Fiscal sustainability in Japan

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Abstract

We investigate fiscal sustainability by providing a dynamic stochastic general equilibrium (DSGE) model that incorporates low interest rates of government bonds relative to the growth rate. We test whether the expected debt-to-GDP ratio stabilizes or increases without bound. We estimate the fiscal policy rule over the past 30 years and simulate the debt-to-GDP ratio under the estimated policy rule. We report that the fiscal policy of Japan is not sustainable in the sense that the debt-to-GDP ratio will increase without bound. We also simulate the debt-to-GDP ratios under alternative fiscal policy rules.

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1. Introduction

Whether government debt is sustainable has become a great concern in the wake of the European fiscal crises in 2010. Japan's government debt outstanding, among others, is close to the double of annual GDP, which is the highest among the developed countries and in its own post-WWII history. Though the Japanese government (Cabined Decision, 2010) declared its target at turning from primary deficit to primary surplus by 2020 and lowering the debt-to-GDP ratio from 2021, the weak economic recovery and unstable political situations are undermining its feasibility and credibility. We investigate the sustainability of Japanese government debt under the fiscal policy observed in the past thirty years and some alternative policy rules.

Investigating fiscal sustainability, however, entails facing a puzzling fact. Interest rates on government bonds have remained quite low relative to the economic growth rate in Japan. Figure 1 illustrates the time series of the Financial bill rate, the interest rate of the long-term bond, and the growth rate for the period of 1981–2009. All figures are measured in real terms in terms of GDP deflator. The averages of the long-term bond and the Financial bill rates are 3.6 percent and 1.9 percent, respectively, while the average of the growth rate is 2.2 percent.

With the annual discount factor that is extensively used in the business cycle literature (namely, $1/1.03$), the exogenous growth model predicts that the interest rate should be above 3 percent. The endogenous growth model predicts that, with log-utility, the interest rate should be higher than the growth rate by at least 3 percent.[2]Understanding a "low" interest rate is crucial to investigating fiscal sustainability.[3]

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[2] This value of the discount factor is equal to the estimate reported in the business cycle literature (see, e.g., Christiano and Eichenbaum, 1992 and Christiano, Eichenbaum and Evans, 2005).

We investigate the low interest rates of government bonds relative to the growth rate by providing a dynamic stochastic general equilibrium (DSGE) model incorporating the AK production technology. To motivate low interest rates, we introduce financial friction into the model by assuming intermediation costs for private lending and borrowing and the heterogeneity in the access to production among agents (e.g., Woodford, 1990 and Bohn, 1999). In addition, to capture the low interest rate on risky bonds, we consider the incomplete bond market where the government can issue only one-period bonds and private agents cannot insure away the income uncertainty.[4]

The introduction of the intermediation cost gives rise to a decline in the economic growth rate and, in addition, the decline in the interest rates. Whether the intermediation cost makes the fiscal policy more sustainable or not depends on the elasticity of substitution on consumption. If it is less than nearly 2, the high intermediation cost improves fiscal sustainability.

We evaluate sustainability by testing whether the debt-to-GDP ratio stabilizes or increases without bound. The approach of checking the intertemporal government budget constraint has a misleading implication for sustainability. In the presence of intermediation costs, any interest rate among various menus of government bonds and their combination is not appropriate for correct discounting. In addition, this criterion of

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[3] This tendency is not specific to Japan. In fact, the average realized real rates of return on government bonds in major OECD countries over the past 30 years have been smaller than the real growth rate (e.g., Blanchard and Weil, 2001).

[4] Some studies cast doubts on the presence of complete bond markets. For example, Marcet and Scott (2009) find the persistency of the data for the US government debt, which is supportive of incomplete markets but is inconsistent with complete markets

fiscal sustainability is derived from the feasibility of tax revenues as well as the non-Ponzi-game conditions of private agents, and thus turns out to be more appropriate than the approach of checking only the intertemporal budget constraint of agents.

Fiscal sustainability depends on the fiscal rule. If we use a fiscal rule that is estimated over the past thirty years, the expected debt-to-GDP ratio would reach 11,5 in 100 years and afterwards would continue to diverge dramatically. The probability that the debt-to-GDP ratio will diverge is greater than 50 percent in 20 years and later, and we have to judge that the Japanese fiscal policy is not sustainable.

The expected debt-to-GDP ratio depends on some key parameters including the financial intermediation cost, the elasticity of intertemporal substitution of consumption, and average real GDP growth rate. The calculated elasticity of intertemporal substitution of consumption is within the range over which the interest rate declines less than the growth rate in response to the reduction of the intermediation cost. In Japan, the high intermediation cost contributes to fiscal sustainability. If the fiscal rule incorporates Bohn's idea that a rational government should increase the primary surplus when the debt-to-GDP ratio is high, sustainability improves. We do not rely on the risk-premium approach to explain low interest rates for two reasons (e.g., Mehra and Prescott, 1985, and Weil, 1989).[5] First, this approach explains only the low interest rate of the safe bond but does not explain low rates of the government bonds as a whole. Secondly, as the literature on the "risk-free rate puzzle" (e.g., Weil, 1989) points out, classes of simple

[5] Abel et al. (1989) and Bohn (1995) provide stochastic growth models in which the risk premium drives down the safe interest rate, often below the economic growth rate. Their argument, along with that of Zilcha (1992), demonstrates that the no-Ponzi condition for the intertemporal government budget constraint holds, even if the safe interest rate is below the growth rate.

utility functions do not succeed in explaining the low interest rate within admissible parameter values.

This paper contributes to the literature on methodology to test fiscal sustainability. One approach uses the intertemporal government budget constraint, including Hamilton and Flavin (1986) and Ahmed and Rogers (1995).[6] Bohn (1995) criticized this approach for the reason that safe government bonds do not reflect correct discounting. Another approach checks the behavior of the debt-to-GDP ratio, including Bohn (1998) and Ball et al. (1998). Bohn (1998) proposed a simple test to check whether the debt-to-GDP ratio displays a mean-reversion property.[7] Ball et al. (1998), in their famous paper entitled "Deficit Gamble," projected future growth rates and interest rates from past data and calculated the probability under which the debt-to-GDP ratio would enter a dangerous zone. Our approach is similar to the latter approach. In our model, when there is a significant intermediation cost, the government can run the Ponzi strategy, but even then, if the debt-to-GDP ratio is constant, the fiscal policy is sustainable.

Recently, some literature has studied fiscal sustainability by applying a dynamic stochastic general equilibrium (DSGE) model. Mendoza and Oviedo (2004, 2006) develop small open economies to investigate how macroeconomic shocks affect government finances and estimate the amount of sustainable public debt in emerging

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[6] Other works include Wilcox (1989), Trehan and Walsh (1991), Hakkio and Rush (1991), Ahmed and Rogers (1995), Uctum and Wickens (2000), and Polito and Wickens (2005).

[7] Bohn (2005) applies his test to the historical data of the US and finds evidence supporting fiscal sustainability (see also Greiner and Kauermann, 2007). Mendoza and Ostry (2008) applies Bohn's test to industrial and emerging countries and finds evidence of fiscal solvency in both types of countries. Galí and Perotti (2003) and Wierts (2007), among others, apply Bohn's test to European countries.

market economies. Arellano (2008) also develops a small open-economy model to study sovereign default risk and its interaction with output and foreign debt. Sakuragawa and Hosono (2009) develop a closed economy model of an exchange economy to test fiscal sustainability of the Japanese economy. Sakuragawa et al (2010) develop a closed economy model of a production economy to test fiscal sustainability of the US.

This paper is also related to the theoretical literature that combines financial frictions with the heterogeneity in the access to production among agents to have implications for the lower interest rate than the growth rate. The literature includes Woodford (1990), Bohn (1999), Kiyotaki and Moore (2008), Hellwig and Lorenzoni (2009) and Kocherlakota (2009).

This paper is organized as follows. In Section 2, we outline the model. In Section 3, we develop the theoretical analysis. In Section 4, we describe the simulation procedure. In Section 5, we investigate the sustainability of the Japanese public debt.

