Optimal Unemployment Insurance over the Business Cycle

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Abstract

This paper characterizes optimal unemployment insurance over the business cycle in a model in which unemployment stems from matching frictions (in booms) and job rationing (in recessions). Job rationing introduces two effects not captured by previous studies. First, job-search efforts have little effect on aggregate unemployment because the number of jobs available is limited, independently of search and matching. Second, while job-search efforts increase the individual probability of finding a job, they create a negative externality by reducing other jobseekers' probability of finding one of the few available jobs. Both effects are captured by the positive and countercyclical wedge between micro-elasticity and macro-elasticity of unemployment with respect to net reward from work. We derive a simple optimal unemployment insurance formula expressed in terms of those two elasticities and risk aversion. The formula coincides with the classical Baily-Chetty formula only when unemployment is low, and macro- and micro-elasticity are (almost) equal. The formula implies that the generosity of unemployment insurance should be countercyclical. We illustrate this result by simulating optimal unemployment insurance over the business cycle in a dynamic stochastic general equilibrium model calibrated with US data.

KEYWORDS: business cycle, unemployment insurance, job rationing.

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1 Introduction

Unemployment insurance (UI) is a key component of social insurance in modern economies, and whether to increase or decrease the generosity of UI during recessions is a critical and controversial public policy question. On the one hand, generous unemployment benefits could discourage job search during recessions and worsen unemployment. On the other hand, high unemployment during recessions does not seem due to a lack of job-search effort but rather to a scarcity of jobs.

To characterize optimal UI over the business cycle, we use the search-and-matching model of Michaillat (2010) in which jobs are endogenously rationed in recessions. In this model, the combination of real wage rigidity and diminishing marginal returns to labor generates job rationing in an economic equilibrium as well as realistic employment fluctuations over the business cycle. Effectively, unemployment therefore stems from two sources: matching frictions in booms and job rationing in recessions. We extend the model of Michaillat (2010) by allowing unemployed workers to choose their job-search efforts, and by introducing UI. We do not allow for deficit spending over the business cycle, so a labor tax finances unemployment benefits period by period.

The textbook model of optimal UI focuses on the trade-off between the insurance value of unemployment benefits and the cost of benefits from reduced job-search effort (Baily 1978; Chetty 2006a).[1] Job rationing introduces two effects that are not captured by the textbook model. First, because unemployment in recessions is primarily due to a lack of jobs, more generous UI has less impact on unemployment in recessions. Hence, the cost of UI benefits is less in recession. Second, job seekers exert a negative externality on other job seekers that becomes important in recessions when few jobs are available. Therefore, as UI benefits discourage job seeking, the value of UI benefits is higher in recession. Therefore, in recessions, the cost of UI from higher unemployment decreases and the value of UI from correcting the job-rationing externality increases. The insurance value of UI from consumption smoothing remains constant over the cycle. Hence, our central result is that optimal UI is more generous in recessions than in expansions.

We begin the analysis with a one-period general-equilibrium model, whose equilibrium matches

[1] Recent studies have considered the issue of optimal UI over the business cycle (for example, Andersen and Svarer 2010, 2011; Kiley 2003; Kroft and Notowidigdo 2010; Moyen and Stahler 2009; Sanchez 2008) but none of those studies capture job rationing. Section 4 discusses in detail how those studies relate and complement our analysis.

the steady state of the dynamic model introduced later. We study the equilibrium of this static model analytically, and represent it diagrammatically in a labor supply-labor demand framework. We characterize optimal unemployment benefits and tax rates across equilibria parameterized by different levels of technology. Our wage-rigidity assumption implies that when technology is high, wages are relatively low, which drives unemployment down ("an expansion"). Conversely, when technology is low, wages are relatively high, which drives unemployment up ("a recession"). We derive a simple optimal UI formula expressed in terms of sufficient statistics that can be empirically estimated: risk aversion, and the micro- and macro-elasticity of unemployment with respect to net reward from work. Micro-elasticity is the elasticity of the probability of unemployment for a single worker whose individual benefits change. Macro-elasticity is the elasticity of aggregate unemployment when UI changes and labor market tightness adjusts. We obtain a formula in terms of these statistics because the macro-elasticity captures the increase in aggregate unemployment caused by UI through lower search effort, while the correction needed for the job-rationing externality is measured by the wedge between micro-elasticity and macro-elasticity. As our formula is expressed in terms of sufficient statistics, it is robust to changes in the primitives of the model.[2]

While UI is acyclical in the standard Baily-Chetty framework, we find that optimal UI is countercyclical, and always higher than in the standard framework. In periods of low unemployment, macro- and micro-elasticity are almost equal and our formula reverts to the Baily-Chetty formula. In periods of high unemployment, the macro-elasticity decreases sharply while the micro-elasticity remains broadly constant. Since only the micro-elasticity enters the Baily-Chetty formula, this formula suggests that optimal UI should remains constant during recessions. In contrast, our formula implies that optimal UI should increase in recessions for two reasons. First, the relevant elasticity is the macro-elasticity instead of the micro-elasticity, as only the macro-elasticity matters for the government budget. Therefore during recessions, when the macro-elasticity is smaller, the optimal replacement rate is higher. Second, the correction of the job-rationing externality depends positively on the wedge between micro- and macro-elasticity. In recessions, the wedge is larger and the optimal replacement rate is even higher.

[2]This parallels Chetty (2006a) who showed that the Baily formula expressed in terms of "sufficient statistics" is quite general and carries over to models in which individuals can partially self-insure.

Next, we use numerical methods to quantify optimal UI in a dynamic stochastic environment that accounts fully for rational expectations of firms and workers, and for the law of motion of unemployment. Technology shocks drive business cycle fluctuations. We calibrate the model with US data. In steady state, the optimal replacement rate b is 72% and the optimal labor tax rate t is 4%. We simulate the time path of optimal unemployment benefits and labor tax in response to a technology shock: a 1% decrease in technology triggers a 5% increase in unemployment. At the same time, optimal UI is more generous: the optimal replacement rate increases by 1%, and the optimal labor tax increases by 8% to balance the government budget. Thus, the countercyclical pattern of optimal UI is quantitatively large.

The paper is organized as follows. Section 2 presents our one-period model to illustrate the key economic mechanisms, obtain optimal UI formulas expressed in terms of sufficient statistics, and characterize theoretically optimal UI over the cycle. Section 3 uses a calibrated DGSE model to obtain realistic numerical simulations. Section 4 reviews the related literature. Section 5 concludes.

2 Static Model

This section presents a one-period model of the labor market that is illustrative of the economic mechanisms at play because its equilibrium can be represented diagrammatically, and relevant because its equilibrium matches that of the dynamic model of Section 3 in which there would be no aggregate shocks and no discounting. We derive a simple optimal unemployment insurance formula that can be expressed in terms of estimable elasticities. We use this formula to show that optimal UI is more generous in recessions than in expansions.

2.1 Economy and equilibrium with unemployment insurance

2.1.1 Labor market

A fraction $1-U$ of all workers are allocated to a job without having to search. One can think of these $1-U$ workers as incumbent, who were already on the job in the past. A fraction U of all workers have to search for a job. One can think of these U workers as unemployed workers, who did not have a job in the past. Unemployed workers exert an average search effort E. Firms open V vacancies to recruit jobseekers. The number of matches is given by a constant-returns matching function $m(E \cdot U, V)$ of aggregate effort $E \cdot U$ and vacancies V, differentiable and increasing in both arguments. Conditions on the labor market are summarized by the labor market tightness

$$\theta \equiv \frac{V}{E \cdot U}.$$

The matching technology is such that not all unemployed workers can find a job, and not all vacancies can be filled. An unemployed worker searching with individual effort e finds a job with probability

$$e \cdot f(\theta) \equiv e \cdot \frac{m(E \cdot U, V)}{E \cdot U} = e \cdot m(1, \theta),$$

and a vacancy is filled with probability

$$q(\theta) \equiv \frac{m(E \cdot U, V)}{V} = m(1/\theta, 1) = \frac{f(\theta)}{\theta}.$$

In a tight market, it is easy for jobseekers to find jobs—the job-finding probability per unit of search effort $f(\theta)$ is high—and difficult for firms to hire workers—the job-filling probability $q(\theta)$ is low. We assume that the matching function is Cobb-Douglas, so that

$$f(\theta) = \omega_m \cdot \theta^{1-\eta}, \ q(\theta) = \omega_m \cdot \theta^{-\eta}, \ \omega_m \in (0, +\infty), \ \eta \in (0, 1).$$

2.1.2 Household

The representative household is composed of a mass one of identical workers with utility that depends on consumption C and job search effort E of the form $u(C) - k(E)$ where $u(.)$ is increasing and concave and $k(.)$ is increasing and convex. To simplify derivations, we assume an isoelastic cost of effort

$$k(E) = \omega_k \cdot \frac{E^{1+\kappa}}{1+\kappa}, \ \omega_k \in (0, +\infty), \ \kappa \in (0, +\infty).$$

Each individual can neither borrow nor save, and consumes all her income each period.[3] When working, an individual earns wage W. The government taxes earnings at rate t to finance unemployment benefits $b \cdot W$ when unemployed. We denote by $C^e = W \cdot (1-t)$ consumption when employed and by $C^u = b \cdot W$ consumption when unemployed. We denote by $\tau = t + b$ the total implicit tax on work and by $\Delta C = C^e - C^u = (1-\tau) \cdot W$ the net reward from work. τ measures the generosity of the UI system and we refer to τ as the net replacement rate in what follows.[4] Our representative household does not provide insurance to its members. Members of the household, however, decide collectively how much to search for jobs. This imposes that unemployed members take into account the effect of their search effort on their probability *of finding a job* conditional on being unemployed, and on their probability *of being unemployed* in the first place. This theoretical construct aims to capture in a one-period model the fact that in a dynamic model, higher search effort increases the probability *of finding a job* in the current period, and decreases the probability *of being unemployed* in the future.

More precisely, the household chooses its labor supply N^s to maximize its aggregate utility. Supplying N^s units of labor provides consumption C^e to N^s household members. The $1-N^s$ unemployed household members consume only C^u. Supplying N^s units of labor is costly: while a fraction $1-s$ of the N^s jobs is filled immediately at no cost, a fraction s of the jobs must be filled through matching on the labor market. The fraction s of jobs that are unfilled aims to capture simply the effects of job turnover and matching frictions in our one-period model. A higher s means more job turnover, and hence more job search.[5] The $1-(1-s) \cdot N^s$ household members unemployed at the beginning of the period must exert search effort E to fill $s \cdot N^s$ vacant jobs. A fraction $E f(\theta)$ of these jobseekers will find a job. Therefore, the required effort is such that

$$E \cdot f(\theta) \cdot [1-(1-s)N^s] = s \cdot N^s, \tag{1}$$

[3] We discuss later on how our results extend to the case with endogenous savings or self-insurance paralleling the analysis of Chetty (2006a).

[4] The gross replacement rate is traditionally defined as $b = C^u/W$ while the net replacement rate is defined as $C^u/C^e = b/(1-t) \simeq b + t = \tau$ when the tax rate t is small. As the unemployment rate U is small relative to the working population, t is also small justifying why we call τ the net replacement rate.

[5] In the dynamic setting of Section 3, s corresponds to the job destruction rate each period. Hence, s is the fraction of employed workers who lose their job each period, and $1-s$ the fraction who retain their job. $1-(1-s)N$ is the number of unemployed workers at the beginning of each period.

which imposes a utility cost $k(E)$ on the $1-(1-s)\cdot N^s$ jobseekers.

Equivalently, the household chooses effort E to maximize its aggregate utility

$$-[1-(1-s)\cdot N^s(E,\theta)]\cdot k(E)+[1-N^s(E,\theta)]\cdot u(C^u)+N^s(E,\theta)\cdot u(C^e),$$

where θ, C^u and C^e are taken as given and the labor supply $N^s(E,\theta)$ is given by

$$N^s(E,\theta)=\frac{1}{\frac{s}{E\cdot f(\theta)}+(1-s)}. \tag{2}$$

This labor supply equation comes directly from (1), and determines how search effort E translates into employment for a given labor market tightness θ. $N^s(E,\theta)$ increases with E and θ.

Let $\Delta u=u(C^e)-u(C^u)$. The optimal search effort E satisfies two equivalent conditions

$$k'(E)\cdot\frac{E}{N^s}=\Delta u+(1-s)\cdot k(E) \tag{3}$$

$$s\cdot\frac{k'(E)}{f(\theta)}+\kappa\cdot(1-s)\cdot k(E)=\Delta u. \tag{4}$$

Equation (4) determines optimal effort $E(\theta,\Delta u)$ as a function of the labor market tightness θ and the UI program Δu. $E(\theta,\Delta u)$ increases with θ and Δu.

To summarize, labor supply $N^s(E(\theta,\Delta u),\theta)$ increases with labor market tightness θ and incentive to search Δu. As an illustration, Figure 1 plots labor supply curves for high (plain line) and low (dotted line) incentive to search Δu in a price θ-quantity N diagram. Indeed, in our model with rigid wages, labor market tightness θ acts as a price to equalize labor supply and labor demand.

2.1.3 Firm

The representative firm produces a consumption good taking price and wage as given.

ASSUMPTION 1 (Diminishing marginal returns to labor)**.** The production function is $F(N,a)=a\cdot N^{\alpha}$, $\alpha\in[0,1)$. $a>0$ is the level of technology that proxies for the position in the business cycle.

To capture the effects of job turnover and matching frictions, we assume that while a fraction $1-s$ of the N^d jobs opened by the firm are filled immediately at no cost, the firm must post vacancies to advertise the fraction s of its N^d jobs that are vacant. Keeping a vacancy open has a cost of $r \cdot a$ units of consumption.[6] The recruiting cost $r \in (0, +\infty)$ captures the resources that firms must spend to recruit workers because of matching frictions. We assume away randomness at the firm level: a firm fills a job with certainty by opening $1/q(\theta)$ vacancies, and thus spends $r \cdot a/q(\theta)$ to fill a job. When the labor market is tighter, a vacancy is less likely to be filled, a firm must post more vacancies to fill a vacant job, and recruiting is more costly.

A firm chooses employment N^d to maximize real profit (the price is normalized to 1)

$$\pi = F(N^d, a) - W \cdot N^d - \frac{r \cdot a}{q(\theta)} \cdot \left(s \cdot N^d\right).$$

The wage W is set once a worker and a firm have matched. Since the vacancy-posting cost and cost of job-search effort are sunk for firms and workers at the time of matching, there are always mutual gains from trade. There is no compelling theory of wage determination in such an environment (Hall 2005; Shimer 2005). In fact in our one-period model, any wage $\in (0, +\infty)$ could be an equilibrium outcome in a labor market with positive employment. That is, the wage would never result in an inefficient allocation of labor from the joint perspective of the worker-firm pair. This property arises because firms start without any employees and the production function satisfies $\lim_{N \to 0} MPL(N) = +\infty$. Given the indeterminacy of the wage in our frictional labor market, we opt to use the Blanchard and Galí (2010) wage schedule.

ASSUMPTION 2 (Wage rigidity). The wage is $W(a) = w_0 \cdot a^{\gamma}$, $w_0 \in (0, +\infty)$, $\gamma \in [0, 1)$.

The parameter γ captures wage rigidity. If $\gamma = 0$, wages are independent of technology and there is complete wage rigidity. If $\gamma = 1$, wages are proportional to technology and there is no wage rigidity. If $\gamma \in [0, 1)$, when technology is high, wages are relatively low, driving unemployment down as in expansions. Conversely, when technology is low, wages are relatively high, driving unemployment up as in recessions.

[6] As we shall see, normalizing costs by the technology level a simplifies the derivations.

From now on, we always denote by F' the marginal product of labor $\partial F/\partial N$. The first-order condition of the firm problem defines implicitly the labor demand $N^d(a,\theta)$ with

$$F'(N^d,a) = W(a) + \frac{s \cdot r \cdot a}{q(\theta)}. \tag{5}$$

Using Assumptions 1 and 2, and dividing by a, we can rewrite (5) as

$$N^d(\theta,a) = \left\{ \frac{1}{\alpha} \left(w_0 \cdot a^{\gamma-1} + \frac{s \cdot r}{q(\theta)} \right) \right\}^{1/(\alpha-1)}. \tag{6}$$

Since $q(\theta)$ decreases in θ and $F'(N,a)$ decreases in N, labor demand $N^d(\theta,a)$ decreases with θ when there are diminishing returns to labor ($\alpha < 1$). Moreover, $N^d(\theta,a)$ increases with a when wages are rigid ($\gamma < 1$). As an illustration, Figure 1 plots labor demand curves for high (left panel) and low (right panel) technology in a price θ-quantity N diagram.

2.1.4 Equilibrium

Given a UI program Δu and technology a, labor market tightness equalizes labor demand to labor supply in equilibrium:

$$N(\Delta u,a) = N^s(E(\theta,\Delta u),\theta) = N^d(\theta,a), \tag{7}$$

where $N(\Delta u,a)$ is equilibrium employment. The equilibrium is illustrated in Figure 1. Equilibrium employment $N(\Delta u,a)$ is given by the intersection of the downward-sloping labor demand curve with the upward-sloping labor supply curve. Labor market tightness acts as a price that equalizes supply and demand in this frictional labor market. If labor supply is above labor demand, supply and demand can be equalized through a reduction in labor market tightness that both reduces hiring costs to increase labor demand N^d, and reduces the job-finding probability as well as optimal search effort to reduce labor supply N^s.

As showed by Michaillat (2010), job rationing results from the combination of diminishing marginal returns to labor ($\alpha < 1$) and wage rigidity ($\gamma < 1$).[7] In our one-period model, these two

[7] Michaillat (2010) defines job rationing as the property of a frictional labor market that does not clear even at the limit when matching frictions disappear.

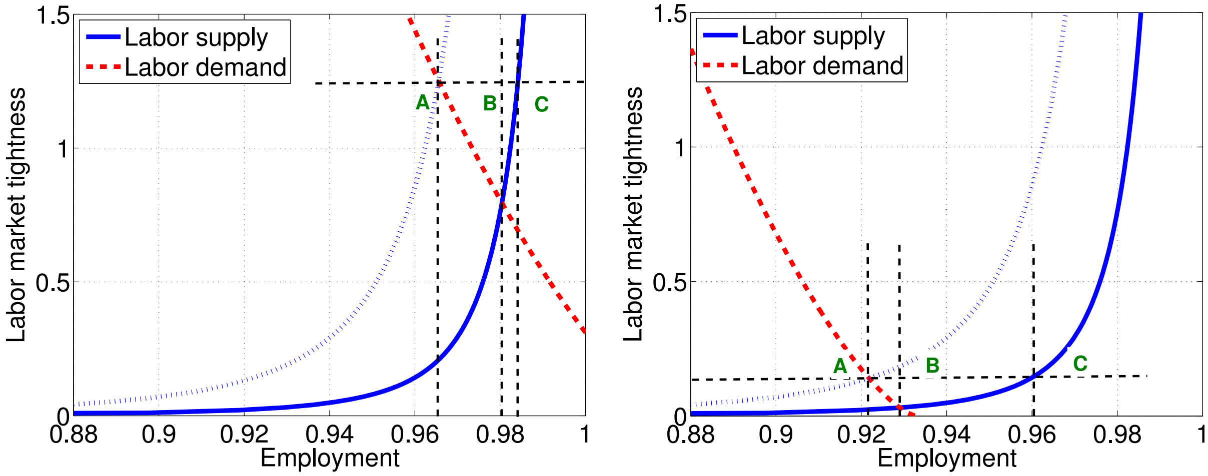

Figure 1: LABOR DEMAND AND LABOR SUPPLY

Notes: The two panels represent labor supply and labor demand for high technology $a = 1.03$ (left) and low technology $a = 0.97$ (right) in our one-period model calibrated in Table 1. The labor supply curves correspond to a net replacement rate $\tau = 72\%$ calibrated in US data (dotted line), and to a low replacement rate $\tau = 50\%$ (plain line). For $\theta \in [0, 1.5]$, labor demand is given by (6) and labor supply is a combination of (2) and (4). With log-utility, a constant τ imposes a constant $\Delta u = -\ln(\tau)$.

assumptions translate into a downward-sloping labor demand curve (because $\alpha < 1$) that shifts down after a negative technology shock (because $\gamma < 1$), as depicted in Figure 1. There is ample historical and empirical evidence in favor of these two assumptions (for example, Michaillat 2010). Furthermore, these two assumptions are necessary to provide a realistic description of business cycle fluctuations in the labor market. Wage rigidity is critical to obtain sufficient unemployment fluctuations in a search-and-matching model (Hall 2005; Shimer 2005). Diminishing returns to labor capture the fact that production inputs (especially capital) do not adjust fully to changes in employment at business cycle frequency.

2.2 Micro-elasticity and macro-elasticity

2.2.1 Definition

We define two elasticities that completely characterize the influence of optimal insurance on unemployment. These elasticities can be estimated empirically. We later express optimal unemployment insurance in terms of these estimable elasticities.

DEFINITION 1. The *macro-elasticity* of unemployment $1-N$ with respect to the net reward from work ΔC is

$$\varepsilon^M = \frac{\Delta C}{1-N} \cdot \frac{dN}{d\Delta C}. \tag{8}$$

It measures the percentage increase in unemployment $1-N$ when the net reward from work decreases by 1%, assuming all other variables adjust. The *micro-elasticity* of unemployment with respect to the net reward from work is

$$\varepsilon^m = \frac{\Delta C}{1-N} \cdot \frac{\partial N^s}{\partial E} \cdot \frac{\partial E}{\partial \Delta u} \cdot \frac{d\Delta u}{d\Delta C}. \tag{9}$$

It measures the percentage increase in unemployment $1-N$ when the net reward from work decreases by 1%, ignoring the effect of the general-equilibrium adjustment of θ on N. Both elasticities are normalized to be positive.

To understand what these elasticities represent, consider a cut in unemployment benefits $d\Delta C > 0$. This change creates variations in all variables dN, $d\theta$, $d\Delta u$, dC^u, and dE so that all equilibrium conditions continue to be satisfied. The variation in effort can be decomposed as $dE = dE_{\Delta u} + dE_{\theta}$, where $dE_{\Delta u} = (\partial E/\partial \Delta u) d\Delta u$ is a partial-equilibrium variation in response to the change in UI, and dE_{θ} is a general-equilibrium adjustment following the change $d\theta$ in tightness. It is useful to represent labor supply (2) and labor demand (6) in a price θ-quantity N diagram as in Figure 1. Using the labor supply equation (2), we have $dN = dN_E + dN_{\theta}$ where $dN_E = (\partial N^s/\partial E) dE_{\Delta u}$ and $dN_{\theta} = (\partial N^s/\partial \theta + (\partial N^s/\partial E)(\partial E/\partial \theta)) d\theta$. $dN_E > 0$ is the increase in aggregate employment due to a positive shift in labor supply, keeping labor market tightness θ constant. The labor supply shifts because the household exerts more job-search effort in response to the cut in unemployment benefits. dN_E is represented by the shift A–C in Figure 1. $dN_{\theta} < 0$ is the reduction in employment that occurs in general equilibrium through a decrease in labor market tightness $d\theta < 0$. dN_{θ} is represented by the shift C–B in Figure 1. As a combination of these two effects, the general-equilibrium increase in employment dN is smaller than the partial-equilibrium increase in labor supply dN_E. dN is represented by the shift A–B in Figure 1. The difference between the micro-effect dN_E and the macro-effect dN is dN_{θ}, which arises from job rationing.

