Welfare Cost of Inflation in Production Networks

Hassan Afrouzi*

Saroj Bhattarai†

Edson Wu◇

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"The views expressed in this paper are those of the authors and do not necessarily represent the views or policies of the Board of Governors of the Federal Reserve System, the Federal Reserve Bank of Minneapolis, or their staff."

* Columbia University, NBER and Federal Reserve Bank of Minneapolis, † UT Austin, ◇ Federal Reserve Board

Motivation

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    - What if we let inflation settle at $\sim 2.5\%$ or $\sim 3\%$?

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  - What if we let inflation settle at $\sim 2.5\%$ or $\sim 3\%$?
- At moderate levels of inflation, welfare costs are negligible and flat
  - "Elusive costs of inflation" in state-dependent pricing models (Burstein and Hellwig, 2008; Nakamura et al., 2018; Blanco, 2021)
  - Standard sticky price model has no input-output linkages
  - Roundabout production amplifies cost of inflation in time-dependent models as dispersion propagates through intermediate inputs (Christiano, 2015)

Research Questions

- How do **production networks** affect the welfare costs of inflation in **state-dependent pricing** models?
    1. Theoretical: What are the **sources** of these welfare costs?
    2. Quantitative: How **large** are these welfare costs?

What We Do

- Production networks model with heterogeneous sectors and state-dependent pricing
- Calibration: US I-O tables; data on moments of price change distribution

What we find

- Theoretically, the welfare cost of inflation in our model is driven by:
  1. **Aggregate misallocation: aggregate *within-sector* misallocation**
  2. **Menu costs paid by firms to adjust prices**
  3. Across-sector misallocation and labor stimulus

What we find

- Theoretically, the welfare cost of inflation in our model is driven by:
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    2. **Menu costs paid by firms to adjust prices**
    3. Across-sector misallocation and labor stimulus
- Quantitatively, in our calibrated *baseline* model:
    - Welfare cost of inflation at 2% is $\approx$ 0.15% (CE units relative to 0% inflation)
    - Welfare cost of inflation 2.7 **times larger** relative to a multi-sector economy
    - Moving from 2% to 4% inflation
        - Cost increases by **21 bps**, compared with **7.87 bps** in the multi-sector economy
        - Menu costs account for **69.5%** of the increase

Related Literature

- Welfare cost of inflation
  - Lucas (2000); Ireland (2009); Benati and Nicolini (2025)
  - Benabou (1992); Diamond (1993); Sara-Zaror (2025)
  - Burstein and Hellwig (2008); Nakamura et al. (2018); Alvarez et al. (2018)
- Random menu costs/Calvo plus models
  - Nakamura and Steinsson (2010); Alvarez and Lippi (2014); Baley and Blanco (2021); Luo and Villar (2021); Alvarez et al. (2016, 2021); Cavallo et al. (2023a, 2024)
- Production networks model with state-dependent pricing
  - Ghassibe and Nakov (2025)
- Welfare cost of inflation in production network, time-dependent pricing model
  - Christiano (2015); Castro (2019)
- **Contribution**: Welfare cost of inflation in a quantitative multi-sector production network model with state-dependent pricing and idiosyncratic shocks

Model

- Time is continuous and runs forever
- n industries indexed by $i \in [n] \equiv \{1, \ldots, n\}$
- A measure of monopolistically competitive intermediate firms in each sector
- A final good producer in each sector packages and sells a sectoral good
- Sectoral goods consumed by households and used for production
- **Objective:** Steady-state welfare comparative statics w.r.t. inflation

- Household

$$\max \int_0^\infty e^{-\rho t}[\log(C_t) - L_t]\mathrm{d}t$$

$$\sum_{i\in[n]} P_{i,t}C_{i,t} + \dot{B}_t \leq W_t L_t + i_t B_t + T_t$$

$$C_t \equiv \prod_{i\in[n]} \left(\frac{C_{i,t}}{\beta_i}\right)^{\beta_i}, \quad P_t \equiv \frac{\sum_{i\in[n]} P_{i,t}C_{i,t}}{C_t}$$

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- Monetary Policy $\{M_t = P_t C_t\}_{t\geq 0}$

$$\dot{M}_t = \pi M_t$$

- **Household**

$$\max \int_0^\infty e^{-\rho t}[\log(C_t) - L_t]\mathrm{d}t$$

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- **Monetary Policy** $\{M_t = P_t C_t\}_{t\geq 0}$

$$\dot{M}_t = \pi M_t$$

- **Final Good** Producer

$$Y^d_{ij,t} = A_{ij,t}\left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta_i} Y_{i,t}$$

$A_{ij,t}$ is ij's taste shock

- **Household**

$$\max \int_0^\infty e^{-\rho t}[\log(C_t) - L_t]\mathrm{d}t$$

$$\sum_{i\in[n]} P_{i,t}C_{i,t} + \dot{B}_t \leq W_t L_t + i_t B_t + T_t$$

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$A_{ij,t}$ is ij's taste shock

- **Intermediate Good** Producers

$$Y^s_{ij,t} = Z_{ij,t}\left(L_{ij,t}/\alpha_i\right)^{\alpha_i} \prod_{k\in[n]} \left(X_{ij,k,t}/a_{ik}\right)^{a_{ik}}$$

$$\mathrm{d}\log(Z_{ij,t}) = \mu_i \mathrm{d}t + \sigma_i \mathrm{d}\mathcal{W}_{ij,t}$$

and $A_{ij,t}Z^{\eta_i - 1}_{ij,t} = 1$, $\mathcal{W}_{ij,t}$ is Wiener

- I-O matrix: $\mathbf{A} \equiv [a_{ik}]_{i,k\in[n]}$

Model – Intermediate Good Producers – Pricing

- Calvo Plus/Random menu costs set-up (Nakamura and Steinsson, 2010; Alvarez and Lippi, 2014; Baley and Blanco, 2021; Alvarez et al., 2022)
  - can pay total menu cost $\tilde{\lambda}_i \chi_i > 0$ (in units of labor) to change prices
  - receives i.i.d free price change opportunities that arrive at Poisson rate $\theta_i > 0$

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  - receives i.i.d free price change opportunities that arrive at Poisson rate $\theta_i > 0$
- Let $N_{ij,t}$ be a Poisson counter with arrival rate θ_i. Nominal adjustment cost is

$$\mathrm{d}\varphi_{ij,t} = \begin{cases} 0 & \text{, if } \mathrm{d}P_{ij,t} = 0 \text{ or } \mathrm{d}N_{ij,t} = 1 \\ W_t \tilde{\lambda}_i \chi_i & \text{, if } \mathrm{d}P_{ij,t} \neq 0 \text{ and } \mathrm{d}N_{ij,t} = 0 \end{cases}$$

where $\mathrm{d}P_{ij,t}$ is price change; $\tilde{\boldsymbol{\lambda}} = (\mathbf{I} - \mathbf{A}')^{-1}\boldsymbol{\beta}$ is cost-based Domar weights (normalization)

Model – Intermediate Good Producers – Pricing

- Firm chooses sequence of prices $(P_{ij,t})_{t=0}^{\infty}$ to solve

$$V_{ij,0} \equiv \max_{(P_{ij,t})_{t=0}^{\infty}} \mathbb{E}\Big\{ \int_0^{\infty} \frac{e^{-\rho t}}{P_t C_t} \Big[\Big\{ \Big((1-\tau_{i,t})P_{ij,t} - \frac{W_t^{\alpha_i} \prod_{k\in[n]} P_{k,t}^{a_{ik}}}{Z_{ij,t}}\Big) A_{ij,t} \Big(\frac{P_{ij,t}}{P_{i,t}}\Big)^{-\eta_i} Y_{i,t} \Big\} \mathrm{d}t - \mathrm{d}\varphi_{ij,t} \Big] \Big\}$$

- Let $x_{ij,t} \equiv \log(P_{ijt}/P^*_{ijt})$ be firm ij's price gap, where P^*_{ijt} is its ideal price ideal price

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- Let $x_{ij,t} \equiv \log(P_{ijt}/P^*_{ijt})$ be firm ij's price gap, where P^*_{ijt} is its ideal price ideal price
- Equilibrium concept: **Recursive stationary equilibrium** Eq. def.
  - x is a *sufficient* state variable for the firm's pricing problem
  - Let $v_i(x) \equiv \frac{V_{i,t}}{\lambda_i Q_i}$ in stationary equilibrium
    - $\lambda_i \equiv \frac{P_{i,t} Y_i}{P_t C}$ is sector i's sales-based Domar weight
    - $Q_i \equiv (1-\tau_i)^{\eta_i} \Big(\frac{\eta_i}{\eta_i - 1} \frac{W_t^{\alpha_i} \prod_{k \in [n]} P_{k,t}^{a_{ik}}}{P_{i,t}} \Big)^{1-\eta_i}$

Model – Intermediate Good Producers – Pricing

- We solve the recursive problem, where the HJB-VI equation is given by

$$\rho v_i(x) = \max\left\{ \underbrace{\rho(v_i(x_i^*) - \phi_i)}_{\text{change}},\ \underbrace{f_i(x) + (\mu_i - \pi)v_i'(x) + \frac{\sigma_i^2}{2}v_i''(x) + \theta_i(v_i(x^*) - v_i(x))}_{\text{no change}} \right\}$$

where $x_i^* \equiv \arg\max_y v_i(y)$; $f_i(x) \equiv \left(e^x - \frac{\eta_i - 1}{\eta_i}\right)e^{-\eta_i x}$; $\phi_i \equiv \frac{\tilde{\lambda}_i \chi_i}{\lambda_i Q_i}$

- Optimal policy: reset point, x_i^*, and Ss bands $\{\underline{x}_i, \overline{x}_i\}$
- Each $i \in [n]$ characterized by $(\phi_i, \theta_i, \sigma_i, \mu_i)$ and can be calibrated *separately*
- For counterfactuals, we recover χ_i from calibrated $(\phi_i, \theta_i, \sigma_i, \mu_i)$ and distribution of price gaps map χ_i and ϕ_i

