How Important Is Mispricing?

April 2011

Aydoğan Altı and Paul C. Tetlock*

University of Texas at Austin and Columbia University

Abstract

There is much controversy over whether risk or mispricing explains observed differences in firms' average stock returns. We contribute to this debate by proposing a quantitative model in which investors' information processing biases cause differences in firms' returns. The model matches many features of empirical data on firm production and asset pricing. We estimate the extent of biases that would reproduce empirical asset pricing anomalies and explore the implications of these biases. Our findings indicate that the magnitude of mispricing is several times larger than observed anomalies suggest. The model also provides novel insights into when and how information processing biases can cause substantial capital misallocation.

---

* We thank UT Austin and Columbia for research support. We appreciate helpful suggestions from Sheridan Titman and seminar participants at Berkeley, Stanford, UT Austin, and UNC Chapel Hill. Please send all correspondence to aydogan.alti@mccombs.utexas.edu and paul.tetlock@columbia.edu.

Many studies show that certain firms have higher average stock returns than others.[1] There is much controversy over whether these firms are riskier or whether they are mispriced, relative to their fundamental values. To investigate this question, several researchers propose, calibrate, and estimate models of risk-based explanations (e.g., Lettau and Wachter (2007) and Liu, Whited, and Zhang (2009)). In contrast, theoretical research on mispricing mainly focuses on qualitative predictions from stylized models.[2] Testing the risk and mispricing explanations on a level playing field requires models of mispricing that make quantitatively realistic predictions.

To fill this void, we develop a quantitative model of mispricing in which investors' information processing biases cause differences in firms' asset returns. The model generates endogenous relationships among firm investment, cash flow, valuation, and asset returns from exogenous variation in firm productivity and investors' information about productivity. We estimate the model parameters by matching key features of firms' production and asset return data. In particular, we identify the magnitude of information processing biases from observed differences in firms' asset returns—i.e., return "anomalies." The model allows us to quantify the investment and asset mispricing implications of these behavioral biases.

This analysis reveals that observed return anomalies are the tip of the iceberg: insofar as behavioral biases cause anomalies, unobservable asset mispricing is much larger than its observable proxies, such as the value anomaly. The reason is that empirical anomalies are poor proxies for the errors in investors' beliefs that determine the degree of mispricing. For example, although a firm's Tobin's Q is a common proxy for mispricing, only some of the variation in Q comes from errors in investors' beliefs. Much of the variation comes from the rational

[1] Respectively, Banz (1981), Bernard and Thomas (1989), and Fama and French (1992) show that stocks with small sizes, positive earnings news, and high ratios of book-to-market equity have positive risk-adjusted returns.
[2] Examples of theories in which errors in investors' expectations lead to return predictability include Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subramanyam (1998), and Hong and Stein (1999).

anticipation of varying firm productivity. Model simulations show that sorting by firms' (true) expected returns to a rational agent produces a difference in expected returns that is three times larger than the difference produced by sorting by firms' Q values.

Our model features managers who determine firm investment decisions and investors who value firm assets have the same beliefs. Firm productivity varies over time, but adjusting the capital stock is costly. Thus, if a firm's expected productivity increases, its manager dynamically responds by gradually increasing the firm's capital stock, generating persistence in the firm's growth opportunities.

Firm productivity is not observable, so agents must estimate it using two pieces of information: cash flow and a soft information signal. Cash flow is a noisy indicator of the firm's current productivity. The soft information signal summarizes all information about productivity that is not reflected in cash flow. In reality, some soft signals such as analyst forecasts or management guidance could be observable, whereas others such as investors' subjective interpretations of news are intangible and practically unobservable.

Agents in the model may suffer from two biases in estimating productivity: overconfidence and excessive extrapolation. Overconfident agents believe the soft signal precision to be higher than it actually is, as in Odean (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Scheinkman and Xiong (2003). Agents who excessively extrapolate believe the persistence of productivity to be higher than it actually is, as in Barberis, Shleifer, and Vishny (1998).[3] Previous research proposes that these two biases could explain why value

[3] Our model also allows for biases in which the agent *under*estimates the signal precision and the persistence of productivity. Our empirical estimates, however, are consistent with overconfidence and excessive extrapolation.

stocks earn higher returns than growth stocks.[4] Accordingly, we use the empirical value anomaly to help us identify the combined magnitude of the biases in our model. We empirically distinguish between the two biases using their opposing predictions for return predictability from cash flows. In estimating firm productivity, an overconfident agent pays more attention to the soft information signal than to cash flow, whereas an extrapolative agent places too much weight on cash flow.

We estimate model parameters using the simulated method of moments (SMM). Simulations based on the SMM parameter estimates closely replicate the relevant features of the empirical data, including firm investment, cash flows, and Tobin's Q, as well as asset return predictability. Twelve of the 13 empirical moments that the eight-parameter model attempts to match are within one standard error of the model's prediction. We cannot reject the hypothesis that the model fits all 13 moments at the 10% level. The SMM parameter estimates suggest that both information processing biases play a role in return predictability.

Our main finding is that asset mispricing can be much larger than its observable proxies, such as return predictability, would suggest. Model simulations allow us to measure empirical quantities that econometricians cannot typically observe, including each firm's true expected returns and mispricing. We find that no observable firm characteristic is able to explain even one quarter of the variance in true expected returns. Importantly, differences in firms' true expected returns of more than 1% per year persist after 10 years.

We investigate various model parameters and alternative specifications to understand the mechanism behind these findings. Experimenting with parameters reveals that the overconfidence bias, not the excessive extrapolation bias, explains why empirical anomalies are

[4] Beyond the theories already cited, empirical evidence consistent with extrapolation appears in Lakonishok, Shleifer, and Vishny (1994), La Porta (1996), La Porta et al. (1997), and Benartzi (2001). Further evidence on investor overconfidence appears in Odean (1999) and Barber and Odean (2001).

poor proxies for investors' beliefs. Extrapolative agents' biases are highly correlated with observable proxies for productivity. Specifically, Tobin's Q captures the variation in mispricing and cash flow can explain expected returns quite well. In contrast, the biases in overconfident agents' beliefs depend on the unobservable soft information signal. Firm characteristics, such as Q, cash flow, and investment, fail to explain the cross section of mispricing and expected returns induced by overconfidence.

Next we compare the return predictability from the benchmark model in which managers have the same biased beliefs as investors to the predictability from an alternative model in which managers are rational but investors remain biased. We find that the rational manager model incorrectly predicts that firm investment only weakly forecasts abnormal asset returns. The benchmark model, however, matches return predictability data well. We focus on this model in analyzing the consequences of managers' investment policies.

The benchmark model with biased managers implies large capital misallocation and inefficient investment. The aggregate value of invested capital is 9.5% lower than it would be if unbiased managers seized control of firms and determined real investment. The aggregate value *added* of invested capital in a steady state economy in which managers are biased is 64.5% lower than it is in a steady state economy in which managers are rational. Underlying these aggregate welfare costs of biased management, there is a wide range of inefficiencies across firms. Firms with high investment rates and low cash flows tend to overinvest the most. Intuitively, managers overinvest when investors are too optimistic about firm productivity, which typically occurs when the soft information signal is high relative to firm cash flows.

Further analysis shows that the excessive extrapolation and overconfidence biases play different roles in capital allocation. An overconfident manager invests too much when the signal is high and too little when it is low. These mistakes are costly, but they may not cause average investment to be too high or low. Consistent with this argument, simulations show that overconfident managers choose more volatile investment rates than rational managers, but their average investment rates are similar. In contrast, a manager that excessively extrapolates productivity trends puts too much value on the firm's future growth opportunities, believing that she can act on trends before they disappear. Consequently, she increases investment today to lower expected adjustment costs. Consistent with this reasoning, simulations indicate that extrapolative managers select higher average investment rates than rational managers. Both biases contribute to welfare costs, but overconfidence plays a greater role in most specifications.

This study's quantitative structural approach contrasts with the reduced form approach in several earlier studies that test the validity of their models' qualitative predictions. Influential examples of this approach include Baker, Stein, and Wurgler (2003), Gilchrist, Himmelberg, and Huberman (2005), and Polk and Sapienza (2009) among many others. These papers focus on how mispricing affects rational managers' investment decisions in the presence of financing constraints or incentives to cater to investors. Our model focuses on the simple case in which financing is frictionless and managers maximize the firm's current market value.

Most studies using structural methods do not jointly model investment behavior and asset prices and even fewer allow for information processing biases. For example, Hennessy and Whited (2007) employ a structural model to estimate the magnitude of corporate financing frictions, rather than asset pricing frictions. Nearly all of the studies that model firms' asset returns and investment decisions, such as Gomes, Yaron, and Zhang (2006) and Liu, Whited, and Zhang (2009), do not consider behavioral biases. For example, Liu, Whited, and Zhang (2009) show that the average stock returns on portfolios formed on size and value could be consistent with the optimality conditions from rational firms' investment choices. In their model, stocks are priced efficiently based on differences in risk and investment is efficient. In our model, stocks are priced inefficiently based on errors in investors' expectations and investment is inefficient.

Two of the only papers to model firms' stock prices and investment in the presence of mispricing are Chirinko and Schaller (2001) and Panageas (2005). Both use aggregate time series data and do not use data on the cross section of firms to estimate parameters, as we do. Panageas (2005) models mispricing using heterogeneous investors and short-sales constraints. In contrast, our model features homogeneous investors with no constraints that cause both pricing and investment inefficiencies. Thus, our insights and analyses are quite different.

An overview of our paper follows. Section I introduces and solves our model of firm investment dynamics and asset prices. Section II describes the empirical identification strategy and methodology. Section III presents the estimation results and evaluates the predictions from two model specifications against empirical benchmarks. Section IV analyzes the extent of mispricing and investment inefficiencies in simulated model economies under various specifications, including one in which managers are rational. Section V discusses and interprets the implications of our results for future research.

I. A Model of Investment and Asset Prices

A. Modeling Framework

This section presents the model that we empirically analyze in later sections. The model builds on the standard dynamic neoclassical framework by allowing for agents to exhibit information processing biases. There are two groups of agents in the economy: investors who price financial assets, and managers who make firms' investment decisions. All agents are riskneutral and have a discount rate of r. For most of the analysis, we assume that investors and managers have homogenous information and beliefs. In this case, a representative investor/manager effectively determines both asset prices and firms' investment policies.

All firms in the model share the same production technology, but their productivity at any given time may differ. Here we describe the model for an individual firm; later in our empirical analysis we consider a cross-section of such firms with differing productivity and capital. Because we do not model tax or financing frictions, capital structure is irrelevant. Without loss of generality, one can assume that investors finance firms using only equity.

We denote the continuous passage of time by t. The firm pays cash flows $d\pi_t$ given by

$$d\pi_t = (f_t dt + \sigma_a d\omega_t^a) m_t^{1-\alpha} K_t^{\alpha}. \tag{1}$$

The firm's capital stock is K_t, and $\alpha \in (0,1]$ is its returns-to-scale parameter. The capital stock evolves according to

$$dK_t = I_t dt - \delta K_t dt, \tag{2}$$

where I_t denotes the firm's instantaneous investment rate and δ is the depreciation rate. To allow for variation in growth opportunities, we adopt the standard assumption that adjusting the capital stock is costly. When the firm invests at rate I_t, it pays the cost of the newly installed capital and a quadratic adjustment cost of

$$\Psi(I_t, K_t) = \frac{\phi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t. \tag{3}$$

In Equation (3), the parameter ϕ determines the cost of adjusting the capital stock.

