Estimating the Impact of Loan Supply Shocks *

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June 2026

Abstract

How much do bank loan supply shocks reduce credit and real activity? Measuring the impact of such shocks requires separating shifts in loan supply from movements in credit demand. We revisit commonly used empirical approaches and show that they do not recover the impact of loan supply shocks on lending within bank-firm relationships, on firms' total borrowing, or on real outcomes. We develop a theoretical framework that decomposes lending responses into a scale effect, which governs total firm borrowing, and a substitution effect, which governs reallocation across lenders. We use the scale-substitution structure to construct estimators of these effects and apply them to the 2011 Spanish debt crisis. The methodology implies substantially smaller effects of loan supply shocks than those generated by standard empirical approaches.

1. Introduction

Disruptions to the financial sector are widely recognized as key drivers of macroeconomic fluctuations, shaping business cycles, amplifying crises, and mediating the transmission of monetary policy (see, e.g., Gertler and Gilchrist (1994); Bernanke and Gertler (1995); Kashyap and Stein (2000); Peek and Rosengren (2000)).

*We thank Dominic Cucic, Nir Jaimovich, Asim Khwaja, David Matsa, Atif Mian, Ramana Nanda, Tarun Ramadorai, Antoinette Schoar, and Jeremy Stein for useful discussions and comments. We thank Kemal Emre Macar for outstanding research assistance. This work has been supported by the Spanish Ministry of Science and Innovation "PID2023-149802NB-I00" through "MCIN/AEI/10.13039/501100011033".

Understanding the effects of loan supply shocks – both on credit allocation and on real economic outcomes – is therefore a central question in economics. At the heart of this question lies a core empirical challenge: disentangling shifts in credit supply from movements in credit demand. In this paper, we revisit this central question. Using a simple theoretical framework, we show that commonly used empirical approaches do not recover the impact of a credit supply shock on the level of lending within a bank-firm relationship, on total firm-level borrowing, or on real outcomes. We then propose an alternative methodology to do so.

A loan supply shock raises two related but distinct questions. The first is the loan-level response: whether, and by how much, the shock reduces lending within an affected bank-firm relationship. This response is informative about shock transmission, bank behavior, and credit reallocation. The second is the firm-level response: how much the firm's total borrowing changes across *all* of its lenders. This is the central object for understanding effects on investment, employment, and output. It is well known that the two responses generally differ because firms borrowing from multiple banks can substitute across lenders.

To analyze these questions, we begin by developing a micro-founded framework with standard ingredients: firms borrow from multiple banks to finance investments, banks face different supply shocks, and firms can substitute borrowing across banks. The framework decomposes the effect of a loan supply shock on lending into two components: a scale effect and a substitution effect. The scale effect captures how a change in the firm's average cost of external finance affects its total borrowing. The substitution effect reflects how a firm reallocates its borrowing across lenders in response to relative changes in banks' financing conditions.[1] We use the framework to show that the effect of a loan supply shock on total firm borrowing is given by the scale effect, whereas the impact of supply shocks on loan-level lending to the firm combines both the scale and substitution effects.

We then analyze two influential strands of the empirical literature on the impact of loan supply shocks: one building on the within-firm comparisons introduced by Khwaja and Mian (2008) (henceforth KM), and another using bank and firm fixed effects following Amiti and Weinstein (2018) (henceforth AW).[2] The key innovation in KM was to tackle the correlation between firm demand and bank supply shocks by

[1] This decomposition is similar in spirit to the Slutsky decomposition in consumer theory, where a price change affects demand both by changing relative prices, inducing substitution across goods, and by changing real purchasing power, affecting the scale of demand.