Value Function Value matching and smooth pasting

Theoretical Results

Steady-State Allocations and Sources of Inefficiencies

- Cost minimization of firms in sector i implies *sectoral production function*:

$$Y_i = \frac{1}{D_i(\pi)} \times \left({}^{L_i}/_{\alpha_i}\right)^{\alpha_i} \prod_{k\in[n]} \left({}^{X_{i,k}}/_{a_{ik}}\right)^{a_{ik}}, \quad D_i(\pi) \equiv \int_0^1 \frac{A_{ij,t}}{Z_{ij,t}} \left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta_i} \mathrm{d}j \geq 1$$

- *Sectoral marginal cost:*

$$\mathrm{MC}_{i,t} \equiv \min_{L_i,(X_{i,k})_{k\in[n]}} W_t L_i + \sum_{k\in[n]} P_{k,t} X_{i,k} \quad s.t. \quad \frac{1}{D_i(\pi)} \times \left({}^{L_i}/_{\alpha_i}\right)^{\alpha_i} \prod_{k\in[n]} \left({}^{X_{i,k}}/_{a_{ik}}\right)^{a_{ik}} \geq 1$$

- *Optimal pricing* of firms implies

$$\mathcal{M}_i(\pi) \equiv \frac{P_{i,t}(W_t, P_{1,t}, \ldots, P_{n,t}; D_i(\pi))}{\mathrm{MC}_{i,t}(W_t, P_{1,t}, \ldots, P_{n,t}; D_i(\pi))} = \frac{\eta_i}{\eta_i - 1} \frac{1}{1 - \tau_i(\pi)} \bar{M}_i(\pi)$$

$\bar{M}_i$

1. Let $MPL \equiv {}^{\partial C}/_{\partial L^p} = Z$, where L^p is total labor used in production;
2. Let $MRS = -U_l/U_c = C$. Then,

$$\frac{MRS}{Z} = \sum_{i \in [n]} \alpha_i \frac{\lambda_i(\pi)}{\mathcal{M}_i(\pi)}$$

$$\log Z(\pi) = -\sum_{i \in [n]} \tilde{\lambda}_i \times \log D_i(\pi) - \Big[\sum_{i \in [n]} \tilde{\lambda}_i \times \log \mathcal{M}_i(\pi) + \log \Big(\sum_{i \in [n]} \alpha_i \frac{\lambda_i(\pi)}{\mathcal{M}_i(\pi)} \Big) \Big]$$

where $\tilde{\boldsymbol{\lambda}} = (\mathbf{I} - \mathbf{A}')^{-1} \boldsymbol{\beta}$; $\quad \boldsymbol{\lambda}(\pi) = (\mathbf{I} - \mathbf{A}' \text{diag}(\mathcal{M}_i^{-1}(\pi)))^{-1} \boldsymbol{\beta}$

- Inflation distorts:
  - Equilibrium labor supply as the factor of production (by changing $\frac{MRS}{Z}$)
  - Aggregate productivity by distorting relative prices within and across sectors

Welfare Cost of Inflation

- $\Lambda(\pi)$ such that $U(C(\pi), L(\pi)) = U(e^{-\Lambda(\pi)}C(0), L(0))$

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Proposition

Let taxes be $(\bar{\tau}_i)_{i\in[n]}$, constant across π. The welfare cost of inflation π is

$$\Lambda(\pi) = \underbrace{\sum_{i\in[n]} \tilde{\lambda}_i \times \left[\log D_i(\pi) - \log D_i(0)\right]}_{\text{within-sector price distortion}} + \underbrace{\sum_{i\in[n]} \tilde{\lambda}_i \chi_i \left[\mathrm{N}_i^{menu}(\pi) - \mathrm{N}_i^{menu}(0)\right]}_{\text{menu cost paid}}$$

$$\underbrace{+ \sum_{i\in[n]} \tilde{\lambda}_i \times \left[\log \mathcal{M}_i(\pi) - \log \mathcal{M}_i(0)\right] + \sum_{i\in[n]} \alpha_i \left[\frac{\lambda_i(\pi)}{\mathcal{M}_i(\pi)} - \frac{\lambda_i(0)}{\mathcal{M}_i(0)}\right]}_{\text{across-sector price distortion + labor stimulus}}$$

where $\mathrm{N}_i^{menu}(\pi)$ measure of firms paying the menu cost in $i \in [n]$ at π

Welfare Cost when Fiscal Policy ensures $\mathcal{M}_i = 1, \forall i \in [n], \pi$

- **Baseline**: Fiscal policy chooses $\tau_i(\pi)$ such that $\mathcal{M}_i(\pi) = 1, \ \forall i \in [n], \pi$
- Then,

$$\frac{MRS}{Z} = 1 \quad \text{and} \quad \log Z(\pi) = -\sum_{i \in [n]} \tilde{\lambda}_i \times \log D_i(\pi)$$

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Proposition

When $\tau_i(\pi)$ such that $\mathcal{M}_i(\pi) = 1, \forall i \in [n], \pi$

$$\Lambda(\pi) = \sum_{i \in [n]} \tilde{\lambda}_i \times \left[\log D_i(\pi) - \log D_i(0)\right] + \sum_{i \in [n]} \tilde{\lambda}_i \chi_i \left[\mathrm{N}_i^{menu}(\pi) - \mathrm{N}_i^{menu}(0)\right]$$

- Sources of welfare cost:
    1. Aggregate misallocation – Domar-weighted sum of log dispersion difference
    2. Menu cost paid

Dispersion at Zero Inflation Steady State

- In the stationary equilibrium

$$\ln D_i(\pi) = \ln\left(\mathbb{E}_{g_i^\pi}\left[e^{-\eta_i x}\right]\right) + \left(\frac{\eta_i}{1-\eta_i}\right)\ln\left(\mathbb{E}_{g_i^\pi}\left[e^{(1-\eta_i)x}\right]\right)$$

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- From Baley and Blanco (2021); Cavallo et al. (2023b)

  Assume (i) profit function $f(x)$ is quadratic; (ii) $\pi = 0$; (iii) $\mu_i = 0$. Then,

$$\ln(D(0)) \approx \frac{\eta}{2} \times \frac{\text{Var}(\Delta p) \times \text{Kur}(\Delta p)}{6}$$

  where $\text{Var}(\Delta p)$ and $\text{Kur}(\Delta p)$ are the variance and kurtosis of price changes

- Targeting them is important for quantification of welfare costs of inflation

Quantitative Results

Baseline Calibration

Parameter	Description	Value	Explanation
n	Number of sectors	65	BEA Summary IO tables
$\mathbf{A}$	Production exp. share		BEA IO tables
$\boldsymbol{\beta}$	Consumption exp. share		BEA IO tables
ρ	Discount rate	0.0034	4% annual
π	Steady-state inflation	$\log(1.02)/12$	2% annual
$\{\tau_i(\pi)\}_{i\in[n]}$	Tax		$\frac{\eta}{\eta-1}\frac{1}{1-\tau_i(\pi)}\bar{M}_i(\pi)=1$
$\{\eta_i\}_{i\in[n]}$	Elasticity of substitution	4	
$\{\phi_i\}_{i\in[n]}$	Menu cost		**Internally Calibrated**
$\{\theta_i\}_{i\in[n]}$	Calvo arrival rate		**Internally Calibrated**
$\{\sigma_i\}_{i\in[n]}$	Std. dev. of firm-level shock		**Internally Calibrated**
$\{\mu_i\}_{i\in[n]}$	Productivity drift		**Internally Calibrated**

Table 1: Model parameters calibration.

Internally Calibrated Parameters and Targets

Data:

- Frequency of price adjustment from Pasten et al. (2020)
- Standard deviation, kurtosis and fraction of positive price change by decile of frequency (Hong et al., 2023)

Targets and methodology:

- Linear interpolation based on frequency to impute sector-specific targets
- For $i \in [n]$, $(\phi_i, \theta_i, \sigma_i, \mu_i)$ to match frequency of price adjustment, standard deviation, kurtosis, and fraction of positive price change

Computer and electronic products | Rental and leasing services... | Wood products | Correlation Freq. × Moments

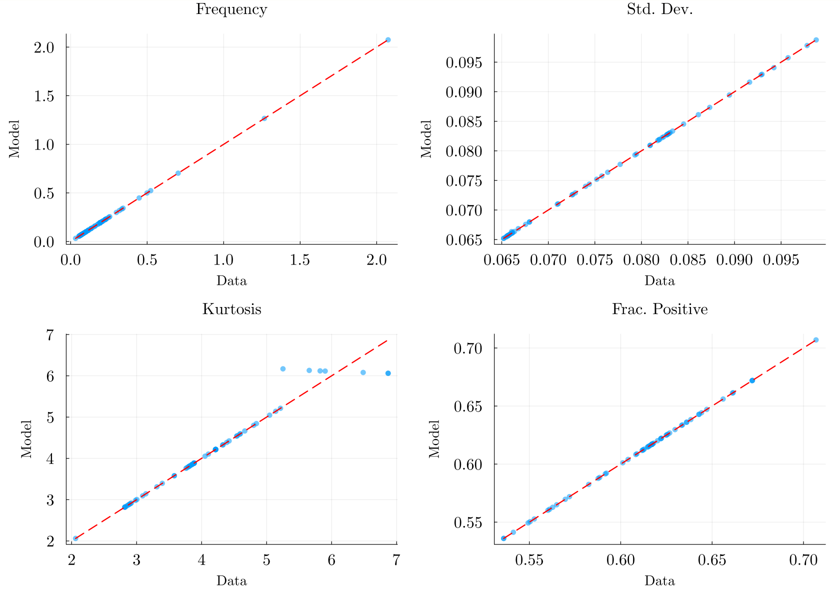

Figure 1: Moments: Data × Model.

