

One-Shot Pricing for Hands-Off-the-Wheel Advertising Markets

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Hands-off-the-Wheel (HOTW) ad campaigns

The image shows two side-by-side screenshots of advertising campaign creation interfaces. The left interface is for Google Ads, and the right is for Meta Ads Manager. Both interfaces have several fields highlighted with colored boxes to indicate specific settings.

Google Ads Interface (Left):

- Budget:** A text box labeled "Set your average daily budget for this campaign" with a dropdown menu set to "US Dollar (USD \$)" and a value of "\$100.00" highlighted with a purple box.
- Bidding:** A section with "Select your bid strategy" and "Pay for". The bid strategy dropdown is set to "Target CPA", and the "Pay for" dropdown is set to "Interactions".
- Target CPA:** A text box labeled "Target CPA" with a value of "\$1.00" highlighted with a green box.
- Start and end dates:** Two dropdown menus for "Start date" and "End date", both set to "Jan 23, 2023" and "Jan 25, 2023" respectively.

Meta Ads Manager Interface (Right):

- Budget:** A section labeled "Budget" with a dropdown menu set to "Daily Budget" and a value of "\$100.00" highlighted with a purple box. The currency is set to "USD".
- Schedule:** A section labeled "Schedule" with "Start date" and "Time" dropdowns. The start date is set to "Jan 23, 2023" and the time is "12:00 AM". The time zone is set to "Eastern Time".
- Optimization for ad delivery:** A section labeled "Optimization for ad delivery" with a dropdown menu set to "Landing Page Views".
- Cost per result goal:** A text box labeled "Cost per result goal" with a value of "\$1.00" highlighted with a green box.
- Meta's goal statement:** A red-bordered box containing the text: "Meta will aim to get the most landing page views and try to keep the average cost around \$1.00. Some results may cost more and some may cost less."

Figure 1 Interfaces on Google Ads (left) and Meta Ads Manager (right) for creating advertising campaigns that allow 1

¹Picture source: Deng, Golrazaei (2023);

²Balseiro, Deng, et al. (2024)

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Meta Ads Manager Interface (Right):

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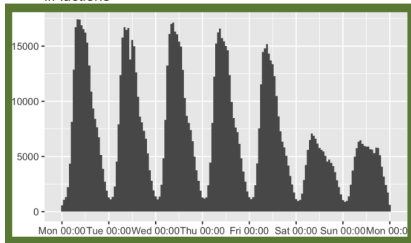
- Advertiser provides two numbers: **budget** b_i and **ROI target** r_i (tCPA).
- The exchange does all other tasks: "autobid" in auctions, prediction of values, pacing
- Most ad revenue at large exchanges now comes from HOTW products.²

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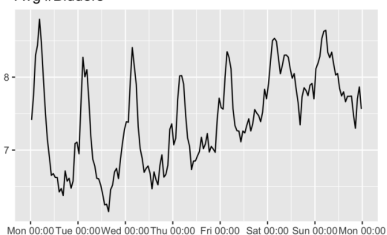
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How does HOTW perform?

