Inelastic Capital

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(b) Increase in the price of capital

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- Paper: Technology asymmetry and labor spec. make capital supply inelastic

Roadmap

Stylized model: two-sector model and specialized labor (Uzawa + Roy)

- → Long-run responses of (p_K, K) to a capital-demand shock
- → Sufficient statistics: factor shares and labor-supply elasticities

Measurement: factor shares and occupation structure (BEA + CPS)

- → Investment goods are labor intensive + require specialized labor
- → Always true (1960–2020), but even more so with rise of intangibles

Quantitative model: 2017–2023 U.S. calibration

- → K responds about 50% less than in the NGM at 10 years
- → Inelastic supply shifts 10 pp of incidence toward capitalists

Stylized model

Environment

1. Technology asymmetry. C and I have different capital shares (Uzawa, 1961)

$$C = A_C K_C^{\alpha_C} L_C^{1-\alpha_C},$$
$$I = A_I K_I^{\alpha_I} L_I^{1-\alpha_I}.$$

2. Labor specialization. Production of C and I uses different skills (Roy, 1951)

$$1 = \left(\pi L_C^{\frac{1+\phi}{\phi}} + (1-\pi) L_I^{\frac{1+\phi}{\phi}} \right)^{\frac{\phi}{1+\phi}}.$$

Neoclassical growth model: $\alpha_C = \alpha_I$ and $\phi \to \infty$

Demand side

- Household problem (conditional on optimal labor supply)

$$
\begin{aligned}
U_0 &= \max_{\{C_t, I_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-1/\psi}}{1-1/\psi}, \\
C_t + p_{K,t} I_t &= w_{C,t} L_{C,t} + w_{I,t} L_{I,t} + (1-\tau) r_{K,t} K_t, \\
K_{t+1} &= (1-\delta) K_t + I_t.
\end{aligned}
$$

where τ is a capital income distortion (tax, preference, ...).

- First-order conditions

$$
\begin{aligned}
C_t^{-1/\psi} &= \beta (1 + r_{t+1}) C_{t+1}^{-1/\psi}, && \text{(Euler)} \\
(1 + r_{t+1}) p_{K,t} &= (1-\tau) r_{K,t+1} + (1-\delta) p_{K,t+1}. && \text{(Capital valuation)}
\end{aligned}
$$

Equilibrium definition

A sequence of allocations (K_C, K_I, K, L_C, L_I) and prices (w_C, w_I, r_K, p_K, r) such that:

1. firms maximize profit given prices;
2. households maximize utility given prices;
3. capital follows its accumulation equation;
4. factor markets clear.

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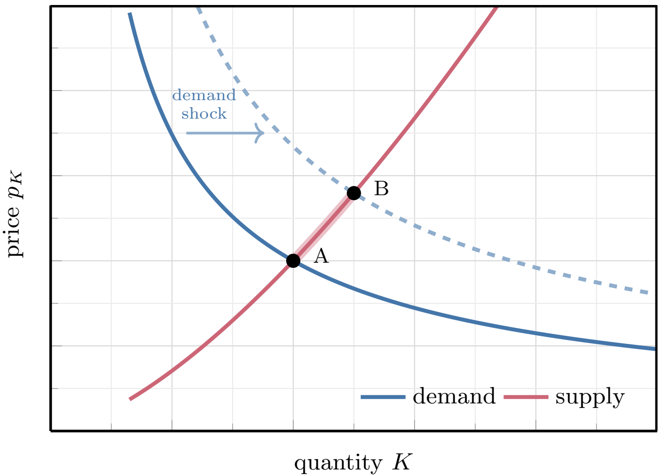
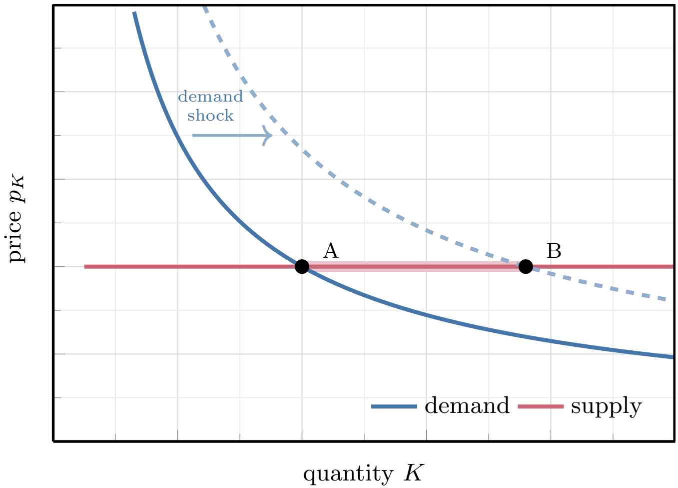
3. capital follows its accumulation equation;
4. factor markets clear.