2. Model

Consider an economy made up of two types of agents that live infinitely, with the number of each normalized to be unity, and the third type of agents that live for two periods and act as intermediaries. We consider heterogeneous agents and financial friction in order to provide implications for low interest rates.[8]

Type E agents have access in all *even* periods to an AK production technology that transforms K_t units of the final good into random $(1+x_{t+1})K_t$ units after one period,

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[8] Aiyagari and Gertler (1991) and Heaton and Lucas (1996) construct models with intermediation costs and heterogeneous agents.

while type O agents have access in all *odd* periods. Two reasons motivate the introduction of the AK model. First, fiscal sustainability is a long-run problem. Second, the AK model enables one to have the positive link between interest and growth rates that is observed in the time series. The rate of return on capital, x_{t+1}, is a random variable that follows a Markov process and takes values in a set, X_t. The history of the economy up to time t is denoted by $h_t = (x_t, x_{t-1}, \ldots)$, which takes values in a set H_t. Denote the probability of a variable, x_{t+1}, given a history h_t, by $\pi(x \mid h_t)$.

To simplify the notation, let there be one representative agent of each type so that the individual income $(1+x_{t+1})K_t$ denotes the aggregate income. There is no population growth. Both types have identical preferences over consumption and maximize $E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\alpha}}{1-\alpha}$, where α is the inverse of the elasticity of substitution of consumptions across periods, β (0< β <1) is the discount factor, and E_0 is the expectation operator. We impose the relevant condition on bounded utility by $E_t\{\beta(1+g_{t+1})^{1-\alpha}\} < 1$, where g_t is the growth rate of the aggregate income argued below. The government spending, G_t, is a constant share of GDP to meet $G_t = zY_t$. The government finances its spending by imposing lump-sum taxes T_t and by issuing public debt.

At each period, finite N agents who act as intermediaries are born and live for two periods. They are endowed with a specific skill of intermediating finance between private agents, and maximize the second-period consumption less the amount of effort exerted by them.

We introduce financial friction by supposing that these agents have to bear a proportional intermediation cost, $\kappa > 0$, per unit of funds. The cost is measured in terms of the loss of effort. One may interpret κ as a cost of monitoring or identifying a borrower, or of verifying credit.

3. Theoretical Analysis

The intermediary issues securities that request the rate of repayment r_t^b to firms and guarantee the rate of return r_t to investors. In a world of competitive intermediation, intermediaries finally have to earn zero profit to satisfy

(1) $$1 + r_t = (1 + r_t^b) \big/ (1 + \kappa)$$

for any x_t. Note that both assets are risky in the sense that the rate of return depends on the productivity.

At the beginning of an even period t, type E agents face a shock x_t, receive capital income $(1 + x_t)K_{t-1}$, repay $(1 + r_t^b)B_{t-1}$, consume C_t, pay taxes T_t, and invest the remaining in the private security W_t and the public bond D_t. They maximize the following value function:

(2) $$V(B_{t-1}, K_{t-1}, x_t) = \max_{C_t, W_t, D_t} \frac{C_t^{1-\alpha}}{1-\alpha} + \beta \sum_{x_{t+1} \in X_{t+1}} \pi(x_{t+1} | h_t)\, \tilde{V}(W_t, D_t, x_{t+1})$$

subject to the budget constraint

(3) $$(1 + x_t)K_{t-1} - (1 + r_t^b)B_{t-1} = C_t + W_t + D_t + T_t .$$

On the other hand, at period t, type O receives interest incomes from the private security $(1+r_t)W_t$ and the public bond $(1+R_t)D_{t-1}$, and transfers Π_t from intermediaries. They consume $\tilde{C}_t$ and invest the remaining in capital to produce in the odd period. They maximize the following value function:

$$\tilde{V}(W_{t-1}, D_{t-1}, x_t) = \max_{\tilde{C}_t, K_t, B_t} \frac{\tilde{C}_t^{1-\alpha}}{1-\alpha} + \beta \sum_{x_{t+1} \in X_{t+1}} \pi(x_{t+1} | h_t)\, V(B_t, K_t, x_{t+1}) \tag{4}$$

subject to

$$\Pi_t + (1+r_t)W_{t-1} + (1+R_t)D_{t-1} + B_t = \tilde{C}_t + K_t, \tag{5}$$

where R_t is the interest rate on the government bond, and $\Pi_t (\equiv \kappa(1+R_t)W_{t-1})$ is the intermediary's profit that is transferred to them. The intermediary is compensated for the loss of effort by income, but the income accruing to the intermediary is transferred to households in a lump-sum fashion.

Assume that the equilibrium has an interior solution. Equilibrium conditions on K_{t+1}, W_{t+1}, B_{t+1}, and D_{t+1}, together with envelope conditions, lead to

$$1 = \beta \sum_{x_{t+1} \in X_{t+1}} \pi(x_{t+1} | h_t)(1 + x_{t+1}) \frac{\{\tilde{C}(h_t)\}^{\alpha}}{\{C(x_{t+1} | h_t)\}^{\alpha}} \tag{6}$$

$$1 = \beta \sum_{x_{t+1} \in X_{t+1}} \pi(x_{t+1} | h_t)\{1 + r(x_{t+1} | h_t)\} \frac{\{C(h_t)\}^{\alpha}}{\{\tilde{C}(x_{t+1} | h_t)\}^{\alpha}} \tag{7}$$

$$1 = \beta \sum_{x_{t+1} \in X_{t+1}} \pi(x_{t+1} | h_t)\{1 + r^b(x_{t+1} | h_t)\} \frac{\{\tilde{C}(h_t)\}^{\alpha}}{\{C(x_{t+1} | h_t)\}^{\alpha}}, \tag{8}$$

and

(9) $$1 = \beta \sum_{x_{t+1} \in X_{t+1}} \pi(x_{t+1} | h_t)\{1 + R(x_{t+1} | h_t)\} \frac{\{C(h_t)\}^{\alpha}}{\{\tilde{C}(x_{t+1} | h_t)\}^{\alpha}} .$$

The market clearing in the good market is expressed as

(10) $$C_t + \tilde{C}_t + K_t + zY_t = (1 + x_t)K_{t-1} .$$

The market clearing in the credit market is expressed as

(11) $$W_t = D_t .$$

Finally, the government's budget constraint is given by

(12) $$D_t = (1 + R_t)D_{t-1} - T_t + G_t .$$

As has been suggested by Barro(1979), Kremers(1989), and Bohn(1991), we impose an additional feasibility constraint restricting the government's taxable income to be limited to some fraction of the aggregate income. We simply call a fiscal policy *feasible* if the tax revenue does not exceed a fraction τ of GDP that is,

(FC) $$T_t \leq \tau Y_t$$

at any time and at any state s, .

The competitive equilibrium is defined as a sequence of nine variables $\{\frac{C_{t+1}}{C_t}, \frac{\tilde{C}_{t+1}}{\tilde{C}_{t+1}}, \frac{K_{t+1}}{K_t}, \frac{W_{t+1}}{W_t}, \frac{B_{t+1}}{B_t}, \frac{D_{t+1}}{D_t}, r_t, r_t^b, R_t\}_{t=0}^{\infty}$, satisfying nine equations (1), (3),(6)–(12), (FC), and relevant non-Ponzi-game (NPG) conditions, given the sequence of random variables $\{x_t\}_{t=0}^{\infty}$ and the sequence of the policy rule $\{T_t, D_t\}_{t=0}^{\infty}$, and given K_0 and D_0.

The fact that two-period-lived intermediaries have no intertemporal consideration simplifies the link among several interest rates. As for the link between x_t and r_t^b, competitive intermediation should lead to $x_t = r^b(x_t)$ for any x_t. Jointly with (1), the investors' rate of return r_t should also be dependent on x_t to satisfy $r_t = (1+r^b(x_t))/(1+\kappa) - 1 \equiv r(x_t)$. Equations (6) and (8) imply that the private security issued by the intermediary and the government bond are perfect substitutes for investors so that, without loss of generality, we may set $R(x_t) = r(x_t)$ for any x_t. We have $R(x_t) < r^b(x_t)$; the government can borrow at a lower rate than private agents. The reason behind this finding is that loans to the government can be monitored with no cost, while loans to private agents need intermediation cost.

We use these features on several interest rates to argue on the non-Ponzi-game (NPG) conditions that formalize the limited willingness of agents to lend. The NPG condition for agents who have access to production at the current period and those who have no access are, respectively,

(NPG1) $$E_t \lim_{s\to\infty} \frac{K_{t+s} - B_{t+s}}{(1+r^b_{t+1})(1+r_{t+2})(1+r^b_{t+3})\ldots} = 0,$$

and

(NPG2) $$E_t \lim_{s\to\infty} \frac{W_{t+s} + D_{t+s}}{(1+r_{t+1})(1+r^b_{t+2})(1+r_{t+3})\ldots} = 0.$$

We assume that both conditions are satisfied. The agent alternates between a lender and a borrower every other period and discounts the future at rate $\sqrt{(1+r_t)(1+r^b_{t+1})} - 1$ (or $\sqrt{(1+r^b_t)(1+r_{t+1})} - 1$).

