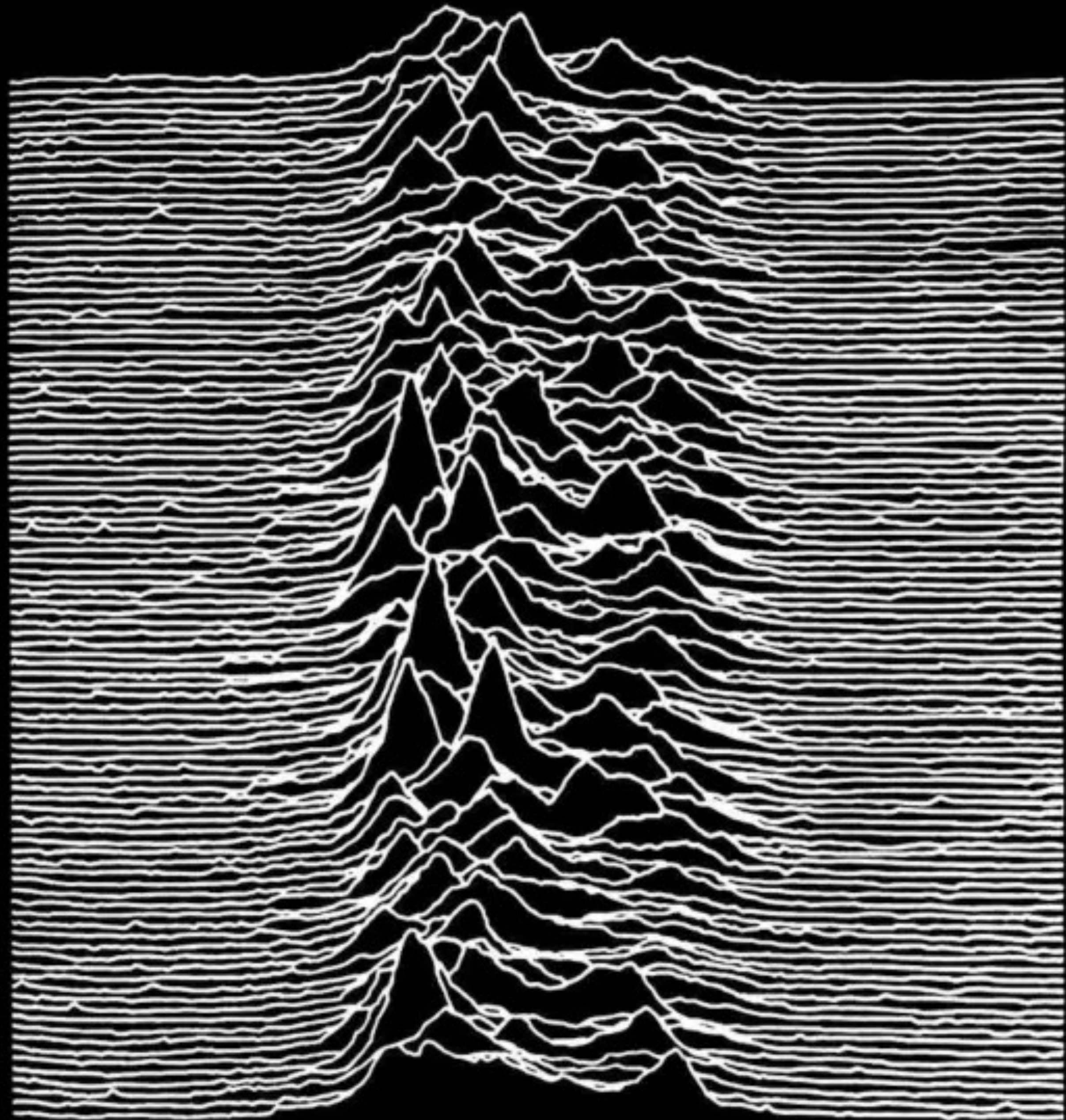


MACROECONOMICS WITH EXCESS DEMANDS

Guido Lorenzoni
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July 2026
NBER-SI



MICROECONOMIC THEORY

ANDREU MAS-COLELL MICHAEL D. WHINSTON
AND JERRY R. GREEN

MICROECONOMIC THEORY

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Michael Woodford

INTEREST
& PRICES

MICROECONOMIC THEORY

ANDREU MAS-COLELL MICHAEL D. WHINSTON
AND JERRY R. GREEN

GE $\leftrightarrow$ Macro

Michael Woodford

INTEREST & PRICES

FOUNDATIONS OF A THEORY
OF MONETARY POLICY

GE ↔ Macro

GE ↔ Macro

■ General Equilibrium Theory...

- ▶ well-developed theory
- ▶ flexible framework
- ▶ applied: Trade, Macro, Public Finance, IO...

$$Z(\bar{p}) = 0$$

equilibrium
(vectors)

GE ↔ Macro

■ General Equilibrium Theory...

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■ New Keynesian Macroeconomics...

- ▶ Success: micro + nominal rigidities...
- ▶ ... GE with many markets but...
- ▶ → gap with GE theory
- ▶ → gap with basic micro 101 intuition

$$Z(\bar{p}) = 0$$

equilibrium
(vectors)

$$U = u(c) - v(n)$$

$$u'(c_t) = \beta R \mathbb{E}_t u'(c_{t+1})$$

$$p_t^* = \mu_t + \sum_{s \geq 0} \beta^s MC_{t+s}$$

GE ↔ Macro

■ General Equilibrium Theory...

- ▶ well-developed theory
- ▶ flexible framework
- ▶ applied: Trade, Macro, Public Finance, IO...

■ Disequilibrium Literature: initial progress GE + price rigidities... ... but challenges → price dynamics + intractable (2^N regimes)

■ New Keynesian Macroeconomics...

- ▶ Success: micro + nominal rigidities...
- ▶ ... GE with many markets but...
- ▶ → gap with GE theory
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**Keynes
&
ISLM**

**General
Equilibrium**
(with Money)

Disequilibrium
(Fixed Price)

**New
Keynesian**

1930-40s

1950s & 1960s

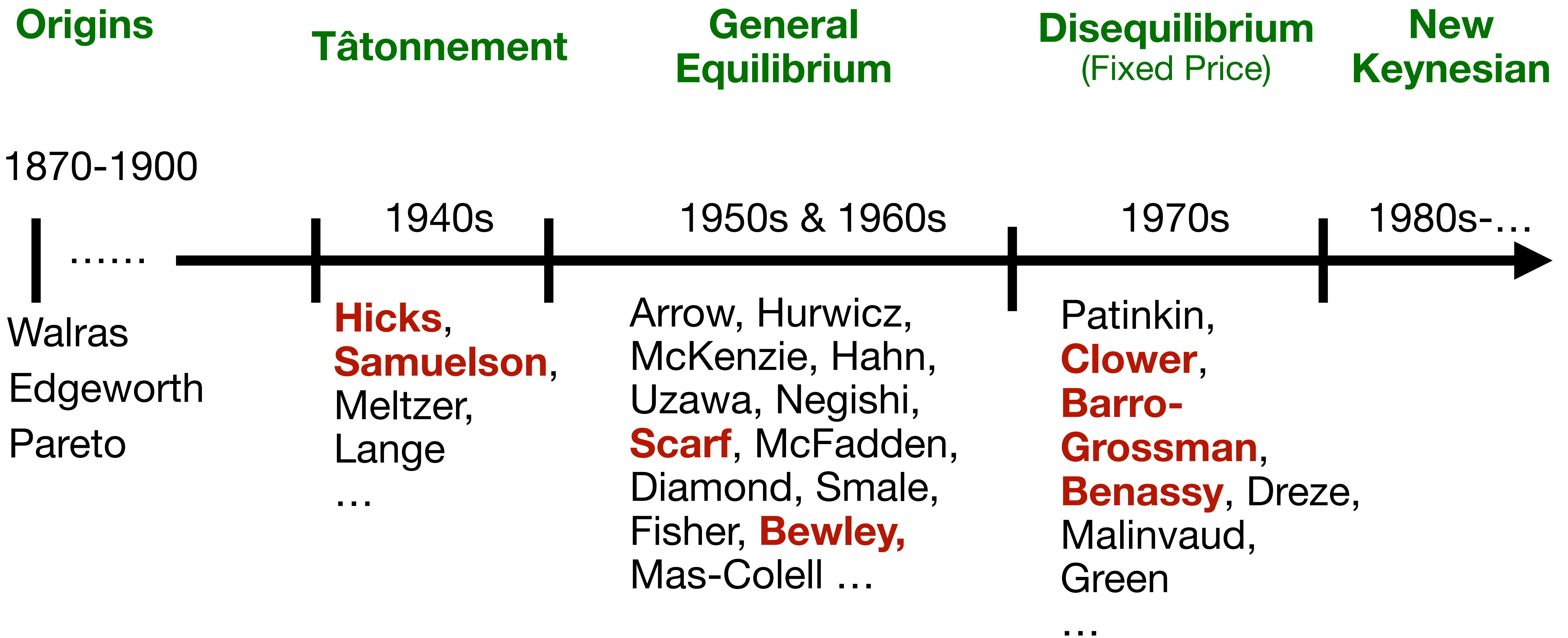
1970s

1980s-present

Keynes
Hicks

Patinkin
Clower

Patinkin,
Clower,
Barro-Grossman,
Benassy, Dreze,
Malinvaud,
Green
...



This Paper

■ This paper...

- ▶ framework developed from previous “Tâtonnement...” paper...
- ▶ ... but focus on macro angle

■ Our framework...

- ▶ quantities and welfare at point in time
- ▶ excess demand function: $Z(p)$
- ▶ price setting → price+wage adjustments

Main Results

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more general $\rightarrow$ more intuitive!

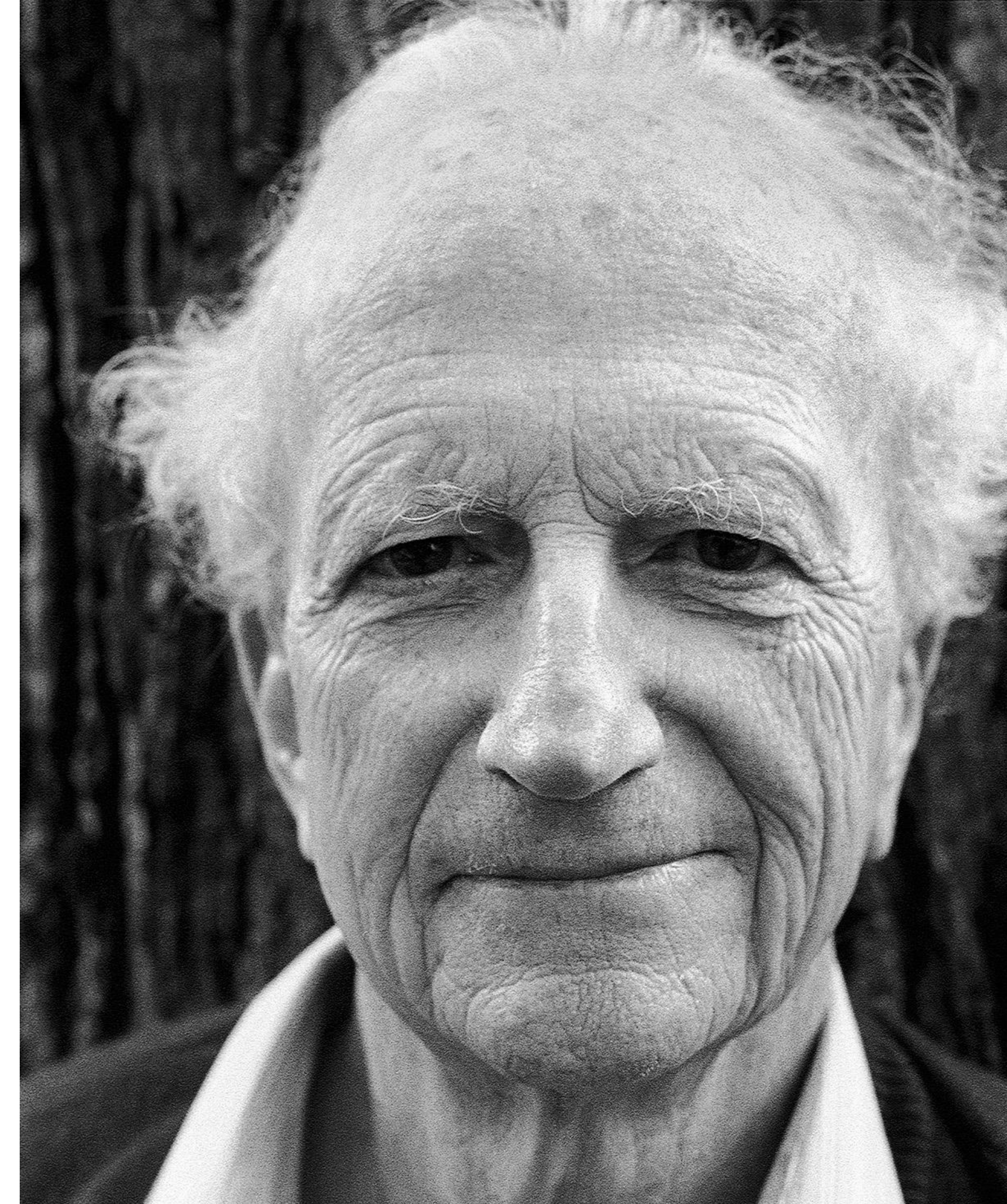
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Micro 101...
“Prices Rise with
Excess Demand”

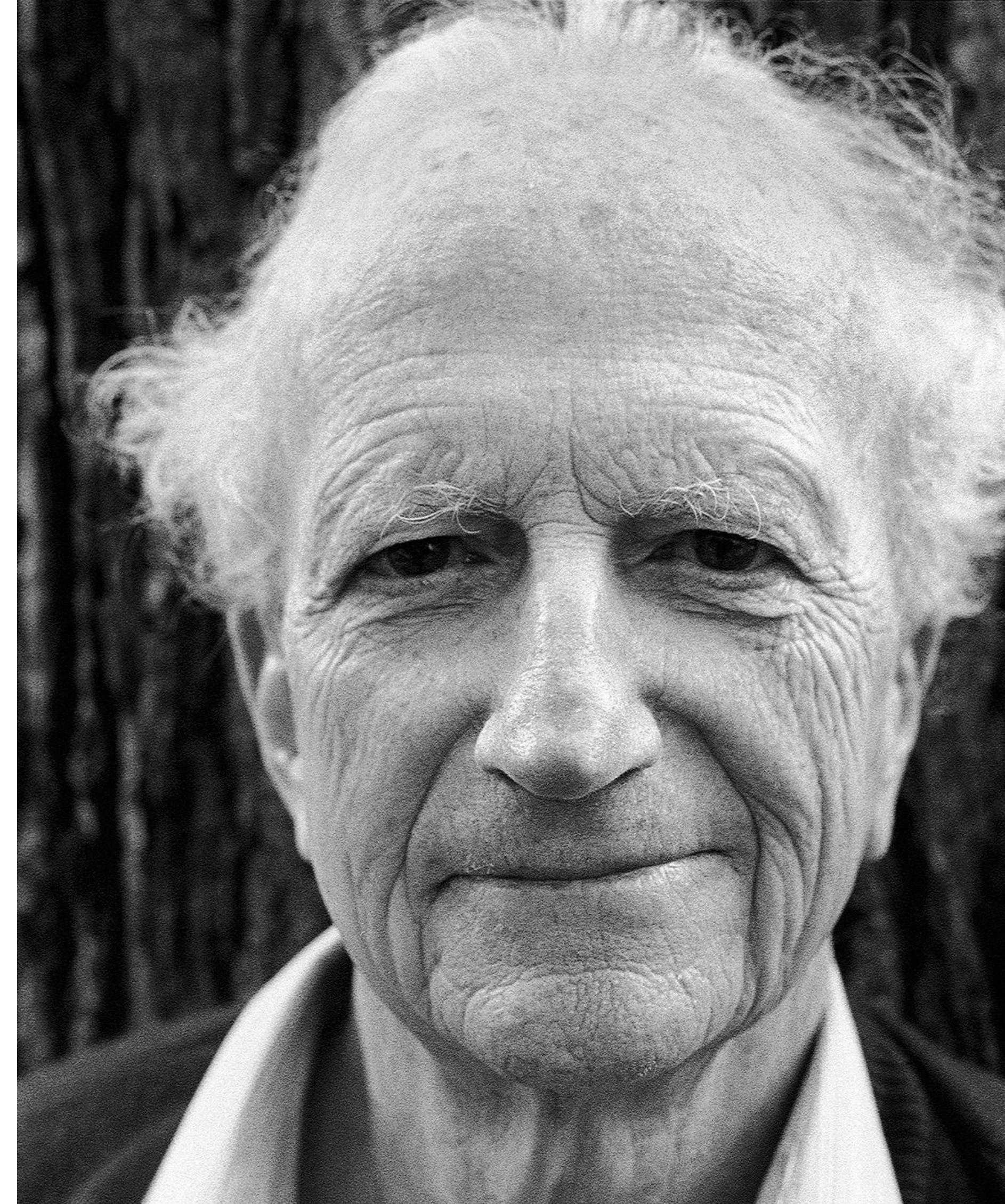
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more general $\rightarrow$ more intuitive!



Heard on the internet...
“Phillips Curves don’t exist
they are dead...”

Micro 101...
“Prices Rise with
Excess Demand”

This Paper

This Paper

- Excess demand approach...
 - ▶ general + simple + intuitive + powerful
 - ▶ takes best of.....
 - disequilibrium GE + NK literatures

This Paper

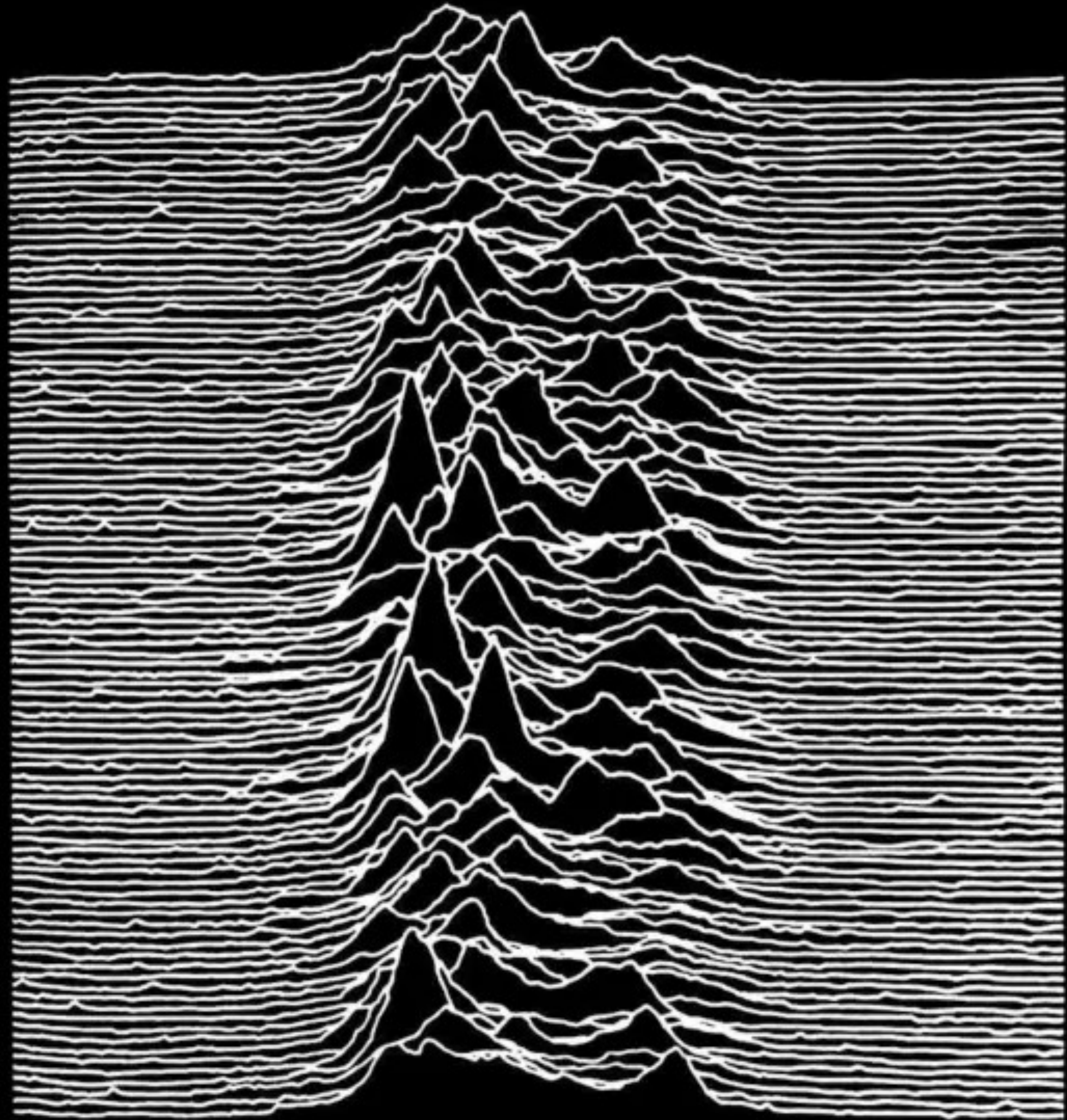
■ Excess demand approach...

- ▶ general + simple + intuitive + powerful
- ▶ takes best of.....
disequilibrium GE + NK literatures

■ Plan...

- ▶ Build Excess Demand approach:
steps by step → from static to dynamic
- ▶ Applications

Static Walrasian Benchmark



0

1

2

3

4

Static GE Model

■ GE = **General** Equilibrium

■ General Primitives...

▶ n goods (goods and factors, many labor etc.)

▶ h household types, general preferences

▶ f firms, general technologies (networks, etc.)

$$x = (x_1, \dots, x_N) \geq 0$$

$$y = (y_1, \dots, y_N) \geq 0$$

$$z = (z_1, \dots, z_N) = x - y$$

Math

$$\Pi^f(P) \equiv \max_{x^f, y^f} P \cdot (y^f - x^f)$$
$$(x^f, y^f) \in Y^f$$

$$D_W^f(P), S_W^f(P)$$
$$D_W^h(P), S_W^h(P)$$

$$D_W(P) = \sum_j D_W^j(P)$$
$$S_W(P) = \sum_j S_W^j(P)$$

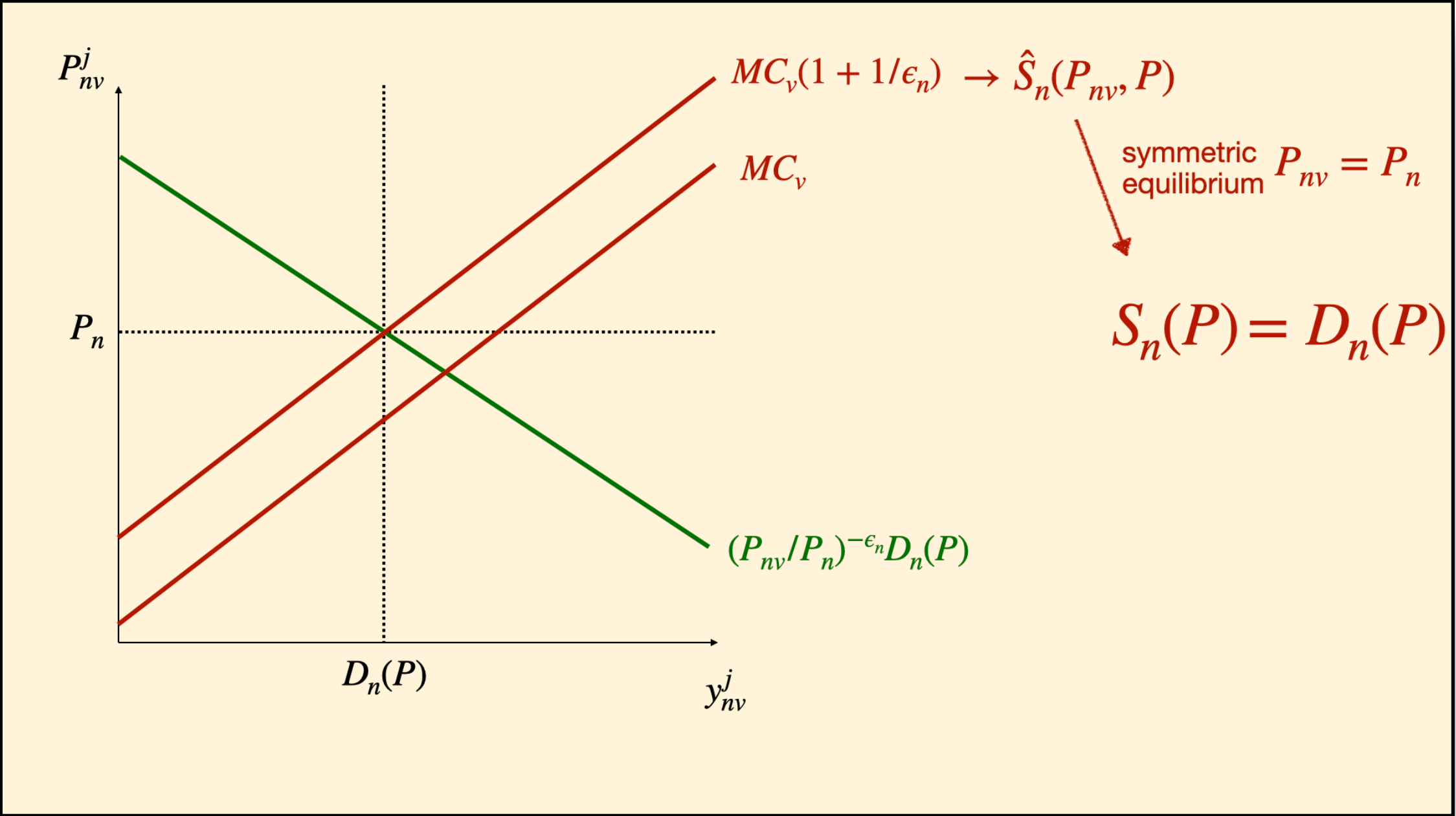
$$D_W(P) = S_W(P)$$

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$$\dot{P}_n = \alpha_n (D_{Wn}(P) - S_{Wn}(P))$$

Samuelson's ad hoc proposal
to capture Walras' idea...

