The Nature of Firm Financing and Growth*

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Abstract

Using the universe of incorporated firms in the United Kingdom, we document that long term debt that is not secured by specific corporate assets (commonly referred to as cash-flow-based debt) is ubiquitous, even among very young firms. Consistent with cash-flow-based lending depending on firms' continuation value, both the timing and the amount of cash-flow-based debt at first financing are strongly related to future performance. In fact, the age at which firms first access cash-flow-based long-term debt is a strong predictor of firm outcomes up to twenty years later. In contrast, the age at which firms access asset-based lending does not predict future performance. To understand how lending based on continuation value impacts firm dynamics, we develop a tractable theory of firm life cycle dynamics with permanent heterogeneity and financing frictions – where access to finance can be based on signals of both current and future performance. The model highlights the importance of financing based on future value in matching the predictability of cash-flow-based financing observed in the data. It also highlights how frictions to financing based on available collateral vs. future value lead to differential selection and hence very different distributions of firms that survive and grow. The degree to which firms can access financing based on future value is therefore fundamentally intertwined with aggregate firm dynamics.

Keywords: Credit Rationing; Cash-Flow-Based Lending; Firm Dynamics; Learning

JEL Codes: D22, D24, E23, L11, L25, L26

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1 Introduction

How firms access external finance over their lifecycle is central to understanding firm growth, resource allocation, and aggregate productivity. A large literature (e.g. Kiyotaki & Moore (1997), Cooley & Quadrini (2001), Ottonello & Winberry (2020), Midrigan & Xu (2014)) emphasizes the role of financial frictions—particularly those tied to collateral constraints—in shaping firm dynamics (e.g. Hopenhayn (1992)). However, this view abstracts from a pervasive feature of real-world financial markets: a large share of financing is based not on collateral, but on investors' expectations of firms' cash flows (Lian & Ma 2021).

For mature firms with slower and more predictable growth, the firm's current cash flows provide a relatively good proxy for its continuation value. For young firms, however, continuation value is more likely to be tied to future growth potential than simply to measures of current performance. This means that frictions to assessing and lending against future value are likely to be much more consequential for young firms than mature ones, and in turn, to impact firm dynamics. Yet, whether—and how— financing based on future value shapes firm dynamics remains largely unexplored.

In this paper, we combine unique longitudinal data on financing and growth for all incorporated firms in the United Kingdom (UK) with a new theoretical framework to explore how the nature of firm financing – specifically debt based on the expectation of future cash flows – relates to the nature of firm growth. This is possible because all incorporated firms in the UK are legally required to file annual financial accounts shortly after incorporation, generating near-universal panel data on firm financing from the earliest stages of firm activity. Matching these financial filings with panel data on firm employment from the UK Census provides a rare view into the evolution of financing and firm dynamics over the lifecycle for the universe of incorporated firms from 2002-2024.

Using this unique data, our paper proceeds in three steps. First, we show that the timing of first access to cash-flow-based long-term debt (LTD) is strongly predictive of subsequent real-outcomes. Firms that obtain cash-flow-based LTD early in life grow significantly larger. For example, those that receive their first cash-flow-based LTD at age 1 have 30% more employment and 40% more revenue at age 20 than those who first receive cash-flow-based LTD age 5. Strikingly, this predictability does not extend to asset-based long-term debt. While collateralizable assets expand borrowing capacity and firm growth, the timing of first access to asset-based LTD has no predictive power for long-run employment, revenue or assets.

These results suggest that cash-flow-based LTD is fundamentally tied to long-term firm growth and firm dynamics in a way that asset-based LTD is not. They also indicate the firms' ability to raise cash-flow-based LTD is based on something richer than just current cashflows, a proxy often used in structural applications with earnings-based constraints. We directly examine this view by regressing cash-flow-based LTD use (on both extensive and intensive margins) against current revenue *and* the present discounted value (PDV) of future cash flows–a measure inspired by the recursive "lifetime revenue" proposed by Jaimovich et al. (2023) as a proxy for firm value in firm dynamics models. Our joint specifications reveal that forward-looking PDV is the dominant predictor of cash-flow-based lending early in the firm life cycle. That is, early financing decisions appear to incorporate information of firms' future performance and value. As a result, models that tie borrowing constraints only to collateral—or even to *current* cashflows—miss an important channel through which financing frictions might impact firm dynamics and aggregate outcomes.

In the second part of our paper, we develop a tractable theory of heterogeneous firms with life cycle dynamics and frictional financing to formalize these mechanisms. The model contrasts financing based on signals of continuation or future firm value relative to more standard collateral or current earnings based borrowing. Our central contribution is to highlight how the presence of permanent (or "ex-ante") heterogeneity–which has received recent attention in the firm dynamics literature, e.g. Sterk et al. (2021),–interacts with constraints to lending based on continuation value–a recent focus in macro-finance.

The model provides a new insight: cash flow-based financing that depends at least in part on *future* value, therefore encoding a permanent component of heterogeneity, is key to rationalizing the empirical relationship between cash-flow-based LTD and future performance we observe in the data. In cases where frictions to financing based on future value are so extreme that financing is based only to currently available collateral or earnings, as commonly assumed in standard models, the lack of long-run selection early in life leads mature firms to appear as if they were ex-ante homogeneous—as in Hopenhayn (1992)— even if structurally they are not. Alternatively, when financing based on future value is feasible, permanent differences in productivity–as in Melitz (2003)— become visible throughout the entire life cycle as firms converge towards different growth trajectories. As a consequence, the combination of financing based on future value and permanent heterogeneity is necessary to generate the unequal growth trajectories that appear with cash-flow-based financing. It arises via a novel long-run selection channel of future "superstar" firms. On the other hand, financing based on assets or current earnings generates the limited long-run inequality that we see among firms financed via asset-based debt.

Because the data provide a comprehensive view of financing from birth, we document that cash-flow-based LTD is not only the dominant form of borrowing overall, but also particularly prevalent early in the lifecycle. This is precisely the stage at which firms' cash flows have not yet converged to more stable trajectories, making frictions to financing based on future value more likely. Consistent with this view and the model's predictions, we find that at the extensive margin, firms in industries where technological features make it harder to predict future success take longer to receive first-cash flow financing. Conditional on receiving financing in the same year, firms in these industries, including (but not limited to) intangible-intensive sectors, grow faster. These patterns are consistent with the interaction between cash-flow-based financing based on future value and selection on ex-ante permanent types our paper identifies.[1]

[1] While we do not take a stance on whether this interaction arises from lenders selectively extending forward-looking credit to firms with the greatest long-run potential when screening is more difficult or whether a la Melitz (2003) tougher financing conditions lead to firms that obtain cash-flow-based financing being positively self-selected on their long-run potential, we present empirical evidence consistent with the former mechanism.

In the third and final part of our paper, we embed financing based on current vs. future value into a structural firm dynamics model that incorporates financing frictions as well as entry and exit among firms. The model incorporates the rich life cycle profitability dynamics a la Sterk et al. (2021), where firms are ex-ante and ex-post heterogeneous. Counterfactuals with a calibration that matches the life cycle profiles of exit rates, firm size and cash-flow-based financing by age in UK data confirm the intuition built in the tractable model about patterns in the data. Early simulations show increasing frictions to financing based on future value could be one of the drivers for why the UK economy has had an aggregate productivity deficit relative to the US that has been worsening in recent decades. Our goal is not to directly explain the US–Europe productivity gap but to study a mechanism that may matter for that gap: the nature of debt markets, in particular, whether lenders can identify and support future high-growth firms early enough.

The model shows that financing based on future value generates a dramatically different distributions of firms—and dramatically different implications for the role of ex-ante predictability—when compared to financing based on current collateral or earnings, as a result of long-run selection. Superstar firms in environments where forward-looking financing abounds can survive periods of negative profitability that are crucial in many industries or among firms that experience negative early shocks, and thrive when mature. Further, the positive effects of future value-based financing are the strongest in sectors where predictability of future paths based on early-signals is the lowest, as is the case in information technology and similar industries. Our work thus highlights the role of cash-flow predictability rather than asset pledgeability in the financing of intangible capital and its implications for firm growth.

Related Literature

Our research speaks to four primary strands of the literature: firm life cycle dynamics, the structural modeling of borrowing constraints in macro-finance, the informational role of screening in entrepreneurial and corporate finance, and the aggregate implications of credit-market misallocation.

A large body of work studies entry, exit, and the evolution of firms over the life cycle under selection and shocks. Foundational dynamic industry models show how selection shapes the size distribution and long-run outcomes when firms face idiosyncratic productivity risk and fixed or adjustment costs (Hopenhayn 1992, Ericson & Pakes 1995, Melitz 2003). Within macro, collateral or net-worth constraints have often been layered onto such frameworks to generate rich firm dynamics (Cooley & Quadrini 2001). More recent empirical work emphasizes that firm growth over the life cycle features a mix of persistence and randomness, with strong selection at early ages and persistent heterogeneity among survivors (Sterk et al. 2021). A parallel literature documents that young, high-potential firms disproportionately account for employment growth and skewness in firm outcomes (Haltiwanger et al. 2013, Decker et al. 2014). We build on Sterk et al. (2021) by studying more complex set of financing constraints that are both asset-based and value-based constraints. A key contribution of our paper is to highlight that permanent heterogeneity among firms has a very important interaction with financial constraints that are explicitly value based and forward looking. The combination of these factors allows for firms to be ex-ante heterogeneous as in Melitz (2003) while retaining ex-post shocks as in Hopenhayn (1992).

Indeed, in macro-finance, financing constraints are commonly modeled as either net-worth or earnings constraints that tie external finance to internal cash flow (Townsend 1979, Holmström & Tirole 1997), and collateral constraints that bind borrowing to pledgeable assets (Kiyotaki & Moore 1997, Cooley & Quadrini 2001). This tradition has delivered key insights into amplification, investment, and firm dynamics. Empirically, cash-flow sensitivity and asset tangibility are used to capture constraints (Hennessy & Whited 2007, Almeida & Campello 2007). Yet a growing corporate finance literature distinguishes asset-based from cash-flow–based lending and shows the latter relies more on firm prospects and information production rather than collateral alone (Lian & Ma 2021). We add this tractable financing margin into a macro-finance framework—forward-looking, value-based lending governed by signals of expected future productivity—and we center this new lens on how young firms are financed from inception. Embedding this margin alongside collateral- and current earnings-based channels in a heterogeneous-firm life-cycle model yields two core results that line up with the data. First, selection under value-based lending is type-dependent: lenders screen on expected long-run potential rather than pledgeable assets, so initial heterogeneity persists into maturity and generates rank-predictability of growth that pure asset- or earnings-based constraints cannot match. Second, due to this type-dependent screening, the nature of selection in the face of frictions implies that surviving firms experience more extreme growth trajectories in environments where financing constraints—and thus screening incentives—are likely to be tighter.

A central theme in corporate and entrepreneurial finance is how lenders produce and act on information to extend credit absent hard collateral. Classic work on relationship lending and soft information shows how screening technologies and organizational design shape credit access for young and opaque firms (Petersen & Rajan 1994, Agarwal & Hauswald 2010). Survey evidence supports the idea that external debt is an important source of financing (Robb & Robinson 2014) and that the role of asset-based financing has decreased (Benmelech et al. 2024). Jang et al. (2025) empirically show private credit lenders may have a comparative advantage in providing the (continuation) value based loans we highlight especially in high-risk, intangible industries (where financiers can run or sell the firm in the event of default). We contribute new, economy-wide evidence that the timing of cash-flow-based LTD access is powerfully predictive of long-run real outcomes, consistent with lenders' forward-looking screening extracting information about firm potential that is not spanned by contemporaneous assets or cash flows. While prior literature may have recognized the importance of static screening for cash-flow based lending (e.g. Jazi & Netto (2026)), our unique contribution is to highlight the interaction with firm dynamics in the presence of permanent heterogeneity.

A large macro development and misallocation literature links financial frictions to dispersion in marginal products and to aggregate TFP losses (Hsieh & Klenow 2009, Buera et al. 2011, Midrigan & Xu 2014, Gopinath et al. 2017). Much of this work models borrowing capacity via net worth or collateral, implicitly favoring tangible asset intensive firms. As production has shifted toward intangible-intensive technologies with weak collateralizability, the gap between firm value and pledgeable assets has widened, lowering debt capacity and altering financing choices (Falato et al. 2022). We show that frictions in value based lending – rather than in collateralized lending per se – can generate sizable misallocation by preventing high potential, intangible intensive firms from obtaining early cash-flow-based LTD. Preliminary calibrated comparisons between the UK and US indicate that relaxing frictions to forward looking lending increases the prevalence of high growth superstar firms, steepens growth by age profiles, and raises aggregate productivity. These results connect with cross-industry evidence that financial development disproportionately benefits sectors more dependent on external finance (Rajan & Zingales 1998) and with firm level evidence that financing and legal environments matter differentially by firm size and opacity (Beck et al. 2005). Our framework highlights that in economies where intangible capital is increasingly central, improvements in screening technologies and in institutions that allow lenders to act on forward looking information can yield first order TFP gains by reallocating credit toward high potential young firms.

The remainder of the paper is organized as follows. Section 2 describes the data. Section 3 presents the empirical patterns. Section 4 shows the tractable firm life cycle model. Section 5 adds a richer calibrated model. Section 6 concludes.

2 Data

We construct a novel panel dataset that merges firm-level real outcomes with comprehensive financing records for the universe of incorporated firms in the United Kingdom. This dataset is the result of a granular link between two primary administrative sources of firm-level data: the Business Structure Database (BSD) from the UK's Office for National Statistics (ONS) (Office for National Statistics 2025), which provides the official record of demographic, employment and turnover variables, and the Bureau van Dijk

(Moody's) FAME dataset, which includes statutory financial filings from Companies House. The merge leverages a cross-walk provided by the ONS that maps the BSD's unique business identifiers (ENTREF) to the Companies House Company Registration Numbers (CRN).

Our empirical setting offers a distinct comparative advantage over traditional administrative datasets, such as the US Longitudinal Business Database (LBD). In the UK, all incorporated entities are legally mandated to file statutory balance sheets within 21 months of incorporation, and these filings are accessible in the public domain. This institutional requirement allows us to observe the financial structure of firms at or near their inception. Consequently, we observe key financial data for the pre-entry or start-up phase – a period that typically remains a black box in other jurisdictions where firms only appear in administrative records after surpassing specific employment or turnover thresholds. Crucially, our data allows us to synchronize the first appearance of a firm in the BSD with its contemporaneous financial position.

Following the firm dynamics literature, we define Age 0 as the year a firm first enters the administrative records—specifically, the point at which it either hires its first employee or registers for VAT. This threshold ensures our lifecycle analysis focuses on economically active firms rather than the broader universe of inactive or shell incorporated entities.

The granularity of the financial information allows for a more detailed decomposition of corporate liabilities than is typically possible with private-firm data. First, we observe standard balance sheet items, allowing us to disaggregate financing into equity, short-term debt, and long-term debt. Second, we exploit the UK's Register of Charges to identify the specific nature of debt contracts. We observe loans using charges or liens registered at Companies House, which reveal both the lender's identity and the underlying collateral. This collateral may consist of charges against specific assets (fixed charges) or over non-specific assets (floating charges). We define LTD lending events as year-on-year increases in LTD. These events are then categorized by their underlying security: increases backed by corporate assets are classified as asset-based, while those without security are classified as cash-flow-based.[2]

Furthermore, we leverage the UK's public register of directors and shareholders to bridge the gap between firm-level outcomes and individual entrepreneurial characteristics. The director register provides identifying information for all directors and managers – including full names and birthdates – which enables the longitudinal tracking of individuals across different firms and over time. We map the shareholder register to the list of directors to identify the type of equity holders (e.g., owner-managers vs. outside investors). This allows for the construction of a comprehensive managerial and ownership history, enabling us to investigate the role of prior entrepreneurial experience.

To comply with data privacy regulations and ONS security protocols, this identification process was conducted in a secure environment prior to the final merge. All individual identifiers were pseudonymized; specifically, names and birthdates were replaced with unique, randomized tracking IDs. This ensures that while we can track the movement and performance of specific serial entrepreneurs and investors across the universe of UK firms, the underlying personal data remains fully anonymized within our research sample. Our final dataset comprises a comprehensive panel of the universe of UK incorporated firms, mapping their real activity, managerial composition, and financing from 2002 through 2024.

We begin by documenting the prevalence and composition of LTD across the firm lifecycle, focusing specifically on the distinction between cash-flow-based and asset-based LTD. A striking feature of the UK corporate universe is the significance of the extensive margin: approximately 50% of firms never access long-term debt markets during our sample period. For those that do, financing is heavily tilted toward cash-flow-based LTD, especially during the early stages of the lifecycle.

As shown in Figure 1a, approximately one-quarter of all firms obtain LTD at start-up (age 0). Among these early borrowers, the overwhelming majority (94%) utilize cash-flow-based LTD, while only 6% rely on asset-based LTD. This dominance persists as firms mature; by age 10, 83% of firms that have levered continue to use cash-flow-based

[2] Our results are robust to alternative definitions—most notably, reclassifying loans secured solely by floating charges (i.e., over a non-specific pool of assets) as cash-flow-based rather than asset-based.

LTD exclusively, while only 2% rely solely on asset-based LTD and 15% utilize both. Even by age 19, 72% of firms maintain exposure only to cash-flow-based LTD. Crucially, this is not merely a small firm phenomenon. Figure 1b shows that in the top decile of firms by employment at start-up, 88% of those with debt at start-up use cash-flow-based LTD. While older firms are more likely to eventually diversify their financing—with 60% of the top decile using both types by age 19—a significant share of even the largest firms never moves beyond cash-flow-based lending.

We focus our analysis on the first lending event for two primary reasons. First, for the majority of firms, the initial entry into the debt market is the most significant financing milestone in their lifecycle. Second, we observe a high degree of path dependency: transitions between debt types are rare. As noted above, nearly three-quarters of firms remain exposed to only one form of debt even after nearly two decades of operation. This suggests that the initial choice of debt type is a fundamental differentiator that characterizes a firm's long-run trajectory.

The modal timing for entering the credit market is at inception. Table 1 displays the count of firms by timing of when they receive their initial LTD financing. We find that 26% of all firms in the universe secure LTD at age 0. Conditional on a firm ever receiving long-term debt, 53% do so at start-up. This propensity to borrow at inception is consistent across both debt categories: 52% of firms that eventually use cash-flow-based LTD and 61% of those that eventually use asset-based LTD secure their first loan at age 0. While the probability of receiving first-time financing decreases as firms age, a significant proportion of survivors obtain their first credit facility later in their lifecycle, providing the temporal variation we exploit to study the effects of debt timing on long-run outcomes.

3 Empirical Patterns

Figure 2 summarizes the main message of this paper linking the nature of firm financing to firm growth: when and whether firms first access financing is closely linked to their long-term trajectories only in the case of cash-flow-based debt. Figure 2 shows that firms that obtain cash-flow-based debt early in life start larger and grow significantly larger than their peers in terms of employment, turnover, and assets. Strikingly, this pattern does not extend to asset-based long-term debt, where firms borrowing earlier may start larger and grow faster around the time of financing, but nevertheless end up in the same growth trajectory. This suggests the presence of a novel long-run selection channel: in the nature of firm financing there is valuable forward-looking information about whether firms have high-growth potential. We further develop further and formally test these insights next.

3.1 Cash-Flow-Based LTD at Start-up to Predict Firm Performance

We first investigate whether the acquisition of Long-Term Debt (LTD) at inception serves as a robust predictor of long-run firm performance, even after conditioning on a comprehensive set of observable start-up characteristics. We show this predictive power is present in the data but only for cash-flow-based LTD.

To do so, we estimate the following cross-sectional specification:

$$\log(\text{Employment}_{i,A}) = \alpha + \beta \mathbb{1}(\text{LTD}_i > 0)_{t=0} + \Gamma \mathbf{X}_{i,0} + \epsilon_i \tag{1}$$

where $A \in \{10, 19\}$ represents the firm's size at age 10 or 19, and $\mathbf{X}_{i,0}$ is a vector of initial observables measured at the time of the firm's first appearance in the BSD. This vector includes combinations of log assets, log employment, log turnover (as a proxy for initial cash flow), and labor productivity.

As reported in Table 2, we find a consistent and statistically significant positive association between initial LTD receipt and future firm size. In our most stringent specification we find that firms that access long-term debt at start-up are approximately 20% larger at age 10 and 19% larger at age 19. This persistent size differential suggests that early-stage LTD issuance contains significant private information regarding a firm's long-run growth potential. This predictability is most concentrated in cash-flow-based LTD while weak for asset-based LTD.

3.2 Timing of Initial Cash-Flow-Based LTD and Firm Performance

We investigate whether the specific age at which a firm first accesses the long-term debt market predicts its long-run scale. To allow for potential non-linearities in this relationship, we estimate a non-parametric specification that regresses log employment on a vector of indicator variables representing the firm's age at its first lending event:

$$\log(\text{Employment}_{i,A}) = \alpha + \sum_{k=1}^{9} \beta_k \mathbb{1}(\text{AgeFirstLTD}_i = k) + \Gamma \mathbf{X}_{i,0} + \epsilon_i \tag{2}$$

where $A \in \{10, 19\}$ denotes the firm's size at age 10 or 19. The term $\mathbb{1}(\text{AgeFirstLTD}_i = k)$ is an indicator for the firm receiving its first long-term debt at age $k \in \{0, \ldots, 9\}$, with age 0 (debt at inception) serving as the omitted baseline category. We estimate this model for the full sample of firms with debt and then separately for those whose first debt event is either cash-flow-based or asset-based LTD. Table 3 presents the results and figure 3 has the corresponding coefficient plots with standard errors.

For the cash-flow-based LTD subsample, we find a large and statistically significant negative coefficient for β: firms that secure cash-flow-based LTD earlier in their lifecycle exhibit significantly steeper growth trajectories. Specifically, firms obtaining their first cash-flow-based LTD at age 1 are, on average, 15% larger by age 19 than firms that first access this financing at age 5.

In stark contrast, these patterns do not extend to asset-based LTD. When estimating Equation 2 for the asset-based LTD subsample, the coefficients on the age of first financing is statistically indistinguishable from zero. We further confirm that this divergence in patterns is not just visible for employment, but also for turnover and assets.

This divergence implies that the forward-looking informational content of early-stage debt is unique to cash-flow-based contracts. To ensure these results are not sensitive to the choice of terminal age, we provide extensive robustness checks in the Appendix. We replicate our primary analysis using alternative outcome variables, specifically log employment 5 years and 10 years after the initial financing event. These results, reported in Figure A5, confirm our results.

3.3 The Forward-looking Nature of Cash-Flow-Based LTD

Next, we explicitly test whether cash-flow-based lending is tied to forward-looking information, and whether this relationship is stronger than its link to current information. To do so, we estimate the following cross-sectional specification:

$$\text{Cash-Flow-Based LTD}_{i,0} = \alpha + \beta_1 log(PDV_{i,0}) + \beta_2 log(Turnover_{i,0}) + \epsilon_{i,0} \tag{3}$$

Cash-Flow-Based $\text{LTD}_{i,0}$ captures cash-flow-based debt's extensive margin–dummy variable equaling one if a firm obtains cash-flow-based LTD, and zero otherwise used in a Probit regression–or its intensive margin–the natural logarithm of cash-flow-based debt, both at age 0. $\log(PDV_{i,0})$ and $\log(\text{Turnover}_{i,0})$ represent the natural logarithms of the present discounted value (PDV) of all future revenues and current revenue at age 0, respectively. Our forward-looking measure PDV is inspired by the recursive "lifetime revenue" instead of cash flows as proposed by Jaimovich et al. (2023) as a proxy for firm value which we implement similarly to them with a discounting of realized revenues that includes sector by age exit rates to account for survival selection and a factor $R = 1$ as a benchmark.

$$PDV_{i \in s,0} = \sum_{a \in \{0,\infty\}} \frac{(1 - P[Exit|s,a])}{R^a} * Turnover_{i,a}$$

Table 4 evaluates whether cash-flow-based long-term debt (LTD) lending at firm inception is primarily driven by forward-looking information (log(PDV)) or current cash flows (log(Turnover)). Looking first at the individual regressions, both variables are positively and significantly associated with cash-flow-based debt across both the extensive margin (Columns 1 and 2) and the intensive margin (Columns 4 and 5). For instance, the univariate probit marginal effects show that a 1% increase in log(PDV) increases the probability of cash-flow-based borrowing by 4.4 percentage points (Column 1), compared to 3.4 percentage points for current turnover (Column 2). Similarly, in the standalone intensive margin OLS regressions, both variables yield strong positive elasticities. Crucially, when both variables are entered jointly in the horse-race specifications, log(PDV) emerges as the dominant predictor across both dimensions. For the intensive margin (Column 6), both coefficients remain positive, but the elasticity of log(PDV) (0.469) is markedly stronger than that of current turnover (0.335). This predictive dominance is also reflected in the model fit, where the standalone log(PDV) model yields a noticeably higher R^2 (10.3%) than the standalone current turnover model (8.5%).