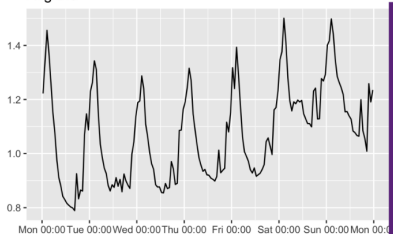
#Auctions



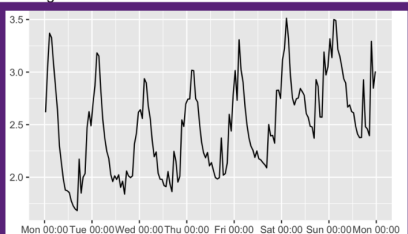
Avg #Bidders



Avg Bid

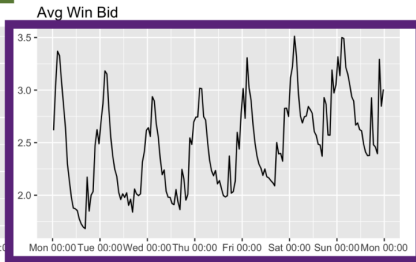
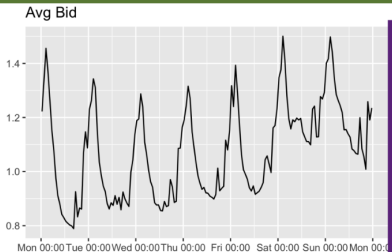
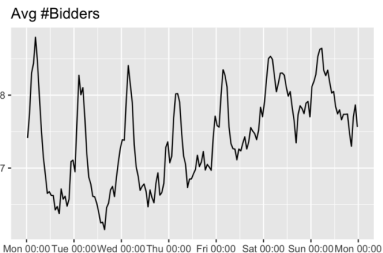
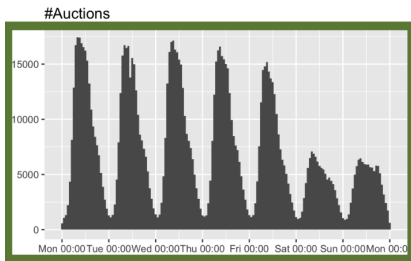


Avg Win Bid



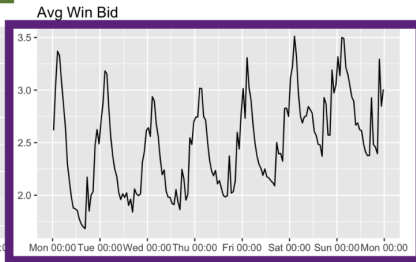
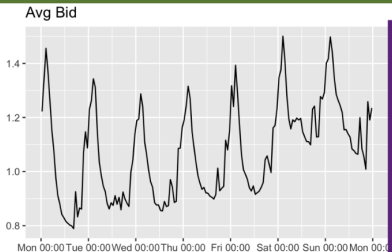
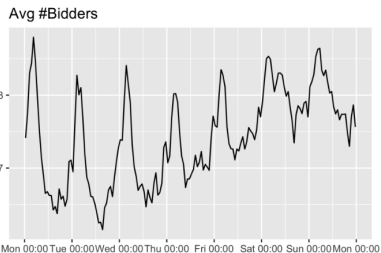
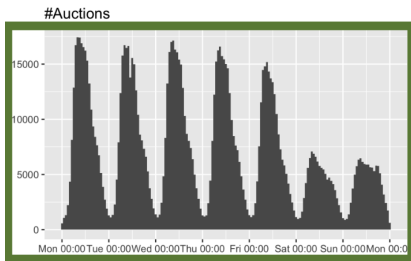
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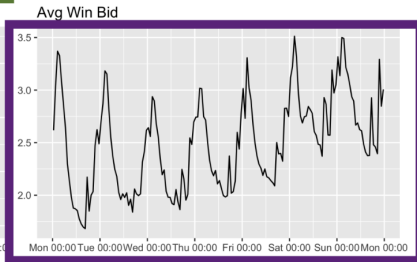
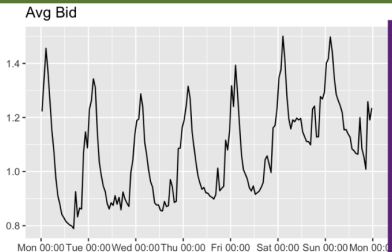
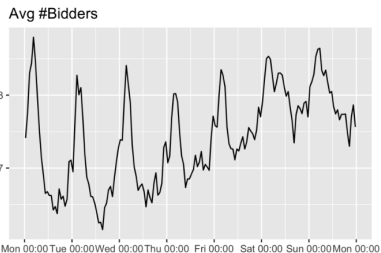
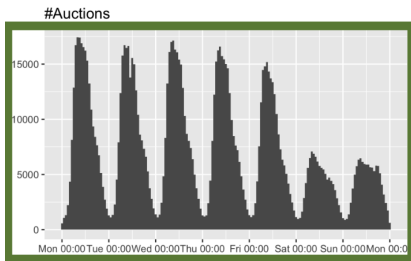
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similarly cyclical:
10-20% price diff within short periods.

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- Suggests arbitrage opportunities?
- Could budgets be better optimized?