2.2.2 Cyclical behavior

Our paper characterizes the optimal unemployment insurance over the business cycle. Proposition 1 is the first step of our analysis. It characterizes the cyclical behavior of micro- and macro-elasticity of unemployment with respect to net reward from work.

PROPOSITION 1 (Cyclical behavior of micro-elasticity and macro-elasticity)**.**

(i)

$$\varepsilon^m \simeq \frac{u'(C^e) \cdot \Delta C}{\Delta u} \cdot \frac{1}{\kappa + 1}, \tag{10}$$

where the approximation is valid for $1 - N << 1$ and $s << (1-N)/N$. Hence, for a given policy $\Delta u = u(C^e) - u(C^u)$, ε^m does not vary systematically with the business cycle.

(ii)

$$\varepsilon^m = \varepsilon^M \cdot \left[1 + \frac{1-\eta}{\eta} \cdot (1-\alpha) \cdot U \cdot \frac{1}{1 - W/F'} \cdot \left(1 + \frac{U}{\kappa}\right)\right] > \varepsilon^M \tag{11}$$

(iii) For a given Δu, $\varepsilon^m / \varepsilon^M > 1$ varies countercyclically (decreases with technology a). In expansions (a is large), this ratio is close to one. In recessions (a is small), this ratio is large.

The proof is provided in appendix. The proposition shows that the micro-elasticity ε^m is approximately constant over the business cycle. Thus, the traditional moral-hazard cost of UI is about constant over the cycle. Moreover, there is a wedge between micro- and macro-elasticity. The macro-elasticity ε^M is smaller because of job rationing, which imposes labor market tightness θ and job-finding probability $f(\theta)$ to adjust downward after a positive shift of the labor supply. Therefore the general-equilibrium increase in aggregate employment following an increase in aggregate search efforts is smaller than the partial-equilibrium increase in the individual probability to find a job following an increase in individual search efforts. Last, the gap between micro and macro-elasticity varies with the business cycle and is small in good times when unemployment is low and largely frictional (as in traditional search models) but large in recessions when unemployment is high and primarily due to job rationing.

Figure 1 illustrates the findings from Proposition 1. The wedge between micro- and macro-elasticity is measured by the distance B–C, which would be positive for any downward-sloping labor demand. The increase in the wedge between micro- and macro-elasticity when technology falls is measured by the increase of the distance B–C between the left panel (high technology) and the right panel (low technology). In the right panel, employment is bounded at $N = 0.93$ because of job rationing, which makes labor demand intercept the x-axis at $N = 0.93$. Even a large positive shift of labor supply would only have a modest positive effect on aggregate employment.

2.2.3 Empirical evidence

Results from the empirical literature on the effects of unemployment benefits on unemployment provide support for the three key positive predictions of our theoretical model: (a) positive wedge between micro- and macro-elasticity, (b) acyclical micro-elasticity, (c) countercyclical macro-elasticity. The labor economic literature focuses primarily on the elasticity of unemployment duration with respect to benefits estimated with micro-data (see Krueger and Meyer (2002) for a survey).[8] Although this literature does not distinguish between micro and macro-elasticity, studies comparing individuals with different benefits in the same labor market estimate primarily micro-elasticities while studies comparing individuals with different benefits across labor markets (for example across US states) estimate macro-elasticities.

First, the classical studies by Moffitt (1985) and Meyer (1990) use the same multi-state multi-year US micro-administrative data but Meyer (1990) includes state fixed effects and hence uses primarily within-state variation in benefits while Moffitt (1985) does not include state fixed effects and hence uses both within- and across-state variation. As a result, Meyer (1990) estimates are closer to a micro-elasticity than Moffitt (1985) estimates. Meyer (1990) finds much higher elasticity estimates than Moffitt (1985).[9] This comparison suggests that the micro-elasticity is larger than the macro-elasticity as in our model.

Second, Schmieder et al. (2010) use sharp variation in unemployment benefits duration by age in Germany and a regression discontinuity approach with population wide administrative data to

[8] A macroeconomic literature uses cross-country and times-series variation to estimate the macro-elasticity of unemployment with respect to benefits. This literature finds a wide range of estimates with no emerging consensus because of both measurement and identification issues (for example, Holmlund 1998; Layard et al. 1991).

[9] See Krueger and Meyer (2002), Table 2.5., p. 2349 for a side by side comparison.

identify compellingly the micro-elasticity of duration with respect to benefits. This is the most credible study to date which is able to estimate the micro-elasticity separately for many years. It shows that the micro-elasticity is almost exactly constant over the business cycle in Germany, as in our model.

Third, Moffitt (1985) estimates how the elasticity of duration with respect to benefits varies with the local state unemployment rate and finds that the disincentive effect of UI declines significantly with the unemployment rate in the state. Using survey data, Kroft and Notowidigdo (2010) also find that the elasticity of unemployment durations with respect to benefits is smaller in high-unemployment than in low-unemployment states. As Moffitt (1985) and Kroft and Notowidigdo (2010) use variation in benefits both across and within states, their estimate likely captures a mix of macro- and micro-elasticities. Arulampalam and Stewart (1995) also find much stronger effects of benefits on durations in Britain in 1978 (low unemployment) than in 1987 (high unemployment). More recently, Valletta and Kuang (2010) also find relatively modest effects of unemployment benefit extensions on average unemployment in the current US great recession.[10] Those results therefore suggest that the macro-elasticity may be countercyclical as in our model.

Finally, it is worth noting that a small body of evidence suggests that search externalities may play a role in the wedge between micro and macro elasticities. Levine (1993) finds that an increase in unemployment benefits for insured unemployed workers results in a reduction of unemployment duration among the uninsured. Burgess and Profit (2001) also find evidence of spillovers due to search externalities across neighboring areas.

While we leave the precise estimation of macro- and micro-elasticities over the business cycle, currently lacking from the empirical literature, for future work, it is useful to confirm that the last two predictions hold as robust stylized facts using the same Continuous Wage and Benefit History (CWBH) data from the early 1980s as in Moffitt (1985) and Meyer (1990).[11] We present complete details about the data and methodology in appendix.

[10] Jurajda and Tannery (2003) also find that UI federal extensions in Pennsylvania in the early 1980s have slightly smaller effects on labor supply in a depressed region of the state (Pittsburgh) than in a less depressed region of the state (Philadelphia). The differential response, however, is much smaller than in the studies just mentioned, maybe because there is substantial mobility across those two cities.

[11] As mentioned above, the first prediction was already validated in this data by the side to side comparison of Moffitt (1985) and Meyer (1990) presented in Krueger and Meyer (2002).

As in Kroft and Notowidigdo (2010), Panel A of Figure 2 shows the survival estimates for the duration of unemployment spells in the CWBH dataset. Spells are broken down by low versus high unemployment regimes as well as by low versus high average benefit regimes.[12] Panel A of figure 2 therefore exploits differences in average level of UI generosity by State and shows that in low unemployment regimes, high average benefits have an large effect on unemployment duration and that this effect seems to disappear in high unemployment regimes. This evidence confirms the robustness of the findings of Kroft and Notowidigdo (2010) based on Current Population Survey data and is also consistent with the findings in Moffitt (1985) or Valletta and Kuang (2010). It suggests that the macro elasticity of UI benefits is decreasing when unemployment is high.

Importantly, this effect disappears by simply shutting down across-State variation in unemployment benefits or over-time-within-State variation, by including State fixed effects and time fixed effects, or State and time fixed effects interacted. We confirm this fact in Panel B of figure 2, in which we display estimated survival functions at the average level of UI benefits, and at 110% of the average level of benefits in both low and high unemployment regimes, fitting a Cox proportional hazard model including State and month fixed effects interacted. The estimated model includes the individual log UI benefit level and controls for age, marital status, number of dependents, race, education, previous wage level, and a 6 pieces exhaustion spline to control for exhaustion spikes due to different potential durations.[13] We estimate this model in low unemployment and high unemployment regimes separately, where low unemployment regime is once again defined as the monthly unemployment rate of the State at the beginning of the spell being below the median unemployment rate of all the States available in the CWBH. By exploiting only within State*month variation in benefits, the model identifies the micro elasticity of duration with respect to benefits. The figure confirms that the baseline survival rate is clearly higher when unemployment rate is high, but that increasing the average benefits of an individual by 10% has the same effect on the survival rate in low or high unemployment rate regimes when we control for local labor market

[12] A spell is in a low unemployment regime if, at the beginning of the spell, the monthly unemployment rate in the State is below the median unemployment rate in all the States available in the CWBH. A spell is in a low average Weekly Benefit Amount (WBA) regime if at the beginning of the spell, the monthly average WBA of the State is below the median average WBA of all the States in the CWBH.

[13] This method is standard and used in Meyer (1990) or Chetty (2008) among others.

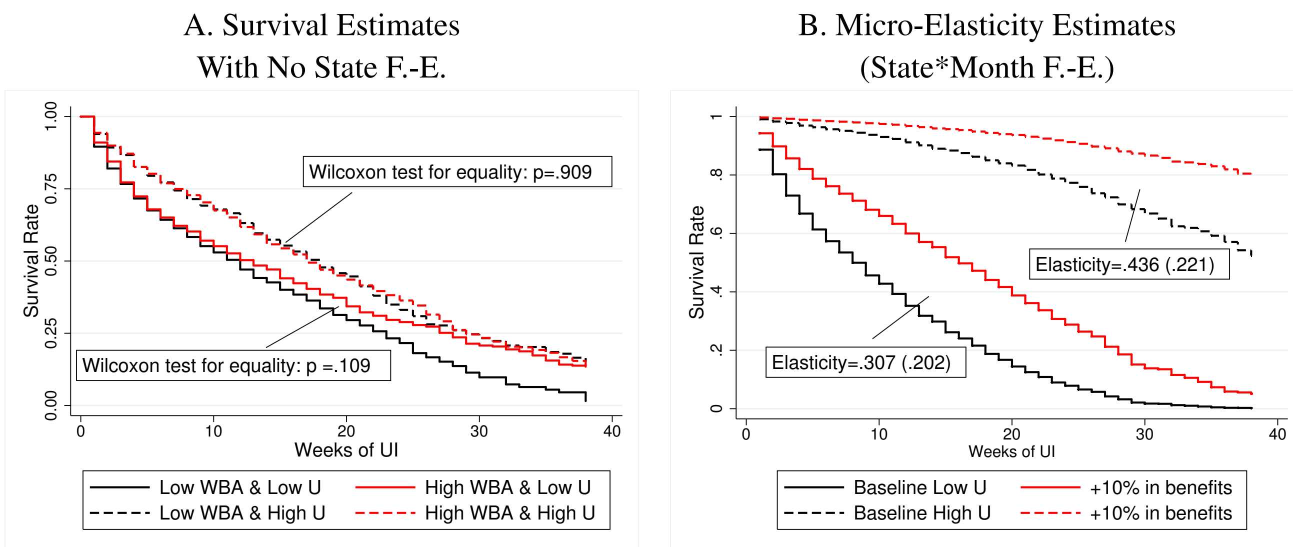

Figure 2: EMPIRICAL EVIDENCE ON MICRO AND MACRO ELASTICITIES

Sources: CWBH

Notes: Panel A displays Kaplan-Meier survival estimates for the duration of unemployment spells broken down by low (plain lines) vs high (dash lines) unemployment regimes and low (dark lines) vs high (red lines) average WBA regimes. A spell is in a low unemployment regime if at the beginning of the spell, the monthly unemployment rate of the State is below the median unemployment rate of all the States in the CWBH. A spell is in a low average WBA regime if at the beginning of the spell, the monthly average WBA of the State is below the median average WBA of all the States in the CWBH. The figure shows that, in low unemployment regimes, high average benefits have a large effect on unemployment duration while this effect disappears in high unemployment regimes.

Panel B displays the baseline survival function fitted at the average level of benefits (blue lines) and at the average level of benefits plus ten percent (red lines) for the duration of unemployment spells broken down by low (plain lines) vs high (dash lines) unemployment regimes. The estimated model includes controls for age, marital status, number of dependents, race, education, previous wage level, and a 6 pieces exhaustion spline to control for exhaustion spikes due to different potential durations. The model also includes the individual log UI benefit level and State and month fixed effects interacted. The model exploits only within State*month variation in benefits and therefore identifies the micro elasticity.

tightness. Therefore, this evidence is consistent with the prediction of the model that the micro elasticity is not significantly different in high and low unemployment regimes.[14]

2.3 Optimal unemployment insurance

Having characterized the cyclical behavior of micro- and macro-elasticity of unemployment with respect to net reward from work, the second step of our analysis of optimal UI over the business

[14]Note that identification is driven by within state within time period variation in UI replacement rates and hence might not be fully exogenous. But to the extent that this bias is the same in low and high unemployment states, it does not invalidate the main finding that the micro elasticity is more or less constant over the business cycle.

cycle consists in deriving a formula that links micro-elasticity ε^m and macro-elasticity ε^M to optimal UI. In the third and last step of the analysis, we infer the behavior of optimal UI over the business cycle from the cyclicality of ε^m and ε^M, and their relationship to optimal UI.

2.3.1 Government problem

The government chooses the net reward from work $\Delta C = C^e - C^u$ to maximize expected utility

$$N^s(E,\theta) \cdot u(C^u + \Delta C) + [1 - N^s(E,\theta)] \cdot u(C^u) - [1 - (1-s) \cdot N^s(E,\theta)] \cdot k(E) \tag{12}$$

where $N^s(E,\theta)$ is given by labor supply (2), $E(\theta,\Delta u)$ is given by the household's optimal choice of effort (4), θ clears the labor market (7), and the government budget constraint is satisfied. For a given ΔC, the government budget constraint pins down C^u:

$$C^u = N \cdot (W - \Delta C). \tag{13}$$

We assume here that benefits are financed entirely out of wages and that the government cannot tax profits to fund benefits.[15] Using the envelope theorem as E is optimized by the household, and denoting by $\bar{u}' = Nu'(C^e) + (1-N)u'(C^u)$ the average marginal utility, the first order condition for the government choice of ΔC is

$$N \cdot u'(C^e) + \bar{u}' \cdot \frac{dC^u}{d\Delta C} + \frac{\partial N^s}{\partial \theta} \cdot \frac{d\theta}{d\Delta C} \cdot [\Delta u + (1-s) \cdot k(E)] = 0. \tag{14}$$

As we shall see, the first two terms are the classical terms of the Baily-Chetty model. The last term is the correction for the job-rationing externality.

2.3.2 Optimal unemployment insurance formulas

Recall that $\Delta C = (1-\tau)W$ and hence $(W - \Delta C)/\Delta C = \tau/(1-\tau)$.

[15] If profits can be fully taxed, then total wages $N \cdot W$ in (13) should be replaced by the sum of wages and profits which is equal to $F(N,a) - (s \cdot N) \cdot r \cdot a/q(\theta)$. This alternative assumption would generate almost identical results and we consider it the general-equilibrium model of Section 3.

PROPOSITION 2 (Optimal UI formulas)**.** *The optimal replacement rate τ satisfies*

$$\frac{\tau}{1-\tau} = \frac{N}{\varepsilon^M} \cdot \frac{u'(C^u) - u'(C^e)}{\bar{u}'} + \left(\frac{\varepsilon^m}{\varepsilon^M} - 1\right) \cdot \frac{\kappa \cdot (\kappa+1)}{(\kappa+U)^2} \cdot \left[\frac{\bar{u}' \cdot \Delta C}{\Delta u}\right]^{-1}. \tag{15}$$

With the approximation that $1-N << 1$ and $s << (1-N)/N$, the optimal formula simplifies to

$$\frac{\tau}{1-\tau} \simeq \frac{1}{\varepsilon^M}\left(\frac{u'(C^u)}{u'(C^e)} - 1\right) + \left(\frac{\varepsilon^m}{\varepsilon^M} - 1\right) \cdot \frac{1}{\frac{u'(C^e) \cdot \Delta C}{\Delta u} - \varepsilon^m}. \tag{16}$$

In both (15) *and* (16)*, the first term on the right-hand-side is the classical Baily-Chetty term while the second term on the right-hand-side is the correction of externality due to job rationing.*

Proposition 2 provides a formula for the generosity of unemployment benefits. Four important points should be noted. First, absent any wedge between macro and micro-elasticity, the second term in the right-hand-side of the formulas (15) and (16) vanishes, and we obtain the Baily-Chetty formula. We express the formula in terms of the elasticity of unemployment with respect to the net rewards from work, instead of the elasticity of unemployment with respect to UI benefits C^u because the latter elasticity cannot be constant (it is zero when UI benefits are zero). This allows us also to have a direct formula for the replacement rate τ instead of an implicit formula as in Baily-Chetty.[16] As in Baily-Chetty, the replacement rate decreases with the elasticity (which measures the moral hazard effect) and increases with the curvature of the utility function (which measures the value of insurance). If utility is linear, then $u'(C^u) = u'(C^e)$ and there should be no insurance.

Second, in the Baily-Chetty term, the relevant elasticity is the macro elasticity ε^M and not the micro elasticity ε^m that has been conventionally use to calibrate optimal benefits in the public economics literature (Chetty 2008; Gruber 1997). This is because what matters in the trade-off is insurance versus aggregate costs in terms of higher unemployment and hence higher unemployment benefits outlays. Most empirical studies measure the duration of unemployment by comparing unemployed workers in the same economy who face different replacement rates. Therefore,

[16] Our convention is consistent with optimal income tax theory which always expresses optimal tax rates as a function of the elasticity of earnings with respect to one minus the marginal tax rate, instead of the elasticity of earnings with respect to the marginal tax rate. The UI problem of Baily-Chetty is effectively isomorphic to an optimal tax problem with two earnings levels (working vs. not working).

those studies measure the micro-level elasticity of unemployment duration with respect to benefits. Hence, when there is a wedge between the micro and macro elasticity, it is no longer appropriate to use the micro-elasticity estimated from those duration studies.

Third, when there is wedge between micro and macro-elasticity, a second term, directly proportional to the difference between the two elasticities, appears in the optimal UI formula. This term is the correction for the externality imposed by job search in the presence of job rationing. Thus, optimal unemployment insurance is higher than in the Baily-Chetty formula to correct for the negative externality. Even in the absence of any concern for insurance (with linear utility and $u'(C^u) = u'(C^e)$), some unemployment insurance should be provided to correct the externality.

Fourth, except for the isoelastic effort functional form, formula (15) does not depend on the functional forms of utility function, production function, or matching function. It is robust to changes in the primitives of the model. The optimal replacement rate can hence be obtained from a few sufficient statistics—micro- and macro-elasticity, curvature of the utility function—that can be empirically estimated. This optimal-policy formula can also be used to assess the current UI system. If the current $\tau/(1-\tau)$ is lower than the right-hand side of formula (15), then increasing the replacement rate is desirable (and conversely).

2.3.3 Intuitive derivation

The proof is obtained by re-arranging terms in (14), and is presented in appendix. To illuminate the key economic mechanisms behind the optimal formulas, we present an intuitive derivation. Consider a small increase $d\Delta C$ in the net reward for work—equivalent to a cut in unemployment benefits. The direct mechanical positive welfare effect on workers is $dS_1 = N \cdot u'(C^e) \cdot d\Delta C$ (first term in (14)). But increasing ΔC requires cutting benefits $C^u = N \cdot (W - \Delta C)$ by $dC^u = -N \cdot d\Delta C + (W - \Delta C) \cdot dN = -N \cdot d\Delta C + (1-N) \cdot [(W - \Delta C)/\Delta C] \cdot \varepsilon^M \cdot d\Delta C$, leading to a welfare loss $dS_2 = -N \cdot \bar{u}' \cdot d\Delta C + (1-N) \cdot [(W-\Delta C)/\Delta C] \cdot \varepsilon^M \cdot \bar{u}' \cdot d\Delta C$ (second term in (14)). In the traditional Baily-Chetty model, those are the only two effects, the optimal UI formula is such that $dS_1 + dS_2 = 0$, and there is only the first term in the right hand side of formulas (15) and (16).

However, in our model, there is a third effect due to job loss resulting from the labor tightness adjustment (third term in (14)). Each job lost reduces social welfare by $u(C^e) - s \cdot k(E) - [u(C^u) - k(E)] = \Delta u + (1-s)k(E)$ as each unemployed person incurs search costs $k(E)$ and a fraction s of the employed had to search and incur costs $k(E)$ as well. The individual optimality condition (3) and the isoelastic assumption for $k(E)$ can be used to rewrite the welfare loss per job as $\Delta u + (1-s)k(E) = \Delta u(\kappa+1)/(\kappa+U)$. As discussed above, a small increase $d\Delta C$ leads to a positive shift in labor supply (more search effort), which leads to a reduction in labor market tightness $d\theta$ in general equilibrium because of job rationing. This reduction $d\theta$ destroys dN_θ jobs through two channels: (i) $(\partial N^s/\partial E)(\partial E/\partial\theta)d\theta$ jobs are destroyed through the reduction in search effort—this reduction, however, does not have any welfare effects by the envelope theorem; and (ii) $(\partial N^s/\partial\theta)d\theta$ jobs are destroyed through a reduction in the job-finding probability. By definition

$$\varepsilon^M - \varepsilon^m = \frac{\Delta C}{1-N} \cdot \left[\frac{\partial N^s}{\partial\theta} + \frac{\partial N^s}{\partial E}\frac{\partial E}{\partial\theta}\right] \cdot \frac{d\theta}{d\Delta C}.$$

But

$$\frac{\partial N^s}{\partial E}\frac{\partial E}{\partial\theta} = \frac{U}{\kappa} \cdot \frac{\partial N^s}{\partial\theta}.$$

Thus, we can show that

$$\frac{\partial N^s}{\partial\theta} d\theta = -d\Delta C \cdot \frac{1-N}{\Delta C} \cdot \frac{\kappa}{\kappa+U} \cdot \left[\varepsilon^m - \varepsilon^M\right].$$

This leads to a welfare loss of $dS_3 = -d\Delta C \cdot (1-N)/\Delta C \cdot [\varepsilon^m - \varepsilon^M] \cdot \Delta u \cdot \kappa(1+\kappa)/(\kappa+U)^2$. This term is negative. It is due to a decrease in job-finding probability (and hence in aggregate employment) when there is more search, which is not internalized by jobseekers. This decrease in job-finding probability is a direct consequence of job rationing.