The most striking evidence of the dominance of the forward-looking continuation value in driving cash-flow-based lending appears in the joint extensive margin regression (Column 3). While the marginal effect of log(PDV) remains robust and rises to 5.2 percentage points, the coefficient on current turnover undergoes a distinct sign flip, becoming negative (-0.014, $p < 0.01$). This sign flip indicates that once a firm's long-term discounted future value is controlled for, higher current turnover does not increase the likelihood of obtaining cash-flow-based credit. We can say that relation between PDV and cash-flow-based lending extensive margin is stronger for younger ages; repeating the estimation for older ages the PDF coefficient becomes smaller and converges towards zero while the coefficient on current turnover increases and becomes positive. Thus, since regressions for older firms tend to show a declining importance of PDV relative to current turnover, consistent with our result that for the study of cash-flow-based forms of debt that are ubiquitous among young firms, proxying for the forward-looking value of young firms is fundamental.

4 A Tractable Firm Life Cycle Model

We build a minimalist theory to guide our understanding of the patterns in the data pointing to long-run selection inherent to cash-flow-based financing.[3] As it turns out, to be able to rationalize many of the patterns seen in the data, we need a forward-looking lending constraint. We show a current earnings-based based borrowing constraint–often used in structural applications–is not enough to capture the data as it resembles a backward looking constraint. Such a forward-looking constraint is appealing because it allows for potential "superstars" to access the financing they need to survive today. As we discuss below, our modeling approach is based on a common notion in the literature–e.g. Jang et al. (2025)–that financiers may be willing to provide continuation value based loans because of the possibility of running or selling the firm in default.

4.1 Selection on Permanent Types with Cash-Flow-Based Financing

We develop a tractable model of value based financing which can be viewed as a blend between Melitz (2003)–a model of permanent heterogeneity–and Hopenhayn (1992)–with temporary heterogeneity which we capture in two periods. Crucially, we allow for financing frictions which interact with permanent and temporary heterogeneity.

Firms live for two periods, and have two different productivities: z in the short-term, and Z in the long-term. Firms survive to the next period if they choose to continue to operate in the first period. Firms hire labor and produce with a Cobb-Douglas technology subject to an operating cost giving rise to the following flow profit function:

$$\text{Profits:} \quad \pi(z_t) = \max_l z_t^{1-\alpha} l^\alpha - wl - f = \kappa z_t - f, \quad z_t \in \{z, Z\}$$

[3] We view our specification as applicable to other forms of financing beyond cash-flow-based long-term debt which may include equity injections; in particular, venture capital investments early on in the firm life cycle are often based on little information on current assets nor cash-flows and all about expectations of future cash-flows embedded in the valuation of firms at the time of financing.

Where the second equality comes from optimizing out labor and defining a constant $\kappa = \kappa(w) = (1-\alpha)(\alpha/w)^{\frac{\alpha}{1-\alpha}}$. We assume the steady-state Z follows a Power Law with exponent ζ and minimum $\underline{Z}$: $Z \sim G(Z) = 1 - (Z/\underline{Z})^{-\zeta}$. The transition from z to Z is governed by a Markov process $F(z, Z)$. Firms borrow and have dividends. Negative dividends are equity. Firms are born without any existing wealth (in the Theory Appendix we extend this to the case where firms can enter with assets b). When a firm borrows it receives a face value $b'/(1+r)$ which they must pay back tomorrow as $-b'$. Thus, b' allows firms to transfer assets intertemporally. The dividend streams are:

$$\text{Short-term:} \quad d_0 = \pi(z) + \frac{b'}{1+r}, \qquad \text{Long-term:} \quad d_1 = \pi(Z) - b'$$

Unconstrained firm dynamics problem. A firm chooses whether to survive or not in every period as well as how much to have as dividends or debt.

$$\text{Value:} \quad v(z, Z) = \max_{d,b'} \left\{ \pi(z) + \frac{b'}{1+r} + \frac{1}{1+r} \left(\mathbb{E}_{Z|z} \max\left\{\pi(Z), 0\right\} - b' \right), 0 \right\}$$

In the case where a firm is not financially constrained, then since $\frac{b'}{1+r} - \frac{b'}{1+r} = 0$ we have that the split between d, b' is indeterminate (a version of Modigliani-Miller). The optimal survival policy naturally depends on the firms' long run type Z as in the extension of Hopenhayn (1992) of Sterk et al. (2021) with ex-ante heterogeneity.

Lemma 1 (Frictionless Economy) Absent financial frictions, the *survival of a firm depends on their long-run type* Z. Firms with $z \geq z^*(Z)$ survive, where the threshold:

$$\text{Frictionless threshold:} \quad z^*(Z) = \begin{cases} \frac{f}{\kappa} & \text{if } Z < Z^* \equiv \frac{f}{\kappa} \text{ firms exits when young} \\ \frac{f}{\kappa} \cdot \frac{2+r}{1+r} - \frac{Z}{1+r} & \text{otherwise firm survives to old age} \end{cases}$$

Proof. See Appendix.

Intuitively, a firm knows itself to be a future "superstar" with high potential Z, it will weather a larger negative shock today.

Def. Current Cashflow-Based Constraint. We assume firms can borrow against a multiple of current cash-flows in an earning based borrowing constraint:

$$\text{Current cash-flow based constraint:} \quad b' \leq \chi_\pi \pi(z)$$

Lemma 2. (Backward-Looking Friction) Under this current-cashflow based constraint, *the short-term exit threshold becomes independent of long-run type Z* – whether the firm is going to be a superstar or not. In essence, the firm viability rule becomes $(1 + \chi_\pi/(1 + r))\pi(z) \geq 0$ instead of $v(z, Z) \geq 0$, erasing the dependence on Z.

$$\text{Backward-looking threshold:} \quad z \geq z_\pi = \frac{f}{\kappa} \quad \perp\!\!\!\perp Z$$

Proof. See Appendix.

Observational equivalence of current cashflow-and asset-based constraints. In the Theory Appendix we extend this result to the case where the firm carries an asset position b at entry that adds to their cash-flows and develop the case of an asset-based constraint that considers the assets a firm can pledge as collateral, which depend on b. The main insight of the current cashflow-based constraint carries through into the asset-based constraint: both types of constraints prevent the forward-looking selection on Z happening in the frictionless case, and are even observationally equivalent in the simple model with $b = 0$.[4] Note that here the threshold rule for survival would also be the case in an economy where no borrowing is allowed, an insight we exploit later.

Def. Continuation Value-Based Constraint. Alternatively, consider the case of a firm with a value based constraint where:

$$\text{Continuation value-based constraint:} \quad b' \leq \chi_v \mathbb{E}_{Z|z} v(Z, b')$$

We interpret χ_v – our parameter of interest – as both the ability to screen firms value or the ability to lend to firms conditional on screening which may be impaired by financial technology or regulatory frictions that we leave unmodeled. We view this as

[4]If $b \neq 0$, $z_j(b) = f/\kappa + \Omega \cdot b$ where the factor Ω varies depending on whether the current cash-flow or asset-based constraint applies, with no dependence on long-run Z.

a theoretical approximation to the financing technology discussed in Lian & Ma (2021) and Jang et al. (2025) that captures the idea that financiers are willing to provide (continuation) value based loans (much like equity investors) because of the possibility of running or selling the firm in the event of default.

Under perfect information, the financier would be able to observe Z. With no information, the lender could try to infer Z from current z or infer this value from a population known to be Power Law. As an illustrative case, when financiers (or the entrepreneur-financier match) have an imperfect process of observing future productivity that follows:

$$z'|z, Z = \begin{cases} z & \text{with probability } \rho \\ Z & \text{with probability } 1-\rho \end{cases}$$

Hence, the process for productivity has effective persistence ρ of past z with a drift related to the long-run Z: $\mathbb{E}\left(z'|z, Z\right) = \left(1-\rho\right) Z + \rho z$.

Proposition 1. The presence of the continuation value-based constraint reintroduces forward-looking selection because borrowing depends on long-run Z and so does the threshold, which is given by:

$$z \geq z_v\left(Z\right) = \frac{f}{\kappa} - \frac{1-\rho}{\left(1+r\right)} \frac{1}{1+1/\chi_v} \max\left\{Z - \frac{f}{\kappa}, 0\right\} \rightarrow_{\chi_v, \rho \rightarrow \infty, 0} z^*(Z)$$

Note that this threshold depends on Z unlike the backward-looking current-cashflow based constraint, but in a way that is muted relative to the frictions in observing and lending against future value. As these frictions disappear, we recover the frictionless threshold, and the slope of the threshold function with respect to Z approaches that of the frictionless economy.[5]

Result 1: survival selection as a function of long-run type. Figure 4 plots all the survival thresholds for today's productivity as a function of future productivity. It highlights visually that Proposition 1, together with the above lemmas, show the

[5]In general, $z \geq z_v^i(Z)$ where the threshold becomes a fixed point of the following equation:

$$z_v^i = \left(1 + \frac{\chi_v}{1+\chi_v} \frac{\mathbb{P}^*\left(z_v^i, Z\right)}{1+r}\right) \frac{f}{\kappa} - \frac{\chi_v}{1+\chi_v} \frac{\mathbb{P}^*\left(z_v, Z\right) \mathbb{E}^*\left(z_v, Z\right)}{1+r}$$

key result that a value-based constraint yields a cutoff that depends on forward-looking productivity Z, like the frictionless cutoff, but with a different intercept and slope. In contrast, current earnings based and no-information value based constraints are isomorphic in the sense that all do not depend on Z at all but with different intercepts. In the Theory Appendix we show this is also the case under a backward-looking asset based constraint.

The intuition for Proposition 1 shown in Figure 4 is simple but powerful. Under the value-based constraint, future Z can enter the valuation today and affect firms' borrowing capacity directly unlike the case of the backward-looking asset/current earnings based constraints. When financiers can only imperfectly observe Z, we have two parameters to shock: ρ, which captures the difficulty of observing Z, and χ_v which captures frictions to lending to seemingly valuable firms after making an inference about the valuation. When $\rho \to 0$ we get the perfect information case.[6] When $\rho \to 1$ we recover a version of the earnings-based constraint is recovered. For ρ in between, the relationship between the threshold and z weakens with information frictions, but remains present in the model to the extent that lenders are able to estimate steady-state Z with some degree of accuracy. Figure 4 makes this point showing that as information becomes more imperfect, the value-based constraint converges to the earnings based constraint. But as the quality of information financiers are able to extract on future Z improves, the threshold schedule begins to resemble the shape of the frictionless environment.

"Superstar" survival. Figure 4 highlights that in the frictionless world, there are future "superstar" firms with productivity so high–$Z \geq Z^* = \frac{f}{\kappa}(2+r)$–that these firms will always be profitable enough to survive regardless of their current draw—i.e. $0 = z^*(Z^*)$. Under the value-based friction there is still a high enough $Z^* > \frac{f}{\kappa}$ such that $z_v(Z^*) = 0$ i.e. where very productive firms will always be able to remain in operation

[6] Under no information about the future profitability of firms we have:

$$z \geq z_{\bar{v}} = \left(1 + \frac{\chi_v}{1+\chi_v}\frac{1}{(1+r)}\frac{(\underline{Z})^{\zeta}\left(\frac{f}{\kappa}\right)^{-\zeta}}{\zeta - 1}\right)\frac{f}{\kappa}$$

regardless of how bad their current conditions are, but higher than in the frictionless economy.

Firm value and entry. To close the model, we assume firms draw their ex-ante long-run type Z at entry, and face a distribution of short-run profitability, which we assume to be normal and only partly linked to Z, inducing a survival probability p_s in the short run.

If firms have access to a cash-flow-based credit line, they are able to borrow based on their future value and face threshold z_v. Otherwise they are only able to borrow based on their current earnings, with observationally equivalent $p_{s,j}$ to a no borrowing constraint. Thus, their ex-ante value becomes:

$$v_j\left(Z\right) = p_s\left(z \geq z_j(Z)\right) \cdot \mathbb{E}_{z \geq z_j(Z)}\left[\pi(z) + \frac{1}{1+r}\max\left\{\pi(Z), 0\right\}\right], \quad j \in \{*, \pi, v\}$$

Result 2: Financing, selection, and firm life cycle growth profiles. Figure 6 shows how average firm employment evolves from entry (Young) to maturity (Old) under two different lending regimes, for firms that start at different initial sizes: i.e. firms that happen to draw different productivity levels at entry z. In the case of ("asset-based") current earnings or asset-based constraints, despite starting at very different sizes, the two firms converge to identical steady-state employment consistent with the view that asset-based lending makes firms appear as if they were ex-ante identical (as in Hopenhayn (1992)) even if they are not. The reason is mechanical: the backward-looking financier sets a single survival threshold that does not depend on where the firm is headed. All firms that make it through selection were screened against the same bar, so the surviving pool has the same average productivity regardless of what the firms looked like at entry. Initial size is completely irrelevant to mature size. In the case of forward-looking value-based lending, the gap that existed at entry persists into maturity. This happens because the value-based lender sets a threshold that varies with the firm's expected future productivity—firms with better prospects face a lower bar. So the two firms, even though they face the same lender, are selected from pools with different thresholds, and therefore different conditional means at maturity. The initial difference in productivity partially maps into difference in size in the long run: as in Melitz (2003), value-based lending results in firms being ex-ante heterogeneous.

Result 3: Cash-flow-based financing constraints and long-run selection. Figure 6 also captures another striking feature of the data, which is that in less constrained industries or where it is potentially harder to lend against the forward looking value of firms (χ_v is low) the average employment growth is faster.[7] Importantly, this is not due to each firms growing faster with financial frictions, rather than due to the distributional consequences of financial frictions. When χ falls, the threshold rises, implying better selected firms that end up growing larger in the steady state as credit tightens. Young firms' employment will also be affected in the richer model where an entrant's size (and not just their survival to old age) depends on the financing technology. Finally, as value-based financing frictions tighten, we should expect to see convergence in firm types in the steady state, providing the insight that worsening of value-based constraints generate less uneven firm distributions in the long run.

4.2 Endogenizing the Extensive Margin of Cash-Flow-Based Lending

The model of the intensive margin of lending based on future value above already induces long-run selection based on endogenous exit. However, a key feature of the data is that the extensive margin of cash-flow-based lending is also based on expectations of future value. The key insight we seek to capture is that due to its forward-looking nature, there is revealed information about a firm's long-run net present value (driven by its permanent type Z) in the timing and identity of who gets cash-flow-based financing. This is not present in the case of asset-based financing or in the case of current earnings based financing.

We assume all firms start without a credit line, and thus, have no access to borrowing. As the previous section highlights, this is equivalent to a case where lending is based

[7] A similar prediction can be obtained when it is harder to assess the value of the firm, or lower ρ in less predictable industries.

only on current earnings in which only firms with $z \geq f/\kappa$ survive.

Access to cash-flow-based lending based on value. In order for a firm to gain value-based financing, there is an imperfect screening process that makes better long-run types get cash-flow-based financing at higher rates (similar results would apply under self-selection).

Lemma 3 (Noisy financing propensity). We characterize an endogenous probability of attaining cash-flow-based financing p_f as a function of long-run Z. Lenders do not observe Z (and hence, $v_v(Z)$) directly, and can only receive a noisy signal of the firm's continuation value which depends on the accuracy of their screening technology.[8]

$$s = v_v(Z) + \sigma_f \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, 1)$$

We assume lenders finance firms if this value is higher than a cutoff $s \geq \tilde{v}$ (which can be viewed as the fixed cost of entering into the relationship and we allow to depend on σ_v). Under the normal parametrization, letting Φ be the CDF of the normal distribution:

$$p_f(Z) = \Pr(s \geq \tilde{v}) = 1 - \Phi\left(x - \frac{1}{\sigma_v} v_1(Z)\right), \quad x = \tilde{v}/\sigma_v$$

Propensity of cash-flow-based financing depends on firm value. One of the contributions of our paper is to take this relationship to the data in order to show that our estimates of the net present value of the firm at the time of first financing are tightly related to the propensity to get cash-flow-based financing early on in life above and beyond measures correlated only with current earnings, see Table 4. In the Theory Appendix we extend this formulation to a richer environment in which lenders have more complex updating rule that shrinks towards the average value in the population $\bar{v}$, which materially only affects the constant x in our functional form.

The ex-ante value of a firm depends on the probability of getting cash-flow-based

[8]This kind of sorting based on noisy signal structure interacting with selection has been widely used in macro literature studying firm heterogeneity over the business cycle: Clementi & Palazzo (2016) look at selection based on short-term productivity signals, while Sedláček & Sterk (2017) and Smirnyagin (2023) study self-selection of young firms with high future growth potential closer to our model.

financing:

$$v(Z) = p_f(Z)v_v(Z) + [1 - p_f(Z)]v_\pi(Z) = v_v(Z) - \Phi\left(x - \frac{1}{\sigma_v}v_v(Z)\right)[v_v(Z) - v_\pi(Z)]$$

Figure 7 shows $p_f(Z)$ and $v(Z)$ as a function of σ_v and χ_v. Inherently, improvements in the screening technology and in the ability to lend conditional on having screened the firm into having a credit line have similar qualitative predictions for value as a function of long-run type Z. Thus, both higher σ_v and a lower χ_v lead to less lending to high-potential firms, preventing them from surviving. Given that the ex-ante value of a firm and aggregate TFP are highly sensitive to the value of high Z types, these changes tend to depress entry as well as productivity.

Further, this model predicts that in an industry with noisier signals σ_v, firms will have lower probabilities of getting cash-flow-based financing and will get financed at an older age. Furthermore, if we see a firm financed in an industry where noise is higher, this formulation delivers that their long-run type Z will tend to be higher on average and that because of this positive selection effect, they will tend to grow faster as in the previous section.

Thus, in the model, firms face a financing constraint based on their value due to both an extensive (who gets the cash-flow-based credit line) and intensive margin (how much cash-flow-based financing firms can access), which can be captured as follows:

$$b'(z, Z) \leq \begin{cases} \frac{1}{1+1/\chi_v} \cdot v_1(Z), & \text{with probability } 1 - \Phi\left(x - \frac{1}{\sigma_v}v_1(Z)\right), \\ \chi_\pi \cdot \pi(z), & \text{with probability } \Phi\left(x - \frac{1}{\sigma_v}v_1(Z)\right). \end{cases}$$

This highlights the relation between firms' long-run types and both the extensive and intensive margin of cash-flow-based debt.

While some of the relationship between accessing financing and long-term outcomes is no doubt likely to be causal, one of our contributions is to document that a number of patterns suggest the important role of selection based on future value which could arise from lender screening or entrepreneur self-selection.

When interpreted as capturing the screening problem of a financier, the above framework generates a lending threshold that depends not only on credit rationing conditional on observing future value (inherently related to χ_v), but also on the precision of the learning process (in this case, of the lender) captured by σ (on the extensive margin) and ρ (on the intensive margin).

Within our framework, we think of comparisons between tangible and intangible sectors, or between familiar versus foreign industries for serial entrepreneurs, as mapping directly to a setting with more uncertainty about a firms' future path. It is natural to hypothesize that screening is inherently more difficult in cases where future value is less observable from current signals, leading to two distinct arguments for empirical verification based on the variation our screening precision parameters.

Test 1: Industry predictability. First, in industries with lower predictability—where ex-ante factors are less tightly coupled with long-run firm outcomes—lenders operate in a noisier informational environment. In these sectors, current earnings z serve as a noisier proxy for future potential Z. To mitigate this heightened uncertainty, lenders apply a more stringent selection filter. Consequently, only firms with exceptionally high long-run (latent) productivity can generate a signal strong enough to overcome the background noise and clear the financing hurdle. Crucially, the signal is based on firm value and not on current profitability z, which as we know from the data could be negatively related to financing propensities. The model therefore predicts that, conditional on attaining cash-flow-based LTD, firms in less predictable industries will start at a smaller initial scale due to tighter credit rationing, but will subsequently exhibit significantly faster growth and higher long-run survival as a direct consequence of this stringent selection on Z.

Empirically, we proxy industry predictability using the cross-industry contribution of the ex-ante component to firm-level employment dispersion from the Sterk et al. (2021) empirical model. We use the estimated values from US data to capture this as an inherent, structural characteristic of each industry. Two pieces of empirical evidence support our hypothesis.

First, Figure A8 shows that firms in less predictable industries (those with a lower ex-ante component contribution) obtain cash-flow-based financing later in their life cycle, implying it is less likely that firms get financed early on. For instance, Figure A8 demonstrates that the mean age at which firms in the accommodation and food sector (a highly predictable industry) secure cash-flow-based LTD is just above age 1, whereas in the information sector (a highly unpredictable industry), the mean age at first issuance is 1.2 years higher.

Second, Figure A9 documents that conditional on obtaining cash-flow-based LTD at the same age, firms in less predictable industries grow faster, ultimately catching up to or surpassing their counterparts in more predictable industries.

Test 2: Serial entrepreneurs. A similar informational friction exists for serial entrepreneurs switching to a new industry, representing a discrete jump in ρ compared to those staying in a familiar sector. While serial entrepreneurs who stay within their original industry can leverage a verifiable track record and industry-specific reputation, those who switch industries present a broken signal to lenders. The soft information and reputation accumulated in the previous sector do not fully transfer, making it harder for lenders to identify high-ability entrepreneurs. Facing this increased noise, lenders again shrink their expectations toward the current signal, requiring switchers to pass a tougher quality test to secure credit. The model thus predicts that switchers will start smaller than their non-switching counterparts but will exhibit steeper growth trajectories and ultimately higher performance, as only the highest-quality entrepreneurs are able to secure financing in a foreign industry. Figure A10 supports this prediction.

Test 3: Industry tangibility. Exploring industries like information technology that we categorize as "intangible" that despite having less tangible assets to collateralize are also more unpredictable and where forward-looking financing constraints are likely tighter, Figure A11 similar patterns linking cash-flow-based financing and long-run selection.

These patterns are critical because they would not emerge in a purely earnings-based constraint model. In a model where lending is determined strictly by current z, firms that start small would not systematically grow faster or eventually outperform larger firms. By highlighting these divergent trajectories for intangibles and serial entrepreneur switchers, we provide empirical evidence that lender screening in cash-flow-based LTD markets effectively filters for latent quality. This selection effect is strongest precisely when the informational environment is most challenging, confirming that lenders engage in learning to identify long-run firm value.

5 Richer Calibrated Model

In this section we embed the financing frictions discussed in our tractable model into the canonical model of firm life cycle dynamics from Sterk et al. (2021) which allows for firms that receive ex-post shocks as in Hopenhayn (1992) to be ex-ante heterogeneous as in Melitz (2003) in a slow moving fashion. While Sterk et al. (2021) study a simple (no-borrowing) friction, we study more complex asset-based (equivalent to a standard current earnings) constraint and continuation value-based constraints with the goal of understanding the empirical patterns discussed earlier and studying their distinct implications.