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- Suggests arbitrage opportunities?
- Could budgets be better optimized?
- We propose a solution: **"one-shot" pricing**

Current HOTW implementation

- The market finds prices in sequence, by running millions of auctions every day.
- Bids are "paced" (shaded down from values), in order to satisfy budgets.³
- Previously, auction bidding elicited private information, to match advertisers' private values to positions on search pages.
- With today's HOTW campaigns, most information established at auction run-time:
 - Advertisers' WTPs, Winning rates (CVR, CPA)
 - Advertisers provide their budgets, ROI constraints⁴
 - Auctions essentially (dynamic) ranking system (like search).

³Kobayashi (2023).

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- Dynamic auctions introduce latency, noise.
 - Pricing dynamics (pacing) without price discovery.
 - Also questionable performance (above): arbitrage opportunities?

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- Dynamic auctions introduce latency, noise.
 - Pricing dynamics (pacing) without price discovery.
 - Also questionable performance (above): arbitrage opportunities?
- *What if we eliminate auctions?* Consider **one-shot approach**, where **exchange calculates prices and allocations in advance**

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One-shot approach

One convex program (Eisenberg–Gale) gives prices and allocations. We solve the program one time, ex-ante

- 1 The prices obey all budget limits and all ROI limits.
- 2 The prices give the maximum revenue of all uniform-price systems.

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- ③ The result is the same as paced first-price auctions using same inputs, but avoid pacing dynamics.
 - Dynamic paced auctions using same inputs provide 90-100% of platform value, 87-96% of advertiser value.

Main results

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 - Dynamic paced auctions using same inputs provide 90-100% of platform value, 87-96% of advertiser value.
- ④ Arbitrage-free: Advertisers' ROI equated across duration of ad campaign.

Setup: the HOTW market

- Advertisers $i \in N = \{1, \dots, n\}$.
- Click types $j \in M = \{1, \dots, m\}$
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 - Maintained in many papers; no diminishing marginal value from ads served to same impression types.
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- $s_j > 0$: supply of type- j clicks (a forecast)

The marketplace knows all these quantities:

- Bidders give b_i and r_i .
- The marketplace learns/predicts other quantities accurately.

Exchange's one-shot Optimization

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- Allocate x_{ij} clicks to advertiser i at uniform price α_j .
 - Restrict to uniform pricing: transparent and explainable
- **Objective:** maximize ad revenue $\sum_i p_i$, where $p_i = \sum_j x_{ij} \alpha_j$ is advertiser i 's total payment.
Subject to constraints:

$$\text{Supply: } \sum_i x_{ij} \leq s_j \quad \forall j$$

$$\text{ROI: } r_i \leq \frac{1}{p_i} \sum_j v_{ij} x_{ij} \quad \forall i$$

$$\text{Budget: } p_i \leq b_i \quad \forall i$$

Incorporating ROI constraints

- The constraint (slide 7) is on the *portfolio*: $\text{ROI}_i \geq r_i$.
- Model the threshold r_i as the return on the advertiser's next-best channel (“outside option”):
 - a unit of *out* costs \$1 and provides $v_{i,\text{out}} = r_i$, so $u_{i,\text{out}} = 1$.

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- Given linear utility and uniform pricing, *out* beats any type with $v_{ij}/\alpha_j < r_i$, so the *reservation price per click* is

$$\alpha_j \leq v_{ij}/r_i \equiv u_{ij} = \underbrace{c_{ij}}_{\text{pCVR}} \times \underbrace{(g_i/r_i)}_{\text{tCPA}_i}$$

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- Demand is **all or nothing**: buy j iff $\alpha_j \leq u_{ij}$; unspent budget goes to *out*. Hence step-like demand.
- The exchange is a multi-product monopolist with supply constraints.

Market equilibrium

This is a "quasi-Fisher market":

- $i = 1, \dots, N$ buyers (bidders) with budgets b_i
- Can spend budgets on $|\mathcal{J}| + 1$ goods (inside impressions + outside options) "quasi"

Define: Competitive equilibrium in Quasi-Fisher Market

Prices $\alpha_j \geq 0$, allocations $x_{ij} \geq 0$, and outside-option quantities $z_i \geq 0$ have these two properties:

- 1 Advertiser optimization: (x_i, z_i) solves $\max \sum_j u_{ij}x_{ij} + r_i z_i$ s.t. $\sum_j \alpha_j x_{ij} + r_i z_i \leq b_i$.
- 2 Market clears: $\sum_i x_{ij} = s_j$ for all j with $\alpha_j > 0$.

