Estimating Fiscal Policy Rules for Japan, US, and UK

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Abstract

This paper estimates fiscal policy feedback rules in Japan, the United States, and the United Kingdom for more than a century, allowing for stochastic regime changes. Estimating a Markov-switching model by the Bayesian method, we find the following: First, the Japanese data clearly reject the view that the fiscal policy regime is fixed, i.e., that the Japanese government adopted a Ricardian or a non-Ricardian regime throughout the entire period. Instead, our results indicate a stochastic switch of the debt-GDP ratio between stationary and nonstationary processes, and thus a stochastic switch between Ricardian and non-Ricardian regimes. Second, our simulation exercises using the estimated parameters and transition probabilities do not necessarily reject the possibility that the debt-GDP ratio may be nonstationary even in the long run (i.e., globally nonstationary). Third, the Japanese result is in sharp contrast with the results for the U.S. and the U.K. which indicate that in these countries the government's fiscal behavior is consistently characterized by Ricardian policy.

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1 Introduction

Recent studies about the conduct of monetary policy suggest that the fiscal policy regime has important implications for the choice of desirable monetary policy rules, particularly, monetary policy rules in the form of inflation targeting (Sims (2005), Benigno and Woodford (2006)). It seems safe to assume that fiscal policy is characterized as "Ricardian" in the terminology of Woodford (1995), or "passive" in the terminology of Leeper (1991), if the government shows strong fiscal discipline. If this is the case, we can design an optimal monetary policy rule without paying any attention to fiscal policy. However, if the economy is unstable in terms of the fiscal situation, it would be dangerous to choose a monetary policy rule independently of fiscal policy rules. For example, some researchers argue that the recent accumulation of public debt in Japan is evidence of a lack of fiscal discipline on the part of the Japanese government, and that it is possible that government bond market participants may begin to doubt the government's intention and ability to repay the public debt. If this is the case, we may need to take the future evolution of the fiscal regime into consideration when designing a monetary policy rule.

Against this background, the purpose of this paper is to estimate fiscal policy feedback rules for Japan, the United States, and the United Kingdom for a period spanning more than a century, so as to gain a deeper understanding of the evolution of fiscal policy regimes. One of the most important features of recent studies on fiscal policy rules is the recognition that fiscal policy regimes are *not* fixed over time, but evolve in a stochastic manner.[1] For example, Favero and Monacelli (2005) and Davig and Leeper (2005) estimate fiscal policy rules for the United States during the postwar period under the assumption that there are two alternative fiscal regimes, i.e. a "passive" and an "active" regime, and that stochastic fluctuations between the two regimes may be characterized by a Markov process. They find that fiscal regime switching occurred fairly frequently: Davig and Leeper (2005) report that there were twelve fiscal regime changes during the period of 1948-2004, while Favero and Monacelli (2005) found that fiscal policy was even more unstable than monetary policy.[2]

However, these pioneering works still have some shortcomings. First, they do not make an empirical distinction between *locally* and *globally* Ricardian policy rules. For example, Favero and Monacelli

[1] A comprehensive list of recent empirical studies on fiscal policy rules is provided by Afonso (2005).

[2] These studies are in sharp contrast with research on fiscal sustainability initiated by Hamilton and Flavin (1986) about two decades ago, which typically investigates whether fiscal variables such as the debt-GDP ratio are characterized by a stationary or a nonstationary process without any break (Trehan and Walsh (1988, 1991), Wilcox (1989), Ahmed and Rogers (1995)).

(2005) specify a locally Ricardian rule and ask whether the U.S. government has followed this rule or deviated from it. However, as pointed out by Bohn (1998) and Canzoneri et al. (2001), the transversality condition may be satisfied even if the debt-GDP ratio does not follow a stationary process, or equivalently, even if a government deviates from a locally Ricardian policy rule. Second, the studies by Davig and Leeper (2005) and Favero and Monacelli (2005) do not pay much attention to governments' tax smoothing behavior. As pointed out by Barro (1986) and Bohn (1998), tax-smoothing behavior may create a negative correlation between public debt and the primary surplus. Without properly controlling for such behavior when estimating a government's reaction function, researchers may easily obtain biased estimates of fiscal policy reactions to a change in public debt. Third, the empirical approach of these studies is based on maximum likelihood estimation and implicitly assumes that the debt-GDP ratio is stationary at least in the long run (i.e., that it is "Harris recurrent"). This condition is satisfied if, for example, the debt-GDP ratio switches between two AR (p) processes, one stationary and the other nonstationary, but the nonstationary regime is not visited too often or for too long (Francq and Zakoian (2001)). However, there is no a priori reason to believe that this condition is indeed satisfied for the debt-GDP ratio; it is possible that a non-Ricardian regime is visited frequently and/or for a long time, depending on the transition probabilities. If this is the case, maximum likelihood estimators will fail to follow a standard normal distribution even asymptotically (Douc et al. (2004)).

We derive an estimating equation based on a model of optimal tax smoothing, paying particular attention to differences between locally and globally Ricardian rules, and then estimate the equation by the Bayesian method. The main findings of the paper are as follows. First, the Japanese data set, covering the period 1885-2004, clearly rejects the view that the fiscal policy regime was fixed throughout the sample period, i.e., that the Japanese government adopted only one policy stance - Ricardian or non-Ricardian - throughout the entire period. Rather, our empirical results suggest that the fiscal policy regime evolved over time in a stochastic manner, and that the debt-GDP ratio is well described by a Markov switching model with two or three states. Specifically, Japanese fiscal policy is characterized by a locally Ricardian rule in 1885-1925 and 1950-1970. The former roughly corresponds to the period when Japan had adopted the gold standard, under which the government was forced to maintain a balanced budget. Japan left the gold standard in 1917. The latter period corresponds to the period of fiscal restructuring just after WWII, when the Japanese government, under the direction of the Supreme Commander for Allied Powers (SCAP) introduced a balanced budget system as part of the so-called "Dodge Line" in order to stop runaway inflation. On the other hand, Japanese fiscal policy is characterized by non-Ricardian rules in 1930-1950 and 1970-2004, suggesting that the Japanese government abandoned fiscal discipline not only during WWII, but also in the most recent period starting in 1970. These empirical results are confirmed as being quite robust to changes in empirical specifications.

Second, given that the Japanese debt-GDP ratio switches between stationary and nonstationary processes, one may wonder to what value the debt-GDP ratio goes to in the long run. To address this question, we conduct stochastic simulation exercises using the estimated transition probabilities, and find that the debt-GDP ratio is quite likely to increase over the next 20 years, but will start declining after that and finally converge to zero. This implies that the debt-GDP process is "globally stationary" (i.e., stationary across regimes), although it may not necessarily be locally stationary (i.e., stationary within each regime).[3] However, we also find that this result is not very robust to changes in the specification of the estimating equation, such as the number of possible "states," and in some cases, we find global nonstationarity.

Third, we apply our methodology to U.S. and U.K. data sets to find that the fiscal behavior of the U.S. government throughout the entire sample period, 1840-2005, may be described as switching between locally Ricardian policy rules, while the behavior of the U.K. government during the entire sample period, 1830-2003, can be characterized as switching between globally Ricardian policy rules. Thus, the U.S. and U.K. results are in sharp contrast with the result for Japan. The U.S. result is consistent with Bohn (1998, 2005), but differs from Favero and Monacelli (2005) who report that U.S. government behavior deviated from Ricardian policy for most of their sample period, 1961-2002.

The remainder of this paper is organized as follows. Sections 2 and 3 explain our empirical approach, while Section 4 explains our data set. Section 5 presents the regression results. Section 6 concludes the paper. A detailed explanation of our data set is provided in Appendix.

[3] The recent behavior of investors in the Japanese government bond market seems to be consistent with this as they show no hesitation to purchase government bonds even though the government is lacking fiscal discipline and rapidly accumulating public debt.

2 Ricardian fiscal policy

2.1 The government's budget constraint

We start by looking at the government's budget constraint. Let us denote the nominal amount of public debt and base money at the end of period t by B_t and M_t. Also, we denote the one-period nominal interest rate starting in period $t-1$ by i_{t-1}, the nominal government expenditure (excluding interest payments) and the nominal tax revenue in period t by G_t and T_t. Then the consolidated flow budget constraint of the government and the central bank takes the following form:

$$M_t + B_t = (1 + i_{t-1})B_{t-1} + M_{t-1} + (G_t - T_t).$$

Dividing both sides of this equation by nominal GDP, Y_t, we obtain:

$$m_t + b_t = \frac{1 + i_{t-1}}{1 + n_t} b_{t-1} + \frac{1}{1 + n_t} m_{t-1} - s_t,$$

where m_t, b_t, s_t, and n_t are defined by

$$m_t \equiv \frac{M_t}{Y_t};\ b_t \equiv \frac{B_t}{Y_t};\ s_t \equiv \frac{T_t - G_t}{Y_t};\ n_t \equiv \frac{Y_t - Y_{t-1}}{Y_{t-1}}.$$

Denoting the total consolidated liabilities by $w_t (\equiv m_t + b_t)$, the transition equation for w_t can be expressed as:

$$w_t - w_{t-1} = \frac{i_{t-1}}{1 + n_t} w_{t-1} - \frac{n_t}{1 + n_t} w_{t-1} - \left[\frac{i_{t-1}}{1 + n_t} m_{t-1} + s_t \right]. \tag{1}$$

Note that $\frac{i_{t-1}}{1+n_t} m_{t-1}$ represents seignorage and that an increase in the primary surplus s_t or seignorage reduces total liabilities. Also note that an increase in the nominal growth rate n_t contributes to lowering total liabilities through the second term on the right-hand side, $-\frac{n_t}{1+n_t} w_{t-1}$, which is sometimes called the "growth dividend" (Bohn (2005)).

Equation (1) can be rewritten as

$$w_t = q_t \left[w_{t+1} + s_{t+1} \right] + \frac{i_t}{1 + i_t} m_t, \tag{2}$$

where q_t represents a discount factor that is defined by

$$q_t \equiv \frac{1 + n_{t+1}}{1 + i_t}.$$

Integrating equation (2) forward from the current period and taking expectations conditional on information available in period t, we obtain a present-value expression of the budget constraint:

$$w_t = s_t + E_t \sum_{j=1}^{T} \left(\prod_{k=0}^{j-1} q_{t+k} \right) s_{t+j} + E_t \sum_{j=0}^{T-1} \left(\prod_{k=0}^{j-1} q_{t+k} \right) \left(\frac{i_{t+j}}{1+i_{t+j}} \right) m_{t+j} + E_t \left(\prod_{k=0}^{T-1} q_{t+k} \right) w_{t+T}.$$

This implies that the transversality condition is given by

$$\lim_{T \to \infty} E_t \left(\prod_{k=0}^{T-1} q_{t+k} \right) w_{t+T} = 0. \tag{3}$$

2.2 Locally Ricardian policy rules

Woodford (1995) proposes that a fiscal policy commitment be called "Ricardian" if it implies that the transversality condition, equation (3), necessarily holds for all possible paths of endogenous variables (in particular, prices). More specifically, Woodford (1995, 1998) proposes two types of Ricardian fiscal policy rule.

The first type, which is referred to as "locally Ricardian," can be expressed as

$$s_t + \frac{i_{t-1}}{1+n_t} m_{t-1} = \left[\lambda_t + \frac{i_{t-1}}{1+n_t} \right] w_{t-1} + \nu_t, \tag{4}$$

where λ_t is a time-varying parameter satisfying $0 < \lambda_t \leq 1$, which represents the government's responsiveness to changes in total liabilities, and ν_t is an exogenous stationary variable. Note that the left-hand side of equation (4) represents the sum of the primary surplus and seignorage. Equation (4) requires the government to create a surplus in period t great enough to cover its interest payment in that period, $\frac{i_{t-1}}{1+n_t} w_{t-1}$.

By substituting (4) into (1), we can fully characterize the dynamics of w_t:

$$w_t = \left[1 - \lambda_t - \frac{n_t}{1+n_t} \right] w_{t-1} - \nu_t. \tag{5}$$

Under the assumption that n_t is an exogenous process (i.e., the government treats n_t as exogenously given when making a fiscal decision in period t),[4] this equation implies that w_t would be a stationary process and thus satisfies the transversality condition if the sum of λ_t and $\frac{n_t}{1+n_t}$ lies between zero and

[4]It is possible that n_t could be an endogenous variable in the sense that the government's fiscal behavior could have non-negligible consequences on the path of n_t. For example, as argued by Woodford (2001) among others, it might be possible that if the government does not react at all to changes in total liabilities (that is, $\lambda_t = 0$), then inflation endogenously emerges ($n_t > 0$), and consequently the coefficient on w_{t-1} in equation (5) becomes less than unity.

unity.[5] Note that the assumption of a locally Ricardian policy requires that $1-\lambda_t$ is smaller than unity, while stationarity of w requires that the coefficient on w_{t-1} in (5) is less than unity. These two conditions are closely related but not identical except for the case of $n_t = 0$.

An alternative specification to equation (4) would be:

$$s_t + \frac{i_{t-1}}{1+n_t} m_{t-1} + \frac{n_t}{1+n_t} w_{t-1} = \left[\hat{\lambda}_t + \frac{i_{t-1}}{1+n_t} \right] w_{t-1} + \nu_t. \tag{6}$$

Note that $\hat{\lambda}_t \equiv \lambda_t + \frac{n_t}{1+n_t}$. Now the government seeks to adjust the sum of the primary surplus, seignorage, and the growth dividend in response to changes in total liabilities. An important difference from equation (4) is that the government reduces the primary surplus when the growth dividend is positive, for example, due to high inflation, and increases it when the growth dividend is negative; on the other hand, equation (4) requires the government to create a primary surplus independently of the level of the growth dividend. It can be easily seen that the transition equation corresponding to (5) is now given by

$$w_t = \left[1 - \hat{\lambda}_t\right] w_{t-1} - \nu_t, \tag{7}$$

and that w_t is a stationary process if $\hat{\lambda}_t$ satisfies the condition that $0 < \hat{\lambda}_t \leq 1$.

Favero and Monacelli (2005) adopt a policy reaction function very close to equation (6). According to their definition, a government with fiscal discipline seeks to keep the primary deficit lower than the "debt-stabilizing deficit", which is given by

$$-\left[\frac{i_{t-1}}{1+n_t} - \frac{n_t}{1+n_t} \right] w_{t-1}.$$

Given this definition, the debt-stabilizing deficit becomes positive if n_t takes a sufficiently large positive value, implying that the government can run a deficit.

2.3 Globally Ricardian policy rules

The idea that the government should maintain a surplus large enough to at least cover interest payments seems to be a useful one from a practical point of view,[6] but the transversality condition

[5] Note that, from an econometric point of view, w_t is a stationary process if the coefficient on w_{t-1} in equation (5) lies between -1 and 1 ($-1 < 1 - \lambda_t - \frac{n_t}{1+n_t} < 1$). However, it seems safe to rule out the possibility that w converges over time to a constant value with oscillation, so that we can concentrate on the condition that the coefficient lies between 0 and 1 ($0 \leq 1 - \lambda_t - \frac{n_t}{1+n_t} < 1$).

[6] If we rewrite equation (4) as $s_t - \frac{i_{t-1}}{1+n_t} b_{t-1} = \lambda_t w_{t-1} + \nu_t$, we see that the rule requires that not the primary surplus but the traditional fiscal surplus (i.e., primary surplus less interest payment) be adjusted in response to a change in total liabilities, which is the idea underlying the Maastricht Treaty and the Stability and Growth Pact. See Woodford (2001) for more on this issue.

does not necessarily require it. Specifically, as shown by Bohn (1998) and Canzoneri et al. (2001), the transversality condition could be satisfied even if the government reacts to an increase in total liabilities by less than the amount needed to cover its interest payments. This is the second type of Ricardian policy, which is referred to as "globally Ricardian."

