Growth at the Resource Frontier*

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July 4th, 2026

Abstract

Much of human progress comes from innovations that expand the frontier of usable resources. A distinctive feature of resources outside the frontier is that property rights are poorly defined. In important historical cases, we document that innovators appropriated such resources in anticipation of extraordinary rents, triggering conflicts with counterparties. These observations do not fit neatly into existing frameworks in which property rights precede innovation, appropriation is purely redistributive, or monopoly rents sustain economic growth. Motivated by these observations, we present a model of growth at the resource frontier that minimally departs from traditional models of expanding varieties. The model rationalizes how property rights emerge as a consequence of innovation, establishes that appropriation rents can incentivize innovation, and ultimately shows that economic growth at the resource frontier frequently comes together with appropriation and conflict.

1 Introduction

Much of human progress comes from transformative innovations that expand the frontier of resources available for economic use. Historically, eras have been defined by new materials, from the Stone Age to the Bronze and Iron Ages. The mastery of nature was a hallmark of Enlightenment culture, becoming a key driver of modern economic growth

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(Mokyr, 2016). Advances in mining and drilling technologies, which transformed underground fossil fuels into sources of energy, ushered in the Industrial Age. In recent decades, the resource frontier has expanded into immaterial and digital domains. Communication technologies converted the electromagnetic spectrum into a medium for radio and television broadcasting, while information technologies have turned data into an input for predicting consumer behavior and training artificial intelligence models. Looking ahead, the search for new resources is poised to move into space.

A distinctive feature of resources outside the frontier is that property rights — who owns the resources and how they can be used — are typically poorly defined. Because these resources had been overlooked or inaccessible prior to the innovations that revealed their value or brought them within reach, it had not yet been worthwhile or even possible to establish ownership and specify rights over their many potential uses.[1]

In important historical cases, innovators appropriated resources outside the frontier in anticipation of extraordinary rents, triggering conflicts with counterparties asserting competing claims. Consider generative AI companies that trained models using online content, leading to disputes over fair use; digital advertising platforms that appropriated consumer-generated data, giving rise to conflicts over ownership and privacy; radio broadcasters that took frequencies of the electromagnetic spectrum, interfering with one another's transmissions; and oil prospectors that captured oil from shared underground reservoirs, resulting in disputes with adjacent landholders. In each case, property rights over these resources emerged through litigation and regulation, often validating the initial acts of appropriation and the rents they generated.

These observations of innovation with appropriation at the resource frontier do not fit neatly into existing frameworks in political economy and economic growth. Property rights were not a precondition for innovation but instead emerged as a consequence, and their ambiguity created opportunities rather than being a hindrance, in contrast to standard political economy accounts of development (Coase, 1960; North, 1981; Acemoglu et al., 2001; La Porta et al., 2008). Moreover, appropriation was intertwined with innovation and value creation, instead of being merely redistributive or extractive (Persson and Tabellini, 1991; Murphy et al., 1993; Acemoglu and Robinson, 2013). Indeed, the expectation of appropriation rents appears to have driven innovation, a source of incentives missing from theories of economic growth where monopoly rents in production sustain

[1] Harold Demsetz argued that property rights emerge when the gains from defining them exceed the costs. In Demsetz (1974)'s account, increases in the value of resources, often due to changes in relative prices, create incentives to define property rights. At the resource frontier, the increase in value arises from the innovative activity itself, which reveals previously overlooked uses of a resource or makes them accessible.

innovation (Romer, 1990; Aghion and Howitt, 1992).

In this paper, we analyze the process of economic growth at the resource frontier. We begin with detailed historical case studies in which innovations that expanded the frontier led to appropriation, property rights disputes, and extraordinary rents, comparing them to innovations which did not. In addition, we present evidence from U.S. law reviews documenting a historical shift in legal attention to appropriation conflicts as the resource frontier expanded from material to immaterial, and later digital, resources. Motivated by these observations, we present a model of endogenous growth at the resource frontier. The model is a minimal, but conceptually important, departure from standard models of expanding varieties (Romer, 1990; Acemoglu, 2008): it allows innovators to discover and appropriate new resource varieties with poorly defined property rights and turn them into new input varieties.[2]

The model clarifies why innovators may choose appropriation and conflict rather than contract with counterparties, implying that property rights emerge as a consequence of innovation rather than being a precondition. The expectation of appropriation rents can therefore incentivize innovation, even in the absence of traditional monopoly rents in production, departing from theories in which such rents are necessary to sustain long-run economic growth. Ultimately, growth at the resource frontier frequently comes together with appropriation and conflict, rather than being separate from or at odds with it, as in accounts where appropriation is purely redistributive or extractive.

Our primary contribution is to document and analyze the process of economic growth at the resource frontier, providing a novel perspective on the political economy of historical development (complementing North, 1981; Acemoglu et al., 2001; Engerman and Sokoloff, 2005; La Porta et al., 2008; Hornbeck, 2010; Dell, 2010) and the drivers of innovation and long-run growth (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992; Jones, 1995). The process of growth at the resource frontier is closely linked to the systematic efforts to understand and exploit the world around us — making ourselves "masters and possessors of nature" (Descartes, 1637) — described in Mokyr (2016) as the cultural foundation of modern economic growth. We identify a distinctive feature of such mastery of nature: resources outside the frontier of human use almost invariably have poorly defined property rights. This feature explains why economic growth at the resource frontier frequently comes together with appropriation and conflict, linking our work to a literature documenting innovation (Beraja et al., 2023a; 2023b) and economic development (Dell and Olken, 2020) alongside extractive institutions.

[2] In standard models, innovations also generate new input varieties but they are produced using *known* resources that already trade in markets with well-defined property rights.

We also contribute to a literature on the economic origins of property rights (Coase, 1960; Demsetz, 1974; Ostrom, 1990) and legal and regulatory institutions (Glaeser and Shleifer, 2002; 2003). Our account emphasizes that, for resources outside the frontier, property rights often emerge as a consequence of innovation, appropriation, and the resolution of conflict via litigation and regulation. This idea also has roots in classical philosophy, bridging John Locke's view that labor and use — in our case, innovation efforts — generate claims to ownership and Thomas Hobbes's view that property becomes legitimate only when defined and enforced by the state.

Our analysis of ambiguity in property rights over resources — and the state as adjudicator — adds to the literature on the economic consequences of legal institutions and property rights regimes (La Porta et al., 1998, 2008; Bowles and Choi, 2019).[3] We show that poorly defined property rights provide incentives — appropriation rents — to innovate at the resource frontier. This relates to recent work linking legal ambiguity to financial innovation (Pistor, 2019) and weak intellectual property rights over ideas to downstream innovation (Williams, 2013; Murray et al., 2016). We discuss the differences between novel ideas and resources outside the frontier in more detail in Section 2.2.

Finally, our work adds to an emerging social science literature that highlights technological breakthroughs in the use of resources. Zuboff (2023) studies how digital technologies turned behavioral data into predictions at the core of surveillance capitalism. Wu (2010) emphasizes how control over communications networks and spectrum shapes access and innovation. Weinzierl (2018) frames space as a new economic frontier whose development depends on unresolved questions of governance and property rights. We provide a framework clarifying the shared features of these advances, and why they so often generate conflict.

2 Case Studies: Innovation, Appropriation, and the Resource Frontier

This section uses case studies to document the process of innovation at the resource frontier and clarify its boundaries. Section 2.1 begins with detailed historical case studies that illustrate defining features of the process: a technological innovation turns a previously unrecognized resource into a productive input, property rights are poorly defined beforehand, innovators appropriate the resource in anticipation of extraordinary rents, and con-

[3] Horwitz (1992) describes historical legal changes in the American judiciary's interpretation of the law (e.g., regarding riparian rights) that supported economic growth in the 19th century.

flicts arise with counterparties asserting competing claims. For comparison, Section 2.2 describes cases where innovation did not lead to conflict, either because the new resource was abundant or non-excludable, property rights were sufficiently well established, or innovation did not expand the resource frontier in the first place.

2.1 Innovation with Appropriation and Conflict

2.1.1 Large Language Models and the Appropriation of Online Content

The origins of artificial intelligence (AI) date back to the 1956 Dartmouth Summer Research Project, marking the field's official birth. For decades, progress was stifled by limited computing power and scarce data to train AI algorithms. But since the early 2000s, the field rapidly advanced, driven by breakthroughs in hardware, computing power, and the explosion of big data online. This progress has recently culminated in the emergence of large language models (LLM's), like ChatGPT and Claude, heralding a transformative era in the industry.

A NOVEL RESOURCE. While training data (e.g., images of faces in training facial recognition software) were previously recognized as valuable inputs into improving AI algorithmic performance, LLM's like ChatGPT relied on a previously overlooked source of data for model training: vast amounts of general online textual content. Such content was available from books, academic articles, websites, and even transcriptions of online video content posted to platforms such as YouTube. This content has proven to be of extraordinary value in training LLM's; freely acquired, it is plausibly a source of substantial economic rents to innovators.[4]

POORLY DEFINED PROPERTY RIGHTS AND APPROPRIATION. Because the value of online content as training data for LLM's was unrecognized, the legal boundaries around the use of such content were loosely defined before the advent of technologies like ChatGPT. Innovators like OpenAI (the company that developed ChatGPT) chose to scrape enormous amounts of online text, appropriating this resource for model training. They have argued that their use of publicly available internet data, including from sources protected by copyright, is legal under the "fair use" doctrine, which allows the use of copyrighted

[4] That freely acquired online content is a source of rents can be seen in contrast to the possibility of payment for corpora of text. This was considered by Meta, in 2023, when the firm discussed the acquisition of Simon & Schuster at a price of $10 per book, with the purpose of acquiring text for model training. See https://www.theguardian.com/books/2024/apr/09/meta-discussed-buying-publisher-simon-schuster-to-train-ai, last accessed March 19, 2026.

material for purposes like research and education.[5]

CONFLICT. However, a wide range of copyright holders, including The New York Times, the Merriam Webster Dictionary, Encyclopedia Britannica, and the Authors' Guild, have filed lawsuits against OpenAI, alleging infringement of copyright that materially harms the plaintiffs.[6] While Anthropic, behind the Claude LLM, has settled a case with authors suing for copyright infringement (agreeing to pay them $1.5 Billion), OpenAI's cases remain unsettled at the time of writing.[7] It is clear that all of the companies that developed LLM's took a calculated gamble by appropriating online text content, banking on the expectation that courts and regulatory bodies would endorse their extensive data usage practices without explicit permissions.

2.1.2 Digital Advertising and the Appropriation of Consumer Data

The late 1990s and early 2000s saw the rise of the commercial internet and, with it, new ways to connect businesses to consumers. Search engines, such as Google, and social networks, most importantly Facebook, became important nodes in the world wide web, providing free services to users and generating a huge amount of user traffic. However, traffic did not directly imply monetization: beyond their breakthrough technical products — Google's search algorithm and Facebook's social media platform — both companies depended on pioneering digital advertising models that allowed companies to market their products to consumers in remarkably targeted ways.

A NOVEL RESOURCE. Prior to the innovations of Google and Facebook, personal data such as search queries, clicks, browsing histories, and social interactions were considered a mere byproduct of online activity, described by Zuboff (2023) as "digital exhaust," and holding little obvious economic value. Google transformed these traces of online behavior into a new factor of production — an input into behavioral prediction — when it introduced AdWords (2000) and later AdSense (2003). This relied on storing and analyzing enormous amounts of user data, allowing advertisers to target audiences with unprecedented precision. Facebook followed a parallel path: its platform initially thrived

[5] See https://openai.com/index/openai-and-journalism/, last accessed March 19, 2026.

[6] See https://www.theguardian.com/books/2025/apr/04/us-authors-copyright-lawsuits-against-openai-and-microsoft-combined-in-new-york-with-newspaper-actions, last accessed March 19, 2026; and https://fortune.com/2026/03/18/dictionaries-suing-openai-chatgpt-copyright-infringement/, last accessed March 19, 2026.

[7] See https://www.theguardian.com/technology/2025/sep/05/anthropic-settlement-ai-book-lawsuit, last accessed March 19, 2026.

by connecting friends and communities, but a key breakthrough came when it leveraged its massive trove of demographic and social data to sell highly targeted advertising.

