Inequality, Business Cycles, and Growth: A Unified Theory of Fiscal Stabilization Policies

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Abstract: Do tax-financed fiscal expansions entail a trade-off between short-run output stabilization and long-run growth? To answer this question, we develop a heterogeneous agent framework with nominal rigidities and endogenous growth. Analytically, we characterize three regimes with potentially opposing effects on output and technology growth, depending on systematic monetary policy and tax incidence. In the U.S., the quantitatively relevant regime features a robust trade-off: a typical fiscal expansion, financed by higher tax progressivity, yields an impact output multiplier above one, a near-zero multiplier after three years, and sizable long-run output losses. Taxing the middle class avoids this trade-off and maximizes welfare.

Keywords: Inequality, Business Cycles, Growth, Fiscal Stabilization Policies.

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1. Introduction

Tax-financed fiscal stimulus programs are a central policy tool for stabilizing economic activity during recessions. Historically, the typical government spending shock in the United States has been financed through greater tax progressivity, with relatively larger tax hikes on high-income households as compared to low-income households. More recently, the 2021 *Build Back Better* proposal followed this regularity: its financing provisions aimed to raise revenues from high-income households directly, through a 5 percent surtax on income above $10 million and an additional 3 percent surtax above $25 million, and more indirectly, through a 15 percent minimum tax on large corporations.

Proponents of this policy design emphasize its *short-run benefits*, arguing that fiscal expansions can stimulate aggregate demand effectively when the resulting tax burden falls on high-income households with low marginal propensities to consume, a key propagation channel in the New Keynesian literature (Kaplan et al., 2018, Auclert et al., 2024, Ferriere and Navarro, 2025). Opponents of this policy design, however, point to sizable *long-run costs*, arguing that greater tax progressivity may dampen entrepreneurial activity, discourage innovation, and weaken future growth by redistributing resources away from innovative entrepreneurs, consistent with central mechanisms in the innovation and growth literature (Jaimovich and Rebelo, 2017, Akcigit et al., 2022, Jones, 2022).[1]

Existing studies that evaluate the efficacy of fiscal stimulus programs rely on models that typically capture one of these contrasting perspectives but not the other. This leaves two unanswered questions: Does this policy design entail a trade-off between short-run stabilization and long-run technology growth? More broadly, how should the tax burden of fiscal stimulus programs be distributed across households to maximize welfare?

This paper attempts to fill this gap by developing a unified theory of fiscal stimulus programs within a Heterogeneous Agent New Keynesian Growth framework (HANK-G). This framework combines three cornerstones of macroeconomic modeling: endogenous technological growth through entrepreneurial innovation, nominal rigidities that generate demand-driven fluctuations in economic activity, and heterogeneous households that differ in their marginal propensities to consume and to innovate.

Through the lens of the HANK-G framework, we show that progressively financed fiscal expansions generate an economically significant and quantitatively robust stabilization trade-off in the U.S. economy: they expand output in the short-run very effectively with fiscal multipliers above one, but lower technology growth leading to sizable drops in the long-run level of output. Therefore, our analysis underscores that the typical U.S. government spending design has focused overly on creating short-run demand stimulus and, by doing so, has systematically overlooked creating conditions for innovation and long-term growth. This finding indicates that the prevailing policy design may not be welfare-maximizing, implying scope for Pareto-improving reforms.

We derive this result in two main steps. As our first contribution, we characterize the key transmission channels of fiscal stimulus programs in an analytically tractable version of the HANK-G framework. As our second contribution, we calibrate an extended

[1] See, for instance, the Tax Foundation's comment on the long-run effects of the *Build Back Better* plan.

version of the model to the U.S. economy, quantify the short- and long-run effects of tax-financed expansions in government spending, and determine the welfare-maximizing tax incidence across the income distribution.

Analytical Framework. We begin by developing an analytical framework that combines elements of the tractable heterogeneous agent New Keynesian model of Bilbiie (2025) with elements of the Keynesian growth model of Benigno and Fornaro (2018). Specifically, there is a continuum of households who consume, work, and face idiosyncratic income risk. Households switch between two states that differ in their access to asset markets, giving rise to a precautionary saving motive. A fraction of households are unconstrained entrepreneurs who self-insure through liquid bonds and own innovative firms that accumulate illiquid innovative capital. Innovative investment increases future technology growth and earns a return premium. The remaining households are hand-to-mouth and consume their disposable income each period. A representative final good firm combines labor with intermediary inputs supplied by entrepreneurs' monopolistic innovative firms. Nominal rigidities arise from the presence of wage stickiness. Finally, fiscal policy operates through transfers and government spending financed by a heterogeneous and non-distortionary tax incidence across household types, while the central bank sets the nominal interest rate according to a Taylor-type feedback rule.

The stationary equilibrium dynamics of this economy are represented by four equations: (i) a dynamic IS equation; (ii) an endogenous growth equation; (iii) a New Keynesian Phillips curve; and (iv) a monetary policy rule. Beyond its analytical tractability, the key novelty of this representation lies in the unified treatment of fiscal stimulus programs within a single framework that remains comparable to existing heterogeneous agent models analyzing their short- and long-run propagation in isolation.

Theoretical Results. We then use our tractable HANK-G framework to study the propagation of a persistent government spending shock that is financed by greater tax progressivity. In the short-run, fiscal stimulus programs raise output whenever the expansion in public spending outweighs any tax-induced decline in private consumption and innovative investment. In the long-run, they are effective only if they also stimulate innovative investment and, through it, the permanent level of output. We show that the distribution of the tax burden across household types and the stance of systematic monetary policy jointly determine the sign and magnitude of short-run and long-run effects.

Progressive tax financing shapes short- and long-run effects through two opposing forces. On the one hand, it channels resources toward households with high spending propensities raising private consumption and output. This short-run expansion in aggregate demand increases the returns to innovation, crowding in innovative investment and increasing permanent household income. On the other hand, it directly reduces the after-tax returns to innovation for top-income entrepreneurs, crowding out innovative investment and depressing permanent household income.

What determines which force prevails is the stance of systematic monetary policy, that is, the aggressiveness with which the central bank responds to higher output and inflation by raising real interest rates and thereby controlling the size of the short-run demand expansion. Its interaction with tax incidence generates three distinct regimes, each with sharply different implications for the efficacy of the stimulus program.

In the first regime, monetary policy is sufficiently accommodative, leading to a sizable short-run output expansion. The resulting surge in aggregate demand is strong enough to crowd in innovative investment, so progressively financed spending shocks raise both short-run output and technological growth relative to the *laissez-faire* allocation. In the second regime, monetary policy is more aggressive. The compressed demand expansion is no longer sufficient to offset the direct disincentive to innovate created by higher taxes on top earners. Economic activity still rises in the short-run, but technology-growth falls and permanent output losses accumulate over time: a short- versus long-run stabilization trade-off emerges. In the third regime, tax incidence is insufficiently progressive and monetary policy sufficiently aggressive. The anticipated decline in permanent household income suppresses current private consumption from the outset, so both short-run output and technology growth contract simultaneously and the fiscal stimulus is ineffective along both margins at once.

Quantitative Analysis. To understand which regime is quantitatively relevant, we calibrate an extended version of the model to the U.S. economy. The quantitative model preserves the core mechanisms of the tractable model, but enriches the environment where needed for empirical discipline. To this end, we relax the spender–entrepreneur dichotomy by incorporating richer idiosyncratic risk in labor productivity, employment status, discount factors, and innovative success. Moreover, we adopt flexible tax and transfer schedules that map more closely to the fiscal instruments used in practice and allow fiscal expansions to be partially debt-financed. We discipline the model to match key moments of income and wealth inequality, realistic short-run consumption responses, and the most up to date empirical estimates of the sensitivity of entrepreneurial innovation to taxation from Akcigit et al. (2022).

We then show that a typical government spending shock, financed by greater tax progressivity, raises short-run output but slows down technology growth, placing the U.S. economy in the regime where short-run stabilization comes at the expense of lower long-run growth. The initial expansion in output, with an annual multiplier of 1.07, is overtaken within just eight quarters by the decline in technology growth, whose annual multiplier is -0.11. This translates into a sizable long-run loss: a persistent 1% increase in government spending reduces the permanent level of output by roughly 0.12%. This stabilization trade-off survives across a range of alternative specifications, including different monetary policy rules, stronger nominal rigidities, alternative earnings incidence assumptions, a lower elasticity of innovation to taxation, and more persistent spending shocks. Despite these adverse long-run effects, utilitarian welfare, measured by the aggregate consumption-equivalent variation (CEV), remains positive overall at 0.37% relative to the *laissez-faire* economy, because the short- to medium-run consumption gains dominate in present value terms the long-run permanent income losses. These welfare gains, however, are unevenly distributed and are concentrated toward the bottom of the income distribution.

Stabilization Trade-off and Tax Incidence In the final part of the paper, we analyze which financing scheme best navigates the trade-off between short-run stabilization and long-run technology growth and maximizes welfare. To this end, we introduce targeted perturbations to the tax schedule that allow the government to allocate the additional tax burden to specific income quantiles. We find that welfare is maximized when the additional tax burden falls on upper-middle-income households, around the 70th percentile of the income distribution. The resulting CEV welfare gain increases by 62%, to 0.60%, relative to the typical progressive financing design. Intuitively, this tax scheme preserves the demand response of high-MPC households at the bottom, and hence yields the largest short-run welfare gains, while limiting the long-run welfare losses from distorting innovation at the top. Concretely, under the welfare-maximizing tax incidence, the annual output multiplier is well above one, reaching 1.22, while the technology-growth multiplier remains slightly positive, at 0.02. Strikingly, welfare gains are positive for every household in the economy, including those directly targeted by the tax incidence, and larger than under the typical progressive financing scheme, implying a Pareto improvement.

Related Literature. This paper contributes to the extensive literature that studies how household heterogeneity affects aggregate outcomes in the short- and long-run. Specifically, we relate to four strands of the literature.

First, our paper contributes to the HANK literature. On the analytical side, our tractable heterogeneous agent model relates to Bilbiie (2008, 2020, 2025), Acharya and Dogra (2020), Broer et al. (2020), Ravn and Sterk (2020), Cantore and Freund (2021), and Debortoli and Galí (2024a). As the source of heterogeneity is based on limited asset markets participation, our HANK-G economy is closest to Bilbiie (2025), however extended by two assets as in Galí et al. (2007) or Bilbiie et al. (2022), a general earnings incidence as in Werning (2015), and sticky wages. Our contribution is to integrate endogenous growth into a tractable HANK framework and derive a novel closed-form, four-equation representation that transparently isolates the key short- and long-run propagation channels of fiscal expansions. On the quantitative side, we relate, among many others, to McKay et al. (2016), Kaplan et al. (2018), Bayer et al. (2019), Auclert (2019), and Auclert et al. (2024) highlighting the role of MPC heterogeneity for aggregate demand. Our quantitative HANK-G model contributes to this literature by integrating heterogeneous innovative entrepreneurs into a HANK environment, so that top incomes emerge endogenously from innovation and household heterogeneity matters not only for aggregate demand but also for aggregate supply.[2]

Second, our paper relates to a growing literature that studies the propagation of aggregate shocks and stabilization policies in business cycles models with endogenous growth. Stadler (1990) and Comin and Gertler (2006) were the first to document medium-term business cycles. Building on their work, Benigno and Fornaro (2018), Anzoategui et al. (2019), Garga and Singh (2021), or Fornaro and Wolf (2025) developed

[2]As technology growth is driven by innovative investment, our paper shares similar propagation mechanisms as Luetticke (2021), Kekre and Lenel (2022) or Melcangi and Sterk (2025), highlighting the role of investment in heterogeneous agent economies.

a Keynesian growth framework with nominal rigidities.[3] Our contribution is to enrich the Keynesian growth framework with household heterogeneity and incomplete markets and, therefore, to provide a comprehensive framework to evaluate the efficacy of tax-financed stimulus programs.

Third, by analyzing the efficacy of tax-financed stimulus programs in a heterogeneous agent incomplete markets framework with nominal rigidities, our paper relates to a large literature that identifies key determinants of fiscal multipliers; see, among others, Oh and Reis (2012), Hagedorn et al. (2019), Bilbiie et al. (2024), Auclert et al. (2024), and Ferriere and Navarro (2025). Most closely related to our work is Ferriere and Navarro (2025), who show that U.S. fiscal multipliers are larger when fiscal expansions are financed with higher tax progressivity. Our paper confirms this amplification mechanism of fiscal expansions in the short-run, but points toward an ambivalent role in the long-run, depending on the joint design of systematic monetary policy and tax incidence. Key to this finding is the negative relationship between personal and corporate income taxes and innovative activity of entrepreneurs. This mechanism is rooted in the seminal empirical work of Akcigit et al. (2016), Moretti and Wilson (2017) and Akcigit et al. (2022) and central to many studies analyzing the effects of top tax changes on entrepreneurial activity, innovation, and inequality; see, for instance, Jaimovich and Rebelo (2017), Aghion et al. (2019) or Jones (2022). Importantly, because these papers abstract from nominal rigidities, they miss the feedback loop through which aggregate demand expansions raise pre-tax returns to innovation. As a result, they may overstate the negative effects of transitory progressive redistribution on long-run technology growth.

Fourth, the HANK-G framework is consistent with a growing empirical literature that documents how discretionary stabilization polices have long-lasting effects on economic activity.[4] Relying on structural vector autoregressions and local projections, Ilzetzki (2024) and Antolin-Diaz and Surico (2025) show that expansions in government spending have positive and long-lasting effects on output, R&D expenditures, and total factor productivity. Cloyne et al. (2025) obtain similar effects for corporate tax cuts.[5] Our results instead point toward a state-dependent role for the short- and long-run effects of government spending shocks with respect to the joint design of systematic monetary policy and tax incidence.

Paper Outline. The paper proceeds as follows. Section 2 develops the analytical HANK-G framework. Section 3 characterizes the three regimes governing the short- and long-run effects of fiscal stimulus programs. Section 4 builds a quantitative extension to the baseline model, calibrates it to the U.S. economy, determines the quantitatively dominant regime, and derives the welfare-maximizing tax-incidence. Section 5 concludes. Analytical derivations, computational details, and additional results are reported in the Online Appendix.

[3] Bianchi et al. (2019), Queralto (2020, 2022), and Fornaro and Wolf (2023) are further related papers.

[4] This strand is motivated and complemented by a vast literature arguing that recessions permanently depress the level of output, i.e., generate *hysteresis* or *scarring*; see, among others, Fatás (2000) and Blanchard et al. (2015). Fatás and Summers (2018) and Cerra et al. (2023) provide a detailed overview.

[5] Related, Moran and Queralto (2018) and Jordá et al. (2024) study the long-run effects of expansionary monetary policy shocks and arrive at a similar conclusion.

2. An Analytical Heterogeneous Agent New Keynesian Growth Economy

We begin with the analytical framework. Our objective is to make transparent the core mechanisms behind the short- and long-run propagation of fiscal expansions. In the HANK-G economy, growth emerges endogenously from innovative investment; nominal rigidities generate demand-driven fluctuations in economic activity; and household heterogeneity shapes aggregate consumption and entrepreneurial innovation. Our central policy focus, the tax incidence of fiscal expansions, provides insurance against income risk while shaping consumption and innovation incentives.

We present the unified framework in Section 2.1, its balanced growth path in Section 2.2, consumption and income inequality across households in Section 2.3, and the four-equation representation that characterizes the model dynamics in Section 2.4. We then derive conditions for local determinacy in Section 2.5 before turning to the paper's main theoretical result in Section 3: the three regimes governing the short- and long-run effects of fiscal stimulus programs.

2.1 Environment

Time is discrete and indexed by $t \in \{1, 2, \dots\}$. The economy is populated by infinitely lived heterogeneous households, final and intermediate good firms, a government, and a monetary authority.

2.1.1 *Households* At each period t, there is a unit mass of households, indexed by $i \in [0, 1]$. Each household derives utility from consumption C_t^i and disutility from labor L_t^i, with preferences represented by the following time-separable utility function:

$$\mathcal{U}(C_t^i, L_t^i) = \log(C_t^i) - \nu \frac{(L_t^i)^{1+\varphi}}{1+\varphi} ,$$

where $\nu > 0$ governs the disutility of labor and $1/\varphi$ is the Frisch labor supply elasticity. Households discount future periods at a constant rate $\beta \in (0, 1)$. They supply differentiated labor services $L_t^i(l)$, where $l \in [0, 1]$ indexes their labor type. While households share identical preferences, they differ in their asset market participation. In particular, households face idiosyncratic shocks that move them between two states: saver (S), and hand-to-mouth (H). Savers hold a portfolio comprising liquid bonds, equity in intermediate-good firms, and illiquid capital from entrepreneurial innovative investment. Because savers own and invest in innovative private businesses, we refer to them as entrepreneurs. By contrast, the remaining households do not participate in illiquid asset markets and rely solely on liquid bonds to buffer income risk. Anticipating the equilibrium solution, their borrowing constraint will bind in each period, implying that they behave as hand-to-mouth consumers. Transitions between states follow a Markov process, with transition probabilities $\mathbb{P}(S_{t+1}|S_t) = s$ and $\mathbb{P}(H_{t+1}|H_t) = h$. In the stationary equilibrium, the mass of hand-to-mouth households is given by $\lambda = \frac{1-s}{2-h-s}$.

At the start of period t, each group – entrepreneurs (S) and hand-to-mouth (H) – pools its members' resources, providing full intra-group insurance against idiosyncratic shocks. Households then observe the aggregate shock and choose to consume and save.

At the end of period t, they learn their asset market status for period $t+1$ and pool liquid bond holdings exclusively within their new peer group. Let B^S_{t+1} (respectively, B^H_{t+1}) denote the *aggregate* real bond holdings carried into period $t+1$ by entrepreneurs (respectively, hand-to-mouth households). By construction, B^S_{t+1} collects the liquid assets of two groups: entrepreneurs who stay in S in $t+1$, a mass $(1-\lambda)s$, and households who move from H in t to S in $t+1$, a mass $\lambda(1-h)$:

$$B^S_{t+1} = (1-\lambda)s\, b^S_{t+1} + \lambda(1-h)\, b^H_{t+1}\,. \tag{1}$$

Similarly, B^H_{t+1} collects the liquid assets of households who remain in the hand-to-mouth state in $t+1$, a mass of λh, and those who move from S in t to H in $t+1$, a mass of $(1-\lambda)(1-s)$:

$$B^H_{t+1} = \lambda h b^H_{t+1} + (1-\lambda)(1-s) b^S_{t+1}\,. \tag{2}$$

Innovative capital is illiquid and cannot be transferred across household types. Entrepreneurs invest an amount $I^S_t(j)$ in each intermediate firm j – firms that are otherwise identical both ex ante and ex post – and these investments upgrade the stock of innovative capital, i.e., the quality of intermediate goods, according to the law of motion

$$A^S_{t+1}(j) = A^S_t(j) + \psi I^S_t(j)\,, \quad \text{with} \quad A_0(j) > 0\,, \tag{3}$$

where $\psi \geq 0$ governs how efficiently innovative investment translates into improvements in quality.[6] Since innovation is carried out only by entrepreneurs, aggregate quality and investment in firm j simply scale with their population share, so that $A_t(j) = (1-\lambda)A^S_t(j)$ and $I_t(j) = (1-\lambda)I^S_t(j)$.

Entrepreneurs Entrepreneurs enter period t with per capita bond holdings of $B^s_t/(1-\lambda)$, carried over from the previous period. These bonds yield a real gross return of R_{t-1}/π_t, where R_{t-1} is the nominal interest rate and $\pi_t = P_t/P_{t-1}$ the gross inflation rate. In addition, entrepreneurs hold innovative capital $A^S_t = \int A^S_t(j)dj$ that earns a real gross return R^A_t, described below. Their capital income also includes equity holdings $\omega^S_t/(1-\lambda)$ in intermediary firms, which pay dividends D_t and can be resold at price q_t. Entrepreneurs pay a per capita lump-sum tax $T^S_t/(1-\lambda)$. They also supply labor L^S_t at the real wage W_t/P_t. These income sources finance consumption C^S_t, investment in innovative capital I^S_t, and financial asset accumulation in the form of bonds b^S_{t+1} and equity shares ω^S_{t+1}. In particular, entrepreneurs choose an allocation

[6] This law of motion generates technology growth based on a linear investment function. For tractability, we assume that investment is always positive, which is ensured under sufficiently small shocks (see Footnote 27 in the main text and, for a relaxation of this assumption, Section E of the Supplementary Appendix of Fornaro and Wolf (2023)). In the extended model of Section 4, we adopt the canonical structure of the entrepreneurial growth literature, featuring decreasing returns to innovation and non-rivalry.

$\mathcal{X}_t^S \equiv \{C_t^S, b_{t+1}^S, A_{t+1}^S, I_t^S, \omega_{t+1}^S\}_{t\geq 0}$ to maximize expected lifetime utility, solving

$$V^S(B_t^S, \omega_t^S, A_t^S) = \max_{\{\mathcal{X}_t^S\}} \mathcal{U}(C_t^S, L_t^S) + \beta \mathbb{E}_t \left[V^S(B_{t+1}^S, \omega_{t+1}^S, A_{t+1}^S) + \frac{\lambda}{1-\lambda} V^H(B_{t+1}^H) \right]$$

subject to

$$C_t^S + I_t^S + b_{t+1}^S + q_t \frac{\omega_{t+1}^S}{1-\lambda} = \frac{W_t}{P_t} L_t^S + R_t^A A_t^S + \frac{R_{t-1}}{\pi_t} \frac{B_t^S}{1-\lambda} + (q_t + D_t) \frac{\omega_t^S}{1-\lambda} - \frac{T_t^S}{1-\lambda},$$

$$b_{t+1}^S \geq 0, \quad \text{and} \quad (1) - (3),$$

where the value function accounts for the possibility that, in period $t+1$, an entrepreneur may become a hand-to-mouth household.[7]

Hand-to-mouth The problem faced by hand-to-mouth households mirrors that of entrepreneurs, except that they lack access to productive asset markets. Since they cannot accumulate innovative capital or equity, their choices are limited to consumption C_t^H and liquid bond savings b_{t+1}^H to maximize expected lifetime utility. In addition to labor income and bond returns, they pay a per capita lump-sum tax T_t^H/λ. Their optimal plan, $\mathcal{X}_t^H \equiv \{C_t^H, b_{t+1}^H\}_{t\geq 0}$, satisfies the following Bellman equation:

$$V^H(B_t^H) = \max_{\{\mathcal{X}_t^H\}} \mathcal{U}(C_t^H, L_t^H) + \beta \mathbb{E}_t \left[V^H(B_{t+1}^H) + \frac{1-\lambda}{\lambda} V^S(B_{t+1}^S, \omega_{t+1}^S, A_{t+1}^S) \right]$$

subject to

$$C_t^H + b_{t+1}^H = \frac{W_t}{P_t} L_t^H + \frac{R_{t-1}}{\pi_t} \frac{B_t^H}{\lambda} - \frac{T_t^H}{\lambda},$$

$$b_{t+1}^H \geq 0, \quad \text{and} \quad (1) - (2).$$

Finally, we solve for the financial market equilibrium by imposing Assumption 1, which specifies the nature of risk-sharing and the asset allocation across households.

ASSUMPTION 1. *(A1.a) there is full insurance among households in the same state, but not across states; (A1.b) stocks and innovative capital are illiquid and cannot be transferred across states; (A1.c) liquid bond holdings are weakly positive, and the positivity constraint of hand-to-mouth agents binds in each period; (A1.d) no bonds are traded in equilibrium.*

$(A1.a)$-$(A1.d)$ jointly ensure a tractable characterization of the asset market equilibrium. Assumptions $(A1.a)$ and $(A1.b)$ imply that all entrepreneurs (respectively, hand-to-mouth households) make identical consumption and saving decisions. Assumption $(A1.c)$ enforces the strict hand-to-mouth constraint by ruling out capital market participation for H households, while Assumption $(A1.d)$ imposes a zero-liquidity limit (see, e.g., Krusell et al. (2011)), eliminating buffer-stock savings for both household types.

[7] The value function $V^H(B_{t+1}^H)$ is scaled by $\lambda/(1-\lambda)$, reflecting that the relevant state variable – aggregate bond holdings – is expressed in terms of total island-level resources. Accordingly, the relative transition probability $\lambda/(1-\lambda)$ maps state transitions into individual probabilities $(s, 1-s)$.

2.1.2 *Labor Market* The labor market exhibits two salient features. First, nominal wage rigidity implies that hours worked are demand-determined. This rigidity generates pro-cyclical firm profits, a feature that proves central to generating pro-cyclical investment in an endogenous growth model. Second, fluctuations in hours worked are heterogeneously distributed across households, implying that labor income inequality is cyclical.

Earnings Incidence Throughout, variables denoted in hat notation represent log-linear deviations from their balanced growth path values. Assumption 2 governs the earnings incidence across household types.

ASSUMPTION 2. *On the balanced growth path, hours worked are equal across household types, such that $L^H = L^S = L$. Off the balanced growth path, however, hours may vary disproportionately across types: $\hat{L}_t^H = \mu \hat{L}_t$, where $\mu \in [0, \overline{\mu})$ and $\overline{\mu} > 1/\lambda$.*

The earnings incidence parameter μ controls the allocation of aggregate hours worked across household types, that is, their respective exposure to business cycle fluctuations.[8] Assumption 2 implies that hours worked by entrepreneurs follow $\hat{L}_t^S = \frac{1-\lambda\mu}{1-\lambda}\hat{L}_t$. When $\mu = 1$, changes in aggregate hours worked are distributed equally across households. When $\mu = 0$, hand-to-mouth households remain at steady-state labor supply, and the full adjustment falls on entrepreneurs. By contrast, when $\mu = \lambda^{-1}$, the full adjustment falls on hand-to-mouth households. In addition, when $\mu \in (0, 1)$, hand-to-mouth households adjust hours underproportionally, while entrepreneurs adjust overproportionally; when $\mu \in (1, \lambda^{-1})$, the opposite holds — hand-to-mouth households bear the bulk of the adjustment, while entrepreneurs adjust only moderately. Finally, for $\mu \in (\lambda^{-1}, \overline{\mu})$, hours worked by hand-to-mouth households rise overproportionally, while entrepreneurs reduce hours worked in response to higher aggregate labor demand.[9]

Nominal Wage Rigidity Aggregate labor input L_t is composed of a continuum of differentiated labor varieties indexed by $l \in [0, 1]$, which are bundled through a CES aggregator with elasticity of substitution $\epsilon_w > 1$. Each labor variety is supplied by a union l that sets the nominal wage $W_t(l)$ subject to quadratic adjustment costs à la Rotemberg (1982). The union maximizes the utility of a quasi-representative household, which is modeled as a composite of hand-to-mouth and entrepreneur types. The effective labor input for each variety is $L_t(l) = \lambda L_t^H(l) + (1-\lambda) L_t^S(l)$. Demand for labor variety l is given

[8]Assumption 2 is a two-type counterpart to the general incidence function $\gamma(i, L_t)$ used by Werning (2015), Auclert and Rognlie (2020), Alves et al. (2020), and Patterson (2023). Individual labor income of the form $(W_t/P_t) \cdot L_t \cdot \gamma(i, L_t)$ maps in our setup to $\mu = 1 + \epsilon_{\gamma,L}^H$, where $\epsilon_{\gamma,L}^H$ is the elasticity of $\gamma(i, L_t)$ with respect to aggregate labor L_t in state H. Consistency requires $\epsilon_{\gamma,L}^S = -[\lambda/(1-\lambda)]\epsilon_{\gamma,L}^H$. Moreover, $\overline{\mu}$ ensures that monetary tightening reduces aggregate demand (Bilbiie, 2008, Bilbiie and Straub, 2013).

[9]Empirically, labor market exposure at the extensive margin, such as cyclical unemployment risk, has been documented in Storesletten et al. (2004), Guvenen et al. (2014), and Kramer (2022). Exposure at the intensive margin, measured through *worker betas*, is explored in Guvenen et al. (2017). Both strands document that low-income workers disproportionately bear hours adjustments to aggregate shocks, pointing toward $\mu > 1$; for monetary policy shocks see, among others, Coglianese et al. (2025), Heathcote et al. (2010), Coibion et al. (2017), Ampudia et al. (2018), or Holm et al. (2021).

by $L_t(l) = (W_t(l)/W_t)^{-\epsilon_w} L_t^d$, where W_t is the aggregate wage index consistent with cost minimization, and L_t^d is total labor demand by firms. Each union l solves

$$\max_{\{W_t(l)\}} \sum_{t=0}^{\infty} \beta^t \left[\log C_t(l) - \nu \frac{(L_t(l))^{1+\varphi}}{1+\varphi} \right]$$

subject to

$$C_t(l) + I_t(l) = \frac{W_t(l)}{P_t} L_t(l) + R_t^A A_t(l) + D_t(l) - T_t(l) - \frac{\theta}{2} \left(\frac{1}{g^A} \frac{W_t(l)}{W_{t-1}(l)} - 1 \right)^2 A_t \,,$$

where adjustment costs scale with aggregate technology A_t by a factor $\theta > 0$, and wage inflation is normalized to steady state technology growth, $g^A > 1$, ensuring that adjustment costs are absent on the balanced growth path. In equilibrium, unions set identical wages, such that $W_t(l) = W_t$ and $L_t(l) = L_t$ for all l.

2.1.3 *Production* The production sector consists of a representative final good firm and a continuum of intermediate-goods producers indexed by $j \in [0,1]$.

Final Good The final good Y_t^G is produced under perfect competition by aggregating differentiated intermediate inputs $X_t(j)$, each of quality $A_t(j)$, according to

$$Y_t^G = L_t^{1-\alpha} \int_j A_t(j)^{1-\alpha} X_t(j)^{\alpha} dj \,,$$

where $\alpha \in (0,1)$ denotes the share of intermediate inputs in production. The final good is sold at the competitive price P_t, while intermediate goods are purchased at variety-specific prices $P_t(j)$. The final good firm chooses demand for labor L_t^d and intermediate inputs $X_t^d(j)$ to maximize profits, taking factor prices as given.

Intermediate Goods Intermediate goods are produced by identical firms, each operating as a monopolist. In each period t, firm j sets a price $P_t(j)$ to maximize nominal profits $\Theta_t^n(j)$, taking as given the demand for its variety. Specifically, $X_t(j)$ is produced by transforming units of the final good at a marginal cost P_t. Firm j's objective is:

$$\max_{\{P_t(j)\}} \Theta_t^n(j) \equiv (P_t(j) - P_t) X_t^d(j) \quad \text{subject to} \quad \frac{P_t(j)}{P_t} = \alpha (L_t^d)^{1-\alpha} A_t(j)^{1-\alpha} (X_t^d(j))^{\alpha-1} \,.$$

The firm's optimal pricing decision yields $\alpha P_t(j) = P_t$, where $1/\alpha > 1$ is the gross markup over marginal cost. Substituting this pricing rule into the demand function yields $X_t(j) = \alpha^{\frac{2}{1-\alpha}} L_t A_t(j)$. Aggregating across varieties, gross output is given by $Y_t^G = \alpha^{\frac{2\alpha}{1-\alpha}} L_t A_t$, with $A_t \equiv \int_0^1 A_t(j) dj$ representing the average quality index. Real profits of firm j are $\Theta_t(j) = \Theta_\alpha L_t A_t(j)$, where $\Theta_\alpha = \alpha^{-1}(1-\alpha)\alpha^{\frac{2}{1-\alpha}}$. Finally, returns to innovation are defined as the ratio of profits to quality, $R_t^A(j) \equiv \Theta_t(j)/A_t(j) = \Theta_\alpha L_t = R_t^A$, which is identical across firms and linear in aggregate labor.

2.1.4 *Government* The government balances a period-by-period budget constraint by financing its spending S_t^G through lump-sum taxes levied on both household types. The allocation of the tax burden is governed by the following fiscal rule:

$$T_t^H = -T_t + \tau_p S_t^G \,, \quad \text{and} \quad T_t^S = T_t + (1-\tau_p) S_t^G \,,$$

where $\tau_p \in [0,1]$ captures the tax incidence across household types. Lower values of τ_p indicate a greater share of the tax burden borne by entrepreneurs.

To simplify the exposition, we begin with a pure redistribution shock from entrepreneurs to hand-to-mouth households, anticipating that it captures qualitatively the same key trade-offs as a progressively financed fiscal expansion. The intuition is straightforward: both hand-to-mouth households and the government feature a marginal propensity to consume equal to unity. As a result, redistribution from entrepreneurs to hand-to-mouth households closely replicates the aggregate effects of a progressively financed fiscal expansion (i.e., $\tau_p = 0$).[10] Formally, the government's budget constraint is balanced in every period when:

$$T_t^H = -T_t^S = -T_t \,, \quad \text{with} \quad T_t = T A_t e^{\hat{\tau}_t} \quad \text{and} \quad T > 0 \,,$$

where $\hat{\tau}_t$ is an exogenous, non-distortive shock that reallocates net transfers between households. The fiscal shock follows an AR(1) process, $\hat{\tau}_t = \rho \hat{\tau}_{t-1} + \epsilon_t$, with persistence $\rho \in [0,1)$ and i.i.d. innovations ϵ_t. A positive realization, $\hat{\tau}_t > 0$, represents progressive redistribution from entrepreneurs to hand-to-mouth households.

2.1.5 *Monetary Authority* We assume that the monetary authority sets the nominal interest rate following a pro-cyclical real interest rate rule of the form

$$R_t = R\mathbb{E}_t \left[\frac{\pi_{t+1}}{\pi}\right] \left(\frac{Y_t/A_t}{Y/A}\right)^{\phi_y} \,, \quad \text{with} \quad \phi_y > 0 \,,$$

where Y_t/A_t denotes stationary net output, as defined below. The parameter $\phi_y > 0$ captures the degree of systematic monetary policy: higher values represent a more "hawkish" policy stance, with stronger real interest rate responses to output fluctuations. In the limit, $\phi_y \to \infty$ corresponds to perfect output stabilization.[11]

2.1.6 *Market Clearing* Aggregate labor and consumption are given by $L_t = \lambda L_t^H + (1-\lambda) L_t^S$ and $C_t = \lambda C_t^H + (1-\lambda) C_t^S$, respectively. Aggregate investment is defined as $I_t \equiv (1-\lambda) \int_j I_t^S(j) dj$, and total intermediate output is $X_t \equiv \int_j X_t(j) dj$. Real dividends, which equal net profits in the final-good sector, are zero in all periods, i.e.,

[10] We analyze tax-financed fiscal expansions in detail in Section 3.

[11] The real interest rate rule is used, among others, in Fornaro and Wolf (2023), Angeletos et al. (2024, 2026), Auclert et al. (2025) or Collard et al. (2025); additionally, Woodford (2011), Bilbiie (2020) and Debortoli and Galí (2024a,b) study the limiting case $\phi_y \to 0$. More recently, Galí (2026) argues that such a rule might be a better indicator of the monetary policy stance and a more useful tool for modeling and assessing U.S. monetary policy. Analytically, the real interest rate rule allows us to cleanly isolate how fiscal policies shape aggregate outcomes in the short- and long-run. We relax this assumption and discuss the role of more standard feedback rules in Section 4; the economic essence is identical.

$D_t = Y_t^G - (W_t/P_t)L_t - (1/\alpha)X_t = 0$. Equity market clearing implies that the entire ownership of firms is held by entrepreneurs, i.e., $\omega_t^S = \omega_{t+1}^S = 1$. Aggregate quality growth is governed by the law of motion:

$$g_{t+1}^A \equiv \frac{A_{t+1}}{A_t} = 1 + \psi \cdot \frac{I_t}{A_t} \, .$$

Using the household budget constraints and the equilibrium condition from the labor union's problem, the aggregate resource constraint can be written as

$$Y_t = C_t + I_t + \frac{\theta}{2}\left(\frac{\pi_t^w}{g^A} - 1\right)^2 A_t \, ,$$

where net output is defined as $Y_t \equiv Y_t^G - X_t = Y_\alpha L_t A_t$, with $Y_\alpha = (1-\alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}}$.

2.2 Balanced Growth Path

A stationary balanced growth path (BGP) consists of a sequence of trending variables and prices $\{C_t, C_t^S, C_t^H, W_t, D_t, T_t^H, T_t^S, Y_t, Y_t^G, X_t, I_t\}$ that are normalized by the level of endogenous technology A_t. Subsequently, we denote stationary variables in small letters and impose without loss of generality $\pi = 1$. We focus on a BGP with positive household consumption and technology growth.

RESULT 1. *The economy admits a unique BGP with strictly positive individual consumption $(c^H, c^S) \in \mathbb{R}_+^2$ and technology growth $g^A > 1$ if $L > L^*$ and $T < T^*$, where*

$$L^* \equiv \frac{\beta^{-1} - 1}{\psi \Theta_\alpha} \, , \qquad T^* \equiv \frac{(1-\lambda)\Theta_\alpha}{\alpha}\left(1 + \alpha\frac{1-\beta}{1-\lambda}\right)L + \frac{1-\beta}{\psi} \, .$$

Result 1 shows that the BGP features strictly positive technology growth, i.e., $g^A > 1$, whenever hours worked are high enough. In this case, intermediate firms operate in a sufficiently large market, raising profits and stimulating innovative investment. The tax bound ensures positive consumption for entrepreneurs. To solve for the approximate equilibrium dynamics, we log-linearize the economy around its BGP.