[2] See, for example, Iyer et al. (2014), Chodorow-Reich (2014), Jiménez et al. (2017), and Greenwald, Krainer and Paul (forthcoming) for KM applications, and Amiti, McGuire and Weinstein (2017), and Alfaro, García-Santana and Moral-Benito (2021) for applications of regressions using both bank and firm fixed effects to estimate the impact of total loan supply shocks. There are, of course, studies which use other approaches to analyze the impact of credit supply shocks. See for example Peek and Rosengren (1997), Kashyap and Stein (2000), Paravisini (2008), Jiménez et al. (2012), Huber (2018), Greenstone, Mas and Nguyen (2020), and Paravisini, Rappoport and Schnabl (2023).

using matched bank-firm credit registry data to compare lending by different banks to the same firm. By including firm fixed effects, the KM regression absorbs unobserved firm-level credit demand.[3] A negative KM estimator is interpreted as evidence that banks transmit adverse supply shocks to borrowers, and its magnitude is often interpreted as the loan-level effect of the shock: i.e., the change in lending within an affected bank-firm relationship.[4] Although the KM methodology is not designed to estimate the effect of supply shocks on total firm borrowing, it has motivated an empirical test that is sometimes used to justify firm-level borrowing and firm-level real effects regressions. This test compares KM estimates with and without firm fixed effects to assess whether loan supply shocks are correlated with firm credit demand (see, e.g., Khwaja and Mian (2008), Chodorow-Reich (2014), Jiménez et al. (2017), Bustos, Garber and Ponticelli (2020)).

We revisit the interpretation of these empirical strategies and show how our framework can be used to construct estimators for the relevant economic objects. We first show that a negative KM coefficient confirms that banks transmit adverse supply shocks to borrowers, but this coefficient does not measure the size of the resulting loan-level effect. Because the firm fixed effect removes the scale response of total borrowing, the remaining variation captures only how the firm reallocates borrowing across lenders exposed to different shocks. The KM coefficient thus recovers the *elasticity of substitution* across lenders, not the change in lending within the affected bank-firm relationship. We further show that the same scale-substitution logic also undermines the comparison of KM estimates with and without firm fixed effects: this comparison does not recover the relevant supply-demand correlation, and therefore cannot justify running the firm-level regressions without adequately controlling for demand.

We continue by using the scale-substitution decomposition to derive new estimators for specific loan supply shocks. The estimators separately identify scale and substitution effects and thereby recover both the total firm-level and loan-level impact of loan supply shocks. The key empirical object is a scale-substitution regression, derived from the model, that relates lending within a bank-firm relationship to two components of the supply shock: the average shock faced by the firm across its lenders (the scale component) and the difference in supply shocks across its lenders (the substitution effect). The regression thus separates the two forces highlighted by the theory. A loan supply shock to a bank thus changes the firm's overall cost of external finance, affecting total borrowing, and it changes the relative cost of borrowing across banks,

[3]As discussed in Paravisini, Rappoport and Schnabl (2023), if firm production can be broken down into various activities, and if banks specialize in financing different activities, firm fixed effects might not adequately address a correlation between demand shocks at the activity level and loan supply shocks.

[4]A standard sentence used in interpreting the coefficient is of the form, "a one standard deviation increase in the [loan supply shifter] reduces bank lending by x%".

inducing substitution across lenders.

As in other loan-level regressions, the scale-substitution regression leaves unobserved firm-level credit demand in the error term. If this demand is correlated with loan supply shocks, OLS cannot be used to consistently estimate the scale and substitution effects. To address this problem, we exploit the fact that the substitution effect can be estimated in two ways. The scale-substitution regression delivers one estimate, which is biased by supply-demand correlation. The KM regression, by contrast, provides a consistent estimate of the same substitution effect because firm fixed effects purge firm-level demand. The difference between these two estimates thus isolates the bias term, allowing us to back out the covariance between loan supply and demand. This, in turn, allows us to correct the biased estimate of the scale effect in the scale-substitution regression, thereby obtaining a measure of the impact of the loan supply shock on total firm borrowing.[5] Combining the scale and substitution effect estimates, we can then also recover the loan-level response to specific loan supply shifters. Finally, we show that the same covariance estimate used to correct the scale-substitution regression also provides a test of whether loan supply and firm credit demand shocks are correlated.

