How Small is Small?

Non-linearities in Heterogeneous Agent Models

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Overview

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  - Match the level and distribution of MPCs in the data

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- BUT: How small is small?
- Our answer: for fiscal transfers, "small" must be *very* small

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Roadmap

1. Partial equilibrium environment

2. Fiscal stimulus in general equilibrium

3. Other shocks and results

Household Consumption-Saving Problem

- Continuum of households with idiosyncratic income shocks

$$\max_{\{c_{i,t},a_{i,t}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t})$$

$$\text{s.t.} \quad c_{i,t} + a_{i,t} = (1+r)a_{i,t-1} + (1-\tau)y_{i,t} + T_t \quad \text{and} \quad a_{i,t} \geq 0$$

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Intertemporal Euler equation:

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Optimization + aggregation ⇒ aggregate consumption function:

$$\mathcal{C}_t(\{T_s\})$$

Fiscal Transfer Policy

[MIT-shock]

Starting from steady state, a one-time unanticipated lump-sum transfer at $t = 0$

$$T_0 = T + \Delta, \qquad T_t = T \ \text{ for } t > 0$$

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Object of interest. Aggregate consumption response:

$$\mathcal{C}_t(\Delta) - \mathcal{C}_t(0)$$

Model Computation: Non-linear vs. Linear Transitions

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Steady-state $\mathcal{C}_0$: endogenous-grid point method for decision rules + aggregation

Model Computation: Non-linear vs. Linear Transitions

Non-linear

- Use household policy to solve exactly consumption response

$$\mathcal{C}_0(\Delta) = \int_Y \int_{-\infty}^{\infty} c(\tilde{x}+\Delta, y)\, \mathrm{d}G(\tilde{x} \mid y)\, \mathrm{d}F(y)$$

cash-on-hand before transfer

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Linear

- Taylor expansion of aggregate consumption around steady state

$$\mathcal{C}_0(\Delta) \approx \mathcal{C}_0(0) + \mathcal{C}_0'(0) \cdot \Delta$$

Slopes computed by finite differences or SSJ toolbox. Then, extrapolate.

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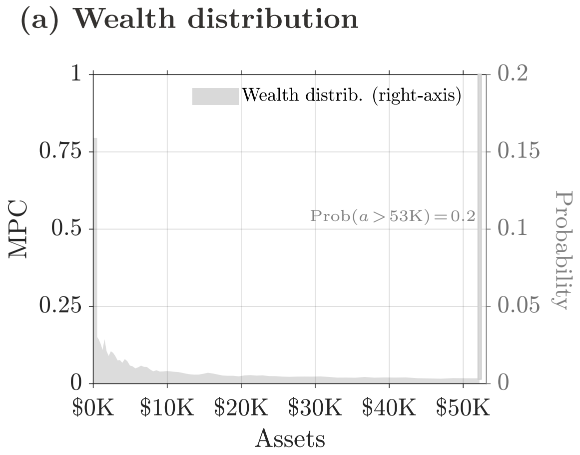
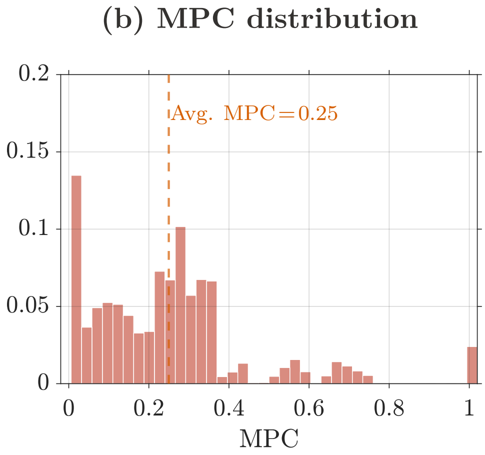
Calibration

- A period = a quarter. $u(c) = \log(c)$
- Earnings process, $y_{i,t}$, calibrated to match second moments of annual earnings (Kaplan and Violante 2022)
- Interest rate 2% annual
- Tax rate $(\tau) = 27.4\%$
- Steady-state transfers = 15% of GDP (≈ \$25K/yr)
- Heterogeneous discount factors:

$$\left.\begin{array}{l}\text{Mean wealth / annual GDP} = 100\% \ (\approx \$166\text{K}) \\ \text{Avg. quarterly MPC (out of \$1,000)} = 25\%\end{array}\right\} \Rightarrow \bar{\beta} = 0.972, \ \sigma_\beta = 0.010$$

Steady-state wealth distribution and MPCs





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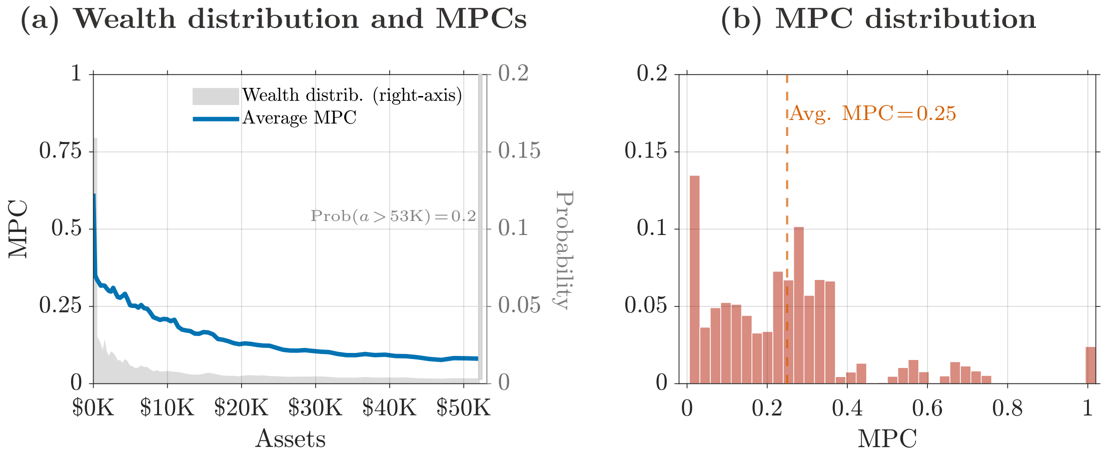

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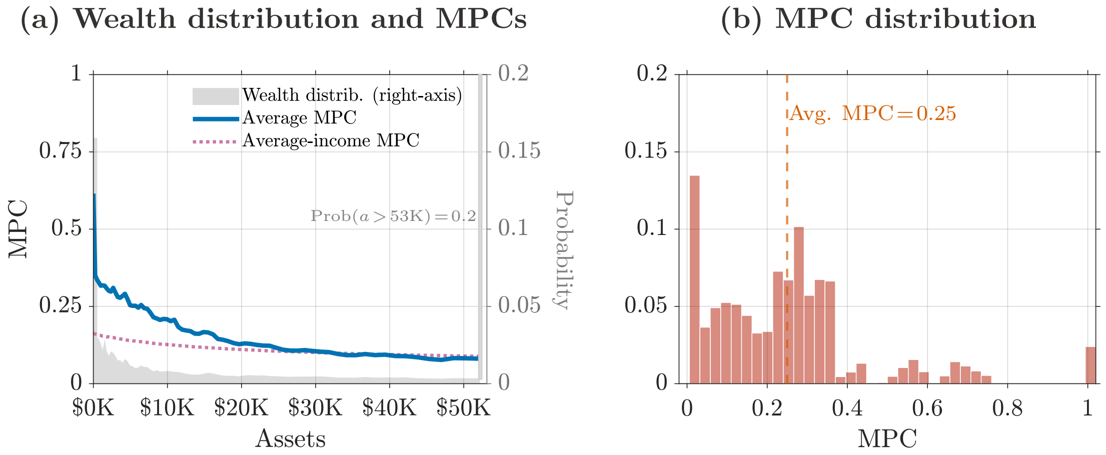

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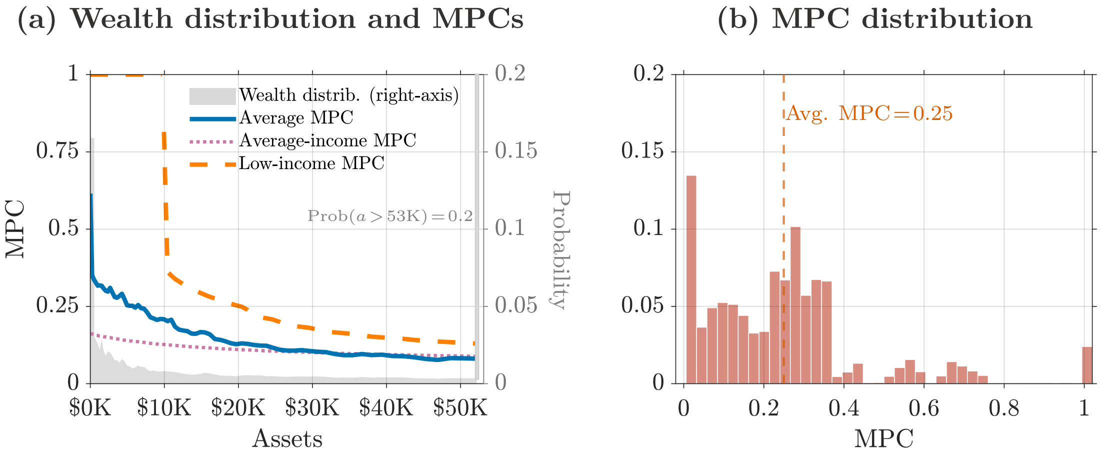
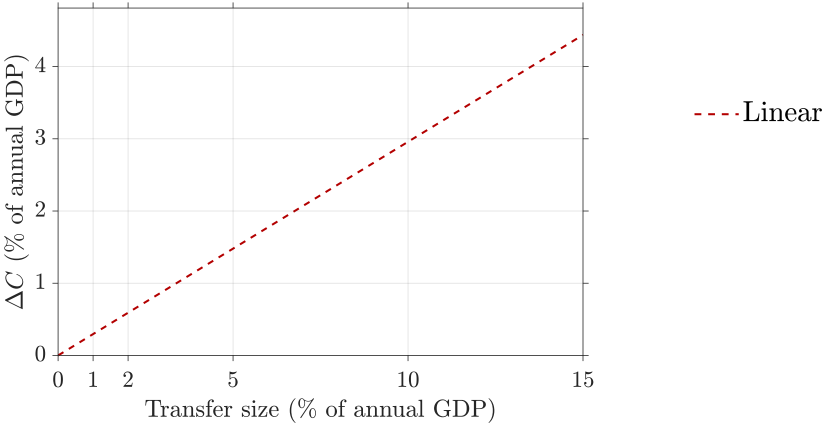