2.3 Inequality Along the Balanced Growth Path

On the BGP, inequality reflects differences in financial market access and income sources. Off the BGP, inequality is instead shaped by heterogeneous earnings exposure to business-cycle fluctuations and fiscal redistribution.

2.3.1 *BGP Inequality* Consumption and income inequalities are measured by the respective S-to-H ratios of consumption and income, i.e., $\Gamma_c \equiv c^S/c^H$ and $\Gamma_y \equiv y^S/y^H$. When the aggregate consumption share $\gamma_c \equiv c/y$ is below one, income inequality exceeds consumption inequality, since $\Gamma_y = \Gamma_c/\gamma_c + \lambda(1-\gamma_c)/[(1-\lambda)\gamma_c]$.[12] For clarity, we

[12] This observation holds generically in a large class of heterogeneous agent models featuring non-homothetic saving behavior; see, for instance, Straub (2019) or Gaillard et al. (2023).

normalize $\Gamma_c = 1$ in the main text, which can be implemented through a transfer T^E from entrepreneurs to hand-to-mouth households, where $0 < T^E < T^*$.[13]

2.3.2 *Cyclical Income Inequality* Income inequality fluctuates over the business cycle because households are exposed to labor-market risk and fiscal redistribution. To make this link transparent, fluctuations in hand-to-mouth consumption can be written as:

$$\hat{c}_t^H = \hat{y}_t^H = \eta\mu\hat{y}_t + (1-\eta)\hat{\tau}_t\,, \quad \text{with} \quad \eta \equiv \frac{\Theta_\alpha L}{\Theta_\alpha L + (\alpha/\lambda)T^E} \in (0,1)\,.$$

The first term, $\mu\hat{y}_t$, captures income exposure to the labor market; the second term, $\hat{\tau}_t$, captures redistribution through fiscal transfers. The relative weight between both income sources depends on the steady-state earnings-to-income ratio η, which falls with higher steady-state transfers T^E. A higher η amplifies the sensitivity of hand-to-mouth consumption to aggregate income while it simultaneously dampens the sensitivity to fiscal transfers. Using the former expression, cyclical income inequality between entrepreneurs and hand-to-mouth households is:

$$\hat{y}_t^S - \hat{y}_t^H = \frac{1}{1-\lambda}\frac{y}{y^S}\left((1-\chi)\hat{y}_t - (1-\eta)\hat{\tau}_t\right), \quad \text{with} \quad \chi \equiv \eta\mu\,.$$

Notably, χ captures how the labor market endogenously redistributes income over the cycle: endogenous redistribution is acyclical when $\chi = 1$, countercyclical when $\chi > 1$, and procyclical when $\chi < 1$. Progressive fiscal redistribution, by contrast, mechanically reduces income inequality. As such, income inequality fluctuates over the cycle unless labor-market redistribution and fiscal redistribution exactly offset each other.

2.4 A Four-Equation HANK-G Representation

We now derive a log-linear four-equation representation of the HANK-G economy and show how household heterogeneity and fiscal redistribution shape its dynamics. It builds on (i) an aggregate demand (IS) equation, (ii) an endogenous growth (EG) equation, (iii) a sticky wage Phillips curve, and (iv) a monetary policy rule.

2.4.1 *Aggregate IS Equation*

PROPOSITION 1. *The aggregate IS equation is given by*

$$\hat{y}_t = \underbrace{\zeta_f \mathbb{E}_t\left[\hat{y}_{t+1}\right]}_{\text{① transitory income}} - \underbrace{\zeta_r\left(\hat{i}_t - \mathbb{E}_t\left[\hat{\pi}_{t+1}\right]\right)}_{\text{② real interest rate}} + \underbrace{\zeta_g \hat{g}_{t+1}^A - \zeta_{g'}\mathbb{E}_t\left[\hat{g}_{t+2}^A\right]}_{\text{③ permanent income}} + \underbrace{\zeta_\tau\hat{\tau}_t - \zeta_{\tau'}\mathbb{E}_t\left[\hat{\tau}_{t+1}\right]}_{\text{④ fiscal redistribution}} \quad \text{(E.1)}$$

where the auxiliary parameters are defined as

$$\zeta_f = 1 + (1-s)\frac{\gamma_c\chi - 1}{1-\lambda\gamma_c\chi}\,, \quad \zeta_r = \frac{(1-\lambda)\gamma_c}{1-\lambda\gamma_c\chi}\,,$$

[13] This normalization isolates the role of heterogeneous income exposure from the propagation effects arising from steady-state consumption inequality; it is inconsequential for our theoretical results.

$$\zeta_g = \frac{(1-\lambda)c^S/y + g^A/(\psi y)}{1-\lambda\gamma_c\chi}\,, \quad \zeta_{g'} = s\frac{g^A}{\psi y}\frac{1}{1-\lambda\gamma_c\chi}\,,$$
$$\zeta_\tau = (1-\eta)\frac{\lambda\gamma_c}{1-\lambda\gamma_c\chi}\,, \quad \zeta_{\tau'} = \zeta_\tau\left(1-\frac{1-s}{\lambda}\right).$$

The first two terms in equation (E.1) from Proposition 1 are standard. The *transitory income channel* ① captures the sensitivity of current demand with respect to future income. Relative to a representative agent economy, income risk ($s < 1$) implies that the IS equation features compounding $\zeta_f > 1$ (respectively, discounting $\zeta_f < 1$) if endogenous income redistribution is sufficiently countercyclical, i.e., $\chi > \gamma_c^{-1}$ (respectively, $\chi < \gamma_c^{-1}$). Intuitively, when $\chi > \gamma_c^{-1}$ entrepreneurs take into account that their consumption would become relatively more exposed to aggregate income fluctuations if they were to transition to the hand-to-mouth state. This reduces their desire for precautionary savings and expands aggregate demand. The *real interest rate channel* ② reflects the elasticity of today's demand with respect to real interest rate changes. A higher degree of endogenous countercyclical income redistribution (χ) raises the real interest rate elasticity ζ_r. This occurs because redistribution toward hand-to-mouth households mechanically increases the economy-wide average MPC, thereby amplifying the general equilibrium forces of real interest rate movements on aggregate demand.[14] In general, innovative investment ($\gamma_c < 1$) attenuates both propagation channels because part of aggregate income is absorbed by investment rather than consumption.

The next two terms are specific to this paper. They capture how endogenous growth and fiscal redistribution feed back into aggregate demand. The *permanent income channel* ③ reflects how current demand responds to changes in both contemporaneous and expected future permanent income. On the one hand, higher permanent income raises entrepreneurs' consumption. On the other hand, since consumption equals output net of innovative investment, innovation diverts resources from entrepreneurs' current and future consumption. Taken together, this generates a positive feedback from $\hat{g}^A_{t+1}$, but a negative feedback from $\mathbb{E}_t[\hat{g}^A_{t+2}]$. The net effect of these opposing forces is positive, so higher permanent income stimulates current aggregate demand, i.e., $\zeta_g - \rho\zeta_{g'} > 0$ because $\rho < 1$ and $s \leq 1$. Finally, the *fiscal redistribution channel* ④ describes the direct effect of exogenous redistribution on current demand. Progressive fiscal redistribution raises the consumption of hand-to-mouth households, but reduces entrepreneurs' contemporaneous and future consumption. Overall, the net effect is positive, and this channel boosts aggregate demand, i.e., $\zeta_\tau - \rho\zeta_{\tau'} > 0$.

2.4.2 *Endogenous Growth (EG) Equation* Proposition 2 shows that innovative investment, and thus technology growth, responds to transitory income, permanent income, and the direct effect of fiscal redistribution.

[14] There exist threshold values $(\underline{\mu}, \overline{\mu})$ with $1 < \underline{\mu} < \overline{\mu}$ such that: (a) the elasticity of aggregate demand w.r.t. real interest rates ζ_r is strictly positive iff $\mu < \overline{\mu}$, and (b) the IS equation features compounding iff $\mu > \underline{\mu}$, and discounting iff $\mu < \underline{\mu}$. The threshold values are given by $\underline{\mu} \equiv 1/(\eta\gamma_c)$ and $\overline{\mu} \equiv 1/(\eta\lambda\gamma_c)$.

PROPOSITION 2. *The endogenous growth equation is*

$$\hat{g}^A_{t+1} = \underbrace{\xi_y \hat{y}_t + \xi_{y'} \mathbb{E}_t\left[\hat{y}_{t+1}\right]}_{\text{⑤ transitory income}} + \underbrace{\xi_{g'} \mathbb{E}_t\left[\hat{g}^A_{t+2}\right]}_{\text{⑥ permanent income}} - \underbrace{\xi_\tau\left(\hat{\tau}_t - \mathbb{E}_t\left[\hat{\tau}_{t+1}\right]\right)}_{\text{⑦ fiscal redistribution}}, \tag{E.2}$$

where we used $\overline{\xi} = 1/\left[g^A(1-\lambda\gamma_c) + \gamma_c(\lambda-1)\right] > 0$ *to define the feedback coefficients:*

$$\xi_y = \overline{\xi} \cdot (g^A - 1)(1 - \lambda\gamma_c\chi),$$
$$\xi_{y'} = \overline{\xi} \cdot (g^A - 1)\left[(1-\lambda)\gamma_c(g^A - \beta)/g^A - (1 - \lambda\gamma_c\chi)\right],$$
$$\xi_{g'} = \overline{\xi} \cdot (1 - \gamma_c) g^A,$$
$$\xi_\tau = \overline{\xi} \cdot (1-\eta)\lambda\gamma_c(g^A - 1).$$

Two forces drive these dynamics: a cost-of-funds effect and a market-size effect.

The *cost-of-funds effect* stems from the stochastic discount factor of entrepreneurs and captures the idea that current technology growth moves inversely with their expected consumption growth. When their consumption decreases, the cost of investment—in terms of forgone consumption—rises. In this sense, redistribution away from entrepreneurs raises their opportunity cost of investing: rather than financing new innovations, they prefer to smooth consumption.

The *market-size effect,* in contrast, affects technology growth through aggregate demand. When future demand is expected to rise, firms anticipate higher sales and greater discounted profits, which in turn stimulate innovative investment. Redistribution toward hand-to-mouth households strengthens this channel because their high marginal propensity to consume translates transfers directly into current demand.

Overall, the cost-of-funds and market-size effects together imply a positive feedback from the *transitory income channel* ⑤ onto technology growth, i.e., $\xi_y + \rho\xi_{y'} > 0$. This sign follows from the parameter restriction $\chi < 1/(\lambda\gamma_c)$, which ensures that aggregate demand responds positively to lower real interest rates. The *permanent income channel* ⑥ captures the forward-looking nature of innovative investment. A higher $\mathbb{E}_t[\hat{g}^A_{t+2}]$ implies higher next period investment. Holding expected income fixed, this lowers entrepreneurs' expected consumption, raises their expected marginal utility, and strengthens entrepreneurs' incentive to smooth consumption by investing more today. Finally, the direct effect of exogenous *fiscal redistributive channel* ⑦ acts purely on the cost of funds effect as progressive fiscal redistribution reduces entrepreneurs' consumption and, therefore, technology growth.

2.4.3 *Phillips Curve and Taylor Rule* In this economy, price and wage inflation are linked through $\hat{\pi}_t = \hat{\pi}^w_t - g^A_t$. Using the unions' optimal wage setting decision, Proposition 3 expresses the New Keynesian Phillips curve in terms of price inflation.

PROPOSITION 3. *The forward-looking price Phillips curve is given by*

$$\hat{\pi}_t = \beta\mathbb{E}_t\left[\hat{\pi}_{t+1}\right] + \kappa_y \hat{y}_t + \kappa_g \hat{g}^A_{t+1} - \hat{g}^A_t, \tag{E.3}$$

where $\kappa_y = \kappa \frac{1+\varphi\gamma_c}{\gamma_c}$*,* $\kappa_g = \beta - \kappa \frac{1-\gamma_c}{\gamma_c} \frac{g^A}{g^A - 1}$*, and* $\kappa = \frac{\epsilon_w - 1}{\theta}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L > 0$.

An expansion in aggregate demand increases labor demand, prompting unions to set higher wages, which translates into higher marginal costs and inflation, i.e., $\kappa_y > 0$. Because $\gamma_c < 1$, this channel becomes stronger than in a model without endogenous growth, since part of the increase in income is spent on innovative investment. Second, higher technology growth has an ambiguous effect on inflation, i.e., $\kappa_g \gtreqless 0$. On the one hand, technology growth strengthens the forward-looking inflation dynamics through $\beta > 0$. On the other hand, it expands the production frontier and reduces marginal costs and inflation, as captured by $\kappa(1-\gamma_c)g^A/[\gamma_c(g^A-1)] > 0$. If the latter channel dominates, the Phillips curve flattens or may even reverse its slope, implying that price inflation responds little to aggregate shocks. Finally, the nominal interest rate is given by:

$$\hat{i}_t = \mathbb{E}_t\left[\hat{\pi}_{t+1}\right] + \phi_y \hat{y}_t \, . \tag{E.4}$$

2.4.4 *Discussion: HANK-G as Unified Framework* The four-equation representation unifies a number of existing frameworks. The model corresponds to (i) a two-agent (TA) Keynesian growth economy with permanent heterogeneity if $s = 1$, and to a representative agent (RA) Keynesian growth economy (Benigno and Fornaro, 2018, Fornaro and Wolf, 2023) if, in addition, $\lambda = 0, \chi = 1$; (ii) it collapses to an alternative tractable HANK model (Bilbiie, 2025) if $\psi = 0$ and $g^A = 1$ in all periods, to a TANK model (Bilbiie, 2008) if, in addition, $s = 1$; and (iii) to a baseline RANK model (Galí, 2015) if $\psi = \lambda = 0, s = \chi = 1$. Absent nominal rigidities, one recovers real model variants: (iv) a flexible price HA growth economy (Cozzi, 2018) if $\theta = 0$; (v) a RA growth economy (Aghion and Howitt, 1992) if $\theta = \lambda = 0, s = \chi = 1$; (vi) a tractable Aiyagari (1994) model if $\theta = \psi = 0$; and (vii) a standard RBC model if $\theta = \psi = \lambda = 0, s = \chi = 1$. HANK-G therefore provides a unified laboratory for studying the short- and long-run effects of fiscal policies while preserving comparability to existing models.

2.5 Determinacy

Before analyzing the transmission of fiscal expansions, we establish conditions under which the model admits a locally determinate equilibrium. This is more than a technical prerequisite. As we show below, the boundaries of the determinacy region directly shape the parameter space in which our three stabilization regimes can arise.

PROPOSITION 4. *Suppose that* $\zeta_f > \underline{\zeta}_f$*, with* $\underline{\zeta}_f < 1$.[15] *The model has a locally determinate solution if and only if*

$$\phi_y > \phi_y^d = \max\left\{\underline{\phi}_1, \underline{\phi}_2\right\} > 0 \, ,$$

[15] This assumption is innocuous and made for expositional purposes only. We provide a complete statement of the Proposition in Appendix B.4.

where the auxiliary parameters are given by:

$$\underline{\phi}_1 = \frac{\zeta_f - 1}{\zeta_r} + \left[1 + (1-s)\frac{\xi_{g'}}{1-\xi_{g'}}\right]\frac{g^A - \beta}{\beta} \,,$$

$$\underline{\phi}_2 = \xi_{g'}\left(\frac{\zeta_f - s}{\zeta_r} + s\frac{g^A - \beta}{\beta}\right) \,.$$

To understand the HANK-G "Taylor-principle", it is instructive to begin with simpler benchmark models. Shutting down the heterogeneous agent and endogenous growth elements, we recover a standard RANK model. In this environment, our pro-cyclical real interest rate rule admits a locally determinate solution whenever $\phi_y > 0$.[16] Moving to a tractable HANK model, determinacy requires $\phi_y > (\tilde{\zeta}_f - 1)/\tilde{\zeta}_r$, where $\tilde{\zeta}_f = \zeta_f|_{\gamma_c=1}$ and $\tilde{\zeta}_r = \zeta_r|_{\gamma_c=1}$. The condition $\phi_y > 0$ is then sufficient for determinacy only when income inequality is procyclical ($\chi < 1$), which yields a discounted DIS equation ($\tilde{\zeta}_f < 1$). When income inequality is instead countercyclical ($\chi > 1$), the DIS equation is compounded ($\tilde{\zeta}_f > 1$) and requires a more aggressive monetary policy response for determinacy.

Since $\underline{\phi}_1 > \underline{\phi}_2$ holds under reasonable calibrations, we now focus on the components of $\underline{\phi}_1$. Relative to HANK, the interaction with endogenous growth acts on four margins. First, endogenous growth ($\gamma_c < 1$) reduces the degree of compounding ($\zeta_f < \tilde{\zeta}_f$), calling for a less aggressive monetary policy. Second, it decreases the real interest rate elasticity ($\zeta_r < \tilde{\zeta}_r$), requiring a more (respectively, less) aggressive monetary policy if $\zeta_f > 1$ (respectively, $\zeta_f < 1$). Beyond these two forces, a pure market-size effect calls for a more aggressive monetary policy as $(g^A - \beta)/\beta > 0$. Finally, a complementarity arises between idiosyncratic income risk ($s < 1$) and the market-size effect ($(g^A - \beta)/\beta > 0$) that requires an even more aggressive monetary policy. Overall, a discounted Euler equation is not sufficient to guarantee determinacy under $\phi_y > 0$ if the complementarity between the market-size effect and income risk is sufficiently strong.

3. On the Ambiguous Effects of Fiscal Stimulus Programs

We now use the HANK-G framework to analyze the transmission of fiscal expansions in both the short and long run. Our analysis proceeds in two steps. First, to isolate the central mechanisms, we consider a temporary increase in transfers from entrepreneurs to hand-to-mouth households, which mimics a progressively financed government spending shock. To anchor our results, we compare the effects across three benchmark economies, each obtained by shutting down one or two key building blocks of the HANK-G framework: a heterogeneous agent economy with exogenous growth and no nominal rigidities (HA), a heterogeneous agent New Keynesian economy with exogenous growth (HANK), and a heterogeneous agent economy with endogenous growth and no nominal rigidities (HA-G). This comparison reveals how the interaction between nominal rigidities and endogenous growth is key to understanding the potentially ambiguous short- and long-run effects of fiscal expansions. In a second step, we then show

[16] This condition mirrors the Taylor principle, $\phi_\pi > 1$, for a standard inflation feedback rule.

how these insights generalize to a government spending shock financed through heterogeneous tax incidence across households.

Throughout this section, the output response is written as $\hat{y}_t = \mathcal{M}_y \hat{\tau}_t$, while the response of technology growth is given by $\hat{g}^A_{t+1} = \mathcal{M}_g \hat{\tau}_t$. The coefficients $\mathcal{M}_y$ and $\mathcal{M}_g$ denote the corresponding impact multipliers, defined by the solution to the system of equations (E.1)-(E.4).

3.1 Three Benchmark Heterogeneous Agent Economies

Proposition 5 characterizes how an exogenous progressive redistribution shock affects aggregate outcomes.

PROPOSITION 5. *A transitory exogenous redistribution from entrepreneurs to hand-to-mouth agents, i.e., $\hat{\tau}_t > 0$, affects output and technology growth as follows.*

(a) HA: $\theta = \psi = 0$. *In an exogenous growth model with flexible wages, progressive redistribution has no effect on output, i.e., $\mathcal{M}_y = 0$.*

(b) HANK: $\psi = 0$. *In an exogenous growth model with sticky wages, progressive redistribution is expansionary, i.e., $\mathcal{M}_y > 0$.*

(c) HA-G: $\theta = 0$. *In an endogenous growth model with flexible wages, progressive redistribution is contractionary in short and long run, i.e., sgn$(\mathcal{M}_y) =$ sgn$(\mathcal{M}_g) < 0$.*

In a heterogeneous agent exogenous growth economy with flexible wages, fiscal redistribution has no impact on output, as the latter is purely supply determined. Specifically, progressive redistribution lowers the labor supply of hand-to-mouth households, whereas it increases the labor supply of entrepreneurs by the same amount. As both effects exactly offset each other and labor is the only input, statement (a) follows.

Nominal rigidities in the form of sticky wages render progressive redistribution expansionary. Intuitively, progressive redistribution from entrepreneurs to hand-to-mouth households raises aggregate demand and, hence, short-run output. This occurs because progressive redistribution increases the income share of high-MPC households, leading to a powerful Keynesian cross as indirect demand effects gain in importance.

In heterogeneous agent endogenous growth economy with flexible wages, progressive redistribution contracts short-run output and long-run technology growth. In this model class, progressive redistribution reduces technology growth and, thus, the long-run production frontier through a negative cost-of-funds effect. Intuitively, entrepreneurs smooth consumption by reducing innovative investment, which lowers permanent income and further depresses current aggregate demand.

3.2 Progressive Redistribution According to HANK-G

PROPOSITION 6. *Let us define $\Phi > 0$. A transitory redistribution from entrepreneurs to hand-to-mouth agents, i.e., $\hat{\tau}_t > 0$, affects output and technology growth as follows:*

$$\mathcal{M}_y = \Phi \cdot \left[\zeta_\tau - \rho\zeta_{\tau'} - (\zeta_g - \rho\zeta_{g'})\Lambda_\tau\right] , \quad \mathcal{M}_g = \Phi \cdot \left[(\zeta_\tau - \rho\zeta_{\tau'})\Lambda_y - (1 + \phi_y\zeta_r - \rho\zeta_f)\Lambda_\tau\right] ,$$

where $\Lambda_y = (\xi_y + \rho\xi_{y'})/(1-\rho\xi_{g'}) > 0$ *and* $\Lambda_\tau = (1-\rho)\xi_\tau/(1-\rho\xi_{g'}) > 0$.

(a) *If* $\rho = 0$ *or* $s = 1$, *progressive redistribution has no effect on output, i.e.,* $\mathcal{M}_y = 0$, *but reduces technological growth, i.e.,* $\mathcal{M}_g < 0$.

(b) *If* $\rho > 0$ *and* $s < 1$, *progressive redistribution increases output, i.e.,* $\mathcal{M}_y > 0$, *but has an ambiguous effect on technological growth, i.e.,* $\mathcal{M}_g \gtreqless 0$ *if*

$$\rho\frac{1-s}{\lambda}\left[\frac{1-\rho}{\gamma_c} + \rho\mathscr{M}\right] - (1-\rho)(\phi_y - \rho\mathscr{M}) \gtreqless 0\,, \tag{4}$$

where we have defined $\mathscr{M} = (g^A - \beta)/g^A$.

Proposition 6 characterizes the impact multipliers of output and technology growth to a progressive redistribution shock. Two channels are at play: a consumption channel that stimulates output $\zeta_\tau - \rho\zeta_{\tau'}$, and technology growth $(\zeta_\tau - \rho\zeta_{\tau'})\Lambda_y$; and an investment channel that contracts output $(\zeta_g - \rho\zeta_{g'})\Lambda_\tau$ and technology growth $(1+\phi_y\zeta_r - \rho\zeta_f)\Lambda_\tau$. Before we explain the intuition behind Proposition 6, let us define the liquidity premium as the difference between the returns on illiquid innovative investment and liquid bond holdings, i.e., $\Delta^{LP}_{t+1} \equiv \hat{R}^A_{t+1} - \hat{r}_t = (1-s)\mathbb{E}_t\left[\hat{c}^S_{t+1} - \hat{c}^H_{t+1}\right]$. Depending on its cyclicality, two cases emerge.

We begin with case (a), in which redistribution is either transitory ($\rho = 0$) or household types are permanent ($s = 1$), eliminating income risk as in a standard TANK model. In this case, progressive redistribution has no effect on output ($\mathcal{M}_y = 0$), but contracts technology growth ($\mathcal{M}_g < 0$). Intuitively, progressive redistribution raises the consumption of hand-to-mouth households one-to-one and, thus, stimulates aggregate consumption. However, progressive redistribution simultaneously decreases entrepreneurs' consumption and innovative investment. As the liquidity premium is acyclical if $\rho = 0$ or $s = 1$, their consumption and investment falls one-to-one with their income loss. Because aggregate output is the sum of private consumption and investment, it must necessarily remain unchanged. In other words, household heterogeneity is necessary but not sufficient to generate short-run effects on output.

Consider now the key case (b) in which redistribution is persistent ($\rho > 0$) *and* households face income risk ($s < 1$). In this scenario, progressive redistribution reduces, ceteris paribus, consumption inequality and, hence, weakens entrepreneurs precautionary saving motives. Therefore, illiquid innovative investment becomes relatively more attractive to entrepreneurs who increase their holdings relative to an environment without precautionary saving motives, generating a fall in the liquidity premium. As a result, innovative investment of entrepreneurs falls less than one to one with their income loss such that the countercyclical liquidity premium stabilizes innovative investment.[17] This leads to an overall increase in output ($\mathcal{M}_y > 0$).

[17] Luetticke (2021) and Kekre and Lenel (2022) document empirically a counter-cyclical liquidity premium, computed as the return on a broad measure of physical capital relative to the federal funds rate, in response to monetary shocks.

Whether innovative investment increases ($\mathcal{M}_g > 0$) or decreases ($\mathcal{M}_g < 0$) is governed by the condition in equation (4) and depends on (i) the persistence of fiscal redistribution (ρ), and (ii) the aggressiveness of systematic monetary policy (ϕ_y).[18] Innovative investment raises (respectively, falls) if progressive redistribution is sufficiently persistent (respectively, sufficiently transitory) and monetary policy is sufficiently accommodative (respectively, sufficiently aggressive). Intuitively, more persistent progressive redistribution generates a more persistent fall in the liquidity premium. This makes it more likely that the expansion in consumption of hand-to-mouth households crowds in innovative investment despite redistributing resources away from entrepreneurs. On the contrary, the more aggressively the central bank raises real interest rates, the more it weakens demand effects, making innovative investment more likely to fall.[19]

3.3 Generalization: Government Spending Shock and Unequal Tax Incidence

We now show how the preceding insights on the potentially opposing responses of output and technology growth extend to a fiscal expansion financed through heterogeneous tax incidence. To recapitulate the modified environment, government spending is denoted by $s_t^G = S_t^G/A_t$, with a zero spending share along the balanced growth path (i.e., $s^G = 0$). It follows an AR(1) process, $\hat{s}_t^G = \rho \hat{s}_{t-1}^G + \epsilon_t^G$, where $\hat{s}_t^G = (s_t^G - s^G)/y$ and ρ now denotes the persistence of government spending. Importantly, the government levies taxes on both household types so that the budget is balanced in every period. The allocation of the tax burden is governed by the following fiscal rule:

$$T_t^H = -T^E A_t + \tau_p S_t^G \,, \quad \text{and} \quad T_t^S = T^E A_t + (1-\tau_p) S_t^G \,,$$

where $\tau_p \in [0,1]$ captures the tax incidence across household types.[20] The value of τ_p determines the nature of the financing scheme, distinguishing three cases. When $\lambda = \tau_p$, both household types contribute proportionally to the financing of the spending shock. When $\tau_p < \lambda$, entrepreneurs finance the majority of the fiscal expansion, corresponding to a progressive financing scheme. Finally, when $\tau_p > \lambda$, hand-to-mouth households bear most of the tax burden, implying a regressive scheme.

PROPOSITION 7. *A government spending shock affects aggregate outcomes as follows:*

(a) *If $\rho = 0$ or $s = 1$, government spending has no effect on output, i.e., $\mathcal{M}_y = 0$, but has a weakly negative effect on technological growth, i.e., $\mathcal{M}_g \leq 0$ if $\tau_p \leq 1$.*

[18] If $s \to 1$, equation (4) reduces to $-(1-\rho)(\phi_y - \rho\mathscr{M}) < 0$, since determinacy requires $\phi_y > \mathscr{M}$ in this case. As a result, we recover case (a) of Proposition 6, i.e., the case of permanent household heterogeneity.

[19] The condition in equation (4) can be rewritten in terms of two policy cutoffs, $\rho \gtreqless \rho^*(\phi_y) \in (0,1)$ and $\phi_y \lesseqgtr \phi_y^*(\rho)$, that we spell out in closed form in Online Appendix B.5.2. Mathematically, both thresholds increase in the other policy parameter, i.e., $\partial\rho^*/\partial\phi_y > 0$ and $\partial\phi_y^*/\partial\rho > 0$.

[20] The restriction $\tau_p \in [0,1]$ implies that both household types contribute, at least weakly, to the financing of government spending. We study the more general scenario $\tau_p \in \mathbb{R}$ in Online Appendix C. This case combines budget-balanced financing with additional redistribution: if $\tau_p < 0$ or $\tau_p > 1$, hand-to-mouth households or entrepreneurs receive a transfer from the counter-party household type that "over-finances" the government spending shock.

(b) *If $\rho > 0$ and $s < 1$, the output multiplier has an ambiguous sign, i.e., $\mathcal{M}_y \gtreqless 0$ if $\tau_p \lesseqgtr \tilde{\tau}_p \in (0,1)$. The sign of technology growth is characterized by three cases:*

(b.1) *For $\phi_y > \phi_y^*$, one obtains $\mathcal{M}_g < 0$.*

(b.2) *For $\phi_y = \phi_y^*$, one obtains $\mathcal{M}_g = 0$ if $\tau_p = 0$ and $\mathcal{M}_g < 0$ if $\tau_p \in (0,1]$.*

(b.3) *For $0 < \phi_y < \phi_y^*$, one obtains $\mathcal{M}_g \gtreqless 0$ if $\tau_p \lesseqgtr \tau_p^*$, where $\tau_p^* < \tilde{\tau}_p < \lambda$.*

Closed form expressions for $\mathcal{M}_y$, $\mathcal{M}_g$, ϕ_y^*, $\tilde{\tau}_p$, and τ_p^* are provided in Appendix B.6.[21]

Statement (a) extends the result of Proposition 6. In the absence of a cyclical liquidity premium, aggregate consumption and investment responses offset each other, leaving output unchanged regardless of tax incidence. In contrast to the earlier case, the response of technology growth now depends on the allocation of the tax burden across households. If $\tau_p = 1$, government spending is entirely financed by hand-to-mouth households, and entrepreneurs' innovative investment remains unchanged. In this case, government spending crowds out private consumption of hand-to-mouth households one-for-one. If $0 < \tau_p < 1$, both household types contribute to financing the spending shock, leading to a decline in both private consumption and innovative investment across households. Finally, if $\tau_p = 0$, entrepreneurs fully finance the fiscal expansion. In this case, consumption of hand-to-mouth households remains unchanged, while government spending crowds out entrepreneurs' consumption and innovative investment one-for-one. As a consequence, innovative investment cannot increase in a TANK economy for any $\tau_p \in [0,1]$.

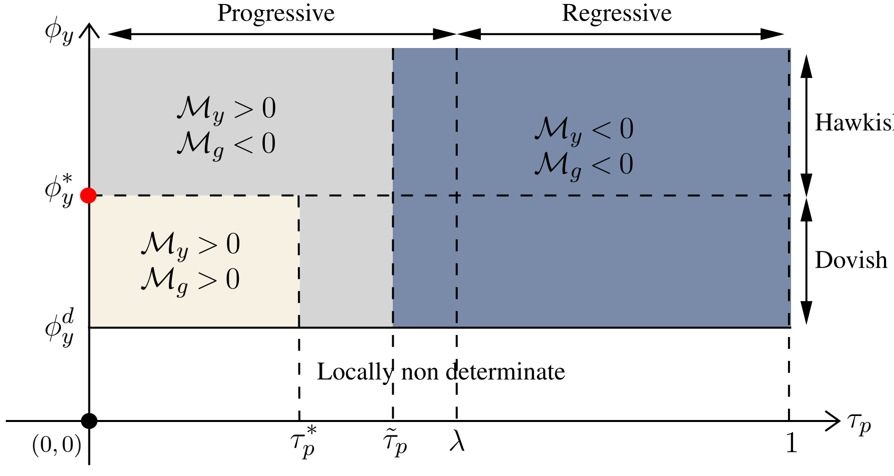
Statement (b) considers a persistent fiscal expansion ($\rho > 0$) in the presence of income risk ($s < 1$). In this environment, the responses of output and technology growth are generally ambiguous. Figure 1 illustrates three distinct regimes that depend on the joint design of systematic monetary policy and tax incidence across household types.

As before, the cyclical liquidity premium allows output to depart from the zero multiplier benchmark. Whether output rises or falls depends on tax incidence. When the financing scheme is sufficiently progressive and hand-to-mouth households bear a sufficiently low tax burden ($\tau_p < \tilde{\tau}_p < \lambda$), the logic of Proposition 6 carries over: the fall in consumption inequality between entrepreneurs and hand-to-mouth households weakens precautionary saving motives, the countercyclical liquidity premium induces innovative investment to decline by less than entrepreneurs' income loss, and output rises. When the financing scheme is instead insufficiently progressive and hand-to-mouth households bear a sufficiently high tax burden ($\tau_p > \tilde{\tau}_p$), precautionary saving motives are strengthened, entrepreneurs reduce their innovative investment, the liquidity premium rises, and output falls.

Regarding the sign of the innovative investment response, three sub-cases emerge. In the first sub-case, monetary policy is sufficiently aggressive ($\phi_y > \phi_y^*$) so that innovative investment will unambiguously fall. This is because the aggregate demand expansion is too weak to crowd in investment. In the second sub-case, monetary policy

[21]The impact multipliers to a government spending shock have an isomorphic analytical form as the ones to a pure fiscal redistribution shock, see Proposition 6 or the corresponding proof in Online Appendix.

FIGURE 1. Stabilization Regimes and Monetary-Fiscal Policy Design ($\rho > 0$ and $s < 1$)



is neither too aggressive nor too accommodative ($\phi_y = \phi_y^*$). This implies that innovative investment remains constant only in the knife-edge case where the entire tax burden falls on entrepreneurs. In this case, the aggregate demand expansion from government spending and hand-to-mouth consumption exactly offsets the disincentives to innovate from higher taxes, leaving aggregate innovative investment unchanged. As soon as hand-to-mouth households contribute to financing the fiscal expansion, demand effects weaken and innovative investment falls. Finally, in the third sub-case, monetary policy is sufficiently accommodative ($0 < \phi_y < \phi_y^*$) and the sign of the technology growth response depends on tax incidence: when financing is sufficiently progressive ($\tau_p < \tau_p^* < \tilde{\tau}_p$), the aggregate demand expansion is strong enough to crowd in innovative investment.

In summary, a tax-financed fiscal expansion gives rise to three distinct stabilization regimes shown on Figure 1. In the first regime, the beige region, a sufficiently progressive financing scheme combined with a dovish systematic monetary policy increases both output and technology growth. In the second regime, the gray region, either a less progressive financing scheme or a more hawkish monetary stance leads to an expansion in output that is accompanied by a decline in technology growth. In the third regime, the blue region, insufficient progressivity in the financing scheme results in a simultaneous contraction in output and technology growth, irrespective of the conduct of systematic monetary policy.

3.3.1 *Short- vs. Long-Run Stabilization Trade-Off* Propositions 6 and 7 show that the multipliers on short-run output and technology growth might have opposite signs. Because any transitory change in technology growth has a permanent effect on the *level* of output, a short- versus long-run stabilization trade-off occurs in this scenario. In the case of a short-run output expansion ($\mathcal{M}_y > 0$) this implies that there exists a time horizon $z^* > 0$ such that $\ln Y_{t+z} > \ln Y_{t+z}^P$ for $z < z^*$ and $\ln Y_{t+z} < \ln Y_{t+z}^P$ for $z > z^*$, where $\ln Y_{t+z}^P$ denotes the logarithm of output in the laissez-faire economy without fiscal intervention at period $t + z$.

COROLLARY 1. *Let us define the ceiling function by $\lceil \cdot \rceil$. The time horizon characterizing the short- versus long-run stabilization trade-off is analytically given by:*

$$z^* = \lceil \hat{z} \rceil \,, \quad \textit{where} \quad \hat{z} \equiv -\frac{\ln\left(1-(1-\rho)\dfrac{\mathcal{M}_y}{\mathcal{M}_g}\right)}{\ln \rho} \,.$$

It can be seen that the tipping point z^* decreases in $\mathcal{M}_y/\mathcal{M}_g$. The underlying mechanism is straightforward: the larger the decline in technological growth relative to the contemporaneous expansion in output, the sooner a progressively financed stimulus induces a reduction in the long-run level of output.

3.3.2 *Output Maximizing Tax Incidence* To close this section, we characterize the tax incidence, $\tau_p^{\max}(z)$, that delivers the largest output response at horizon $t+z$ following a government spending shock in period t.