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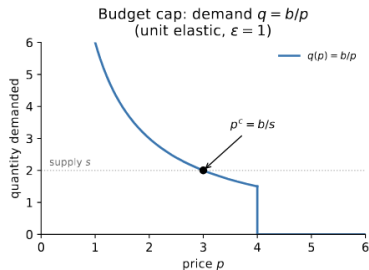
Although platform is monopolist, **her optimum is at the competitive (market-clearing) price (!)**

Corollary 7: Revenue Optimality Among Uniform Prices (Finster, Ginsberg, Lock)

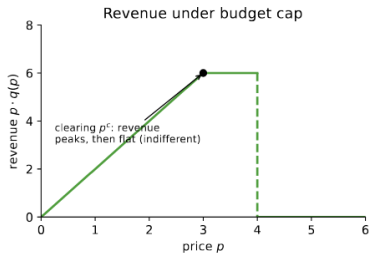
The equilibrium prices α^* maximize exchange revenue $\sum_{i \in N} p_i$

- among *all* uniform per-type price vectors, not only market-clearing ones.

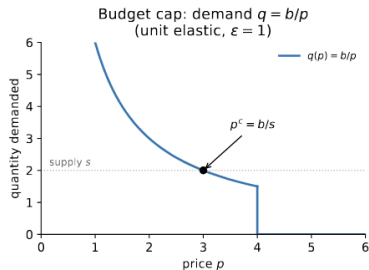
Why market clearing is revenue-optimal (intuition)



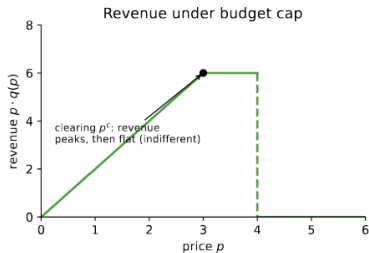
- With budget-constrained demand, revenue extraction from bidders limited.
 - Pic shows demand $q = b/p$
- Start at MC price (dot). An increase in price cannot increase revenue and leaves units unsold.



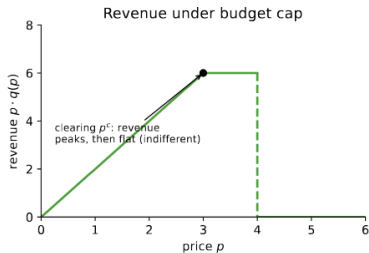
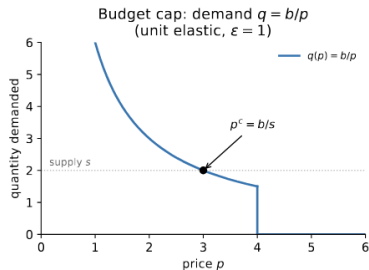
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- Start at MC price (dot). An increase in price cannot increase revenue and leaves units unsold.
 - Elasticity $\varepsilon = 1$ uniformly, or $MR \leq 0$
- Warning: this result needs linear values. It is not correct for values that decrease at the margin.^a

^aFinster, Goldberg, Lock (2025).

Computing equilibrium: augmented EG program

"Eisenberg-Gale Theorem"

The CE allocation and prices of a (quasi) Fisher market are characterized in the EG program^a

$$\max_{x,z} \sum_{i \in N} \left[b_i \log \left(\sum_j u_{ij} x_{ij} + r_i z_i \right) - r_i z_i \right] \quad \text{s.t.} \quad \sum_i x_{ij} \leq s_j \quad \forall j$$

^aJain and Vazirani give the theorem for Fisher markets and provide efficient algorithms.

- Primal solution x^* is equilibrium allocation. Dual multipliers provide equilibrium prices α^* .
- The size of the problem depends on $n + m$. It does not depend on the number of impressions.
 - Optimization in *convex*: Number of algorithms available.

Characterization of Optimal Prices/Allocation

The first-order (KKT) conditions of the EG program characterize the optimal prices/allocation.

