Multi-Asset Market Making

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Motivation

"A market maker's success depends on . . . accurately responding to relevant market data in similar and **correlated instruments***."*

— Virtu Financial, IPO Filing (SEC Form S-1, 2014)

- Each trade has two sides:
  - **Liquidity demand** (e.g., market order): pays the spread
    - E.g., 1€ = \$1.17; Ask \$1.18, Bid \$1.16 $\implies$ spread = \$0.02
  - **Liquidity supply** (e.g., limit order): earns the spread; loses from **inventory**

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- **This paper:** how does a dealer manage inventory **across correlated assets**?

Modeling Liquidity Demand & Supply

- **Liquidity Demand:** Modern Portfolio Theory (MPT) (Markowitz, Sharpe, Tobin)
  - N risky assets (+ 1 risk-free), 1 period
  - Inputs: expected returns, standard deviations, correlations
  - Results: efficient frontier, tangency portfolio, optimal portfolio
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  - Results: efficient frontier, tangency portfolio, optimal portfolio
  - If all investors agree on inputs $\implies$ CAPM
- **Liquidity Supply:** How do market makers behave?
  - N risky assets, infinitely many periods
  - Inputs: standard deviations, correlations
  - Results: **?**

Market Makers (Dealers) and Prices

- Market makers supply liquidity by setting the bid and ask quotes at which others trade:
    - How do they set their quotes?
    - How do they manage inventory in multiple assets?
    - How much **cross-hedging** is there?
    - Which assets do they choose to make markets in?

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- Do market makers affect prices?
  - Hendershott & Menkveld (2014) measure "price pressures" in **one asset**: a $100,000 inventory shock causes a 0.98% price pressure for small-cap stocks, 0.02% for large-cap
  - Does cross-hedging reduce price pressures?
  - How does cross-stock correlation vary with frequency?

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- **This paper:** dynamic model in N correlated assets + tests on TSX message-level data

Model

- Discrete time $t = 0, 1, 2, \ldots$; N **risky assets**, fundamentals $v_{n,t}$ follow a random walk; Δv has covariance Σ_v
- (Monopolist) dealer posts ask a_t and bid b_t in each asset; aggregate market orders satisfy:

$$Q_t^b = \frac{k}{2}(v_t - a_t) + \ell + \varepsilon_t^b, \quad \text{with} \quad \varepsilon_t^b \overset{IID}{\sim} \mathcal{N}(0, \Sigma_L/2)$$

$$Q_t^s = \frac{k}{2}(b_t - v_t) + \ell + \varepsilon_t^s, \quad \text{with} \quad \varepsilon_t^s \overset{IID}{\sim} \mathcal{N}(0, \Sigma_L/2)$$

$\implies$ 3 components: elastic + inelastic + random

- Inventory: $x_{t+1} = x_t + Q_t^s - Q_t^b$
- Dealer maximizes expected profit (time discount $\beta < 1$), penalized by **quadratic inventory cost** (**risk aversion** γ)

Optimal Quotes

- **Proposition 1:** In equilibrium, the dealer sets:

$$a_t = p_t + h, \qquad b_t = p_t - h, \qquad p_t = v_t - \lambda x_t$$

where $h = k^{-1}\ell$, and $\lambda = k^{-1}\Lambda$ is the $N \times N$ **pricing matrix**, with Λ the unique symmetric positive definite solution of:

$$\Lambda^2 + \Lambda\left(S + \frac{1-\beta}{\beta} I_N\right) = S, \quad \text{with} \quad S = \gamma k \Sigma_v$$

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- Price pressure: $p_t = v_t - \lambda x_t$
  - Diagonal λ_{ii}: own-stock price pressure
    - One-dimensional intuition (Ho–Stoll 1981): buy imbalance $\Longrightarrow$ $x_t < 0$ $\Longrightarrow$ dealer raises a_t, b_t to discourage buyers & encourage sellers $\Longrightarrow$ p_t rises
  - Off-diagonal λ_{ij}: cross-stock price pressure = **cross-hedging**

Cross-Hedging

- When $\beta \to 1$, the pricing matrix λ converges to:

$$\lambda = \frac{k^{-1}}{2}\Big((S^2 + 4S)^{1/2} - S\Big), \quad \text{with} \quad S = \gamma k \Sigma_v$$