Poorly Defined Property rights and appropriation. The extraordinary rents from digital advertising hinged on the absence of clearly defined property rights over personal data: companies like Google and Facebook did not need to compensate users for the use of their personal data, they simply appropriated it, selling it to third parties, and exploiting it to offer opportunities to advertisers. Because user data was not previously recognized as a valuable resource, property rights governing its use were not clearly established.

Conflict. However, consumers eventually demanded digital privacy and property rights over their online behavioral data. Google has been sued for billions of dollars in the US and UK (and ordered to pay consumers hundreds of millions of dollars) for privacy violations.[8] Facebook paid users hundreds of millions of dollars and was fined $5 billion by the US Federal Trade Commission for violating user privacy as a result of the Cambridge Analytica scandal.[9] Regulators, particularly in Europe, have moved to assert users' rights over their data, notably, the General Data Protection Regulation. However, Google has successfully argued that "blocking it from collecting users' account-related data would 'cripple' an analytics service relied on by millions of app developers."[10] Intensive collection and exploitation of user data continues to generate enormous revenue streams for Google and Facebook (we discuss the value of user data in further detail, below).

2.1.3 Radio and the Utilization of the Electromagnetic Spectrum

The second half of the 19th century saw James Clerk Maxwell propose the existence of electromagnetic waves in the 1860s, and Heinrich Hertz produce radio waves in the 1880s. The "father of radio," Guglielmo Marconi transmitted Morse code across the Atlantic Ocean using radio waves in 1901, and the first commercial radio station, Pittsburgh's KDKA, broadcast in October 1920.

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[8] See https://www.legalfutures.co.uk/latest-news/supreme-court-strikes-down-3bn-google-data-protection-claim; https://www.reuters.com/sustainability/boards-policy-regulation/google-defeats-bid-billions-dollars-new-penalties-us-privacy-class-action-2026-01-30/, both last accessed March 19, 2026.

[9] See https://www.theguardian.com/technology/2025/jul/17/mark-zuckerberg-meta-settlement-privacy, last accessed March 19, 2026.

[10] https://www.reuters.com/sustainability/boards-policy-regulation/google-defeats-bid-billions-dollars-new-penalties-us-privacy-class-action-2026-01-30/, last accessed March 19, 2026.

A NOVEL RESOURCE. When radio emerged in the early 20th century, the core innovation lay not only in the technical development of transmitters and receivers, but also in the recognition that the electromagnetic spectrum itself could serve as a factor of production. Airwaves, once considered a scientific curiosity with little economic significance, became the indispensable medium through which music, news, entertainment, political content, and advertising, could be transmitted to mass audiences.

POORLY DEFINED PROPERTY RIGHTS AND APPROPRIATION. The early years of broadcasting were marked by the rapid entry of local broadcasters, and disorder (e.g., Wu, 2010). By the early 1920s, thousands of amateur and commercial stations transmitted without coordination, often using the same frequencies. Signals collided, programs overlapped, and listeners were left with interference. The problem was not technological, but institutional: broadcasters appropriated frequencies simply by using them, because property rights over the airwaves were undefined.

CONFLICT. It was not ex ante obvious how property rights over the spectrum would be allocated. Indeed, a prominent Illinois Circuit Court case involving the owners of The Chicago Tribune and WGN radio (Tribune Co. v. Oak Leaves Broadcasting Station, 1926) granted property rights on the basis of priority. However, the federal government, with an interest in regulating the resource (and to the benefit of the incumbent broadcasters, such as RCA) responded with the Radio Act of 1927, creating the Federal Radio Commission (FRC) and granting it authority to allocate frequencies "as public convenience, interest, or necessity requires." Acquisition of a license to broadcast was associated with substantial rents. Hazlett (1998, pp. 278-279) writes, "'Public interest' licensing effectively awarded incumbents zero-priced rights, while excluding new entry, thus enhancing license values due to artificial scarcity constraints. Political players (incumbent politicians, bureaucrats, and public interest advocates) shared in the rents by being brought 'into the loop' on broadcast license (and renewal) applications."

2.1.4 Drilling and the Capture of Oil from Underground Fields

Before the industry's emergence in the 1860s, oil was already known as a product; however, it was not obtained in sufficient quantities to support a large-scale industry. This changed in 1859, when Edwin Drake drilled the first oil well in Titusville, Pennsylvania. The industry grew quickly, with Ida Tarbell (1904, p. 21) writing, "By the opening of 1872 [oil men] had produced nearly 40,000,000 barrels of oil, and had raised their product to the fourth place among the exports of the United States, over 152,000,000 gallons going abroad in 1871. (...) As for the market, they had developed it until it included almost every country of the earth — China, the East and West Indies, South America and Africa. Over forty different European ports received refined oil from the United States in 1871."

A NOVEL RESOURCE. In her History of the Standard Oil Company, Tarbell (1904, p. 3) describes the transformation of part of Northwestern Pennsylvania from "little better than a wilderness" into "a bustling trade centre, where towns elbowed each other for place, into which three great trunk railroads had built branches, and every foot of whose soil was fought for by capitalists." This was due, quite simply, to "the discovery and development of a new raw product, petroleum."

POORLY DEFINED PROPERTY RIGHTS AND APPROPRIATION. After Drake's successful well, Yergin (1991, p. 29) describes the ensuing oil boom: "What followed immediately was like a gold rush. The flats in the narrow valley of Oil Creek were quickly leased, and by November of 1860, fifteen months after Drake's discovery, about seventy-five wells were producing, with many more dry holes scarring the earth." The rush to drill occurred despite inherent ambiguity of property rights over underground oil: the boundaries of an underground oil field, and the oil extracted by a well, will not respect the boundaries established by above-ground property rights (see Daintith, 2010). Oil drilling and extraction occurred despite this ambiguity: oil prospectors appropriated the resource even if it was extracted from fields lying beneath land owned by their neighbors.

CONFLICT. Unsurprisingly, disputes arose over the extraction of oil from fields with ambiguous property rights. By the late 19th century, the "Rule of Capture" was applied by US courts to assign property rights to the owner of the well from which oil was extracted. The 1889 decision of the Pennsylvania Supreme Court in Westmoreland & Cambria Natural Gas Co. v. De Wit unambiguously stated: "Water and oil, and still more strongly gas, may be classed by themselves, if the analogy be not too fanciful, as minerals *ferae naturae* (of a wild nature). In common with animals, and unlike other minerals, they have the power and the tendency to escape without the volition of the owner. (...) They belong to the owner of the land, and are part of it, so long as they are on or in it, and are subject to his control; but when they escape, and go into other land, or come under another's control, the title of the former owner is gone. Possession of the land, therefore, is not necessarily possession of the gas. If an adjoining, or even a distant, owner, drills his own land, and taps your gas, so that it comes into his well and under his control, it is no longer yours, but his."[11]

2.1.5 Complementary Inputs

At times, innovation does not directly transform an overlooked resource but instead requires the use of a complementary input that had previously been ignored, underutilized, or treated as a commons. If innovation is interpreted narrowly as the core technical breakthrough, the complementary input may appear peripheral. But if innovation is understood more broadly — as the commercialization, scaling, and deployment of a technology — then the complementary input becomes central to value creation.

Under this broader interpretation, the complementary input is the "resource outside the frontier." Its prior neglect implies that property rights were poorly defined, and its sudden importance creates scope for appropriation and conflict. The relevant innovation lies as much in recognizing how to mobilize this input as in the underlying technology itself.

A salient example is the use of overhead urban space for electricity distribution. In the late 19th century, the new electric lighting companies in New York City distributed electricity using overhead wires that appropriated urban space (a physical, albeit immaterial resource) which was not subject to obvious property rights at the time. Sullivan (1995) describes the ensuing "Overhead Wire Panic" of the 1880s, when "the streets of the city were darkened by dangerous, overhead electric wires." This generated intense legal opposition and a regulatory response that was contested by the electric companies.[12] Multiple legislative efforts were made to force wires underground, but as Hooley (1959) writes, "little public utility legislation was so earnestly contested, so persistently ignored, and so voluminously litigated as the compulsory underground wiring statutes affecting that city which were enacted during the latter part of the nineteenth century." In this case, the legal conflict eventually went against the appropriator: in 1889, the New York Supreme Court dissolved an injunction preventing the City from cutting down dangerous overhead lines, with thousands of feet of wire cut that year — and a substantial (if temporary) decline in electric lighting as a result (Sullivan, 1995).[13]

[11] See Kramer and Anderson (2005).

[12] Hooley (1959) writes that, "no sooner were power lines erected than litigation commenced concerning the manner in which they had been constructed and the various routing patterns that had been effected in different local areas."

[13] This example of the appropriation of "spatial commons" by innovators is not unique. The exploitation of airspace has been necessary for commercial aviation and more recently for the use of drones in the "low-altitude economy" (Maher, 2017). Positions in Low Earth Orbit have also been appropriated by satellite companies, such as Starlink (Weinzierl and Rosseau, 2025).

2.2 Innovation without Conflict

Not all innovations lead to appropriation and conflict. The cases below clarify the boundaries of the process we study. Conflict may be absent when the appropriated resource is abundant or non-excludable, when property rights are sufficiently well established that innovators choose to contract, or when the innovation does not expand the resource frontier in the first place. Anticipating the model below, these cases share a common feature: they generate no expected appropriation rents, and are therefore equivalent from the perspective of innovation incentives (see Section 4.3.2).

Appropriation of Abundant or Non-excludable Resources. In some cases, innovations expand the resource frontier but involve overlooked resources that are abundant or effectively non-excludable. Consider solar energy panels that converted sunlight into electric power. Resource appropriation (of sunlight) did not lead to conflict, and rents arose primarily from product innovation — protected by trade secrets or patents — rather than from control over the resource itself. Therefore, a core mechanism emphasized in this paper, whereby expected rents from appropriation sustain innovation, plays a limited role.

Contracting over New Resources. In some cases, innovators expand the resource frontier but choose to contract with counterparties — rather than appropriation and conflict — because a counterparty has a strong property right claim over the resource beforehand. Consider the development of plastics. DuPont was amongst the first to realize that petroleum refinery waste could be used in the production of synthetic materials. While these byproducts of refining were not recognized as being valuable and commanded no market price, they clearly belonged to oil companies such as Conoco. As a result, DuPont was forced to rely on contracting (rather than appropriation) to acquire crucial petroleum-derived inputs. The most dramatic instance of such contracting was the company's 1981 purchase of the oil company Conoco for 7 billion dollars — the largest ever corporate merger at the time. Time magazine described DuPont's motives: "Du Pont's interest in Conoco is understandable. Petroleum is the raw material for some 80 percent of its products. Like all chemical makers, DuPont has been badly hurt by the surge in oil prices since 1973. Now DuPont will have its own private supply of crude."[14]

[14] Time magazine article reference: "History's Biggest Merger: Du Pont-Conoco" by Charles Alexander, JUL 20, 1981

Online at: https://time.com/archive/6700538/historys-biggest-merger-du-pont-conoco/, last accessed July 2, 2026.

Process and Quality Improvements, and Romerian Innovations. Many important innovations do not expand the resource frontier. In turn, they do not generate conflicts over property rights regarding new resources or appropriation rents. First, innovations which are primarily process or quality improvements, such as the assembly line and Schumpeterian innovations (Aghion et al., 2014). Second, technologies that create new input varieties out of existing, well known resources — what we refer to as Romerian innovations. Examples include steel from iron and coal, computers, early internet technologies, and operating systems. In all these cases, resources are already traded in markets with established property rights, and innovation rents arise through monopoly power or intellectual property protection.[15]

New Ideas versus Resources. The resources we study are either rival, or, when non-rival, excludable. Ideas therefore sit somewhat outside our main cases of interest, even though they share some features with them. Consider first a genuinely novel idea about how to use an existing input (for example, a Romerian innovation). In that case, the natural presumption is that the discoverer owns the idea.[16] This corresponds to a limiting case in our model of Section 5 in which property rights are well defined beforehand. There is no ambiguity, and hence no central role for appropriation or conflict. Consider next a new idea about how to use an existing idea. If appropriation of the existing idea is allowed, then its non-rivalry and non-excludability imply that others can use the existing idea too. As a result, there can be neither rents from appropriating it, nor conflict. This case thus resembles our previous cases of appropriation of abundant or non-excludable resources.[17]

[15] While Romerian innovation can generate speculative rents — particularly when innovators acquire inputs before others update beliefs about their value — these rents stem from informational asymmetries rather than from contested ownership. The innovator can rent or purchase the resource outright; conflict is not intrinsic to the innovation process.

