A Model of Leveraged Bubbles

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Motivation: Bubbles & Recessions

Jordà et al. 2015; Dell'Aricchia et al. 2011; Greenwood et al. 2020

Empirical facts:

1. Asset-price booms are not always accompanied by elevated credit growth.
2. Small real effects of bursting asset-price booms absent elevated credit.
3. Credit expansion & asset price booms $\rightarrow$ even deeper (2x more), longer recessions when prices burst.

Leveraged Bubbles

Call constellation of elevated credit and prices a *leveraged bubble*, following Jordà et al. (2015), vis-à-vis an *unleveraged bubble* when there is only an asset-price boom.

This paper

What we do

- Develop a simple model to capture and study the real effects of leveraged bubble dynamics, consistent with empirical findings→ emphasis on non-linearities.
- Characterize optimal policy to address bubble externalities on top of usual asset price externalities in the presence of borrowing constraints tied to collateral values.

How we do it

- Augment Bianchi and Mendoza (2018, JPE) macro model—featuring occasionally binding borrowing constraints—with a stochastic rational asset price bubble.
    - The bubble **grows endogenously** and raises collateral values when it expands, relaxing borrowing limits.
    - When the bubble **bursts**, collateral value collapses, borrowing constraints tighten, and deleveraging amplifies the downturn.
- Calibrate the model to match bubble facts in Jordà, Schularick, and Taylor (2015, JME) and solve for the global non-linear dynamics of bubbles and credit.

Preview of results

Positive results

- **Leveraged vs. unleveraged downturns:** output declines are about **1.5 times larger** when the bursting of a bubble takes place after high debt accumulation episode.
- **Nonlinear amplification:** when a bubble collapse coincides with a negative productivity shock, output falls by **about 11.7%**, compared to **4%** for a bubble burst alone and **6.1%** for a productivity shock alone.
- **Persistence:** leveraged-bubble recessions feature slower recoveries due to persistently binding borrowing constraints.

Normative results

- **Two externalities:** agents do not internalize how borrowing affects (i) the price of capital and (ii) the growth path of the bubble, and hence collateral values and the tightness of the borrowing constraint.
- **Optimal policy:** a Pigouvian borrowing tax addresses externalities and substantially **mitigates severe recessions**.
- **Welfare gains:** optimal policy delivers welfare gains of about **0.3% of permanent consumption**.

Related Literature

1. Borrowing constraints and endogenous financial crises

Kiyotaki and Moore (1997); Lorenzoni (2008); Mendoza (2010); Geanokoplos (2010); Jermann and Quadrini (2012); Brunnermeier and Sannikov (2014)

- We explore the effect of leveraged and unleveraged bubbles on output, and consider downturns without exogenous tightening of financial conditions.

2. Rational bubble macro models

Kocherlakota (2009); Farhi and Tirole (2012); Martin and Ventura (2012); Galí (2014); Miao and Wang (2018); Biswas et al. (2020); Allen et al. (2022)

- We focus on non-linear joint dynamics of bubbles and credit rather than on long-run growth, steady-state, or linearized dynamics.
- We solve the model globally to match stylized facts of (bursting) leveraged bubbles and derive the optimal policy.

3. Optimal macroprudential policy

Bianchi (2011); Benigno et al. (2013); Bianchi and Mendoza (2018); Davila and Korinek (2018); Jeanne and Korinek (2020)

- We characterize bubble externalities separately from fundamental-price pecuniary externalities.

Economy

- Small open economy with an infinitely-lived representative agent.
  - Owns firm that produces a consumption good using labor, capital, and intermediate inputs.
- The agent can borrow externally by pledging capital and bubble holdings as collateral.
  - World interest rate and pledgeability of collateral are exogenously fixed.
- Capital is in aggregate fixed supply of 1.
- The bubble does not yield dividends but relaxes borrowing constraints.

Bubble Process

- The bubble follows a two-state Markov process governed by indicator variable $\mathcal{I}_t$.
  - Bubble survives with probability π; otherwise it bursts and a new bubble emerges.

$$q_t^b = \begin{cases} u & \text{if } \mathcal{I}_t = 0 \quad \text{w.p.} \quad 1-\pi \\ b_t & \text{if } \mathcal{I}_t = 1 \quad \text{w.p.} \quad \pi \end{cases}$$

- Every new bubble is in aggregate fixed supply of 1.
  - New bubbles are distributed equally among all agents at the beginning of the period they emerge.
  - New bubbles are always exogenously valued at the same price u (could be relaxed).
- If a previous-period bubble survives, its price b_t grows endogenously with economic conditions in a way consistent with rational expectations.
- The transversality condition does not exclude the bubble: The bubble has value because it serves as collateral when the borrowing constraint binds *infinitely often*.