Aggregate consumption response



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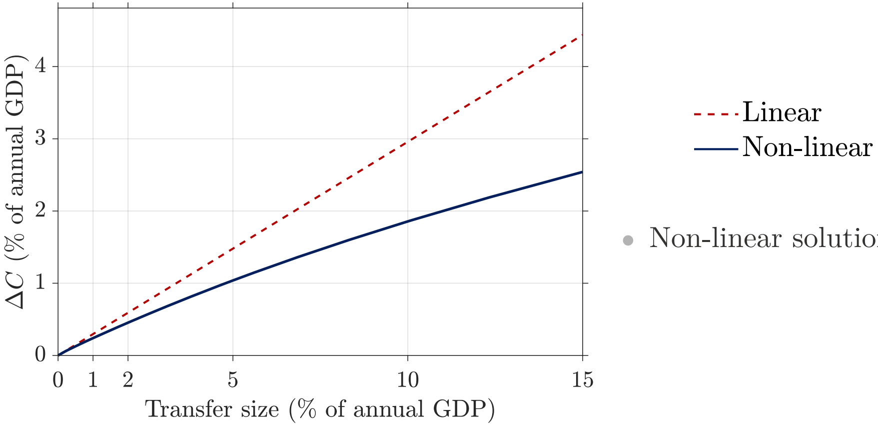

- Non-linear solution is concave

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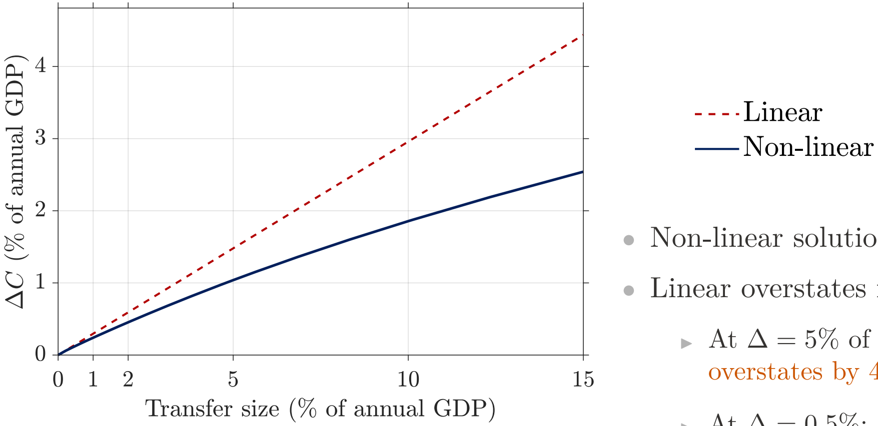

- Non-linear solution is concave
- Linear overstates response:
  - At $\Delta = 5\%$ of GDP ($\approx \$8K$): overstates by 42%
  - At $\Delta = 0.5\%$: overstates by 17%

Understanding the Gap: Two Sources of Concavity

- Concavity of the individual consumption function
- Households moving on and off the borrowing constraint

Understanding the Gap: Two Sources of Concavity

$$\mathcal{C}_0(\Delta) = \int_Y \int_{-\infty}^{\infty} c(\tilde{x} + \Delta, y)\, \mathrm{d}G(\tilde{x} \mid y)\, \mathrm{d}F(y)$$

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Initial distribution (before transfers)

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Initial distribution (before transfers)

- Denote $\bar{x}(y)$ cash-on- hand at which the borrowing constraint stops binding
  - Household constrained if $\tilde{x} \leq \bar{x}(y) - \Delta$

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$$\mathcal{C}_0(\Delta) = \int_Y \left[\underbrace{\int_{-\infty}^{\bar{x}(y)-\Delta} c(\tilde{x}+\Delta, y)\,\mathrm{d}G(\tilde{x} \mid y)}_{\text{constrained}} + \underbrace{\int_{\bar{x}(y)-\Delta}^{\infty} c(\tilde{x}+\Delta, y)\,\mathrm{d}G(\tilde{x} \mid y)}_{\text{unconstrained}} \right] \mathrm{d}F(y)$$

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MPC has a discontinuity at $\bar{x}(y)$.

$$c_x(x, y) = \begin{cases} 1, & x < \bar{x}(y) \quad \text{(constrained)}, \\ c_x^+(x, y) < 1, & x > \bar{x}(y) \quad \text{(unconstrained)}. \end{cases}$$

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$$\mathcal{C}_0'(0) = \bar{\Lambda} \times \underbrace{1}_{\text{MPC}=1} + (1-\bar{\Lambda}) \times \underbrace{\mathbb{E}\left[c_x^+ \mid \tilde{x} > \bar{x}(y)\right]}_{\text{unconstrained MPC}}$$

where $\bar{\Lambda} \equiv \int_Y G(\bar{x}(y) \mid y)\,\mathrm{d}F(y)$ is the steady-state constrained share;

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Linear solution simply extrapolates steady-state aggregate MPC

- Changes in the constrained share are irrelevant to first order
  - As if constrained households maintain MPC $= 1$ regardless of Δ

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$$\mathcal{C}_0''(0) = \Lambda'(0)\, \mathbb{E}\left[\left(\quad 1 - c_x^+ \quad \right) \mid \tilde{x} = \bar{x}(y) \right] + \ (1-\bar{\Lambda})\, \mathbb{E}\left[c_{xx}^+ \mid \tilde{x} > \bar{x}(y)\right]$$

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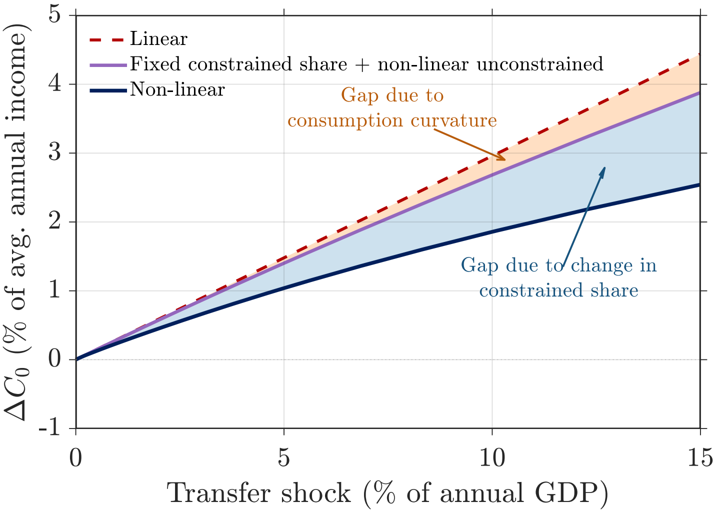
change in constrained share

$$\mathcal{C}_0''(0) = \underbrace{\Lambda'(0)\,\mathbb{E}\left[\left(\overbrace{1-c_x^+}^{\text{Drop in MPC at Kink}}\right) \mid \tilde{x} = \bar{x}(y)\right]}_{\substack{\text{movement off- the constraint}\\ <0}} + \underbrace{(1-\bar{\Lambda})\,\mathbb{E}\left[c_{xx}^+ \mid \tilde{x} > \bar{x}(y)\right]}_{\substack{\text{individual consumption curvature}\\ <0}}$$

The aggregate consumption response is concave in transfer size.

Decomposing the Linear-Non-linear Gap



Alternative decomposition

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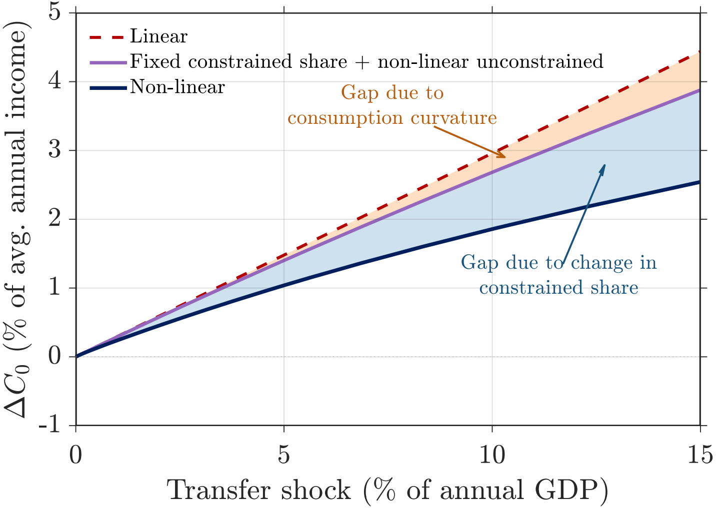
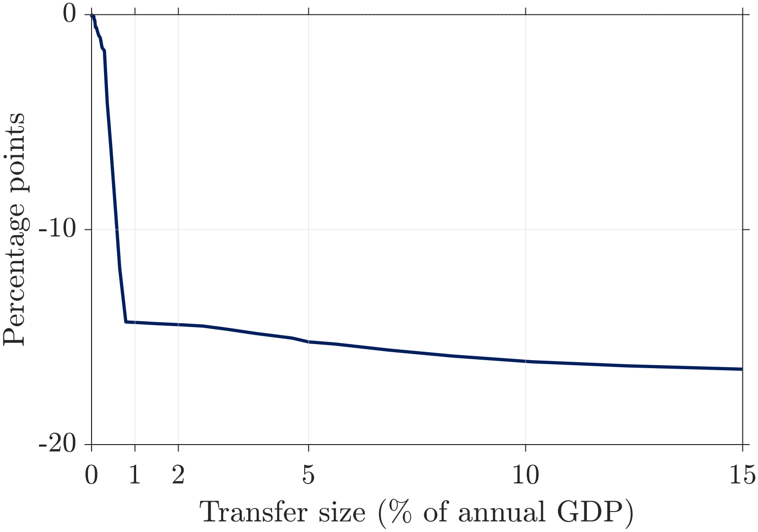


