. Solid line is the technology component (due to neutral and investment specific technology shocks), dashed line is the data.
7.4 Wage rigidity and the transmission of neutral shocks

Historical decompositions indicate that technology shocks alone are able to reproduce the cyclical fluctuations in unemployment, vacancies and finding probabilities in the US, and, amongst European countries most prominently Sweden, followed by Germany. Estimates of wage rigidity are highest for these two countries, which hints that in our empirical model wage rigidity plays an important role in solving the so called Shimer puzzle, in line with the conclusions by Hall (2005), Shimer (2005a) and Shimer (2010). Although the degree of wage rigidity seems well identified from our data, this observation is certainly limited by the absence of compensation measures to estimate the model. Bearing this caveat in mind, we perform a simple counterfactual exercise to distill the contribution of wage rigidity in propagating the effects of neutral technology in the US. Using the same methodology underlying our historical decompositions, we condition on the estimated history of neutral shocks only and ask how the time series of our observables would change if the degree of wage rigidity $\theta$ was half of its estimated value. All other parameters are left unchanged. Notice that the change in $\theta$ leaves unaffected all steady state quantities. Comparing the historical and the counterfactual resulting series driven by only neutral technology shocks we find that the cyclical properties of all series remain qualitatively unchanged, but volatility declines considerably when there is less wage rigidity.

This can be seen clearly from Table 8 that reports the standard deviations with low wage rigidity, $\theta = 0.28$, estimated wage

Table 8: Changing the level of wage rigidity in the US

<table>
<thead>
<tr>
<th>US Standard Deviations</th>
<th>$\theta = 0.56$</th>
<th>$\theta = 0.28$</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment</td>
<td>8.23</td>
<td>4.89</td>
<td>8.23</td>
</tr>
<tr>
<td>Vacancy</td>
<td>12.32</td>
<td>6.98</td>
<td>13.45</td>
</tr>
<tr>
<td>Finding Probability</td>
<td>3.18</td>
<td>1.85</td>
<td>4.85</td>
</tr>
<tr>
<td>Separation Probability</td>
<td>0.89</td>
<td>0.60</td>
<td>2.80</td>
</tr>
<tr>
<td>GDP</td>
<td>0.87</td>
<td>0.76</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Notes: Standard deviation of counterfactual paths when only US neutral technology shocks are fed through the model at the estimated mode, $\theta = 0.56$, and when $\theta = 0.28$. All remaining parameters are unchanged.

---

24 A more thorough analysis of transmission mechanisms and identification is provided in Justiniano, Lopez-Salido, and Michelacci (2010) using a richer model and dataset in the US.

25 A comparison of the historical path relative to that in figure 3 is available upon request and omitted due to space considerations.
rigidity $\theta = 0.56$ and in the data. With $\theta = 0.28$ the volatility of unemployment, vacancies and finding probability would be roughly $3/5$ of the value obtained under the estimated value of $\theta$.

7.5 Contribution of shocks to business cycles

The conclusions from the visual inspection of Figure 3 are confirmed by performing a formal decomposition of the business cycle variance of the observables. Figures 4 reports the contribution of each shock to the variance of observables in each country at business cycle frequencies, which correspond to cycles between 6 and 32 quarters, as in Stock and Watson (1999). We compute the spectral density implied by the state space representation of the DSGE model and decompose the variation of its diagonal elements within the frequency band associated with business cycle fluctuations. In this case we report decompositions at the quarterly horizon for all series.

To save space we focus on unemployment in panel (a), vacancy in panel (b), the flow probabilities in panel (c) and (d), GDP in panel (e) and investment in panel (f). In each panel a country corresponds to a different bar. A more detailed variance decomposition is reported in Table 9 in the Appendix. Panel (e) shows that neutral technology shocks account for roughly $1/2$ of the business cycle variability in GDP across all countries. Investment specific shocks explain between 20 and 30 percent of cyclical movements in GDP, with shares being somewhat higher in the US and Norway. In general matching and job destruction shocks have a limited role in GDP and investment fluctuations, although their combined contribution is relatively larger in Great Britain and France. Decompositions for consumption (omitted due to space considerations) also reveal a high degree of homogeneity in the driving forces of national accounts data. While neutral shocks are important for both investment and consumption in all countries, investment specific shocks explain about half of investment variability, and discount factor shocks account for $1/3$ of consumption fluctuations.