Welfare Cost of Inflation: Role of Production Networks

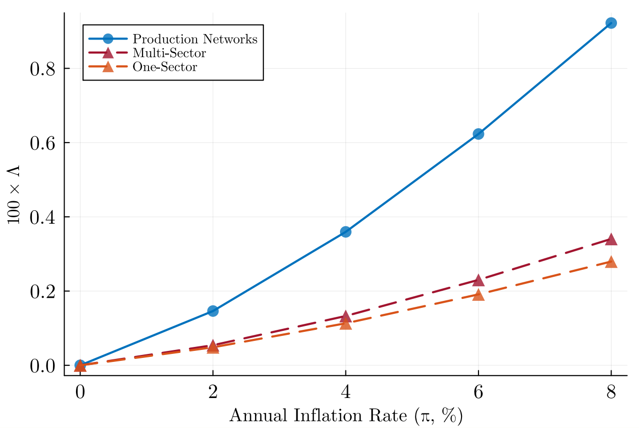

Figure 2: $\sum_i \lambda_i = 2.59$. One-sector Moments

1. At 2%, production networks alone amplify the welfare cost significantly
2. Slope is also steeper

Welfare Cost of Inflation at 2%

- Multi-sector (w/o production networks)

$$\Lambda(\pi) = \underbrace{\underbrace{\mathbb{E}_{\beta}[\Delta \log D_i(\pi)]}_{\approx 0.013} + \underbrace{\mathbb{E}_{\beta}[\chi_i \Delta \mathrm{N}_i^{menu}(\pi)]}_{\approx 0.041}}_{=0.054\%}$$

π_{ss}	Prod. Networks	Multi-Sector	Ratio
2.0	0.1462	0.0541	2.70
4.0	0.3595	0.1328	2.71
6.0	0.6232	0.2301	2.71
8.0	0.9221	0.3405	2.71

Table 2: $100 \times \Lambda(\pi)$. $\sum_{i\in[n]} \lambda_i = 2.59$

Multi $\times$ One | $\lambda_i \times \Delta \log D_i$ | $\beta_i \times \Delta \log D_i$ | Robustness 0.66 $\times$ GOS | $\eta_i = 6$

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$$\Lambda(\pi) = \underbrace{\underbrace{\mathbb{E}_\beta[\Delta \log D_i(\pi)]}_{\approx 0.013} + \underbrace{\mathbb{E}_\beta[\chi_i \Delta N_i^{menu}(\pi)]}_{\approx 0.041}}_{=0.054\%}$$

- Production networks

$$\Lambda(\pi) = \underbrace{\underbrace{\left(\sum_{i\in[n]} \lambda_i\right)}_{\approx 2.59}\left(\underbrace{\mathbb{E}_\lambda[\Delta \log D_i(\pi)]}_{\approx 0.0148} + \underbrace{\mathbb{E}_\lambda[\chi_i \Delta N_i^{menu}(\pi)]}_{\approx 0.041}\right)}_{=0.146\%}$$

Multi $\times$ One | $\lambda_i \times \Delta \log D_i$ | $\beta_i \times \Delta \log D_i$ | Robustness 0.66 $\times$ GOS | $\eta_i = 6$

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Table 3: $100 \times \Lambda(\pi)$. $\sum_{i\in[n]} \lambda_i = 2.59$

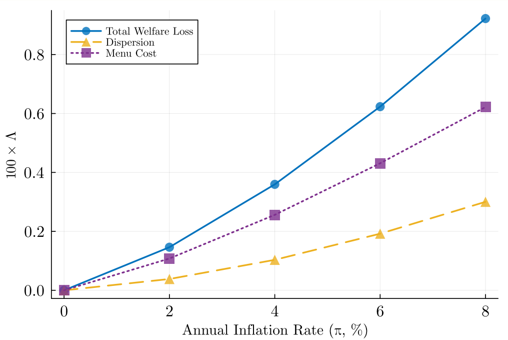
- Multi-sector (w/o production networks)

$$\Delta\Lambda(\pi) = \underbrace{\mathbb{E}_\beta[\Delta \log D_i(\pi)]}_{\approx 0.023} + \underbrace{\mathbb{E}_\beta[\chi_i \Delta \mathrm{N}_i^{menu}(\pi)]}_{\approx 0.056} = 0.079\%$$

- Production networks

$$\underbrace{\Delta\Lambda(\pi) = \underbrace{\Big(\sum_{i\in[n]} \lambda_i\Big)}_{\approx 2.59}\Big(\underbrace{\mathbb{E}_\lambda[\Delta \log D_i(\pi)]}_{\approx 0.025} + \underbrace{\mathbb{E}_\lambda[\chi_i \Delta \mathrm{N}_i^{menu}(\pi)]}_{\approx 0.057}\Big)}_{=0.213\%}$$

Welfare Cost of Inflation: Decomposition



π_{ss}	Dispersion	Menu Cost	Menu Cost Share
2.0	0.0384	0.1079	73.8
4.0	0.1034	0.2561	71.2
6.0	0.1921	0.4311	69.2
8.0	0.3003	0.6218	67.4

- At 2%, bulk of welfare cost due to menu cost
- Going from 2% to 4% inflation, $\approx$ 70% of increase due to menu cost

Baseline Calibration: $\frac{\text{Total menu cost paid}}{\text{Total sales}} = 0.228\%$, table comparison multi-sector

Welfare Cost of Inflation: Sectoral Disaggregation

Pct\π	0.0	2.0	4.0	6.0	8.0
20	0.0065	0.0067	0.0069	0.0071	0.0074
40	0.0072	0.0073	0.0074	0.0076	0.0079
60	0.0087	0.0087	0.0089	0.0092	0.0096
80	0.0093	0.0094	0.0097	0.0100	0.0105

(a) $\log$(Dispersion)

Pct\π	0.0	2.0	4.0	6.0	8.0
20	0.0190	0.0228	0.0274	0.0326	0.0382
40	0.0280	0.0341	0.0414	0.0495	0.0584
60	0.0393	0.0493	0.0617	0.0757	0.0905
80	0.0879	0.1069	0.1381	0.1714	0.2022

(b) Fraction of non-free price changes

Table 4: Moments of the sectoral price-change distribution across steady-state inflation

- Not only dispersion increases, but also fraction of non-free price changes

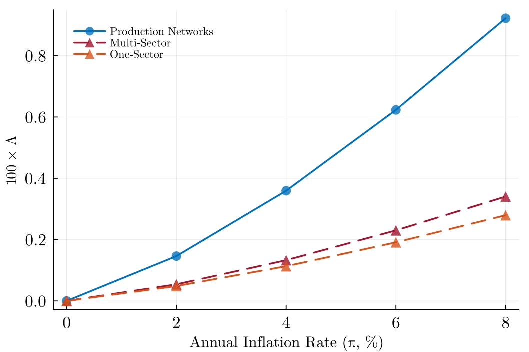
Other moments

Further Analysis

- Role of (fiscal policy) ensuring sectoral markups are one
- Heterogeneous η_i

Welfare Cost of Inflation: $\tau_i(\pi) = \bar{\tau}_i$

- What if $\tau_i(\pi) = \bar{\tau}_i$, where $\bar{\tau}_i$ such that $\mathcal{M}_i(0.02) = 1$, $\forall i \in [n]$, π ?
  - Equilibrium labor supply and sectoral relative prices are distorted



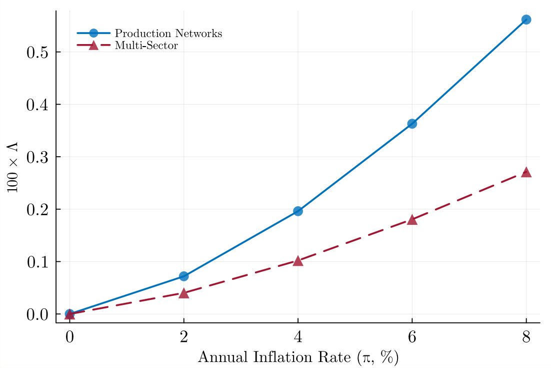
π_{ss}	Production Networks	Multi-Sector	(1)/(2)	One-Sector	(1)/(4)
2.0	0.1462	0.0541	2.70	0.0485	3.01
4.0	0.3595	0.1328	2.71	0.1132	3.18
6.0	0.6232	0.2301	2.71	0.1911	3.26
8.0	0.9221	0.3405	2.71	0.2796	3.30

Figure 3: $100 \times \Lambda(\pi)$.

- Amplification relative to multi-sector and one-sector same as baseline
- Why? New terms offset each other Useful approx. Decomposition

Welfare Cost of Inflation: $\tau_i(\pi) = 0$, $i \in [n]$, π

- Now, what if $\tau_i(\pi) = 0$, $\forall i \in [n]$, π ?



π_{SS}	Production Networks	Multi-Sector	Ratio
2.0	0.0719	0.0403	1.78
4.0	0.1961	0.1020	1.92
6.0	0.3630	0.1805	2.01
8.0	0.5616	0.2712	2.07

- At 2%, amplification relative to multi-sector still significant (1.78$\times$)
- Slope still steeper relative to multi-sector: 12.5 $\times$ 6.1 bps ($\pi = 2\% \to 4\%$)
- Welfare cost is smaller than baseline (at 2%, $\approx$ 14.5bps)

Welfare Cost of Inflation: Baseline $\times$ $\tau_i(\pi) = 0$, $i \in [n]$, π

π_{SS}	Total	Dispersion	Menu Cost
2.0	0.1462	0.0384	0.1079
4.0	0.3595	0.1034	0.2561
6.0	0.6232	0.1921	0.4311
8.0	0.9221	0.3003	0.6218

Table 5: $\tau_i(\pi)$ s.t. $\mathcal{M}_i(\pi) = 1$, $i \in [n]$, π

π_{SS}	Total	Dispersion	Menu Cost	$\approx$ Labor Stimulus
2.0	0.0719	0.0381	0.0578	-0.0241
4.0	0.1961	0.1030	0.1371	-0.0439
6.0	0.3630	0.1915	0.2306	-0.0592
8.0	0.5616	0.2995	0.3327	-0.0705

Table 6: $\tau_i(\pi) = 0$, $\forall i \in [n]$, π

Lower welfare losses under $\tau_i(\pi) = 0$ (w.r.t. baseline) due to

1. lower calibrated menu costs
    - Baseline subsidies remove market power, making firms larger
    - Larger firms require higher menu costs to match the same moments
2. labor stimulus