5.1 Setup

The economy is populated by firms that produce differentiated goods using a linear technology with a fixed operating cost: $y_i = n_i - f$. Each good has a demand function with an ideosyncratic demand shifter φ_i which we can be interpreted as the stochastic customer capital a firm accumulates over time: $y_i = \varphi_i \left(p_i/P\right)^{-\xi} Y$. The price and output indices depend on the endogenous set of firms active in equilibrium Ω_t. The price index is $P \equiv \left[\int_{i\in\Omega_t} \varphi_i p_i^{1-\xi} di\right]^{\frac{1}{1-\xi}}$ and the aggregate quality-adjusted output index $Y \equiv \left[\int_{i\in\Omega_t} \varphi_i^{\frac{1}{\xi}} y_i^{1-\frac{1}{\xi}} di\right]^{\frac{\xi}{\xi-1}}$.

We set $W = 1$ as the numeraire and let $w = 1/P$ be the real wage. The budget constraint must hold $\int_{i\in\Omega_t} p_i y_i di = WN + \Pi$. The later reflects the flow profits of each firms: $\pi_i = p_i y_i - W n_i$. Firms ideal hiring decision tracks productivity:

$n_i = \varphi_i \left(\xi/(\xi-1)\right)^{-\xi} w^{-\xi} Y + f$. Letting $\chi \equiv (\xi-1)^{\xi-1}/\xi^{\xi}$ we can express profits as: $\pi_i = \varphi_i \chi w^{-\xi} Y - f$. We let firms borrow or lend in a financial asset b_i. We assume firms start with a distribution of assets b_0. The value of the firm net of their asset position is:

$$V_j(a,b) = \max\left\{0, E\left[\pi\left(a'\right) + \beta(1-\delta)\int_{\varphi(a',b')\geq\varphi_j^*(a,b)} V_k\left(a',b'\right) dF\left(a' \mid a, k|j\right)\right]\right\}$$

Note that firms can have financing types with transition probabilities embedded in F which we describe below. Where firms have a bankruptcy threshold. We follow Sterk et al. (2021) in allowing assets to accumulate via the budget constraint: $b' = (1+r)\left(b + \pi\left(a'\right)\right)$.[9] Note that initially in the firms' life $\pi\left(a'\right) < 0$ due to the presence of the costs of operations and low initial productivity. The economy is segmented in markets (e.g. information technology sectors) that differ in their productivity process or financing environment (e.g. future cash-flow based financing might be more prevalent). The presence of the constraint affects the forward looking value of firms. Note the baseline Sterk et al. (2021) impose an exogenous borrowing constraint $b_i \geq -\chi_0$ where $\chi_0 \geq 0$ (in their solution imposing essentially no-borrowing $\chi_0 = 0$). We instead allow for the constraint to evolve endogenously with the rest of the firms' problem and study its economic consequences. Like Sterk et al. (2021) assume, as we did in the tractable model, we assume that if at any point in time the firm cannot meet this limit, it is forced to exit (i.e. default). That is, financing frictions work primarely through an extensive margin of exit selection.

We allow for several financing arrangements:

Baseline type: No borrowing. Firms start with a no borrowing constraint: $b' \leq 0$. This is the original constraint studied in Sterk et al. (2021).

Current earnings-based borrowing constraint. Firms are allowed to borrow up to a multiple of their current operating profits:

[9]This emerges from imposing a non-negative dividend constraint that binds with equality:

$$d = \pi(a') + b - b'/(1+r) \geq 0$$

$$b' \leq \chi_\pi \pi(a)$$

This is a standard implementation in the literature using cash-flow based constraints following the work of Lian & Ma (2021).

Value-based lending. If allowed to access cash-flow-based financing, firms can borrow up to a multiple of their value. The intuition is that the lender may be able to recover the intangible value of the firm in the event of default, inherently operating the firm as an entrant with the same technology and current assets. We impose that the firm and the lender have the same expectations emphasizing uncertainty rather than relying on information asymmetry. This constraint inherently acts as a discount rate wedge.

$$b_i' \leq \chi_u \mathbb{E} V\left(a_i', b_i'\right)$$

As a result, relatively high long-run value firms will be less likely to exit, but at the same time, temporary shocks that drive down the value of firms will affect exit. As discussed above, this constraint is meant to reflect the continuation-value based lending that appears for example in the case of private credit Jang et al. (2025) where financiers may take over the firm or sell it in the market for firms, and thus, be willing to lend against the value of the firm.

The rest of the model is more standard and follows the tradition of the firm dynamics literature. We impose a free entry condition under fixed entry costs f^e and an entrant distribution of a, b. Firms start to borrow–if allowed–when their current cash-flows turn negative (which often happens early in life) and is feasible. The endogenous distribution of firms over productivity and assets and financing types evolves according to the exogenous productivity process and the endogenous exit and entry process, as well as the accumulation of assets among firms. Further, we enforce labor market clearing with fixed labor supply, where part of the labor demand goes to production and part towards investment into paying the entry and operating costs. In this richer version of the model, aggregate productivity is endogenous and is tied to both the wage and the number of entrants, both endogenous objects solved in general equilibrium to solve free entry and labor market clearing jointly. Note because the risk-free asset can also be held by households $r = 1/\beta - 1$.

We can also allow for an endogenous extensive margin of financing. We can allow for transition as in the previous section to transition into having access to the cash-flow-based value-based borrowing constraint rather than no-borrowing with probability:

$$p(v) = 1 - \Phi\left(\hat{x} - \frac{v}{\sigma_s}\right)$$

We can also allow entrants to transition into a financing type at entry after drawing their long-run type and before drawing anything else using the same $p(v)$ above, hence getting that some entrants can have cash-flow-based financing from the start. However, because results on this extensive margin are still preliminary, and because the presence of ex-post risk in the stochastic productivity process described below already plays a similar role, we focus on financing types as being permanent types being drawn at birth in the main text.

Given it's importance, we describe the productivity process used in depth below.

5.2 Calibration and Counterfactual Exercises

We follow Sterk et al. (2021) and let firms have a productivity process that has ex-post (temporary) and ex-ante (permanent) heterogeneity driven by multiple underlying components defining the firm's productivity state $a_{i,t} = (\theta_i, u_{i,t}, v_{i,t} + w_{i,t}, z_{i,t})$, where:

$$\ln \varphi_i = u_{i,t} + v_{i,t} + w_{i,t} + z_{i,t}$$

Firms have a permanent "ex-ante" type θ_i they draw at birth and converge to as long as they choose to remain in operations (otherwise they lose this firm-specific intangible).

$$u_{i,t} = \rho_u u_{i,t-1} + \theta_i, u_{i,-1} \sim \text{iid}\left(\mu_{u^e}, \sigma^2_{u^e}\right), \quad \theta_i \sim \text{iid}\left(\mu_{\theta^e}, \sigma^2_{\theta^e}\right)$$

Note the presence of u makes firms technologically to arrive at heterogeneous speeds to their long-run potential. In addition, firms experience rich ex-post shocks that are also mean reverting, and thus, can driven them away for extended periods of time from their long-run targets, and if particularly unlucky, force them to decline.

$$v_{i,t} = \rho_v v_{i,t-1}, \quad w_{i,t} = \rho_w w_{i,t-1} + \varepsilon_{i,t}, \quad z_{i,t} \sim \text{iid}\left(0, \sigma_z^2\right). \quad \varepsilon_{i,t} \sim \text{iid}\left(0, \sigma_\varepsilon^2\right)$$

We calibrate this rich productivity process, constraint and probability of cash-flow-based financing access in the model jointly to match average size-by-age, average exit-by-age, the variance-covariance matrix of firm growth rates as proposed in Sterk et al. (2021), and average probability of getting cash-flow-based financing by age profiles for the UK economy in Figure 8.

In this model, the lack of predictability of firms early in life despite high-permanent productivity arises from the combination of temporary uncertainty due to the ex-post shocks and slow moving convergence to temporary types. When necessary, we recalibrate the model's parameters to match those across different sectors using empirical parameters provided in Sterk et al. (2021), where (for example) information technology turns out to be technologically more unpredictable than food and accommodation. A key intuition we carry into the counterfactuals is that in information technology, firms are more ex-ante unpredictable (in the language of the previous section, short run z can be quite different from long-run Z). In contrast, firm in food and accommodation have a short-run z that is very predictive of long run Z.

While we do not directly target this in the calibration, the model with the forward-looking (value-based) constraints matches the patterns of cash-flow-based lending: firms that borrow earlier converge to higher long-run points as shown in Figure 9. We also have shown that with the backward-looking (asset based) constraints (where $b' \leq \chi_b b$) the patterns of asset-based lending emerge: firms converge to the same point regardless of when they start borrowing; differences in initial positions map to differences in asset-values used for collateral, not to long-run differences in firm quality as with cash-flow-based financial frictions.[10] This resembles the patterns found in the tractable model but in a richer environment where firms can live for more than two periods and with a much richer firm dynamics environment. Further untargeted, we find that lending leads to lower predictability of future size relative to the initial size of firms.

With the calibrated model, we can study the effects of differences in value-based financial frictions, i.e. changing χ for each financing constraint type. Part of the motivation is to think of differences across sectors (e.g. IT vs food/accommodation) or regions (US vs Europe). However, this is also motivated by a key development in cash-flow-based financing within the UK over the past 20 years: the average amount of long-term debt that firms receive at the time of financing has dramatically decreased, both in absolute terms and relative to initial firm size. Figure A7 shows that the cash-flow-based debt-to-current turnover has decreased by 10 percentage points, and log cash-flow-based debt has decreased by about 60%. In our model, tightening financial frictions lead to (as expected) depressed aggregate output and productivity mainly due to their effect on the number of firms in the economy, see Figures B4 and B5. As the value-based constraint tightens, exit rate rises but at the same time it crowds out higher future productivity firms due to the negative selection mechanism described in the tractable model. This leads to slower growth rates of employment and predictability (since firms are less selected based on their long-run types), resembling the differences between the UK and the US that motivated our analyses. In fact, we believe our results might provide insights into part of the drivers behind the productivity gap between Europe and the UK and the patterns seen across sectors like information technology vs food and accommodation.

We repeat the exercise of comparing an economy with current earnings and future cash-flow based borrowing constraints. We find that for the same degree of ex-ante heterogeneity in productivity, the value-based or future cash-flows based constraint generates a wider long-run inequality among firms than an asset-based or current cash-flow based constraint as shown in Figure 12. This is natural given the fact that the life cycle

[10] However, the effects relative to an asset-based constraint economy are subtle due to the fact that the value-based constraint may induce a higher level of baseline efficiency (and thus, minimize the efficiency gains).

paths of firms stay ranked throughout their lives as opposed to converging to the same point. This difference in long-run inequality between the two economies only arises with ex-ante heterogeneity: with only ex-post heterogeneity, the two economies are virtually indistinguishable. We are currently exploring these and other patterns further and will present results in a future version of the paper.

5.3 Discussion of Simulation Results

While both types of constraints (current vs future cash flow based) depress entry and aggregate productivity, our counterfactuals highlight several key differences:

Value-based financing and the equilibrium distribution of firms. As shown in Figure 12, relative to a current-earnings based constraint, dispersion is widened under value-based constraints because smaller firms are allowed to survive early on in life and they grow to be large, "superstars" later on in life. This shows up as a wider dispersion overall but more compressed early on in life. In contrast, financing based on current cash flows has very different implications for the distribution of firms over the life cycle. In the short-run firms are young, there are more firms being "cut" out of funding because they had negative early on draws. However, when firms are more mature, the fact that selection was done on a margin that is less correlated with future performance implies a distribution of firms that is shifter to the left in the long-run. Thus, financing based on future rather than current cash flows is necessary to allow a large mass of potentially viable superstars to survive in the future which tend to have worse cash-flows in the short term. We view this as related to the mechanisms discussed in Manso (2016).

Predictability and equilibrium impacts of value-based financing frictions. As seen in Figures 11 and 12 the degree of predictability of a sector (or an economy) drives which type of constraint–has bigger effects on the death rates of superstars that are so crucial for aggregate productivity (as discussed in Sterk et al. (2021)). Economies with more predominant current earnings-based borrowing favor superstar survival in sectors that are very predictable, like food and accommodation. In contrast, economies where forward-looking lending is more prevalent should see more survival of superstars in unpredictable sectors like information technology. As highlighted by these graphs, superstars in very predictable sectors thrive with financing based on current cash-flows, while those in unpredictable ones do not because their first years of cash-flows are not reflective of their long-run trajectories. Anecdotally, if one believes that Europe relies more heavily on the former compared to the later, in might rationalize why we see superstars in the predictable accommodation and food sector (e.g. Michelin starred restaurants) versus the information technology giants that abound in the US. This might be reflective of the higher degree of forward-looking financing in the US for more unpredictable business models and sectors relative to the more conservative European environment.

Forward looking financing driving overall predictability. Higher value-based frictions decrease the overall level of predictability by impeding the long-run selection that drives predictability. Selection on the wrong margin lowers predictability, selection on the right margin increases predictability. Our view is that the nature of firm financing could help explain differences between Sterk et al. (2021) in the US and Jaimovich et al. (2023) in Europe (using Orbis Spain). While the former find that in US Census data, highly heterogeneous ex-ante types are quite predictable of firms' future performance, the later find using Orbis Spain that firms performance prospects are quite compressed; we view this as a symptom of the predominance of forward-looking financing in the US (which leads to divergence of long-run paths) and a prevalence of lending against current cash-flows in Europe (Which leads to convergence in the long-run).

Value based financing and the survival of potential superstars. Value-based constraints tend to add impatience relative to the frictionless economy. They prevent the survival of potential superstars that take too long to reach their optimal size even if they show signs of early promise. In contrast, earnings-based constraints screen out firms that would deliver high cash flows relatively quickly (and be screened in by value based constraints) as long as they have an initial negative draw.

Our results point to a sector's life cycle "predictability" of firms profitability shocks rather than simply "pledgeability" of their assets as a key driver of the interplay between financing and firm growth.

6 Conclusion

We document that the timing of firms' access to cash-flow-based long-term debt is a powerful predictor of their long-run performance. Firms that obtain this form of financing earlier go on to grow larger and follow more unequal trajectories than otherwise similar firms, while the timing of access to asset-based debt has little relation to long-run outcomes. To interpret these facts, we develop a tractable framework in which financing decisions depend both on collateral and on signals about firms' future value. The model helps explain why forward-looking financing is associated with strong predictability in the data and highlights how frictions in different credit markets shape firm dynamics. Simulating a richer, calibrated model shows that reducing frictions to forward looking financing in the UK would lead to more US-like unequal firm dynamics, including a larger number of "superstar firms".

Our results suggest that the structure of financing constraints plays a central role in determining which firms grow and survive, with important implications for aggregate outcomes and for the reduced-form life cycle dynamics observed in the data. Policies or market developments that ease access to forward-looking financing can foster faster growth and a greater prevalence of high-performing firms, but may also increase inequality in firm size and performance in the long-run. More broadly, understanding how different forms of credit interact with firm heterogeneity is essential for evaluating the consequences of financial development for economic dynamism.

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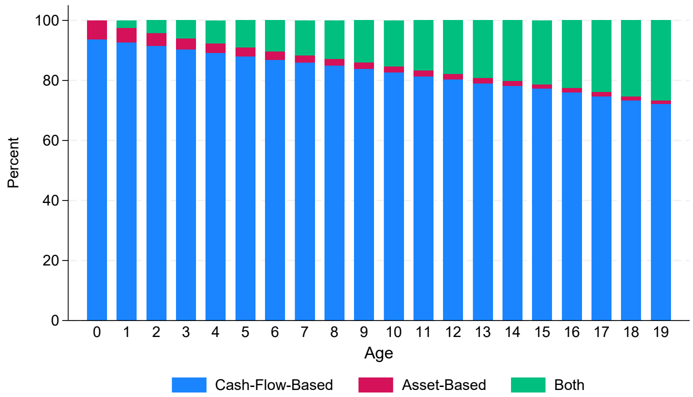
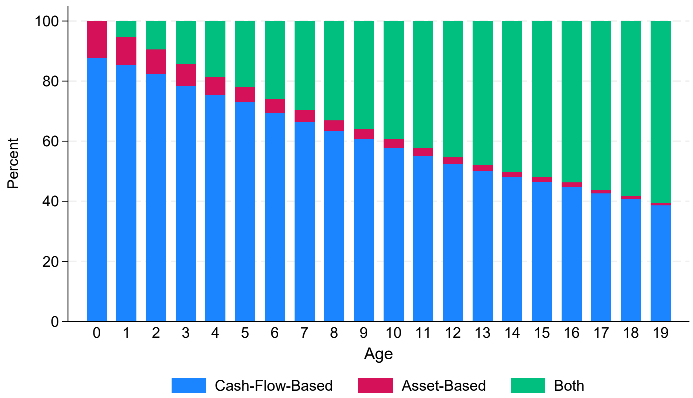
Figures

Figure 1: Use of LTD types over the lifecycle

(a) Full sample



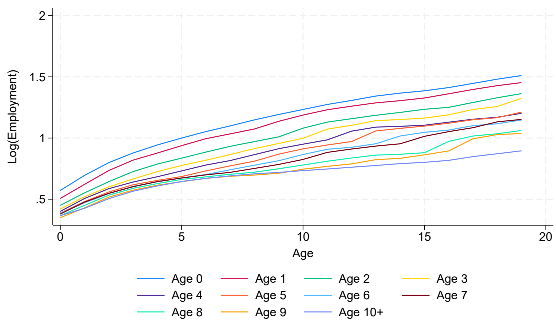
(b) Top 10%



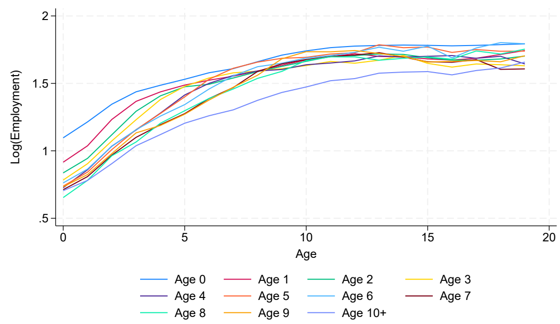
This figure plots the conditional distribution of Long-Term Debt (LTD) types by firm age. The sample is restricted to firms that have accessed the LTD market at least once by the indicated age. Panel 1a presents results for the full universe of incorporated firms. Panel 1b restricts the sample to the top 10% of firms by employment within each age cohort. Both indicates that a firm has utilized both debt instruments (cash-flow-based and asset-based) at some point between incorporation and the specified age. All figures are based on statutory financial filings from Companies House merged with administrative records from the Business Structure Database.

Figure 2: Firm dynamics by initial LTD type, age, and age of initial financing

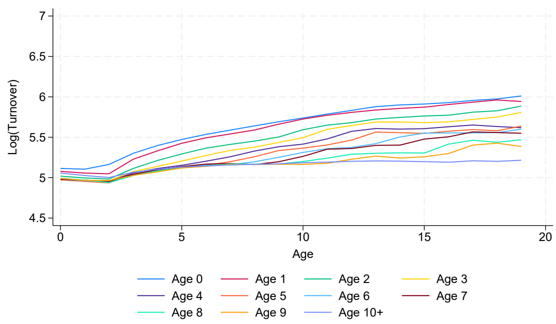
(a) Log(Employment) - cash-flow-based LTD



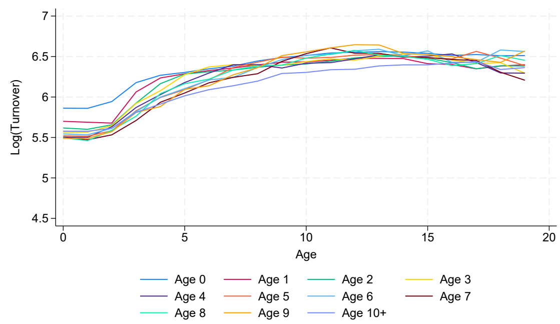
(b) Log(Employment) - asset-based LTD



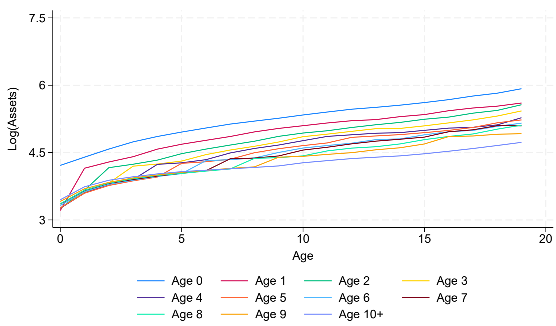
(c) Log(Turnover) - cash-flow-based LTD



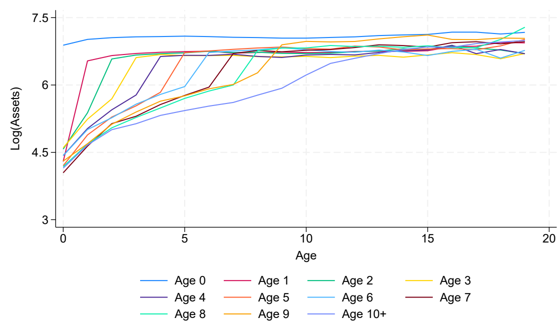
(d) Log(Turnover) - asset-based LTD



(e) Log(Assets) - cash-flow-based LTD



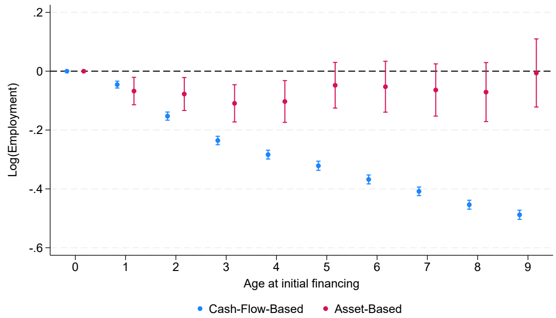
(f) Log(Assets) - asset-based LTD



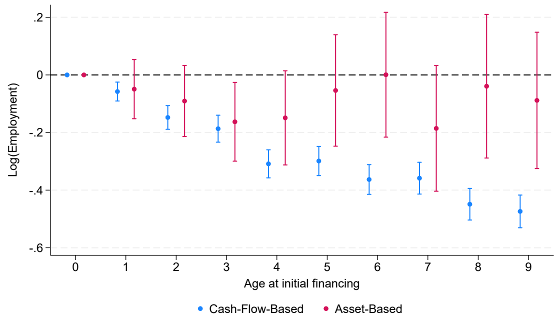
These panels illustrate the evolution of mean log employment, turnover and assets from age 0 through age 19, stratified by the specific age at which a firm first asset-based long-term debt. Age 10+ includes firms that receive their initial LTD between ages 10 and 19. Panels 2a, 2c and 2e display trajectories for firms whose initial financing was cash-flow-based LTD, while panels 2b, 2d and 2f display trajectories for those whose initial financing was asset-based. The figures utilize an unbalanced panel of survivors, where the mean at each age is calculated across all firms in the respective cohort that remain active in the Business Structure Database (BSD). This visualization highlights the persistent quality ladder associated with early-stage cash-flow lending, a pattern that is notably absent in the collateralized debt subsample.