Figure



$$\Pi^f(P) \equiv \max_{x^f, y^f} P \cdot (y^f - x^f)$$

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$$D_W^f(P), S_W^f(P)$$



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**Samuelson's ad hoc proposal
to capture Walras' idea...**

$$\Pi^f \equiv \max_{z \in Y^f} -p \cdot z$$

$$\Pi^f \equiv \max - p \cdot z$$



$$z \in Y^f$$

$$Z^{**f}(p)$$

$$\Pi^f \equiv \max - p \cdot z$$



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$$\downarrow$$

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$$\uparrow$$

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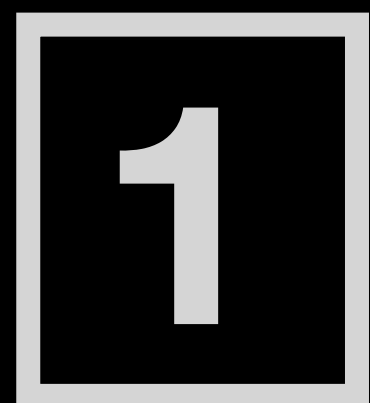
$$p \cdot z^h \leq p \cdot a^h + \sum_f \omega^{h,f} \Pi^f$$

$$Z^{**}(p) = 0$$

N equations
 N unknowns

Static Market Power

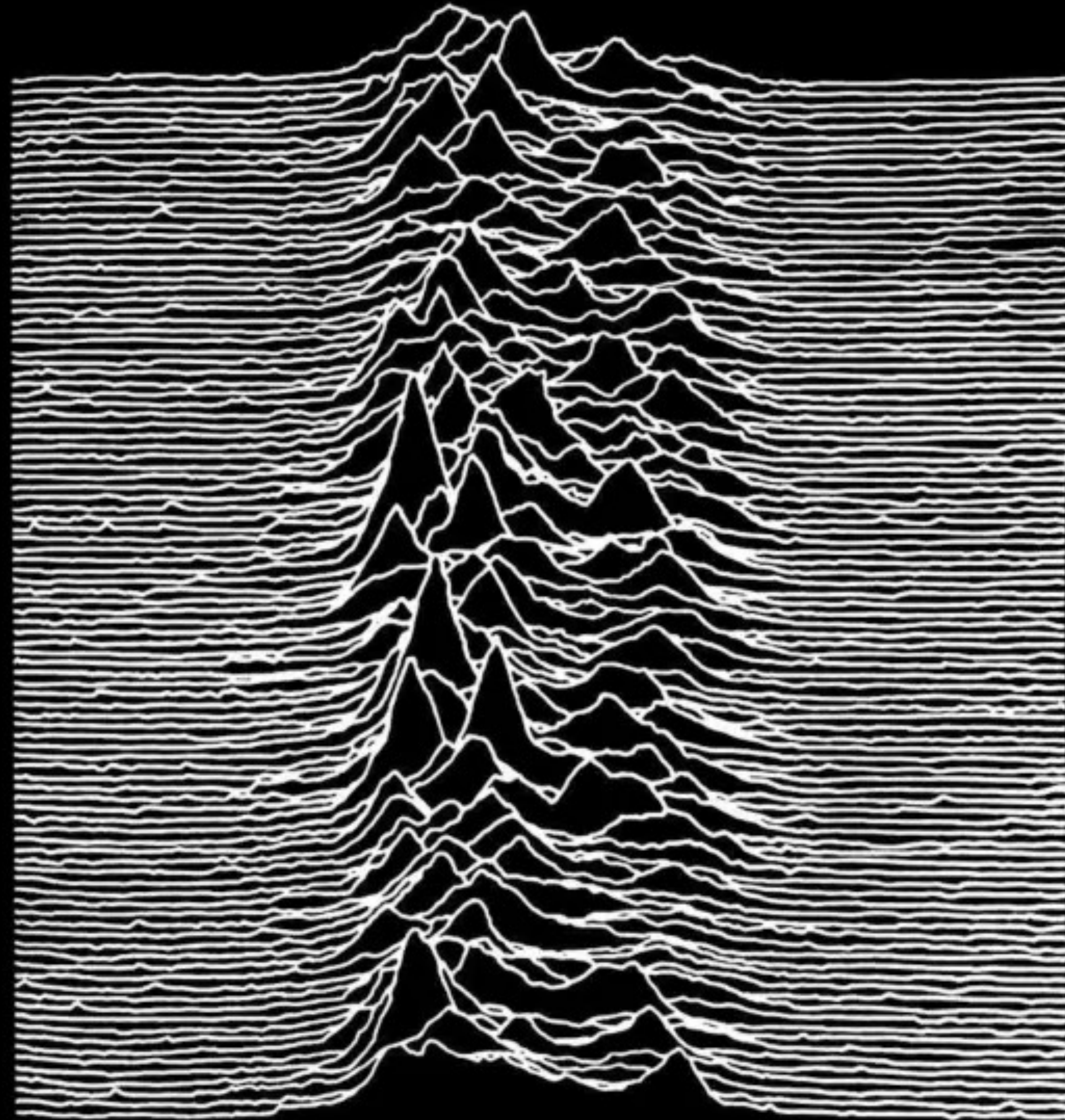
0



2

3

4



Adding Market Power

$$z_{un}(\omega) + z_{vn}(\omega) = 0$$

Adding Market Power

■ Each market $n \rightarrow$ varieties $\omega \in [0,1]$

► **price-taking undifferentiated (u)** vs. **price-setting differentiated (v = varieties)**

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► **undifferentiated side n** : CES aggregator over varieties with elasticity ϵ_n
(other non-CES aggregators possible, Kimball, Matsuyama, etc.)

$$\Pi^f(p) \equiv \max_{z_u, z_{vn}, p_{vn}} \quad - (p \cdot z_u^f + \textcolor{blue}{p_{vn} z_{vn}})$$

$$(z_u^f, \textcolor{blue}{z_{vn}}) \in Y^f$$

$$\textcolor{blue}{z_{vn}} = - (p_{vn}/p_n)^{-\epsilon_n} z_{un}$$

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Need prices and quantities

p and $z_{un} \dots$

... or do we really? ;)

$$\begin{aligned}\max_{z_u, z_{vn}, p_{vn}} \quad & U^h(z_u^h, z_{vn}) \\ p \cdot z_u^h + p_{vn} z_{nv} &\leq p \cdot a^h + \sum_f \omega^{h,f} \Pi^f \\ z_{vn} &= - (p_{vn}/p_n)^{-\epsilon_n} z_{un}\end{aligned}$$

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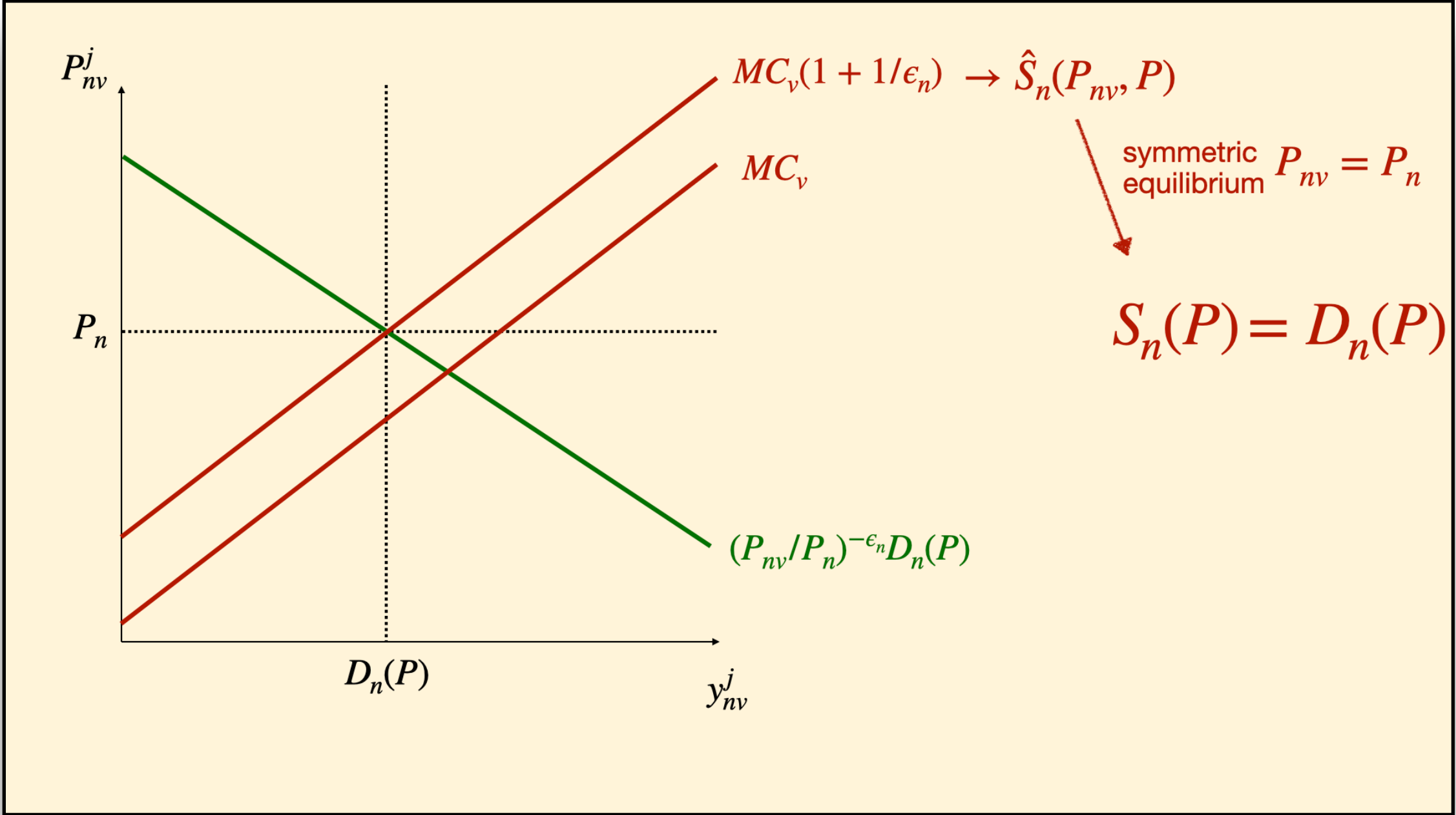
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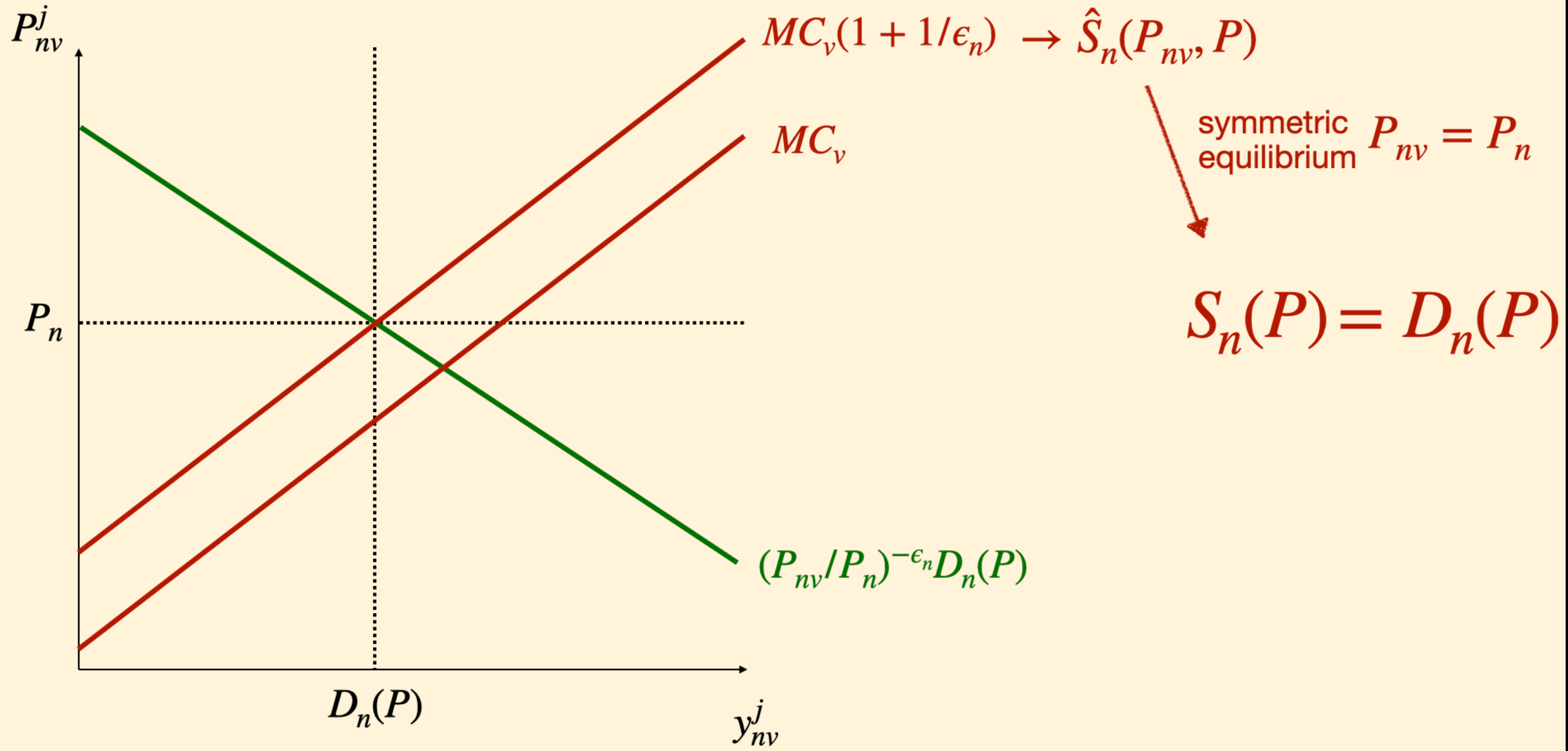
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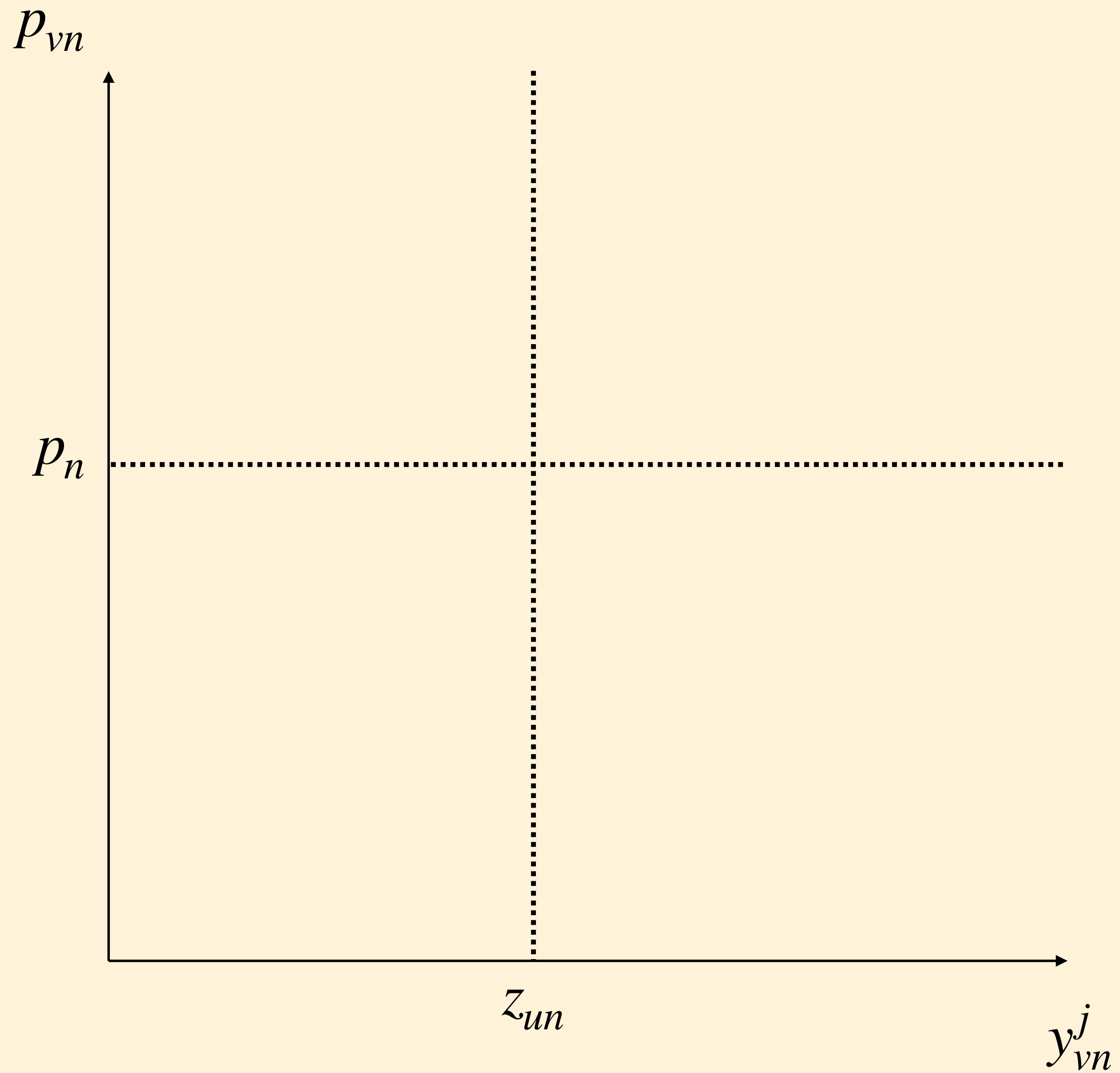
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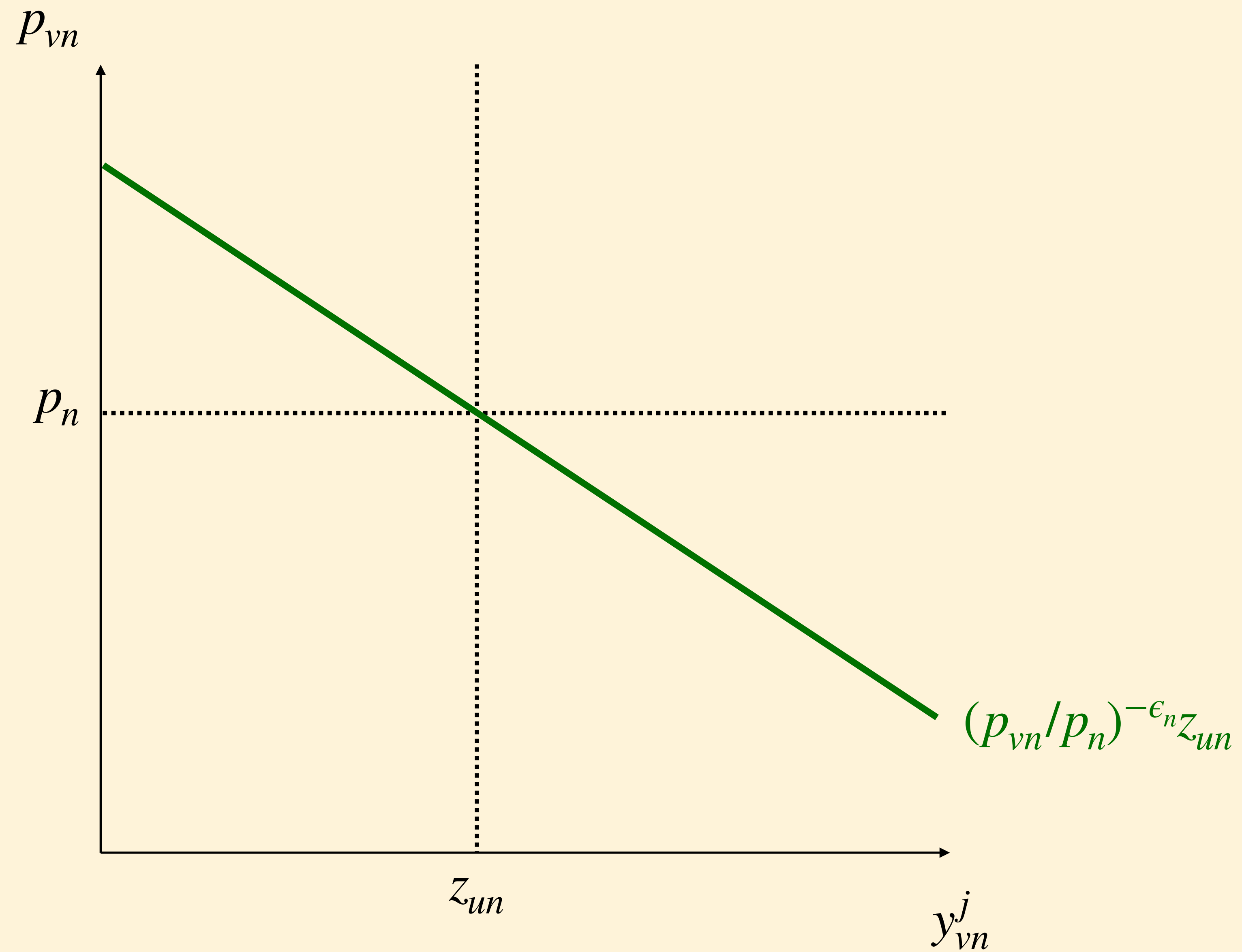
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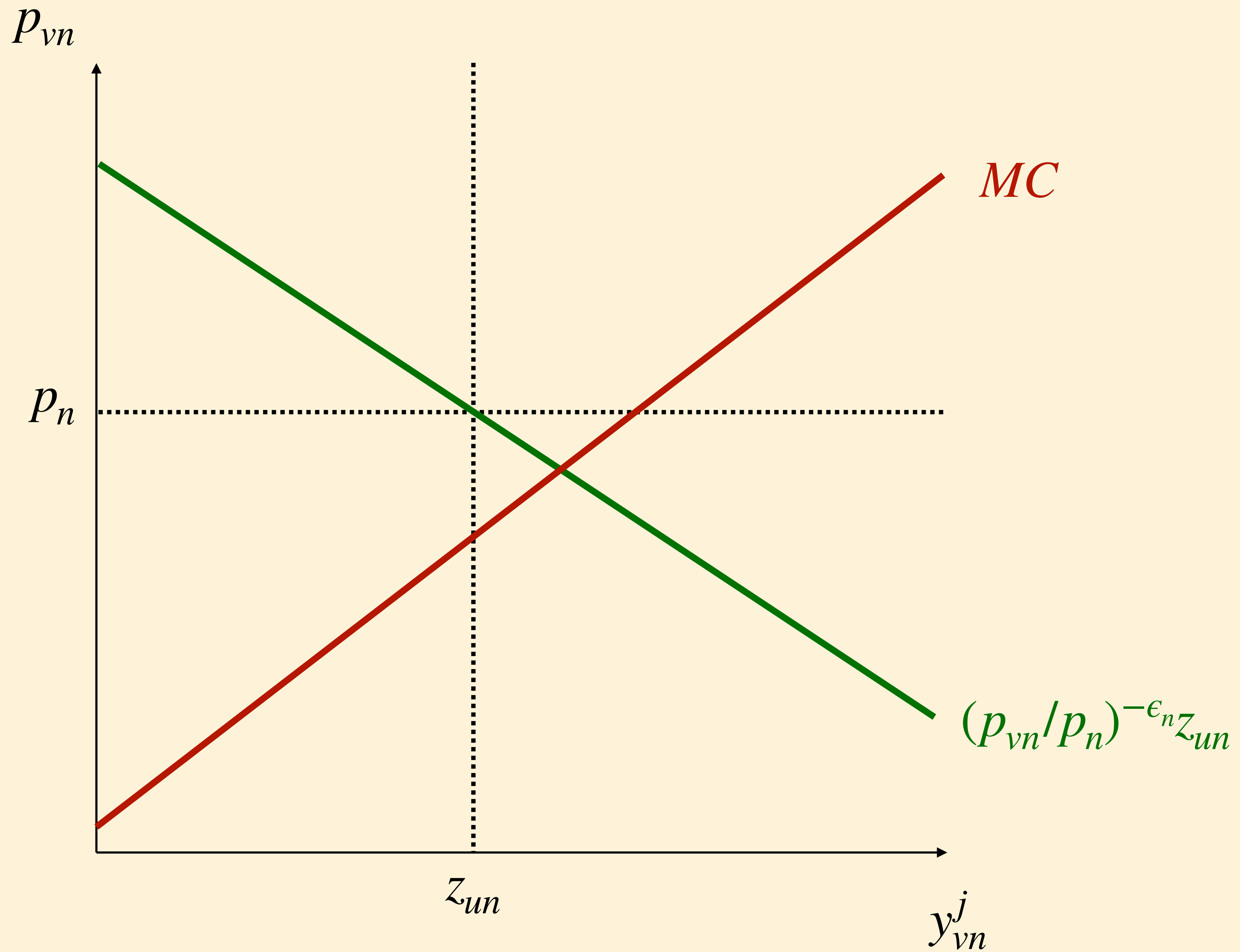
Figure