PROPOSITION 8. *The output maximizing tax incidence $\tau_p^{max}(z)$ is given by:*

(a) *If $\rho = 0$ or $s = 1$, one obtains $\tau_p^{max}(0) \in [0,1]$ and $\tau_p^{max}(z) = 1 \; \forall z \geq 1$.*

(b) *If $\rho > 0$ and $s < 1$, two cases emerge depending on the threshold $\overline{\phi}_y > \phi_y^*$:*

(b.1) *If $0 < \phi_y \leq \overline{\phi}_y$, one obtains $\tau_p^{max}(z) = 0 \; \forall z \geq 0$.*

(b.2) *If $\phi_y > \overline{\phi}_y$, one obtains $\tau_p^{max}(0) = 0$. Moreover, there exists $\tilde{z} > 0$ such that $\tau_p^{max}(z) = 0 \; \forall z < \tilde{z}$, $\tau_p^{max}(z) \in [0,1]$ for $z = \tilde{z}$ and $\tau_p^{max}(z) = 1 \; \forall z > \tilde{z}$.*

Statement (a) of Proposition 8 is a direct implication of statement (a) of Proposition 7. As the impact output multiplier is zero, any tax incidence $\tau_p \in [0,1]$ maximizes the level of output on impact. However, as the level of output for any future period is determined by technology growth, it follows that the output-maximizing tax incidence must place the entire tax burden on hand-to-mouth households. The absence of precautionary saving motives or a fully transitory spending shock therefore calls for a regressive financing scheme of fiscal expansions.

Statement (b) characterizes the output-maximizing tax incidence for a persistent spending shock in the presence of precautionary saving motives, where the signs of the output and technology growth responses depend on the monetary-fiscal interaction. Two sub-cases emerge. The first sub-case (b.1) calls for progressive financing. Specifically, when monetary policy is sufficiently accommodative ($\phi_y \leq \overline{\phi}_y$), both the output and the technology growth multiplier are strictly decreasing in τ_p. Consequently, a tax incidence that places the entire tax burden on entrepreneurs maximizes the level of output at any time horizon following the government spending shock. The second sub-case (b.2) underscores the tension between a progressive and a regressive tax incidence. In this case, the output multiplier continues to decrease in τ_p, whereas the technology growth multiplier increases in τ_p. As a result, a progressive tax incidence maximizes short-run output gains, whereas a regressive tax incidence maximizes long-run output gains. Between these two forces, there exists a time horizon $z = \tilde{z}$ at which they exactly offset each other, so any tax incidence maximizes the level of output.

Taking stock, our analysis provides new analytical insights into the tension between progressively and regressively financed fiscal expansions. We highlight that the resolution of this trade-off hinges on four main propagation channels: the persistence of the fiscal expansion, idiosyncratic income risk, tax incidence, and systematic monetary policy. Whenever the fiscal expansion is persistent and households face income risk, output can increase in the short- and long-run if monetary policy is sufficiently accommodative and the bulk of the tax burden falls on entrepreneurs. The combination of these forces generates a powerful stabilization regime.

3.3.3 *Back-of-the-Envelope Calculation* To assess how likely each of the three regimes is, we now complement the analytical characterization with a simple back-of-the-envelope calculation.

The model period is one quarter. We set $\beta = 0.995$, implying an annual interest rate of 4%. The share of hand-to-mouth households is $\lambda = 0.35$, the probability of remaining an entrepreneur is $s = 0.96$, and the degree of countercyclical earnings inequality is $\chi = 1.50$, consistent with the parameterization in Bilbiie (2020) and the matching multiplier estimates in Patterson (2023).[22] We calibrate transfers from entrepreneurs to hand-to-mouth households, T^E, such that balanced-growth-path consumption inequality is absent, i.e., $\Gamma_c = 1$. We set α to generate a R&D expenditure-to-GDP ratio equal to 3%, which provides a conservative lower bound for total innovative investment. This target implies a value of $\gamma_c = 0.97$, such that income inequality along the balanced-growth-path exceeds consumption inequality by five percentage points, i.e., $\Gamma_y = 1.05$. The efficacy of innovative investment (ψ) is chosen to target a quarterly steady-state growth rate of 0.50 percentage points, corresponding to $g^A = 1.005$. In addition, we set labor dis-utility (ν) to normalize hours worked to $L = 1$. As our benchmark policy experiment we consider a 1% increase in government spending with persistence $\rho = 0.90$ and progressive tax incidence $\tau_p = 0.00$. This assumption is in line with Ferriere and Navarro (2025), who show that the average tax rate of the bottom 50% of the income distribution, as opposed to the top 50%, varies little following a typical government spending shock in the U.S. economy. Systematic monetary policy is parameterized as $\phi_y = 0.075$, consistent with the moderate real interest rate response to government spending shocks documented empirically; see, for example, Wolf (2023), Hack et al. (2026) and Galí (2026).

Under this parameterization, the model exhibits locally stable equilibrium dynamics if the real interest rate rule is weakly procyclical, that is, if $\phi_y > \phi_y^d = 0.06$. In addition, we obtain $\phi_y^* = 0.07$, implying that the no-trade-off regime – characterized by positive responses of both output and technology growth – emerges only over a narrow region of the parameter space. Output increases whenever $\tau_p < \tilde{\tau}_p = 0.22$, whereas technology growth increases for $0.06 < \phi_y < 0.07$ provided that $\tau_p < \tau_p^* \in [0.00, 0.05]$. Under the

[22] The earnings incidence parameter is identified by $\mu = 1 + \frac{cov(mpc,\upsilon)}{\beta\lambda}$, where $cov\,(mpc, \upsilon)$ denotes the covariance between individual MPCs and the elasticity of individual earnings with respect to aggregate earnings. Empirical estimates of the matching multiplier for the U.S. economy imply $\hat{cov}\,(mpc, \upsilon) > 0$.

benchmark calibration, the model generates an output multiplier of $\mathcal{M}_y = 1.18 > 0$ and a technology growth multiplier of $\mathcal{M}_g = -0.03 < 0$, implying a regime with short-run versus long-run stabilization trade-off. The horizon beyond which the level of output permanently declines is approximately four years, corresponding to $z^* = 16$, while the associated long-run output loss equals -0.30 percent.

3.4 Discussion and Additional Results

In this section, we place our analysis in the context of the literature, clarify the specific margins along which our approach differs, and outline promising extensions.

3.4.1 *Physical vs. Innovative Capital* The core of our HANK-G framework features two assets, liquid bonds and illiquid innovative capital. In terms of the underlying structure, our model thus shares similarities to the standard two asset HANK models with liquid bonds and illiquid physical capital (Galí et al. (2007), Bilbiie et al. (2022), Kaplan et al. (2018), Auclert (2019)) that are well-suited to study short-run dynamics in consumption, investment, and output. Compared to these models, our framework with innovative capital offers a distinct yet complementary perspective. First, innovative capital generates a unit root in the technology level such that any transitory shock that alters innovative investment will permanently alter the level of output. Second, as consumption and investment are driven by changes in permanent income, the endogenous growth setup further strengthens the indirect propagation effects of transitory policy shocks relative to an exogenous growth setup. Finally, a key advantage of our framework lies in its analytical tractability. Because innovative returns are linear in output, we are able to provide a closed-form solution to a two asset THANK economy.

3.4.2 *Equivalence between Fiscal and Monetary Policy* It is well known that, in one-asset HANK models, fiscal transfers across households are macroeconomically equivalent to a monetary stimulus in their effects on consumption and output (see, among others, Seidl and Seyrich (2023), Bilbiie et al. (2024), Wolf (2025)). This equivalence breaks down in the HANK-G framework because fiscal transfers also operate through an investment channel: they appear both in the DIS equation and the endogenous-growth (EG) equation. Essentially, in this framework, a monetary expansion raises both output and technology growth and the redistributive effects of the monetary stimulus weaken or amplify the consumption or investment channel without reversing their signs. As a result, a short- vs long-run stabilization trade-off does not occur for a monetary stimulus. We defer a full analysis of monetary policy in HANK-G to a companion paper.

3.4.3 *Departure from FIRE* There is growing empirical evidence that supports a departure from the full information rational expectations (FIRE) benchmark (see, among others, Coibion and Gorodnichenko (2015) or Roth et al. (2023)). To account for this, we follow Gabaix (2020) and Pfäuti and Seyrich (2022) and introduce bounded rationality in the form of cognitive discounting, where $\overline{m} \in [0, 1]$ denotes the degree of cognitive discounting. By generating a discounted IS equation, this extension rules out the "Catch-22" (Bilbiie, 2025) in the HANK-G framework: cognitive discounting solves the forward guidance puzzle for output, while counter-cyclical income inequality leads to amplification. Economically, cognitive discounting acts on household expectations and, hence, effectively weakens the market size effect. One can show that Propositions 6 - 8 continue to hold by replacing the persistence of the respective fiscal policy shock with $\overline{m}\rho < \rho$.

4. Quantitative Analysis

The analytical results so far reveal three possible regimes for how fiscal expansions affect output and technology growth, depending on both tax incidence and the stance of systematic monetary policy. To assess which regime is empirically relevant, we now complement our analysis with a quantitative evaluation of tax-financed fiscal expansions in the U.S. economy. The extended model preserves the same backbone as the theoretical model, enriching the environment only where needed for empirical discipline along three margins: richer household heterogeneity, an innovation process aligned with the entrepreneurial growth literature, and realistic tax and transfer schedules.

We first present the model and its calibration, and then study its quantitative properties. Our main finding is that a typical postwar U.S. fiscal expansion – financed by greater income tax progressivity – falls into the regime characterized by a stark and robust short- versus long-run stabilization trade-off. We further show that placing the tax incidence on the upper middle class maximizes welfare by preserving short-run output gains without sacrificing long-run growth.[23]

4.1 Extended HANK-G Model and Calibration

We normalize variables by aggregate technology to keep the household problem and cross-sectional distribution on a time-invariant state space.

Households Unlike the theory, we distinguish between *ex ante* and *ex post* household heterogeneity. Specifically, a time-invariant share Λ_E of households has the ex ante ability to become entrepreneurs, while the remaining share $1 - \Lambda_E$ are workers. Households now face idiosyncratic shocks to labor productivity h_{it}, employment status $e_{it} \in \{0, 1\}$, and discount factor β_{it}, each governed by a Markov process. They supply efficiency units of labor $L_{it} = e_{it} h_{it} \ell(h_{it})$, where $\ell(h_{it})$ denotes hours worked when employed ($e_{it} = 1$). In steady state, hours are identical across productivity groups, while employment transitions within each group are pinned down by structural unemployment and separation rates. Off steady state, we extend Assumption 2: unions allocate aggregate labor-demand fluctuations across productivity types via $\Delta L_t^h = \mu(h) \Delta L_t$, and split each group's adjustment between hours and employment using the weights $\phi(h)$ and $1 - \phi(h)$, respectively. Employment adjustment operates through changes in the transition probabilities between employment and unemployment.

Workers can save in liquid assets $b_{it} \equiv B_{it}/A_t \geq 0$, while entrepreneurs can also invest $x_{it} \equiv I_{it}/A_t$ to accumulate a stock of illiquid innovations, $a_{it} \equiv A_{it}/A_t$. Households

[23] For expositional purposes, we keep the description of the stationary extended model concise and refer the interested reader to Section D of the Online Appendix for details.

die with probability p_d and are replaced by newborns who inherit their parents' ex ante entrepreneurial ability. Newborns draw their employment status e_{it}, discount factor β_{it}, and labor productivity h_{it} from the corresponding Markov processes. Bequests are fully taxed, and innovations are lost when the entrepreneur dies.

With zero distributed dividends, household income before personal taxes is the sum of labor income, entrepreneurial income net of corporate taxes, and bond income

$$y_{it} = [(1-e_{it})\varsigma + e_{it}]\, w_t h_{it} \ell_{it} + \mathbb{I}_{it}^E (1-\tau_c) R_t^A a_{it} + r_{t-1} b_{it} \,,$$

where $w_t \equiv W_t/A_t$ is the real wage, ς denotes the unemployment replacement rate if unemployed $(e_{it}=0)$, $\mathbb{I}_{it}^E$ indicates the entrepreneurial type with return $R_t^A = \Theta_\alpha L_t$, τ_c is the corporate tax rate, and $r_{t-1} = R_{t-1}/\pi_t - 1$ is the net real interest rate between $t-1$ and t. The household budget constraint is

$$c_{it} + x_{it} + g_{t+1}^A b_{it+1} = y_{it} - \mathcal{T}(y_{it}) + b_{it} \,,$$

where $c_{it} \equiv C_{it}/A_t$ and $\mathcal{T}(y_{it})$ is the tax-and-transfer schedule. In addition, $m_t(a_{it}, b_{it}, \mathbf{s}_{it})$ denotes the measure of households, where $\mathbf{s}_{it} = (h_{it}, e_{it}, \beta_{it}, \mathbb{I}_{it}^E)$ is the vector of exogenous states.

Entrepreneurial Innovation Only a fraction of entrepreneurs will hold a positive illiquid stock of innovations, $a_{it} > 0$, depending on realizations of idiosyncratic shocks. In particular, by investing an amount x_{it}, entrepreneurs generate a successful innovation with probability $\zeta(x_{it}) = 1 - \exp(-\gamma x_{it})$. Upon success, the stock of innovations increases by a fixed step ηA_t, where $\eta > 0$ and $A_t \equiv \int A_{it} dm_t^E(a_{it}, b_{it}, \mathbf{s}_{it})$ denotes aggregate innovations, with m_t^E representing the measure of households with entrepreneurial ability.[24]

Moreover, we incorporate an idiosyncratic investment shock $\iota_{it+1} \sim F_\iota$, which scales previously realized innovations and generates heterogeneity in entrepreneurial income. This modeling device captures the idea that innovative investment does not always translate one-for-one into realized innovation, and that large innovations can occur even with modest investment. The relative stock of innovations evolves as

$$a_{it+1} = \frac{\iota_{it+1}(1-\delta^A) a_{it} + \eta\, d_{it+1}}{g_{t+1}^A} \,,$$

where g_{t+1}^A denotes aggregate gross technology growth, δ^A is the depreciation rate of innovation capital and $d_{i,t+1} \in \{0,1\}$ indicates whether the innovation step is realized. Aggregating the individual laws of motions for $a_{i,t+1}$ determines the aggregate stock of innovations and thus overall technology growth g_{t+1}^A.

[24]The key difference between the analytical and quantitative models lies in the returns to scale of innovative investment. In the analytical model, the law of motion can be equivalently rewritten as $A_{t+1}(j) = \zeta(x_{it})\big[A_t(j) + \eta A_t\big] + \big(1 - \zeta(x_{it})\big) A_t(j)$, with a linear success probability $\zeta(x_{it}) = (\psi/\eta) x_{it}$. The quantitative model departs from this benchmark by introducing decreasing returns to innovation effort, with $\zeta(x_{it}) = 1 - \exp(-\gamma x_{it})$. Balanced growth is nevertheless preserved because each successful innovation raises knowledge by a step proportional to aggregate technology, ηA_t, through non-rivalry.

Government and Monetary Policy The government issues bonds and collects tax revenues. Income taxes and transfers are of the form specified by Ferriere et al. (2023), whereby the average tax rate and transfer are given by:

$$\tau(y_{it}) = \exp\left(\log(\vartheta_t)\left(y_{it}/y\right)^{-2\varrho_t}\right), \quad \text{and} \quad T(y_{it}) = \underline{T}y\frac{2\exp\left\{-\xi\left(y_{it}/y\right)\right\}}{1+\exp\left\{-\xi\left(y_{it}/y\right)\right\}}.$$

The parameter $\varrho_t \in (-\infty, 1)$ governs tax progressivity, ϑ_t controls the average tax rate, and y is the steady-state mean income in the economy. The tax schedule is linear when $\varrho_t = 0$, progressive when $\varrho_t \in (0,1)$, and regressive when $\varrho_t < 0$. For transfers, the parameter $\underline{T}$ denotes transfers to a household with zero income as a multiple of steady-state mean income, while ξ determines how quickly transfers phase out with income. When $\xi = 0$, transfers are lump-sum. As ξ becomes larger, transfers phase out faster. The total tax-and-transfer schedule is $\mathcal{T}(y_{it}) = \tau(y_{it})y_{it} - T(y_{it})$.

Total revenues of the government are the sum of income taxes net of transfers, bequest taxes ($p_d b_{it}$), and corporate taxes. They are used to finance an exogenous level of government spending s_t^G, interest payments on bonds $r_{t-1}b_t^G$, and unemployment benefits. The government budget is:

$$g_{t+1}^A b_{t+1}^G + \int [\mathcal{T}(y_{it}) + p_d b_{it} - (1-e_{it})\varsigma w_t h_{it}\ell_{it}] dm_t(a_{it}, b_{it}, \mathbf{s}_{it}) + \tau_c R_t^A = (1+r_{t-1})b_t^G + s_t^G.$$

Furthermore, monetary policy sets the nominal interest rate according to a standard Taylor rule, which depends on price inflation and output. With a slight abuse of notation relative to the analytical setup, the rule has the following form:

$$i_t = r + \phi_\pi(\pi_t - 1) + \phi_y(y_t/y - 1).$$

Calibration The model is calibrated to the quarterly BGP of the U.S. economy.

Households Households have utility $\mathcal{U}(c_{it}, e_{it}\ell_{it}) = \frac{c_{it}^{1-\sigma}}{1-\sigma} - \nu\frac{(e_{it}\ell_{it})^{1+\varphi}}{1+\varphi}$ with effective discount factor $\widetilde{\beta}_{it} = \beta_{it}(g_{t+1}^A)^{1-\sigma}$.[25] We set the inverse EIS to $\sigma = 1.5$ and the inverse Frisch elasticity to $\varphi = 1$. The labor disutility parameter ν is chosen so that employed households work $\ell = 1/3$ on the BGP, and the death probability is $p_d = 0.005$.

To generate heterogeneous MPCs (Carroll et al., 2017), households differ in their discount factor, with $\beta_{it} \in \{\beta_L, \beta_M, \beta_H\}$. Transitions are restricted to adjacent types and follow Krusell and Smith (1998) and Ferriere and Navarro (2025) with persistence $p_\beta = 0.995$, $\beta_L = \beta_M - \Delta^\beta$, and $\beta_H = \beta_M + \Delta^\beta$. We set $\beta_M = 0.97$ and $\Delta^\beta = 0.026$ to match an average MPC out of transitory income of 0.18 and a debt-to-GDP ratio of 40%.

The Markov process for labor productivity h_{it} is discretized with 10 states so that it approximates an AR(1) process in logs, i.e., $\log(h_{i,t+1}) = \rho_h \log(h_{it}) + \epsilon_{i,t+1}^h$, where $\epsilon_{i,t+1}^h \sim \mathcal{N}(0, \sigma_h^2)$, with $\sigma_h^2 = 0.04$ and $\rho_h = 0.973$, consistent with Storesletten et al. (2004). In addition, we normalize $\int_i h_{it} di = 1$.

[25] Note that the disutility of labor in the non-stationary formulation scales with $A_t^{1-\sigma}$.

To calibrate BGP labor market features and individual exposure, we use CPS panel data from 1989–2022 for individuals aged 20–65, grouped into wage bins consistent with the model's h-heterogeneity. For each h-group, we set steady-state job-finding rates to their long-run CPS averages and back out steady-state separation rates to match long-run average unemployment rates by group. Regarding labor-market exposure, we calibrate the earnings-incidence function $\mu(h_{it})$ to match the cyclicality of total hours by wage bin in the CPS. We then use the weights $\phi(h_{it})$ to split this labor adjustment between hours per worker and employment, again matching the observed employment-hours decomposition. This calibration implies that low-wage workers are more exposed to aggregate fluctuations through both employment and hours worked. Finally, implementing the extensive employment margin requires taking a stand on how employment fluctuations are split between separations and job finding. We set the share of unemployment fluctuations due to separations to $\phi^s \approx 0.3$, following Krusell et al. (2017). The labor market parameter values are reported in Table D.1 in Online Appendix D.3.

Technology and Innovation The parameter γ governs how efficiently innovative investment translates into successful innovation: a higher γ raises the success probability for a given x_{it}, lowering the effective cost of realized innovations. This parameter is difficult to discipline because the empirical literature on inventor behavior mainly measures outcomes such as patents, citations, and inventor mobility, rather than latent innovative investment. For instance, Akcigit et al. (2022) report elasticities of patents P, or citations, with respect to personal net-of-tax rates $1-\tau$, such as the marginal net-of-tax rate faced by earners at the 90th percentile, $1 - \text{MTR90}$. We denote this elasticity by $\varepsilon_{P,1-\tau}$.

Our calibration strategy therefore proceeds in two steps. First, following the patent-R&D investment literature, we use the elasticity of patents with respect to innovative investment, $\varepsilon_{P,x}$, to map model investment responses into patent responses. The relevant model counterpart is therefore $\varepsilon_{P,1-\tau} = \varepsilon_{P,x} \times \varepsilon_{x,1-\tau}$, where $\varepsilon_{x,1-\tau}$ is the model elasticity of innovative investment with respect to the net-of-tax rate $1-\tau$. We set $\varepsilon_{P,x} = 0.5$, which is close to the average estimate in the patent-R&D investment literature (Hausman et al., 1984, Kortum and Lerner, 2000, Blundell et al., 2002). Second, we discipline the patent response to personal income taxation using the individual-inventor evidence in Akcigit et al. (2022). For the U.S. benchmark, following Jones (2022), we focus on individual responses rather than macro elasticities, since the latter also include geographic reallocation of innovators across states. In particular, we use the individual elasticity of patents with respect to $1 - \text{MTR90}$, reported in Table C.37 of the Online Appendix of Akcigit et al. (2022). This elasticity is about 0.3 for patents and 0.15 for citations.[26] In the model, we choose $\gamma = 0.44$ so that the resulting elasticity $\varepsilon_{P,1-\text{MTR90}}$ equals 0.23.

[26] This estimate is lower than the headline elasticities in Akcigit et al. (2022) because the latter use the marginal tax rate actually faced by each inventor, whereas Table C.37 uses the economy-wide MTR90. This distinction matters because many regular inventors are not directly subject to top marginal tax rates, so their response to MTR90 is muted. Consistently, regular inventors have relatively low elasticities, around 0.18 or below, while star inventors, defined as the top 10% of the cumulative patent distribution, are more directly exposed to high tax incentives and have much larger elasticities, around 0.8. Since idea production is highly skewed, Jones (2022) argues that the relevant aggregate elasticity is an idea-share-weighted average of regular and star inventors' responses, which he estimates to lie around $\varepsilon_{P,1-\text{MTR90}} \in [0.24, 0.40]$.

We do so by permanently changing MTR90 through tax progressivity ϱ and evaluating the behavioral response of innovative investment at the initial distribution.[27] Under this calibration, the model also implies an elasticity with respect to the corporate net-of-tax rate, $\varepsilon_{P,1-\tau_c}$, of 0.18, which falls within the range documented by Akcigit et al. (2022), between 0.14 for patents and 0.26 for citations. In Section 4.3.1, we also consider two alternative calibrations: one targeting a lower elasticity of 0.1, and another incorporating mobility effects with an elasticity of 0.5, at the lower end of their macro estimates. The latter is more relevant for smaller open economies, where international reallocation of entrepreneurs and inventors may be more prevalent.

We set the share of potentially innovative entrepreneurs to $\Lambda_E = 0.10$, which roughly corresponds to the share of active entrepreneurs in the U.S. Survey of Consumer Finances (2010). The quarterly depreciation rate of innovation is set to $\delta^A = 0.03$ (about 12.6% annually), a value that lies between the estimate in de Rassenfosse and Jaffe (2018) for patented inventions—based on the revenue streams generated by innovations—and the BEA convention of 15% annual depreciation for R&D expenditures. We assume that the innovation shock ι is Gaussian, i.e., $F_\iota(\iota) \sim \mathcal{N}(1, \sigma_\iota^2)$, where $\sigma_\iota^2 = 0.0225$ generates a Pareto tail for income of about 1.7, consistent with Piketty and Saez (2014). Finally, given γ, the step size $\eta = 0.32$ pins down an annual growth rate of 2%.

Nominal Rigidity The labor share is set to $\alpha = 0.4$. We calibrate the NKPC by setting $\epsilon_w = 7$, which corresponds to a union's markup of 16.7%. The wage adjustment cost θ is 200 to match a wage Phillips curve slope, ϵ_w/θ, of 0.035.

Fiscal and Monetary Policy We borrow the progressivity of the tax system and transfer parameters from Ferriere et al. (2023).[28] In particular, the steady-state progressivity of the income tax schedule is $\varrho = 0.08$, while $\vartheta = 0.7$ implies an average labor income tax rate of about 23%. Transfers at minimum income are $\underline{T} = 8.3\%$ of average income, with a phase-out parameter $\xi = 4.22$. The corporate tax rate is set to its statutory level, $\tau_c = 0.21$. The UI replacement rate ς is set to 40% of previous labor income, the average across U.S. states between 1990 and 2020. The implied government spending to output ratio is roughly 25%. Finally, monetary policy follows a standard Taylor rule with coefficients $\phi_\pi = 1.5$ on inflation and $\phi_y = 0$ on output. The real interest rate on the BGP is $r = 1\%$.

4.2 Inspecting the Model's Properties

Innovation, Income, and Wealth Inequality We now examine how well the model matches key features of the data. We begin with the innovation-inequality relationship. Table 1 compares the income and wealth distributions in the data and the model.

The benchmark model (2nd row) generates a pronounced concentration of income and wealth, largely driven by the strong endogenous sorting of entrepreneurs into the top of the distribution. Entrepreneurs account for 45% of the wealthiest 5% of households. The model is therefore consistent with the causal link between innovation and

[27] The response is naturally smaller for transitory tax changes, as in the main policy experiment below.

[28] The latter incorporate SNAP, housing assistance, the refunded Additional Child Tax Credit (ACTC), the refundable part of the EITC, state credits, and other welfare transfers.

TABLE 1. The Distribution of Income and Wealth

	Income, top share (in %)				Wealth, top share (in %)			
	1%	10%	20%	50%	1%	10%	20%	50%
US data [a]	17	42	58	85	38	73	86	98
Benchmark model	15	38	52	80	35	71	85	99
No entrepreneurs ($\Lambda_E = 0$) [c]	2	19	35	68	11	56	77	98
No innovation shock ($\sigma_\iota^2 = 0$) [b]	12	36	51	79	17	57	75	97

[a] Statistics for income and wealth come from the World Inequality Database (WID).
[b] Recalibrated to match an annual growth rate of 2%.
[c] Recalibrated to match the same average MPC and debt-to-GDP ratio.

top income shares documented by Aghion et al. (2019). According to their estimates, a 1% increase in patents per capita increases the top 1% income share by ∼0.2%.

It is well established that self-employed and business owners are disproportionately represented among the rich. Cagetti and De Nardi (2006), for example, show that among the top 5% of households, 68% are business owners (39% when restricted to the self-employed). Consistent with their findings, removing entrepreneurs from the model (3rd row) sharply compresses the income and wealth distributions. A key distinction in our framework, however, is that while 10% of the population are entrepreneurs, only those with a positive stock of innovations ($a > 0$) qualify as innovative. By this criterion, just 4.5% of the population are innovators—a small minority. According to the Global Entrepreneurship Monitor, about 11% of U.S. adults aged 18–64 manage a new business. Roughly one-third of them offer products or services that are new to at least some customers and face little or no direct competition. This implies that about 3.9% of adults can be classified as innovative entrepreneurs. Using the 2017 U.S. Annual Business Survey, about 43% of active for-profit firms reported an innovation between 2015 and 2017. Combined with the SCF estimate that 13.3% of adults own a business, and assuming one firm per owner, this implies that at most 5.7% of adults own innovative firms.

Finally, removing the idiosyncratic shocks to innovation (4th row), ι, compresses the distributions of income and wealth, underscoring the central role of idiosyncratic investment risk, or luck, as opposed to the level of investment alone, as an engine of concentration in our calibration.

Consumption Responses We now turn to the demand properties of the model and report the model-implied distribution of MPCs across household types and along the income and wealth distributions. We compute the MPC as the marginal change in consumption resulting from a small change in liquid wealth, $\text{MPC}(a_{it}, b_{it}, \mathbf{s}_{it}) = \frac{\partial c(a_{it}, b_{it}, \mathbf{s}_{it})}{\partial b_{it}}$. We find that unemployed individuals have the highest MPCs (0.38 on average), followed by employed workers (0.16) and, finally, entrepreneurs (0.04). This pattern is consistent with Carroll et al. (2017), Ganong and Noel (2019), and Kekre (2023), who show that MPCs are highest among the unemployed because borrowing constraints and low liquid wealth make their consumption highly sensitive to income changes. In contrast, entrepreneurs—having accumulated wealth over time—display the lowest marginal propensity to consume.

TABLE 2. Quarterly MPC across Groups, Wealth Quantiles, and Income Quantiles

GROUP	MPC	MEAN WEALTH	WEALTH Q	MPC	INCOME Q	MPC
Overall Population	0.18	1.00	0–20%	0.39	0–20%	0.47
Unemployed	0.38	0.76	20–40%	0.18	20–40%	0.21
Employed	0.16	1.01	40–60%	0.13	40–60%	0.11
Entrepreneurs	0.04	10.15	60–80%	0.08	60–80%	0.08
			80–100%	0.04	80–100%	0.04

4.3 Tax-Financed Fiscal Expansions

Propositions 6 and 7 show that tax-financed fiscal expansions can generate opposing short- and long-run effects, depending on the tax incidence and the stance of systematic monetary policy. Together, these forces give rise to three stabilization regimes. We now quantify which of these regimes is relevant in practice under our benchmark calibration.

The economy initially lies on its BGP. As our main policy experiment, we implement an unexpected government spending shock equal to 1% of steady-state government spending, i.e. $\epsilon_0^G = 0.01 s^G$, which then gradually reverts to its BGP level with persistence $\rho = 0.85$. After the shock, agents have perfect foresight.

Our benchmark financing scheme follows the design of a typical U.S. government spending shock that is empirically financed through a combination of debt issuance and higher personal income tax progressivity (Ferriere and Navarro, 2025). Government debt evolves in response to a spending shock according to

$$b_{t+1}^G - b^G = \phi_B (d_t^B - d^B) \, ,$$

where $d_t^B = s_t^G + (1 + r_{t-1}) b_t^G - \tau_c R_t^A - \int [p_d b_{it} - T(y_{it}) - \varsigma w_t h_{it} \ell_{it}] \, dm_t(a_{it}, b_{it}, \mathrm{s}_{it})$ denotes the fiscal deficit before personal income taxes. When $\phi_B = 0$, government debt is constant and the fiscal expansion is fully financed through higher personal income taxes. When $\phi_B > 0$, part of the government spending shock is financed through debt. The remaining financing comes from personal income taxes. Specifically, we assume that tax progressivity evolves according to

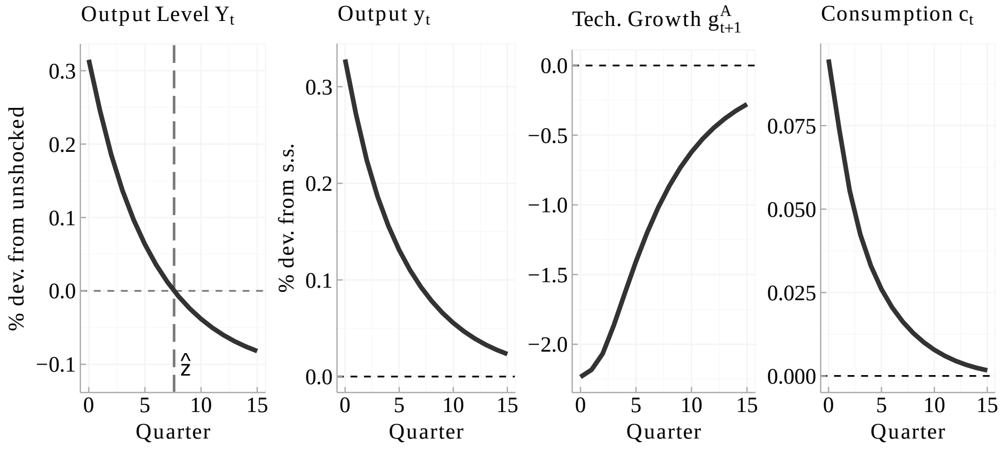
$$\varrho_t - \varrho = \phi_\varrho (g_t - g) \, ,$$

where ϕ_ϱ governs the increase in tax progressivity following the fiscal stimulus. The residual financing need is then absorbed each period by adjusting ϑ_t, i.e. the average tax rate, to satisfy the government budget constraint.

We discipline the two financing margins using the evidence in Ferriere and Navarro (2025). We set the debt-financing parameter to $\phi_B = 0.5$, in line with the average deficit response to a government spending shock, and the benchmark progressivity-response parameter to $\phi_\varrho = 1.3$ so that the cumulative Average Tax Rate (ATR) response over the transition is approximately zero for the bottom 50% of the income distribution, as in the data. This implies that most of the financing burden is borne by the top 50%, with the ATR rising further toward the top of the distribution.

Impulse Response Functions (IRFs) Figure 2 displays the resulting IRFs for the level of output Y_t, normalized output y_t, technology growth g^A_{t+1}, and consumption c_t.[29] Financing the fiscal expansion through higher progressivity produces a clear stabilization trade-off: the short-run expansion is at the cost of weaker technology growth thereafter. In particular, private consumption rises on impact, but taxation compresses the net returns to innovation, leading to a decline in entrepreneurial investment and thus technology growth. As a result, the annual output multiplier, reported in Table 3, is $\mathcal{M}_y = 1.07$, while the technology-growth multiplier is $\mathcal{M}_g = -0.11$. The switch time $\hat{z}$ defined in Corollary 1 occurs after roughly eight quarters: beyond that horizon, the output effect turns negative, implying a permanent output loss of 0.12% following a persistent 1% increase in government spending at time $t = 0$.

FIGURE 2. Aggregate Effects of Government Spending Shock



Note: Impulse responses to a 1% shock to government spending financed with higher tax progressivity.

4.3.1 *Sensitivity Analysis of the Short- and Long-run Tradeoff* We then evaluate the robustness of the stabilization trade-off across alternative calibrations of the key determinants highlighted in Propositions 5, 6, and 7: the strength of nominal rigidities, the persistence and financing of the fiscal expansion, the specification of the Taylor rule, the elasticity of the innovation channel, and the earnings incidence across households. Table 3 summarizes our findings.

Part A of Table 3 reveals that the stabilization trade-off is a robust feature of the U.S. economy: changing core model parameters affects its magnitude but not its presence. First, stronger nominal rigidities make wages and prices adjust more sluggishly, amplifying the initial demand push: the impact multiplier climbs to 1.29, long-run losses are smaller, and the switch time is pushed to eleven quarters. Because it requires a larger and more persistent tax increase, greater shock persistence slightly weakens the

[29] We provide the complete set of impulse response functions in Online Appendix D.4.

TABLE 3. Sensitivity Analysis: Government Spending Shock

SPECIFICATION	PARAMETERS	MULTIPLIER[a]		LONG-RUN DEVIATION[b]	SWITCH TIME $\hat{z}$
		$\mathcal{M}_y$	$\mathcal{M}_g$		
Baseline		1.07	-0.11	-0.12	7.60
A. Core Parameters					
Higher nominal rigidities	$\theta = 400$	1.29	-0.06	-0.07	10.56
Higher shock persistence	$\rho_g = 0.90$	0.98	-0.14	-0.22	7.38
Taylor rule	$\phi_y = 0.10$	0.94	-0.14	-0.15	6.16
No earnings incidence	$\mu(h) = 1.00$	1.08	-0.11	-0.12	7.66
Low innovation elasticity[c]	$\varepsilon_{P,1-\tau} = 0.10$	1.20	0.01	-0.03	17.79
High innovation elasticity[c]	$\varepsilon_{P,1-\tau} = 0.50$	0.98	-0.19	-0.21	5.16
B. Financing Parameters					
No debt financing	$\phi_B = 0.00$	0.96	-0.17	-0.17	6.02
Higher progressivity increase	$\phi_\rho = 2.00$	1.06	-0.15	-0.16	6.37
Adjust tax scaling only	$\phi_\rho = 0.00$	1.10	-0.03	-0.03	12.41
Welfare-maximizing incidence	Section 4.4.3	1.22	0.02	0.06	NA
C. Model Environment					
HANK	fixed g^A	1.19	0.00	0.00	NA
HA	fixed g^A, $\theta = 0$	0.64	0.00	0.00	NA
HA-G	$\theta = 0$	0.56	-0.22	-0.22	3.07

[a] The multipliers are reported for the annual cumulative responses. They are computed as $\mathcal{M}_y = \sum_{t=1}^{t=4} dy_t / \sum_{t=1}^{t=4} ds_t^G$ and $\mathcal{M}_g = \sum_{t=1}^{t=4} dg_{t+1}^A / \sum_{t=1}^{t=4} ds_t^G$.

[b] The long-run deviation is computed as $Y_T / Y_T^P - 1$ and reported in percent, where T is the horizon by which the economy has converged to its balanced growth path.