We investigate sustainability conditions for the fiscal policy. We argue that evaluating sustainability in terms of the intertemporal budget constraint is not appropriate in two respects. First, when agents discount the future at a higher rate than the government, checking the government budget constraint does not establish a condition for sustainability. The discounted value of debt that the government can earn by a Ponzi strategy is

(PG) $$E_t \lim_{s\to\infty} \frac{D_{t+s}}{(1+R_{t+1})(1+R_{t+2})...(1+R_{t+s})...} -$$

which is derived from the government budget constraint. On the other hand, the government cannot run a Ponzi strategy so long as agents do not find it optimal to be on the lending side of the Ponzi scheme. The latter condition is expressed as

$E_t \lim_{s\to\infty} \frac{D_{t+s}}{(1+r_{t+1})(1+r^b_{t+2})(1+r_{t+3})...} = 0$, which is a weaker condition than the one under

which (PG) converges to zero since, by using (1) and $r_t = R^b_t$, we rewrite the agent's NPG condition as

(NPG3) $$E_t \lim_{s\to\infty} \frac{D_{t+s}}{(1+\kappa)^{s/2}(1+R_{t+1})(1+R_{t+2})...(1+R_{t+s})...} = 0,$$

when there is an intermediation cost.

Secondly, even checking the agent's NPG condition is not sufficient to establish a condition for sustainability. The condition (NPG3) restricts the debt to grow no faster than the rate of discount (multiplied by the term $(1+\kappa)^{1/2}$), but when the interest rate is greater than the economic growth rate, it permits the debt to grow faster than the economy as whole. An unbounded debt-output ratio cannot be ruled out only by the NPG condition (e.g., McCallum, 1984).

We impose an additional constraint of feasibility, (FC). At this stage it makes sense to define fiscal sustainability more narrowly. A fiscal policy rule is defined to be sustainable if the following two conditions are satisfied. First, the NPG condition of private agents, (NPG3), is to be satisfied. Second, the government's budget is to be feasible, i.e., (FC) is satisfied at any time and at any state. We have the following.

Proposition: If fiscal policy is sustainable, then $E_t \frac{D_{t+s}}{Y_{t+s}}$ is bounded above.

Proof. See Appendix A.

The intuition behind the proof is as follows. Suppose that the debt-to-GDP ratio grows unboundedly. The government will either let debt grow further without increasing taxes, or constraint debt by increasing taxes. In the former case, debt will eventually exceed the amount that the private agents are willing to lend to the government (NPG). In the latter case, tax revenues required will exceed the limit of taxability (FC). To satisfy both (NPG) and (FC), the debt-to-GDP ratio must be bounded above.

An example of the deterministic economy shows that the sustainable fiscal policy requires $\frac{D_t}{Y_t} < \frac{(\tau - z)(1+g)}{R-g}$. Suppose, for example, that $R-g = 0.01$, $z = 0.2$, $\tau = 0.5$, and $g = 0.02$, then the upper bound becomes $D_t / Y_t < 30.6$, which is extremely high so that the latter condition seems virtually not restrictive if the debt-to-GDP ratio is stable. Therefore, we may safely say that if the debt-to-income ratio does not grow unboundedly but converges to some level, the fiscal policy is sustainable.

Let $C_t / \tilde{C}_t \equiv \theta_t$ denote the consumption ratio between two different types of agents. Limiting focus on an economy with θ_t being constant through time, we have the consumption growth rate as

(13) $$\frac{C(x_{t+1}|h_t)}{C(h_t)} = \frac{\tilde{C}(x_{t+1}|h_t)}{\tilde{C}(h_t)} \equiv 1 + g(x_{t+1}|h_t).$$

We use (13) to rewrite (7) as

(14) $$1 = \beta \sum_{X_t} \pi(x_{t+1}|h_t)\{1 + g(x_{t+1}|h_t)\}^{-\alpha} \theta^{\alpha} \{1 + r(x_{t+1})\},$$

which embodies the relationship between the growth and interest rates. On the other hand, (8), (9), (13), and (1) jointly imply $\theta = (1+\kappa)^{1/2\alpha}$. In the presence of an intermediation cost, agents consume more when they receive income and consume less when they do not. On the other hand, plugging (1) and $\theta = (1+\kappa)^{1/2\alpha}$ into (14), we have the following:

(15) $$1 = \beta \sum_{X_t} \pi(x_{t+1}|h_t)\{1 + g(x_{t+1}|h_t)\}^{-\alpha} (1 + x_{t+1})(1+\kappa)^{-1/2}.$$

Equations (14) and (15) fully determine the sequence of the growth and interest rates as the stochastic variable x_t evolves.

We explicitly solve (14) and (15). We denote the transition probability as $\pi(x_{t+1} = \lambda_j | x_t = \lambda_i) = \phi_{ij}$ in the set $X = \{x_1, ..., x_n\}$. We denote $g(x_{t+1} = x_j | h_t) = g_j$, which implies that the growth rate of consumption depends only on the current rate of return on capital. This arises from the fact that the production technology is the "AK" type and the fact that the proportion of government expenditure in output is constant. Accordingly, we rewrite (15) as

(16) $$1 = \beta \sum_{j=1}^{n} \phi_{ij} (1 + g_j)^{-\alpha} (1 + x_j)(1+\kappa)^{-1/2} \quad \text{for } i = 1, ..., n.$$

We rewrite (14) similarly as

$$1 = \beta \sum_{j=1}^{N} \phi_{ij} (1+g_j)^{-\alpha} \theta^{\alpha} (1+r_j) \text{ for } i = 1,...,n \, . \tag{17}$$

Equations (16) and (17) constitute $2n$ equations for solving n growth rates and n interest rates. We show the case of $n = 2$, with $X = \{x_1, x_2\}$. We solve four variables $\{g_1, g_2, r_1, r_2\}$ from the following four equations:

$$\begin{bmatrix} (1+g_1)^{-\alpha}(1+x_1) \\ (1+g_2)^{-\alpha}(1+x_2) \end{bmatrix} = \beta^{-1}(1+\kappa)^{\frac{1}{2}} \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \tag{18}$$

and

$$\begin{bmatrix} (1+r_1)(1+g_1)^{-\alpha} \\ (1+r_2)(1+g_2)^{-\alpha} \end{bmatrix} = \beta^{-1}(1+\kappa)^{-\frac{\twoheaduparrow}{\twoheadrightarrow}} \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{19}$$

Appendix B provides the procedure for solving the general case.

It is interesting to investigate how a change in κ influences g_{t+1} and r_{t+1}, and their relationship. Equation (18) implies that a one percent increase in κ leads to a decline in approximately $1/(2\alpha)$ percent of g_{t+1} for each state. On the other hand, (19), together with (18), implies that a one percent increase in κ leads to a decline of one percent of r_t for each state. If α is sufficiently small (less than 0.5), or equivalently, the elasticity of substitution of consumptions is sufficiently large, the growth rate declines more sharply than the interest rate as κ increases. Figure 2A illustrates the case for $\alpha = 0.4$.[9] By contrast, if α is sufficiently large, the growth rate declines less than the interest rate for higher intermediation costs. Figure 2B illustrates the case for $\alpha = 1$(log-utility). Whether

[9] In Figure 2, we set $x^a = 0.033$. See Table 2 for other parameter values.

higher intermediation costs make fiscal sustainability difficult depends on the elasticity of substitution of consumption.