Representative agent's problem

$$\max_{c_t, l_t, v_t, L_{t+1}, k_{t+1}, x_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - G(l_t))$$

s.t. **Budget constraint:**

$$c_t \leq \underbrace{y_t}_{\text{Output}} - \underbrace{p^v v_t}_{\text{Intermediate good purchases}} + \underbrace{\frac{L_{t+1}}{R} - L_t}_{\text{Net borrowing}} - \underbrace{q_t^k (k_{t+1} - k_t)}_{\text{Net capital purchases}} - \underbrace{q_t^b (x_{t+1} - 1 - (x_t - 1)\mathcal{I}_t)}_{\text{Net bubble purchases}}$$

Borrowing constraint:

$$L_{t+1}/R + \theta p^v v_t \leq m\left(q_t^k k_t + q_t^b (1 + (x_t - 1)\mathcal{I}_t)\right)$$

- Output $y_t = z_t k_t^{\alpha_k} v_t^{\alpha_v} l_t^{\alpha_l}$; k_t: capital, v_t: intermediate good; l_t: labor.
- L_{t+1}: new borrowing; R: interest rate.
- q_t^k and q_t^b: price of capital and bubble ; x_{t+1}: new bubble holdings.
- m: pledgeable portion of capital and the bubble; θ: working capital funding.

Recursive Equilibrium & Bubble Growth

- **State variables.** Absent a bubble the state variables are (z_t, L_t); with the bubble additional states arise $(\mathcal{I}_t, q_t^b)$.

- **Indeterminacy.** The Euler condition for the bubble is consistent with multiple bubble values.

$$q_t^b U_{c,t} = \beta\pi \tilde{E}_t \left(U_{c,t+1} b_{t+1} (1 + m\mu_{t+1}) \right), \quad \text{where } \tilde{E}_t(\cdot) = E_t(\cdot|\mathcal{I}_{t+1} = 1)$$

- **Simplifying assumption.** Zero innovations in the price of the bubble:

$$b_t = \tilde{E}_{t-1}[b_t]$$

- **Bubble growth.**

$$\frac{b_{t+1}}{q_t^b} = \frac{1}{\pi \tilde{E}_t M_{t+1} (1 + m\mu_{t+1})}$$

- Bubble growth affects the entire expected future path of the borrowing limit.

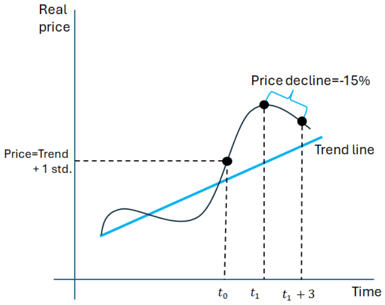
Calibration and Solution Method

Solution Method

- Mendoza and Villalvazo (2020) fixed-point iteration algorithm.
  - Two endogenous states: (L_t, b_t).
- Global solution, occasionally binding constraints.

Calibration

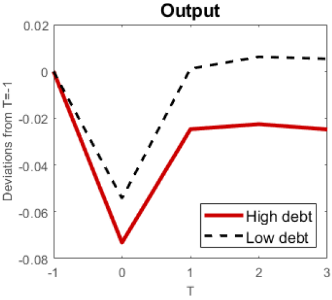
- Baseline parameters: Bianchi and Mendoza (2018).
- Bubble process $\rightarrow$ Match stylized facts in Jorda, Schularick, and Taylor (2015):
  - Identify JST bubble episodes in simulated economy.
  - Target Duration = 3.2 yrs and Amplitude = 17.3%.
  - Set $u = 0.002$ (new bubble value) and $\pi = 0.67$ (survival probability).

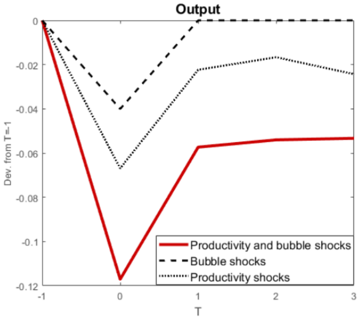


JST Bubble Episode:

- *Start* (t_0): $q_t > \bar{q} + \sigma(q)$.
- *End* (t_1): 15% decline over 3 years.

Debt levels, type of recessions, and bubble crashes





- Recessions following leveraged bubbles are roughly 1.5 times deeper.
- Recoveries are slower when debt accumulation is high.