Globally Ricardian policy can be expressed as

$$s_t + \frac{i_{t-1}}{1+n_t} m_{t-1} = \gamma_t w_{t-1} + \nu_t, \tag{8}$$

where γ_t is a time-varying parameter satisfying $0 < \gamma_t \leq 1$. Note that equations (4) and (6) require the government to generate a primary surplus that is sufficient to cover its interest payments in each period. Here, however, the government can now issue additional debt to pay interest on the existing debt at the beginning of that period. Under this policy rule, the dynamics of w_t are now given by

$$w_t = \left[1 - \gamma_t - \frac{n_t}{1+n_t} + \frac{i_{t-1}}{1+n_t}\right] w_{t-1} - \nu_t \tag{9}$$

or

$$w_t = \left[\frac{1}{q_{t-1}} - \gamma_t\right] w_{t-1} - \nu_t, \tag{10}$$

which implies that the transversality condition (equation (3)) is satisfied if $0 < \gamma_t \leq \frac{1}{q_{t-1}}$.[7] Note that this condition does not necessarily guarantee that w_t is a stationary process; in fact, it allows w_t to grow forever, but at a rate lower than the interest rate in each period. In that sense, a globally Ricardian rule imposes a weaker condition on government behavior than a locally Ricardian rule.

Bohn (1998, 2005) adopts a policy reaction function very close to equation (8) and looks at U.S. data to determine whether γ_t is positive.[8] Equation (8) is an appropriate estimating equation when the government adopts a globally Ricardian policy or when it actually adopts a locally Ricardian policy but interest rates do not fluctuate much during the sample period. In the latter case, we would be able to empirically distinguish between a locally and a globally Ricardian policy just by looking at whether the estimated coefficient on w_{t-1} is greater than the sample average of the nominal interest rate. However, if the government adopts a locally Ricardian policy and fluctuations in interest rates are not small, then Bohn's specification may not be appropriate. For example, the estimated coefficient on w_{t-1} may become biased towards zero if fluctuations in interest rates are quite large during the sample period while those in public debt are negligibly small.

[7] Again, we rule out the possibility that the coefficient on w_{t-1} in (9) or (10) is below zero.

[8] However, Bohn (1998, 2005) does not consider the possibility that the fiscal regime evolves over time in a stochastic manner.

3 Estimation method

3.1 Estimating equations

Given the two definitions of Ricardian fiscal policy above, we estimate an equation of the form

$$b_t = \begin{cases} \mu_0 + (\alpha_0 + \eta_t) b_{t-1} + u_{0t}, & \text{if} \quad S_t = 0 \\ \\ \mu_1 + (\alpha_1 + \eta_t) b_{t-1} + u_{1t}, & \text{if} \quad S_t = 1 \end{cases} \tag{11}$$

where $u_{it} = \varepsilon_{it} - \nu_t$ with $\varepsilon_{it} \sim i.i.d.N(0, \sigma_i^2)$. $\{S_t \in (0,1)\}$ is a two-state Markov chain with transition probabilities $p_{ij} = \Pr(S_t = j \mid S_{t-1} = i)$. Note that, given that the current regime is i, the expected average duration of staying in the same regime is $(1 - p_{ii})^{-1}$. Also, note that we use public debt issued by the government b_t as the dependent variable rather than the total liabilities w_t, assuming that the amount of base money m_t is small relative to the public debt and that fluctuations play a much less important role in seignorage than in the primary surplus.

We specify four different estimation equations based on different definitions of the observable variables, η_t and ν_t.

Specification 1 $\eta_t = \nu_t = 0$: This is the benchmark case in which no exogenous variables are included. Hence, b_t follows a simple Markov-switching AR(1) process.

Specification 2 $\eta_t = 0$, and $\nu_t = -g_t^m$: This is a case in which government tax smoothing behavior is incorporated through g_t^m (military expenditures relative to GDP). As pointed out by Barro (1986) and Bohn (1998), the government's tax-smoothing behavior may create a negative correlation between public debt and the primary surplus. To illustrate this, consider a situation in which the government increases its expenditures, but only temporarily (such as in the case of a war). The government could increase taxes simultaneously by the same amount as the increase in expenditures, but it is costly to change marginal tax rates over time, since doing so increases the excess burden of taxation. Recognizing this, an optimizing government would seek to smooth marginal tax rates over time. This implies that a temporary increase in government expenditures would lead to a decrease in the primary surplus and an increase in public debt. Bohn (1998) argues that such a negative correlation between the primary surplus and public debt should be properly controlled for when estimating the government's reaction function; otherwise researchers may easily obtain imprecise estimates of fiscal policy reactions to an increase in public debt. Bohn (1998, 2005) shows that empirical results for the U.S. sharply differ depending on whether or not temporary government expenditures are included as an independent variable, while Iwamura et al. (2005) report a similar finding for Japan during the post-war period.

Specification 3 $\eta_t = -\frac{n_t}{1+n_t}$, and $\nu_t = -g_t^m$: This specification corresponds to equation (5) with $\alpha_i = 1 - \lambda_i$. Note that when n_t is very close to 0, specification 3 reduces to specification 2. This condition might hold in a very stable economy without any experience of high inflation, but unfortunately, this is not the case for Japan, which experienced three-digit inflation rates just after the end of WWII. Of course, Japan is not an exception, and one can easily find other examples in which the accumulation of public debt leads to uncontrollably high inflation. For such countries, specifications 2 and 3 are not identical.

Specification 4 $\eta_t = -\frac{n_t}{1+n_t} + \frac{i_{t-1}}{1+n_t}$, and $\nu_t = -g_t^m$: This corresponds to equation (9) with $\alpha_i = 1 - \gamma_i$. This specification differs from specification 3 in that interest payments, $\frac{i_{t-1}}{1+n_t}$, are included in η_t, reflecting the fact that the government is not required to create surplus to cover its interest payments. Note that a globally Ricardian policy requires α_i to be less than unity, implying that, when n_t is always equal to zero, b_t could continue to grow forever, but at a rate lower than the borrowing cost in each period.

3.2 Estimation

We estimate equation (11) by employing a Bayesian approach via the Gibbs sampler instead of a classical approach based on maximum likelihood estimation. The Bayesian approach has the following advantages. First, the maximum likelihood estimator (MLE) has the potential disadvantage that inference on S_t is conditional on the estimates of the unknown parameters. We estimate the parameters of the model and then make inferences on S_t conditional on the estimates of the parameters as if we knew for certain the true values of the parameters. In contrast, the Bayesian approach allows both the unknown parameters and S_t to be random variables. Therefore, inference on S_t is based on the joint distribution of the parameters and S_t (see Kim and Nelson (1999)).

Second, for the Markov switching models, the likelihood is often not uni-modal but multi-modal. Therefore, numerical algorithms such as Expectation Maximization (EM) and Newton-Rapson algorithms sometimes converge to a local maximum on the likelihood surface. This is a typical problem encountered with data in practice, regardless of which optimization algorithms are used. Maddala and Kim (1998) argue that the maximum likelihood estimation method is fragile as multiple local maxima are often found.

Third, MLE follows a non-standard limiting distribution when the process is nonstationary in the long run (or globally nonstationary). To our knowledge, such limiting distributions have not been derived for Markov-switching models. On the other hand, the Bayesian method can approximate the joint and marginal distributions of the parameters and S_t via a Markov chain Monte Carlo (MCMC) simulation method such as the Gibbs sampler. The method is valid even when the observed process exhibits non-stationarity (or explosive) behavior in the long run. To illustrate this point, let us suppose there are two fiscal policy regimes: one is a stable regime in which the debt-GDP ratio is characterized by a stationary process, and the other one is an unstable regime in which the debt-GDP ratio is characterized by a nonstationary process. Note that the mere existence of an unstable regime does not necessarily imply global instability: The system could still be globally stable if the unstable regime is not visited too often or for too long. In this sense, the transition probabilities of the Markov chain are important determinants of global stability or instability. On the other hand, as shown by Francq and Zakoian (2001), it is possible that the system is globally unstable even when both of the two regimes are stable. An important point to be emphasized here is that it would not be appropriate to employ MLE if it is uncertain whether the system is globally stable.[9]

3.3 MCMC simulation

The first time the Gibb sampler was used in a Bayesian analysis of Markov switching models was in the study by Albert and Chib (1993). The Gibbs sampler is used to approximate the joint and marginal distributions of the parameters of interest from the conditional distributions of the subsets of parameters given the other parameters (see Kim and Nelson (1999) for an introduction to Gibbs sampling). It is useful in this case because the joint distributions are difficult to obtain.

We follow Kim and Nelson (1999) to estimate a model of the form:

[9] An alternative empirical framework to study fiscal regime shifts would be to use the methodology proposed by Bai and Perron (1998), in which a multiple linear regression model with l breaks (or $l+1$ regimes) is examined within the classical framework. However, this approach requires the process to be weakly stationary in each regime. Therefore, their method cannot be applied in our context.

$$b_t^* = \begin{cases} \mu_0 + \alpha_0 b_{t-1} + \varepsilon_{0t}, & \text{if} \quad S_t = 0 \\ \mu_1 + \alpha_1 b_{t-1} + \varepsilon_{1t}, & \text{if} \quad S_t = 1 \end{cases}$$

where $b_t^* = b_t - \eta_t b_{t-1} + \nu_t$ and $\varepsilon_{it} \sim i.i.d.N(0, \sigma_i^2)$ for $i = 0, 1$ with $\sigma_{S_t}^2 = \sigma_0^2(1 + h_1 S_t)$ and $h_1 > 0$. $\{S_t \in (0, 1)\}$ is a two-state Markov chain with transition probabilities $p_{ij} = \Pr(S_t = j \mid S_{t-1} = i)$. Note that the two states are assumed to be identified not by α_{S_t} but by $\sigma_{S_t}^2$, simply because we want to know if there is any difference between the two states in terms of α_{S_t}.

3.3.1 Prior distributions

Next we describe the choice of priors for the unknown parameters. Let $\widetilde{h}_1 = 1 + h_1$ with $h_1 > 0$. Then the priors are the following:

$$\begin{aligned} \mu_i &\sim N(\psi, \omega^{-1}), \;\; \alpha_i \sim N(\phi, c^{-1}), \\ \sigma_0^2 &\sim IG(\frac{\upsilon}{2}, \frac{\delta}{2}), \;\; \widetilde{h}_1 \sim IG(\frac{\upsilon}{2}, \frac{\delta}{2})_{1(\widetilde{h}_1 > 1)}, \\ p_{11} &\sim beta(u_{11}, u_{10}), \;\; p_{00} \sim beta(u_{00}, u_{01}) \end{aligned}$$

The parameters used are $\psi = 0$, $\omega = 25$, $\phi = 0$, $c = 1$, $(\upsilon, \delta) = (0, 0)$, $u_{00} = u_{11} = 8$, and $u_{10} = u_{01} = 2$. Hence the prior of σ_i^2 is non-informative. The other parameters are chosen so that the priors are informative but relatively diffused. The means and standard deviations of the prior distributions are presented in the following table.

Priors for the parameters

	Distribution	Mean	Std. Dev. (??)
μ_i	Normal	0.00	0.20
α_i	Normal	0.00	1.00
p_{ii}	Beta	0.80	0.12
σ_0^2	Inverted Gamma	—	—
$\widetilde{h}_1$	Inverted Gamma	—	—

3.3.2 Computational algorithm

The needed posterior conditional distributions for implementing Gibbs sampling are easily obtained from the priors and the assumptions of the data generating process. The following steps 1 through 5 are iterated to obtain the joint and marginal distributions of the parameters of interest.

Step 1: Generate p_{11} and p_{00} conditional on $\widetilde{S}_T = (S_1, ..., S_T)$. Let n_{ij} refer to the total number of transitions from state i to j, which can be counted from $\widetilde{S}_T$. Then

$$\begin{aligned} p_{11} \mid \widetilde{S}_T &\sim beta(u_{11} + n_{11}, u_{10} + n_{10}) \\ p_{00} \mid \widetilde{S}_T &\sim beta(u_{00} + n_{00}, u_{01} + n_{01}) \end{aligned}$$

Step 2: Generate μ_i conditional on $\widetilde{S}_T$, σ_i^2, and α_i. We have the regression $y_t = \mu_i + \varepsilon_{it}$ where $y_t = b_t^* - \alpha_i b_{t-1}$ for $t \in \{t : S_t = i\}$. Hence, the posterior distribution is $\mu_i \sim N(\psi_*, \omega_*^{-1})$ where

$$\omega_* = \sum_{t \in \{t:S_t=i\}} 1/\sigma_i^2 + \omega,\ \psi_* = \omega_*^{-1} \left[\sum_{t \in \{t:S_t=i\}} y_t/\sigma_i^2 + \omega\psi \right]$$

Step 3: Generate α_i conditional on $\widetilde{S}_T$, σ_i^2, and μ_i. Let $d_t^* = b_t^* - \mu_i$, then we have the regression $d_t^* = \alpha_i b_{t-1} + \varepsilon_{it}$ for $t \in \{t : S_t = i\}$. Hence, the posterior distribution is $\alpha_i \sim N(\phi_{i*}, c_{i*}^{-1})$ where

$$c_{i*} = \sum_{t \in \{t:S_t=i\}} b_{t-1}^2/\sigma_i^2 + c,\ \phi_{i*} = c_{i*}^{-1} \left[\sum_{t \in \{t:S_t=i\}} b_{t-1} d_t^*/\sigma_i^2 + c\phi \right]$$

Step 4: Generate σ_0^2 and σ_1^2 conditional on $\widetilde{S}_T$, μ_i, and α_i. We first generate σ_0^2 conditional on h_1 and then generate $\widetilde{h}_1 = 1 + h_1$ to indirectly generate σ_1^2. Conditional on h_1, the posterior distribution of σ_0^2 is as follows:

$$\sigma_0^2 \sim IG\left(\frac{\upsilon_{0*}}{2}, \frac{\delta_{0*}}{2}\right)$$

where

$$\begin{aligned} \upsilon_{0*} &= \upsilon + T \\ \delta_{0*} &= \delta + RSS_0 + RSS_1/(1 + h_1) \end{aligned}$$

with $RSS_i = \sum_{t \in \{t:S_t=i\}} (b_t^* - \mu_i - \alpha_i b_{t-1})^2$. Conditional on σ_0^2, the posterior distribution of $\widetilde{h}_1 = 1 + h_1$ is as follows:

$$\widetilde{h}_1 \sim IG\left(\frac{\upsilon_{1*}}{2}, \frac{\delta_{1*}}{2}\right)_{1(\widetilde{h}_1 > 1)}$$

where

$$\begin{aligned} \upsilon_{1*} &= \upsilon + T_1 \\ \delta_{1*} &= \delta + RSS_1/\sigma_0^2 \end{aligned}$$

with $T_1 = \sum_{t=1}^{T} S_t$. Once $\widetilde{h}_1$ is obtained, we can calculate σ_1^2.

Step 5: Generate $\widetilde{S}_T = (S_1, ..., S_T)$ conditional on the other parameters. This is conducted using multi-move Gibbs sampling, which was first introduced by Carter and Kohn (1994) in the context of a state-space model. Here the procedure for generating $\widetilde{S}_T$ using the multi-move Gibbs-sampling is the same as that in Kim and Nelson (1999).

We iterate steps 1 through 5 $M + N$ times and discard the realizations of the first M iterations but keep the last N iterations to form a random sample of size N on which statistical inference can be made. M must be sufficiently large so that the Gibbs sampler converges. Also, N must be large enough to obtain the precise empirical distributions. Taking these aspects into consideration, we set $M = 5000$ and $N = 10000$.

4 Data

We construct a data set covering the period 1885-2004 for Japan, 1840-2005 for the United States, and 1830-2003 for the United Kingdom.[10]

4.1 Japan

Public debt Public debt is defined as the amount of gross debt issued by the central and local governments at the end of each fiscal year.[11] To convert the figures reported in various budget documents into a format consistent with the SNA, we make adjustments by excluding the amount of debt issued under the Colonial Special Accounts and the Public Enterprise Special Accounts, both of which are outside the general government according to the SNA definition.[12]

Nominal GDP A single data set covering the entire sample period is not available, so that we collect data from various sources and link them in a consistent way. For the period after FY1936, we use a data set produced by the Japanese government (various versions of the SNA), while for the

[10]See Appendix for details. All data we use are available upon request.