At the optimum, the sum of the three terms $dS_1 + dS_2 + dS_3$ is zero leading to formula (15). When $1 - N << 1$, then $N \simeq 1$ and hence $\bar{u}' \simeq u'(C^e)$. Furthermore, using the approximation for $\varepsilon^m \simeq (u'(C^e) \cdot \Delta C/\Delta u)/(\kappa+1)$ from Proposition 1, we can obtain formula (16) from formula (15).

2.3.4 Optimal unemployment insurance over the business cycle

Propositions 1 and 2 imply that the optimal net replacement rate is countercyclical, both through the Baily term and the through the externality term. The Baily term is higher in recessions because the macro-elasticity is smaller. The externality term is higher in recessions because the wedge between micro- and macro-elasticity grows during recessions. Formally, we can state the following proposition (the proof is presented in appendix).

PROPOSITION 3 (Cyclical behavior of optimal net replacement rate)**.** *Assume log-utility $u(C) = \ln(C)$. Assume that the approximated formulas (10) for ε^m and (16) for τ are valid at the equilibrium (i.e., technology a is high enough such that $1-N << 1$ and $s << (1-N)/N$). Then the optimal net replacement rate τ is countercyclical (i.e., decreases with technology a).*

This proposition proves that the optimal UI is more generous in recessions than in expansions in the sense that the net replacement rate $\tau = C^u/C^e$ is countercyclical. We show in Section 2.5 that, in our calibrated one-period model, the optimal UI is more generous in recessions than in expansions in a more general sense: the replacement rate $b = C^u/W$ and even the consumption of unemployed workers C^u are countercyclical.

2.4 Extensions and special cases

In this section, we explore how the model could be extended to include partial self-insurance by workers, and to allow wages to respond to the level of unemployment benefits. We explain why our optimal UI formula, expressed in terms of sufficient statistics, would remain valid with these extensions. We also consider three special cases—no job rationing, constant job-finding probability, no wage rigidity—to illustrate the economic mechanisms behind our model.

2.4.1 Savings and self-insurance

Chetty (2006a,b) shows that the simple Baily formula carries over to models with savings, borrowing constraints, private insurance arrangements, or search and leisure benefits of unemployment.

To a large extent, the same generalizations apply to our model and formulas (15) and (16) carry over with minor modifications.

As an illustration, suppose that unemployed workers can increase their consumption with home production. We assume that home production generates additional consumption $h - g(h)$ where $g(h)$ is a convex and increasing function representing costs of home producing h. Let $\tilde{C}^u = C^u + h - g(h)$ be the total consumption of unemployed workers. Individuals choose E and h to maximize

$$-[1-(1-s)N^s(E,\theta)]\,k(E) + (1-N^s(E,\theta)) \cdot u(C^u + h - g(h)) + N^s(E,\theta) \cdot u(C^e),$$

and the government chooses the net rewards from work $\Delta C = C^e - C^u$ to maximize expected utility

$$N^s(E,\theta)u(C^u + \Delta C) + (1-N^s(E,\theta))u(C^u + h - g(h)) - [1-(1-s)N^s(E,\theta)] \cdot k(E)$$

where both E and h is chosen optimally by individuals, and subject to the same constraints as in our original problem. Hence, the first order condition for the government problem is exactly identical and formulas (15) and (16) carry over simply by replacing C^u by $\tilde{C}^u$ in each of the utility and marginal utility expressions $u(C^u)$ and $u'(C^u)$.

Although the structure of the formula does not change, the consumption smoothing benefit term $u'(\tilde{C}^u)/u'(C^e) - 1$ in the first term of formulas (15) and (16) is smaller if individuals can partly self insure, using for example home production. In the extreme case where individuals can fully self-insure and smooth consumption absent a UI program, $u'(\tilde{C}^u)/u'(C^e) = 1$ and there is no reason to have a UI program for insurance purposes. This point was first noted in Baily (1978) and then generalized by Chetty (2006a). It was also used in the calibration of the Baily formula by Gruber (1997) who estimated empirically that each dollar of UI benefits increase consumption by \$0.30 when unemployed (instead of dollar for dollar as in our basic model). To keep our numerical illustrations simple, we rule out partial insurance. Thus, our optimal replacement rate is on the high side. We leave more elaborate simulations with partial self-insurance for future work.

2.4.2 Wage response to unemployment insurance

Our model implicit assumes that wages are not affected by UI. In particular, wages do not rise if unemployment benefits become more generous. This assumption is supported by empirical evidence (for example, Holmlund 1998; Layard et al. 1991). Nonetheless, wages may respond positively to benefits as higher benefits strengthen the bargaining power of workers. If we assume that $W(\Delta C)$, an additional term arises in the first-order condition (14) of the government as a change in ΔC affects the government budget constraint through its effect on W. However, this effect is artificial: it arises because we have assumed that the government cannot tax profits and affecting wages through benefits in an indirect way to tax profits. If we assume, as in the dynamic model of Section 3, that the government can fully tax profits, this effect disappears and optimal formulas (15) and (16) are unaffected. Effectively, W disappears from the government problem when the government controls C^u and C^e and total resources in the economy. The fact that W depends on ΔC, however, affects macro-elasticity ε^M as changes in wages affect labor demand: if a lower ΔC increases W, it will not only reduce jobseekers' incentives to search for jobs, but also reduce firms incentives to recruit. Thus a lower ΔC would increase unemployment more than what is predicted by our model, and the macro-elasticity ε^M would be higher than in the model. Hence, our optimal net replacement rate would again be on the high side. Nevertheless formulas (15) and (16), expressed in terms of sufficient statistics, would remain valid.

2.4.3 No job rationing ($\alpha = 1$)

This model with $\alpha = 1$ can generate large employment fluctuations (Hall 2005). But since diminishing marginal returns to labor are absent, it does not exhibit job rationing and all unemployment is frictional (Michaillat 2010). Intuitively, the number of jobs is not limited, but solely driven by job-search efforts. Thus, an unemployed worker searching for a job does not impose any negative externality on other jobseekers. Moreover, since unemployment solely results from matching frictions, increasing search efforts can always reduce unemployment. Accordingly, the optimal net replacement rate remains constant over the business cycle, and is lower than in our model.

This argument can be clarified using Figure 1. In the figure, the labor demand curve (6) would be horizontal because of constant marginal returns to labor. The marginal product of labor is independent from employment and as a result, the amount of labor demanded by firms is independent from labor market tightness. The points B and C would be superposed, and there would be no job-rationing externality. Moreover, the shift A–C would be the same in both panels, which means that the macro-elasticity ε^M would remain constant over the cycle.

Formally, Proposition 1 shows that $\varepsilon^m = \varepsilon^M$, confirming that there is no job-rationing externality and the macro-elasticity is broadly constant over the cycle in this model. In that case, the traditional Baily-Chetty formula applies, and Proposition 2 shows that the optimal net replacement rate satisfies approximately

$$\frac{\tau}{1-\tau} \simeq \frac{1}{\varepsilon^m}\left(\frac{u'(C^u)}{u'(C^e)} - 1\right). \tag{17}$$

Thus, the optimal net replacement rate τ is constant over the business cycle, and lower than in our baseline model.

2.4.4 Constant job-finding probability ($\eta = 1$)

In the model with $\eta = 1$, the job-finding probability $f(\theta) = \omega_m$ is constant. Hence, there is no negative job-rationing externality, and macro- and micro-elasticity are equal. Jobs may be rationed but equilibrium employment is directly determined by the labor supply equation (2),while labor demand equation (6) pin downs equilibrium labor market tightness. Since workers' search behavior solely determines employment independently of firms' behavior, the policy trade-off is independent from technology, and the optimal UI remains constant over the business cycle.

In Figure 1, the labor supply curve (2) would be vertical the job-finding probability $f(\theta) = \omega_m$ is independent from labor market tightness θ. Thus, θ does not enter labor supply (2), and does not affect the optimal provision of search effort (4). The points B and C would be superposed, and there would be no job-rationing externality. Moreover, the shift A–C would be the same in both panels, which means that the macro-elasticity ε^M would remain constant over the cycle.

When $\eta = 1$, Proposition 1 shows that $\varepsilon^m = \varepsilon^M$, and that the macro-elasticity ε^M is broadly constant over the cycle. The Baily-Chetty formula (17) applies, and the optimal replacement rate τ is constant over the business cycle.

2.4.5 No wage rigidity ($\gamma = 1$)

If there wages are fully flexible ($\gamma = 1$), fluctuations in technology do not lead to any variation in the other variables because both firm's and household's problem are invariant to technology. Hence, equilibrium and problem of the government are independent of technology, and the optimal UI is independent of technology as well.

Formally, technology a drops out of the labor demand equation (6) and labor market tightness θ and employment N are independent of a. While this model generates a wedge between micro- and macro-elasticity, and the externality term is present in the optimal UI formula (15), the optimal net replacement rate is constant over the "business cycle" because employment N, unemployment U, labor market tightness θ, elasticities ε^m and ε^M remain constant. Consumptions C^e and C^u do vary in proportion with technology, but these variations cancel out with a logarithmic utility function.

2.5 Numerical illustration

We now illustrate our theoretical results numerically. Table 1 presents the calibration of the model. Since we calibrate the parameters in the dynamic model, we only present the calibration strategy in Section 3.3, after formally introducing the dynamic model. Although these numerical results are obtained in a one-period model abstracting from dynamics, they are broadly consistent with those obtained in Section 3.4 when we simulate our dynamic stochastic general equilibrium model.

Figure 3 displays in six panels, as a function of unemployment: (a) job-search effort E, (b) labor market tightness θ, (c) labor tax $t = 1 - C^e/W$, (d) replacement rate $b = C^u/W$, (e) net replacement rate (total implicit tax on work) $\tau = t + b$, and (f) consumptions C^e and C^u of employed and unemployed workers. These curves are parameterized by technology a, which drives the business cycle: when unemployment increases from 4% to 11%, technology decreases from 1.04 to 0.96.

Panel (e) confirms that the net replacement rate is countercyclical, i.e., increases with unemployment. Quantitatively, the effect is significant as the net replacement rate increases from 68% to 88% when the unemployment rate increases from 4% to 11%. Panels (c) and (d) show that both components of the net replacement rate—replacement rate and labor tax—are countercycli-

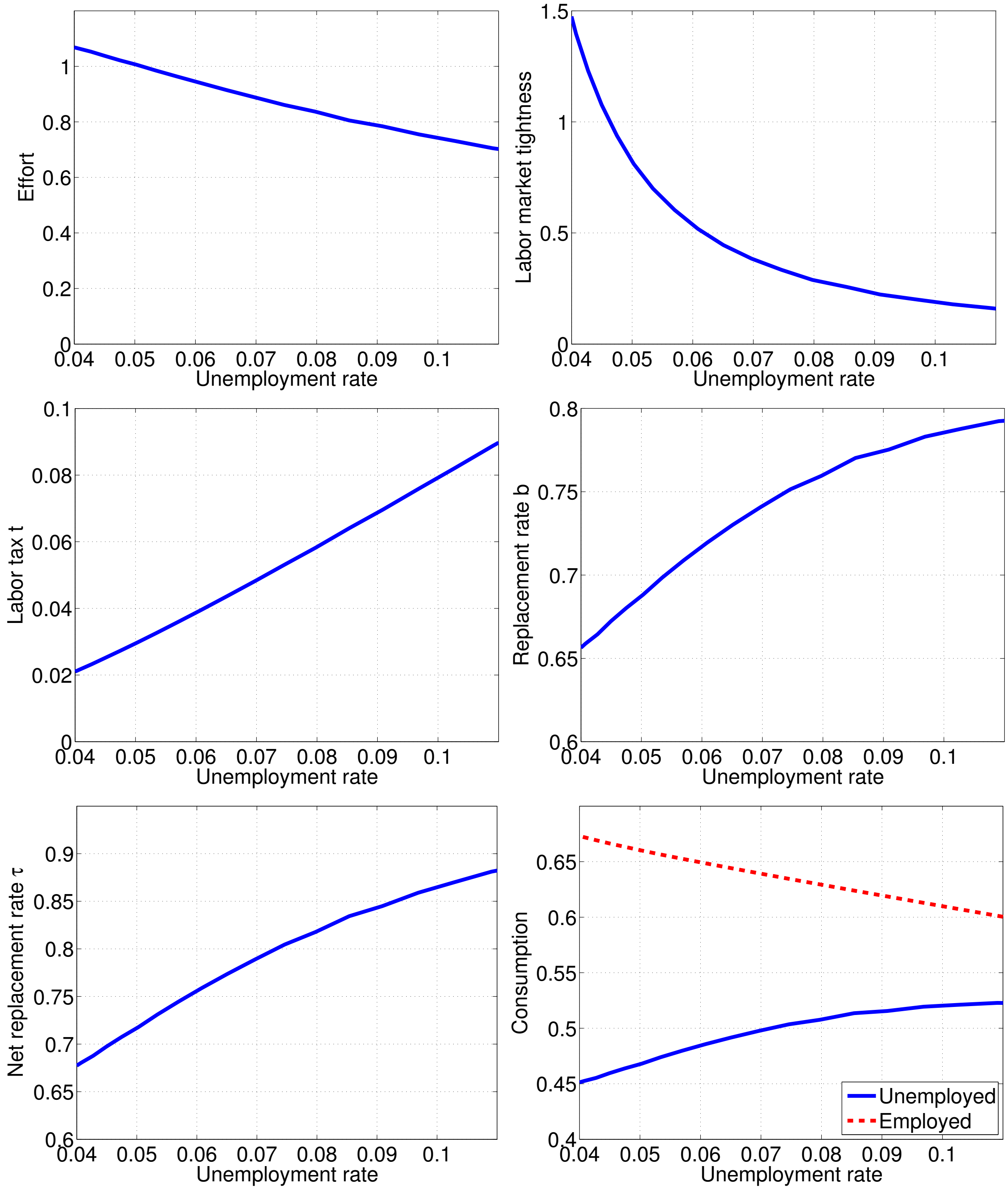

Figure 3: EQUILIBRIUM OUTCOMES AND OPTIMAL UI PROGRAM

Notes: All panels are obtained with our one-period model calibrated in Table 1. The net replacement rate τ is obtained with formula (15), and plotted against unemployment U. Each point (U, τ) corresponds to a different underlying technology level a. U spans $[0.04, 0.11]$ for $a \in [0.96, 1.04]$. For a given a and τ, we combine definition $\tau = 1 - (C^e - C^u)/W(a)$ with equilibrium conditions (2), (4), (7), and (13) to solve for effort E, labor market tightness θ, unemployment $U = 1 - (1 - s)N$, consumption C^u for unemployed workers, and consumption C^e for employed workers. Replacement rate is $b = C^u/W(a)$, labor tax is $t = 1 - C^e/W(a)$.

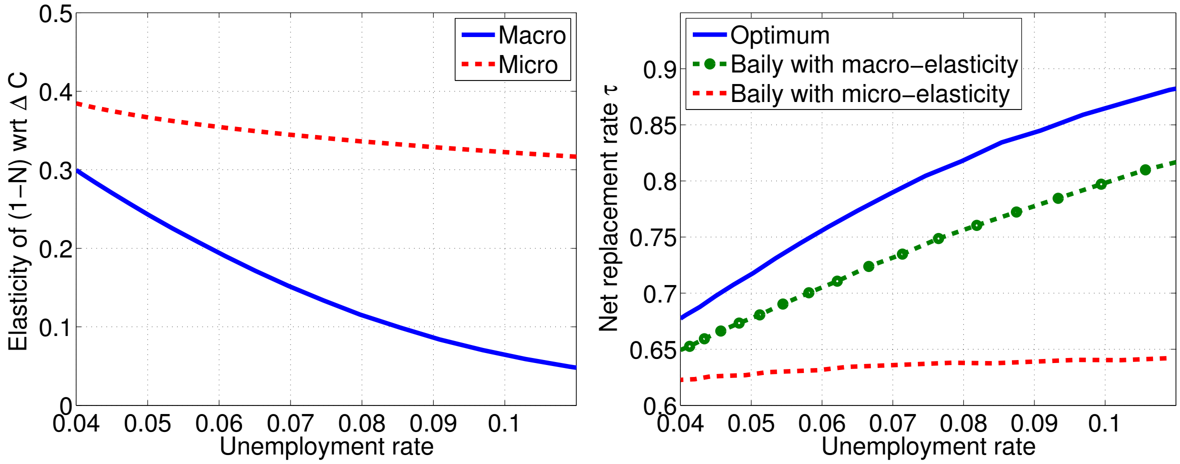

Figure 4: MICRO-ELASTICITY, MACRO-ELASTICITY, AND NET REPLACEMENT RATES.

Notes: Both panels are obtained with our one-period model calibrated in Table 1. The left panel plots, as a function of unemployment, the elasticities of unemployment $1-N$ with respect to net reward from work $\Delta C = C^e - C^u$, obtained with $\tau = 76\%$, which is the optimal net replacement rate with technology $a = 1$. The macro-elasticity ε^M (blue, solid line) is defined by (8) and computed in appendix. The micro-elasticity ε^m (red, dashed line) is defined by (9) and can be obtained from ε^M using (11). Each pair of points $[(u,\varepsilon^m),(u,\varepsilon^M)]$ corresponds to a different underlying technology level a. The right panel plots net replacement rates as a function of unemployment. The red (dashed) line is the net replacement rate τ obtained with the Baily-Chetty formula using the micro-elasticity ε^m: $\tau/(1-\tau) = N/\varepsilon^m \cdot [u'(C^u) - u'(C^e)]/\bar{u}'$. The green (dashed with circles) line is the net replacement rate τ obtained with the Baily-Chetty formula using the macro-elasticity ε^M: $\tau/(1-\tau) = N/\varepsilon^M \cdot [u'(C^u) - u'(C^e)]/\bar{u}'$. The blue (solid) line is the net replacement rate τ obtained with our optimal formula (15). Each point (u,τ) corresponds to a different underlying technology level a. Broadly, in the three cases, unemployment U spans $[0.04, 0.11]$ for $a \in [0.95, 1.05]$.

cal. Hence, in recessions, it is desirable to increase taxes substantially, not only to finance benefits to a larger fraction of the population that is unemployed, but also to finance benefits that are more generous for each unemployed worker (relative to the prevailing wage). Panel (f) shows that UI is more generous in recessions because the gap between consumption of employed and unemployed workers shrink, and because the consumption of unemployed workers even increase. Panels (a) and (b) show that both effort and labor market tightness are procyclical, as they decrease with unemployment—panel (b) represents a Beveridge curve.

Figure 4 displays the micro-elasticity ε^m and the macro-elasticity ε^M of unemployment with respect to net reward from work, as a function of unemployment for a constant net replacement rate $\tau = 76\%$, which is the optimal net replacement rate with technology $a = 1$. The figure confirms the three theoretical results from Proposition 1. First, micro-elasticity is close to constant over the business cycle: it remains on a narrow range from 0.32 to 0.39. Second and in contrast, macro-elasticity varies substantially over the business cycle: it increases from 0.05 when unemployment is 11% to 0.30 when unemployment is 4%, a six-fold increase. Third, macro-elasticity is always smaller than micro-elasticity although the gap is quite small in expansions.

Figure 4 also displays the net replacement rate τ obtained from three alternative formulas, as a function of unemployment. The first curve is the optimal net replacement rate, obtained with the optimal UI formula (15). The second graph is the net replacement rate obtained from a Baily-Chetty formula, similar to (15) but excluding the externality term. This second replacement rate is lower than the optimum, and the discrepancy is more important in recessions as the externality term depends on the wedge between micro- and macro-elasticity, which is larger in recessions. The third graph is the net replacement rate obtained from a Baily-Chetty formula, similar to (15) but excluding the externality term and replacing macro-elasticity by micro-elasticity. This replacement rate is almost constant over the business cycle: it varies within a narrow range from 62% to 64%, because the micro-elasticity is almost constant over the business cycle. The latter graph is the standard simulation presented in the public economics literature (for example, Gruber 1997). Figure 4 shows that job rationing changes the picture substantially in recession.[17]

Finally, Figure 5 compares our main calibration to an alternative calibration with $\alpha = 1$, that is, a calibration with constant returns to labor and no job rationing as in the influential study of Hall (2005). The left panel confirms that when $\alpha = 1$, micro- and macro-elasticity, which are identical, vary little over the business cycle. The right panel confirms that the optimal net replacement rate is almost constant over the business cycle in the Hall (2005) model whereas it varies substantially in our model.

[17] We measure the welfare gains from using the optimal UI formula (15) instead of Baily-Chetty formulas by computing the percentage-increase in certainty-equivalent consumption C^{eq}, defined by $u(C^{eq}) \equiv Nu(C^e) + (1-N)u(C^u) - [1-(1-s)N]\,k(E)$. The welfare gains are negligible in expansions when micro- and macro-elasticity are almost equal. But the gains are substantial in recessions: when unemployment reaches 10%, the welfare gain is 1.5% compared to the Baily-Chetty formula with micro-elasticity and 0.25% compared to the Baily-Chetty formula with macro-elasticity.

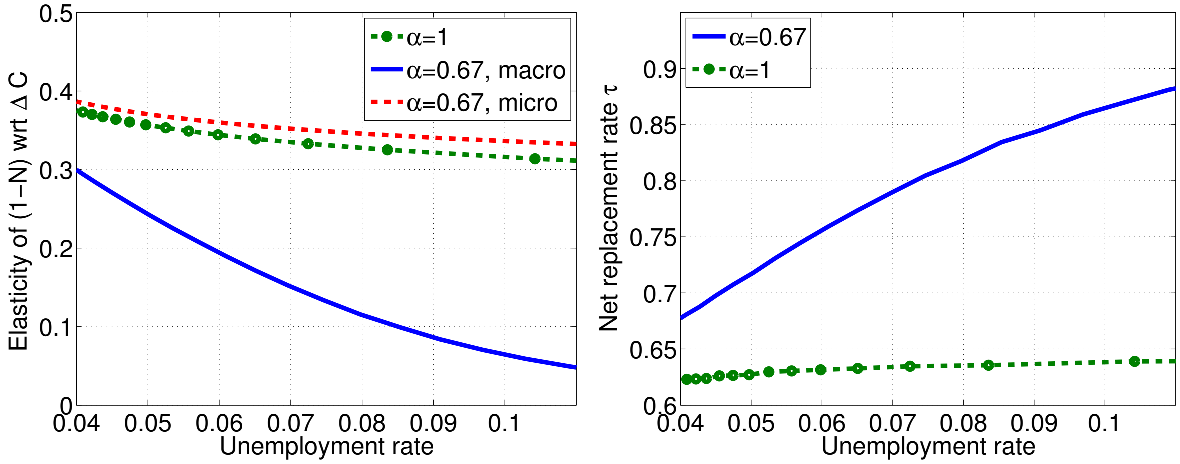

Figure 5: OPTIMAL UI IN MODEL WITH ($\alpha = 0.67$) AND WITHOUT ($\alpha = 1$) JOB RATIONING

Notes: Both panels are obtained with our one-period model calibrated in Table 1 (only α varies). The left panel plots, as a function of unemployment, the elasticities of $1-N$ with respect to $\Delta C = C^e - C^u$ obtained with $\tau = 76\%$, which is the optimal net replacement rate with technology $a = 1$. The macro-elasticities ε^M are defined by (8) and computed in appendix. The micro-elasticities ε^m are defined by (9) and can be obtained from the macro-elasticities using (11). When $\alpha = 1$, $\varepsilon^M = \varepsilon^m$ so we report only one elasticity. Each point (u, ε) corresponds to a different underlying technology level a. The right panel plots, as a function of unemployment, the net replacement rate τ obtained from formula (15). Each point (u, τ) corresponds to different a. With $\alpha = 0.67$, unemployment U spans $[0.04, 0.11]$ for $a \in [0.96, 1.04]$. With $\alpha = 1$, U spans $[0.04, 0.11]$ for $a \in [0.985, 1.02]$.