Baseline comparison

Heterogeneous η_i, $i \in [n]$: η_i to match skewness

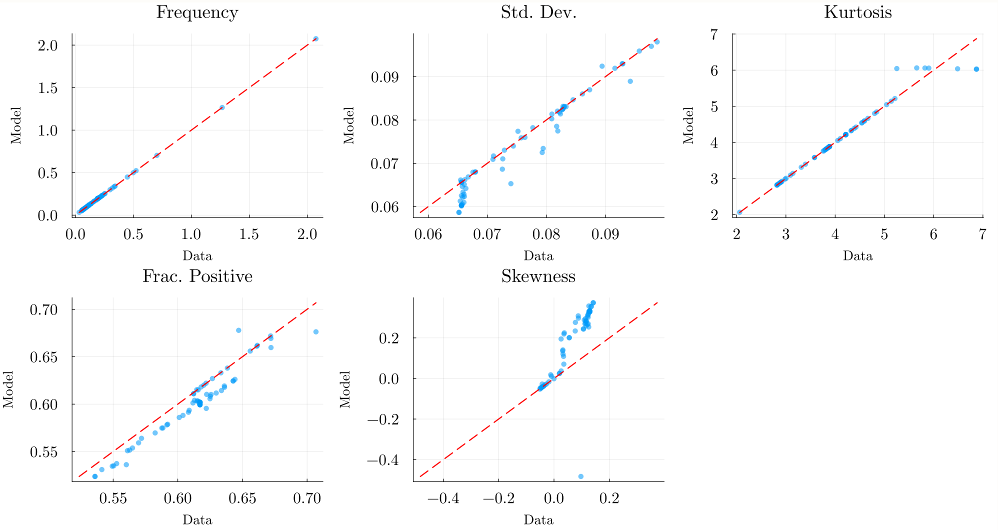
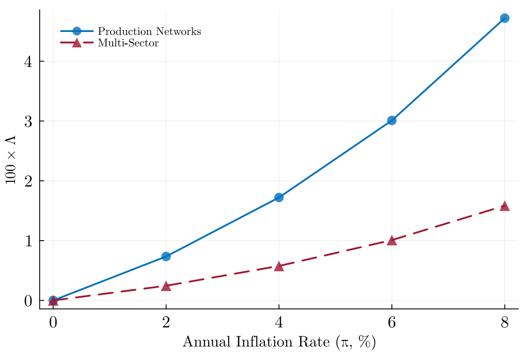


Figure 4: Moments: Data $\times$ Model. η_i histogram

Welfare Cost of Inflation: Heterogeneous $\eta_i, i \in [n]$

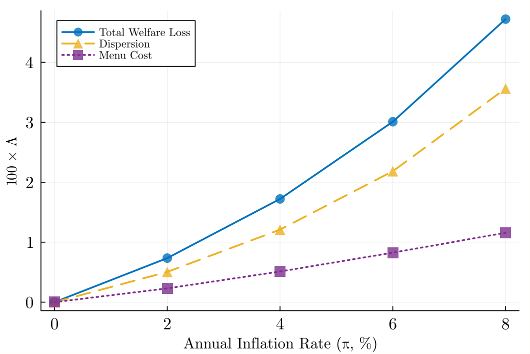


π_{SS}	Production Networks	Multi-Sector	Ratio
2.0	0.7347	0.2454	2.99
4.0	1.7210	0.5761	2.99
6.0	3.0091	1.0089	2.98
8.0	4.7202	1.5841	2.98

- At 2%, large amplification relative to multi-sector Amplification at 2%
- Slope is still steeper relative to multi-sector: 100×33bps ($\pi = 2\% \rightarrow 4\%$)
- At 2%, welfare cost is substantially larger than baseline ($5\times$)

Slope

Welfare Cost of Inflation Decomposition: Heterogeneous $\eta_i, i \in [n]$



π_{ss}	Dispersion	Menu Cost	Menu Cost Share
2.0	0.5044	0.2302	31.3
4.0	1.2095	0.5115	29.7
6.0	2.1838	0.8253	27.4
8.0	3.5622	1.1580	24.5

- Dispersion accounts for bulk of welfare cost

Additional Results

Additional Results

- Welfare cost of nominal rigidities welfare nominal rigidities
- Different calibration of Domar weights 0.66 GOS
- Role of sector-specific productivity drift zero drift
- Comparison with pure time-dependent pricing Calvo Plus vs Pure Calvo
- Comparison with pure state-dependent pricing Calvo vs Pure Menu
- One-sector economy (for comparison with previous literature) One-sector analysis

Conclusion

Conclusion

- Production networks *significantly* amplify welfare cost of inflation
  - At 2%, welfare cost is around $2.7\times$ larger relative to a multi-sector economy
  - When moving from 2% to 4% inflation,
    - Cost increases by 21 bps, compared to 7.9 bps in the multi-sector economy
    - Menu costs account for 69.5% of the increase
- Future work
  - General consumption aggregator and production functions
  - Transition dynamics

Appendix

- So far, welfare cost relative to zero-inflation steady state. What about relative to the efficient steady state?

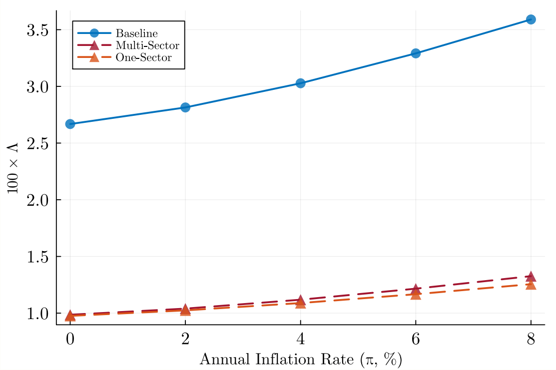
Proposition

Let $\tau_i(\pi)$ such that $\mathcal{M}_i(\pi) = 1$, $\forall i \in [n], \pi$. The welfare cost of inflation π relative to the efficient steady state is

$$\Lambda^{\mathrm{nr}}(\pi) = \sum_{i\in[n]} \tilde{\lambda}_i \log D_i(\pi) + \sum_{i\in[n]} \tilde{\lambda}_i \chi_i \mathrm{N}_i^{menu}(\pi)$$

where $\tilde{\boldsymbol{\lambda}} = (\mathbf{I} - \mathbf{A}')^{-1}\boldsymbol{\beta}$

Welfare Cost of Nominal Rigidities

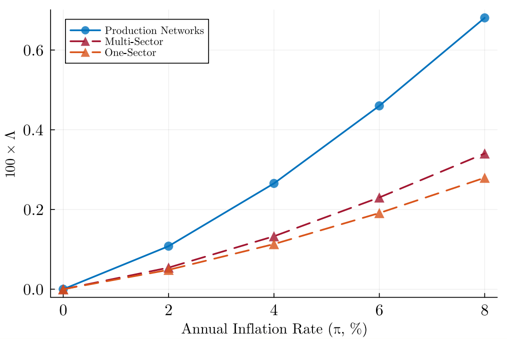


π_{ss}	Baseline	Multi-Sector	(1)/(2)	One-Sector	(1)/(4)
0.0	2.6679	0.9857	2.71	0.9760	2.73
2.0	2.8142	1.0399	2.71	1.0245	2.75
4.0	3.0274	1.1186	2.71	1.0891	2.78
6.0	3.2911	1.2159	2.71	1.1670	2.82
8.0	3.5901	1.3263	2.71	1.2555	2.86

- Even at 0% steady-state inflation, there is substantial welfare loss
- Amplification results are similar to baseline

Back

Different calibration of Domar weights

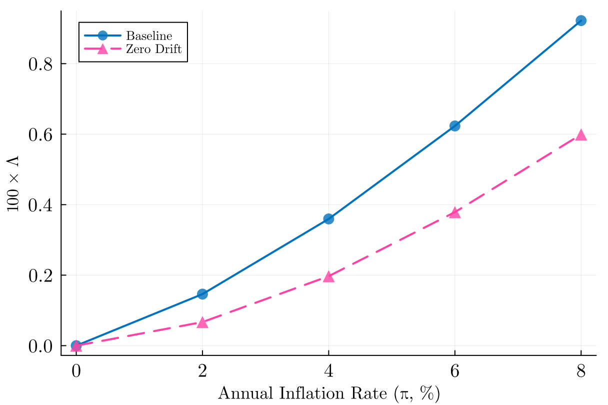


π_{SS}	Production Networks	Multi-Sector	(1)/(2)	One-Sector	(1)/(4)
2.0	0.1080	0.0541	1.99	0.0485	2.22
4.0	0.2654	0.1328	2.00	0.1132	2.34
6.0	0.4601	0.2301	2.00	0.1911	2.41
8.0	0.6808	0.3405	2.00	0.2796	2.43

Figure 5: $\sum_i \lambda_i = 1.928$. $\frac{\text{Gross Output}}{\text{GDP}} = \frac{\text{Total Intermediate}+\text{Compensation of Employees}+0.66\times\text{GOS}}{\text{Compensation of Employees}+0.66\times\text{GOS}}$ Back

- Baseline: $\frac{\text{Gross Output}}{\text{GDP}} = \frac{\text{Total Intermediate}+\text{Compensation of Employees}}{\text{Compensation of Employees}}$
- Assumption: $\text{GOS}_i + \text{Taxes on production and imports, less subsidies}_i = 0, \; i \in [n]$
- Here: Add 0.66×'Gross Operating Surplus' to 'Compensation of Employees' Alternative

- $\mu_i = 0, \forall i \in [n]$: Drop fraction of positive price change as target



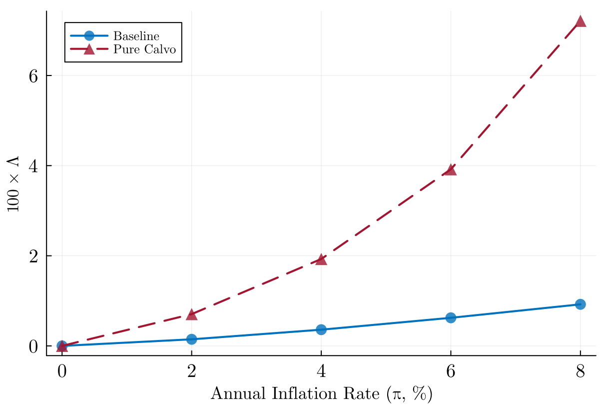
π_{ss}	Baseline	Zero Drift	Ratio
2.0	0.1462	0.0669	2.18
4.0	0.3595	0.1971	1.82
6.0	0.6232	0.3788	1.65
8.0	0.9221	0.5992	1.54

Table 7: Welfare cost in % : $100 \times \Lambda(\pi)$.