- Output losses: Bubble (-4%) + Productivity (-6.1%) $<$ Both together (-11.7%).
- Bursting leveraged bubbles + fundamental weakness = severe and persistent recessions.

Time-consistent constrained planner

$$V(\Xi, L, b) = \max_{\{c, L', v, \ell, q^k, b'\}} U(c - G(\ell)) + \beta E_{\Xi' | \Xi} V(\Xi', L', b')$$

$c \leq y - p^v v + L'/R - L$ resource constraint

$L'/R + \theta p^v v \leq m \left(q^k + \mathcal{I}\, b + (1 - \mathcal{I}) u \right),$ collateral constraint

$q^k U_c(c - G(\ell)) = \beta E_{\Xi' | \Xi} H^k(\Xi', L', b')$ capital-price implementability

$(\mathcal{I}\, b + (1 - \mathcal{I}) u) U_c(c - G(\ell)) = \beta \pi \widetilde{E}_{\Xi' | \Xi} H^b(\Xi', L', b')$ bubble-growth implementability.

- Planner internalizes the effect of consumption and borrowing decisions on the price of capital and the size of the bubble, and hence on the path of current and future borrowing constraints.

Externalities private agents do not internalize

- The capital-price externality operates *contemporaneously* through the current- and next-period borrowing constraints.

$$\overbrace{-\xi_t U_{c,t}}^{\text{Shadow value of concurrent implementability constraint}} + \overbrace{m\mu_t^P}^{\text{Shadow value of relaxing concurrent borrowing constraint}} = 0,$$

- The bubble-growth externality operates through *all* future borrowing constraints because it also affects the future state.

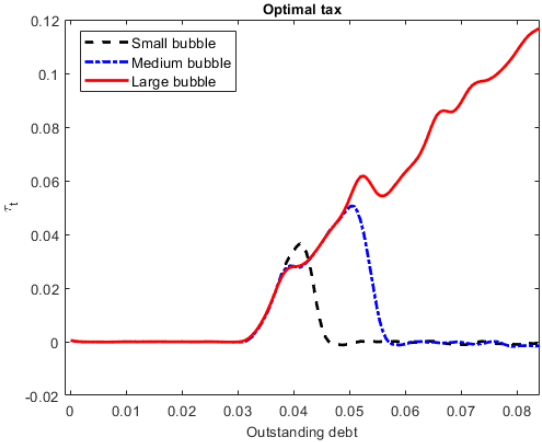
$$\overbrace{-\beta\pi\tilde{E}_t\phi_{t+1}U_{c,t+1}}^{\text{Shadow value of relaxing future borrowing constraint}} + \overbrace{\beta\pi m\tilde{E}_t\mu_{t+1}^P}^{\text{Shadow value of relaxing future borrowing constraint}} \overbrace{+\xi_t\beta\pi\tilde{E}_t\mathcal{H}_{b,t+1}^k + \phi_t\pi\beta\tilde{E}_t\mathcal{H}_{b,t+1}^b}^{\text{Effect on bubble state on expectational terms on current prices}} = 0$$

- Assume the current borrowing constraint is slack:
  - Higher borrowing today tightens borrowing constraints in the future, suppressing consumption and the future capital price, which further tightens the constraint.
  - Higher borrowing today increases current consumption which suppresses the growth of the bubble and, thus, decreases its ability to relax future binding constraints.

Optimal borrowing tax implementing planner's solution

$$\tau_t = \underbrace{\text{Bubble growth externality}}_{\text{New externality}} + \underbrace{\text{Fundamental price externality}}_{\text{Standard}}$$

- Tax increases with debt and bubble size.
- Declines when borrowing constraint binds contemporaneously.



Summary statistics: Laissez-faire versus planner's solution

	Laissez-faire		**Optimal policy**	
PANEL A	mean	st. dev	mean	st. dev
Output (GDP)		2.5%		2.3%
Consumption		1.7%		1.3%
Credit growth	0.7%	11.4%	0.1%	4.7%
PANEL B				
Incidence of a recession	20.3%		30.2%	
Output decline in a recession	-6.5%		-3.3%	
Incidence of a severe recession	4.4%		0.03%	
Output decline in a severe recession	-11.3%		-13.2%	
PANEL C				
Tax on borrowing			2.2%	1.3%
Welfare gains			0.3%	

Notes: A recession is defined as a period with an output decline compared to the period before. A severe recession is a recession with an output decline of -10% or more.