The variable m_t in Equation (1) denotes the economy-wide component of the cash flow process, which grows geometrically at a rate of g according to

$$\frac{dm_t}{m_t} = gdt. \tag{4}$$

We define the firm-specific component of the cash flow process in Equation (1) as

$$da_t \equiv f_t dt + \sigma_a d\omega_t^a. \tag{5}$$

The term f_t in Equation (5) is the firm's *productivity*, which reverts to its long-term mean of $\overline{f}$ according to the law of motion

$$df_t = -\lambda(f_t - \bar{f})dt + \sigma_f d\omega_t^f. \tag{6}$$

The parameter λ measures the extent of mean reversion in productivity. We set the long-term mean of productivity $\overline{f}$ to one without loss of generality. The second term in Equation (5), $\sigma_a d\omega_t^a$, represents noise in cash flows that is uncorrelated with the firm's true productivity.

Because agents in the economy do not observe productivity f_t directly, they estimate it using two available pieces of information. First, agents observe the realized cash flow rate da_t in Equation (5), which is informative about f_t. Second, agents observe a *soft information signal* s_t that is correlated with the innovations in f_t. The signal s_t summarizes all productivity-related information, such as subjective interpretations of news events, other than cash flow realizations. We assume that the signal s_t evolves according to

$$ds_t = \eta d\omega_t^f + \sqrt{1-\eta^2}\, d\omega_t^s. \tag{7}$$

The $d\omega_t^s$ term represents noise in the signal. Equation (7) ensures that the variance of the signal is equal to one for any value of signal informativeness, $\eta \in [0,1]$. We define signal precision as

$$\theta \equiv \frac{\eta}{\eta + \sqrt{1-\eta^2}}. \tag{8}$$

A value of one (zero) for θ represents a fully revealing (uninformative) signal.[5] In summary, there are three jointly independent sources of randomness in the model: $d\omega_t^f$ (the firm's productivity innovation), $d\omega_t^a$ (the cash flow noise), and $d\omega_t^s$ (the signal noise).

B. Information Processing

We model two information processing biases, *overconfidence* and *cash flow extrapolation*. Much like prior models of overconfidence, we assume that agents believe the signal informativeness to be $\eta_B > \eta$ and hence the signal precision to be $\theta_B > \theta$. The difference $\theta_B - \theta$ then measures how overconfident the agents are. We model the cash flow extrapolation bias by assuming that agents believe the productivity mean reversion parameter in Equation (6) to be $\lambda_B < \lambda$. The difference $\lambda - \lambda_B$ measures agents' excessive extrapolation of cash flow.[6]

As agents observe cash flow and signal realizations, they revise their beliefs about productivity f_t using Bayes' rule, given their possibly incorrect beliefs (λ_B and η_B) about the productivity and signal processes. Let $\widehat{f}_t$ denote the conditional estimate of f_t given information available at time t. Then the law of motion of $\widehat{f}_t$ is

$$d\widehat{f}_t = -\lambda_B\left(\widehat{f}_t - \overline{f}\right)dt + \sigma_f \eta_B \underbrace{ds_t}_{\equiv d\overline{\omega}_t^1} + \frac{\gamma_B}{\sigma_a}\underbrace{\frac{da_t - \widehat{f}_t dt}{\sigma_a}}_{\equiv d\overline{\omega}_t^2}, \tag{9}$$

where $d\overline{\omega}_t^1 \equiv ds_t$ and $d\overline{\omega}_t^2 \equiv \left(da_t - \widehat{f}_t dt\right)/\sigma_a$ are standard Brownian motions, and γ_B is the steady-

[5] The modeling of the soft information signal follows Scheinkman and Xiong (2003).
[6] In estimating the model, we allow for θ_B and λ_B to be higher or lower than θ and λ, respectively. As we show below, however, the empirical estimates indicate overconfidence ($\theta_B > \theta$) and cash flow extrapolation ($\lambda_B < \lambda$).

state variance of the estimation error $\hat{f}_t - f_t$. The variance γ_B is the solution to

$$\underbrace{\frac{\sigma_f^2}{2\lambda_B}}_{\text{variance of} f} = \underbrace{\frac{1}{2\lambda_B}\left[\left(\sigma_f \eta_B\right)^2 + \left(\frac{\gamma_B}{\sigma_a}\right)^2\right]}_{\text{variance of } \hat{f}} + \underbrace{\gamma_B}_{\text{variance of estimation error}}, \tag{10}$$

which gives

$$\gamma_B = \sigma_a \left(-\lambda_B \sigma_a + \sqrt{\left(\lambda_B \sigma_a\right)^2 + \left(1 - \left(\eta_B\right)^2\right)\sigma_f^2} \right). \tag{11}$$

The learning process for a rational agent satisfies equations identical to (9) through (11), except that we replace η_B and λ_B with their rational counterparts, η and λ.[7]

C. The Firm's Optimization Problem

A manager who shares the same beliefs and information as investors determines the firm's investment policy. The manager sets investment to maximize firm value:

$$V(K_t, \hat{f}_t, m_t) = \max_I \; E_t \left(\int_{u=t}^{\infty} e^{-r(u-t)} \left[\hat{f}_u m_u^{1-\alpha} K_u^{\alpha} - I_u - \Psi\left(I_u, K_u\right) \right] du \right), \tag{12}$$

subject to the evolution of capital, productivity, and the state of the economy:

$$\begin{aligned} dK_u &= I_u du - \delta K_u du, \\ d\hat{f}_u &= -\lambda_B \left(\hat{f}_u - \bar{f} \right) du + \sigma_f \eta_B d\bar{\omega}_u^1 + \frac{\gamma_B}{\sigma_a} d\bar{\omega}_u^2, \\ \frac{dm_u}{m_u} &= g du. \end{aligned} \tag{13}$$

Firm value is the expected discounted value of future *net* cash flows—*i.e.*, cash flows minus investment and adjustment costs—according to the manager's beliefs and information.

[7] We assume that initial productivity f_0 is normally distributed with mean $\bar{f}$ and variance $\sigma_f^2/(2\lambda)$. There is also an initial signal $s_0 = f_0 + \varepsilon_0$, where ε_0 is normally distributed with mean zero and variance $v_0 = \gamma\sigma_f^2/\left(\sigma_f^2 - 2\gamma\lambda\right)$. To ensure the model starts at the steady state, we choose the initial signal variance such that the biased agents perceived initial estimation error variance matches their perceived steady-state estimation error variance γ_B.

The Hamilton-Jacobi-Bellman equation for this maximization is

$$rV(K_t,\widehat{f}_t,m_t)=\max_I\left\{\begin{array}{l}\widehat{f}_t m_t^{1-\alpha}K_t^{\alpha}-I-\Psi\left(I_t,K_t\right)+\left(I-\delta K_t\right)V_K\\ -\lambda_B\left(\widehat{f}_t-\overline{f}\right)V_f+gm_tV_m+\dfrac{1}{2}\left(\left(\sigma_f\eta_B\right)^2+\left(\dfrac{\gamma_B}{\sigma_a}\right)^2\right)V_{ff}\end{array}\right\}. \tag{14}$$

Substituting adjustment costs from Equation (3) and maximizing with respect to *I* yields

$$\frac{I_t}{K_t}=\delta+\frac{V_K-1}{\phi}. \tag{15}$$

Equation (15) is the standard neoclassical investment policy: the firm invests at a rate that exceeds the replacement of depreciated capital if and only if the *marginal* q, V_K, exceeds one. The rate of adjustment of the capital stock is inversely related to adjustment costs φ.

Substituting Equation (15) into Equation (14) and suppressing the t subscripts to simplify notation, we obtain a partial differential equation that characterizes the firm's value *V*:

$$\begin{aligned}rV&=\widehat{f}m^{1-\alpha}K^{\alpha}-\delta K+\frac{1}{2\varphi}\left(V_K-1\right)^2K\\&\quad-\lambda_B\left(\widehat{f}-\overline{f}\right)V_f+gmV_m+\frac{1}{2}\left(\left(\sigma_f\eta_B\right)^2+\left(\frac{\gamma_B}{\sigma_a}\right)^2\right)V_{ff}.\end{aligned} \tag{16}$$

We solve for firm value *V* in Equation (16) and the resulting investment policy *I* in Equation (15) using the numerical methods described in the Appendix.

II. Empirical Estimation of the Model

A. Identification and Moment Selection

We use both calibration and estimation to determine the values of the 11 model parameters. We calibrate three parameters—g, r, and δ—to be equal to the values of three empirical moments and estimate the other eight parameters. We use value-weighted moments to calibrate economy-wide parameters and equal-weighted moments to calibrate firm-level parameters. The discount rate (r) is equal to the sum of the annualized averages of the real risk-free rate of interest (1.26%) and the equal-weighted excess return on firm assets (5.91%). We use the yield on one-month nominal Treasury Bills deflated by the Consumer Price Index (CPI) as the risk-free rate. We set the average growth rate of economy-wide dividends (g) equal to the average of value-weighted growth in firm assets (1.40%). The depreciation rate of firm assets (δ) is the equal-weighted depreciation rate on firm assets (2.96%). We provide detailed definitions of firm asset returns, depreciation rates, and other empirical quantities in Section II.B.

We select 15 additional moments to ensure that they collectively provide sufficient information about the values of the other eight model parameters. Strictly speaking, each of the technology and information processing parameters affects all of the moments, implying that all moments are informative about all parameters. Ten moments based on firms' production processes and cash flows primarily help us identify firms' six perceived technology parameters: α, ϕ, σ_a, σ_f, θ_B, and λ_B. If all agents were rational, these ten moments would be sufficient for identifying the six parameters above. Five additional moments based on differences in firms' expected asset returns mainly help us identify the magnitude of the two overreaction biases, which arise from the discrepancy between θ and θ_B and between λ and λ_B. Our point estimates of the eight model parameters above are the values that produce model behavior that most closely matches the 15 cross-sectional moments described below.

We use the averages of firms' Tobin's Q and cash flows to identify the return-to-scale parameter (α). As alpha decreases below one, firms' average cash flows and the values of their growth opportunities (Q values) increase. We infer the adjustment cost parameter (ϕ) from the coefficient on Q in a regression of investment on Q and cash flow. High adjustment costs cause firm investment to respond less to growth opportunities. The volatility of firm asset returns, cash flow, investment, and Q jointly provide information about the volatility of productivity and cash flows (σ_f and σ_a). A high response of firms' asset returns to their cash flows implies that agents perceive the signal about productivity (θ_B) to be highly informative. The persistence in firms' cash flows helps us infer the true and perceived rates of mean reversion in firm productivity (λ and λ_B).

We use differences in firms' average asset returns to identify the differences between the true and perceived productivity and signal processes. Recall that the difference λ and λ_B measures the excessive extrapolation of trends in productivity, while the difference between θ_B and θ captures the magnitude of overconfidence in the precision of the soft productivity signal. An increase in either of these biases produces an increase in the value anomaly. We measure the short-term (long-term) value anomaly using the difference between the one-year (years two and three) average returns of the firms in the top and bottom Q deciles. When forming portfolios based on Q and other accounting variables, we use data from each firm's most recently ended fiscal year, allowing for a three-month reporting lag. We rebalance all portfolios monthly.

To distinguish which of the two biases produces the observed value anomaly, we exploit their conflicting predictions about return predictability coming from firms' cash flows. If investors excessively extrapolate from current cash flow information, they will be too optimistic about firms that have high cash flows, implying such firms will have low future returns. In contrast, if investors are overconfident in their soft productivity signal and place too little weight on cash flow information, they will be too pessimistic about firms with high cash flows, implying such firms will have high future returns. Thus, the extent to which firms with high current cash flows exhibit high future returns reveals the relative importance of the extrapolation and overconfidence biases. To quantify this, we compute the difference between the one-year average returns of firms in the top and bottom deciles of cash flows.

One test of the model is whether it can replicate the observed relationships between managers' investment decisions and investors' valuations of firms. In some specifications, we use two additional return predictability moments as over-identifying restrictions to test this aspect of the model's performance. These two moments are the difference between the one-year (years two and three) average returns of the firms in the top and bottom investment deciles.