- Heterogeneous drift amplifies welfare cost

Back

Pure Calvo vs. Calvo Plus

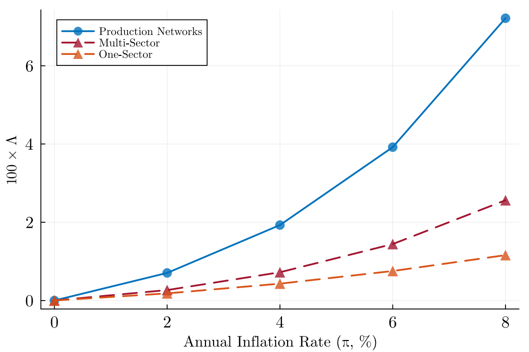


π_{SS}	Baseline	Pure Calvo	Ratio
2.0	0.1462	0.7050	0.21
4.0	0.3595	1.9281	0.19
6.0	0.6232	3.9190	0.16
8.0	0.9221	7.2135	0.13

Table 8: $100 \times \Lambda(\pi)$

- Much higher welfare cost of inflation in the pure Calvo model

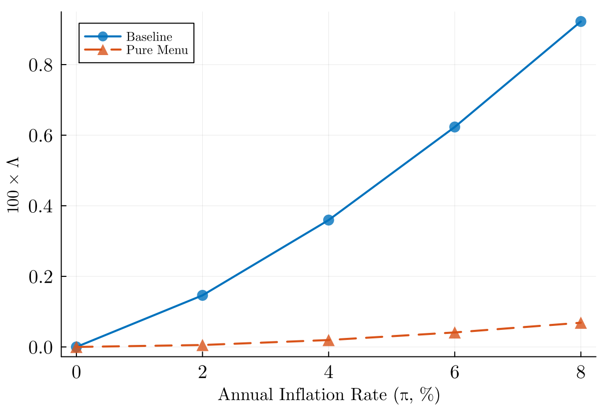
Pure Calvo: Production Networks × Multi-Sector × One-Sector



π_{ss}	Production Networks	Multi-Sector	(1)/(2)	One-Sector	(1)/(4)
2.0	0.7050	0.2672	2.64	0.1821	3.87
4.0	1.9281	0.7220	2.67	0.4310	4.47
6.0	3.9190	1.4428	2.72	0.7540	5.20
8.0	7.2135	2.5624	2.82	1.1598	6.22

- Amplification compared to multi-sector is same
- Amplification compared to one-sector is higher

Pure Menu vs. Calvo Plus

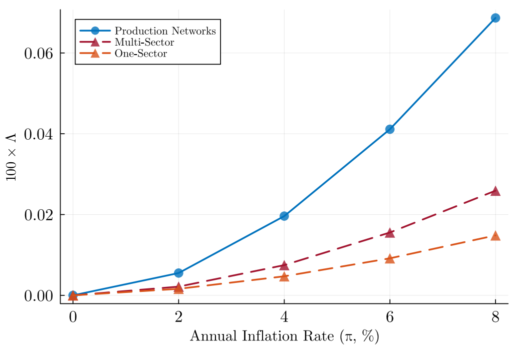


π_{ss}	Baseline	Pure Menu	Ratio
2.0	0.1462	0.0055	26.48
4.0	0.3595	0.0196	18.33
6.0	0.6232	0.0411	15.16
8.0	0.9221	0.0687	13.43

Table 9: $100 \times \Lambda(\pi)$

- Much lower welfare cost of inflation in the pure menu cost model

Pure Menu: Production Networks × Multi-Sector × One-Sector

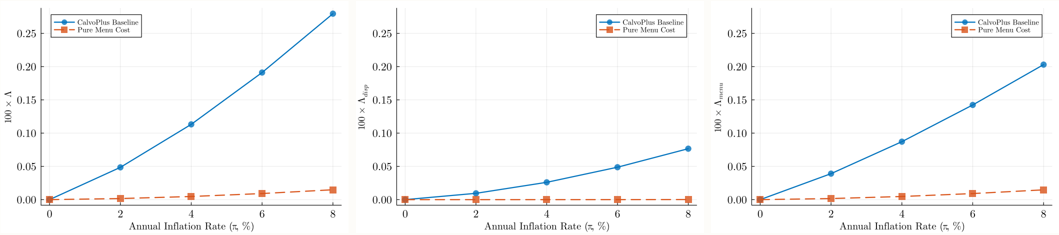


π_{SS}	Production Networks	Multi-Sector	(1)/(2)	One-Sector	(1)/(4)
2.0	0.0055	0.0022	2.56	0.0016	3.42
4.0	0.0196	0.0075	2.62	0.0047	4.16
6.0	0.0411	0.0156	2.64	0.0092	4.48
8.0	0.0687	0.0259	2.65	0.0148	4.63

- Amplification compared to multi-sector is same
- Amplification compared to one-sector is higher

Back

One-Sector: Welfare Decomposition (Calvo Plus × Pure Menu)



(a) Total Welfare (b) Dispersion (c) Menu Cost

Figure 6: One-Sector. Calvo Plus × Pure Menu.

One-Sector Pure Menu Cost

Statistic\ π	0.0	2.0	4.0	6.0	8.0
$\log$ (Dispersion)	0.00191	0.00191	0.00191	0.00191	0.00191
Frequency	0.15815	0.15950	0.16208	0.16576	0.17041
Std. Dev. of price changes	0.07541	0.07400	0.07166	0.06859	0.06498
Kurtosis of price changes	1.05159	1.23185	1.54735	2.01149	2.64524
Fraction of positive price changes	0.55642	0.61703	0.67347	0.72463	0.76993

Back

Welfare Cost of Inflation (Calvo Plus): Multi-Sector × One-Sector

π_{ss}	Multi-Sector	One-Sector	Ratio
2.0	0.0541	0.0485	1.12
4.0	0.1328	0.1132	1.17
6.0	0.2301	0.1911	1.20
8.0	0.3405	0.2796	1.22

Table 10: Welfare cost in % : $100 \times \Lambda(\pi)$. Multi-sector × One-sector

Back

Flexible Sector

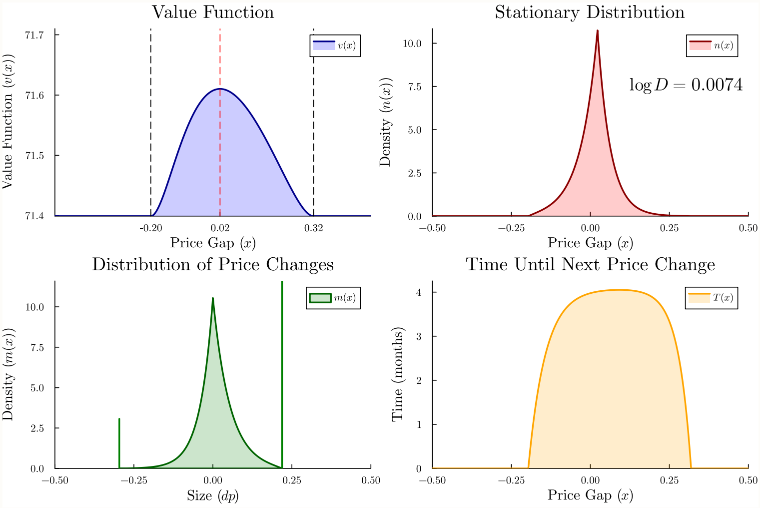


Figure 7: Wood products. FPA = 0.2493; Standard deviation = 0.066; Kurtosis = 4.80; Fra. Pos.: 0.571. Back

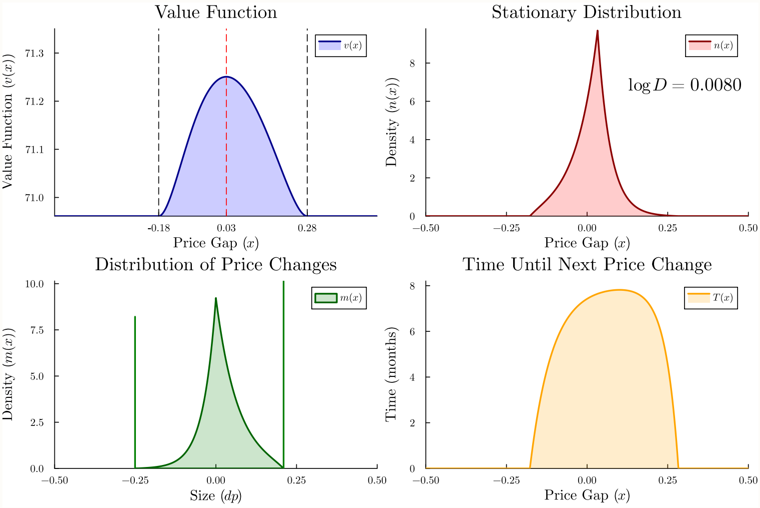


Figure 8: Rental and leasing services and lessors of intangible assets. FPA = 0.1307; Standard deviation = 0.0739; Kurtosis = 3.85; Fra. Pos: 0.633. Back

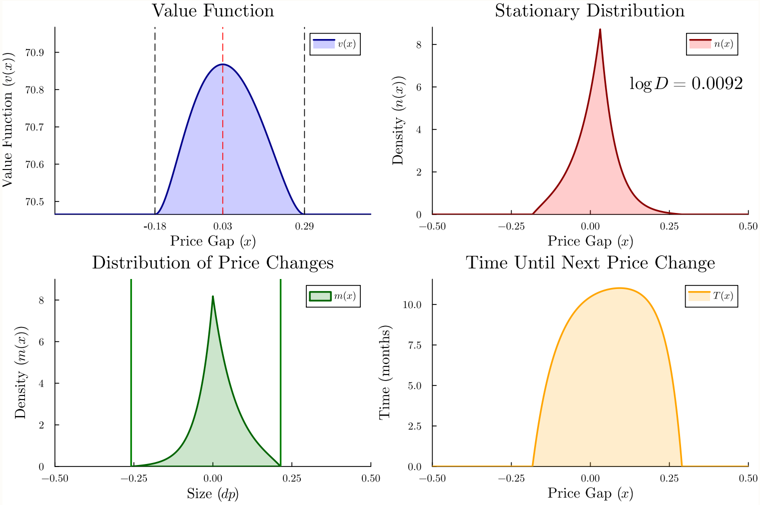


Figure 9: Computer and electronic products. FPA = 0.0937; Standard deviation = 0.0823; Kurtosis = 3.759; Fra. Pos.: 0.6117. Back