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- **Corollary 1:** With diagonal $k = k_0 I_N$ (per-asset demand):
  Small $\gamma \implies$ **cross-hedging**:

$$\lambda \approx \sqrt{\tfrac{\gamma}{k_0}}\, \Sigma_v^{1/2}$$

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  - *Intuition:* dealer mean-reverts each inventory to zero so quickly that hedging across assets has no time to be valuable

Two Assets: Pricing Matrix

- **Corollary 2:** Two assets with correlation a, diagonal k:

$$\Sigma_v = \sigma_v^2 \begin{bmatrix} 1 & a \\ a & 1 \end{bmatrix}, \quad k = k_0 I_2.$$

Then the pricing matrix is:

$$\lambda = \frac{1}{k_0(m_1+1)(m_2+1)} \begin{bmatrix} m_1+m_2+2 & m_1-m_2 \\ m_1-m_2 & m_1+m_2+2 \end{bmatrix}$$

$$m_1 = \left(\frac{4}{\gamma k_0 \sigma_v^2 (1-a)} + 1\right)^{1/2}, \quad m_2 = \left(\frac{4}{\gamma k_0 \sigma_v^2 (1+a)} + 1\right)^{1/2}$$

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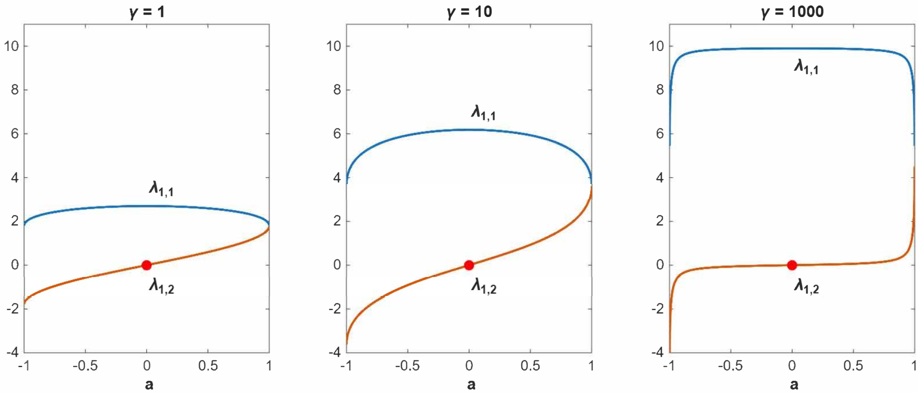
- Note: off-diagonal λ_{12} is proportional to $m_1 - m_2$:
  - $a = 0 \implies m_1 = m_2 \implies$ no cross-hedging
  - $a > 0 \implies m_1 > m_2 \implies$ **cross-hedging**

Pricing Coefficients vs Correlation & Risk Aversion



$\lambda_{1,1}$ (own) and $\lambda_{1,2}$ (cross) vs correlation $a = \text{corr}(\Delta v_1, \Delta v_2)$, for risk aversion $\gamma = 1$ (left), 10 (center), 1000 (right). ($\sigma_v = 1$, $k_0 = 0.1$)

- $\lambda_{1,2}$ **rises with** a: cross-hedging strengthens with correlation
- $|\lambda_{1,2}|$ **rises, then falls with** γ: need risk aversion to cross-hedge, but very high γ eliminates cross-hedging

Two Assets: Numerical Example

- Set $\gamma = 1$, $\sigma_v = 1$, $k_0 = 0.1$, $a = 0.6 \implies$ pricing matrix:

$$\lambda = \begin{bmatrix} 2.54 & \mathbf{0.73} \\ \mathbf{0.73} & 2.54 \end{bmatrix}$$

- Suppose $v_t = [10, 10]'$. If $x_t = [-1, +1]' \implies$

$$p_t = \begin{bmatrix} 10 \\ 10 \end{bmatrix} - \begin{bmatrix} 2.54 & 0.73 \\ 0.73 & 2.54 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 11.81 \\ 8.19 \end{bmatrix}$$