[c] The low- and high-innovation-elasticity parameterizations are $(\gamma, \eta) = (8.05, 0.05)$ and $(0.10, 1.30)$, respectively, with γ pinning down the innovation-tax elasticity and η matching a 2% annual growth rate.

short-run expansion but substantially strengthens disincentives to invest and thus the long-run contraction: output eventually falls by more than 0.2% in the long run, and the switch time arrives earlier. A stronger output-stabilization motive in the Taylor rule makes monetary policy lean more aggressively against the fiscal expansion: the impact multiplier falls to 0.94, and the economy experiences permanent output losses slightly earlier, after seven quarters. In contrast, shutting down heterogeneous business-cycle exposure through an equal earnings-incidence barely affects the results, suggesting that it is not a major driver of the stabilization trade-off.

To assess how sensitive the stabilization trade-off is to the strength of the innovation response to taxation, we change the degree of decreasing returns in the success probability through γ, while choosing η to match a 2% annual growth rate. With a low innovation-tax elasticity, $\varepsilon_{P,1-\tau} = 0.10$, which we interpret as the lower bound of the micro estimates in Akcigit et al. (2022) and Jones (2022), long-run losses are muted and the switch time is delayed, to eighteen quarters. By contrast, a higher elasticity, set to $\varepsilon_{P,1-\tau} = 0.50$, calibrated to the lower end of the macro estimates in Akcigit et al. (2022) and therefore incorporating mobility effects across U.S. states, substantially amplifies the long-run cost: output falls by 0.21%, and the switch time arrives after six quarters. We interpret this mobility-inclusive calibration as informative for small open economies, such as European countries, where innovators may be more able to relocate internationally in response to taxation.

In Part B of Table 3, we examine how the financing of the fiscal expansion shapes the results, mirroring our findings in Proposition 7. First, shutting down deficit financing lowers the output multiplier by weakening the demand expansion and, as a result, leads to slightly larger long-run losses. We then turn to the role of transitional tax adjustment by considering a larger increase in tax progressivity. This margin is quantitatively more consequential, particularly for the technology growth multiplier, which falls further. However, even when progressivity is held fixed, so that the entire adjustment operates through higher deficits and higher average taxes via the scaling factor ϑ_t, the economy still exhibits a short-run versus long-run trade-off, although a much weaker one. The next section examines this margin in greater detail by analyzing the welfare-maximizing tax incidence.

Finally, Part C of Table 3 varies the underlying model environment itself, providing a quantitative counterpart to the mechanisms isolated in Proposition 5. In a standard HANK model with exogenous technology growth, short-run output multipliers are higher than in the baseline HANK-G model, but long-run deviations disappear entirely and no switch time exists, since fiscal interventions have no lasting effect on technology growth. In a standard HA model without endogenous growth or nominal rigidities, the short-run output multiplier declines to 0.64, while long-run effects again vanish. As before, a switch time does not exist. By contrast, in the HA-G model, shutting down nominal rigidities weakens the demand response—lowering the impact multiplier to 0.56—while long-run innovation contracts sharply, with a multiplier of -0.22. The result is a sizable permanent long-run output contraction of about -0.22%, with losses occurring already after four quarters.

4.4 The Stabilization Trade-Off: The Role of Tax Incidence

We now examine the role of tax incidence more thoroughly. In Proposition 7, we have characterized the tax incidence that maximizes the output response at different horizons following a fiscal expansion. We follow the same logic here by studying how the output and technology-growth multipliers, as well as welfare, vary with tax incidence. Specifically, we allow the government to finance additional spending by allocating the tax burden to particular income quantiles, while holding fixed the deficit-financed share of the fiscal expansion.

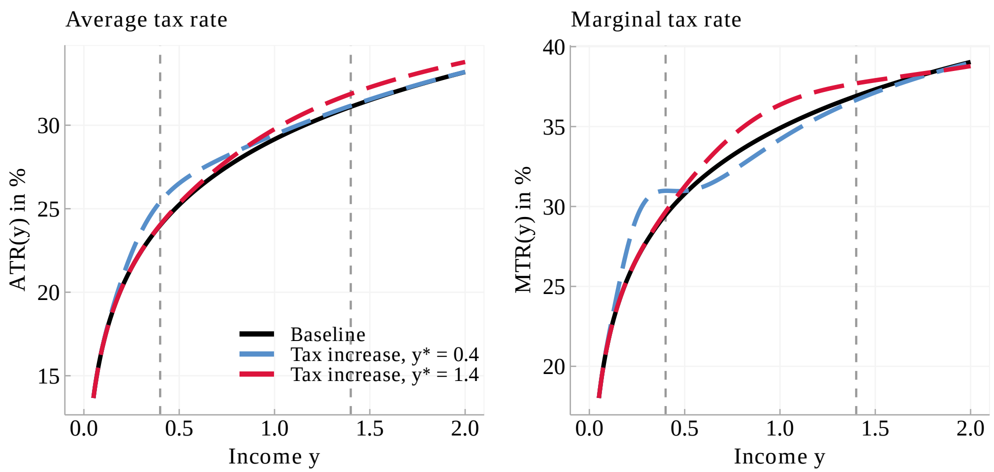
4.4.1 *A Flexible Tax-Incidence Schedule* Concretely, we implement a perturbation to the tax schedule of the following form:

$$\tau(y_{it}) = \exp\left(\log(\vartheta_t)\left(\frac{y_{it}}{y}\right)^{-2\varrho}[1-\theta_t\varphi(y_{it})]\right), \quad \varphi(y_{it}) = \exp\left(-\frac{\left(\ln y_{it} - \ln y^*\right)^2}{2\sigma^2}\right).$$

The function $\varphi(\cdot)$ creates a smooth, localized shift in the tax schedule, and allows us to concentrate the additional tax burden on specific parts of the income distribution. The time-varying scale parameter θ_t is set endogenously to raise a fixed amount of additional revenue. The location parameter y^* pins down where the extra tax burden falls—for example around a chosen income quantile—while σ controls how far the adjustment spreads. A larger σ spreads the increase in taxation to a broader range of households with corresponding income levels centered around y^*. Households sufficiently far from y^* experience essentially no change in their tax bill.

Figure 3 displays the effects of two localized tax adjustments used to finance an increase in government spending. The first panel plots the resulting average tax rate (ATR), and the second panel the corresponding marginal tax rate (MTR), which adjusts locally around each threshold to accommodate the perturbation. The two reforms concentrate the additional tax burden at different parts of the income distribution: one around $y^* = 0.4$ (roughly the 55th percentile) and the other around $y^* = 1.5$ (about the 95th percentile). Both use the same dispersion, $\sigma = 0.5$, illustrating how shifting the adjustment from middle- to high-income households changes the overall incidence of the reform.

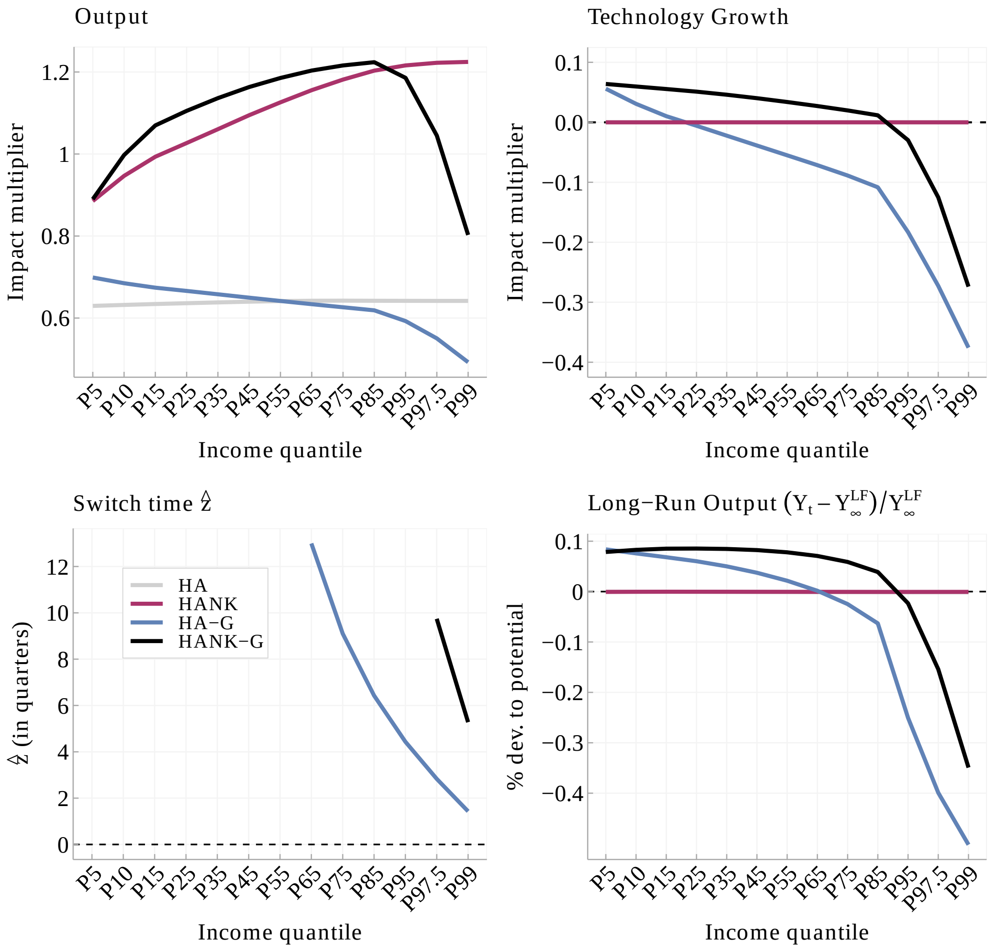
FIGURE 3. Local Perturbation of Tax Schedule



4.4.2 *Tax Incidence, Output, and Technology Growth* Equipped with the above tax-incidence toolkit, we now analyze a government that finances its spending shock by shifting the tax burden across the income distribution through adjustments in y^*. Specifically, we vary the affected portion of the distribution from the 5th to the 99th percentile and set $\sigma = 0.4$ throughout. The lower end of this range corresponds to financing through reductions in transfers to low-income households, whereas the upper end corresponds to a highly progressive adjustment concentrated among top earners.

Figure 4 gathers the results. The top panels report the annual multiplier of normalized output (y_t) and the technology growth rate (g^A_{t+1}). The bottom left panel reports the switch time $\hat{z}$, while the bottom right panel shows the long-run deviation of output from the laissez-faire, unshocked allocation. In each panel, we compare outcomes across four model environments—the baseline HANK-G economy, a standard HANK model with

FIGURE 4. Government Spending Shock and Heterogeneous Tax Incidence



Note: The top panels display the annual normalized output and technology growth multipliers following a 1% government spending shock as a function of the targeted income quantile. The bottom left panel shows the switching time $\hat{z}$, if it exist, while the bottom right panel shows the long-run deviation from the laissez faire economy, both as a function of the targeted income quantile.

exogenous growth, the HA-G economy without nominal rigidities, and an HA economy with exogenous growth but without nominal rigidities.

The results mirror those in Proposition 5 for the redistribution shock, but now emerge from a flexible tax-incidence design. We begin with the most intuitive environment: the HA economy. In this economy, the normalized output multiplier varies little with tax incidence—and any short-run effects vanish entirely within a decade. This stands in sharp contrast to the HANK economy, where the strength of the demand response is dictated by the MPC channel. In that environment, shifting the tax burden toward the top of the income distribution delivers the largest impact and cumulative multipliers. The picture changes drastically in the HA-G economy. With endogenous growth but no nominal rigidities, tax incidence matters for long-run outcomes: placing the tax burden on top earners generates the largest declines in technology growth. In this model, taxing higher-income households reduces innovative investment most strongly, generating a pronounced negative multiplier on technology growth, consistent with Jaimovich and Rebelo (2017) and Jones (2022). A small exception arises when the incidence falls only on the bottom of the distribution, in which case the expansion in market size temporarily dominates and generates a small positive response in technology growth. In the absence of a New Keynesian household MPC channel, shifting the tax burden toward the top always lowers the short-run output multiplier by weakening innovative investment.

Once both mechanisms operate jointly—nominal rigidities and endogenous growth—the HANK-G economy delivers a much richer pattern. As in the HANK economy, the short-run demand response is larger when the tax burden falls on the very top of the income distribution. But with endogenous technology growth, the same incidence generates the smallest technology-growth multiplier, which is negative. These two forces pull in opposite directions, producing a hump-shaped response of output. In this environment, taxing the upper middle class emerges as the incidence scheme that best balances the stabilization trade-off: it maximizes short-run fiscal multipliers without sacrificing long-run technology growth.

Finally, long-run output relative to the unshocked path declines as the tax burden moves up the income distribution in the HA-G economy. In HANK-G, by contrast, the short-run demand expansion makes the long-run response initially increase slightly, then flatten, and only eventually decline as the tax burden shifts toward higher-income households. Consistent with this pattern, a switch time emerges in HA-G for any tax incidence above the 60th percentile of income, whereas in HANK-G it appears only when the tax burden is concentrated within the top 5 percent.[30]

4.4.3 *Welfare and Decomposition* Because gains or losses in technology growth materialize only gradually, welfare measures may rank tax policies differently than output multipliers do. To account for this distinction, we now compute household welfare across tax-incidence patterns and in the baseline with progressive financing.

For this purpose, we compare at date 0 the full transition induced by the government spending shock with the initial stationary balanced-growth path without shock. Let $k=0$ denote the unshocked economy and $k=1$ the shocked economy, respectively. Because aggregate technology growth evolves according to $A_{t+1}^k = g_{t+1}^{A,k} A_t^k$, every temporary deviation in the growth rate $g_{t+1}^{A,k}$ has a permanent effect on the level of aggregate technology, output, and therefore on lifetime utility.

Let U_{it}^k denote household i's normalized period utility along path k, including unions' utility costs, and let $\mathcal{B}_{it}^k$ denote the cumulative discount factor net of growth but inclusive of survival probabilities. Aggregate utilitarian welfare along path k is

$$W^k = \int \mathbb{E}_0 \left[\sum_{t=0}^{\infty} Q_t^k \mathcal{B}_{it}^k U_{it}^k \right] dm_0(a_{i0}, b_{i0}, \mathbf{s}_{i0}) \,,$$

[30] The switch time at the 95th percentile tax incidence is relatively large and not visible in the plot.

where $Q_t^k \equiv \left(\prod_{s=0}^{t-1} g_{s+1}^{A,k}\right)^{1-\sigma}$, with $Q_0^k = 1$. To isolate the short-run component, we construct a counterfactual welfare measure that evaluates the allocation path of the shocked economy using the technology-growth rates of the unshocked economy:

$$\widetilde{W}^{1|0} = \int \mathbb{E}_0 \left[\sum_{t=0}^{\infty} Q_t^0 \mathcal{B}_{it}^1 U_{it}^1 \right] dm_0(a_{i0}, b_{i0}, \mathbf{s}_{i0}) \,.$$

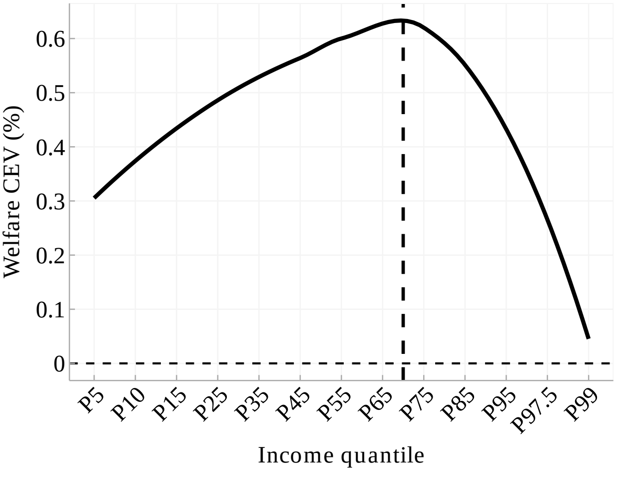
This measure removes the welfare effects arising from permanent changes in the level of technology. We then use consumption-equivalent variation (CEV) to measure both the total welfare effect and its short-run component. Given our utility function, this yields:

$$\Delta\text{CEV}^{\text{TOT}} = \left(1 + \frac{W^1 - W^0}{\mathcal{C}^0}\right)^{\frac{1}{1-\sigma}} - 1\,, \qquad \Delta\text{CEV}^{\text{SR}} = \left(1 + \frac{\widetilde{W}^{1|0} - W^0}{\mathcal{C}^0}\right)^{\frac{1}{1-\sigma}} - 1\,,$$

where $\mathcal{C}^0$ is the discounted utility weight of consumption along the unshocked path.[31]

Figure 5 reports total welfare as a function of the income quantile on which the tax burden falls. It shows that taxing the upper middle of the income distribution, around the 70th percentile, is the welfare-maximizing way to finance the fiscal expansion.

FIGURE 5. Welfare as a Function of Tax Incidence



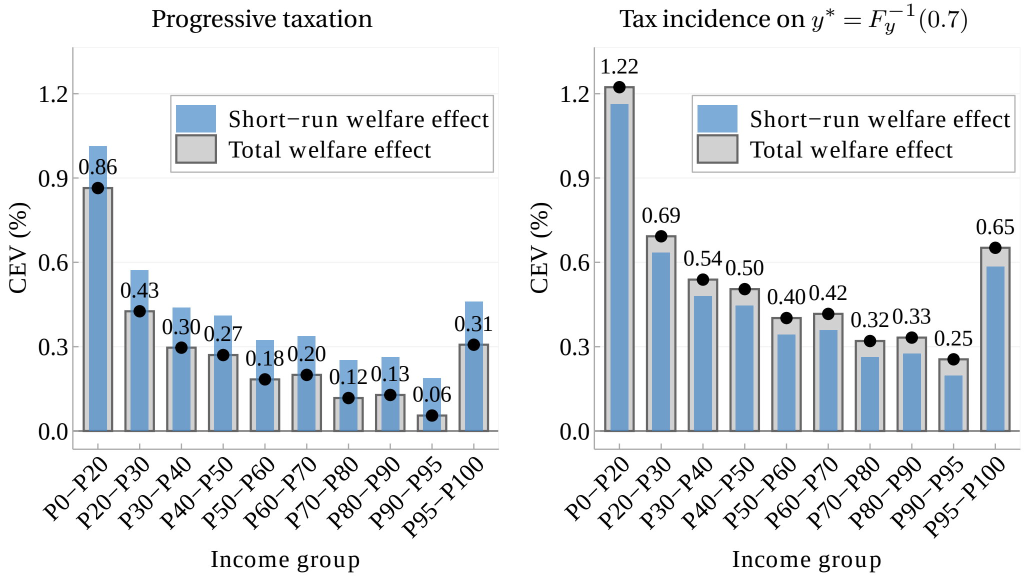
The last row of Part B in Table 3 reports the corresponding multipliers and long-run output deviation under the welfare-maximizing tax incidence: the output multiplier reaches 1.22, while the technology growth multiplier remains positive at 0.02, generating a long-run output gain of 0.06% following a persistent 1% increase in government spending at time $t = 0$. Quantitatively, this welfare-maximizing tax incidence delivers a consumption-equivalent welfare gain of 0.6% relative to the laissez-faire economy, compared with 0.37% under the baseline progressivity experiment. Put differently, shifting

[31] The full derivation of these welfare calculations is provided in Online Appendix D.5.

the financing burden toward the upper-middle class raises welfare gains by about 62% relative to the typical U.S. fiscal policy design with progressive financing.

Moreover, Figure 6 reports the decomposition of welfare across income groups for the baseline experiment with progressive financing and the welfare-maximizing tax incidence. Strikingly, in both cases, even the groups that bear the additional tax burden experience welfare gains from the fiscal expansion. However, relative to the case of progressive financing, the welfare-maximizing tax incidence benefits every individual across the entire state space. This implies that a tax incidence on the upper-middle class constitutes a Pareto improvement.

FIGURE 6. Welfare Gains and Decomposition along Income Distribution



Note: The left panel shows the welfare decomposition for the progressively financed fiscal expansion, while the right panel shows the decomposition for a tax incidence around the 70th percentile.

The decomposition into short-run and total welfare effects is also informative. In the case of a progressively financed fiscal expansion, long-run technology losses reduce overall welfare substantially. By contrast, under the welfare-maximizing tax incidence, technology growth remains positive, implying that the overall welfare effect exceeds the short-run effect. In other words, this tax incidence delivers the largest short-run output gains while also generating positive long-run welfare gains through higher technology growth. This is precisely why taxing the upper middle class is welfare-maximizing.

5. Conclusion

In this paper, we develop a unified theory of tax-financed fiscal expansions within a HANK-G framework that incorporates household heterogeneity, nominal rigidities, and endogenous technological growth. We use this framework to show that both their short- and long-run effects depend critically on the interaction between two central policy margins: systematic monetary policy and the incidence of taxation.

In the first part of the paper, we use a tractable version of the HANK-G framework to characterize three distinct stabilization regimes, each with sharply different effects on output and technology growth. When monetary policy is sufficiently dovish, a progressively financed fiscal expansion raises output in both the short and long run. When monetary policy is instead sufficiently hawkish, the same policy increases short-run output but induces permanent long-run output losses by suppressing innovative investment. Finally, when tax progressivity is weak, output declines in both the short and long run, regardless of the stance of systematic monetary policy. To evaluate the quantitative relevance of these three regimes, the second part of the paper uses an extended version of the HANK-G model to analyze a typical U.S. government spending shock financed through an increase in tax progressivity. Across various specifications, the regime featuring a trade-off between short-run output stabilization and long-run technology growth emerges as a robust outcome. When the additional tax burden is shifted toward upper-middle income households, this trade-off disappears: output rises, technology growth increases, and welfare increases for all household groups.

Overall, this paper provides a first step toward understanding the interaction of household heterogeneity, business cycles with nominal rigidities, and long-run technological growth. Our analysis points to several directions for future research. First, exploring the normative implications of the framework—particularly the optimal monetary-fiscal policy mix and the role of taxes and social insurance as automatic stabilizers—appears especially promising. Second, while the middle-class tax proposal is conceptually appealing, its political feasibility requires further investigation. Finally, further structural work is needed to identify the potentially opposing short- and long-run effects of fiscal stabilization policies. Taken together, these considerations highlight the scope for continued theoretical, quantitative, and empirical work within the HANK-G framework.

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ONLINE APPENDIX (NOT FOR PUBLICATION)

Inequality, Business Cycles, and Growth

A Unified Theory of Fiscal Stabilization Policies

The Online Appendix is organized as follows. Appendix A contains detailed derivations of the analytical HANK-G framework, Appendix B provides proofs, and Appendix C incorporates supplementary theoretical results. Finally, Appendix D contains a full description of the quantitative model environment, details the model calibration, shows impulse response functions to further variables, derives the welfare criterion and explains the computational algorithm.

APPENDIX A: Analytical HANK-G Model - Derivations

A.1 Competitive Equilibrium

In Appendix A.1, we characterize the first order conditions to the individual household, firm, and union maximization problems.

A.1.1 *Final and Intermediate Good Firms* The objective of the competitive final good producer is to maximize profits, i.e.,

$$\max_{\{L_t, X_t(j)\}} P_t L_t^{1-\alpha} \int_0^1 A_t(j)^{1-\alpha} X_t(j)^{\alpha} dj - P_t(j) X_t(j) - W_t L_t \,,$$

with corresponding first-order conditions:

$$W_t = (1-\alpha) P_t L_t^{-\alpha} \int_0^1 A_t(j)^{1-\alpha} X_t(j)^{\alpha} dj \,,$$

$$P_t(j) = \alpha P_t L_t^{1-\alpha} A_t(j)^{1-\alpha} X_t(j)^{\alpha-1} \,.$$

Using both conditions, the price-setting problem of an intermediate firm j becomes

$$\max_{\{P_t(j)\}} \Theta_t^n(j) = (P_t(j) - P_t) X_t^d(j) = (P_t(j) - P_t) \left(\frac{P_t(j)}{\alpha P_t} \right)^{\frac{1}{\alpha-1}} L_t A_t(j) \,.$$

The first order condition to this problem is

$$\left(\frac{P_t(j)}{\alpha P_t} \right)^{\frac{1}{\alpha-1}} + \frac{1}{\alpha-1} (P_t(j) - P_t) \left(\frac{P_t(j)}{\alpha P_t} \right)^{\frac{1}{\alpha-1}-1} \frac{1}{\alpha P_t} = 0 \,,$$

which can be rearranged to $\alpha P_t(j) = P_t$. As a result, one obtains $X_t(j) = \alpha^{\frac{2}{1-\alpha}} L_t A_t(j)$ and consequently $Y_t^G = \alpha^{\frac{2\alpha}{1-\alpha}} L_t A_t$. As such, nominal profits are $\Theta_t^n(j) = \Theta_\alpha P_t L_t A_t(j)$, where $\Theta_\alpha \equiv \alpha^{-1} (1-\alpha) \alpha^{\frac{2}{1-\alpha}}$. Finally, real profits are given by $\Theta_t(j) = \Theta_\alpha L_t A_t(j)$ and returns to innovation $R_t^A(j) = \Theta_t(j)/A_t(j) = \Theta_\alpha L_t = R_t^A$ for each j.

A.1.2 *Households* The first-order conditions for savers are

$$
\begin{aligned}
\mathcal{U}_c(C_t^S, L_t^S) &= \Lambda_t^S \,, \\
\Lambda_t^S &= \beta \mathbb{E}_t \left[(1-\lambda) s V_B^S(B_{t+1}^S, \omega_{t+1}^S, A_{t+1}^S) + \lambda(1-s) V_B^H(B_{t+1}^H) \right] + \Xi_t^S \,, \\
\Lambda_t^S \frac{q_t}{1-\lambda} &= \beta \mathbb{E}_t \left[V_\omega^S(B_{t+1}^S, \omega_{t+1}^S, A_{t+1}^S) \right] \,, \\
\Lambda_t^S &= \psi \Omega_t^S \,, \\
\Omega_t^S &= \beta \mathbb{E}_t \left[V_A^S(B_{t+1}^S, \omega_{t+1}^S, A_{t+1}^S) \right] \,,
\end{aligned}
$$

together with the complementary slackness condition $\Xi_t^S b_{t+1}^S = 0$, with $\Xi_t^S \geq 0$. Note that Λ_t^S, Ω_t^S and Ξ_t^S are Lagrange multipliers associated with the budget constraint, the LOM of capital, and the individual bond constraint. The Envelope theorem implies

$$
\begin{aligned}
V_B^S(B_t^S, \omega_t^S, A_t^S) &= \frac{\Lambda_t^S}{1-\lambda} \frac{R_{t-1}}{\pi_t} \,, \\
V_\omega^S(B_t^S, \omega_t^S, A_t^S) &= \frac{\Lambda_t^S}{1-\lambda} (q_t + D_t) \,, \\
V_A^S(B_t^S, \omega_t^S, A_t^S) &= \Lambda_t^S R_t^A + \Omega_t^S \,.
\end{aligned}
$$

Moreover, the first-order conditions for hand-to-mouth households are

$$
\begin{aligned}
\mathcal{U}_c(C_t^H, L_t^H) &= \Lambda_t^H \,, \\
\Lambda_t^H &= \beta \mathbb{E}_t \left[\lambda h V_B^H(B_{t+1}^H) + (1-\lambda)(1-h) V_B^S(B_{t+1}^S, \omega_{t+1}^S, A_{t+1}^S) \right] + \Xi_t^H \,,
\end{aligned}
$$

together with the complementary slackness condition $\Xi_t^H b_{t+1}^H = 0$, where $\Xi_t^H \geq 0$. The Envelope theorem, in turn, implies

$$
V_B^H(B_t^H) = \frac{\Lambda_t^H}{\lambda} \frac{R_{t-1}}{\pi_t} \,.
$$

Using the previous expressions, saver and hand-to-mouth FOC's can be rewritten as

$$
\begin{aligned}
\frac{1}{C_t^i} &= \Lambda_t^i \,, \quad \text{for} \quad i \in \{S, H\} \,, \\
\Lambda_t^H &= \beta \mathbb{E}_t \left[\frac{R_t}{\pi_{t+1}} \left(h \Lambda_{t+1}^H + (1-h) \Lambda_{t+1}^S \right) \right] + \Xi_t^H \,, \\
\Lambda_t^S &= \beta \mathbb{E}_t \left[\frac{R_t}{\pi_{t+1}} \left(s \Lambda_{t+1}^S + (1-s) \Lambda_{t+1}^H \right) \right] + \Xi_t^S \,, \\
\Lambda_t^S q_t &= \beta \mathbb{E}_t \left[\Lambda_{t+1}^S (q_{t+1} + D_{t+1}) \right] \\
\Lambda_t^S &= \psi \Omega_t^S \,, \\
\Omega_t^S &= \beta \mathbb{E}_t \left[\Lambda_{t+1}^S R_{t+1}^A + \Omega_{t+1}^S \right] \,.
\end{aligned}
$$

Assumption 1 implies $\Xi_t^H > 0$ and $\Xi_t^S = 0$. As a result, H households are strictly hand-to-mouth and consume their disposable income in every period, while the Euler equation of savers hods with equality, i.e.,

$$\frac{1}{C_t^S} = \beta \mathbb{E}_t \left[\frac{R_t}{\pi_{t+1}} \left(s \frac{1}{C_{t+1}^S} + (1-s) \frac{1}{C_{t+1}^H} \right) \right] .$$

Finally, the optimal innovative investment decision is given by

$$1 = \beta \mathbb{E}_t \left[\frac{C_t^S}{C_{t+1}^S} \left(1 + \psi R_{t+1}^A \right) \right] .$$

A.1.3 *Union Wage Setting* Differentiated labor inputs are bundled according to a CES aggregator that has the usual form

$$L_t = \left(\int_0^1 L_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}} dl \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} , \quad \text{with} \quad \epsilon_w > 1 .$$

The final good firm's cost minimization problem is

$$\min_{\{L_t(l)\}_l} \int_0^1 W_t(l) L_t(l) \qquad \text{subject to} \qquad L_t = \left(\int_0^1 L_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}} dl \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} .$$

Taking the first order conditions and following the standard steps provides us with the following labor demand for variety l

$$L_t(l) = \left(\frac{W_t(l)}{W_t} \right)^{-\epsilon_w} L_t^d ,$$

where the aggregate wage index is defined by

$$W_t \equiv \left(\int_0^1 W_t(l)^{1-\epsilon_w} dl \right)^{\frac{1}{1-\epsilon_w}} .$$

The maximization objective of union l is then given by

$$\max_{\{W_t(l)\}} \sum_{t=0}^{\infty} \beta^t \left[\log C_t(l) - \nu \frac{(L_t(l))^{1+\varphi}}{1+\varphi} \right]$$

subject to

$$L_t(l) = \left(\frac{W_t(l)}{W_t} \right)^{-\epsilon_w} L_t^d ,$$

$$C_t(l) + I_t(l) = \frac{W_t(l)}{P_t} L_t(l) + R_t^A A_t(l) + D_t(l) - T_t(l) - \frac{\theta}{2} \left(\frac{1}{g^A} \frac{W_t(l)}{W_{t-1}(l)} - 1 \right)^2 A_t ,$$

where we have imposed the equilibrium condition for shares, i.e., $\omega_t = 1\ \forall t$, respectively bonds $b^S_{t+1} = b^H_{t+1} = 0$. The nominal labor income of the average household is

$$W_t(l)L_t(l) = \frac{W_t(l)^{1-\epsilon_w}}{W_t^{-\epsilon_w}} L_t^d \, .$$

The first-order condition is

$$\frac{1}{C_t(l)}\left[\frac{(1-\epsilon_w)L_t(l)}{P_t} - \theta\left(\frac{1}{g^A}\frac{W_t(l)}{W_{t-1}(l)} - 1\right)\frac{1}{g^A}\frac{A_t}{W_{t-1}(l)}\right] + \nu\epsilon_w \frac{L_t(l)^{1+\varphi}}{W_t(l)}$$
$$+\beta\theta\mathbb{E}_t\left[\frac{1}{C_{t+1}(l)}\left(\frac{1}{g^A}\frac{W_{t+1}(l)}{W_t(l)} - 1\right)\frac{1}{g^A}\frac{W_{t+1}(l)}{(W_t(l))^2}A_{t+1}\right] = 0\, .$$

As unions face identical first-order conditions, we presuppose a symmetric equilibrium with $W_t(l) = W_t$ and, thus, $L_t = L_t(l)$. Multiplying through with W_t and defining gross nominal wage inflation as $\pi_t^w \equiv \frac{W_t}{W_{t-1}}$ provides us with

$$\left(\frac{\pi_t^w}{g^A} - 1\right)\frac{\pi_t^w}{g^A} - \beta\mathbb{E}_t\left[\frac{C_t}{C_{t+1}}\frac{A_{t+1}}{A_t}\left(\frac{\pi_{t+1}^w}{g^A} - 1\right)\frac{\pi_{t+1}^w}{g^A}\right] = \frac{\epsilon_w}{\theta}L_t\left(\nu L_t^{\varphi}\frac{C_t}{A_t} - \frac{\epsilon_w - 1}{\epsilon_w}\frac{W_t}{P_t A_t}\right)\, .$$

A.1.4 *Inflation Dynamics* By substituting the expression for $X_t(j)$ into the final good producer's first order condition for labor, we obtain

$$P_t = \frac{1}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}}\frac{W_t}{A_t}\, ,$$

such that price inflation, wage inflation, and technology growth are related as follows

$$\pi_t = \frac{\pi_t^w}{g_t^A}\, .$$

A.2 Stationary Equilibrium

After normalizing all trending variables by the level of aggregate technology, the stationary equilibrium is described by the following equations.