Figure 3: Differences in firm size by age and type of initial LTD financing

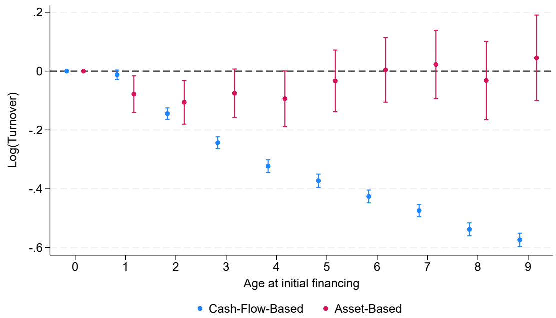
(a) Log(Employment) - Age 10



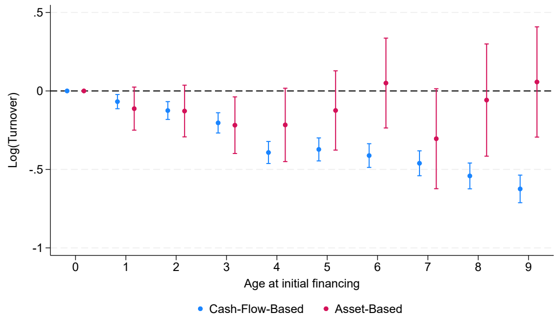
(b) Log(Employment) - Age 19



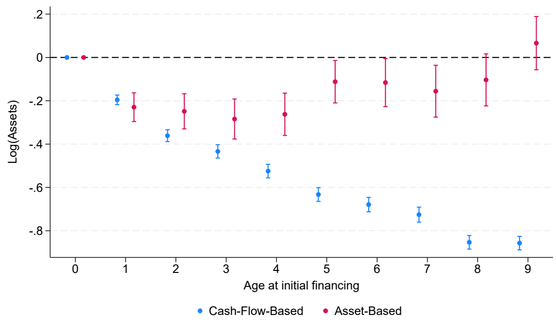
(c) Log(Turnover) - Age 10



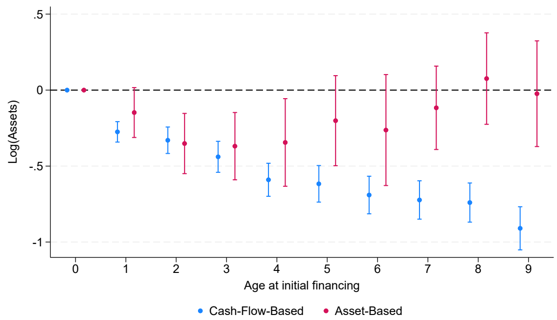
(d) Log(Turnover) - Age 19



(e) Log(Assets) - Age 10

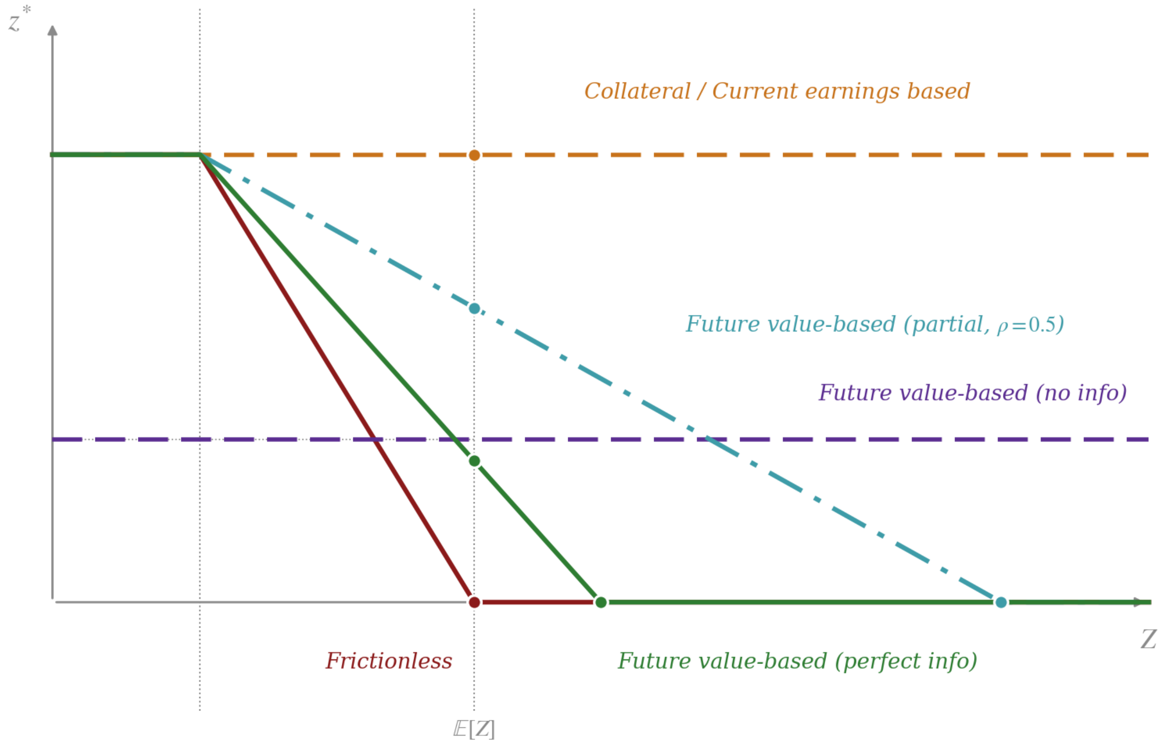


(f) Log(Assets) - Age 19



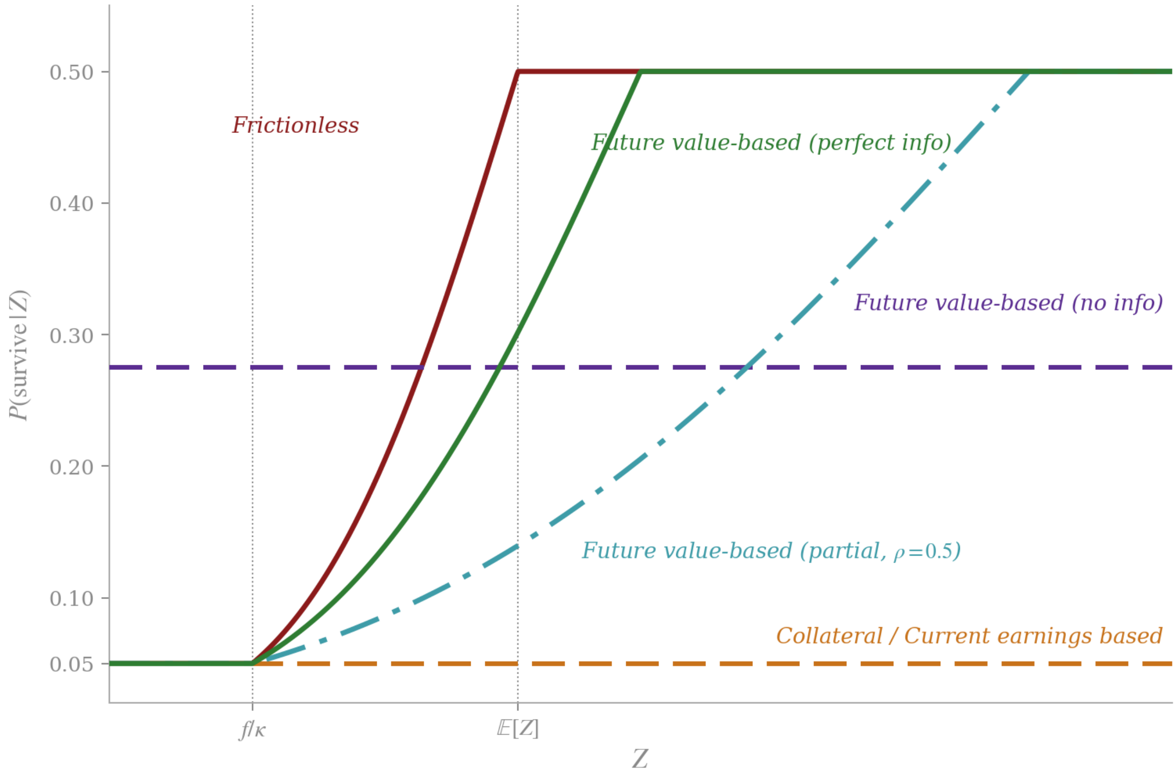
These panels illustrate relative differences in mean log employment, turnover and assets at ages 10 and 19 relative to firms financed with same LTD type at age 0. Estimations are based on equation 2. Panels 3a, 3c and 3e display differences at age 10, while panels 3b, 3d and 3f display differences at age 19. Standard errors use a 90 percent significane level. The figures utilize an unbalanced panel of survivors, where the mean at each age is calculated across all firms in the respective cohort that remain active in the Business Structure Database (BSD). This visualization highlights the persistent quality ladder associated with early-stage cash-flow lending, a pattern that is notably absent in the collateralized debt subsample.

Figure 4: Types and survival under different forms of lending



Today's survival threshold tracks (imperfectly) future steady-state profitability under valued-based lending but not under current earnings-based lending (equivalent to asset-based lending in our setup). The better the information on the firms' long-run value, the more the schedule approaches the one from the frictionless financing economy.

Figure 5: Types and survival under different forms of lending



Today's survival probability rises with future steady-state profitability under valued-based lending but not under current earnings-based lending (equivalent to asset-based lending in our setup). The better the information on the firms' long-run value, the more the schedule approaches the one from the frictionless financing economy, where survival strongly tracks future profitability.

Figure 6: Asset (value) based lending makes firms appear ex-ante homogenous (heterogeneous)

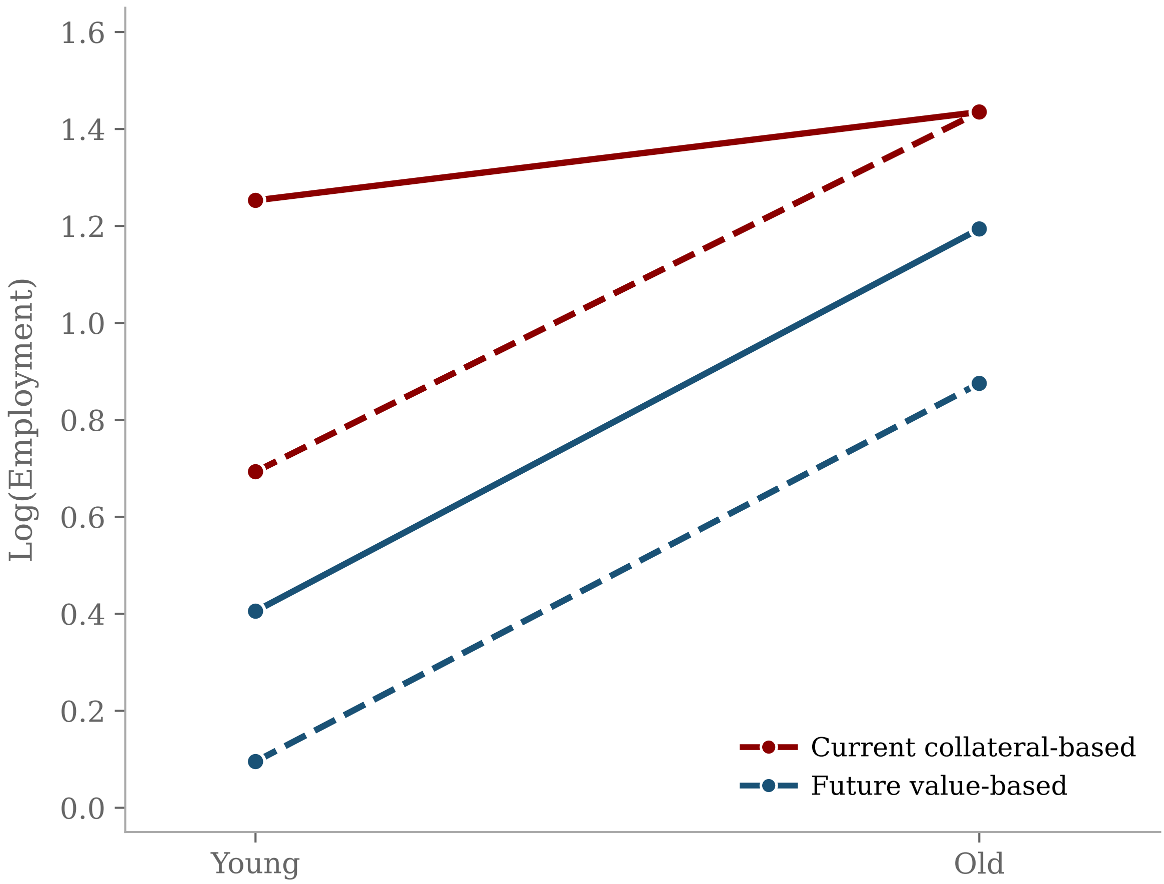


Figure 7: Model with noisy value-based financing

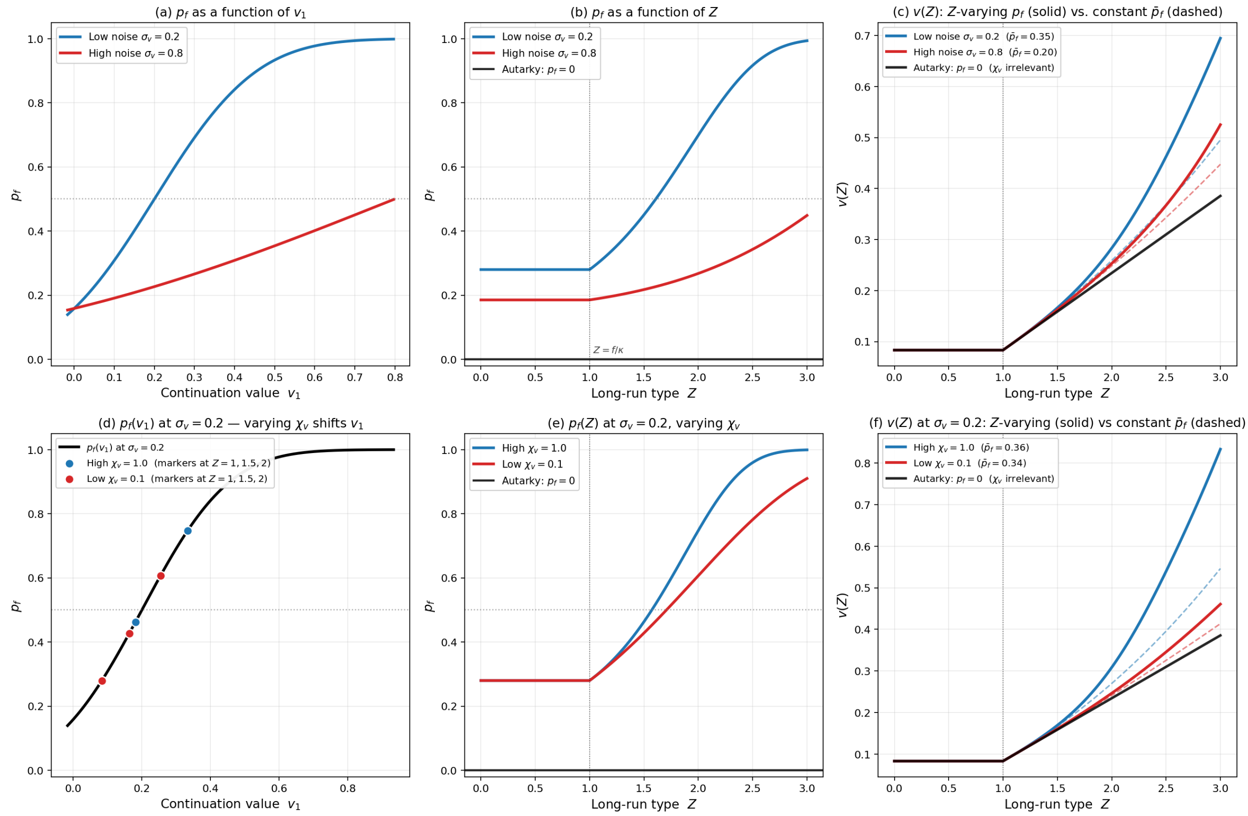


Figure 8: Baseline results with a value based constraint

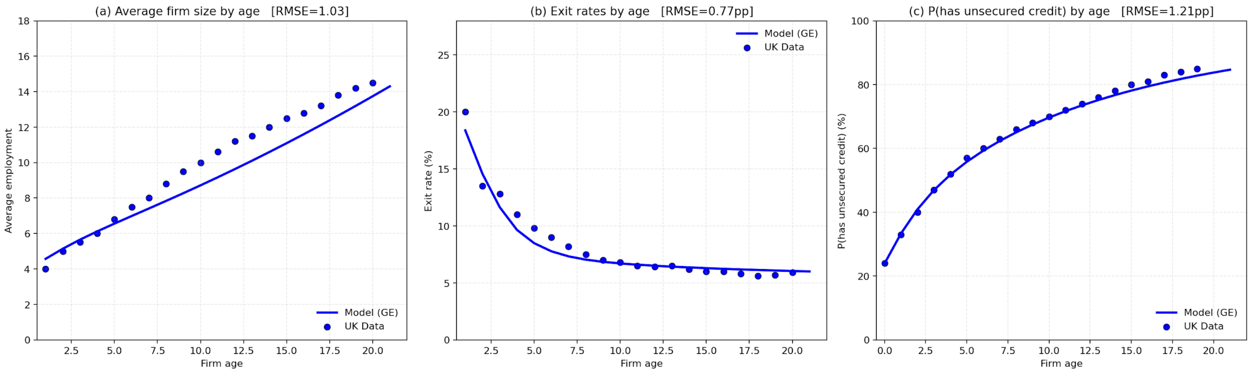


Figure 9: Firms that get cash-flow-based financed earlier in the model converge to higher long-run paths as in the data

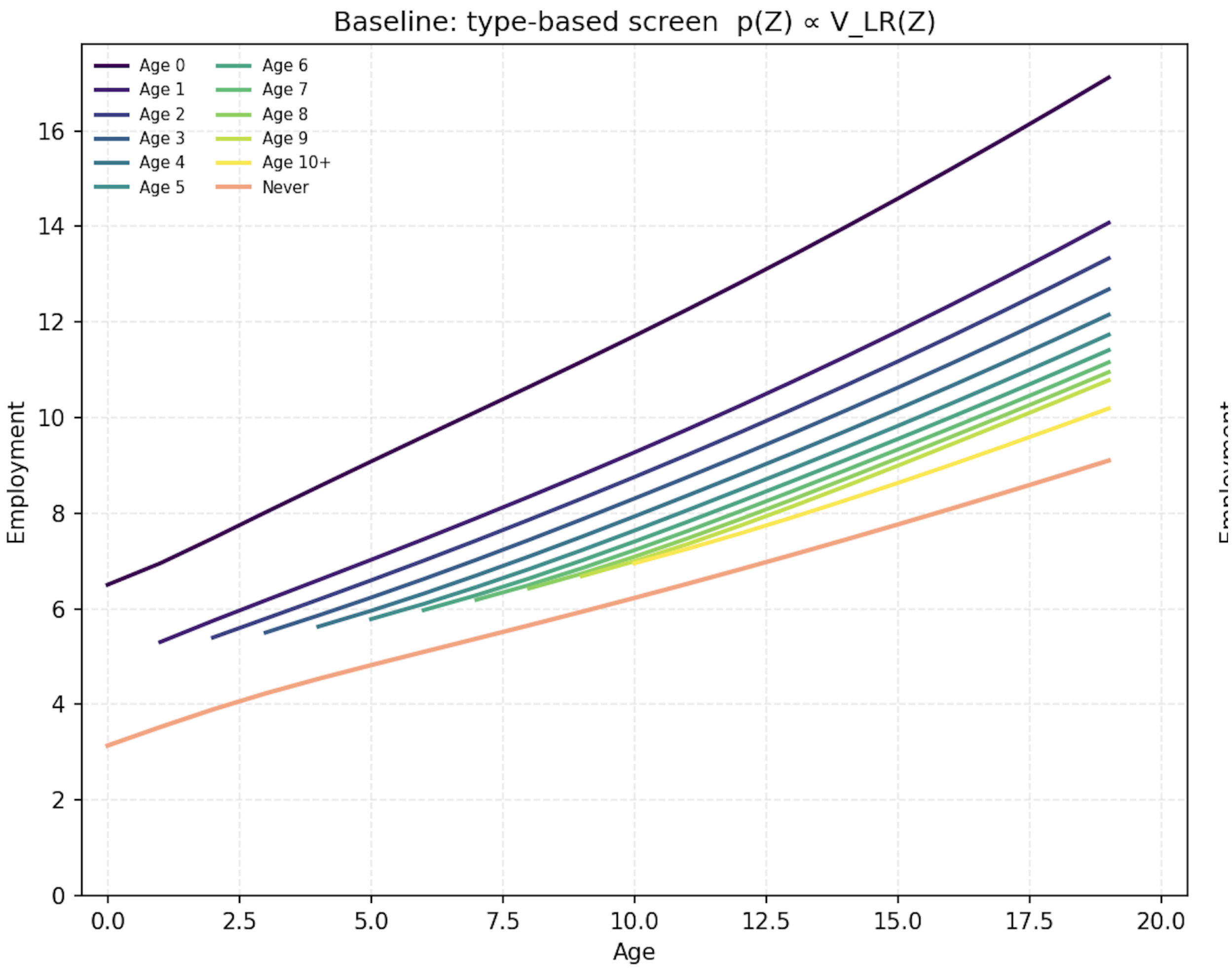


Figure 10: Size Distribution by Age by Financing Regime

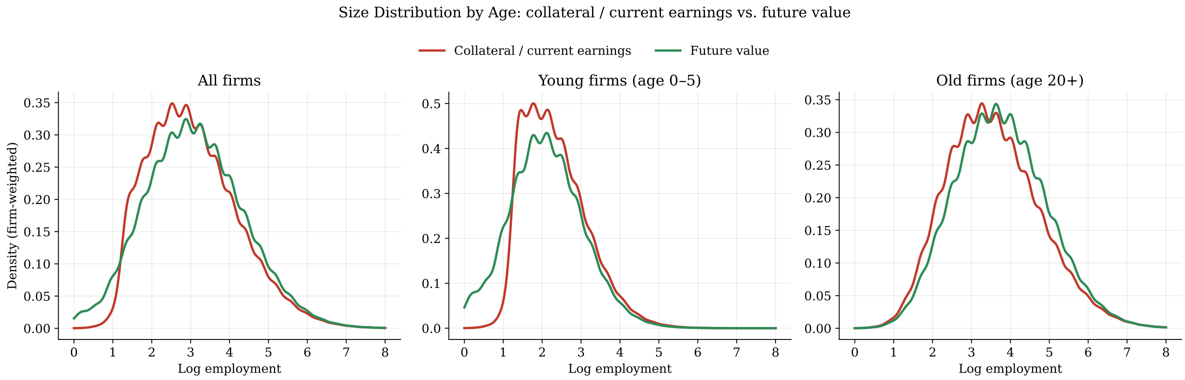


Figure 11: Relaxing current earnings based frictions across sectors by predictability