  If instead $x_t = [-1, 0]' \implies$

$$p_t = \begin{bmatrix} 10 \\ 10 \end{bmatrix} - \begin{bmatrix} 2.54 & 0.73 \\ 0.73 & 2.54 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 12.54 \\ 10.73 \end{bmatrix}$$

- Long position in (correlated) asset 2 partially hedges short position in asset 1 $\implies$ p_1 rises less (11.81 vs 12.54)
  - Substitution rate: $\lambda_{12}/\lambda_{11} = 0.73/2.54 \approx 29\%$: inventory in asset 2 absorbs about 29% of asset-1's price pressure

Two Assets: Hedging vs Correlations

- If correlation rises from $a = 0.6$ to 0.8, pricing matrix changes:

$$\lambda_{(a=0.6)} = \begin{bmatrix} 2.54 & 0.73 \\ 0.73 & 2.54 \end{bmatrix} \quad \rightarrow \quad \lambda_{(a=0.8)} = \begin{bmatrix} 2.38 & 1.06 \\ 1.06 & 2.38 \end{bmatrix}$$

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- **Cross-hedging $\lambda_{1,2}$ rises**: $0.73 \rightarrow 1.06$
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- **Cross-hedging $\lambda_{1,2}$ rises**: $0.73 \to 1.06$
  - Substitution rate $\lambda_{12}/\lambda_{11}$: $29\% \to 45\%$ (reaches 100% as a converges to 1)
- **Own-hedging $\lambda_{1,1}$ falls**: $2.54 \to 2.38$
  - At $a = 1$, all entries of λ equal 1.79 $\implies$ assets are perfect substitutes; less own-hedging needed as a rises

Inventory Half-Life

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$$\left(\frac{m_1 - 1}{m_1 + 1}\right)^T + \left(\frac{m_2 - 1}{m_2 + 1}\right)^T = 1$$

  - Common inventory $(x_1 + x_2)$ decays at rate $\frac{m_2-1}{m_2+1}$
  - Spread inventory $(x_1 - x_2)$ decays at rate $\frac{m_1-1}{m_1+1}$ (slower; cross-hedging works on the spread)
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- Approximation when γ is small:

$$T \approx \frac{m}{\sqrt{1+a} + \sqrt{1-a}}, \qquad m = \frac{1}{\sqrt{\gamma k_0}\, \sigma_v}$$

$\implies$ T depends on correlation a

Inventory Half-Life: Multi-Asset Bias

- Recall: $T_a \approx \frac{m}{\sqrt{1+a}+\sqrt{1-a}}$
  - $a = 0$: $T_{a=0} \approx \frac{m}{2}$
  - $a = 1$: $T_{a=1} \approx \frac{m}{\sqrt{2}} \approx \sqrt{2}\, T_{a=0}$
  - $\implies$ Half-life is $\approx$ 41% larger at $a = 1$

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- If we estimate γ by observing T (fixing a), then:

$$\gamma_{a=1} = 2\,\gamma_{a=0}$$

$\implies$ **Single-asset model underestimates γ by 50%**

Asset Selection vs Correlation

- Dealers also pick assets. *Which correlation do they prefer?*
- Dealer's **normalized value** (steady-state profit per period) W = the $\beta \to 1$ limit of value function $V(x) = W_0 - x'\alpha x$:

$$W = \lim_{\beta \to 1} (1-\beta) V(x) = \underbrace{\ell' k^{-1} \ell}_{\text{spread profits}} - \underbrace{\alpha \circ \Sigma_L}_{\text{inventory penalty}}$$

where $\alpha^{-1} = \Lambda^{-1} - k$ and $\Sigma_L = L_0 \begin{bmatrix} 1 & b \\ b & 1 \end{bmatrix}$

- **Inventory penalty** depends on *two* correlations:
  - $a = \text{corr}(\Delta v_1, \Delta v_2)$ enters α: **cross-hedging channel**
  - $b = \text{corr}(\varepsilon_1^s - \varepsilon_1^b, \varepsilon_2^s - \varepsilon_2^b)$ enters Σ_L: **liquidity-shocks channel**
- **Corollary 3** ($N = 2$, $k = k_0 I$): If $n_1 = m_1(1-a), n_2 = m_2(1+a)$,

$$W = k_0(\ell_1^2 + \ell_2^2) - \frac{\gamma \sigma_v^2 L_0}{2} \big(n_1(1-b) + n_2(1+b) + 2(1+ab) \big)$$

- γ small vs γ large give **opposite** predictions in a

High Risk Aversion: Diversification Wins

- When γ **is large**:

$$W \approx k_0(\ell_1^2 + \ell_2^2) - 2\gamma L_0 \sigma_v^2 \, \mathbf{(1{+}ab)} - \frac{2L_0}{k_0}$$

Dominant term $\propto -(1 + ab)$: **decreasing in** a when $b > 0$.