[11]The data for the debt issued by the central government are taken from various documents published by the Ministry of Finance, including "Japanese Government Bonds Statistics," "The Financial History of the Meiji and Taisho Period in Japan," the "Annual Report on Japanese Government Bonds Statistics," and "Budget Statistics," while the data for the debt issued by local governments is taken from documents issued by the Ministry of Finance and the Ministry of Home Affairs (Ministry of Internal Affairs and Communications), including "The Financial History of the Meiji and Taisho Period in Japan," "Local Government Bonds Statistics," the "Annual Publication on Local Public Finance," and the "Annual Report on Local Public Finance Statistics."

[12]We use various definitions of the general government: For 1885-1954, we use the definition by the Economic Counsel Board, for 1995-1969, the OLD SNA, for 1970-1979, the 68SNA, and for 1980-2004, the 93SNA. Note that these definitions slightly differ from each other, because special accounts held by the central government and business accounts held by local governments are sometimes classified as part of the general government and sometimes not.

period before FY1935, we basically use Ohkawa et al. (1974). However, since data are completely missing for the final stage of WWII (FY1944 and 1945), we estimate the real GDP in these two years by using the index of industrial production and the index of agriculture, forestry and fishery production,[13] and the GDP deflator by using the agricultural price index, the production goods price index, and the consumer price index.

Government interest payments The data for government interest payments for FY1885 to 1929 are taken from Emi and Shionoya (1966) for FY1885 to 1929, while those for FY1952 to 2004 are from various documents published by the government, including the "White Paper on National Income," the "Annual Report on National Income Statistics," and the "Annual Report on National Accounts." As for the period between FY 1930 and FY1951, we estimate interest payments closely following the methodology adopted by Emi and Shionoya (1966).

Military expenditure For the years after FY1947, we use the figures referred to as "National Defense and Related Affairs" in various issues of the "Settlement of General Account Revenues and Expenditures" published by the Ministry of Finance. The data for FY1946 are taken from Economic Counsel Board (1954), while for the years before FY1946, we use the data from Emi and Shionoya (1966).

As for military spending during wartime, that is, FY1937 to FY1945, we define this as expenditures spent only by the forces at home, and do *not* include expenditures spent by the forces overseas. This is consistent with our definition of public debt in which those debts issued under the five Colonial Special Accounts (namely, the Chosen Government, Taiwan Government, Kwantung Office, Karafuto Office, and Nanyo Office) are *not* included.[14]

[13] This methodology closely follows the one used by the Japanese central bank in its various publications on financial and economic activities around the end of the WWII (see, for example, Bank of Japan (1950)).

[14] However, as one might imagine, a non-negligible portion of expenditures spent by the forces *overseas* was financed by the central government through the issue of public debt, especially at the final stage of WWII. Ideally, this portion should be included in our definition of military expenditure, but we do not do so because reliable figures for that portion are not available. However, to see how sensitive our empirical results are to this treatment of military expenditures, we created an alternative series of military expenditures using a tentative estimate by Emi and Shionoya (1966) for military spending by the forces overseas that were financed by the central government through the Colonial Special Accounts, and repeated the same empirical exercise as in Section 5. We were able to confirm that the basic empirical findings are not sensitive to the definition of military spending.

4.2 The U.S. and the U.K.

For the United States, the data are taken from the "Historical Statistics of the United States" (Carter et al. (2006)) and the "Historical Tables, Budget of the United States Government" published by the Office of Management and Budget. For the United Kingdom, the data sources are the "British Historical Statistics" (Mitchell (1988)), the "Annual Abstract of Statistics" published by the Office for National Statistics, and the Public Sector Finances Databank by HM Treasury.

5 Empirical results

5.1 Preliminary analysis

The trend in the debt-GDP ratio for Japan, the U.S., and the U.K. is shown in Figure 1. We see that there are three major periods of debt accumulation in Japan. The first period, 1904 to 1905, is the period of the Russo-Japanese War (1904-1905). Reflecting a substantial increase in military expenditure, the debt-GDP ratio increased to over 50 percent at the end of 1905; however, it started to decrease again right after the end of the war and the decline continued until, in 1918, the debt-GDP ratio had returned to the pre-war level. Given that there was no remarkable growth dividend during this period (the nominal growth rate in 1906-1915 was 5.4 percent per year on average), one can see that this downward trend mainly came from fiscal reconstruction, including substantial spending reductions.[15] As pointed out by many researchers, the government during this period had a strong political will to restore budget balance so as to avoid the risk of a massive outflow of gold under the gold standard system.

The second phase of debt accumulation was 1920 to 1944, i.e., the period that includes WWII. The increase in the debt-GDP ratio accelerated following the outburst of war with China in 1937, and the ratio eventually reached 1.8 when the war ended in 1945. However, as can be seen in Figure 1, the debt-GDP ratio dropped precipitously right after the end of the war, all the way to a level very close to zero. This is an episode of inflationary erosion of the debt, or "partial default," due to hyper-inflation during this period.[16]

Finally, the most recent phase of debt accumulation started in the early 1970s and continues until today. A series of reforms in the social security system, including the introduction of indexation in the

[15]Although Japan won the war, it received no war reparations from Russia.

[16]The rate of inflation in terms of the GDP deflator was 273 percent in 1945, 175 percent in 1946, and 154 percent in 1947.

public pension system, have been implemented since the Tanaka administration declared a changeover to the welfare state in 1973. This accumulation of debt continued until the government finally started fiscal reconstruction in the latter half of the 1980s, including a substantial cut in spending and the introduction of a consumption tax in 1989. However, the debt-GDP ratio started to increase again in the 1990s, at least partially due to the collapse of the asset price bubble in the early 1990s.

Turning to the U.S. and the U.K., we see that the main cause of debt accumulation was increases in military expenditures during wartime. Specifically, the U.S. debt-GDP showed a rapid and substantial increase in 1861-66, 1916-19, and 1941-46, respectively corresponding to the Civil War, WWI, and WWII periods. The debt-GDP ratio for the U.K. is also characterized by three spikes, created by the Napoleonic War, WWI, and WWII. A notable difference with the Japanese data is that in both of these countries there was no major inflation comparable to Japan's hyper-inflation in 1945-47. It should also be noted that the U.S. and the U.K. have never experienced an uncontrollable accumulation of public debt during peacetime, which again is in sharp contrast with the Japanese experience since the early 1970s.

Table 1 presents a standard ADF test for the debt-GDP ratio in Japan, the U.S., and the U.K. Specifically, we run an AR (p) regression of the form

$$b_t = \mu + \alpha b_{t-1} + \sum_{j=1}^{p-1} \phi_j \triangle b_{t-j} + u_t \tag{12}$$

for the entire sample period with no break. We repeat this with various lag lengths ($p = 1$ to 10) to find that the estimates of α are very close to unity, and thus the null hypothesis $H_0 : \alpha = 1$ cannot be rejected for each of the three countries.

But does this necessarily imply that the debt-GDP ratio follows a unit root process throughout the entire sample period? In order to examine this, we conduct a rolling regression of equation (12) with a window of 40 years; for example, the estimated value for 1925 is from a regression conducted over the period 1885-1925. The lag length is set to $p = 2$. The results are shown in Figure 2 and show for Japan that the estimate of α fluctuates substantially in a range from 0.6 to 1.1, suggesting that the time-series properties of the debt-GDP ratio changed significantly at least several times. Specifically, the estimate of α shows a sharp rise during WWII, indicating that the Japanese government abandoned fiscal discipline during this period. On the other hand, α shows a sharp decline during the period just after the war, probably reflecting the fact that hyper-inflation during that period quickly reduced the real value of public debt. It should also be noted that the value of α has stayed very close to (or slightly above) unity since the latter half of the 1980s, suggesting that the debt-GDP ratio has been following a unit root process or even an explosive process during this most recent period.

In contrast with the Japanese result, the estimates of α for the U.S. and the U.K. do not show large fluctuations. Basically, the estimates stay below unity, except that the U.S. estimate shows a sharp rise during WWII.[17]

5.2 Empirical results for Japan

Table 2 presents the regression results for Japan obtained from a two-state model. Panel A of the table shows a benchmark regression in which no exogenous variables are included (namely, specification 1). The estimate of α in regime 0 is 0.517, indicating that the debt-GDP ratio is characterized by a stationary process that converges to its mean quite quickly. On the other hand, the estimate of α in regime 1 is 1.116. Since its lower bound (1.067) exceeds unity, we cannot reject the null that the debt-GDP ratio follows an explosive process. Figure 3 presents the estimated probability of regime 1 in each year of the sample period, as well as the estimated coefficient on b_{t-1}, which is calculated as a weighted average of the coefficients in regimes 0 and 1, with the estimated probabilities of each regime being used as a weight. The shaded area represents the 95 percent confidence interval. Figure 3 shows that the years except 1945-1970 fall under regime 1 and that the coefficient on b_{t-1} exceeds unity except during the period 1945-1970.

Panel B of Table 2 shows the results of a similar regression, but this time we added military expenditures as an exogenous variable (specification 2). Again, the debt-GDP ratio is characterized by a stationary process for regime 0 and an explosive process for regime 1. The estimated coefficient on b_{t-1}, shown in Figure 3, looks quite similar to the previous case, except that the coefficient is now lower than unity in 1890-1905 (the period of the Sino-Japanese and the Russo-Japanese Wars) and 1915-1920 (the period of WWI).

Panel C of Table 2 reports the regression result for the case in which military expenditure and the growth dividend, $-\frac{n_t}{1+n_t}b_{t-1}$, are included as exogenous variables (specification 3). Again, we see that regime 0 is characterized by a stationary process and regime 1 by an explosive process. But a

[17]In addition, α shows a sharp decline during high inflation periods (the 1920s in the U.S. and the U.K., and the 1950s in the U.K.), again probably reflecting the fact that the real value of public debt quickly decreased due to high inflation.

notable difference from the previous two specifications is that the estimate of α in regime 0 is now much closer to unity, indicating that convergence to its mean is much slower. Specifically, the estimate of α in specification 1 (0.5177) implies that the debt-GDP ratio declines to half of its initial value after about 1.05 years, while the one in specification 3 (0.9178) implies that the half-life is 8.08 years. The surprisingly quick decline in the debt-GDP ratio found in specifications 1 and 2 mainly reflects the fact that the debt-GDP ratio fell very quickly during the hyper-inflation period in 1945-47. This problem is now fixed by properly controlling for the growth dividend. Figure 3 now shows that the probability of regime 1 is close to unity in 1930-1950 and 1970-2004, while the probability of regime 0 is high in 1885-1925 and 1950-1970. These results suggest that the former periods are characterized by a lack of fiscal discipline, while the latter periods are characterized by a locally Ricardian rule.

Finally, Panel D of Table 2 reports the results for the case in which military expenditure and $\left(\frac{i_{t-1}}{1+n_t} - \frac{n_t}{1+n_t}\right) b_{t-1}$ are included as exogenous variables (specification 4). The results are basically the same as those for specification 3, except that the estimates of α in regimes 0 and 1 are both lower, confirming that the assumption of a globally Ricardian policy is weaker than that of a locally Ricardian policy.

In sum, we find that the Japanese government made several large changes with respect to its fiscal behavior over the past 120 years. Specifically, Japanese fiscal policy is characterized by a locally Ricardian rule in 1885-1925 and 1950-1970. The former largely corresponds to the period in which Japan had adopted the gold standard under which the government was forced to maintain a balanced budget until Japan left the gold standard in 1917, following the same move by the core countries of the system.[18] The second period follows the fiscal restructuring ushered in in December 1948, when SCAP instructed the Japanese government to implement a balanced budget in order to stop runaway inflation. [19] On the other hand, Japanese fiscal policy is characterized by a non-Ricardian rule in 1930-1950 and 1970-2004, suggesting that the Japanese government abandoned fiscal discipline not only during WWII, but also in the most recent period starting in 1970.

5.3 Sensitivity analysis

AR (2) model The baseline regressions reported in Table 2 assume that the government adjusts the primary surplus in period t in response to a change in public debt at the beginning of period

[18]See Shizume (2001) for more on the Japanese government's fiscal behavior during the gold standard period.

[19]For details on the "Dodge Line," see, for example, Cohen (1950) and Yamamura (1967).

t. Given that we use annual data, this seems to be a good approximation to actual policy making. However, as often pointed by researchers and practitioners, it usually takes more than one year before fiscal decisions are finally made. If this is the case, our baseline specification may not be appropriate. To address this potential problem, we change the lag structure of our estimating equation (equation (11)) to

$$b_t = \begin{cases} \mu_0 + \alpha_0 \sum_{k=1}^{K} \omega_{0k} b_{t-k} + \eta_t b_{t-1} + u_{0t}, & \text{if} \quad S_t = 0 \\ \mu_1 + \alpha_1 \sum_{k=1}^{K} \omega_{1k} b_{t-k} + \eta_t b_{t-1} + u_{1t}, & \text{if} \quad S_t = 1 \end{cases}$$

where ω_{0k} and ω_{1k} are parameters satisfying $\sum_{k=1}^{K} \omega_{ik} = 1$ and representing the lag structure of fiscal decision making.[20] The estimating equation is now given by

$$b_t = \begin{cases} \mu_0 + (\alpha_0 + \eta_t) b_{t-1} + \sum_{k=1}^{K-1} \theta_{0k} \triangle b_{t-k} + u_{0t}, & \text{if} \quad S_t = 0 \\ \mu_1 + (\alpha_1 + \eta_t) b_{t-1} + \sum_{k=1}^{K-1} \theta_{1k} \triangle b_{t-k} + u_{1t}, & \text{if} \quad S_t = 1 \end{cases}$$

where $\theta_{ik} \equiv -\alpha_i \omega_{i,k+1}$. We conducted a lag search to end up with $K = 2$. The results of the regressions using this equation are reported in Table 3 and Figure 4 and are basically the same as before. In addition, the coefficient on $\triangle b_{t-1}$, denoted by θ in Table 3, is very close to zero in each regime, indicating that the AR (1) specification is not a binding constraint.

Net public debt The baseline regressions use *gross* public debt issued by the central and local governments rather than *net* debt. This is based on the assumption that governments own only a small amount of financial assets and that fluctuations in the amount of financial assets over time are insubstantial. However, as pointed out by Broda and Weinstein (2004), Japan's public sector, through its social security funds, holds non-negligible amounts of financial assets. According to their estimate, *net* debt held by the Japanese public sector at the end of FY2002 was equivalent to 64 percent of GDP, while the corresponding *gross* figure was 161 percent. Obviously, the difference is not trivial.

To evaluate how sensitive the baseline results are, we re-estimate our equations replacing gross debt with net debt. The net debt data we use here are the data published by the Economic and Social Research Institute (ESRI), Cabinet Office, which cover the general government, including the central

[20] Note that this specification differs from a partial adjustment model, such as the one adopted by Favero and Monacelli (2005), in that the coefficient on lagged values of b depends only on the current regime (and not on past regimes).

and local governments and the social security funds. Unfortunately, however, the ESRI data cover only the postwar period starting in 1955, so that the estimation is conducted only for this shorter sample period.

Figure 5 compares Japanese general government gross and net debt. Although the difference between the two in terms of the vertical distance is indeed substantial, we still see a common long-term trend: Namely, both start to increase around 1970 and basically continue to rise over the next 35 years. Comparing the estimation result reported in Table 4 with the baseline result (Table 2), we see no change in that the debt-GDP ratio is characterized by a stationary process in regime 0 and a nonstationary process in regime 1. We may therefore safely conclude that our baseline results are not particularly sensitive to the definition of public debt.

Somewhat interestingly, however, if one looks closely at Figure 6, one can see a substantial decline in the probability of regime 1 during the latter half of the 1980s. Correspondingly, the coefficient on b_{t-1} fell below unity during this period in specification 3 and more clearly in specification 4. The latter half of the 1980s famously is a period of fiscal reconstruction during which the Japanese government intensively cut expenditure to achieve the target of "no net issuance of government bonds."[21] One may interpret the decline in the coefficient on b_{t-1} during this period as reflecting the restoration of fiscal discipline.[22] However, the coefficient on b_{t-1} started to increase again in the early 1990s and has remained very close to unity since.