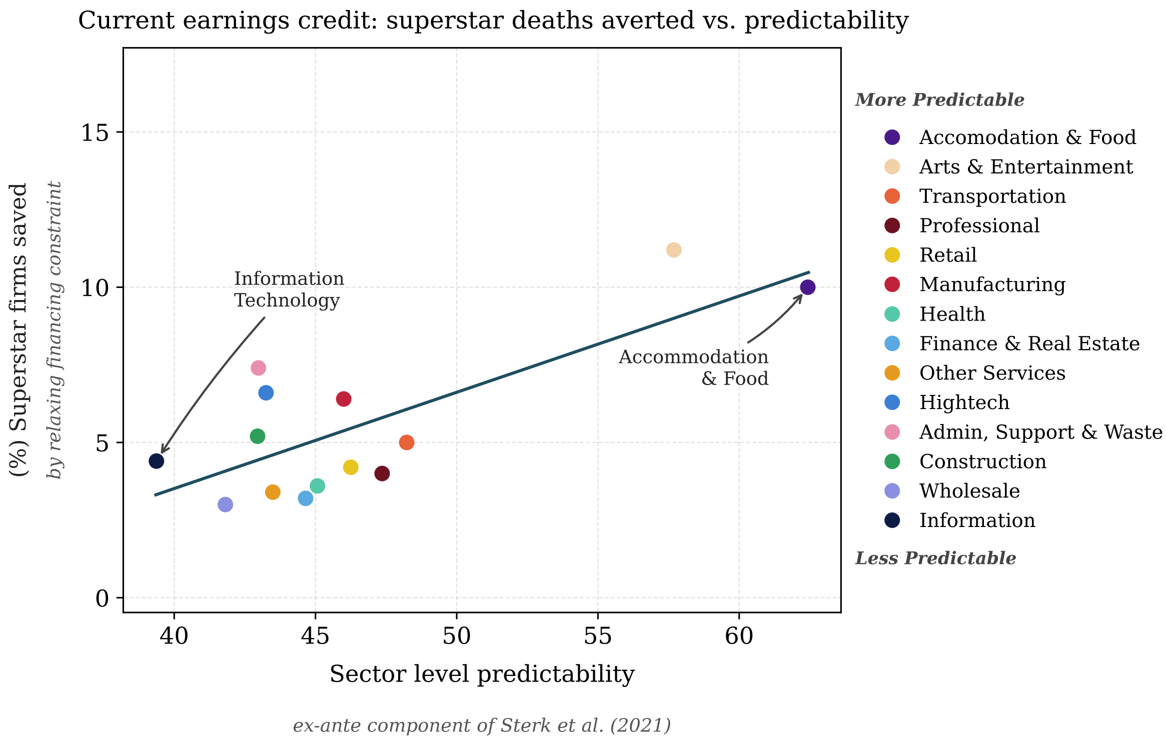
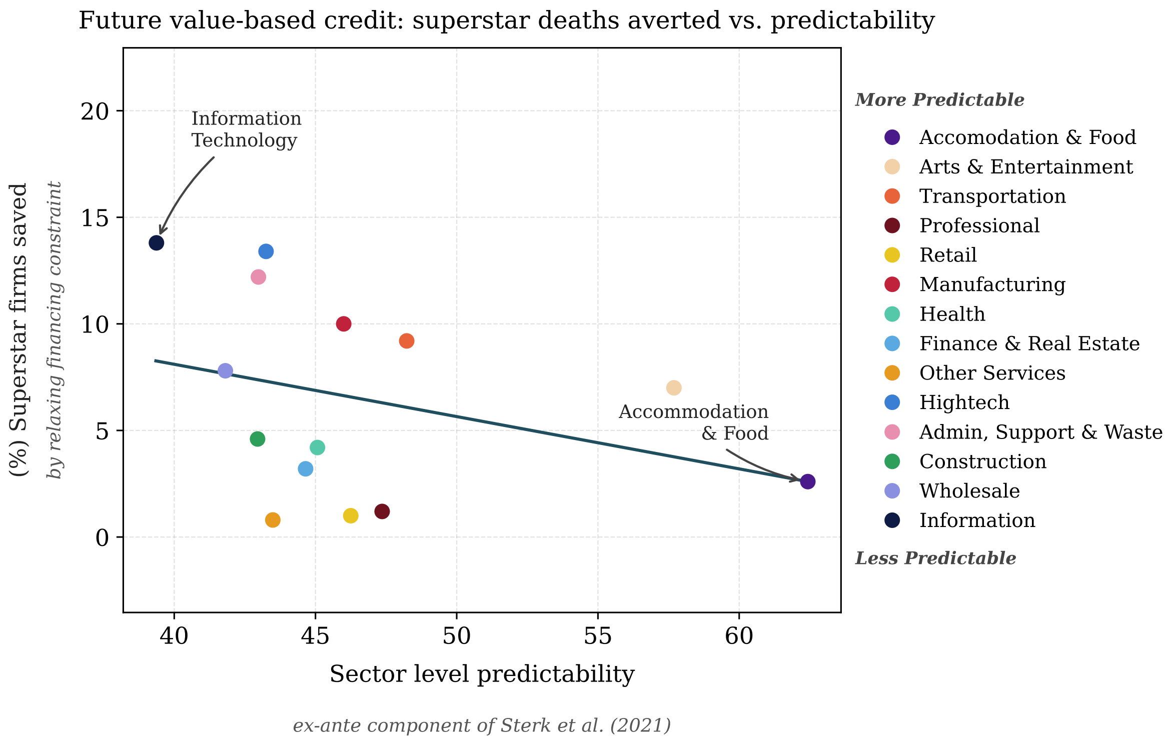


Figure 12: Relaxing future value based frictions across sectors by predictability



Tables

Table 1: Cohort Summary Statistics: Full, Cash-Flow-Based, and Asset-Based Samples

		Age 0				**Age 10**			
Cohort	N	Emp.	LT Debt	Assets	Turn.	Emp.	LT Debt	Assets	Turn.
Panel A: Full Sample									
0	930,824	0.604	2.370	4.384	5.161	1.283	3.498	5.514	5.814
1	275,901	0.527	–	3.275	5.110	1.227	2.985	5.232	5.783
2	151,648	0.469	–	3.322	5.048	1.131	2.824	5.095	5.664
3	104,151	0.436	–	3.316	5.013	1.043	2.769	5.001	5.562
4	75,580	0.414	–	3.308	5.001	0.994	2.540	4.897	5.480
5	57,524	0.399	–	3.306	4.992	0.958	2.372	4.788	5.431
6	42,601	0.398	–	3.359	4.998	0.904	2.193	4.719	5.369
7	31,640	0.390	–	3.379	4.998	0.860	2.063	4.666	5.316
8	24,035	0.375	–	3.386	5.007	0.812	2.068	4.520	5.246
9	18,999	0.361	–	3.441	5.007	0.779	1.994	4.508	5.212
10+	57,810	0.375	–	3.468	5.071	0.754	1.777	4.326	5.216
Never	1,860,996	0.349	–	2.295	4.868	0.504	–	3.711	4.869
Panel B: Firms with initally cash-flow-based LT debt									
0	872,041	0.572	2.157	4.215	5.115	1.233	3.245	5.338	5.738
1	261,518	0.506	–	3.210	5.078	1.188	2.790	5.099	5.726
2	144,083	0.450	–	3.257	5.019	1.081	2.583	4.936	5.594
3	99,533	0.420	–	3.257	4.987	0.998	2.538	4.855	5.494
4	72,528	0.401	–	3.261	4.980	0.950	2.356	4.766	5.415
5	55,397	0.387	–	3.271	4.972	0.912	2.172	4.656	5.366
6	41,155	0.385	–	3.322	4.979	0.865	2.008	4.602	5.312
7	30,571	0.379	–	3.356	4.981	0.825	1.880	4.557	5.264
8	23,266	0.366	–	3.360	4.991	0.779	1.937	4.427	5.200
9	18,396	0.349	–	3.413	4.992	0.745	1.869	4.418	5.164
10+	56,222	0.366	–	3.450	5.057	0.734	1.675	4.274	5.186
Panel C: Firms with initially asset-based LT debt									
0	58,783	1.096	5.530	6.887	5.862	1.741	5.479	7.044	6.513
1	14,383	0.916	–	4.296	5.699	1.674	4.931	6.722	6.435
2	7,565	0.837	–	4.574	5.617	1.663	4.888	6.701	6.407
3	4,618	0.784	–	4.602	5.566	1.632	4.888	6.637	6.438
4	3,052	0.727	–	4.429	5.505	1.638	4.658	6.668	6.419
5	2,127	0.738	–	4.202	5.521	1.693	4.957	6.811	6.480
6	1,446	0.762	–	4.428	5.544	1.688	4.884	6.786	6.517
7	1,069	0.712	–	4.045	5.501	1.677	4.818	6.774	6.536
8	769	0.654	–	4.181	5.496	1.670	4.815	6.825	6.481
9	603	0.734	–	4.305	5.488	1.735	4.934	6.971	6.558
10+	1,588	0.705	–	4.149	5.578	1.473	4.989	6.223	6.304

This table presents descriptive statistics for the firm universe at two distinct lifecycle milestones: inception (Age 0) and maturity (Age 10). The data is partitioned into three panels: Panel A includes the full sample of firms; Panel B is restricted to firms whose initial long-term debt was cash-flow-based; and Panel C is restricted to firms whose initial long-term debt was asset-based. N denotes the total number of unique firms in each subsample. All other variables—including employment, long-term debt, total assets, and turnover—are reported as mean log-transformed values. The mean at each age is calculated across all firms in the respective cohort that remain active in the Business Structure Database (BSD).

Table 2: Predictive Power of Initial Debt and Startup Characteristics for Long-run Scale

Dependent Variable:	Log Employment (Age 10)			Log Employment (Age 19)		
	(1)	(2)	(3)	(4)	(5)	(6)
Initial Long-Term Debt	0.4749***	0.1949***	0.2839***	0.4993***	0.1857***	0.2727***
	(0.0034)	(0.0030)	(0.0033)	(0.0095)	(0.0091)	(0.0097)
Log Employment (Age 0)		0.5358***			0.4503***	
		(0.0028)			(0.0079)	
Log Turnover (Age 0)		0.0593***			0.0959***	
		(0.0022)			(0.0062)	
Log Total Assets (Age 0)		0.0499***	0.1286***		0.0459***	0.1307***
		(0.0008)	(0.0100)		(0.0023)	(0.0027)
Log Productivity (Age 0)			-0.0590***			-0.0297***
			(0.0025)			(0.0068)
N	560,412	560,412	560,412	84,504	84,504	84,504
R-squared	0.0389	0.2768	0.0976	0.0347	0.2166	0.0822

This table reports OLS regressions based on equation 1. Dependent variables are employment at Age 10 (Columns 1–3) and Age 19 (Columns 4–6). Columns 1 and 4 present the baseline relationship; Columns 2 and 5 include full initial scale controls; Columns 3 and 6 isolate the predictive power of debt relative to initial assets and productivity. Robust standard errors are reported in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 3: Long-run Firm Outcomes by Age of Initial Long-Term Debt

	Log(Employment)		Log(Turnover)		Log(Assets)	
	(1) Cash-flow-based	(2) Asset-based	(3) Cash-flow-based	(4) Asset-based	(5) Cash-flow-based	(6) Asset-based
Panel A: Outcomes at Age 10						
Age of First Debt = 1	-0.0455***	-0.0674***	-0.0124	-0.0783**	-0.1956***	-0.2296***
	(0.0060)	(0.0238)	(0.0081)	(0.0317)	(0.0113)	(0.0338)
Age of First Debt = 2	-0.1527***	-0.0776***	-0.1445***	-0.1058***	-0.3611***	-0.2487***
	(0.0071)	(0.0286)	(0.0098)	(0.0380)	(0.0140)	(0.0413)
Age of First Debt = 3	-0.2355***	-0.1092***	-0.2436***	-0.0754*	-0.4338***	-0.2842***
	(0.0073)	(0.0323)	(0.0103)	(0.0420)	(0.0156)	(0.0471)
Age of First Debt = 4	-0.2835***	-0.1029***	-0.3231***	-0.0941*	-0.5247***	-0.2623***
	(0.0077)	(0.0362)	(0.0110)	(0.0482)	(0.0161)	(0.0498)
Age of First Debt = 5	-0.3214***	-0.0478	-0.3725***	-0.0334	-0.6328***	-0.1121**
	(0.0079)	(0.0396)	(0.0114)	(0.0535)	(0.0162)	(0.0499)
Age of First Debt = 6	-0.3679***	-0.0527	-0.4260***	0.0040	-0.6793***	-0.1160**
	(0.0077)	(0.0442)	(0.0111)	(0.0559)	(0.0170)	(0.0565)
Age of First Debt = 7	-0.4083***	-0.0639	-0.4742***	0.0226	-0.7257***	-0.1558**
	(0.0075)	(0.0453)	(0.0108)	(0.0593)	(0.0177)	(0.0610)
Age of First Debt = 8	-0.4538***	-0.0710	-0.5379***	-0.0320	-0.8532***	-0.1037*
	(0.0077)	(0.0511)	(0.0112)	(0.0680)	(0.0160)	(0.0612)
Age of First Debt = 9	-0.4880***	-0.0061	-0.5735***	0.0448	-0.8571***	0.0661
	(0.0079)	(0.0591)	(0.0116)	(0.0743)	(0.0159)	(0.0627)
N	390,246	33,283	389,534	33,142	299,460	27,198
R-squared	0.0216	0.0008	0.0161	0.0005	0.0235	0.0040
Panel B: Outcomes at Age 19						
Age of First Debt = 1	-0.0577***	-0.0494	-0.0680***	-0.1128	-0.2748***	-0.1477*
	(0.0167)	(0.0524)	(0.0231)	(0.0700)	(0.0343)	(0.0837)
Age of First Debt = 2	-0.1477***	-0.0907	-0.1249***	-0.1281	-0.3300***	-0.3516***
	(0.0210)	(0.0629)	(0.0290)	(0.0840)	(0.0445)	(0.1011)
Age of First Debt = 3	-0.1867***	-0.1628**	-0.2035***	-0.2182**	-0.4390***	-0.3690***
	(0.0239)	(0.0697)	(0.0329)	(0.0920)	(0.0520)	(0.1129)
Age of First Debt = 4	-0.3087***	-0.1491*	-0.3923***	-0.2165*	-0.5901***	-0.3446**
	(0.0248)	(0.0834)	(0.0358)	(0.1192)	(0.0554)	(0.1470)
Age of First Debt = 5	-0.2989***	-0.0539	-0.3726***	-0.1244	-0.6168***	-0.2012
	(0.0258)	(0.0987)	(0.0373)	(0.1289)	(0.0614)	(0.1511)
Age of First Debt = 6	-0.3631***	0.0006	-0.4118***	0.0502	-0.6904***	-0.2630
	(0.0263)	(0.1105)	(0.0385)	(0.1460)	(0.0631)	(0.1862)
Age of First Debt = 7	-0.3585***	-0.1857*	-0.4609***	-0.3044*	-0.7231***	-0.1165
	(0.0281)	(0.1113)	(0.0405)	(0.1625)	(0.0644)	(0.1399)
Age of First Debt = 8	-0.4489***	-0.0393	-0.5415***	-0.0582	-0.7401***	0.0759
	(0.0280)	(0.1272)	(0.0419)	(0.1824)	(0.0657)	(0.1535)
Age of First Debt = 9	-0.4738***	-0.0885	-0.6243***	0.0572	-0.9096***	-0.0237
	(0.0289)	(0.1207)	(0.0448)	(0.1792)	(0.0724)	(0.1774)
N	55,671	8,089	55,539	8,058	30,531	4,402
R-squared	0.0129	0.0013	0.0105	0.0017	0.0180	0.0059

This table reports OLS regressions based on equation 2. Dependent variables are log employment, log turnover, and log total assets measured at firm Age 10 (Panel A) and Age 19 (Panel B). The independent variables are dummy indicators for the age at which a firm first asset-based long-term debt, with Age 0 (inception) as the omitted reference category. Odd-numbered columns report results for the cash-flow-based LTD sample, while even-numbered columns report results for the asset-based LTD sample. Robust standard errors are reported in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 4: Predictive Power of Firm Value and Current Cash Flows for Cash-Flow-Based LTD at Age 0

Dependent Variable:	Extensive Margin (Probit)			Intensive Margin (Log cash-flow-based LTD)		
	(1)	(2)	(3)	(4)	(5)	(6)
Log PDV	0.157***		0.186***	0.648***		0.469***
	(0.0117)		(0.010)	(0.034)		(0.046)
Log Turnover (Age 0)		0.117***	-0.049***		0.746***	0.335***
		(0.017)	(0.009)		(0.048)	(0.059)
N	2,751,248	2,751,248	2,751,248	575,250	575,250	575,250
(Pseudo-)R-squared	0.023	0.007	0.024	0.103	0.085	0.114
Marginal Effects:						
Log PDV	0.044***		0.052***			
	(0.003)		(0.003)			
Log Turnover (Age 0)		0.034***	-0.014***			
		(0.004)	(0.0027)			

This table reports OLS regressions of cash-flow-based LTD Use at age 0 on the present discounted value (PDV) of cash flows (turnover) and current cash flows at age 0. Dependent variables are an indicator that captures the extensive margin (Columns 1–3) and log cash-flow-based LTD that captures the intensive margin (Columns 4–6). Standard errors are clustererd by industry. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

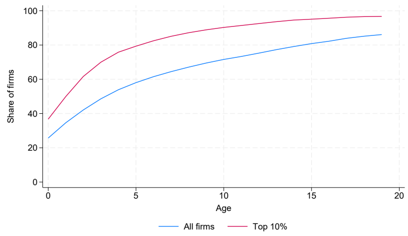
Appendix

A Empirical Appendix

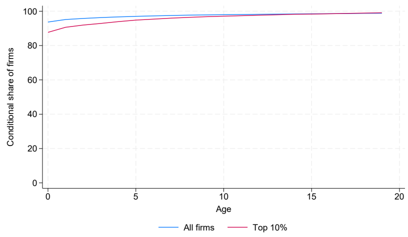
A.1 LTD Prevalence

Figure A1: Prevalence of LT debt by age

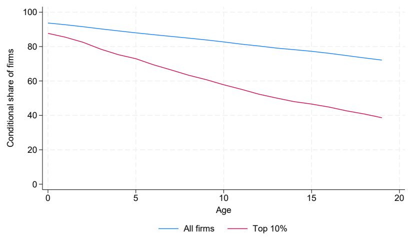
(a) Share of firms that have had any LT debt by age



(b) Share of firms that have had some cash-flow-based LT debt by age (conditional on having had any LT debt)

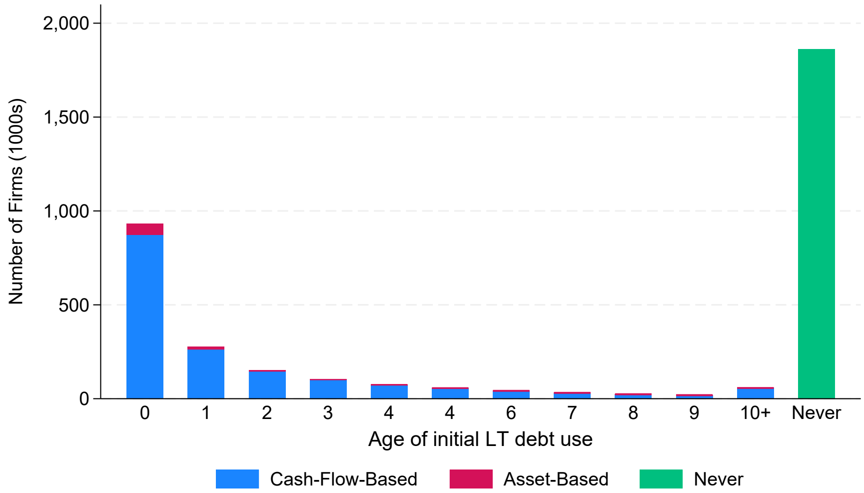


(c) Share of firms that have had cash-flow-based LT debt only by age (conditional on having had any LT debt)



These panels illustrate the lifecycle evolution of credit market participation and debt composition using administrative records from the BSD and Companies House (2002–2024). Each plot presents two series: the full sample of incorporated firms (blue line) and the top 10% of firms by employment at each respective age (red line). Panel A1a plots the cumulative share of firms that have accessed any form of long-term debt (LTD) by the indicated age. Panels A1b and A1c restrict the sample to firms with at least one LTD contract. Panel A1b shows the share of borrowers that have utilized cash-flow-based LTD at least once, while Panel A1c shows the share that has relied exclusively on cash-flow-based LTD throughout their lifecycle.

Figure A2: Distribution of age of initial LT debt use

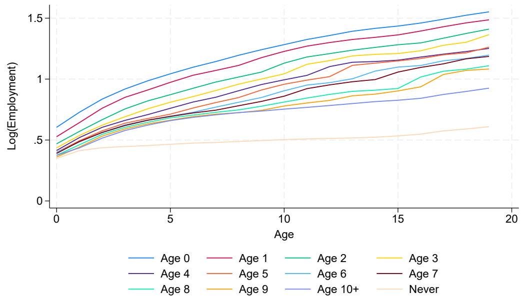


This figure illustrates the lifecycle timing of first-time Long-Term Debt (LTD) issuance for the universe of UK incorporated firms (2002–2024). The vertical axis represents the absolute number of firms entering the credit market at each specific age. For each age cohort, stacked bars decompose these counts into firms whose first lending event was cash-flow-based versus asset-based LTD. The Non-User category denotes the total count of firms that remained unfinanced throughout the sample period. The data is constructed by merging administrative records from the Business Structure Database (BSD) with Companies House records.

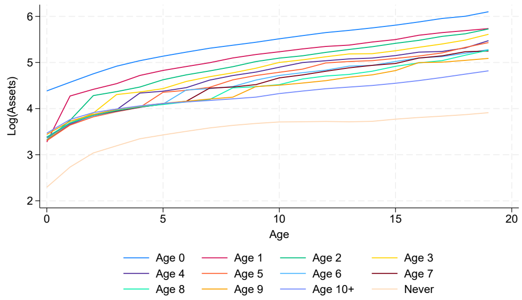
A.2 LTD and Firm Dynamics

Figure A3: Firm dynamics by age and first use of LT debt

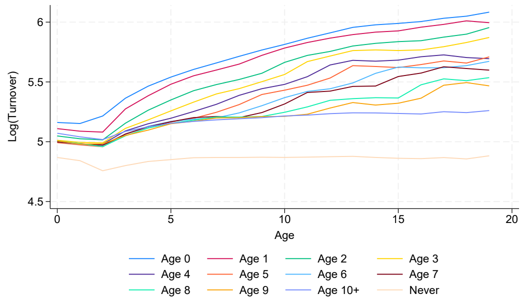
(a) Log(assets) by first use of LT debt



(b) Log(assets) by first use of LT debt



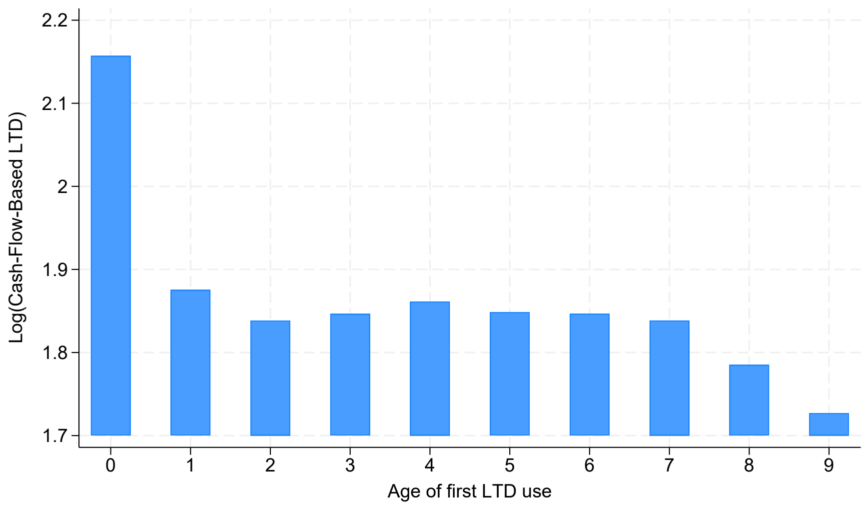
(c) Log(turnover) by first use of LT debt



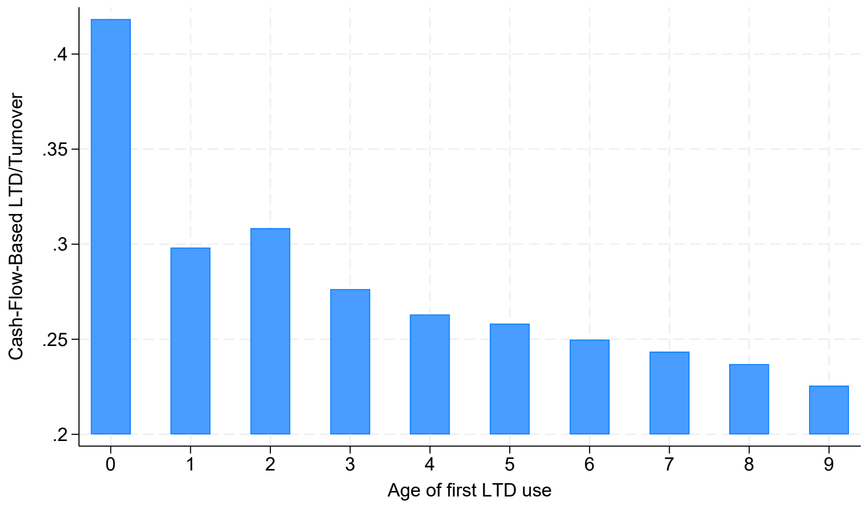
These panels illustrate the evolution of firm dynamics from age 0 through age 19, stratified by the specific age at which a firm first asset-based long-term debt. Age 10+ includes firms that receive their initial LTD between ages 10 and 19. Panel A3a has mean log employment as outcome. Panel A3b has mean log assets as outcome. Panel A3c has mean log turnover as outcome. The figures utilize an unbalanced panel of survivors, where the mean at each age is calculated across all firms in the respective cohort that remain active in the Business Structure Database (BSD). This visualization highlights the persistent quality ladder associated with early-stage LTD lending in aggregate, driven by cash-flow-based LTD lending.