  - Dealer kills inventory quickly $\implies$ no time for cross-hedging:
  - Steady-state $x_t \approx \varepsilon^s_{t-1} - \varepsilon^b_{t-1}$ (last period's demand shock).
  - Per-period cost $\approx \gamma \, \mathrm{E}[x' \Sigma_v x] \; = \; \gamma \, \Sigma_v \circ \Sigma_L \; = \; 2\gamma L_0 \sigma_v^2 (1 + ab)$

- When $b > 0$, a high-γ dealer behaves like a **Markowitz investor** and prefers low (or negative) correlation a

Low Risk Aversion: Cross-Hedging Wins

- When γ **is small**:

$$W \approx k_0(\ell_1^2 + \ell_2^2) - \gamma^{\frac{1}{2}} L_0 \sigma_v k_0^{-\frac{1}{2}} \left((1-a)^{\frac{1}{2}}(1-b) + (1+a)^{\frac{1}{2}}(1+b) \right)$$

  $W = W(a)$ is **U-shaped in** a, with local maxima at $a = \pm 1$

- **Asymmetry** when $b > 0$: $W(-1) > W(+1)$.
  - In sum (s) and spread (d) coordinates, $s, d = (x_1 \pm x_2)/\sqrt{2}$:
    Σ_v eigenvalues: $(1+a)\sigma_v^2$ on s, $(1-a)\sigma_v^2$ on d
    Σ_L eigenvalues: $(1+b)L_0$ on s, $(1-b)L_0$ on d
  - At $a = +1$: $(1-a) = 0$, only s contributes; variance on s: $(1+b)L_0 \implies$ **Large penalty**
    At $a = -1$: $(1+a) = 0$, only d contributes; variance on d: $(1-b)L_0 \implies$ **Small penalty**

  $\implies$ **Theoretical optimum**: $a = -1$ (when $b > 0$)

- However, in practice $a > 0 \to$ next slide

Low Risk Aversion: In Practice

- In practice $a > 0$ (stocks positively correlated) and $b > 0$ (commonality in liquidity). Restrict to $a \in (0, 1]$.
- Within $(0, 1]$, W has an **interior minimum** at $a^* = 2b/(1 + b^2)$ $\Longrightarrow$ Optimum is at an endpoint:
  - At $a = 0$: cost spread across both axes (no cross-hedging). Cost factor: 2
  - At $a = 1$: full cross-hedging (cost only on s); but s-variance is $(1 + b)L_0$. Cost factor: $\sqrt{2}\,(1 + b)$
- **Threshold**: $\sqrt{2}\,(1 + b^*) = 2 \quad \Longrightarrow \quad b^* = \sqrt{2} - 1 \approx 0.41$
  - $b < 0.41$: **cross-hedging wins** $\Longrightarrow$ dealer picks $a \to 1$ (highly correlated)
  - $b > 0.41$: **diversification wins** $\Longrightarrow$ dealer picks $a \to 0$ (uncorrelated $>$ correlated)
- Equity b: ~ 0.1–0.3 cross-sector, ~ 0.5 same-sector, > 0.7 during market stress $\Longrightarrow$ **regime-switching prediction**

Price Correlation: One Period

- How does dealer behavior shape **observed** price correlations?
- We consider Σ_L diagonal, to isolate the dealer's effect from the common liquidity demand ($b > 0$)
- **Corollary 5** ($N = 2$, $k = k_0 I$, $\Sigma_L = L_0 I$):

$$L_0 \gg k_0 \sigma_v^2 \gg 1/\gamma \quad \Longrightarrow \quad \rho_p = \mathrm{Corr}(\Delta p_{1,t}, \Delta p_{2,t}) \approx 0$$