Automatic stabilizers Recent empirical studies on fiscal policy rules emphasize the importance of automatic stabilizers in explaining fluctuations in the fiscal surplus/deficit (see Taylor (2000), Auerbach (2003), and Bohn (1998)). For example, Taylor (2000), using U.S. data for 1960-1999, finds that the cyclical surplus was highly correlated with fluctuations in the output gap but this was not necessarily the case for the structural surplus. To control for this effect in our regression exercise, we add the output gap to the estimating equations. Specifically, we closely follow Barro (1986) and Bohn (1998) by introducing a new variable, $YVAR_t$, which is defined as $YVAR_t \equiv (1 - Y_t/Y_t^*)(G_t^*/Y_t)$, where Y_t is the real GDP, Y_t^* is its trend component estimated by HP filter,

[21] See, for example, Ihori et al. (2001) for more on fiscal reform efforts during this period.

[22] Figure 6 shows that the coefficient on b_{t-1} during the latter half of the 1980s is slightly below unity but not statistically different from unity in specification 3, while it is significantly smaller than unity in specification 4. This implies that the fiscal regime during this period is characterized not by a locally Ricardian but a globally Ricardian rule. This result is perfectly consistent with the fact that the government indeed aimed at "no net issuance of government bonds" but had little intention of going further than that, i.e., it had no intention to reduce the debt-GDP ratio to a lower level or even zero.

and G_t^* is the trend component of real government spending. The regression results presented in Table 5 show that the coefficient on $YVAR$ is around 0.7 and significantly different from zero in both specifications, indicating that automatic stabilizers did play an important role even in the Japanese case. However, the coefficient of main interest to us, α, is almost the same as before, suggesting that the baseline result is not sensitive to whether we control for the output gap or not.

No restriction on the coefficient on interest payments As we can see from equations (4) and (8), the sole difference between locally and globally Ricardian rules is what kind of restriction we impose on the coefficient on interest payments $\frac{i_{t-1}}{1+n_t}w_{t-1}$. Locally Ricardian rules impose the restriction that the coefficient should be equal to unity, while globally Ricardian rules impose the restriction that it should be zero. The former corresponds to specification 3, while the latter corresponds to specification 4. An important implication of these restrictions, whether the coefficient should be zero or unity, is that these specifications allow a switching only between locally Ricardian rules and other rules (i.e., rules that do not belong to locally Ricardian rules) in the case of specification 3, and a switching only between globally Ricardian rules and the other rules in the case of specification 4. These specifications would be inappropriate if, for example, policy switching occurs between locally and globally Ricardian rules.

To deal with this potential problem, we conduct a similar regression as before but now do not impose an a priori restriction on the coefficient on interest payments. Specifically, we add a new independent variable $\frac{i_{t-1}}{1+n_t}b_{t-1}$ to equation (11) with

$$\eta_t = -\frac{n_t}{1+n_t};\ \nu_t = -g_t^m.$$

The coefficient on the new independent variable should be close to zero if the true rule is well approximated by a locally Ricardian rule, and it should be unity in the case of a globally Ricardian rule. The results are shown in Table 6 and indicate that the estimated coefficient is 0.628 in regime 0 (the stationary regime) and 0.506 in regime 1 (the nonstationary regime). More importantly, the lower bound in regime 0 is 0.235, rejecting the null of zero, while the upper bound in regime 0 is slightly lower than unity (0.990), again rejecting the null of unity. This means that the true rule is not well approximated by the two extremes (i.e., locally and globally Ricardian rules) but is located between them. The same results can be seen for regime 1.[23] However, the estimated values of α in Table 6

[23]These results suggest that neither empirical studies focusing only on locally Ricardian rules nor those focusing only

tend to fall between those obtained in specifications 3 and 4 of Table 2, confirming that the main results regarding fiscal policy behavior in Table 2 hold without any substantial modifications.

Three-state model The robustness of the findings in Table 2 are examined in Table 7 by extending the analysis to a three state model. Panel A, which reports the regression results for specification 3, shows that regime 0 is characterized by a stationary process ($\alpha = 0.926$), regime 1 by an explosive process ($\alpha = 1.081$), and regime 2 by another highly explosive process ($\alpha = 1.313$). Figure 9 shows that the periods falling under regime 1 in Figure 3 are again classified as regime 1,[24] suggesting that the number of regimes allowed in Table 2 (namely, two regimes) is not an inappropriate description of the true model. These results, together with the results for specification 4, more or less confirm the earlier findings: (1) the periods 1885-1920 and 1950-1970 fall under regime 0 (a regime with fiscal discipline); (2) the period 1920-1950 falls under regime 1 (a regime without fiscal discipline).

5.4 Are debt ratios globally stationary or nonstationary?

The regression analysis in this section seeks to determine whether the debt-GDP ratio follows a stationary process within a regime. However, as we discussed earlier, even if the ratio is stationary within a regime, this does not necessarily imply that it is stationary in the long run. This is simply because regime changes occur stochastically in accordance with transition probabilities. Thus, what we need to know is where the debt-GDP ratio is headed in the long run given the estimated transition probabilities, or, put differently, we need to know whether its distribution converges over time to a certain distribution. A process is said to be globally stationary if the distribution converges to a certain distribution over time, while stationarity within a regime is called local stationarity. Global stationarity implies that the effect of policy shocks on the debt-GDP ratio becomes smaller and smaller over time and finally disappears in the long run. Investors in government bonds markets are interested in whether this global stationarity is satisfied or not, and policymakers, especially central banks, are interested in this property when designing monetary policy rules.

Francq and Zakoian (2001) obtain a result regarding the relationship between local and global stationarity that is of some interest in the present context, namely that local stationarity is neither a necessary nor a sufficient condition for global stationarity. For example, suppose there are two regimes

on globally Ricardian rules employ an appropriate estimating equation.

[24]The exceptions are 1944 and 1970-1980, years in which the debt-GDP ratio recorded an extremely high growth rate, so that they are classified as regime 2.

and one satisfies local stationary while the other does not. Even in this combination, the process could be globally stationary. On the other hand, even if each of the two regimes satisfies local stationarity, this does not necessarily imply global stationarity.[25]

As we saw in Table 2, the regression results using a two-state model show that one regime satisfies (local) stationarity while the other one does not. Also, as we saw in Table 7, the regression results using a three-state model indicate that one regime satisfies (local) stationarity, but the other two do not. Given these results, one may wonder if they imply global stationarity or nonstationarity. To address this issue, we conduct the following simulation exercise. We generate a time series of b_t using

$$b_t = \begin{cases} \hat{\mu}_0 + \left(\hat{\alpha}_0 - \frac{n_t}{1+n_t}\right) b_{t-1} + u_{0t}, & \text{if} \quad S_t = 0 \\ \hat{\mu}_1 + \left(\hat{\alpha}_1 - \frac{n_t}{1+n_t}\right) b_{t-1} + u_{1t}, & \text{if} \quad S_t = 1 \end{cases}$$

for a two-state model and the corresponding equation for a three-state model. Here, parameters with a hat represent the values estimated in the earlier regressions. More specifically, we randomly draw policy shocks and policy regimes using the parameters and transition probabilities obtained from the regression of specification 3 and generate a replication for the time series of the debt-GDP ratio over 1000 years for various paths of the nominal growth rate (n_t) that are exogenously determined. We repeat this process 5000 times to obtain a distribution of the debt-GDP ratio in every year of the 1000 years. We can say that the debt-GDP process is globally stationary if this distribution is stable over time; otherwise it is globally nonstationary.

Table 8 reports the first, second, and third quantiles of the simulated distribution with $T = 500$ (500 years later) and $T = 1000$ for the two-state and three-state models. In Panel A it is assumed that the initial regime is a stationary one (i.e., $S_0 = 0$), and that the debt-GDP ratio in period 0 is zero. On the other hand, in Panel B, it is assumed that the initial regime is a nonstationary one ($S_0 = 1$) and that the initial debt-GDP ratio is unity (100 percent). The simulation results from the two-state model show that the distribution is stable over time, irrespective of the initial conditions and the assumed values of nominal growth rates (n), clearly indicating that the debt-GDP ratio satisfies global stationarity. On the other hand, the results from the three-state model show that the distributions with $T = 500$ and $T = 1000$ differ significantly for the case of $n = 0.00$, 0.03, and 0.06,

[25] Gali (2006) provides a clear and interesting discussion of the implications of Francq and Zakoian's (2001) result on the determinacy of an equilibrium in a monetary economy.

implying that the process is globally nonstationary.[26]

Figure 10 presents a similar simulation conducted to forecast the future path of the debt-GDP ratio over the next 100 years. To make the initial condition as close to the current situation in Japan as possible, we assume that the initial regime is $S_0 = 1$ and that the debt-GDP ratio in period 0 is 1.7, which is the actual figure at the end of 2004. According to the result from the two-state model with 3 percent nominal growth, the "third quantile" line goes up until it reaches 3 with $T = 20$, indicating that a further increase in the debt-GDP ratio is quite likely to occur over the next 20 years. After that, however, the debt-GPD ratio enters a declining trend as a result of the switch to a stationary regime and then converges to a quite narrow (and probably tolerable) band within 100 years. On the other hand, the result from the three-state model with 3 percent nominal growth shows that the median of the distribution increases quite quickly to reach an unrealistic and intolerable level within 50 years, and that its variance increases over time, clearly indicating global nonstationarity.

5.5 Empirical results for the U.S.

Table 9 presents the regression results for the United States using a two state model. Results for specification 3, presented in Panel A, indicate that each of the regimes, 0 and 1, is characterized by a stationary process. This implies that the U.S. government's fiscal behavior during the sample period can be described as a switching between locally Ricardian policy rules. If we turn to the results for specification 4, presented in Panel B, they again indicate that each regime, 0 and 1, satisfies stationarity, implying that U.S. fiscal policy is characterized by a switching between globally Ricardian rules.

These results suggest that the U.S. government's fiscal behavior consistently has been very close to locally Ricardian policy throughout the entire sample period. In fact, the estimated coefficient on b_{t-1}, presented in Figure 11, consistently and statistically significantly remains below unity. If we compare these results with those reported in previous studies on U.S. fiscal policy, we find some similarities. Bohn (2005), for example, regressed the U.S. primary surplus on public debt for a sample period from 1793 to 2003 and reports that the OLS estimate of the coefficient on public debt is positive and significantly different from zero when tax smoothing effects are properly controlled for. Bohn interprets

[26]However, when n goes up to 0.10, the distributions with $T = 500$ and $T = 1000$ become identical, suggesting that sufficiently high nominal growth could make the debt-GDP ratio globally stationary. The threshold for nominal growth rates is about 8 percent, which is lower than the sample average (13.7 percent).

this result as providing evidence for a globally Ricardian rule; but since the estimated coefficient is typically greater than the average interest rate level, this could be interpreted as suggesting a rule that is even locally Ricardian. Bohn (1998) conducts a similar exercise using data for 1916-1995 and finds that the coefficient on public debt is significantly positive not only for the entire sample period, but also for five sub-sample periods, including the postwar period. These results reported by Bohn (1998, 2005) are consistent with ours.

Favero and Monacelli (2005) estimate an equation that is very close to our specification 1 (equation (6)) using the maximum likelihood method and report that U.S. government behavior has been deviating from Ricardian policy for most of the entire sample period (1961-2002), except that it was close to a locally Ricardian rule during the period of 1995-2001. Although their results cannot be directly compared to ours because the empirical methodologies differ in several respects, we still attempt to do so by adjusting our sample period to theirs. Panels C and D of Table 9 show the results of a regression that is similar to that underlying Panels A and B, but that now uses data for the postwar period. The regression results indicate that the estimate of α in regime 0 is less than unity, suggesting that it is a stationary regime as before, but that the upper bound of α in regime 1 slightly exceeds unity, so that we fail to reject the null of a unit root. Fluctuations in the estimated coefficient on b_{t-1}, presented in Figure 11, show that it has been slightly higher than unity since 1975, implying the possibility that the U.S. government started to deviate from Ricardian policy around 1975. However, the figure clearly shows that the estimated coefficient on b_{t-1} is consistently less than unity during the period before 1975 and that there is no evidence for the return to Ricardian policy around 1995 that Favero and Monacelli (2005) detected. Thus, there are certain inconsistencies between their results and ours. [27]

Given that U.S. fiscal policy is characterized by switching between stationary regimes, we may apply a model with multiple breaks, as proposed by Bai and Perron (1998), to the U.S. data. This model does not require researchers to assume that policy regime switching is a recurrent phenomenon, and that it has a Markov property. This is an important advantage, but on the other hand it requires the debt process to be weakly stationary in each regime, so that we cannot apply it to the Japanese data. The regression results reported in Table 10 show that regime changes occur four times (i.e., there are five different regimes) with both specifications 3 and 4. According to the result for specification

[27] We also estimated specification 1, which is very close to the estimating equation employed by Favero and Monacelli (2005), for the entire sample period as well as for the postwar period, but found that both regimes are stationary ones.

4, the estimate of α is slightly higher than unity during the wartime period (regime 3, 1917-1943) but is significantly smaller than unity in the other four regimes. These results may be interpreted as confirming our earlier results obtained from the Markov switching regression.[28]

5.6 Empirical results for the U.K.

Table 11 presents the regression results for the United Kingdom using a two-state model. Results for specification 3 indicate that regime 0 is characterized by a stationary process, while regime 1 is characterized by a unit root process (the upper bound of α slightly exceeds unity). On the other hand, results for specification 4 indicate that both regime 0 and regime 1 are characterized by a stationary process, implying that the U.K. government's fiscal behavior is characterized by switching between globally Ricardian rules.

6 Conclusion

This paper estimated fiscal policy feedback rules in Japan, the United States, and the United Kingdom for more than a century and allowing for stochastic regime changes. By estimating a Markov switching model by the Bayesian method, we arrived at the following findings. First, the Japanese data clearly reject the view that the fiscal policy regime has been fixed, i.e., that the Japanese government has adopted a regime that is either Ricardian or non-Ricardian throughout the entire period. Rather, our results indicate a stochastic switch of the debt-GDP ratio between stationary and nonstationary processes and thus a stochastic switch between Ricardian and non-Ricardian regimes. Specifically, Japanese fiscal policy was characterized by a locally Ricardian rule in 1885-1925 and 1950-1970 but by a non-Ricardian rule in 1930-1950 and 1970-2004. Second, through simulation exercises using the estimated parameters and transition probabilities, we showed that the debt-GDP ratio may be nonstationary even in the long run (i.e., globally nonstationary). Third, the Japanese result stands in sharp contrast with the results for the U.S. and the U.K., which indicate that in these countries, government fiscal behavior has been consistently characterized by Ricardian policy.

[28] However, the results for specification 3 are not very informative since α exceeds unity in three out of the five regimes. This result may be interpreted as evidence against applying the Bai-Perron method even to the U.S. data.

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Table 1: Unit Root Tests

	Japan		U.S.		U.K.	
p	α	t-stat	α	t-stat	α	t-stat
1	0.9990	-0.02	0.9868	-1.04	0.9864	-1.17
2	0.9607	-1.31	0.9784	-2.09	0.9798	-2.21
3	0.9702	-0.94	0.9818	-1.74	0.9800	-2.15
4	0.9729	-0.80	0.9804	-1.86	0.9817	-1.94
5	0.9756	-0.68	0.9808	-1.79	0.9825	-1.84
6	0.9691	-0.82	0.9806	-1.78	0.9813	-1.94
7	0.9720	-0.71	0.9815	-1.67	0.9853	-1.54
8	0.9824	-0.43	0.9817	-1.63	0.9819	-1.91
9	0.9716	-0.67	0.9839	-1.42	0.9826	-1.82
10	0.9740	-0.58	0.9797	-1.81	0.9827	-1.78

Note: We conduct the standard ADF tests, $b_t = \mu + \alpha b_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta b_{t-j} + u_t$, with various lag length. The null hypothesis is $\alpha = 1$ and the 10% critical value is -2.57 when the sample size is 100 (see Hamilton (1994)). The sample periods for Japan, the U.S. and the U.K. are 1885-2004, 1840-2005, and 1830-2003, respectively.