Uzawa's capital-intensity condition $\alpha_C > \alpha_I$ ensures global stability.

Long-run equilibrium



Long-run equilibrium: NGM special case



Long-run comparative static

$$\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)} = \underbrace{\frac{\alpha_C - \alpha_I}{1-\alpha_I}}_{\text{technology asymmetry}} + \underbrace{\frac{1}{1+\phi}\frac{1-\alpha_C}{\alpha_C}\frac{\lambda_K}{\lambda_C}}_{\text{labor specialization}} \qquad > 0$$

$$\frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau)} = \frac{1}{\lambda_L}\left[1 - \underbrace{(\alpha_C - \alpha_I)\left(\frac{\lambda_C}{1-\alpha_I} + \frac{\lambda_I}{\alpha_C}\right)}_{\text{technology asymmetry}} - \underbrace{\frac{1}{1+\phi}\frac{1-\alpha_C}{\alpha_C}\lambda_K}_{\text{labor specialization}}\right] \qquad \leq \frac{1}{\lambda_L}.$$

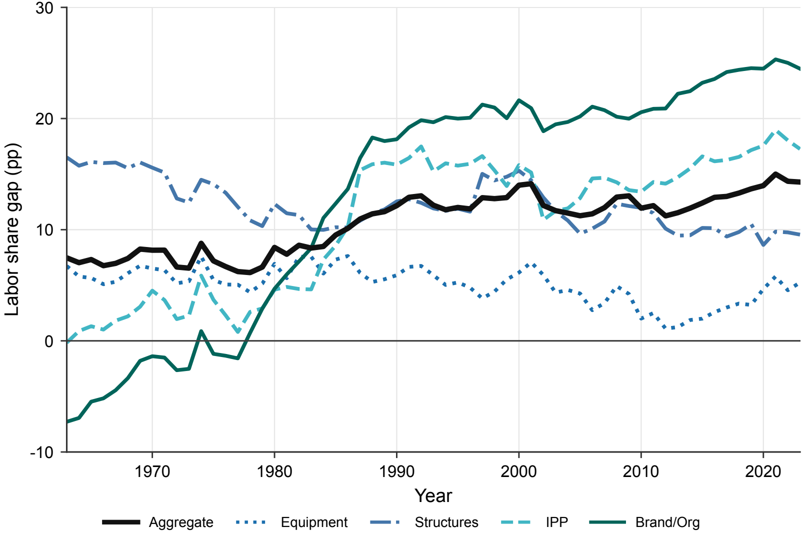
Sufficient statistics: factor shares (α, λ) and labor-supply elasticity ϕ

Measurement

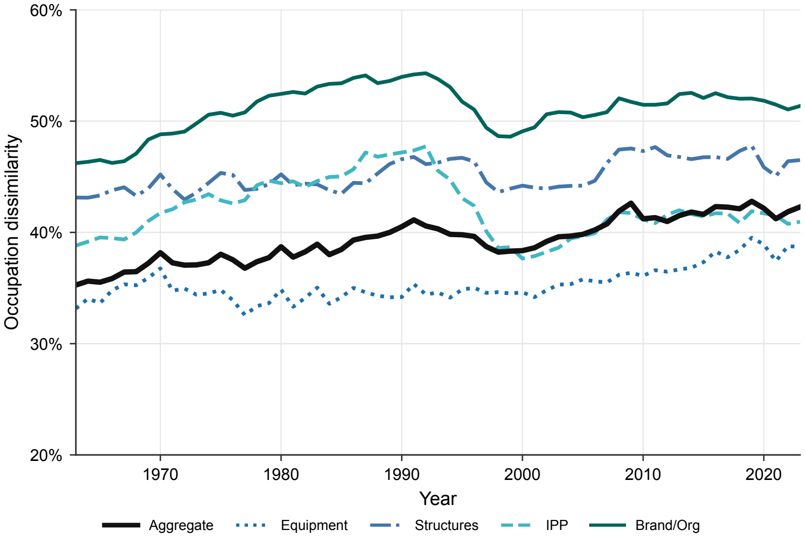
Methodology

- Factor shares and occupation structure for consumption and four investment types
- BEA Use–Make tables: consolidate factor use; capitalize organizational services
- CPS wages and transitions: occupation structure, mobility, and cross-wage elasticities