Figure A4: Initial debt scale and intensity for cash-flow-based LTD entrants

(a) Log(LTD)



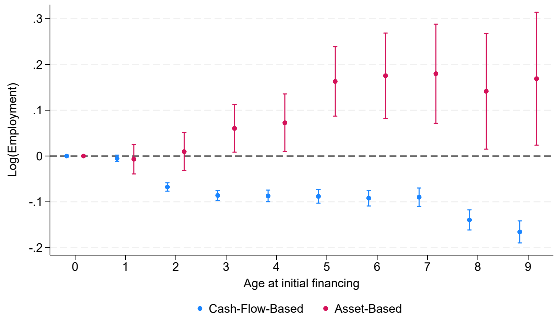
(b) LTD/Turnover



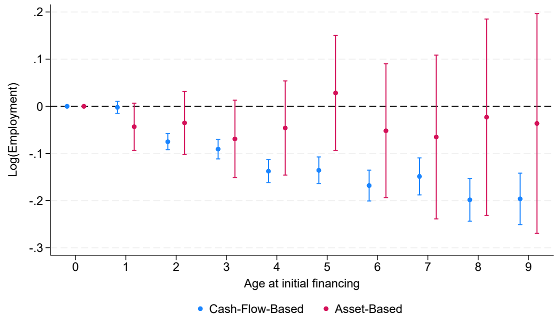
These figures illustrate the scale and intensity of credit at the moment of first-time debt issuance. The sample is restricted to firms whose initial long-term debt (LTD) was cash-flow-based. Panel A4a plots the natural logarithm of the initial LTD amount, representing the absolute scale of the loan. Panel A4b plots the ratio of initial LTD to annual turnover, representing credit intensity relative to the firm's contemporaneous operations. Results are grouped by the firm's age at the time of its first financing event. All variables are measured in the year of debt origination.

Figure A5: Differences in firm size by age and type of initial LTD financing

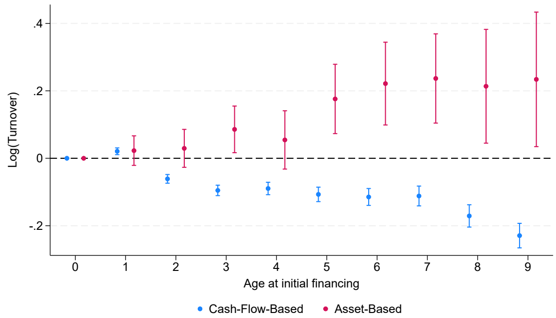
(a) Log(Employment) - 5 years after initial financing



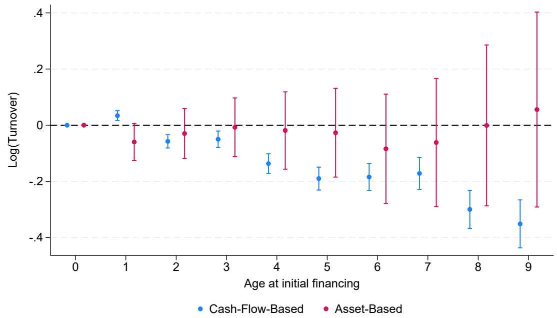
(b) Log(Employment) - 10 years after initial financing



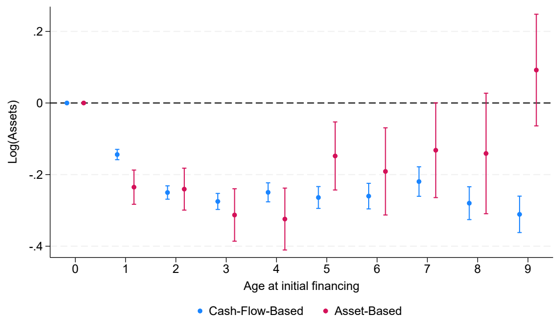
(c) Log(Turnover) - 5 years after initial financing



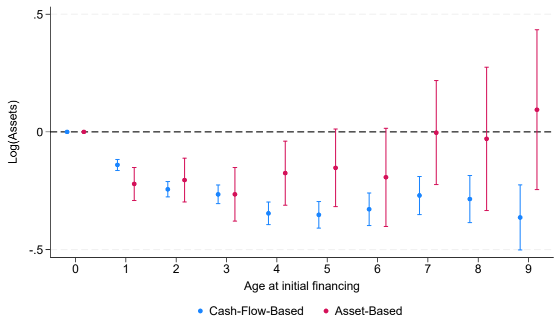
(d) Log(Turnover) - 10 years after initial financing



(e) Log(Assets) - 5 years after initial financing



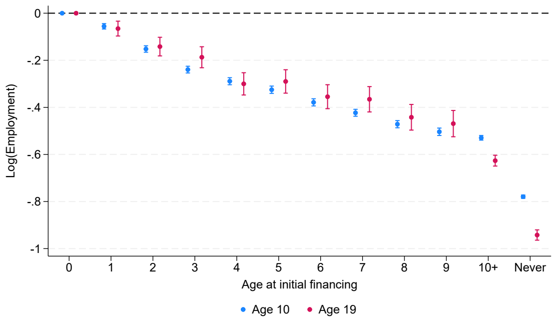
(f) Log(Assets) - 10 years after initial financing



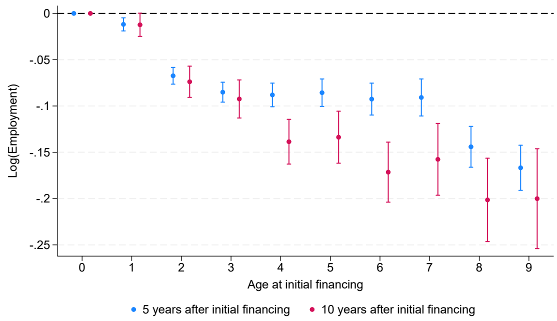
These panels illustrate relative differences in mean log employment, turnover and assets 5 and 10 years after initial financing relative to firms financed with same LTD type at age 0. Estimations are based on equation 2. Panels A5a, A5c and A5e display differences at age 10, while panels A5b, A5d and A5f display differences at age 19. Standard errors use a 90 percent significance level. The figures utilize an unbalanced panel of survivors, where the mean at each age is calculated across all firms in the respective cohort that remain active in the Business Structure Database (BSD). This visualization highlights the persistent quality ladder associated with early-stage cash-flow lending, a pattern that is notably absent in the collateralized debt subsample.

Figure A6: Differences in firm size by age and type of initial LTD financing

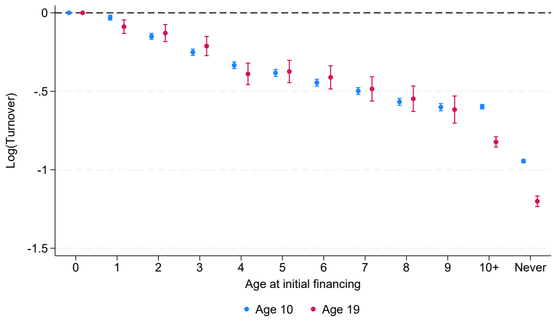
(a) Log(Employment) - Age 10 and 19



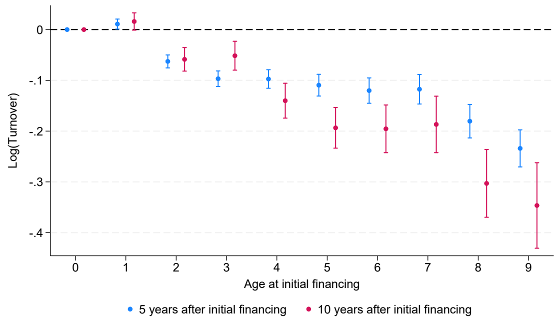
(b) Log(Employment) - 5 and 10 years after initial financing



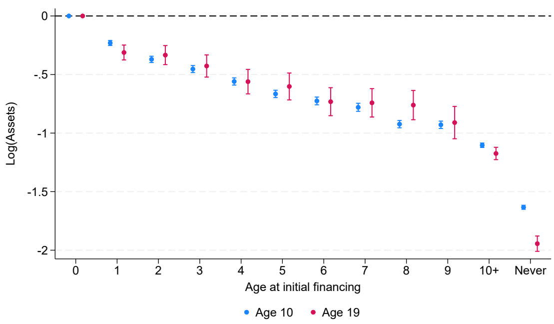
(c) Log(Turnover) - Age 10 and 19



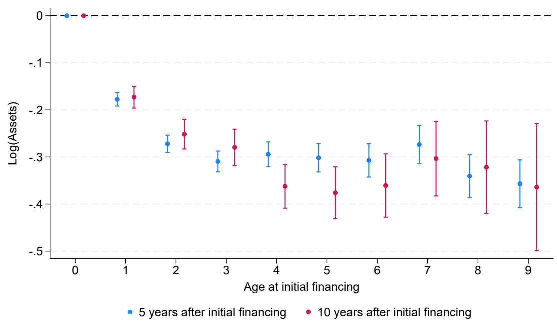
(d) Log(Turnover) - 5 and 10 years after initial financing



(e) Log(Assets) - Age 10 and 19



(f) Log(Assets) - 5 and 10 years after initial financing

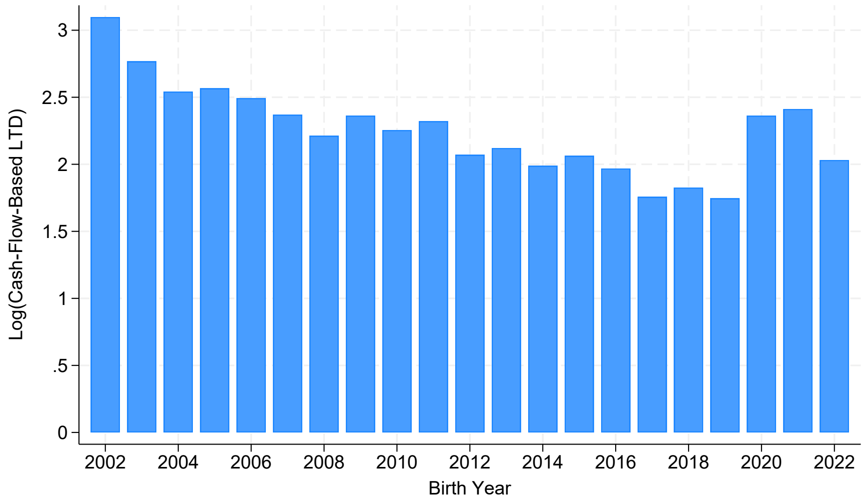


These panels illustrate relative differences in mean log employment, turnover and assets relative to firms financed with same LTD type at age 0. Estimations are based on equation 2. Panels A6a, A6c and A6e display differences at ages 10 and 19, while panels A6b, A6d and A6f display differences 5 and 10 years after initial financing. Standard errors use a 90 percent significance level. The figures utilize an unbalanced panel of survivors, where the mean at each age is calculated across all firms in the respective cohort that remain active in the Business Structure Database (BSD). This visualization highlights the persistent quality ladder associated with early-stage LTD lending.

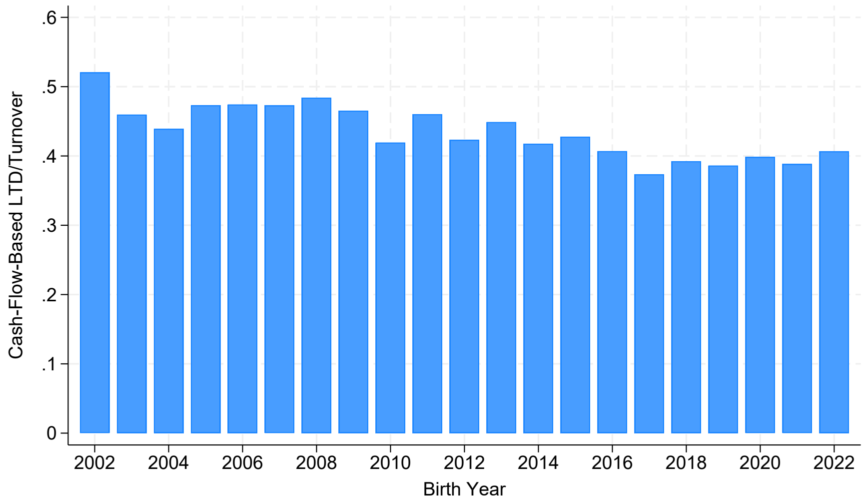
A.3 Time Series Evidence

Figure A7: Intensive margin of cash-flow-based lending at start-up over time

(a) Log(cash-flow-based LTD) at age 0 over time



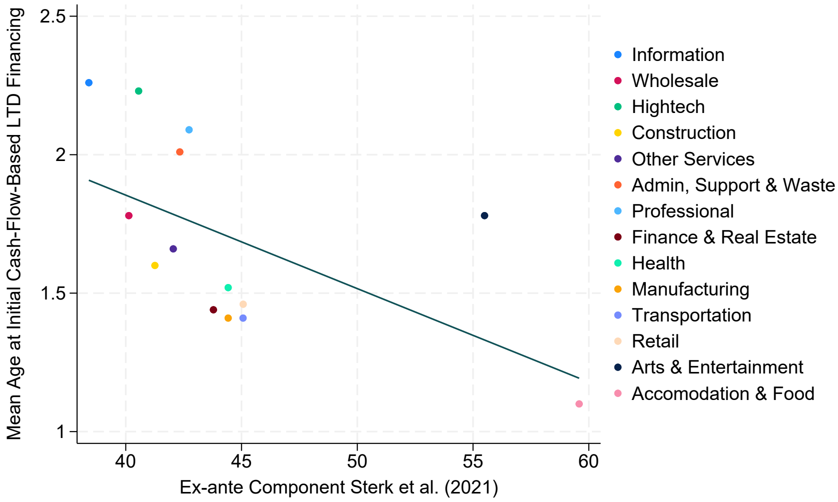
(b) Cash-Flow-Based LTD/Turnover at age 0 over time



These figures illustrate the scale and intensity of credit at the moment of first-time debt issuance over time. The sample is restricted to firms whose initial long-term debt (LTD) was a cash-flow-based contract (cash-flow-based LTD). Panel A7a plots the natural logarithm of the initial LTD amount, representing the absolute scale of the loan. Panel A7b plots the ratio of initial LTD to annual turnover, representing credit intensity relative to the firm's contemporaneous operations. Results are grouped by the firm's age at the time of its first financing event. All variables are measured in the year of debt origination.

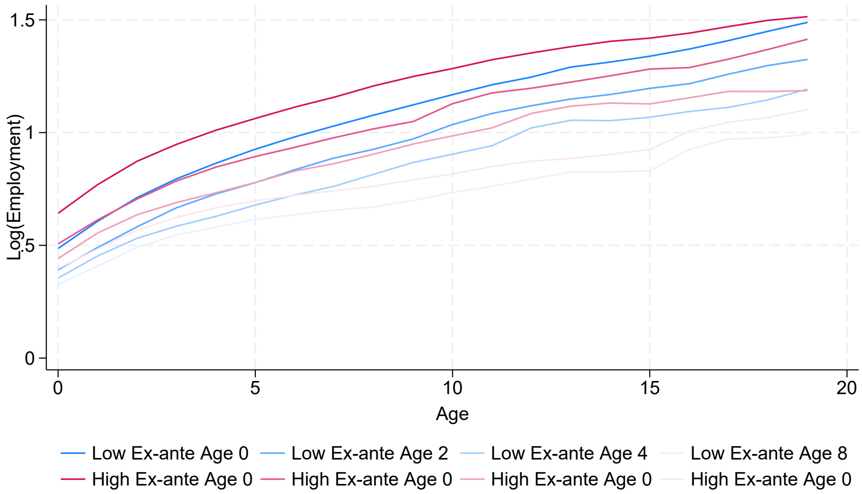
A.4 Heterogeneity by Industry

Figure A8: Industry-level mean age at initial cash-flow-based LTD financing and Sterk et al. (2021) ex-ante component



This figure plots the mean age at initial cash-flow-based LTD financing and Sterk et al. (2021) ex-ante component by industry. The visualization demonstrates that firms in industries with a low ex-ante component obtain cash-flow-based LTD financing later in their life-cycle.

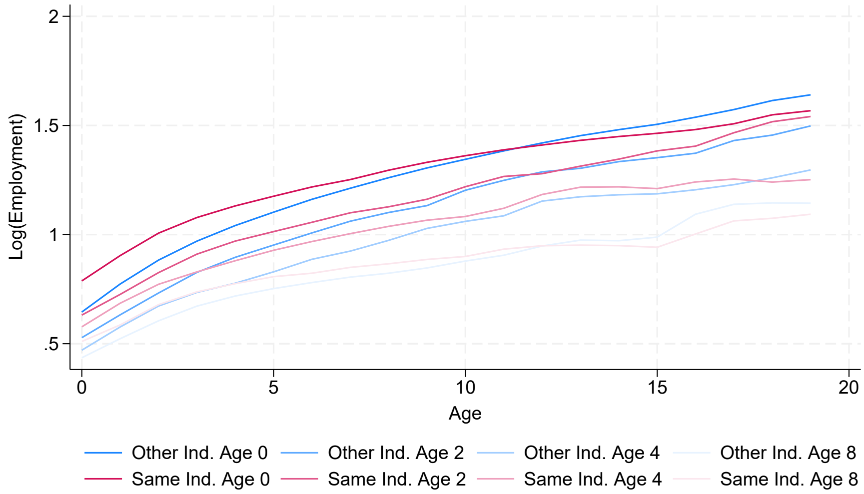
Figure A9: Log(employment) by first use of cash-flow-based LT debt and industry-level Sterk et al. (2021) ex-ante component



This figure plots mean log employment trajectories from Age 0 through Age 19, stratified by industry-level Sterk et al. (2021) ex-ante component and the age of initial long-term debt (LTD) financing. Red lines represent entrepreneurs starting businesses in industries with high ex-ante component (at or above median), while blue lines denote those entering industries with low ex-ante component (below median). Separate trajectories are shown for cohorts receiving their first cash-flow-based LTD at ages 0, 2, 4, and 8. The visualization demonstrates that, conditional on securing financing at the same age, entrants in low ex-ante industries (blue) typically enter at a smaller initial scale but exhibit significantly steeper growth slopes, eventually catching up or surpassing their high ex-ante counterparts (red). The figures utilize an unbalanced panel of survivors, where the mean at each age is calculated across all firms in the respective cohort that remain active in the Business Structure Database (BSD).

A.5 Heterogeneity by Entrepreneur Experience

Figure A10: Log(employment) by first use of LT debt and serial entrepreneur experience



This figure plots mean log employment trajectories from Age 0 through Age 19 for serial entrepreneurs, stratified by industry-specific experience and the age of initial long-term debt (LTD) financing. Red lines represent entrepreneurs starting businesses in industries where they possess prior experience, while blue lines denote those entering unfamiliar industries. Separate trajectories are shown for cohorts receiving their first LTD at ages 0, 2, 4, and 8. The visualization demonstrates that, conditional on securing financing at the same age, cross-industry entrants (blue) typically enter at a smaller initial scale but exhibit significantly steeper growth slopes, eventually surpassing their industry-experienced counterparts (red). The figures utilize an unbalanced panel of survivors, where the mean at each age is calculated across all firms in the respective cohort that remain active in the Business Structure Database (BSD).

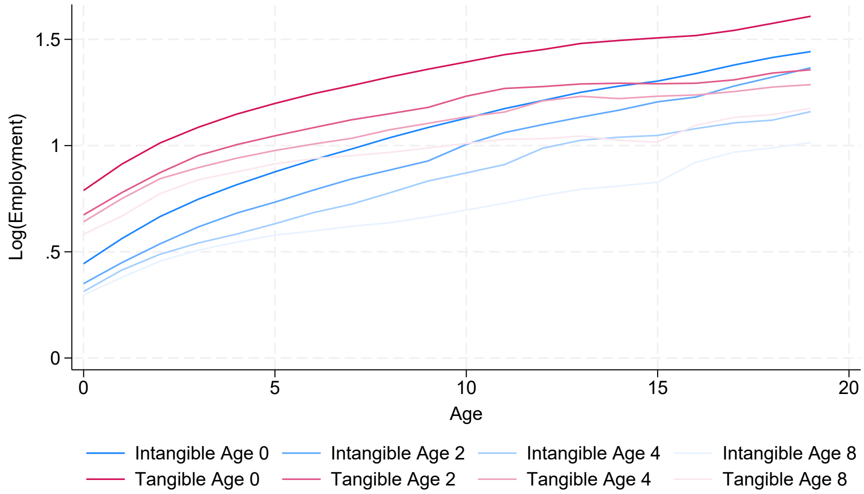


Figure A11: Log(employment) by first use of LT debt and industry tangibility

This figure plots mean log employment trajectories from Age 0 through Age 19, stratified by industry tangibility and the age of initial long-term debt (LTD) financing. Red lines represent high tangible industries, while blue lines denote low tangible industries. Separate trajectories are shown for cohorts receiving their first LTD at ages 0, 2, 4, and 8. The visualization demonstrates that, conditional on securing financing at the same age, intangible firms (blue) typically enter at a smaller initial scale but exhibit significantly steeper growth slopes than tangible firms (red). The figures utilize an unbalanced panel of survivors, where the mean at each age is calculated across all firms in the respective cohort that remain active in the Business Structure Database (BSD).

B Theory Appendix

B.1 Tractable Model with Existing Assets and Asset-Based Lending

There are two periods. Firms have two different productivities today z and tomorrow Z. Firms survive to the next period if they choose to continue to operate in the first period. Firms hire labor and produce with a Cobb-Douglas technology subject to an operating cost giving rise to the following flow profit function:

$$\pi(z) = \max_{l} z^{1-\alpha} l^{\alpha} - wl - f = \kappa z - f$$

Where the second equality comes from optimizing out labor and defining a constant $\kappa = \kappa(w) = (1-\alpha)(\alpha/w)^{\frac{\alpha}{1-\alpha}}$. We assume the steady-state Z follows a Power Law with exponent ζ and minimum $\underline{Z}$: $Z \sim G(Z) = 1 - (Z/\underline{Z})^{-\zeta}$. The transition from z to Z is governed by a Markov process $F(z, Z)$. Firms borrow and have dividends. Negative dividends are equity. Firms are born with a stock of debt b. If b is positive, they need to pay it back. b negative are assets firms receive. If the firm exits at any point they receive back their net asset positions. When a firm borrows it receives a face value $b'/(1+r)$ which they must pay back tomorrow as $-b'$.

$$d_0 = \pi(z) - b + \frac{b'}{1+r}, \quad d_1 = \pi(Z) - b'$$

Unconstrained decision. A firm chooses whether to survive or not in every period as well as how much to have as dividends or debt.

$$v(z, Z, b) = \max_{d, b'} \left\{ \pi(z) - b + \frac{b'}{1+r} + \frac{1}{1+r} \left(\mathbb{E}_{Z|z} \max\{\pi(Z), 0\} - b' \right), -b \right\}$$

In the case where a firm is not financially constrained, then since $\frac{b'}{1+r} - \frac{b'}{1+r} = 0$ we have that the split between d, b' is indeterminate (a version of Modigliani-Miller). Regardless, the survival choice is independent of b, and the value function:

$$v(z, Z, b) = \max_{d, b'} \left\{ \pi(z) + \frac{1}{1+r} \mathbb{E}_{Z|z} \pi(Z) \right\} - b = v(z, Z) - b$$

Lemma 1. Absent financial frictions, if Z is know to firms, firms will survive today if $(z|Z)$ is such that:

$$z^*(Z) = \begin{cases} \frac{f}{\kappa} & \text{if } Z < Z^* \equiv \frac{f}{\kappa} \text{ firms exits when young} \\ \frac{f}{\kappa} \cdot \frac{2+r}{1+r} - \frac{Z}{1+r} & \text{otherwise firm survives to old age} \end{cases}$$

Intuitively, if the firm knows itself to be a future superstar it will weather a stronger negative z today. Given this formulation, there are future superstar firms with productivity so high $Z \geq Z^* = \frac{f}{\kappa}(2+r)$ such that $0 = z^*(Z^*)$ and hence, will find it always profitable to survive regardless of their current draw.