  Price returns nearly **uncorrelated** even when fundamentals are correlated ($a > 0$). Why

  - High $\gamma \implies$ dealer kills each inventory in almost very period (no cross-hedging) $\implies \lambda$ becomes approximately diagonal $\implies$ quotes move with *own-asset* inventory only
  - $\Sigma_L = L_0 I \implies$ inventories independent across assets

  $\implies$ Price moves are uncorrelated

- **Dealer's footprint**: at short horizons, observed correlation $<$ fundamental correlation a (next slide: Epps effect)

The Epps Effect

- Price correlation depends on **horizon**. With high γ:
  - *One period*: $\rho_p \approx 0$ (dealer's footprint: previous slide)
  - *Many periods*: $\rho_p \to a$ (fundamental correlation)
- Why? Dealer's quote moves are transient (mean-reverting with inventory) $\implies$ fundamental shocks Δv accumulate $\implies$ At long horizon the fundamentals dominate.

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- Why? Dealer's quote moves are transient (mean-reverting with inventory) $\implies$ fundamental shocks Δv accumulate $\implies$ At long horizon the fundamentals dominate.
- **Epps (1979)**: log-price correlation increases with horizon, eventually stabilizes:

	Pairs of Stocks					
Interval	*AMC-Chrysler*	*AMC-Ford*	*AMC-GM*	*Chrysler-Ford*	*Chrysler-GM*	*Ford-GM*
10 minutes	001	.009	– 009	– 014	007	055
20 minutes	009	018	011	017	026	118
40 minutes	.006	012	.014	041	040	197
One hour	–.043	.057	064	.023	065	.294
Two hours	029	.060	094	.112	129	383
Three hours	031	.158	111	361	518	519
One day	–.067	.170	078	342	442	571
Two days	– 020	223	186	336	449	572
Three days	– 098	203	.100	334	542	645

Five Testable Predictions

- From the pricing matrix λ:
  - **Fact 1. Cross-hedging exists**
    Off-diagonal $\lambda_{ij} > 0$ when $\text{corr}(\Delta v_i, \Delta v_j) > 0$
  - **Fact 2. Cross-hedging rises with correlation**
    λ_{ij} increases in a
  - **Fact 3. Own-hedging falls with correlation**
    λ_{ii} decreases in a (a substitution effect)
  - **Fact 4. Risk aversion reduces cross-hedging**
    $\gamma \uparrow \Rightarrow \lambda$ becomes diagonal
  - **Fact 5. The Epps effect**
    Return correlations are lower at higher frequencies

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  - **Fact 5. The Epps effect**
    Return correlations are lower at higher frequencies
- We test all five on **message-level TSX data** from January 13, 2006

Data: TSX STAMP Internal Feed

- **Trade-by-trade dealer inventory** from the Toronto Stock Exchange
  STAMP 4.0.3 Internal Feed, January 13, 2006
  - 8.2 million timestamped messages, sub-second resolution
  - Full order-book reconstruction with complete order lifecycle
  - `UserId` on every message $\Longrightarrow$ dealer inventory reconstructed trade by trade
- Regular Trading Hours (09:30–16:00 EST): 337,303 trades, $6.54B volume, 3,169 symbols

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- **Active universe:** 722 symbols meeting liquidity thresholds
  - $\geq$ 50 daily trades, $\geq$ \$50,000 daily volume
  - Valid NBBO $\geq$ 80% of session, median spread $\leq$ 500 bps

Market Maker Populations

- Two main populations of market makers:
  - **Designated Market Makers (DMMs):** broker-dealers with formal specialist obligations for assigned symbols
    - Identified via `AccountType=ST` and `RTAutofill` flags
    - 254 users, 1,832 (user, symbol) pairs
    - Portfolio assignment is **exogenous** to the trading day
  - **Endogenous Market Makers (EMMs):** voluntary two-sided quoters identified from quoting behavior
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    - Portfolio composition is **endogenous** (self-selection)
- Median characteristics (DMM | EMM):
  - Trade-to-quote ratio: 2.20 | 0.63
  - Inventory half-life (trades): 7.6 | 10.8
  - Symbols quoted (full portfolio): 9.5 | 18.0