[16] This is different from natural resources — for instance, airspace, seabeds, or extraterrestrial resources — where often the legal presumption is that the sovereign (or humanity as a whole) has a claim.

[17] A final case is one in which an innovator discovers an idea about how to use an existing idea, successfully appropriates that idea, and is then able to exclude others from using it. One example is when a patent aggregator acquires existing intellectual property and assembles it into a portfolio that can be used to block others. We view such cases of appropriation of intellectual property as having had limited impact on historical innovation; rather such appropriation is often studied in the context of "patent trolls" who are not innovators themselves (Cohen et al., 2019).

3 Primary Empirical Questions

The process described in the preceding sections raises three primary empirical questions. First, is the expansion of the resource frontier systematically associated with appropriation and conflict, or do the case studies just presented merely capture isolated or idiosyncratic legal conflicts arising following the emergence of specific technologies? Second, are appropriation rents economically meaningful for appropriating firms? Third, is the expansion of the resource frontier quantitatively important for economic growth?

These questions are substantial enough that each warrants a separate, rigorous investigation. Here, we take a more modest step. We present evidence bearing on each question, while pointing toward an empirical agenda for future work on growth at the resource frontier.

3.1 Is the expansion of the resource frontier systematically associated with appropriation and conflict?

The case studies presented above illustrate how prominent innovations that expanded the resource frontier resulted in appropriation and conflict. But one may wonder how systematic the experience is of appropriation and conflict at the resource frontier. Perhaps the legal conflicts over resource appropriation described in the case studies were relatively unusual, isolated, or unimportant episodes.

We next present descriptive evidence from U.S. law reviews that bears on this question. Rather than focus on specific technologies, we expand the scope of our analysis to three more general "waves" that expanded the resource frontier: (i) the exploitation of material resources like oil, gas, and minerals in the late nineteenth and early twentieth centuries; (ii) the incorporation into human use of immaterial resources like the spectrum and airspace through the middle of the twentieth century; and (iii) the use of digital resources like data in recent decades.

To identify legally salient disputes over resource appropriation in these domains, we examine the publications of 10 of the leading law reviews in the United States.[18] Specifically, we aim to identify articles that analyze legal disputes, doctrines, or regulatory interventions related to appropriation, restricting attention to those in which appropriation concerns a specific resource. To do so, we conduct full-text searches of the articles published in these reviews and search for keywords that capture the relevant resources (e.g.,

[18] These are, by date of foundation: UPenn, Harvard, Yale, Columbia, Michigan, California, Virginia, NYU, Chicago, and Stanford.

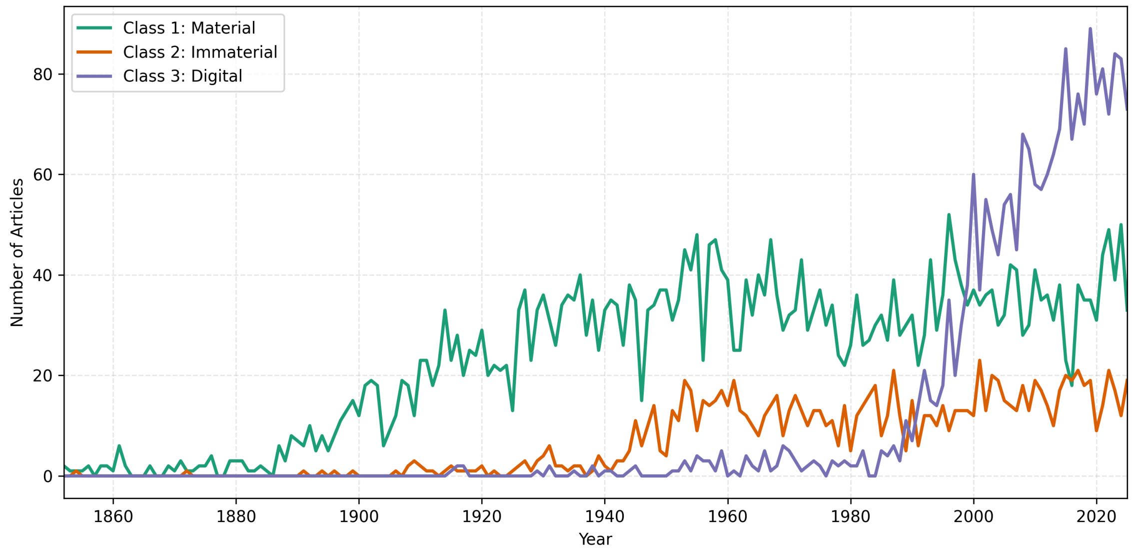

Figure 1: Waves of Appropriation and Conflict Articles in U.S. Law Reviews

"mining claim" or "oil pool" for material resources, "broadcast frequency" for immaterial, and "training data" for digital) as well as keywords that indicate a conflict specifically over the resource (e.g., "appropriation," "first possession," and "right of access") rather than other conflicts, such as labor disputes in a particular sector. Appendix 3.1 describes in detail the construction of the dataset and the classification of articles.

Figure 1 plots the number of articles discussing resource appropriation by year for each resource class. It therefore captures when legal scholarship focused on problems associated with appropriation of different types of resources, rather than merely the timing of the underlying technological innovations themselves.

The evidence reveals historical shifts in legal attention to conflicts over appropriation. Material resources dominate the legal literature on appropriation in the late nineteenth and early twentieth centuries, with a sharp rise around the turn of the century. This period corresponds to intense legal attention to conflicts over minerals, land, chemicals, and other physical resources, as resource extraction in support of industrialization outpaced the development of property rights. Immaterial resources — such as spectrum or airspace — rise later and become an important focus of appropriation-related legal scholarship in the second half of the twentieth century. Beginning in the 1990s and accelerating thereafter, legal scholarship has increasingly focused on the appropriation of data, information, and digital traces.

Overall, the figure shows that legal attention to appropriation conflicts has moved with the resource frontier: from material resources to immaterial resources, and later to digital resources. This pattern is consistent with one of the paper's central themes: as innovations bring new classes of resources into economic use, disputes over their appropriation begin to arise.

3.2 Are appropriation rents economically meaningful?

Even if innovations that expand the resource frontier are often associated with appropriation and conflict, the resulting appropriation rents may be economically modest. Appropriation disputes could concern resources that are legally salient but ultimately account for only a small share of economic value. In that case, appropriation rents would not be a central incentive for innovation; the relevant incentive would instead come primarily from rents in product markets.

To assess the quantitative importance of appropriation rents, we estimate the share of industry earnings attributable to appropriation of newly valuable resources in two of our case studies: digital advertising and broadcasting. We focus on these cases because there is available empirical evidence that allows us to perform simple and transparent back-of-the-envelope calculations. These calculations are not intended to provide a complete accounting of innovation incentives, which is difficult even for traditional monopoly rents in production. Rather, they ask whether appropriation rents are plausibly first-order in settings where they can be measured transparently. The details of the derivations are provided in Appendix A.1.

DIGITAL ADVERTISING. Evidence from Kraft et al. (2023) suggests that targeted online advertisements, relying on user-generated data, command a price premium of roughly 50 percent relative to non-targeted ads, and that 74 percent of all ads are targeted. Combining these pieces of evidence, we estimate that the share of advertising revenue attributable to user-generated data is 27 percent. To express this share in terms of earnings, we scale by net operating margins. For companies like Google or Meta, net operating margins are on the order of 30 to 40 percent.[19] Thus, we estimate that more than 2/3 of operating earnings in digital advertising can be traced to data-enabled targeting. Therefore, the appropriation of user data appears to be a primary source of economic rents for digital advertisers.

BROADCASTING. In a 2017 reverse auction, the FCC asked broadcasters to sell back frequencies that they had been allocated for free decades earlier. Based on data from the

[19] See https://www.alphaspread.com/comparison/nasdaq/meta/vs/nasdaq/googl.

auction, Soloveichik (2024) calculates that the value of such spectrum corresponds to approximately 6 percent of the net present value of industry revenue. To express this value in terms of earnings, we scale by net operating margins. Industry evidence indicates that margins for broadcasters are on the order of 9 percent (Damodaran, 2017). Thus, we estimate that roughly 2/3 of earnings are attributable to the frequencies that these broadcasters have been allocated for free decades earlier. As in digital advertising, the appropriation of an initially unpriced resource — in this case, electromagnetic spectrum — appears to be a primary source of economic rents.

3.3 Is the expansion of the resource frontier important for economic growth?

There is no doubt that improvements in production processes, product quality, and Romerian innovations have substantially contributed to economic growth in the past. One naturally wonders whether expansions of the resource frontier have been important as well. From petrochemicals to AI, the case studies already suggest that important sectors of the economy were built on the discovery and use of resources that had previously been economically marginal or unavailable. We next provide a simple quantification that gives a sense of the magnitudes involved.

As in the model we develop in Section 4, we assume that GDP is a constant-elasticity-of-substitution (CES) aggregator over the value added of different industries used as inputs. We divide industries into two groups. Frontier industries are those that our case studies identify as having become economically relevant primarily through expansions of the resource frontier. Non-frontier industries are all remaining industries, which we treat as pre-existing relative to the frontier expansions we study. Specifically, aggregate GDP in year t is

$$Y_t = \left(\sum_{i \in \mathrm{NF}} \alpha_i y_{it}^{\frac{\omega-1}{\omega}} + \sum_{i \in \mathrm{F}} \alpha_i y_{it}^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}},$$

where y_{it} is real value added in industry i, α_i is its weight, $\omega \in [0, \infty]$ is the elasticity of substitution across industries, and "F" and "NF" denote the sets of frontier and non-frontier industries, respectively.

Our baseline classification includes two frontier industries as measured by the Bureau of Economic Analysis (BEA): (i) broadcasting and telecommunications; and (ii) data processing, internet publishing, and other information services. These industries are intended to capture the emergence of the electromagnetic spectrum and data as economically important resources. We also report results for a broader classification that adds a third industry: computer systems design and related services. We begin the exercise in 1963, the first year in which the BEA reports value added for these industries.

Next, we ask how much lower GDP growth would have been from 1963 to 2025 if frontier industries had remained at their 1963 real value-added levels, while all non-frontier industries evolved as observed. The exercise is therefore not a full general-equilibrium counterfactual. It holds fixed the observed growth of non-frontier industries — as well as the elasticity of substitution — and measures the importance of the frontier industries' realized post-1963 growth.[20]

The counterfactual GDP growth Y^c_{2025}/Y_{1963} can be written as

$$\frac{Y^c_{2025}}{Y_{1963}} = \frac{Y_{2025}}{Y_{1963}} \left(\sum_{i \in \mathrm{NF}} s_{i,2025} + \sum_{i \in \mathrm{F}} s_{i,2025} \left(\frac{y_{i,2025}}{y_{i,1963}} \right)^{-\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}},$$

where $s_{i,2025}$ is the value-added share in GDP of industry i.[21] We evaluate the expression for $\omega \in [0.6, 1]$. This range is consistent with the multisector growth and structural-transformation literature, which typically estimates or calibrates substitution elasticities below one across broad sectoral aggregates, including Ngai and Pissarides (2007), Herrendorf et al. (2014), and Comin et al. (2021).

Actual GDP in 2025 was about 6 times its 1963 value ($Y_{2025}/Y_{1963} = 6$). Under the baseline classification, counterfactual GDP growth Y^c_{2025}/Y_{1963} falls to 3.21 for $\omega = 0.6$ and 5.22 in the Cobb-Douglas limit $\omega \to 1$. Under the broader classification, which also includes computer systems design and related services, counterfactual GDP growth falls to 2.02 and 4.76 for the same values of ω. These results are economically meaningful. They imply that frontier industries accounted for between 7.8 and 61 percent of the increase in

[20] On the one hand, the exercise may understate the importance of innovations that expanded the resource frontier because it omits many discoveries of new resources that raised productivity in previously existing industries or enabled growth in industries not included in our classification. On the other hand, it may overstate the contribution of the industries we include because other industries might have grown faster had these industries not emerged. We therefore interpret the exercise as an indicator of plausible magnitudes rather than as a full general-equilibrium counterfactual.