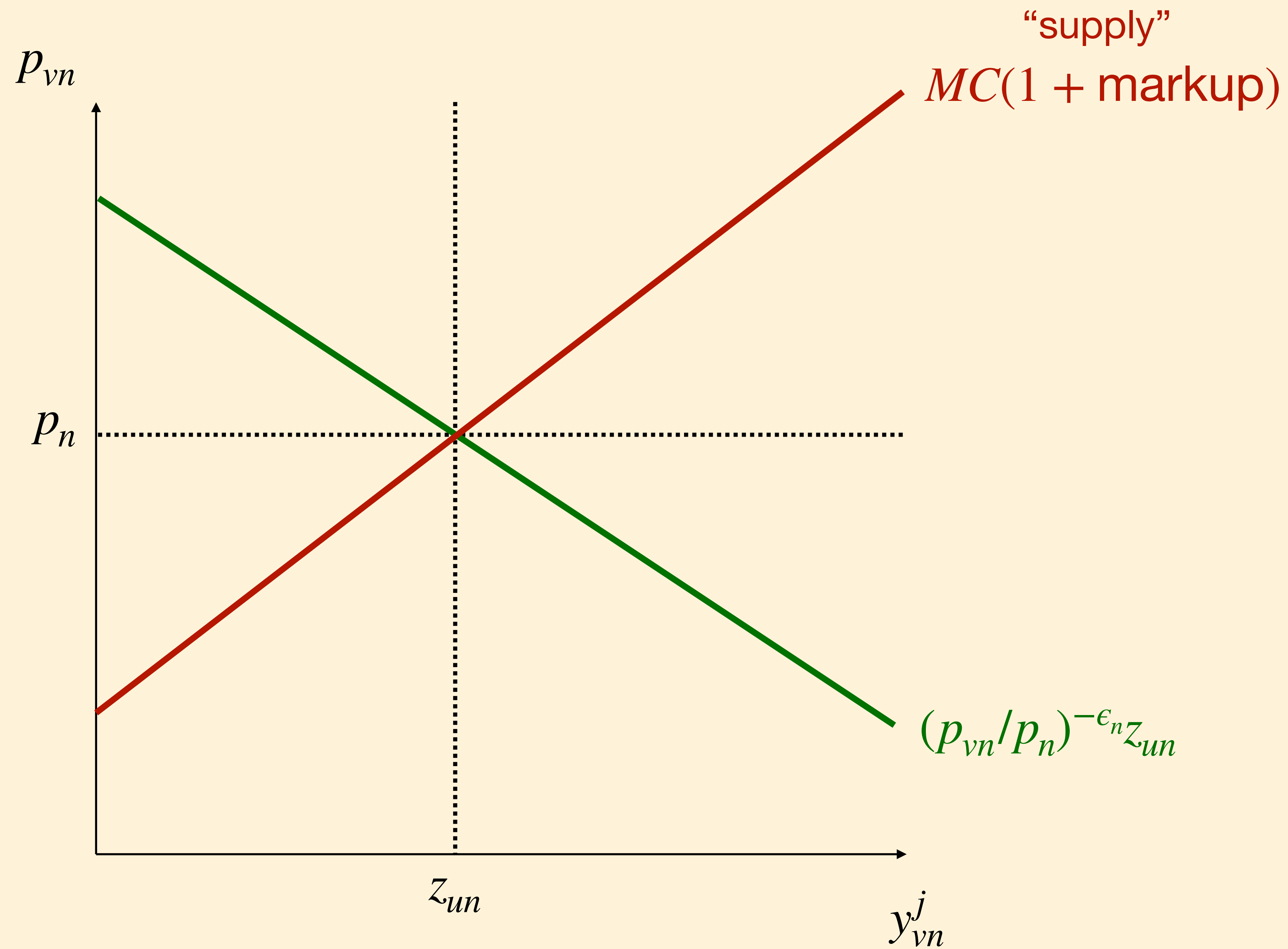


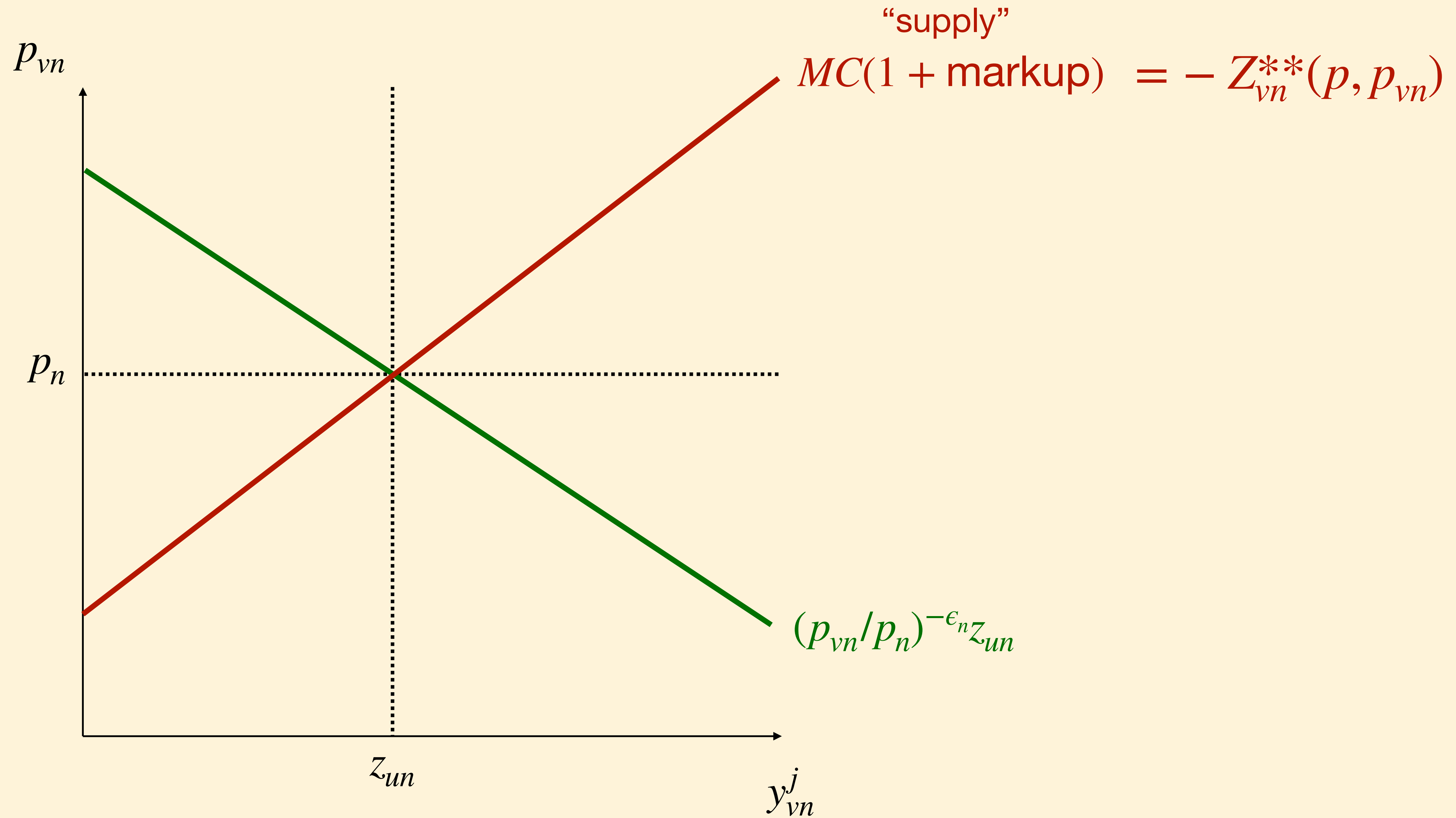


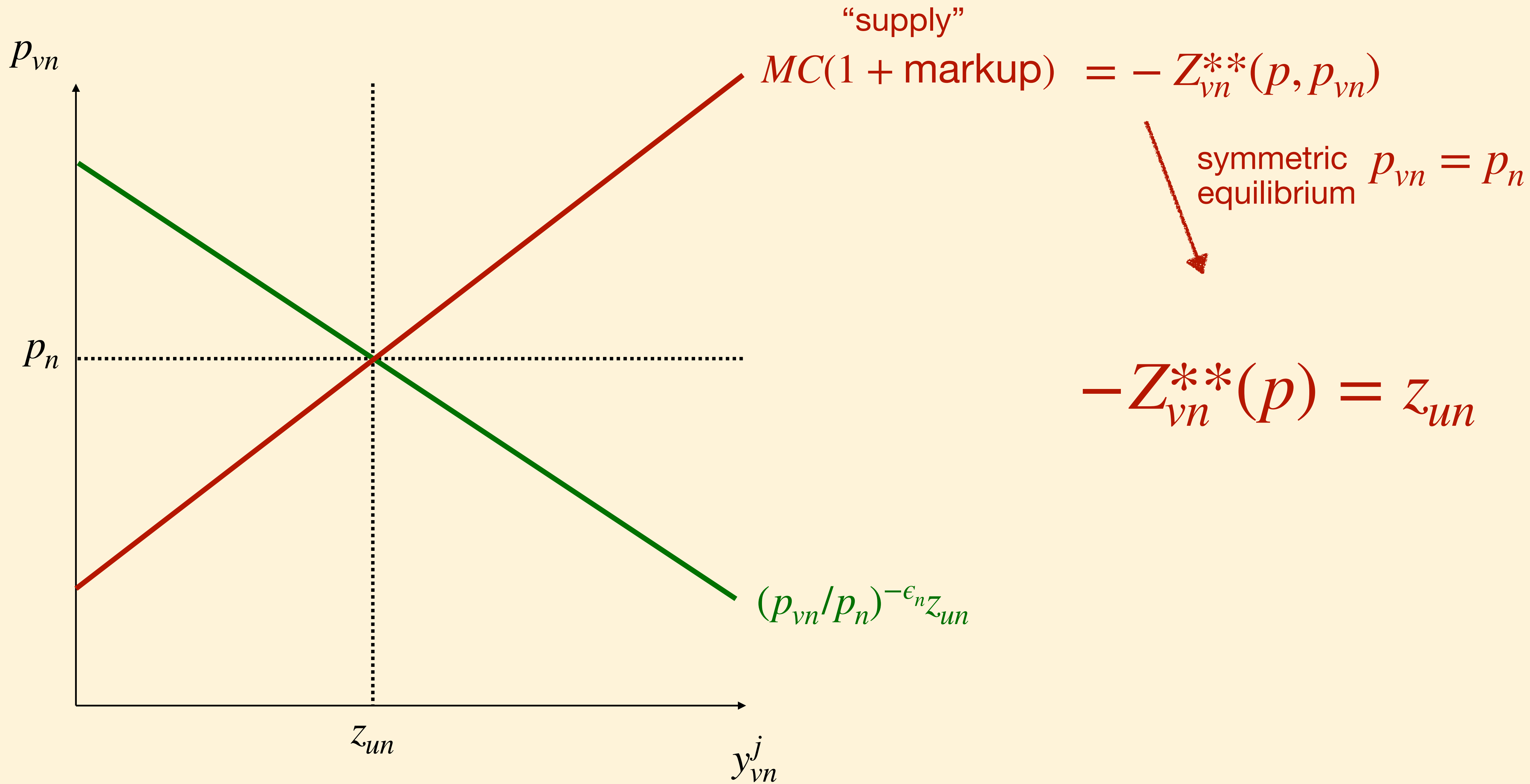












$$\begin{aligned}\Pi^f(p) &\equiv \max_{(z_u^f, z_{vn}) \in Y^f} - (p \cdot z_u^f + p_{vn} z_{vn}) \\ z_{vn} &= - (p_{vn}/p_n)^{-\epsilon_n} z_{un}\end{aligned}$$

“as if” competitive...
(+ markup omitted
here for simplicity)

$$\begin{aligned}\max U^h(z_u^h, z_{vn}) \\ p \cdot z_u^h + p_{vn} z_{vn} &\leq p \cdot a^h + \sum_f \omega^{h,f} \Pi^f(p) \\ z_{vn} &= - (p_{vn}/p_n)^{-\epsilon_n} z_{un}\end{aligned}$$

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“as if” competitive...
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$$Z^{**}(p) \equiv Z_{un}^{**}(p) + Z_{vn}^{**}(p) \longrightarrow \boxed{Z^{**}(p) = 0}$$

...exactly as if Walrasian!
(new result)

$$\begin{aligned} &\max U^h(z_u^h, z_{vn}^h) \\ &p \cdot z_u^h + p_n z_{vn}^h \leq p \cdot a^h + \sum_f \omega^{h,f} \Pi^f(p) \end{aligned}$$

Excess Demands 1.0

■ Monotone Pricing = optimal price increasing in demand

▶ automatic for firms

▶ households?

Excess Demands 1.0

■ Monotone Pricing = optimal price increasing in demand

▶ automatic for firms

▶ households?

Proposition.

Monotone Pricing

$$\Rightarrow Z^{**h}(p) = Z_u^{**h}(p) + Z_v^{**h}(p)$$

$$\hat{U}^j(z_u^j, z_{vn}, I^j)$$

(optional outside good I^j)

$$\hat{U}^j(z_u^j, z_{vn}, I^j)$$

(optional outside good I^j)

$$p \cdot z_u^j + p_{vn} z_{vn} + I^j \leq p \cdot a^j + \sum_f \omega^{j,f} \Pi^f$$

$$z_{vn} = - \left(p_{vn}/p_n \right)^{-\epsilon_n} z_{un}$$

$$V^{*j}(p, z_{vn}, p_{vn}) = \max_{z_u^j, I^j} \hat{U}^j(z_u^j, z_{vn}, I^j) \qquad \text{(optional outside good } I^j)$$

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$$\rightarrow Z_u^{*j}(p, z_{vn}, p_{vn})$$

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$$\rightarrow Z_u^{*j}(p, z_{vn}, p_{vn}) \rightarrow Z_u^{*j}(p, z_{vn}) \quad \text{when } p_{vn} = p_n$$

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$$V^{*j}(p, z_{vn}, p_{vn}) = \max_{z_u^j, I^j} \hat{U}^j(z_u^j, z_{vn}, I^j) \qquad \text{(optional outside good } I^j)$$

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$$\max_{z_{vn}, p_{vn}} V^{*j}(p, z_{un}, p_{vn}) \qquad z_{vn} = - (p_{vn}/p_n)^{-\epsilon_n} z_{un}$$

$$\rightarrow (p_{vn}^{**}(p, z_{un}), z_{vn}^{**}(p, z_{un}))$$

$$V^{*j}(p, z_{vn}, p_{vn}) = \max_{z_u^j, I^j} \hat{U}^j(z_u^j, z_{vn}, I^j) \quad \text{(optional outside good } I^j)$$

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$$\rightarrow (p_{vn}^{**}(p, z_{un}), z_{vn}^{**}(p, z_{un}))$$

increasing
in z_{un}

$$V^{*j}(p, z_{vn}, p_{vn}) = \max_{z_u^j, I^j} \hat{U}^j(z_u^j, z_{vn}, I^j) \quad (\text{optional outside good } I^j)$$

$$p \cdot z_u^j + p_{vn} z_{vn} + I^j \leq p \cdot a^j + \sum_f \omega^{j,f} \Pi^f$$

$$\rightarrow Z_u^{*j}(p, z_{vn}, p_{vn}) \rightarrow Z_u^{*j}(p, z_{vn}) \quad \text{when } p_{vn} = p_n$$

$$\max_{z_{vn}, p_{vn}} V^{*j}(p, z_{un}, p_{vn}) \quad z_{vn} = - (p_{vn}/p_n)^{-\epsilon_n} z_{un}$$

$$\rightarrow (p_{vn}^{**}(p, z_{un}), z_{vn}^{**}(p, z_{un})) \rightarrow Z_{vn}^{**}(p, p_{vn})$$

increasing in z_{un} **Price Setting → As If Walrasian Demand**

$$V^{*j}(p, z_{vn}, p_{vn}) = \max_{z_u^j, I^j} \hat{U}^j(z_u^j, z_{vn}, I^j) \quad (\text{optional outside good } I^j)$$

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$$\rightarrow Z_u^{*j}(p, z_{vn}, p_{vn}) \rightarrow Z_u^{*j}(p, z_{vn}) \quad \text{when } p_{vn} = p_n$$



Aggregating...

$$Z_u^*(p, z_v)$$

$$Z_v^{**}(p)$$

$$Z_u^{**}(p)$$

$$\max_{z_{vn}, p_{vn}} V^{*j}(p, z_{un}, p_{vn}) \quad z_{vn} = - (p_{vn}/p_n)^{-\epsilon_n} z_{un}$$

$$\rightarrow (p_{vn}^{**}(p, z_{un}), z_{vn}^{**}(p, z_{un})) \rightarrow Z_{vn}^{**}(p, p_{vn})$$

increasing
in z_{un}

Price Setting → As If Walrasian Demand

$$V^{*j}(p, z_{vn}, p_{vn}) = \max_{z_u^j, I^j} \hat{U}^j(z_u^j, z_{vn}, I^j) \quad (\text{optional outside good } I^j)$$

$$p \cdot z_u^j + p_{vn} z_{vn} + I^j \leq p \cdot a^j + \sum_f \omega^{j,f} \Pi^f$$

$$\rightarrow Z_u^{*j}(p, z_{vn}, p_{vn}) \rightarrow Z_u^{*j}(p, z_{vn}) \quad \text{when } p_{vn} = p_n$$



Aggregating...

$$Z_u^*(p, z_v)$$

$$Z_v^{**}(p)$$

$$Z_u^{**}(p)$$

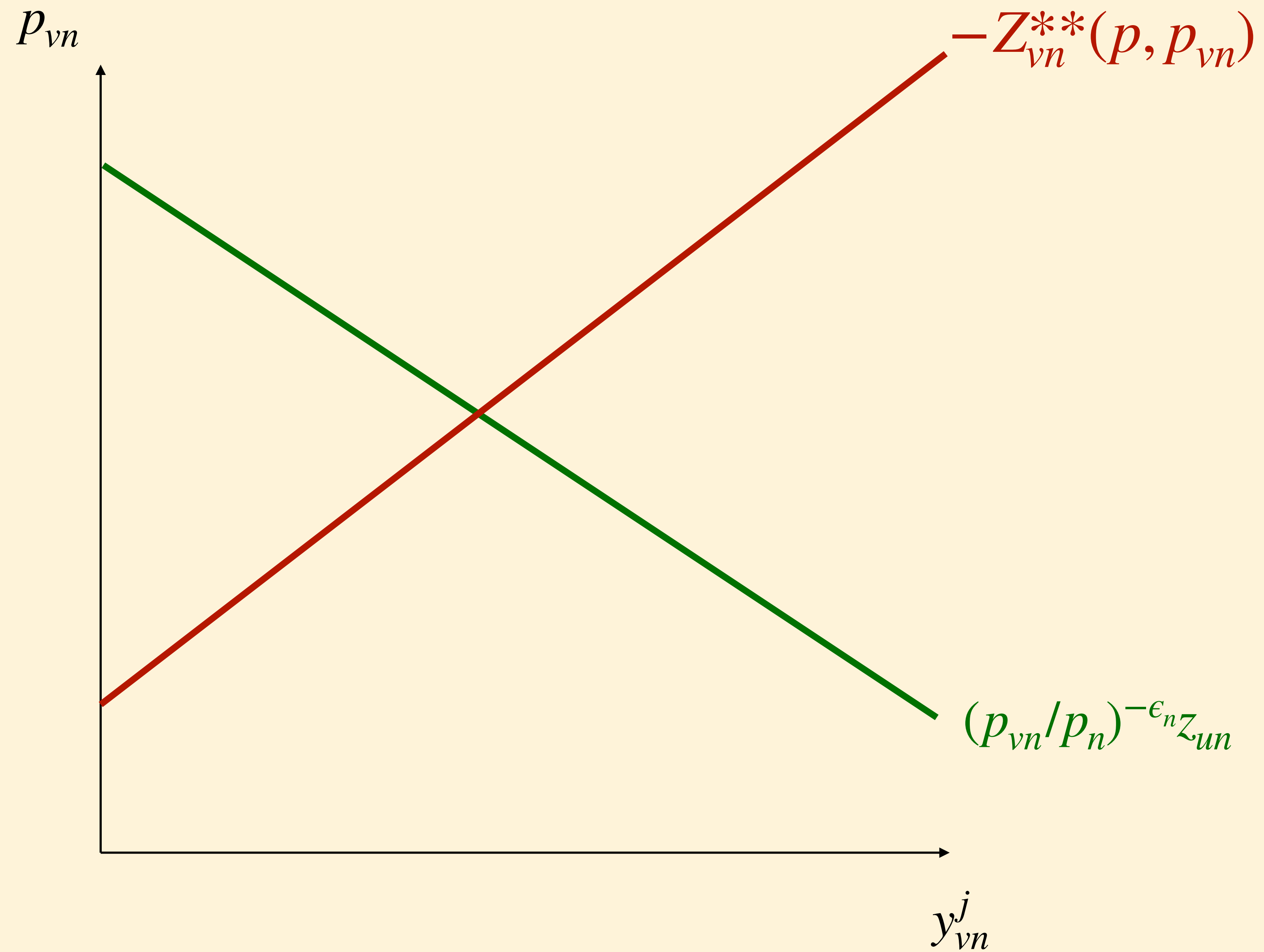
$$\max_{z_{vn}, p_{vn}} V^{*j}(p, z_{un}, p_{vn}) \quad z_{vn} = - (p_{vn}/p_n)^{-\epsilon_n} z_{un}$$

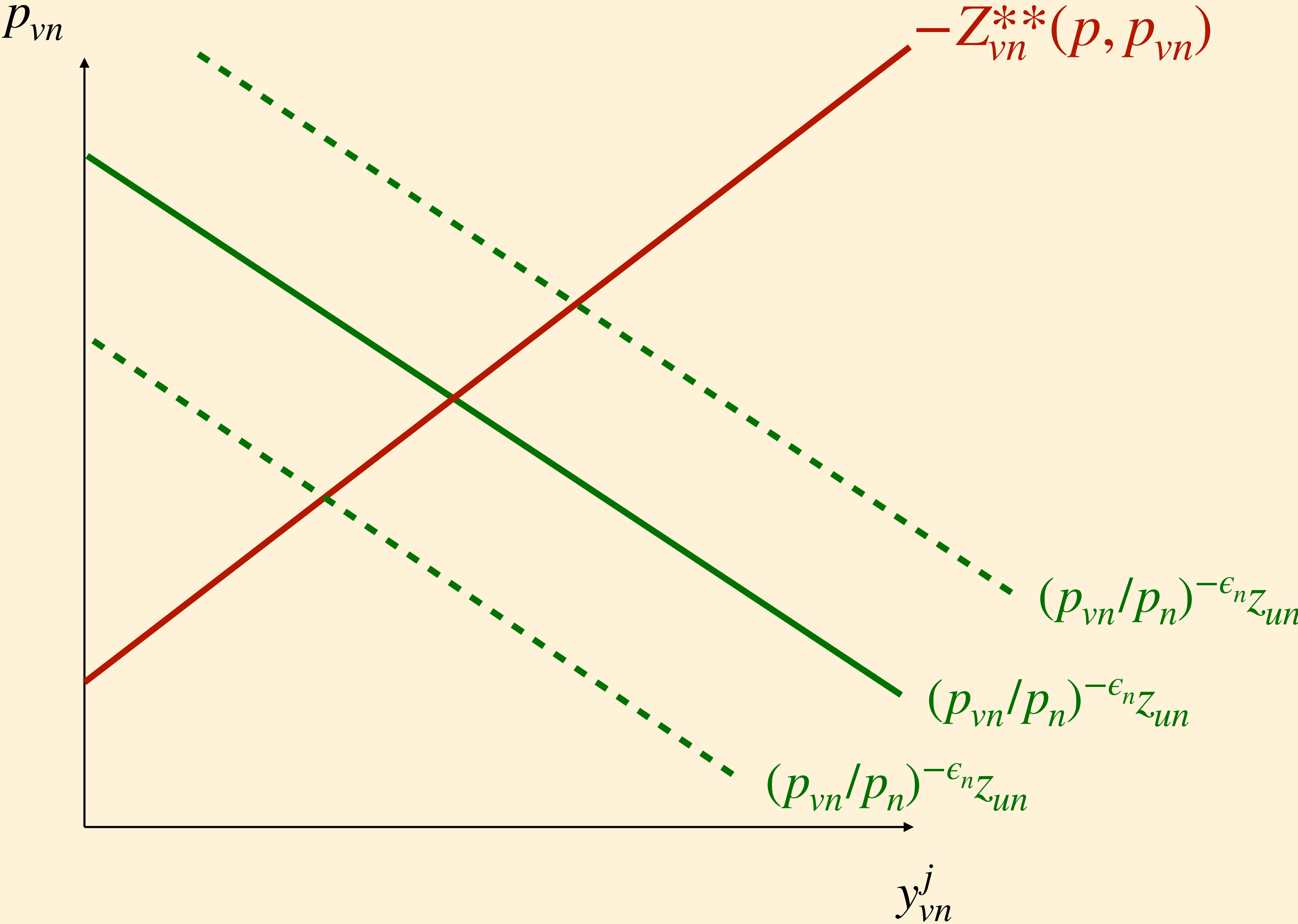
$$\rightarrow (p_{vn}^{**}(p, z_{un}), z_{vn}^{**}(p, z_{un}))$$

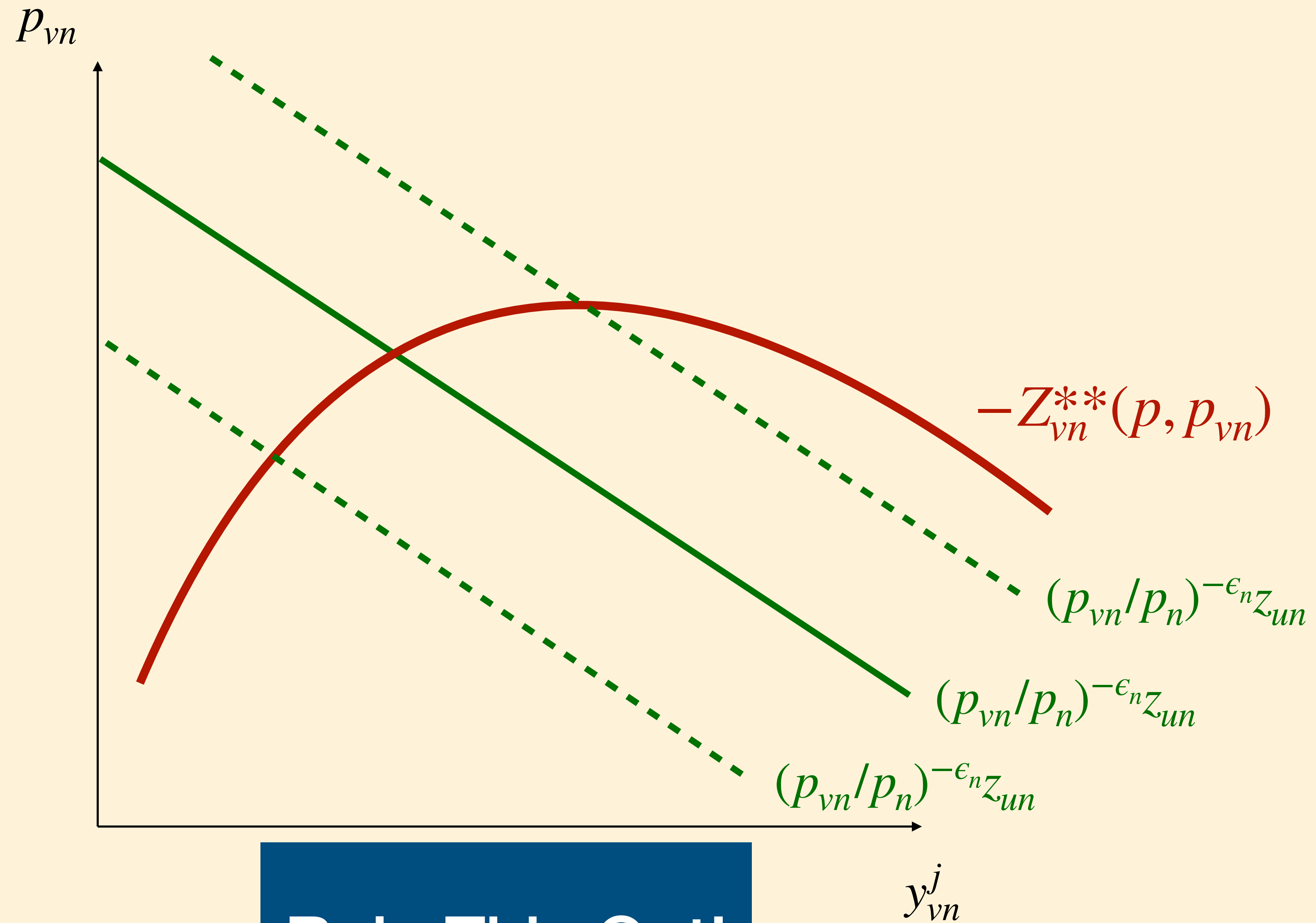
increasing
in z_{un}

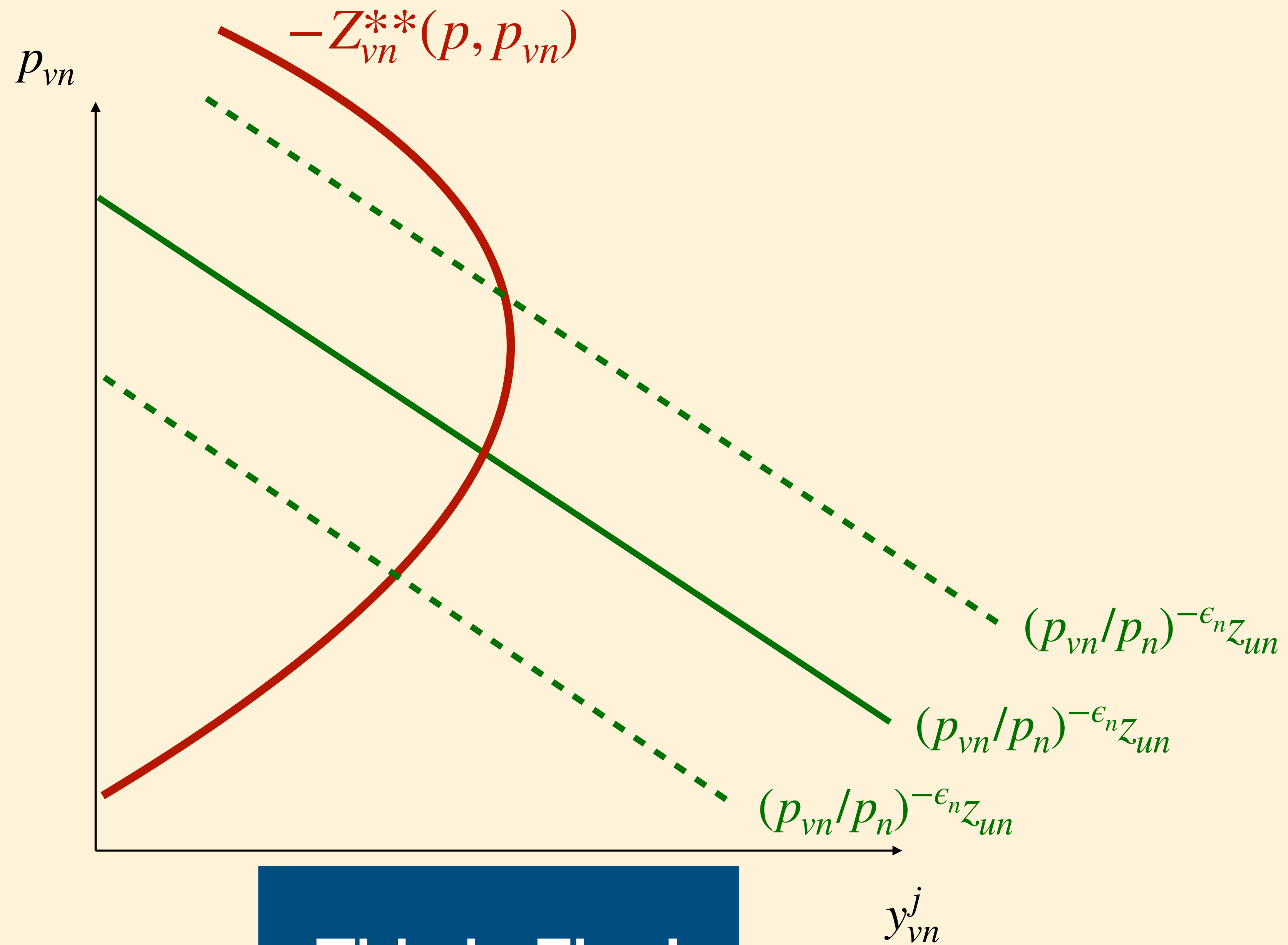
$$\rightarrow Z_{vn}^{**}(p, p_{vn}) \rightarrow Z_{vn}^{**}(p, p_{vn})$$