B. Empirical Methodology for Estimating Moments

Our goal in constructing an empirical sample is to provide an appropriate testing ground for the model in Section I. The basis for the sample is the universe of all public US firms from 1968 through 2006. We restrict the sample to firms that have common stocks trading on the NYSE, Nasdaq, or AMEX and those with accounting information.

Because the model considers a firm in its steady state, all sample firms must have at least 60 months of continuous stock price data. The accounting information necessary for the identification of key model parameters includes data on market equity, book equity, book debt, cash, property, plant, and equipment (PP&E), cash flows, investment, sales, and total assets for each of the last two years. The sample excludes firms with negative values of market equity, book debt, PP&E, sales, and non-cash assets. It also excludes public utilities, regulated firms, and financial firms—those with Standard Industry Classification codes in the ranges 4900-4999, 9000-9999, or 6000-6999—because our model of the capital stock may not apply to these firms. We also exclude firms with stock prices less than $1 and those without trading volume in the most recent month because these firms' stock returns may contain significant measurement error.

Computing the empirical moments requires two types of raw inputs: accounting and asset market variables. Each firm's accounting variables include book equity, book debt, total assets, Tobin's Q, investment, cash flows, and depreciation. We define book equity as in Davis, Fama, and French (2000), book debt as in Kaplan and Zingales (1997), total assets using the non-cash asset definition in Cooper, Gulen, and Schill (2008). We do not include cash in firm assets because firms in the model in Section I do not require any cash. Thus, we also subtract cash from firm value when computing Q, which is (market equity + book debt – cash) / (non-cash assets). Market equity is shares outstanding times the stock price at the end of the firm's fiscal year. Investment is the annual growth rate of assets. Cash flow is after-tax operating income before depreciation and before research and development expenses, divided by assets. That is, we treat research and development as an investment, not an expense. Depreciation is the annual change in the difference between gross capital and PP&E, divided by assets.

We compute monthly asset returns for each firm as the weighted average of equity and debt returns, using the firm's capital structure weights. We use standard CRSP data to measure the equity return. A lack of firm-specific bond return data presents a challenge for measuring debt returns. For the years 1973 through 2006, we use the monthly return on the Lehman Brothers investment grade bond index as a proxy for firms' debt returns. For the years 1968 to 1972, when the Lehman index is unavailable, we use the equal-weighted monthly return on US Treasuries with maturities between five and 10 years from the CRSP US Monthly Treasury Database. From 1973 to 2006, a regression of the investment grade returns on the Treasury index return yields a coefficient of 1.00 with a standard error of 0.03 and an R^2 of 83%. The correlation of 0.91 and regression coefficient near 1.0 suggest that risk-free rates between five and 10 years are a reasonable proxy for investment grade bond returns in most economic climates.

We estimate each empirical moment using the time series average of cross-sectional estimates following Fama and MacBeth (1973). For the five return anomaly moments, such as the one-year Tobin's Q anomaly, we use the monthly difference between the equal-weighted returns of the firms in the top and bottom deciles of the accounting variable. In forming portfolios, we allow for a reporting lag of three months for accounting variables. We compute the averages and standard deviations of variables such as Q, cash flows, investment, and returns using monthly cross-sectional estimates. We winsorize Q, cash flow, and investment at the 5% level to reduce the influence of outliers. For returns, we winsorize at the 5% level only for computing return volatility, not for the other return moments.

The point estimate for each empirical moment is the time series average of the cross-sectional estimates. We use the time series autocorrelation and standard deviation of the annual moments as the basis for computing standard errors. The annual estimates of return anomalies are the monthly anomaly moments multiplied by 12. We model the yearly time series of each annual moment as a first-order autoregressive process. For moments with an autoregressive coefficient that is significantly different from zero, the delta method provides the moment's standard error. For other moments, we use the Newey-West (1987) standard error with the Newey-West (1994) recommendation for the number of yearly lags. The covariance matrix (W) of the 15 annual moments is the element-wise product of the moments' correlation matrix and the outer product of the standard error vector.

C. Estimation Procedure

The goal of the SMM estimation is to select model parameters that produce simulated model moments that are as close as possible to the empirical moments above. We compute the distance between the model and empirical moments using an SMM criterion function, which assumes a quadratic form. Let *m* and *d* be the 15 by 1 vectors of simulated model and empirical data moments, respectively. We select model parameters to minimize the SMM criterion function, $(m - d)^{T} W^{-1}(m - d)$. This function uses the inverted covariance matrix to weight the moments, which is efficient under the conditions described in Cochrane (2000). We use a nonlinear least squares routine to find the parameters that minimize this SMM criterion function.

Section II.B summarizes the empirical computation of *d* and *W*. The first step in computing the vector of simulated model moments (*m*) is to obtain numerical approximations of the firm value and policy functions, using methods described in the Appendix. Next, we apply these numerical approximations over discrete time intervals of 40 periods per year to generate simulated sample paths of model firms. We generate simulated data for a cross section of 1,000 firms for 500 years. For each firm, we draw the initial values of true productivity and agents' productivity estimate from their respective steady-state distributions.[8] To allow the capital stock to reach its steady-state distribution, we discard the first 100 years of data. We use the remaining 400 years of data to compute the model counterparts of empirical data moments and the resulting SMM criterion. Because there are 39 years of empirical data, using 400 years of simulated data corresponds to more than 10 simulated samples.

III. Estimation Results

[8] For convenience, we assume productivity shocks are independent across firms. Because there are many firms in this economy that exhibit predictable and independent asset returns, a rational arbitrageur could obtain an infinitely high risk-free return. Of course, there are no rational agents in our model. Moreover, introducing productivity shocks that are correlated across firms, even if they are not priced, would eliminate the possibility of risk-free arbitrage and would not affect the qualitative conclusions below.

This section analyzes our model's ability to match the empirical moments described in Section II.B. The first estimation relies on 13 moments, excluding the two moments measuring return predictability from firm investment. We use all moments in estimating the 15-moment model.

A. Empirical Moments

The first two columns in Panel A of Table 1 show the estimates and standard errors of the 15 empirical moments. The average Tobin's Q of 1.58 and average cash flow of 13.2% suggest that the average firm has diminishing returns to scale and valuable growth opportunities. The cross-sectional standard deviation of Q of 0.937 may seem high relative to average Q. More than 5% of firms have Q values exceeding 4.38, which increases the dispersion in Q. The persistence of cash flow is 0.790, which is higher than we would obtain if we used fixed effects. The investment sensitivities to Q and cash flow of 0.067 and 0.660 are similar to the estimates in other studies. The Q coefficient is slightly higher than the values reported in Kaplan and Zingales (1997) because we use a univariate specification to measure this coefficient.

[Insert Table 1 here.]

The last five rows in Panel A of Table 1 report the difference in asset returns between the top and bottom decile portfolios sorted on cash flow, Q, and investment at different time horizons. The significantly negative Q and investment anomalies are consistent with the well-known value, investment, and asset growth effects in equity returns found in Fama and French (1992), Titman, Wei, and Xie (2004), and Cooper, Gulen, and Schill (2008), respectively. The annualized return differences from sorts on Q and investment are -6.38% and -6.71%, respectively, one year after portfolio formation. These anomalies decline somewhat to annualized averages of -4.56% and -2.42% in years two and three after portfolio formation. The positive but statistically insignificant one-year return predictability of 1.39% from cash flow shows that post-earnings announcement drift is not large in our sample. Nevertheless, this coefficient is informative about the relative importance of the two information processing biases, as discussed in Section II.A. Some anomaly estimates appear slightly smaller than standard estimates because we measure anomalies using firms' asset returns, not just their equity returns.

Table 1 also presents the results from minimizing of the SMM criterion function in the 13-moment estimation and the 15-moment estimation that includes two investment anomaly moments. The last two columns in Panel A report the two models' predictions for each of the 15 moments. The last two rows in these columns summarize how the two models fit the data, using χ^2 statistics with degrees of freedom of five and seven. The $\chi^2(5)$ value of 8.88 for the 13-moment model shows that most of the simulated model moments match the 13 empirical moments quite closely. Formally, the *p*-value of 0.114 implies that we cannot reject the hypothesis that the five over-identifying restrictions are valid. The $\chi^2(7)$ value of 16.27 and *p*-value of 0.023 for the 15-moment model suggests a slightly worse fit, implying that the we can reject the over-identifying restrictions at the 5% level. In both estimations, the standard errors of all eight parameters are small relative to the parameter values, indicating that there is enough statistical power to identify each parameter.

Figure 1 graphically summarizes the fit of the model in the 13-moment and 15-moment estimations. The height of each vertical bar represents the deviation of the model's predicted moment from each empirical moment, as a percentage of the empirical moment. The model matches the technology moments more closely than the return predictability moments because the estimates of return anomalies are less precise. Visually, this implies that the ten leftmost bars in Figure 1 are smaller in magnitude than the five rightmost bars.

[Insert Figure 1 here.]

The 13-moment and 15-moment models both match most moments quite well. For the ten technology moments across both estimations, excluding the standard deviation of investment, the largest percentage model error is 16%. The two prediction errors for the volatility of firm investment are -42% and -43%, meaning that the models predict too little investment volatility. One likely reason is that both models do not include fixed adjustment costs, which could produce lumpy and hence more volatile firm investment. Such adjustment costs also imply that empirical investment is a noisy reflection of agents' beliefs about productivity.

Although both models qualitatively match the empirical return anomalies well, there are economically significant prediction errors for some anomalies. The 13-moment model predicts short-term and long-term value anomalies that closely match the data. It does, however, predict a one-year investment anomaly that is too large by 61%, though this moment is not used in the estimation. The lumpy investment data discussed above could explain this apparent shortcoming of the model. None of the other model errors is statistically significant at the 5% level because they are not large relative to the standard errors on return anomalies.

The main goal of the 15-moment estimation is to match the investment anomaly better, which it achieves. However, the 15-moment model fails to reproduce most of the empirical value anomaly, so we view this model as a conservative estimate of the importance of mispricing. Although neither estimation matches the cash flow anomaly well in percentage terms, both models' prediction errors are statistically insignificant at even the 10% level.

The rows in Panel B of Table 1 show the parameter estimates for both models that produce the best fit to the data, along with parameters' standard errors. The estimates of the technology parameters are reasonable given the empirical data. The returns to scale (α) estimates of 0.849 and 0.852 in the two models are both significantly below one, implying diminishing returns scale. The adjustment cost parameters (ϕ) of 2.90 and 2.88 are smaller than the estimates in Gomes, Yaron, and Zhang (2006), who use aggregated investment data which tends to be less volatile. The two estimates of mean reversion in productivity (λ) of 0.230 and 0.163 are low because cash flow is quite persistent. The last two columns in Panel A of Table 1 show that both models match this feature of the data well.

Panel B reports the information processing parameter estimates. The perceived quality of the soft information signal (θ_B) is higher at 0.936 in the 13-moment model than its value of 0.810 in the 15-moment model. These high precisions of the soft signal enable both models to match the weak response of market returns to cash flows. The true qualities of the soft signal (θ) are 0.458 and 0.491 in the 13- and 15-moment models, respectively, which are significantly lower than the perceived precisions of 0.936 and 0.810. This implies that agents overreact to the soft information signal, especially in the 13-moment estimation. The model adjusts this parameter to match the negative return predictability from Tobin's Q, which it does well in the 13-moment model. Because there is less overconfidence in the 15-moment model, this model predicts a smaller investment anomaly. This occurs at the expense of matching the value anomaly.