1% transfer	
Constrained share	93%
Curvature	7%

5% transfer	
Constrained share	82%
Curvature	18%

Alternative decomposition

Change in share of constrained households



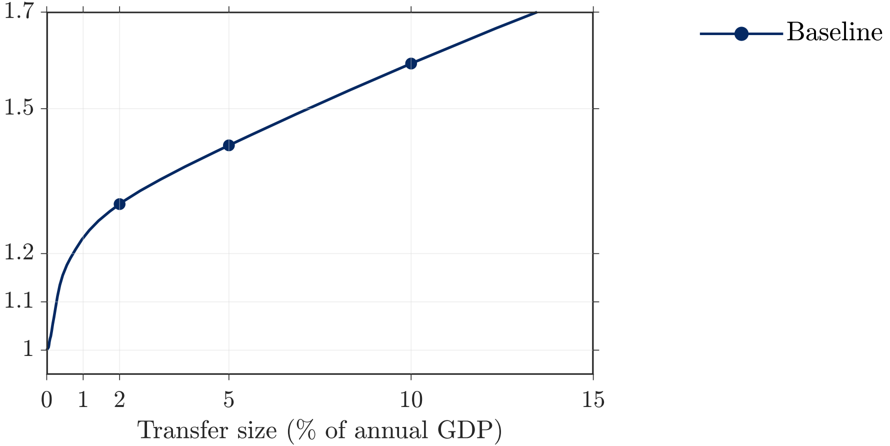
Taking stock: Why linear-method overstates?

- Linear solution, changes in constrained share have zero first-order effects
  - Effectively, as if constrained households permanently MPC=1
- Non-linear solution is highly concave
  - Key: Drop in MPC at kink $\times$ Share of households becoming unconstrained

$\Rightarrow$ Significant gaps between linear and non-linear solution, even for small shocks

How small is small? Robustness of P.E. Results

Linear/Non-linear Consumption Response



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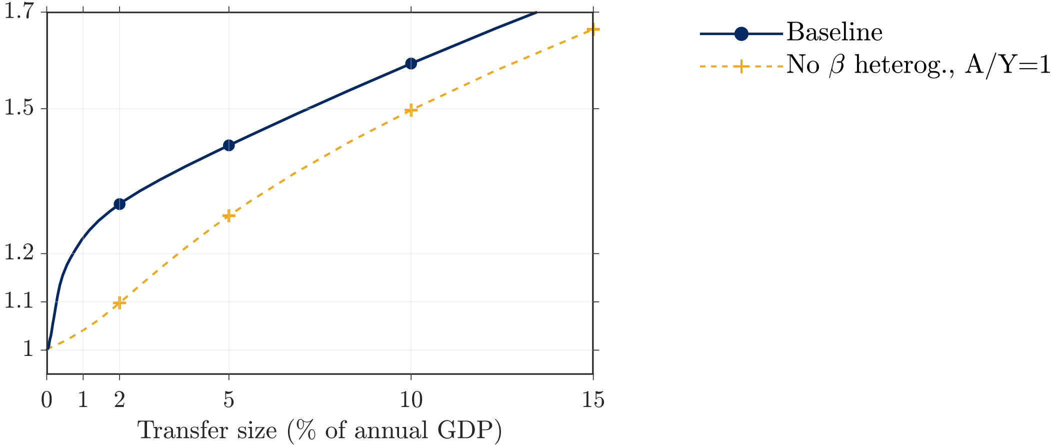

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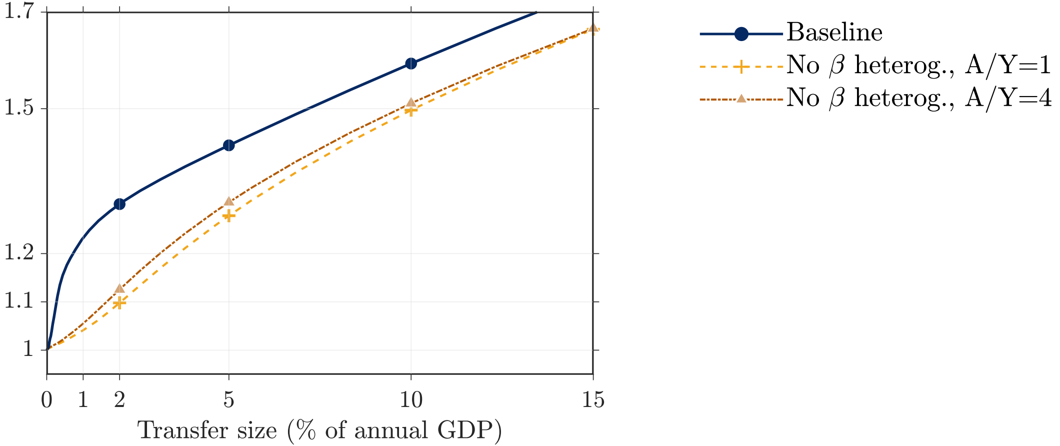

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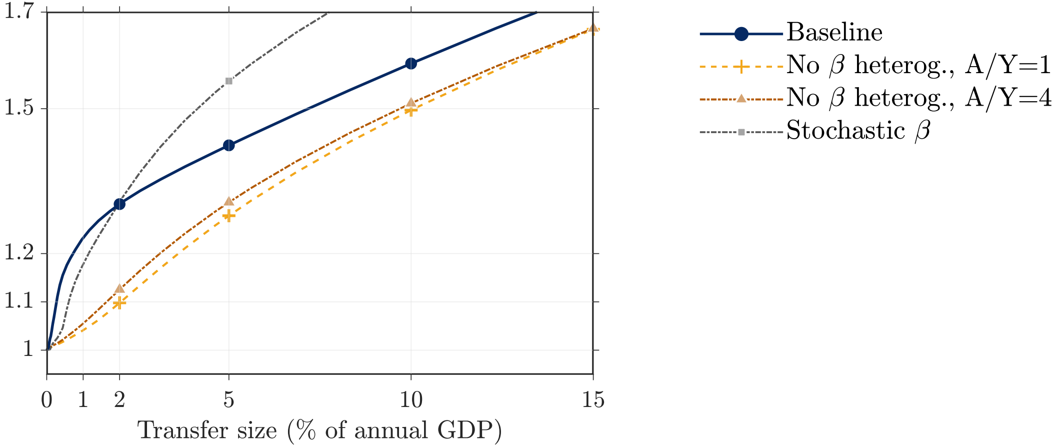

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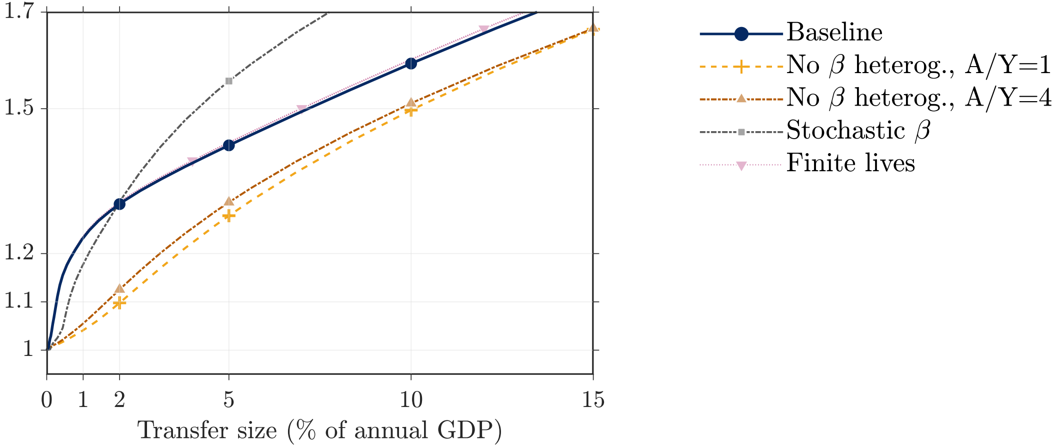

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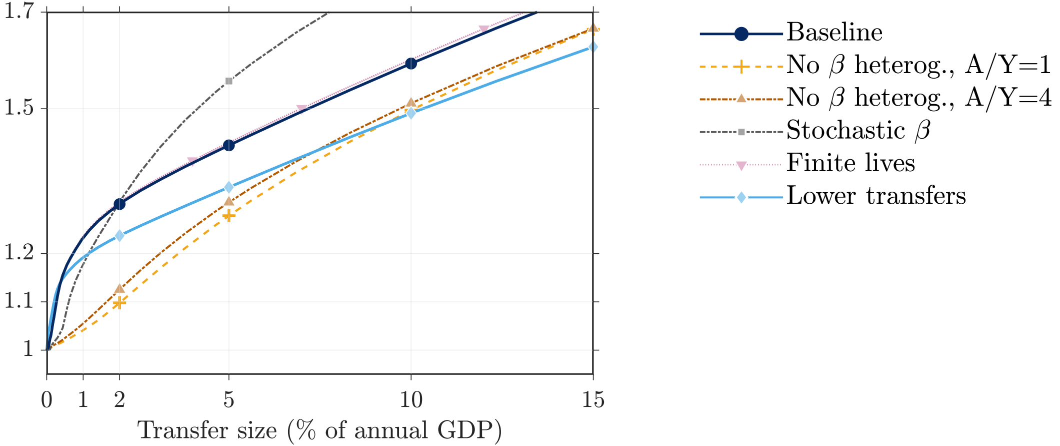