4. Calibration

In this section and the next, we simulate the model to investigate the fiscal sustainability of Japan. Our methodology is to update the future fiscal variables by introducing a specified fiscal rule into the developed DSGE model and to simulate the debt-to-GDP ratio. The driving force of the growing economy is the rate of return on capital, which determines the growth rate and the interest rate. We specify the stochastic process for the logarithm of the gross rate of return on capital by discretizing a simple AR(1) process with nine states $(n=9)$. The AR(1) process is described, with the serial correlation coefficient ρ and the average $\log(1+x^a)$, as

(20) $$\log(1+x_{t+1}) = \rho\log(1+x_t) + (1-\rho)\log(1+x^a) + e_{t+1},$$

where e_{t+1} for all *t* are random shocks that are independent and identically distributed as a normal distribution with standard deviation σ_e. Once three parameters, x^a, ρ, and σ_e, have been set, following the method developed by Tauchen and Hussey (1991), we construct the nine states $\{x_1,....,x_9\}$ and the transition probability ϕ_{ij} $(i, j=1,...9)$. We use (16) and (17) to solve for 18 variables (g_j, r_j) $(j=1,...9)$, given the specified (x_j, ϕ_{ij}) (see Sakuragawa, Hosono and Sano, 2010 for details). We use (20) to obtain the stochastic process for the GDP growth rate as a discretized version of an AR(1) process:

(21) $$\log(1+g_{t+1}) = \rho\log(1+g_t) + (1-\rho)\log(1+g) + \varepsilon_{t+1},$$

where $\varepsilon_{t+1}(\equiv e_{t+1}/\alpha)$ is a random shock with the standard deviation of $\sigma_\varepsilon(\equiv \sigma_e/\alpha)$ and g is the steady-state growth rate.

The debt-to-GDP ratio evolves from (12) as

$$d_{t+1} = \frac{(1+R_{t+1})}{(1+g_{t+1})} d_t - s_{t+1}, \tag{22}$$

where d_t and s_t are debt and primary surplus divided by GDP, respectively. These two equations, together with a fiscal policy rule that determines s_{t+1}, provide the full system. Before going on to the fiscal policy rule, we choose parameter values. Data used to set parameters is described in Appendix C.

First, we choose the preference parameters, β and α. We set the annual discount factor β to 1/1.02=0.980. The inverse of the elasticity of intertemporal substitution, α, plays a central role in relating the interest rate to the growth rate, as captured by (19). To set α, we note from (19) that

$$\log(1+R_j) = \alpha \log(1+g_j) + \gamma_j, \tag{23}$$

where γ_j is the logarithm of the *j*th element of the right-hand side in (19). We regressed the nominal government bond yield on the nominal GDP growth rate using OLS. The sample used for the estimation below covers the period of 1981-2009 except for otherwise mentioned. The estimation result is

$$R_t = \underset{(0.003)}{0.023} + \underset{(0.077)}{0.565} g_t, \quad Adj.R^2 = 0.651, \tag{24}$$

where the numbers in parentheses are standard errors. Following (23), we set α at 0.565, implying that the elasticity of intertemporal substitution is set at 1.770. It is noteworthy that this figure is in the region of parameters in which a rise in the intermediation cost can improve fiscal sustainability.

Next, we specify the technology parameters, ρ and σ_e. We obtain the OLS estimation result as

(25) $$g_{t+1} = 0.002 + 0.780 g_t, \quad Adj.R^2 = 0.413.$$

$$(0.006) \quad (0.175)$$

We set ρ at 0.780 and σ_ε at 0.01996, where the latter is the root mean squared error of the regression. Given the chosen σ_ε and α, we set σ_e at 0.01128. We set the average return to capital, x^a at a value that yields 1 percent average GDP growth rate given the chosen values of β, α, and κ in (16). We find that x^a =0.033. The average GDP growth rate over the whole sample period of 1981-2009 is 2.2 percent, but this figure seems to be too high for predicting the future growth rate, especially when we consider the rapid population aging in Japan. We choose the average growth rate to be unity 1 considering that the average GDP growth rate over the period of 1990-2009 is 1.1 percent.[10]

Third, we set the financial intermediation cost, κ, at 0.015, the average of net interest margins between the bank loans and the bank deposits over the period of 2000–2009. We choose the average over the last decade because, as Figure 3 shows, the net interest margins tended to decline over the last two decades especially at a high rate in the 1990s.

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[10] One percent GDP growth rate roughly corresponds to the “conservative” scenario of the government’s mid-term forecast up to 2020 (Cabinet Office, 2010).

We complete the model by specifying the government's fiscal policy rule. The fiscal rule is interpreted as a consequence of the conflict of interests among many pressure groups and so changes little unless the political situation changes drastically. Based on this idea, we specify the fiscal rule by the regression. We regress the primary balance as a proportion of GDP on the real GDP growth rate and the one-period lagged primary balance. The GDP growth rate is expected to capture the business cycle effects. When the economic boom comes, an increase in tax revenues improves the fiscal stance. Actually, Figure 4 illustrates a positive correlation between the two. The lagged variable captures the persistency of the government expenditure and tax revenues. To be specific, we describe a fiscal policy rule as

(26) $$s_t = \gamma_0 + \gamma_1 s_{t-1} + \gamma_2 g_t .$$

Using the sample of 1981-2008[11], we obtain the regression result as

(27) $$s_t = -0.021 + 0.658 s_{t-1} + 0.577 g_t ,\quad Adj.R^2 = 0.759.$$

$$\quad (0.006)\ (0.160)\quad (0.104)$$

Based on (27), we set γ_0=–0.021, γ_1=0.658, and γ_2=0.577.

We simulate the model recursively by generating $(g_{t+1}, R_{t+1}, s_{t+1})$ for the stochastic process of x_{t+1}, and obtaining d_{t+1}, given the starting value of d_t.

Table 1 summarizes the parameters that we use for the baseline calibration. The simulation procedure is described in Appendix D.

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[11] The data of primary balance is available only up to 2008. See footnote 2.

5. Simulation Results

A. Baseline Forecasts

Table 2 shows the expected debt-to-GDP ratio and the probability that the debt-to-GDP ratio exceeds d_{2009} (=1.792). Under the baseline parameters, the average interest rate is 1.8 percent and the average growth rate is 1.0 percent, with the gap of 0.8 percentage points. The average primary surplus is -4.5 percent of GDP. The expected debt-to-GDP ratio reaches 11.5 in 100 years and continues to increase afterwards, suggesting that debt is not sustainable.

The path of the expected debt-to-GDP ratio and the probability that the debt-to-GDP ratio will exceed its initial value are highly sensitive to the intermediation cost. If we set $\kappa = 0.0$, the average interest rate and the GDP growth increase to 3.33 percent and 2.34 percent, respectively[12]. The gap between the average interest rate and the average growth rate rises, and the expected debt-to-GDP ratio increases more rapidly than in the benchmark case, reaching 13.1 in 100 years (the second row of Table 2). The smaller intermediation costs make the fiscal sustainability more difficult. This is because the estimated elasticity of intertemporal substitution of consumption is below two.

The sensitivity of the interest rate to the growth rate also depends on the intertemporal rate of substitution. If we return the average real GDP growth rate to 1.0 percent, as in the benchmark case, but set the inverse of the elasticity of intertemporal substitution at one (the log-utility case), the average real interest rate becomes 2.26

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[12] In Sections 5A and 5B (Tables 2 and 3), we adjust γ_0 so that the average primary surplus is the same as in the benchmark case (-4.5 percent of GDP).

percent, making the sustainability conditions more difficult to meet: the expected debt-to-GDP ratio reaches 14.7 in 100 years.

B. Alternative GDP Growth Rates

In the baseline case, we have assumed that the long-run average real GDP growth rate is 1 percent. Here we assume higher real GDP growth rates to see if the high growths help restore sustainability. Specifically, we increase the average return on capital, x^a, to attain the 2 percent and 3 percent average GDP growth rates. The other parameters are set as in the baseline case[13]. The 2 percent growth rate corresponds to the "Growth Strategy Scenario" in the government's mid-term forecast up to 2020 (in Cabinet Office, 2010; Cabinet Decision, 2010), It is also close to the average real GDP growth rate over the period of 1981-2009 (2.2 percent). The second and third rows of Table 3 show that debt-to-GDP ratio grows more slowly as the average growth rate increases: while in the case of 1 percent growth rate (in the baseline case) it reaches 11.5 in 100 years, the corresponding figures in the cases of 2 percent and 3 percent growth rates are 8.6 and 6.5, respectively. As the average real GDP growth rate increases, the average real interest rate also rises, but the gap shrinks, resulting in a lower expected debt-to-GDP ratio. Actually, the average interest rate in the case of 2 percent growth rate is 2.37 percent, with the gap of 0.37 percentage points. In the case of the 3 percent GDP growth rate, the gap becomes negative (the average interest rate is 2.94 percent). It is notable, however, that even if the economy grows at 3 percent, debt is not sustainable

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[13] We adjust γ_0 so that the average primary surplus is the same as in the benchmark case (-4.5 percent of GDP) for each average growth rate (see footnote 3).

given the current fiscal policy rule because the interest rates increase with the GDP growth rates,.