Price Setting → As If Walrasian Demand

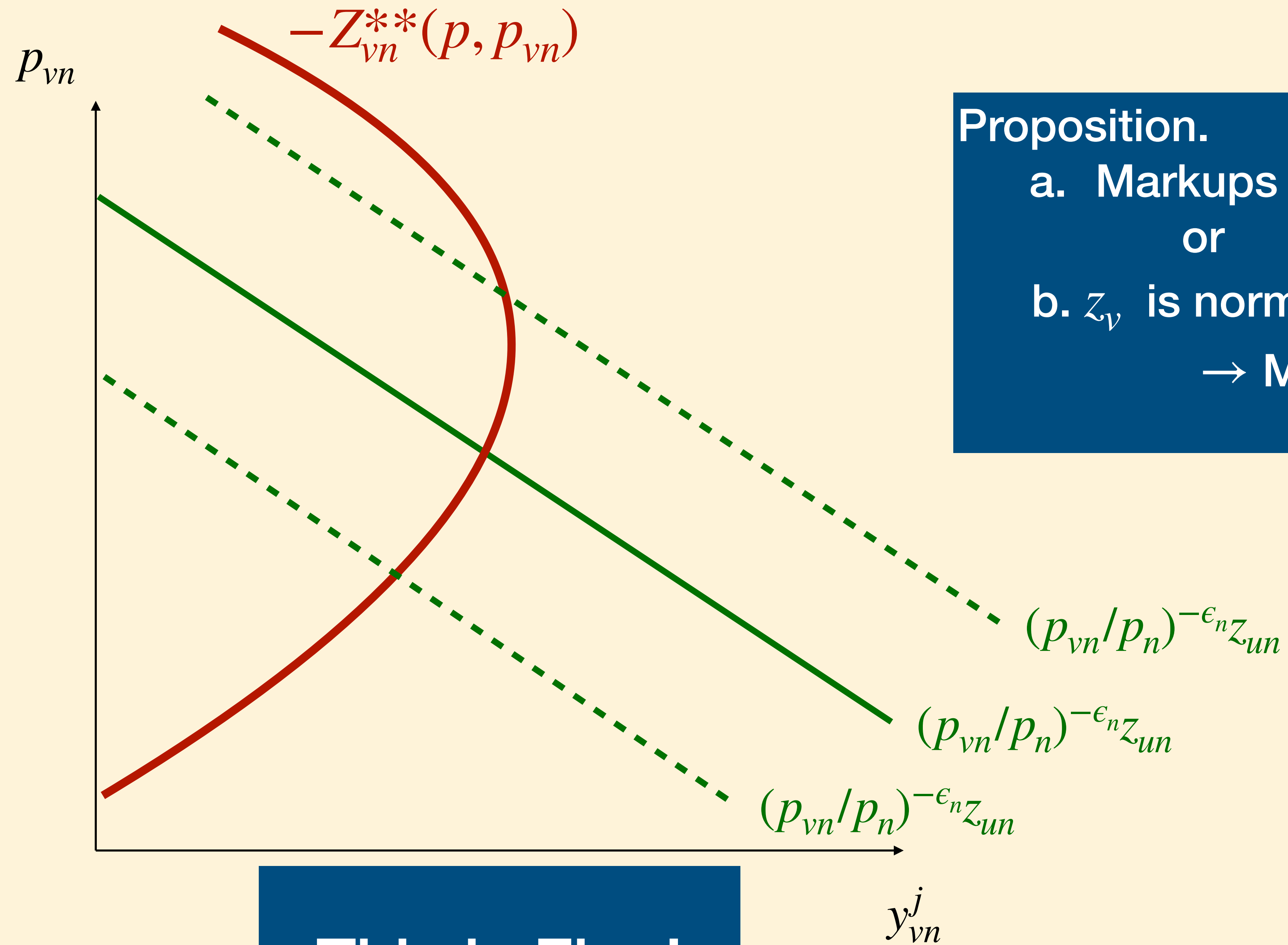








This is Fine!



Proposition.
 a. Markups are not too large
 or
 b. z_v is normal
 → Monotone Pricing

This is Fine!

Excess Demands 1.0

■ Monotone Pricing = optimal price increasing in demand

► automatic for firms

► households?

Proposition.

Monotone Pricing

→

$$Z^{**h}(p) = Z_u^{**h}(p) + Z_v^{**h}(p)$$

Proposition.

a. Markups are not too large
or

b. z_v is normal

⇒ Monotone Pricing

**Weak Sufficient
Condition!**

Static Price Frictions

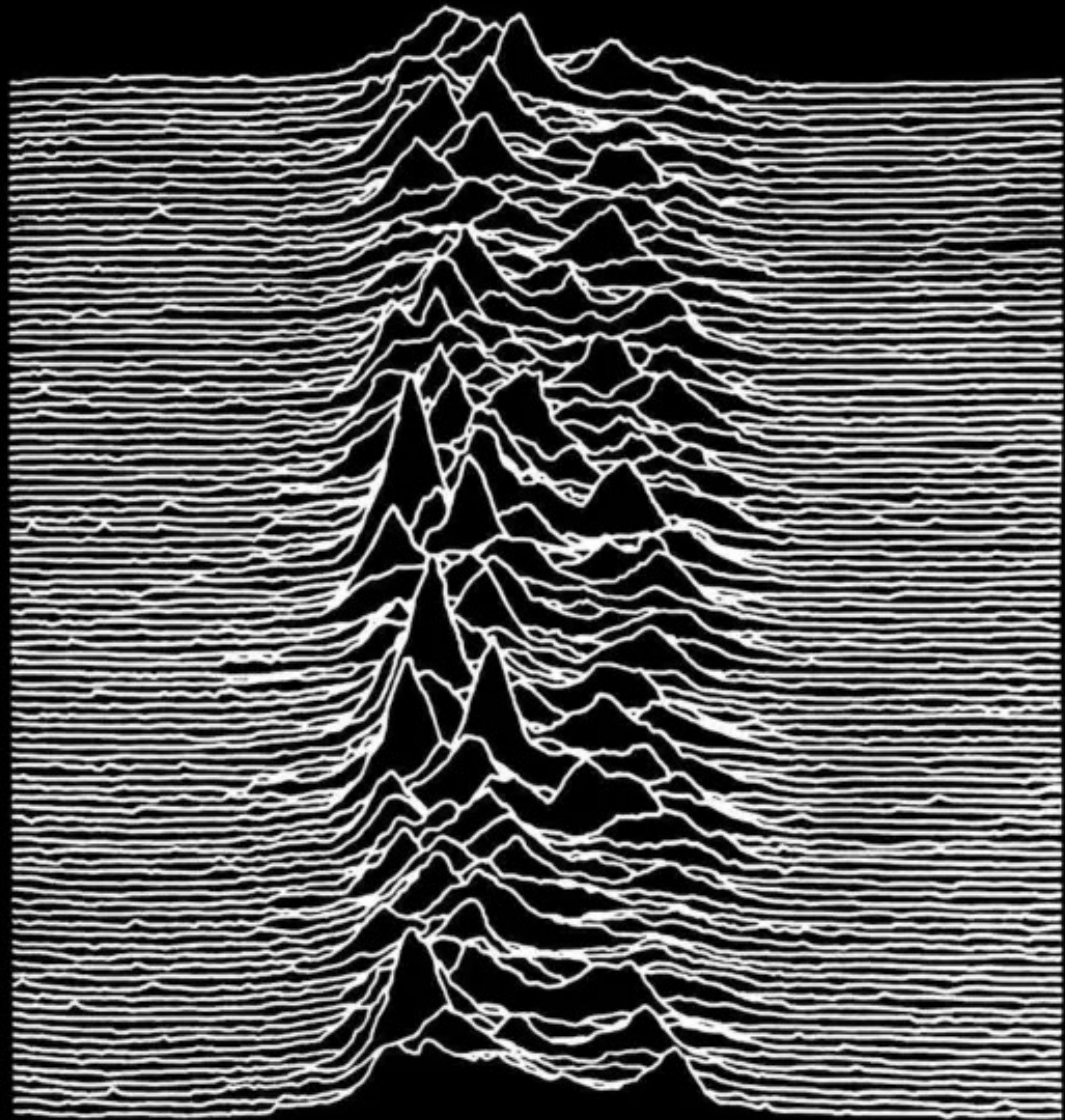
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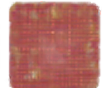
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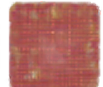
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Non-Walrasian Equilibria

Non-Walrasian Equilibria

 **Coming...** Price Frictions + Dynamics

 **First....** Price Frictions

Non-Walrasian Equilibria

- **Coming...** Price Frictions + Dynamics

- **First....** Price Frictions

- Non-Walrasian (temporary) equilibrium...

 - ▶ price pressure market n $\leftrightarrow$ excess demand $\tilde{z}_n = z_{un} + Z_{vn}^{**}(p)$

Non-Walrasian Equilibria

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 - ▶ solve equilibrium z_{un} or $\tilde{z}_n$ quantities at given prices p ...

Non-Walrasian Equilibria

- **Coming...** Price Frictions + Dynamics

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 - ▶ solve equilibrium z_{un} or $\tilde{z}_n$ quantities at given prices p ...

 - ▶ equilibrium excess demand $\neq$ Walrasian excess demand...

Non-Walrasian Equilibria

■ **Coming...** Price Frictions + Dynamics

■ **First....** Price Frictions

■ Non-Walrasian (temporary) equilibrium...

▶ price pressure market n $\leftrightarrow$ excess demand $\tilde{z}_n = z_{un} + Z_{vn}^{**}(p)$

▶ solve equilibrium z_{un} or $\tilde{z}_n$ quantities at given prices p ...

▶ equilibrium excess demand $\neq$ Walrasian excess demand...

$$Z(p) = (I - A)^{-1} Z^{**}(p) \quad (A = \text{Spillover Matrix})$$

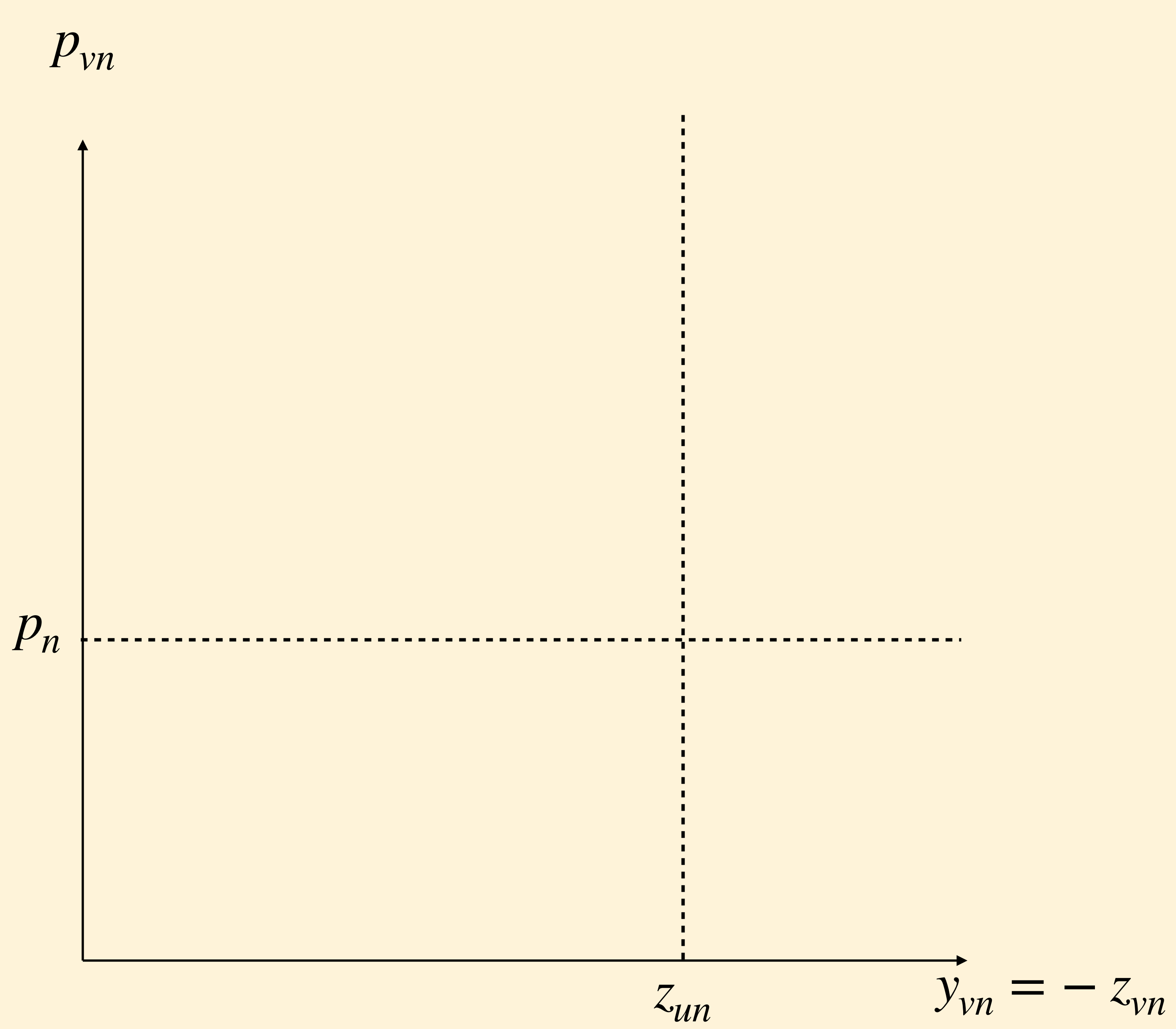
Price Pressure and Excess Demand

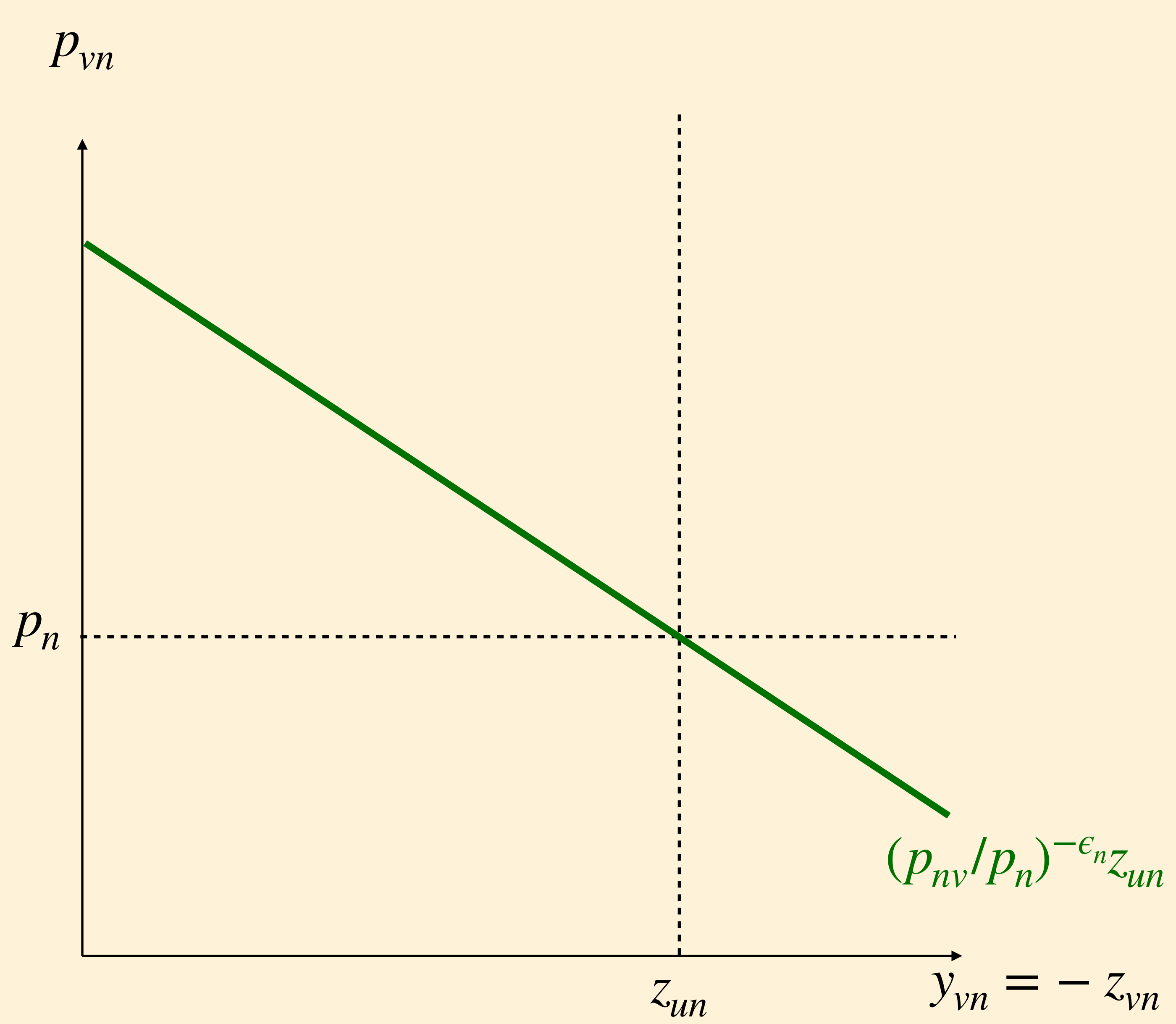
■ Price deviator: $p_{vn} \neq p_n$

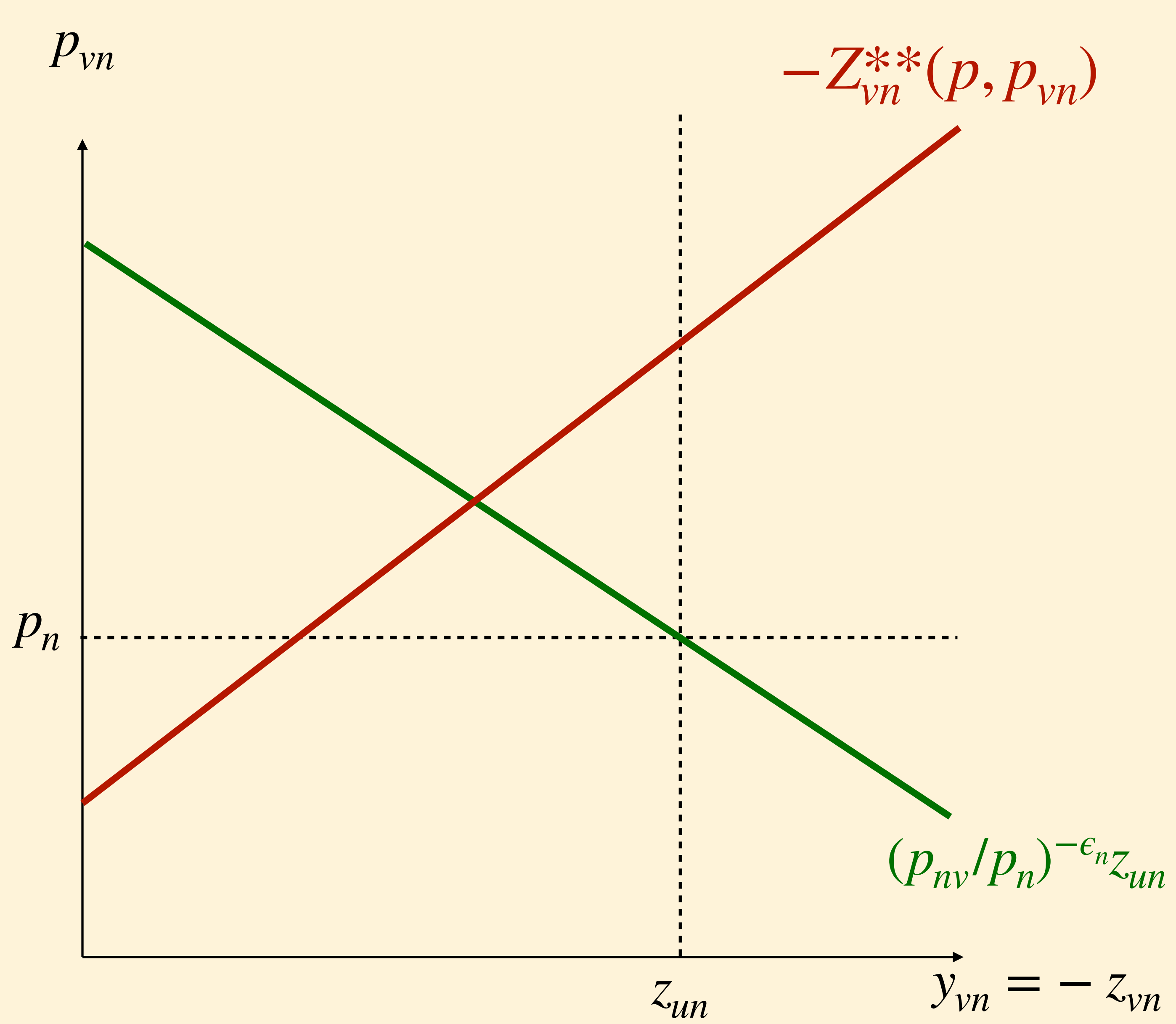
■ Market clearing...

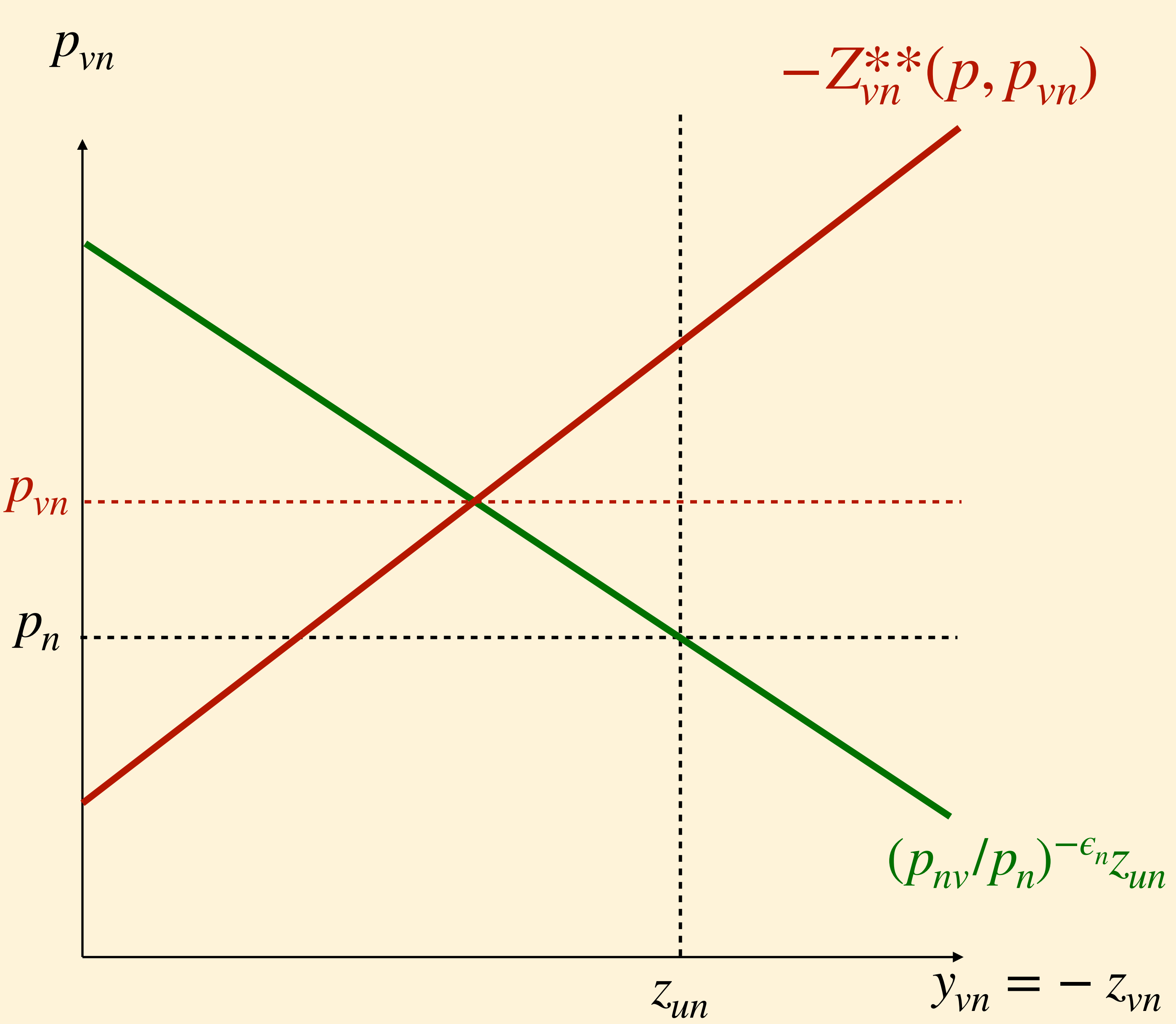
$$(p_{vn}/p_n)^{-\epsilon_n} z_{un} + Z_{vn}^{**}(p, p_{vn}) = 0$$