[21] At an interior optimal allocation, the CES weights α_i and value-added shares in GDP s_i are related by

$$\alpha_i \left(\frac{y_{i,2025}}{Y_{2025}} \right)^{\frac{\omega-1}{\omega}} = s_{i,2025}.$$

We use this relationship to recover the CES weights from observed 2025 value-added shares. We then substitute these weights into the CES aggregator and compute a counterfactual GDP Y^c_{2025} in which frontier industries are set back to their 1963 real value-added levels, while non-frontier industries remain at their observed 2025 levels.

log GDP between 1963 and 2025, depending on the classification and elasticity used.

4 A Model of Growth at the Resource Frontier

This section presents a model of endogenous growth at the resource frontier that formalizes the process we have just described — namely, that innovation expands the resource frontier by creating productive inputs out of previously unrecognized resources with poorly defined property rights, and that innovators may choose appropriation in anticipation of extraordinary rents instead of contracting with counterparties beforehand. We keep the model as close as possible to standard growth models with expanding varieties (Romer, 1990; Acemoglu, 2008) so that the differences are stark and easy to see.

4.1 Environment

The economy is populated by innovators, a final goods representative firm, and a representative household. Innovators can expand the resource frontier. They discover an idea about how to use a previously unrecognized resource for production. A discovery creates a new resource variety j that can be used to produce a new input variety j. At a point in time, the measure of resource and input varieties is A_t. The representative firm uses the inputs to produce the final good consumed by the household.

Unrecognized resources do not have established property rights before they are discovered and turned into an input. As such, they do not trade in markets at the time of discovery and can become contested afterwards. The innovator thus has two options. First, they can choose to appropriate the resource. A member of the representative household may have a claim over the resource. We will call this member a *counterparty* from now on. The counterparty may challenge the appropriating innovator. The dispute over property rights is settled as a result of litigation in the courts or by the government dictating laws and regulations, assigning ownership of the resource. Alternatively, the innovator can choose to contract, recognizing the counterparty's claim and purchasing the resource. The resource trades in a competitive market at price q_{jt} after either conflict is resolved or contracting occurs.

4.2 Representative Firm and Household

The final good Y_t is produced by a representative firm. The firm demands input varieties y_{jt} and produces with the aggregate production function

$$Y_t = \left(\int_0^{A_t} (y_{jt})^\gamma \, dj \right)^{\frac{1}{\gamma}}.$$

The firm maximizes profits

$$\Pi_t = Y_t - \int_0^{A_t} p_{jt} y_{jt} dj,$$

given input prices p_{jt}.

The representative consumer maximizes discounted utility over the consumption of final goods

$$\int_0^\infty e^{-\rho t} \frac{(C_t)^{1-\sigma}}{1-\sigma} dt,$$

subject to the budget constraint

$$C_t + \dot{a}_t \leq r_t a_t + \int_0^{A_t} q_{jt} X_{jt} dj,$$

where a_t is the value of the portfolio of all firms, r_t is the return on the portfolio, and X_{jt} are service flows from ownership of existing resource variety j with price q_{jt}.

4.3 Innovators

The innovator can use all existing resources to discover new resource (and input) varieties — to expand the resource frontier. We assume existing resources are perfect substitutes in such investment and the technology is linear. Given investment $\tilde{x}_{it}$ of resource i, the innovator discovers $\dot{A}_t$ new varieties according to

$$\dot{A}_t = \lambda \int_0^{A_t} \tilde{x}_{it} di. \tag{4.1}$$

The assumption of perfect substitutability immediately implies that all resources will trade at the same price $q_{it} = q_t$ in equilibrium, so we drop the index i from now on.[22]

[22] The assumption also allows for long-run growth to be sustained (Section B.1), similar to the "lab model" in Acemoglu (2008), since the flow of new varieties becomes linear in A_t along a balanced growth path. If the flow was concave instead, long-run growth could be sustained by assuming A_t has an external effect on the productivity of innovation, as in Romer (1990).

Production of input variety j uses the corresponding resource. The production technology is linear: x_{jt} units of the resource delivers

$$y_{jt} = x_{jt}$$

units of the input.[23] The flow profits are

$$\pi_{jt} = \left(p_{jt} - q_t\right) x_{jt}.$$

We assume that the firm sets a price p_{jt} equal to a constant markup μ over marginal cost $p_{jt} = (1+\mu)\, q_t$. This allows us to easily capture two cases. First, given the representative firm CES production function, the economy with $\mu = \frac{1-\gamma}{\gamma}$ corresponds to the case of monopolistic competition, where $0 < \gamma \leq 1$. Innovators obtain an infinite patent for their differentiated input variety j and charge monopoly prices, as in traditional models of expanding input varieties (Romer, 1994). Second, the economy with $\mu = 0$ corresponds to the case of perfect competition. In this case, other firms cannot be excluded from using the new idea that a previously unknown resource is valuable in production. Nonrivalry implies that the idea diffuses, and the threat of firm entry makes prices fall to marginal cost.

4.3.1 Appropriation and Conflict

After a discovery, the innovator can choose to appropriate the resource with poorly defined property rights. If they do, counterparties with potential claims can challenge them. Motivated by our case studies in Section 2, we assume this process takes time and the outcome of the dispute over property rights is uncertain.

The counterparty is initially unaware of the innovation and resource appropriation. They learn at some future point in time, an event that follows a Poisson process with arrival rate δ. The counterparty initiates a dispute at that point. The dispute is resolved in favor of the innovator with probability θ, in which case they keep the resource they have appropriated. Otherwise, ownership goes to the counterparty and, in addition, the innovator must pay ν as a penalty (or in damages) for appropriation.[24] The probability

[23] It is easy to extend the model for production to use all available resources, instead of just one variety.

[24] Conflict is zero-sum under these assumptions. One could have assumed that conflict was costly to society. For example, κ units of the resource could be lost in the process of assigning property rights (e.g., the cost of litigation or implementing regulation). Moreover, resolving conflicts may take time, following a Poisson process with arrival rate α. The rate α would capture how fast institutions are in resolving disputes; for example, disputes may take longer to resolve because courts are sluggish or regulations are slow to be implemented.

θ and penalty ν are exogenous for now. We endogenize them later when considering optimal policy and political strategies innovators may use to shape disputes (Section 7.2).

The rate δ captures how quickly it becomes known that an innovation has turned a previously unrecognized resource into a productive input, and that the innovator has appropriated it. In expectation, the innovator enjoys the fruits from appropriation for a period of length $1/\delta$ where property rights remain poorly defined. The probability θ captures the extent to which the resolution of the dispute is expected to favor the innovator. For example, θ would be higher when court precedent regarding similar disputes favored the innovator in the past, and θ would be lower when past laws already assign some rights to the counterparty. Alternatively, θ could be higher when the innovation creates a more powerful constituency of consumers that support it, as in Section 7.2.

In all, choosing to appropriate the resource and resolve the conflict over property rights afterwards creates the chance for extraordinary rents if the dispute is resolved in favor of the innovator. But it risks expropriation, penalties, damages, or outright business failure when it does not. Such is the innovator's *gamble*. Formally, the expected value for an innovator that chooses to appropriate a resource and begin producing at time t is

$$\mathcal{V}_{it}^{\text{conflict}} = \mathcal{D}_{it} + \mathcal{R}_t,$$

where

$$\mathcal{D}_{it} \equiv \int_t^\infty e^{-\int_t^s r_v dv} \pi_{is} ds$$

are traditional rents from production of the new input variety, and

$$\mathcal{R}_t \equiv \underbrace{\int_t^\infty e^{-\int_t^s r_v dv} e^{-\delta(s-t)} q_s ds}_{\text{Informational rents}} + \underbrace{\int_t^\infty e^{-\int_t^s r_v dv} \left(1 - e^{-\delta(s-t)}\right) (\theta - (1-\theta)\nu) q_s ds}_{\text{Institutional rents}}$$

are expected rents from appropriation of the new resource.[25]

[25] The expected value under conflict is obtained as follows:

$$\begin{aligned}
\mathcal{V}_{it}^{\text{conflict}} &= \int_t^\infty e^{-\int_t^s r_v dv} p_{is} x_{is} ds + \int_t^\infty \delta e^{-\delta(s-t)} \left(\int_t^s e^{-\int_t^n r_v dv} (1 - x_{in}) q_n dn \right) ds \\
&\quad + \int_t^\infty \delta e^{-\delta(s-t)} \left(\theta \int_s^\infty e^{-\int_t^n r_v dv} (1 - x_{in}) q_n dn - (1-\theta) \int_s^\infty e^{-\int_t^n r_v dv} (\nu + x_{in}) q_n dn \right) ds \\
&= \int_t^\infty e^{-\int_t^s r_v dv} \pi_{is} ds + \int_t^\infty \delta e^{-\delta(s-t)} \left(\int_t^s e^{-\int_t^n r_v dv} q_n dn + \int_s^\infty e^{-\int_t^n r_v dv} (\theta - (1-\theta)\nu) q_n dn \right) ds \\
&= \int_t^\infty e^{-\int_t^s r_v dv} \pi_{is} ds + \int_t^\infty e^{-\int_t^s r_v dv} e^{-\delta(s-t)} q_s ds + \int_t^\infty e^{-\int_t^s r_v dv} \left(1 - e^{-\delta(s-t)}\right) (\theta - (1-\theta)\nu) q_s ds,
\end{aligned}$$

where the last line follows from a change in the order of integration. It is worth noting that, in the

The expression for appropriation rents $\mathcal{R}_t$ shows two sources of rents for the innovator at the time of appropriation t. First, the innovator enjoys *informational rents* because it takes time for the counterparty to become aware of the innovation and the resource appropriation in order to initiate the dispute — a period of length $1/\delta$ in expectation. Second, the innovator enjoys *institutional rents* because they expect to gain $\theta - (1-\theta)\,\nu$ units of resources at the time of resolution. Institutional rents may be positive or negative, depending on the penalties or damages paid ν and the probability θ of winning the dispute.

4.3.2 Contracting

Instead of appropriating the resource, the innovator may choose to recognize the counterparty's claim and contract with them to purchase the resource. Recognition means that the innovator treats the counterparty as the owner entitled to sell the resource. That is, once the claim is recognized, the resource trades as an owned asset.

We assume that recognized owners behave competitively, so that the purchase price of the resource equals its competitive value $\int_t^{\infty} e^{-\int_t^s r_v dv} q_s ds$. Thus, under contracting, the innovator retains only the rents from producing the new input variety, and the value of contracting is their present discounted value:

$$\mathcal{V}_{it}^{\text{contract}} = \mathcal{D}_{it}.$$

The assumption of competitive counterparties has the advantage of making the comparison with standard models of expanding varieties transparent. When innovators contract for resources at competitive prices, innovation is rewarded only through rents from producing new input varieties, exactly as in the standard environment (see Section 6.2). From an incentives perspective, innovations that expand the resource frontier and lead to contracting are therefore equivalent to innovations with no appropriation rents, including those involving abundant or non-excludable resources, process and quality improvements, and Romerian innovations (Section 2.2).

Formally, the choice between contracting and conflict therefore reduces to the sign of expected appropriation rents $\mathcal{R}_t$. By contracting, the innovator gives up the appropriation gamble: it forgoes the upside from controlling the resource before counterparties

derivation above, the implicit assumption is that the innovator can sell the service flows of the resource at price q_n as soon as the resource is appropriated — gaining $(1-x_{in})\,q_n$ in revenues. Alternatively, one could have assumed that a market for the resource only appears once the dispute is resolved and property rights are well established, and the innovator can only use the appropriated resource for its own production before that.

become aware of its value and from having the dispute resolved in the innovator's favor, but also avoids the downside from losing ownership or paying penalties. The innovator contracts whenever $\mathcal{R}_t \leq 0$, and chooses appropriation and conflict whenever $\mathcal{R}_t > 0$.