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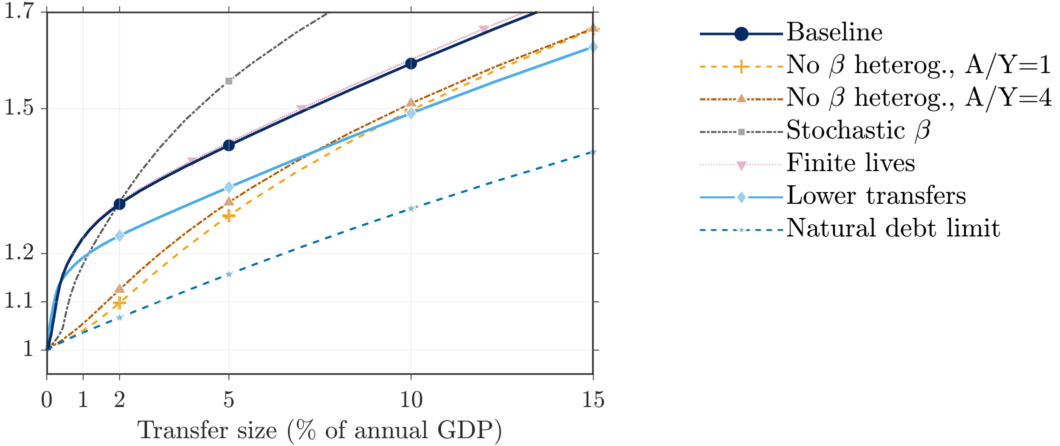

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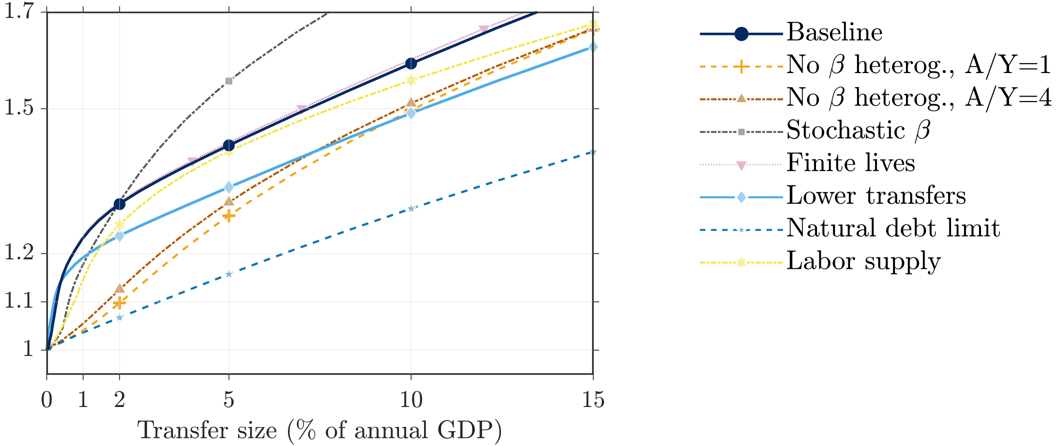

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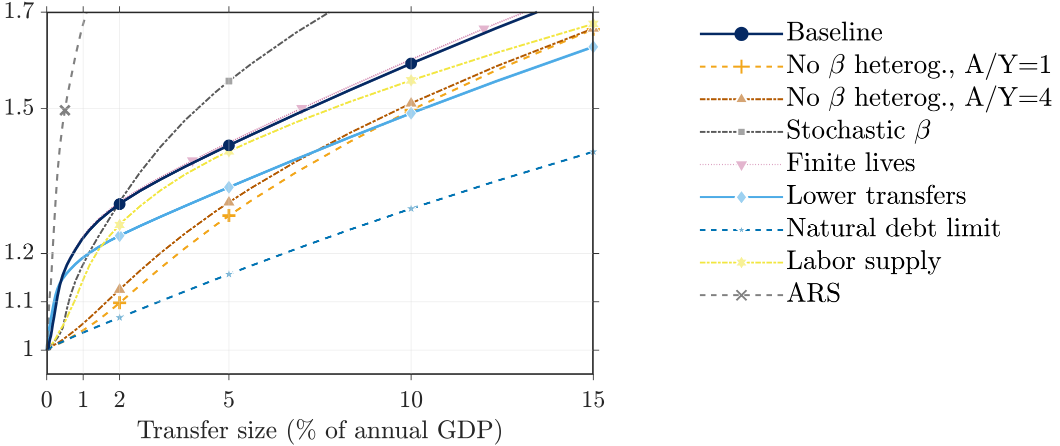
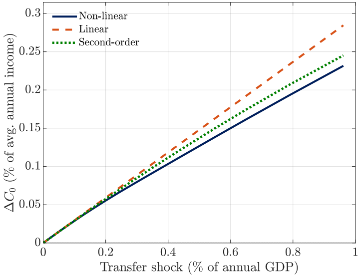
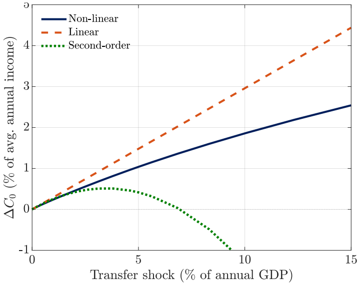


Do higher-order local methods help?

Narrow range (0–1%)



Full range (0–15%)



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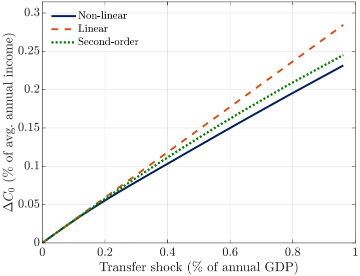

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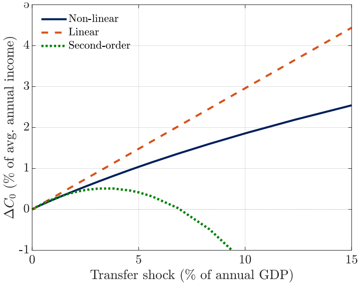

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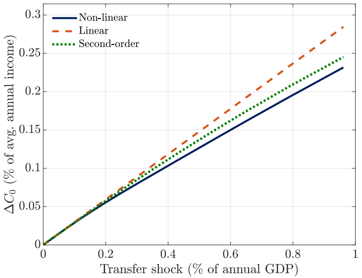
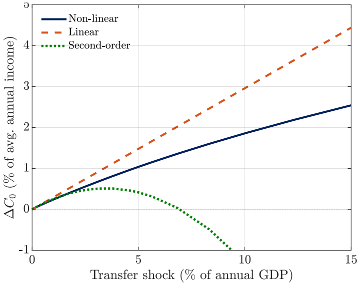


Full range (0–15%)



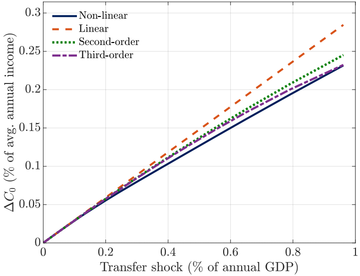
$$\mathcal{C}_0''(0) = \underbrace{\Lambda'(0)\,\mathbb{E}\big[(1-c_x^+) \mid \tilde{x} = \bar{x}(y)\big]}_{\text{movement off constraint } <0} + \underbrace{(1-\bar{\Lambda})\,\mathbb{E}\big[c_{xx}^+ \mid \tilde{x} > \bar{x}(y)\big]}_{\text{individual curvature } <0}$$

$$\Lambda'(0) = -\int_Y g(\bar{x}(y) \mid y)\,\mathrm{d}F(y) < 0$$

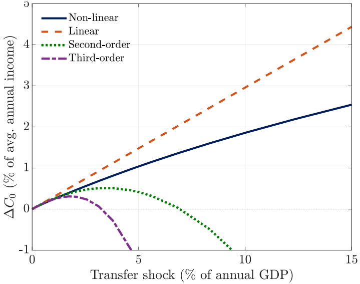
Second-order extrapolates change in constrained share at constant rate $\Rightarrow$ overshoots for small Δ, *understating* aggregate response

Do higher-order local methods help?

Narrow range (0–1%)



Full range (0–15%)

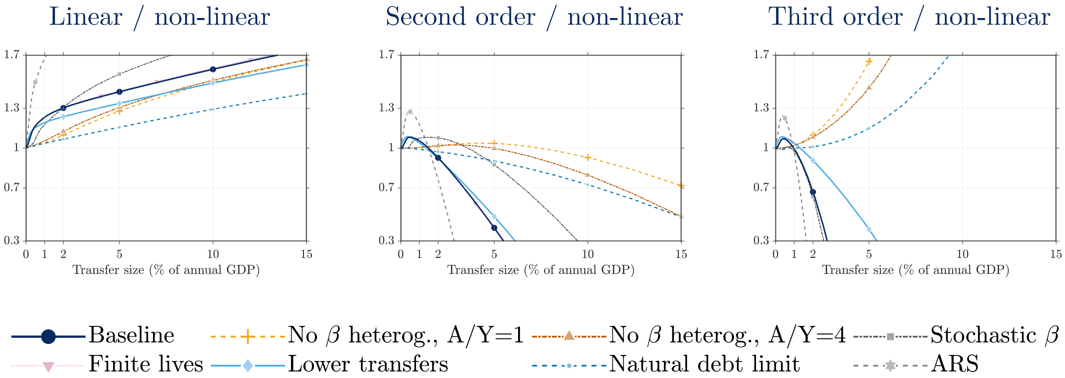


$$\mathcal{C}_0'''(0) = \underbrace{\Lambda''(0)\,\widetilde{\mathbb{E}}\big[(1-c_x^+) \mid \tilde{x} = \bar{x}(y)\big]}_{\text{curvature of constrained share}} - \Lambda'(0)\,\mathbb{E}\big[c_{xx}^+ \mid \tilde{x} = \bar{x}(y)\big] + (1-\bar{\Lambda})\,\mathbb{E}\big[c_{xxx}^+ \mid \tilde{x} > \bar{x}(y)\big]$$