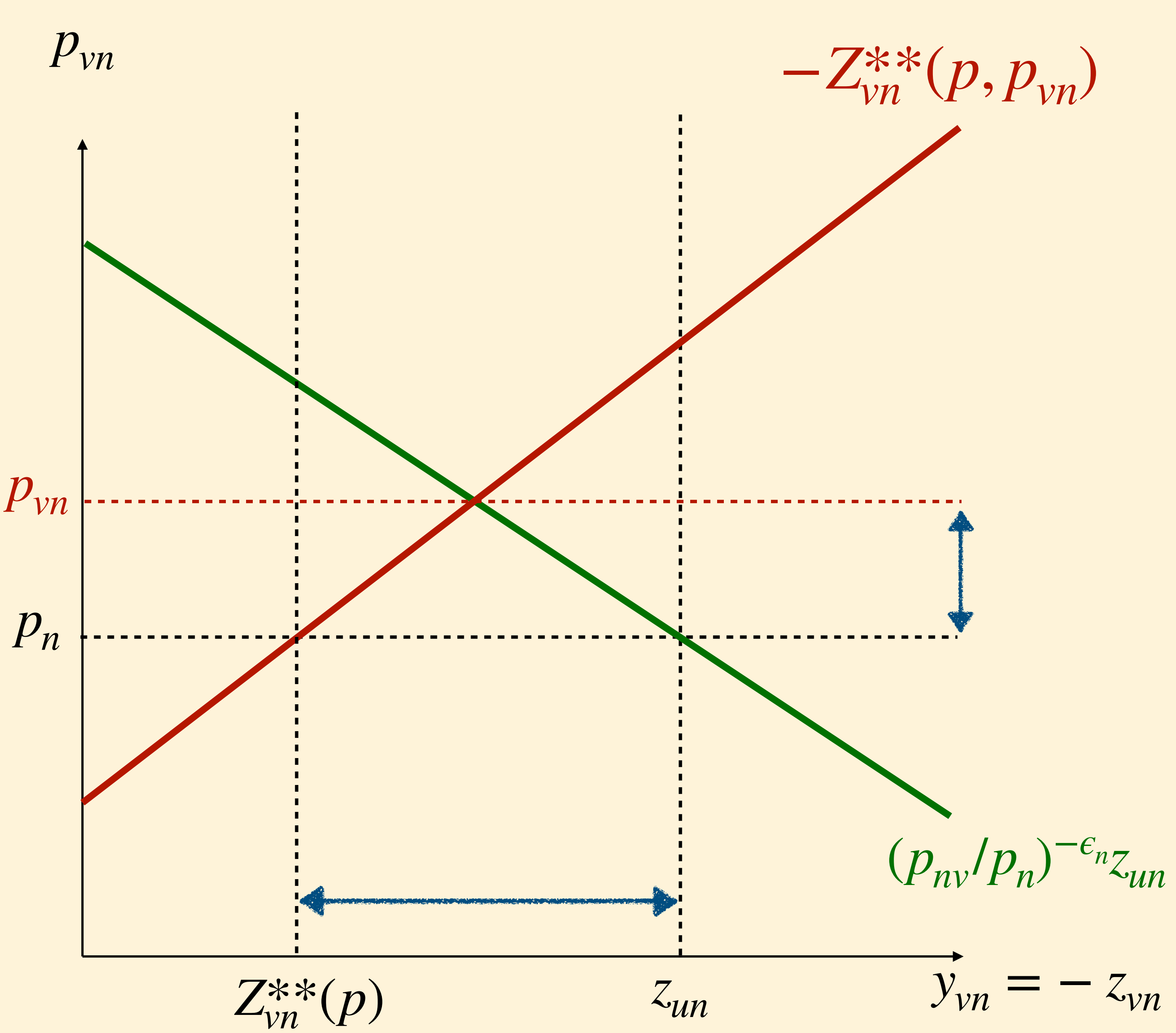
$$Z_v^{**}(p)$$











Price Pressure and Excess Demand

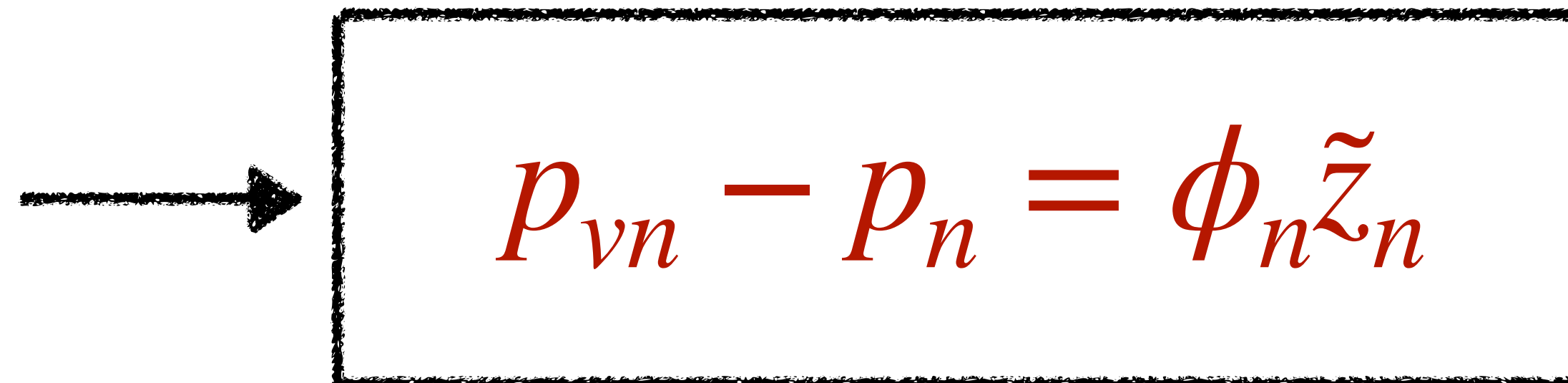
■ Price deviator: $p_{vn} \neq p_n$

■ Market clearing...

$$(p_{vn}/p_n)^{-\epsilon_n} z_{un} + Z_{vn}^{**}(p, p_{vn}) = 0$$

■ Excess (Effective) Demand...

$$\tilde{z}_n \equiv z_{un} + Z_{vn}^{**}(p)$$


$$p_{vn} - p_n = \phi_n \tilde{z}_n$$

(nonlinear in paper)

New Keynesian Literature:
excess demand $\neq$ “gaps”)

Disequilibrium Literature:
did not have this

Equilibrium Quantities at Fixed Prices

$$Z_u^*(p, z_v)$$

$$Z_v^{**}(p)$$

Equilibrium Quantities at Fixed Prices

■ Fixed price: $p_{vn} = p_n$

$$z_u = Z_u^*(p, z_v)$$

$$z_{un} + z_{vn} = 0$$

$$\rightarrow z_u = Z_u^*(p, -z_u)$$

$$Z_u^*(p, z_v)$$

$$Z_v^{**}(p)$$

Equilibrium Quantities at Fixed Prices

■ Fixed price: $p_{vn} = p_n$

$$z_u = Z_u^*(p, z_v)$$

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$$\rightarrow z_u = Z_u^*(p, -z_u)$$

$$\rightarrow \tilde{z} = Z^{**}(p) + A\tilde{z}$$

↓
spillover
matrix

$$Z_u^*(p, z_v)$$

$$Z_v^{**}(p)$$

Equilibrium Quantities at Fixed Prices

$$Z_u^*(p, z_v)$$

$$Z_v^{**}(p)$$

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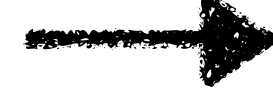
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↓
spillover
matrix



Effective
Demand

Notional
Demand

$$Z(p) = (I - A)^{-1} Z^{**}(p)$$

Equilibrium Quantities at Fixed Prices

$$Z_u^*(p, z_v)$$

$$Z_v^{**}(p)$$

■ Fixed price: $p_{vn} = p_n$

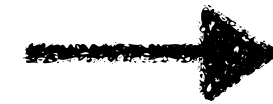
$$z_u = Z_u^*(p, z_v)$$

$$z_{un} + z_{vn} = 0$$

$$\rightarrow z_u = Z_u^*(p, -z_u)$$

$$\rightarrow \tilde{z} = Z^{**}(p) + A\tilde{z}$$

spillover
matrix



Effective
Demand

Notional
Demand

$$Z(p) = (I - A)^{-1} Z^{**}(p)$$

$$r(A) < 1$$

$$(I - A)^{-1} = I + A + A^2 + \dots$$

Leontief Inverse

$$Z(p) = (I - A)^{-1} Z^{**}(p)$$

$$Z(p) = (I - A)^{-1} Z^{**}(p)$$

$$p_{nv} - p_n = \phi_n Z_n(p)$$

(nonlinear in paper)

Static Phillips Curve

Static Phillips Curve

■ Price Pressure $\leftrightarrow$ Equilibrium price adjustment? **Yes.**
Later dynamic, now static setting...

► fraction θ_n $\rightarrow$ reset prices optimally

► fraction $1 - \theta_n$ $\rightarrow$ exogenous prices $p_{vn} = p_n$

► index $\hat{p}_n = (1 - \theta_n)p_n + \theta_n p_{vn}^{**}$

Static Phillips Curve

- Price Pressure $\leftrightarrow$ Equilibrium price adjustment? **Yes.**
Later dynamic, now static setting...
- ▶ fraction θ_n $\rightarrow$ reset prices optimally
- ▶ fraction $1 - \theta_n$ $\rightarrow$ exogenous prices $p_{vn} = p_n$
- ▶ index $\hat{p}_n = (1 - \theta_n)p_n + \theta_n p_{vn}^{**}$
- Optimal price reset $\rightarrow p_{vn}^{**} - p_n = \gamma_n Z_n(\hat{p})$

Static Phillips Curve

- Price Pressure $\leftrightarrow$ Equilibrium price adjustment? **Yes.**
Later dynamic, now static setting...

► fraction θ_n $\rightarrow$ reset prices optimally

► fraction $1 - \theta_n$ $\rightarrow$ exogenous prices $p_{vn} = p_n$

► index $\hat{p}_n = (1 - \theta_n)p_n + \theta_n p_{vn}^{***}$

- Optimal price reset $\rightarrow p_{vn}^{***} - p_n = \gamma_n Z_n(\hat{p})$

- Define: $\pi_n \equiv \hat{p}_n - p_n$

Proposition.

$$\pi_n = \theta_n \gamma_n Z_n(p)$$

Dynamic Price Adjustment

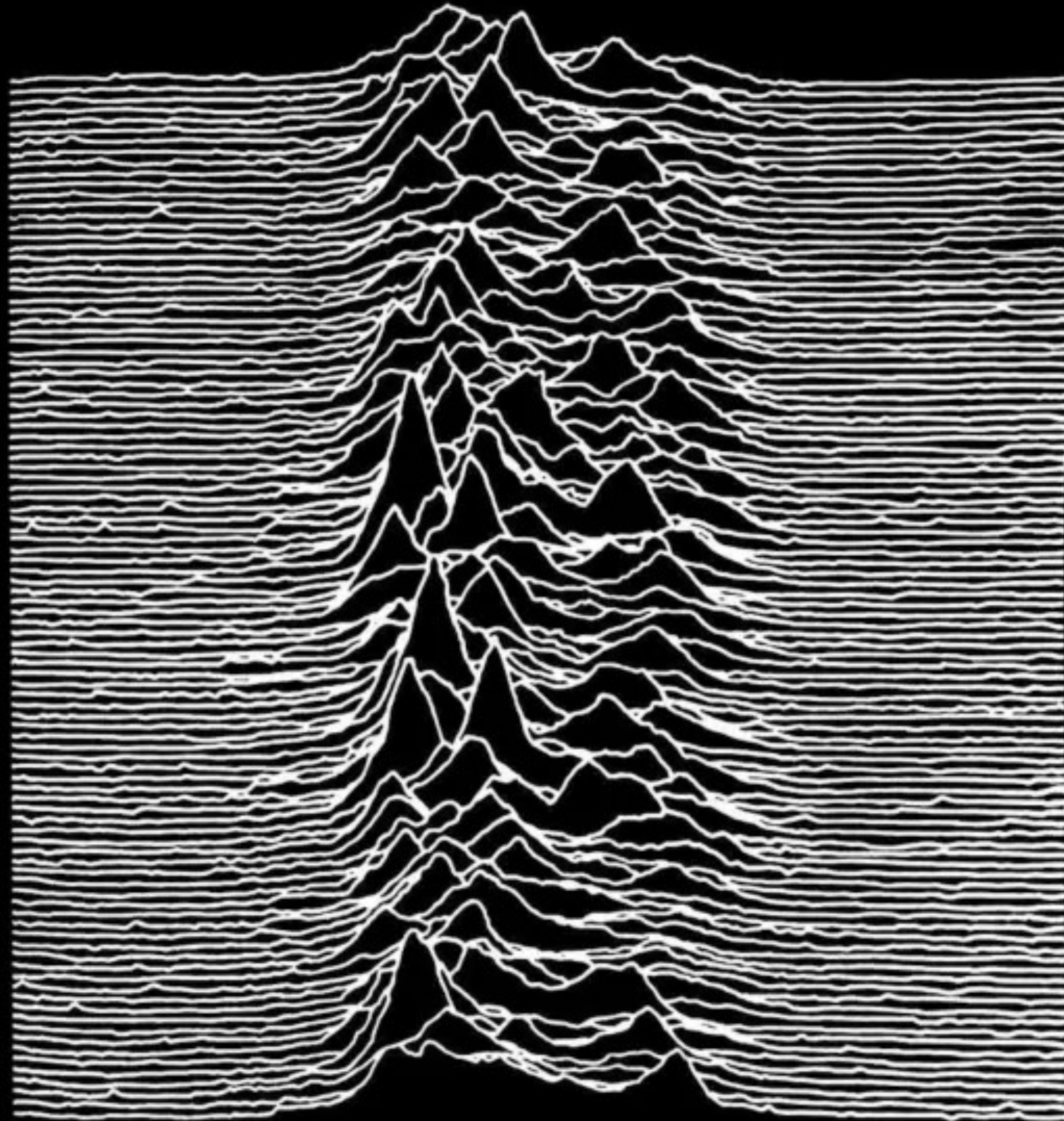
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Adding Dynamics

■ Static GE $\rightarrow$ **Dynamic GE...**

► repeated GE spot each $t \rightarrow$ previous GE model natural reference

► intertemporal choices $+$ add price frictions à la Calvo

Adding Dynamics

■ Static GE → **Dynamic GE...**

▶ repeated GE spot each t → previous GE model natural reference

▶ intertemporal choices + add price frictions à la Calvo

▶ NK model but...

setup	→ more general (GE tradition)
approach	→ excess demand (GE tradition)

$$\Pi^f \equiv \max \qquad \qquad \qquad - (p \cdot z_u^f + p_{vn}z_{vn})$$

$$(z_{ut}^f,z_{vnt}) \in Y^f$$

$$z_{vnt} = - \left(p_{vnt}/p_{nt} \right)^{-\epsilon_n} z_{unt}$$

$$\max \qquad \qquad \qquad U^h(z_u^h,z_{vn})$$

$$p \cdot z_u^h + p_{vn}z_{vn} \qquad \leq p \cdot a^h + \sum_f \omega^{h,f} \Pi^f$$

$$z_{vnt} = - \left(p_{vnt}/p_{nt} \right)^{-\epsilon_n} z_{unt}$$

$$\Pi^f \equiv \max - \mathbb{E} \sum_0^\infty \textcolor{blue}{Q}_{\textcolor{blue}{t}} (p_t \cdot z_{ut}^f + p_{vnt} z_{vnt})$$

$$(z_{ut}^f,z_{vnt})\in Y^f$$

$$z_{vnt} = -\left(p_{vnt}/p_{nt}\right)^{-\epsilon_n} z_{unt}$$

$$\textcolor{blue}{Q}_t = \frac{1}{(1+i_0)(1+i_1)\cdots(1+i_{t-1})}$$

$$\max \qquad U^h(z_u^h,z_{vn})$$

$$p\cdot z_u^h+p_{vn}z_{vn}\qquad\leq p\cdot a^h+\sum_f\omega^{h,f}\Pi^f$$

$$z_{vnt} = -\left(p_{vnt}/p_{nt}\right)^{-\epsilon_n} z_{unt}$$

$$\Pi^f \equiv \max - \mathbb{E} \sum_0^\infty \textcolor{blue}{Q}_t (p_t \cdot z_{ut}^f + p_{vnt} z_{vnt})$$

$$(z_{ut}^f,z_{vnt}) \in Y^f$$

$$z_{vnt} = - \left(p_{vnt}/p_{nt}\right)^{-\epsilon_n} z_{unt}$$

$$\textcolor{blue}{Q}_t = \frac{1}{(1+i_0)(1+i_1)\cdots(1+i_{t-1})}$$

$$\max \mathbb{E} \sum_0^\infty \textcolor{blue}{\beta}^t U^h(z_{ut}^h,z_{vnt})$$

$$p \cdot z_u^h + p_{vn} z_{vn} \qquad \leq p \cdot a^h + \sum_f \omega^{h,f} \Pi^f$$

$$z_{vnt} = - \left(p_{vnt}/p_{nt}\right)^{-\epsilon_n} z_{unt}$$

$$\Pi^f \equiv \max - \mathbb{E} \sum_0^\infty \textcolor{blue}{Q}_{\textcolor{blue}{t}} (p_t \cdot z_{ut}^f + p_{vnt} z_{vnt})$$

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$$\mathbb{E} \sum_0^\infty \textcolor{blue}{Q}_{\textcolor{blue}{t}} (p_t \cdot z_{ut}^h + p_{vnt} z_{vnt}) \leq p_0 \cdot a_0^h + \sum_f \omega^{h,f} \Pi^f$$

$$z_{vnt} = -\left(p_{vnt}/p_{nt}\right)^{-\epsilon_n} z_{unt}$$

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$$(z_{ut}^f, z_{vnt}) \in Y^f$$

$$z_{vnt} = - (p_{vnt}/p_{nt})^{-\epsilon_n} z_{unt}$$

s.t. Calvo pricing friction

$$Q_t = \frac{1}{(1 + i_0)(1 + i_1) \cdots (1 + i_{t-1})}$$

$$\max \mathbb{E} \sum_0^\infty \beta^t U^h(z_{ut}^h, z_{vnt})$$

$$\mathbb{E} \sum_0^\infty Q_t(p_t \cdot z_{ut}^h + p_{vnt}z_{vnt}) \leq p_0 \cdot a_0^h + \sum_f \omega^{h,f} \Pi^f$$

$$z_{vnt} = - (p_{vnt}/p_{nt})^{-\epsilon_n} z_{unt}$$

s.t. Calvo pricing friction

$$Q_t = \beta^t$$

For now...

$$\begin{aligned}\Pi^f \equiv \max & - \sum_0^\infty \beta^t [p_t \cdot z_{ut} + p_{vnt} y_{vnt}] \\ & (z_{ut}, z_{vnt}) \in Y^f\end{aligned}$$

$Q_t = \beta^t$
For now...

$$\max \sum_0^\infty \beta^t U^h(z_{ut}, z_{vnt})$$

$$\sum_0^\infty \beta^t [p_t \cdot z_{ut} + p_{nv} z_{nvt} - p_t \cdot a^h] \leq \sum_f \omega^{h,f} \Pi^f$$

$$\Pi^f \equiv \max - \sum_0^\infty \beta^t [p_t \cdot z_{ut} + p_{vnt} y_{vnt}]$$

$$(z_{ut}, z_{vnt}) \in Y^f$$



(Lagrangian)

$$L^j = \mu^j \sum_0^\infty \beta^t \left[\frac{1}{\mu^j} U^j(z_{ut}, z_{vnt}) - p_t \cdot z_{ut} - p_{vnt} z_{vnt} \right]$$



μ^j = Lagrange multiplier on agent j budget constraint

$$\max \sum_0^\infty \beta^t U^h(z_{ut}, z_{vnt})$$

$$\sum_0^\infty \beta^t [p_t \cdot z_{ut} + p_{nv} z_{nvt} - p_t \cdot a^h] \leq \sum_f \omega^{h,f} \Pi^f$$

$Q_t = \beta^t$
For now...

$$\Pi^f \equiv \max - \sum_0^\infty \beta^t [p_t \cdot z_{ut} + p_{vnt} y_{vnt}]$$

$$(z_{ut}, z_{vnt}) \in Y^f$$



(Lagrangian)

$$L^j = \mu^j \sum_0^\infty \beta^t \left[\frac{1}{\mu^j} U^j(z_{ut}, z_{vnt}) - p_t \cdot z_{ut} - p_{vnt} z_{vnt} \right]$$



only this matters at a point in time...

μ^j = Lagrange multiplier on agent j budget constraint

$Q_t = \beta^t$
For now...

$$\max \sum_0^\infty \beta^t U^h(z_{ut}, z_{vnt})$$

$$\sum_0^\infty \beta^t [p_t \cdot z_{ut} + p_{nv} z_{nvt} - p_t \cdot a^h] \leq \sum_f \omega^{h,f} \Pi^f$$

$$\frac{1}{\mu^j} U^j(z_{ut}, z_{vnt}) - p_t \cdot z_{ut} - p_{vnt} z_{vnt}$$

“as if” utility
as in
static analysis...



$$\hat{U}^j = \frac{1}{\mu^j} U^j(z_u, z_{vn}) + I^j$$

“as if”
quasilinear
outside good



“as if” utility
as in
static analysis...



$$\hat{U}^j = \frac{1}{\mu^j} U^j(z_u, z_{vn}) + I^j$$

“as if”
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outside good



$Z(p)$
 $Z_u^*(p, z_v)$
 $Z_v^{**}(p)$

“as if” utility
as in
static analysis...

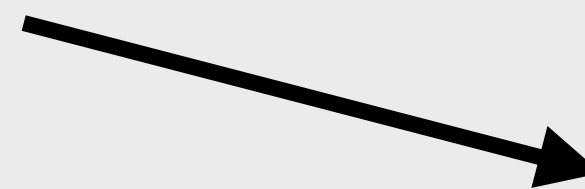


$$\hat{U}^j = \frac{1}{\mu^j} U^j(z_u, z_{vn}) + I^j$$

“as if”
quasilinear
outside good



except now...



Frisch Demands

“As If” Representative Agent
with Quasilinear Utility

Removes income effects
along path

$$\begin{aligned} Z(p) \\ Z_u^*(p, z_v) \\ Z_v^{**}(p) \end{aligned}$$

**Skip Details of
Dynamic Pricing Problem**

**But similar to static
+ standard NK math magic**



$$\begin{aligned} Z(p) \\ Z_u^*(p, z_v) \\ Z_v^{**}(p) \end{aligned}$$

Proposition.

$$\pi_{nt} = d_n Z_n(p_t) + \beta \mathbb{E}_t \pi_{nt+1}$$

$$(d_n = \theta_n(1 + \theta_n/\rho)\phi_n)$$



$$Z(p)$$

$$Z_u^*(p, z_v)$$

$$Z_v^{**}(p)$$

Excess Demand Drives Price Adjustment

Proposition.

$$\pi_{nt} = d_n Z_n(p_t) + \beta \mathbb{E}_t \pi_{nt+1}$$

$$(d_n = \theta_n(1 + \theta_n/\rho)\phi_n)$$



$$Z(p)$$

$$Z_u^*(p, z_v)$$

$$Z_v^{**}(p)$$

Excess Demand $\neq$ Output Gap Drives Price Adjustment

Proposition.

$$\pi_{nt} = d_n Z_n(p_t) + \beta \mathbb{E}_t \pi_{nt+1}$$

$$(d_n = \theta_n(1 + \theta_n/\rho)\phi_n)$$



$$Z(p)$$

$$Z_u^*(p, z_v)$$

$$Z_v^{**}(p)$$

Excess demand in market n only!

**Excess Demand $\neq$ Output Gap
Drives Price Adjustment**

Proposition.

$$\pi_{nt} = d_n Z_n(p_t) + \beta \mathbb{E}_t \pi_{nt+1}$$

$$(d_n = \theta_n(1 + \theta_n/\rho)\phi_n)$$



$Z(p)$

$Z_u^*(p, z_v)$

$Z_v^{**}(p)$

Excess demand in market n only!

**Excess Demand $\neq$ Output Gap
Drives Price Adjustment**

Proposition.

$$\pi_{nt} = d_n Z_n(p_t) + \beta \mathbb{E}_t \pi_{nt+1}$$

$$(d_n = \theta_n(1 + \theta_n/\rho)\phi_n)$$

$$\begin{aligned} &Z(p) \\ &Z_u^*(p, z_v) \\ &Z_v^{**}(p) \end{aligned}$$



$$\rightarrow \pi_{nt} = d_n \sum_{s=0}^{\infty} (1 - \beta) \beta^s Z_n(p_{t+s})$$

$$\pi_{nt} = d_n z_{nt} + \beta \mathbb{E}_t \pi_{nt+1}$$

Applications

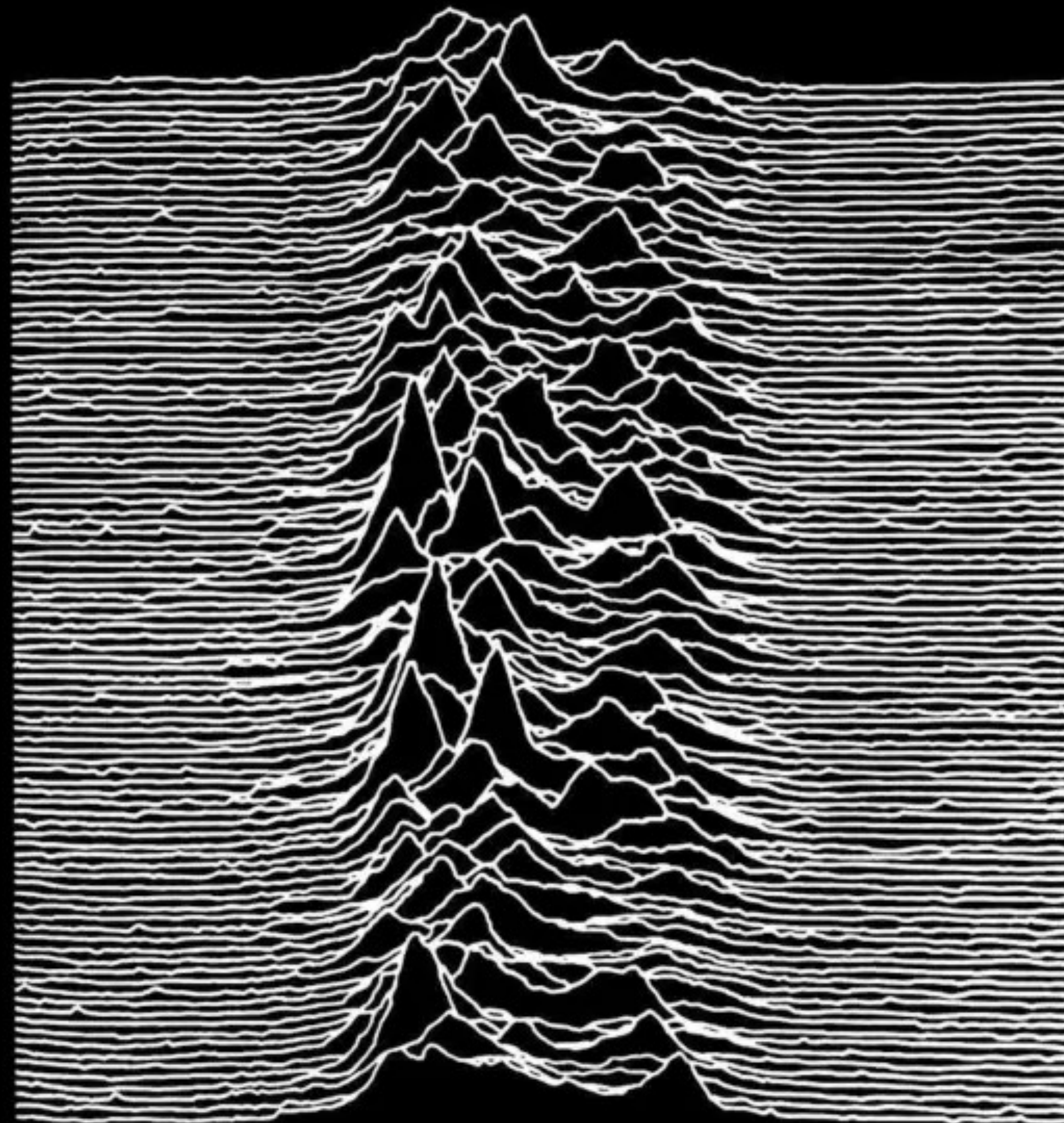
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$$\pi_{nt} = d_n Z_n(p_t) + \beta \mathbb{E}_t \pi_{nt+1}$$

Macro with Excess Demands...

- Excess Demand $\neq$ Gap

- Monetary Policy Rules

 - ▶ Taylor Rules + Determinacy

 - ▶ ZLB

- Shocks $\rightarrow$ Primitives + Policy

- Government Spending example (g_t)

- Extensions...