C. Alternative Fiscal Policy Rules

The above results show that government debt is not sustainable under a wide range of parameters given the current fiscal policy rule. In this subsection, we consider alternative fiscal policy rules. The parameters except for the fiscal policy rule are the same as in the baseline case.

First, we examine whether the government's fiscal reconstruction target is sufficient to restore sustainability. The government (Cabinet Decision, 2010) decided that it aims at turning from the current primary deficit of the central and local governments to a primary surplus by fiscal year 2020. Taking this target into consideration, we assume that the primary balance as a proportion of GDP linearly increases and reaches zero in 2020 and that the average primary balance is zero after 2020.The first row of Table 4A reports the simulation result, showing that the expected debt-to-GDP ratio reaches 5.1 in 100 years and continues to increase afterwards.

A simple way to restore sustainability would be to raise the average primary surplus by raising the value of γ_0 in the rule (26). We find that the primary surplus that is 1.96 percent of GDP on average is enough to stabilize the expected debt-to-GDP ratio at its initial value and, thus, to make debt sustainable[14].

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[14] See Appendix E for the simple calculation concerning how much consumption tax rate will be required if primary surplus of 2 percent of GDP is to be achieved only by the increase by the consumption tax rate.

A more flexible and maybe more interesting way to restore sustainability is to change the fiscal policy rule. As Bohn (1998) states, a rational government should increase the primary surplus when the debt-to-GDP ratio is high. We incorporate his idea into the fiscal rule by assuming that the primary surplus-to-GDP ratio depends on the lagged value debt-to-GDP ratio as well. To be specific, we have

(28) $$s_t = \gamma_0 + \gamma_1 s_{t-1} + \gamma_2 g_t + \gamma_3 d_{t-1}.$$

We use the same parameters for $\gamma_1 (= 0.658)$ and $\gamma_2 (= 0.577)$ as in the baseline case, and set γ_3 arbitrarily.[15] To compare with the benchmark case, we adjust γ_0 so that the average primary surplus is –4.5 percent of GDP as in the baseline case if d_{t-1} is at its value as of 2010.

The second to fourth rows of Table 4A reports the expected debt-to-GDP ratios for three values of γ_3: 0.001, 0.01, and 0.1. We find that a sufficiently large positive response to the debt-to-GDP ratio stabilizes the debt-to-GDP ratio and thus to makes government debt sustainable. If γ_3 is very high ($\gamma_3 = 0.1$), the debt-to-GDP ratio stabilizes almost at 2. Table 4B reports the expected primary surplus-to-GDP ratio, showing that as γ_3 increases, the average primary surplus-to-GDP ratio is initially larger but eventually becomes smaller. In the case of $\gamma_3 = 0.1$, the average primary surplus is about 1.8 percent of GDP.

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[15] We estimated (28) and found that the coefficient on d_{t-1} was positive but not significant. Figure 5 illustrates no significant positive relationship between s_t and d_{t-1}, though it does not control for the GDP growth rate.

Discussion; Consumption Tax Rate Required to Restore Sustainability

If fiscal sustainability is to be restored only by raising consumption tax rate, how much increase in tax rate is needed?

Eq. (27) implies that the long-run primary deficit ($\frac{\gamma_0+\gamma_1 g}{1-\gamma_2}$) is 4.5percent of GDP if the average GDP growth rate is 1 percent ($g=0.01$). We can regard this long-run deficit as structural (i.e., cyclicality- and momentum- adjusted) one. On the other hand, to achieve fiscal sustainability under the 1 percent average GDP growth rate, the primary surplus must be 2.0 percent of GDP. The difference between the structural and targeted primary balance (6.5 percentage points) is a necessary primary surplus to be raised either by a decrease in expenditures or an increase in revenues.

The consumption tax revenue as a proportion of nominal GDP is 2.0 percent on average during the period of 1997-2009, when the tax rate was 5 percent. If we do not take into consideration possible effects of the consumption tax on consumption and economic growth, we can suppose that raising consumption tax by one percentage point contributes to a 0.4 percentage increase in primary surplus-to-GDP ratio. Under this assumption, we can mechanically compute that consumption tax rate must be increased by 16 percentage points, i.e., from 5 to 21 percent, to raise primary surplus-to-GDP ratio by 6.5 percentage points. Note that if we do not take into consideration negative effects of consumption tax rates on consumption or increases in social security expenditures associated with population aging. If we do so, consumption tax rates required will be higher than 21 percent.

Appendix A: Proof of Proposition

Suppose that $E_t \lim_{s\to\infty} \frac{D_{t+s}}{Y_{t+s}} = \infty$. Then, for some realizations of the states, $\lim_{s\to\infty} \frac{D_{t+s}}{Y_{t+s}} = \infty$. Below we limit our attention to those exploding paths. The growing debt-to-GDP ratio should imply that the public debt will crowd out private lending and at some date (denoted by T), the credit market will disappear. Agents start financing investment only by their net worth. The NPG condition of agents as of time T is then given by

(NPG4) $$E_T \lim_{s\to\infty} \frac{D_{T+s}}{(1+R_{T+1})(1+R_{T+2})...(1+R_{T+s})...} = 0 .$$

Let g_t^D denote the growth rate of debt. Then, (NPG4) implies that $E_T \lim_{s\to\infty}\left(\frac{1+g_{T+s}^D}{1+R_{T+s}}\right) < 1$, which, in turn, implies that $\lim_{s\to\infty}(R_{T+s} - g_{T+s}^D) > \delta$ for some $\delta > 0$ for some realizations of the states. In addition, given that x_t is bounded above, R_t is bounded above (see Appendix B for the relationship between x_t and R_t) and so is g_t^D. Let $\bar{g}^D$ denote the upper bound of g_t^D.

We can rewrite (FC) as

(FC') $$\frac{T_t}{Y_t} = \frac{R_t - g_t^D}{1+g_t^D}\frac{D_t}{Y_t} + z < \tau$$

for all period t and all the realizations of the states. For $R_t > g_t^D$, (FC') can also be rewritten as $\frac{D_t}{Y_t} < \frac{1+g_t^D}{R_t - g_t^D}(\tau - z)$. The RHS is bounded by $\frac{1+\bar{g}^D}{\delta}(\tau - z)$, which contradicts that $\frac{D_{t+s}}{Y_{t+s}}$ grows unboundedly. Q.E.D.

Appendix B. General Solution for Growth and Interest Rates

We derive the equilibrium growth and interest rates. We first solve for $\{g_1, g_2,,, g_n\}$. Letting G_α denote the $n \times 1$ vector with its j th element of $(1+g_j)^{-\alpha}$, Φ denote the $n \times n$ matrix with the (i, j) element of $\phi_{i,j}$, $\tilde{X}$ denote the $n \times n$ diagonal matrix with the (j, j) element of $1+x_j$, and I denote the $n \times 1$ unit vector, we can rewrite (16) as $I = \beta(1+\kappa)^{-\frac{1}{2}} \Phi \tilde{X} G_\alpha$, which leads to

(A1) $$G_\alpha = \beta^{-1}(1+\kappa)^{\frac{1}{2}} \tilde{X}^{-1} \Phi^{-1} I \cdot$$

Next we solve for $\{r_1, r_2,,, r_n\}$. Letting P denote the $n \times 1$ vector with its jth element of $(1+r_j)$ and $\tilde{G}_\alpha$ denote the $n \times n$ diagonal matrix with the (j, j) element of $(1+g_j)^{-\alpha}$, we can rewrite (17) as $I = \beta\theta^\alpha \Phi \tilde{G}_\alpha P$, which leads to

(A2) $$P = \beta^{-1}\theta^{-\alpha} \tilde{G}_\alpha^{-1} \Phi^{-1} I = \beta^{-1}(1+\kappa)^{-\frac{\uparrow}{\twoheadrightarrow}} \tilde{G}_\alpha^{-1} \Phi^{-1} I ,$$

where the second equality comes from (14). Letting X denote the $n \times 1$ vector with its j th element of $1+x_j$, we can rewrite (16) as $I = \beta(1+\kappa)^{-\frac{1}{2}} \Phi \tilde{G}_\alpha X$. Substituting this into (A2), we obtain

(A3) $$P = (1+\kappa)^{-1} X \ .$$

Finally, we compute the steady-state values of g_t and r_t. Let the vector λ denote the stationary distribution of Φ. Then the steady state values of g_t and r_t are obtained by $g = \sum_{j=1}^{n} g_j \lambda_j$ and $r = \sum_{j=1}^{n} r_j \lambda_j$, respectively, where λ_j is the j th element of λ.