Turning to unemployment (panel a), it is clear that there is substantially more heterogeneity in its cyclical drivers, compared to national accounts data, in line with the evidence from Figure 3. Neutral shocks capture about $2/3$ of cyclical fluctuations in unemployment in the US and Sweden, roughly 40 percent in Germany and Norway, but less than 20 percent in France and Great Britain. Conversely, matching shocks are far more important in these last two countries—contributing almost half of the cycle in unemployment. Furthermore, almost 50 percent of the cyclical variance in unemployment is due to job destruction shocks in France, with smaller shares elsewhere, especially the US.
Figure 4: Variance covariance decomposition at business cycle frequencies, cross-country comparison

(a) Unemployment

(b) Vacancy

(c) Finding probability

(d) Separation probability

Notes: Percentage of variance explained by each shock in each country at business cycle frequency (cycles from 6 to 32 quarters). Decomposition are based on spectral density after correcting for aliasing.
Cross-country differences in the sources of labor market fluctuations are also evident in the job finding probability (panel c). As in the case of unemployment, neutral shocks contribute fifty percent or more to the business cycle of the job finding probability in the US, but play a far more muted role in Norway, Great Britain and France. On the other hand, matching shocks account for the bulk of the cyclical variability in the finding probability in France and Great Britain, yet less than 20 percent in the US. Although idiosyncratic shocks are quite volatile, their explanatory power over the business cycle is limited to roughly 10 percent.

Regarding the separation probability, its cyclical variation is driven mainly by job destruction shocks in all countries, with a couple of notable exceptions (panel d). In Norway, matching shocks are the dominant source of its fluctuation. Meanwhile, idiosyncratic shocks are superfluous everywhere except Sweden, where their contribution reaches 30 percent.

In summary, focusing on the labor market, we find that neutral shocks explain
the bulk of cyclical variations in unemployment, vacancies and finding probability in
the US, Sweden and to lesser extent Germany. Broadly speaking matching and job
destruction shocks are less important for unemployment and finding probabilities in the
US and Sweden than in Norway, the UK and France, with Germany being somewhere
in between. Finally, the contribution of discount factor and aggregate demand shocks
to labor market variables is fairly limited.

7.6 Cross-country comparisons in transmission mechanism

We now compare impulse responses to shocks across countries. We organize the pre-
sentation by shock, with aggregate demand and discount factor shocks omitted given
their small contribution to business cycle variability, particularly in the labor market.
We focus on the same variables as in Figures 3 and 4. The discussion is useful to
clarify the comovement properties induced by each shock and to better understand the
source of identification in the data. With full information methods it is not possible
to claim with certainty that specific correlations are responsible for identification. Yet
comparing the correlations generated by each shock with those in the data can help to
understand why a particular disturbance plays a more important role in one country
than in another.

Figure 5a reports the impulse response to a unitary neutral technology shock, $\epsilon_a$.
Different lines in each box corresponds to a different country. Qualitatively, impulse
responses are very similar across countries. Neutral technology shocks work exactly
as in the canonical textbook search model discussed in Pissarides (2000) leading to
an increase in finding rates and a decrease in the separation rate, expanding output
while inducing unemployment and vacancies to move in opposite direction. As shown
in Table 1, large correlations with these signs—positive between finding and vacancies,
negative for both with unemployment—are found in the US, Sweden and to lesser extent
Germany, which helps explain why in these countries neutral shocks are prominent
contributors to cyclical labor market fluctuations. The comovements between vacancies
and both finding and the unemployment rate is instead substantially weaker in France,
where neutral shocks play a limited role in explaining the last two series.