Strategic bargaining. The competitive benchmark captures cases in which claims are sufficiently diffuse, counterparties are price-takers, or equivalent resources can be purchased from many counterparties, so that bargaining power is limited. In other cases, however, a single party — or a small number of parties — may have a strong claim over the resource. With this in mind, in Appendix B.2, we study an alternative environment in which the counterparty behaves strategically and the innovator can offer a contract without fully recognizing the counterparty's claim. The contract price is then determined by bilateral bargaining. A key force is a form of hold-up generated by information revelation: by approaching the counterparty, the innovator reveals that the resource is valuable and that the counterparty may have a claim over it. The counterparty can then demand compensation based on the value of rejecting the offer and challenging appropriation later, while the innovator's willingness to pay is limited by the appropriation rents lost through disclosure. We show conditions under which contracting remains feasible, including conditions under which the resource's competitive value lies in the contracting set.

4.4 Market Clearing

Market clearing in the input market requires that

$$y_{jt} = x_{jt},$$

and in the final goods market

$$C_t = Y_t = \int_0^{A_t} p_{jt} y_{jt} di.$$

Resource j yields a one-unit service flow at each point in time, which can be allocated to producing input j, or innovation. Market clearing in the resource market thus requires

$$x_{jt} + \tilde{x}_{jt} = 1.$$

Lastly, the value of the household's firm portfolio is

$$a_t = \int_0^{A_t} \mathcal{V}_{jt} dj,$$

where $\mathcal{V}_{jt}$ is the discounted value of profits from input variety j.

5 Equilibrium Characterization

This section characterizes when and why appropriation and conflict arise, and derives an expression for the economy's growth rate along a balanced growth path.

5.1 Balanced Growth Path

We now solve for the equilibrium of our model along a balanced growth path (BGP) where all variables grow at constant rates. The following proposition establishes sufficient conditions for conflict or contracting to arise along a BGP, and characterizes the growth rate of varieties g_A in each case.

We assume throughout that $\rho < \lambda\mu$ and $\frac{\sigma-1}{\gamma}+1>0$, which guarantee that the growth rate g_A remains positive and that the interest rate r is above the growth rate, and that $\lambda < \left(1+\frac{\mu}{\frac{\sigma-1}{\gamma}+1}\right)(\rho+\delta)$, which guarantees that the growth rate g_A is unique (see Appendix B.1.1). We normalize the initial resource price q to 1.

Proposition 1. (Balanced growth path) *Conflict arises along a balanced growth path if*

$$((1-\theta)(1+\nu)-1)\,\delta < \bar{\chi}$$

for a cutoff $\bar{\chi}$ defined in Appendix B.1.2, whereas contracting arises otherwise.

The growth rate of varieties is the solution to

$$g_A = \frac{\lambda\left(\mu+\mathcal{R}\left(g_A\right)\right)-\rho}{\frac{\sigma-1}{\gamma}+1+\mu}, \tag{5.1}$$

where the expected appropriation rents are positive

$$\mathcal{R}\left(g_A\right) \equiv 1-\frac{\delta}{\rho+\delta+\left(\frac{\sigma-1}{\gamma}+1\right)g_A}\left(1-\theta\right)\left(1+\nu\right)>0$$

when conflict arises and $\mathcal{R}\left(g_A\right)=0$ when contracting does.

Proof. See Appendix B.1. □

The first part of the proposition shows a sufficient condition for appropriation rents to be positive in expectation so that conflict arises along a balanced growth path. Intuitively, the condition holds when institutional rents are large in expectation — either because the probability θ that the innovator wins the dispute over property rights is high enough or the penalty ν from losing is low enough — or when informational rents are large — the rate δ at which counterparties learn about the innovation and appropriation of the resource is low enough.

Regarding the growth rate g_A, there are two key equations that characterize innovation and growth. First, optimality of the innovator investments in discovering resource varieties requires that returns to an additional variety are equal to its marginal cost $\lambda \mathcal{V}_t = q_t$. This implies that

$$\lambda \left(\frac{\pi}{q} + \mathcal{R}\left(g_A\right) \right) = \tilde{r}$$

where profits over marginal cost are $\frac{\pi}{q} = \mu x$, and the effective discount rate is $\tilde{r} \equiv r - \frac{\dot{q}_t}{q_t} = \rho + \left(\frac{\sigma - 1}{\gamma} + 1 \right) g_A$ from the household's Euler equation and goods market clearing. Second, the accumulation equation together with market clearing in each resource market implies that the growth rate g_A of varieties is

$$g_A = \lambda \left(1 - x\right).$$

Together, these conditions imply the BGP growth rate g_A in Proposition 1.

6 Implications of Growth at the Resource Frontier

This section presents the main implications of our model of growth at the resource frontier, contrasting with those of existing frameworks in political economy and economic growth.

6.1 Property Rights Emerge as a Consequence of Innovation

Much work emphasizes the cost of insecure property rights for economic growth, and well-established property rights over ideas and new products as a precondition for innovation (Coase, 1960; North, 1981; Acemoglu et al., 2001; Engerman and Sokoloff, 2005; La Porta et al., 2008; Hornbeck, 2010; Dell, 2010).

We qualify this view in two ways. First, when it comes to growth at the resource frontier, firms innovate precisely because property rights over overlooked or inaccessible resources are poorly defined — expecting rents from appropriation. Second, property rights emerge as a consequence of innovation, not a precondition. Formally, there is a period of time — of length $1/\delta$ on average — where property rights remain poorly defined after innovation and appropriation have taken place, but before a counterparty learns about them and initiates a dispute. Property rights become established only after the dispute is resolved. For example, it took multiple lawsuits over decades to establish the "rule of capture" in the case of oil extraction from underground reserves (Section 2.1.4); lawsuits regarding the appropriation of online content by developers of LLMs are still ongoing several years since these models were first released.

The above changes the role of the state even in typical market economies. The state does not only protect property rights over ideas and new inputs, it allocates property rights over resources outside the frontier as well.

6.2 Appropriation Rents Incentivize Innovation and Sustain Growth

From models of expanding input variety (Romer, 1990; Jones, 1995) or capital vintages (Jovanovic and Yatsenko, 2012) to models of creative destruction (Aghion and Howitt, 1992; Grossman and Helpman, 1991), traditional models of innovation-driven growth share two key features. First, innovations turn *known* resources that trade in markets with well-defined property rights into new input varieties, capital vintages or higher-quality goods. Second, innovations grant firms monopoly rents from production — for example, because of patents over new inputs — that justify their costs.

Equation 5.1 makes these statements precise when contracting arises. In this case, our model reduces to a standard growth model. In particular, the growth rate coincides with that of a "lab-model" of expanding varieties (Acemoglu, 2008):

$$g_A = \frac{\lambda\mu - \rho}{\frac{\sigma-1}{\gamma} + 1 + \mu}.$$

This is a direct consequence of assuming that contracting occurs at the resource's competitive price; in the standard model, existing resources trade in competitive markets. As a result, absent production rents from new input varieties — when the markup $\mu = 0$ — the economy neither innovates nor grows in the long-run.

In our model of growth at the resource frontier, however, innovation turns previously unrecognized resources into new input varieties, and traditional monopoly rents in the output market are not required to incentivize innovation and sustain long-run growth. Instead, expected rents $\mathcal{R}$ from appropriating unrecognized resources may suffice.[26] To

[26] Obviously, this is not to claim that *all* economic growth in history has been supported by appropriation

see this clearly, the growth rate in 5.1 when markups $\mu = 0$ is

$$g_A = \frac{\lambda \mathcal{R}(g_A) - \rho}{\frac{\sigma - 1}{\gamma} + 1}.$$

Therefore, the prospect of securing ownership over the resource — and thereby earning appropriation rents $\mathcal{R}$ — is enough to generate positive long-run growth $g_A > 0$. Appropriation rents are therefore not merely a legal or redistributive concern: they can incentivize innovation at the resource frontier alongside the traditional monopoly rents that have been the focus of the literature on economic growth.

The calculations in Section 3.2 suggest that this source of incentives was quantitatively important in digital advertising and radio broadcasting. Interpreted as a share

$$s \equiv \frac{\mathcal{R}}{\mu + \mathcal{R}}$$

of total innovator rents, the implied values of s were on the order of two-thirds. In these cases of innovation at the resource frontier, appropriation rents thus appear to have been of the same order of magnitude as — and plausibly larger than — traditional rents from production.

Poorly defined does not mean weak property rights. It is worth clarifying that our finding that the opportunity to appropriate incentivizes innovation does not imply that innovation would be greater in fragile states or economies with weak property rights enforcement, more generally. The appropriation we study is specific to resources outside of the frontier, which intrinsically have poorly defined property rights beforehand. Crucially, innovators at the resource frontier expect *strong* property rights protection upon the resolution of conflict. Without it, innovators cannot expect any appropriation rents, as others could in turn appropriate the resource from them. In addition, in fragile states, the conflict over property rights might not be zero-sum, as assumed above (footnote 24), but rather negative sum. In this case, the costs of adjudicating legal disputes to establish property rights may be so high that appropriation rents are dissipated.

rents, but rather to highlight their importance alongside traditional monopoly rents.

6.3 Growth at the Resource Frontier Often Comes with Appropriation and Conflict

The appropriation of resources is typically seen as being purely redistributive or even extractive and predatory (Persson and Tabellini, 1991; Murphy et al., 1993; Acemoglu and Robinson, 2013). In these accounts, appropriation and conflict are therefore either separate from or at odds with economic growth.

In our model, growth at the resource frontier frequently comes together with appropriation and conflict. They are not incidental to the growth process, but intrinsically linked to it.[27] To show this, we conduct a large-scale simulation in the spirit of Monte Carlo sensitivity analysis in applied general equilibrium models (Harrison and Vinod, 1992). We draw all model parameters jointly from uniform distributions over specified supports and, for each draw, impose basic feasibility and structural restrictions to ensure a well-defined balanced growth path. We then classify the equilibrium regime using the model's sufficient condition for conflict in Proposition 1, and compute the implied growth rate g_A. This produces a cross-sectional dataset of model economies, each with an associated growth rate and a binary indicator for whether appropriation and conflict arise.

Figure 2 bins parameterizations by the realized growth rate and computes, within each bin, the share of parameterizations that exhibit appropriation and conflict. We find a strong, monotonic relationship: the share of parameterizations exhibiting appropriation and conflict rises sharply with the growth rate and then flattens at high levels of g_A. This relationship does not mean that growth mechanically causes conflict. Rather, across parameterizations, high growth rates arise when the expected rents from securing ownership of newly valuable resources are large. Since these same rents also make appropriation and conflict more likely, the model predicts that growth and conflict should often appear together when innovations expand the resource frontier.

The figure gives a model-based interpretation of the historical waves of appropriation and legal conflict documented in Section 3.1. Legal conflicts were more likely to arise when innovations expanded the resource frontier — from material resources to immaterial and digital domains — precisely because these were times when the expected rents from appropriating newly valuable resources were unusually large. Waves of conflict are thus not separate from the innovation process, but are a recurring feature of growth at the resource frontier.

[27] From equation (5.1), one can already see that a higher probability θ will be associated with higher appropriation rents — and so higher growth together with conflict.