- (a) Budget identity: $\sum_j \alpha_j^* x_{ij}^* + r_i z_i^* = b_i \quad \forall i$. Each budget divides between impressions and the outside good.
 - (b) ROI for each click: $x_{ij}^* > 0 \Rightarrow v_{ij}/\alpha_j^* \geq r_i$. Types that we allocate obey the ROI rule.
 - (c) **Budget-binding bidders** ($z_i^* = 0$) earn higher ROI than their target r_i .
 - (d) **ROI-binding bidders** ($z_i^* > 0$) divide their budget between inside and outside goods, and $v_{ij}/\alpha_j^* = r_i$.
-
- **Existence (Cor. 5)**. An equilibrium with ROI conditions exists.
 - **Unique prices (Cor. 6)**. The prices α^* are unique, if each type has value to at least one advertiser.

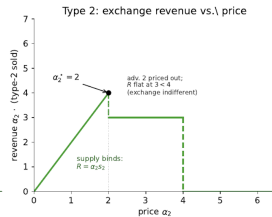
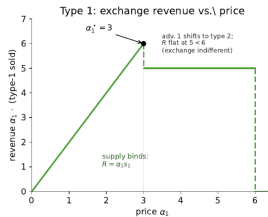
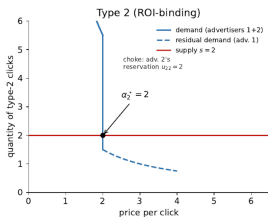
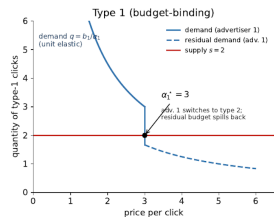
Illustrative Example: two advertisers, two click types

	v_{i1}	v_{i2}	r_i	b_i	
Advertiser 1	12	8	2	\$9	$s_1 = s_2 = 2$
Advertiser 2	2	4	2	\$2	

- Values divided by r_i : $u_{11} = 6$, $u_{12} = 4$, $u_{21} = 1$, $u_{22} = 2$
- Equilibrium: $\alpha^* = (\$3, \$2)$; $x_{11}^* = 2$, $x_{12}^* = 1.5$, $x_{22}^* = 0.5$, $z_2^* = 0.5$
Advertiser 1 buys both types; Advertiser 2 only type 2.
- Advertiser 1 is budget-binding ($\text{ROI} = (24 + 12)/(6 + 3) = 4 > 2$). No outside spend.
Advertiser 2 is ROI-binding ($\text{ROI} = 2$, \$1 goes to the outside good).
- Revenue of the exchange: $\alpha_1^* s_1 + \alpha_2^* s_2 = \$10$

Example: demand and revenue

Demand and revenue illustration around equilibrium point show the same "fixed budget" features as previously:



Left: demand is equal to supply on the flat segment. Right: revenue from one price is maximum at α_j^* . At a higher price, the marginal advertiser exits.

Additional results: equivalence to paced first-price auctions

At HOTW equilibrium, define the following auxiliary quantity:

$$\mu_i = \frac{b_i}{W_i^*}, \quad W_i^* \equiv \sum_j u_{ij} x_{ij}^* + r_i z_i^*.$$

- For ROI-binding bidders: $\mu_i = 1$. Budget-binding: $\mu_i < 1$.

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- For ROI-binding bidders: $\mu_i = 1$. Budget-binding: $\mu_i < 1$.
- μ_i is an *equilibrium* number, unknown ex-ante.

In dynamic paced first-price auctions, pacing multipliers corresponding to μ_i control bids

- Auto-marks down bids from values to titrate budget spend across time.
- μ_i is learned and adjusted over time. Learning is slow and has errors. (See data later.)

Proposition 10 (Equivalence)

CE allocation and prices in HOTW Market coincide with the outcome of paced first-price auctions with multipliers $\mu_i = b_i/W_i^*$, where $\mu_i u_{ij} = \alpha_j^*$ for all j with $x_{ij}^* > 0$. (Conitzer et al., 2022a, 2022b)

Time-varying supply: no-arbitrage across periods

Return to motivating picture from head of talk.

- In practice, campaigns run over days or weeks: b_i, r_i are "low frequency"
- But traffic and values (CTR) change predictably by hour: s_i, v_{ij} are "high frequency".

Time-varying supply: no-arbitrage across periods

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- Straightforward to extend HOTW market to this setting:
 - A good is a pair (click type, period): we add a time index t .
 - Allow budget to be fungible between periods, spent endogenously over time.