Saver Euler Equation	$\frac{g^A_{t+1}}{c^S_t} = \beta \mathbb{E}_t \left[\frac{R_t}{\pi_{t+1}} \left(s \frac{1}{c^S_{t+1}} + (1-s) \frac{1}{c^H_{t+1}} \right) \right]$
Growth Equation	$g^A_{t+1} = \beta \mathbb{E}_t \left[\frac{c^S_t}{c^S_{t+1}} \left(1 + \psi R^A_{t+1}\right) \right]$
Real Wage	$w_t = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}$
NK Wage Phillips Curve	$\left(\frac{\pi^w_t}{g^A} - 1\right) \frac{\pi^w_t}{g^A} - \beta \mathbb{E}_t \left[\frac{c_t}{c_{t+1}} \left(\frac{\pi^w_{t+1}}{g^A} - 1 \right) \frac{\pi^w_{t+1}}{g^A} \right] = \frac{\epsilon_w}{\theta} L_t \left(\nu L^\varphi_t c_t - \frac{\epsilon_w - 1}{\epsilon_w} w_t \right)$
Fiscal Transfers	$t^H_t = -t^S_t$
Individual Hours Worked	$L^H_t = (1-\mu)L + \mu L_t\,, \quad L^S_t = \frac{\lambda(\mu-1)}{1-\lambda} L + \frac{1-\lambda\mu}{1-\lambda} L_t$
Aggregate Hours Worked	$L_t = \lambda L^H_t + (1-\lambda) L^S_t$
HtM Consumption	$c^H_t = w_t L^H_t - \frac{t^H_t}{\lambda}$
Saver Consumption	$c^S_t = w_t L^S_t + \frac{R^A_t + d_t - \iota^A_t - t^S_t}{1-\lambda}$
Dividends	$d_t = 0$
Innovative Return	$R^A_t = \Theta_\alpha L_t$
Aggregate Consumption	$\lambda c^H_t + (1-\lambda) c^S_t = c_t$
LOM Technology Growth	$g^A_{t+1} = 1 + \psi \iota^A_t$
Monetary Policy Rule	$R_t = R \mathbb{E}_t \left[\frac{\pi_{t+1}}{\pi} \right] \left(\frac{Y_t/A_t}{Y/A} \right)^{\phi_y}$
Intermediary Inputs	$x_t = \alpha^{\frac{2}{1-\alpha}} L_t$
Gross Output	$y^G_t = \alpha^{\frac{2\alpha}{1-\alpha}} L_t$
Output	$y_t = Y_\alpha L_t$
Resource Constraint	$y_t = c_t + \iota^A_t + \frac{\theta}{2} \left(\frac{\pi^w_t}{g^A} - 1 \right)^2$
Inflation Dynamics	$\pi_t = \frac{\pi^w_t}{g^A_t}$

A.3 Steady State

We consider a full employment steady state ($L > 0$) with positive technology growth (see the proof of Result 1). We assume $\pi = 1$, which implies $\pi^w = g^A$. To begin with, we determine hours worked L using the following equations:

$$g^A = \beta\,(1 + \psi \Theta_\alpha L) \ ,$$
$$\nu L^\varphi c = \frac{\epsilon_w - 1}{\epsilon_w}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} \ ,$$
$$Y_\alpha L = c + \frac{g^A - 1}{\psi} \ .$$

As a result, hours worked are implicitly characterized by

$$\nu L^\varphi \left((Y_\alpha - \beta\Theta_\alpha)\,L + \frac{1-\beta}{\psi} \right) = \frac{\epsilon_w - 1}{\epsilon_w}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} \ . \tag{5}$$

To see whether there exists a unique $L > 0$ that satisfies (5), notice that

$$Y_\alpha - \beta\Theta_\alpha = \alpha^{\frac{2\alpha}{1-\alpha}}(1-\alpha^2)\left(1 - \beta\frac{\alpha}{1+\alpha}\right) \ , \tag{6}$$

which is strictly positive because of $\alpha \in (0,1)$ and $\beta < 1$. The left-hand side of (5) is, therefore, strictly increasing in L. Moreover, it is zero for $L = 0$. As a result, there exists a unique positive L that satisfies (5). Based on L, one can determine all remaining steady state values recursively. Steady-state hand-to-mouth and saver consumption, for instance, are given by

$$c^H = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L - \frac{t^H}{\lambda}$$
$$c^S = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L \left(1 + \alpha\frac{1-\beta}{1-\lambda}\right) + \frac{1-\beta}{(1-\lambda)\psi} - \frac{t^S}{1-\lambda} \ .$$

Steady-state hand-to-mouth and saver consumption are positive if

$$t^H < \bar{t}^H \equiv \lambda \cdot (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L > 0 \,,$$
$$t^S < \bar{t}^S \equiv (1-\lambda)\cdot(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L \left(1 + \alpha\frac{1-\beta}{1-\lambda}\right) + \frac{1-\beta}{\psi} > 0 \,.$$

There exists a lump-sum redistribution t^E from savers to hand-to-mouth households that equates their steady state consumption levels, i.e., $\Gamma_c = c^S/c^H = 1$. It is given by

$$t^E = \lambda\cdot(1-\beta)\left[\alpha(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L + \frac{1}{\psi}\right] < \bar{t}^S \ .$$

Finally, steady-state growth is positive, i.e., $g^A > 1$, if

$$\beta\,(1+\psi\Theta_\alpha L) > 1 \quad \Leftrightarrow \quad L > \frac{\beta^{-1} - 1}{\psi\Theta_\alpha} \ .$$

A.4 Log-Linear Stationary Equilibrium

Let us define the following auxiliary parameters:

$$\tilde{s} \equiv \frac{s}{s+(1-s)\Gamma_c}\,, \quad \mathscr{M} \equiv \frac{g^A-\beta}{g^A}\,, \quad \kappa \equiv \frac{\epsilon_w - 1}{\theta}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}L\,, \quad \gamma_c \equiv \frac{c}{y}\,.$$

Log-linearizing the conditions from Appendix A.2 around the steady state, we obtain[32]

Saver Euler Equation	$\hat{c}_t^S = -(\hat{i}_t - \mathbb{E}_t[\hat{\pi}_{t+1}]) + \tilde{s}\mathbb{E}_t\left[\hat{c}_{t+1}^S\right] + (1-\tilde{s})\mathbb{E}_t\left[\hat{c}_{t+1}^H\right] + \hat{g}_{t+1}^A$
Growth Equation	$\hat{g}_{t+1}^A = \hat{c}_t^S - \mathbb{E}_t\left[\hat{c}_{t+1}^S\right] + \mathscr{M}\mathbb{E}_t\left[\hat{R}_{t+1}^A\right]$
Real Wage	$\hat{w}_t = 0$
NK Wage Phillips Curve	$\hat{\pi}_t^w = \beta\mathbb{E}_t\left[\hat{\pi}_{t+1}^w\right] + \kappa\left(\varphi\hat{L}_t + \hat{c}_t\right)$
Fiscal Transfers	$\hat{t}_t^H = -\hat{t}_t^S = -\hat{\tau}_t$
Individual Hours Worked	$\hat{L}_t^H = \mu\hat{L}_t\,, \quad \hat{L}_t^S = \frac{1-\lambda\mu}{1-\lambda}\hat{L}_t$
Aggregate Hours Worked	$\hat{L}_t = \lambda\hat{L}_t^H + (1-\lambda)\hat{L}_t^S$
Hand-to-Mouth Consumption	$\hat{c}_t^H = \frac{wL}{c^H}\left(\hat{w}_t + \hat{L}_t^H\right) - \frac{1}{\lambda}\frac{t^H}{c^H}\hat{t}_t^H$
Saver Consumption	$\hat{c}_t^S = \frac{wL}{c^S}\left(\hat{w}_t + \hat{L}_t^S\right) + \frac{1}{1-\lambda}\left(\frac{R^A}{c^S}\hat{R}_t^A + \frac{y}{c^S}\hat{d}_t - \frac{\iota^A}{c^S}\hat{\iota}_t^A - \frac{t^S}{c^S}\hat{t}_t^S\right)$
Dividends	$\hat{d}_t = 0$
Innovative Return	$\hat{R}_t^A = \hat{L}_t$
Aggregate Consumption	$\lambda c^H\hat{c}_t^H + (1-\lambda)c^S\hat{c}_t^S = c\hat{c}_t$
LOM Technology Growth	$\hat{g}_{t+1}^A = \frac{g^A-1}{g^A}\hat{\iota}_t^A$
Monetary Policy Rule	$\hat{i}_t = \mathbb{E}_t[\hat{\pi}_{t+1}] + \phi_y\hat{y}_t$
Intermediary Inputs	$\hat{x}_t = \hat{L}_t$
Gross Output / Output	$\hat{y}_t^G = \hat{y}_t = \hat{L}_t$
Resource Constraint	$\hat{y}_t = \gamma_c\hat{c}_t + (1-\gamma_c)\hat{\iota}_t^A$
Inflation Dynamics	$\hat{\pi}_t = \hat{\pi}_t^w - \hat{g}_t^A$

[32] We express variables as log deviations from steady state, i.e., $\hat{x}_t = \log x_t - \log x$. We log-linearize the gross rates, which are approximately equal to the net rates. One exception are dividends, which equal zero. We express them as absolute deviations from steady state, relative to steady state output: $\hat{d}_t = (d_t - d)/y$.

APPENDIX B: Analytical Model - Proofs

B.1 Balanced Growth Path - Steady State

B.1.1 *Proof Result 1*

PROOF. The first part is similar to the proof of Proposition 1 in Fornaro and Wolf (2023). A necessary condition for individual saver and hand-to-mouth consumption to be positive is that aggregate consumption is positive. This is the case if

$$c = (Y_\alpha - \beta\Theta_\alpha)\, L + \frac{1-\beta}{\psi} > 0\,,$$

which holds true as $\beta\frac{\alpha}{1+\alpha} < 1$. From Appendix A.3, we know that there exists a positive employment steady state $L > 0$ satisfying

$$\nu L^{\varphi}\left((Y_\alpha - \beta\Theta_\alpha)\, L + \frac{1-\beta}{\psi}\right) = \frac{\epsilon_w - 1}{\epsilon_w}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\,, \tag{7}$$

as the left-hand side is strictly increasing in L and takes a zero value for $L = 0$. Provided that $t^H < \bar{t}^H$ and $t^S < \bar{t}^S$ steady state consumption of saver and hand-to-mouth households is positive. Additionally, a positive steady-state growth is guaranteed by

$$L > \frac{\beta^{-1} - 1}{\psi\Theta_\alpha} > 0\,, \tag{8}$$

which is the condition stated in the Proposition. As the left hand side of (7) strictly increases in ν, one can write $L(\nu)$ with $L'(\nu) < 0$. As $\lim_{\nu\to 0} L(\nu) = \infty$ and $\lim_{\nu\to\infty} L(\nu) = 0$, there exists by continuity a unique ν^* such that (8) is satisfied for all $\nu < \nu^*$.[33] □

B.2 Inequality On and Off the Balanced Growth Path

B.2.1 *Inequality on the BGP* To link consumption and income inequality, we use the resource constraint

$$y = \lambda y^H + (1-\lambda)y^S = \lambda c^H + (1-\lambda)c^S + \frac{g^A - 1}{\psi}\,.$$

Rearranging the previous equation by using $c^H = y^H$, we obtain

$$y^S = c^S + \frac{1}{1-\lambda}\frac{g^A-1}{\psi}$$

$$\Leftrightarrow \Gamma_y = \frac{c^S}{y^H} + \frac{1}{1-\lambda}\frac{g^A-1}{\psi}\frac{1}{y^H} = \Gamma_c + \frac{1}{1-\lambda}\frac{g^A-1}{\psi y}\frac{y}{y^H} = \Gamma_c + \frac{1-\gamma_c}{1-\lambda}\frac{y}{y^H} = \Gamma_c + \frac{1-\gamma_c}{1-\lambda}\left(\lambda + (1-\lambda)\Gamma_y\right)$$

$$\Leftrightarrow \Gamma_y = \frac{\Gamma_c}{\gamma_c} + \frac{\lambda}{1-\lambda}\frac{1-\gamma_c}{\gamma_c}\,.$$

As a result, this implies $\Gamma_y > \Gamma_c$ if $\gamma_c < 1$.

[33] The wage NKPC also admits a steady state with $L = 0$ that leads, however, to negative technology growth $g^A < 1$ and positive hand-to-mouth consumption only if $t^H < 0$. It is therefore ruled out, i.e., the positive employment steady state is the unique steady state consistent with positive technology growth.

B.2.2 *Cyclical Hand-to-Mouth Consumption* Hand-to-mouth consumption writes

$$c_t^H = w_t L_t^H - \frac{t_t^H}{\lambda} = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L_t^H + \frac{T}{\lambda} e^{\hat{\tau}_t} \; .$$

Log-linearization provides us with

$$\hat{c}_t^H = \frac{1}{c^H}\left(wL^H \hat{L}_t^H + \frac{T}{\lambda}\hat{\tau}_t \right) = \frac{wL^H}{c^H}\mu \hat{L}_t + \frac{1}{c^H}\frac{T}{\lambda}\hat{\tau}_t = \eta\mu\hat{y}_t + (1-\eta)\hat{\tau}_t \; ,$$

where we have defined the auxiliary parameter η by

$$\eta \equiv \frac{wL^H}{c^H} = \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L + \frac{T}{\lambda}} \; ,$$

where $\eta \in (0,1)$ if $T > 0$.

B.2.3 *Cyclical Income Inequality* The aggregate income identity is given by

$$\lambda y_t^H + (1-\lambda) y_t^S = y_t \; ,$$

which can be written in log-linear terms as

$$\hat{y}_t^S = \frac{y\hat{y}_t - \lambda y^H \hat{y}_t^H}{(1-\lambda)y^S} \; .$$

Subtracting hand-to-mouth income results in

$$\hat{y}_t^S - \hat{y}_t^H = \frac{y\hat{y}_t - (\lambda y^H + (1-\lambda)y^S)\hat{y}_t^H}{(1-\lambda)y^S} = \frac{1}{1-\lambda}\frac{y}{y^S}\left(\hat{y}_t - \hat{y}_t^H\right) \; .$$

Finally, substituting for $\hat{y}_t^H = \eta\mu\hat{y}_t + (1-\eta)\hat{\tau}_t$ yields

$$\hat{y}_t^S - \hat{y}_t^H = \frac{1}{1-\lambda}\frac{y}{y^S}\left((1-\eta\mu)\hat{y}_t - (1-\eta)\hat{\tau}_t\right) \; ,$$

which corresponds to the expression stated in the main text.

B.3 Four-Equation Representation

B.3.1 *Proof Proposition 1*

PROOF. We proceed in two steps: first, we compute consumption of savers $\hat{c}_t^S$ as a function of output $\hat{y}_t$ and technology growth $\hat{g}_{t+1}^A$. Second, we substitute consumption of hand-to-mouth and saver households into the Euler equation. To begin with, we have

$$\hat{c}_t^S = \frac{c\hat{c}_t - \lambda c^H \hat{c}_t^H}{(1-\lambda)c^S} = \left(1 + \frac{\lambda}{1-\lambda}\frac{1}{\Gamma_c}\right)\hat{c}_t - \frac{\lambda}{1-\lambda}\frac{1}{\Gamma_c}\hat{c}_t^H \; .$$

The aggregate resource constraint implies $\hat{y}_t = \gamma_c \hat{c}_t + (1-\gamma_c)\frac{g^A}{g^A-1}\hat{g}^A_{t+1}$. Substituting in for $\hat{c}_t$ and $\hat{c}^H_t$, we obtain

$$\begin{aligned}\hat{c}^S_t &= \left(1+\frac{\lambda}{1-\lambda}\frac{1}{\Gamma_c}\right)\left(\frac{\hat{y}_t}{\gamma_c}-\frac{1-\gamma_c}{\gamma_c}\frac{g^A}{g^A-1}\hat{g}^A_{t+1}\right)-\frac{\lambda}{1-\lambda}\frac{1}{\Gamma_c}\left(\eta\mu\hat{y}_t+(1-\eta)\hat{\tau}_t\right)\\ &= \frac{(1-\lambda)\Gamma_c+\lambda-\lambda\gamma_c\eta\mu}{(1-\lambda)\Gamma_c\gamma_c}\hat{y}_t-\frac{(1-\lambda)\Gamma_c+\lambda}{(1-\lambda)\Gamma_c}\frac{1-\gamma_c}{\gamma_c}\frac{g^A}{g^A-1}\hat{g}^A_{t+1}-\frac{\lambda(1-\eta)}{(1-\lambda)\Gamma_c}\hat{\tau}_t\\ &= \mathcal{E}_y\hat{y}_t-\mathcal{E}_g\hat{g}^A_{t+1}-\mathcal{E}_\tau\hat{\tau}_t\ ,\end{aligned}$$

where $\mathcal{E}_y, \mathcal{E}_g$ and $\mathcal{E}_\tau$ denote partial equilibrium elasticities of $\hat{c}^S_t$ with respect to aggregate income, endogenous technology growth and transfers, i.e.,

$$\mathcal{E}_y \equiv \frac{(1-\lambda)\Gamma_c+\lambda-\lambda\gamma_c\chi}{(1-\lambda)\Gamma_c\gamma_c}\ ,\quad \mathcal{E}_g \equiv \frac{(1-\lambda)\Gamma_c+\lambda}{(1-\lambda)\Gamma_c}\frac{1-\gamma_c}{\gamma_c}\frac{g^A}{g^A-1}\ ,\quad \mathcal{E}_\tau \equiv \frac{\lambda(1-\eta)}{(1-\lambda)\Gamma_c}\ .$$

The saver Euler equation is given by

$$\hat{c}^S_t = \tilde{s}\mathbb{E}_t\left[\hat{c}^S_{t+1}\right]+(1-\tilde{s})\mathbb{E}_t\left[\hat{c}^H_{t+1}\right]-\left(\hat{i}_t-\mathbb{E}_t\left[\hat{\pi}_{t+1}\right]\right)+\hat{g}^A_{t+1}\ .$$

Substituting in for the consumption of saver and hand-to-mouth households results in

$$\begin{aligned}\mathcal{E}_y\hat{y}_t-\mathcal{E}_g\hat{g}^A_{t+1}-\mathcal{E}_\tau\hat{\tau}_t = \tilde{s}\mathbb{E}_t\left[\mathcal{E}_y\hat{y}_{t+1}-\mathcal{E}_g\hat{g}^A_{t+2}-\mathcal{E}_\tau\hat{\tau}_{t+1}\right]+(1-\tilde{s})\mathbb{E}_t\left[\chi\hat{y}_{t+1}+(1-\eta)\hat{\tau}_{t+1}\right]\\ -\left(\hat{i}_t-\mathbb{E}_t\left[\hat{\pi}_{t+1}\right]\right)+\hat{g}^A_{t+1}\ ,\end{aligned}$$

which can be rearranged to

$$\begin{aligned}\hat{y}_t = \left(\tilde{s}+(1-\tilde{s})\frac{\chi}{\mathcal{E}_y}\right)\mathbb{E}_t\left[\hat{y}_{t+1}\right]-\frac{1}{\mathcal{E}_y}\left(\hat{i}_t-\mathbb{E}_t\left[\hat{\pi}_{t+1}\right]\right)+\frac{1+\mathcal{E}_g}{\mathcal{E}_y}\hat{g}^A_{t+1}-\tilde{s}\frac{\mathcal{E}_g}{\mathcal{E}_y}\mathbb{E}_t\left[\hat{g}^A_{t+2}\right]\\ +\frac{\mathcal{E}_\tau}{\mathcal{E}_y}\hat{\tau}_t-\frac{\mathcal{E}_\tau}{\mathcal{E}_y}\left(\tilde{s}-(1-\tilde{s})\frac{1-\eta}{\mathcal{E}_\tau}\right)\mathbb{E}_t\left[\hat{\tau}_{t+1}\right]\ .\end{aligned}$$

The former equation can be restated as

$$\hat{y}_t = \zeta_f\mathbb{E}_t\left[\hat{y}_{t+1}\right]-\zeta_r\left(\hat{i}_t-\mathbb{E}_t\left[\hat{\pi}_{t+1}\right]\right)+\zeta_g\hat{g}^A_{t+1}-\zeta_{g'}\mathbb{E}_t\left[\hat{g}^A_{t+2}\right]+\zeta_\tau\hat{\tau}_t-\zeta_{\tau'}\mathbb{E}_t\left[\hat{\tau}_{t+1}\right]\ ,$$

where the auxiliary parameters are defined as follows:

$$\begin{aligned}\zeta_f \equiv& 1+(1-\tilde{s})\left(\frac{\chi}{\mathcal{E}_y}-1\right)=1+(1-\tilde{s})\frac{(1-\lambda)\Gamma_c\gamma_c\chi-((1-\lambda)\Gamma_c+\lambda-\lambda\gamma_c\chi)}{(1-\lambda)\Gamma_c+\lambda-\lambda\gamma_c\chi}\\ =& 1+(1-\tilde{s})\frac{(1-\lambda)\Gamma_c+\lambda}{(1-\lambda)\Gamma_c+\lambda-\lambda\gamma_c\chi}(\gamma_c\chi-1)=1+(1-\tilde{s})\frac{\gamma_c\chi-1}{1-\lambda\chi y^H/y}\ ,\end{aligned}$$

with the last equality is due to $\gamma_c = \left((1-\lambda)\Gamma_c+\lambda\right)\frac{y^H}{y}$.

$$\zeta_r \equiv \frac{1}{\mathcal{E}_y} = \frac{(1-\lambda)\Gamma_c\gamma_c}{(1-\lambda)\Gamma_c+\lambda-\lambda\gamma_c\chi}\ ,$$

$$\zeta_g \equiv \frac{1+\mathcal{E}_g}{\mathcal{E}_y} = \frac{\dfrac{(1-\lambda)\Gamma_c\gamma_c(g^A-1)}{(1-\lambda)\Gamma_c\gamma_c(g^A-1)} + \dfrac{(1-\lambda)\Gamma_c+\lambda}{(1-\lambda)\Gamma_c}\dfrac{1-\gamma_c}{\gamma_c}\dfrac{g^A}{g^A-1}}{\dfrac{(1-\lambda)\Gamma_c+\lambda-\lambda\gamma_c\chi}{(1-\lambda)\Gamma_c\gamma_c}}$$

$$= \frac{(1-\lambda)\Gamma_c\gamma_c(g^A-1) + ((1-\lambda)\Gamma_c+\lambda)(1-\gamma_c)g^A}{(g^A-1)\left[(1-\lambda)\Gamma_c+\lambda-\lambda\gamma_c\chi\right]} = \frac{\dfrac{(1-\lambda)\Gamma_c\gamma_c}{(1-\lambda)\Gamma_c+\lambda} + \dfrac{(1-\gamma_c)g^A}{g^A-1}}{1-\lambda\chi y^H/y}$$

$$= \frac{(1-\lambda)c^S/y + g^A/(\psi y)}{1-\lambda\chi y^H/y},$$

where the last equality follows from $1-\gamma_c = \frac{g^A-1}{\psi y}$.

$$\zeta_{g'} \equiv \tilde{s}\frac{\mathcal{E}_g}{\mathcal{E}_y} = \tilde{s}\frac{g^A}{g^A-1}\frac{((1-\lambda)\Gamma_c+\lambda)(1-\gamma_c)}{(1-\lambda)\Gamma_c+\lambda-\lambda\gamma_c\chi} = \tilde{s}\frac{g^A}{\psi y}\frac{1}{1-\lambda\chi y^H/y},$$

$$\zeta_\tau = \frac{\mathcal{E}_\tau}{\mathcal{E}_y} = \frac{\lambda\gamma_c(1-\eta)}{(1-\lambda)\Gamma_c+\lambda-\lambda\gamma_c\chi},$$

$$\zeta_{\tau'} = \frac{\mathcal{E}_\tau}{\mathcal{E}_y}\left(\tilde{s}-(1-\tilde{s})\frac{1-\eta}{\mathcal{E}_\tau}\right) = \frac{\lambda\gamma_c(1-\eta)}{(1-\lambda)\Gamma_c+\lambda-\lambda\gamma_c\chi}\left(\tilde{s}-(1-\tilde{s})\frac{(1-\lambda)\Gamma_c}{\lambda}\right).$$

The expressions stated in the main text follow directly from setting $\Gamma_c = 1$. This concludes the proof of Proposition 1. □

B.3.2 *Proof Proposition 2*

PROOF. From Appendix A.4 the endogenous technology growth equation is given by

$$\hat{g}^A_{t+1} = \hat{c}^S_t - \mathbb{E}_t\left[\hat{c}^S_{t+1}\right] + \mathscr{M}\mathbb{E}_t\left[\hat{R}^A_{t+1}\right].$$

Substituting in for cyclical saver consumption, $\hat{c}^S_t$, we obtain

$$\hat{g}^A_{t+1} = \mathcal{E}_y\hat{y}_t - \mathcal{E}_g\hat{g}^A_{t+1} - \mathcal{E}_\tau\hat{\tau}_t - \mathbb{E}_t\left[\mathcal{E}_y\hat{y}_{t+1} - \mathcal{E}_g\hat{g}^A_{t+2} - \mathcal{E}_\tau\hat{\tau}_{t+1}\right] + \mathscr{M}\mathbb{E}_t\left[\hat{y}_{t+1}\right],$$

where we have used that $\hat{R}^A_t = \hat{L}_t = \hat{y}_t$. Collecting terms provides us with

$$\hat{g}^A_{t+1} = \xi_y\hat{y}_t + \xi_{y'}\mathbb{E}_t\left[\hat{y}_{t+1}\right] + \xi_{g'}\mathbb{E}_t\left[\hat{g}_{t+2}\right] - \xi_\tau\left(\hat{\tau}_t - \mathbb{E}_t\left[\hat{\tau}_{t+1}\right]\right),$$

where we have the following auxiliary variables

$$\xi_y \equiv \frac{\mathcal{E}_y}{1+\mathcal{E}_g}, \quad \xi_{y'} \equiv \frac{\mathscr{M}-\mathcal{E}_y}{1+\mathcal{E}_g}, \quad \xi_{g'} \equiv \frac{\mathcal{E}_g}{1+\mathcal{E}_g}, \quad \xi_\tau \equiv \frac{\mathcal{E}_\tau}{1+\mathcal{E}_g}.$$

Substituting for the expressions of $\mathcal{E}_y, \mathcal{E}_g, \mathcal{E}_\tau$, and $\mathscr{M}$ and setting $\Gamma_c = 1$ provides us with the expressions in the main text. This concludes the proof of Proposition 2. □

B.3.3 *Proof Proposition 3*

PROOF. From Appendix A.4, the log-linear NKPC is given by

$$\hat{\pi}_t^w = \beta \mathbb{E}_t \left[\hat{\pi}_{t+1}^w\right] + \kappa \left(\varphi \hat{L}_t + \hat{c}_t\right) .$$

Substituting in for $\hat{L}_t$ and $\hat{c}_t$ results in

$$\begin{aligned}
\hat{\pi}_t^w &= \beta \mathbb{E}_t \left[\hat{\pi}_{t+1}^w\right] + \kappa \left(\varphi \hat{y}_t + \frac{\hat{y}_t - (1-\gamma_c)\dfrac{g^A}{g^A - 1}\hat{g}_{t+1}^A}{\gamma_c}\right) \\
&= \beta \mathbb{E}_t \left[\hat{\pi}_{t+1}^w\right] + \kappa \frac{1+\varphi\gamma_c}{\gamma_c}\hat{y}_t - \kappa \frac{1-\gamma_c}{\gamma_c}\frac{g^A}{g^A-1}\hat{g}_{t+1}^A .
\end{aligned}$$

As $\hat{\pi}_t = \hat{\pi}_t^w - \hat{g}_t^A$, one finally obtains

$$\begin{aligned}
\hat{\pi}_t &= \beta \mathbb{E}_t \left[\hat{\pi}_{t+1}\right] + \kappa \frac{1+\varphi\gamma_c}{\gamma_c}\hat{y}_t + \left(\beta - \kappa \frac{1-\gamma_c}{\gamma_c}\frac{g^A}{g^A-1}\right)\hat{g}_{t+1}^A - \hat{g}_t^A \\
&= \beta \mathbb{E}_t \left[\hat{\pi}_{t+1}\right] + \kappa_y \hat{y}_t + \kappa_g \hat{g}_{t+1}^A - \hat{g}_t^A ,
\end{aligned}$$

where the auxiliary parameters are defined as follows

$$\kappa_y \equiv \kappa \frac{1+\varphi\gamma_c}{\gamma_c} , \qquad \text{and} \qquad \kappa_g \equiv \beta - \kappa \frac{1-\gamma_c}{\gamma_c}\frac{g^A}{g^A-1} .$$

□

B.4 Determinacy

B.4.1 *Proof Proposition 4*

PROOF. To derive conditions under which the model has a determinate, locally unique rational expectations equilibrium, one can neglect all exogenous shocks. Based on Propositions 1-3, the dynamic system can thus be rewritten as

$$\begin{aligned}
\hat{y}_t &= \zeta_f \mathbb{E}_t \left[\hat{y}_{t+1}\right] - \zeta_r \hat{r}_t + \zeta_g \hat{g}_{t+1}^A - \zeta_{g'} \mathbb{E}_t \left[\hat{g}_{t+2}^A\right] , \\
\hat{g}_{t+1}^A &= \Phi_y \hat{y}_t + \Phi_{y_+} \mathbb{E}_t \left[\hat{y}_{t+1}\right] + \Phi_{g_+} \mathbb{E}_t \left[\hat{g}_{t+2}^A\right] , \\
\hat{r}_t &= \phi_y \hat{y}_t .
\end{aligned}$$

We proceed in three steps. First, we derive a reduced form system for the forward-looking variables $\hat{y}_t$ and $\hat{g}_{t+1}^A$. Second, we determine signs of the eigenvalue conditions. Third, we state conditions on ϕ_y ensuring that both eigenvalues lie inside the unit circle.

Step 1: Derivation Reduced Form System.

Substituting the real interest rate rule into the aggregate IS equation, we obtain

$$\hat{y}_t = \zeta_f \mathbb{E}_t\left[\hat{y}_{t+1}\right] - \phi_y \zeta_r \hat{y}_t + \zeta_g \hat{g}^A_{t+1} - \zeta_{g'} \mathbb{E}_t\left[\hat{g}^A_{t+2}\right]$$
$$\Leftrightarrow \hat{y}_t = \eta_0 \mathbb{E}_t\left[\hat{y}_{t+1}\right] + \eta_1 \hat{g}^A_{t+1} - \eta_2 \mathbb{E}_t\left[\hat{g}^A_{t+2}\right] ,$$

where we have defined the following auxiliary parameters

$$\eta_0 = \frac{\zeta_f}{1+\phi_y\zeta_r} , \quad \eta_1 = \frac{\zeta_g}{1+\phi_y\zeta_r} , \quad \eta_2 = \frac{\zeta_{g'}}{1+\phi_y\zeta_r} .$$

Substituting the former equation into the endogenous growth equation, one obtains

$$\hat{g}^A_{t+1} = \Phi_y\left(\eta_0 \mathbb{E}_t\left[\hat{y}_{t+1}\right] + \eta_1 \hat{g}^A_{t+1} - \eta_2 \mathbb{E}_t\left[\hat{g}^A_{t+2}\right]\right) + \Phi_{y+} \mathbb{E}_t\left[\hat{y}_{t+1}\right] + \Phi_{g+} \mathbb{E}_t\left[\hat{g}^A_{t+2}\right]$$
$$\Leftrightarrow \hat{g}^A_{t+1} = \frac{\eta_0\Phi_y + \Phi_{y+}}{1-\eta_1\Phi_y} \mathbb{E}_t\left[\hat{y}_{t+1}\right] + \frac{\Phi_{g+} - \eta_2\Phi_y}{1-\eta_1\Phi_y} \mathbb{E}_t\left[\hat{g}^A_{t+2}\right] .$$

Substituting this expression back into the output equation, we obtain

$$\hat{y}_t = \eta_0 \mathbb{E}_t\left[\hat{y}_{t+1}\right] + \eta_1\left(\frac{\eta_0\Phi_y + \Phi_{y+}}{1-\eta_1\Phi_y} \mathbb{E}_t\left[\hat{y}_{t+1}\right] + \frac{\Phi_{g+} - \eta_2\Phi_y}{1-\eta_1\Phi_y} \mathbb{E}_t\left[\hat{g}^A_{t+2}\right]\right) - \eta_2 \mathbb{E}_t\left[\hat{g}^A_{t+2}\right]$$
$$= \left(\frac{\eta_0 - \eta_0\eta_1\Phi_y + \eta_0\eta_1\Phi_y + \eta_1\Phi_{y+}}{1-\eta_1\Phi_y}\right) \mathbb{E}_t\left[\hat{y}_{t+1}\right] + \left(\frac{\eta_1\Phi_{g+} - \eta_1\eta_2\Phi_y - \eta_2 + \eta_1\eta_2\Phi_y}{1-\eta_1\Phi_y}\right) \mathbb{E}_t\left[\hat{g}^A_{t+2}\right]$$
$$= \frac{\eta_0 + \eta_1\Phi_{y+}}{1-\eta_1\Phi_y} \mathbb{E}_t\left[\hat{y}_{t+1}\right] + \frac{\eta_1\Phi_{g+} - \eta_2}{1-\eta_1\Phi_y} \mathbb{E}_t\left[\hat{g}^A_{t+2}\right] .$$

As a result, the recursive system of equations is purely forward-looking and can be written in a more compact form as

$$\begin{pmatrix} \hat{y}_t \\ \hat{g}^A_{t+1} \end{pmatrix} = \mathbf{A}_T \begin{pmatrix} \hat{y}_{t+1} \\ \hat{g}^A_{t+2} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \mathbb{E}_t \begin{pmatrix} \hat{y}_{t+1} \\ \hat{g}^A_{t+2} \end{pmatrix} ,$$

where the auxiliary statistics are given by

$$A_{11} = \frac{\eta_0 + \eta_1\Phi_{y+}}{1-\eta_1\Phi_y} , \quad A_{12} = \frac{\eta_1\Phi_{g+} - \eta_2}{1-\eta_1\Phi_y} , \quad A_{21} = \frac{\eta_0\Phi_y + \Phi_{y+}}{1-\eta_1\Phi_y} , \quad A_{22} = \frac{\Phi_{g+} - \eta_2\Phi_y}{1-\eta_1\Phi_y} .$$

Step 2: Sign Eigenvalue Conditions.

The dynamic system is determinate if and only if both eigenvalues of $\mathbf{A}_T$ lie inside the unit circle. Following Bullard and Mitra (2002) and LaSalle (1986), this applies if

$$|A_{11}A_{22} - A_{12}A_{21}| < 1 , \quad |A_{11} + A_{22}| < 1 + A_{11}A_{22} - A_{12}A_{21} .$$

To make progress, we first rearrange $A_{11}A_{22} - A_{12}A_{21}$ to

$$\begin{aligned}
A_{11}A_{22} - A_{12}A_{21} &= \frac{(\eta_0 + \eta_1\Phi_{y+})(\Phi_{g+} - \eta_2\Phi_y) - (\eta_1\Phi_{g+} - \eta_2)(\eta_0\Phi_y + \Phi_{y+})}{(1-\eta_1\Phi_y)^2} \\
&= \frac{\eta_0\Phi_{g+} - \eta_0\eta_2\Phi_y + \eta_1\Phi_{y+}\Phi_{g+} - \eta_1\eta_2\Phi_y\Phi_{y+} - \eta_0\eta_1\Phi_y\Phi_{g+} - \eta_1\Phi_{y+}\Phi_{g+} + \eta_0\eta_2\Phi_y + \eta_2\Phi_{y+}}{(1-\eta_1\Phi_y)^2} \\
&= \frac{\eta_0\Phi_{g+} - \eta_1\eta_2\Phi_y\Phi_{y+} - \eta_0\eta_1\Phi_y\Phi_{g+} + \eta_2\Phi_{y+}}{(1-\eta_1\Phi_y)^2} \\
&= \frac{\eta_0\Phi_{g+}(1-\eta_1\Phi_y) + \eta_2\Phi_{y+}(1-\eta_1\Phi_y)}{(1-\eta_1\Phi_y)^2} \\
&= \frac{\eta_0\Phi_{g+} + \eta_2\Phi_{y+}}{1-\eta_1\Phi_y}\,.
\end{aligned}$$

Before we verify the previous conditions conditions, we state two intermediary results.

RESULT 2. *It holds that sgn*$(A_{11}A_{22} - A_{12}A_{21}) =$ *sgn*(ϕ_y).

To prove the result, we first simplify the denominator of $A_{11}A_{22} - A_{12}A_{21}$, i.e.,

$$1 - \eta_1\Phi_y = 1 - \frac{\zeta_g}{1+\phi_y\zeta_r}\frac{\mathcal{E}_y}{1+\mathcal{E}_g} = \frac{\phi_y\zeta_r}{1+\phi_y\zeta_r}\,.$$

Regarding the numerator, we obtain

$$\eta_0\Phi_{g+} + \eta_2\Phi_{y+} = \frac{\zeta_f\Phi_{g+} + \zeta_{g'}\Phi_{y+}}{1+\phi_y\zeta_r} = \frac{1}{1+\phi_y\zeta_r}\left[\zeta_f\frac{\mathcal{E}_g}{1+\mathcal{E}_g} + \tilde{s}\frac{\mathcal{E}_g}{\mathcal{E}_y}\frac{\mathscr{M} - \mathcal{E}_y}{1+\mathcal{E}_g}\right]\,.$$

Note that the term in brackets is positive due to

$$\zeta_f\frac{\mathcal{E}_g}{1+\mathcal{E}_g} + \tilde{s}\frac{\mathcal{E}_g}{\mathcal{E}_y}\frac{\mathscr{M} - \mathcal{E}_y}{1+\mathcal{E}_g} = \tilde{s}\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\frac{\mathscr{M}}{\mathcal{E}_y} + (1-\tilde{s})\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\frac{\chi}{\mathcal{E}_y} > 0\,.$$

Therefore, we have

$$A_{11}A_{22} - A_{12}A_{21} = \frac{1}{\phi_y\zeta_r}\left[\zeta_f\frac{\mathcal{E}_g}{1+\mathcal{E}_g} + \tilde{s}\frac{\mathcal{E}_g}{\mathcal{E}_y}\frac{\mathscr{M} - \mathcal{E}_y}{1+\mathcal{E}_g}\right]$$

such that $\text{sgn}(A_{11}A_{22} - A_{12}A_{21}) = \text{sgn}(\phi_y)$. Additionally, we have the following result.

RESULT 3. $A_{11} + A_{22}$ *is positive if* $\phi_y > \underline{\phi}_y$, *where* $\underline{\phi}_y \equiv -\frac{1-\tilde{s}}{\zeta_r} - \frac{1+\mathcal{E}_g}{\mathcal{E}_g}\left(\frac{\zeta_f - 1}{\zeta_r} + \mathscr{M}\right)$.

First, recognize that we have

$$A_{11} + A_{22} = \frac{\eta_0 + \eta_1\Phi_{y+} + \Phi_{g+} - \eta_2\Phi_y}{1-\eta_1\Phi_y}\,.$$

The numerator can be rewritten to

$$\eta_0 + \eta_1\Phi_{y+} + \Phi_{g+} - \eta_2\Phi_y = \frac{1}{1+\phi_y\zeta_r}\left[\zeta_f + \zeta_g\Phi_{y+} + (1+\phi_y\zeta_r)\Phi_{g+} - \zeta_{g'}\Phi_y\right]$$

$$= \frac{1}{1+\phi_y\zeta_r}\left[\zeta_f + \frac{\mathscr{M}-\mathcal{E}_y}{\mathcal{E}_y} + (1+\phi_y\zeta_r)\frac{\mathcal{E}_g}{1+\mathcal{E}_g} - \tilde{s}\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\right]$$
$$= \frac{1}{1+\phi_y\zeta_r}\left[\zeta_f - 1 + \frac{\mathscr{M}}{\mathcal{E}_y} + (1-\tilde{s})\frac{\mathcal{E}_g}{1+\mathcal{E}_g} + \phi_y\zeta_r\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\right].$$

As a result, we have

$$A_{11} + A_{22} = \frac{1}{\phi_y\zeta_r}\left[\zeta_f - 1 + \frac{\mathscr{M}}{\mathcal{E}_y} + (1-\tilde{s})\frac{\mathcal{E}_g}{1+\mathcal{E}_g} + \phi_y\zeta_r\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\right].$$

Note that the former expression is positive if

$$\zeta_f - 1 + \frac{\mathscr{M}}{\mathcal{E}_y} + (1-\tilde{s})\frac{\mathcal{E}_g}{1+\mathcal{E}_g} + \phi_y\zeta_r\frac{\mathcal{E}_g}{1+\mathcal{E}_g} > 0\,,$$

which is satisfied if $\phi_y > \max\left\{0, \underline{\phi}_y\right\}$.