3 Dynamic Model

This section presents a dynamic stochastic extension of our one-period model. We calibrate the model using micro- and macro-data for the US labor market. We move beyond the comparative-static results of Proposition 3 by computing impulse response functions of labor market variables and optimal UI in the fully dynamic model.[18]

3.1 Economy and equilibrium with unemployment insurance

The stochastic process for technology $\{a_t\}_{t=0}^{+\infty}$ drives economic fluctuations. The history of technology realizations is $a^t = (a_0, a_1, \dots, a_t)$. Together with the initial employment N_{-1} in the representative firm, a^t fully describes the state of the economy in period t.

The labor market is similar to that in the one-period model. The only difference are the flows of

[18]A byproduct of the quantitative analysis, presented in appendix, is to verify that the calibrated model describes well the US labor market.

workers between employment and unemployment. At the end of period $t-1$, a fraction $s \in (0,1)$ of the N_{t-1} existing worker-job matches is exogenously destroyed. Workers who lose their job become unemployed, but can search for a new job immediately. At the beginning of period t, U_t unemployed workers are looking for a job:

$$U_t = 1 - (1-s) \cdot N_{t-1}. \tag{18}$$

We assume that the government fully taxes profits, taxes or subsidizes labor income, and subsidizes unemployed workers. We also impose period-by-period budget balance. Thus, it is as if the government could directly choose consumption for unemployed workers C_t^u and consumption for employed workers C_t^e subject to the resource constraint in the economy.

DEFINITION 2. A *government policy* is a collection of stochastic processes $\{C_t^e, C_t^u\}_{t=0}^{+\infty}$ that satisfy the government budget constraint for all t and all a^t:

$$F(N_t, a_t) = N_t \cdot C_t^e + (1-N_t) \cdot C_t^u + \frac{r \cdot a_t}{q(\theta_t)} \cdot [N_t - (1-s) \cdot N_{t-1}]. \tag{19}$$

The t element of the government policy must be measurable with respect to (a^t, N_{-1}).

We impose period-by-period budget balance to rule out the possibility that the government uses deficit spending to shift resources intertemporally from good times to bad times and smooth workers' consumption. This assumption is natural as individuals can neither save nor borrow to smooth consumption over time. Moreover, this assumption allows us to focus on within-period insurance-efficiency trade-off. Labor tax t_t and replacement rate b_t (and net replacement rate $\tau_t = t_t + b_t$) can be recovered from the government policy and equilibrium employment. Let

$$C_t \equiv \frac{1}{N_t} \cdot \left(F(N_t, a_t) - \frac{r \cdot a_t}{q(\theta_t)} \cdot [N_t - (1-s) \cdot N_{t-1}] \right)$$

be the hypothetical consumption of employed workers if only they were able to consume, and unemployed workers did not have access to consumption (that is, if there were no UI and all the profits of firms were distributed to employed workers). Then we obtain $t_t = 1 - C_t^e / C_t$ and

$b_t = C_t^u / C_t$.

In the static model, we assumed that members of the household decided collectively how much to search for jobs. The representative-household construct forced unemployed workers, in the static model, to internalize the effect of their search effort on their probability of finding a job conditional on being unemployed, and on their probability of being unemployed in the first place. In our dynamic model, higher search efforts increase the probability of finding a job in the current period, and naturally decrease the probability of being unemployed in the future. We do not need to resort to the representative-household construct, and therefore focus on a representative individual.

DEFINITION 3. Given the government policy $\{C_t^e, C_t^u\}_{t=0}^{+\infty}$, and labor market tightness $\{\theta_t\}_{t=0}^{+\infty}$, the *representative individual's problem* is to choose a stochastic process for search effort $\{E_t\}_{t=0}^{+\infty}$ to maximize the expected utility

$$\mathbb{E}_0 \left[\sum_{t=0}^{+\infty} \delta^t \cdot \left\{ (1 - N_t^s) \cdot u(C_t^u) + N_t^s \cdot u(C_t^e) - \left[1 - (1-s) \cdot N_{t-1}^s\right] \cdot k(E_t) \right\} \right], \tag{20}$$

subject to the law of motion of the probability to be employed in period t for all t,

$$N_t^s = \left[1 - (1-s) \cdot N_{t-1}^s\right] \cdot E_t \cdot f(\theta_t) + (1-s) \cdot N_{t-1}^s. \tag{21}$$

$\delta \in (0,1)$ is the discount factor. The time t element of household's choice must be measurable with respect to (a^t, N_{-1}).

The optimal effort function therefore satisfies the following Euler equation

$$\left\{ \frac{k'(E_t)}{f(\theta_t)} - \delta \cdot (1-s) \cdot \mathbb{E}\left[\frac{k'(E_{t+1})}{f(\theta_{t+1})} \right] \right\} + \kappa \cdot \delta \cdot (1-s) \cdot \mathbb{E}\left[k(E_{t+1})\right] = u(C_t^e) - u(C_t^u). \tag{22}$$

As explained in Section 2, in our frictional labor market, there is no compelling theory of wage determination. Given the indeterminacy of the wage, we opt to use the Blanchard and Galí (2010) wage schedule, as in the static model.

DEFINITION 4. A *wage process* is a stochastic process $\{W_t\}_{t=0}^{+\infty}$ defined for all t and all a^t by

$$W_t = w_0 \cdot a_t^{\gamma}, \;\; \gamma \in [0,1). \tag{23}$$

Firms are owned by risk-neutral entrepreneurs who maximize profits.

DEFINITION 5. Given wage, labor market tightness, and technology processes $\{W_t, \theta_t, a_t\}_{t=0}^{+\infty}$, the *representative firm's problem* is to choose a stochastic process for employment and hiring $\left\{N_t^d, H_t\right\}_{t=0}^{+\infty}$ to maximize

$$\mathbb{E}_0 \left[\sum_{t=0}^{+\infty} \delta^t \cdot \left\{ F(N_t^d, a_t) - W_t \cdot N_t^d - \frac{r \cdot a_t}{q(\theta_t)} \cdot \left[N_t^d - (1-s) \cdot N_{t-1}^d \right] \right\} \right]. \tag{24}$$

The time t element of a firm's choice must be measurable with respect to (a^t, N_{-1}).

The definition of the firm's problem implicitly assumes that endogenous layoffs never occur, such that for all t $N_t^d \geq (1-s) \cdot N_{t-1}^d$. Since layoffs are rules out by the equilibrium definition 7, this assumption is without loss of generality. Employment N_t^d is determined by the following first-order condition (as in equilibrium $N_t^d < 1$):

$$F'(N_t^d, a_t) = W_t + \frac{r \cdot a_t}{q(\theta_t)} - \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{r \cdot a_{t+1}}{q(\theta_{t+1})} \right] \tag{25}$$

This equation implies that the representative firm hires labor until marginal revenue from hiring equals marginal cost. The marginal revenue is the marginal product of labor F'. The marginal cost is the sum of the wage W_t, the cost of hiring a worker $r \cdot a_t / q(\theta_t)$, minus the discounted cost of hiring next period $\delta \cdot (1-s) \cdot \mathbb{E}_t \left[r \cdot a_{t+1} / q(\theta_{t+1}) \right]$.

Individuals decide how much labor to supply through their choice of job-search effort. Firms decide how much labor to employ through hiring. Wages follow an exogenous stochastic process given by (23), and cannot equalize supply and demand in this frictional labor market. Thus, as in the static model, labor market tightness acts as a price that equalizes supply and demand of labor.

DEFINITION 6. A *labor market tightness process* is a stochastic process $\{\theta_t\}_{t=0}^{+\infty}$ such that the demand for labor $\{N_t^d\}_{t=0}^{+\infty}$ by firms equals the supply of labor $\{N_t^s\}_{t=0}^{+\infty}$ by the household: for all t and all a^t,

$$N_t = N_t^d = N_t^s. \tag{26}$$

The t element of the labor market tightness must be measurable with respect to (a^t, N_{-1}).

We can now define an equilibrium with unemployment insurance.

DEFINITION 7. Given initial employment N_{-1}, a stochastic process $\{a_t\}_{t=0}^{+\infty}$ for technology, an *equilibrium with unemployment insurance* is a collection of stochastic processes $\{E_t, N_t\}_{t=0}^{+\infty}$, a government policy, a wage process, and a labor market tightness process that solve the household and firm problems. Moreover, the wage process satisfies the condition that no worker-employer pair has an unexploited opportunity for mutual improvement. The wage should neither interfere with the formation of an employment match that generates a positive bilateral surplus, nor cause the destruction of such a match .

An equilibrium with unemployment insurance is a collection of stochastic processes $\{C_t^e$, C_t^u, W_t, E_t, N_t, $\theta_t\}_{t=0}^{+\infty}$ that satisfies equations (22), (25), (19), (23), (26). We can also derive a sufficient condition for the wage process to always respect the (private) efficiency of all worker-employer matches. This condition would be exactly the same as the one derived by Michaillat (2010): it imposes a lower bound on wage rigidity γ (which depends on α and s) such that inefficient layoffs do not occur with a high enough probability.[19]

3.2 Government's problem and optimal equilibrium

The unemployment insurance program is history contingent—it is fully contingent on the history of realizations of shocks— and it is taken as given by firms and household. Moreover, we follow Chari et al. (1991) and Aiyagari et al. (2002) and assume that an institutional arrangement exists through which the government can bind itself to the policy plan.

[19] We find that if $\gamma \geq 0.5$, wages are flexible enough to avoid inefficient separations with probability below 1 percent. In other words, inefficient layoffs cannot occur with a negative technology shock of amplitude below 2.3 standard deviations. This sufficient condition is independent from government policy.

DEFINITION 8. The *government's problem* is to choose a government policy to maximize social welfare

$$\mathbb{E}_0\left[\sum_{t=0}^{+\infty}\delta^t\cdot\{(1-N_t)\cdot u(C_t^u)+N_t\cdot u(C_t^e)-[1-(1-s)\cdot N_{t-1}]\cdot k(E_t)\}\right] \tag{27}$$

over all equilibria with unemployment insurance. An *optimal equilibrium* is an equilibrium that attains the maximum of (27).

The optimal equilibrium is fully described in Proposition A1 in appendix. In particular, we can describe the first-order conditions and constraints of the government's problem in the absence of aggregate shocks. In that case, the optimal equilibrium converge to a constant equilibrium that is characterized by Proposition 4 (proof presented in appendix).

PROPOSITION 4 (Equivalence with one-period model)**.** *The steady-state solution of the government's problem in the dynamic model in the absence of aggregate shocks converges to the solution of the government's problem in the one-period model when the discount factor* δ *converges towards* 1. *In particular, the optimal approximated formula* (16) *continues to apply in the steady-state of the dynamic model when* $1-N << 1$, $s << 1-N$, *and* $1-\delta << 1$.

This proposition implies that the static model presented Section 2 is the limiting case of the steady-state of the dynamic model when there is no discounting. This implies that the same economic mechanisms determine the steady-state of the dynamic model. Therefore, the remaining of the section highlights the dynamics of the model, which could not be analyzed in the static model.

Note also that we make the simplifying assumption that benefits do not depend on the length of the unemployment spells. As we shall see in Section 4, there is a wide literature analyzing the optimal path of benefits overtime, independently of the business cycle issues we focus on here.

3.3 Calibration

We calibrate all parameters at a weekly frequency.[20] Table 1 summarizes the calibrated parameters.

[20] A week is 1/4 of a month and 1/12 of a quarter.

Table 1: PARAMETER VALUES IN SIMULATIONS.

	Interpretation	Value	Source
$\bar{a}$	Steady-state technology	1	Normalization
$\bar{e}$	Steady-state effort	1	Normalization
s	Separation rate	0.95%	JOLTS, 2000–2010
δ	Discount factor	0.999	Corresponds to 5% annually
ω_m	Efficiency of matching	0.19	JOLTS, 2000–2010
η	Elasticity of job-filling	0.7	Petrongolo and Pissarides (2001)
γ	Real wage rigidity	0.5	Pissarides (2009), Haefke et al. (2008)
c	Recruiting costs	0.21	0.32× steady-state wage
w_0	Steady-state real wage	0.67	Matches steady-state unemployment of 5.9%
α	Returns to labor	0.67	Matches labor share of 0.66
σ	Risk aversion	1	Chetty (2006b)
κ	Search elasticity	1.8	Meyer (1990)
ω_k	Searching cost	0.49	Matches $\bar{e} = 1$ for $t = 15\%$ and $b = 60\%$

Separation rate: We estimate the job destruction rate from the seasonally-adjusted monthly series for total separation rate in all nonfarm industries constructed by the BLS from the Job Openings and Labor Turnover Survey (JOLTS) for the for the December 2000–June 2010 period.[21] The average separation rate is 0.037, so $s = 0.0093$ at weekly frequency.

Recruiting costs: We estimate the recruiting cost from microdata gathered by Barron et al. (1997) who find that on average, the flow cost of opening a vacancy amounts to 0.098 of a worker's wage. This number accounts only for the labor cost of recruiting. Silva and Toledo (2005) account for other recruiting expenses such as advertising costs, agency fees, and travel costs, to find that 0.42 of a worker's monthly wage is spent on each hire. Unfortunately, they do not report recruiting times. Using the average monthly job-filling rate of 1.3 in JOLTS, 2000–2010, the flow cost of recruiting could be as high as 0.54 of a worker's wage. We calibrate recruiting cost as 0.32 of a worker's wage, the midpoint between the two previous estimates.[22]

[21] December 2000–June 2010 is the longest period for which JOLTS is available. Comparable data are not available before this date.

[22] Using the average unemployment rate and labor market tightness in JOLTS, we find that 0.89 percent of the total wage bill is spent on recruiting.

Matching function: We picked a Cobb-Douglas matching function. We now set $\eta = 0.7$. Both assumptions are reasonable in light of empirical results surveyed by Petrongolo and Pissarides (2001). To estimate the matching efficiency ω_m, we use steady-state relationships and the normalization $\bar{e} = 1$ to find

$$\omega_m = \frac{s}{1-s} \cdot \frac{1-U}{U} \cdot \theta^{\eta-1}$$

We use the seasonally-adjusted, monthly series for the number of vacancies from JOLTS, 2000–2010, and the seasonally-adjusted, monthly unemployment level computed by the BLS from the Current Population Survey (CPS) over the same period, to compute labor market tightness and unemployment. We find $\theta = 0.47$ and $U = 5.9\%$. The resulting estimate of the matching efficiency at weekly frequency is $\omega_m = 0.19$.

Wage rigidity: Next we calibrate the elasticity γ of wages with respect to technology based on estimates obtained from panel data recording wages of individual workers. These microdata are more adequate because they are less prone to composition effects than aggregate data. The survey of the literature by Pissarides (2009) places the productivity-elasticity of wages of existing jobs in the 0.2–0.5 range in US data. A recent study by Haefke et al. (2008) estimates the elasticity of wages of job movers with respect to productivity using panel data for US workers. For a sample of production and supervisory workers over the period 1984–2006, they obtain a productivity-elasticity of total earnings of 0.7. Their estimate, however, is an upper bound on the elasticity of wages as they do not control for the cyclical composition of jobs.[23,24] Therefore, we set $\gamma = 0.5$, a reasonable mid-point in the range of available evidence.

Diminishing marginal returns to labor: So far, we have estimated parameters from microdata or aggregate data, independently of the model. We now calibrate the remaining parameters to

[23] Workers may accept lower-paid, stop-gap jobs in recessions, and move to better jobs during expansions, biasing the estimated elasticity upwards.

[24] 0.7 is an estimate of the elasticity of wages with respect to labor productivity Y/N, whereas γ is the elasticity of wages with respect to technology $a = Y/N^{\alpha}$. While technology and productivity are highly correlated, productivity is less volatile than technology and therefore an estimate of the elasticity of wages with respect to technology would be below 0.7.

match key moments estimated in the data. We calibrate the production function parameter α such that the steady state of the model matches average labor market tightness $\overline{\theta} = 0.47$ and average labor share $\overline{ls} = 0.66$ in US data. We find that $\alpha = 0.67$.[25]

Wage level: We target a steady-state unemployment rate of $U = 5.9\%$, so we calibrate the wage w_0 to obtain a steady-state employment $\overline{n} = 0.95$, and a steady-state labor share of $\overline{ls} = 0.66$, which imposes $\overline{ls} = w_0 \cdot \overline{n}^{1-\alpha}$. We find $w_0 = 0.67$. Hence, the recruiting cost is $r = 0.32 \cdot w_0 = 0.22$.

Utility function: We choose risk aversion $\sigma = 1$ such that $u(\cdot) = \ln(\cdot)$, which is on the low side of the most compelling estimates Chetty (2006b) but is often used in macro-economic calibration. A lower risk aversion implies a lower value of insurance and hence lower optimal unemployment benefits. Therefore, our risk aversion parameter is conservative.

We choose $\kappa = 1.8$ to match the micro-elasticity of unemployment with respect to benefits estimated in the empirical micro-economic literature. This literature consistently finds large elasticities of duration with respect to benefits levels. For example, the widely cited study by Meyer (1990) estimates an elasticity of 0.9, and this elasticity is used in optimal UI simulations using the Baily formula by Gruber (1997).[26] We normalize the steady-state search effort $\overline{e}$ to 1. For the US, we assume unemployment benefits $b = 60\%$ and labor tax $t = 15\%$, in line with the literature (Chetty 2006b; Gruber 1997).[27] With $\kappa = 1.8$ and $\sigma = 1$, we obtain $\omega_k = 0.49$. With this calibration, we find $\varepsilon^m \simeq 0.36$. The elasticity of unemployment with respect to *benefits* (instead of net reward from work) is $\frac{C^u}{1-N}\frac{\partial(1-N)}{\partial C^u} \simeq 0.9$ in line with Meyer (1990).

There remains considerable uncertainty about some of the parameters and our model abstracts from a number of relevant issues–many of which are explored in the earlier literature. Therefore, this exercise illustrates the magnitudes one could expect from our rationing theory, and how such

[25] We can show that the labor share $\overline{ls} \equiv (\overline{w} \cdot \overline{n})/\overline{y}$ is related to α through the firm's optimality condition by $\overline{ls}\left(s \cdot \frac{0.32}{q(\theta)} + 1\right) = \alpha$. So α is slightly larger than the labor share because of the recruiting costs.

[26] This elasticity is conceptually close to a micro-elasticity because it either controls for state unemployment rates or uses state fixed effects.

[27] The UI payroll tax itself is on the order of 3% and hence much smaller than 15% but workers pay a much higher tax rate than unemployed workers because (a) social security taxes do not apply to UI benefits, (b) federal and state income taxes are progressive and workers have substantially higher incomes than the unemployed.

magnitudes vary with key parameters.

3.4 Impulse response to unexpected and transitory technology shock

To determine the equilibrium of the model for a given UI program, and to solve the government's problem, we log-linearize the model around the steady state with $\overline{a} = 1$. The appendix describes the log-linear model in detail. We assume that the log-deviation of technology $\check{a}_t \equiv d\ln(a_t)$ (which represents the percentage-deviation of technology from steady-state) follows an AR(1) process: $\check{a}_{t+1} = \rho\check{a}_t + z_{t+1}$ where z_t is an innovation to technology. We estimate this AR(1) process in US data. We construct log technology as a residual $\log(a) = \log(Y) - \alpha \cdot \log(N)$. Output Y and employment N are seasonally-adjusted quarterly real output and employment in the nonfarm business sector constructed by the Bureau of Labor Statistics (BLS) Major Sector Productivity and Costs (MSPC) program. The sample period is 1964:Q1–2009:Q2. To isolate fluctuations at business cycle frequency , we follow Shimer (2005) and take the difference between log technology and a low frequency trend—a Hodrick-Prescott (HP) filter with smoothing parameter 10^5. We estimate detrended log technology as an AR(1) process: $\log(a_{t+1}) = \rho \cdot \log(a_t) + z_{t+1}$ with $z_{t+1} \sim N(0,\nu^2)$. With quarterly data, we obtain an autocorrelation of 0.897 and a conditional standard deviation of 0.0087, which yields $\rho = 0.991$ and $\nu = 0.0026$ at weekly frequency.

We solve the government's problem by log-linearization. The log-linear system has three state variables: employment N, as well as the Lagrange multipliers on the household's and firm's optimality conditions. These multipliers impose that the government keep track of the promises made in the previous period to job-searching workers and recruiting firms. The steady state of the optimal equilibrium is $\overline{u} = 6.1\%$, $\overline{v/u} = 0.49$, $\overline{\tau} = 76\%$, $\overline{e} = 0.93$. To confirm the comovements of technology with unemployment insurance in a fully dynamic model, we compute the impulse response functions (IRFs) in the log-linear model.