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 - A good is a pair (click type, period): we add a time index t .
 - Allow budget to be fungible between periods, spent endogenously over time.
- (In contrast, paced auctions typically divide up budget uniformly over campaign duration. Arguably artificially creates price cycles.)

No-arbitrage (Prop. 12)

In the one-shot equilibrium, each advertiser gets the same ROI in every active period and click type:

$$v_{ijt}/\alpha_{jt}^* = r_i \cdot W_i^*/b_i \geq r_i, \quad \text{for all active } (j, t)$$

No gains from moving budget over time. Prices α_{jt}^* can be different.

Proof of concept: real auction data

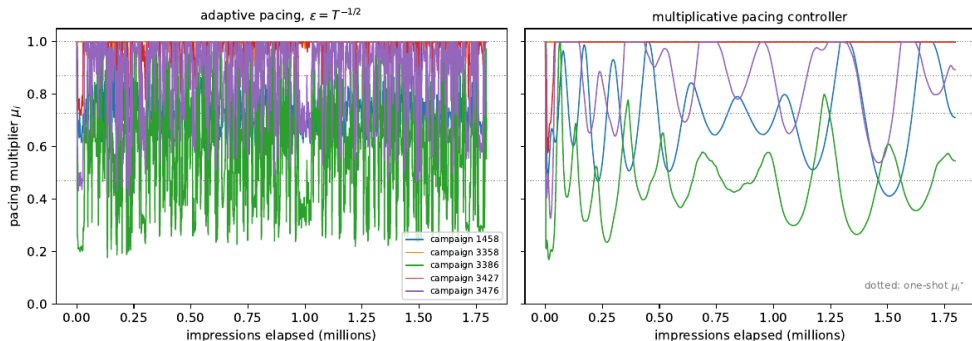
Next we consider some simulations using real digital ad market data: iPinYou RTB logs, 6 June 2013.⁵

- 5 advertisers; 1.82M impressions; 1,162 clicks; $m = 2,471$ click types (exchange $\times$ region $\times$ user-tag profile); $s_j \in [50, \approx 23,000]$. Many types have small supply.
- These are not HOTW campaigns; some data guesswork needed to apply here.
- Solve EG problem (dual) using proportional response.⁶ Time: 0.1 seconds for 5 bidders; less than 1 minute for $n = 1,000$.
- **Optimality (Cor. 7), a strong test:** Confirm platform revenue optimized at CE prices (vv. $\approx 32,000$ other uniform price vectors).
- **Robustness to supply uncertainty:** use prices from 6/6 with supply of 6/7 (mean error across types = 22.1%)
 - Revenue is 98.1% of the revenue with perfect information.
 - Resolving equilibrium during the day creates churn: revenue +9.2% with 24 new solutions, but advertiser value -76.7%, with volatile ROI.

⁵Liao and Kroer use the same data (2024, 2025).

⁶Zhang (2011), proportional response dynamics in Fisher markets.

Compare One-Shot vs. Paced Auctions

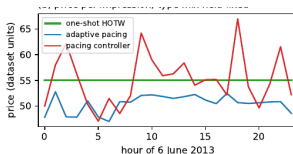
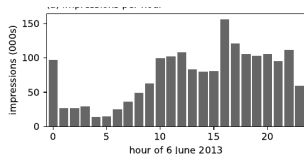


Simulate paced 1PAs with obs'd sequence of 1.80M impressions, using two rules to update multipliers:⁷

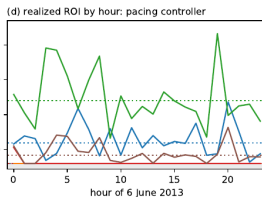
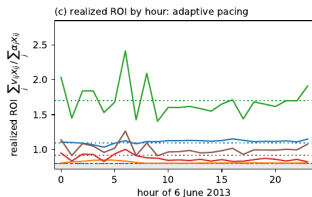
- Learning takes 1/3 day: multipliers come to 5% of μ_i^* after 646k impressions. Sensitive to stepsize.
- Advertisers get 87–96% of one-shot value; Exchange gets 90-100%.