Step 3: Derivation of HANK-GS Taylor Principle

The first determinacy condition, i.e., $|A_{11}A_{22} - A_{12}A_{21}| < 1$ is satisfied if

$$\phi_y > \frac{\mathcal{E}_g}{1+\mathcal{E}_g}\left[\frac{\zeta_f - \tilde{s}}{\zeta_r} + \tilde{s}\mathscr{M}\right].$$

The second determinacy condition, i.e., $|A_{11} + A_{22}| < 1 + A_{11}A_{22} - A_{12}A_{21}$ is satisfied if

$$\frac{1}{\phi_y\zeta_r}\left[\zeta_f - 1 + \frac{\mathscr{M}}{\mathcal{E}_y} + (1-\tilde{s})\frac{\mathcal{E}_g}{1+\mathcal{E}_g} + \phi_y\zeta_r\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\right] < 1 + \frac{1}{\phi_y\zeta_r}\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\left[\zeta_f - \tilde{s} + \tilde{s}\frac{\mathscr{M}}{\mathcal{E}_y}\right],$$

which can be rearranged to

$$\phi_y > \frac{\zeta_f - 1}{\zeta_r} + [1 + (1-\tilde{s})\mathcal{E}_g]\,\mathscr{M}\,.$$

As a result, model dynamics are locally determinate if

$$\phi_y > \max\left\{\underline{\phi}_y, \frac{\mathcal{E}_g}{1+\mathcal{E}_g}\left[\frac{\zeta_f - \tilde{s}}{\zeta_r} + \tilde{s}\mathscr{M}\right], \frac{\zeta_f - 1}{\zeta_r} + [1 + (1-\tilde{s})\mathcal{E}_g]\,\mathscr{M}\right\}.$$

Note that we recover the case stated in the main text when $\underline{\phi}_y \le 0$, which applies if $\zeta_f > \underline{\zeta}_f = 1 - (1-\tilde{s})\frac{\mathcal{E}_g}{1+\mathcal{E}_g} - \zeta_r\mathscr{M} < 1$. □

B.5 Progressive Redistribution According to HANK-G

B.5.1 *Proof Proposition 5*

PROOF. Subsequently, we provide a proof for every statement of the proposition.

Statement (a): In a HA model ($\theta = \psi = 0$) with flexible wages, the labor supply decision of unions is characterized by

$$\nu L_t^{\varphi} c_t = \frac{\epsilon_w - 1}{\epsilon_w}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} ,$$

which equals to $\varphi \hat{L}_t + \hat{c}_t = 0$ in log-linear terms. As $\hat{y}_t = \hat{L}_t = \hat{c}_t$ applies, the labor supply equation implies $\mathcal{M}_y = 0$. The aggregate IS equation then provides us with the associated (real) natural interest rate.[34]

Statement (b): In a HANK model ($\psi = 0$), the equilibrium dynamics are summarized by

$$\hat{y}_t = \zeta_f \mathbb{E}_t[\hat{y}_{t+1}] - \zeta_r\left(\hat{i}_t - \mathbb{E}_t[\hat{\pi}_{t+1}]\right) + \zeta_\tau \hat{\tau}_t - \zeta_{\tau'} \mathbb{E}_t[\hat{\tau}_{t+1}] ,$$
$$\hat{i}_t = \mathbb{E}_t[\hat{\pi}_{t+1}] + \phi_y \hat{y}_t ,$$

Note that $(\zeta_f, \zeta_r, \zeta_\tau, \zeta_{\tau'})$ are now defined under $\gamma_c = 1$ as $\hat{y}_t = \hat{c}_t$ applies. The impact output multiplier is given by

$$\mathcal{M}_y = \frac{\zeta_\tau - \rho\zeta_{\tau'}}{1 + \phi_y \zeta_r - \rho \zeta_f} .$$

The sign of the denominator is positive if $\phi_y > (\rho_\tau \zeta_f - 1)/\zeta_r$, which is for $\rho \in [0,1)$ guaranteed by local determinacy, i.e., $\phi_y > (\zeta_f - 1)/\zeta_r$. As a result, the sign of $\mathcal{M}_y$ is determined by the sign of the numerator. We obtain

$$\zeta_\tau - \rho\zeta_{\tau'} = \frac{\mathcal{E}_\tau}{\mathcal{E}_y}\left(1 - \rho\left[\tilde{s} - (1-\tilde{s})\frac{1-\eta}{\mathcal{E}_\tau}\right]\right) = \frac{\lambda(1-\eta)}{1-\lambda\chi}\left(1 - \rho\left[\tilde{s} - (1-\tilde{s})\frac{(1-\lambda)\Gamma_c}{\lambda}\right]\right) ,$$

where the second equality follows from substituting in for $\mathcal{E}_y$ and $\mathcal{E}_\tau$. As a result, we have $\zeta_\tau - \rho\zeta_{\tau'} > 0$ where the positive sign follows as $\eta \in (0,1)$ and $\chi < 1/\lambda$. Consequently, $\mathcal{M}_y > 0$ holds true.

Statement (c): In a HA-G model with flexible wages ($\theta = 0$), the equilibrium dynamics are summarized by

$$\hat{g}^A_{t+1} = \frac{\mathcal{E}_y}{1+\mathcal{E}_g}\hat{y}_t + \frac{\mathscr{M} - \mathcal{E}_y}{1+\mathcal{E}_g}\mathbb{E}_t[\hat{y}_{t+1}] + \frac{\mathcal{E}_g}{1+\mathcal{E}_g}\mathbb{E}_t\left[\hat{g}^A_{t+2}\right] - \frac{\mathcal{E}_\tau}{1+\mathcal{E}_g}\left(\hat{\tau}_t - \mathbb{E}_t[\hat{\tau}_{t+1}]\right) ,$$
$$\hat{y}_t = \gamma_c \hat{c}_t + (1-\gamma_c)\frac{g^A}{g^A - 1}\hat{g}^A_{t+1} ,$$
$$0 = \varphi \hat{L}_t + \hat{c}_t .$$

[34] Note that the result does not rely on the assumptions regarding union wage setting. Specifically, in an alternative setup in which households choose their individual labor supply, we would obtain in log-linear terms $\varphi \hat{L}^i_t + \hat{c}^i_t = \hat{w}_t$, for $i \in \{H, S\}$. Aggregating over household types and abstracting from steady state consumption inequality yields, as above, $\varphi \hat{L}_t + \hat{c}_t = \hat{w}_t = 0$.

Using $\hat{y}_t = \hat{L}_t$, output and technology multipliers are given by

$$\mathcal{M}_y = \frac{1-\gamma_c}{1+\varphi\gamma_c}\frac{g^A}{g^A-1}\mathcal{M}_g \,,$$
$$\mathcal{M}_g = \frac{\mathcal{E}_y(1-\rho)+\rho\mathscr{M}}{1+\mathcal{E}_g(1-\rho)}\mathcal{M}_y - \frac{\mathcal{E}_\tau(1-\rho)}{1+\mathcal{E}_g(1-\rho)} \,.$$

The first equation implies $\text{sgn}(\mathcal{M}_y) = \text{sgn}(\mathcal{M}_g)$. Substituting the first into the latter equation provides us with

$$\left(1 - \frac{1-\gamma_c}{1+\varphi\gamma_c}\frac{g^A}{g^A-1}\frac{\mathcal{E}_y(1-\rho)+\rho\mathscr{M}}{1+\mathcal{E}_g(1-\rho)}\right)\mathcal{M}_g = -\frac{\mathcal{E}_\tau(1-\rho)}{1+\mathcal{E}_g(1-\rho_\tau)} \,.$$

The sign of the right-hand side is negative. As a result, $\text{sgn}(\mathcal{M}_g) \geq 0$ if

$$(1+\varphi\gamma_c)\left(g^A-1\right)(1+\mathcal{E}_g(1-\rho)) \geq (1-\gamma_c)g^A\left(\mathcal{E}_y(1-\rho)+\rho\mathscr{M}\right) \,,$$
$$\Leftrightarrow \frac{(1+\varphi\gamma_c)\left(g^A-1\right)}{(1-\gamma_c)g^A} \geq \frac{\mathcal{E}_y(1-\rho)+\rho\mathscr{M}}{1+\mathcal{E}_g(1-\rho)} \,. \tag{9}$$

The derivative of the right hand side with respect to ρ is given by

$$\frac{\partial\left(\dfrac{\mathcal{E}_y(1-\rho)+\rho\mathscr{M}}{1+\mathcal{E}_g(1-\rho)}\right)}{\partial\rho} = \frac{\mathscr{M}(1+\mathcal{E}_g)-\mathcal{E}_y}{[1+\mathcal{E}_g(1-\rho)]^2} \,.$$

There are two sub-cases. First, when $\mathscr{M}(1+\mathcal{E}_g)-\mathcal{E}_y \geq 0$, equation (9) holds for $\rho \in (0,1)$ if

$$\frac{(1+\varphi\gamma_c)\left(g^A-1\right)}{(1-\gamma_c)g^A} > \mathscr{M} \quad \Leftrightarrow \quad \frac{1+\varphi\gamma_c}{1-\gamma_c}\left(g^A-1\right) > g^A-\beta \,.$$

The previous equality can be rearranged to

$$g^A > 1 + \frac{1-\gamma_c}{\gamma_c}\frac{1-\beta}{1+\varphi} \,.$$

The resource constraint implies $\gamma_c = 1-(g^A-1)/(\psi Y_\alpha L)$. Substituting into the previous condition yields

$$g^A > 1 + \frac{\dfrac{g^A-1}{\psi Y_\alpha L}}{1-\dfrac{g^A-1}{\psi Y_\alpha L}}\frac{1-\beta}{1+\varphi} \,,$$

which can be rearranged to

$$(g^A-1)\left(1-\frac{g^A-1}{\psi Y_\alpha L}\right) > \frac{g^A-1}{\psi Y_\alpha L}\frac{1-\beta}{1+\varphi}$$

$$\Leftrightarrow \quad 1 > \frac{1}{\psi Y_\alpha L}\left(g^A - 1 + \frac{1-\beta}{1+\varphi}\right)$$
$$\Leftrightarrow \quad 1 + \psi Y_\alpha L - \frac{1-\beta}{1+\varphi} > g^A \,.$$

Using $g^A = \beta(1+\psi\Theta_\alpha L)$, we can rewrite the previous equation as

$$\beta(1+\psi\Theta_\alpha L) < 1 + \psi Y_\alpha L - \frac{1-\beta}{1+\varphi}$$
$$\Leftrightarrow \quad (Y_\alpha - \beta\Theta_\alpha)\psi L > (1-\beta)\left[\frac{1}{1+\varphi} - 1\right] ,$$

which is always satisfied as $Y_\alpha - \beta\Theta_\alpha > 0$ and the right hand side is strictly negative. Second, when $\mathscr{M}(1+\mathcal{E}_g) - \mathcal{E}_y < 0$, equation (9) holds on $\rho \in (0,1)$ if

$$\frac{(1+\varphi\gamma_c)\left(g^A - 1\right)}{(1-\gamma_c)g^A} > \frac{\mathcal{E}_y}{1+\mathcal{E}_g} = \frac{(1-\lambda)\Gamma_c + \lambda - \lambda\gamma_c\chi}{(1-\lambda)\Gamma_c\left(1 - \frac{\gamma_c}{g^A}\right) + \lambda(1-\gamma_c)}\,\frac{g^A - 1}{g^A} \,.$$

After some tedious algebra, the condition can be rewritten as

$$(1-\lambda)\Gamma_c\gamma_c\left(1 - \frac{1}{g^A}\right) + \left[(1-\lambda)\Gamma_c\left(1 - \frac{\gamma_c}{g^A}\right) + \lambda(1-\gamma_c)\right]\varphi\gamma_c > -\lambda\gamma_c(1-\gamma_c)\chi \,,$$

which is satisfied as $0 \le \chi \le 1/(\lambda\gamma_c)$. □

B.5.2 *Proof Proposition 6*

PROOF. Under the full HANK-G economy with a real interest rate rule, equilibrium dynamics are summarized by

$$\hat{y}_t = \zeta_f \mathbb{E}_t\left[\hat{y}_{t+1}\right] - \phi_y\zeta_r\hat{y}_t + \zeta_g\hat{g}^A_{t+1} - \zeta_{g'}\mathbb{E}_t\left[\hat{g}^A_{t+2}\right] + \zeta_\tau\hat{\tau}_t - \zeta_{\tau'}\mathbb{E}_t\left[\hat{\tau}_{t+1}\right] ,$$
$$\hat{g}^A_{t+1} = \frac{\mathcal{E}_y}{1+\mathcal{E}_g}\hat{y}_t + \frac{\mathscr{M} - \mathcal{E}_y}{1+\mathcal{E}_g}\mathbb{E}_t\left[\hat{y}_{t+1}\right] + \frac{\mathcal{E}_g}{1+\mathcal{E}_g}\mathbb{E}_t\left[\hat{g}^A_{t+2}\right] - \frac{\mathcal{E}_\tau}{1+\mathcal{E}_g}\left(\mathbb{E}_t\left[\hat{\tau}_t\right] - \mathbb{E}_t\left[\hat{\tau}_{t+1}\right]\right) .$$

A. Output and Technology Multipliers.

The impact multipliers are

$$\left(1 + \phi_y\zeta_r - \rho\zeta_f\right)\mathcal{M}_y = \left(\zeta_g - \rho\zeta_{g'}\right)\mathcal{M}_g + \zeta_\tau - \rho\zeta_{\tau'} ,$$
$$\mathcal{M}_g = \Lambda_y\mathcal{M}_y - \Lambda_\tau ,$$

where we have defined the auxiliary parameters

$$\Lambda_y \equiv \frac{\mathcal{E}_y(1-\rho) + \rho\mathscr{M}}{1+\mathcal{E}_g(1-\rho)} > 0 \,, \qquad \Lambda_\tau \equiv \frac{\mathcal{E}_\tau(1-\rho)}{1+\mathcal{E}_g(1-\rho)} > 0 \,.$$

As a result, the output and technology multiplier becomes

$$\mathcal{M}_y = \frac{-\left(\zeta_g - \rho\zeta_{g'}\right)\Lambda_\tau + \zeta_\tau - \rho\zeta_{\tau'}}{1 + \phi_y\zeta_r - \rho\zeta_f - \left(\zeta_g - \rho\zeta_{g'}\right)\Lambda_y}\,,$$

$$\mathcal{M}_g = \frac{(\zeta_\tau - \rho_\tau\zeta_{\tau'})\Lambda_y - \left(1 + \phi_y\zeta_r - \rho\zeta_f\right)\Lambda_\tau}{1 + \phi_y\zeta_r - \rho\zeta_f - \left(\zeta_g - \rho\zeta_{g'}\right)\Lambda_y}\,.$$

B. Sign Denominator Multipliers.

Next, we show that the denominators of both multipliers are strictly positive if the Taylor principle applies. To see this more clearly, we write the dynamic system as

$$\begin{pmatrix} \hat{y}_t \\ \hat{g}^A_{t+1} \end{pmatrix} = \mathbf{A}_T \begin{pmatrix} \mathbb{E}_t\left[\hat{y}_{t+1}\right] \\ \mathbb{E}_t\left[\hat{g}^A_{t+2}\right] \end{pmatrix} + \mathbf{B}_T\hat{\tau}_t\,,$$

where

$$\mathbf{A}_T \equiv \frac{1}{1-\eta_1\Phi_y}\begin{pmatrix} \eta_0 + \eta_1\Phi_{y_+} & \eta_1\Phi_{g_+} - \eta_2 \\ \eta_0\Phi_y + \Phi_{y_+} & \Phi_{g_+} - \eta_2\Phi_y \end{pmatrix}\,,$$

$$\mathbf{B}_T \equiv \frac{1}{1-\eta_1\Phi_y}\begin{pmatrix} B_{11} \\ B_{21} \end{pmatrix} = \frac{1}{1-\eta_1\Phi_y}\begin{pmatrix} \eta_3 - \eta_1\Phi_\tau + \rho\left[\eta_1\Phi_{\tau_+} - \eta_4\right] \\ \eta_3\Phi_y - \Phi_\tau + \rho\left[\Phi_{\tau_+} - \eta_4\Phi_y\right] \end{pmatrix}\,.$$

Note that the additional auxiliary parameters are defined according to

$$\Phi_\tau = \Phi_{\tau_+} = \frac{\mathcal{E}_\tau}{1+\mathcal{E}_g}\,, \quad \eta_3 = \frac{\zeta_\tau}{1+\phi_y\zeta_r}\,, \quad \eta_4 = \frac{\zeta_{\tau'}}{1+\phi_y\zeta_r}\,.$$

As $\mathbb{E}_t\left[\hat{y}_{t+1}\right] = \rho\hat{y}_t$ and $\mathbb{E}_t\left[\hat{g}^A_{t+2}\right] = \rho\hat{g}^A_{t+1}$ hold on the equilibrium path, we have

$$\begin{aligned}
\begin{pmatrix} \hat{y}_t \\ \hat{g}^A_{t+1} \end{pmatrix} &= \left(\mathbf{I}_2 - \rho\mathbf{A}_T\right)^{-1}\mathbf{B}_T\hat{\tau}_t \\
&= \frac{1}{(1-\rho A_{11})(1-\rho A_{22}) - \rho^2 A_{12}A_{21}}\begin{pmatrix} 1-\rho A_{22} & \rho A_{12} \\ \rho A_{21} & 1-\rho A_{11} \end{pmatrix}\mathbf{B}_T\hat{\tau}_t \\
&= \frac{1}{(1-\rho A_{11})(1-\rho A_{22}) - \rho^2 A_{12}A_{21}}\frac{1}{1-\eta_1\Phi_y}\begin{pmatrix} (1-\rho A_{22})B_{11} + \rho A_{12}B_{21} \\ \rho A_{21}B_{11} + (1-\rho A_{11})B_{21} \end{pmatrix}\hat{\tau}_t\,.
\end{aligned}$$

After some tedious algebra we have

$$\begin{aligned}
(1-\rho A_{22})B_{11} + \rho A_{12}B_{21} &= \left(1 - \rho\frac{\Phi_{g_+} - \eta_2\Phi_y}{1-\eta_1\Phi_y}\right)\left(\eta_3 - \eta_1\Phi_\tau + \rho\left[\eta_1\Phi_{\tau_+} - \eta_4\right]\right) \\
&\quad + \rho\frac{\eta_1\Phi_{g_+} - \eta_2}{1-\eta_1\Phi_y}\left(\eta_3\Phi_y - \Phi_\tau + \rho\left[\Phi_{\tau_+} - \eta_4\Phi_y\right]\right) \\
&= (\eta_3 - \rho\eta_4)(1-\rho\Phi_{g_+}) - (\eta_1 - \rho\eta_2)(\Phi_\tau - \rho\Phi_{\tau_+})\,.
\end{aligned}$$

Substituting in for the auxiliary parameters yields

$$
\begin{aligned}
(1-\rho A_{22})B_{11}+\rho A_{12}B_{21} &= \frac{1}{1+\phi_y\zeta_r}\left\{(\zeta_\tau-\rho\zeta_{\tau'})\left(1-\rho\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\right)-\left(\zeta_g-\rho\zeta_{g'}\right)\frac{(1-\rho)\mathcal{E}_\tau}{1+\mathcal{E}_g}\right\} \\
&= \frac{1}{1+\phi_y\zeta_r}\frac{1+(1-\rho)\mathcal{E}_g}{1+\mathcal{E}_g}\left\{(\zeta_\tau-\rho\zeta_{\tau'})-\left(\zeta_g-\rho\zeta_{g'}\right)\Lambda_\tau\right\} \,.
\end{aligned}
$$

As $1-\eta_1\Phi_y>0$ applies, this implies that $\text{sgn}(\mathcal{M}_y)=\text{sgn}((\zeta_\tau-\rho\zeta_{\tau'})-\left(\zeta_g-\rho\zeta_{g'}\right)\Lambda_\tau)$ if $\det(\mathbf{I}_2-\rho\mathbf{A}_T)>0$. To show the latter statement, note that

$$
\begin{aligned}
A_{11}A_{22}-A_{12}A_{21} &= \frac{1}{\phi_y\zeta_r}\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\left[\zeta_f-\tilde{s}+\tilde{s}\frac{\mathscr{M}}{\mathcal{E}_y}\right]>0\,, \\
A_{11}+A_{22} &= \frac{1}{\phi_y\zeta_r}\left[\zeta_f-1+\frac{\mathscr{M}}{\mathcal{E}_y}+(1-\tilde{s})\frac{\mathcal{E}_g}{1+\mathcal{E}_g}+\phi_y\zeta_r\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\right]>0\,.
\end{aligned}
$$

where the strict inequalities are due to the determinacy conditions. Moreover, we rewrite

$$
\begin{aligned}
\det(\mathbf{I}_2-\rho\mathbf{A}_T) &= (1-\rho A_{11})(1-\rho A_{22})-\rho^2A_{12}A_{21} \\
&= 1-\rho\left(A_{11}+A_{22}\right)+\rho^2\left(A_{11}A_{22}-A_{12}A_{21}\right)\,.
\end{aligned}
$$

As $A_{11}+A_{22}>0$ applies, we obtain

$$
\begin{aligned}
\det(\mathbf{I}_2-\rho\mathbf{A}_T) &= 1-\rho\left(A_{11}+A_{22}\right)+\rho^2\left(A_{11}A_{22}-A_{12}A_{21}\right) \\
&> 1-\rho\left(1+A_{11}A_{22}-A_{12}A_{21}\right)+\rho^2\left(A_{11}A_{22}-A_{12}A_{21}\right) \\
&= (1-\rho)\left[1-\rho\left(A_{11}A_{22}-A_{12}A_{21}\right)\right] \\
&> 0\,,
\end{aligned}
$$

where the first strict inequality is due to the determinacy condition $A_{11}+A_{22}<1+A_{11}A_{22}-A_{12}A_{21}$ and the second strictly inequality is due to the determinacy condition $A_{11}A_{22}-A_{12}A_{21}<1$. As a result, this implies that

$$
\begin{aligned}
\text{sgn}(\mathcal{M}_y) &= \text{sgn}((\zeta_\tau-\rho\zeta_{\tau'})-\left(\zeta_g-\rho\zeta_{g'}\right)\Lambda_\tau)\,, \\
\text{sgn}(\mathcal{M}_g) &= \text{sgn}((\zeta_\tau-\rho_\tau\zeta_{\tau'})\Lambda_y-\left(1+\phi_y\zeta_r-\rho\zeta_f\right)\Lambda_\tau)\,,
\end{aligned}
$$

which is equivalent to stating

$$
1+\phi_y\zeta_r-\rho\zeta_f-\left(\zeta_g-\rho\zeta_{g'}\right)\Lambda_y>0\,.
$$

C. Sign Numerator Multipliers.

The sign of (the numerator of) $\mathcal{M}_y$ is

$$
\begin{aligned}
\text{sgn}\left(\mathcal{M}_y\right) &= \zeta_\tau-\rho\zeta_{\tau'}-\left(\zeta_g-\rho\zeta_{g'}\right)\Lambda_\tau \\
&= \frac{\mathcal{E}_\tau}{\mathcal{E}_y}-\rho\frac{\mathcal{E}_\tau}{\mathcal{E}_y}\left(\tilde{s}-(1-\tilde{s})\frac{1-\eta}{\mathcal{E}_\tau}\right)-\left[\frac{1+\mathcal{E}_g}{\mathcal{E}_y}-\rho\tilde{s}\frac{\mathcal{E}_g}{\mathcal{E}_y}\right]\frac{\mathcal{E}_\tau(1-\rho)}{1+\mathcal{E}_g(1-\rho)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\mathcal{E}_\tau}{\mathcal{E}_y} \frac{1}{1+\mathcal{E}_g(1-\rho)} \left\{ (1+\mathcal{E}_g(1-\rho)) \left[1 - \rho \left(\tilde{s} - (1-\tilde{s}) \frac{1-\eta}{\mathcal{E}_\tau} \right) \right] - (1-\rho)\left(1+\mathcal{E}_g - \rho\tilde{s}\mathcal{E}_g\right) \right\} \\
&= \frac{\mathcal{E}_\tau}{\mathcal{E}_y} \frac{\rho}{1+\mathcal{E}_g(1-\rho)} \left\{ 1 - (1+\mathcal{E}_g(1-\rho)) \left[\tilde{s} - (1-\tilde{s}) \frac{1-\eta}{\mathcal{E}_\tau} \right] + (1-\rho)\tilde{s}\mathcal{E}_g \right\} \\
&= \frac{\mathcal{E}_\tau}{\mathcal{E}_y} \frac{\rho}{1+\mathcal{E}_g(1-\rho)} \left\{ 1 - \tilde{s} - (1+\mathcal{E}_g(1-\rho)) \left[-(1-\tilde{s}) \frac{1-\eta}{\mathcal{E}_\tau} \right] \right\} \\
&= \frac{\mathcal{E}_\tau}{\mathcal{E}_y} \frac{\rho(1-\tilde{s})}{1+\mathcal{E}_g(1-\rho)} \left\{ 1 + \left[1+\mathcal{E}_g(1-\rho)\right] \frac{1-\lambda}{\lambda}\Gamma_c \right\} .
\end{aligned}
$$

It follows that $\operatorname{sgn}(\mathcal{M}_y) = 0$ if $\rho = 0$ or $\tilde{s} = 1$ and $\operatorname{sgn}(\mathcal{M}_y) > 0$ otherwise. Regarding the sign of the technology growth multiplier, we obtain

$$
\begin{aligned}
\operatorname{sgn}(\mathcal{M}_g) &= (\zeta_\tau - \rho_\tau\zeta_{\tau'})\Lambda_y - \left(1+\phi_y\zeta_r - \rho\zeta_f\right)\Lambda_\tau \\
&= \left\{ \frac{\mathcal{E}_\tau}{\mathcal{E}_y} - \rho\frac{\mathcal{E}_\tau}{\mathcal{E}_y} \left(\tilde{s} - (1-\tilde{s}) \frac{1-\eta}{\mathcal{E}_\tau} \right) \right\} \frac{\mathcal{E}_y(1-\rho) + \rho\mathscr{M}}{1+\mathcal{E}_g(1-\rho)} - \left(1+\phi_y\zeta_r - \rho\zeta_f\right) \frac{\mathcal{E}_\tau(1-\rho)}{1+\mathcal{E}_g(1-\rho)} \\
&= \frac{\mathcal{E}_\tau}{\mathcal{E}_y} \frac{1}{1+\mathcal{E}_g(1-\rho)} \left\{ \left[1 - \rho \left(\tilde{s} - (1-\tilde{s}) \frac{1-\eta}{\mathcal{E}_\tau} \right) \right] \left[\mathcal{E}_y(1-\rho) + \rho\mathscr{M}\right] - (1-\rho)\left(1+\phi_y\zeta_r - \rho\zeta_f\right)\mathcal{E}_y \right\} .
\end{aligned}
$$

The sign of the previous expression is determined by the term in brackets, i.e.,

$$
\begin{aligned}
\operatorname{sgn}(\mathcal{M}_g) &= \left[1 - \rho \left(\tilde{s} - (1-\tilde{s}) \frac{1-\eta}{\mathcal{E}_\tau} \right) \right] \left[\mathcal{E}_y(1-\rho) + \rho\mathscr{M}\right] - (1-\rho)\left(1+\phi_y\zeta_r - \rho\zeta_f\right)\mathcal{E}_y \\
&= \left[1 - \rho + \rho(1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda} \right] \left[\mathcal{E}_y(1-\rho) + \rho\mathscr{M}\right] - (1-\rho)\left(1+\phi_y\zeta_r - \rho\zeta_f\right)\mathcal{E}_y \\
&= (1-\rho)\mathcal{E}_y \left\{ 1 - \rho + \rho(1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda} - \left(1+\phi_y\zeta_r - \rho\zeta_f\right) \right\} \\
&+ \left[1 - \rho + \rho(1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda} \right] \rho\mathscr{M} \\
&= \rho(1-\rho)\mathcal{E}_y \left\{ \zeta_f - 1 + (1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda} \right\} - (1-\rho)\phi_y \\
&+ \left[1 - \rho + \rho(1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda} \right] \rho\mathscr{M} \\
&= \rho(1-\rho)\mathcal{E}_y \left\{ (1-\tilde{s}) \left(\frac{\chi}{\mathcal{E}_y} - 1 \right) + (1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda} \right\} - (1-\rho)\phi_y \\
&+ \left[1 - \rho + \rho(1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda} \right] \rho\mathscr{M} \\
&= \rho(1-\rho)(1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda\gamma_c} - (1-\rho)\phi_y + \left[1 - \rho + \rho(1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda} \right] \rho\mathscr{M} .
\end{aligned}
$$

This equation defines a cutoff value ϕ_y^* such that $\operatorname{sgn}(\mathcal{M}_g) \gtreqless 0$ if $\phi_y \lesseqgtr \phi_y^*$. It is given by

$$
\phi_y^* = \rho(1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda\gamma_c} + \left[1 + (1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda} \frac{\rho}{1-\rho} \right] \rho\mathscr{M} .
$$

It is straightforward to verify that ϕ_y^* is strictly increasing in ρ. Alternatively, the former equation defines a second order polynomial in ρ, i.e., $a\rho^2 + b\rho + c$, where we have the following auxiliary parameters

$$
\begin{aligned}
a &\equiv -(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\left(\frac{1}{\gamma_c}-\mathscr{M}\right)-\mathscr{M}<0\,,\\
b &\equiv (1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda\gamma_c}+\phi_y+\mathscr{M}>0\,,\\
c &\equiv -\phi_y<0\,,
\end{aligned}
$$

where the first inequality follows from $\gamma_c \in (0,1)$ and $\mathscr{M} \in (0,1)$ and the second and third from $\phi_y \geq 0$. The determinant of the second order polynomial is

$$
\Delta = b^2-4ac=\left(\mathscr{M}-\phi_y+(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda\gamma_c}\right)^2+4\phi_y(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\mathscr{M}>0\,.
$$

As such, the second order polynomial has two real valued roots that are given by

$$
\tilde{\rho}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\,,\quad \text{with}\quad \rho^{min}=\frac{-b+\sqrt{b^2-4ac}}{2a}<\rho^{max}=\frac{-b-\sqrt{b^2-4ac}}{2a}\,,
$$

where the inequality is due to $a<0$. Note that $0<\rho^{min}<1$ as

$$
\begin{aligned}
&-b+\sqrt{b^2-4ac}<0 \;\Leftrightarrow\; 4ac>0\,,\\
&-b+\sqrt{b^2-4ac}>2a \;\Leftrightarrow\; \sqrt{b^2-4ac}>2a+b \;\Leftrightarrow\; 0>a(a+b+c)\,,
\end{aligned}
$$

where the last inequality is satisfied as $a<0$ and

$$
a+b+c=(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\mathscr{M}>0\,.
$$

Moreover, we have $\rho^{max}>1$ due to

$$
-b-\sqrt{b^2-4ac}<2a \;\Leftrightarrow\; -\sqrt{b^2-4ac}<2a+b \;\Leftrightarrow\; 0>a(a+b+c)\,.
$$

As a result, the persistence cutoff is

$$
\rho^*=\frac{-\left[(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda\gamma_c}+\phi_y+\mathscr{M}\right]+\sqrt{\left(\mathscr{M}-\phi_y+(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda\gamma_c}\right)^2+4\phi_y(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\mathscr{M}}}{2\left[-(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\left(\frac{1}{\gamma_c}-\mathscr{M}\right)-\mathscr{M}\right]}\,.
$$

The comparative statics of ρ^* with respect to ϕ_y are given by

$$
\frac{\partial\rho^*}{\partial\phi_y}=\frac{-1+\frac{\Delta^{-\frac{1}{2}}}{2}\left[-2\left(\mathscr{M}-\phi_y+(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda\gamma_c}\right)+4(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\mathscr{M}\right]}{2\left[-(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\left(\frac{1}{\gamma_c}-\mathscr{M}\right)-\mathscr{M}\right]}\,.
$$

As a result, we have $\frac{\partial \rho^*}{\partial \phi_y} > 0$ if

$$\Delta^{\frac{1}{2}} > 2(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\mathscr{M} - \left(\mathscr{M} - \phi_y + (1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda\gamma_c}\right) .$$

This condition is obviously satisfied when the right hand side is weakly negative. In the reverse case, we can rewrite

$$\Delta > \left[2(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\mathscr{M} - \left(\mathscr{M} - \phi_y + (1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda\gamma_c}\right)\right]^2 ,$$

which can be simplified to

$$0 > (1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\left(\mathscr{M} - \frac{1}{\gamma_c}\right) - \mathscr{M} ,$$

which holds true as $\gamma_c \in (0,1)$ and $\mathscr{M} < 1$. □

B.6 Generalization: Government Spending Shock and Unequal Tax Incidence

B.6.1 *Proof Proposition 7*

PROOF. The proof proceeds in several steps. First, we derive expressions for hand-to-mouth and saver consumption. Second, we provide the new DIS and EG equations. Third, we compute output and technology multipliers before we finally derive conditions that determine their respective sign.

A. Policy Functions. Under this formulation, we obtain for HtM consumption

$$c_t^H = w_t L_t^H - \frac{t_t^H}{\lambda} = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L_t^H + \frac{T}{\lambda} - \frac{\tau_p}{\lambda} s_t^G .$$

Log-linearization provides us with

$$\hat{c}_t^H = \frac{wL^H}{c^H}\hat{L}_t^H - \frac{\tau_p}{\lambda}\frac{y}{c^H}\hat{s}_t^G = \frac{wL^H}{c^H}\mu\hat{L}_t - \frac{\tau_p}{\lambda}\frac{y}{c^H}\hat{s}_t^G = \eta\mu\hat{y}_t - \frac{\tau_p}{\lambda}\Theta\hat{s}_t^G ,$$

where η is defined as before, $\Theta \equiv \lambda + (1-\lambda)\Gamma_y$, and $\hat{s}_t^G = (s_t^G - s^G)/y$. The aggregate resource constraint now reads

$$\hat{y}_t = \gamma_c \hat{c}_t + (1-\gamma_c)\frac{g^A}{g^A-1}\hat{g}_{t+1}^A + \hat{s}_t^G$$

and saver consumption becomes

$$\begin{aligned}\hat{c}_t^S &= \left(1 + \frac{\lambda}{1-\lambda}\frac{1}{\Gamma_c}\right)\left(\frac{\hat{y}_t - \hat{s}_t^G}{\gamma_c} - \frac{1-\gamma_c}{\gamma_c}\frac{g^A}{g^A-1}\hat{g}_{t+1}^A\right) - \frac{\lambda}{1-\lambda}\frac{1}{\Gamma_c}\left(\eta\mu\hat{y}_t - \frac{\tau_p}{\lambda}\Theta\hat{s}_t^G\right) \\ &= \mathcal{E}_y\hat{y}_t - \mathcal{E}_g\hat{g}_{t+1}^A - \mathcal{E}_s\hat{s}_t^G ,\end{aligned}$$

where $\mathcal{E}_y, \mathcal{E}_g$ and $\mathcal{E}_s$ are

$$\mathcal{E}_y \equiv \frac{(1-\lambda)\Gamma_c + \lambda - \lambda\gamma_c\chi}{(1-\lambda)\Gamma_c\gamma_c} ,$$

$$\mathcal{E}_g \equiv \frac{(1-\lambda)\Gamma_c + \lambda}{(1-\lambda)\Gamma_c} \frac{1-\gamma_c}{\gamma_c} \frac{g^A}{g^A - 1} ,$$

$$\mathcal{E}_s \equiv \frac{1}{(1-\lambda)\Gamma_c} \left(\frac{\lambda + (1-\lambda)\Gamma_c}{\gamma_c} - \Theta\tau_p \right) .$$

B. DIS and EG Equations. The saver Euler equation is given by

$$\hat{c}_t^S = \tilde{s}\mathbb{E}_t \left[\hat{c}_{t+1}^S \right] + (1-\tilde{s})\mathbb{E}_t \left[\hat{c}_{t+1}^H \right] - \hat{r}_t + \hat{g}_{t+1}^A .$$

Substituting in for the consumption of saver households, $\hat{c}_t^S$, results in

$$\mathcal{E}_y\hat{y}_t - \mathcal{E}_g\hat{g}_{t+1}^A - \mathcal{E}_s\hat{s}_t^G = \tilde{s}\mathbb{E}_t \left[\mathcal{E}_y\hat{y}_{t+1} - \mathcal{E}_g\hat{g}_{t+2}^A - \mathcal{E}_s\hat{s}_{t+1}^G \right] + (1-\tilde{s})\mathbb{E}_t \left[\chi\hat{y}_{t+1} - \frac{\tau_p}{\lambda}\Theta\hat{s}_{t+1}^G \right] - \hat{r}_t + \hat{g}_{t+1}^A ,$$

which can be rearranged to

$$\hat{y}_t = \left(\tilde{s} + (1-\tilde{s})\frac{\chi}{\mathcal{E}_y} \right) \mathbb{E}_t \left[\hat{y}_{t+1} \right] - \frac{1}{\mathcal{E}_y}\hat{r}_t + \frac{1+\mathcal{E}_g}{\mathcal{E}_y}\hat{g}_{t+1}^A - \tilde{s}\frac{\mathcal{E}_g}{\mathcal{E}_y}\mathbb{E}_t \left[\hat{g}_{t+2}^A \right] + \frac{\mathcal{E}_s}{\mathcal{E}_y}\hat{s}_t^G - \frac{\mathcal{E}_s}{\mathcal{E}_y}\left(\tilde{s} + (1-\tilde{s})\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s} \right) \mathbb{E}_t \left[\hat{s}_{t+1}^G \right] .$$

The former equation can be restated as

$$\hat{y}_t = \zeta_f \mathbb{E}_t \left[\hat{y}_{t+1} \right] - \zeta_r \hat{r}_t + \zeta_g \hat{g}_{t+1}^A - \zeta_{g'} \mathbb{E}_t \left[\hat{g}_{t+2}^A \right] + \zeta_s \hat{s}_t - \zeta_{s'} \mathbb{E}_t \left[\hat{s}_{t+1}^G \right] ,$$

where $\zeta_f, \zeta_r, \zeta_g, \zeta_{g'}$ are as before and the new auxiliary parameters are given by:[35]

$$\zeta_s = \frac{\mathcal{E}_s}{\mathcal{E}_y} ,$$

$$\zeta_{s'} = \frac{\mathcal{E}_s}{\mathcal{E}_y} \left(\tilde{s} + (1-\tilde{s})\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s} \right) .$$

Similarly, substituting the consumption policy function into the growth equation yields

$$\hat{g}_{t+1}^A = \frac{\mathcal{E}_y}{1+\mathcal{E}_g}\hat{y}_t + \frac{\mathscr{M} - \mathcal{E}_y}{1+\mathcal{E}_g}\mathbb{E}_t \left[\hat{y}_{t+1} \right] + \frac{\mathcal{E}_g}{1+\mathcal{E}_g}\mathbb{E}_t \left[\hat{g}_{t+2}^A \right] - \frac{\mathcal{E}_s}{1+\mathcal{E}_g}\left(\mathbb{E}_t \left[\hat{s}_t^G \right] - \mathbb{E}_t \left[\hat{s}_{t+1}^G \right] \right) .$$

[35] These expressions can be further simplified (see Appendix C: Additional Theoretical Results) using the relation $\gamma_c\Theta = 1$, which holds true as $\Gamma_c = 1$ implies $\Gamma_y = (1-\lambda\gamma_c)/[(1-\lambda_c)\gamma_c]$.