Figure 6 aims to detail the response of policy variables to a negative technology shock of one percent in the optimal equilibrium, and to compare them to responses in an equilibrium with constant replacement rate $\tau = 72\%$ (corresponding to the US economy). In the optimal equilibrium, both tax rate t and replacement rate b increase slowly after the adverse shock, which drives the

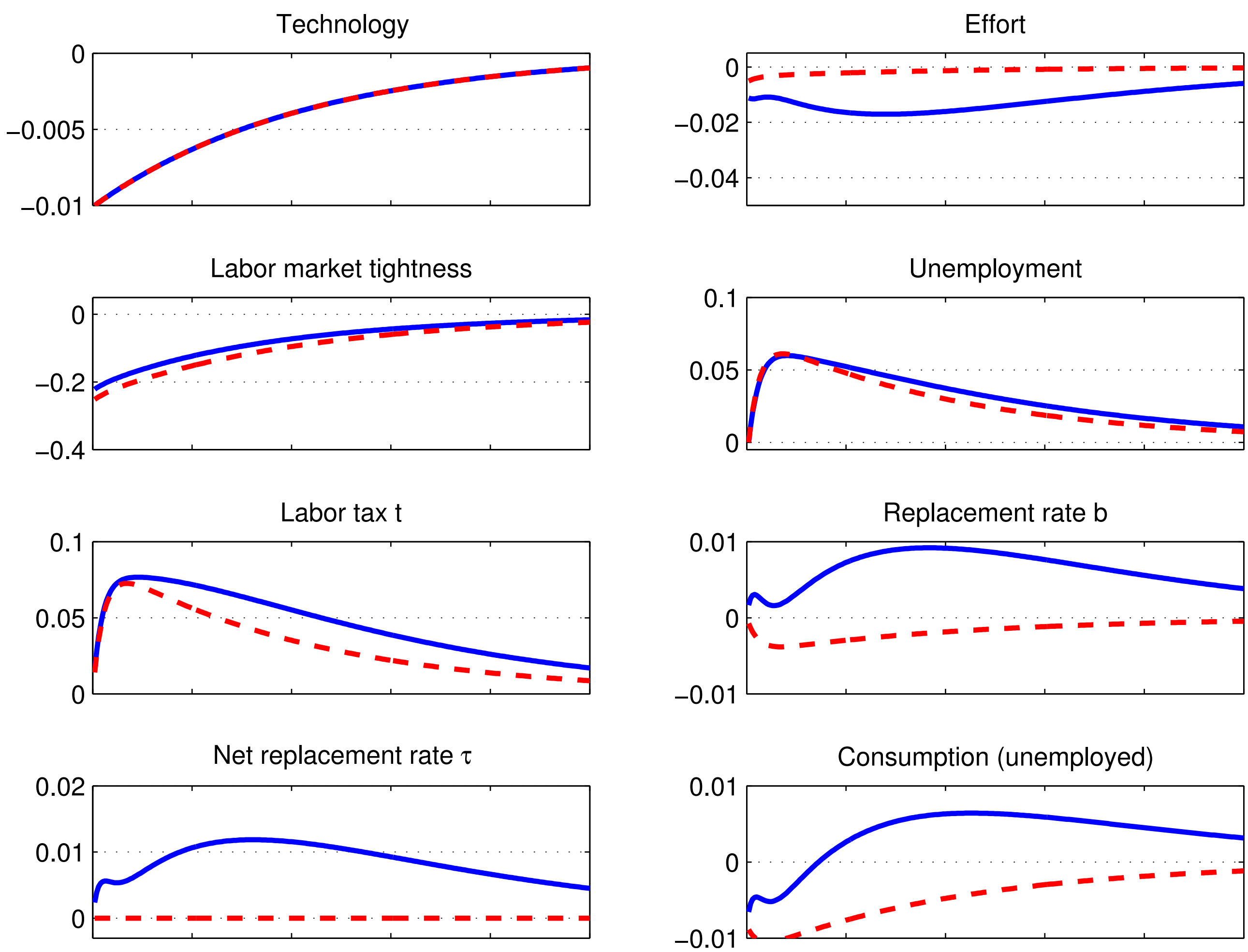

Figure 6: RESPONSE OF OPTIMAL UI PROGRAM AND EQUILIBRIUM OUTCOMES TO A NEGATIVE TECHNOLOGY SHOCK

Notes: This figure displays impulse response functions (IRFs), which represent the logarithmic deviation from steady state for each variable. IRFs are obtained by imposing a negative technology shock $z_1 = -0.01$ to the log-linear model (about 4 times the standard deviation 0.0026). The time period displayed on the x-axis is 250 weeks. The blue (solid) line IRFs are responses of the optimal equilibrium. The red (dashed) IRFs are a useful benchmark: the responses of the equilibrium with a constant net replacement rate $\tau = 72\%$. The complete log-linear system is described in appendix.

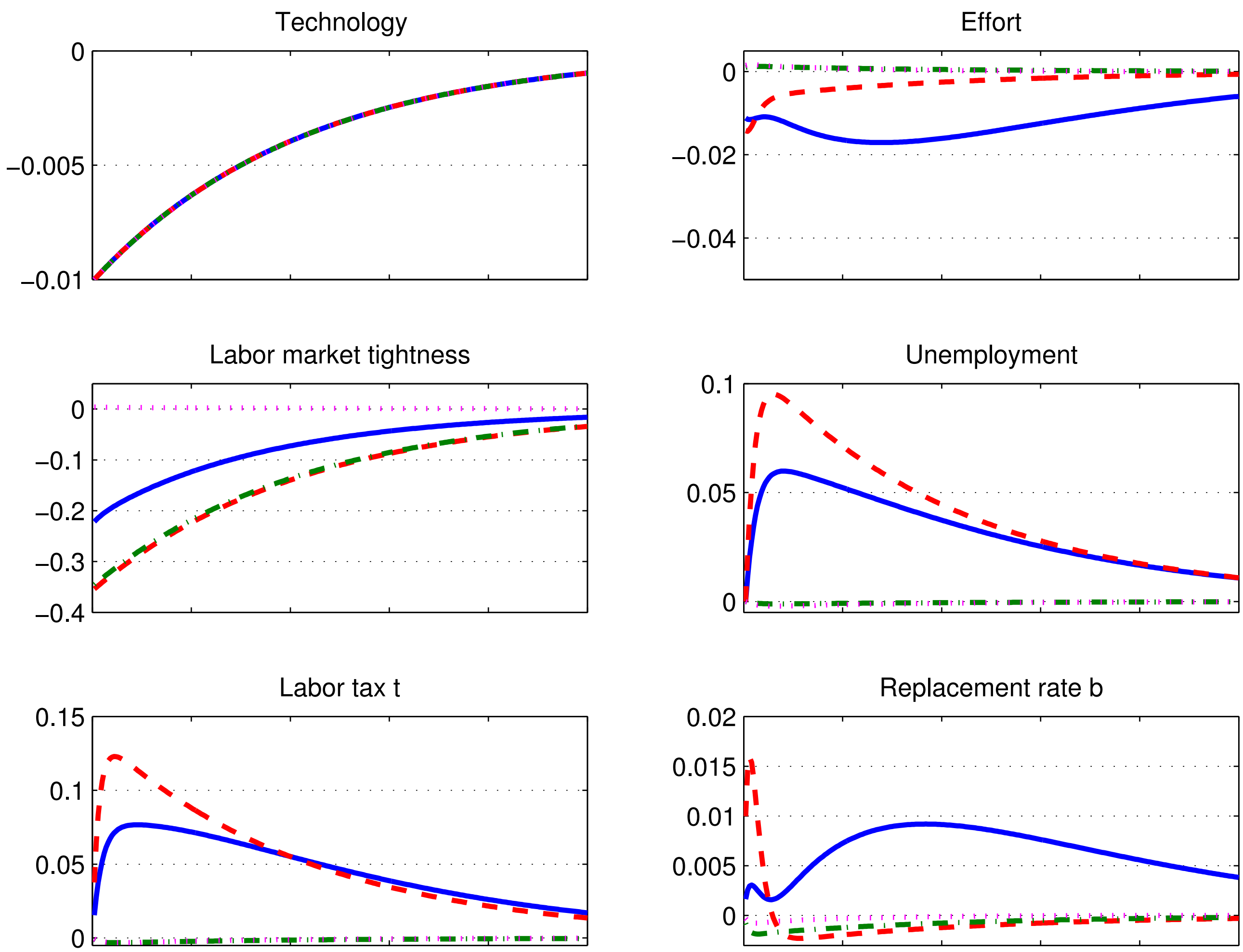

Figure 7: COMPARISON OF RESPONSES OF OPTIMAL UI PROGRAM TO A NEGATIVE TECHNOLOGY SHOCK ACROSS MODELS

Notes: This figure displays impulse response functions (IRFs), which represent the logarithmic deviation from steady state for each variable. IRFs are obtained by imposing a negative technology shock $z_1 = -0.01$ to the log-linear model (about 4 times the standard deviation 0.0026). The time period displayed on the x-axis is 250 weeks. The blue (solid) IRFs are in our base model ($\alpha = 0.67, \eta = 0.7, \gamma = 0.5$). The red (dashed) IRFs are in a model with $\alpha = 1$ (no diminishing returns to labor). The green (dot-dashed) IRFs are in a model with $\eta = 1$. The magenta (dotted) IRFs are in a model with $\gamma = 1$ (no wage rigidity). The complete log-linear system is described in appendix.

increase in the net replacement rate τ. On impact, the net replacement rate increases slightly, and it builds steadily for 80 weeks. At its peak, the net replacement rate τ increases by about 1.3%. Consumption of unemployed workers C^u drops on impact as a consequence of lower income per worker and then rises. C^u becomes higher than its steady-state level after 40 weeks as a consequence of a higher replacement rate. It then remains above its steady-state level until the economy converges back to the steady state. To summarize, the impulse responses confirms that the optimal UI is more generous in response to an adverse economic shock: when unemployment peaks, net replacement rate, replacement rate, and even consumption of unemployed workers are above their steady-state levels. In the economy with constant net replacement rate, labor tax increases and replacement rate decreases in response to a negative shock, to maintain budget balance. As a consequence, and combined with the lower wage, C^u drop on impact after a negative technology shock, before slowly recovering to its steady-state level.

Figure 6 also displays the behavior of labor market variables. Unemployment builds slowly and peaks after about 30 weeks. The labor market tightness $\theta = V/(U \cdot E)$ drops on impact, reflecting the reduction in hiring by firms. Search efforts drop on impact and decrease further over time, in response to both higher benefits and lower labor market tightness. Compared to an economy with constant net replacement rate, higher net replacement rate reduces search efforts. While a higher replacement rate does not increase the amplitude of the peak of unemployment (around week 50), it delays the recovery and imposes higher unemployment than in the economy with constant net replacement rate between week 50 and week 250.

Comparing Figure 6 to Figure 3 suggests that results in the dynamic and static frameworks are broadly consistent. In the dynamic framework, an increase in unemployment from 6% to 7%—that is, a 15% increase from steady state, about 3 times the increase displayed in Figure 6—should be accompanied by an increase in the net replacement rate τ from 76% to 80%—that is, a 4% increase from steady state. This increase is consistent with the slope of the plot of net replacement rate as a function of unemployment rate on Figure 3: when unemployment increases from 6% to 7%, the net replacement rate increases broadly from 75% to 79%.

Next, we compare the dynamic behavior of our baseline model with that of three variants: a model without job rationing ($\alpha = 1$), a model in which job-finding probability $f(\theta)$ is constant ($\eta = 1$), and a model without wage rigidity ($\gamma = 1$). These models are calibrated following the strategy described in Section 3.3. The steady-state optimal equilibria differ across models. The steady-state equilibrium does not depend on wage rigidity γ, since wage rigidity only affects the dynamics of the model. Thus, the model with $\gamma = 1$ has the same steady-state optimal equilibrium as our baseline model. In a model with $\alpha = 1$, as explained in Section 2.4.3, there is no job-rationing externality, macro- and micro-elasticity coincide, and the Baily-Chetty formula (17) applies. Hence, it is socially optimal to reduce the net replacement rate to $\overline{\tau} = 56\%$, increasing search efforts to $\overline{e} = 1.21$ and reducing unemployment to $\overline{u} = 4.9\%$. In a model with $\eta = 1$, as explained in Section 2.4.5, there is no job-rationing externality either, and macro- and micro-elasticity also coincide. Hence, it is optimal to reduce the net replacement rate to $\overline{\tau} = 59\%$, increasing efforts to $\overline{e} = 1.18$ and reducing unemployment to $\overline{u} = 5.0\%$.

Figure 7 compares the IRFs across the four models. The dynamics of the optimal equilibrium differ starkly across models. The dynamics of our baseline model are described above. We reproduce them as a benchmark. When $\gamma = 1$, the optimal equilibrium does not respond to technology shocks because wage and recruiting cost are fully flexible. In particular, replacement rate and labor tax remain constant. When $\eta = 1$, the UI program does not respond to technology shocks because the policy trade-off is independent from technology, as discussed in Section 2.4.5. As effort and unemployment are solely determined by $u(C^e) - u(C^u)$, they do not fluctuate either. Only labor market tightness θ responds to the technology shock so that labor demand equals labor supply. When $\alpha = 1$, as pointed out in Michaillat (2010), unemployment responds more strongly to a technology shock than in our model with $\alpha < 1$. The optimal replacement rate jumps on impact before decreasing rapidly to its steady-state level: in order to smooth recruiting, the government reduces unemployed workers' search effort on impact when firms substitute recruiting intertemporally from the future to the present to take advantage of a slack labor market. Labor tax builds up and remains above steady state for a long time to maintain a balanced budget each period when unemployment is above steady state.

4 Relation to the Literature

Our paper is related to a large normative literature that analyzes optimal UI theoretically and numerically.[28] Following the work of Baily (1978), a theoretical literature in public economics and macroeconomics has studied optimal UI in search models in which there is a trade-off between insurance and incentives to search (see Fredriksson and Holmlund (2006) for a recent survey). Papers have also analyzed how optimal benefits should vary over the duration of the unemployment spell (Hopenhayn and Nicolini 1997; Kocherlakota 2004; Shavell and Weiss 1979; Shimer and Werning 2008). Studies have simulated optimal UI in calibrated models considering various UI tools (Davidson and Woodbury 1997; Fredriksson and Holmlund 2001; Hansen and Imrohoroglu 1992; Lentz 2009; Wang and Williamson 1996, 2002). Other papers have characterized optimal UI when unemployment benefits distort wages (Cahuc and Lehmann 2000; Coles and Masters 2006). However, none of these papers take business cycle fluctuations into account.

A number of papers have also considered models with externalities and their consequences for optimal UI. Diamond (1981) shows that, if the distribution of job offerings becomes more attractive when there are more vacancies and more unemployment, then the steady-state equilibrium is not efficient and UI can restore efficiency by making workers more selective in the jobs they accept. Acemoglu (2001) develops a model of noncompetitive labor markets in which good and bad jobs coexist, and in which UI can shift employment toward good jobs and improve efficiency. Marimon and Zilibotti (1999) develop a model in which UI reduces employment but also helps workers to get a suitable job. These three papers assume risk neutrality so UI is just a subsidy for searching longer and improving the quality of job-worker matches. Acemoglu and Shimer (1999) show that, with risk aversion, UI induces workers to seek high-wage jobs with high unemployment risk, and hence improves both risk sharing and output. Spinnewijn (2010) extends the Baily model to the case where unemployed workers have biased beliefs regarding future employment, which calls for corrective UI over and above the traditional Baily formula. Kroft (2008) considers a model of

[28] There is also a large positive literature that studies empirically and theoretically the effects of UI on the behavior and outcomes of jobseekers, unemployment duration, unemployment rate, wages, and other aspects of the labor market. Section 2.2.3 presents some of the empirical studies. (for example, Andolfatto and Gomme 1996; Brown and Ferrall 2003; Gomes et al. 2001; Ljungqvist and Sargent 1998, 2008) present theoretical and calibrated analyzes of the effect of UI on aggregate labor market variables.

optimal UI with endogenous take-up driven in part by social interactions that create an externality. He extends the Baily-Chetty formula and shows that the macro-elasticity is the relevant one and that the externality requires an additional correction to the formula. In contrast to these studies, our paper zooms on an externality due to endogenous job rationing that is inherently tied to the business cycle.

A few recent papers study optimal UI over the business cycle when the government faces a trade-off between insurance and job-search incentives.[29] Kiley (2003) and Sanchez (2008) use partial equilibrium models in which benefits are posited to have less distortionary effects in downturns than in booms. In contrast, we construct a model in which such a pattern arises endogenously in general equilibrium. Using general equilibrium models with matching frictions in the labor market, Andersen and Svarer (2010, 2011) and Moyen and Stahler (2009) find countercyclical optimal benefits when the government is not constrained to balance its budget each period, but faces an intertemporal budget constraint instead. In these models, optimal UI is countercyclical because the government uses UI to smooth workers' consumption over the cycle.[30] In contrast, we impose a period-by-period budget balance so that the government cannot use UI for intertemporal consumption smoothing through deficit spending. In spite of this restriction, we find that optimal UI is countercyclical. Kroft and Notowidigdo (2010) propose a model, close in spirit to the traditional Baily model, in which the elasticity of unemployment duration with respect to benefits, and accordingly optimal unemployment benefits, may vary over the business cycle. Since the cyclicality of elasticity ambiguously depends on the parameters of the model, they propose an empirical estimation. Variations of optimal UI come from variations in the micro-elasticity in their Baily formula. In contrast, in our model, the micro-elasticity of unemployment with respect to net reward from work is roughly constant; the countercyclicality of optimal UI comes from the procyclicality of the macro-elasticity, which arises from the presence of job rationing.

[29] Other papers study optimal UI over the business cycle when the government faces a trade-off between providing insurance to unemployed workers and providing recruiting incentives to firms. In these papers, the policy problem studied is different from ours: there is no moral hazard, because workers search for jobs with constant effort; but UI distorts the recruiting behavior of firms (for example, Beaudry and Pages 2001; Costain and Reiter 2005).

[30] In fact, Andersen and Svarer (2010) find that optimal benefits should be *procyclical* when they derive comparative statics in the static version of the model (in which risk sharing through intertemporal substitution of consumption is not possible).

5 Conclusion

This paper analyzes optimal unemployment insurance over the business cycle. We model unemployment as the result from matching frictions (in good times) and job rationing (in bad times). Our model captures the intuitive notion that jobs are scarce during a recession, while retaining the core structure of standard search models. Our central result is that the optimal replacement rate is higher during recessions. We prove this result theoretically with a simple optimal UI formula expressed in terms of micro- and macro-elasticity of unemployment with respect to net reward from work, and risk aversion. Numerical simulations of our model calibrated with US data show that variations of the optimal replacement rate are quantitatively large over the cycle.

There are a variety of models with job rationing. Here, we present only one possible source of job rationing: the combination of real wages that only partially adjust to productivity shocks with diminishing marginal returns to labor. We showed that our optimal UI formula can be expressed in terms of sufficient statistics, and that the cyclical behavior of these statistics drove the properties of optimal UI. Since the three fundamental properties of our sufficient statistics—ε^m is acyclical, the wedge $(\varepsilon^m - \varepsilon^M)$ is positive, and the wedge $(\varepsilon^m - \varepsilon^M)$ is countercyclical—are robust to the origin of job rationing, the countercyclicality of the optimal replacement rate is a general property, independent from the specific source of job rationing.

This paper is a first attempt at providing a general-equilibrium framework to study optimal unemployment insurance over the business cycle. Our analysis should be extended in various directions in future work. First and most important, our key economic mechanism hinged crucially on a positive and countercyclical gap between micro- and macro-elasticity. Although there is a large empirical literature on the effects of unemployment insurance on unemployment duration, to our knowledge, no study has estimated separately micro- and macro-elasticity, and the cyclicality of gap between the two. While the existing literature provides evidence consistent with our model and we have indeed shown basic preliminary evidence in this direction, we think that providing more compellingly identified empirical evidence is the most urgent step to test the validity of our normative predictions, and provide most realistic numerical simulations solidly grounded on those estimated elasticities. Conceptually, this test is also important to distinguish between models of unemployment fluctuations without job rationing ($\alpha = 1$ as in Hall (2005)) and models with job rationing ($\alpha < 1$ as in Michaillat (2010)), which have very different policy implications.

Second, the model is simplistic in that there are only technology shocks. Future work should explore how other shocks (such as demand shocks or financial disturbances) influence optimal UI. We conjecture that our reduced-form formulas expressed in terms of the micro- and macro-elasticity of unemployment are likely to carry over to a model with other shocks. A gap between the two elasticities will continue to be a symptom of job rationing.

Finally, we could allow a broader and more realistic set of UI tools. In most OECD countries, the government chooses both level and duration of UI. Indeed, in the United States and other countries, the debate about the generosity of UI benefits during recessions focuses primarily on the duration of benefits. Our analysis could be fruitfully extended to a setting in which more generous unemployment insurance implies both higher and longer unemployment benefits.