Labor share gap



Occupation dissimilarity index



Quantitative model

Ingredients

- Four capital types; five final uses; 3×66 occupation cells; land $(N = 4,\ \ F = 199)$

- Capital adjustment costs

$$\mathrm{d}\log\left(\frac{I_{sn,t}}{K_{sn,t}}\right) = \frac{1}{\theta_n}\,\mathrm{d}\log\left(\frac{p_{K_{sn},t}}{p_{I_n,t}}\right)$$

p_{I_n} is the investment price, $p_{K_{sn}}$ the value of installed capital, and θ_n the adjustment-cost parameter.

- Labor-supply dynamics

$$\mathrm{d}\log L_{f,t+1} = (1-\nu)\,\mathrm{d}\log L_{f,t} + \nu\eta\,\mathrm{d}\log\frac{p_{L_f,t}}{p_{L,t}},$$

$$\mathrm{d}\log L_{fo,t+1} = \mathrm{d}\log L_{f,t+1} + (1-\mu_f)\,\mathrm{d}\log\frac{L_{fo,t}}{L_{f,t}} + \mu_f\sum_{o'}\phi_{foo'}\,\mathrm{d}\log\frac{p_{L_{fo'},t}}{p_{L_f,t}}.$$

$p_{L_{fo}}, p_{L_f}, p_L$ are human-wealth indexes; $\nu, \eta, \mu_f, \phi_{foo'}$ govern cohort, education, occupation, and cross-wage responses.

Calibration

Parameter	Value	Targeted moment	Reference
α, λ, δ	Table 1	sectoral factor/output shares	BEA-KLEMS
ν	0.03	cohort replacement rate of 3% annual	Adão et al. (2024)
η	0.4	education choice elasticity of 0.4	Adão et al. (2024)
μ_f	(0.08, 0.07, 0.06)	annual occupation transition speeds	CPS
$\phi_{foo'}$	Table A1	occupation cross-elasticities	CPS
θ_n	(0.17, 0.04, 0.27, 0.27)	quadratic adjustment cost of 1.5	Chodorow-Reich et al. (2026)

Table 1: factor shares   Table 2: model equations   Table A1: occupation elasticities