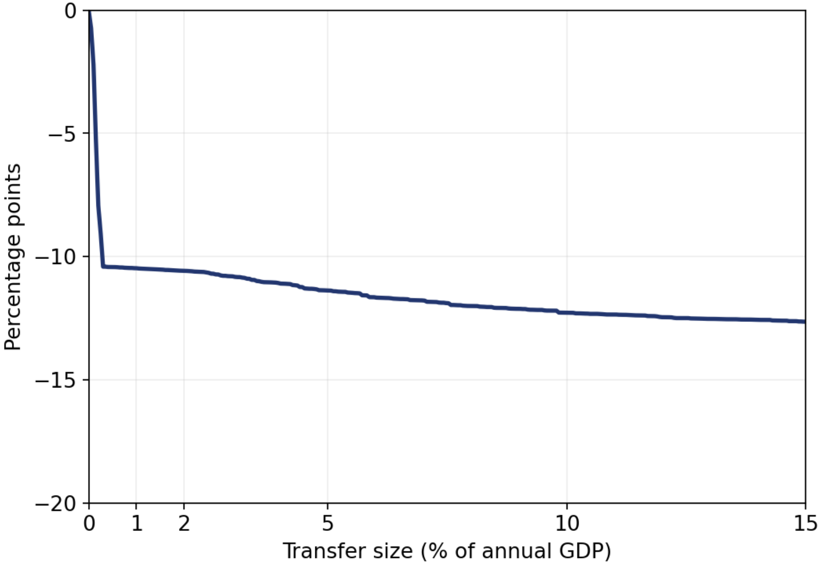
$$\Lambda''(0) = \int_Y g'(\bar{x}(y) \mid y)\,\mathrm{d}F(y) > 0$$

Third-order overshoots *further*

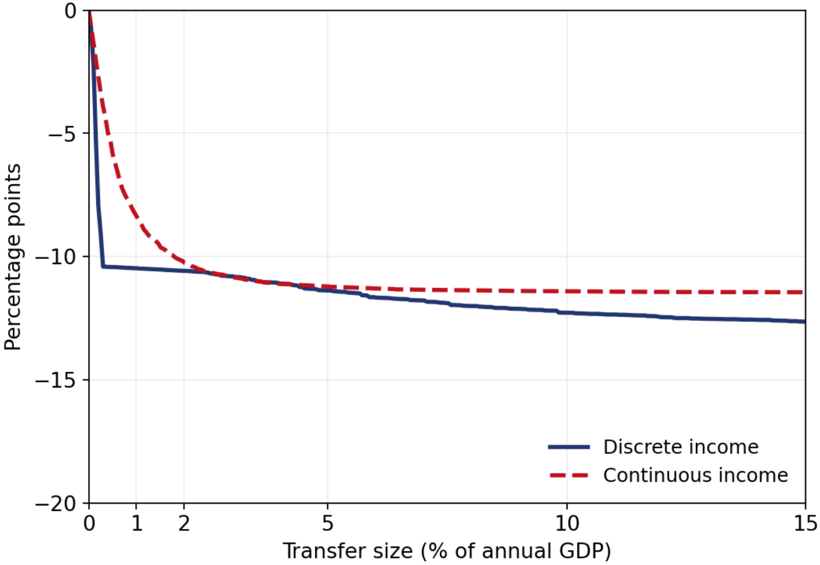
Do Higher-order approximations help?



Change in Share of Constrained Households

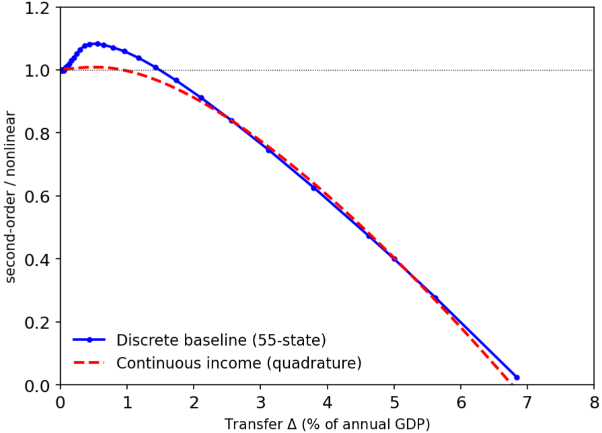


Change in Share of Constrained Households



Robustness to Discreteness of Income Process

Second-order/Non-Linear



A continuous income process yields a smoother higher-order approximation, but the approximation remains accurate only for small shocks.

Roadmap

1. Partial equilibrium model and calibration
2. Fiscal stimulus in general equilibrium
3. Other shocks

General Equilibrium Model

PE + production, labor supply, nominal rigidities, fiscal/monetary rules.

General Equilibrium Model

- **Households:**
  - Utility flow, $u(c_{it}) - v(n_{it})$ with $v' > 0, v'' > 0$
  - Income $(1-\tau)\left(\frac{W_t}{P_t} e_{it} n_{it} + d_{i,t}\right)$

General Equilibrium Model

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  - Utility flow, $u(c_{it}) - v(n_{it})$ with $v' > 0, v'' > 0$
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  - Labor union with sticky wages (linearized)

$$\pi_t = \kappa\left(v'(N_t) - \frac{1-\tau}{C_t}\right) + \bar{\beta}\,\pi_{t+1}, \qquad \text{(Phillips Curve)}$$

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- **Firms and Technology:**

$$\left.\begin{array}{l} \text{Production: } Y_t = N_t \\ \text{Flexible prices} \end{array}\right\} \Rightarrow W_t = P_t, \quad d_{i,t} = 0$$

General Equilibrium Model (ctd)

- Government budget:

$$B_t + \tau Y_t = G + T_t + (1 + r_{t-1})B_{t-1}$$

- Fiscal rule ($t > 0$)

$$T_t = T - \phi_B \bar{r}(B_{t-1} - \bar{B})$$

- Monetary policy:

$$i_t = \bar{i} + \phi_\pi(\pi_t - \bar{\pi})$$

Calibration

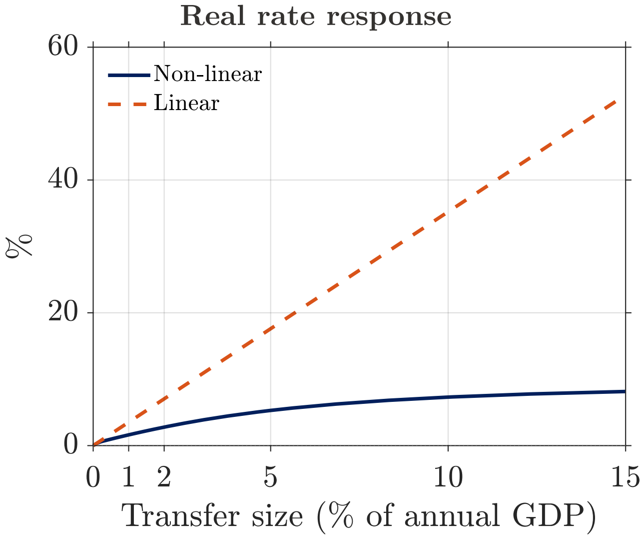
Additional Parameters for GE Model

Parameter	Value	Description
$\bar{B}$	4	Steady-state government debt=100% GDP
κ	0.01	Phillips curve slope
ν	1	Frisch elasticity of labor supply
ϕ_π	1.5	Taylor coefficient
ϕ_B	8	Fiscal rule
G	0.104	Government spending
T	0.15	Government transfers

Computation:

- Linear: sequence space Jacobian (ABRS)
- Non-linear: quasi-Newton algorithm

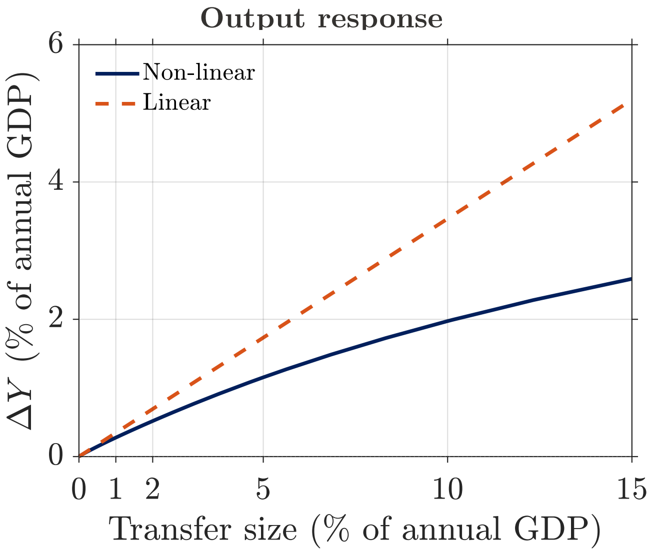
Flexible-Price Allocation



- No output effects
- Linear methods overstate increase in demand and rise in r

With household labor choice $\rightarrow$

HANK



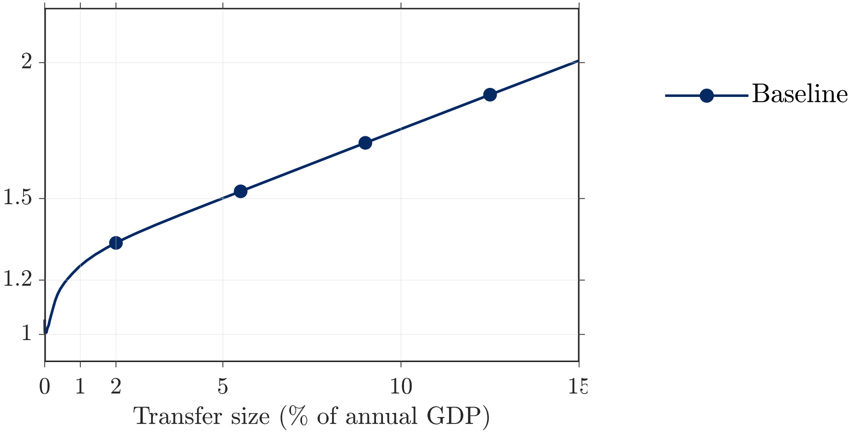
- At $\Delta = 2\%$ GDP: linear overstates by 34%
- At $\Delta = 5\%$ GDP: linear overstates by 50%