Another way for the model to produce negative return predictability from Q is to set the perceived mean reversion in productivity (λ_B) lower than the true mean reversion (λ). Indeed, the parameter estimates in both models satisfy this excessive extrapolation condition. The overconfidence bias is strong relative to the extrapolation bias in both models in order to match the positive empirical estimate of post-earnings announcement drift. Although both models produce slightly too much return predictability from cash flow, the prediction errors are not statistically significant. The extrapolation bias in the 15-moment model is smaller than the 13-

moment model's bias to more closely match the empirical investment anomaly. Overall, though, most of the parameter estimates in the two models are quite similar.

Next we gauge the combined economic magnitude and plausibility of the two biases. The last three rows in Panel B of Table 1 compare the volatility of the error in estimating productivity for a rational agent and for a biased agent, along with the standard deviation of true productivity. In the 13-moment (15-moment) model, the biased agent's perceived estimation error is 0.057 (0.151), which is 78% (49%) lower than the rational agent's perceived error of 0.253 (0.298). This calculation suggests the biases are large. However, the difference in the biased and rational agents' perceived volatilities of 0.196 is only 26% of the magnitude of the volatility in true productivity (0.766). This comparison suggests such biases may be difficult to detect and correct.

IV. Analysis of the Simulated Economy

This section analyzes the properties of simulated model economies that mimic real-world firm behavior and asset return predictability, using parameters such as those in Table 1. Using simulated data allows us to explicitly measure many features of the economy that econometricians cannot typically observe. Our discussion focuses on the 13-moment estimation because it produces a simulated value anomaly that is closer to the empirical value anomaly. We view the value anomaly as more critical for identification than the investment anomaly, which may be a poorer reflection of agents' beliefs because of fixed adjustment costs, as noted earlier.

We divide the analysis into four subsections. First, we analyze firms' expected returns and asset mispricing from the perspective of a rational investor. This investor evaluates firms' expected cash flows, knowing that managers with information processing biases select firm investment and that biased investors determine asset prices. Second, we distinguish the individual impacts of the two behavioral biases, overconfidence and excessive extrapolation, on mispricing. Third, to assess how return predictability depends on managerial behavior, we simulate an economy in which rational managers choose firm investment but biased investors still determine asset prices. Fourth, we analyze investment inefficiencies by contrasting economic outcomes, such as firm value and capital allocation, when biased and rational managers choose firm investment.

A. Analysis of Mispricing with Biased Investors and Managers

Empirical asset pricing research often uses the predictability in average *realized* returns as a proxy for mispricing. Instead, we use two direct measures with natural interpretations that are unavailable to empiricists: firms' *true* expected returns and values from the perspective of an agent with rational expectations conditional on investors' information.[9, 10] First, we analyze the predictability in firms' true (abnormal) expected returns (minus the risk-free rate). Second, we compare firms' observed asset prices to their true firm values. We define *mispricing* as the firm's true value divided by its market value, so that a ratio below one implies price exceeds value. This definition makes the market efficiency notion proposed in Black (1986) directly observable: "we might define an efficient market as one in which price is within a factor of 2 of value" (p. 533). In his view, efficiently priced firms are ones with mispricing values between 0.5 and 2.0.

[Insert Table 2 here.]

[9] We estimate firms' expected returns for up to 10 years from the perspective of a rational agent by computing the firms' average returns, including both price changes and dividends, based on 5,000 simulated sample paths of the economy. Although there is no aggregate risk in our model, we use abnormal returns in these computations to facilitate interpretation. We compute a firm's abnormal return using its raw return minus the risk-free rate.

[10] True firm value is the rationally expected present value of the firm's cash flows discounted at the appropriate rate (r). True firm value is a solution to a partial differential equation analogous to Equation (16), except that it includes an extra state variable representing the rational estimate of productivity. We solve this equation numerically using the same method described in the Appendix.

Panel A in Table 2 reports summary statistics of true one-year abnormal expected returns and mispricing. The summary statistics in Panel A reveal that the standard deviations of one-year expected returns and mispricing are both higher in the 13-moment model (5.8% and 17.3%) than in the 15-moment model (4.8% and 11.2%), implying less efficient pricing in the 13-moment case. In both models, average value-weighted returns are lower than equal-weighted returns, indicating that larger firms have lower expected returns, as explained below.

Panel A shows that the equal-weighted average mispricing (*i.e.*, value-to-price) ratio is 0.7444 in the 13-moment model, meaning that the average firm is substantially overpriced. This occurs because investors and managers overestimate the value of firms' growth options. Both information processing biases cause agents to think their information about future productivity is more precise than it actually is. Consequently, they expect firms to make better-informed investment decisions than they actually will, which leads agents to overprice firms' assets.

Panels B and C in Table 2 show the distribution of true one-year abnormal expected returns and mispricing for portfolios sorted on firm characteristics. The first (second) column in Panel B shows that simulations of the 13-moment (15-moment) model produce large variation in expected returns: the top-to-bottom decile difference in firms' expected returns is 20.4% (16.1%). The distribution of expected returns is smooth and positively skewed, much like empirical data on firms' realized returns. Importantly, columns three to eight show that the *majority* of the differences in portfolios' true expected returns remain unexplained by observable variables, such as Q, investment, and cash flow. This comment applies to both models. The investment anomaly in the 13-moment model is the most able to explain differences in expected returns, but this occurs because the investment anomaly is counterfactually large in this model.

Panel C shows that market prices often deviate widely from firm values. The top-to-bottom decile spread in mispricing is 60.7% in the 13-moment estimation and it is 39.3% even in the 15-moment estimation, which predicts a value anomaly that is too small. The 13-moment estimates show that the average firm in the bottom mispricing decile is inefficiently overpriced, even by Black's (1986) loose criterion. Interestingly, in the 13-moment model, Q does a better job in explaining mispricing than it does in explaining expected returns. The reason is that Q reflects a long-term component of mispricing that is not eliminated in the next year.

[Insert Table 3 here.]

Table 3 displays estimates from cross-sectional regressions predicting one-year abnormal expected returns in the 13-moment and 15-moment simulations in Panels A and B, respectively. The regressors include five observable firm characteristics: Q, investment, the log of market capitalization (size), cash flow, and one-year asset returns. The bottom row in each panel reports the R^2 statistics from the regressions.

The low R^2 statistics reveal that the ability of each of the individual firm characteristics to explain true abnormal expected returns is quite low. The R^2s from the univariate regressions in Panel A range from 0.0023 to 0.2313. The main reason for the low R^2s is that the econometrician cannot directly measure the soft information signal to which investors overreact. This is why using data from the simulated economy is necessary to measure the true extent of mispricing. Regression (8) shows that the joint explanatory power of the five firm characteristics is much greater than the sum of their univariate explanatory power. Mechanically, this occurs because cash flow and investment are negatively correlated and predict returns with opposite signs. Intuitively, only the joint specification is able to separately identify the two distinct information processing biases, which have different implications for post-earnings announcement drift but similar implications for the value and investment anomalies. The success of this joint model in our simulations may have an empirical counterpart: Chen, Novy-Marx, and Zhang (2010) argue that an asset pricing model with factors representing firm investment, cash flow, and market risk outperforms the widely used Fama-French (1993) three-factor model.

In the simulations of both models, the return predictability coefficients on firm size in the univariate Regression (4) and the multivariate Regression (8) are negative. Although the estimations do not include moments based on the empirical size anomaly, the model predicts a negative size premium. This is consistent with the empirical findings in Banz (1981) and Fama and French (1992). In the model, size is a noisy proxy for investor beliefs, which are excessively volatile and exhibit mean reversion, leading to a negative size premium. In Regressions (1) to (3), the return predictability coefficients on Q and investment are negative, while the coefficient on cash flow is positive. These findings are consistent with the values reported in Tables 1.

The contemporaneous coefficient on past one-year returns in predicting one-year true expected returns is close to zero in both panels in Table 3, showing that the models do not predict positive return momentum of the sort identified empirically in Jegadeesh and Titman (1993). This is interesting because there is "earnings momentum" in the model, meaning that cash flow positively predicts returns, because overconfident agents underreact to cash flow signals. This effect alone would produce return momentum because cash flow is positively associated with returns. There is, however, an offsetting effect that comes from the positive correlation between the soft information signal and returns. Agents overreact to the soft signal, which could cause negative autocorrelation in returns. The two countervailing effects roughly offset at the empirical parameter estimates in these specifications.[11]

[11] Naturally, including moments that measure return momentum in the estimations could lead to model parameters that would produce positive momentum.

In Figures 2 and 3, we compare firms' true expected returns to expected returns conditional on two observable variables (Tobin's Q and investment) and one unobservable variable (mispricing) at horizons up to 10 years. The eight lines in the figures show expected returns for firms ranked in the top and bottom deciles based on their true three-month expected returns, mispricing, Q values, and investment rates.[12] The top and bottom lines depict sorts on true expected returns. The next two outermost lines represent sorts on mispricing, while the four innermost lines show sorts on investment and Q. Because Q and investment negatively predict returns, firms in the top deciles of these two observable characteristics are in the bottom deciles of expected returns. Figures 2 and 3 are identical except that Figure 2 uses parameter estimates from the 13-moment model, whereas Figure 3 uses estimates from the 15-moment model.

[Insert Figures 2 and 3 here.]

Importantly, both figures show that sorts based on true three-month expected returns produce much larger differences in future returns than the two sorts based on the observable Q and investment variables. In the 13-moment model in Figure 2, the difference between firms' one-year expected returns is 20.4% using sorts based on three-month expected returns, whereas the difference is only 6.9% (10.7%) using sorts based on Q (investment). In this sense, the observable return anomalies vastly understate the true differences in expected returns. The observable firm characteristics do an even worse job of explaining true expected returns in the 15-moment estimates in Figure 3. Remarkably, in both figures, differences in firms' expected returns of more than 1% per year persist beyond the 10-year investment horizon. Clearly, empirical analyses face challenges in detecting return predictability at such long horizons.

There are two more noteworthy features of Figures 2 and 3. First, using the two observable variables, one can forecast a significant amount of positive return predictability but

[12] We use three months as a realistic short investment horizon.

very little negative return predictability. The firms ranked in the top Q decile have expected abnormal returns of -1%, which is much closer to zero than the -8% expected abnormal return for firms with the lowest three-month expected returns. In contrast, firms ranked in the bottom Q decile exhibit highly positive returns of 6% per year, which captures about half of the 12% expected return for firms ranked in the top decile of three-month expected returns. This asymmetry in observable return predictability applies to the investment anomaly, too.

Second, sorting on true mispricing does not predict future returns as well as sorting on true expected returns. This occurs because mispricing in some firms disappears more rapidly than in other firms. The expected rate of convergence to fundamentals depends on past realizations of the soft signal and cash flows.

B. Distinguishing the Impact of Overconfidence and Extrapolation

This subsection analyzes the separate impacts of the overconfidence and excessive extrapolation biases on mispricing. Using the parameter estimates from the 13-moment model in Table 1 as a starting point, we consider two parameter changes.[13] First, we eliminate agents' overconfidence by setting perceived signal quality equal to true signal quality ($\theta_B = \theta = 0.458$), while retaining all other parameter values, including the excessive extrapolation parameters ($\lambda_B = 0.143$ and $\lambda = 0.230$). Second, we eliminate agents' excessive extrapolation by setting $\lambda_B = \lambda = 0.230$, while retaining all other parameter values, including the overconfidence parameters ($\theta = 0.936$ and $\theta_B = 0.458$). We simulate the behavior of economies with these two sets of parameter estimates to isolate the impact of the two individual biases.

[13] In unreported analyses, we consider variations in parameters starting from the parameter estimates in the 15-moment model in Table 1 and obtain qualitatively similar findings.