Identification: Inventory Shocks and Sell Ratio

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- **Sell ratio shift:** $\Delta = \text{sell ratio}_{\text{post}} - \text{sell ratio}_{\text{pre}}$
  - $\Delta > 0$ after a buy shock $\implies$ dealer tilts toward selling (correct direction for inventory management)
  - Measured in shocked stock (**own**) and other qualified stocks (**cross**)

Fact 1: Cross-Hedging Exists

- Sell ratio shift at 300s after buy shocks (dollar-weighted):

		Same stock		Other qualified	
Population	N	Δ	t	Δ	t
Panel A: DMMs					
All buy shocks	3,810	+.413	15.0	+.184	7.2
Involuntary shocks	1,950	+.498	18.1	+.221	6.9
Panel B: EMMs (Set B)					
All buy shocks	7,809	+.173	22.6	+.021	4.3
Involuntary shocks	2,977	+.253	17.2	+.021	2.0

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- **Cross-stock shift is significantly positive in every population**
  - Range across populations: +.021 to +.221
  - Larger for involuntary shocks $\Longrightarrow$ causal direction is right

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- **(2) Larger for involuntary shocks:** 25–50% larger response than active unwinds $\implies$ shock is a genuine position problem, not a chosen trade
- **(3) Passive, not aggressive:** cross-stock shift appears only in **new passive limit orders**
  - Aggressive trades show **momentum** (more buying), not hedging
  - Mechanism: **quote adjustment**, as the model predicts

Fact 2: Cross-Hedging $\uparrow$ with Correlation

- For each shock, compute **average portfolio correlation**: mean monthly return correlation (36 months, Compustat) between shocked stock and other qualified stocks
- Regress cross-stock sell ratio shift on this correlation:

	DMM		EMM (Set B)	
	β	t	β	t
Avg. portfolio correlation	$+0.055$	0.49	$+0.098^{*}$	1.81
Involuntary shocks only	$+0.141$	1.62	$+0.203^{***}$	3.21

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- Coefficient is **positive in both populations**, significant for EMMs and for the cleanest specification (involuntary shocks)
  - A 1-SD increase in correlation (≈ 0.15) raises the cross-stock shift by $\sim$ 29% of its mean
  - Pooled coefficient: $+0.084$ ($t = 2.40$)

Fact 3: Own-Hedging Substitutes

- Same regression, but dependent variable is the **own-stock** sell ratio shift. Theory predicts $\beta < 0$:

	β	t
Panel A: DMMs (assignment $\approx$ exogenous)		
All buy shocks	-0.155	-1.96
Involuntary shocks only	-0.192	-1.86
Panel B: EMMs (voluntary $\implies$ self-selection)		
All buy shocks	$+0.175$	$+2.39$
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- **EMMs:** opposite sign $\implies$ self-selection (high-correlation EMMs are more active on every dimension); substitution shows up where identification is clean

Fact 4: Risk Aversion and Cross-Hedging

- Proxies for dealer risk aversion γ:
  - Inventory **half-life** (shorter $\implies$ higher γ): cleanest proxy
  - Zero crossings, inverse user size, composite index: confounded by activity level
- Cross-stock shift T3−T1 (top tercile minus bottom):

	DMM		EMM (Set B)	
Risk aversion proxy	T3−T1	t	T3−T1	t
Half-life (short = high RA)	-0.059^{***}	-7.0	-0.041^{***}	-5.1
Zero crossings (more = high RA)	$+0.052^{***}$	$+5.4$	$+0.029^{***}$	$+3.8$
Inverse user size (small = high RA)	$+0.085^{***}$	$+9.2$	$+0.061^{***}$	$+7.1$
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- Half-life proxy, **correct sign**: higher γ $\implies$ **less** cross-hedging
- Other proxies wrong-signed: confounded by **activity level**

Fact 5: The Epps Effect in TSX Data

- Compute VWAP return correlations across active-universe stock pairs:
  - Intraday (5, 15, 30, 60 minutes): from STAMP feed
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- **Take-aways:** market makers **do** cross-hedge; single-asset models **underestimate** dealer risk aversion (up to 50%); the mechanism is **quote adjustment**, not aggressive trading