⁷Adaptive pacing: Balseiro and Gur (2019). Second rule: a multiplicative controller for spend.

Time-varying supply

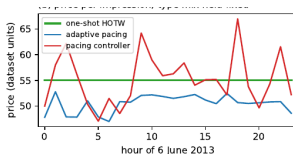
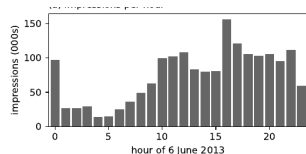


- Volatile hourly supply (top-left)

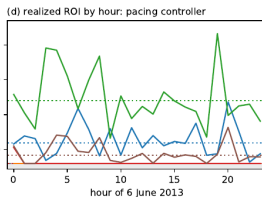
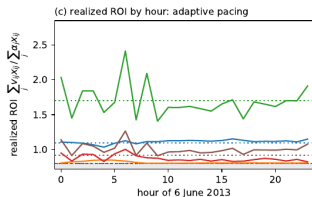


(e) relative spread of realized ROI across the 24 hours

Time-varying supply

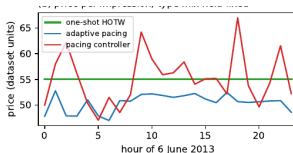
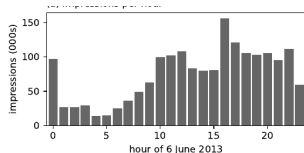


- Volatile hourly supply (top-left)
- Flat ROI for HOTW; volatile ROI for pacing rules (bottom row)

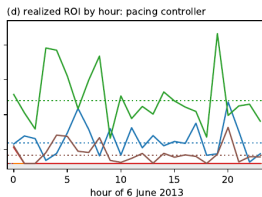
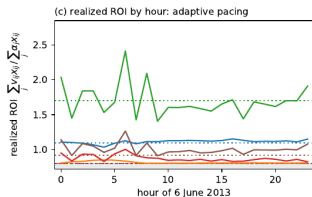


(e) relative spread of realized ROI across the 24 hours

Time-varying supply



- Volatile hourly supply (top-left)
- Flat ROI for HOTW; volatile ROI for pacing rules (bottom row)
- Flat prices for HOTW, volatile under pacing (top right)



(e) relative spread of realized ROI across the 24 hours

Conclusions

Results of this paper.

- One-shot pricing in HOTW Market presents feasible via characterization and computation of competitive equilibrium in a quasi-Fisher market
 - Advertisers' budget constraints eliminate platform's market power
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Thank you!

Backup: auction equivalence (math)

Proposition (Auction equivalence, Prop. 10)

Multipliers are $\mu_i = b_i/W_i^*$, with $W_i^* = \sum_j u_{ij}x_{ij}^* + r_iz_i^*$. We have $\mu_i \in (0, 1]$. We have $\mu_i = 1$ if and only if the ROI condition binds. If $x_{ij}^* > 0$, then $\mu_i u_{ij} = \alpha_j^* = \max_k \mu_k u_{kj}$. The result is the same as a paced first-price auction with multipliers μ_i .

Corollary (Ex-ante computability, Cor. 11)

We read the multipliers from the convex program. We do not need an auction. We do not need an iterative search.

Incentives: the two systems use the same reports (b_i, r_i) . A true budget report is weakly dominant. A false report can gain a factor of 2 at maximum.⁸

⁸Branzei et al. (2014), Fisher market game; Chen, Deng, Tang, Zhang (2016), strategic behavior in Fisher markets.

Backup: dual, computation

Dual problem (Prop. 8). $\min_{p \geq 0, \beta \in (0,1]^n} \sum_j p_j s_j - \sum_i b_i \log \beta_i$ with the conditions $p_j \geq u_{ij} \beta_i \forall i, j$. Strong duality is true. We get $p^* = \alpha^*$ and $\beta_i^* = b_i / W_i^*$. If $z_i^* > 0$, then $\beta_i^* = 1$. The dual problem has $n + m$ variables only.

Algorithms. (i) A primal–dual method that increases prices in steps (an algorithmic call auction; no strategic bids);⁹ (ii) tâtonnement, that is, gradient descent on the dual problem;¹⁰ (iii) proportional response: the method is decentralized, with error $O(1/t)$.¹¹ The data section uses method (iii).