Figure 5a also suggests some important differences across countries in the quanti-
tative effects of neutral technology shocks on labor market variables. These elasticities
are in turn largest in Sweden (the country with the largest fluctuations in labor mar-
et variables in the data) followed by the US. In contrast, cross-country differences
in the response of GDP, investment and consumption (not shown) are comparatively
smaller. This said, caution is required when comparing the magnitude of impulse re-
responses as they represent log deviations from (sometimes very) different steady states (particularly when considering labor market variables). While this explains part of the variation in magnitudes, differences are also evident across countries with similar levels of unemployment and labor flows. For instance, the elasticities of unemployment, finding and vacancy rates are lowest for France, even compared to countries (such as the UK and Germany) with similar unemployment rate and worker flow probabilities.

The responses to a unitary Marginal Efficiency of Investment shock, $\epsilon_\varphi$, are shown in Figure 5b. MEI shocks are qualitatively similar to neutral shocks, but with stronger impact effects on investment and a negative co-movement between consumption and investment—due to well-known substitution effects. This last observation suggests that allowing for non separable preferences may substantially help the transmission of MEI shocks, as suggested by Eusepi and Preston (2009). Finally, the propagation of MEI shocks relies partly on strong responses in capital utilization, thereby rationalizing the large utilization elasticities estimated in all countries.

Figure 5c presents the responses to a unitary matching shock, $\epsilon_m$. These disturbances have the distinctive property of generating a positive co-movement in finding and separation probabilities. This helps explain the importance of matching shocks in Norway, where the correlation between these two series is large and positive (Table 1). When the separation rate is endogenous, the comovement between unemployment and vacancies induced by matching shocks may be positive or negative, as noted by Hosios (1994). This can be seen by comparing the impulse responses in the US and Germany, where vacancies fall, with those in France and Great Britain in which unemployment and vacancies move in the same direction. These considerations suggest first that it is difficult to infer the importance of matching shocks by just looking at the properties of the Beveridge curve and, second, that having worker flow probabilities in the set of observables is important for identification. In general, matching shocks generate large movements in finding probabilities loosely correlated with the level of vacancies. Since the correlation between finding probability and vacancies is particularly low in France, Norway and the UK, this helps explaining why in these countries matching shocks account for a large share of the fluctuations in the job finding rate and unemployment.

Finally, Figure 5e traces out the effects of a unitary impulse in job destruction shocks, $\epsilon_\lambda$. These disturbances drive any fluctuations in job separation probabilities that are not due to fluctuations in the net value of a job. On impact, they tend to generate a strong positive co-movement between the separation probability and unemployment. The empirical correlation between these two series is relatively higher in Germany and France, which helps partly explain why job destruction shocks are fairly
important for these countries.

Figure 5: Cross country comparison in impulse responses to a neutral technology shock

Notes: Impulse response to a unitary shock in $\epsilon_a$ in all countries.
Figure 5b (continued): Impulse responses to an MEI shock

Notes: Impulse response to a unitary shock in $\epsilon$ in all countries.
Figure 5c (continued): Impulse responses to an matching shock

Notes: Impulse response to a unitary shock in $\epsilon_m$ in all countries.
Figure 5d (continued): Impulse responses to a job destruction shock

Notes: Impulse response to a unitary shock in $\epsilon_\lambda$ in all countries.
8 Conclusions

In this paper we have set up a real business cycle model with search and matching frictions driven by several shocks. The model nests full Nash Bargaining and wage rigidity as special cases and includes other transmission mechanisms suggested by the literature for the propagation and amplification of disturbances. The model is estimated using full information methods, allowing for mixed frequency data, to study the properties of unemployment, vacancies, unemployment flows, output, consumption and investment in six different OECD countries. We have focused on the ability of technology shocks to generate cyclical fluctuations in line with the data. Our main finding is that while technology shocks are the key driving force of national accounts variables in all countries, their contribution to labor market fluctuations vary substantially both between the two sides of the Atlantic and across countries within Europe. Technology shocks alone replicate remarkably well the volatility in vacancies, unemployment and finding probabilities observed in the US. But their success is mixed in Europe where matching shocks and job destruction shocks play a substantially more important role than in the US.