Table 2: Two-State Model for Japan

Panel A: Specification 1

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	0.0355	0.0512	0.0669	-0.0549	-0.0267	0.0023
α	0.4783	0.5177	0.5529	1.0674	1.1168	1.1649
σ^2	0.0006	0.0010	0.0018	0.0037	0.0050	0.0068
p_{11}	0.9193	0.9658	0.9941			
p_{00}	0.7804	0.9078	0.9811			

Panel B: Specification 2

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	0.0380	0.0535	0.0688	-0.0863	-0.0591	-0.0321
α	0.3785	0.4134	0.4476	1.0424	1.0821	1.1233
σ^2	0.0005	0.0009	0.0015	0.0024	0.0033	0.0044
p_{11}	0.8978	0.9564	0.9900			
p_{00}	0.7706	0.8984	0.9792			

Panel C: Specification 3

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0133	0.0036	0.0164	-0.0300	0.0073	0.0476
α	0.8681	0.9178	0.9762	1.0167	1.0641	1.1110
σ^2	0.0003	0.0005	0.0007	0.0022	0.0033	0.0049
p_{11}	0.8552	0.9378	0.9867			
p_{00}	0.8778	0.9448	0.9864			

Panel D: Specification 4

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0067	0.0056	0.0169	-0.0524	-0.0150	0.0193
α	0.8126	0.8550	0.8998	1.0103	1.0536	1.1003
σ^2	0.0003	0.0004	0.0006	0.0022	0.0033	0.0050
p_{11}	0.8631	0.9440	0.9876			
p_{00}	0.8921	0.9469	0.9831			

Note: The transition probability, p_{ij}, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled "LB" and "UB" refer to the lower and upper bound of the 95% confidence interval and the columns labeled "Mean" refer to the mean of the marginal distribution of the parameter.

Table 3: AR(2) Model

Panel A: Specification 3

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0088	0.0069	0.0185	-0.0298	0.0075	0.0383
α	0.8572	0.9004	0.9578	1.0151	1.0558	1.1028
θ	0.0218	0.1227	0.3150	-0.0099	0.1009	0.1999
σ^2	0.0003	0.0004	0.0006	0.0020	0.0030	0.0045
p_{11}	0.8782	0.9454	0.9883			
p_{00}	0.8752	0.9412	0.9827			

Panel B: Specification 4

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0058	0.0079	0.0189	-0.0489	-0.0125	0.0248
α	0.8007	0.8429	0.8933	0.9979	1.0449	1.0871
θ	0.0317	0.1213	0.2190	-0.0024	0.0854	0.1745
σ^2	0.0002	0.0004	0.0005	0.0019	0.0030	0.0045
p_{11}	0.8689	0.9467	0.9855			
p_{00}	0.8868	0.9439	0.9876			

Note: The transition probability, p_{ij}, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled "LB" and "UB" refer to the lower and upper bound of the 95% confidence interval and the columns labeled "Mean" refer to the mean of the marginal distribution of the parameter.

Table 4: Net Public Debt

Panel A: Specification 3

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0333	-0.0213	-0.0144	0.0075	0.0276	0.0415
α	0.7131	0.9227	0.9916	0.9916	1.0311	1.0737
σ^2	0.0001	0.0002	0.0003	0.0002	0.0005	0.0010
p_{11}	0.7812	0.9040	0.9783			
p_{00}	0.7321	0.8799	0.9674			

Panel B: Specification 4

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0276	-0.0211	-0.0140	-0.0009	0.0202	0.0356
α	0.7961	0.8822	0.9412	0.9770	1.0185	1.0572
σ^2	0.0001	0.0002	0.0003	0.0003	0.0005	0.0009
p_{11}	0.7855	0.9052	0.9788			
p_{00}	0.7159	0.8701	0.9583			

Note: The transition probability, p_{ij}, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled "LB" and "UB" refer to the lower and upper bound of the 95% confidence interval and the columns labeled "Mean" refer to the mean of the marginal distribution of the parameter.

Table 5: Automatic Stabilizers

Panel A: Specification 3

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
The coefficient on $YVAR$	0.4700	0.7765	1.1077	0.3547	0.7559	1.1602
μ	-0.0103	0.0031	0.0163	-0.0263	0.0152	0.0564
α	0.8696	0.9192	0.9631	1.0117	1.0598	1.1073
σ^2	0.0003	0.0005	0.0007	0.0024	0.0039	0.0061
p_{11}	0.8603	0.9397	0.9879			
p_{00}	0.8903	0.9482	0.9851			

Panel B: Specification 4

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
The coefficient on $YVAR$	0.4811	0.8034	1.1901	0.3282	0.7032	1.1213
μ	-0.0073	0.0067	0.0188	-0.0514	-0.0099	0.0323
α	0.8054	0.8505	0.9008	0.9992	1.0483	1.0948
σ^2	0.0003	0.0004	0.0006	0.0025	0.0040	0.0062
p_{11}	0.8626	0.9423	0.9880			
p_{00}	0.8796	0.9459	0.9875			

Note: The transition probability, p_{ij}, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled "LB" and "UB" refer to the lower and upper bound of the 95% confidence interval and the columns labeled "Mean" refer to the mean of the marginal distribution of the parameter.

Table 6: No Restriction on the Coefficient on Interest Payments

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
The coefficient on interest payments	0.2351	0.6283	0.9907	0.1046	0.5064	0.8778
μ	-0.0088	0.0052	0.0174	-0.0431	-0.0062	0.0336
α	0.8228	0.8772	0.9367	1.0155	1.0617	1.1075
σ^2	0.0003	0.0004	0.0006	0.0022	0.0033	0.0048
p_{11}	0.8668	0.9420	0.9884			
p_{00}	0.8826	0.9454	0.9850			

Note: The transition probability, p_{ij}, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled "LB" and "UB" refer to the lower and upper bound of the 95% confidence interval and the columns labeled "Mean" refer to the mean of the marginal distribution of the parameter.

Table 7: Three-State Model

Panel A: Specification 3

	Regime 0			Regime 1			Regime 2		
	LB	Mean	UB	LB	Mean	UB	LB	Mean	UB
μ	-0.0448	-0.0018	0.0232	-0.0533	-0.0241	0.0193	-0.3370	-0.0425	0.0438
α	0.8518	0.9261	1.0657	1.0428	1.0819	1.1122	1.1850	1.3136	1.5440
σ^2	0.0001	0.0003	0.0006	0.0005	0.0007	0.0011	0.0006	0.0081	0.0154
p_{00}	0.7937	0.9111	0.9783						
p_{01}	0.0053	0.0560	0.1523						
p_{10}	0.0018	0.0388	0.1067						
p_{11}	0.8357	0.9235	0.9796						
p_{20}	0.0015	0.0666	0.2667						
p_{21}	0.0149	0.1353	0.3272						

Panel B: Specification 4

	Regime 0			Regime 1			Regime 2		
	LB	Mean	UB	LB	Mean	UB	LB	Mean	UB
μ	-0.0002	0.0153	0.0262	-0.0731	-0.0602	-0.0469	-0.0624	-0.0266	0.0051
α	0.7958	0.8283	0.8673	1.0625	1.0840	1.1013	1.1971	1.2783	1.3412
σ^2	0.0001	0.0003	0.0004	0.0004	0.0006	0.0008	0.0005	0.0014	0.0032
p_{00}	0.7839	0.8994	0.9698						
p_{01}	0.0103	0.0607	0.1548						
p_{10}	0.0062	0.0457	0.1155						
p_{11}	0.8498	0.9261	0.9739						
p_{20}	0.0015	0.0556	0.1903						
p_{21}	0.0317	0.1393	0.3257						

Note: The transition probability, p_{ij}, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled "LB" and "UB" refer to the lower and upper bound of the 95% confidence interval and the columns labeled "Mean" refer to the mean of the marginal distribution of the parameter.

Table 8: Globally Stationary or Nonstationary?

Panel A: $S_0 = 0$ and $b_0 = 0$

	Two-State Model			Three-State Model		
Quantile	First	Median	Third	First	Median	Third
0% Growth						
T=500	-0.1471	0.0006	0.1525	-1.1E+10	-3.1117	1.4E+10
T=1000	-0.1593	-0.0007	0.1497	-2.5E+21	-2.7E+07	2.6E+21
3% Growth						
T=500	-0.0751	0.0005	0.0772	-3.6E+04	0.0414	4.9E+04
T=1000	-0.0790	-0.0011	0.0786	-5.9E+09	0.6536	1.1E+10
6% Growth						
T=500	-0.0569	0.0003	0.0574	-4.2741	-0.0016	4.4939
T=1000	-0.0591	-0.0012	0.0584	-16.665	0.0085	20.506
10% Growth						
T=500	-0.0476	0.0001	0.0463	-0.1160	0.0008	0.1205
T=1000	-0.0478	-0.0006	0.0466	-0.1175	0.0010	0.1192
13.7% Growth						
T=500	-0.0423	-0.0003	0.0413	-0.0579	0.0013	0.0615
T=1000	-0.0428	-0.0004	0.0403	-0.0598	-0.0001	0.0592

Panel B: $S_0 = 1$ and $b_0 = 1$

	Two State Model			Three State Model		
Quantile	First	Median	Third	First	Median	Third
0% Growth						
T=500	-0.1476	0.0017	0.1623	6.1E+08	2.9E+11	8.9E+13
T=1000	-0.1524	-0.0028	0.1515	1.0E+19	5.2E+22	1.8E+26
3% Growth						
T=500	-0.0800	-0.0007	0.0780	131.32	1.7E+05	7.2E+07
T=1000	-0.0780	-0.0007	0.0751	4.2E+05	1.3E+10	9.2E+13
6% Growth						
T=500	-0.0573	-0.0003	0.0582	-0.2212	0.4199	121.10
T=1000	-0.0575	0.0006	0.0585	-1.8212	0.2312	359.11
10% Growth						
T=500	-0.0481	-0.0007	0.0467	-0.1082	0.0011	0.1195
T=1000	-0.0458	0.0007	0.0467	-0.1131	0.0006	0.1172
13.7% Growth						
T=500	-0.0427	-0.0009	0.0416	-0.0603	-0.0009	0.0574
T=1000	-0.0400	0.0008	0.0413	-0.0573	0.0013	0.0600

Note: We randomly draw policy shocks and policy regimes using the parameters obtained from regressions of specification 3 and generate 5000 replications for the time series of the debt-GDP ratio (1000 years) for various paths of the nominal growth rate (n_t), which are exogenously determined. The figures in the table represent the first, second, and third quantiles of the simulated distribution with T=500 (i.e., 500 years later) and T=1000. The average growth rate over the entire sample was 13.7 percent.

Table 9: Two-State Model for the U.S.

Panel A: Specification 3, 1840-2005

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0039	-0.0013	0.0014	0.0055	0.0242	0.0458
α	0.8734	0.8805	0.8885	0.9025	0.9393	0.9760
σ^2	0.00003	0.00007	0.0001	0.0004	0.0007	0.0010
p_{11}	0.8448	0.9161	0.9702			
p_{00}	0.8635	0.9287	0.9704			

Panel B: Specification 4, 1840-2005

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0031	-0.0012	0.0008	0.0062	0.0277	0.0518
α	0.8448	0.8526	0.8594	0.8432	0.8811	0.9165
σ^2	0.00003	0.00005	0.00007	0.0007	0.0010	0.0014
p_{11}	0.8101	0.8922	0.9530			
p_{00}	0.8637	0.9243	0.9644			

Panel C: Specification 3, 1948-2004

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0426	-0.0111	0.0114	-0.0182	0.0085	0.0332
α	0.8478	0.8887	0.9441	0.9261	0.9699	1.0189
σ^2	0.00005	0.0001	0.0002	0.0002	0.0004	0.0006
p_{11}	0.8250	0.9273	0.9849			
p_{00}	0.7771	0.9068	0.9777			

Panel D: Specification 4, 1948-2004

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0522	-0.0300	-0.0073	-0.0443	-0.0245	-0.0057
α	0.8492	0.8907	0.9285	0.9330	0.9653	1.0004
σ^2	0.00005	0.0001	0.0002	0.0001	0.0002	0.0004
p_{11}	0.8379	0.9292	0.9833			
p_{00}	0.7636	0.9024	0.9794			

Note: The transition probability, p_{ij}, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled "LB" and "UB" refer to the lower and upper bound of the 95% confidence interval and the columns labeled "Mean" refer to the mean of the marginal distribution of the parameter.

Table 10: Multiple Break Tests for the U.S.

	Panel A: Specification 3		Panel B: Specification 4	
	μ	α	μ	α
Regime 1	-0.0093	0.9555	-0.0120	0.9167
1840-1872	(0.0026)	(0.0240)	(0.0028)	(0.0256)
Regime 2		1.0009		0.9781
1873-1916		(0.0265)		(0.0283)
Regime 3		1.0469		1.0264
1917-1943		(0.0125)		(0.0133)
Regime 4		0.9071		0.8868
1944-1972		(0.0057)		(0.0061)
Regime 5		1.0135		0.9450
1973-2004		(0.0079)		(0.0084)

Note: The constant term is imposed to be identical across regimes. The maximum number of breaks is 5 with ϵ=0.15. Figures in parentheses denote standard errors.

Table 11: Two-State Model for the U.K.

Panel A: Specification 3

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	0.0837	0.1026	0.1220	-0.0341	-0.0264	-0.0180
α	0.8043	0.8213	0.8369	0.9884	0.9955	1.0019
σ^2	0.0003	0.0004	0.0005	0.0004	0.0005	0.0006
p_{11}	0.9128	0.9544	0.9817			
p_{00}	0.6603	0.8175	0.9248			

Panel B: Specification 4

	Regime 0			Regime 1		
	LB	Mean	UB	LB	Mean	UB
μ	-0.0033	0.0692	0.0952	-0.0411	-0.0336	-0.0239
α	0.7986	0.8202	0.8686	0.9640	0.9709	0.9773
σ^2	0.0003	0.0004	0.0005	0.0003	0.0004	0.0006
p_{11}	0.9027	0.9474	0.9816			
p_{00}	0.6323	0.7900	0.9199			

Note: The transition probability, p_{ij}, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled "LB" and "UB" refer to the lower and upper bound of the 95% confidence interval and the columns labeled "Mean" refer to the mean of the marginal distribution of the parameter.

Figure 1: Public Debt (Relative to Nominal GDP)