Proof. See Appendix.

Asset-Based Constraint. Firms decide whether to exit today or not and whether to borrow to weather the shock given the following problem subject two a non-equity issuance constraint $d \geq 0$ and a borrowing constraint on b' based on the assets the firm can pledge as collateral.

$$v\left(z, Z, b\right) = \max_{d,b'} \left\{ d + \frac{1}{1+r} \mathbb{E}_{Z|z} v(Z, b'), -b \right\}$$

$$v(Z, b') = \max\left\{\pi(Z) - b', -b'\right\}$$

$$d = \pi(z) - b + \frac{b'}{1+r} \geq 0$$

$$b' \leq \chi b$$

Lemma 2. Given an asset based constraint, the survival threshold depends on b only:

$$z \geq z_b(b) \equiv \frac{f}{\kappa} + \bar{\Gamma}\frac{b}{\kappa} \equiv \frac{f}{\kappa\left(w\right)} + \left(1 - \frac{\chi_b}{1+r}\right)\frac{b}{\kappa\left(w\right)}$$

and thus, the asset-based constraint prevents the forward-looking selection on Z happening in the frictionless case.

Proof. See Appendix.

Current Cashflow-Based Constraint. If instead of an the last asset-based constraint above, firms can borrow against a multiple of current cash-flows in an earning based borrowing constraint, we will have:

$$b' \leq \chi_\pi \pi(z)$$

Lemma 3. The current earnings based constraint is essentially isomorphic to an asset based one. In essence, the exit threshold still depends on b and is independent on Z – whether the firm is going to be a superstar or not – and only depends on their current assets.

$$z \geq z_\pi(b) = \frac{f}{\kappa} + \bar{\Gamma}_\pi \frac{b}{\kappa}, \quad \text{s.t.} \quad \bar{\Gamma}_\pi = \left(\frac{1+r}{1+r+\chi_\pi}\right)$$

Proof. See Appendix.

Value-Based Constraint.

Alternatively, consider the case of a firm with a value based constraint where:

$$b' \leq \chi_v \mathbb{E}_{Z|z} v(Z, b')$$

We interpret χ_v – our parameter of interest – as both the ability to screen firms value or ability to lend to firms conditional after screening which may be impaired by financial technology or regulation frictions we leave unmodeled.

Under perfect information, the financier would be able to observe Z. With no information, the lender could try to infer Z from current z or infer this value from a population known to be Power Law.

As an illustrative case, when financiers have an imperfect process of observing future productivity that follows:

$$z'|z, Z = \begin{cases} z & \text{with probability } \rho \\ Z & \text{with probability } 1-\rho \end{cases}$$

Hence, the process for productivity has effective persistence ρ of past z with a drift related to the long-run Z: $\mathbb{E}\left(z'|z, Z\right) = (1-\rho) Z + \rho z$.

Proposition 1.

Under a perfect information value based constraint, we can drop the expectation because Z is known and the threshold depends on both b and Z.

$$z \geq z_v\left(b, Z\right) = \frac{b}{\kappa} + \frac{f}{\kappa} - \frac{1}{(1+r)} \frac{\chi_v}{1+\chi_v} \max\left\{Z - \frac{f}{\kappa}, 0\right\}$$

Hence, the value-based constraint reintroduces forward-looking selection because borrowing depends on future Z. Note that there is still a high enough value of $Z^* > \frac{f}{\kappa}$ such that $z_v\left(b, Z^*\right) = 0$ i.e. where very productive firms will always be able to remain in operation regardless of how bad their current conditions are, although in general higher than in the frictionless economy. Further, as $\chi_v \to \infty$ the slope of the threshold function with respect to Z approaches that of the frictionless economy.

Under no information about the future profitability of firms we have:

$$z \geq z_{\bar{v}}(b) = \bar{\Gamma}_v \frac{f}{\kappa} + \frac{b}{\kappa}, \quad \bar{\Gamma}_v = 1 + \frac{\chi_v}{1+\chi_v} \frac{1}{(1+r)} \frac{\left(\underline{Z}\right)^{\zeta} \left(\frac{f}{\kappa}\right)^{-\zeta}}{\zeta - 1} < 1$$

That is, we recover a version of the asset-based constraint with $\bar{\Gamma}_b = 1$ and a $\bar{\Gamma}_v$ that amplifies the effect of the fixed cost. In this extreme case, however, we would expect convergence to the same point like the case of the asset-based constraint. Then, the extent to which the degree of selection depends on long run Z could be informative about the lending technology.

Figure 4 and Figure 5 plots all the survival thresholds and survival probabilities for today's productivity as a function of future productivity. It highlights visually that Proposition 1 with Lemmas 1-4 together show the powerful result that a perfect information value-based constraint yields a cutoff that depends on forward-looking productivity Z type, like the frictionless cutoff, but crucially with different slopes. In contrast, backward asset based, current earnings based, and no-information value based constraints are all isomorphic in the sense that all do not depend on Z at all. If assets b are introduced, as we do in the Theory Appendix, the intercepts will also change

Proposition 2. Lenders have an imperfect technology to infer Z given data on z (we assume for tractability b cannot be used in the inference). Then $z \geq z_v^i(b, Z)$ where the threshold becomes a fixed point of the following equation.

$$z_v^i = \frac{b}{\kappa} + \left(1 + \frac{\chi_v}{1+\chi_v}\frac{\mathbb{P}^*\left(z_v^i, Z\right)}{1+r}\right)\frac{f}{\kappa} - \frac{\chi_v}{1+\chi_v}\frac{\mathbb{P}^*\left(z_v, Z\right)\mathbb{E}^*\left(z_v, Z\right)}{1+r}$$

Under the simplification with ρ, the threshold collapses to:

$$z_v(Z,b) = \begin{cases} \frac{f}{\kappa} + \left[1 + \rho\frac{1}{1+r}\frac{\chi_v}{1+\chi_v}\right]^{-1}\frac{b}{\kappa}, & \text{if } Z \leq \frac{f}{\kappa}, \\ \frac{f}{\kappa} + \left[1 + \rho\frac{1}{1+r}\frac{\chi_v}{1+\chi_v}\right]^{-1}\left[\frac{b}{\kappa} - \frac{1}{1+r}\frac{\chi_v}{1+\chi_v}(1-\rho)\left(Z - \frac{f}{\kappa}\right)\right], & \text{if } \frac{f}{\kappa} < Z < \frac{f}{\kappa} + \frac{\frac{b}{\kappa}}{\frac{1}{1+r}\frac{\chi_v}{1+\chi_v}(1-\rho)}, \\ \frac{f}{\kappa} + \frac{b}{\kappa} - \frac{1}{1+r}\frac{\chi_v}{1+\chi_v}(1-\rho)\left(Z - \frac{f}{\kappa}\right), & \text{if } Z \geq \frac{f}{\kappa} + \frac{\frac{b}{\kappa}}{1+r}\frac{\chi_v}{1+\chi_v}(1-\rho). \end{cases}$$

Proof. See Appendix.

The intuition for Proposition 1 is simple but powerful. Under the value-based constraint, future Z can enter the valuation today and affect firms' borrowing capacity. When financiers can only imperfectly observe Z, we have two parameters to shock: ρ or how difficult is it to observe Z and χ_v even after making an inference about the valuation, how easy is it to lend to lend to seemingly valuable firms. There are three key regions: (1) when $Z \leq f/\kappa$, there's no possible future since the firm will not continue. z_v is flat in Z, (b) when $Z > f/\kappa$, the promise of future cashflows help the firm borrow today. z_v falls with Z, (c) eventually, $z_v \leq f/\kappa$. This means that the firm will survive even if their current cashflows are negative.

When $\rho \to 0$ we get the perfect information case. When $\rho \to 1$ we recover a version of the earnings-based constraint is recovered. For ρ in between, the relationship between the threshold and z weakens with information frictions, but remains present in the model to the extent that lenders are able to estimate steady-state Z with some degree of accuracy. Figure 4 makes this point showing that as information becomes more imperfect, the value-based constraint converges to the earnings based constraint. But as the quality of information financiers are able to extract on future Z improves, the threshold schedule resembles more and more the shape of the frictionless environment.

B.2 Proof of Lemma 1

Proof. See Appendix.

$$\kappa z - f + \frac{1}{1+r}\mathbb{E}_{Z|z}\max\left\{\kappa Z - f, 0\right\} \geq 0$$

Only productive enough firms will survive tomorrow if

$$Z > Z^* \equiv \frac{f}{\kappa}$$

$$\mathbb{E}_{Z|z}\max\left\{\kappa Z - f, 0\right\} = \Pr\left(Z \geq Z^*\right)\mathbb{E}_{Z|Z\geq Z^*,z}\left(\kappa Z - f\right)$$

$$\kappa z - f\left(1+\frac{\Pr\left(Z\geq Z^{*}\right)}{1+r}\right)+\frac{Pr\left(Z\geq Z^{*}\right)}{1+r}\mathbb{E}_{Z|Z\geq Z^{*},z}\kappa\left(Z\right)\geq 0$$

$$z+\frac{Pr\left(Z\geq Z^{*}\right)}{1+r}\mathbb{E}_{Z|Z\geq Z^{*},z}\left(Z\right)\geq\frac{f}{\kappa}\left(1+\frac{\Pr\left(Z\geq Z^{*}\right)}{1+r}\right)$$

Hence their threshold

$$z\geq z^{*}\left(z,Z\right)=\frac{f}{\kappa}\left(1+\frac{Pr\left(Z\geq Z^{*}\right)}{1+r}\right)-\frac{Pr\left(Z\geq Z^{*}\right)}{1+r}\mathbb{E}_{Z|Z\geq\frac{f}{\kappa},z}Z$$

Thus, future superstars Z will find it easier to survive today (will have a lower threshold $z^{*}\left(z,Z\right)$).

If Z is unknown to the firm we have that $Z\sim G(Z)$ is Pareto, this simplifies to:

$$\Pr\left(Z\geq Z^{*}\right)=\left(\frac{\underline{Z}}{Z^{*}}\right)^{\zeta}=\left(\frac{\kappa\underline{Z}}{f}\right)^{\zeta}$$

$$\mathbb{E}\left[Z\mid Z\geq Z^{*}\right]=\frac{\zeta}{\zeta-1}Z^{*}=\frac{\zeta}{\zeta-1}\frac{f}{\kappa}$$

$$\begin{aligned}
z^{*}(z,Z)&=\frac{f}{\kappa}\left(1+\frac{\Pr\left(Z\geq Z^{*}\right)}{1+r}\right)-\frac{\Pr\left(Z\geq Z^{*}\right)}{1+r}\cdot\frac{\zeta}{\zeta-1}\frac{f}{\kappa}\\
&=\frac{f}{\kappa}\left[1+\frac{\Pr\left(Z\geq Z^{*}\right)}{1+r}\left(1-\frac{\zeta}{\zeta-1}\right)\right]\\
&=\frac{f}{\kappa}\left[1-\frac{\Pr\left(Z\geq Z^{*}\right)}{(\zeta-1)(1+r)}\right]\\
&=\frac{f}{\kappa}\left[1-\frac{\left(\frac{\kappa\underline{Z}}{f}\right)^{\zeta}}{(\zeta-1)(1+r)}\right]
\end{aligned}$$

However, if Z is known to the firm,

$$\mathbb{E}\left[Z\mid Z\geq Z^{*},z\right]=Z$$

$$\Pr\left(Z\geq Z^{*}\right)=\mathbf{1}\left[Z\geq Z^{*}\right]$$

Hence, the threshold becomes Z dependent

$$z^{*}(Z)=\frac{f}{\kappa}\left(1+\frac{\mathbf{1}\left[Z\geq Z^{*}\right]}{1+r}\right)-\frac{\mathbf{1}\left[Z\geq Z^{*}\right]}{1+r}Z$$

Thus, we have that:

$$z^{*}\left(Z\right)=\begin{cases}\frac{f}{\kappa(w)} & \text{if } Z<Z^{*}\\ \frac{f}{\kappa(w)}\cdot\frac{2+r}{1+r}-\frac{Z}{1+r} & \text{if } Z\geq Z^{*}\end{cases}$$

Q.E.D.

B.3 Proof of Lemma 2

Proof of Lemma 2. Plug in:

$$v\left(z,Z,b\right)=\max_{d,b'}\left\{\pi(z)-b+\frac{b'}{1+r}+\frac{1}{1+r}\mathbb{E}_{Z|z}\left[\max\left\{\pi(Z)-b',0\right\}\right],-b\right\}$$

If $d>0$ is optimal because the firm has very high z, the firm is not financially constrained.

If $d=0$ then, firms that have negative cashflows will borrow more even without existing debt.

$$b'=\left(1+r\right)\left(b-\pi(z)\right)$$

Firms will be able to borrow up to:

$$\left(1+r\right)\left(b-\pi(z)\right)\leq\chi b$$

Then, with the borrowing constraint, the following firms will not be able to survive and will exit.

The survival threshold of future superstars Z is independent of Z and crucially now depends on b.

$$\begin{gathered}(1+r)(b-(\kappa z-f))\leq\chi b\\ (1+r)b-(1+r)\kappa z+(1+r)f\leq\chi b\\ (1+r-\chi)b+(1+r)f\leq(1+r)\kappa z\\ z\geq\frac{f}{\kappa}+\left(1-\frac{\chi}{1+r}\right)\frac{b}{\kappa}\end{gathered}$$

$$z\geq z_b\left(b\right)=\frac{f}{\kappa}+\left(1-\bar{\Gamma}_b\right)\frac{b}{\kappa},\quad\text{s.t.}\quad\bar{\mathit{\Gamma}}_b=\left(1-\frac{\chi_b}{1+r}\right)$$

The main insight is that the asset-based constraint kills forward-looking selection i.e. regardless of future productivity Z.

B.4 Proof of Lemma 3

Proof of Lemma 3.

$$v\left(z,Z,b\right)=\max_{d,b'}\left\{\pi(z)-b+\frac{b'}{1+r}+\frac{1}{1+r}\mathbb{E}_{Z|z}v(Z,b'),-b'\right\}$$

If dividends bind, the earnings based constraint is slack (and the firm survives) as long as:

$$\left(1+r\right)\left(b-\pi(z)\right)=b'\leq\chi_\pi\pi(z)$$

Hence,

$$b \leq \frac{1 + r + \chi_\pi}{(1 + r)} \pi(z)$$

Thus, an earnings based constraint is essentially isomorphic to an asset based one with a different constant.

$$z \geq z_\pi(b) = \frac{f}{\kappa} + \left(\frac{1 + r}{1 + r + \chi_\pi} \right) \frac{b}{\kappa}$$

$$z \geq z_\pi(b) = \frac{f}{\kappa} + \bar{\Gamma}_\pi \frac{b}{\kappa}, \quad \text{s.t.} \quad \bar{\Gamma}_\pi = \left(\frac{1 + r}{1 + r + \chi_\pi} \right)$$

QED.

B.5 Proof of Proposition 1 and Corollary 1

Proof of Proposition 1 & Corollary 1.

Perfect information.

$$\mathbb{E}_{Z|z} v(Z, b') = \max \{\pi(Z), 0\} - b'$$

$$b' \leq \chi_v [\max \{\pi(Z), 0\} - b']$$

$$b' \leq \frac{\chi_v}{1 + \chi_v} \max \{\kappa Z - f, 0\} = \max \left\{ \frac{\chi_v}{1 + \chi_v} (\kappa Z - f), 0 \right\}$$

Whenever $d = 0$, the constraint will be slack if

$$b' = (1 + r)(b - \pi(z)) = (1 + r)(b + f - \kappa z) \leq \frac{\chi_v}{1 + \chi_v} \max \{\kappa Z - f, 0\}$$

Or

$$z \geq z_v(b, Z) = \frac{b}{\kappa} + \frac{f}{\kappa} - \frac{1}{(1 + r)} \frac{\chi_v}{1 + \chi_v} \max \left\{ Z - \frac{f}{\kappa}, 0 \right\}$$

Imperfect information.

In the general case when Z is not perfectly known, we have:

$$b' \leq \chi_v \mathbb{E}_{Z|z} \max \{\pi(Z) - b', -b'\}$$

Regardless, if the firm has $d = 0$, then again:

$$b' = (1 + r)(b - \pi(z))$$

if on top of that

$$b' \leq \frac{\chi_v}{1 + \chi_v} \mathbb{E}_{Z|z} \max \{\pi(Z), 0\}$$

Thus,

$$b' = (1+r)\left(b - \pi(z)\right) \leq \frac{\chi_v}{1+\chi_v}\mathbb{E}_{Z|z}\max\left\{\pi(Z), 0\right\}$$

$$b' = (1+r)\left(b - \kappa z\right) \leq \frac{\chi_v}{1+\chi_v}\mathbb{E}_{Z|z}\max\left\{\pi(Z), 0\right\}$$

$$z \geq z_v(z, Z, b) = \frac{b}{\kappa} + \frac{f}{\kappa} - \frac{\chi_v}{1+\chi_v}\frac{1}{(1+r)}\frac{\mathbb{E}_{Z|z}\max\left\{\pi(Z), 0\right\}}{\kappa}$$

Inherently the term is the expected value of a call option with uncertain future price Z and strike price $\frac{f}{\kappa}$.

$$\frac{\mathbb{E}_{Z|z}\max\left\{\pi(Z), 0\right\}}{\kappa} = \mathbb{E}_{Z|z}\max\left\{Z - \frac{f}{\kappa}, 0\right\}$$

Note that as before we have $Z^* = f/\kappa(w)$ and

$$\Pr\left(Z \geq Z^*\right) = \left(\frac{\underline{Z}}{Z^*}\right)^{\zeta} = \left(\frac{\kappa\underline{Z}}{f}\right)^{\zeta}$$

$$\mathbb{E}\left[Z \mid Z \geq Z^*\right] = \frac{\zeta}{\zeta-1}Z^* = \frac{\zeta}{\zeta-1}\frac{f}{\kappa}$$

$$Pr\left(Z \geq Z^*\right)\mathbb{E}\left[Z \mid Z \geq Z^*\right] = \frac{\zeta}{\zeta-1}\left(\underline{Z}\right)^{\zeta}\left(Z^*\right)^{1-\zeta}$$

$$\mathbb{E}_{Z|z}\max\left\{\pi(Z), 0\right\} = \kappa\mathbb{P}\left[Z \geq Z^*\right]\mathbb{E}\left[Z \mid Z \geq Z^*\right] - \mathbb{P}\left[Z \geq Z^*\right]f$$

$$z \geq z_v(z, Z, b) = \frac{b}{\kappa} + \frac{f}{\kappa} + \frac{\chi_v}{1+\chi_v}\frac{\mathbb{P}\left[Z \geq Z^*\right]f}{(1+r)}\frac{f}{\kappa} - \frac{\chi_v}{1+\chi_v}\frac{Pr\left(Z \geq Z^*\right)\mathbb{E}\left[Z \mid Z \geq Z^*\right]}{(1+r)}$$

$$z \geq z_v(z, b) = \frac{b}{\kappa} + \frac{f}{\kappa} + \frac{\chi_v}{1+\chi_v}\frac{\left(\underline{Z}\right)^{\zeta}\left(\frac{f}{\kappa}\right)^{1-\zeta}}{(1+r)} - \frac{\chi_v}{1+\chi_v}\frac{\zeta}{\zeta-1}\frac{\left(\underline{Z}\right)^{\zeta}\left(\frac{f}{\kappa}\right)^{1-\zeta}}{(1+r)}$$

$$z \geq z_v(b) = \frac{b}{\kappa} + \left(1 + \frac{\chi_v}{1+\chi_v}\frac{1}{(1+r)}\frac{\left(\underline{Z}\right)^{\zeta}\left(\frac{f}{\kappa}\right)^{-\zeta}}{\zeta-1}\right)\frac{f}{\kappa}$$

$$z \geq z_v(b) = \frac{b}{\kappa} + \bar{\Gamma}_v\frac{f}{\kappa}$$

$$\bar{\Gamma}_v = \left(1 + \frac{\chi_v}{1+\chi_v}\frac{1}{(1+r)}\frac{\left(\underline{Z}\right)^{\zeta}\left(\frac{f}{\kappa}\right)^{-\zeta}}{\zeta-1}\right)$$

Consider an imperfect signal for a firms' Z.

$$b' \leq \frac{\chi_v}{1+\chi_v}\mathbb{E}^*_{Z|z}\max\left\{\pi(Z), 0\right\}$$

$$b' \leq \frac{\chi_v}{1+\chi_v}\left(\mathbb{P}^*_{Z|z}\kappa\mathbb{E}_{Z|z}Z - \mathbb{P}^*_{Z|z}f\right)$$

$$b' \leq \frac{\chi_v}{1+\chi_v}\left(\mathbb{P}^*\left(z,Z,b\right)\kappa\mathbb{E}^*\left(z,Z,b\right) - \mathbb{P}^*\left(z,Z,b\right)f\right)$$

Hence

$$b' = \left(1+r\right)\left(b+f-\kappa z\right) \leq \frac{\chi_v}{1+\chi_v}\left(\mathbb{P}^*\left(z,Z,b\right)\kappa\mathbb{E}^*\left(z,Z,b\right) - \mathbb{P}^*\left(z,Z,b\right)f\right)$$

Thus, as long as $z \geq z_v^i(z,Z,b)$ we have:

$$z \geq \frac{b}{\kappa} + \frac{f}{\kappa} + \frac{\chi_v}{1+\chi_v}\frac{\mathbb{P}^*\left(z,Z,b\right)}{\left(1+r\right)}\frac{f}{\kappa} - \frac{\chi_v}{1+\chi_v}\frac{1}{\left(1+r\right)}\mathbb{P}^*\left(z,Z,b\right)\mathbb{E}^*\left(z,Z,b\right)$$

where z_v is implicitly defined by:

$$z_v = \frac{b}{\kappa} + \left(1 + \frac{\chi_v}{1+\chi_v}\frac{\mathbb{P}^*\left(z_v,Z,b\right)}{1+r}\right)\frac{f}{\kappa} - \frac{\chi_v}{1+\chi_v}\frac{\mathbb{P}^*\left(z_v,Z,b\right)\mathbb{E}^*\left(z_v,Z,b\right)}{1+r}$$

Tractable example.

With probability $1-\rho$ the lender is right and they get Z. With probability ρ today was the right one z.

$$z'|z,Z = \begin{cases} z & \text{with probability } \rho \\ Z & \text{with probability } 1-\rho \end{cases}$$

Hence, the expected productivity of a firms is like an AR(1) Markov process that has persistence ρ of past z with a drift related to the long-run Z.

$$\mathbb{E}\left(z'|z,Z\right) = \left(1-\rho\right)Z + \rho z$$

Intuitively, ρ captures the degree at which lenders (and the firm) look at today vs tomorrow when trying to infer where the firm is going to be in their steady state.