We next turn from the approach building on Khwaja and Mian (2008) to a second approach to measuring the impact of loan supply shocks: the methodology introduced by Amiti and Weinstein (2018), which estimates loan-level regressions with bank and firm fixed effects. Rather than focusing on an observed, specific supply shifter, such as a deposit shock or real-estate exposure, an important conceptual contribution of the approach is its focus on capturing the impact of *total* bank supply shocks, which reflect the full change in a bank's cost and, consequently, its loan supply. In the fixed effects regression, the AW approach interprets the bank fixed effect as capturing the bank's total, idiosyncratic loan supply shock (i.e., the total shock not common among all banks) and the firm fixed effect as capturing the firm's total, idiosyncratic loan demand shock. These estimated fixed effects are then used in regressions of firm real outcomes.

Using our framework, we show that the bank and firm fixed effects do not, in general, correspond to the supply and demand shocks they are meant to measure. The reason is again the scale-substitution structure – the scale effect captures the impact of the supply shocks on total firm borrowing, and therefore part of the bank supply shock is captured by the firm fixed effect rather than the bank fixed effect. This, in turn, implies that regressions of real outcomes on bank and firm fixed effects misestimate the impact of loan

[5] Jiménez et al. (2020) start from the observation that comparing KM regressions estimated with and without firm fixed effects may be informative about the relationship between loan demand and loan supply. They formalize this idea to construct a correction for firm-level regressions and use it to estimate the effect of loan supply shocks on total firm borrowing. Using our framework, however, we show that the resulting estimator does not recover the covariance between loan demand and loan supply. As a result, the correction does not identify the impact of loan supply shocks on total firm borrowing (see Section 6.2 for further details).

supply shocks.

We then show how our scale-substitution decomposition can be used to recover each bank's total loan supply shock, together with each firm's loan demand shock. We obtain these shocks by appropriately combining the bank and firm fixed effects from lending regressions with the scale and substitution elasticities estimated from the analysis of a specific supply shifter. The recovered shocks allow us to estimate the effects of total loan supply shocks on total firm borrowing, loan-level lending, and firm real outcomes.

We apply our estimation framework to the 2011 Spanish debt crisis, a period when Spanish banks exposed to real estate experienced negative supply shocks following the collapse of the real estate market. Using banks' pre-crisis real estate exposure as a loan-supply shifter, we find that interpreting the KM coefficient as measuring the loan-level effect overstates this effect by approximately 50%. We also find that the effect on total firm borrowing is approximately half of the loan-level effect, indicating substantial substitution across lenders. Finally, analyzing the debt crisis, we show that methodologies in the spirit of AW that use bank fixed effects as a proxy for total loan supply shocks overstate the real effects of these shocks on firm-level outcomes by up to 100%.

Our analysis provides a unified interpretation of important empirical approaches used to study loan supply shocks with matched bank-firm data. The literature has long recognized that within-firm comparisons use substitution across lenders to isolate variation in bank loan supply from borrower demand. We show that this substitution margin is not merely a source of identification; it is the economic object identified by the KM coefficient, which recovers an elasticity of substitution across lenders. Distinguishing this substitution elasticity from the scale response of total firm borrowing is key for the analysis. The scale-substitution decomposition provides a common structure for interpreting existing estimators and constructing the estimators developed in this paper. It shows how to recover firm-level borrowing and real effects, why tests employed in the literature do not recover the relevant supply-demand correlation, and how bank and firm fixed effects in the AW methodology should be adjusted to identify total supply and demand shocks.

The paper proceeds as follows. Section 2 provides our theoretical framework introducing the scale and substitution effects, and linking them to the objects of interest – the impact of loan supply shocks at the total firm-level and at the loan-level. Section 3 introduces the empirical counterpart to the model. Section 4 re-examines existing methodologies for estimating the impact of loan supply shocks. Section 5 uses our framework to present a methodology to estimate the impact of loan supply shocks – both observable supply shifters and total supply shocks – on loan-level lending and total firm-level borrowing. Section 6 provides additional extensions. Finally, Section 7 applies our methodology to the case of the 2011 Spanish debt crisis. Section 8 concludes.