C. Output and Technology Multipliers

As a result, the impact multipliers are

$$\left(1+\phi_y\zeta_r-\rho\zeta_f\right)\mathcal{M}_y=\left(\zeta_g-\rho\zeta_{g'}\right)\mathcal{M}_g+\zeta_s-\rho\zeta_{s'}\,,$$
$$\mathcal{M}_g=\Lambda_y\mathcal{M}_y-\Lambda_s\,,$$

where we have defined the auxiliary parameters

$$\Lambda_y\equiv\frac{\mathcal{E}_y(1-\rho)+\rho\mathscr{M}}{1+\mathcal{E}_g(1-\rho)}>0\,,\qquad\Lambda_s\equiv\frac{\mathcal{E}_s(1-\rho)}{1+\mathcal{E}_g(1-\rho)}\geq 0\,.$$

As a result, the output and technology multiplier becomes

$$\mathcal{M}_y=\frac{\zeta_s-\rho\zeta_{s'}-\left(\zeta_g-\rho\zeta_{g'}\right)\Lambda_s}{1+\phi_y\zeta_r-\rho\zeta_f-\left(\zeta_g-\rho\zeta_{g'}\right)\Lambda_y}\,,$$
$$\mathcal{M}_g=\frac{(\zeta_s-\rho\zeta_{s'})\Lambda_y-\left(1+\phi_y\zeta_r-\rho\zeta_f\right)\Lambda_s}{1+\phi_y\zeta_r-\rho\zeta_f-\left(\zeta_g-\rho\zeta_{g'}\right)\Lambda_y}\,,$$

where, in case of $\Gamma_c=1$, the auxiliary variables are given by:

$$\zeta_s=\frac{1-\tau_p}{1-\lambda\gamma_c\chi}\,,$$
$$\zeta_{s'}=\left(s+(1-s)\frac{1-\lambda}{\lambda}\frac{\tau_p}{1-\tau_p}\right)\zeta_s\,,$$
$$\xi_s=\frac{(g^A-1)(1-\tau_p)}{g^A(1-\lambda\gamma_c)+\gamma_c(\lambda-1)}\,,$$
$$\Lambda_s=(1-\rho)\xi_s/(1-\rho\xi_{g'})\,.$$

D. Sign Output and Technology Multipliers.

The sign of $\mathcal{M}_y$ is

$$\begin{aligned}
\operatorname{sgn}(\mathcal{M}_y)&=\zeta_s-\rho\zeta_{s'}-\left(\zeta_g-\rho\zeta_{g'}\right)\Lambda_s\\
&=\frac{\mathcal{E}_s}{\mathcal{E}_y}-\rho\frac{\mathcal{E}_s}{\mathcal{E}_y}\left(\tilde{s}+(1-\tilde{s})\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s}\right)-\left[\frac{1+\mathcal{E}_g}{\mathcal{E}_y}-\rho\tilde{s}\frac{\mathcal{E}_g}{\mathcal{E}_y}\right]\frac{\mathcal{E}_s(1-\rho)}{1+\mathcal{E}_g(1-\rho)}\\
&=\frac{\mathcal{E}_s}{\mathcal{E}_y}\frac{1}{1+\mathcal{E}_g(1-\rho)}\left\{(1+\mathcal{E}_g(1-\rho))\left[1-\rho\left(\tilde{s}+(1-\tilde{s})\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s}\right)\right]-(1-\rho)\left(1+\mathcal{E}_g-\rho\tilde{s}\mathcal{E}_g\right)\right\}\\
&=\frac{\mathcal{E}_s}{\mathcal{E}_y}\frac{\rho}{1+\mathcal{E}_g(1-\rho)}\left\{1-(1+\mathcal{E}_g(1-\rho))\left[\tilde{s}+(1-\tilde{s})\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s}\right]+(1-\rho)\tilde{s}\mathcal{E}_g\right\}\\
&=\frac{\mathcal{E}_s}{\mathcal{E}_y}\frac{\rho}{1+\mathcal{E}_g(1-\rho)}\left\{1-\tilde{s}-(1+\mathcal{E}_g(1-\rho))\,(1-\tilde{s})\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s}\right\}\\
&=\frac{\mathcal{E}_s}{\mathcal{E}_y}\frac{\rho(1-\tilde{s})}{1+\mathcal{E}_g(1-\rho)}\left\{1-[1+\mathcal{E}_g(1-\rho)]\,\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s}\right\}
\end{aligned}$$

$$= \frac{1}{\mathcal{E}_y} \frac{\rho(1-\tilde{s})}{1+\mathcal{E}_g(1-\rho)} \left\{ \mathcal{E}_s - [1+\mathcal{E}_g(1-\rho)] \frac{\tau_p}{\lambda} \Theta \right\} .$$

It follows that $\text{sgn}(\mathcal{M}_y) = 0$ if $\rho = 0$ or $\tilde{s} = 1$. In all other cases, the sign of the output multiplier is determined by the sign of the term in brackets, i.e.,

$$\begin{aligned}
\mathcal{E}_s - [1+\mathcal{E}_g(1-\rho)] \frac{\tau_p}{\lambda} \Theta &= \frac{1}{(1-\lambda)\Gamma_c} \left(\frac{\lambda + (1-\lambda)\Gamma_c}{\gamma_c} - \Theta \tau_p \right) - [1+\mathcal{E}_g(1-\rho)] \frac{\Theta \tau_p}{\lambda} \\
&= \frac{\lambda + (1-\lambda)\Gamma_c}{(1-\lambda)\Gamma_c \gamma_c} - \left(\frac{1}{(1-\lambda)\Gamma_c} + \frac{1+\mathcal{E}_g(1-\rho)}{\lambda} \right) \Theta \tau_p \, ,
\end{aligned}$$

which is strictly positive if

$$\tau_p < \tilde{\tau}_p \equiv \frac{1}{\Theta} \frac{\dfrac{\lambda + (1-\lambda)\Gamma_c}{(1-\lambda)\Gamma_c \gamma_c}}{\dfrac{1}{(1-\lambda)\Gamma_c} + \dfrac{1+\mathcal{E}_g(1-\rho)}{\lambda}} \, .$$

In the case of $\Gamma_c = 1$, we obtain

$$\tau_p < \frac{1}{\Theta} \frac{\dfrac{1}{\gamma_c}}{\dfrac{1}{\lambda} + (1-\rho)\dfrac{1-\lambda}{\lambda}\mathcal{E}_g} = \frac{\lambda}{1 + (1-\rho)\dfrac{1-\gamma_c}{\gamma_c}\dfrac{g^A}{g^A - 1}} < 1 \, .$$

To determine the sign of the technology growth multiplier, we compute

$$\begin{aligned}
\text{sgn}\left(\mathcal{M}_g\right) &= (\zeta_s - \rho\zeta_{s'})\Lambda_y - \left(1 + \phi_y \zeta_r - \rho\zeta_f\right)\Lambda_s \\
&= \left[\frac{\mathcal{E}_s}{\mathcal{E}_y} - \rho \frac{\mathcal{E}_s}{\mathcal{E}_y} \left(\tilde{s} + (1-\tilde{s}) \frac{\tau_p}{\lambda} \frac{\Theta}{\mathcal{E}_s} \right) \right] \frac{\mathcal{E}_y(1-\rho) + \rho\mathscr{M}}{1+\mathcal{E}_g(1-\rho)} - \left(1 + \phi_y \zeta_r - \rho\zeta_f\right) \frac{\mathcal{E}_s(1-\rho)}{1+\mathcal{E}_g(1-\rho)} \\
&= \frac{\mathcal{E}_s}{\mathcal{E}_y} \frac{1}{1+\mathcal{E}_g(1-\rho)} \left\{ \left[1 - \rho \left(\tilde{s} + (1-\tilde{s}) \frac{\tau_p}{\lambda} \frac{\Theta}{\mathcal{E}_s} \right) \right] [\mathcal{E}_y(1-\rho) + \rho\mathscr{M}] - (1-\rho)\left(1 + \phi_y \zeta_r - \rho\zeta_f\right)\mathcal{E}_y \right\} .
\end{aligned}$$

We can simplify the term in brackets further to

$$\begin{aligned}
&\left[1 - \rho \left(\tilde{s} + (1-\tilde{s}) \frac{\tau_p}{\lambda} \frac{\Theta}{\mathcal{E}_s} \right) \right] [\mathcal{E}_y(1-\rho) + \rho\mathscr{M}] - (1-\rho)\left(1 + \phi_y \zeta_r - \rho\zeta_f\right)\mathcal{E}_y \\
=& \left[1 - \rho + \rho(1-\tilde{s}) \left(1 - \frac{\tau_p}{\lambda} \frac{\Theta}{\mathcal{E}_s} \right) \right] [\mathcal{E}_y(1-\rho) + \rho\mathscr{M}] - (1-\rho)\left(1 + \phi_y \zeta_r - \rho\zeta_f\right)\mathcal{E}_y \\
=& (1-\rho)\mathcal{E}_y \left\{ 1 - \rho + \rho(1-\tilde{s}) \left(1 - \frac{\tau_p}{\lambda} \frac{\Theta}{\mathcal{E}_s} \right) - \left(1 + \phi_y \zeta_r - \rho\zeta_f\right) \right\} \\
+& \left[1 - \rho + \rho(1-\tilde{s}) \left(1 - \frac{\tau_p}{\lambda} \frac{\Theta}{\mathcal{E}_s} \right) \right] \rho\mathscr{M} \\
=& \rho(1-\rho)\mathcal{E}_y \left\{ \zeta_f - 1 + (1-\tilde{s}) \left(1 - \frac{\tau_p}{\lambda} \frac{\Theta}{\mathcal{E}_s} \right) \right\} - (1-\rho)\phi_y \\
+& \left[1 - \rho + \rho(1-\tilde{s}) \left(1 - \frac{\tau_p}{\lambda} \frac{\Theta}{\mathcal{E}_s} \right) \right] \rho\mathscr{M}
\end{aligned}$$

$$=\rho(1-\rho)\mathcal{E}_y\left\{(1-\tilde{s})\left(\frac{\chi}{\mathcal{E}_y}-1\right)+(1-\tilde{s})\left(1-\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s}\right)\right\}-(1-\rho)\phi_y$$

$$+\left[1-\rho+\rho(1-\tilde{s})\left(1-\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s}\right)\right]\rho\mathscr{M}$$

$$=\rho(1-\rho)(1-\tilde{s})\left\{\chi-\frac{\tau_p\Theta}{\lambda}\frac{\mathcal{E}_y}{\mathcal{E}_s}\right\}-(1-\rho)\phi_y+\left[1-\rho+\rho(1-\tilde{s})\left(1-\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s}\right)\right]\rho\mathscr{M}\,.$$

As a result, the sign is given by (multiplying by $\mathcal{E}_s$ from above)

$$\rho(1-\rho)(1-\tilde{s})\left\{\chi\mathcal{E}_s-\frac{\tau_p\Theta}{\lambda}\mathcal{E}_y\right\}-(1-\rho)\phi_y\mathcal{E}_s+\left[(1-\rho)\mathcal{E}_s+\rho(1-\tilde{s})\left(\mathcal{E}_s-\frac{\tau_p\Theta}{\lambda}\right)\right]\rho\mathscr{M}\,.$$

Collecting terms yields

$$[\rho(1-\rho)(1-\tilde{s})\chi+(1-\rho\tilde{s})\rho\mathcal{M}-(1-\rho)\phi_y]\,\mathcal{E}_s-\rho(1-\tilde{s})\frac{(1-\rho)\mathcal{E}_y+\rho\mathcal{M}}{\lambda}\Theta\tau_p\,.$$

Substituting in for $\mathcal{E}_s$ delivers finally an equation of the form $\Xi_1-\Xi_2\Theta\tau_p$, where

$$\Xi_1\equiv\frac{\lambda+(1-\lambda)\Gamma_c}{(1-\lambda)\Gamma_c\gamma_c}\left(\rho(1-\rho)(1-\tilde{s})\chi+(1-\rho\tilde{s})\rho\mathcal{M}-(1-\rho)\phi_y\right),$$

$$\Xi_2\equiv\rho(1-\tilde{s})\frac{(1-\rho)\mathcal{E}_y+\rho\mathcal{M}}{\lambda}+\frac{1}{(1-\lambda)\Gamma_c}\left(\rho(1-\rho)(1-\tilde{s})\chi+(1-\rho\tilde{s})\rho\mathcal{M}-(1-\rho)\phi_y\right).$$

From here the following case distinction applies:

1. $\phi_y>\overline{\phi}_y$ implies $\Xi_2<0$ and $\Xi_1<0$. Thus, we have $\mathcal{M}_g\gtreqless 0$ if $\tau_p\gtreqless\frac{1}{\Theta}\frac{\Xi_1}{\Xi_2}>0$.
2. $\phi_y=\overline{\phi}_y$ implies $\Xi_2=0$ and $\Xi_1<0$. Thus, we have $\mathcal{M}_g<0$.
3. $\overline{\phi}_y>\phi_y>\underline{\phi}_y$ implies $\Xi_2>0$ and $\Xi_1<0$. Thus, we have $\mathcal{M}_g\gtreqless 0$ if $\tau_p\lesseqgtr\frac{1}{\Theta}\frac{\Xi_1}{\Xi_2}<0$.
4. $\phi_y=\underline{\phi}_y$ implies $\Xi_2>0$ and $\Xi_1=0$. Thus, we have $\mathcal{M}_g\gtreqless 0$ if $\tau_p\lesseqgtr 0$.
5. $\overline{\phi}_y>\underline{\phi}_y>\phi_y$ implies $\Xi_2>0$ and $\Xi_1>0$. Thus, we have $\mathcal{M}_g\gtreqless 0$ if $\tau_p\lesseqgtr\frac{1}{\Theta}\frac{\Xi_1}{\Xi_2}>0$.

Statement (a). In the case of $\tilde{s}=1$, i.e. for a TANK model, we obtain

$$\Xi_1=\frac{\lambda+(1-\lambda)\Gamma_c}{(1-\lambda)\Gamma_c\gamma_c}(1-\rho)\Big(\rho\mathcal{M}-\phi_y\Big)<0\,,\quad \Xi_2=\frac{1}{(1-\lambda)\Gamma_c}(1-\rho)\Big(\rho\mathcal{M}-\phi_y\Big)<0\,,$$

where the signs follows from the determinacy condition. Moreover, for $\rho=0$, we obtain

$$\Xi_1=-\frac{\lambda+(1-\lambda)\Gamma_c}{(1-\lambda)\Gamma_c\gamma_c}\phi_y<0\,,\quad \Xi_2=-\frac{1}{(1-\lambda)\Gamma_c}\phi_y<0\,.$$

As a result, we obtain for both cases that $\mathcal{M}_g\gtreqless 0$ if

$$\tau_p\gtreqless\frac{1}{\Theta}\frac{\Xi_1}{\Xi_2}=\frac{\lambda+(1-\lambda)\Gamma_c}{\Theta\gamma_c}=\frac{1}{\gamma_c}\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda+(1-\lambda)\Gamma_y}=1\,.$$

As a result, demand effects are never strong enough in a TANK economy, such that technology can only increase if one redistributes sufficiently towards savers, i.e., we are in a regressive regime.

Statement (b). In the first case $\phi_y > \overline{\phi}_y$, the tax threshold is given by

$$\tau_p = \frac{\lambda+(1-\lambda)\Gamma_c}{\Theta\gamma_c} \frac{\frac{1}{(1-\lambda)\Gamma_c}\left(\rho(1-\rho)(1-\tilde{s})\chi+(1-\rho\tilde{s})\rho\mathcal{M}-(1-\rho)\phi_y\right)}{\frac{1}{(1-\lambda)\Gamma_c}\left(\rho(1-\rho)(1-\tilde{s})\chi+(1-\rho\tilde{s})\rho\mathcal{M}-(1-\rho)\phi_y\right)+\rho(1-\tilde{s})\frac{(1-\rho)\mathcal{E}_y+\rho\mathcal{M}}{\lambda}},$$

which can be rearranged to

$$\tau_p = 1 - \frac{\rho(1-\tilde{s})\frac{(1-\rho)\mathcal{E}_y+\rho\mathcal{M}}{\lambda}}{\frac{1}{(1-\lambda)\Gamma_c}\left(\rho(1-\rho)(1-\tilde{s})\chi+(1-\rho\tilde{s})\rho\mathcal{M}-(1-\rho)\phi_y\right)+\rho(1-\tilde{s})\frac{(1-\rho)\mathcal{E}_y+\rho\mathcal{M}}{\lambda}} > 1\,,$$

which implies $\mathcal{M}_g < 0$ on $\tau_p \in [0,1]$. Additionally, $\phi_y = \overline{\phi}_y$ implies $\mathcal{M}_g < 0$. Moreover, for $\overline{\phi}_y > \phi_y > \underline{\phi}_y$ we have $\mathcal{M}_g < 0$ if $\tau_p > \frac{1}{\Theta}\frac{\Xi_1}{\Xi_2}$, which is satisfied throughout on $\tau_p \in [0,1]$ as $\frac{1}{\Theta}\frac{\Xi_1}{\Xi_2} < 0$ in this case. For $\phi_y = \underline{\phi}_y = \phi_y^*$, we obtain $\mathcal{M}_g = 0$ if $\tau_p = 0$ and $\mathcal{M}_g < 0$ if $\tau_p \in (0,1]$. Finally, $\overline{\phi}_y > \underline{\phi}_y > \phi_y$ implies $\mathcal{M}_g \gtreqless 0$ if $\tau_p \lesseqgtr \tau_p^* = \frac{1}{\Theta}\frac{\Xi_1}{\Xi_2} \in (0,1)$. To show that $\tau_p^* < \tilde{\tau}_p$ note that the technology growth multiplier writes $\mathcal{M}_g = \Lambda_y\mathcal{M}_y - \Lambda_s$, where $\Lambda_y > 0$ and

$$\begin{aligned}
\Lambda_s = \frac{(1-\rho)\mathcal{E}_s}{1+\mathcal{E}_g(1-\rho)} &= \frac{(1-\rho)}{1+\mathcal{E}_g(1-\rho)}\frac{1}{(1-\lambda)\Gamma_c}\left(\frac{\lambda+(1-\lambda)\Gamma_c}{\gamma_c} - \Theta\tau_p\right)\\
&= \frac{(1-\rho)}{1+\mathcal{E}_g(1-\rho)}\frac{\Theta}{(1-\lambda)\Gamma_c}(1-\tau_p) \geq 0\,.
\end{aligned}$$

As a result, $\mathcal{M}_g$ can only be strictly positive if $\mathcal{M}_y$ is strictly positive. This implies $\tau_p^* = \frac{1}{\Theta}\frac{\Xi_1}{\Xi_2} < \tilde{\tau}_p < 1$, which concludes the proof. □

B.6.2 *Proof Corollary 1*

PROOF. The output response at period $t+z$ is $\ln\left(\frac{y_{t+z}}{y}\right) = \rho^z\mathcal{M}_y$, respectively the endogenous technology growth response $\ln\left(\frac{g^A_{t+z+1}}{g^A}\right) = \rho^z\mathcal{M}_g$. As a result, we have

$$\begin{aligned}
\ln y_{t+z} &= \ln y + \rho^z\mathcal{M}_y\,,\\
\ln Y_{t+z} &= \ln A_{t+z} + \ln y + \rho^z\mathcal{M}_y\,,\\
\ln g^A_{t+z+1} &= \ln g^A + \rho^z\mathcal{M}_g\,,\\
\ln A_{t+z+1} &= \ln A_{t+z} + \ln g^A + \rho^z\mathcal{M}_g\,.
\end{aligned}$$

Consequently, we can rewrite

$$\ln Y_{t+z} = \ln A_{t+z} + \ln y + \rho^z\mathcal{M}_y$$

$$= \ln A_{t+z-1} + \ln g^A + \rho^{z-1}\mathcal{M}_g + \ln y + \rho^z \mathcal{M}_y$$

$$= \ln A_t + \sum_{i=0}^{z-1} \rho^i \mathcal{M}_g + z \ln g^A + \ln y + \rho^z \mathcal{M}_y \,.$$

In the absence of shocks, we have $\ln\left(\frac{y_{t+z}^{NS}}{y}\right) = 0$ such that we can state

$$\ln y_{t+z}^{NS} = \ln y \;\Leftrightarrow\; \ln Y_{t+z}^{NS} = \ln A_{t+z}^{NS} + \ln y = \ln A_t + z \ln g^A + \ln y \,.$$

Using the former expression, we can rewrite

$$\ln Y_{t+z} = \begin{cases} \ln Y_{t+z}^{NS} + \sum\limits_{i=0}^{z-1} \rho^i \mathcal{M}_g + \rho^z \mathcal{M}_y & \text{for} \quad \rho > 0 \\ \ln Y_{t+z}^{NS} + \mathcal{M}_g \quad \text{for} \quad \rho = 0 \end{cases}$$

such that the permanent output loss under a persistent shock with $|\rho| < 1$ is

$$\lim_{z\to\infty} \left(\ln Y_{t+z} - \ln Y_{t+z}^{NS}\right) = \frac{\mathcal{M}_g}{1-\rho} \,.$$

In the case $\mathcal{M}_y > 0$ and $\mathcal{M}_g < 0$, a short- vs. long-run stabilization trade-off occurs. Using the relation that $\sum\limits_{i=0}^{z-1} \rho^i = \frac{1-\rho^z}{1-\rho}$, the time span $\hat{z}$ is thus implicitly defined by

$$\frac{1-\rho^{\hat{z}}}{1-\rho}\mathcal{M}_g + \rho^{\hat{z}}\mathcal{M}_y = 0 \quad \Leftrightarrow \quad \rho^{\hat{z}}\left((1-\rho)\mathcal{M}_y - \mathcal{M}_g\right) = -\mathcal{M}_g \,,$$

which can finally be rearranged to

$$\rho^{\hat{z}} = \frac{1}{1-(1-\rho)\dfrac{\mathcal{M}_y}{\mathcal{M}_g}} \quad \Leftrightarrow \quad \hat{z} = -\frac{\ln\left(1-(1-\rho)\dfrac{\mathcal{M}_y}{\mathcal{M}_g}\right)}{\ln(\rho)} > 0 \,.$$

Applying the ceiling function $\lceil\cdot\rceil$ delivers the expression for z^* stated in the main text. □

B.6.3 *Proof Proposition 8*

PROOF. We begin with statement (a). For $\rho = 0$, Proposition 7 implies $\mathcal{M}_y = 0$. As a result, any tax incidence maximizes the impact output level $\ln Y_t$, i.e., $\tau_p^{\max}(0) \in [0,1]$. For any $z > 0$, the output level can be written as $\ln Y_{t+z} = \ln A_t + z \ln g^A + \ln y + \mathcal{M}_g$. Maximizing the level of output is therefore equivalent to maximizing the technology growth multiplier $\mathcal{M}_g$. Importantly, only its numerator depends on τ_p in a linear way, i.e., $\Xi_1 - \Xi_1\Theta\tau_p$, where $\Xi_1 < 0$ and $\Xi_2 < 0$. As a result, $\mathcal{M}_g$ and $\ln Y_{t+z}$ are maximized under $\tau_p^{\max}(z) = 1\;\forall z > 0$. Similarly, when $\tilde{s} = 1$, we obtain for any ρ that $\mathcal{M}_y = 0$ such that $\tau_p^{\max}(0) \in [0,1]$ follows. For any $z > 0$, however, the output level is now given by

$$\ln Y_{t+z} = \ln A_t + z \ln g^A + \ln y + \sum_{i=0}^{z-1} \rho^i \mathcal{M}_g + \rho^z \mathcal{M}_y \ ,$$

which is again equivalent to maximizing $\mathcal{M}_g$ with respect to τ_p. As before, only the numerator of $\mathcal{M}_g$ depends in a linear way on τ_p, i.e., $\Xi_1 - \Xi_1\Theta\tau_p$, where $\Xi_1 < 0$ and $\Xi_2 < 0$ and the result follows.

To prove statement (b), recall that the denominators of $\mathcal{M}_y$ and $\mathcal{M}_g$ are equal and independent of τ_p. Moreover, their numerators are given by

$$\mathrm{NU}(\mathcal{M}_y) = \frac{1}{\mathcal{E}_y} \frac{1}{1+\mathcal{E}_g(1-\rho)} \rho(1-\tilde{s}) \left[\Lambda_1 - \Lambda_2\Theta\tau_p\right] \ ,$$

$$\mathrm{NU}(\mathcal{M}_g) = \frac{1}{\mathcal{E}_y} \frac{1}{1+\mathcal{E}_g(1-\rho)} \left[\Xi_1 - \Xi_2\Theta\tau_p\right] \ ,$$

where Ξ_1 and Ξ_2 are defined in the proof of Proposition 7. The other both auxiliary parameters are

$$\Lambda_1 = \frac{\lambda + (1-\lambda)\Gamma_c}{(1-\lambda)\Gamma_c\gamma_c} > 0 \, , \quad \Lambda_2 = \frac{1}{(1-\lambda)\Gamma_c} + \frac{1+\mathcal{E}_g(1-\rho)}{\lambda} > 0 \, .$$

Additionally, using the rules for geometric series, we can rewrite

$$\ln Y_{t+z} = \ln A_t + z \ln g^A + \ln y + \rho^z \mathcal{M}_y + \frac{1-\rho^z}{1-\rho} \mathcal{M}_g \, .$$

Consequently, we obtain

$$\mathrm{sgn}\left(\frac{\partial \ln Y_{t+z}}{\partial \tau_p}\right) = -\rho^z \rho (1-\tilde{s})\Lambda_2 - \frac{1-\rho^z}{1-\rho}\Xi_2 \, .$$

From here, we distinguish two cases. In the first case, $0 < \phi_y \leq \overline{\phi}_y$ implies $\Xi_2 \geq 0$. As a result, sgn $\left(\frac{\partial \ln Y_{t+z}}{\partial \tau_p}\right) < 0$ such that $\tau_p^{\max}(z) = 0 \; \forall z \geq 1$. Moreover, maximizing $\ln Y_t$ is achieved by maximizing $\mathcal{M}_y$, resulting in $\tau_p^{\max}(0) = 0$. Taken together, we have $\tau_p^{\max}(z) = 0 \; \forall z \geq 0$. In the second case, $\phi_y > \overline{\phi}_y$ implies $\Xi_2 < 0$. As a result, the overall sign is ambiguous, i.e.,

$$\mathrm{sgn}\left(\frac{\partial \ln Y_{t+z}}{\partial \tau_p}\right) = -\frac{\rho(1-\rho)(1-\tilde{s})\Lambda_2 - \Xi_2}{1-\rho} \left[\rho^z + \frac{\Xi_2}{\rho(1-\rho)(1-\tilde{s})\Lambda_2 - \Xi_2}\right] \gtreqless 0 \, .$$

The term in brackets is zero for $z = \tilde{z}$, where

$$\tilde{z} = \frac{\ln\left(-\dfrac{\Xi_2}{\rho(1-\rho)(1-\tilde{s})\Lambda_2 - \Xi_2}\right)}{\ln \rho} > 0 \, .$$

As a consequence, we obtain sgn $\left(\frac{\partial \ln Y_{t+z}}{\partial \tau_p}\right) < 0$ and thus $\tau_p^{\max}(z) = 0 \; \forall \; 0 < z < \tilde{z}$, sgn $\left(\frac{\partial \ln Y_{t+z}}{\partial \tau_p}\right) = 0$ and thus $\tau_p^{\max}(\tilde{z}) \in [0,1]$, and sgn $\left(\frac{\partial \ln Y_{t+z}}{\partial \tau_p}\right) > 0$ and thus $\tau_p^{\max}(z) =$

$1\ \forall z > \tilde{z}$. As before, for in the case $z = 0$ maximizing $\ln Y_t$ is achieved by maximizing $\mathcal{M}_y$, resulting in $\tau_p^{\max}(0) = 0$. This completes the proof. $\square$

APPENDIX C: Additional Analytical Results

The following proposition presents results on the aggregate effects of a government spending shock in the case of $\tau_p \in \mathbb{R}$ and, thus, generalizes the proposition presented in the main text. The corresponding proof is contained in the one of Proposition 7 and provided in Appendix B.

PROPOSITION 9. *The impact multipliers to output and technology growth following a government spending shock are given by:*

$$\mathcal{M}_y = \frac{\zeta_s - \rho\zeta_{s'} - (\zeta_g - \rho\zeta_{g'})\Lambda_s}{1 + \phi_y\zeta_r - \rho\zeta_f - (\zeta_g - \rho\zeta_{g'})\Lambda_y}\ , \quad \mathcal{M}_g = \frac{(\zeta_s - \rho\zeta_{s'})\Lambda_y - (1 + \phi_y\zeta_r - \rho\zeta_f)\Lambda_s}{1 + \phi_y\zeta_r - \rho\zeta_f - (\zeta_g - \rho\zeta_{g'})\Lambda_y}\ ,$$

where we have used $\Theta = \lambda + (1-\lambda)\Gamma_y$ to define the new auxiliary variables

$$\begin{aligned}
\zeta_s &= \frac{1 - \gamma_c\tau_p\Theta}{1 - \lambda\gamma_c\chi}\ , \\
\zeta_{s'} &= \left(s + (1-s)\frac{1-\lambda}{\lambda}\frac{\tau_p\Theta}{1/\gamma_c - \Theta\tau_p}\right)\zeta_s\ , \\
\xi_s &= (g^A - 1)\frac{1 - \gamma_c\tau_p\Theta}{g^A(1-\lambda\gamma_c) + \gamma_c(\lambda - 1)}\ , \\
\Lambda_s &= (1-\rho)\xi_s/(1-\rho\xi_{g'})\ .
\end{aligned}$$

(a) *If $\rho = 0$ or $s = 1$, government spending has no effect on output, i.e., $\mathcal{M}_y = 0$, but has an ambiguous effect on technological growth, i.e., $\mathcal{M}_g \gtreqless 0$ if $\tau_p \gtreqless 1/(\Theta\gamma_c)$.*

(b) *If $\rho > 0$ and $s < 1$, government spending has ambiguous effects on output, i.e., $\mathcal{M}_y \gtreqless 0$ if*

$$\tau_p \lesseqgtr \tilde{\tau}_p \equiv \frac{1}{\Theta}\frac{\lambda/\gamma_c}{1 + (1-\rho)\dfrac{1-\gamma_c}{\gamma_c}\dfrac{g^A}{g^A - 1}} \in (0,1)\ .$$

Regarding the response of technology growth, there are three regimes:

(R1) *For $\phi_y > \overline{\phi}_y$, one obtains $\mathcal{M}_g \gtreqless 0$ if $\tau_p \gtreqless \tau_p^*$, where $\tau_p^* \in \mathbb{R}_+$. Additionally, for $\phi_y = \overline{\phi}_y$, one obtains $\mathcal{M}_g < 0$.*

(R2) *For $\overline{\phi}_y > \phi_y > \underline{\phi}_y$, one obtains $\mathcal{M}_g \gtreqless 0$ if $\tau_p \lesseqgtr \tau_p^{**}$, where $\tau_p^{**} \in \mathbb{R}_-$.*

(R3) *For $\overline{\phi}_y > \underline{\phi}_y > \phi_y$, one obtains $\mathcal{M}_g \gtreqless 0$ if $\tau_p \lesseqgtr \tau_p^{***}$, where $\tau_p^{***} \in \mathbb{R}_+$. Additionally, for $\phi_y = \underline{\phi}_y$, one obtains $\mathcal{M}_g \gtreqless 0$ if $\tau_p \lesseqgtr 0$.*

APPENDIX D: Quantitative Model

This appendix provides the complete specification of the extended HANK-G model whose key features are summarized in Section 4 of the main text. Appendix D.1 describes the full model environment, Appendix D.2 states the definition of the BGP equilibrium, Appendix D.3 details the labor-market splitting rule and the calibration of the incidence functions, Appendix D.4 provides the impulse response functions, and Appendix D.5 derives the welfare criterion. Finally, Appendix D.6 explains the computational algorithm, while Appendix D.7 presents additional results on debt financing.

D.1 Full Model Environment

Since the main text presents a condensed version of the model environment (see Section 4), we spell out the full specification below. As in the main text, the model is stated in stationary form.

D.1.1 *Households* Household i's utility depends on consumption c_{it}, hours worked ℓ_{it}, and employment status $e_{it} \in \{0,1\}$ according to:[36]

$$\mathcal{U}(c_{it}, e_{it}\ell_{it}) = \frac{c_{it}^{1-\sigma}}{1-\sigma} - \nu \frac{(e_{it}\ell_{it})^{1+\varphi}}{1+\varphi}\,.$$

A time-invariant share Λ_E of households are *potential entrepreneurs*; the remainder are workers. Both types supply labor hours ℓ_{it}, face idiosyncratic labor income shocks to productivity h_{it} and employment status e_{it}, and differ in their discount factor β_{it}. Workers save only in liquid assets $b_{it} \equiv B_{it}/A_t \geq 0$. Entrepreneurs additionally accumulate an illiquid stock of normalized innovation capital $a_{it} \equiv A_{it}/A_t > 0$, representing knowledge, patents, and entrepreneurial skills, which depreciates at rate δ^A. Note that the union's utility cost is added to the flow utility above when computing welfare.

Agents die with probability p_d each period and are replaced by newborns with zero wealth who inherit their parent's entrepreneurial type, while β and h follow their Markov processes. Bequests are fully taxed; innovations vanish upon death.

The vector of exogenous states is $\mathbf{s}_{it} \equiv (\beta_{it}, h_{it}, e_{it}, \mathbb{I}_i^E)$, where $\mathbb{I}_i^E = 1$ flags a potential entrepreneur. The joint distribution over assets and exogenous states is $m_t(a_{it}, b_{it}, \mathbf{s}_{it})$.

Idiosyncratic heterogeneity and Markov processes. We now details the process behind the three dimensions of persistent heterogeneity: the discount factor β_{it}, idiosyncratic labor productivity h_{it}, and employment status e_{it}.