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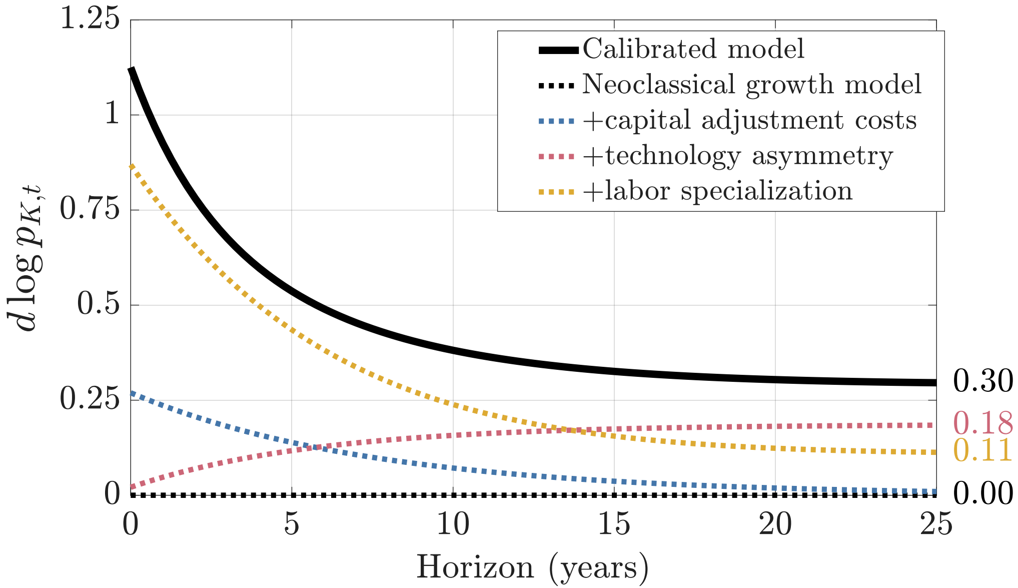
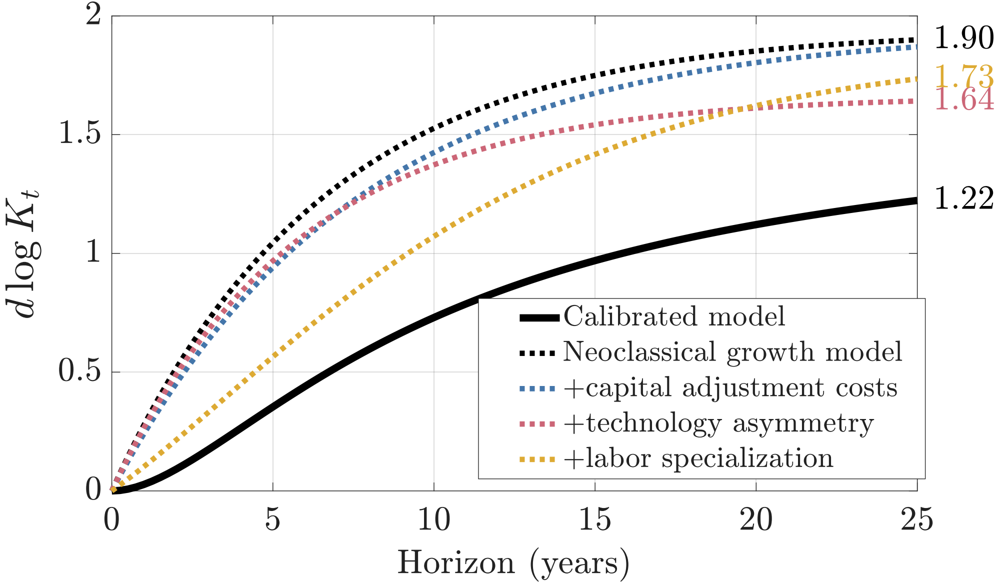
Next. Simulate a permanent, unanticipated demand shock $\mathrm{d}\log(1-\tau) = 1$ on prices and quantities



Calibrated model
Neoclassical growth model
+capital adjustment costs
+technology asymmetry
+labor specialization
$d \log p_{K,t}$
1.25
1
0.75
0.5
0.25
0
0
5
10
15
20
25
Horizon (years)
0.30
0.18
0.11
0.00



$d \log K_t$
Horizon (years)
0
0.5
1
1.5
2
0
5
10
15
20
25
1.90
1.73
1.64
1.22
Calibrated model
Neoclassical growth model
+capital adjustment costs
+technology asymmetry
+labor specialization

	On impact	Medium-run		Long-run	
	$\mathrm{d}\log p_{K,\mathbf{0}}$	$\mathrm{d}\log p_{K,\mathbf{10}}$	$\mathrm{d}\log K_{\mathbf{10}}$	$\mathrm{d}\log p_{K,\infty}$	$\mathrm{d}\log K_{\infty}$
Calibrated model	1.12	0.38	0.73	0.30	1.51
Neoclassical growth model	0.00	0.00	1.53	0.00	1.94
Demand-side sensitivities					
intertemporal elasticity $\psi = 0.5$	$+3$	-6	-8	0	0
intertemporal elasticity $\psi = 2$	-3	$+4$	$+5$	0	0
spender–saver $\omega = 0.25$	$+4$	0	0	0	0
spender–saver $\omega = 0.50$	$+9$	-1	0	0	0
discount factor $\rho = 0.98$	-1	$+1$	0	$+3$	$+12$
discount factor $\rho = 0.95$	-4	$+2$	$+1$	$+10$	$+35$
Supply-side extensions					
imported inputs $\sigma_M = 3$	$+3$	-2	$+11$	-2	$+30$
imported inputs $\sigma_M = 5$	$+9$	-1	$+19$	-2	$+60$
capital–labor elasticity $\sigma_{KL} = 0.5$	-18	-10	-24	-5	-72
capital–labor elasticity $\sigma_{KL} = 1.5$	$+11$	$+7$	$+16$	$+5$	$+67$
occupation transition speeds $2 \times \mu_f$	-1	-2	$+1$	0	0
occupation transition speeds $10 \times \mu_f$	-2	-3	$+3$	0	0
occupation elasticities $2 \times \phi_{foo'}$	-1	-3	$+1$	-2	$+3$
occupation elasticities $0.5 \times \phi_{foo'}$	$+1$	$+2$	-1	$+2$	-2
Alternative data construction					
no capitalization of org. services	-11	-6	-4	-10	$+2$
capitalization of financial services	$+6$	$+1$	$+3$	0	$+6$
consolidate government sector	-1	-3	$+1$	-5	$+1$
common industry-tech. assumption	$+1$	-2	$+1$	-5	$+7$