Our analysis should be extended along several dimensions. For instance, the presentation here abstracts from an in-depth analysis of the merits and shortcomings of competing transmission mechanisms and their role in shaping differences across countries. Relatedly, it would be interesting to understand why matching and job destruction shocks matter more in Europe than in the US. Looking at the effects of different labor market institutions on the cyclical behavior of the labor market seems to be an obvious first step in trying to answer these questions.

The theoretical framework could also be expanded to model flows in and out of the labor force whose importance over the business cycle is likely to vary by country. In addition, specifying more general preferences that allow for consumption hours complementarity might matter for results, as emphasized in Shimer (2010), particularly considering some of the model’s difficulties in matching the comovement in consumption. One might also want to incorporate an intensive margin of labor supply, to increase the menu of possible relevant shocks, or, to allow for a richer structure in the way information about shocks gets revealed to agents in the economy—as in the case of “news” shocks (Schmitt-Grohe and Uribe 2008). Finally, although our cross-country analysis has been feasible thanks to the availability of annual comparable data on workers flows, it might be useful to specify the model at the monthly, instead of quarterly frequency. This modification will inherently limit the cross-country dimension of
the analysis but is particularly relevant for the US, where transitions occur at very high frequency.

In our view, the most compelling priority in expanding the model is to incorporate direct measures of wages into the analysis and to better characterize the cyclical properties of the labor share. After all this is one of the key insights provided by Hagedorn and Manovskii (2008) and Hornstein, Krusell, and Violante (2005). Of course in explaining the cyclical properties of the labor share one should build on Haefke, Sonntag, and Van-Rens (2007) and Pissarides (2009) to recognize that wages in new and ongoing jobs have very different implications for the allocation of resources in search models. For example, and for given workers’ bargaining power, the allocation of resources is unaffected by whether bargaining occurs continuously over time or just at the start of the employment relationship by setting wages through long-term wage contracts. However, different ways of splitting the surplus over the life of a match can matter for the cyclical properties of the labor share. Moreover, the labor share exhibits rich cyclical dynamics that, while informative, may prove challenging to model. In the US, for instance, its correlation is negative with contemporaneous output, but positive with output lagged three quarters or more—what Ríos-Rull and Santealalia-Llopís (2010) refer to as the overshooting property of the labor share. The former correlation suggests that some wage rigidity is present in the US data, while the latter correlation indicates that wages do respond to aggregate shocks, albeit with a lag. We are currently working an a monthly model of wage determination to account for the cyclical properties of the labor share in the US, which also incorporates some of the model extensions discussed above.
References


Figure 2b (continued): Cross-correlogram of the model in France

Notes: Theoretical correlation of the model in France. The solid line corresponds to the data the dotted line to the model. Dashed line are 95 percent confidence interval
Figure 2c (continued): Cross-correlogram of the model in Germany

Notes: Theoretical correlation of the model in Germany. The solid line corresponds to the data the dotted line to the model. Dashed line are 95 percent confidence interval
Figure 2d (continued): Cross-correlogram of the model in Norway

Notes: Theoretical correlation of the model in Norway. The solid line corresponds to the data the dotted line to the model. Dashed line are 95 percent confidence interval.
Figure 2e (continued): Cross-correlogram of the model in Sweden

Note: Data (solid), model median (solid dotted) and [5,95] posterior bands (dashed)

Notes: Theoretical correlation of the model in Sweden. The solid line corresponds to the data, the dotted line to the model. Dashed line are 95 percent confidence interval.
Figure 2f (continued): Cross-correlogram of the model in the UK

Notes: Theoretical correlation of the model in UK. The solid line corresponds to the data, the dotted line to the model. Dashed line are 95 percent confidence interval.
Table 9: Variance covariance decomposition at business cycle frequencies, cross-country comparison

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Notes: Percentage of variance explained by each shock at periodicity 6-32 quarters. Decompositions controls for aliasing.
Table 9: Variance Covariance Decomposition (continued)

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