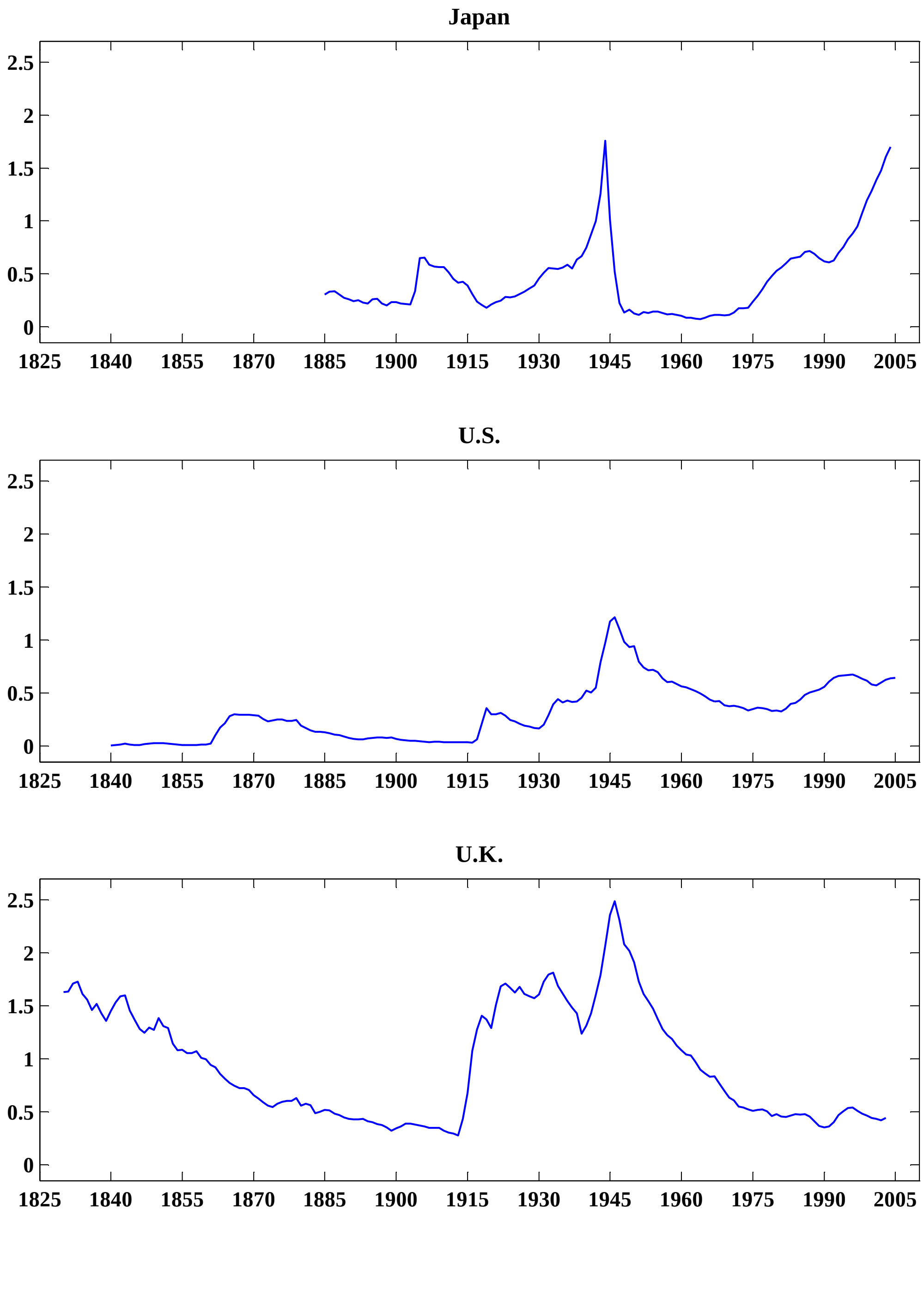


Figure 2: Coefficient on b_{t-1} Estimated from Rolling Regressions

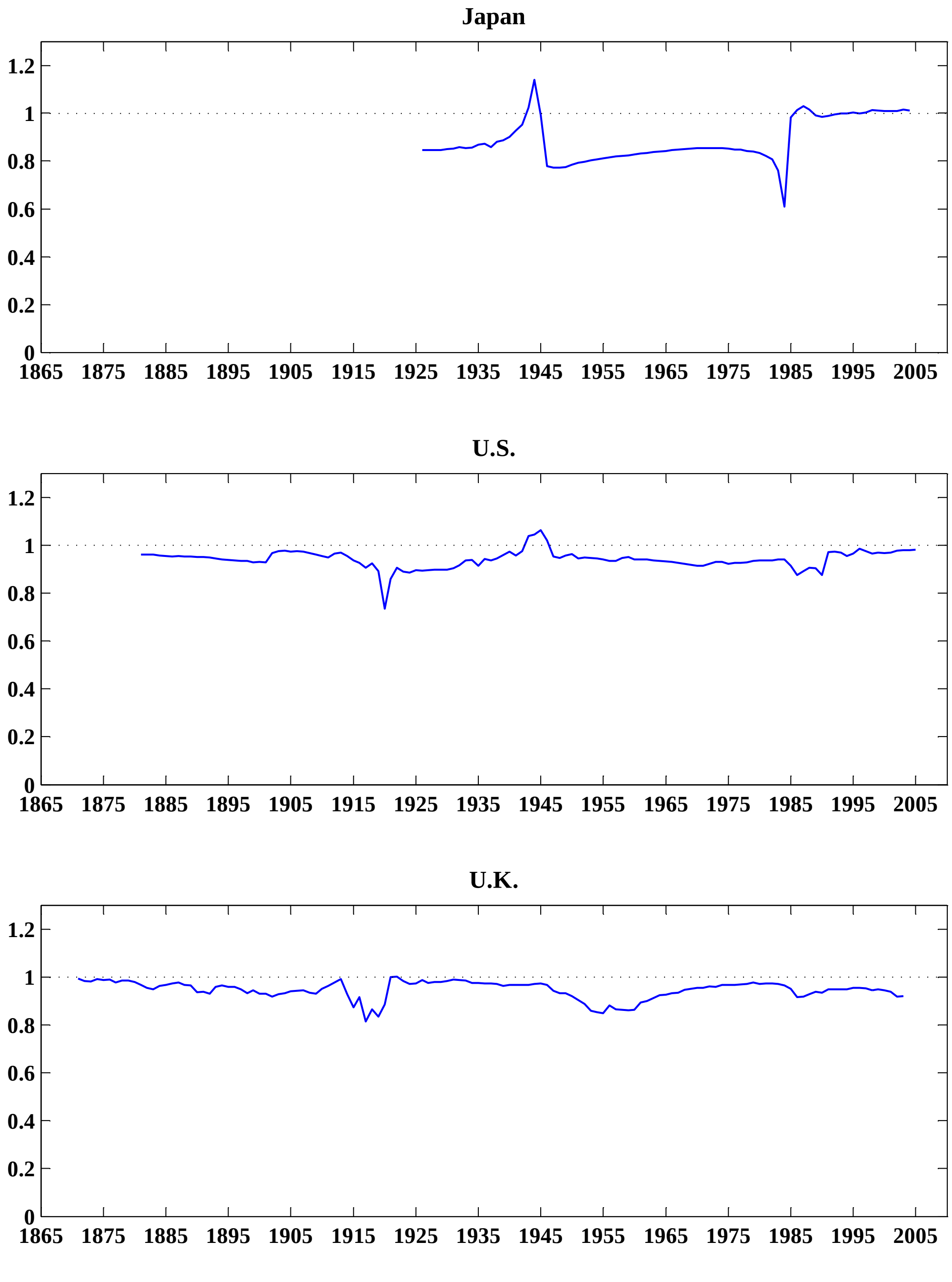


Figure 3: Two-State Model for Japan

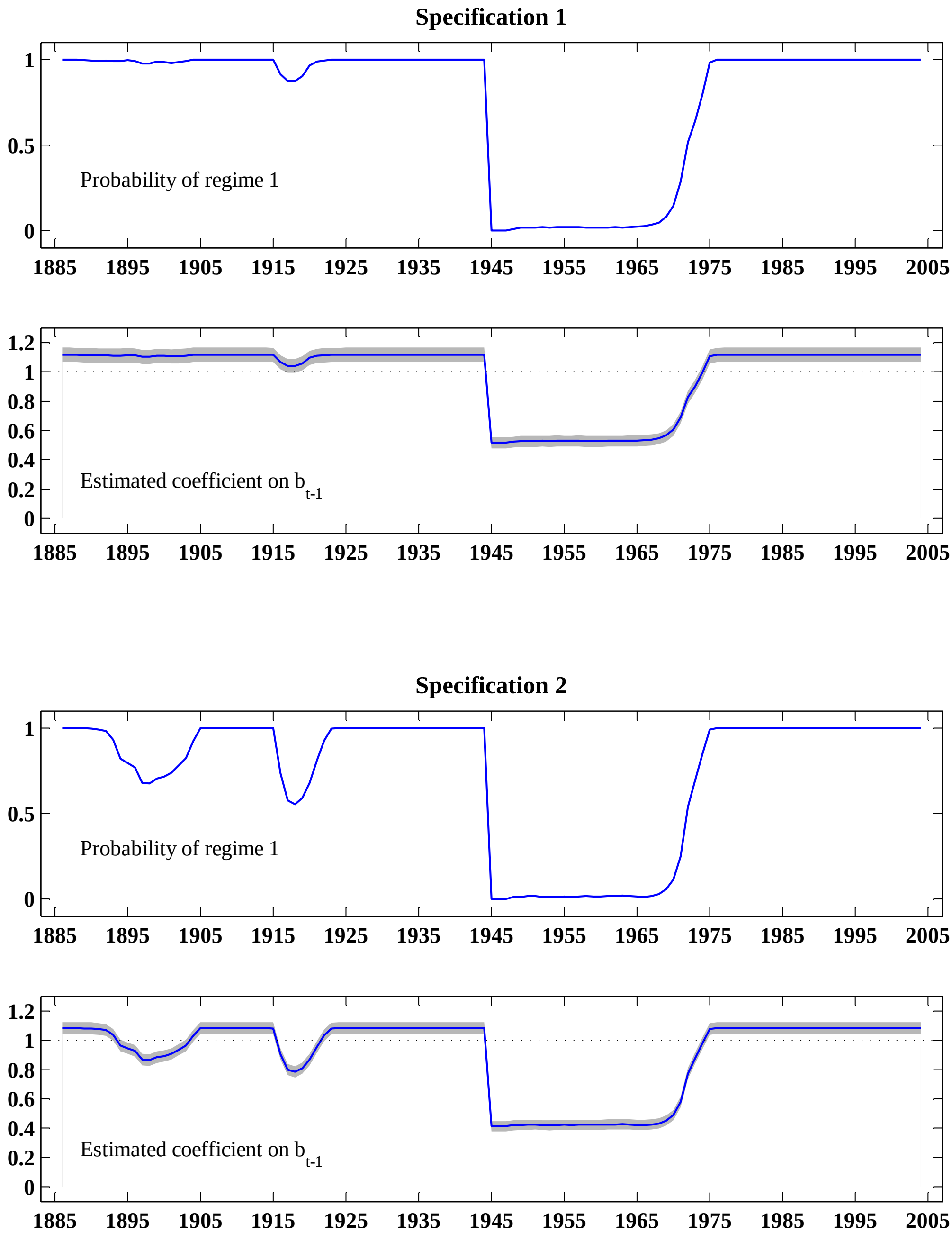
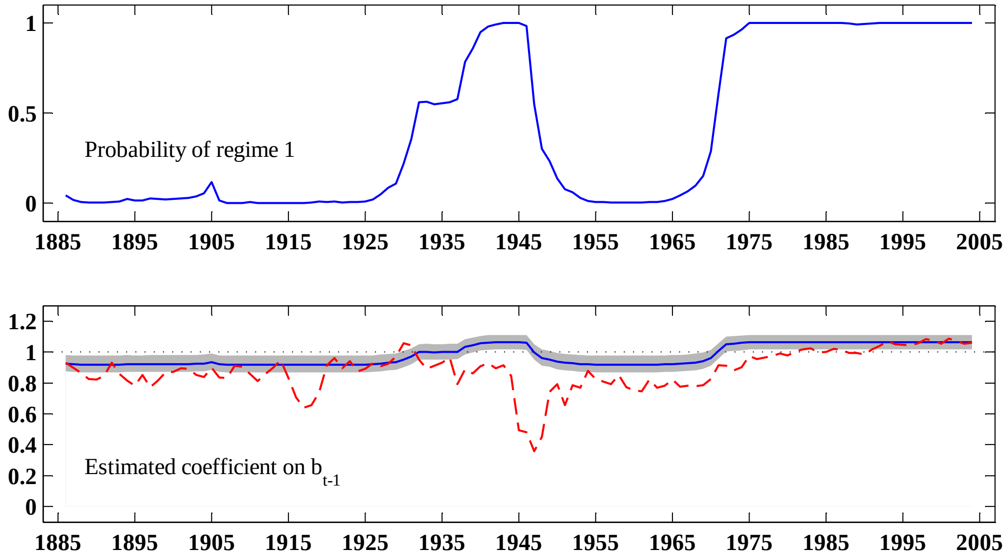


Figure 3: Two-State Model for Japan—Continued

Specification 3



Specification 4

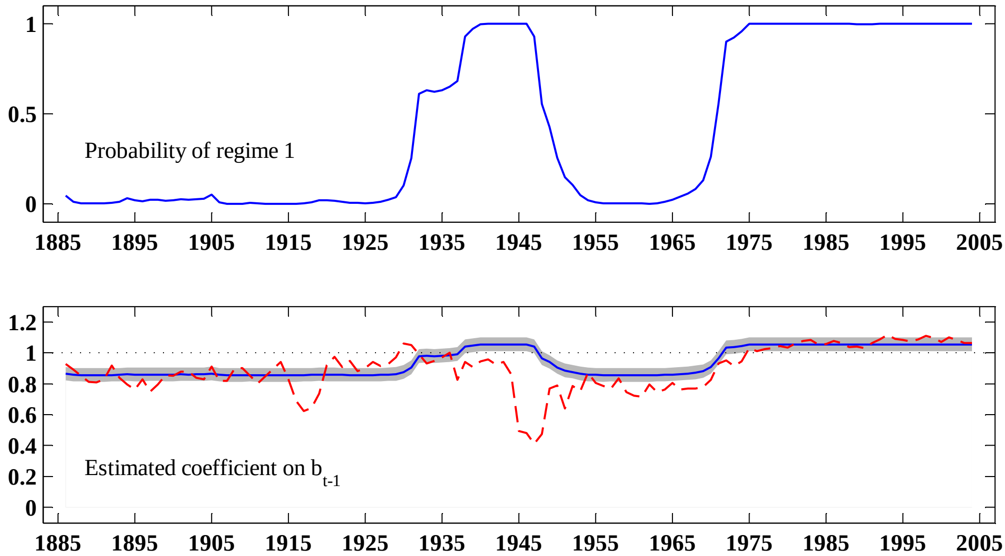


Figure 4: AR(2) Model

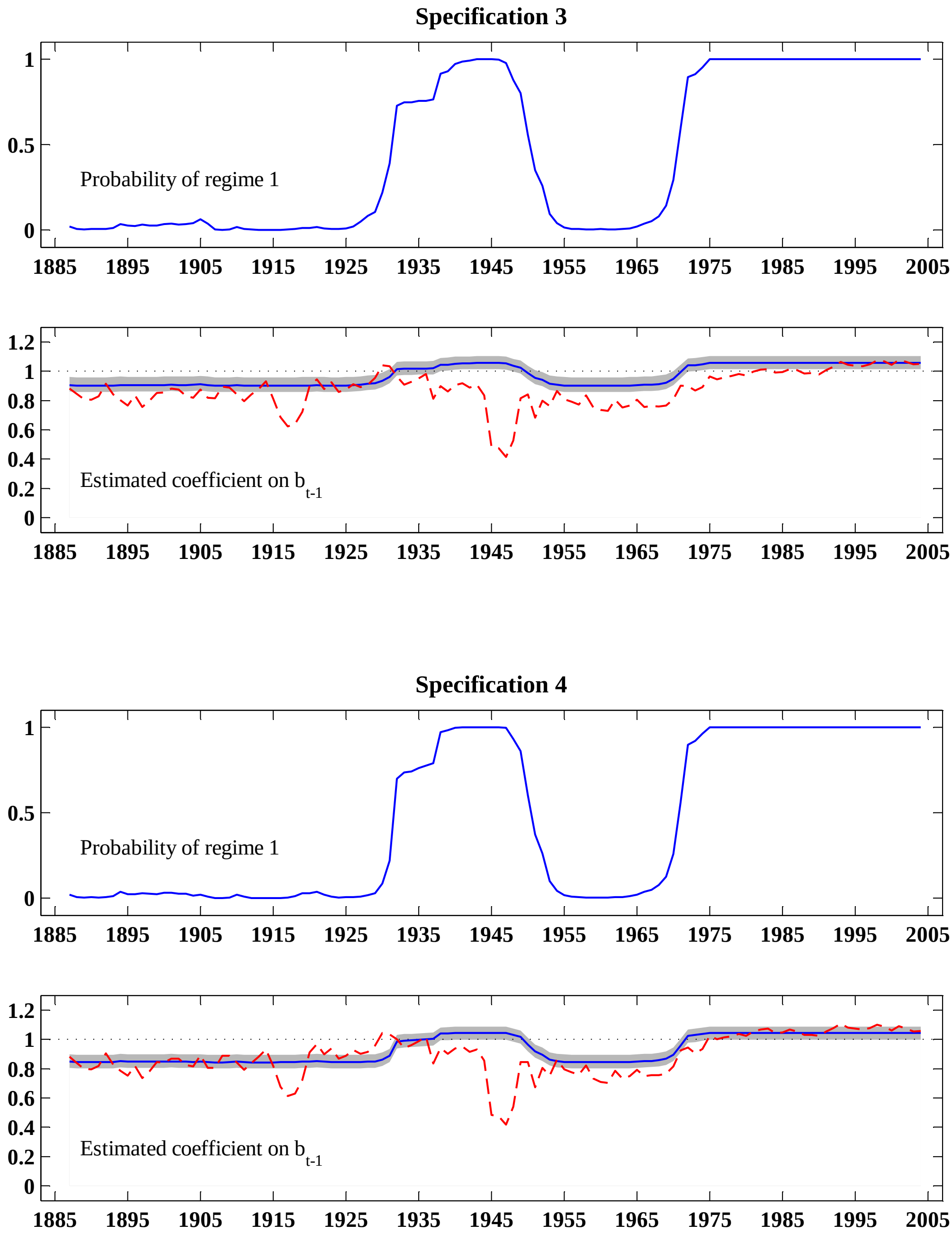
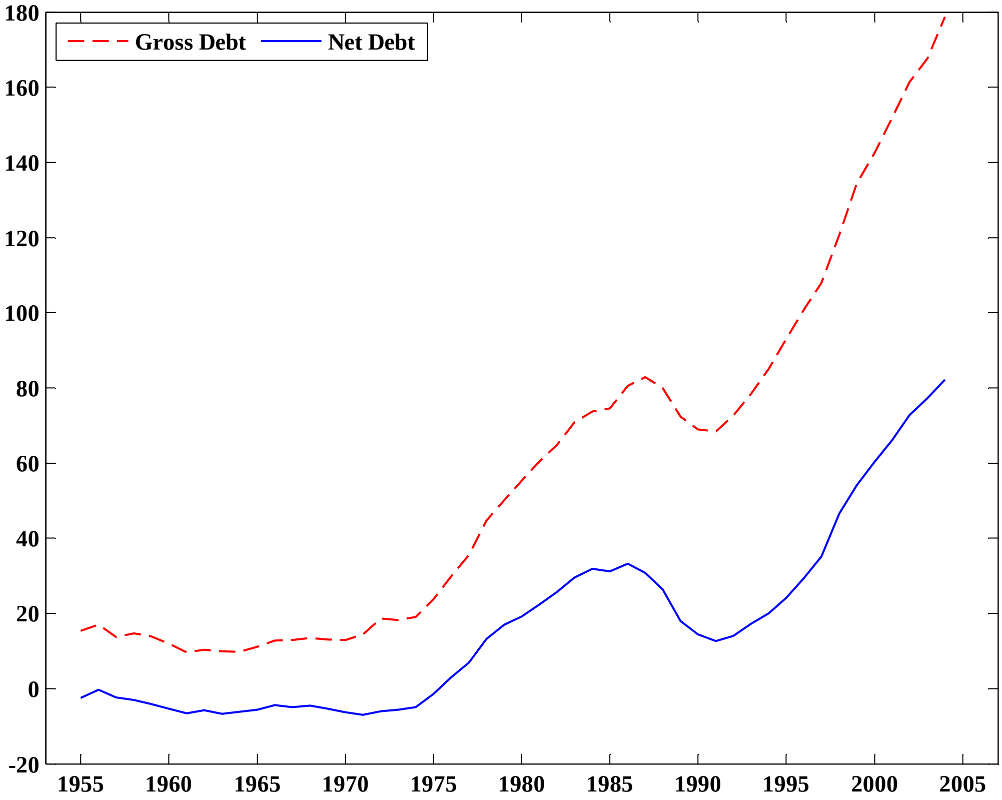