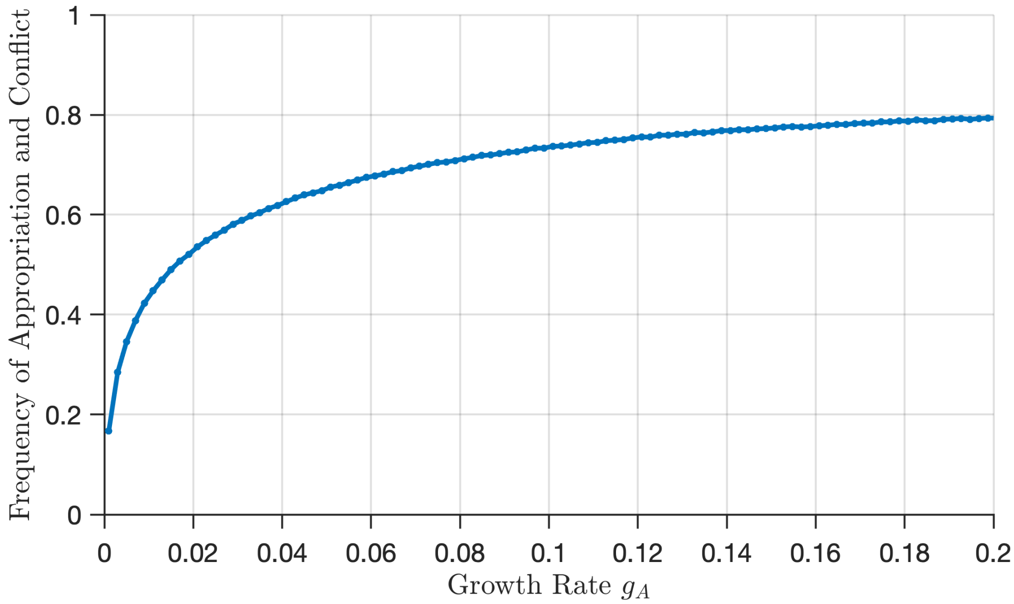

Figure 2: Frequency of Appropriation and Conflict, conditional on Growth

Note: The figure bins parameterizations by realized g_A and, for each bin, plots the frequency of parameterizations where the sufficient condition for conflict in Proposition 1 is met. All parameters are drawn from uniform distributions, with: $\sigma \in [0,6], \rho \in [0,1/2], \gamma \in [0,1], \mu \in [0,2(1-\gamma)/\gamma], \lambda \in [0,2], \delta \in [0,2], \theta \in [0,1]$, and $\nu \in [0,2\mu]$. The intervals are chosen to span a broad range of parameters, while centering markup μ around the monopolistic-competition markup $(1-\gamma)/\gamma$ and penalty ν around full expropriation of profits.

7 Political Economy of Growth at the Resource Frontier

So far, we have treated the institutional environment governing appropriation and conflict as given. In particular, the parameters θ and ν summarize how conflicts are resolved, with θ capturing the probability that innovators succeed in securing ownership and ν penalties or damages imposed on innovators when appropriation fails.

We now ask how these parameters are determined. We first study the benchmark case of a benevolent planner choosing the institutional environment to maximize welfare. We then turn to political influence, where innovators, consumers, incumbents, and the state may have conflicting incentives over how permissive institutions governing appropriation should be.

7.1 Benevolent Planner

Household utility along a balanced growth equilibrium is

$$U = \frac{1}{1-\sigma} \frac{\left(1 - \frac{1}{\lambda} g_A\right)^{1-\sigma}}{\rho + \frac{\sigma-1}{\gamma} g_A},$$

where we have normalized the initial number of varieties to 1. A higher growth rate thus increases future consumption, but crowds-out current consumption because of resources spent in discovering new varieties — at a cost of $\frac{1}{\lambda}$. A benevolent planner that maximizes household utility would choose the optimal growth rate

$$g_A^* = \frac{1}{\sigma} (\lambda - \rho\gamma)$$

that balances the forces just described.

Appendix B.3 derives the optimal choice of probability θ and penalty ν to implement g_A^* or achieve the highest utility possible. If the optimal growth rate g_A^* is below the equilibrium growth rate under contracting in Proposition 1, then the planner optimally chooses a probability θ low enough and penalty ν high enough so that contracting arises in equilibrium — formally, $(1-\theta)(1+\nu) \geq \bar{\chi}$ in Proposition 1. In this case, the planner cannot implement g_A^* exactly with only these instruments. If the optimal growth rate g_A^* is instead above the equilibrium growth rate under contracting, then the planner chooses a probability θ high enough and penalty ν low enough so that conflict arises in equilibrium $(1-\theta)(1+\nu) < \bar{\chi}$ and the parameters satisfy

$$(1-\theta)(1+\nu) = \max\left\{\mu - \frac{1-\gamma}{\gamma}, 0\right\} \left(1 - \frac{1}{\lambda} g_A^*\right) \frac{\rho + \delta + \left(\frac{\sigma-1}{\gamma} + 1\right) g_A^*}{\delta}.$$

It is worth noting that the case of monopolistic competition $\mu = \frac{1-\gamma}{\gamma}$ implies that $\theta = 1$ *exactly* implements the optimal growth rate — the government should allow full appropriation. An economy with even lower markups (as with perfect competition $\mu = 0$) implies a corner solution with $\theta = 1$. Otherwise, there is an interior solution $\theta < 1$ if markups are higher than under monopolistic competition $\mu > \frac{1-\gamma}{\gamma}$.

These stark prescriptions should not be read as a general case for unconstrained appropriation. The model abstracts from several forces that would push the planner toward a lower probability of appropriation (or toward higher penalties ν). First, the production of resource service flows may itself be costly. For example, in the case of digital advertising or generative AI, behavioral data or original text and images must be created. Because these costs are borne by users and digital creators and not by innovators, full appropriation overstates the social value of frontier innovation. Second, conflict may be negative-sum: it creates litigation costs, disruption, or deadweight losses beyond those captured by the resource costs of discovering new varieties $\frac{1}{\lambda}$. Finally, societies may place intrinsic weight on limiting conflict or disruptive appropriation, even when it raises growth. Differences in legal and political cultures — some more celebratory of disruptive innovators, others more skeptical of conflictual change — would then map into different socially preferred values of θ and ν. One example is the contrast often drawn between the United States and Europe in attitudes toward disruption, innovation, and the social costs of conflict a (Acemoglu et al., 2017); another is the relation between religion and innovation (Bénabou et al., 2015).

7.2 Political Influence

The planner's problem treats θ and ν as policy instruments. In practice, however, these parameters are themselves shaped by political influence. Innovators that benefit from appropriating newly valuable resources have strong incentives to affect how the state resolves disputes over ownership. In earlier eras, this could mean shaping the use of force; today, it more often means influencing courts, legislation, and regulation in ways that raise the probability θ that appropriation is upheld or that lower penalties ν.

One route is direct political influence. Innovators may lobby, seek regulatory capture, or engage in the traffic of influence to secure favorable rules. These efforts can raise private returns by increasing θ or lowering ν, but they may also be socially wasteful when they distort the allocation away from what would be optimal from the benevolent's planner perspective.

A second route is to shape the trajectory of innovation itself. Firms may design technologies so that they generate broad consumer surplus or serve politically influential constituencies, thereby increasing the political support for permissive appropriation. This strategy can be socially valuable when it lowers markups μ, as greater consumer surplus from weaker monopolistic distortions results from increasing θ (or lowering ν) beyond what would be optimal otherwise. For example, in digital advertising, estimates of the large consumer surplus generated by Google's search engine have been used to support arguments for antitrust leniency (Varian, 2011).

This strategy also has risks. When firms adapt technologies to state priorities, they may make those technologies more valuable to state power itself, including through dual-use applications that blur civilian, military, and surveillance purposes (Beraja et al., 2023a). Political support may then come not only from consumers, but also from governments that value the technology for strategic or coercive ends. Thus, political influence can raise θ or lower ν either by shaping formal rules directly or by changing the political constituencies that benefit from frontier innovation. Recent disputes over frontier AI firms' relationships with the U.S. defense establishment illustrate this tension: OpenAI reached an agreement to deploy its systems in classified defense settings, while Anthropic became embroiled in a dispute over contractual limits on military uses of its models (OpenAI, 2026; Amodei, 2026).

8 Conclusions

Economic thought has long emphasized innovation as a central source of economic growth, with well-defined property rights preceding innovation and enabling firms to capture rents that incentivize discovery. A parallel tradition has pointed to the deleterious effects of the appropriation and extraction of resources.

Yet, from the Stone Age to the Digital Age, much of human progress has been driven by innovations that expand the frontier of usable resources, where property rights have typically been poorly defined. In prominent historical cases, we document that innovators appropriated resources outside the frontier — expecting extraordinary rents and triggering conflicts with counterparties advancing competing claims. Motivated by these observations, we developed a model of growth at the resource frontier. The model rationalizes how property rights emerge as a consequence of innovation, establishes that expected rents from appropriation can sustain long-run growth, and ultimately shows that growth at the resource frontier is frequently accompanied by appropriation and conflict.

These results suggest a different view of how economies grow and of the relationship between property rights and innovation. Rather than being a precondition for innovation, property rights can arise from the resolution of disputes following innovations expanding the resource frontier. Moreover, innovators may act before ownership is clearly defined in anticipation of rents from appropriation. As a result, appropriation and conflict are not incidental to, or at odds with, the process of economic growth. They can be central to how new resources are incorporated into the economy.

In the near future, we see the same forces shaping further expansions of the resource frontier in two dimensions. The green energy transition and US–China geopolitical rivalry have sharply increased the value of deep-sea minerals.[28] As extraction technologies make these resources accessible, they are turning the deep sea into a new frontier of conflict over property rights.[29] Outer space is quickly becoming the "final frontier." Recent innovations are making resources on the Moon, asteroids, and beyond contestable (Weinzierl, 2018), with legal ambiguity over resource ownership and extraction from space likely leading to international conflict.[30]

Our analysis of economic growth at the resource frontier raises important questions. For policymakers, a central challenge is how to balance the opportunities for growth from resource appropriation against the costs of conflict and the downstream consequences of wealth accumulation and concentration driven by appropriation rents. For academics, there is a clear need for systematic empirical study of growth at the resource frontier. We currently have only a partial understanding of the contribution to aggregate growth from innovations that expand the frontier of usable resources, and even less evidence on its effects on conflict, law, and the evolution of property rights, making this a promising area for future research.

[28] See, for example, the discussions in the International Energy Agency (2021) report and in Latourette et al. (2025).

[29] Under UNCLOS, mineral resources in the international seabed area are treated as the common heritage of humankind and are governed through the International Seabed Authority, but recent efforts to pursue seabed mining outside this framework have exposed unresolved legal and geopolitical tensions (United Nations, 1982; International Seabed Authority, 2025).

[30] For example, the Outer Space Treaty's prohibition on national appropriation coexists uneasily with domestic laws, such as the U.S. Commercial Space Launch Competitiveness Act, that recognize rights over extracted space resources (United Nations, 1967; United States Congress, 2015).

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Online Appendix for:
Growth at the Resource Frontier

This online appendix contains details on the empirical evidence, and the proofs and derivations of all theoretical results for the article "Growth at the Resource Frontier."

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded by "A." or "B." refer to the main article.

A Empirical Evidence in Section 3

A.1 Calculations of Rents from Appropriation

A.1.1 Digital Advertising

Let p^u denote the price of untracked (contextual) ads, p^t the price of tracked (targeted) ads, and mc the marginal cost of an ad. Likewise, let a^u and a^t denote the corresponding ad quantities, with $a = a^u + a^t$. We can thus write earnings for an advertising platform as

$$\begin{aligned} Earnings &= p^u * a^u + p^t * a^t - mc * a \\ &= \underbrace{(p^u - mc) * a}_{\text{Pure profits } \pi} + \underbrace{(p^t - p^u) * a^t}_{\text{Rents from data } \mathcal{R}} \end{aligned}$$

The first term captures baseline operating profits that would obtain absent data-driven targeting. The second term isolates rents from data, i.e., the incremental revenue generated by tracking relative to contextual advertising.

Two pieces of evidence discipline this decomposition. Kraft et al. (2023) finds that targeted advertisements command a price premium of roughly 50 percent relative to non-targeted ads, and that 74 percent of all of ads are targeted. Combining these pieces of evidence, we estimate that the share of advertising revenue attributable to user-generated data is

$$\alpha \equiv \frac{\text{Revenue from targeting}}{\text{Total Revenue}} = \frac{(p^t - p^u) * a^t}{p^u * a^u + p^t * a^t} = \frac{\left(\frac{p^t}{p^u} - 1\right) * \frac{a^t}{a^u}}{\frac{p^t}{p^u} * \frac{a^t}{a^u} + 1} = \frac{0.5 * \frac{0.74}{1-0.74}}{1.5 * \frac{0.74}{1-0.74} + 1} \approx 0.27.$$

Moreover, assuming net operating margins $\mu = \frac{\text{Earnings}}{\text{Total Revenue}}$ are 40 percent in the industry, the share of earnings attributable to data-enabled targeting is $\alpha/\mu \approx 2/3$.[1] The value is even higher assuming a margin of 30 percent.

A.1.2 Broadcasting

We begin with estimates reported in Soloveichik (2024). In the 2017 FCC reverse auction, UHF broadcasters received \$10.05 billion in exchange for relinquishing 14 spectrum units in the 600 MHz band (Gilroy, 2017). Using industry revenue data and standard assumptions, they estimate a net present value of approximately \$12 billion per spectrum unit; implying that the value of spectrum corresponds to 6.15 percent of the net present value

[1] See https://www.alphaspread.com/comparison/nasdaq/meta/vs/nasdaq/googl.

of industry revenue. Equivalently, the flow value of the spectrum is 6.15 percent of the flow of revenues.