Appendix C. Data

1. Primary balance is obtained from the current and capital accounts of the system of national accounts (*93SNA*, Economic and Social Research Institute) of the general government as follows:

   Primary balance = (Taxes on products and imports + Current taxes on income and wealth + Social burdens + Other current transfers received + Fixed capital depreciation + Capital transfer received) **–** (Subsidy + Social benefit except for social transfers in kind + Other current transfers paid + Final consumption + Gross fixed capital formation + Increases in inventories + Net purchase of land + Capital transfers paid).

   The primary balance data based on *93 SNA* is available only up to 2008. To estimate the primary balance in 2009 and 2010, we used the government's estimate that the primary balances of the central and local governments are -8.1 percent and -6.4 percent of nominal GDP in fiscal years 2009 and 2010, respectively (Cabinet Office, *Midterm Economic and Fiscal Forecasts*, June 22, 2010). To convert the government's estimate to the primary balances of the general government (i.e., the total of the central government, local governments, and the social security funds), we assumed that the primary balance of the social security funds as a proportion of nominal GDP as of fiscal year 2008 (-1.1 percent of nominal GDP) did not change constant up to 2010.

2. Nominal and real GDP are based on *93 SNA*, which is obtained from the website of Economic and Social Research Institute.

Nominal and real GDP data are available up to 2009. For the real GDP growth rate in 2010, we used the government's estimate for fiscal year 2010 (2.6 percent, Cabinet Office, 2010).

3. Interest rate margin is the difference between the deposit rate and the lending rate, both of which are obtained from IMF's *International Financial Statistics*.
4. Real yield on financial bills and government bonds are nominal yields on each asset minus the change in GDP deflator. Those data are obtained from IMF's *International Financial Statistics*.
5. Government debt is total debt minus financial bills outstanding. Data source of government debt is *Flow of Funds*, obtained from the website of Bank of Japan.

Appendix D: Simulation Procedure

1. Construction of the initial value of d

In the benchmark case and the other cases except for *Fiscal Management Strategy* case, we conduct the stochastic simulation from year 2011. To do so, we need to construct the initial value of d_{2010}. To do so, we first construct R_{2010} by assuming perfect foresight and substituting g_{2010} into the deterministic version of (14),

$$\text{(A1)} \qquad \log(1+R_{2010}) = \alpha \log(1+g_{2010}) - \frac{1}{2}\log(1+\kappa) - \log\beta \ .$$

Then, we substitute R_{2010}, g_{2010}, d_{2009} and s_{2010} into (22) to get d_{2010} (=1.860).

In the case of *Fiscal Management Strategy*, we conduct the stochastic simulation from year 2021. For year 2010, we follow the same procedure as the benchmark case above. For the period from 2011 to 2020, by assuming that g_t =0.02 (constant) and that s_t linearly increases from s_{2000} (=-0.075) to $s_{2020}=0$, we construct d_{2020} using the similar formulae of (A1) recursively.

2. Stochastic Simulation

Step 1. We generate a series of x_{t+1} for 1000 periods starting from the initial value drawn from the stationary distribution of x_t .

Step 2. Given a series of x_{t+1}, we obtain g_{t+1} and R_{t+1} from (A2) and (A5).

Step 3. Given a series of g_{t+1}, we construct s_{t+1} recursively from the fiscal policy rule (26) with a starting value of s_{2000}.

Step 4. We construct d_{t+1} by substituting R_{t+1}, g_{t+1}, and s_{t+1} into (22) with a starting value of d_t as of year 2010.

Step 5. We repeat Steps 1–4 N times to obtain the distribution of d_{t+1}. Indexing each series by i, the expected value of d_t and the probabilities that d_t exceeds its critical values $\bar{d}$ are computed as $E[d_t] = \frac{1}{N}\sum_{i=1}^{N} d_{i,t}$ and $\Pr ob\left[d_t \geq \bar{d}\right] = \frac{1}{N}\sum_{i=1}^{N} I\left(d_{i,t} \geq \bar{d}\right)$, where $\bar{d} = 1.792$, $N = 10000$, and $I\left(d_{i,t} \geq \bar{d}\right) = \begin{cases} 1 & if \quad d_{i,t} \geq \bar{d} \\ 0 & otherwise \end{cases}$.

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Table 1. Paramters

Preference		
β	discount factor	0.980
α	inverse of elasticity of intertemporal substitution	0.565
Technology		
x^a	average return to capital	0.033
ρ	serial correlation of return to capital	0.780
σ	standard deviation of error term in return to capital	0.011
Financial Intermediation		
κ	financial intermediation cost	0.015
Fiscal Policy Rule		
γ_0	Constant	−0.021
γ_1	coefficient on previous−year primary surplus/GDP	0.658
γ_2	coefficient on GDP growth rate	0.577

Table 2. Expected debt-to-GDP ratio under alternative parameters

After	20 years	50 years	100 years	500 years	1000 years
baseline	3.43	5.57	11.54	572.74	48260.43
	(94.7%)	(98.1%)	(99.7%)	(100.0%)	(100.0%)
no intermediation cost	3.55	5.92	13.09	1175.99	228257.75
	(95.5%)	(98.4%)	(99.8%)	(100.0%)	(100.0%)
log utility	3.67	6.28	14.68	2535.90	1213105.53
	(99.8%)	(100.0%)	(100.0%)	(100.0%)	(100.0%)

1. Numbers in the parentheses are the probabilities that the debt-to-GDP ratio exceeds its value as of year 2009 (1.762)

Table 3. Expected debt-to-GDP ratio under alternative growth rates

After	20 years	50 years	100 years	500 years	000 years
GDP Growth=1% (baseline)	3.43	5.57	11.54	572.74	48260.43
	(94.7%)	(98.1%)	(99.7%)	(100.0%)	(100.0%)
GDP Growth=2%	3.18	4.78	8.60	105.97	1124.80
	(92.4%)	(96.5%)	(99.2%)	(100.0%)	(100.0%)
GDP Growth=3%	2.95	4.14	6.51	27.41	57.28
	(89.1%)	(93.5%)	(97.8%)	(100.0%)	(100.0%)

1. Numbers in the parentheses are the probabilities that the debt-to-GDP ratio exceeds its value as of year 2009 (1.762)

Table 4. Alternative fiscal policy rules

A. Expected debt-to-GDP ratio

After	20 years	50 years	100 years	500 years	1000 years
Average primary surplus/GDP=0	2.54	3.24	5.10	186.89	15579.56
	(87.7%)	(81.1%)	(81.7%)	(89.2%)	(89.8%)
Bohn's rule γ_3 = 0.001	3.39	5.29	10.13	196.29	4148.03
	(94.7%)	(97.9%)	(99.7%)	(100.0%)	(100.0%)
Bohn's rule γ_3 = 0.01	3.07	3.85	4.58	4.96	4.94
	(94.5%)	(97.5%)	(98.9%)	(99.3%)	(99.4%)
Bohn's rule γ_3 = 0.1	2.08	2.09	2.08	2.09	2.09
	(88.9%)	(89.4%)	(89.0%)	(89.2%)	(88.8%)

1. Numbers in the parentheses are the probabilities that the debt-to-GDP ratio exceeds its value as of year 2009 (1.762)

B. Expected Primary Surplus-to-GDP Ratio

After	20 years	50 years	100 years	500 years	1000 years
Bohn's rule γ_3 = 0.001	−3.99%	−3.43%	−2.11%	51.44%	1187.22%
Bohn's rule γ_3 = 0.01	−1.30%	1.14%	3.34%	4.54%	4.65%
Bohn's rule γ_3 = 0.1	1.78%	1.98%	1.87%	1.88%	1.83%