Validation

Paper	Capital	Sample	Quantity variable	Price variable	Data	Model
Goolsbee (1998)	Equip.	1962–1988	Equip. output	Output price	0.48	0.80
			Equip. output	Output price	0.31	0.36
House–Shapiro (2008)	Equip. & struct.	2001–2006	Investment purchases	Pre-tax investment price	0.12	0.85
House et al. (2022)	Equip.	1959–2009	Equip. purchases	Equip. purchase price	-0.07	0.66
			Worker hours	Hourly wage	0.40	0.19
	Struct.		Struct. investment	Struct. price	0.38	0.68

Tax experiment

Tax incidence

$\tau = 0$	Workers				Capitalists				Government	Aggregate
	Total	HS	Col.	Post-col.	Total	Tang.	Intan.	Land	Total	Total
Calibrated model	0.65	0.26	0.22	0.17	0.35	0.20	0.09	0.06	-1.00	0.00
Neoclassical growth model	0.75	0.33	0.23	0.19	0.25	0.09	0.07	0.09	-1.00	0.00
+capital adjustment costs	0.73	0.32	0.23	0.19	0.27	0.10	0.08	0.09	-1.00	0.00
+technology asymmetry	0.76	0.33	0.23	0.20	0.24	0.08	0.07	0.09	-1.00	0.00
+labor specialization	0.70	0.30	0.22	0.18	0.30	0.12	0.09	0.09	-1.00	0.00

Tax incidence

$\tau = 0.2$	Workers				Capitalists				Government	Aggregate
	Total	HS	Col.	Post-col.	Total	Tang.	Intan.	Land	Total	Total
Calibrated model	0.93	0.38	0.31	0.24	0.35	0.21	0.07	0.07	-1.00	0.28
Neoclassical growth model	1.17	0.51	0.36	0.30	0.31	0.11	0.09	0.11	-1.00	0.48
+capital adjustment costs	1.13	0.49	0.35	0.29	0.33	0.12	0.10	0.11	-1.00	0.46
+technology asymmetry	1.14	0.49	0.35	0.30	0.26	0.09	0.07	0.11	-1.00	0.40
+labor specialization	1.07	0.46	0.33	0.28	0.33	0.12	0.10	0.11	-1.00	0.40

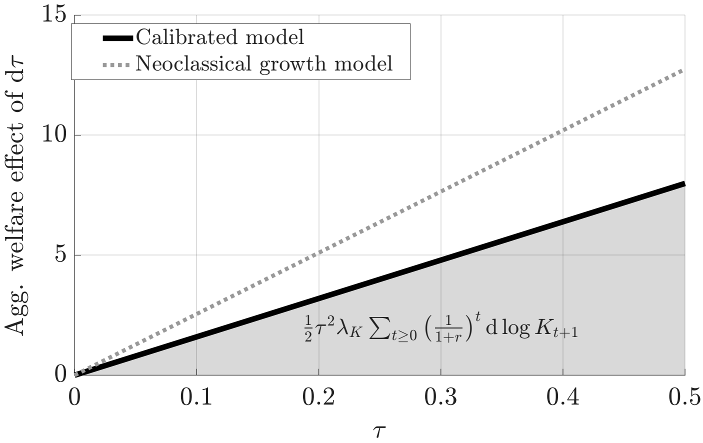
Worker incidence by occupation

Occupation	Log wage change (annuity value)	Occupation	Log wage change (annuity value)
Executives and Managers	1.03	Administrative Support	0.61
Engineering and Computing	0.97	Agriculture, Logging, Extraction	0.60
Mechanics and Construction	0.92	Sales, All	0.59
Health and Science Technicians	0.79	Fire, Police, and Guards	0.56
Management Related	0.77	Teachers, Postsecondary	0.51
Scientists	0.73	Food, Cleaning, Personal Services	0.49
Fabricators and Material Handlers	0.70	Doctors and Lawyers	0.48
Precision Manufacturing	0.68	Teachers and Librarians	0.43
Vehicle Operators	0.67	Nurses and Health Services	0.29
Manufacturing Operators	0.66		
Aggregate			0.67