Using a 9 percent net operating margin (Damodaran, 2017), we compute that the share of operating earnings that can be attributed to the spectrum is roughly two-thirds (6/9).

A.2 Construction of Figure 1

To construct a time series of legal engagement with property rights disputes across varying resource domains, we conduct the following exercise.

We first define a set of 10 leading law reviews that have long publication histories. These are the *University of Pennsylvania Law Review (American Law Register)*, founded in 1852; *Harvard Law Review*, founded in 1887; *Yale Law Journal*, founded in 1891; *Columbia Law Review*, founded in 1901; *Michigan Law Review*, founded in 1902; *California Law Review*, founded in 1912; *Virginia Law Review*, founded in 1913; *New York University Law Review*, founded in 1924; *University of Chicago Law Review*, founded in 1933; and *Stanford Law Review*, founded in 1948.

We then identify articles over the entire period covered by these law reviews that discuss resources in three domains: material, immaterial, and digital. To do so, we search the full text of all articles for the following "resource keywords," which were generated using human judgment complemented by LLMs.

For the material domain, the keywords are: unappropriated land, homestead, land grant, cession, Indian title, aboriginal title, frontier land, public lands, preemption right, squatter claim, water right, watercourse, navigable water, riparian land, prior appropriation, beneficial use, river flow, usufructuary right, irrigation water, mining claim, mineral patent, lode, placer, mineral right, subsurface right, ore body, oil pool, common pool, reservoir, oil concession, and natural gas.

For the immaterial domain, the keywords are: radio frequency, radio spectrum, broadcast frequency, airwave, electromagnetic spectrum, wavelength, ether, clear channel, spectrum allocation, bandwidth, airspace, navigable airspace, overflight path, sovereign airspace, ad coelum, air route, and navigation easement.

For the digital domain, the keywords are: training data, training corpus, datasets, mass digitization, database, web content, user data, personal data, biometric data, social graph, behavioral surplus, digital footprint, server access, API access, website data, screen scraping, data scraping, web crawling, copyrighted work, public domain works, hot news, database right, and trespass to chattels.

This generates a set of law review articles that touch on resources in the three relevant domains that experienced technological change in the last 150 years.

We further refine this set by searching for those articles that specifically relate to *disputes* over the resources themselves, rather than, for example, antitrust cases or labor cases relating to relevant sectors of the economy. To do so, we search the full text of the articles for the following "property rights dispute" keywords: appropriation, first possession, rule of capture, discovery doctrine, dispute, conflict, interference, sovereignty, exclusive right, allocation, right of access, claim, ownership, property right, access right, jurisdiction, control, dominion, title, litigation, and injunction.

We then count the articles classified as resource dispute articles, disaggregated by resource domain: material, immaterial, and digital. We plot article counts by resource domain in Figure 1 of the paper.

B Proofs and Derivations

B.1 Proof of Proposition 1

We consider a symmetric equilibrium and drop all i indices from now on. Firm profits are

$$\pi_t = \mu (1+\mu)^{-\frac{1}{1-\gamma}} (q_t)^{-\frac{\gamma}{1-\gamma}} C_t,$$

and the first order conditions for the representative firm and household are

$$\begin{aligned} c_t &= (p_t)^{-\frac{1}{1-\gamma}} C_t \\ 1 &= A_t (p_t)^{-\frac{\gamma}{1-\gamma}} \\ r_t &= \rho + \sigma \frac{\dot{C}_t}{C_t}, \end{aligned}$$

where we have normalized the price of aggregate consumption good to 1.

Innovator's optimal investment implies that the prices of all factor varieties are equalized $q_{it} = q_t$ and that the returns to an additional variety are equal to its marginal cost

$$\lambda \mathcal{V}_t = q_t.$$

The market clearing conditions are

$$\begin{aligned}
a_t &= A_t \mathcal{V}_t \\
1 &= \tilde{x}_t + x_t \\
c_t &= x_t \\
\dot{A}_t &= \lambda A_t \tilde{x}_t.
\end{aligned}$$

Next, we solve for the balanced growth path. We denote by g_q, g_A, and g_C the growth rates of factor prices q, varieties A, and consumption C along a BGP. The equilibrium conditions result in

$$\begin{aligned}
q_0 &= (A_0)^{\frac{1-\gamma}{\gamma}} \frac{1}{1+\mu} \\
g_q &= \frac{1-\gamma}{\gamma} g_A \\
C_0 &= x\,(A_0)^{\frac{1}{\gamma}} \\
g_C &= \frac{1}{\gamma} g_A \\
r &= \rho + \sigma g_C \\
g_A &= \lambda\,(1-x) \\
\lambda \mathcal{V} &= q \\
\pi &= \mu q x
\end{aligned}$$

$$\begin{aligned}
q_0 &= (A_0)^{\frac{1-\gamma}{\gamma}} \frac{1}{1+\mu} \\
\frac{1}{(1+\mu)^{\frac{2-\gamma}{1-\gamma}}} &= \frac{q_0 x}{p_0 C_0}
\end{aligned}$$

We normalize the initial price level $q_0 = 1$ from now on. Given expected rents from appropriation under conflict

$$\mathcal{R} = 1 - \frac{\delta}{r - g_q + \delta}\,(1-\theta)\,(1+\nu)\,,$$

we can write the value for the innovator as

$$\mathcal{V} = \frac{\pi + \mathbb{I}_{\text{conflict}} \mathcal{R}}{r - g_q}.$$

Together, with the innovator's optimality condition $\lambda \mathcal{V}_t = q_t$ and that conflict is optimal if and only if appropriation rents are positive, the above implies that the growth rate of varieties is the solution to

$$g_A = \frac{\lambda\left(\mu + \mathcal{R}\left(g_A\right)\right) - \rho}{\frac{\sigma-1}{\gamma} + 1 + \mu},$$

where $\mathcal{R}\left(g_A\right) = \max\left\{1 - \frac{\delta}{\rho+\delta+\left(\frac{\sigma-1}{\gamma}+1\right)g_A}\left(1-\theta\right)\left(1+\nu\right), 0\right\}$.

B.1.1 Sufficient conditions for Unique BGP with $g_A > 0$ and $r > g_q$

Define the function

$$H\left(g_A\right) = g_A - \frac{\lambda\left(\mu + \mathcal{R}\left(g_A\right)\right) - \rho}{\frac{\sigma-1}{\gamma} + 1 + \mu},$$

so that the solution to $H\left(g_A\right) = 0$ is the BGP growth rate.

It is easy to see that the assumptions that $\rho < \lambda\mu$ and $\frac{\sigma-1}{\gamma} + 1 > 0$ guarantee that $g_A > 0$ and that the interest rate satisfies $r = \rho + \frac{\sigma}{1-\gamma} g_q > g_q$.

In the case of contracting with $\mathcal{R}\left(g_A\right) = 0$, the equilibrium is unique and the growth rate is

$$g_A = \frac{\lambda\mu - \rho}{\frac{\sigma-1}{\gamma} + 1 + \mu}.$$

In the case of conflict with $\mathcal{R}\left(g_A\right) > 0$, the equilibrium is unique if and only if $H'\left(g_A\right) > 0$, since $\lim_{g_A \to \infty} = \infty$ and $H\left(0\right) < 0$ under the maintained assumptions. A sufficient condition is thus

$$\begin{aligned} H'\left(g_A\right) &= 1 - \frac{\lambda}{\frac{\sigma-1}{\gamma}+1+\mu} \frac{\left(\frac{\sigma-1}{\gamma}+1\right)\left(1-\mathcal{R}\left(g_A\right)\right)}{\rho+\delta+\left(\frac{\sigma-1}{\gamma}+1\right)g_A}, \\ &> 1 - \frac{\lambda}{\frac{\sigma-1}{\gamma}+1+\mu} \frac{\frac{\sigma-1}{\gamma}+1}{\rho+\delta} > 0. \end{aligned}$$

Or equivalently,

$$\lambda < \left(1 + \frac{\mu}{\frac{\sigma-1}{\gamma}+1}\right)\left(\rho+\delta\right).$$

B.1.2 Sufficient Conditions for Positive Appropriation Rents and Conflict

Conflict arises if and only if there are positive appropriation rents $\mathcal{R}\left(g_A\right) > 0$, whereas contracting arises otherwise. We can find a sufficient condition as follows.

If contracting arises, then necessarily

$$((1-\theta)(1+\nu)-1)\,\delta \geq \rho + \left(\frac{\sigma-1}{\gamma}+1\right)\frac{\lambda\mu-\rho}{\frac{\sigma-1}{\gamma}+1+\mu} \equiv \bar{\chi}$$

for the growth rate $g_A = \frac{\lambda\mu-\rho}{\frac{\sigma-1}{\gamma}+1+\mu}$ under contracting to be consistent with $\mathcal{R}(g_A)=0$.

Moreover, if the opposite holds $\delta((1-\theta)(1+\nu)-1) < \bar{\chi}$, then $\mathcal{R}(g_A) > 0$ since

$$\begin{aligned}
\mathcal{R} &= 1 - \frac{\delta}{\rho+\delta+\left(\frac{\sigma-1}{\gamma}+1\right)g_A}(1-\theta)(1+\nu) \\
&= 1 - \frac{\delta}{\delta+\left(\frac{\sigma-1}{\gamma}+1\right)\frac{\lambda\mathcal{R}(g_A)}{\frac{\sigma-1}{\gamma}+1+\mu}+\bar{\chi}}(1-\theta)(1+\nu) \\
&= \frac{\left(\frac{\sigma-1}{\gamma}+1\right)\frac{\lambda\mathcal{R}(g_A)}{\frac{\sigma-1}{\gamma}+1+\mu}+\bar{\chi}-((1-\theta)(1+\nu)-1)\,\delta}{\delta+\left(\frac{\sigma-1}{\gamma}+1\right)\frac{\lambda\mathcal{R}(g_A)}{\frac{\sigma-1}{\gamma}+1+\mu}+\bar{\chi}} \\
&> \frac{\left(\frac{\sigma-1}{\gamma}+1\right)\frac{\lambda\mathcal{R}(g_A)}{\frac{\sigma-1}{\gamma}+1+\mu}}{\delta+\left(\frac{\sigma-1}{\gamma}+1\right)\frac{\lambda\mathcal{R}(g_A)}{\frac{\sigma-1}{\gamma}+1+\mu}+\bar{\chi}} \geq 0.
\end{aligned}$$

B.2 Strategic Counterparties

Here, we consider an alternative environment where counterparties behave strategically when offered a contract, and show conditions under which the resource's competitive price is in the contracting set.

B.2.1 Innovator's Contract Offer

Instead of choosing appropriation and conflict, the innovator can attempt to contract with the counterparty. The innovator offers to pay the counterparty for recognition of ownership of the resource, and both parties commit not to re-negotiate or dispute afterwards. Specifically, the contract negotiates a one-time payment $\tilde{p}_t$ at time t. The conflict option remains available to both parties — it is their outside option.

The expected value of contracting for the innovator is

$$\mathcal{V}_{it}^{\text{contract}} = \mathcal{D}_{it} + \int_t^\infty e^{-\int_t^s r_v dv} q_s ds - \tilde{p}_t.$$

As for the counterparty, they may choose to accept the contract or reject it. The existence of the resource is initially unknown to the counterparty, or equivalently, they believe it is useless. By offering the contract, the innovator reveals to the counterparty that a new resource exists, that it has some productive value, and, importantly, that they may have a claim over it.[2] That is, the innovator gives away the informational rents from the counterparty remaining unaware for some period of time — it is as if the arrival rate was $\delta \to \infty$. In keeping the gambling metaphor, the innovator inevitably *tips their hand* when trying to contract beforehand.