 - ▶ Non-Ricardian consumers

 - ▶ non-Calvo pricing

 - ▶ non-rational expectations, e.g. learning

Excess Demand $\neq$ Output Gap

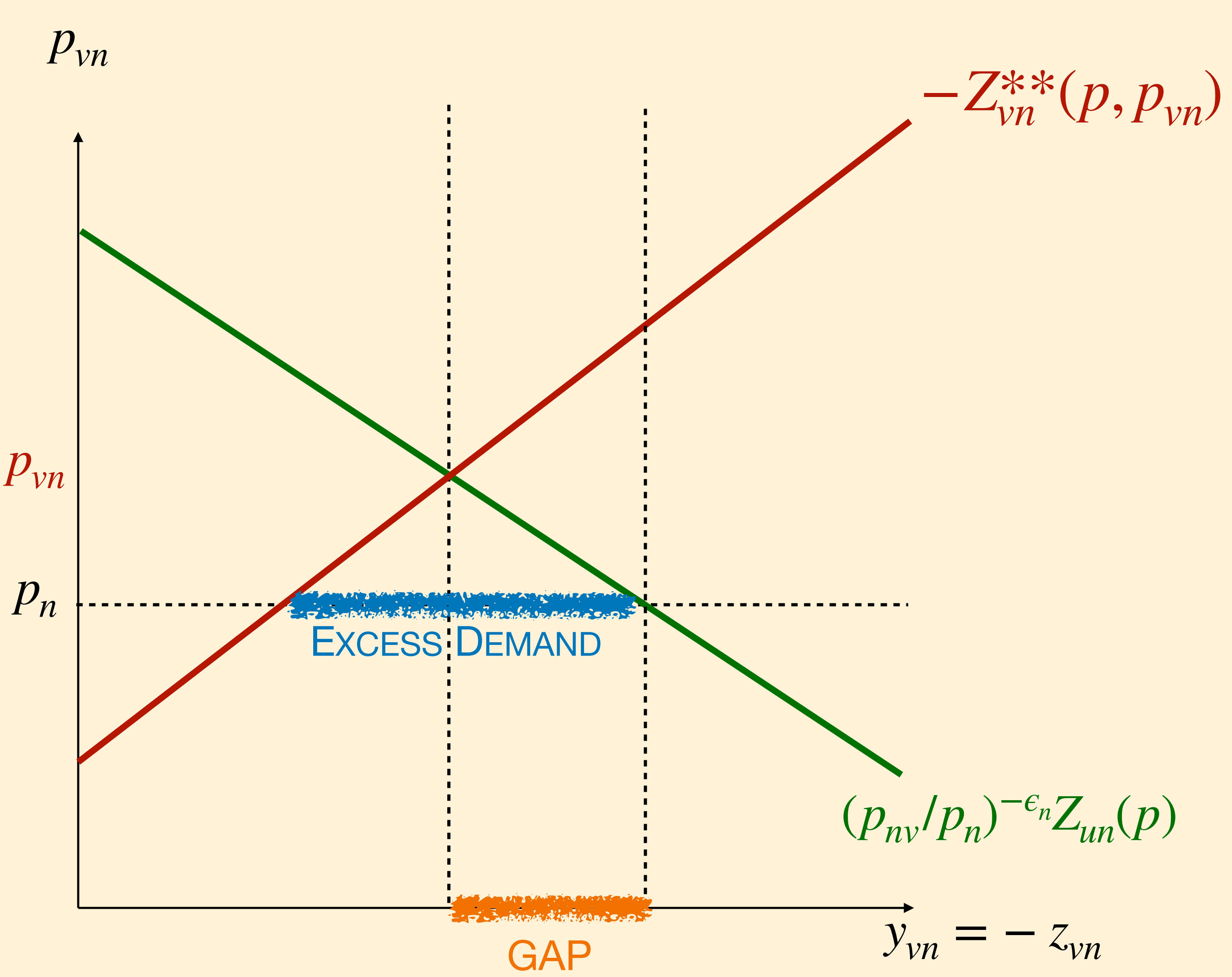
Excess Demand $\neq$ Output Gap

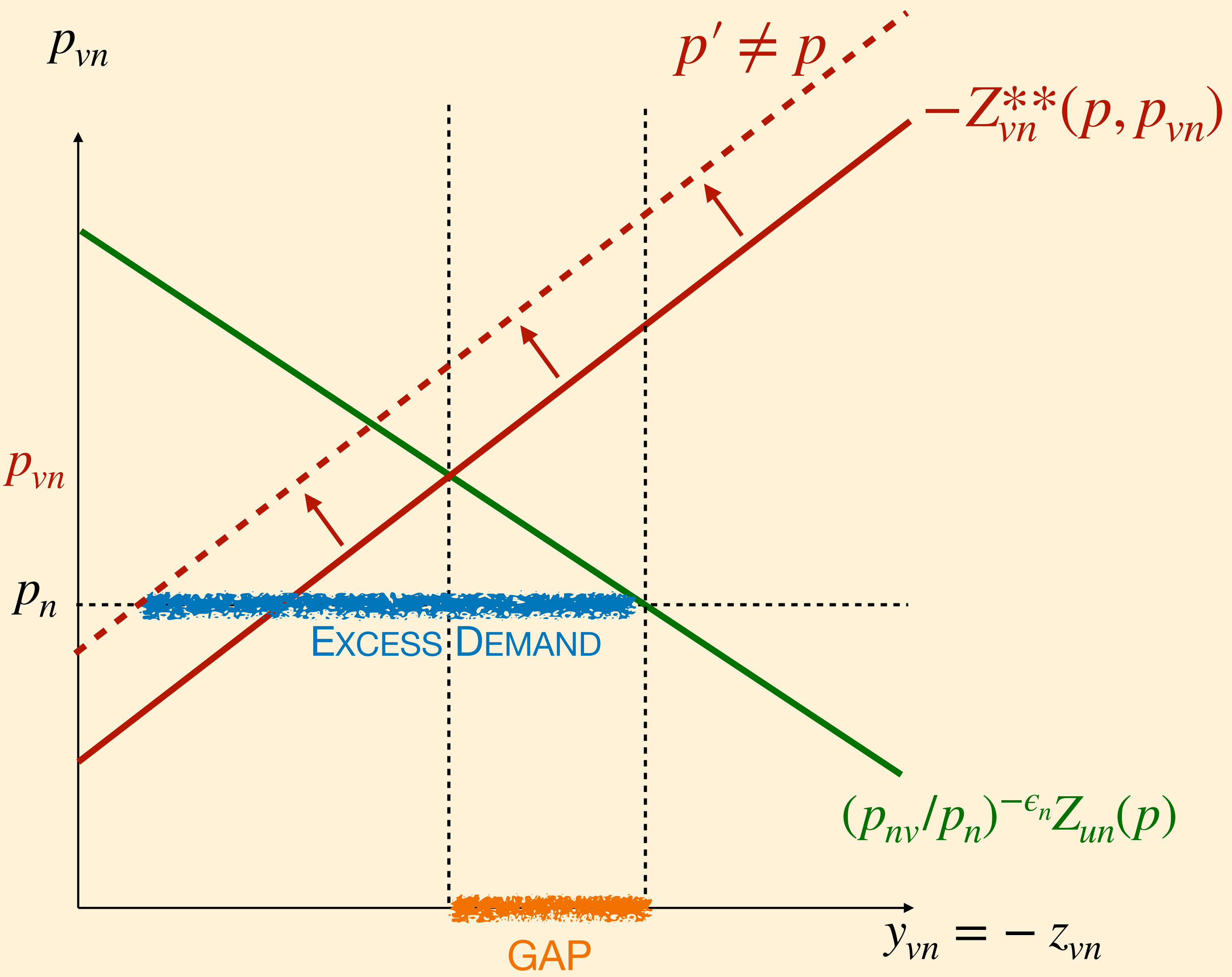
■ Gap...

- ▶ $N = 2$ textbook single final good + flex wages
→ output gap $\sim$ excess demand
- ▶ $N \geq 3$ “**output gap**” as aggregate measure (e.g. Rubbo, 2023)
→ assumptions homothetic + identical preferences+ CRS
→ plus: all other prices!
- ▶ $N \geq 3$ what about an output gap in each market? (non-aggregate)

$$z_n^{\text{gap}} = z_{un} - \bar{z}_{un} \neq \tilde{z}_n$$

not equal to excess demand! (next figure...)





Macro with Excess Demands...

- Excess Demand $\neq$ Gap

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 - ▶ Taylor Rules + Determinacy

 - ▶ ZLB

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- Aggregation

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 - ▶ Non-Ricardian consumers

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Monetary Policy

Taylor Rule $\longrightarrow i_t = \phi \omega \cdot \pi_t + \bar{i}_t$

$$\phi > 0 \quad \omega \in \mathbb{R}^N$$

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$$\hat{Q}_t = Q_t \beta^t = 1$$

$$\bar{\mu} = 1$$

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$$\begin{array}{l} z_t = Z(p_t) \\ \hat{Q}_t = Q_t \beta^t = 1 \\ \bar{\mu} = 1 \end{array} \longrightarrow \begin{array}{l} 0 = Z(\bar{p}) \\ p_t \rightarrow p_\infty \sim \bar{p} \in \mathbb{R}^N \end{array}$$

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Just as before!

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 - ▶ at $\phi = 0$ do we always have stability?
 - ▶ if stable, can such monetary policy obtain determinacy?
- No and no, in general, but...
- ... some positive results...
 - ▶ “Tâtonnement” paper $\rightarrow$ conditions for stability
 - ▶ more general monetary policies allowing $\omega_n < 0$
or
constraints under which $\omega \geq 0$ works

Production Economies

■ Production economy

$$N = N_c + N_\ell$$

▶ N_c final + intermediate goods

▶ N_ℓ labor types

■ Production is CRS (w.l.o.g.)

■ Pricing: monopolistic...

▶ firms set price of goods p_c

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(Afrouzi Bhattacharai is special case of 1)

Partitioned Production Economies

- Partitioned production economy...
- ▶ N_c partitions
- ▶ each partition $\rightarrow$ single final good

Partitioned Production Economies

■ Partitioned production economy...

▶ N_c partitions

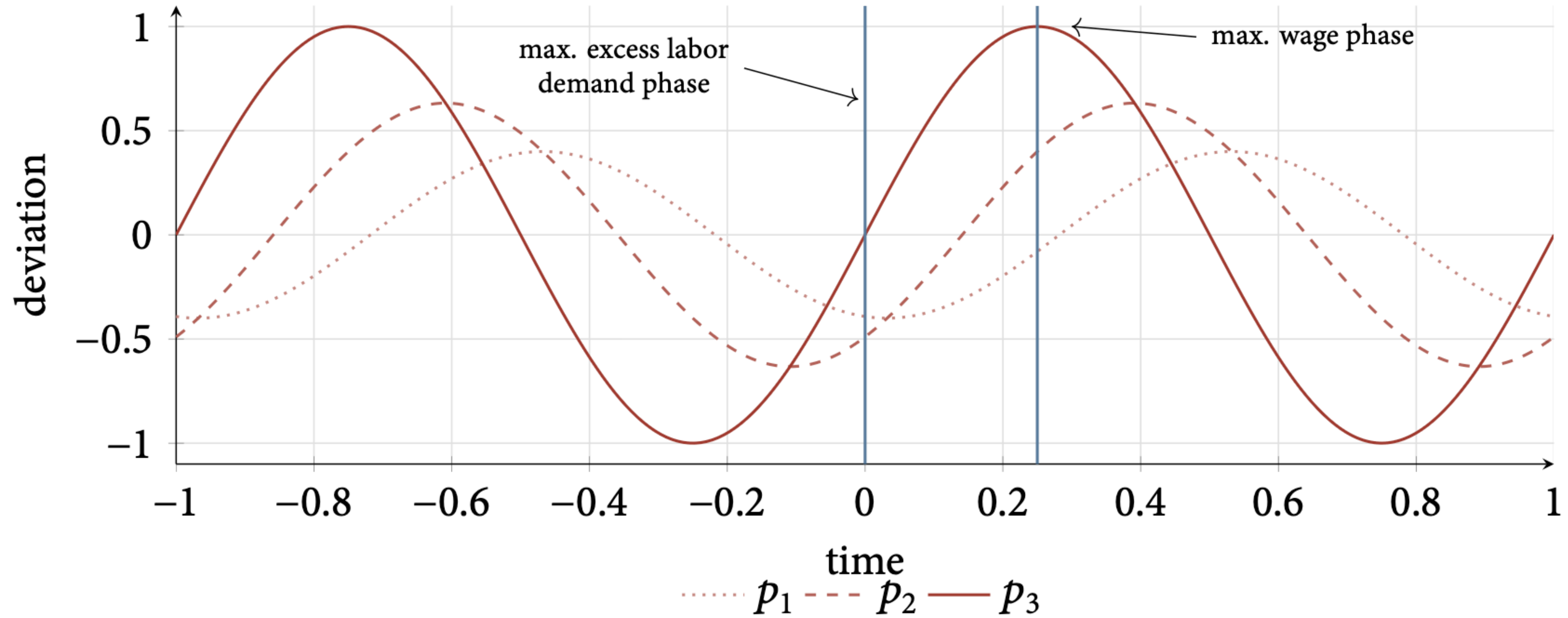
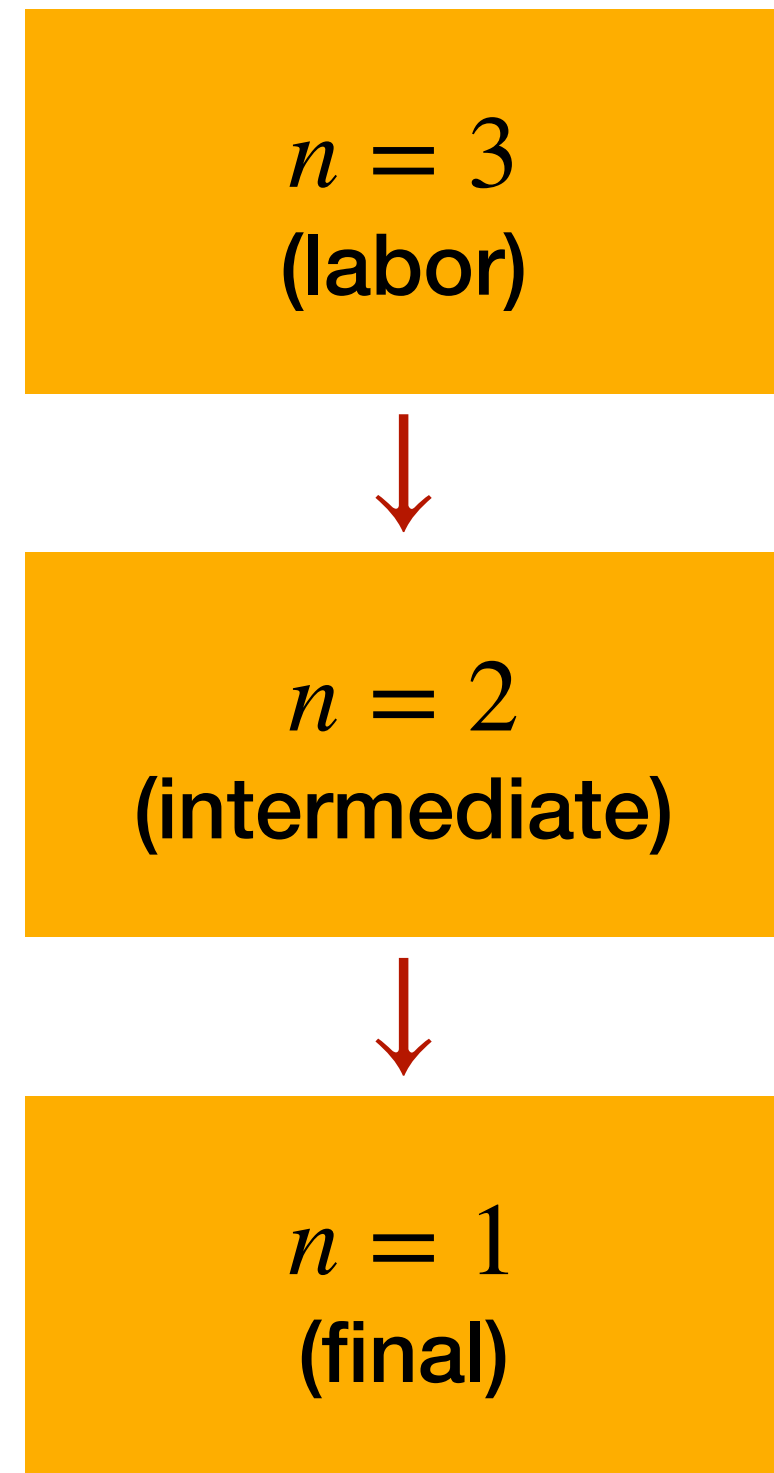
▶ each partition $\rightarrow$ single final good

Proposition.

Within each partition....

1. No input-output $\rightarrow$ Stable
2. Dedicated exclusive labor *or* a Leontief input-output
 $\rightarrow$ stable or oscillatory instability (no explosive)

Production Chain Cycle: 3 goods



Proposition. [Monetary Policy → Determinacy]

Exists ω and ϕ

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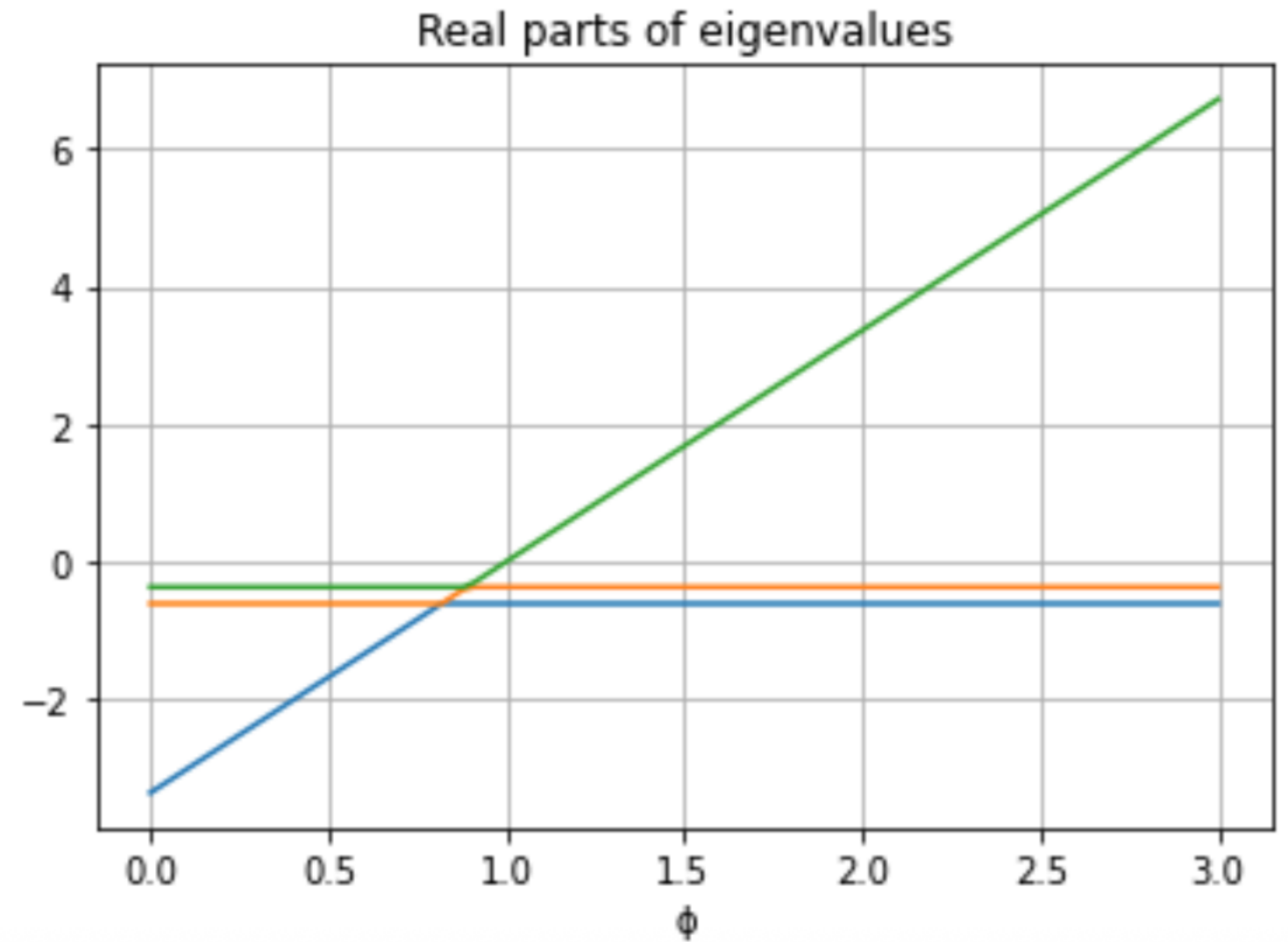
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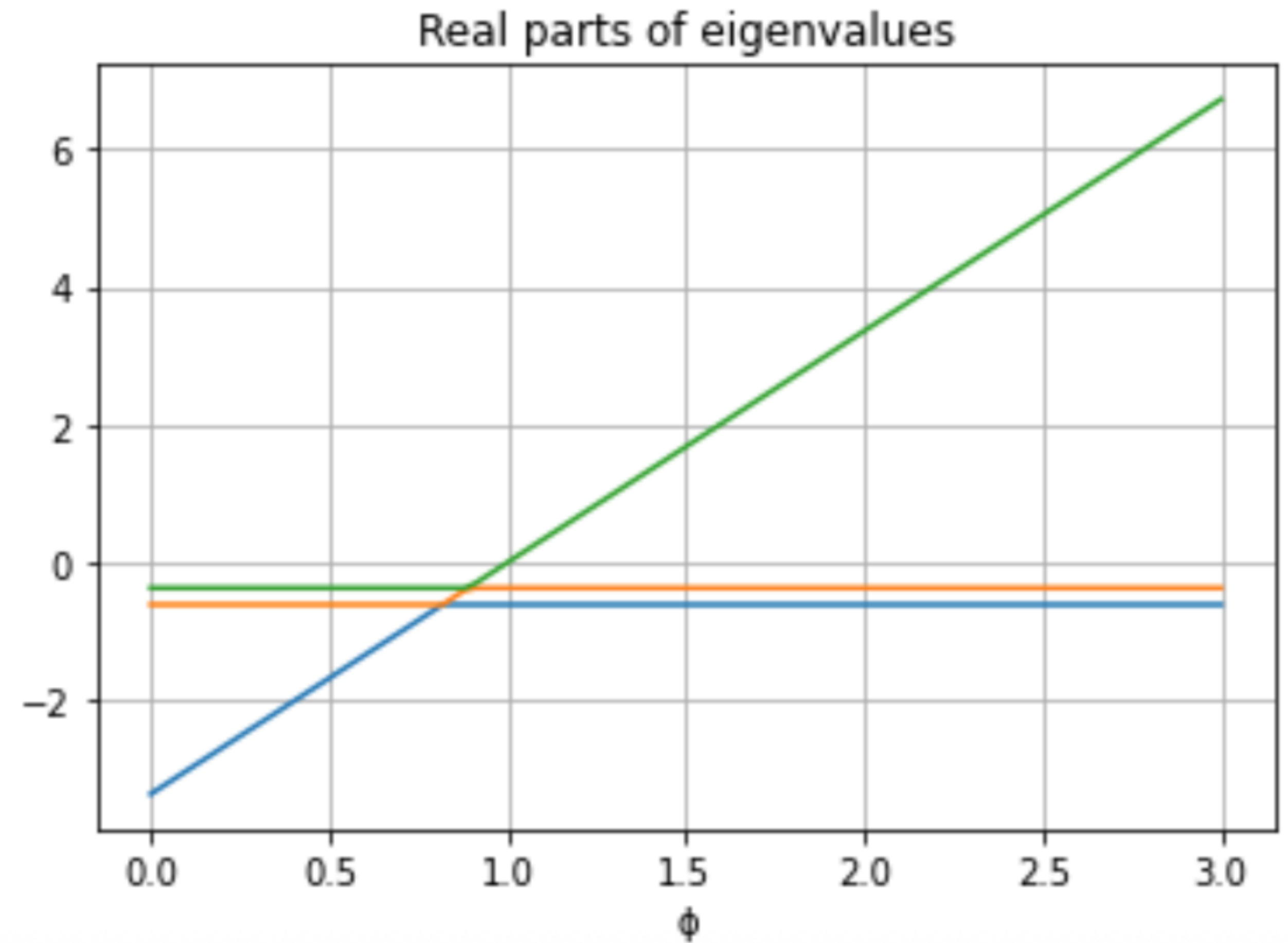
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- Next: show Taylor principle for arbitrary or CPI ω fails to work