4Q response →

Impulse responses →

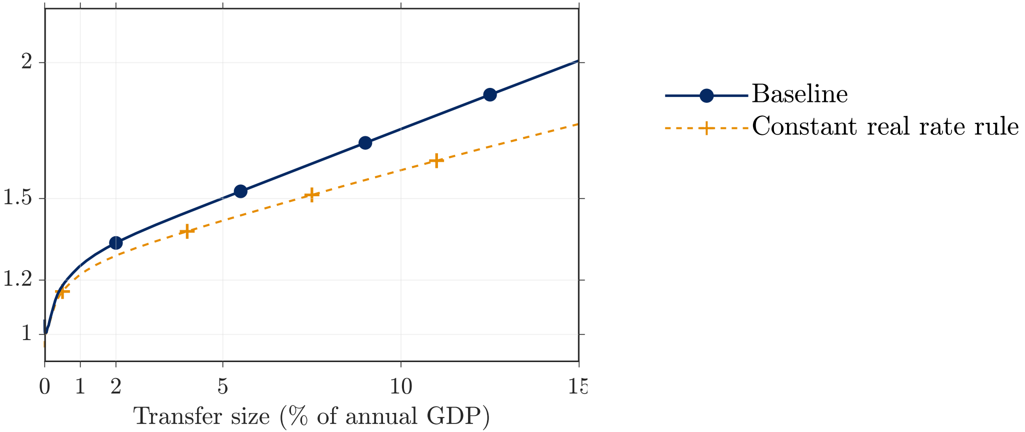
How small is small? Robustness in HANK

Linear/Non-linear Output Response



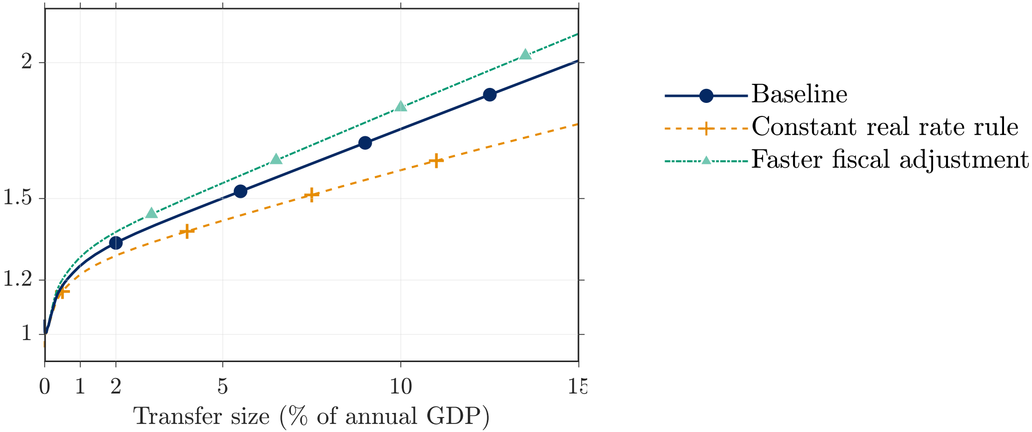
How small is small? Robustness in HANK

Linear/Non-linear Output Response



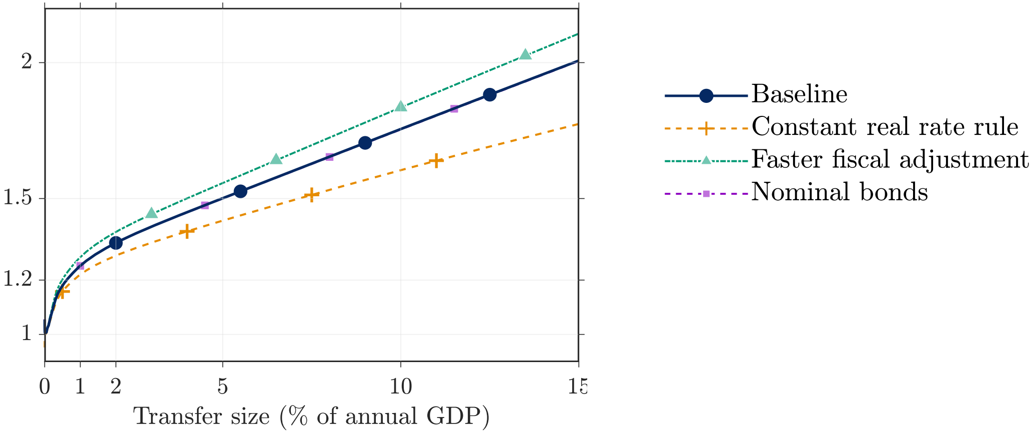
How small is small? Robustness in HANK

Linear/Non-linear Output Response



How small is small? Robustness in HANK

Linear/Non-linear Output Response



How small is small? Robustness in HANK

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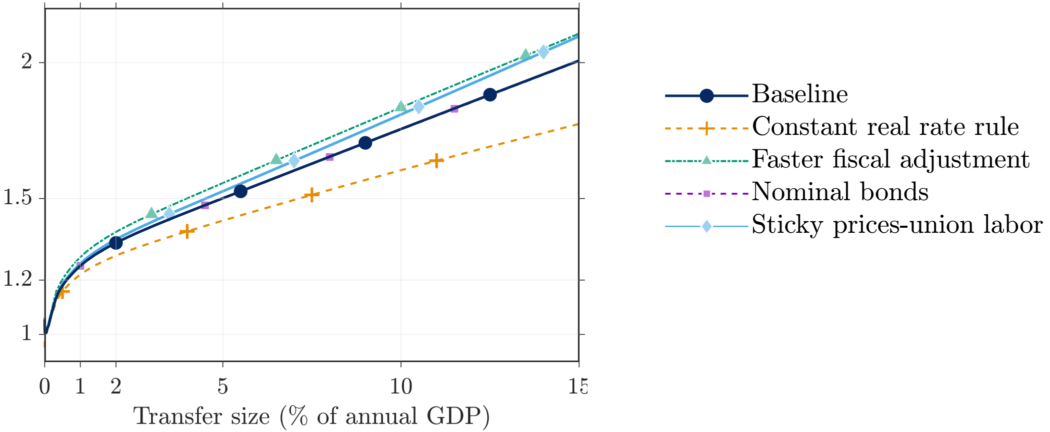

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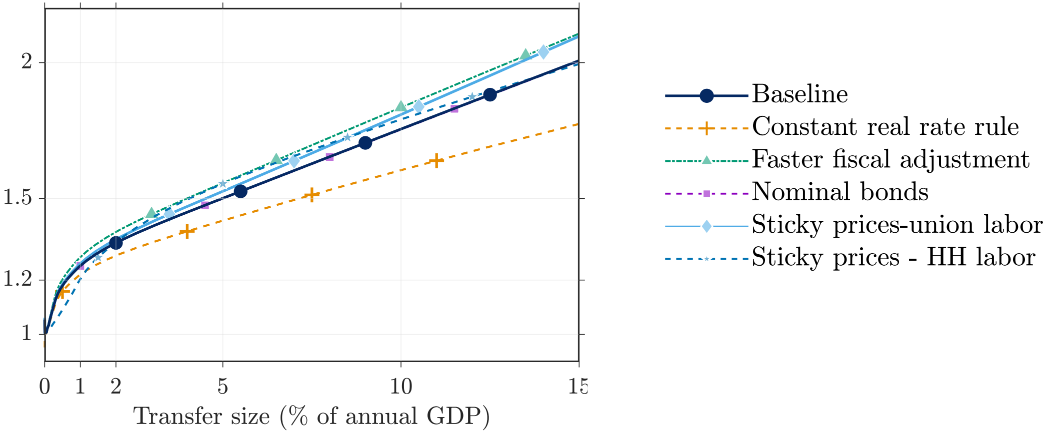

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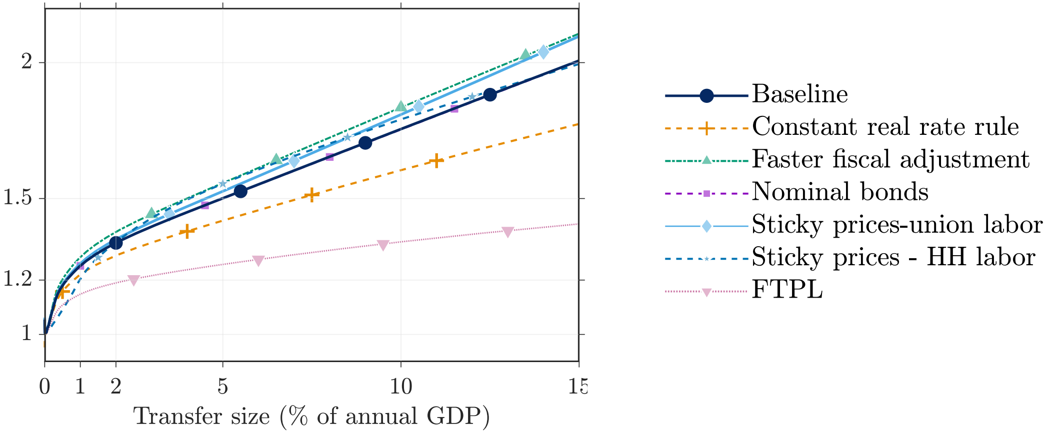
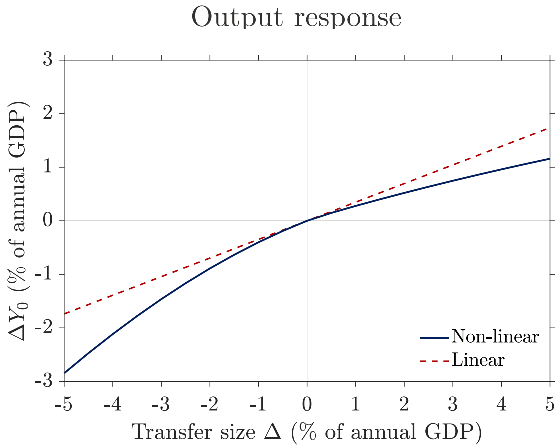


Other results

1. Fiscal contractions vs. expansions go →
2. Gap in persistence go →
3. Targeted transfers go →
4. Anticipated transfers go →
5. Other shocks go →

Fiscal contractions vs. expansions

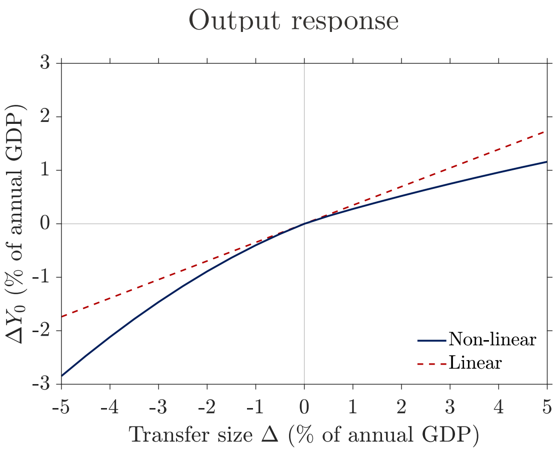


Concavity: linear overstates expansions, understates contractions.