Conclusion

- Recessions following leveraged asset price bubbles are deeper and more persistent than those following unleveraged booms.
- Time-consistent optimal policy addresses both fundamental price and bubble growth externalities, and reduces the effect of leveraged-bubble recessions.

Back-up slides

Euler Equations

- **Borrowing decision:**

$$U_{c,t}(1-\mu_t) = \beta R E_t U_{c,t+1}$$

Stochastic discount factor lower than $1/R$.

- **Price of capital:**

$$q_t^k U_{c,t} = \beta E_t U_{c,t+1} \left(F_{k,t+1} + q_{t+1}^k (1 + m\mu_{t+1}) \right)$$

Discounted dividends and future price of capital incorporating collateral premium.

- **Bubble growth:**

$$q_t^b U_{c,t} = \beta \pi \tilde{E}_t \left(U_{c,t+1} b_{t+1} (1 + m\mu_{t+1}) \right)$$

Discounted dividends and future price of capital incorporating collateral premium.

Bubble existence

- A rational bubble needs to satisfy the Euler condition, i.e., being priced appropriately in equilibrium.

- But it also needs to satisfy the transversality condition

$$\lim_{T\to\infty} E_t \beta^T U_{c,t+T} b_{t+T} x_{t+T} = \lim_{T\to\infty} E_t \beta^T U_{c,t+T} b_{t+T} = 0.$$

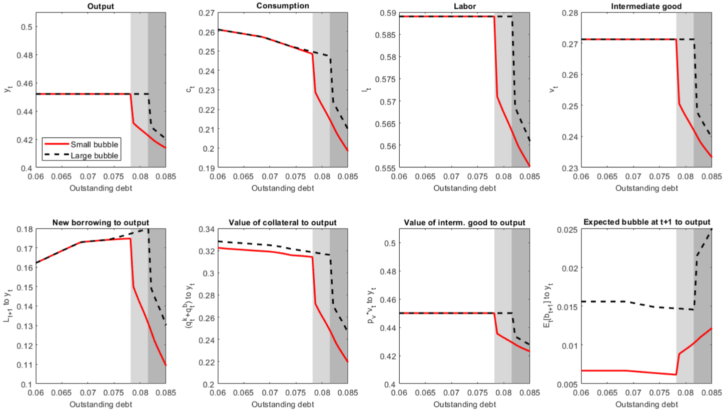
- Iterating the Euler equation we get

$$q_t^b = \lim_{T\to\infty} E_t \beta^T \Pi_{i=1}^T (1 + m\mu_{t+i}) \frac{U_{c,t+T}}{U_{c,t}} b_{t+T}.$$

- A rational bubble can exist if the borrowing constraint binds infinitely often, $\lim_{t\to\infty} sup \mu_t > 0$.

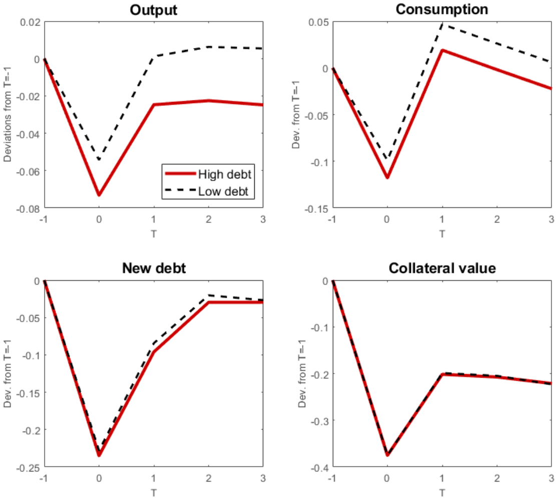
- This is true for sufficiently low m.

Policy Functions



- Larger bubbles support higher debt without triggering the borrowing constraint.
- Binding constraint: consumption, intermediate inputs, labor, and output fall.
- Expected future bubble value increases nonlinearly once the constraint binds.

Event Analysis: The Aftermath of Bubble Crashes



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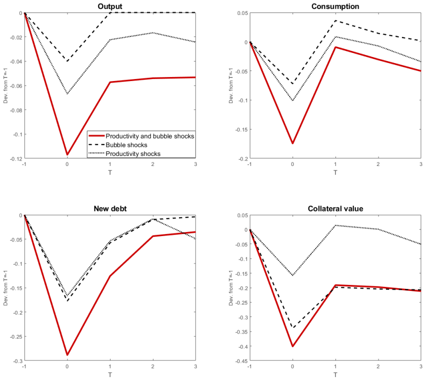
Types of Recessions: The Role of Bubbles and Fundamentals



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