Figure 1

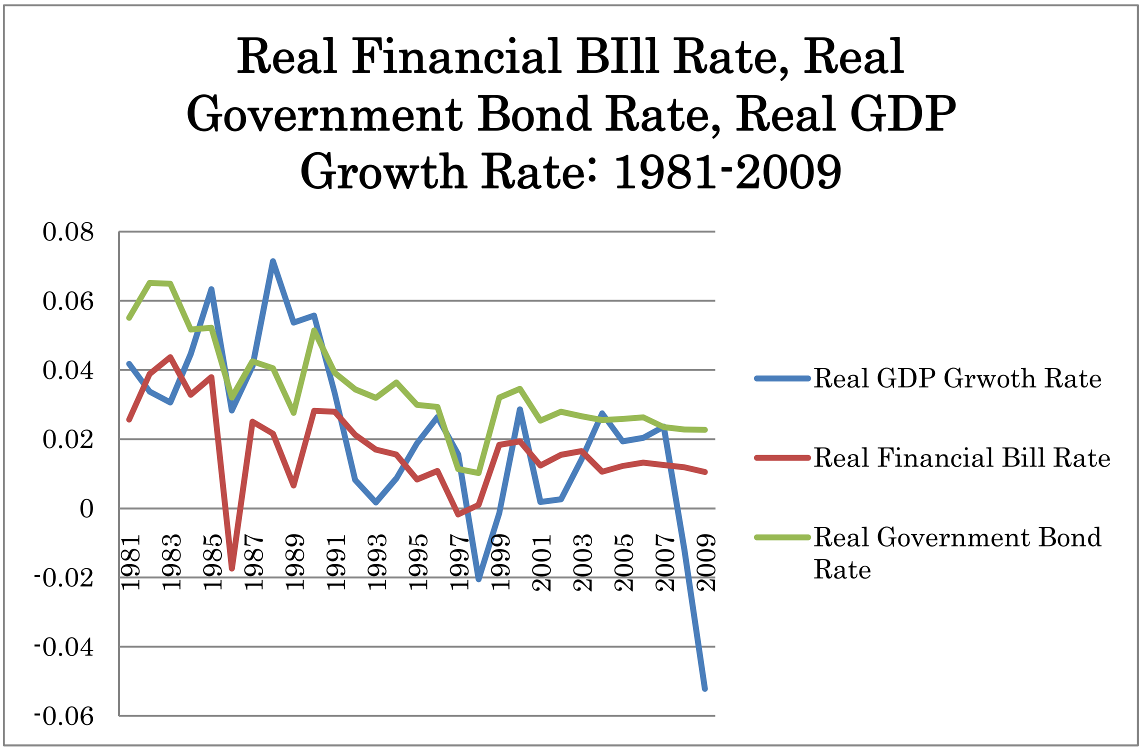

Real Financial BIll Rate, Real Government Bond Rate, Real GDP Growth Rate: 1981-2009
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
1981
1983
1985
1987
1989
1991
1993
1995
1997
1999
2001
2003
2005
2007
2009
Real GDP Grwoth Rate
Real Financial Bill Rate
Real Government Bond Rate