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Appendix (not for publication)

A Proofs

A.1 Proof of Proposition 1

By definition, we have:

$$\varepsilon^M - \varepsilon^m = \frac{\Delta C}{1-N} \cdot \left[\frac{\partial N^s}{\partial \theta} + \frac{\partial N^s}{\partial E}\frac{\partial E}{\partial \theta}\right] \cdot \frac{d\theta}{d\Delta C}. \tag{A1}$$

The supply equation (2), $N^s(\theta, E) = E \cdot f(\theta)/[s + (1-s) \cdot E \cdot f(\theta)]$ implies that $U = 1 - (1-s) \cdot N = s/[s + (1-s) \cdot E \cdot f(\theta)]$, and hence

$$\frac{\partial N^s}{\partial E} = \frac{s \cdot f(\theta)}{[s + (1-s) \cdot E \cdot f(\theta)]^2} = U \cdot \frac{N}{E}, \tag{A2}$$

$$\frac{\partial N^s}{\partial \theta} = \frac{s \cdot E \cdot f'(\theta)}{[s + (1-s) \cdot E \cdot f(\theta)]^2} = U \cdot (1-\eta) \cdot \frac{N}{\theta}, \tag{A3}$$

where $1 - \eta = \theta f'(\theta)/f(\theta)$ is the elasticity of $f(\theta)$ with respect to θ which is constant with a Cobb-Douglas matching function. So we can rewrite (A1) as

$$\varepsilon^M - \varepsilon^m = \frac{\Delta C}{1-N} \cdot U \cdot \frac{N}{\theta} \cdot \left[1 - \eta + \frac{\theta}{E}\frac{\partial E}{\partial \theta}\right] \cdot \frac{d\theta}{d\Delta C}.$$

Using the labor demand equation (6), $F'(N) = W(a) + \frac{s \cdot r \cdot a}{q(\theta)}$, we have $F'' \cdot dN = -d\theta \cdot s \cdot r \cdot aq'(\theta)/q(\theta)^2 = (d\theta/\theta) \cdot (F' - W) \cdot \eta$ where $\eta = -\theta q'(\theta)/q(\theta)$ is minus the elasticity of $q(\theta)$. Therefore, $d\theta/dN = [F''/(F'-W)]\,(\theta/\eta) = -\,[(1-\alpha)/N]\,[F'/(F'-W)]\,(\theta/\eta)$ where $1 - \alpha = -NF''/F'$ is minus the elasticity of F' and constant in the Cobb-Douglas case. Hence, we have

$$\frac{N}{\theta}\frac{d\theta}{dN} = -\frac{1-\alpha}{\eta} \cdot \frac{F'}{F'-W}$$

$$\frac{d\theta}{d\Delta C} = \frac{d\theta}{dN} \cdot \frac{dN}{d\Delta C} = -\frac{1-\alpha}{\eta} \cdot \frac{F'}{F'-W} \cdot \frac{\theta}{N} \cdot \frac{1-N}{\Delta C} \cdot \varepsilon^M.$$

Finally, the individual first-order condition (4) for E defines implicitly $E(\Delta u, \theta)$ with

$$\frac{\Delta u}{E} \cdot \frac{\partial E}{\partial \Delta u} = \frac{U}{\kappa} + \frac{1-U}{\kappa+1}, \tag{A4}$$

$$\frac{\theta}{E} \cdot \frac{\partial E}{\partial \theta} = (1-\eta) \cdot \frac{U}{\kappa}. \tag{A5}$$

Combining those equations, we obtain

$$\varepsilon^M - \varepsilon^m = -\frac{1-\eta}{\eta} \cdot (1-\alpha) \cdot U \cdot \frac{1}{1-W/F'} \cdot \left(1+\frac{U}{\kappa}\right) \cdot \varepsilon^M.$$

This proves item (ii) in Proposition 1. We define

$$R(a,\Delta u) \equiv \varepsilon^m/\varepsilon^M = 1+\frac{1-\eta}{\eta} \cdot (1-\alpha) \cdot U \cdot \frac{1}{1-W/F'} \cdot \left(1+\frac{U}{\kappa}\right).$$

We have

$$\frac{dN}{d\Delta C} = \frac{\partial N^s}{\partial E}\frac{dE}{d\Delta C} + \frac{\partial N^s}{\partial \theta}\frac{d\theta}{d\Delta C} = \frac{N \cdot U}{E}\frac{dE}{d\Delta C} - \frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}\frac{dN}{d\Delta C},$$

and hence,

$$\frac{dN}{d\Delta C} = \frac{N \cdot U/E}{1+\frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}} \cdot \frac{dE}{d\Delta C},$$

$$\frac{d\theta}{d\Delta C} = -\frac{1-\alpha}{\eta}\frac{F'}{F'-W} \cdot \frac{\theta}{N} \cdot \frac{dN}{d\Delta C} = \frac{-\frac{1-\alpha}{\eta}U\frac{F'}{F'-W}(\theta/E)}{1+\frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}} \cdot \frac{dE}{d\Delta C}.$$

Therefore using (A5) and (A4):

$$\frac{dE}{d\Delta C} = \frac{\partial E}{\partial \Delta u}\frac{d\Delta u}{d\Delta C} + \frac{\partial E}{\partial \theta}\frac{d\theta}{d\Delta C} = \left(\frac{U}{\kappa}+\frac{1-U}{\kappa+1}\right)\frac{E}{\Delta u}\frac{d\Delta u}{d\Delta C} - \frac{\frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}\frac{U}{\kappa}}{1+\frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}} \cdot \frac{dE}{d\Delta C},$$

which implies

$$\frac{dE}{d\Delta C} = \frac{\left(1+\frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}\right)\left(\frac{U}{\kappa}+\frac{1-U}{\kappa+1}\right)}{1+\frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}\left(1+\frac{U}{\kappa}\right)} \cdot \frac{E}{\Delta u} \cdot \frac{d\Delta u}{d\Delta C},$$

$$\frac{dN}{d\Delta C} = \frac{\frac{U \cdot N}{\Delta u}\left(\frac{U}{\kappa}+\frac{1-U}{\kappa+1}\right)}{1+\frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}\left(1+\frac{U}{\kappa}\right)} \cdot \frac{d\Delta u}{d\Delta C}.$$

Now, we have

$$\frac{d\Delta u}{d\Delta C} = \frac{d(u'(C^u+\Delta C)-u'(C^u))}{d\Delta C} = u'(C^e)+\Delta u'\frac{dC^u}{d\Delta C}.$$

Using $C^u = N(W - \Delta C)$, this implies

$$\frac{d\Delta u}{d\Delta C} = \bar{u}' + \Delta u'(W - \Delta C)\frac{dN}{d\Delta C}$$

$$\frac{dN}{d\Delta C} = \frac{\frac{NU}{\Delta u}\left(\frac{U}{\kappa} + \frac{1-U}{\kappa+1}\right)\left(\bar{u}' + \Delta u'(W - \Delta C)\frac{dN}{d\Delta C}\right)}{1 + \frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}\left(1 + \frac{U}{\kappa}\right)},$$

$$\frac{dN}{d\Delta C} = \frac{NU \cdot \frac{\bar{u}'}{\Delta u}\left(\frac{U}{\kappa} + \frac{1-U}{1+\kappa}\right)}{1 + \frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}\left(1 + \frac{U}{\kappa}\right) - \frac{\Delta u' \cdot (W-\Delta C)}{\Delta u} \cdot NU \cdot \left(\frac{U}{\kappa} + \frac{1-U}{1+\kappa}\right)}.$$

Therefore,

$$\varepsilon^M = \frac{\frac{NU}{1-N} \cdot \frac{\bar{u}' \cdot \Delta C}{\Delta u}\left(\frac{U}{\kappa} + \frac{1-U}{1+\kappa}\right)}{1 + \frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}\left(1 + \frac{U}{\kappa}\right) - \frac{\Delta u' \cdot (W-\Delta C)}{\Delta u} \cdot NU \cdot \left(\frac{U}{\kappa} + \frac{1-U}{1+\kappa}\right)}$$

$$\varepsilon^M = \frac{\bar{u}' \cdot \Delta C \cdot [U/(1-N)]}{(\kappa+1) \cdot R(a, \Delta u) \cdot \Delta u \cdot \frac{1}{N} \cdot \left[1 + \frac{U}{\kappa}\right]^{-1} - \Delta u' \cdot (W - \Delta C) \cdot U}$$

Finally, using (ii) in Proposition 1, we have,

$$\varepsilon^m = \frac{\bar{u}' \cdot \Delta C \cdot [U/(1-N)]}{(\kappa+1) \cdot \Delta u \cdot \frac{1}{N} \cdot \left[1 + \frac{U}{\kappa}\right]^{-1} - \Delta u' \cdot (W - \Delta C) \cdot U \cdot R(a, \Delta u)^{-1}}$$

Using the approximation, $1 - N << 1$ and $s << (1-N)/N$, we have $U = 1 - (1-s)N << 1$, $U/\kappa << 1$, $1 - N \simeq 1$, and $\bar{u}' \simeq u'(C^e)$. The second term in the denominator in ε^m is negligible (relative to the first term). Furthermore, $U/(1-N) = 1 - sN/(1-N) \simeq 1$ as $s << (1-N)/N$, implying $\varepsilon^m \simeq [u'(C^e) \cdot \Delta C/\Delta u]/(\kappa+1)$ and proving (i).

We show that $\partial R/\partial a < 0$ to prove (iii) in the proposition. We first state a lemma describing the response of the equilibrium to a change in technology (comparative statics) for a given UI Δu.

Let $T \equiv F'(N,a)/(F'(N,a) - W(a))$. Using the firm's optimal recruiting behavior (5), we can write

$$T(N, \theta) = \frac{F'(N,a)}{F'(N,a) - W(a)} = \frac{F'(N,a)}{s \cdot r \cdot a} q(\theta(a)) = \frac{\alpha}{s \cdot r} \cdot N^{\alpha-1} \cdot q(\theta).$$

LEMMA A1. *Fix the UI program $\Delta u > 0$. Let $a > 0$. In equilibrium, we have the following comparative-static results: $dN/da > 0$, $dU/da < 0$, $dE/da > 0$, $d\theta/da > 0$, and $dT/da < 0$.*

Proof. For a given UI program Δu, a worker's optimal search behavior (4) implicitly defines search effort as a function $E(\theta)$ such that $\partial E/\partial\theta > 0$. Firm's optimal recruiting behavior (6) implicitly defines labor demand as a function $N^d(a, \theta)$ such that $\partial N^d/\partial a > 0$ and $\partial N^d/\partial\theta < 0$. Equation (2) defines labor supply as a function $N^s(E(\theta), \theta)$ such that $\partial N^s/\partial E > 0$ and $\partial N^s/\partial\theta > 0$—that is, $dN^s/d\theta > 0$. The equilibrium condition $N^s(\theta) = N^d(a, \theta)$ implicitly defines labor market tightness as a function $\theta(a)$. Differentiating this condition with respect to a yields

$$\frac{dN^s}{d\theta} \cdot \frac{d\theta}{da} = \frac{\partial N^d}{\partial a} + \frac{\partial N^d}{\partial \theta} \cdot \frac{d\theta}{da}$$
$$\frac{d\theta}{da} = \frac{\partial N^d}{\partial a} \cdot \left[\frac{dN^s}{d\theta} - \frac{\partial N^d}{\partial \theta}\right]^{-1}.$$

Thus $d\theta/da > 0$. In equilibrium, $N(a) = N^s(\theta(a))$ so $dN/da > 0$ and $dU/da = -(1-s)(dN/da) < 0$. Since $E(a) = E(\theta(a))$, $dE/da > 0$. Since $\partial T/\partial\theta < 0$ and $\partial T/\partial N < 0$, $dT/da < 0$. □

Using Lemma A1, we can immediately conclude that $\partial R/\partial a < 0$.

A.2 Proof of Proposition 2

First, using $C^u = N(W - \Delta C)$,

$$\frac{dC^u}{d\Delta C} = (1-N) \cdot \frac{\tau}{1-\tau} \cdot \varepsilon^M - N.$$

Second, using the optimality condition (3), and the isoelastic assumption for $k(E)$, we can write

$$\Delta u + (1-s) \cdot k(E) = \Delta u \cdot \frac{\kappa + 1}{\kappa + U}.$$

Lastly, the combination of (A1), (A2), (A3), and (A5) yields

$$\frac{\partial N^s}{\partial \theta} \cdot \frac{d\theta}{d\Delta C} = \left(\varepsilon^M - \varepsilon^m\right) \cdot \frac{1-N}{\Delta C} \cdot \frac{\kappa}{\kappa + U}.$$

Reshuffling these terms in (14) and dividing the equation by $(1-N) \cdot \varepsilon^M \cdot \overline{u}'$ yields (15).

A.3 Proof of Proposition 3

Consider optimality condition (15). It can be written as

$$Q(\tau) = Z(a, \tau) \tag{A6}$$

with $a \in (0, +\infty)$ and $\tau \in [0, 1]$. For any a, we assume that (A6) admits a unique solution $\tau^*(a)$. Equivalently, we assume that $Q(\tau)$ and $Z(a, \tau)$ cross only once for $\tau \in [0, 1]$.

LEMMA A2. $\lim_{\tau \to 1} Q(\tau) = +\infty$ *and for any* $a > 0$, $\lim_{\tau \to 1} Z(a, \tau) = M < +\infty$

Proof. We consider two cases.

First case: $C^e/C^u \to K > 1$ Then $\Delta u = \ln(C^e/C^u) \to \ln(K) > 0$. In that case all variables are $\in (0,+\infty)$. Moreover, $\Delta C, \Delta u, \Delta u'$ are bounded away from zero. Accordingly, the elasticities ε^m and $\varepsilon^M \in (0,+\infty)$. Then $\lim_{\tau\to 1} Z(a,\tau) \in (0,+\infty)$.

Second case: $C^e/C^u \to 1$ Then $\Delta u = \ln(C^e/C^u) \to 0$, which complicates the analysis. We need to prove that $Q(a,\tau)$ converges to a finite limit. Since $\Delta u \to 0$, $U \to 1$, $E \to 0$, $N \to 0$, $\theta \to +\infty$. Hence $R(a,\Delta u) \to R \equiv 1 + (1-\eta)/\eta(1-\alpha)(\kappa+1)/\kappa$. Budget constraint imposes $(1-N)bW + N(1-t)W = NW$, or $t = b(1-N)/N$. Since $\tau = t + b$, $\tau = b/N$, so that $C^u = \tau NW$ and $C^e = [1-(1-N)\tau]W$. When $\tau \to 1$, $C^u \sim NW$ and $C^e \sim NW$. We have $U/(1-N) \to 1$, $(\kappa+1)(1+U/\kappa)^{-1} \to \kappa$, $N(W-\Delta C) \sim NW$, $\bar{u}' \sim u'(C^u) = 1/C^u$, $\bar{u}'\Delta C \sim \Delta C/C^u$, $\Delta u = \ln(C^e/C^u) \sim C^e/C^u - 1 = \Delta C/C^u$, $-\Delta u' = \Delta C/(C^e \cdot C^u)$ so that $-\Delta u' \cdot NW \sim \Delta C/C^u$. Accordingly, $\varepsilon^M/N \sim 1/(\kappa R+1)$. Moreover, $-\Delta u'/\bar{u}' \to 0$ when $\tau \to 1$, $(\varepsilon^m/\varepsilon^M - 1)\kappa(\kappa+1)/(\kappa+U)^2 \to (1-\eta)/\eta(1-\alpha)$, and $\bar{u}'\Delta C/\Delta u \sim 1$. Hence, $\lim_{\tau\to 1} Z(a,\tau) \in (0,+\infty)$. □

LEMMA A3. *Let $a > 0$ and let $\tau^*(a)$ be the unique solution to (A6). For all $\tau < \tau^*(a)$, $Q(\tau) < Z(a,\tau)$ and for all $\tau > \tau^*(a)$, $Q(\tau) > Z(a,\tau)$.*

Proof. Using the results from Lemma A2 and the single-crossing assumption. □

As the government budget is $b(1-N)W = tNW$, $1-N << 1$ implies that $t << 1$ and hence $C^u/C^e = b/(1-t) \simeq b + t = \tau$. Therefore, $\Delta u = \ln(C^e/C^u) = -\ln(\tau)$. We denote again $R(a,\tau) = \varepsilon^m/\varepsilon^M$. Using the approximation(10) for ε^m from Proposition 1, we can write the micro-elasticity as a function of τ:

$$\varepsilon^m(\tau) \simeq -\frac{1}{\kappa+1}\frac{1-\tau}{\ln(\tau)}.$$

Therefore, the approximated optimal formula (16) can be rewritten as:

$$\frac{\tau}{1-\tau} \simeq \frac{1}{\varepsilon^m(\tau)}\left\{R(a,\tau)\cdot\frac{1-\tau}{\tau} + \frac{1}{\kappa}(R(a,\tau)-1)\right\}.$$

We write the equilibrium condition as $Q(\tau) = \hat{Z}(a,\tau)$. From Proposition 1, we know that $\partial R(a,\tau)/\partial a < 0$ for all $\tau \in [0,1]$. We can use the result from Proposition 1 because the partial derivative with respect to a taking Δu as given is the same as the partial derivative with respect to a taking τ as given, since Δu depends *only on* τ *and not on* a. Therefore $\partial\hat{Z}/\partial a < 0$ for all τ.

Consider a decrease in technology from a to $a' < a$. $Q(\tau^*(a)) = \hat{Z}(a,\tau^*(a)) < \hat{Z}(a',\tau^*(a))$. Lemma A3 (which applies to $\hat{Z}$ if a' close enough to a, when our approximations are valid) implies that $\tau^*(a) < \tau^*(a')$. Thus, $\partial\tau^*/\partial a < 0$.

B Derivation of the Optimal Allocation in the Dynamic Model

B.1 Firm and household problem

The unconditional probability of observing an history a^t is given by the probability measure $\mu_t(a^t)$.

Representative firm: Endogenous layoffs never occur in equilibrium so the Lagrangian of the firm problem is

$$\mathcal{L} = \sum_{t=0}^{+\infty} \delta^t \sum_{a^t} \mu_t(a^t) \cdot \left\{ F(N_t^d, a_t) - W_t \cdot N_t^d - \frac{r \cdot a_t}{q(\theta_t)} \cdot \left[N_t^d - (1-s) \cdot N_{t-1}^d \right] \right\}.$$

I assume that the firm maximization problem is concave and admits an interior solution (which will always be the case in equilibrium). Immediately, we can show that employment N_t^d is determined by first-order condition (25).

Representative household: The Lagrangian of the household's problem is

$$\sum_{t=0}^{+\infty} \delta^t \sum_{a^t} \mu_t(a^t) \cdot \Big\{ - \left[1 - (1-s)N_{t-1}^s\right] \cdot k(E_t) + (1 - N_t^s) \cdot u(C_t^u) + N_t^s \cdot u(C_t^e)$$
$$+ A_t \left\{ \left[1 - (1-s) \cdot N_{t-1}^s\right] \cdot E_t f(\theta_t) + (1-s) \cdot N_{t-1}^s - N_t^s \right\} \Big\},$$

where $N_t^s(a^t)$ is the probability to be employed in period t after period a^t and $\{A_t(a^t)\}$ is a collection of Lagrange multipliers. The first-order condition with respect to effort in the current period e_t gives:

$$k'(E_t) = f(\theta_t) \cdot A_t.$$

The first-order condition with respect to expected employment status N_t^s yields

$$A_t = [u(C_t^e) - u(C_t^u)] + \delta(1-s)\mathbb{E}_t\left[k(E_{t+1})\right] + \delta \cdot (1-s) \cdot \mathbb{E}_t\left[A_{t+1}\left(1 - E_{t+1} f(\theta_{t+1})\right)\right]$$
$$\frac{k'(E_t)}{f(\theta_t)} = [u(C_t^e) - u(C_t^u)] + \delta \cdot (1-s) \cdot \mathbb{E}_t\left[\frac{k'(E_{t+1})}{f(\theta_{t+1})}\right] - \delta \cdot (1-s)(\kappa+1) \cdot \mathbb{E}_t\left[k(E_{t+1})\right] + \delta(1-s)\mathbb{E}_t\left[k(E_{t+1})\right]$$

Thus, the optimal effort function therefore satisfies the Euler equation (22).

B.2 Government's problem

The maximization of the government is over a collection of sequences $\{N_t(a^t), E_t(a^t), \theta_t(a^t), C_t^e(a^t), C_t^u(a^t),\ \forall a^t\}_{t=0}^{+\infty}$. We can form a Lagrangian:

$$\begin{aligned}
&\sum_{t=0}^{+\infty}\delta^t \sum_{a^t}\mu_t(a^t)\cdot\Bigg\{(1-N_t)\cdot u(C_t^u)+N_t\cdot u(C_t^e)-[1-(1-s)N_{t-1}]\cdot k(E_t)\\
&+A_t\left[F(N_t,a_t)-N_tC_t^e-(1-N_t)C_t^u-\frac{r\cdot a_t}{q(\theta_t)}[N_t-(1-s)\cdot N_{t-1}]\right]\\
&+B_t\left[[u(C_t^e)-u(C_t^u)]-\frac{k'(E_t)}{f(\theta_t)}+\delta(1-s)\mathbb{E}_t\left[\frac{k'(E_{t+1})}{f(\theta_{t+1})}\right]-\kappa\delta(1-s)\mathbb{E}_t\left[k(E_{t+1})\right]\right]\\
&+C_t\left[F'(N_t,a_t)-W_t-\frac{r\cdot a_t}{q(\theta_t)}+\delta(1-s)\mathbb{E}_t\left[\frac{r\cdot a_{t+1}}{q(\theta_{t+1})}\right]\right]\\
&+D_t\left[(1-(1-s)\cdot N_{t-1})\cdot E_t f(\theta_t)+(1-s)\cdot N_{t-1}-N_t\right]\Bigg\}
\end{aligned}$$

where $\{A_t(a^t), B_t(a^t), C_t(a^t), D_t(a^t),\ \forall a^t\}_{t=0}^{+\infty}$ are sequences of Lagrange multipliers, and

$$\mathbb{E}_t\left[X_{t+1}\right]=\sum_{a_{t+1}}\frac{\mu_{t+1}(a^t,a_{t+1})}{\mu_t(a^t)}X_{t+1}(a^t,a_{t+1})$$

is conditional expectation operator. Let $B_{-1}\equiv 0$ and $C_{-1}\equiv 0$. We rewrite the Lagrangian as:

$$\begin{aligned}
&\sum_{t=0}^{+\infty}\delta^t \sum_{a^t}\mu_t(a^t)\cdot\Bigg\{(1-N_t)\cdot u(C_t^u)+N_t\cdot u(C_t^e)-[1-(1-s)N_{t-1}]\cdot k(E_t)\\
&+A_t\left[F(N_t,a_t)-N_tC_t^e-(1-N_t)C_t^u-\frac{r\cdot a_t}{q(\theta_t)}[N_t-(1-s)\cdot N_{t-1}]\right]\\
&+B_t\left[[u(C_t^e)-u(C_t^u)]-\frac{k'(E_t)}{f(\theta_t)}\right]+B_{t-1}\cdot(1-s)\left[\frac{k'(E_t)}{f(\theta_t)}-\kappa\cdot k(E_t)\right]\\
&+C_t\left[F'(N_t,a_t)-W_t-\frac{r\cdot a_t}{q(\theta_t)}\right]+C_{t-1}\cdot(1-s)\left[\frac{r\cdot a_t}{q(\theta_t)}\right]\\
&+D_t\left[(1-(1-s)\cdot N_{t-1})\cdot E_t\cdot f(\theta_t)+(1-s)\cdot N_{t-1}-N_t\right]\Bigg\}
\end{aligned}$$

The first-order conditions of the government's problem with respect to $N_t(a^t)$ for $t \geq 0$ are

$$
\begin{aligned}
0 &= u(C_t^e) - u(C_t^u) + \delta(1-s)\mathbb{E}_t\left[k(E_{t+1})\right] \\
&\quad - D_t + (1-s)\mathbb{E}_t\left[D_{t+1}\cdot(1-E_{t+1}f(\theta_{t+1})\right] \\
&\quad + C_t \cdot F''(N_t, a_t) \\
&\quad + A_t\left\{F'(N_t,a_t) - (C_t^e - C_t^u) - \frac{ra_t}{q(\theta_t)}\right\} + (1-s)\delta\mathbb{E}_t\left[A_{t+1}\cdot\frac{ra_{t+1}}{q(\theta_{t+1})}\right] \\
D_t &= u(C_t^e) - u(C_t^u) + \delta(1-s)\mathbb{E}_t\left[k(E_{t+1})\right] + (1-s)\mathbb{E}_t\left[D_{t+1}\cdot(1-E_{t+1}f(\theta_{t+1})\right] \\
&\quad + C_t\cdot F''(N_t,a_t) + A_t\left\{W(a_t) - (C_t^e - C_t^u)\right\} + (1-s)\delta\mathbb{E}_t\left[(A_{t+1}-A_t)\cdot\frac{ra_{t+1}}{q(\theta_{t+1})}\right].
\end{aligned}
$$