If the counterparty rejects the contract and the gamble is profitable, the innovator would appropriate the resource anyway and settle the conflict over property rights afterwards. That is, whenever $\lim_{\delta\to\infty} V_{it}^{\text{conflict}} \geq 0$ the innovator cannot commit not to appropriate the resource and use it to produce the new input. In this case, the act of offering the contract improves the counterparty's outside option: it gives them the option to reject the contract and resolve the conflict over property rights afterwards, obtaining the expected value $\mathcal{U}_s^{\text{conflict}}$. Otherwise, when the gamble is not profitable, the counterparty's outside option is simply zero; the innovator would never appropriate the resource if the counterparty rejects the contract.

In all, the value of a contract for the counterparty is

$$\mathcal{U}_t^{\text{contract}} = \max\left\{\tilde{p}_t, \mathbb{I}_{\lim_{\delta\to\infty} \mathcal{V}_{it}^{\text{conflict}} \geq 0} \mathcal{U}_t^{\text{conflict}}\right\}.$$

B.2.2 Conflict or Contract?

To develop intuition about the economics of the problem, we begin by considering a partial equilibrium of the model with a constant interest rate r, and where the stream of profits π_{it}, resource prices q_{it} and contract prices $\tilde{p}_s$ grow at a constant rate g. The next section endogenizes all variables in general equilibrium. In particular, we characterize the equilibrium along a balanced growth path and provide sufficient conditions for conflict to arise, and for the resource's competitive price to be in the contracting set.

[2] We assume that the information about the counterparty's chance of winning an eventual dispute $1-\theta$ is either common knowledge or it is "hard information" that is credibly disclosed to them. Likewise, the counterparty learns about the true productive value of the resource after being offered a contract, as captured by the stream of profits π_{it} and prices q_{is}. That is, there is no information asymmetry and the counterparty is neither pessimistic nor overly optimistic. Asymmetric information would create additional scope for contracting when the counterparty is pessimistic, and reduce the scope when they are overoptimistic.

In partial equilibrium, we obtain the values under conflict

$$\mathcal{V}^{\text{conflict}} = \max\left\{\frac{\pi+q}{r-g} - \frac{\delta}{r-g+\delta}\mathcal{U}^{\text{conflict}}, 0\right\}$$
$$\mathcal{U}^{\text{conflict}} = \frac{1}{r-g}\left(1-\theta\right)\left(1+\nu\right)q$$

and under contract

$$\mathcal{V}^{\text{contract}} = \max\left\{\frac{\pi+q}{r-g} - \tilde{p}, \lim_{\delta\to\infty}\mathcal{V}^{\text{conflict}}\right\}$$
$$\mathcal{U}^{\text{contract}} = \max\left\{\tilde{p}, \mathbb{I}_{\lim_{\delta\to\infty}\mathcal{V}^{\text{conflict}}\geq 0}\mathcal{U}^{\text{conflict}}\right\}.$$

Figure B.1 shows the contract prices $\tilde{p}$ that the innovator is willing to offer (blue region) and the counterparty is willing to accept (red region) as a function of the counterparty's utility under conflict $\mathcal{U}^{\text{conflict}}$. The purple region depicts when contracting arises; outside of this region conflict arises instead.

Consider first the case where $V^{\text{conflict}} < 0$ so that the innovator's gamble is not profitable — $\mathcal{U}^{\text{conflict}}$ is higher than $\frac{r-g+\delta}{\delta}\frac{\pi+q}{r-g}$ in the figure. The counterparty is willing to accept any positive price $\tilde{p} \geq 0$ since their outside option is nil. The innovator is willing to offer any price which makes the innovation profitable under the contract — that is, any $\tilde{p}$ lower than $\frac{\pi+q}{r-g}$ in the figure. As a result, there is always a price at which both parties are willing to contract — the purple region in the figure. Therefore, conflict does not arise and ownership is always established via contracting.

Consider next the case where $\mathcal{V}^{\text{conflict}} \geq 0$ so that the innovator's gamble is profitable — $\mathcal{U}^{\text{conflict}}$ is lower than $\frac{r-g+\delta}{\delta}\frac{\pi+q}{r-g}$ in the figure. The innovator is now only willing to offer contract prices that are low enough; lower than $\tilde{p} : \mathcal{V}^{\text{conflict}} = \mathcal{V}^{\text{contract}}$ which makes them indifferent — the blue line in the figure. The counterparty's outside option remains nil when $\mathcal{U}^{\text{conflict}}$ is still higher than $\frac{\pi+q}{r-g}$: they accept any positive price $\tilde{p} \geq 0$. The reason is that, if they rejected the contract, the innovator would walk away instead of choosing to innovate under conflict, since $\lim_{\delta\to\infty}\mathcal{V}^{\text{conflict}} < 0$ in this case. Again, there is always a price at which both parties are willing to contract — the purple region in the figure.

Finally, when $\mathcal{U}^{\text{conflict}}$ is lower than $\frac{\pi+q}{r-g}$, the counterparty's outside option improves to $\mathcal{U}^{\text{conflict}}$ because, if they rejected the contract, the innovator would still choose to resolve the dispute over property rights via conflict. Thus, the counterparty only accepts contract prices that are high enough; above the price $\tilde{p} = \mathcal{U}^{\text{conflict}}$ which makes them indifferent — the red line in the figure. In this case, there is no possible contract that the innovator is willing to offer and the counterparty accept. Therefore, conflict arises and

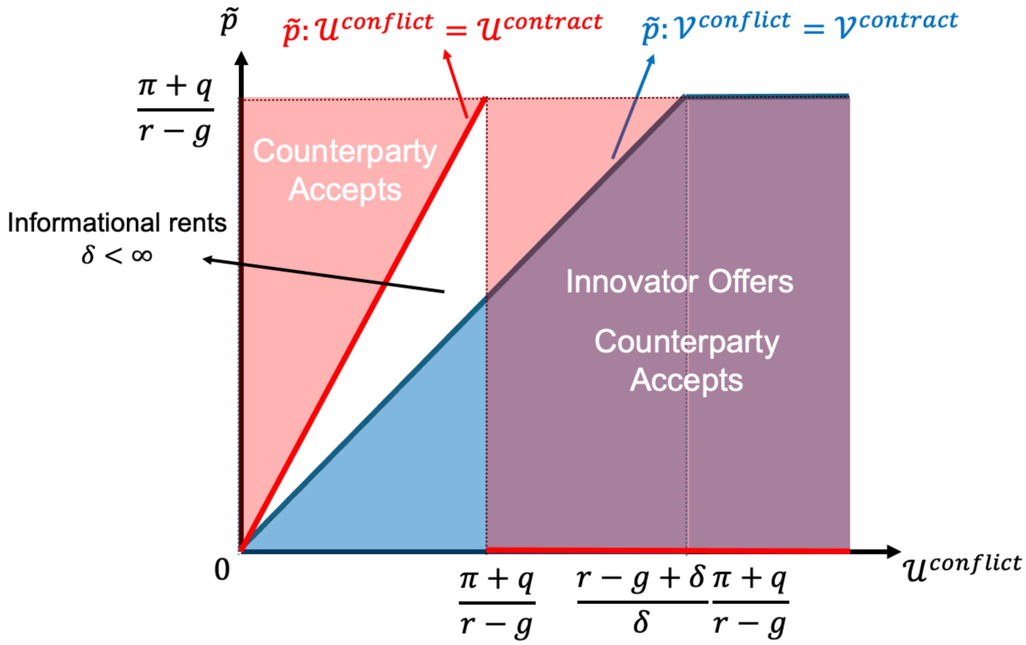

Figure B.1: Conflict v. Contract

property rights are established after the time at which innovation and appropriation had taken place. The reason is that, by offering the contract, the innovator gives away their informational advantage and allows the counterparty to extract some rents. Graphically, the informational advantage just described implies that the indifference line of the innovator — the blue line in the figure — is flatter than that of the counterparty's — the red line. As a result, there is a region where neither does the innovator wish to offer a contract nor would the counterparty accept it — the white region in the figure.

THE ROLE OF INFORMATION REVELATION. Central to conflict arising is the assumption that the act of offering a contract reveals valuable information to the counterparty. That is, that the innovator tips their hand. To see this, suppose that the contract revealed no information about how valuable the new resource was, so the counterparty keeps believing that it is useless — they believe profits π and price q are such that $\pi + q = 0$ and thus expect $\mathcal{V}^{\text{conflict}} = 0$. Then, conflict never arises: the innovator can offer to contract at a zero price $\tilde{p}$ and the counterparty would accept it. Similarly, suppose that $\delta \to \infty$ so that the innovator had no informational advantage. Then, there is always a contract price $\tilde{p} = \mathcal{U}^{\text{conflict}}$ that parties would agree on.

B.2.3 Sufficient conditions for Contracting at the Resource's Competitive Price

Along a balanced growth path, contracting arises if

$$\left(r - g_q\right) \mathcal{U}^{\text{conflict}} \equiv (1-\theta)(1+\nu) > \pi + 1$$

and the resource's competitive price $\tilde{p} = \frac{1}{r - g_q}$ is in the contracting set if

$$\frac{\delta}{r - g_q + \delta}(1-\theta)(1+\nu) \geq 1.$$

Using the expression for $\mathcal{V}$ and the fact that $1 = \lambda \mathcal{V}$, these are equivalent to

$$\begin{aligned}
(1-\theta)(1+\nu) &> \frac{\rho + \left(\frac{\sigma-1}{\gamma} + 1\right) g_A + \lambda}{\lambda} \\
(1-\theta)(1+\nu) &\geq \frac{\rho + \left(\frac{\sigma-1}{\gamma} + 1\right) g_A + \delta}{\delta}.
\end{aligned}$$

Therefore, if $\delta > \lambda$, the first inequality implies that the second is satisfied too. Moreover, replacing with the BGP growth rate g_A, we have that the first inequality becomes

$$(1-\theta)(1+\nu) > 1 + \frac{\left(\frac{\sigma-1}{\gamma} + 1 + \frac{\rho}{\lambda}\right)\mu}{\frac{\sigma-1}{\gamma} + 1 + \mu}.$$

In all, we have shown that, if $(1-\theta)(1+\nu) > 1 + \frac{\left(\frac{\sigma-1}{\gamma}+1+\frac{\rho}{\lambda}\right)\mu}{\frac{\sigma-1}{\gamma}+1+\mu}$ and $\delta > \lambda$, then contracting arises along a balanced growth path and the resource's competitive price is in the contracting set.

B.3 Optimal Policy

Household utility is

$$\begin{aligned}
U &= \int_0^\infty e^{-\rho t} \frac{\left(C_0 e^{g_C t}\right)^{1-\sigma}}{1-\sigma} dt \\
&= \frac{(C)^{1-\sigma}}{1-\sigma} \int_0^\infty e^{-\left(\rho + \frac{\sigma-1}{\gamma} g_A\right)t} t dt \\
&= \frac{A^{\frac{1-\sigma}{\gamma}}}{1-\sigma} \frac{\left(1 - \frac{1}{\lambda} g_A\right)^{1-\sigma}}{\rho + \frac{\sigma-1}{\gamma} g_A}.
\end{aligned}$$

Therefore, the optimal growth rate is

$$g_A^* = \frac{1}{\sigma}\left(\lambda - \gamma\rho\right).$$

If the optimal growth rate is below the equilibrium growth rate under contracting, then any combination of θ and ν such that contracting arises $(1-\theta)(1+\nu) \geq \underline{\chi}$ is optimal for the planner. The reason is that the equilibrium growth rate under contracting is below the growth rate under conflict for any θ and ν. The planner cannot perfectly implement the optimal growth rate in this case, and thus chooses to implement the closest growth rate possible. Otherwise, if the optimal growth rate is above the equilibrium growth rate under contracting, then any combination of θ and ν such that conflict arises $(1-\theta)(1+\nu) < \underline{\chi}$ and that satisfies

$$(1-\theta)(1+\nu) = \left(\mu - \frac{1-\gamma}{\gamma}\right)\left(1 - \frac{1}{\lambda} g_A^*\right)\frac{\rho + \delta + \left(\frac{\sigma-1}{\gamma} + 1\right) g_A^*}{\delta}$$

would implement g_A^* when $\mu \geq \frac{1-\gamma}{\gamma}$, since $1 - \frac{1}{\lambda} g_A^* > 0$ whenever aggregate consumption is strictly positive along a BGP. Lastly, if $\mu < \frac{1-\gamma}{\gamma}$, then $\theta = 1$ is optimal since it implements the closest possible growth rate to the optimal growth rate.