Figure 2 A.　Γ↫ 0.4<0.5

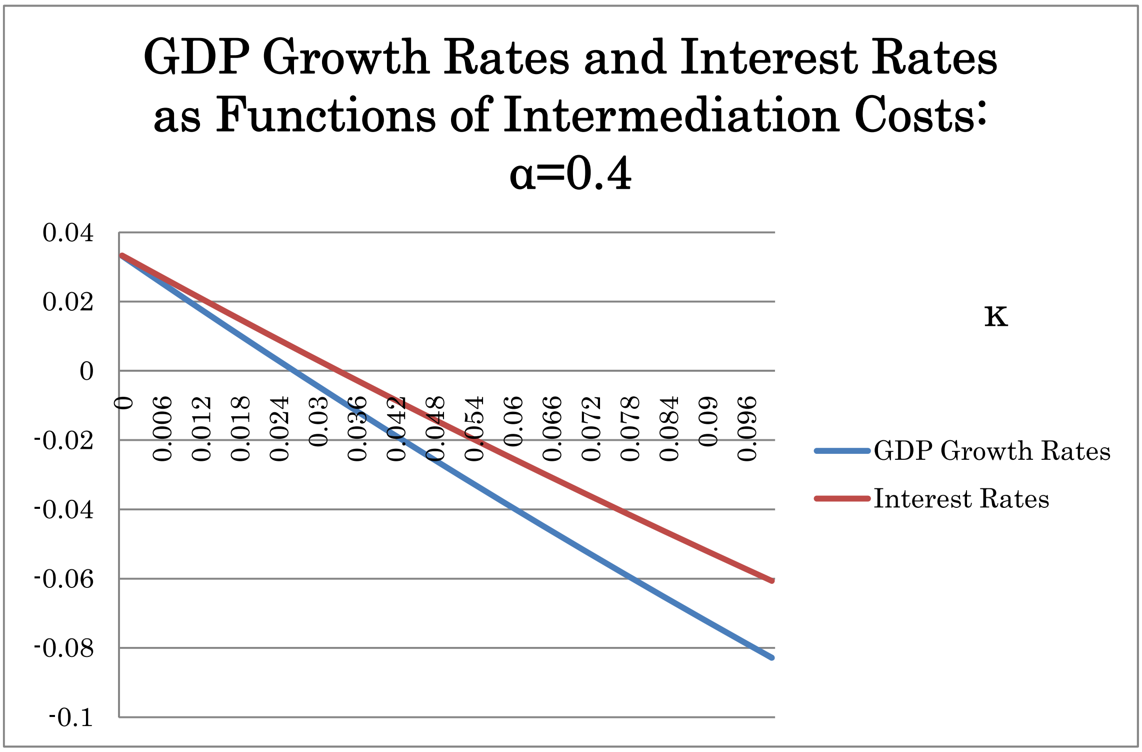


Figure 2B.　Γ=1>0.5

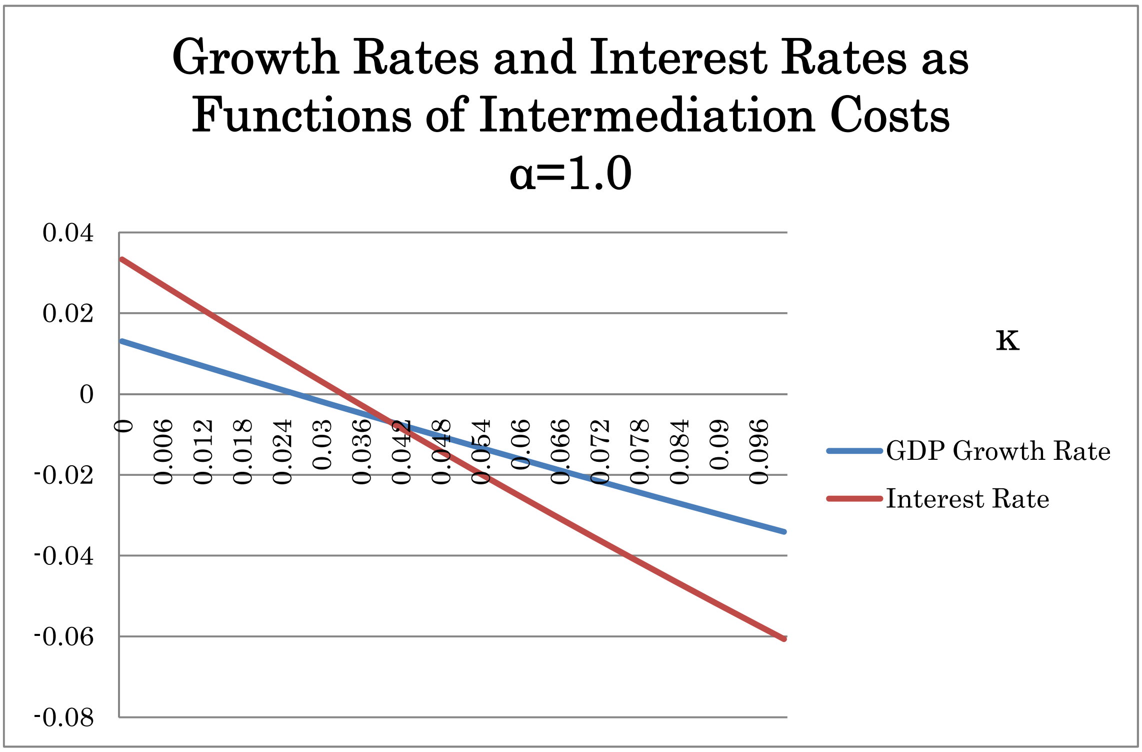


Figure 4

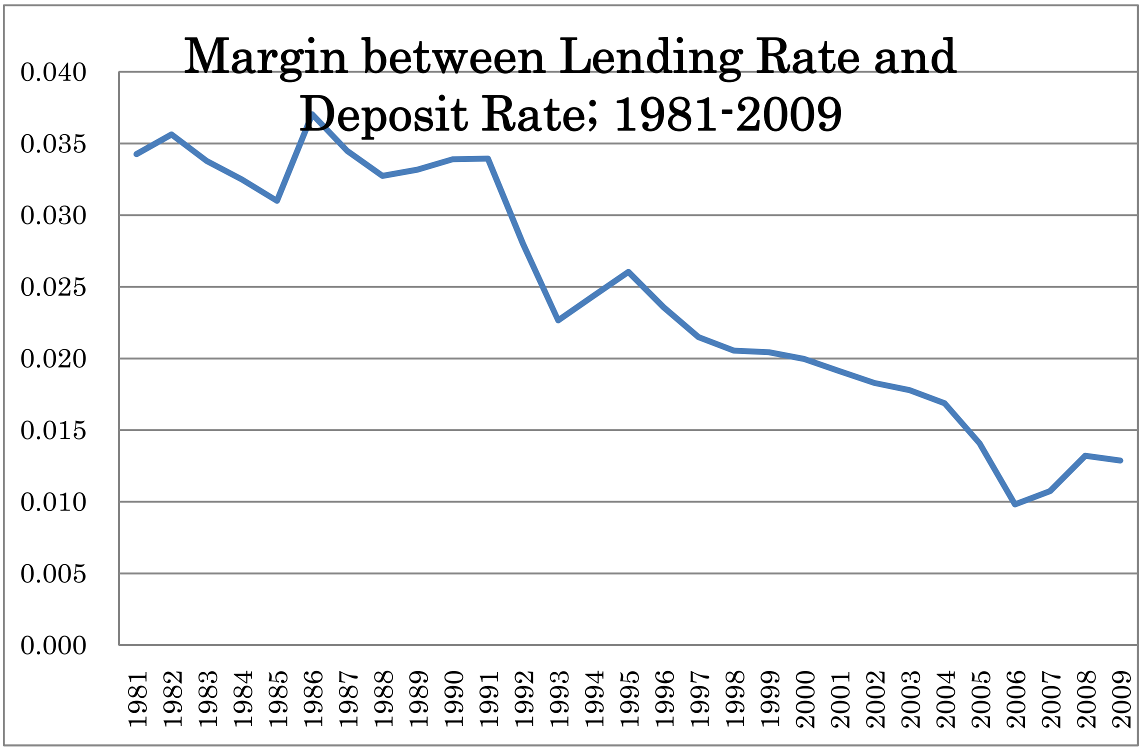

Margin between Lending Rate and Deposit Rate; 1981-2009
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009

Figure 5

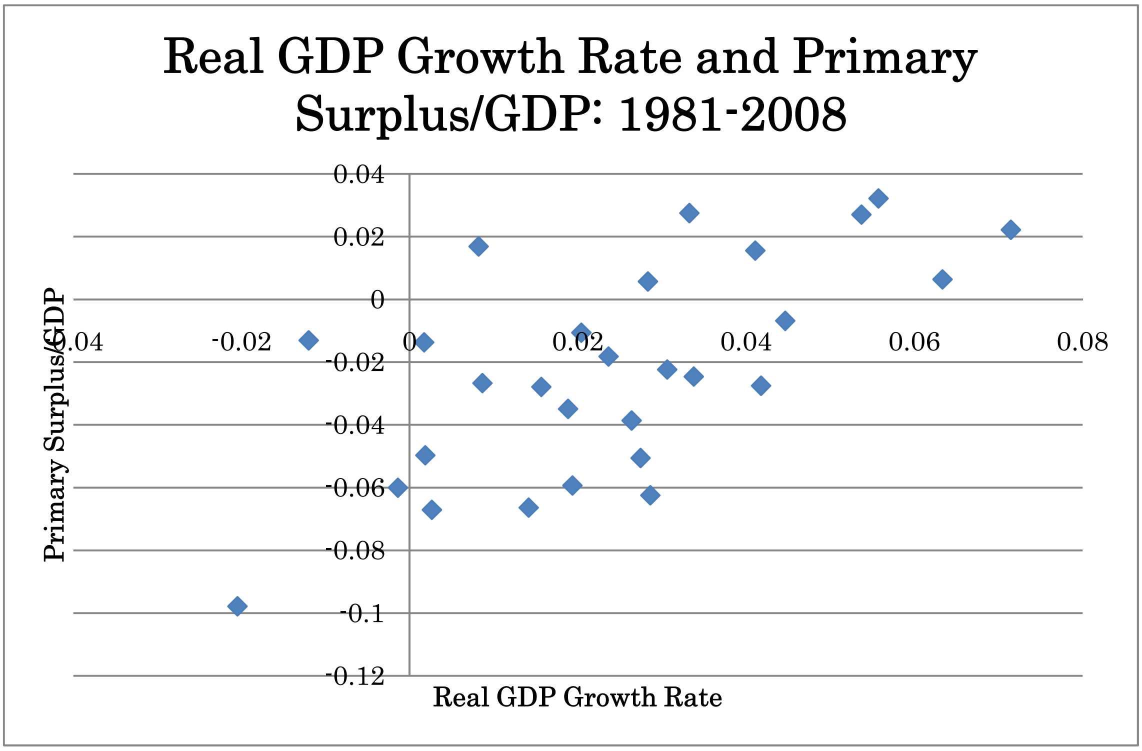
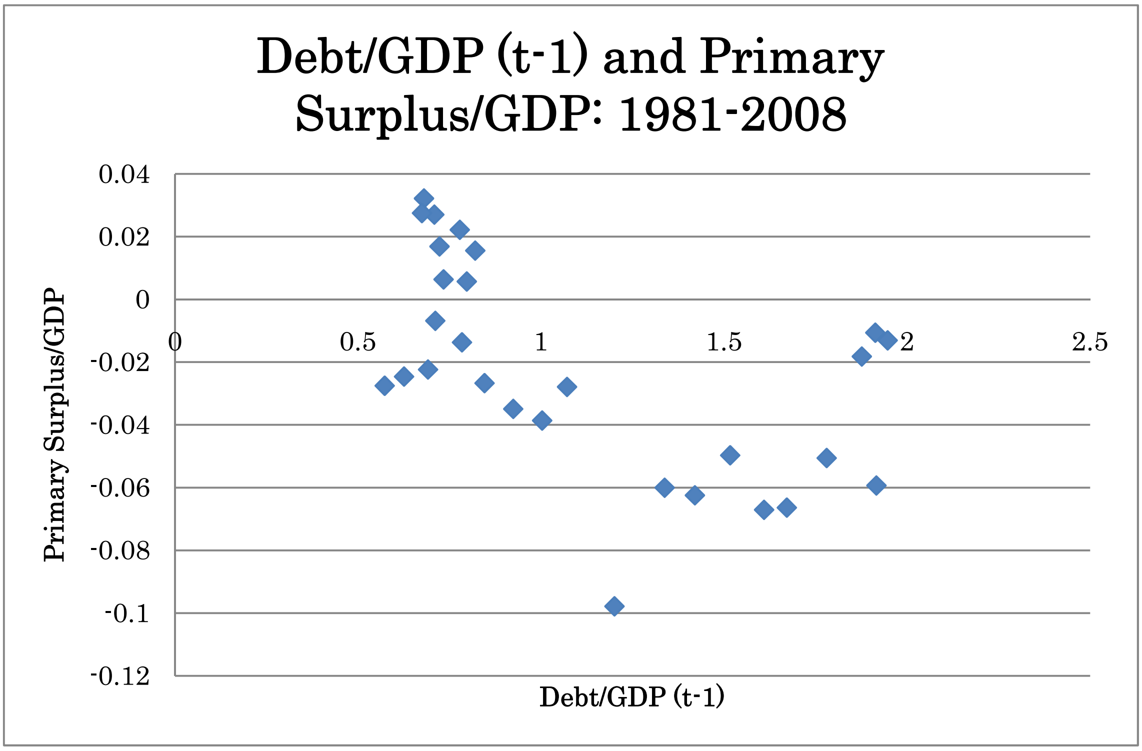

Real GDP Growth Rate and Primary Surplus/GDP: 1981-2008
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
-0.04
-0.02
0
0.02
0.04
0.06
0.08
Primary Surplus/GDP
Real GDP Growth Rate

Figure 6



Debt/GDP (t-1) and Primary Surplus/GDP: 1981-2008
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
0
0.5
1
1.5
2
2.5
Primary Surplus/GDP
Debt/GDP (t-1)