$\bar{M}_i(\pi)$

$$\frac{P_{i,t}}{\mathrm{MC}_{i,t}} = \frac{1}{(1-\tau_i)} \frac{\eta_i}{\eta_i - 1} \bar{M}_i(\pi)$$

where

$$\bar{M}_i(\pi) \equiv \frac{\int e^{(1-\eta_i)x_{ijt}} \mathrm{d}j}{\int e^{-\eta_i x_{ijt}} \mathrm{d}j}$$

and is a function of π as the stationary distribution depends on π

Back

Kolmogorov-Forward equation

The stationary distribution $g_i(x)$ is given by

$$-(\mu_i - \pi)\frac{\mathrm{d}g_i(x)}{\mathrm{d}x} + \frac{\sigma_i^2}{2}\frac{\mathrm{d}^2 g_i(x)}{\mathrm{d}x^2} - \theta_i g_i(x) = 0, \quad \forall x \in (\underline{x}, \bar{x}), \quad x \neq x_i^*$$

where

1. $\int_{\underline{x}_i}^{\bar{x}_i} g_i(x)\mathrm{d}x = 1$
2. $g_i(\underline{x}_i) = g_i(\bar{x}_i) = 0$
3. $g(x^*)_- = g(x^*)_+$

Back

Value matching conditions:

$$v(\underline{x}) = v(x^*) - \phi, \qquad v(\bar{x}) = v(x^*) - \phi$$

Smoothing pasting conditions:

$$v'(\underline{x}) = 0, \qquad v'(\bar{x}) = 0$$

Optimality of x^*:

$$v'(x^*) = 0$$

Back

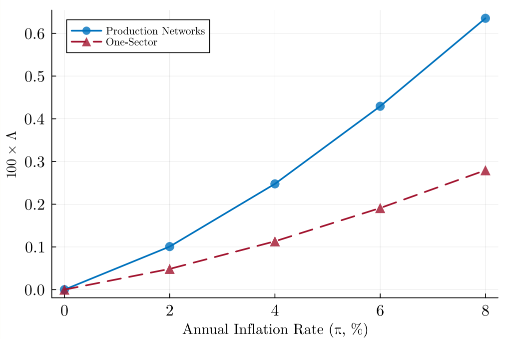
Value Function – Pricing Problem

$$v(x) = \max \left\{ \max_y v(y) - \phi_i, f_i(x_i)\mathrm{d}t + (1 - \rho dt)\left[(1 - \theta_i \mathrm{d}t)\mathbb{E}\left[v(x + dx)\right] + \theta_i \mathrm{d}t \times \max_y v(y)\right]\right\}$$

$$\mathrm{d}x_{ij,t} = (\mu_i - \pi)\mathrm{d}t + \sigma_i \mathrm{d}\mathcal{W}_{ij,t}$$

Back

Robustness: Adding 'Gross Operating Surplus' to 'Compensation of Employees'



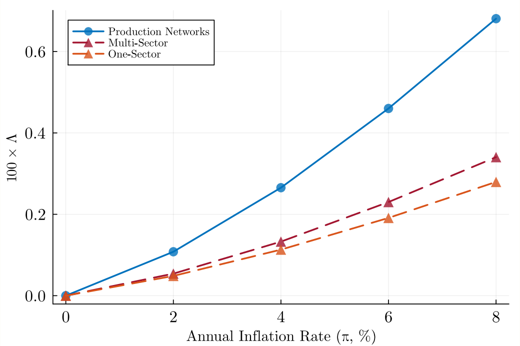
π_{SS}	Production Networks	One-Sector	Ratio
2.0	0.1008	0.0485	2.08
4.0	0.2477	0.1132	2.19
6.0	0.4293	0.1911	2.25
8.0	0.6353	0.2796	2.27

Figure 10: $\sum_i \lambda_i = 1.805$. $\frac{\text{Gross Output}}{\text{GDP}} = \frac{\text{Total Intermediate+Compensation of Employees+GOS}}{\text{Compensation of Employees+GOS}}$

- Baseline: $\frac{\text{Gross Output}}{\text{GDP}} = \frac{\text{Total Intermediate+Compensation of Employees}}{\text{Compensation of Employees}}$
- Assumption: $\text{GOS}_i + \text{Taxes on production and imports, less subsidies}_i = 0, \; i \in [n]$

Back

Robustness: Adding 'Gross Operating Surplus' to 'Compensation of Employees'



π_{ss}	Production Networks	Multi-Sector	(1)/(2)	One-Sector	(1)/(4)
2.0	0.1080	0.0541	1.99	0.0485	2.22
4.0	0.2654	0.1328	2.00	0.1132	2.34
6.0	0.4601	0.2301	2.00	0.1911	2.41
8.0	0.6808	0.3405	2.00	0.2796	2.43

Figure 11: $\sum_i \lambda_i = 1.928$. $\frac{\text{Gross Output}}{\text{GDP}} = \frac{\text{Total Intermediate}+\text{Compensation of Employees}+0.66\times\text{GOS}}{\text{Compensation of Employees}+0.66\times\text{GOS}}$

- Baseline: $\frac{\text{Gross Output}}{\text{GDP}} = \frac{\text{Total Intermediate}+\text{Compensation of Employees}}{\text{Compensation of Employees}}$
- Assumption: $\text{GOS}_i + \text{Taxes on production and imports, less subsidies}_i = 0, \;\; i \in [n]$

Back Alternative

Relationship between λ_i and $\log D_i(0.02) - \log D_i(0.00)$

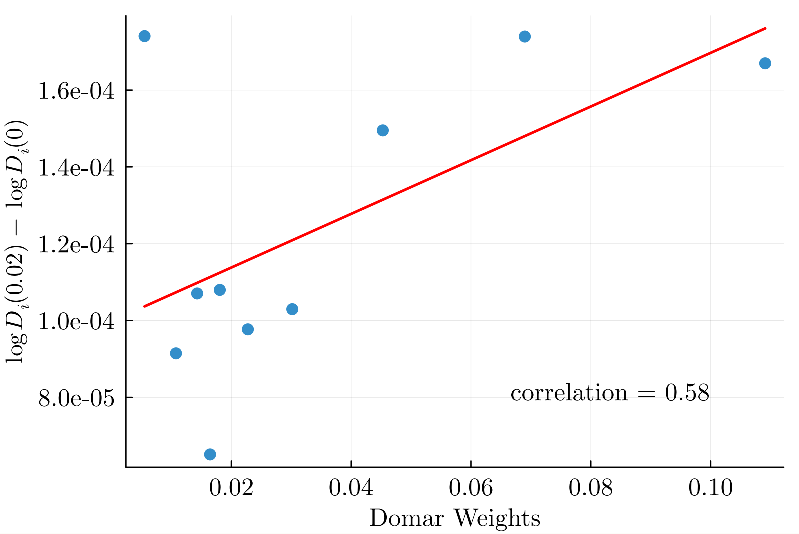


Figure 12: Binscatter plot $\lambda_i \times \log D_i(0.02) - \log D_i(0)$. 10 bins. Back

Relationship between λ_i and $\log D_i(0.02) - \log D_i(0.00)$

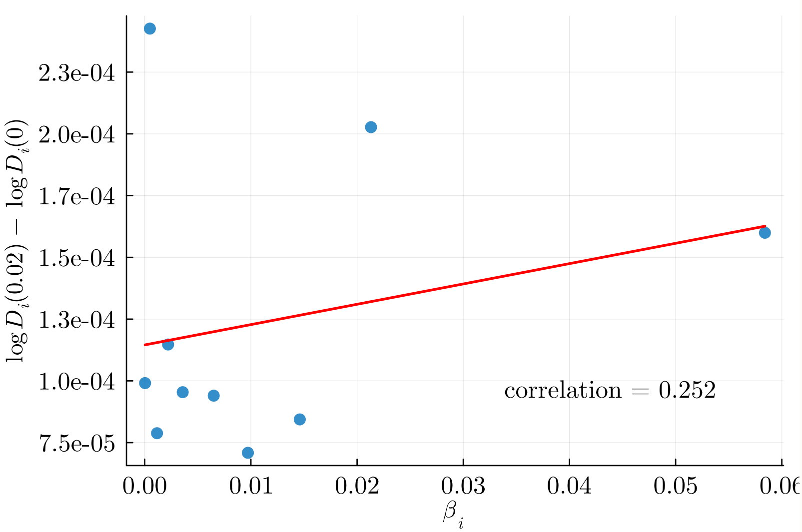


Figure 13: Binscatter plot $\beta_i \times \log D_i(0.02) - \log D_i(0)$. 10 bins. Back

Relationship between λ_i and $\log D_i(0.02) - \log D_i(0.00)$

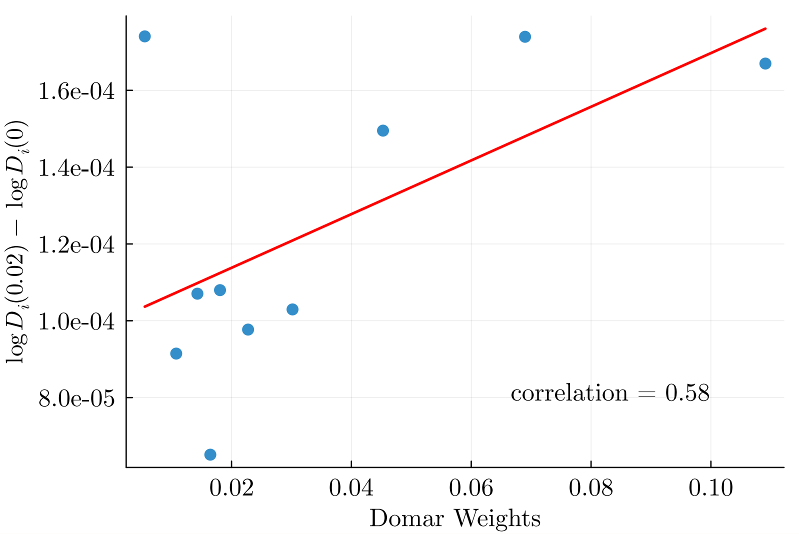


Figure 14: Binscatter plot $\lambda_i \times \log D_i(0.02) - \log D_i(0)$. 10 bins. Back