Table 4 reports the moments from the simulation in which agents excessively extrapolate, the simulation in which agents are overconfident, and a fully rational economy without both biases. For comparison, the last two columns in Table 4 redisplay the moments in the 13-moment model with both biases and the empirical moments. Many of the production-related moments in Table 4 are quite similar in the biased and rational economies. As shown in Section IV.D, this does *not* imply that biases have no real effects, but it does mean that production moments play a small role in identifying biases. The exception to this is Tobin's Q, which differs significantly in the biased and rational economies because Q depends on asset prices. The bottom rows in Table 4 show that the extrapolation and overconfidence biases both contribute to the value and investment anomalies, whereas the two biases have opposing impacts on the cash flow anomaly.

[Insert Table 4 here.]

Tables 5 analyzes the mispricing properties of both one-bias simulations in more detail. The two panels show the equal-weighted mispricing (rational value / market price) and expected returns for portfolios consisting of firms sorted into deciles using firm characteristics. All sorting characteristics, such as Q and cash flow, are observable, except true mispricing. Our analysis focuses on top-to-bottom decile spreads in mispricing and expected returns.

[Insert Table 5 here.]

One striking fact in Panel A in Table 5 is that the simulation with excessive extrapolation only produces a spread in mispricing of 25.3% and just one observable variable, Tobin's Q, captures nearly all of this spread (25.2%). In contrast, the simulation with just overconfident agents produces a much larger spread in true mispricing (50.8%) and no observable variable, such as Q or investment, captures even half of the spread in mispricing. The reason is that overconfident agents' errors in expectations depend on their soft information signal, which is not easily observable, whereas one can partly infer the extrapolative agents' mistaken beliefs based on Q, which summarizes past productivity trends.

Panel B in Table 5 shows that cash flow negatively predicts returns in the extrapolative investor simulation and positively predicts returns in the overconfident investor simulation. In the 13-moment model in Table 1, these two effects roughly offset, producing a small positive cash flow anomaly. An overconfident investor ignores public cash flow signals in favor of his soft information signal, whereas an extrapolative investor puts too much weight on past trends in cash flow, thinking that such trends will continue.

C. Analysis of Mispricing with Rational Managers and Biased Investors

This subsection investigates whether a model with rational managers and biased investors can match the empirical data. We assume that a rational manager selects investment to maximize true firm value, rather than investors' perception of firm value. A rational manager uses the true mean reversion rate of productivity and the true signal quality to forecast productivity. As a result, this forecast differs from investors' forecasts. Biased investors know that the manager is not pursuing their ideal policy and think that this reduces firm value. The differing beliefs of investors and managers imply that the firm value function in Equation (16) depends on an additional state variable that represents the rational managers' beliefs. We solve for the value function using the polynomial approximation described in the Appendix.[14]

We analyze the steady state properties of the economy with rational managers in the two panels in Tables 6. Panel A compares the simulated empirical moments for the two rational manager economies to the simulated moments in otherwise identical economies where the managers exhibit biases. The third and sixth columns in Panel A compute the percentage changes

[14] The approximation now includes a third state variable with an polynomial approximation degree of 10.

in each steady state moment in moving from the irrational manager economy to the rational manager economy.

[Insert Table 6 here.]

The main result in Panel A is that the one-year value anomaly increases by as much as twofold, while the one-year investment anomaly becomes negligible in the economy with rational managers. The value anomaly arises when investors are too optimistic about growth firms' productivity prospects, leading to high Q values and low future returns. When rational managers run such growth firms, they do not exercise as many growth options as investors would like. Thus, a greater proportion of the firms' asset prices come from overpriced unexercised growth options when managers are rational. As a result, the rationally managed growth firms future returns are more negative, as realized productivity disappoints the biased investors.

The investment anomaly, however, arises when high managerial investment coincides with excessive investor optimism. These two events coincide the most in an economy where managers and investors share the same optimism. This intuition and the simulations suggest that the rational manager model produces an investment anomaly that is much smaller than the value anomaly. Empirically, we observe value and investment anomalies of comparable size. Accordingly, aside from the analysis in Table 6, we focus on the specifications where managers and investors have the same beliefs, which seem to fit the return predictability data. Models in which managers are rational but simply act as if they have the same biases—*e.g.*, because they cater to investors' biased beliefs—would fit the data equally well.

The bottom four rows in Panel B in Table 6 provide further insights into mispricing in the rational manager and biased manager economies. The equal-weighted mean of mispricing is much closer to one in the economy where the rational manager runs the firm—*e.g.*, 0.875 vs.

0.753 in the 13-moment model. This reduction in overpricing occurs because biased investors think that the rational manager chooses a suboptimal investment policy, whereas a biased manager selects the optimal policy. For the same reason, changes in the degree of disagreement between the rational manager and biased investors produce volatility in mispricing. This is why the volatility of mispricing is higher in the rational manager economy—*e.g.*, 0.246 vs. 0.173.

D. Analysis of Investment Inefficiencies

Finally, we examine the benchmark model's implications for capital allocation. The model enables us to quantify inefficiencies in investment using several measures that are unavailable to empirical researchers. Specifically, we compare firm values, capital stocks, and investment rates when a manager with information processing biases selects investment and when a rational manager with no biases selects investment. From the earlier simulations, we know firm values, capital stock, and investment rates in the biased economy. We simulate these quantities in the hypothetical rational economy by setting both information processing biases equal to zero, while holding all other model parameters constant. That is, we set the perceived mean reversion in productivity (λ_B) equal to the true mean reversion (λ) and the perceived quality of the productivity signal (θ_B) equal to the true quality of the signal (θ). We refer to the quantities, such as firm values, from these two simulations as "biased" and "rational" quantities.

Table 7 reports several comparisons between the biased and rational economies in the 13-moment and 15-moment estimations. To compare firm values, we adopt the perspective of a rational agent who properly computes the expected discounted value of future cash flows. Value added is the difference between the firm's value (V) and the cost of its capital stock (K). The firm value per unit of capital is *V/K*, which is equal to average Q in the rational economy but differs from average Q in the biased economy because market prices differ from rational firm value.

[Insert Table 7 here.]

Panel A in Table 7 shows that behavioral biases cause large capital misallocation in the steady state. In the 13-moment estimation, the steady state aggregate value added in the biased economy is only 35.5% as large as value added in the rational economy, as shown in the top row. In the 15-moment estimation in which behavioral biases are smaller, the aggregate value added in the biased economy is larger at 92.0% of the rational economy's value added. In both models, the biased economy has a larger capital stock and lower value per unit of capital (*V/K*). The intuition is that extrapolative managers perceive future investment opportunities to be more persistent than they are, so they install more capital to minimize future adjustment costs.

The top row in Panel A shows that the aggregate value added in the economies with only overconfidence and only extrapolation are 81.9% and 90.7%, respectively, of the rational economy's value added. The combined value added reductions of 18.1% and 9.3% from the individual biases are much less than the value added reduction from both biases together (64.5%). One explanation is that a second distortion in an economy often causes greater harm than the first distortion because changes in investment policy have a small first-order impact on firm value when investment policy is close to the optimal rational policy. A complementary explanation is that overconfidence exacerbates the tendency for managers who excessively extrapolate to increase firm size, as shown in the last two rows of Panel A. An overconfident extrapolative manager is more certain than an (otherwise rational) extrapolative manager that he can act on growth opportunities before they dissipate.

Panel B of Table 7 compares the marginal investment rates chosen by a biased versus a rational manager. Here we analyze the investment choices of each manager just after being hired to run a firm whose capital stock has been installed by a biased manager. This comparison differs somewhat from the steady state comparisons in Panel A because adjustment costs prevent a rational manager from instantly changing the capital stock and reaching a new steady state.

The positive values in the top row of Panel B in Table 7 show that rational managers would invest much less than biased managers, on average, which is consistent earlier findings. In the 13-moment model, the median of overinvestment is 1.5% of the capital stock. The standard deviation of overinvestment is large at 6.9%, implying that many firms do not invest enough in any given time period. Both overinvestment and underinvestment are common because managers react too much to noise in the soft information signal, which could be positive or negative.

The last two rows in Panel B of Table 7 show that an instant takeover of an irrational managed firm by a rational manager would create substantial value for the typical firm. The median biased firm is worth only 88.5% (97.2%) as much as the rationally managed firm in the 13-moment (15-moment) model. This 11.5% (2.8%) marginal value reduction from irrational management is much less than the value added reductions in the steady state partly because rational managers incur large costs in adjusting the firm's capital stock to the optimal level. A more mundane reason is that value added is much smaller than firm value. The median value reduction from biased management is greater than the aggregate reduction. Because a greater proportion of a small firm's value consists of growth opportunities, which are the source of agents' biases, biases reduce the value of small firms by a larger percentage.

The second and third columns in both panels in Table 7 show the separate impact of each bias on firm value and real investment. The first row in Panel B also reveals that extrapolative managers exhibit higher median investment rates than rational managers by 1.4%. As noted earlier, an extrapolative manager puts too much value on the firm's future growth opportunities, believing that she can act on productivity shocks before they dissipate. Thus, she increases investment today to lower expected adjustment costs. In contrast, there is no median bias (0.0%) in an overconfident manager's investment rate.

The second row in Panel B shows that firms' investment rates are much more volatile in the simulation with overconfident agents (5.1%) than in the simulation with extrapolative agents (2.7%). In the overconfidence simulation, the unobserved volatility in the soft information signal must be large to produce the observable value anomaly because noise in the signal is only partially correlated with Tobin's Q. The extrapolation bias does not need to be as large because past productivity trends are more directly related to Q and thus the observable value anomaly. Overall, both biases contribute significantly to capital misallocation, but overconfidence usually plays a larger quantitative role.

[Insert Table 8 here.]

Lastly, we analyze the cross section of managerial overinvestment, as measured by the difference in marginal investment rates of biased and rational managers. Table 8 displays the results from regressions of overinvestment on Q, investment, size, cash flow, and one-year returns. The best predictor of overinvestment is a firm's observable investment rate. In the 13-moment (15-moment) model, a firm's investment rate accounts for 44% (8%) of the variation in overinvestment, as shown in Regression (2). The intuition is that managerial overreaction to information, coming from both overconfidence and excessive extrapolation, makes investment decisions too volatile. As a result, firms that invest a lot tend to invest too much. This is why investment positively predicts overinvestment in Regressions (2), (6), (7), and (8) in Table 8.

Similarly, biased managers of firms with valuable growth opportunities overreact to positive productivity information and invest too much relative to a rational manager. Excessive optimism about growth explains the positive size and Q coefficients in Regressions (1) and (4). Holding Q and investment constant, managers underreact to the cash flow signal, which explains the negative cash flow coefficient in Regression (7). In summary, this analysis suggests that firms with high investment rates and relatively low cash flows tend to invest too much.

V. Concluding Discussion

This study is one of the first to quantify the asset pricing and investment distortions implied by behavioral theories of information processing biases. Our first finding is that observed asset return anomalies understate the inefficiencies in firms' asset prices and investment decisions. Intuitively, econometricians cannot accurately measure investors' information sets, which makes the efficient market hypothesis difficult or impossible to test, as argued in Summers (1986) and Hansen and Richard (1987). Conversely, any observed evidence of asset return anomalies could represent the tip of an iceberg of true mispricing. We use a dynamic structural model to quantify the size of this iceberg.

The analysis shows that the overconfidence bias causes unobserved mispricing to be larger than observable return anomalies. The mechanism is that overconfident investors have incorrect beliefs that depend on an unobservable signal, making it difficult to see the true extent of mispricing. Other behavioral biases, such as excessive extrapolation, do not share this feature.

We also find that a model with rational managers and irrational investors fails to match a prominent empirical feature of return predictability: the large investment anomaly. This suggests that models in which managers exhibit information processing biases, such as our benchmark specification in which managers' and investors' biases are identical, are promising to explore. Future work could test whether more nuanced managerial biases fit the empirical data significantly better than the biases in our benchmark model.