Hence, the firm's problem:

$$v\left(z,Z,b\right) = \max_{d,b'}\left\{d + \frac{1}{1+r}\mathbb{E}_{z'|z,Z}\max\left\{\pi(z') - b', -b'\right\}, -b\right\}$$

$$d = \pi(z) - b + \frac{b'}{1+r} \geq 0$$

$$b' \leq \chi_v\mathbb{E}_{z'|z,Z}\max\left\{\pi(z') - b', -b'\right\}$$

$$b' \leq \frac{\chi_v}{1+\chi_v}\mathbb{E}_{z'|z,Z}\max\left\{\pi(z'), 0\right\}$$

Thus, the firms' value function:

$$v\left(z,Z,b\right)=\max_{d,b'}\left\{d+\frac{1}{1+r}\left[\rho\max\left\{\pi(z),0\right\}+(1-\rho)\max\left\{\pi(Z),0\right\}\right]-\frac{b'}{1+r},-b\right\}$$

Given the uncertainty about whether the firms signal about Z is the right one or whether they're going to remain at z, we have:

$$b'\leq\frac{1}{1+\frac{1}{\chi_v}}\left[\rho\max\left\{\pi(z),0\right\}+(1-\rho)\max\left\{\pi(Z),0\right\}\right]$$
$$b'\leq\frac{1}{1+\frac{1}{\chi_v}}\left[\rho\max\left\{\kappa z-f,0\right\}+(1-\rho)\max\left\{\kappa Z-f,0\right\}\right]$$

Hence

$$b'=(1+r)\left(b+f-\kappa z\right)\leq\frac{1}{1+\frac{1}{\chi_v}}\left[\rho\max\left\{\kappa z-f,0\right\}+(1-\rho)\max\left\{\kappa Z-f,0\right\}\right]$$

Thus, as long as $z\geq z_v^i(z,Z,b)$ we have:

$$\left(b+f-\kappa z\right)\leq\frac{1}{1+r}\frac{1}{1+\frac{1}{\chi_v}}\left[\rho\max\left\{\kappa z-f,0\right\}+(1-\rho)\max\left\{\kappa Z-f,0\right\}\right]$$

$$\kappa z-f\geq b-\frac{1}{1+r}\frac{1}{1+\frac{1}{\chi_v}}\left[\rho\max\left\{\kappa z-f,0\right\}+(1-\rho)\max\left\{\kappa Z-f,0\right\}\right]$$

$$z\geq\frac{b}{\kappa}+\frac{f}{\kappa}-\frac{1}{(1+r)}\frac{1}{1+\frac{1}{\chi_v}}\left[\rho\max\left\{z-\frac{f}{\kappa},0\right\}+(1-\rho)\max\left\{Z-\frac{f}{\kappa},0\right\}\right]$$

Thus, at the threshold of current productivity $z=z_v=z_v\left(b,Z\right)$ for which the firm finds it profitable to survive is:

$$z_v=\frac{f}{\kappa}+\frac{b}{\kappa}-\frac{1}{(1+r)}\frac{1}{1+\frac{1}{\chi_v}}\left[\rho\max\left\{z_v-\frac{f}{\kappa},0\right\}+(1-\rho)\max\left\{Z-\frac{f}{\kappa},0\right\}\right]$$

Note that $z_v-\frac{f}{\kappa}>0$ ("static" active region)

$$\kappa z-f\geq b-\frac{1}{1+r}\frac{1}{1+\frac{1}{\chi_v}}\left[\rho\left(\kappa z-f\right)+(1-\rho)\max\left\{\kappa Z-f,0\right\}\right]$$
$$\left(\kappa z-f\right)\left(1+\rho\frac{1}{1+r}\frac{1}{1+\frac{1}{\chi_v}}\right)\geq b-\frac{1}{1+r}\frac{(1-\rho)}{1+\frac{1}{\chi_v}}\left[\max\left\{\kappa Z-f,0\right\}\right]$$

$$z \geq \frac{f}{\kappa} + \frac{1}{\left(1+\rho\frac{1}{1+r}\frac{1}{1+\frac{1}{\chi_v}}\right)}\frac{b}{\kappa} - \frac{(1-\rho)\frac{1}{1+r}\frac{1}{1+\frac{1}{\chi_v}}}{\left(1+\rho\frac{1}{1+r}\frac{1}{1+\frac{1}{\chi_v}}\right)}\left[\max\left\{Z-\frac{f}{\kappa},0\right\}\right]$$

$$\frac{b}{\kappa} \geq \frac{1}{(1+r)}\frac{1}{1+\frac{1}{\chi_v}}\left[\rho\max\left\{z_v-\frac{f}{\kappa},0\right\} + (1-\rho)\max\left\{Z-\frac{f}{\kappa},0\right\}\right]$$

This happens as long as b is such that the firms' assets are such that:

$$(1+r)\,b \geq \frac{\chi_v}{1+\chi_v}\left[\rho\max\{\kappa z_v - f, 0\} + (1-\rho)\max\{\kappa Z - f, 0\}\right]$$

Hence, in this case

$$z_v - \frac{f}{\kappa} = \frac{b}{\kappa} - \frac{1}{(1+r)}\frac{1}{1+\frac{1}{\chi_v}}\left[\rho\left(z_v-\frac{f}{\kappa}\right) + (1-\rho)\max\left\{Z-\frac{f}{\kappa},0\right\}\right]$$

$$\left(z_v-\frac{f}{\kappa}\right)\left[1+\rho\frac{1}{(1+r)}\frac{1}{1+\frac{1}{\chi_v}}\right] = \frac{b}{\kappa} - \frac{1}{(1+r)}\frac{(1-\rho)}{1+\frac{1}{\chi_v}}\left[\max\left\{Z-\frac{f}{\kappa},0\right\}\right]$$

$$z_v = z_v(b,Z) = \frac{f}{\kappa} + \left[1+\rho\frac{1}{(1+r)}\frac{1}{1+\frac{1}{\chi_v}}\right]^{-1}\left[\frac{b}{\kappa} - \frac{1}{(1+r)}\frac{(1-\rho)}{1+\frac{1}{\chi_v}}\left[\max\left\{Z-\frac{f}{\kappa},0\right\}\right]\right]$$

$$z \geq \frac{f}{\kappa} + \frac{1}{\left(1+\rho\frac{1}{1+r}\frac{1}{1+\frac{1}{\chi_v}}\right)}\frac{b}{\kappa} - \frac{(1-\rho)\frac{1}{1+r}\frac{1}{1+\frac{1}{\chi_v}}}{\left(1+\rho\frac{1}{1+r}\frac{1}{1+\frac{1}{\chi_v}}\right)}\left[\max\left\{Z-\frac{f}{\kappa},0\right\}\right]$$

Otherwise, we have that if at the threshold: $z_v - \frac{f}{\kappa} \leq 0$ then

$$z_v = \frac{f}{\kappa} + \frac{b}{\kappa} - \frac{1}{(1+r)}\frac{(1-\rho)}{1+\frac{1}{\chi_v}}\max\left\{Z-\frac{f}{\kappa},0\right\}$$

Hence, to summarize, we have that, the general threshold rule simplifies to:

$$z_v = \frac{f}{\kappa} + \left[1+\rho\frac{1}{(1+r)}\frac{\chi_v}{1+\chi_v}\right]^{-\mathbf{1}\left\{z_v-\frac{f}{\kappa}\geq 0\right\}}\left[\frac{b}{\kappa} - \frac{(1-\rho)}{(1+r)}\frac{\chi_v}{1+\chi_v}\max\left\{Z-\frac{f}{\kappa},0\right\}\right]$$

When $\rho \to 0$ we get the perfect information case:

$$z \geq z_v(b,Z) = \frac{b}{\kappa} + \frac{f}{\kappa} - \frac{1}{(1+r)}\frac{\chi_v}{1+\chi_v}\max\left\{Z-\frac{f}{\kappa},0\right\}$$

While with $\rho \to 1$ we get that $z = Z$ constant and we can drop the option value altogether and we recover:

$$z_v = \frac{f}{\kappa} + \left(1 + \left[1 + \frac{1}{(1+r)} \frac{\chi_v}{1+\chi_v}\right]^{-\mathbf{1}\left\{z_v - \frac{f}{\kappa} \geq 0\right\}}\right) \frac{b}{\kappa}$$

Note that this case is one where the earnings-based constraint is recovered, i.e.

$$(1+r)(b + f - \kappa z) = b' \leq \frac{1}{1+\frac{1}{\chi_v}} [\max\{\kappa z - f, 0\}]$$

In summary, there are three key regions:

- When $Z \leq f/\kappa$, there's no possible future since the firm will not continue. z_v is flat in Z.
- When $Z > f/\kappa$, the promise of future cashflows help the firm borrow today. z_v falls with Z.
- Eventually, $z_v \leq f/\kappa$. This means that the firm will survive even if their current cashflows are negative.

$$z_v(Z,b) = \begin{cases} \frac{f}{\kappa} + \left[1 + \rho \frac{1}{1+r} \frac{\chi_v}{1+\chi_v}\right]^{-1} \frac{b}{\kappa}, & \text{if } Z \leq \frac{f}{\kappa}, \\ \frac{f}{\kappa} + \left[1 + \rho \frac{1}{1+r} \frac{\chi_v}{1+\chi_v}\right]^{-1} \left[\frac{b}{\kappa} - \frac{1}{1+r} \frac{\chi_v}{1+\chi_v} (1-\rho)\left(Z - \frac{f}{\kappa}\right)\right], & \text{if } \frac{f}{\kappa} < Z < \frac{f}{\kappa} + \frac{\frac{b}{\kappa}}{\frac{1}{1+r}\frac{\chi_v}{1+\chi_v}(1-\rho)}, \\ \frac{f}{\kappa} + \frac{b}{\kappa} - \frac{1}{1+r} \frac{\chi_v}{1+\chi_v} (1-\rho)\left(Z - \frac{f}{\kappa}\right), & \text{if } Z \geq \frac{f}{\kappa} + \frac{\frac{b}{\kappa}}{1+r} \frac{\chi_v}{1+\chi_v}(1-\rho). \end{cases}$$

The switch occurs when

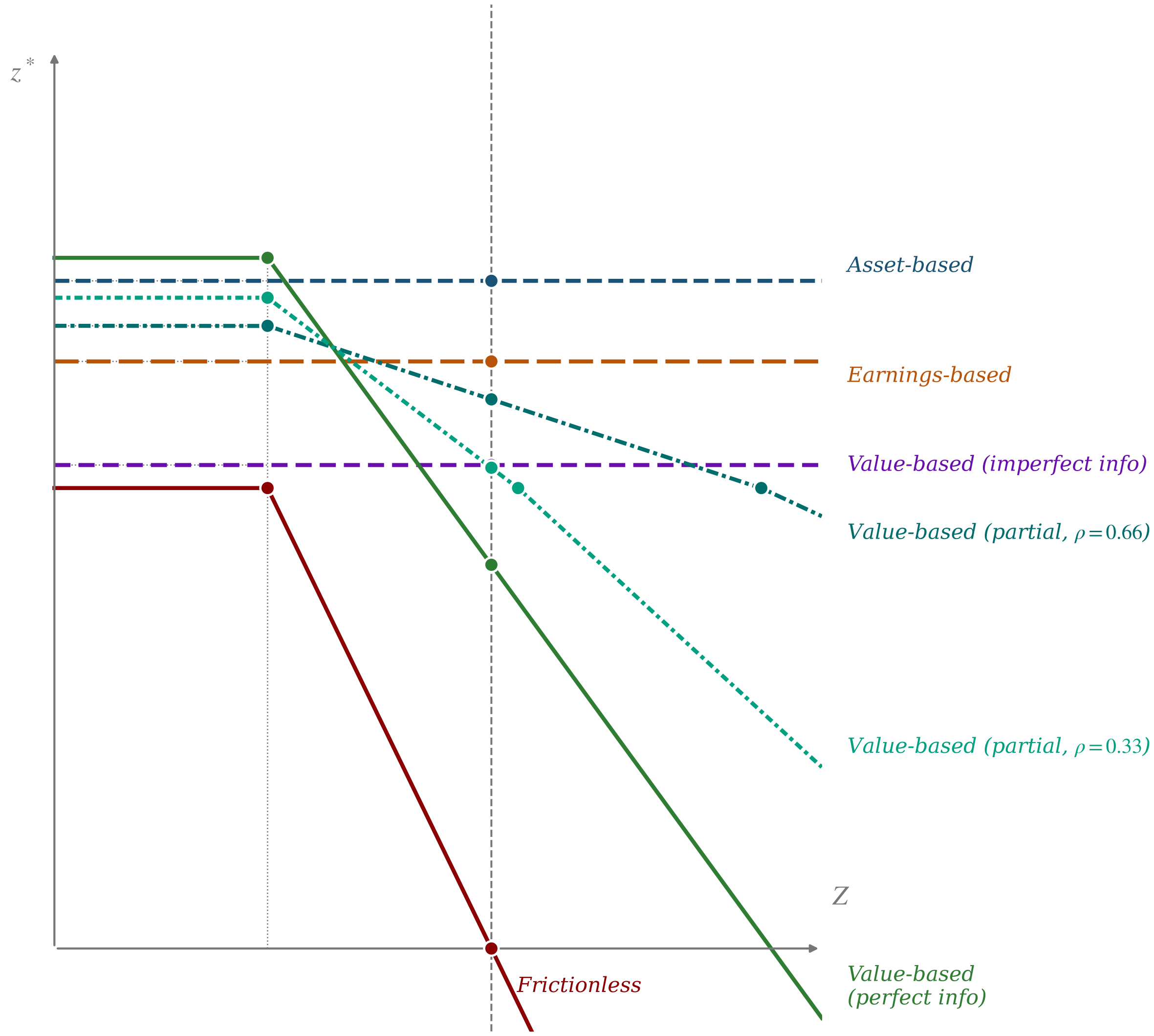
$$z_v = \frac{f}{\kappa}$$

That happens when:

$$Z = \frac{f}{\kappa} + \frac{\frac{b}{\kappa}}{\frac{1}{1+r}\frac{\chi_v}{1+\chi_v}(1-\rho)}$$

Thus, for firms that will be bad, $\partial z_v / \partial Z = 0$. For more "regular firms" $\partial z_v / \partial Z = -\frac{1}{1+r}\frac{\chi_v}{1+\chi_v}(1-\rho) / \left(1 + \rho \frac{1}{1+r}\frac{\chi_v}{1+\chi_v}\right)$. While for future "superstars" $\partial z_v / \partial Z = -\frac{1}{1+r}\frac{\chi_v}{1+\chi_v}(1-\rho)$. This means that if clearly the firm will be very good, the slope becomes steeper like in the frictionless world. However, note that the slope will still be less steep that in the perfect information case due to $(1-\rho)$.

Figure A12: Types and survival under different forms of lending



Today's survival threshold tracks future steady-state productivity under valued-based lending but not under asset-based lending.

Figure B1: TFP and financial frictions: no collateral limit

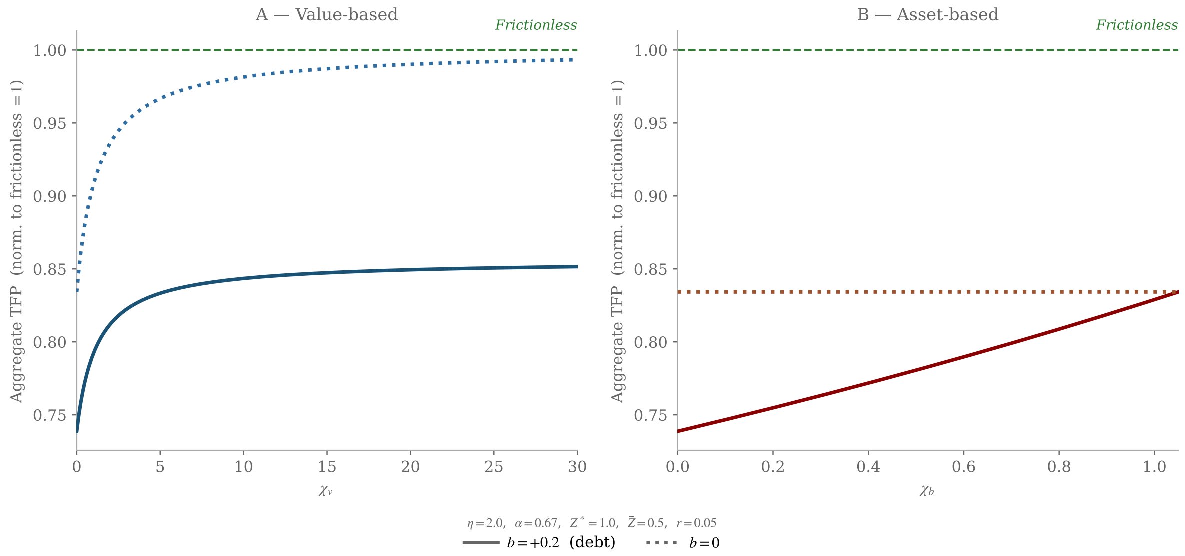


Figure B2: Survival impact of changes in frictions χ and ρ under value-based financing

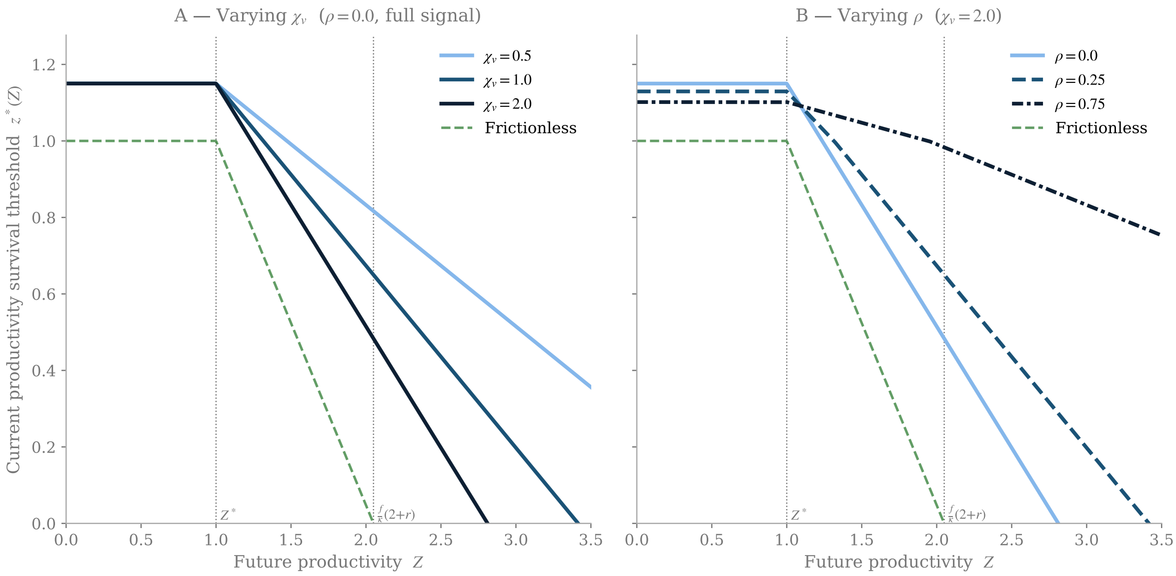


Figure B3: TFP impact of changes in frictions χ and ρ under value-based financing

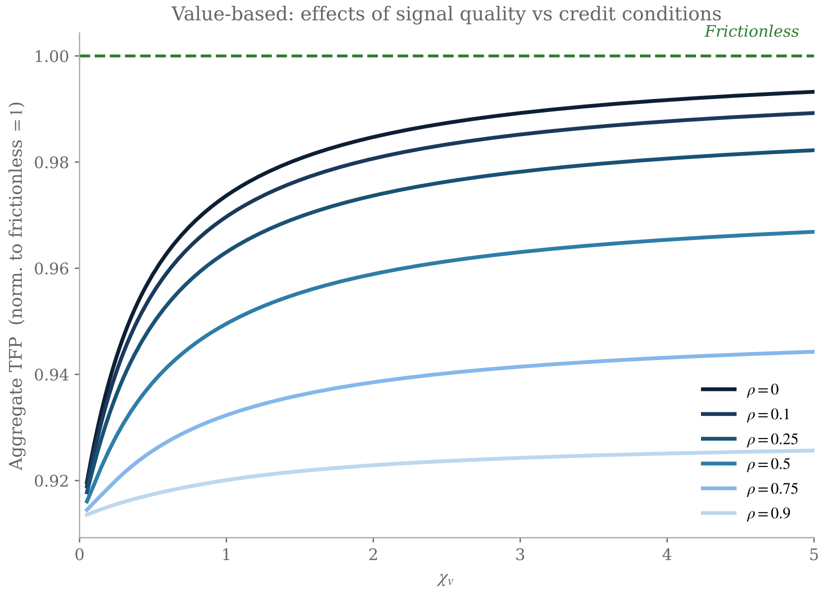


Figure B4: Decline in value based χ and TFP

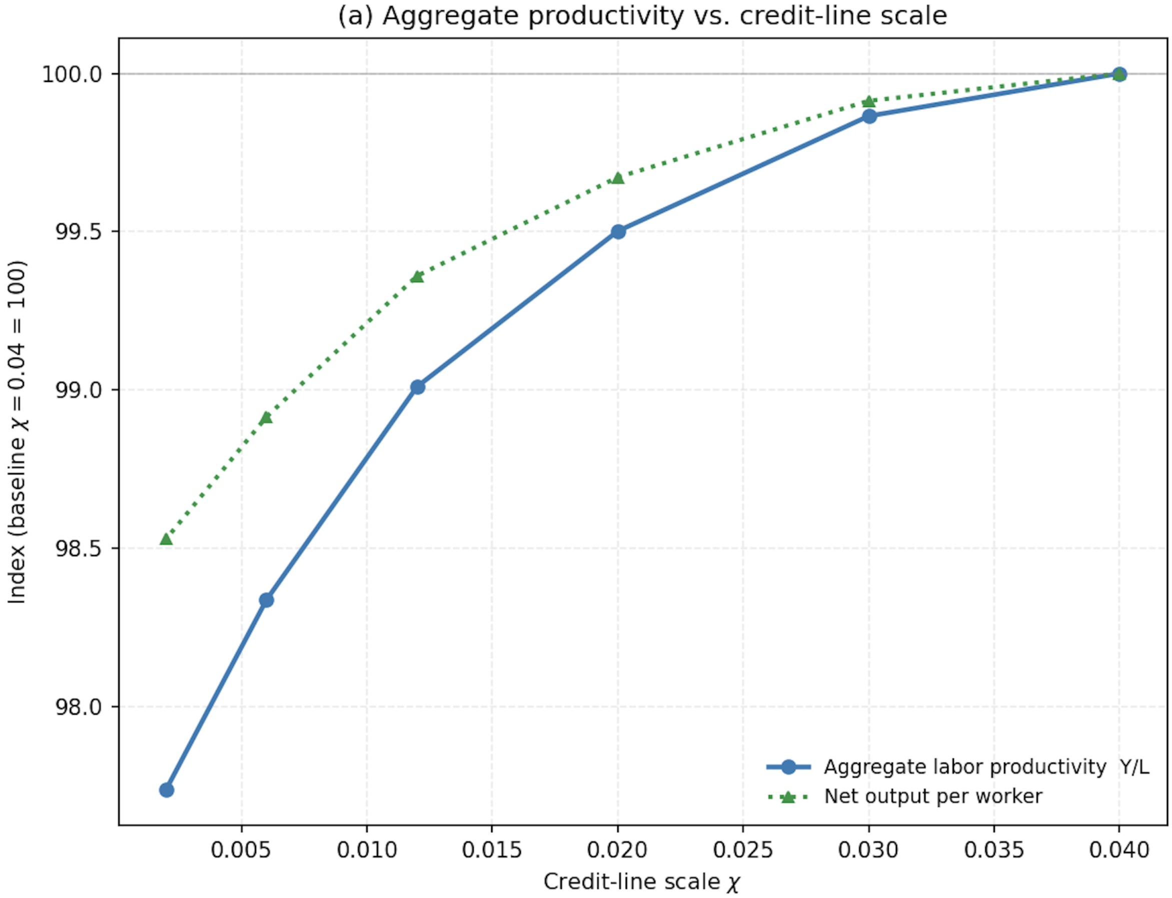


Figure B5: Decline in value based χ and Mass of Firms