Capitalist incidence by capital type

Capital type	Post-tax rental rate (annuity value)	Marginal cost (annuity value)	Cash flow (annuity value)
Land	1.17	–	1.17
Structures	0.61	0.29	0.46
Equipment	0.37	0.18	0.27
IPP	0.47	0.38	0.23
Brand/Org	0.53	0.52	0.21
Aggregate	0.54	0.32	0.35

Harberger triangle



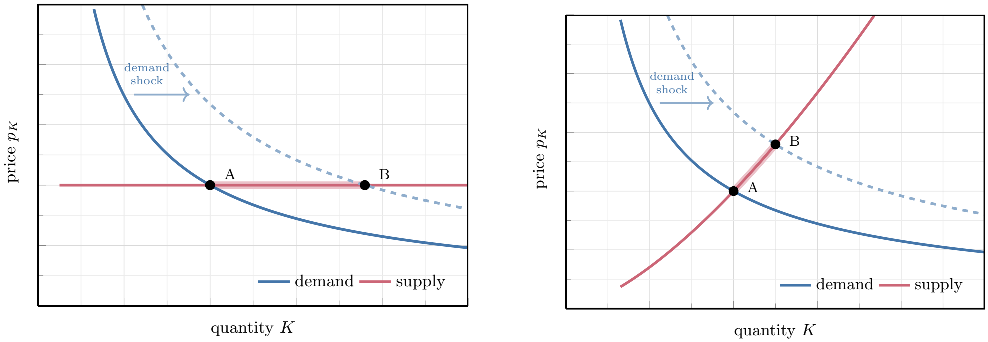
Inelastic capital $\iff$ small Harberger triangle

Conclusion

- Capital supply lies between the neoclassical model and an endowment economy
  - Capital prices absorb 1/3 of the long-run demand shock
  - The channels are 1/3 labor specialization and 2/3 technology asymmetries
- Inelastic capital shifts the incidence of capital taxes
  - A 10 pp shift from workers to capitalists
  - Within labor, executives/managers win; within capital, landowners win

Appendix

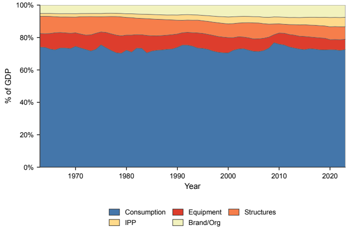
Figure 1: Equilibrium in the capital market after a shift in capital demand



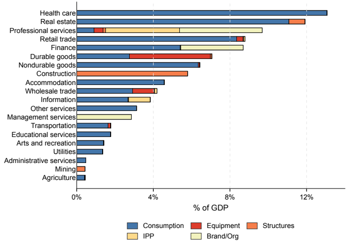
(a) Neoclassical-growth model: $\alpha_C = \alpha_I$, $\phi = \infty$

(b) Stylized model: $\alpha_C > \alpha_I$ or $\phi < \infty$

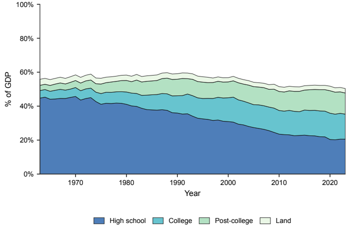
Figure 2: Composition of U.S. value added and labor inputs