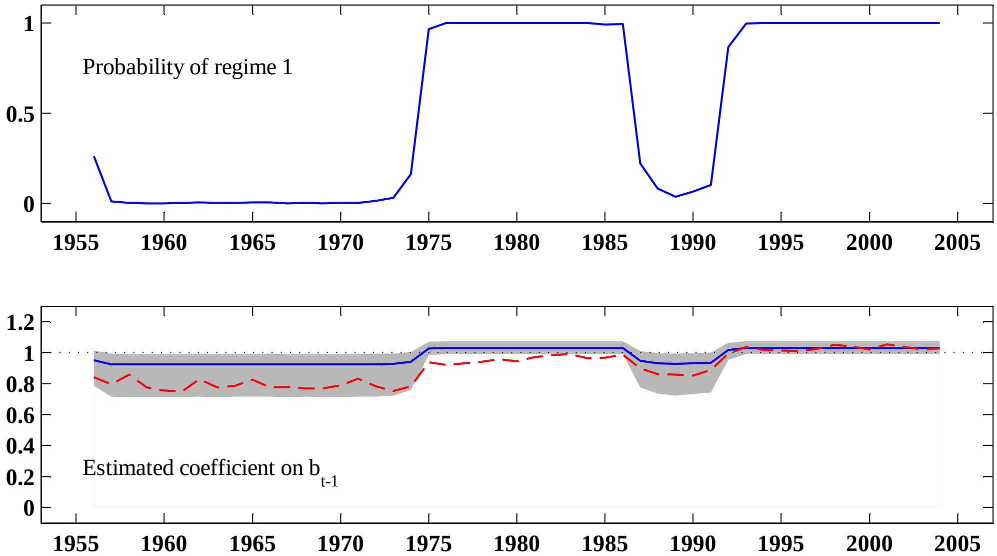
Figure 5: Net Public Debt in Japan (Relative to Nominal GDP)



Sources: Economic Planning Agency, *Report on National Accounts*, 1955 to 1969, and Cabinet Office, *Annual Report on National Accounts*.