2. Understanding the Impact of Loan Supply Shifters: A Simple Model

In this section we introduce a simple model which analyzes how loan supply shocks affect lending. In considering supply shocks, we will distinguish between what we will call the "total supply shock", which measures the total change in the cost of borrowing from a given bank, and a "specific supply shifter", which is a particular variable that shifts loan supply, but does not necessarily capture the entire shift in loan supply. Examples of such loan supply shifters in the literature include banks' exposure to the interbank lending market and the degree of credit line drawdowns experienced by banks. Empirically, total supply shocks are not directly observed, while supply shifters, by definition, are.

The model will introduce two key parameters of interest governing the impact of loan supply shocks: the impact on total firm-level borrowing, as well as the impact on loan-level borrowing, i.e., on the firm's level of borrowing from each of its lenders. In doing so, we consider both the impact of a specific loan supply shifter as well as the impact of the total supply shock.

2.1. Model Setup

Consider a firm raising capital for a project. For simplicity, assume that the firm has no internal funds but has the option of raising external finance from two different sources of financing, which we call banks.[6] For ease of exposition, the model setup and all derivations are shown for the case of two banks. The general case of N banks is shown in the appendix and is applied in the empirical section of the paper.

Let L_j denote the amount borrowed from bank $j \in \{1,2\}$, so that total borrowing (and hence total investment) is $L = L_1 + L_2$. Investing an amount L in the project generates cash flow with present value given by $R(L)$, which is assumed to be concave.[7]

External borrowing is costly, with the deadweight cost associated with raising an amount L_j from bank j given by $a_j c(L_j)$, with $c(L_j) = L_j^\rho$ and $\rho > 1$.[8] The variables a_j shift the cost of external finance obtained from bank j, capturing bank-level credit supply shocks. Borrowing amounts L_1 and L_2 is thus associated with a deadweight loss of $\widetilde{C}(L_1, L_2) = \left(a_1 L_1^\rho + a_2 L_2^\rho\right)$. These deadweight cost functions capture in a reduced form way financial frictions such as those having to do with information or moral hazard frictions. In what follows, we analyze the elasticity of loan-level lending, L_j, and the elasticity of total firm borrowing, L, to loan supply shocks, as captured by variation in a_j.

[6] This assumption is easily generalizable for the case where the firm can partially rely on internal funds to invest in the project.

[7] The results hold for the case where R is locally concave at the optimal level of investment.

[8] $c(L_j)$ is thus increasing and convex as in Stein (2003).

In deciding its level of investment, the firm maximizes the second-best NPV, inclusive of the external financing costs:

$$\begin{aligned}\max_{L_1,L_2}&\left\{R(L_1+L_2)-(L_1+L_2)-\widetilde{C}(L_1,L_2)\right\}\\ &\text{with } \widetilde{C}(L_1,L_2)=\left(a_1L_1^{\rho}+a_2L_2^{\rho}\right).\end{aligned} \tag{1}$$

As is standard, this problem can be written in two steps. The firm chooses total borrowing, L:

$$\max_{L}\left\{R(L)-L-C(L)\right\}, \tag{2}$$

with $C(L)$, the cost minimization function associated with (1):

$$C(L):=\min_{L_1,L_2}\left\{a_1L_1^{\rho}+a_2L_2^{\rho}\,|L_1+L_2=L\right\}. \tag{3}$$

Solving (3), it is straightforward to show (see appendix) that the cost minimization function associated with borrowing a total amount L across both banks is given by:

$$C(L)=\kappa_C L^{\rho}, \text{ with } \kappa_C:=\left(a_1^{\frac{1}{1-\rho}}+a_2^{\frac{1}{1-\rho}}\right)^{1-\rho}. \tag{4}$$

2.2. Substitution vs. Scale Effects

The parameter ρ pins down the elasticity of substitution between borrowing from the two banks, where this substitution elasticity is equal to $\frac{1}{\rho-1}$. To see this, note that the ratio of the first-order conditions of the cost minimization problem implies that the optimal borrowing from each bank satisfies:

$$\log\left(\frac{L_1}{L_2}\right)=-\frac{1}{\rho-1}\log\left(\frac{a_1}{a_2}\right),$$

which, taking a_1/a_2 as the relative costs of bank 1 and bank 2 lending, yields $\frac{1}{\rho-1}$ as the definition of the elasticity of substitution.