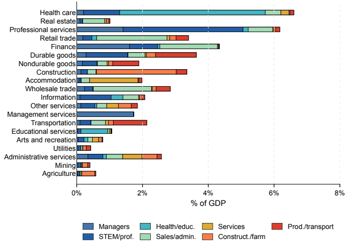
(a) Final uses over time



(b) Final uses by industry

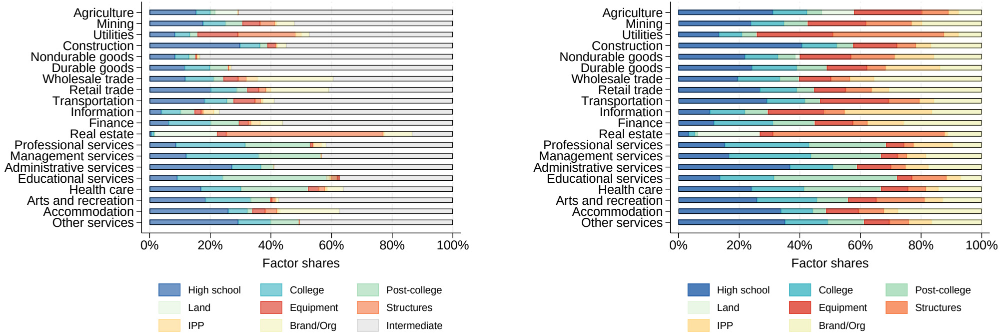


(c) Labor and land shares



(d) Occupation labor content by industry

Figure 3: Direct and total factor requirements by industry



(a) Direct-requirements factor shares

(b) Total-requirements factor shares

Table 1: Factor shares by sector (2014–2023)

	$C+I$ share	Factor shares							
		Labor			Land	Capital			
		HS	College	Post-college		Equipment	Structures	IPP	Brand/Org
$C+I$	1.00	0.22	0.15	0.12	0.03	0.12	0.15	0.09	0.12
C	0.73	0.21	0.13	0.12	0.04	0.12	0.19	0.07	0.12
I	0.27	0.24	0.20	0.15	0.00	0.12	0.06	0.12	0.12
$I_{\text{Equipment}}$	0.07	0.23	0.16	0.10	0.00	0.14	0.07	0.16	0.14
$I_{\text{Structures}}$	0.08	0.37	0.12	0.07	0.00	0.16	0.09	0.05	0.14
I_{IPP}	0.05	0.15	0.26	0.21	0.00	0.08	0.04	0.17	0.09
$I_{\text{Brand/Org}}$	0.08	0.17	0.28	0.24	0.00	0.07	0.04	0.11	0.09

◂ Calibration

Figure 4: Occupation labor content by final use (2014–2023)

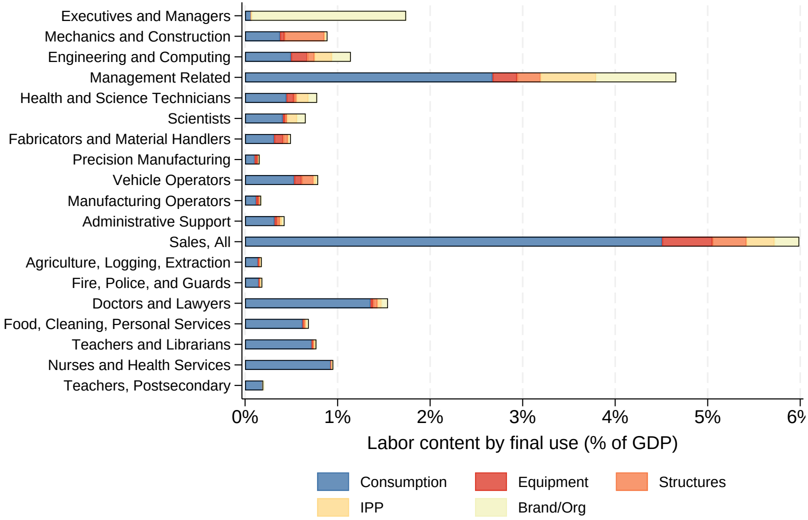
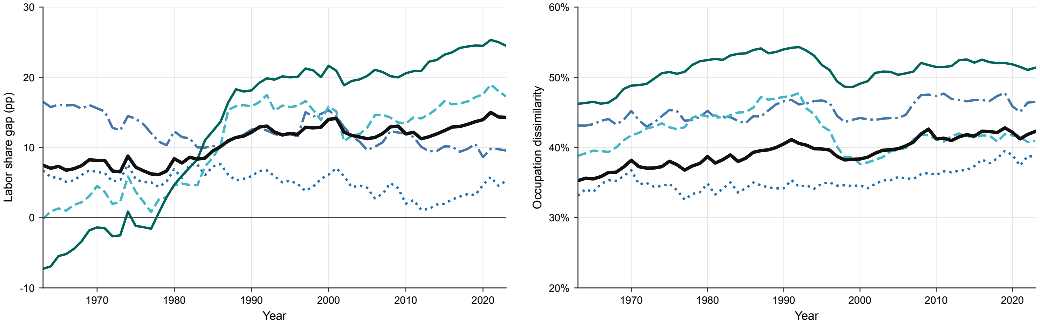