Figure 6: Net Public Debt

Specification 3



Specification 4

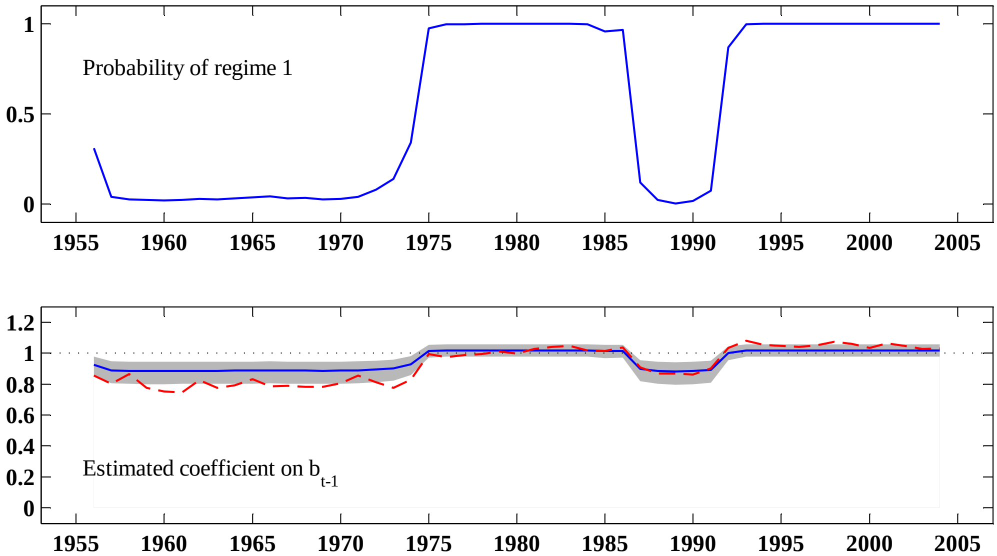


Figure 7: Automatic Stabilizers

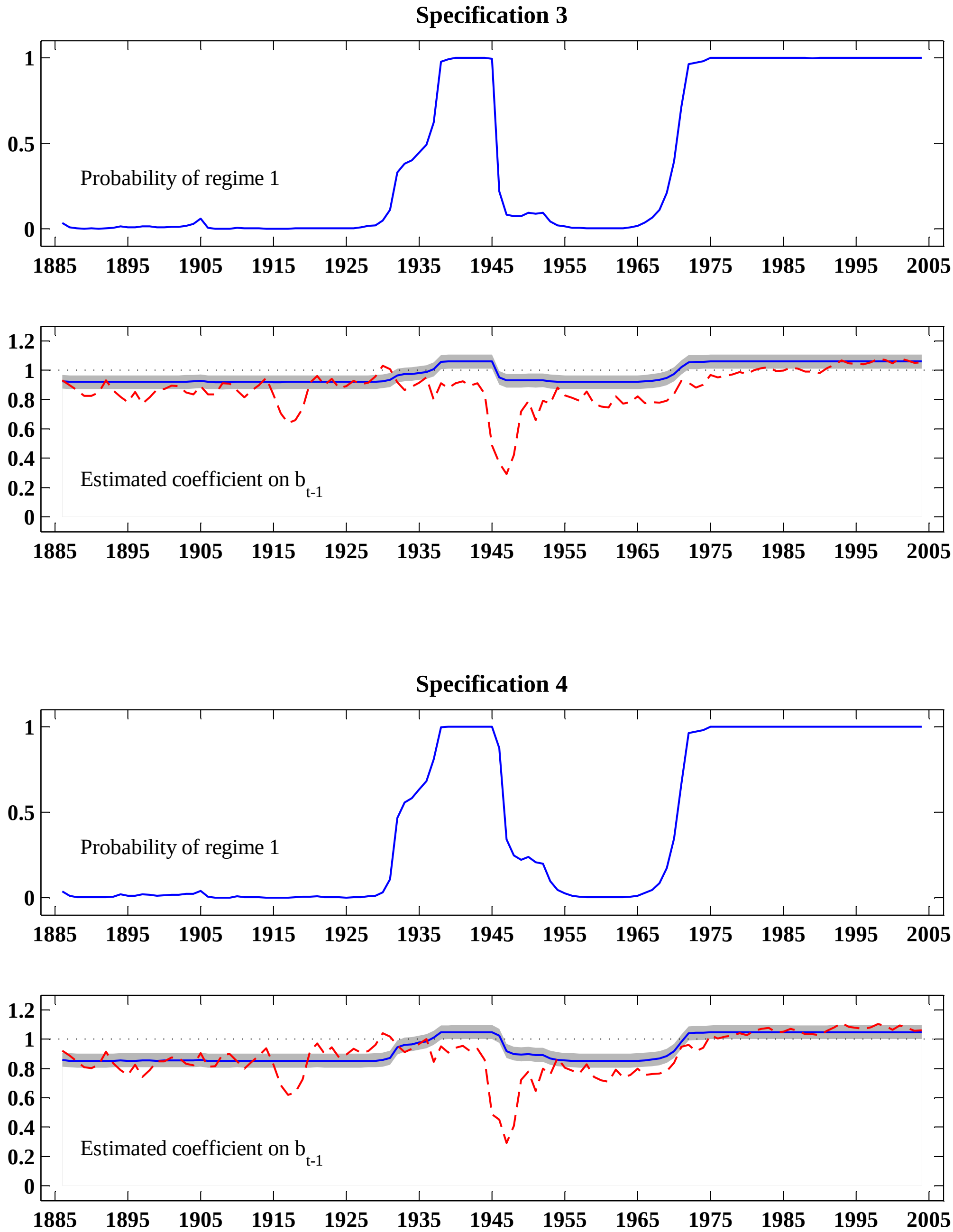


Figure 8: No Restriction on the Coefficient on Interest Payments

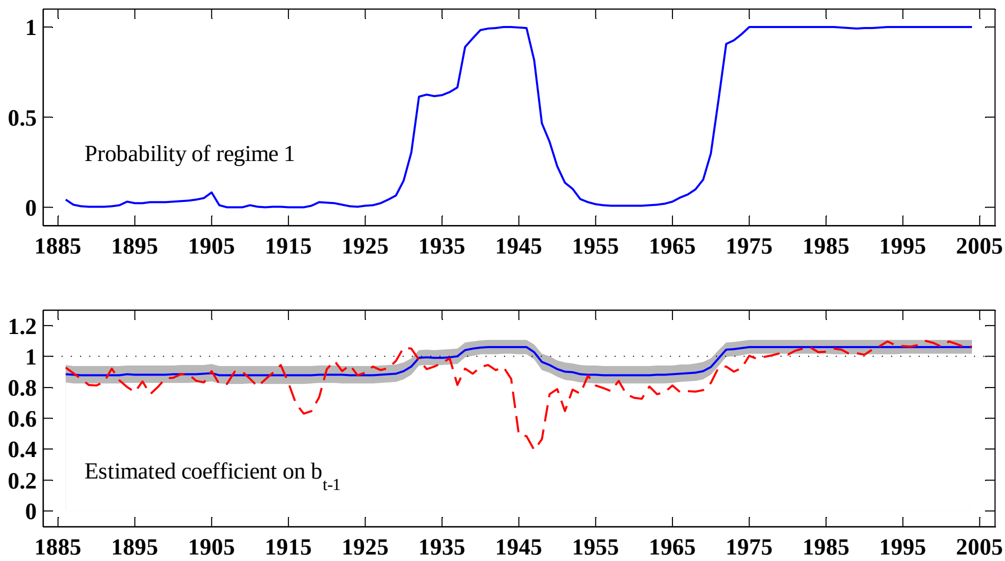


Figure 9: Three-State Model for Japan

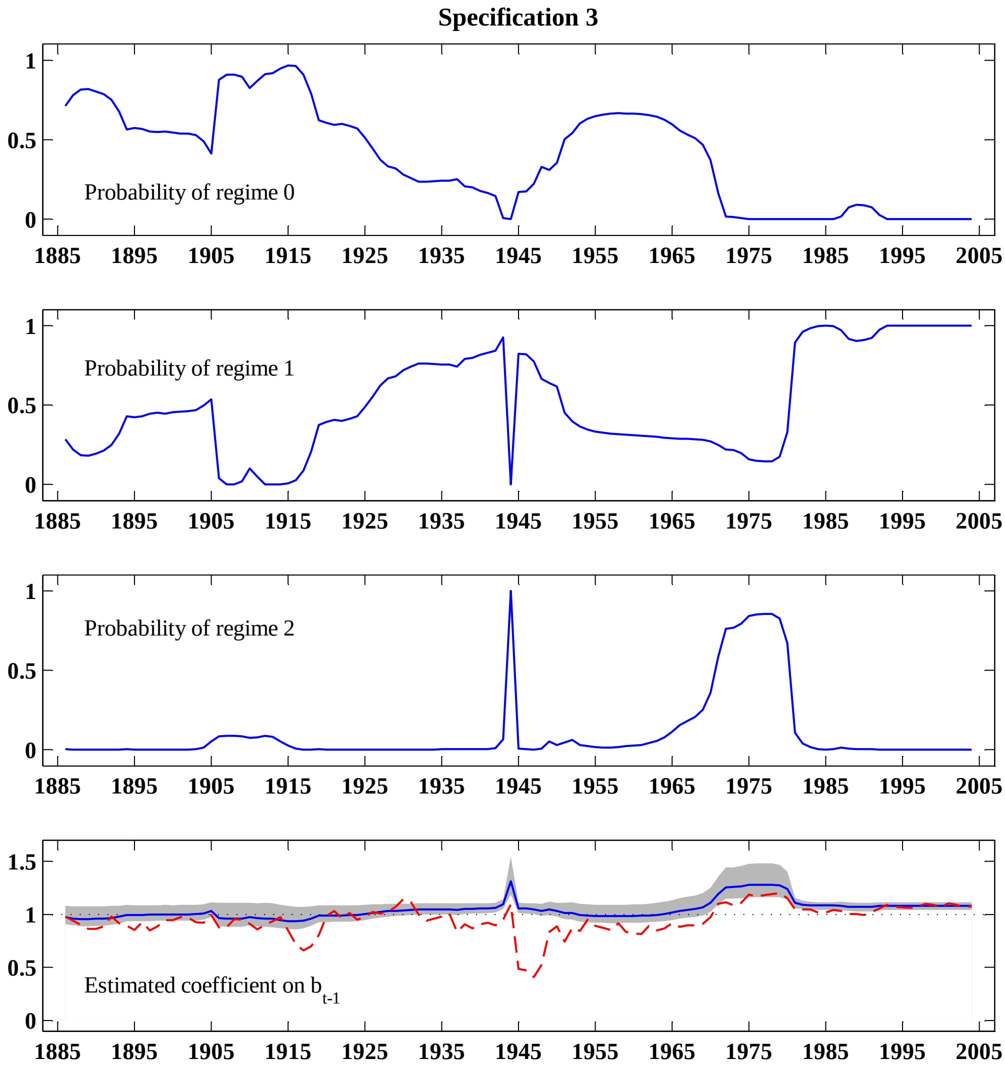


Figure 9: Three-State Model for Japan—Continued

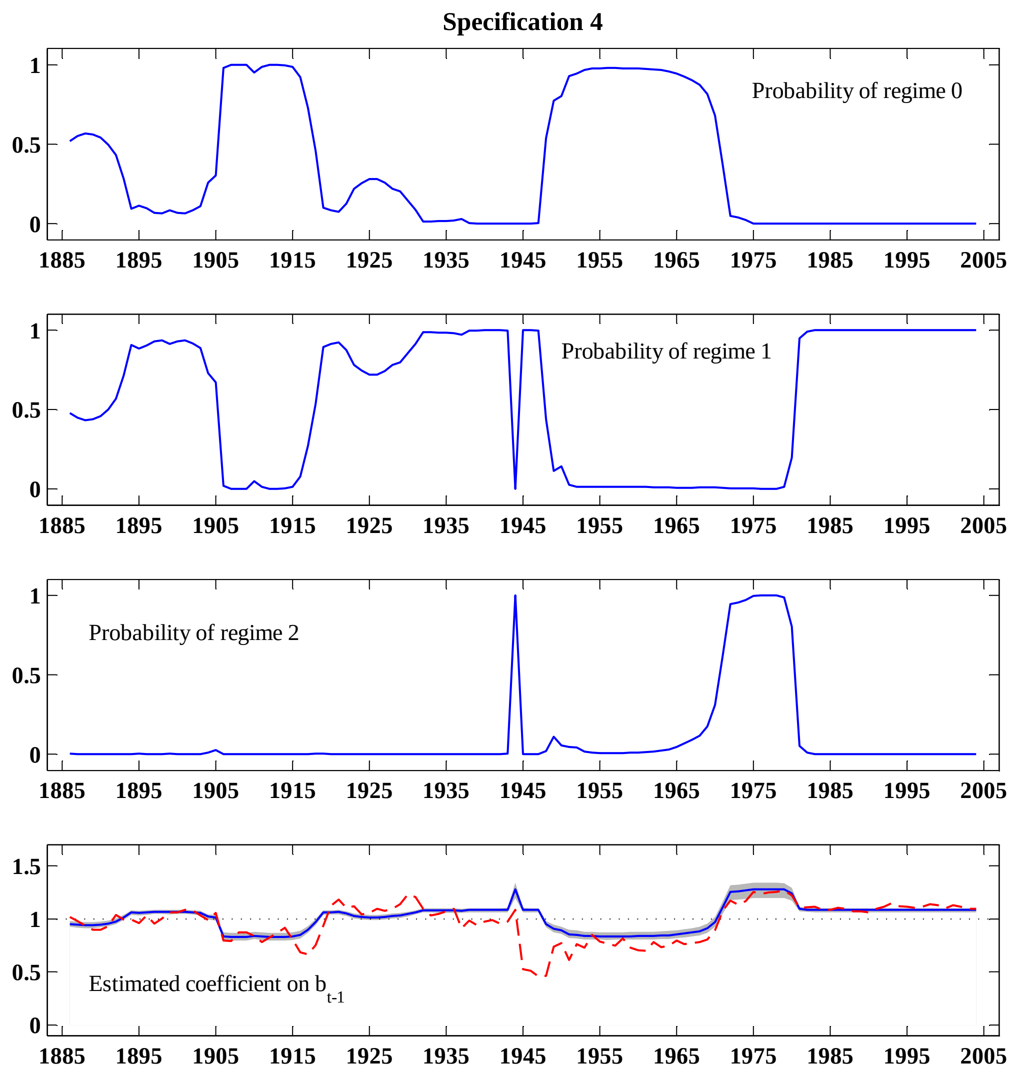
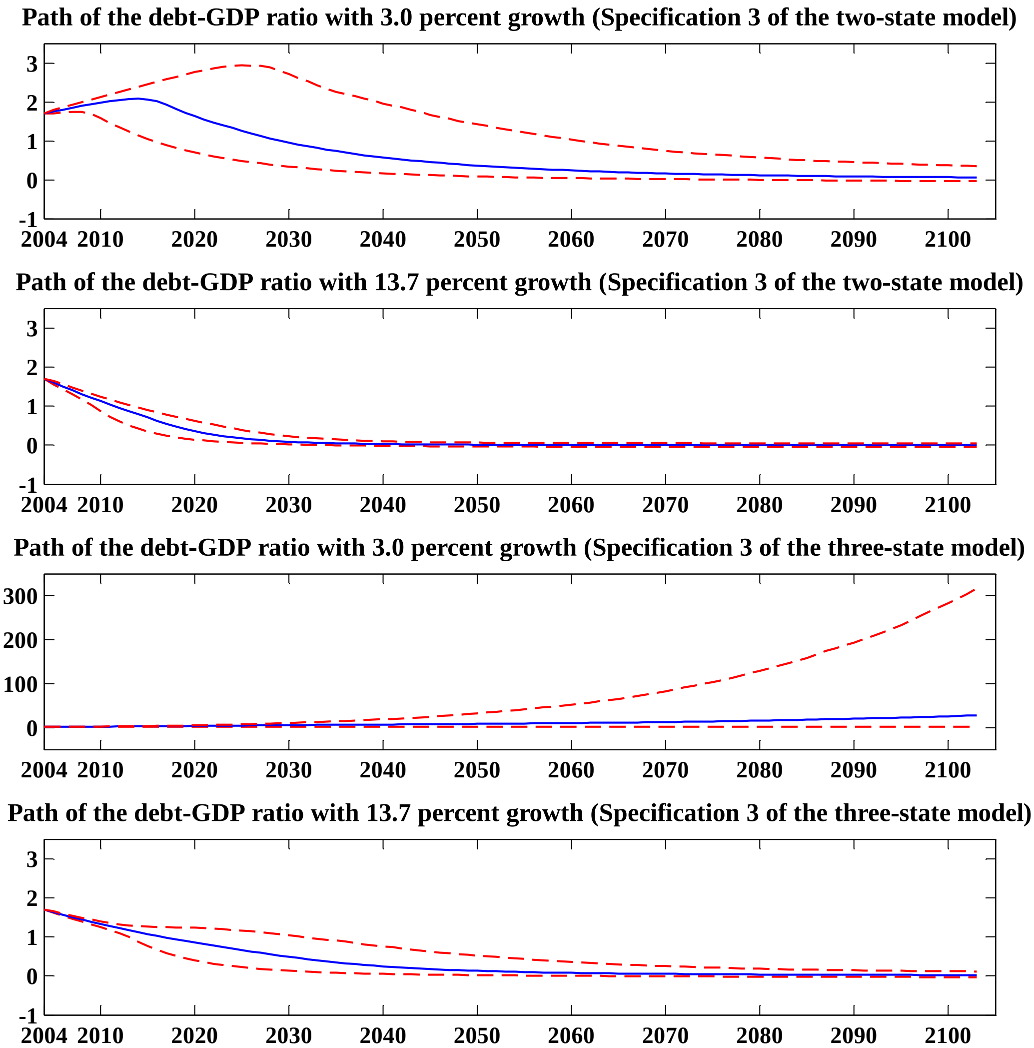


Figure 10: Globally Stationary or Nonstationary?



Note: The data of size 120 are generated from Specification 3 using estimated values with $b_0 = 1.7$ and $S_0 = 1$. In all cases, we replicate this procedure 5000 times to compute the first, second, and third quantiles.

Figure 11: Two-State Model for the U.S.

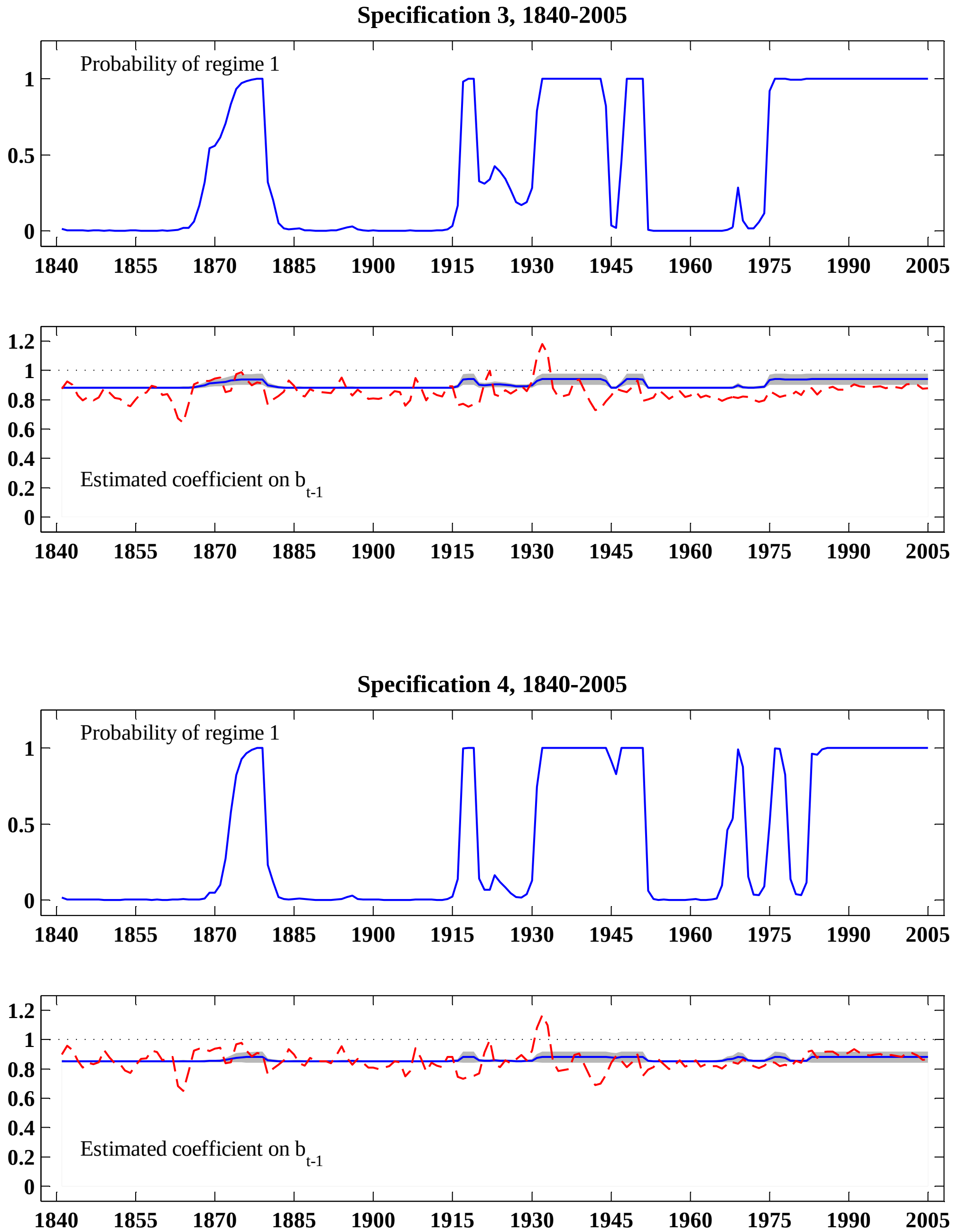
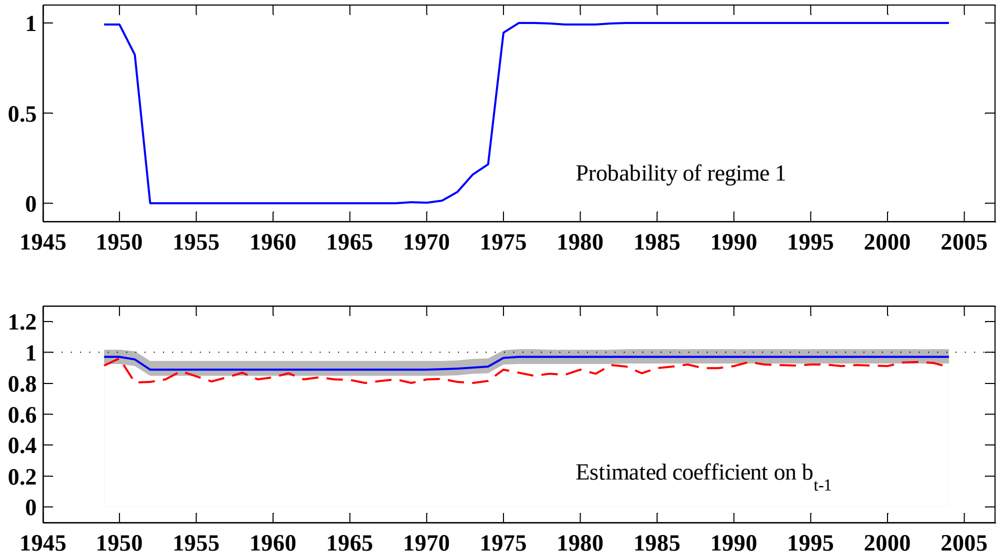


Figure 11: Two-State Model for the U.S.—Continued

Specification 3, 1948-2004



Specification 4, 1948-2004

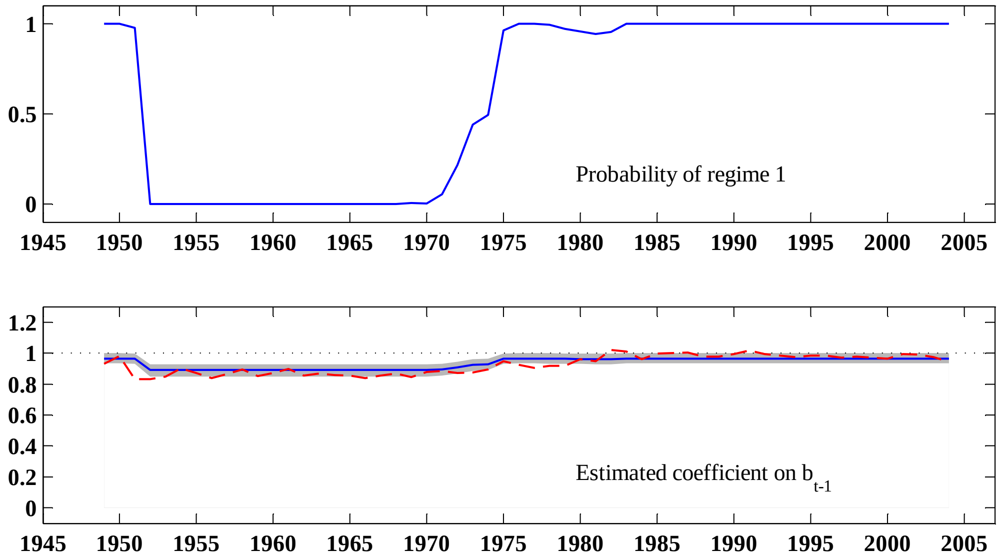
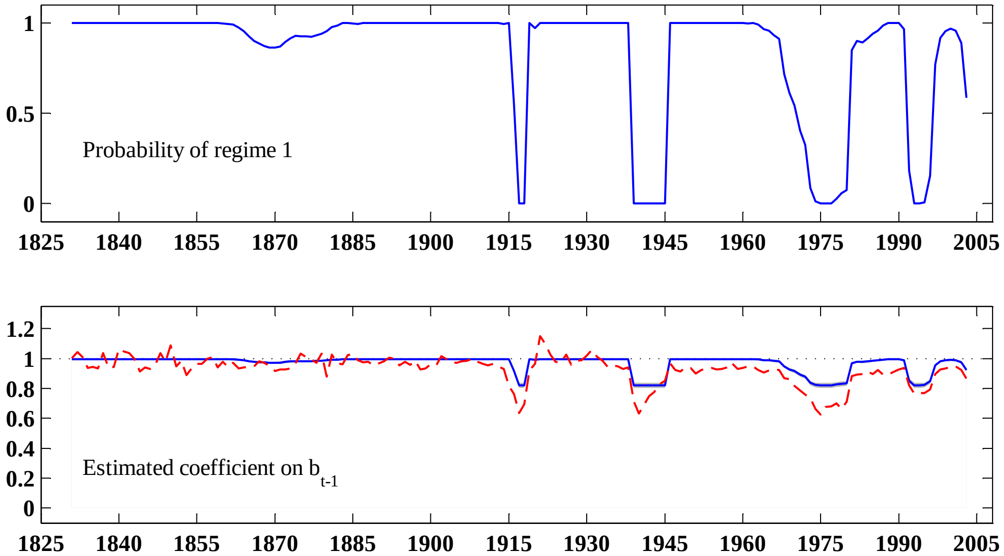


Figure 12: Two-State Model for the U.K.

Specification 3



Specification 4