Relationship between λ_i and $\log D_i(0.02) - \log D_i(0.00)$

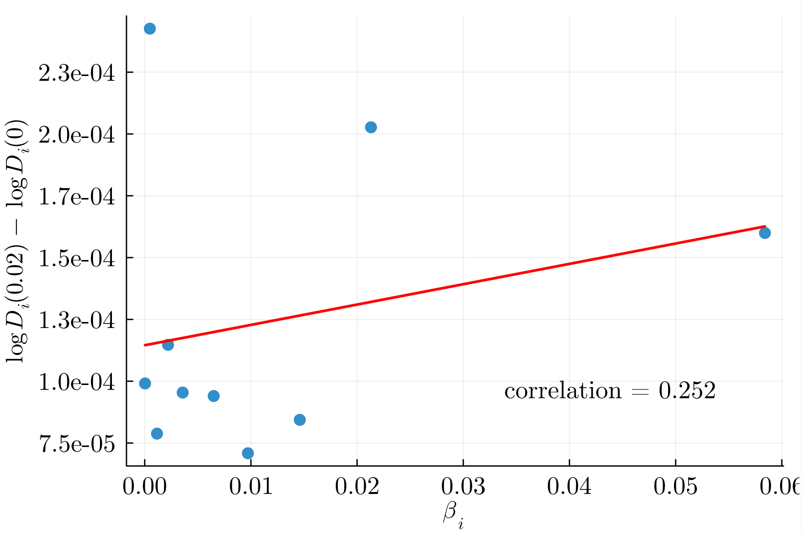


Figure 15: Binscatter plot $\beta_i \times \log D_i(0.02) - \log D_i(0)$. 10 bins. Back

Equilibrium Definition

A **recursive stationary equilibrium with steady-state inflation** π consists of allocations $\mathcal{A}_h = \{C, L, B, (C_i)_{i\in[n]}\}$, $\mathcal{A}_{final} = \{(Y_i, Y_{ij}^d)_{i\in[n]}\}$, value and policy functions, $\{(v_i(x))_{i\in[n]}, (\{x_i^*, \underline{x_i}, \bar{x}_i\})_{i\in[n]}\}$ for monopolistically competitive firms, a distribution of price gaps $(\mathrm{G}_i(x))_{i\in[n]}$, monetary and fiscal policies $\mathcal{A}_g = \{M_t, T, (\tau_i)_{i\in[n]}\}_{t\geq 0}, {}^{\dot{M}_t}/_{M_t} = \pi$, and aggregate relative prices $\mathcal{P} = \{({}^{P_{i,t}}/_{W_t})_{i\in[n]}, {}^{P_t}/_{W_t}, i\}_{t\geq 0}$ such that $\mathcal{P}, \mathcal{A}_h$, and $\mathcal{A}_{final}$ are time-invariant and

1. Given $\mathcal{P}$ and $\mathcal{A}_g, \mathcal{A}_h$ solves the household's problem
2. Given $\mathcal{P}$ and $\mathcal{A}_g, \mathcal{A}_{final}$ solves the final good price problem
3. Given $\mathcal{P}, \mathcal{A}_g$, and $\mathcal{A}_{final}$, $(v_i(x))_{i\in[n]}$ and $(\{x_i^*, \underline{x_i}, \bar{x}_i\})_{i\in[n]}$ solve HJB-VI
4. $(\mathrm{G}_i(x))_{i\in[n]}$ is stationary
5. Labor, bonds, and final sectoral goods markets clear and government budget constraint is satisfied

Back

Solving for the Value Function and Optimal Policies

- Let $u_i(x) \equiv v_i(x) - v_i(x_i^*)$ so that $u_i(x_i^*) = 0$. The value function becomes:

$$-\frac{\sigma_i^2}{2}u_i''(x) - (\mu_i - \pi)u_i'(x) + (\rho + \theta_i)u_i(x) = e^{(1-\eta_i)x} - \frac{\eta_i - 1}{\eta_i}e^{-\eta_i x} - \rho v_i(x_i^*)$$

- **2nd order nonhomogeneous ODE** $\Rightarrow$ General Solution: $u_i(x) = u_{i,h}(x) + u_{i,p}(x)$
  - *Homogeneous:* $u_{i,h}(x) = c_1 e^{r_1 x} + c_2 e^{r_2 x}$ (where $r_{1,2}$ are characteristic roots)
  - *Particular:* $u_{i,p}(x) = A_i e^{(1-\eta_i)x} + B_i e^{-\eta_i x} + C_i$
- **Endogenous Boundaries:** We solve for the six unknowns $(c_1, c_2, C_i, \underline{x}_i, \overline{x}_i, x_i^*)$ using the value-matching and smooth-pasting conditions:

$$\begin{aligned} \text{Value matching:} &\quad u_i(\underline{x}_i) = -\psi_i, \quad u_i(\overline{x}_i) = -\psi_i, \quad u_i(x_i^*) = 0 \\ \text{Smooth pasting:} &\quad u_i'(\underline{x}_i) = 0, \quad u_i'(\overline{x}_i) = 0, \quad u_i'(x_i^*) = 0 \end{aligned}$$

- Once system is solved numerically, $v_i(x_i^*) = -\frac{\rho+\theta_i}{\rho}C_i$.

◂ Back to Solution Method

Stationary Distribution via Kolmogorov-Forward Equation

- The stationary distribution $g_i(x)$ of price gaps satisfies the KFE

$$\frac{\sigma_i^2}{2} g_i''(x) - (\mu_i - \pi) g_i'(x) - \theta_i g_i(x) = 0, \quad \forall x \neq x_i^*$$

- **Piecewise Solution:** Solved across the two policy regions using roots r_1, r_2:

$$g_i(x) = \begin{cases} A_1 e^{r_1 x} + A_2 e^{r_2 x} & \text{for } \underline{x}_i \leq x < x_i^* \\ B_1 e^{r_1 x} + B_2 e^{r_2 x} & \text{for } x_i^* < x \leq \overline{x}_i \end{cases}$$

- **Linear System:** The four constants are solved analytically via the linear system $M\mathbf{c} = \mathbf{b}$ where $\mathbf{c} = [A_1, A_2, B_1, B_2]^T$, pinned down by:
  - *Absorbing Barriers:* $g_i(\underline{x}_i) = 0$ and $g_i(\overline{x}_i) = 0$
  - *Continuity at Target:* $g_i(x_i^*)_- = g_i(x_i^*)_+$
  - *Unit Probability Mass:* $\int_{\underline{x}_i}^{\overline{x}_i} g_i(x)\, dx = 1$

Back to Solution Method

Relationship χ_i and ϕ_i

Let $g_i^\pi(x)$ be the density of the stationary distribution at rate of inflation π

$$\chi_i = \frac{\lambda_i}{\tilde{\lambda}_i} \times (1 - \tau_i) \times \frac{1}{\mathbb{E}_{g_i^\pi(x)}\left[e^{(1-\eta_i)x}\right]} \times \phi_i$$

When taxes are such that $\mathcal{M}_i(\pi) = 1$, $\lambda_i = \tilde{\lambda}_i$ and

$$\chi_i = \frac{\eta_i}{\eta_i - 1} \frac{1}{\mathbb{E}_{g_i^\pi(x)}\left[e^{-\eta_i x}\right]} \phi_i$$

Back

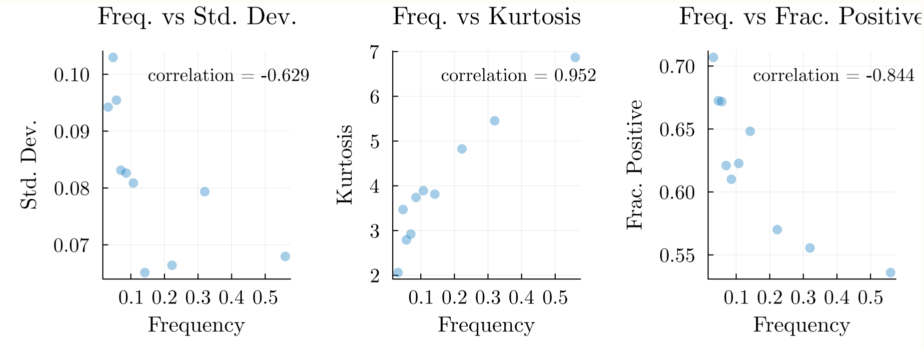


Figure 16: Data. Relationship between Frequency and other moments Back

Pct\π	0.0	2.0	4.0	6.0	8.0
20	0.0648	0.0657	0.0672	0.0690	0.0710
40	0.0679	0.0680	0.0691	0.0696	0.0722
60	0.0782	0.0794	0.0796	0.0809	0.0831
80	0.0836	0.0830	0.0842	0.0869	0.0902

(a) Standard deviation

Pct\π	0.0	2.0	4.0	6.0	8.0
20	3.5621	3.1322	2.8568	2.4326	2.0979
40	4.2391	3.8547	3.4599	3.0925	2.7849
60	4.5046	4.2155	3.9143	3.6229	3.3541
80	4.8333	4.6105	4.3751	4.2432	4.0233

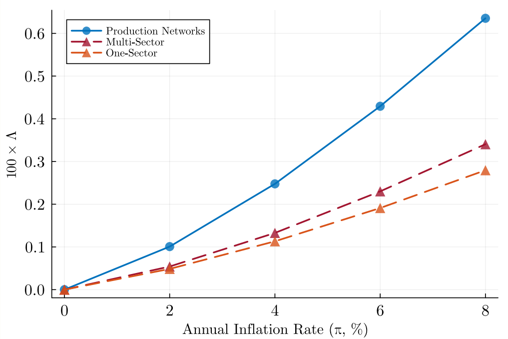
(b) Kurtosis

Pct\π	0.0	2.0	4.0	6.0	8.0
20	0.5346	0.5866	0.6226	0.6560	0.6867
40	0.5437	0.6148	0.6591	0.6970	0.7306
60	0.5606	0.6177	0.6799	0.7330	0.7712
80	0.5714	0.6360	0.6992	0.7596	0.8140

(c) Fraction of positive price change

Back

Different calibration of Domar weights



π_{ss}	Production Networks	Multi-Sector	(1)/(2)	One-Sector	(1)/(4)
2.0	0.1008	0.0541	1.86	0.0485	2.08
4.0	0.2477	0.1328	1.86	0.1132	2.19
6.0	0.4293	0.2301	1.87	0.1911	2.25
8.0	0.6353	0.3405	1.87	0.2796	2.27

Figure 17: $\sum_i \lambda_i = 1.805$. $\frac{\text{Gross Output}}{\text{GDP}} = \frac{\text{Total Intermediate+Compensation of Employees+GOS}}{\text{Compensation of Employees+GOS}}$

- Here: Add 'Gross Operating Surplus' to 'Compensation of Employees'

Back

Median Moments

Statistic	Value
Frequency	0.1595
Std. Dev.	0.0734
Kurtosis	3.8871
Frac. Pos.	0.6170

Table 12: One-sector economy moments.