Figure 5: Technology asymmetry and occupational dissimilarity



(a) Technology asymmetry

(b) Occupational dissimilarity

Aggregate Equipment Structures IPP Brand/Org

Table 2: Linearized model equations (intratemporal block)

Intratemporal block	Equation		
C pricing	0	$=$	$\sum_m \alpha_{CK_m} \,\mathrm{d}\log r_{K_{Cm}} + \sum_{fo} \alpha_{CL_{fo}} \,\mathrm{d}\log w_{fo}$
I_n pricing	$\mathrm{d}\log p_{I_n}$	$=$	$\sum_m \alpha_{I_n K_m} \,\mathrm{d}\log r_{K_{I_n m}} + \sum_{fo} \alpha_{I_n L_{fo}} \,\mathrm{d}\log w_{fo}$
K_{sn} demand	$\mathrm{d}\log K_{sn}$	$=$	$\mathrm{d}\log Y_s + \sum_{m \neq n} \alpha_{sK_m} \,\mathrm{d}\log \frac{r_{K_{sm}}}{r_{K_{sn}}} + \sum_{fo} \alpha_{sL_{fo}} \,\mathrm{d}\log \frac{w_{fo}}{r_{K_{sn}}}$
L_{sfo} demand	$\mathrm{d}\log L_{sfo}$	$=$	$\mathrm{d}\log Y_s + \sum_m \alpha_{sK_m} \,\mathrm{d}\log \frac{r_{K_{sm}}}{w_{fo}} + \sum_{f'o' \neq fo} \alpha_{sL_{f'o'}} \,\mathrm{d}\log \frac{w_{f'o'}}{w_{fo}}$
L_{fo} clearing	$\mathrm{d}\log L_{fo}$	$=$	$\sum_s \frac{\lambda_s \alpha_{sL_{fo}}}{\lambda_{L_{fo}}} \,\mathrm{d}\log L_{sfo}$

◂ Calibration

Table 2: Linearized model equations (capital and consumption)

Intertemporal block	Equation		
K_{sn} valuation	$\mathrm{d}\log((1+r_{t+1})p_{K_{sn},t})$	$=$	$\frac{\lambda_{K_{sn}}}{\lambda_{W_{sn}}}\mathrm{d}\log((1-\tau_{n,t+1})r_{K_{sn},t+1}) - \frac{\lambda_{I_{sn}}}{\lambda_{W_{sn}}}\mathrm{d}\log p_{I_n,t+1} + \frac{1}{1+r}\mathrm{d}\log p_{K_{sn},t+1}$
K_{sn} accumulation	$\mathrm{d}\log K_{sn,t+1}$	$=$	$\big(1-(1-\theta_n)\delta_n\big)\mathrm{d}\log K_{sn,t} + (1-\theta_n)\delta_n\,\mathrm{d}\log I_{sn,t}$
I_{sn} investment–q	$\mathrm{d}\log I_{sn,t}$	$=$	$\mathrm{d}\log K_{sn,t} + \frac{1}{\theta_n}\mathrm{d}\log\frac{p_{K_{sn},t}}{p_{I_n,t}}$
C Euler equation	$\mathrm{d}\log C_t$	$=$	$\mathrm{d}\log C_{t+1} - \psi\,\mathrm{d}\log(1+r_{t+1})$

Table 2: Linearized model equations (labor dynamics)

Intertemporal block	Equation		
L_f accumulation	$\mathrm{d}\log L_{f,t+1}$	$=$	$(1-\nu)\,\mathrm{d}\log L_{f,t} + \nu\eta\,\mathrm{d}\log \frac{p_{L_f,t}}{p_{L,t}}$
L_{fo} accumulation	$\mathrm{d}\log L_{fo,t+1}$	$=$	$\mathrm{d}\log L_{f,t+1} + (1-\mu_f)\,\mathrm{d}\log \frac{L_{fo,t}}{L_{f,t}} + \mu_f \sum_{o'} \phi_{foo'}\,\mathrm{d}\log \frac{p_{L_{fo'},t}}{p_{L_f,t}}$
L_{fo} valuation	$\mathrm{d}\log p_{L_{fo},t}$	$=$	$\frac{r+\mu_f}{1+r}\,\mathrm{d}\log w_{fo,t+1} - \mathrm{d}\log(1+r_{t+1}) + \frac{1-\mu_f}{1+r}\,\mathrm{d}\log p_{L_{fo},t+1}$

p_{L_f} and p_L are income-share-weighted value indexes across occupations and education groups.