Note: Baseline HANK model

Other results

Fiscal contractions vs. expansions



Concavity: linear overstates expansions, understates contractions.

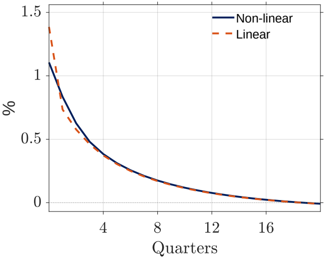
Non-linearity slightly larger for contractions:

Note: Baseline HANK model

Other results

Linear method understates persistence

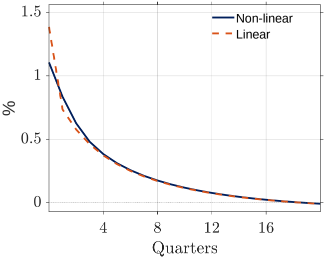
(a) Output



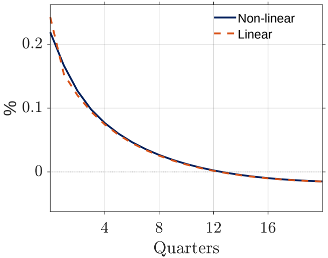
Note: Impulse responses in the HANK to a 1% of annual GDP transfer.

Linear method understates persistence

(a) Output



(b) Real rate



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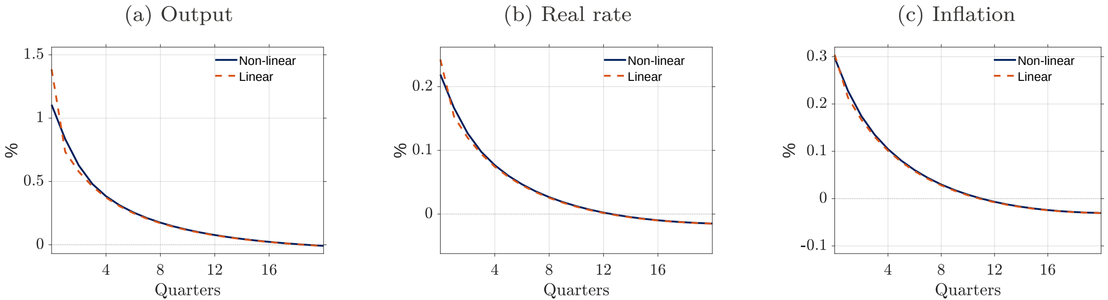
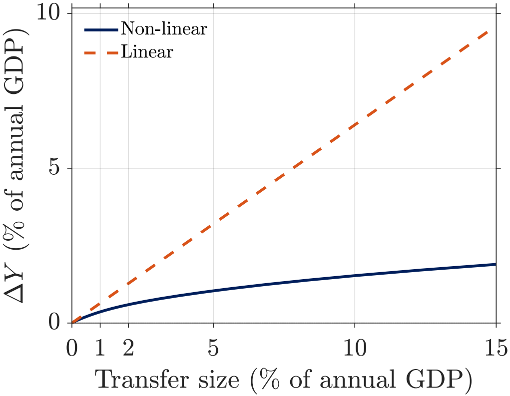


Note: Impulse responses in the HANK to a 1% of annual GDP transfer.

Targeted Transfers vs. Untargeted Transfers

◂ Other results

(a) Output response (bottom 20%)

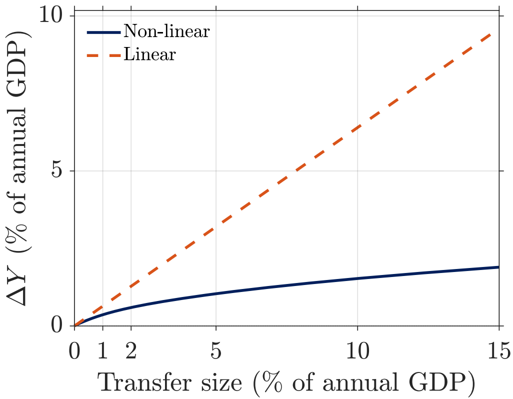


(b) Linear / non-linear

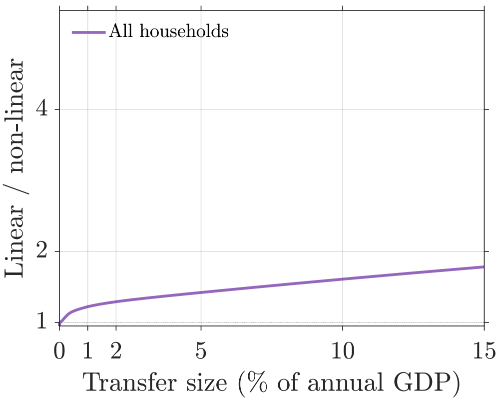
Targeted Transfers vs. Untargeted Transfers

◂ Other results

(a) Output response (bottom 20%)



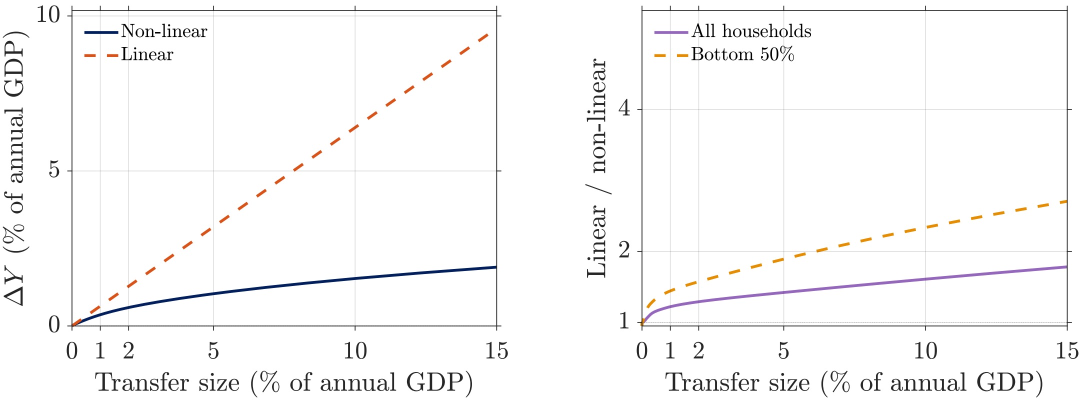
(b) Linear / non-linear



Targeted Transfers vs. Untargeted Transfers

◂ Other results

(a) Output response (bottom 20%)   (b) Linear / non-linear



Targeted Transfers vs. Untargeted Transfers

Other results

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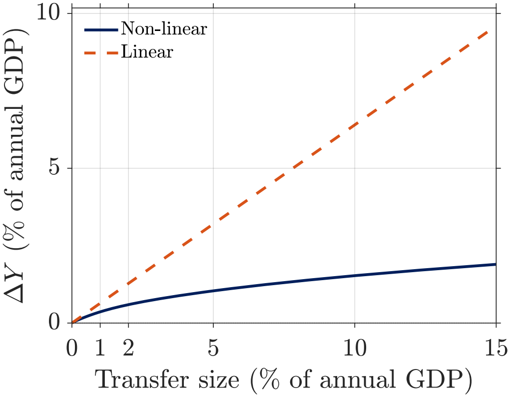
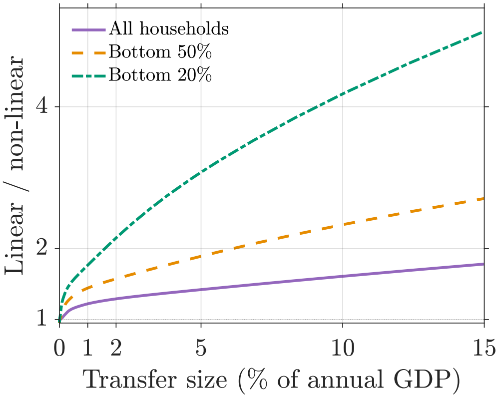


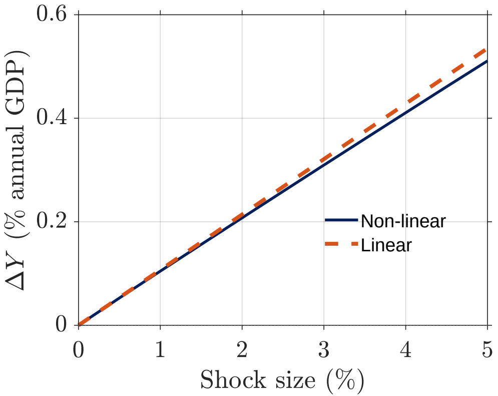
(b) Linear / non-linear



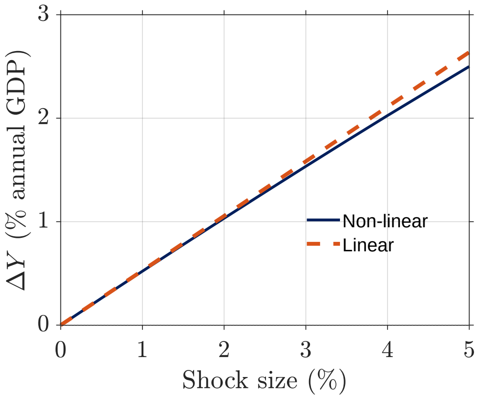
Non-linearities for monetary policy and TFP

◂ Other results

(a) Monetary (rate cut)



(b) TFP



▸ Tax-rate cut

▸ Discount-factor (β) shock

Conclusions

- First-order methods significantly overstate the effects of fiscal transfers
  - Key: higher transfers shift households out of the borrowing constraint, so the aggregate MPC falls with transfer size