1. *Discount factor.* To generate a realistic cross-sectional distribution of marginal propensities to consume (Carroll et al., 2017), the discount factor takes three values, $\beta_{it} \in \{\beta_L, \beta_M, \beta_H\}$, with $\beta_L = \beta_M - \Delta^\beta$ and $\beta_H = \beta_M + \Delta^\beta$. Transitions are

[36] The corresponding non-stationary utility is $A_t^{1-\sigma}\left[\frac{c_{it}^{1-\sigma}}{1-\sigma} - \nu\frac{(e_{it}\ell_{it})^{1+\varphi}}{1+\varphi}\right]$, so the disutility of labor $\nu A_t^{1-\sigma}$ scales with technology.

restricted to adjacent types, so the Markov transition matrix $\mathbb{P}^\beta$ satisfies

$$\mathbb{P}^\beta = \begin{array}{c} \\ \beta_L \\ \beta_M \\ \beta_H \end{array} \begin{array}{c} \begin{array}{ccc} \beta_L & \beta_M & \beta_H \end{array} \\ \begin{pmatrix} p_\beta & 1-p_\beta & 0 \\ \frac{1-p_\beta}{2} & p_\beta & \frac{1-p_\beta}{2} \\ 0 & 1-p_\beta & p_\beta \end{pmatrix} \end{array}$$

2. *Labor productivity.* The logarithm of idiosyncratic productivity follows an AR(1) process with autocorrelation ρ_h and innovation ε^h_{it}:

$$\log h_{i,t+1} = \rho_h \log h_{it} + \varepsilon^h_{it} \, , \qquad \varepsilon^h_{it} \sim \mathcal{N}(0, \sigma_h^2) \, .$$

   We normalize $\int_i h_{it}\, di = 1$. In the numerical implementation this process is discretized using the standard Rouwenhorst method into the ten productivity groups $\{h_1, \dots, h_{10}\}$ described in Table D.1, with transition matrix $\mathbb{P}^h(h_{t+1} \mid h_t)$.

3. *Employment status.* Employment $e_{it} \in \{0,1\}$ evolves according to a type-h transition matrix that is allowed to vary over time:

$$\mathbb{P}^e_t(\cdot \mid h_{it}) = \begin{array}{c} \\ e_t = 0 \\ e_t = 1 \end{array} \begin{array}{c} \begin{array}{cc} e_{t+1} = 0 & e_{t+1} = 1 \end{array} \\ \begin{pmatrix} 1 - \mathbb{P}^e_t(1 \mid 0; h_{it}) & \mathbb{P}^e_t(1 \mid 0; h_{it}) \\ \mathbb{P}^e_t(0 \mid 1; h_{it}) & 1 - \mathbb{P}^e_t(0 \mid 1; h_{it}) \end{pmatrix} \end{array} ,$$

   where $\mathbb{P}^e_t(1 \mid 0; h_{it})$ is the job-finding probability and $\mathbb{P}^e_t(0 \mid 1; h_{it})$ is the separation probability, both depending on productivity type h_{it}. Their steady-state values and cyclical calibration are described in Appendix D.3.

D.1.2 *Entrepreneurial Innovation.* By investing an amount $x_{it} \equiv I_{it}/A_t$, entrepreneurs generate innovations with success probability

$$\zeta(x_{it}) = 1 - \exp(-\gamma x_{it}) \, .$$

A successful innovation raises the entrepreneur's knowledge stock by a fixed step ηA_t, where $\eta > 0$ is the step size and A_t the aggregate stock of innovation. Independently of the investment decision, realized innovations are scaled by an idiosyncratic success shock $\iota_{it+1} \sim F_\iota$, which generates heterogeneity in innovation outcomes and hence in entrepreneurial income: investment does not always translate one-for-one into a realized innovation, and large realizations can occur even with modest investment.

Let $d_{i,t+1} \in \{0,1\}$ indicate whether the innovation step is realized. Then normalized innovation capital evolves as

$$a_{it+1} = \frac{\iota_{it+1}(1-\delta^A) a_{it} + \eta \, d_{it+1}}{g^A_{t+1}} \, ,$$

where g^A_{t+1} captures aggregate gross technology growth (obsolescence of the innovation relative to the average one) and δ^A is the depreciation rate of innovation capital. Equivalently, by case distinction:

$$a_{it+1} = \begin{cases} \iota_{it+1} \dfrac{a_{it}(1-\delta^A)+\eta}{g^A_{t+1}}, & \text{with prob. } \zeta(x_{it}), \\ \iota_{it+1} \dfrac{a_{it}(1-\delta^A)}{g^A_{t+1}}, & \text{with prob. } 1-\zeta(x_{it}). \end{cases}$$

D.1.3 *Income.* With zero distributed dividends, household income before personal taxes is the sum of labor income, entrepreneurial income net of corporate taxes, and bond income

$$y_{it} = [(1-e_{it})\varsigma + e_{it}]\, w_t h_{it} \ell_{it} + \mathbb{I}^E_{it}(1-\tau_c) R^A_t a_{it} + r_{t-1} b_{it}\,, \tag{10}$$

where $w_t \equiv W_t/A_t$ is the real wage, ς denotes the unemployment replacement rate if unemployed ($e_{it}=0$), $\mathbb{I}^E_{it}$ indicates the entrepreneurial type with return $R^A_t = \Theta_\alpha L_t$, τ_c is the corporate tax rate, and $r_{t-1} = R_{t-1}/\pi_t - 1$ is the net real interest rate between $t-1$ and t. The household budget constraint is

$$c_{it} + x_{it} + g^A_{t+1} b_{it+1} = y_{it} - \mathcal{T}(y_{it}) + b_{it}\,,$$

where $c_{it} \equiv C_{it}/A_t$ and $\mathcal{T}(y_{it})$ is the tax-and-transfer schedule.

D.1.4 *Household Problem.* The household's recursive problem is

$$V(a_{it}, b_{it}, \mathbf{s}_{it}) = \max_{\substack{c_{it}, x_{it}, \\ b_{it+1} \geq 0}} \mathcal{U}(c_{it}, e_{it}\ell_{it}) + \widetilde{\beta}_{it} \mathop{\mathbb{E}}_{\substack{h_{it+1}, \beta_{it+1}, \\ e_{it+1}}} \Big[\mathbb{I}^E_i V^x\big(a_{it}, b_{it+1}, \mathbf{s}_{it+1} | x_{it}\big) + (1-\mathbb{I}^E_i) V\big(0, b_{it+1}, \mathbf{s}_{it+1}\big) \Big],$$

subject to $\quad c_{it} + x_{it} + g^A_{t+1} b_{it+1} = y_{it} - \tau(y_{it}) + T(y_{it}) + b_{it}\,, \quad$ and $\quad$ (10) ,

where $\widetilde{\beta}_{it} = \beta_{it}(1-p_d)(g^A_{t+1})^{1-\sigma}$ is the growth-normalized discount factor. The entrepreneurial continuation value from innovating is

$$V^x\big(a_{it}, b_{it+1}, \mathbf{s}_{it+1} \mid x_{it}\big) = \int_\iota \Bigg[(1-\zeta(x_{it})) V\bigg(\frac{\iota a_{it}(1-\delta^A)}{g^A_{t+1}}, b_{it+1}, \mathbf{s}_{it+1} \bigg) + \zeta(x_{it}) V\bigg(\frac{\iota a_{it}(1-\delta^A)+\eta}{g^A_{t+1}}, b_{it+1}, \mathbf{s}_{it+1} \bigg) \Bigg] dF_\iota(\iota).$$

D.1.5 *Unions and Phillips Curve* Nominal rigidity is introduced through union wage setting. Each union k chooses its wage W_{kt} to maximize the utility of a quasi-representative member, subject to quadratic wage adjustment costs that enter utility directly:

$$\sum_{t=0}^{\infty} \left(\prod_{s=0}^{t} \widetilde{\beta}_s \right) \left[\big(u(C_t) - \nu(L_{kt})\big) - \frac{\theta}{2} \left(\frac{W_{kt}}{W_{k,t-1}} - g^A \right)^2 \right],$$

where the demand for employment tasks is $L_{kt} = \left(\frac{W_{kt}}{W_t}\right)^{-\epsilon_w} L_t$ and $W_t = \left(\int W_{kt}^{1-\epsilon_w} dk\right)^{\frac{1}{1-\epsilon_w}}$ is the price index for aggregate employment services. Under the above specification, the Phillips curve is given by

$$\frac{\epsilon_w}{\theta} L_t \left(\nu'(L_t) - u'(C_t) w_t \frac{(\epsilon_w - 1)}{\epsilon_w} \left(1 - \frac{\partial \mathcal{T}(Y_t)}{\partial Y_t}\right)\right) = \pi_t^w (\pi_t^w - g^A) - \widetilde{\beta}_t \mathbb{E}_t \left[\pi_{t+1}^w (\pi_{t+1}^w - g^A)\right] .$$

The steady-state is characterized by

$$L = (\nu')^{-1} \left(u'(C) w_t \frac{\epsilon_w - 1}{\epsilon_w} \left(1 - \frac{\partial \mathcal{T}(Y)}{\partial Y}\right)\right) .$$

D.1.6 *Labor-Market Splitting Rule.* Off the balanced growth path, unions distribute labor adjustments across productivity types and between the intensive and extensive margins according to a simple splitting rule. For each productivity type h, total efficiency-weighted labor is given by

$$L_t^h = h(1 - u_t^h) \ell_t^h .$$

We assume that deviations of labor at type h from its steady-state level satisfy

$$\Delta L_t^h \equiv L_t^h - \overline{L}^h = \mu(h) \big(L_t - \overline{L}\big) , \qquad \text{with} \qquad \int_h \mu(h) \, dF_h(h) = 1 ,$$

where $\overline{L}$ and $\overline{L}^h$ denote steady-state aggregate labor and steady-state labor for productivity type h, respectively. The function $\mu(h)$ therefore governs how aggregate labor fluctuations are distributed across productivity groups.

Within each productivity type, all employed workers supply the same number of hours, and steady-state hours are common across types, so that ℓ_t^h is type-specific off steady state while $\overline{\ell}^h \equiv \overline{\ell}$ in the steady state. A lottery then determines which households are employed and which are unemployed.

We further assume that unions split changes in labor demand for type h between hours per worker and the employment rate. In particular,

$$\Delta \ell_t^h = \frac{\phi(h)}{1 - \overline{u}^h} \cdot \frac{\Delta L_t^h}{h} , \qquad \Delta u_t^h = -\frac{1 - \phi(h)}{\ell_t^h} \cdot \frac{\Delta L_t^h}{h} , \tag{11}$$

where $\phi(h) \in [0, 1]$ measures the share of labor adjustment absorbed by hours per worker, while $1 - \phi(h)$ measures the share absorbed by changes in employment.[37]

Unemployment for type h must also be consistent with flows into and out of employment. Letting $\mathbb{P}_t^e(\cdot \mid \cdot ; h)$ denote transition probabilities conditional on productivity

[37] These expressions follow from the accounting identity

$$\frac{\Delta L_t^h}{h} = (1 - \overline{u}^h) \Delta \ell_t^h - \ell_t^h \Delta u_t^h .$$

h, the law of motion for unemployment can be written as

$$\Delta u_{t+1}^{h} = \int_i \Big(\underbrace{\mathbb{P}_t^e(e_{i,t+1}=0 \mid e_{it}=1; h_{it}) \mathbb{1}_{e_{it}=1}}_{\text{job separations among workers of type } h} - \underbrace{\mathbb{P}_t^e(e_{i,t+1}=1 \mid e_{it}=0; h_{it}) \mathbb{1}_{e_{it}=0}}_{\text{jobs found among unemployed of type } h} \Big) \mathbb{P}^h(h \mid h_{it}) \, di \,. \tag{12}$$

This expression makes clear that unemployment risk for a given productivity group reflects both separation and job-finding flows, including transitions from other productivity states.[38]

To discipline the decomposition of unemployment fluctuations into separations and job-finding, let $p_t^u(h)$ denote the separation rate that rationalizes the observed change in unemployment for type h, holding job-finding rates fixed at their steady-state values. We then parameterize actual separations as

$$\mathbb{P}_t^e(e'=0 \mid e=1; h) = \phi^s p_t^u(h) + (1-\phi^s)\overline{\mathbb{P}}^e(e'=0 \mid e=1; h) \,, \quad \forall h \,,$$

where $\phi^s \in [0,1]$ controls the share of unemployment variation accounted for by separations. Job-finding rates are then determined residually so as to match the implied unemployment dynamics for each productivity group. This parsimonious structure allows the model to reproduce both the level and the persistence of unemployment risk across workers with different labor income exposure.

D.1.7 *Government and Monetary Policy* The government issues bonds and collects tax revenues. Income taxes and transfers are of the form specified by Ferriere et al. (2023), whereby the average tax rate and transfer are given by:

$$\tau(y_{it}) = \exp\left(\log(\vartheta_t)\,(y_{it}/y)^{-2\varrho_t}\right) \,, \quad \text{and} \quad T(y_{it}) = \underline{T} y \frac{2\exp\{-\xi\,(y_{it}/y)\}}{1+\exp\{-\xi\,(y_{it}/y)\}} \,.$$

The parameter $\varrho_t \in (-\infty, 1)$ governs tax progressivity, ϑ_t controls the average tax rate, and y is the steady-state mean income in the economy. The tax schedule is linear when $\varrho_t = 0$, progressive when $\varrho_t \in (0,1)$, and regressive when $\varrho_t < 0$. For transfers, the parameter $\underline{T}$ denotes transfers to a household with zero income as a multiple of steady-state mean income, while ξ determines how quickly transfers phase out with income. When $\xi = 0$, transfers are lump-sum. As ξ becomes larger, transfers phase out faster. The total tax-and-transfer schedule is $\mathcal{T}(y_{it}) = \tau(y_{it})y_{it} - T(y_{it})$.

Total revenues of the government are the sum of income taxes net of transfers, bequest taxes ($p_d b_{it}$), and corporate taxes. They are used to finance an exogenous level of government spending s_t^G, interest payments on bonds $r_{t-1}b_t^G$, and unemployment benefits. The government budget is:

$$g_{t+1}^A b_{t+1}^G + \int [\mathcal{T}(y_{it}) + p_d b_{it} - (1-e_{it})\varsigma w_t h_{it} \ell_{it}] dm_t(a_{it}, b_{it}, \mathbf{s}_{it}) + \tau_c R_t^A = (1+r_{t-1})b_t^G + s_t^G \,.$$

[38] No terms involving population shares appear here because the law of motion is written in integral form over individuals rather than in group-mass form. The integration measure already accounts for the mass of each origin productivity group, while $\mathbb{P}^h(h \mid h_{it})$ maps those individuals into destination type h.

Furthermore, monetary policy sets the nominal interest rate according to a Taylor rule, which depends on price inflation and output:

$$i_t = r + \phi_\pi (\pi_t - 1) + \phi_y (y_t/y - 1) \ .$$

D.2 Definition Balanced Growth Path Equilibrium

DEFINITION 1. Given a stock of innovations A_t, a *balanced growth path general equilibrium* is a sequence of prices $\{P_t, W_t, \pi_t, \pi_t^w, r_{t-1}, i_t\}$, sequences of taxes and transfers, a gross technology growth rate g_{t+1}^A, individual policy functions $\{c_{it}(a_{it}, b_{it}, \mathbf{s}_{it}),$ $b_{it+1}(a_{it}, b_{it}, \mathbf{s}_{it}),$ $x_{it}(a_{it}, b_{it}, \mathbf{s}_{it})\}$, a measure of households over states $m_t(a_{it}, b_{it}, \mathbf{s}_{it})$, and aggregate variables $\{y_t, L_t, s_t^G, b_t^G\}$ such that:

1. Households, unions, and firms optimally solve their respective programs.
2. Monetary and fiscal policy follow their rules and constraints.
3. Goods markets clear:
$$s_t^G + \int [c(a_{it}, b_{it}, \mathbf{s}_{it}) + x(a_{it}, b_{it}, \mathbf{s}_{it})] \, dm_t(a_{it}, b_{it}, \mathbf{s}_{it}) = y_t \ .$$
4. Bond markets clear: $B_t \equiv \int_i b_{it} \, di = B_t^G$.
5. The measure of households is time-invariant.
6. On the balanced growth path, aggregate technology growth is obtained by aggregating the individual law of motion for innovation capital. Let $\Pr(d_{it+1} = 1) = \zeta(x_{it})$, and let $\iota_{it+1} \sim F_\iota$ be an idiosyncratic mean-one shock. At the individual level, one obtains:
$$A_{i,t+1} = (1 - \delta^A) A_{it} \iota_{it+1} + \eta A_t d_{it+1}.$$
Aggregating over households with entrepreneurial ability gives
$$A_{t+1} = (1 - \delta^A) \iint_\iota A_{it} \iota \, dF_\iota(\iota) \, dm_t^E(a_{it}, b_{it}, \mathbf{s}_{it}) + \eta A_t \int \zeta\big(x_{it}(a_{it}, b_{it}, \mathbf{s}_{it})\big) \, dm_t^E(a_{it}, b_{it}, \mathbf{s}_{it}).$$
Since ι is iid and satisfies $\int_\iota \iota \, dF_\iota(\iota) = 1$ and that there is a continuum of entrepreneurs in each state, idiosyncratic innovation success risk washes out in the aggregate, so the measure of successful innovations is given by the integral of success probabilities. Hence,
$$A_{t+1} = (1 - \delta^A) A_t + \eta A_t \int \zeta\big(x_{it}(a_{it}, b_{it}, \mathbf{s}_{it})\big) \, dm_t^E(a_{it}, b_{it}, \mathbf{s}_{it}).$$
Dividing by A_t yields
$$g_{t+1}^A \equiv \frac{A_{t+1}}{A_t} = (1 - \delta^A) + \eta \int \zeta\big(x_{it}(a_{it}, b_{it}, \mathbf{s}_{it})\big) \, dm_t^E(a_{it}, b_{it}, \mathbf{s}_{it}).$$

On the balanced growth path, $m_t^E = m^E$ and policy functions are stationary, so $g_{t+1}^A = g^A$ is constant.

D.3 Labor-Market Calibration and Incidence Functions

D.3.1 *Steady-State Employment* We use panel data from the *Current Population Survey* (CPS) over 1989–2022, restricting the sample to individuals aged 20–65. Workers are sorted into wage bins that map to the model's productivity groups h. Because wages are not observed for unemployed individuals, we impute latent wages using observable characteristics (sex, age, education level, occupation, etc.).

For each h-group, the steady-state job-finding rate, $\bar{\mathbb{P}}^e(e_{it} = 1 \mid e_{it} = 0; h_{it})$, is set equal to its long-run average in the CPS. Given these values, the steady-state separation rate, $\bar{\mathbb{P}}^e(e_{it} = 0 \mid e_{it} = 1; h_{it})$, is chosen so that the model matches the long-run unemployment rate of each group observed in the CPS.

D.3.2 *Cyclical Incidence* The CPS also provides empirical counterparts for fluctuations in total employment, ΔL_t, average hours by productivity group, $\Delta \ell_t^h$, and total hours by productivity group, $\Delta L_t^h/h$. We use these series to discipline the two objects governing cyclical labor-market incidence. From equations (11), we compute

$$\phi(h) = \frac{\Delta \ell_t^h (1 - \bar{u}^h)}{\Delta L_t^h / h}\,, \qquad \mu(h) = \frac{\Delta L_t^h}{\Delta L_t}\,.$$

The parameter $\phi(h)$ measures the share of labor adjustment absorbed by hours per worker rather than by employment. Its value ranges from 0.17 to 0.32, with lower values for low-income groups, indicating that labor adjustment at the bottom of the wage distribution occurs primarily through the extensive margin. The incidence function $\mu(h)$ ranges from 0.28 to 1.61 and is highest for the lowest wage bins, implying that low-wage workers bear a disproportionate share of aggregate employment fluctuations.

To discipline cyclical unemployment risk across groups, we use equation (12) to infer the separation rates that are consistent with the observed fluctuations in unemployment, while keeping job-finding rates fixed at their steady-state values. We denote these implied separation rates by $p_t^u(h)$. By construction, they represent the changes in separation rates that would exactly reproduce the observed variation in unemployment for group h if job-finding rates did not move.

Because part of unemployment fluctuations in the data also reflects variation in job-finding rates, we do not impose $p_t^u(h)$ directly as the actual separation rate. Instead, following Krusell et al. (2017), we assume that actual separation rates are given by

$$\mathbb{P}_t^e(e = 0 \mid e = 1; h) = \phi^s p_t^u(h) + (1 - \phi^s)\bar{\mathbb{P}}^e(e = 0 \mid e = 1; h)\,, \qquad \forall h\,, \tag{13}$$

where ϕ^s measures the share of unemployment fluctuations accounted for by separations. We set $\phi^s \approx 0.3$, so that job separations explain roughly one third of total unemployment variation. Given these separation rates, job-finding rates are then solved residually from equation (12) so that the model matches the observed unemployment dynamics across h-groups.

Table D.1 reports the resulting incidence measures together with the steady-state labor-market statistics by h-group. Note that steady-state unemployment rates take into account movements across h-groups.

TABLE D.1. Labor Market Statistics Across h-Groups

		Incidence		Steady-state		
h-group	Wage bin	$\mu(h)$	$\phi(h)$	$\bar{u}^h$	$\bar{\mathbb{P}}^e(e=1\mid e=0;h)$	$\bar{\mathbb{P}}^e(e=0\mid e=1;h)$
h_1	$[0.000, 0.089]$	1.61	0.25	0.136	0.510	0.090
h_2	$[0.089, 0.174]$	1.41	0.22	0.104	0.497	0.061
h_3	$[0.174, 0.275]$	1.11	0.20	0.063	0.492	0.034
h_4	$[0.275, 0.385]$	1.15	0.18	0.045	0.487	0.025
h_5	$[0.385, 0.500]$	1.18	0.17	0.043	0.481	0.024
h_6	$[0.500, 0.615]$	1.23	0.22	0.042	0.475	0.022
h_7	$[0.615, 0.725]$	0.77	0.25	0.033	0.471	0.015
h_8	$[0.725, 0.826]$	0.64	0.28	0.027	0.467	0.012
h_9	$[0.826, 0.911]$	0.42	0.30	0.023	0.464	0.010
h_{10}	$[0.911, 1.000]$	0.28	0.32	0.021	0.461	0.009

D.4 Impulse Response Functions

Figure D.1 shows the remaining impulse response following the government spending shock in the baseline experiment of the core paper.

D.5 Welfare Criterion

We evaluate welfare at the onset of a deterministic MIT shock by comparing the full transition path of the shocked economy to the initial stationary balanced-growth path. Let $k \in \{0, 1\}$ index the economy, where $k = 0$ denotes the unshocked economy and $k = 1$ the shocked economy. Aggregate technology evolves according to

$$A^k_{t+1} = g^{A,k}_{t+1} A^k_t \,, \quad A^1_0 = A^0_0 \equiv A_0 \,.$$

Thus, any temporary deviation in the growth rate $g^{A,k}_{t+1}$ has a permanent effect on the level of aggregate technology, output, and lifetime utility.

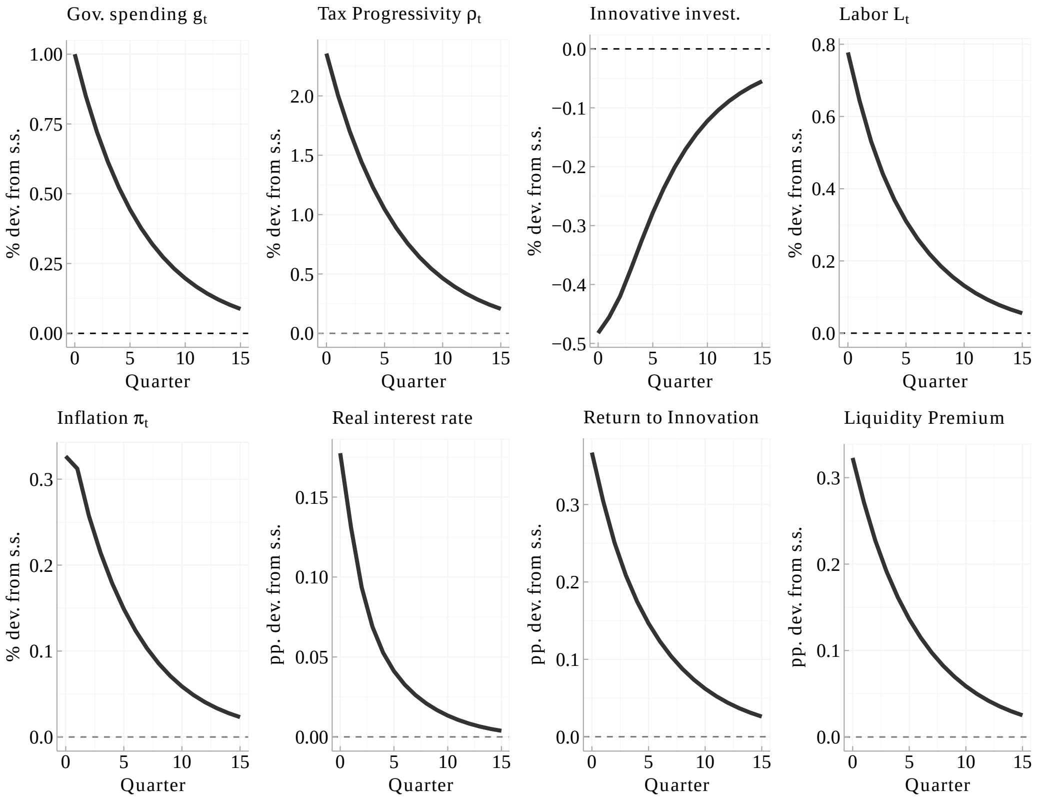
Period utility for household i is

$$u(C_{it}, L_{it}; A_t) = \frac{C_{it}^{1-\sigma}}{1-\sigma} - A_t^{1-\sigma} \left[\nu \frac{L_{it}^{1+\varphi}}{1+\varphi} + \frac{\theta}{2} \left(\frac{W_{\kappa t}}{W_{\kappa, t-1}} - g^A \right)^2 \right] .$$

This specification ensures that the household problem can be written in stationary form after normalizing by aggregate technology. Defining normalized consumption by

$$c^k_{it} \equiv \frac{C^k_{it}}{A^k_t} \,,$$

FIGURE D.1. Aggregate Effects of Government Spending Shock



Note: IRFs to a 1% shock to government spending financed with higher tax progressivity.

period utility becomes

$$u(C_{it}^k, L_{it}^k; A_t^k) = (A_t^k)^{1-\sigma}\left[\frac{(c_{it}^k)^{1-\sigma}}{1-\sigma} - \nu\frac{(L_{it}^k)^{1+\varphi}}{1+\varphi} - \frac{\theta}{2}\left(\frac{W_{\kappa t}}{W_{\kappa,t-1}} - g^A\right)^2\right]$$
$$= (A_t^k)^{1-\sigma}U(c_{it}^k, L_{it}^k)\,.$$

We measure the welfare effect of the shock using consumption-equivalent variation (CEV), defined as the uniform proportional change in consumption along the unshocked path that makes households indifferent between the shocked and unshocked economies. Since aggregate technology evolves according to $A_{t+1}^k = g_{t+1}^{A,k}A_t^k$, define the cumulative technology-growth weight

$$Q_t^k \equiv \left(\prod_{s=0}^{t-1} g_{s+1}^{A,k}\right)^{1-\sigma}, \qquad Q_0^k = 1\,.$$

Let U_{it}^k denote household i's normalized period utility along path k, including unions' utility costs, and let $\mathcal{B}_{it}^k$ denote the cumulative discount factor excluding growth. Aggregate utilitarian welfare along path k is

$$W^k = \sum_{t=0}^{\infty} Q_t^k \int \mathbb{E}_0 \left[\mathcal{B}_{it}^k U_{it}^k\right] dm_0(a_{i0}, b_{i0}, \mathbf{s}_{i0}) \,.$$

This representation separates the welfare effects of normalized allocations, captured by U_{it}^k, from the level effects induced by cumulative technology growth, captured by Q_t^k.

The discounted utility weight of consumption along the unshocked path is

$$\mathcal{C}^0 \equiv \sum_{t=0}^{\infty} Q_t^0 \int \mathbb{E}_0 \left[\mathcal{B}_{it}^0 \frac{(c_{it}^0)^{1-\sigma}}{1-\sigma}\right] dm_0(a_{i0}, b_{i0}, \mathbf{s}_{i0}) \,.$$

The total welfare effect of the shock is therefore

$$\Delta \mathrm{CEV}^{\mathrm{TOT}} = \left(1 + \frac{W^1 - W^0}{\mathcal{C}^0}\right)^{\frac{1}{1-\sigma}} - 1 \,.$$

A positive value of $\Delta\mathrm{CEV}^{\mathrm{TOT}}$ indicates that the shocked transition raises welfare relative to the initial balanced-growth path, while a negative value indicates a welfare loss.

D.5.1 *Decomposition* To isolate the component of welfare that comes from transitional allocations rather than from changes in the path of aggregate technology, we construct a counterfactual welfare measure that keeps the allocation path of the shocked economy but evaluates it using the technology-growth weights of the unshocked economy:

$$\widetilde{W}^{1|0} = \sum_{t=0}^{\infty} Q_t^0 \int \mathbb{E}_0 \left[\mathcal{B}_{it}^1 U_{it}^1\right] dm_0(a_{i0}, b_{i0}, \mathbf{s}_{i0}) \,.$$

This object removes the welfare effects that operate through changes in the level of aggregate technology while preserving the shocked economy's normalized allocation path.

The short-run component of welfare is then measured by

$$\Delta \mathrm{CEV}^{\mathrm{SR}} = \left(1 + \frac{\widetilde{W}^{1|0} - W^0}{\mathcal{C}^0}\right)^{\frac{1}{1-\sigma}} - 1 \,.$$

Equivalently, the total welfare effect can be decomposed as

$$W^1 - W^0 = \underbrace{\left(\widetilde{W}^{1|0} - W^0\right)}_{\text{short-run component}} + \underbrace{\left(W^1 - \widetilde{W}^{1|0}\right)}_{\text{long-run component}} \,.$$

The first term captures the welfare effect of the shocked allocation path when evaluated under the unshocked technology-growth path. The second term captures the additional welfare effect generated by changes in the level of aggregate technology. Thus, even temporary deviations in technology growth can have persistent welfare consequences through their cumulative effect on aggregate technology.

D.6 Computational Algorithm

We solve for the stationary equilibrium by iterating on aggregate prices and quantities until all equilibrium conditions are satisfied.

1. Construct the state-space grid over liquid assets b, normalized innovation capital a, idiosyncratic productivity h, discount factors β, employment status e, and entrepreneurial type $\mathbb{I}^E$.

2. Guess aggregate objects $\{r^b, L, g^A\}$, together with the fiscal variables implied by the steady-state government budget constraint. Given these objects, compute wages, entrepreneurial returns, taxes, transfers, and employment transition probabilities.

3. Conditional on aggregate prices and policies, solve the household problem. We use the endogenous grid method for the consumption-saving decision. For households with entrepreneurial ability, innovation investment x is chosen jointly with savings. The optimal choice of x trades off the current utility cost of investment against the expected continuation-value gain from increasing the probability of a successful innovation.

4. Given the policy functions for consumption, savings, and innovation investment, construct the transition matrix $\mathbf{M}$ induced by the exogenous Markov processes, employment transitions, death and replacement, and the endogenous policy rules. The stationary distribution $\mathcal{G}$ is obtained by iterating

$$\mathcal{G}' = \mathbf{M}\mathcal{G}$$

until convergence.

5. Aggregate individual decisions using the stationary distribution. In particular, compute aggregate assets, labor, output, taxes and transfers, innovation investment, and aggregate technology growth from

$$g^A = (1 - \delta^A) + \eta \int \zeta\big(x(a, b, \mathbf{s})\big)\, dm^E(a, b, \mathbf{s})\,.$$

6. Update the guessed aggregate objects using the equilibrium residuals. The bond return is updated to clear the asset market, labor and wages are updated using the labor-market and Phillips-curve conditions, and technology growth is updated using the aggregate innovation law of motion. We repeat steps 2–5 until all equilibrium conditions are satisfied up to a pre-specified tolerance.

D.6.1 *Transitional Dynamics* We solve transition paths using a standard MIT-shock algorithm over a finite horizon T, chosen sufficiently long so that the economy is arbitrarily close to its terminal balanced-growth path by date T.

We begin by guessing sequences for aggregate prices and quantities,

$$\left\{ r_t^b, w_t, L_t, g_{t+1}^A \right\}_{t=0}^{T},$$

together with the paths of exogenous shocks and fiscal instruments implied by the experiment. Given these sequences, we compute taxes, transfers, employment transition probabilities, and the corresponding paths of income and returns.

Taking the guessed aggregate paths as given, we solve the household problem backward from date T to date 0, using the terminal steady-state value function as the final condition. This backward step delivers time-dependent policy functions for consumption, savings, and innovation investment at each date along the transition.

Given these policy functions, we simulate the distribution of households forward. Starting from the initial stationary distribution $\mathcal{G}_0$, the policy rules and exogenous transition matrices imply a sequence of distributions

$$\{\mathcal{G}_t\}_{t=1}^{T} .$$

This forward step yields the model-implied paths for aggregate savings, labor, output, tax revenues, transfers, innovation investment, and technology growth.

We then compare these model-implied paths with the guessed aggregate paths. The bond return is updated to clear the asset market at each date. The path of technology growth is updated using the aggregate innovation law of motion,

$$g_{t+1}^{A} = (1-\delta^{A}) + \eta \int \zeta\big(x_t(a,b,\mathbf{s})\big)\, dm_t^{E}(a,b,\mathbf{s}) .$$

Wages and wage inflation are updated so as to satisfy the Phillips curve, while labor-market transitions are updated consistently with the labor-splitting rule. Fiscal variables are updated according to the closure rule of the experiment, either period by period or in present value when debt absorbs temporary imbalances.

We iterate on the backward household problem, the forward distribution, and the aggregate updating step until the guessed and model-implied paths coincide up to a pre-specified tolerance.

D.7 Additional Results: Debt Financing and Lump-Sum Repayment

We now ask whether *pure* instantaneous debt financing can attenuate the stabilization trade-off. The idea is simple. If the government issues debt on impact, fiscal consolidation is postponed while the short-run demand expansion, and the associated response of innovation, occurs immediately. If the growth response is sufficiently strong, higher future output can partly offset the repayment burden. In this sense, public debt can become self-financing, echoing recent results Angeletos et al. (2024) but reinforced here as endogenous technology growth helps repay outstanding debt permanently.

We implement debt financing by allowing the government to issue debt on impact and to repay it gradually through lump-sum transfer adjustments. Once repayment starts, the transfer adjustment follows

$$T_{t+j} = \rho_T^{j} T_t, \qquad j \geq 0 ,$$

where $\rho_T \in (0,1)$ governs the speed of fiscal consolidation. The initial adjustment T_t is chosen to satisfy the government budget constraint in present value.

Let $\mathcal{P}_t^{def,T}$ denote the primary balance net of the lump-sum transfer adjustment. Using the present-value government budget constraint over a horizon of length v, the impact transfer adjustment satisfies

$$T_t = \frac{\frac{B^G}{A}\left(g^A_{t+v} - (1+r^b_{t-1})\prod_{s=1}^{v-1}\frac{1+r^b_{t-1+s}}{g^A_{t+s}}\right) - \left(\sum_{j=0}^{v-2}\mathcal{P}^{def,T}_{t+j}\prod_{s=j+1}^{v-1}\frac{1+r^b_{t-1+s}}{g^A_{t+s}} + \mathcal{P}^{def,T}_{t+v-1}\right)}{\sum_{j=0}^{v-2}\rho_T^j\prod_{s=j+1}^{v-1}\frac{1+r^b_{t-1+s}}{g^A_{t+s}} + \rho_T^{v-1}}.$$

This expression pins down the transfer adjustment required to keep the intertemporal government budget constraint satisfied, taking as given the paths of interest rates, technology growth, and primary balances excluding the lump-sum adjustment. Intuitively, T_t makes the discounted value of future transfer adjustments exactly offset the debt issued during the transition.

Table D.2 summarizes the results when the persistent government-spending increase studied in the core paper is repaid with transfer adjustments that decay at rate $\rho_T = 0.95$. Multipliers are positive and no switch time appears because output remains above the laissez-faire path throughout the transition.

TABLE D.2. Debt Financing and Lump-Sum Repayment

FINANCING SCHEME	$\mathcal{M}_y$	$\mathcal{M}_g$	LONG-RUN DEV.	z^*
Debt and lump-sum repayment	1.21	0.02	0.05	NA

Many hybrid financing schemes, combining debt issuance with targeted tax incidence, are also possible. For example, the government could initially finance the shock with debt to harness the short-run demand expansion, and then concentrate the eventual consolidation on the segments of the income distribution that generate the smallest drag on innovation. This exploration lies beyond the scope of this paper.