Our model generates return predictability from behavioral biases alone, whereas in fact some return predictability may come from risk premiums. This paper does not test the relative merits of these explanations. Rather, our goal is to understand the implications of the behavioral explanation. Future models could combine both return predictability mechanisms. Although many of the qualitative insights here would apply to such models, there could be novel and unanticipated interactions between behavioral and risk-based channels.

One need not interpret the parameter estimates from our model as constant over time. It is possible or even likely that the asset return predictability patterns in the next 40 years will be quite different from predictability patterns observed in the past 40 years. As the economy's latent structural parameters change, behavioral biases may dissipate or be replaced by new biases. Furthermore, rational agents may learn only gradually about such changes, generating return patterns that are predictable only in hindsight. Under these alternative interpretations, our model still provides a useful ex-post description of the underlying mispricing and investment inefficiencies that occurred over the past 40 years. This description of the past may or may not apply to the future. Thus, our main contribution is not the specific parameter estimates but the modeling framework and the general insights it provides.

Appendix: Numerical Solution of the Model

In this Appendix, we solve for the firm's value function V. The partial differential equation in Equation (16) characterizes firm value as a function of three state variables: capital stock K_t, productivity estimate $\widehat{f}_t$, and state of the economy m_t. Now we introduce a variable transformation that reduces the number of state variables in Equation (16) to two. We define

$$X \equiv m^{-1}K, \tag{A1}$$

so that Equation (16) simplifies to

$$\begin{aligned}(r-g)J &= \widehat{f}X^{\alpha} - \delta X + \frac{1}{2\varphi}(J_X - 1)^2 X - gXJ_X \\ &\quad -\lambda_B\left(\widehat{f} - \overline{f}\right)J_f + \frac{1}{2}\left(\left(\sigma_f \eta_B\right)^2 + \left(\frac{\gamma_B}{\sigma_a}\right)^2\right)J_{ff},\end{aligned} \tag{A2}$$

where

$$J\left(X, \widehat{f}\right) = m^{-1}V\left(K, \widehat{f}, m\right). \tag{A3}$$

Thus, one can solve for the function J in Equation (A2) and then use Equation (A3) to obtain the value function V. By differentiating both sides of Equation (A3) with respect to K, one obtains

$$V_K = J_X. \tag{A4}$$

Equation (A4) states that marginal q, V_K, equals the derivative of J with respect to X.

We use a dual numerical approximation strategy to solve for functions J and J_X. First, we approximate J_X. Differentiating both sides of Equation (A2) with respect to X, we obtain

$$rJ_X = \alpha\hat{f}X^{\alpha-1} - \delta + \frac{1}{2\varphi}\left(J_X - 1\right)^2 + \left[\frac{J_X - 1}{\varphi} - g\right]XJ_{XX}$$
$$-\lambda_B\left(\hat{f} - \bar{f}\right)J_{fX} + \frac{1}{2}\left(\left(\sigma_f\eta_B\right)^2 + \left(\frac{\gamma_B}{\sigma_a}\right)^2\right)J_{ffX}. \tag{A5}$$

We solve for J_X in Equation (A5) using a Chebyshev polynomial approximation. We provide only a brief overview of this procedure—see Judd (1998) for details. Chebyshev polynomials are a family of orthogonal functions that constitute a basis for the space of continuous functions.

We approximate J_X as a sum of products of Chebyshev polynomials:

$$J_X\left(X, \hat{f}\right) = \sum_{m=0}^{M}\sum_{n=0}^{N} a_{m,n}T_m\left(2\frac{X - X_L}{X_H - X_L} - 1\right)T_n\left(2\frac{\hat{f} - f_L}{f_H - f_L} - 1\right). \tag{A6}$$

In Equation (A6), M and N are integers that specify approximation degrees, $\left\{a_{m,n}\right\}$ is a set of polynomial coefficients, and $\{T_m\}$ and $\{T_n\}$ are Chebyshev polynomials.[15] The approximation applies over intervals $X \in \left[X_L, X_H\right]$ and $\hat{f} \in \left[f_L, f_H\right]$, which we choose such that the realized state variables X and $\hat{f}$ attain values outside the bounds with less than 1% probability. To solve for the unknown polynomial coefficients $\left\{a_{m,n}\right\}$, we write Equation (A5) in terms of the Chebyshev approximation and its partial derivatives. Then we minimize the sum of squared differences between the right- and left-hand sides of the equation over a grid of $\left(X, \hat{f}\right)$ points using a nonlinear least squares routine.

Next, we use a similar polynomial approximation for the function J:

$$J\left(X, \hat{f}\right) = \sum_{m=0}^{M}\sum_{n=0}^{N} b_{m,n}T_m\left(2\frac{X - X_L}{X_H - X_L} - 1\right)T_n\left(2\frac{\hat{f} - f_L}{f_H - f_L} - 1\right). \tag{A7}$$

[15] We use approximation degrees $M = 80$ and $N = 10$.

Equation (A7) differs from Equation (A6) only in the polynomial coefficients $\{b_{m,n}\}$. The partial differential equation that characterizes J in Equation (A2) is linear in J and its partial derivatives, except for the term that includes $(J_X - 1)^2$. Because we already know the numerical approximation for J_X, we can use this function to compute $(J_X - 1)^2$. Finally, using the computed value of J_X and the Chebyshev approximations for J and its partial derivatives in Equation (A7), we express Equation (A2) as a linear system of equations in coefficients $\{b_{m,n}\}$, which we can easily solve by matrix inversion. The resulting solution for J allows us to compute the firm's value function and its optimal investment policy using Equations (A3) and (15), respectively.

References

Baker, Malcolm, Jeremy Stein, and Jeffrey Wurgler, 2003, When does the market matter? Stock prices and the investment of equity-dependent firms, *Quarterly Journal of Economics* 118, 969-1005.

Banz, Rolf W., 1981, The relationship between return and market value of common stocks, *Journal of Financial Economics* 9, 3-18.

Barber, Brad, and Terrance Odean, 2001, Boys will be boys: Gender, overconfidence, and common stock investment, *Quarterly Journal of Economics* 116, 261–292.

Barberis, Nicholas, Andrei Shleifer, and Robert Vishny, 1998, A model of investor sentiment, *Journal of Financial Economics* 49, 307–343.

Benartzi, Shlomo, 2001, Excessive extrapolation and the allocation of 401(k) accounts to company stock, *Journal of Finance* 56, 1747-1764.

Bernard, Victor L., and Jacob K. Thomas, 1989, Post-earnings-announcement drift: Delayed price response or risk premium? *Journal of Accounting Research* 27, 1-36.

Black, Fischer, 1986, Noise, *Journal of Finance* 41, 529-543.

Chen, Long, Robert Novy-Marx, and Lu Zhang, 2010, An alternative three-factor model, Ohio State University working paper.

Chirinko, Robert S., and Huntley Schaller, 2001, Business fixed investment and "bubbles": The Japanese case, *American Economic Review* 91, 663-680.

Cochrane, John H., 2001, *Asset Pricing*, Princeton University Press.

Cooper, Michael J., Huseyin Gulen, and Michael J. Schill, 2008, Asset growth and the cross-section of stock returns, *Journal of Finance* 63, 1609-1651.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, 1998, Investor psychology and security market under- and overreactions, *Journal of Finance* 53, 1839-1886.

Davis, James L., Eugene F. Fama, and Kenneth R. French, 2000, Characteristics, covariances, and average returns: 1929 to 1997, *Journal of Finance* 55, 389-406.

Fama, Eugene F., and James D. MacBeth, 1973, Risk, return, and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607-636.

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427-465.

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3-56.

Gilchrist, Simon, Charles P. Himmelberg, and Gur Huberman, 2005, Do stock price bubbles influence corporate investment? *Journal of Monetary Economics* 52, 805-827.

Gomes, João, Amir Yaron, and Lu Zhang, 2006, Asset pricing implications of firms' financing constraints, *Review of Financial Studies* 19, 1321-1356.

Hansen, Lars P., and Steven F. Richard, 1987, The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models, *Econometrica* 55, 587-613.

Hennessy, Christopher A., and Toni M. Whited, 2007, How costly is external financing? *Journal of Finance* 62, 1705-1745.

Hong, Harrison, and Jeremy C. Stein, 1999, A unified theory of underreaction, momentum trading, and overreaction in asset markets, *Journal of Finance* 54, 2143-2184.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance* 48, 65-91.

Judd, Kenneth L., 1998, Numerical methods in economics, The MIT Press.

Kaplan, Steven N., and Luigi Zingales, 1997, Do investment-cash flow sensitivities provide useful measures of financing constraints? *Quarterly Journal of Economics* 112, 169-215.

La Porta, Rafael, Josef Lakonishok, Andrei Shleifer, and Robert W. Vishny, 1997, Good news for value stocks: Further evidence on market efficiency, *Journal of Finance* 52, 859-874.

Lakonishok, Josef, Andrei Shleifer, and Robert Vishny, 1994, Contrarian investment, extrapolation, and risk, *Journal of Finance* 49, 1541-1578.

Liu, Laura Xiaolei, Toni M. Whited, and Lu Zhang, 2009, Investment-based expected stock returns, *Journal of Political Economy* 117, 1105-1139.

Newey, Whitney K., and Kenneth D. West, 1987, A simple positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix estimator, *Econometrica* 55, 703-708.

Newey, Whitney K., and Kenneth D. West, 1994, Automatic lag selection in covariance matrix estimation, *Review of Economic Studies* 61, 631-653.

Odean, Terrance, 1998, Volume, volatility, and profit when all traders are above average, *Journal of Finance* 53, 1887-1934.

Odean, Terrance, 1999, Do investors trade too much? *American Economic Review* 89, 1279-1298.

Panageas, Stavros, 2005, The neoclassical theory of investment in speculative markets, University of Chicago working paper.

Polk, Christopher, and Paola Sapienza, 2009, The stock market and corporate investment: A test of catering theory, *Review of Financial Studies* 22, 187-217.

Scheinkman, Jose A., and Wei Xiong, 2003, Overconfidence and speculative bubbles, *Journal of Political Economy* 111, 1183-1219.

Summers, Lawrence H., 1986, Does the stock market rationally reflect fundamental values? *Journal of Finance* 41, 591-601.

Titman, Sheridan, K.C. John Wei, and Feixue Xie, 2004, Capital investment and stock returns, *Journal of Financial and Quantitative Analysis* 39, 677-700.