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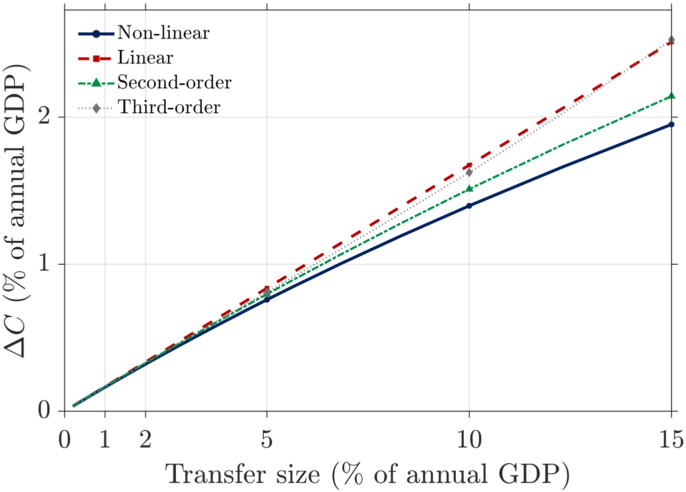
Conclusions

- First-order methods significantly overstate the effects of fiscal transfers
  - Key: higher transfers shift households out of the borrowing constraint, so the aggregate MPC falls with transfer size
- For monetary policy, linear methods are accurate. But HA $\approx$ RA
- When transmission works through departures from Ricardian equivalence, first-order methods are quantitatively unreliable
  - Higher-order local approx. improves for narrow shocks, but deteriorate quickly

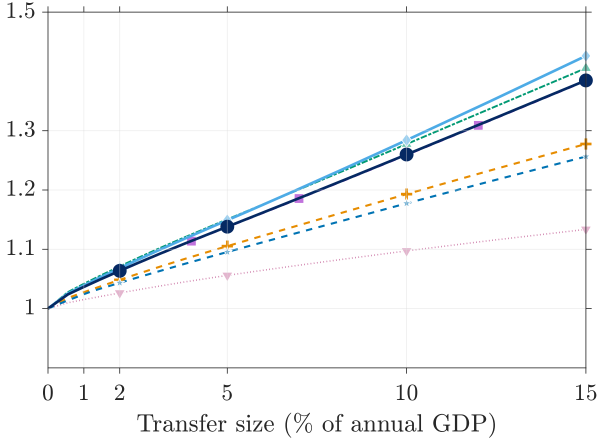
Appendix

Additional figures

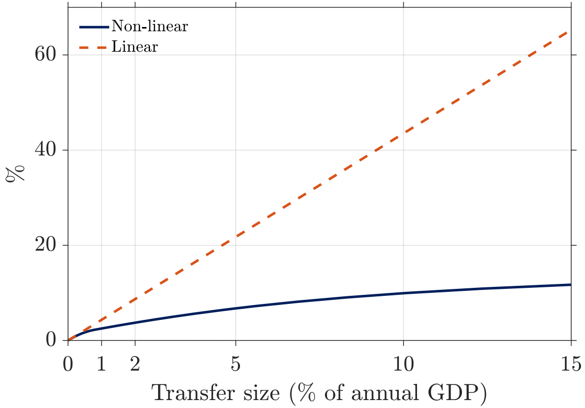
Unconstrained: higher-order local approx.



GE: first-year linear-to-non-linear ratio



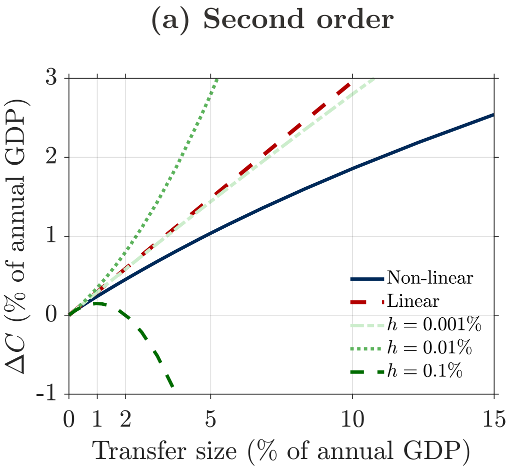
GE: flexible prices with household labor choice

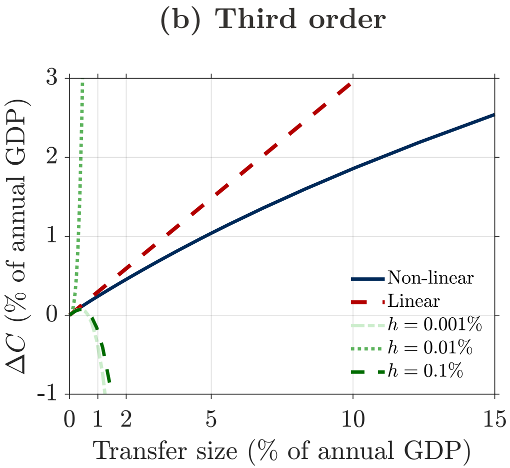


- Households choose labor freely (no union)
- Output effects via labor-supply incentives
- Real-rate response remains highly concave

Back

Higher-order with finite differences



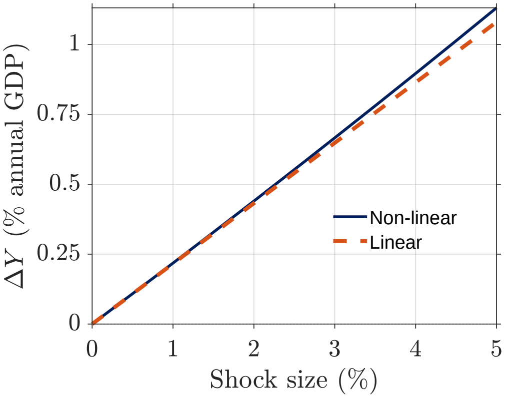


Extremely sensitive to step size and poorly behaved even for small shocks.

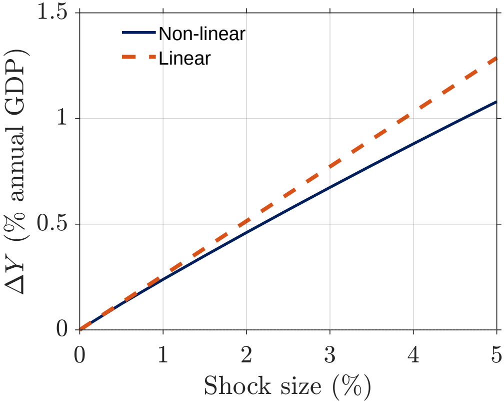
Back

Other shocks: discount factor and labor tax

(a) Discount-factor shock



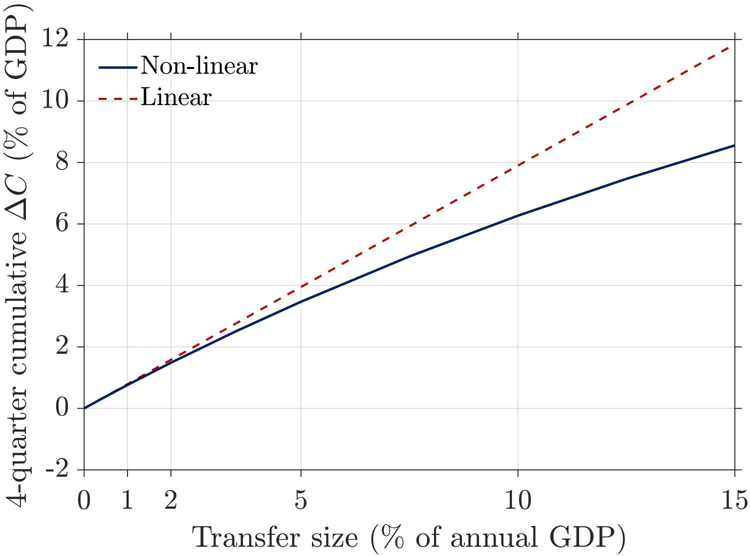
(b) Labor-tax cut



A τ cut shifts fewer resources to low-income, constrained households.

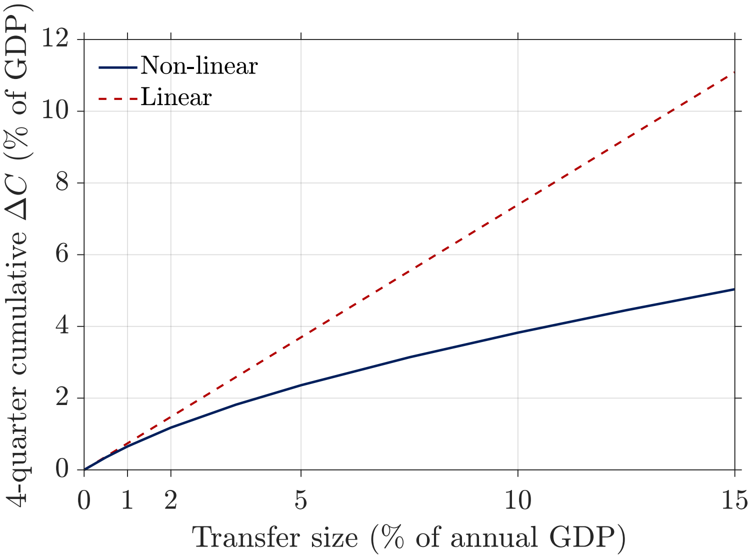
4-quarter cumulative response (Baseline HANK)

Back



4-quarter cum. response (bottom-20% targeted)

Back



Alternative decomposition

Back

Exact non-linear constrained share + first-order Taylor for the unconstrained:

$$\mathcal{C}_0^{\text{hyb}}(\Delta) = \int_Y \left[\underbrace{\int_{-\infty}^{\bar{x}(y)-\Delta} c(\tilde{x}+\Delta, y)\,\mathrm{d}G(\tilde{x} \mid y)}_{\text{exact constrained share, MPC=1}} + \underbrace{\int_{\bar{x}(y)-\Delta}^{\infty} \left[c(\tilde{x}, y) + c_x^{+}(\tilde{x}, y)\,\Delta\right] \mathrm{d}G(\tilde{x} \mid y)}_{\text{unconstrained: first-order Taylor}} \right] \mathrm{d}F(y)$$