Example: Three good economy



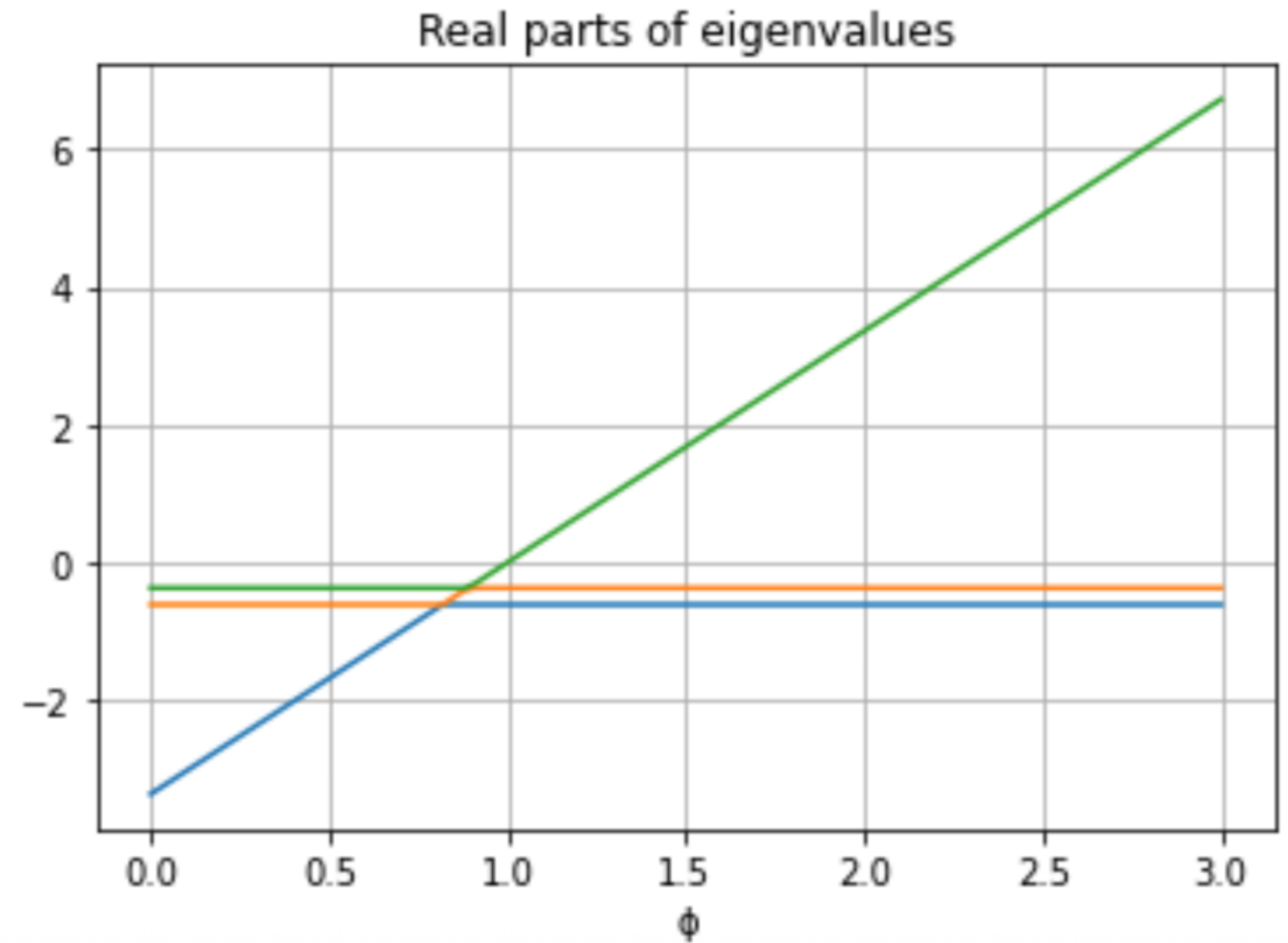
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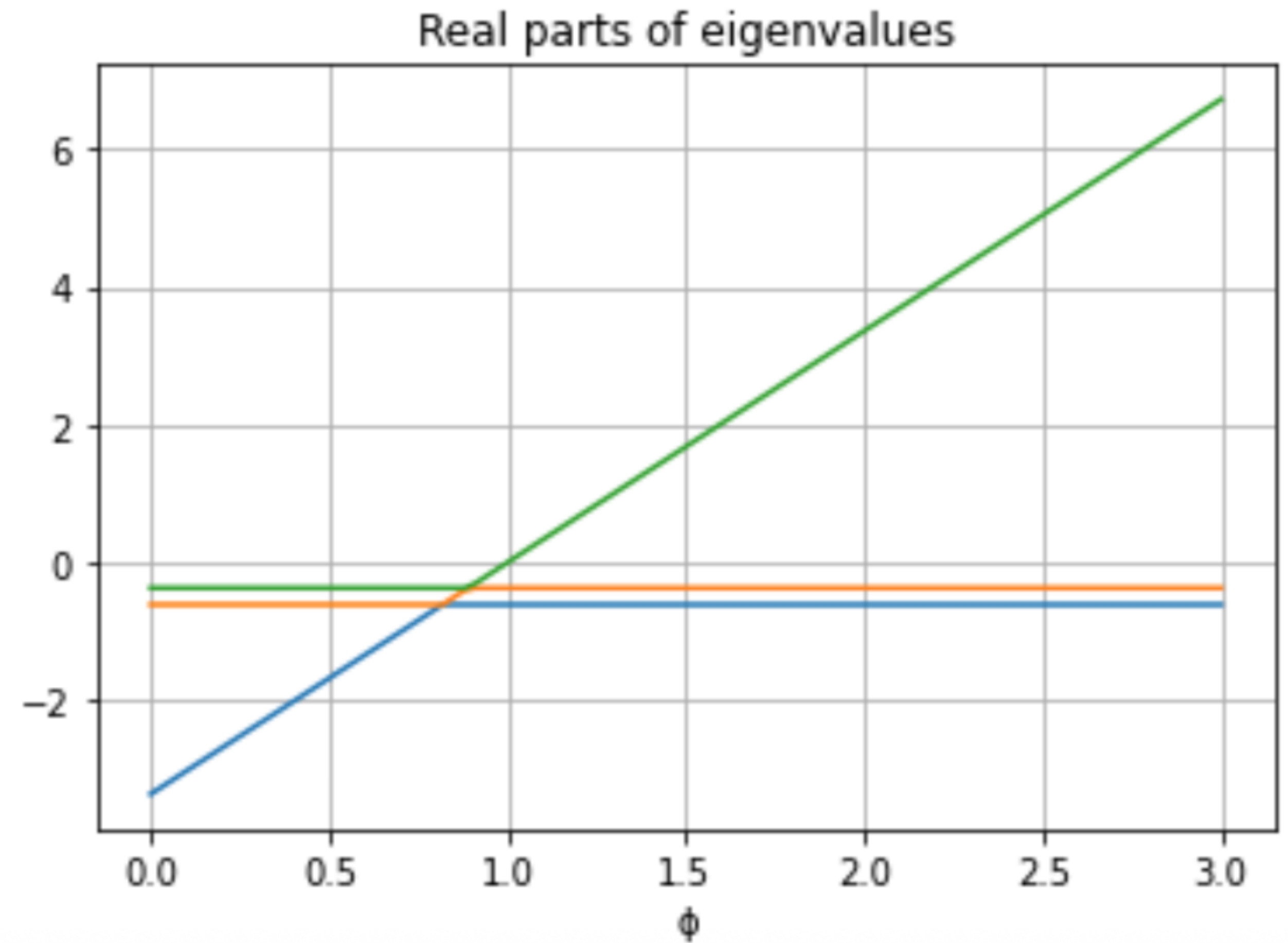
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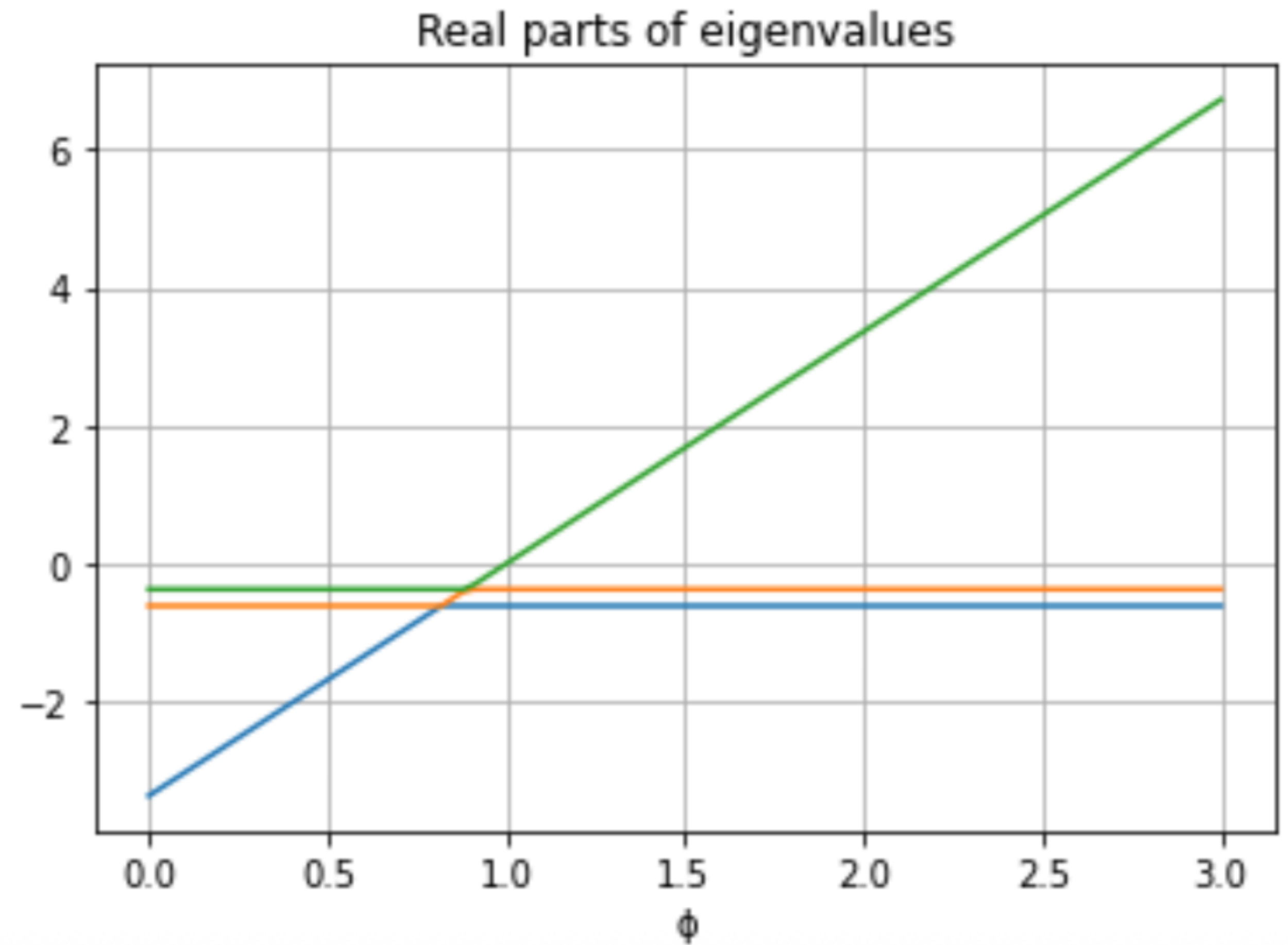
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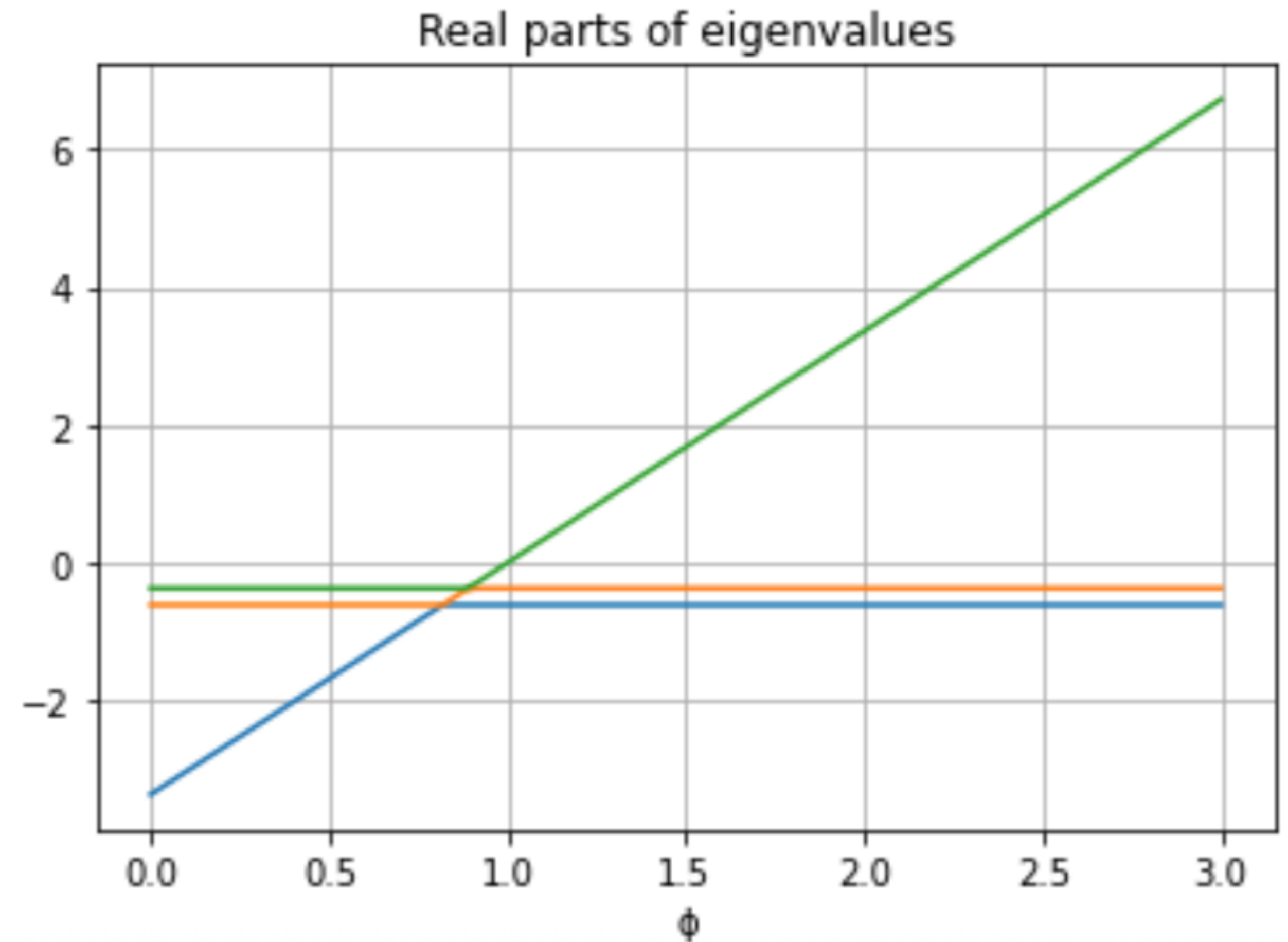
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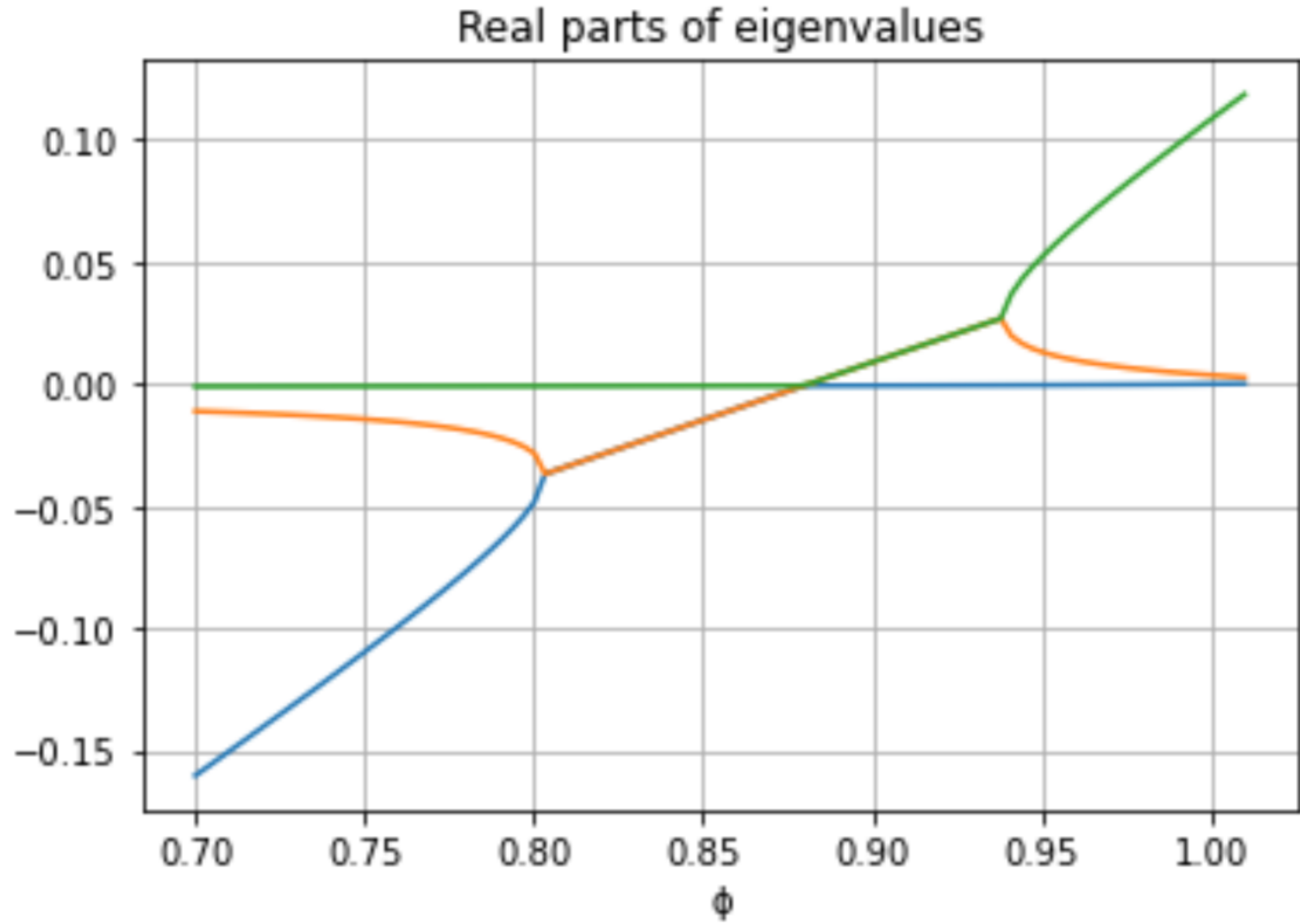


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 - ▶ two eigenvalues remain unchanged
 - ▶ other rises and crosses 0 at $\phi = 1$!