The first-order conditions of the government's problem with respect to $E_t(a^t)$ for $t \geq 0$ are

$$
\begin{aligned}
0 &= -U_t\cdot k'(E_t) - B_t\frac{k''(E_t)}{f(\theta_t)} + (1-s)B_{t-1}\frac{k''(E_t)}{f(\theta_t)} - \kappa(1-s)B_{t-1}k'(E_t) + D_t\cdot U_t\cdot f(\theta_t) \\
0 &= -U_t\cdot k'(E_t) + \frac{k''(E_t)}{f(\theta_t)}\left((1-s)B_{t-1} - B_t\right) - \kappa(1-s)B_{t-1}k'(E_t) + D_t\cdot U_t\cdot f(\theta_t) \\
0 &= -(\kappa+1)U_t\cdot k(E_t) + \kappa\frac{k'(E_t)}{f(\theta_t)}\left((1-s)B_{t-1} - B_t\right) - \kappa(\kappa+1)(1-s)B_{t-1}k(E_t) + D_t\cdot\cdot E_t\cdot U_t\cdot f(\theta_t) \\
0 &= -\frac{D_t\cdot H_t}{(\kappa+1)k(E_t)} + U_t + \kappa\frac{1}{E_t f(\theta_t)}\left[B_t - (1-s)B_{t-1}\right] + \kappa(1-s)B_{t-1},
\end{aligned}
$$

where $B_{-1} = 0$. The first-order conditions of the government's problem with respect to $C_t^e(a^t)$ for $t \geq 0$ are

$$
A_t = u'(C_t^e)\cdot\left(1 + \frac{B_t}{N_t}\right).
$$

The first-order conditions of the government's problem with respect to $C_t^u(a^t)$ for $t \geq 0$ are

$$
A_t = u'(C_t^u)\cdot\left(1 - \frac{B_t}{1-N_t}\right).
$$

The first-order conditions of the government's problem with respect to $\theta_t(a^t)$ for $t \geq 0$ are

$$
\begin{aligned}
0 &= -A_t \cdot \eta \cdot \frac{r \cdot a_t}{f(\theta_t)} \cdot H_t + (1-\eta) B_t \frac{k'(E_t)}{\theta_t \cdot f(\theta_t)} - (1-\eta)(1-s) \cdot B_{t-1} \frac{k'(E_t)}{\theta_t \cdot f(\theta_t)} \\
&\quad - C_t \cdot \eta \cdot \frac{r \cdot a_t}{f(\theta_t)} + C_{t-1} \cdot (1-s) \cdot \eta \frac{r \cdot a_t}{f(\theta_t)} + D_t \cdot U_t \cdot (1-\eta) \cdot E_t q(\theta_t) \\
0 &= -A_t \cdot \frac{r \cdot a_t}{q(\theta_t)} \cdot H_t + \frac{1-\eta}{\eta} \frac{k'(E_t)}{f(\theta_t)} \left[B_t - (1-s) \cdot B_{t-1}\right] - \frac{r \cdot a_t}{q(\theta_t)} \left[C_t - (1-s) \cdot C_{t-1}\right] + D_t U_t \cdot \frac{1-\eta}{\eta} \cdot E_t f(\theta_t) \\
0 &= H_t \cdot \left(-A_t \cdot \frac{r \cdot a_t}{q(\theta_t)} + D_t \frac{1-\eta}{\eta}\right) + \frac{1-\eta}{\eta} \frac{k'(E_t)}{f(\theta_t)} \left[B_t - (1-s) \cdot B_{t-1}\right] - \frac{r \cdot a_t}{q(\theta_t)} \left[C_t - (1-s) \cdot C_{t-1}\right] \\
0 &= H_t \cdot \left(-A_t \cdot r \cdot a_t + D_t q(\theta_t) \frac{1-\eta}{\eta}\right) + \frac{1-\eta}{\eta} \frac{k'(E_t)}{\theta_t} \left[B_t - (1-s) \cdot B_{t-1}\right] - r \cdot a_t \left[C_t - (1-s) \cdot C_{t-1}\right],
\end{aligned}
$$

where $B_{-1} = 0$ and $C_{-1} = 0$. The following proposition summarizes the results.

PROPOSITION A1 (Optimal equilibrium)**.** *The optimal equilibrium $\{C_t^e, C_t^u, \theta_t, N_t, E_t\}_{t=0}^{+\infty}$ and the sequences of Lagrange multipliers from the government's problem $\{A_t, B_t, C_t, D_t\}_{t=0}^{+\infty}$ are characterized by the following constraints $\forall t \geq 0$:*

$$0 = F(N_t, a_t) - N_t \cdot C_t^e - (1-N_t) \cdot C_t^u - \frac{r \cdot a_t}{q(\theta_t)} \cdot H_t \tag{A7}$$

$$0 = \left[u(C_t^e) - u(C_t^u)\right] - \frac{k'(E_t)}{f(\theta_t)} + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{k'(E_{t+1})}{f(\theta_{t+1})}\right] - \kappa \cdot \delta \cdot (1-s) \cdot \mathbb{E}_t \left[k(E_{t+1})\right] \tag{A8}$$

$$0 = F'(N_t, a_t) - W(a_t) - \frac{r \cdot a_t}{q(\theta_t)} + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{r \cdot a_{t+1}}{q(\theta_{t+1})}\right] \tag{A9}$$

$$0 = U_t \cdot E_t \cdot f(\theta_t) - H_t, \tag{A10}$$

and the following first-order conditions with respect to N_t, C_t^e, C_t^u, E_t, θ_t respectively, $\forall t \geq 0$:

$$
\begin{aligned}
D_t &= u(C_t^e) - u(C_t^u) + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[k(E_{t+1})\right] + (1-s) \cdot \mathbb{E}_t \left[D_{t+1} \cdot (1 - E_{t+1} f(\theta_{t+1}))\right] \\
&\quad + C_t \cdot F''(N_t, a_t) + A_t \left[W(a_t) - (C_t^e - C_t^u)\right] + (1-s) \cdot \delta \cdot \mathbb{E}_t \left[(A_{t+1} - A_t) \cdot \frac{r a_{t+1}}{q(\theta_{t+1})}\right]
\end{aligned} \tag{A11}
$$

$$0 = -\frac{D_t \cdot H_t}{(\kappa+1) k(E_t)} + U_t + \kappa \frac{1}{E_t f(\theta_t)} \left[B_t - (1-s) B_{t-1}\right] + \kappa (1-s) B_{t-1} \tag{A12}$$

$$A_t = \left[\frac{N_t}{u'(C_t^e)} + \frac{1-N_t}{u'(C_t^e)}\right]^{-1} \tag{A13}$$

$$B_t = N_t \cdot (1-N_t) \cdot \left[\frac{1}{u'(C_t^e)} - \frac{1}{u'(C_t^u)}\right] \cdot A_t \tag{A14}$$

$$0 = H_t \left[D_t q(\theta_t) \frac{1-\eta}{\eta} - A_t r a_t\right] + \frac{1-\eta}{\eta} \frac{k'(E_t)}{\theta_t} \left[B_t - (1-s) B_{t-1}\right] - r a_t \left[C_t - (1-s) C_{t-1}\right] \tag{A15}$$

where $B_{-1} = 0$ and $C_{-1} = 0$.

COROLLARY A1 (Equivalence with one-period model)**.** *The optimal equilibrium in the dynamic model in the absence of aggregate shocks converges to the solution of the government's problem in the one-period model when the discount factor δ converges towards* 1.

Proof. The incentive-compatibility constraint in the one-period model is given by (3). Notice that, using $Ef(\theta)U = s \cdot N$,

$$\begin{aligned}
k'(E)\frac{E}{N} &= \frac{k'(E)}{f(\theta)}\frac{Ef(\theta)U}{U \cdot N} = s\frac{k'(E)}{f(\theta)}\frac{1}{U} \\
-(1-s)k(E) &= \kappa(1-s)k(E) - (1+\kappa)(1-s)k(E) \\
&= \kappa(1-s)k(E) - (1-s)\frac{k'(E)}{f(\theta)}\frac{Ef(\theta)U}{U} \\
&= \kappa(1-s)k(E) - s\frac{k'(E)}{f(\theta)}\frac{(1-s)N}{U} \\
k'(E)\frac{E}{N} - (1-s)k(E) &= s\frac{k'(E)}{f(\theta)}\left[\frac{1}{U} - \frac{(1-s)N}{U}\right] + \kappa(1-s)k(E) = s\frac{k'(E)}{f(\theta)} + \kappa(1-s)k(E)
\end{aligned}$$

So the incentive-compatibility constraint in the one-period model can be rewritten as (4). We can form a Lagrangian:

$$\begin{aligned}
\mathcal{L} = &-(1-(1-s)N) \cdot k(E) + (1-N) \cdot u(C^u) + N \cdot u(C^e) \\
&+ A\left[F(N,a) - NC^e - (1-N)C^u - \frac{r \cdot a}{q(\theta)}s \cdot N\right] \\
&+ B\left[[u(C^e) - u(C^u)] - s\frac{k'(e)}{f(\theta)} - \kappa(1-s)k(e)\right] \\
&+ C\left[F'(N,a) - W(a) - \frac{s \cdot r \cdot a}{q(\theta)}\right] \\
&+ D\left[(1-(1-s) \cdot N) \cdot Ef(\theta) - s \cdot N\right]
\end{aligned}$$

By inspection, it appears that the constant equilibrium solving the system of equations described in Proposition A1 for $\delta = 1$ and $a_t = a \forall t$ also solves constraints and first-order conditions associated with the maximization of the Lagrangian in the one-period model.If both optimization problems are convex, then they admit the same unique solution. □

B.3 Impulse response to unexpected, transitory, technology shock

We first characterize the steady state of the model, and then describe the log-linearized equilibrium conditions around this steady state. $\bar{x}$ denotes the steady-state value of variable X_t. The steady-state optimal equilibrium $\left\{\overline{c_e}, \overline{c_u}, \overline{n}, \overline{\theta}, \overline{A}, \overline{B}, \overline{C}, \overline{D}\right\}$ is characterized by Corollary **??** when $a = \overline{a} = 1$.

Moreover $\overline{h} = s\overline{n}$ and $\overline{u} = 1-(1-s)\overline{n}$. $\check{x}_t \equiv d\log(X_t)$ denotes the logarithmic deviation of variable X_t. The equilibrium is described by the following system of log-linearized equations:

- Definition of labor market tightness:

$$\check{u}_t + \check{e}_t + (1-\eta)\cdot\check{\theta}_t - \check{h}_t = 0$$

- Definition of unemployment:

$$\check{u}_t + \zeta\cdot\check{n}_{t-1} = 0$$

  where $\zeta = \frac{1-\overline{u}}{\overline{u}}$.

- Law of motion of employment (A10):

$$(1-s)\cdot\check{n}_{t-1} + s\cdot\check{h}_t - \check{n}_t = 0$$

- Resource constraint (A7):

$$\check{a}_t + \alpha\check{n}_t - \left\{q_1\cdot\left(\check{h}_t + \eta\cdot\check{\theta}_t + \check{a}_t\right) + q_2\cdot\left\{p_1\left(\check{n}_t + \check{ce}_t\right) + p_2\left(-\nu\check{n}_t + \check{cu}_t\right)\right\}\right\} = 0,$$

  with $q_1 = \frac{r}{q(\overline{\theta})}\cdot s\cdot\overline{n}^{1-\alpha}$, $p_1 = \frac{\overline{n}\overline{c_e}}{(1-\overline{n})\overline{c_u}+\overline{n}\overline{c_e}}$, $\nu = \frac{\overline{n}}{1-\overline{n}}$, $q_2 = 1-q_1$, and $p_2 = 1-p_1$.

- Firm's Euler equation (A9):

$$-\check{a}_t + (1-\alpha)\cdot\check{n}_t + r_1\cdot\gamma\cdot\check{a}_t + r_2\cdot\left(\eta\cdot\check{\theta}_t + \check{a}_t\right) + r_3\mathbb{E}_t\left[\eta\cdot\check{\theta}_{t+1} + \check{a}_{t+1}\right] = 0$$

  with $r_1 = w_0\cdot\frac{1}{\alpha\cdot\overline{a}}\cdot\overline{n}^{1-\alpha}$, $r_2 = \frac{c}{q(\overline{\theta})}\cdot\frac{1}{\alpha}\cdot\overline{n}^{1-\alpha}$, and $r_3 = 1-r_1-r_2$.

- Productivity shock:

$$\check{a}_t = \rho\cdot\check{a}_{t-1} + z_t$$

- Household's Euler equation (A8):

$$\begin{aligned}\varepsilon_e s_1\check{ce}_t + \varepsilon_u s_2\check{cu}_t - \Bigg\{ t_2\left[\frac{1}{1-\delta(1-s)}\left[\kappa\check{e}_t - (1-\eta)\check{\theta}_t\right] - \frac{\delta(1-s)}{1-\delta(1-s)}\mathbb{E}\left[\kappa\check{e}_{t+1} - (1-\eta)\check{\theta}_{t+1}\right]\right] \\ + t_1(1+\kappa)\mathbb{E}\left[\check{e}_{t+1}\right]\Bigg\} = 0\end{aligned}$$

  where we define the elasticity of $u(\cdot)$ around steady-state $\varepsilon_i = \frac{d\ln(u(x))}{d\ln(x)}\Big|_{x=\overline{c_i}}$ and $s_1 = u(\overline{ce})/\overline{\Delta u}$, $s_2 = 1-s_1$, $t_2 = 1-t_1$, and $t_1 = \frac{\kappa\delta(1-s)k(\overline{e})}{\overline{\Delta u}}$.

- Lagrangian A_t defined by equation (A13):

$$\check{A}_t + u_1\left(\check{n}_t - \varepsilon'_e\check{ce}_t\right) + u_2\left(-\nu\check{n}_t - \varepsilon'_u\check{cu}_t\right) = 0$$

where we define the elasticity of $u(\cdot)$ around steady-state $\varepsilon_i' = \frac{d\ln(u'(x))}{d\ln(x)}\Big|_{x=\overline{c_i}}$ and where $u_1 = \frac{\overline{n}/u'(\overline{c_e})}{\overline{n}/u'(\overline{c_e})+(1-\overline{n})/u'(\overline{c_u})}$, and $u_2 = 1-u_1$.

- Lagrangian B_t defined by equation (A14):

$$\check{B}_t - \left[(1-\nu)\check{n}_t + \check{A}_t - \left(\varepsilon_e'\check{c}_{et}\right) - \left(\varepsilon_u'\check{c}_{ut}\right) + \left\{\varepsilon_e' v_1 \check{c}e_t + \varepsilon_u' v_2 \check{c}_{ut}\right\}\right] = 0$$

where $v_1 = \frac{u'(\overline{c_e})}{u'(\overline{c_e})-u'(\overline{c_u})}$, and $v_2 = 1-v_1$.

- Lagrangian D_t defined by equation (A12):

$$\check{D}_t + \check{u}_t + (1-\eta)\check{\theta}_t - \kappa\check{e}_t - \left[w_2\check{u}_t + w_3\check{B}_{t-1} - w_4\left[(1-\eta)\check{\theta}_t + \check{e}_t - \left\{\frac{1}{s}\check{B}_t - \frac{1-s}{s}\check{B}_{t-1}\right\}\right]\right] = 0$$

where $w_1 = \frac{\overline{u}\cdot\overline{D}\cdot f(\overline{\theta})}{k'(\overline{e})}$, and $w_2 = \overline{u}/w_1$, $w_3 = \kappa\cdot(1-s)\cdot\overline{B}/w_1$, $w_4 = 1-w_2-w_3$.

- Lagrangian C_t defined by equation (A15):

$$\check{h}_t + x_4\left(\check{A}_t + \check{a}_t\right) + x_5\left(-\eta\check{\theta}_t + \check{D}_t\right) - x_6\left[-\check{\theta}_t + \kappa\check{e}_t + \frac{1}{s}\check{B}_t - \frac{1-s}{s}\check{B}_{t-1}\right] - x_7\left[\check{a}_t + \frac{1}{s}\check{C}_t - \frac{1-s}{s}\check{C}_{t-1}\right] = 0$$

where $x_1 = \overline{A}r - q(\overline{\theta})\cdot\overline{D}\cdot\frac{1-\eta}{\eta}$, $x_2 = \frac{1-\eta}{\eta}\cdot s\cdot\overline{B}\cdot\frac{k'(\overline{e})}{\overline{\theta}}$, $x_3 = \cdot s\cdot r\cdot\overline{C}$, and $x_4 = \overline{A}\cdot r/x_1$, $x_5 = 1-x_4$, $x_6 = x_2/(x_1\overline{h})$, $x_7 = 1-x_6$.

- Optimality condition (A11) with respect to N_t

$$\begin{aligned}\check{D}_t - \Big\{ & y_1\left(\varepsilon_e z_1 \check{c}e_t + \varepsilon_u z_2 \check{cu}_t\right) + y_2(1+\kappa)\mathbb{E}\left[\check{e}_{t+1}\right] + y_3\mathbb{E}\left[\check{D}_{t+1} - z_6\left(\check{e}_{t+1} + (1-\eta)\check{\theta}_{t+1}\right)\right] \\ & + y_4\left(\check{C}_t + \check{a}_t + (\alpha-2)\check{n}_t\right) + y_5\left(\check{A}_t + \left\{z_3\gamma\check{a}_t + z_4\check{c}e_t + z_5\check{cu}_t\right\}\right) + y_6\mathbb{E}\left[\check{A}_{t+1} - \check{A}_t\right]\Big\} = 0\end{aligned}$$

where ε_i is defined as above and $z_1 = \frac{u(\overline{c_e})}{u(\overline{c_e})-u(\overline{c_u})}$, $y_1 = \frac{u(\overline{c_e})-u(\overline{c_u})}{\overline{D}}$, $y_2 = \delta\cdot(1-s)\cdot\frac{k(\overline{e})}{\overline{D}}$, $y_3 = (1-s)\cdot\left(1-\overline{e}f(\overline{\theta})\right)$, $z_3 = \frac{w_0}{w_0-(\overline{c_e}-\overline{c_u})}$, $z_4 = -\frac{\overline{c_e}}{w_0-(\overline{c_e}-\overline{c_u})}$, $y_4 = -\alpha\cdot(1-\alpha)\cdot\frac{\overline{C}\cdot\overline{n}^{\alpha-2}}{\overline{D}}$, $z_6 = \frac{\overline{e}f(\overline{\theta})}{1-\overline{e}f(\overline{\theta})}$, $y_6 = (1-s)\cdot\frac{\overline{A}}{\overline{D}}\cdot\frac{r\overline{a}}{q(\overline{\theta})}$, and $z_2 = 1-z_1$, $z_5 = 1-z_3-z_4$, $y_5 = 1-y_1-y_2-y_3-y_4$.

Once we have solved the log-linear system, we can recover steady-states $\overline{c},\overline{b},\overline{t},\overline{\tau}$ and log-deviations of the policy variables, as explained in Section 3.1:

$$\begin{aligned}\check{c}_t &= -\check{n}_t + \left(a_1\check{y}_t + a_2\left(\check{a}_t + \eta\check{\theta}_t\right) + \check{h}_t\right)\\ \check{t}_t &= -b_1\left(\check{c}_{et} - \check{c}_t\right)\\ \check{b}_t &= \check{c}_{ut} - \check{c}_t\\ \check{\tau}_t &= c_1\check{t}_t + c_2\check{b}_t\end{aligned} \tag{A16}$$

where $a_1 = \overline{y}/(\overline{nc})$, $a_2 = 1 - a_1$, $b_1 = (\overline{c_e}/\overline{c})/\overline{t}$, $c_1 = \overline{t}/\overline{\tau}$, $c_2 = 1 = c_1$.

B.3.1 Log-linear model under constant UI program

In that case τ is constant, and the government does not pick the UI program optimally. In the log-linear system, we eliminate the 4 Lagrange multipliers $\check{A}_t, \check{D}_t, \check{C}_t, \check{D}_t$ and 4 log-linear equations that give these multipliers. We also replace the equation giving the optimal UI program by an equation that ensures that τ remain constant: $\check{\tau}_t = 0$, where $\check{\tau}_t$ is a linear function of the log-deviations in the system, as described by (A16).

C Validity of the Model

In this section, we verify that the model provides a sensible description of reality by comparing important simulated moments to their empirical counterparts. We simulate a model in which the net replacement rate $\tau = 72\%$ is constant over time. This model describes an economy in which the UI program does not respond systematically to the business cycle (tax rate and replacement ratio adjust automatically to ensure budget balance). This net replacement rate allows to keep the same incentives to search $\Delta u = u(C^e) - u(C^u)$ as in the US economy, while having a balanced UI budget. Given the design of our calibration, the steady state of this model matches average US data very well: $\overline{u} = 5.9\%$, $\overline{v/u} = 0.47$, $\overline{e} = 1$.

We focus on second moments of the unemployment rate U, the vacancy/unemployment ratio V/U, real wage W, output Y, and technology a. Table A1 presents empirical moments in US data for the 1964:Q1–2009:Q2 period. Unemployment rate, output, and technology are described above. The real wage is quarterly, average hourly earnings for production and nonsupervisory workers in the nonfarm business sector constructed by the BLS Current Employment Statistics (CES) program, and deflated by the quarterly average of monthly Consumer Price Index (CPI) for all urban households, constructed by BLS. To construct a vacancy series for the 1964–2009 period, we merge the vacancy data for the nonfarm sector from JOLTS for 2001–2010, with the Conference Board help-wanted advertising index for 1964–2001.[31] We take the quarterly average of the monthly vacancy-level series, and divide it by employment to obtain a vacancy-rate series. We construct labor market tightness as the ratio of vacancy to unemployment. All variables are seasonally-adjusted, expressed in logs, and detrended with a HP filter of smoothing parameter 10^5.

Next, we perturb our log-linear model with i.i.d. technology shocks $z_t \sim N(0, 0.0026)$. We obtain weekly series of log-deviations for all the variables. We record values every 12 weeks for quarterly series (Y, W, a). We record values every 4 weeks and take quarterly averages for monthly series (U, U/V). We discard the first 100 weeks of simulation to remove the effect of initial conditions. We keep 50 samples of 182 quarters (2,184 weeks), corresponding to quarterly

[31] The Conference Board index measures the number of help-wanted advertisements in major newspapers. It is a standard proxy for vacancies (for example, Shimer 2005). The merger of both datasets is necessary because JOLTS began only in December 2000 while the Conference Board data become less relevant after 2000, owing to the major role played by the Internet as a source of job advertising.

data from 1964:Q1 to 2009:Q2. Each sample provides estimates of the means of model-generated data. We compute standard deviations of estimated means across samples to assess the precision of model predictions. Table A2 presents the resulting simulated moments. Simulated and empirical moments for technology are similar because we calibrate the technology process to match the data. All other simulated moments are outcomes of the mechanics of the model.