Table 1: The Fit of the Benchmark Model to the Data

Panel A: Comparing Empirical Moments and Simulated Moments in Two Estimations

Moments	Empirical Estimate	Standard Error	13-Moment Model	15-Moment Model
Mean Tobin's Q	1.5801	0.3696	1.6818	1.6317
Mean Cash Flow	0.1317	0.0333	0.1129	0.1102
Std Dev of Q	0.9370	0.2987	0.9166	0.9649
Std Dev of Abnormal Return	0.2108	0.0252	0.2029	0.1997
Std Dev of Cash Flow	0.0940	0.0206	0.0879	0.1040
Std Dev of Investment	0.1686	0.0327	0.0976	0.0968
Persistence of Cash Flow	0.7901	0.0170	0.8018	0.8066
Abn Ret Sensitivity to Cash Flow	1.7569	0.1485	1.7742	1.7593
Investment Sensitivity to Q	0.0666	0.0213	0.0690	0.0635
Investment Sensitivity to Cash Flow	0.6603	0.0544	0.6409	0.6551
Cash Flow Anomaly – Year 1	0.0139	0.0143	0.0176	0.0408
Tobin's Q Anomaly – Year 1	-0.0638	0.0235	-0.0663	-0.0265
Tobin's Q Anomaly – Years 2 and 3	-0.0456	0.0205	-0.0576	-0.0169
Investment Anomaly – Year 1 *	-0.0671	0.0139	-0.1077	-0.0353
Investment Anomaly – Years 2 and 3 *	-0.0242	0.0040	-0.0880	-0.0225
χ^2 *statistic*			8.88	16.27
χ^2 *p-value*			(0.114)	(0.023)

* Investment anomalies not included in estimation of the 13-moment model

Table 1: The Fit of the Benchmark Model to the Data (Continued)

Panel B: Two Sets of Parameter Estimates in the Benchmark Model

	13-Moment Estimation		15-Moment Estimation	
Parameters	Estimate	Standard Error	Estimate	Standard Error
Returns-to-Scale α	0.8493	0.0244	0.8515	0.0088
Adjustment Cost φ	2.8968	0.3998	2.8839	0.4119
Productivity Shock σ_f	0.5200	0.0581	0.5180	0.1106
Noise in Cash Flow σ_a	0.1780	0.0491	0.2667	0.0349
True Mean Reversion λ	0.2302	0.0393	0.1629	0.0309
Biased Mean Reversion λ_B	0.1434	0.0235	0.1508	0.0245
True Signal Quality θ	0.4576	0.0672	0.4908	0.0512
Biased Signal Quality θ_B	0.9364	0.0629	0.8095	0.0394
Std Dev of Estimation Error, Rational	0.2527		0.2976	
Std Dev of Estimation Error, Biased	0.0566		0.1507	
Std Dev of Productivity	0.7664		0.9076	

Table 2: The Cross Section of Expected Returns and Mispricing

Panel A: Summary Statistics for Expected Returns and Mispricing

Summary Statistic	Value-Weighted Mean		Equal-Weighted Mean		Median		Standard Deviation	
Moments in Model	13	15	13	15	13	15	13	15
One-Year Expected Returns	-0.0133	-0.0027	0.0067	0.0029	0.0006	0.0015	0.0582	0.0484
Mispricing	0.7444	0.9553	0.7528	0.9227	0.7321	0.9207	0.1731	0.1120

Panel B: Comparing True and Observed One-Year Expected Returns

Sorting Variable	1-Yr Expected Return		Q		Investment		Cash Flow	
Moments in Model	13	15	13	15	13	15	13	15
Lowest Decile	-0.0811	-0.0789	0.0611	0.0244	0.0776	0.0259	-0.0056	-0.0199
2	-0.0498	-0.0458	0.0228	0.0109	0.0368	0.0136	0.0024	-0.0090
3	-0.0331	-0.0297	0.0121	0.0054	0.0207	0.0083	0.0034	-0.0042
4	-0.0189	-0.0165	0.0033	0.0034	0.0102	0.0056	0.0055	-0.0016
5	-0.0058	-0.0044	0.0010	0.0006	0.0037	0.0020	0.0051	0.0046
6	0.0075	0.0073	-0.0044	-0.0008	-0.0051	-0.0002	0.0086	0.0056
7	0.0222	0.0197	-0.0064	-0.0026	-0.0099	-0.0028	0.0101	0.0071
8	0.0396	0.0335	-0.0068	-0.0036	-0.0158	-0.0059	0.0109	0.0115
9	0.0634	0.0517	-0.0081	-0.0041	-0.0211	-0.0076	0.0123	0.0133
Highest Decile	0.1231	0.0918	-0.0074	-0.0051	-0.0298	-0.0105	0.0149	0.0210

Table 2: The Cross Section of Expected Returns and Mispricing (Continued)

Panel C: Comparing True and Observed Mispricing

Sorting Variable	Mispricing		Q		Investment		Cash Flow	
Moments in Model	13	15	13	15	13	15	13	15
Lowest Decile	0.4866	0.7318	1.0027	0.9912	0.9490	0.9464	0.7084	0.8251
2	0.5817	0.8106	0.8763	0.9620	0.8614	0.9459	0.7569	0.8878
3	0.6348	0.8487	0.8272	0.9471	0.8187	0.9403	0.7634	0.9068
4	0.6778	0.8796	0.7879	0.9381	0.7859	0.9355	0.7681	0.9171
5	0.7176	0.9072	0.7617	0.9265	0.7634	0.9282	0.7628	0.9319
6	0.7576	0.9338	0.7295	0.9184	0.7331	0.9224	0.7676	0.9365
7	0.8010	0.9618	0.6994	0.9080	0.7132	0.9157	0.7670	0.9410
8	0.8535	0.9935	0.6686	0.8983	0.6828	0.9069	0.7580	0.9507
9	0.9239	1.0356	0.6266	0.8858	0.6455	0.9000	0.7499	0.9561
Highest Decile	1.0935	1.1245	0.5483	0.8517	0.5758	0.8850	0.7269	0.9733

Table 3: Cross-Sectional Regressions of One-Year Expected Returns on Firm Characteristics Using Simulated Data

Panel A: Simulated Data based on the 13-Moment Estimates

Regressions	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Q	-0.0102					0.0062	0.0098	-0.0033
Investment		-0.0280				-0.0313	-0.0713	-0.0584
Cash Flow			0.0056				0.0532	0.0612
Size				-0.0272				-0.0294
Past One-year Return					-0.0016			-0.0056
R^2	0.0323	0.2313	0.0105	0.2185	0.0023	0.2397	0.6454	0.8040

Panel B: Simulated Data based on the 15-Moment Estimates

Regressions	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Q	-0.0016					0.0003	0.0067	0.0039
Investment		-0.0098				-0.0098	-0.0575	-0.0677
Cash Flow			0.0116				0.0572	0.0844
Size				-0.0073				-0.0298
Past One-year Return					0.0003			-0.0085
R^2	0.0024	0.0418	0.0583	0.0234	0.0009	0.0419	0.5167	0.7857

Table 4: Simulated Moments in 13-Moment Models with Different Biases

Bias in the Model	Extrapolation	Overconfidence	No Biases	Both Biases	Empirical
Mean Tobin's Q	1.5965	1.4510	1.445	1.6818	1.5801
Mean Cash Flow	0.1125	0.1185	0.1198	0.1129	0.1317
Std Dev of Q	0.5087	0.3041	0.2979	0.9166	0.9370
Std Dev of Abnormal Return	0.2026	0.1560	0.1644	0.2029	0.2108
Std Dev of Cash Flow	0.0868	0.0907	0.0907	0.0879	0.0940
Std Dev of Investment	0.0866	0.0653	0.0643	0.0976	0.1686
Persistence of Cash Flow	0.7959	0.8032	0.7975	0.8018	0.7901
Abn Ret Sensitivity to Cash Flow	3.0180	1.4084	2.4307	1.7742	1.7569
Investment Sensitivity to Q	0.1344	0.1832	0.1854	0.0690	0.0666
Investment Sensitivity to Cash Flow	0.7077	0.2292	0.4443	0.6409	0.6603
Cash Flow Anomaly – Year 1	-0.0826	0.0648	-	0.0176	0.0139
Tobin's Q Anomaly – Year 1	-0.0540	-0.0480	-	-0.0663	-0.0638
Tobin's Q Anomaly – Years 2 and 3	-0.0409	-0.0301	-	-0.0576	-0.0456
Investment Anomaly – Year 1 *	-0.0747	-0.0518	-	-0.1077	-0.0671
Investment Anomaly – Years 2 and 3 *	-0.0552	-0.0327	-	-0.0880	-0.0242

* Investment anomalies not included in estimation of the 13-moment model

Table 5: True Mispricing and One-Year Expected Returns in 13-Moment Models with Different Biases

Panel A: True Mispricing in Portfolios Sorted by Firm Characteristics

Sorting Variable	Mispricing		Q		Investment		Cash Flow	
Bias in the Model	Extrapolat.	Overconf.	Extrapolat.	Overconf.	Extrapolat.	Overconf.	Extrapolat.	Overconf.
Lowest Decile	0.7123	0.7112	0.9649	1.0343	0.9246	1.0266	0.8998	0.8145
2	0.7606	0.8043	0.9075	0.9893	0.8920	0.9912	0.8840	0.8818
3	0.7856	0.8529	0.8814	0.9684	0.8723	0.9728	0.8690	0.9074
4	0.8062	0.8925	0.8606	0.9634	0.8564	0.9632	0.8556	0.9318
5	0.8248	0.9283	0.8425	0.9504	0.8419	0.9499	0.8432	0.9471
6	0.8425	0.9637	0.8250	0.9419	0.8293	0.9415	0.8323	0.9645
7	0.8606	1.0009	0.8064	0.9339	0.8140	0.9299	0.8191	0.9823
8	0.8815	1.0443	0.7857	0.9209	0.7977	0.9201	0.8056	0.9959
9	0.9079	1.1009	0.7610	0.9032	0.7781	0.9076	0.7869	1.0137
Highest Decile	0.9657	1.2196	0.7128	0.8827	0.7405	0.8867	0.7511	1.0504

Panel B: One-Year Expected Returns in Portfolios Sorted by Firm Characteristics

Sorting Variable	1-Yr Expected Return		Q		Investment		Cash Flow	
Bias in the Model	Extrapolat.	Overconf.	Extrapolat.	Overconf.	Extrapolat.	Overconf.	Extrapolat.	Overconf.
Lowest Decile	-0.0372	-0.0740	0.0382	0.0330	0.0458	0.0341	0.0491	-0.0343
2	-0.0232	-0.0439	0.0172	0.0157	0.0239	0.0189	0.0266	-0.0150
3	-0.0155	-0.0282	0.0098	0.0083	0.0144	0.0115	0.0163	-0.0080
4	-0.0091	-0.0157	0.0043	0.0067	0.0075	0.0075	0.0090	-0.0011
5	-0.0029	-0.0043	-0.0002	0.0025	0.0024	0.0028	0.0026	0.0029
6	0.0035	0.0075	-0.0034	-0.0002	-0.0027	-0.0006	-0.0030	0.0079
7	0.0103	0.0198	-0.0062	-0.0024	-0.0077	-0.0045	-0.0084	0.0126
8	0.0183	0.0339	-0.0092	-0.0062	-0.0126	-0.0081	-0.0141	0.0161
9	0.0296	0.0527	-0.0110	-0.0108	-0.0183	-0.0121	-0.0208	0.0206
Highest Decile	0.0554	0.0938	-0.0147	-0.0153	-0.0279	-0.0181	-0.0325	0.0299

Table 6: Summarizing the Rational Manager Model's Fit to the Data

Panel A: Comparing Simulated Moments in the Rational and Biased Manager Models

Simulated Model	13-Moment			15-Moment		
Manager Type	Rational Manager	Biased Manager	% Change	Rational Manager	Biased Manager	% Change
Mean Tobin's Q	1.7167	1.6818	2.1%	1.6445	1.6317	0.8%
Mean Cash Flow	0.1176	0.1129	4.2%	0.1117	0.1102	1.4%
Std Dev of Q	0.5560	0.9166	-39.3%	0.7641	0.9649	-20.8%
Std Dev of Abnormal Return	0.2693	0.2029	32.7%	0.2853	0.1997	42.9%
Std Dev of Cash Flow	0.0905	0.0879	2.9%	0.1048	0.1040	0.7%
Std Dev of Investment	0.0640	0.0976	-34.4%	0.092	0.0968	-5.0%
Persistence of Cash Flow	0.7972	0.8018	-0.6%	0.8042	0.8066	-0.3%
Abn Ret Sensitivity to Cash Flow	1.8432	1.7742	3.9%	1.8553	1.7593	5.5%
Investment Sensitivity to Q	0.0751	0.0690	8.9%	0.0722	0.0635	13.7%
Investment Sensitivity to Cash Flow	0.5647	0.6409	-11.9%	0.6839	0.6551	4.4%
Cash Flow Anomaly – Year 1	0.0092	0.0176	-47.7%	0.046	0.0408	12.9%
Tobin's Q Anomaly – Year 1	-0.1339	-0.0663	101.9%	-0.0291	-0.0265	10.0%
Tobin's Q Anomaly – Years 2 and 3	-0.1208	-0.0576	109.8%	-0.0155	-0.0169	-8.5%
Investment Anomaly – Year 1	-0.0037	-0.1077	-96.6%	0.0237	-0.0353	-167.2%
Investment Anomaly – Years 2 and 3	-0.0353	-0.0880	-59.9%	0.0093	-0.0225	-141.3%