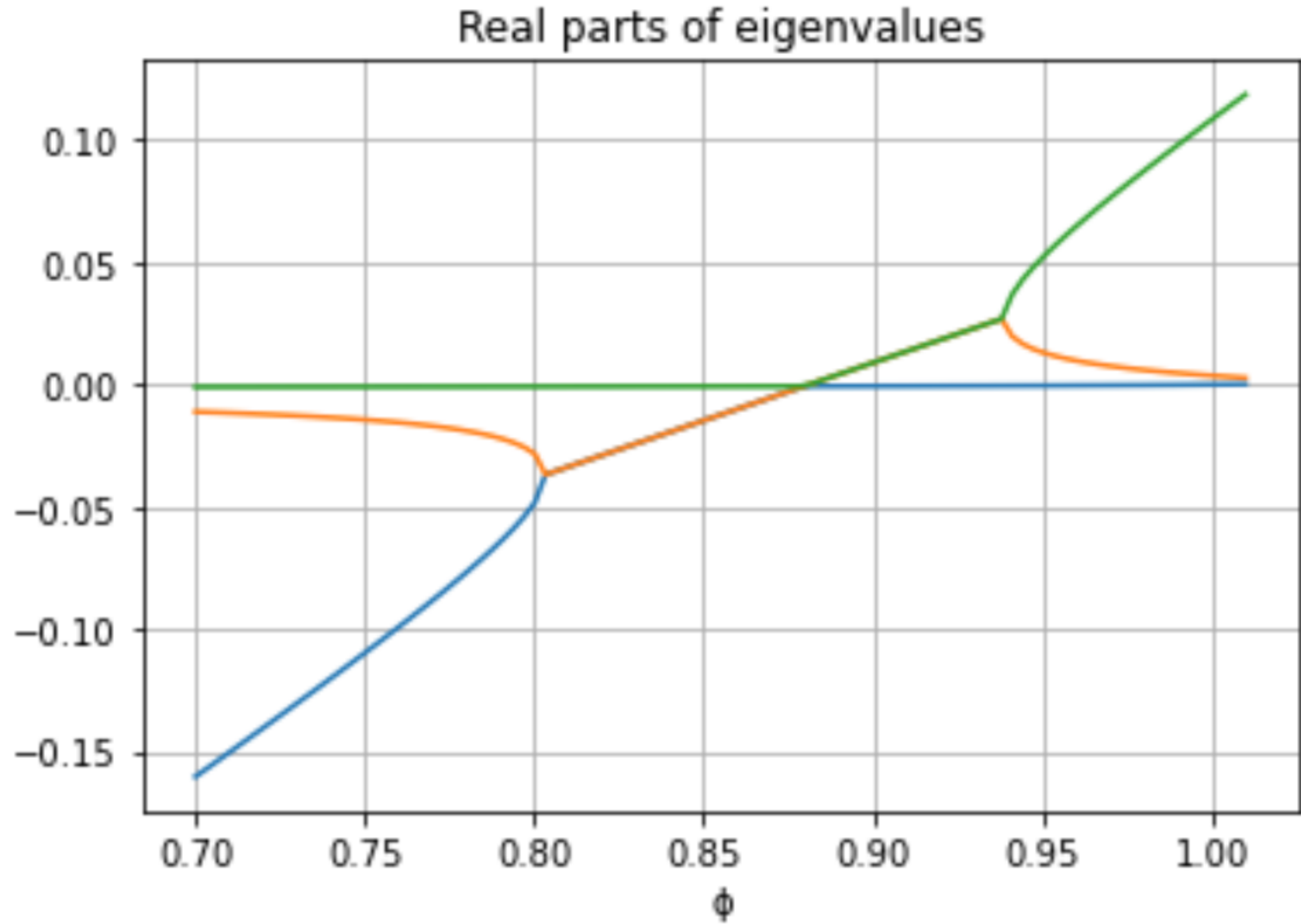


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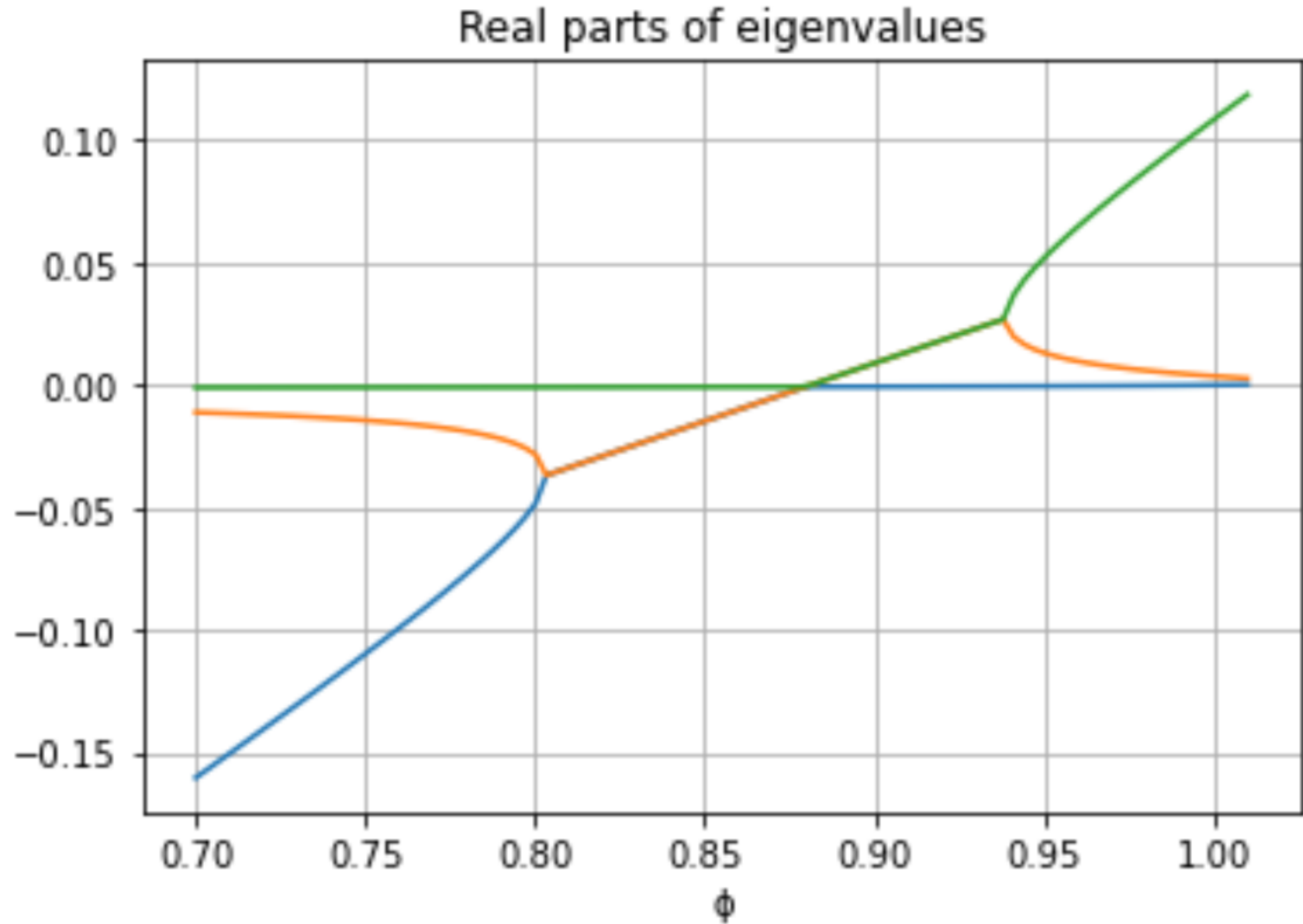
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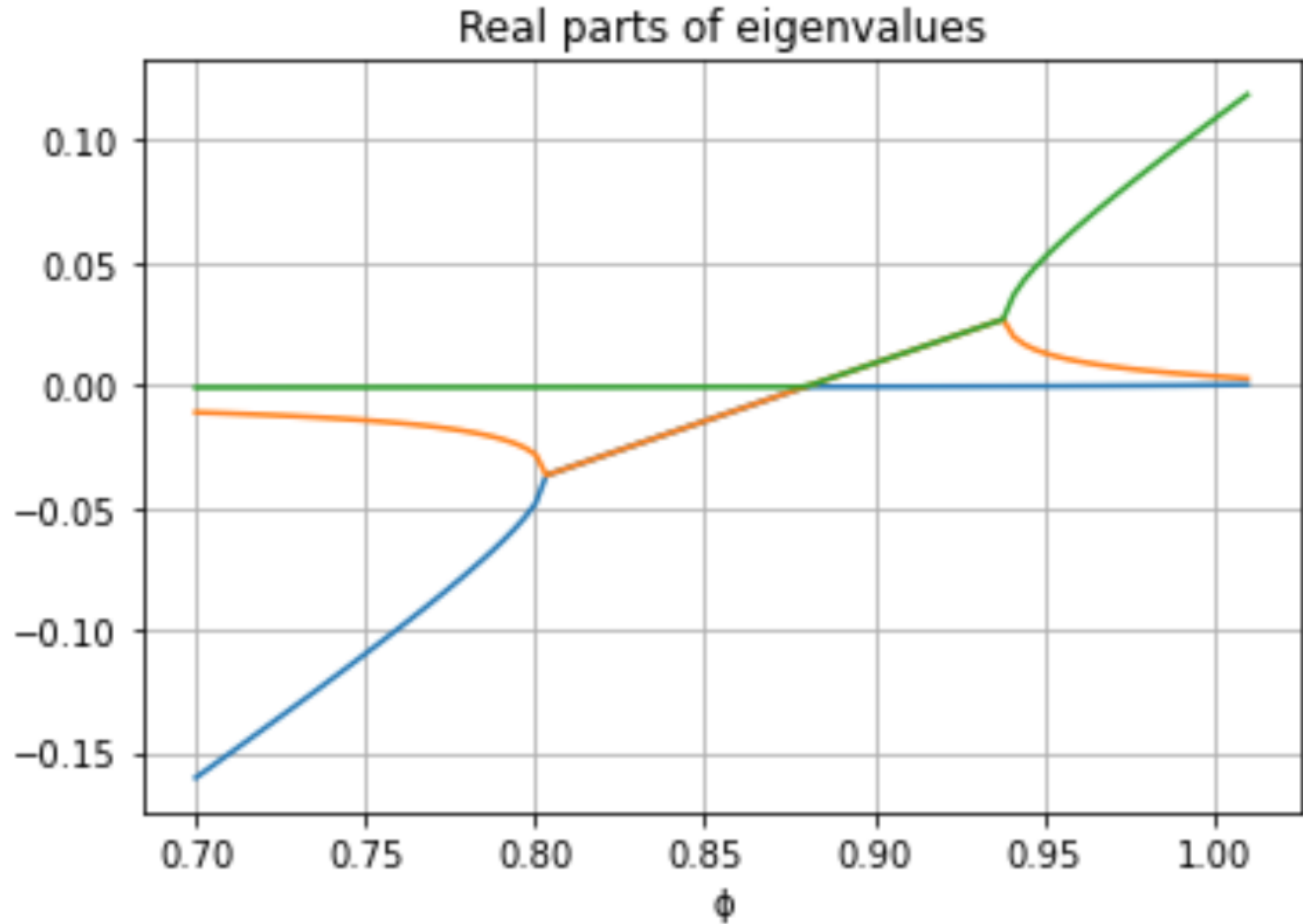
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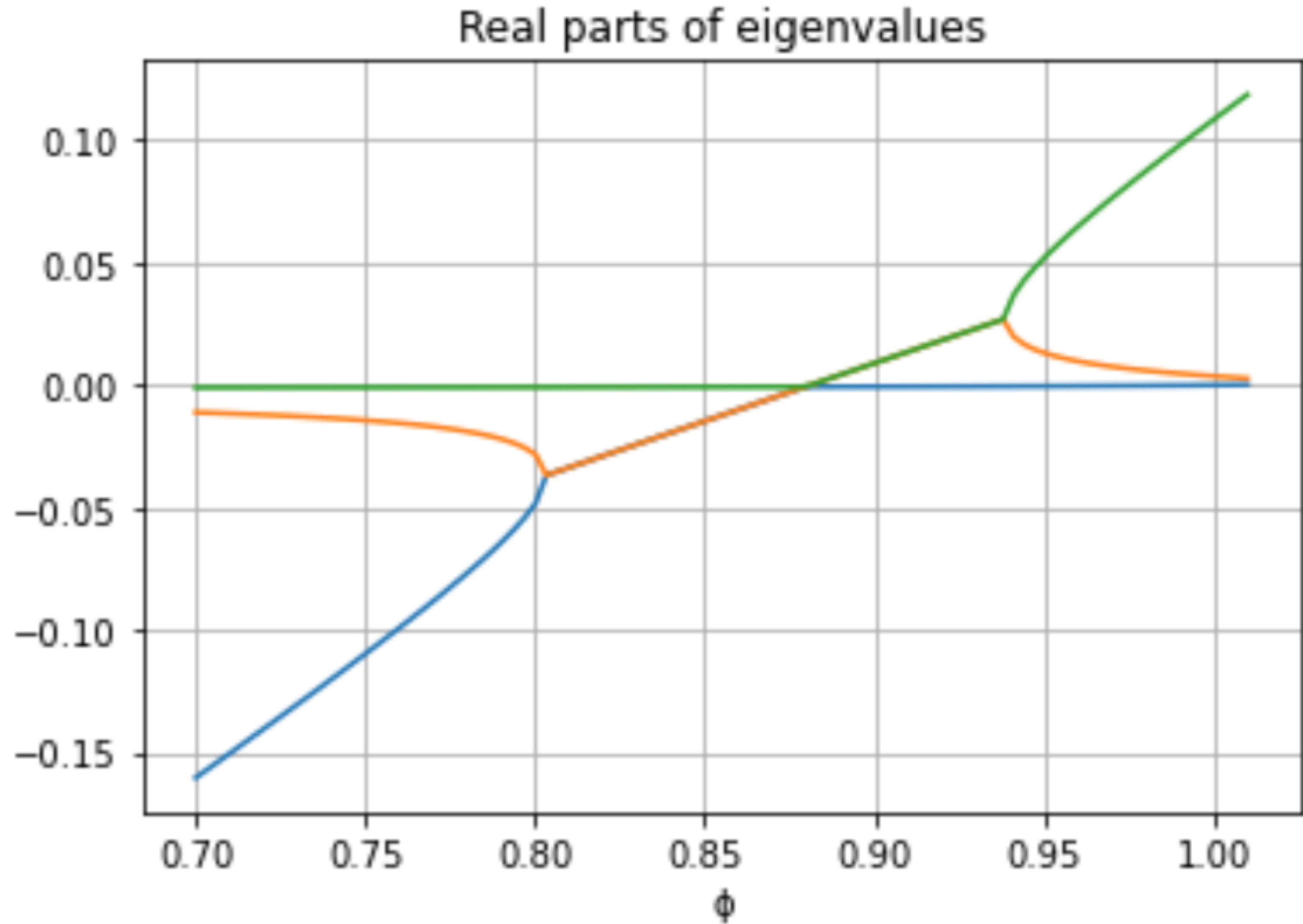
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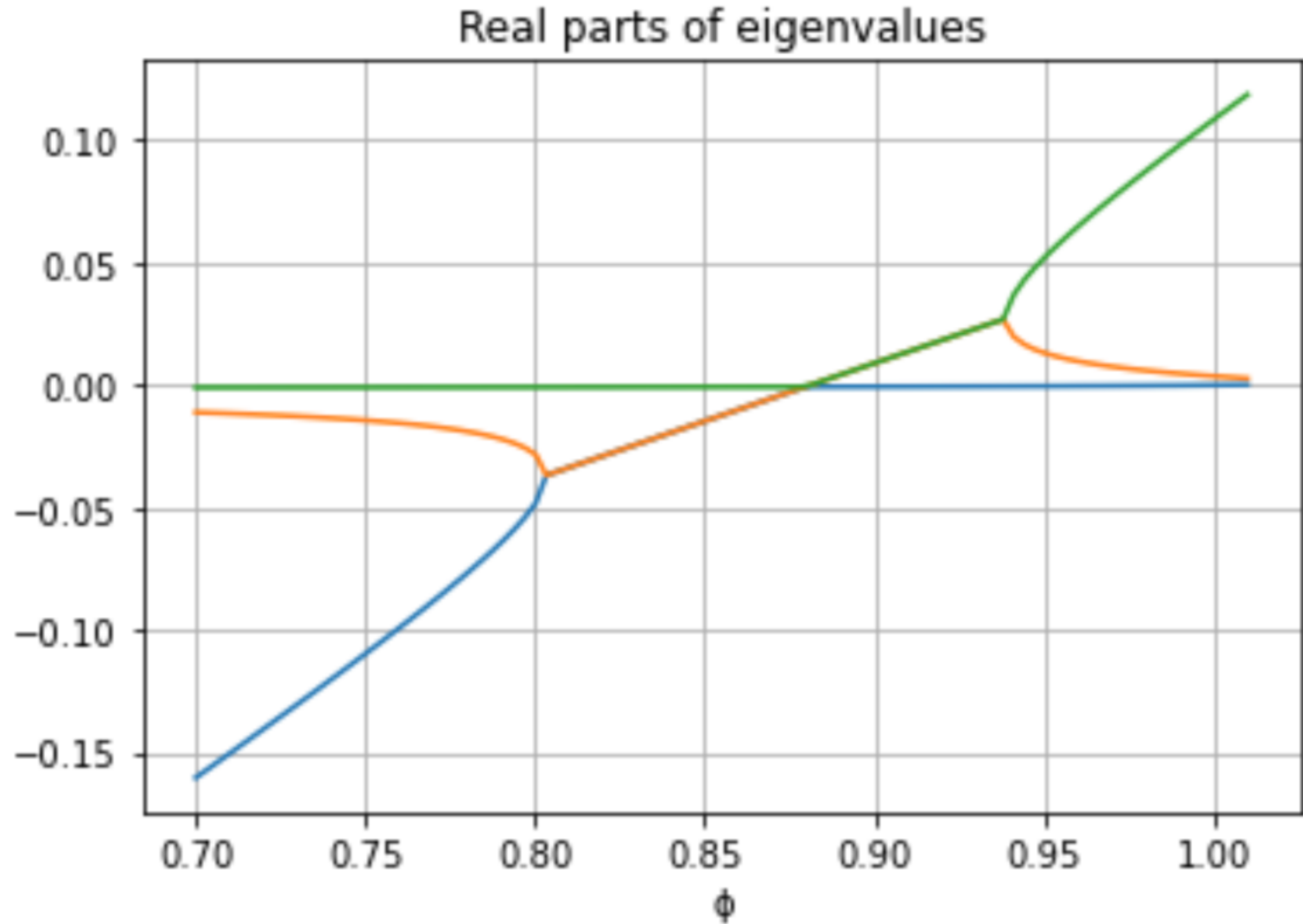
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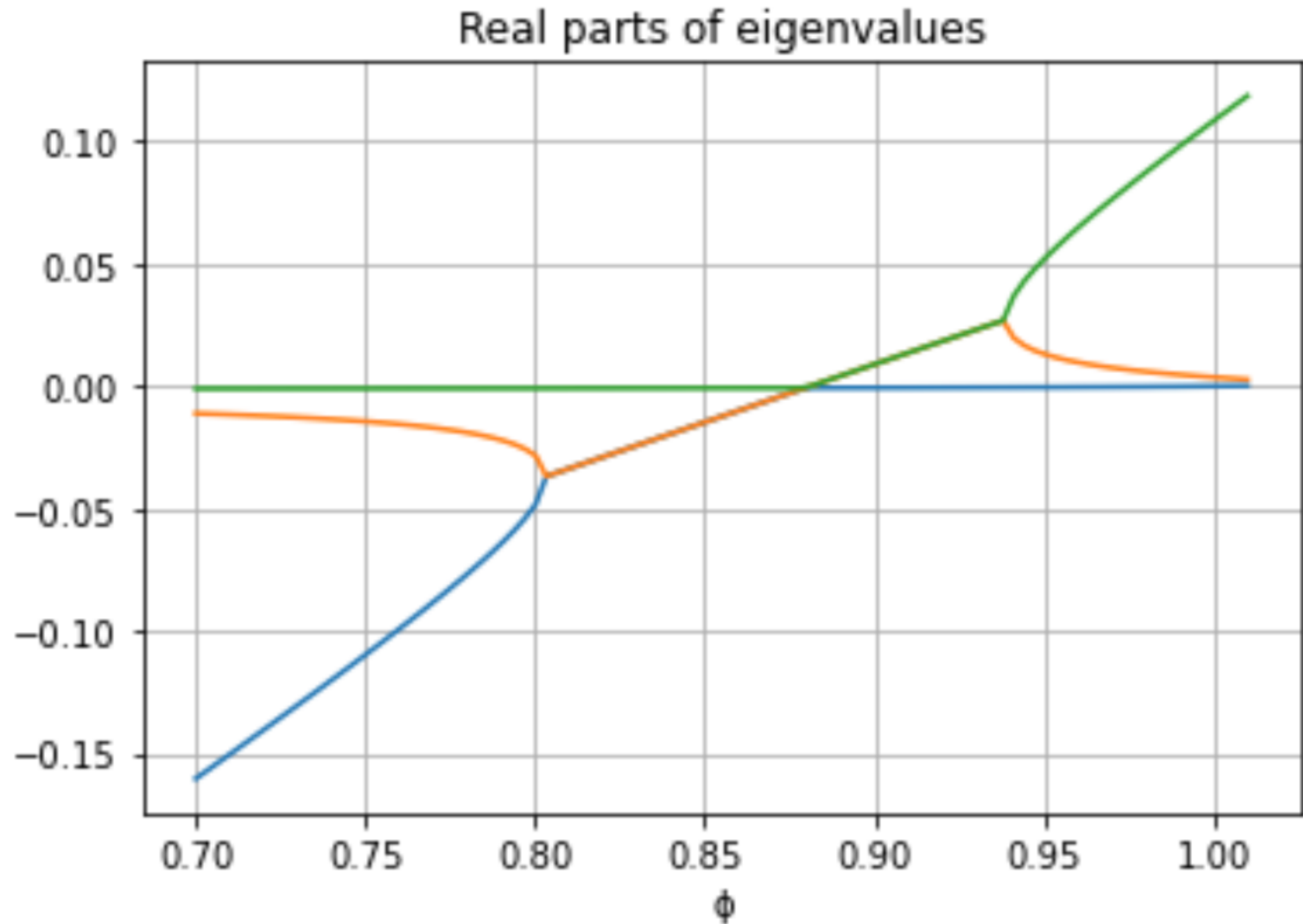
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- Taylor rule with weights $\neq$ left eigenvector
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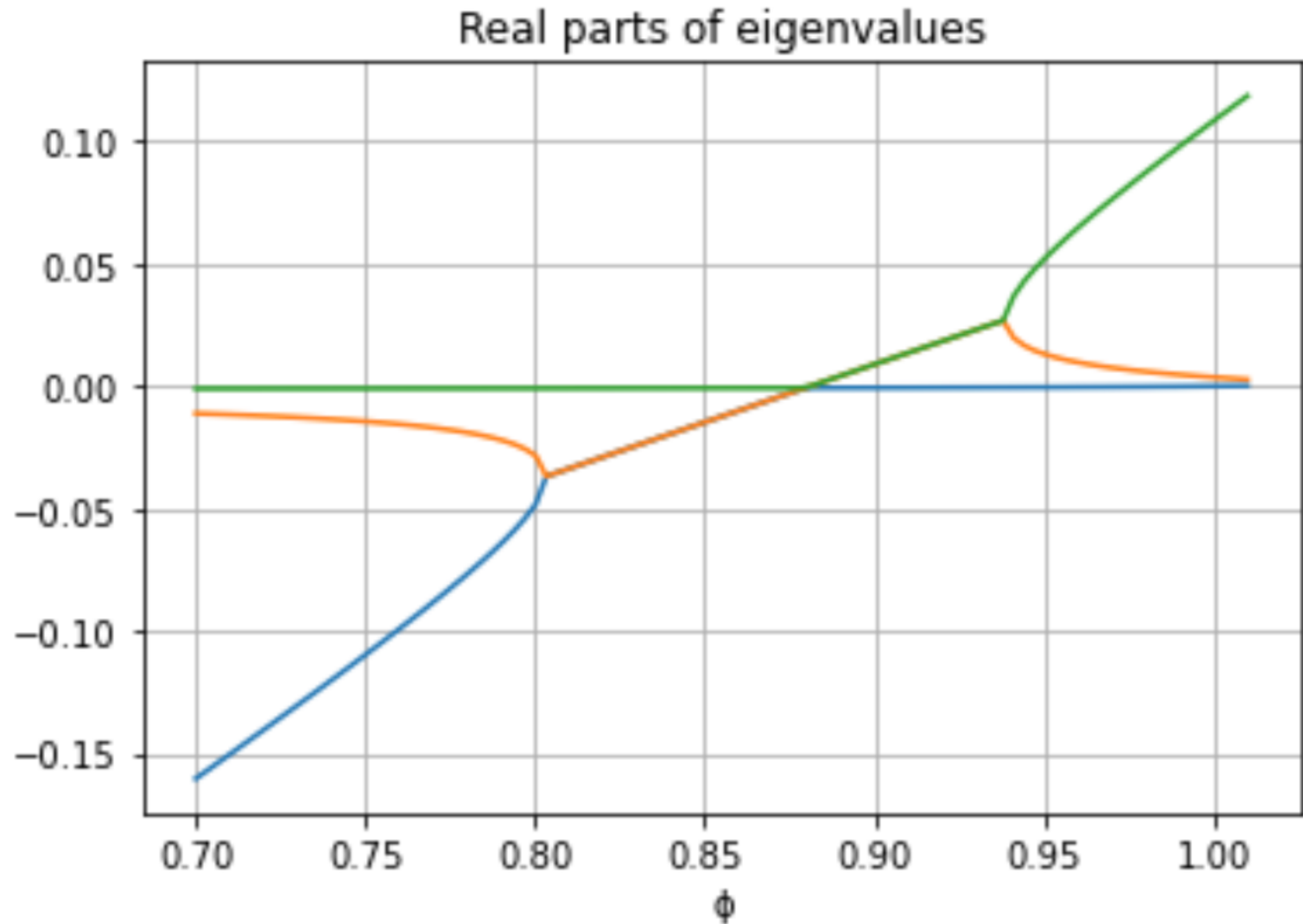
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 - ▶ complex conjugate pair of eigenvalues emerge
 - ▶ common real part pair crosses 0 around $\phi = 0.857$
- No ϕ with two negative and one positive eigenvalues! 😓



Macro with Excess Demands...

- Excess Demand $\neq$ Gap
- Monetary Policy Rules
 - ▶ Taylor Rules + Determinacy
 - ▶ ZLB
- Shocks $\rightarrow$ Primitives + Policy
- Aggregation
- Extensions...
 - ▶ Non-Ricardian consumers
 - ▶ non-Calvo pricing
 - ▶ non-rational expectations, e.g. learning

■ Monetary Policy $Q_t[p_t]$ time dependent

► for $t \in [0, T]$ $\rightarrow Q_t[]$ flat

► for $t > T$ $\rightarrow Q_t[]$ Taylor Rule (determinacy)

■ Solve backwards...

$$\pi_t = Z_t(p_t) + \beta \mathbb{E} \pi_{t+1}$$

$$(\pi_t = CM_t p_t + \beta \mathbb{E} \pi_{t+1})$$

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Super
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Keynesian
Cross!

Shocks

■ More generally...

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$$\rightarrow Z(p_t; g_t) = (I - A)^{-1}(Z^{**}(p_t, \theta_t))$$

■ Note: changes in $A \rightarrow$ no first order effects!

Super
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 - ▶ inflation in each market $\leftarrow$ single aggregate output gap + all other prices!

$$\pi_{n,t} = d_n g_n(y_t, p_t) + \beta \mathbb{E} \pi_{n,t+1}$$

- Our approach $\rightarrow$ general + immediate!

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Endogenous “aggregate output gap”!

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- ▶ TANK: fraction λ Hand-to-Mouth...

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Extensions

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Excess Demand Approach Applies

Extensions

■ Behavioral

- ▶ Non-maximizing households

- ▶ Non-rational expectations...

 - households

 - price setters

- ▶ Excess Demand approach unchanged!

■ (Note: Tâtonnement: considers myopic price setters)

Extensions

Extensions

■ Non-Calvo...

- ▶ time dependent (e.g. Taylor): vintages of past prices
- ▶ menu cost: distribution of prices
- ▶ as usual → more complicated

Extensions

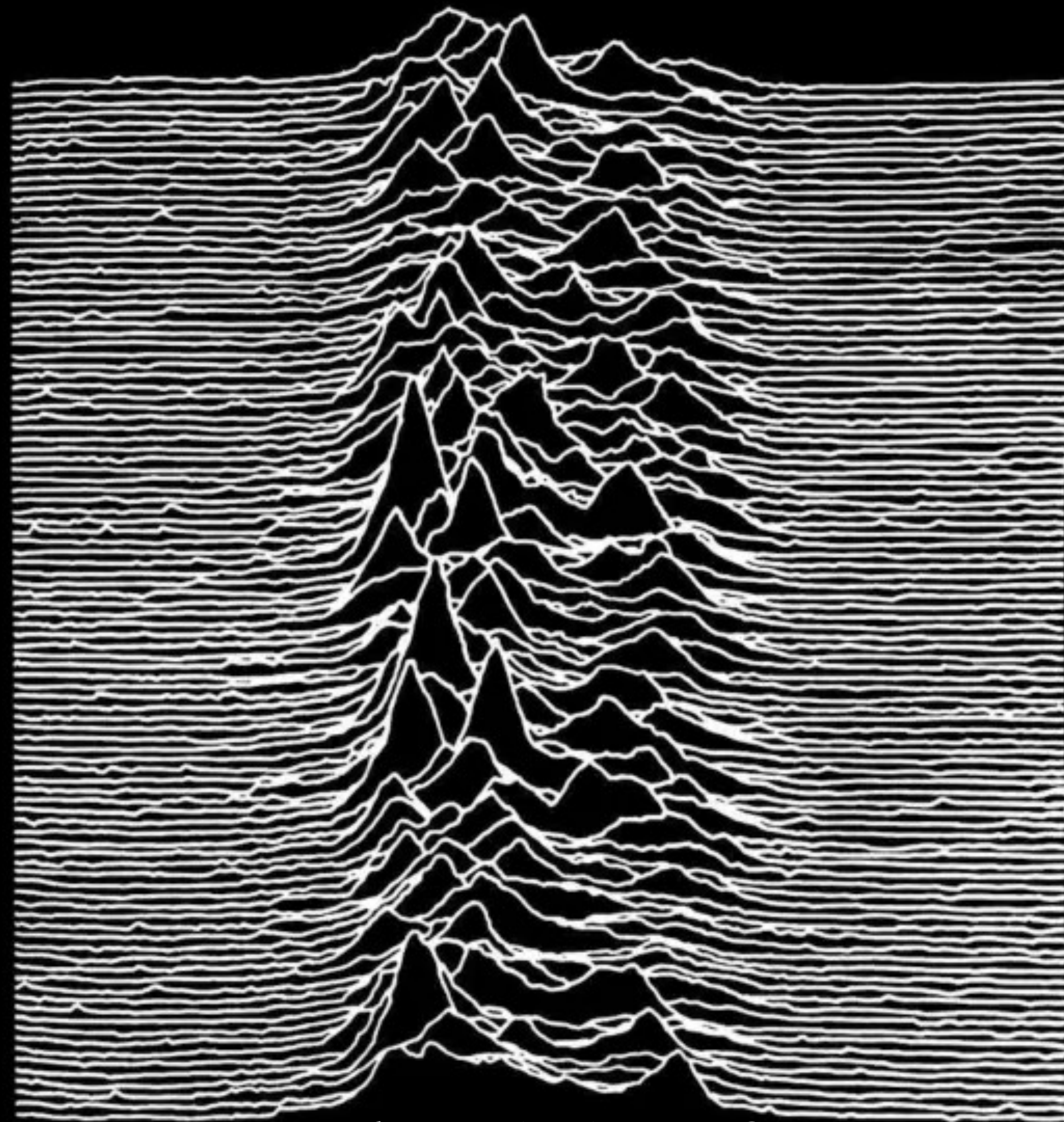
■ Non-Calvo...

- ▶ time dependent (e.g. Taylor): vintages of past prices
- ▶ menu cost: distribution of prices
- ▶ as usual → more complicated

■ Excess demands...?

- ▶ as forcing variable $\{\tilde{z}_t\}$
- ▶ linearized approximations

Conclusions



$$\pi_{nt} = d_n Z_n(p_t) + \beta \mathbb{E}_t \pi_{nt+1}$$

**Keynes
&
ISLM**

**General
Equilibrium**
(with Money)

Disequilibrium
(Fixed Price)

**New
Keynesian**

1930-40s

1950s & 1960s

1970s

1980s-present

Keynes
Hicks

Patinkin
Clower

Patinkin,
Clower,
Barro-Grossman,
Benassy, Dreze,
Malinvaud,
Green
...

MICROECONOMIC THEORY

ANDREU MAS-COLELL MICHAEL D. WHINSTON
AND JERRY R. GREEN

GE $\leftrightarrow$ Macro

Michael Woodford

INTEREST & PRICES

FOUNDATIONS OF A THEORY
OF MONETARY POLICY

MICROECONOMIC THEORY

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INTEREST
& PRICES

Main Results

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more general $\rightarrow$ more intuitive!

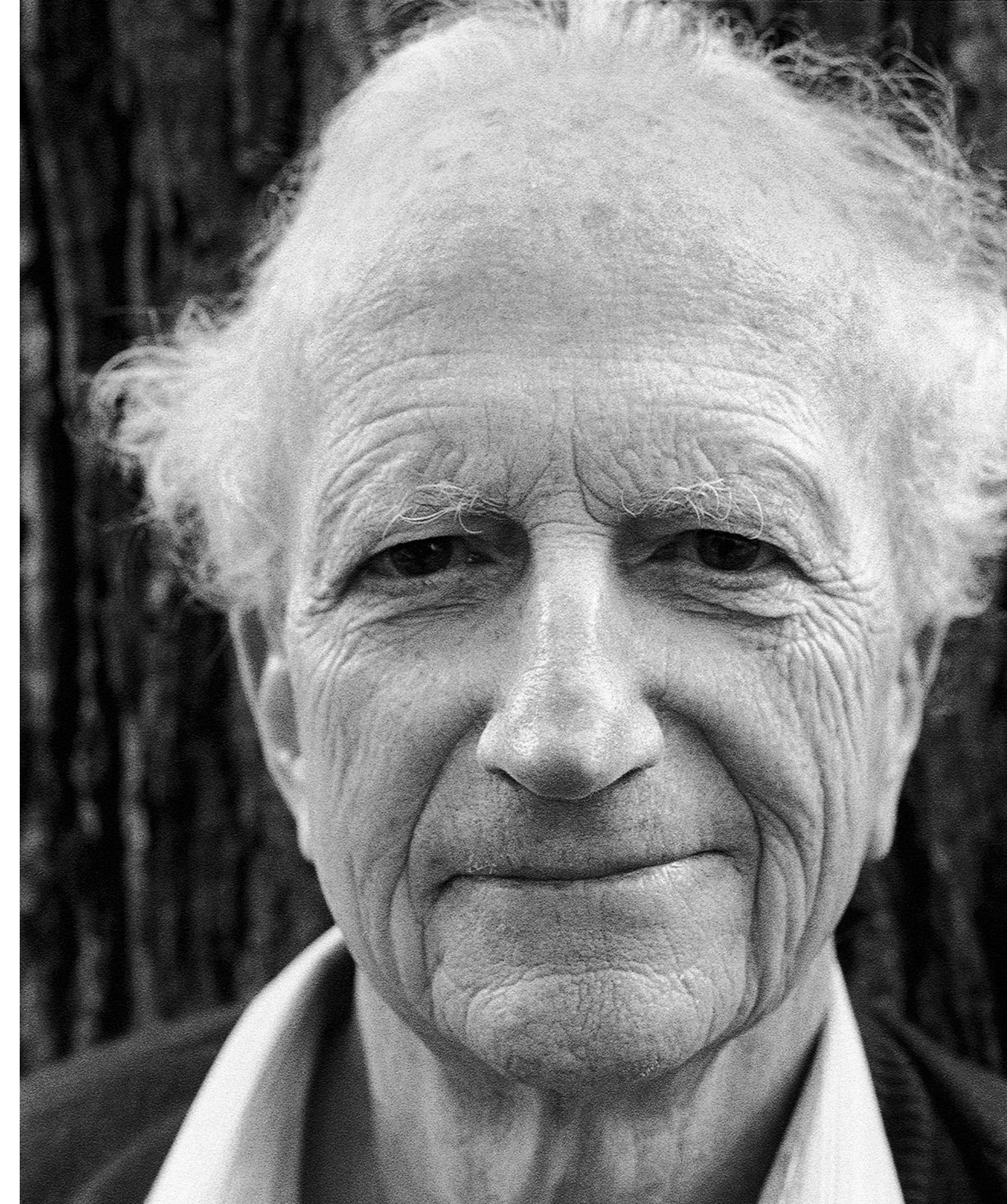
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Micro 101...
“Prices Rise with
Excess Demand”

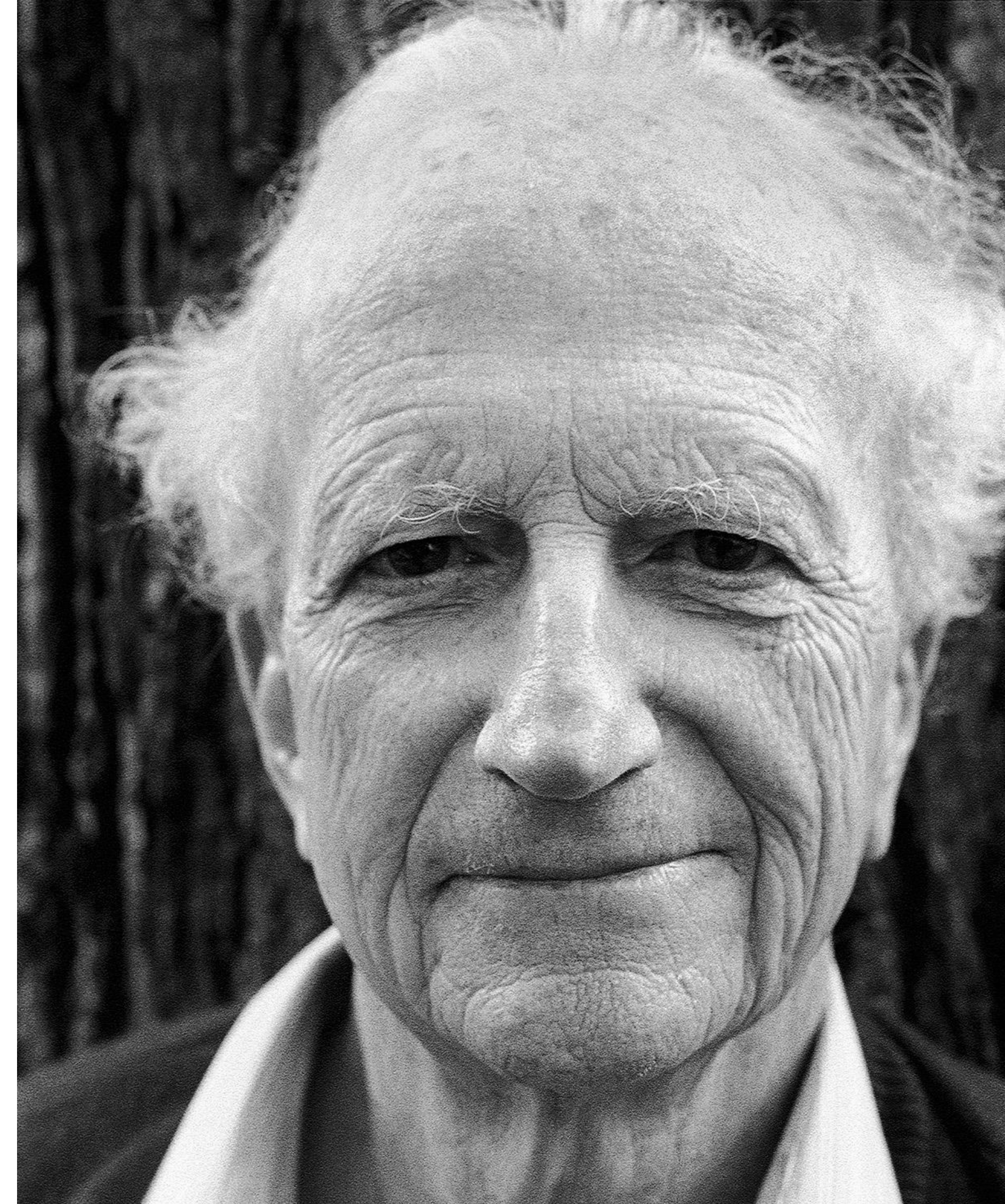
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more general $\rightarrow$ more intuitive!



Heard on the internet...
“Phillips Curves don’t exist
they are dead...”

Micro 101...
“Prices Rise with
Excess Demand”

Fernando Email This Morning

■ Value of alternative perspectives?

Richard Feynman: "Suppose we have two such theories...
...they both agree with experiment to the same extent...

So two theories... may be mathematically identical...

However, for psychological reasons, in order to guess new theories, **these two things are very far from equivalent.**"

- ▶ Feynman goes on to talk about **three** ways to write Newton's gravitational theories, and how one gets to general relativity, while the others do not suggest it.
- ▶ He does not say—he was modest?— but his own path integral is such an example!

Thank you!

$$\pi_{nt} = d_n z_{nt} + \beta \mathbb{E}_t \pi_{nt+1}$$