⁹Devanur, Papadimitriou, Saberi, Vazirani (2008).

¹⁰Cole, Fleischer (2008); Cheung, Cole, Devanur (2020).

¹¹Birnbaum, Devanur, Xiao (2011).

Backup: delivery rule, incentives

- Impressions arrive in sequence. We use the solution (x^*, α^*) with a quota rule (we track the difference from x_{ij}^*) or with random assignment (probability x_{ij}^*/s_j).
- We do not calculate new prices during the day. We do not change multipliers when impressions arrive.
- Incentives are the same in the two systems (same reports (b_i, r_i)). A true budget report is weakly dominant. A false report about values can gain a factor of 2 at maximum. The result also applies to r_i , because $u_{ij} = v_{ij}/r_i$.

No-arbitrage pricing

- Each advertiser gets the same ROI across all (j, t) where $x_{ijt}^* > 0$.
- Hence,
- ROI of each advertiser is constant across period, but not necessarily prices. No-arbitrage is not the same as constant prices.

Example:

- 2 advertisers, each w/ $b_i = 2$
- 3 periods
- 1 type: $s_1 = 4, s_2 = 2, s_3 = 4$.

	v_{i1}	v_{i2}	v_{i3}	r_i	b_i
Advertiser 1	1	1	0	2	2
Advertiser 2	0	2	2	2	2

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Equilibrium:

- Allocation: $x_{11}^* = 4, x_{12}^* = 1, x_{22}^* = 1, x_{23}^* = 4, (z_1^* = z_2^* = 0)$
- Prices: $\alpha_1^* = \alpha_2^* = \alpha_3^* = \$0.4$,

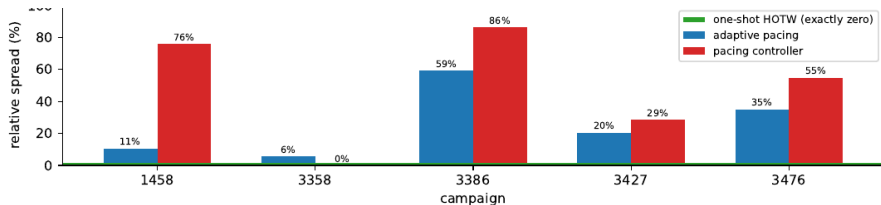
Backup: robustness

More advertisers: We make new advertisers from the models we estimate ($n = 5 \rightarrow 1,000$; the market is 200 times larger). Learned multipliers do not come to the correct values in one day if $n > 5$. The largest error grows from 27% to 385%. More advertisers means more numbers that must agree. But the value for advertisers changes little: 0.98 / 0.91 of one-shot value at $n = 1,000$. Advertisers are safe. The cost of learning is a less efficient allocation, not a large loss.

Prices one day early: We use prices from 6 June with the supply of 7 June. The mean error for each type is 22.1%. Some types have large errors.

- Revenue is 98.1% of the revenue with perfect information. We sell all inventory. If a type has less supply, the outside option moves the budget to other types.
- A change of all prices by one factor does not help. The errors are different for different types. They are not one change of level.
- If we solve the program again during the day: revenue increases by 9.2% with 24 new solutions. But the value for advertisers decreases to 76.7%. And the ROI is not the same at all times (range up to 114%) — Figure 7 of the paper.
- Guidance: set prices one day early. Do not change them during the day. A correction in the day gives small revenue and takes value from advertisers.

Intra-day: ROI spread across hours



1458 3358 3386 3427 3476 learned pacing one-shot HOTW ROI target $r_t = 0.8$ (shaded: below target)

Range of ROI in 24 hours, for each campaign. Compare with the first slide. One-shot: zero (less than 10^{-9}). Learned pacing: 6–59% (adaptive), up to 86% (controller). Arbitrage between hours is zero.

What does the day of auctions add?

- Theory: the one-shot allocation is the same allocation that a paced auction can reach. We lose nothing if we calculate it directly.
- Data (6 June 2013): learned pacing gives 87–96% of the one-shot value. The ROI range in 24 hours is 6–86%, not 0.
- Revenue: the clearing prices are the best uniform prices. Learning does not give more (32,000 tests, none better).

Result: we lose nothing. The ROI is the same at all hours of the day.