Table 6: Summarizing the Rational Manager Model's Fit to the Data (Continued)

Panel B: Comparing Expected Returns and Mispricing in the Rational and Biased Manager Models

Simulated Model	13-Moment		15-Moment	
Manager Type	Rational Manager	Biased Manager	Rational Manager	Biased Manager
Value-weighted Mean Expected Return	-0.0090	-0.0133	-0.0019	-0.0027
Equal-weighted Mean Expected Return	0.0068	0.0067	0.0021	0.0029
Median Expected Return	0.0012	0.0006	0.0007	0.0015
Std Dev of Expected Returns	0.0730	0.0582	0.0592	0.0484
Value-weighted Mean Mispricing	0.8250	0.7444	0.9558	0.9553
Equal-weighted Mean Mispricing	0.8751	0.7528	0.9560	0.9227
Median Mispricing	0.8331	0.7321	0.9460	0.9207
Std Dev of Mispricing	0.2456	0.1731	0.1490	0.1120

Table 7: Comparing the Real Effects of Information Processing Biases in Different Models

Panel A: Steady State Comparisons

Estimation	13-Moment			15-Moment
Model	Both Biases	Extrapolation Only	Overconfidence Only	Both Biases
Aggregate value added V – K, biased / rational	0.355	0.907	0.819	0.920
Aggregate value per unit of capital V/K, biased	1.077	1.221	1.311	1.304
Aggregate value per unit of capital V/K, rational	1.385	1.385	1.385	1.375
Median firm value per unit of capital V/K, biased	1.139	1.269	1.344	1.382
Median firm value per unit of capital V/K, rational	1.424	1.424	1.424	1.466
Aggregate capital stock K, biased / rational	1.820	1.567	1.007	1.135
Median firm capital stock K, biased / rational	1.518	1.460	1.015	1.097

Panel B: Rational Manager Takes Control of the Firm Run by the Biased Manager

Estimation	13-Moment			15-Moment
Model	Both Biases	Extrapolation Only	Overconfidence Only	Both Biases
Median marginal investment rate, biased – rational	0.015	0.014	0.000	0.004
Std dev marginal investment rate, biased – rational	0.069	0.027	0.051	0.038
Aggregate firm value, biased / rational	0.905	0.979	0.956	0.983
Median firm value, biased / rational	0.885	0.975	0.949	0.972

Table 8: Cross-Sectional Regressions of the Biased Manager's Overinvestment

Note: Biased Manager's Overinvestment = Biased Manager's Marginal Inv. Rate – Rational Manager's Marginal Inv. Rate

Panel A: Overinvestment in Simulated Data based on the 13-Moment Parameter Estimates

Regression	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Q	0.0154					-0.0119	-0.0155	-0.0031
Investment		0.0460				0.0523	0.0911	0.0787
Cash Flow			0.0083				-0.0515	-0.0630
Size				0.0391				0.0328
Past One-year Return					0.0103			0.0131
R^2	0.0542	0.4426	0.0152	0.3199	0.0237	0.4650	0.7351	0.9338

Panel B: Overinvestment in Simulated Data based on the 15-Moment Parameter Estimates

Regression	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Q	0.0022					0.0001	-0.0048	-0.0026
Investment		0.0109				0.0109	0.0477	0.0555
Cash Flow			-0.0064				-0.0443	-0.0673
Size				0.0077				0.0248
Past One-year Return					0.0018			0.0079
R^2	0.0066	0.0831	0.0286	0.0418	0.0032	0.0841	0.5430	0.8480

Figure 1: Percentage Modeling Errors = (Simulated Moment / Empirical Moment) – 1

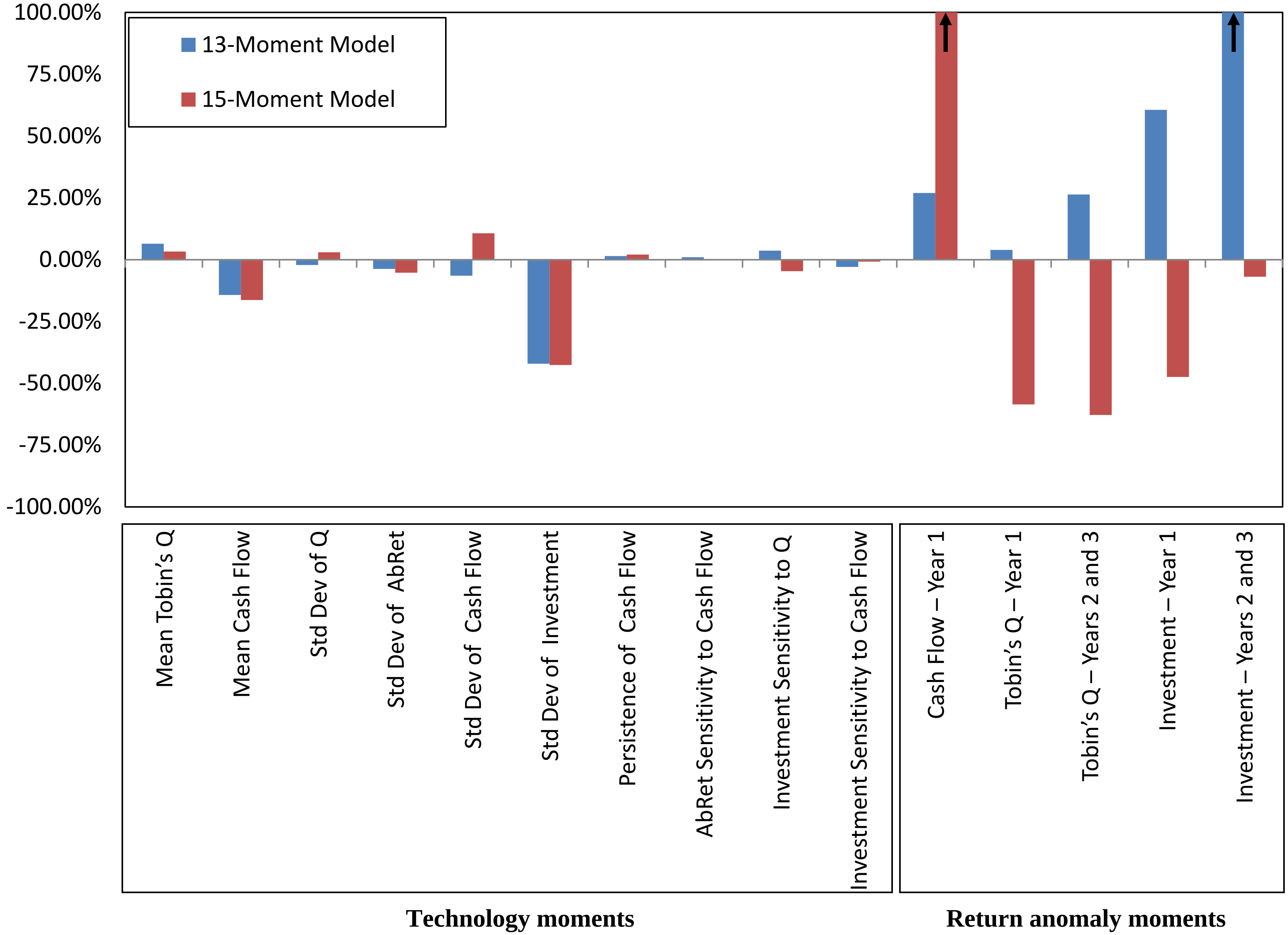

Figure 2: Expected abnormal returns of top and bottom decile portfolios

(Using 13-moment estimates)

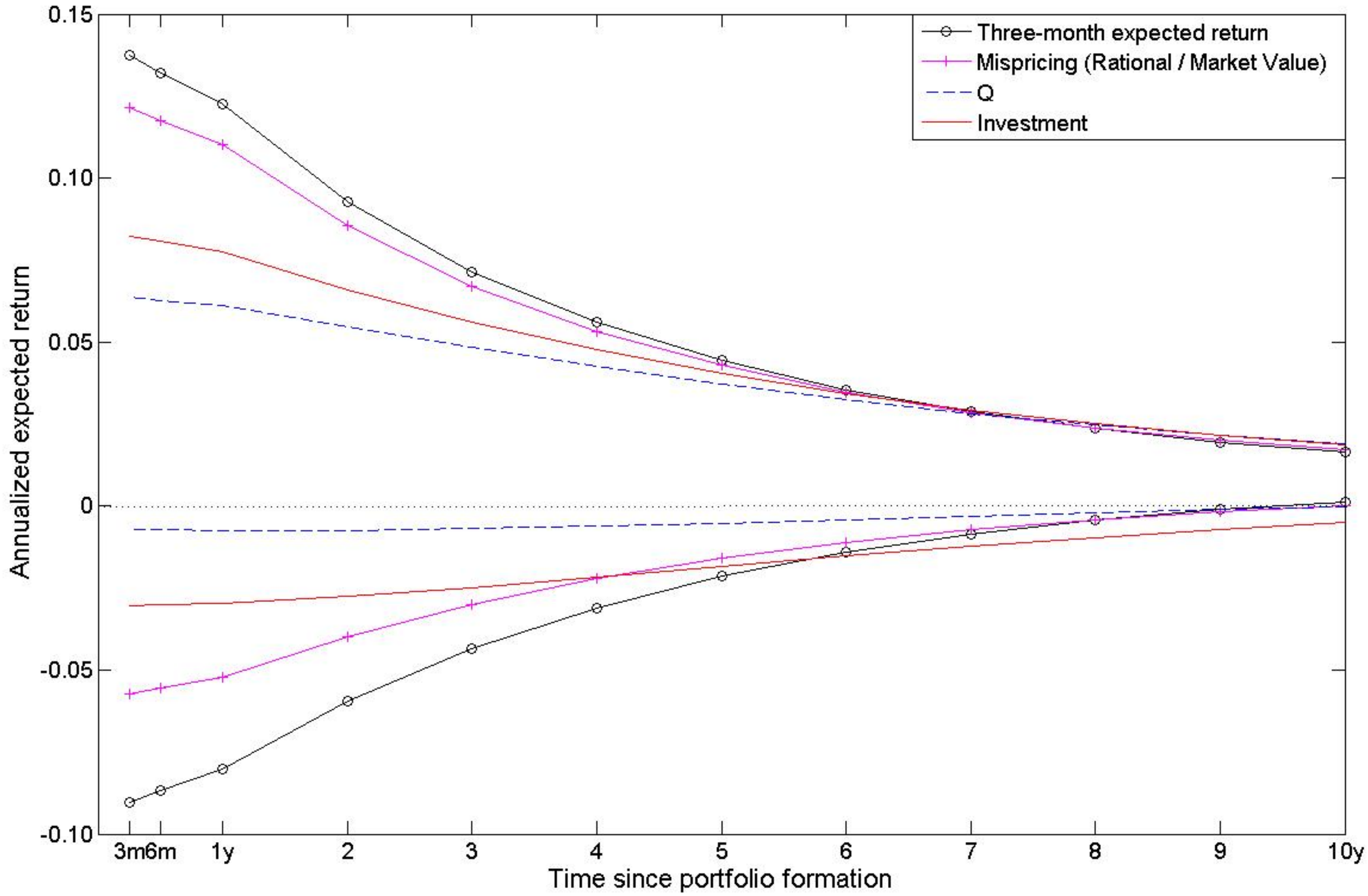

Figure 3: Expected abnormal returns of top and bottom decile portfolios

(Using 15-moment estimates)