The fit of the model is good along several critical dimensions. First, the model amplifies technology shocks as much as observed in the data because the simulated standard deviation of unemployment (0.126), output (0.024) and of the vacancy-unemployment ratio (0.441) are comparable to the standard deviations estimated in the data (0.168, 0.029, and 0.344, respectively). The response of wages to technology shocks in the model and the data are quite close. A 1-percent decrease in technology decreases wages by 0.7 percent in the data, and 0.5 percent in our model. Third, simulated and empirical slopes of the Beveridge curve are almost identical. The slope, measured by the correlation of unemployment with vacancy, is -0.98 in the model and -0.89 in the data. Last, autocorrelations of all variables in the model match the data. As highlighted by Michaillat (2010), however, labor market variables and wages are too highly correlated with technology.[32]

D Robustness Checks

In this section, we evaluate the sensitivity of the results to our calibration. We examine how the the dynamic behavior of the model changes when we modify the calibration of the parameters shaping the utility function (σ, κ) and of the parameters influencing job rationing (α, γ). We first change the calibration of the utility function and study IRFs with less risk aversion $(\sigma = 0.5)$, more risk aversion $(\sigma = 2)$, a more elastic effort function $(\kappa = 0.9)$, and a more inelastic effort function $(\kappa = 3.6)$.Table A3 shows that the calibration of the scaling parameters $(\omega_m, \omega_k, w_0)$ is affected by the change in the utility parameters, because the scaling parameters are calibrated to match moments that depend both on scaling parameters *and* utility parameters. Since parameters differ, the steady-state optimal equilibrium also differ across these scenarios, as described in Table A3. Figure A1 shows, however, that the qualitative behavior of the model with these different calibrations remains unchanged. Quantitatively, the net replacement rate increases more after an adverse shock when workers are less risk-averse. The steady-state net replacement rate $\bar{\tau}$, however, is lower. The converse is true when workers are more risk-averse. A change in the elasticity κ of the search cost $k(e)$ has a small effect on the optimal UI. A higher κ slightly reduces the optimal increase in τ.

Next we change the calibration of parameters determining job rationing and study IRFs with more wage rigidity $(\gamma = 0.25)$, less wage rigidity $(\gamma = 0.75)$, more diminishing marginal returns to labor $(\alpha = 0.5)$, and less diminishing marginal returns to labor $(\alpha = 0.84)$. Table A3 describes the different calibrations and steady states across these scenarios. As shown in Figure A2, the qualitative behavior of the model with these different calibrations remains unchanged. Quantitatively, the net replacement rate increases more after an adverse shock when wages are more rigid or the production function has more diminishing marginal returns to labor . The converse is true when wages

[32]Demand shocks, financial disturbances, and nominal rigidities—absent from the model but empirically important—could explain these discrepancies.

Table A1: SUMMARY STATISTICS, QUARTERLY US DATA, 1964–2009.

	U	V/U	W	Y	a
Standard Deviation	0.168	0.344	0.021	0.029	0.019
Autocorrelation	0.914	0.923	0.950	0.892	0.871
Correlation	1	-0.968	-0.239	-0.826	-0.478
	–	1	0.220	0.828	0.479
	–	–	1	0.512	0.646
	–	–	–	1	0.831
	–	–	–	–	1

Notes: All data are seasonally adjusted. The sample period is 1964:Q1–2009:Q2. Unemployment rate U is quarterly average of monthly series constructed by the BLS from the CPS. Vacancy rate V is quarterly average of monthly series constructed by merging data constructed by the BLS from the JOLTS and data from the Conference Board, as detailed in the text. Vacancy-unemployment ratio V/U is the ratio of vacancy to unemployment. Real wage W is quarterly, average hourly earnings of production and non-supervisory workers in the private sector, constructed by the BLS CES program, and deflated by the quarterly average of monthly CPI for all urban households, constructed by BLS. Y is quarterly real output in the nonfarm business sector constructed by the BLS MSPC program. $\log(a)$ is computed as the residual $\log(Y) - \alpha \cdot \log(N)$ where N is quarterly employment in the nonfarm business sector constructed by the BLS MSPC program. All variables are reported in log as deviations from an HP trend with smoothing parameter 10^5.

Table A2: SIMULATED MOMENTS WITH TECHNOLOGY SHOCKS AND CONSTANT UI PROGRAM

	U	V/U	W	Y	a
Standard deviation	0.126	0.441	0.009	0.024	0.018
	(0.023)	(0.076)	(0.002)	(0.004)	(0.003)
Autocorrelation	0.936	0.909	0.877	0.894	0.877
	(0.023)	(0.031)	(0.040)	(0.035)	(0.040)
Correlation	1	-0.977	-0.985	-0.991	-0.985
	–	1	0.974	0.972	0.974
	–	–	1	0.999	1.000
	–	–	–	1	0.999
	–	–	–	–	1

Notes: Results from simulating the log-linearized model under constant UI program such that $\tau = 72\%$ with stochastic technology. All variables are reported as logarithmic deviations from steady state. Simulated standard errors (standard deviations across 50 simulations) are reported in parentheses.

Table A3: STEADY-STATE OPTIMAL EQUILIBRIUM ACROSS CALIBRATIONS

Calibration	Parameter values			Steady state			
	ω_m	ω_k	w_0	$\overline{\tau}$	$\overline{u}$	$\overline{e}$	$\overline{v/u}$
From Table 1	0.19	0.49	0.67	76%	6.1%	0.93	0.49
$\gamma \in [0,1]$	0.19	0.49	0.67	76%	6.1%	0.93	0.49
$\alpha = 1$	0.19	0.49	0.99	56%	4.9%	1.21	0.57
$\eta = 1$	0.15	0.49	0.67	59%	5.0%	1.18	0.40
$\sigma = 0.5$	0.19	0.74	0.67	76%	6.2%	0.92	0.49
$\sigma = 2$	0.19	0.69	0.67	80%	6.6%	0.81	0.52
$\kappa = 0.9$	0.19	0.65	0.67	76%	6.2%	0.90	0.49
$\kappa = 3.6$	0.19	0.41	0.67	78%	6.1%	0.94	0.49
$\alpha = 0.5$	0.19	0.49	0.50	79%	6.2%	0.90	0.50
$\alpha = 0.84$	0.19	0.49	0.84	68%	5.7%	1.05	0.47

are more flexible or marginal returns to labor do not diminish as much with employment. Furthermore, unemployment and vacancy-unemployment ratio respond much more to a technology shock when wages are more rigid (lower γ).

E Empirical Evidence from the CWBH

In this section, we give some details about the empirical evidence presented in section 2.2.3. Data used is from the Continuous Wage and Benefit History (CWBH) dataset used in Moffitt (1985) or Meyer (1990). The dataset records employment and unemployment history for a sample of males in 11 States from 1978 to 1983. The advantage of CWBH data is that it is administrative data with accurate information on weeks of UI receipt, pre-unemployment earnings, the level of UI benefits, and the potential duration of benefits over time. Our data contains 3,365 unemployment spells. Since we do not observe individuals after their benefits lapse, we censor their unemployment spells at the time they exhaust their benefits.

To investigate how micro and macro elasticities vary over the business cycle, we begin by defining for each spell whether it faces a low or a high unemployment regime in a way similar to Kroft and Notowidigdo (2010)[33]. For each State*month pair, we use the monthly CPS unemployment rate and compute the median unemployment rate for that given month among all States in the CPS. A spell is defined as being in a low unemployment regime if the monthly unemployment rate in the State at the beginning of the spell is below the median unemployment rate[34].

[33] Except they consider yearly base time periods instead of monthly base time periods.

[34] We also tested using the median unemployment rate in all US States, and using quarters instead of months as base time periods, without any loss of generality.

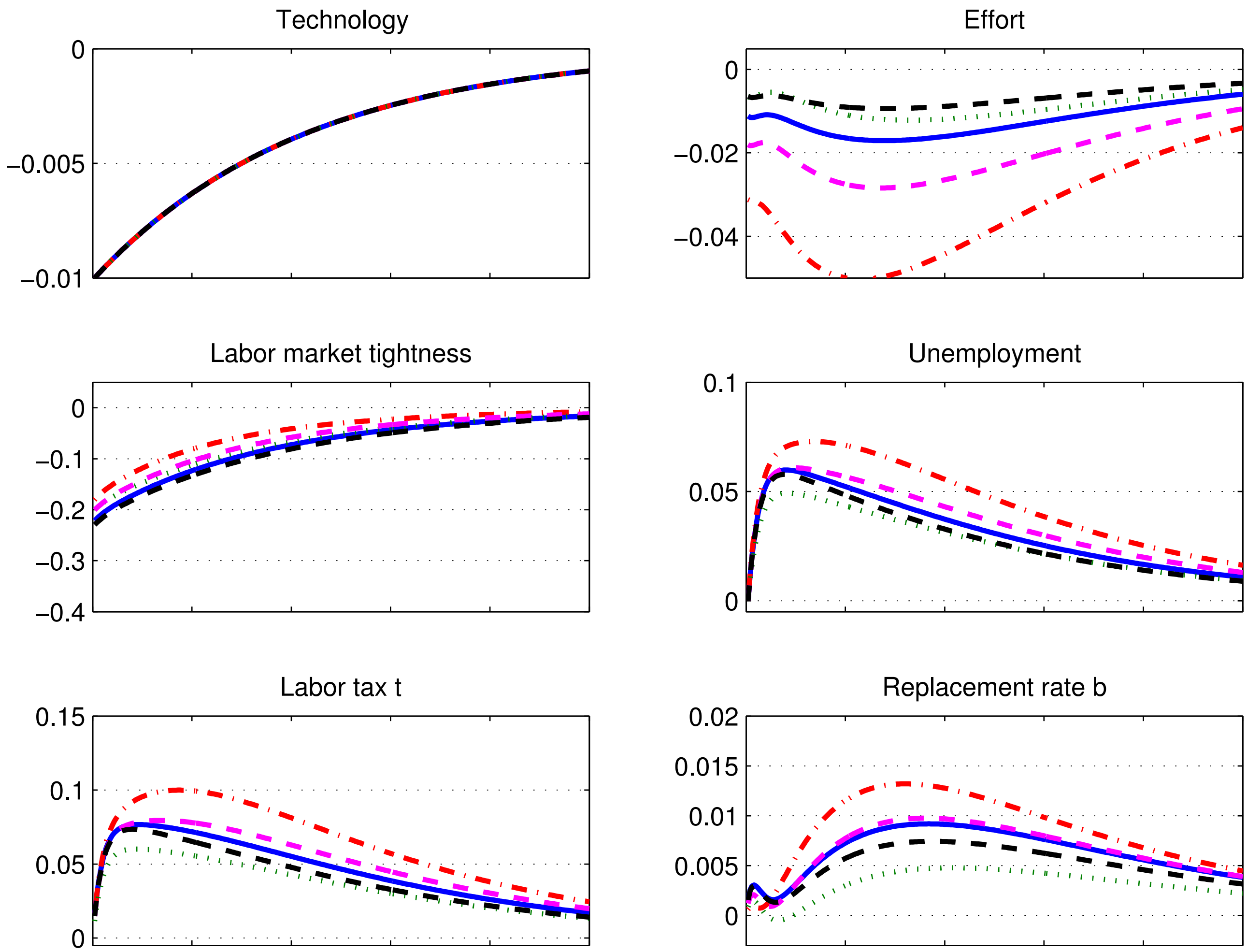

Figure A1: RESPONSES OF OPTIMAL UI TO A NEGATIVE TECHNOLOGY SHOCK FOR ALTERNATIVE UTILITY CALIBRATIONS

Notes: This figure displays impulse response functions (IRFs), which represent the logarithmic deviation from steady state for each variable. IRFs are obtained by imposing a negative technology shock $z_1 = -0.01$ to the log-linear model (about 4 times the standard deviation 0.0026). The time period displayed on the x-axis is 250 weeks. The blue IRFs are in our base model ($\sigma = 1, \kappa = 1.8$). The red (dot-dashed) IRFs are in a model with $\sigma = 0.5$ (less risk aversion). The green (dotted) IRFs are in a model with $\sigma = 2$ (more risk aversion). The magenta (dashed) IRFs are in a model with $\kappa = 0.9$ (larger micro-elasticity). The black (dashed) IRFs are in a model with $\kappa = 3.6$ (smaller micro-elasticity).

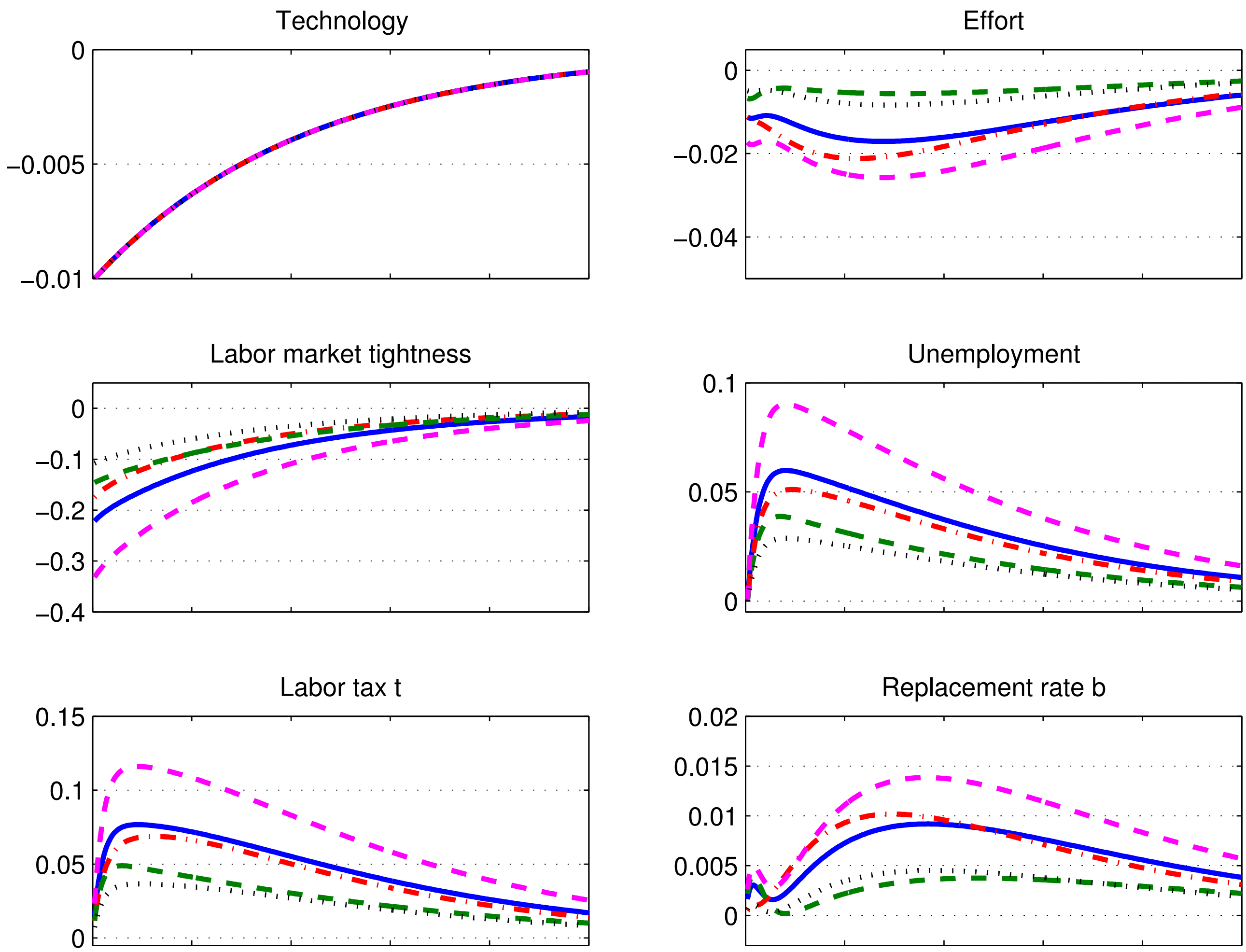

Figure A2: RESPONSES OF OPTIMAL UI TO A NEGATIVE TECHNOLOGY SHOCK FOR ALTERNATIVE JOB-RATIONING CALIBRATIONS

Notes: This figure displays impulse response functions (IRFs), which represent the logarithmic deviation from steady state for each variable. IRFs are obtained by imposing a negative technology shock $z_1 = -0.01$ to the log-linear model (about 4 times the standard deviation 0.0026). The time period displayed on the x-axis is 250 weeks. The blue IRFs are in our base model ($\sigma = 1, \kappa = 1.8$). The red (dot-dashed) IRFs are in a model with $\alpha = 0.5$ (more diminishing returns to labor). The green (dashed) IRFs are in a model with $\alpha = 0.84$ (less diminishing returns to labor). The magenta (dashed) IRFs are in a model with $\gamma = 0.25$ (more wage rigidity). The black (dotted) IRFs are in a model with $\gamma = 0.75$ (less wage rigidity).

We then begin by confirming the evidence found in Kroft and Notowidigdo (2010) that unemployment spells tend to be more negatively affected by the average level of benefits when unemployment is low than when unemployment is high. As discussed in section 2.2.3, this evidence is suggestive that the macro elasticity varies with labor market conditions. To do so, we display Kaplan-Meier survival estimates for spells in low and high unemployment regimes and break them down given the generosity of average benefits in the State in which the spell takes place. To determine high versus low generosity regimes, we compute the average weekly benefit amount (WBA) level for each State*month pair, and compute for a given month the median average WBA among all States in the CWBH. A spell is defined as being in a low generosity regime if the monthly average WBA in the State at the beginning of the spell is below the median average WBA. The results are presented in panel A of figure 2.

The evidence presented in panel A of figure 2 is totally nonparametric. To assess the robustness of our finding to compositional effects of the population of unemployed that may vary over the business cycle, we estimated a Cox proportional hazard model controlling for age, marital status, number of dependents, race, education, previous wage level, and a 6 pieces exhaustion spline to control for exhaustion spikes due to different potential durations. We estimated this model in low unemployment and high unemployment regimes separately, and stratified by high versus low generosity States. Formally, we fitted a model of the form $h(t|X,Z=j)=h_j(t)exp(X\beta)$ where the baseline hazard is allowed to vary across levels of generosity ($Z=1$ or $Z=0$). Importantly, we do not include State and month fixed effects among the regressors X to exploit variations across States and over time of average WBA, therefore capturing the macro elasticity. Figure A3 displays the baseline survival estimates from these regressions and confirm the findings presented in panel A of figure 2. In low unemployment regimes, high benefit States exhibit a significantly higher baseline unemployment survival rate, while there is no significant difference in baseline survival in high unemployment regimes between high and low benefit States.

Then, in the same Cox proportional hazard model, we introduce State and month fixed effects interacted, to fully control for across-States and over-time variations in UI generosity and therefore focus on the micro elasticity of duration with respect to benefits. We fitted a model of the form $h(t|X,Z=j)=h_0(t)exp(X\beta)$ estimated separately for low and high unemployment regimes, and in which we include the log of the individual level of benefits. Once we retrieved the estimated baseline hazard, we can easily plot the survival function estimated at the mean of the log benefit and at 110% of the mean of log benefit for all spells in the sample, as done in panel B of figure 2. The difference between low and high unemployment regimes disappears: at the micro level, the effect of higher benefits is the same in low and high unemployment regimes, with a micro elasticity of duration with respect to benefits of around .3 to .4. Notice that variation left for identification in these specifications with State and month fixed effects interacted is small, and smaller than that used by Meyer (1990) on the same dataset, thus explaining fairly large standard errors and slightly lower point estimates for the micro elasticity than the elasticities presented by Meyer (1990).

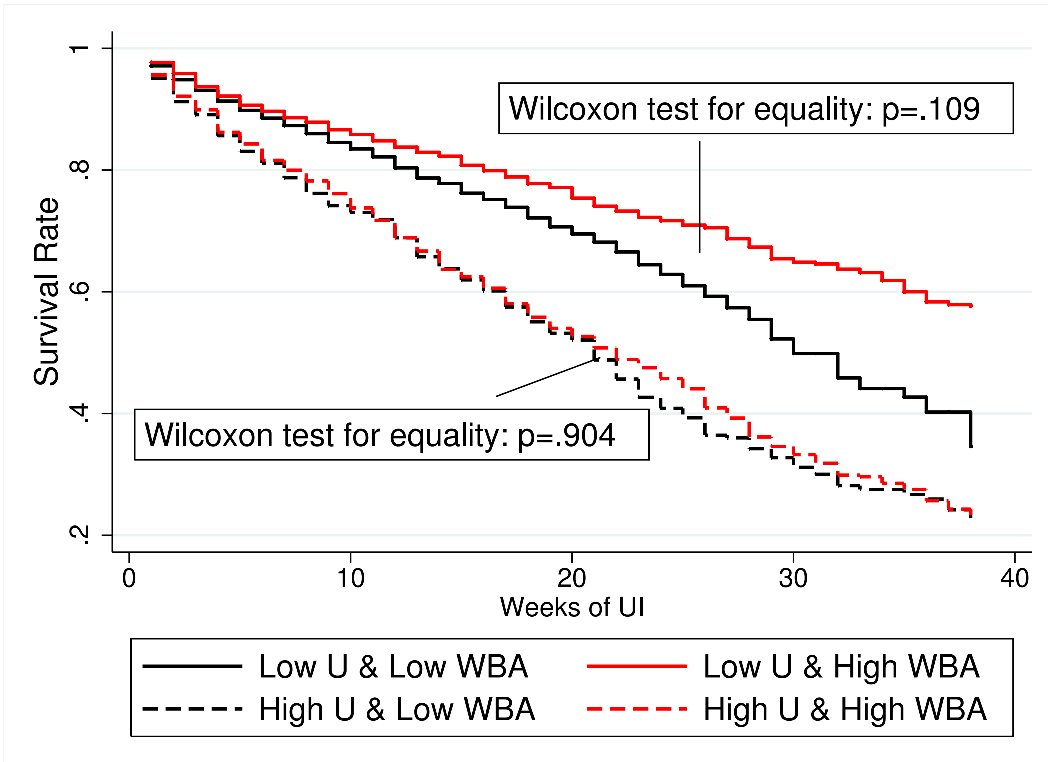

Figure A3: SURVIVAL ESTIMATES WITH NO STATE & TIME FIXED-EFFECTS
*Source:*CWBH
Notes: The figure displays the baseline survival function for the duration of unemployment estimated separately for high unemployment regimes (dark lines) and low unemployment regimes (red lines) and stratified by low (plain lines) vs high (dash lines) unemployment regimes. The estimated model includes controls for age, marital status, number of dependents, race, education, previous wage level, and a 6 pieces exhaustion spline to control for exhaustion spikes due to different potential durations. The model does not control for State fixed effects and exploits across-State and over-time variation in UI generosity, thus capturing the macro elasticity.