Back

- Up to first order, around 2% inflation, with τ_i such that $\mathcal{M}_i(0.02) = 1$

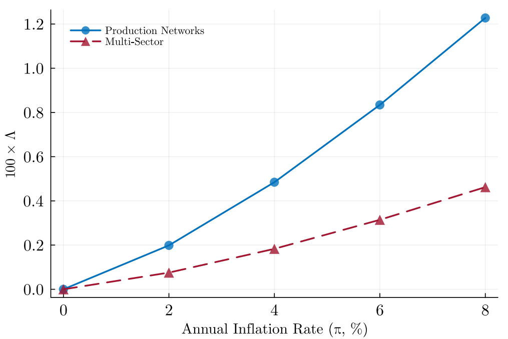
$$\sum_{i \in [n]} \alpha_i \frac{\lambda_i(\pi)}{\mathcal{M}_i(\pi)} \approx 1 - \sum_{i \in [n]} \tilde{\lambda}_i \log \mathcal{M}_i(\pi)$$

- Therefore,

$$\sum_{i \in [n]} \alpha_i \Big[\frac{\lambda_i(\pi)}{\mathcal{M}_i(\pi)} - \frac{\lambda_i(0)}{\mathcal{M}_i(0)} \Big] \approx - \sum_{i \in [n]} \tilde{\lambda}_i \Big[\log \mathcal{M}_i(\pi) - \log \mathcal{M}_i(0) \Big]$$

Back

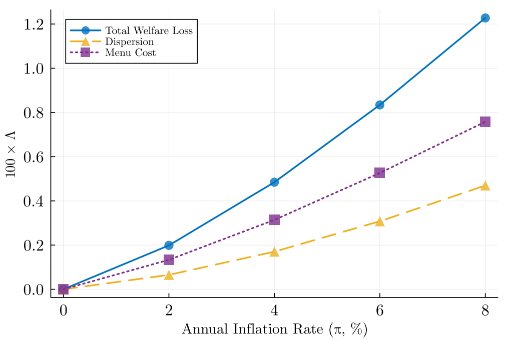
Welfare Cost of Inflation: $\eta_i = 6, i \in [n]$



π_{ss}	Production Networks	Multi-Sector	Ratio
2.0	0.1987	0.0755	2.63
4.0	0.4844	0.1830	2.65
6.0	0.8342	0.3144	2.65
8.0	1.2275	0.4621	2.66

Back

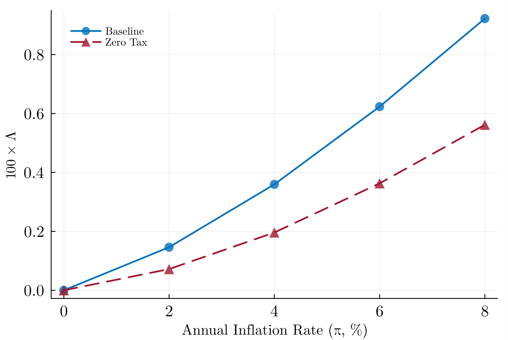
Welfare Cost of Inflation Decomposition: $\eta_i = 6, i \in [n]$



π_{ss}	Dispersion	Menu Cost	Menu Cost Share
2.0	0.0653	0.1334	67.2
4.0	0.1703	0.3141	64.8
6.0	0.3079	0.5263	63.1
8.0	0.4696	0.7579	61.7

Back

Welfare Cost of Inflation: Baseline $\times$ $\tau_i(\pi) = 0$, $i \in [n]$, π



π_{ss}	Baseline	Zero Tax	Ratio
2.0	0.1462	0.0719	2.03
4.0	0.3595	0.1961	1.83
6.0	0.6232	0.3630	1.72
8.0	0.9221	0.5616	1.64

- Welfare cost is *smaller* than in the baseline

Back

Welfare Cost of Inflation: $\tau_i(\pi) = \bar{\tau}_i,\ i \in [n],\ \pi$, where $\bar{\tau}_i$ such that $\mathcal{M}_i(0.02) = 1$

π_{ss}	Total	Dispersion	Menu Cost
2.0	0.1462	0.0384	0.1079
4.0	0.3595	0.1034	0.2561
6.0	0.6232	0.1921	0.4311
8.0	0.9221	0.3003	0.6218

Table 13: Baseline: $\tau_i(\pi)$ s.t. $\mathcal{M}_i(\pi) = 1$, $i \in [n]$, π

π_{ss}	Total	Dispersion	Menu Cost	X-Sec. Disp. + Labor
2.0	0.1462	0.0381	0.1081	-0.0000
4.0	0.3595	0.1029	0.2566	-0.0000
6.0	0.6232	0.1914	0.4318	0.0000
8.0	0.9221	0.2993	0.6228	0.0000

Table 14: $\tau_i(\pi) = \bar{\tau}_i$ s.t. $\mathcal{M}_i(0.02) = 1$, $i \in [n]$, π

- *Composition* of welfare cost changes a bit
  - Recall: welfare relative to zero-inflation SS
- New terms offset each other

Back

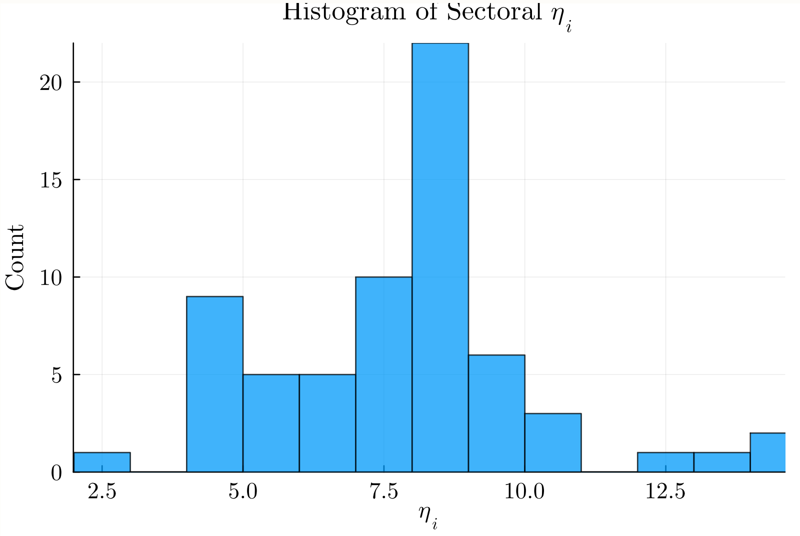


Figure 18: Histogram of calibrated η_i. Mean = 7.81. Median = 8.11. Back

Welfare Cost of Inflation: Decomposition

π_{SS}	Dispersion	Menu Cost	Menu Cost Share
2.0	0.0384	0.1079	73.8
4.0	0.1034	0.2561	71.2
6.0	0.1921	0.4311	69.2
8.0	0.3003	0.6218	67.4

Table 15: Production Networks

π_{SS}	Dispersion	Menu Cost	Menu Cost Share
2.0	0.0133	0.0408	75.4
4.0	0.0362	0.0966	72.8
6.0	0.0676	0.1626	70.6
8.0	0.1060	0.2345	68.9

Table 16: Multi-Sector

Between economies:

- Ratio of Dispersion roughly constant (≈ 2.85)
- Ratio of Menu cost roughly constant (≈ 2.65)

Back

WELFARE COST OF INFLATION AT 2%, HETEROGENEOUS η_i

π_{ss}	Production Networks	Multi-Sector	Ratio
2.0	0.7347	0.2454	2.99
4.0	1.7210	0.5761	2.99
6.0	3.0091	1.0089	2.98
8.0	4.7202	1.5841	2.98

Table 17: $\sum_i \lambda_i = 2.59$

Back

- Multi-sector (w/o production networks)

$$\Lambda(\pi) = \underbrace{\mathbb{E}_\beta[\Delta \log D_i(\pi)]}_{\approx 0.163} + \underbrace{\mathbb{E}_\beta[\chi_i \Delta N_i^{menu}(\pi)]}_{\approx 0.082} = 0.245\%$$

- Production networks

$$\Lambda(\pi) = \underbrace{\underbrace{\Big(\sum_{i\in[n]} \lambda_i\Big)}_{\approx 2.59}\Big(\underbrace{\mathbb{E}_\lambda[\Delta \log D_i(\pi)]}_{\approx 0.195} + \underbrace{\mathbb{E}_\lambda[\chi_i \Delta N_i^{menu}(\pi)]}_{\approx 0.089}\Big)}_{=0.735\%}$$

Welfare Cost of Inflation: Moving from 2% to 4% inflation, Heterogeneous η_i

π_{ss}	Production Networks	Multi-Sector	Ratio
2.0	0.7347	0.2454	2.99
4.0	1.7210	0.5761	2.99
6.0	3.0091	1.0089	2.98
8.0	4.7202	1.5841	2.98

Table 18: $\sum_i \lambda_i = 2.59$

- Multi-sector (w/o production networks)

$$\Delta\Lambda(\pi) = \underbrace{\mathbb{E}_\beta[\Delta \log D_i(\pi)]}_{\approx 0.229} + \underbrace{\mathbb{E}_\beta[\chi_i \Delta \mathrm{N}_i^{menu}(\pi)]}_{\approx 0.100} = 0.330\%$$

- Production networks

$$\underbrace{\Delta\Lambda(\pi) = \Big(\underbrace{\sum_{i\in[n]} \lambda_i}_{\approx 2.59}\Big)\Big(\underbrace{\mathbb{E}_\lambda[\Delta \log D_i(\pi)]}_{\approx 0.272} + \underbrace{\mathbb{E}_\lambda[\chi_i \Delta \mathrm{N}_i^{menu}(\pi)]}_{\approx 0.108}\Big)}_{=0.986\%}$$

Back

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