The impact on total firm borrowing Next, we consider the impact of changes to the cost of external finance on total firm borrowing. Denoting log changes over time (between periods t and $t+1$) with the Δ operator and log-linearizing the first-order conditions, we show in the appendix that to the first order the response of total firm lending across both banks to changes in the cost of external finance is given by:

$$\Delta\log L=\theta\left(s_1\Delta\log a_1+s_2\Delta\log a_2\right), \tag{5}$$

where $s_j=\frac{L_j}{L}$ is the pre-shock lending share of bank j.[9]

[9] Note that by the nature of the approximation, the shares s_j are determined by the *pre-shock* levels of lending.

Equation (5) shows that total lending is determined by the weighted average of the loan supply shocks across the two banks, with θ the elasticity that captures how the change in total lending responds to the weighted average change in lending costs.[10] We call this effect (capturing how average loan supply shocks impact total firm lending) the 'scale effect'.

What determines the size of θ? We show in the appendix that:

$$\theta = \frac{1}{\eta_{G',L} - \eta_{C',L}} = \frac{1}{\left(\frac{1+C'}{C'}\right)\eta_{R',L} - \eta_{C',L}}, \tag{6}$$

where $G(L) = R(L) - L$ is the first-best NPV of investment (exclusive of the external financing cost) when investing an amount L, $G' = \frac{dG(L^*)}{dL}$, $R' = \frac{dR(L^*)}{dL}$ and $C' = \frac{dC(L^*)}{dL}$ are the marginal NPV, the marginal PV and the marginal external finance cost at the optimal level of borrowing, L^*, respectively, and we use η to denote the partial elasticity operator.[11] As would be expected, the elasticity of total borrowing to financing costs is negative, since marginal PV (R') declines with L while marginal external finance cost (C') rises with L.[12] As is intuitive, θ, the elasticity of total firm borrowing and investment to external finance costs, declines in absolute value when external financing costs become relatively less important (i.e. when $\frac{C'}{1+C'}$ declines). Indeed, when the relative importance of marginal external financing cost declines to zero, the elasticity of investment to external financing cost goes to zero as well.

The first equality in equation (6) shows that θ depends on two forces: (1) the elasticity of the marginal financing cost, $\eta_{C',L}$, and (2) the elasticity of the project's marginal NPV, $\eta_{G',L}$. To gain intuition on these two forces, it is useful to examine the problem through the firm's FOC, which pins down total firm borrowing: $G'(L) = C'(L)$. Consider an upward shift in the marginal cost curve $C'(L)$, which results from a change in the cost parameters a_j. As a result, the firm will reduce total borrowing, L, until the FOC is restored. Now, if the marginal NPV, $G'(L)$, declines quickly with L – i.e., $\eta_{G',L}$ is large in absolute value – then a relatively small change in borrowing generates a large increase in marginal present value, implying that the firm reduces borrowing only modestly. Put differently, under these circumstances, demand for credit will be relatively inelastic, and hence the upward shift in the marginal financing cost curve will not affect total firm borrowing, L, by much; The magnitude of θ will thus be low. Analogously, if the marginal financing cost $C'(L)$ rises steeply with L – i.e., $\eta_{C',L}$ is large – then restoring the FOC requires a smaller reduction in L, and hence the magnitude of θ will be small.

[10] Alternatively, it is the exact elasticity of lending to the weighted geometric mean of the cost.

[11] Formally, given a function y and a variable x, we denote the partial elasticity of y with respect to x by $\eta_{y,x} := \frac{\partial y}{\partial x}\frac{x}{y}$.

[12] Concavity of $R(L)$ at L^* ensures that $\eta_{R',L} < 0$, ruling out knife-edge cases such as linear $R(L)$ where the project has infinite NPV.

The loan-level effect Turning to the borrowing of the firm from a specific bank, we show in the appendix that to the first order the response of lending from a specific bank j to changes in the cost of external finance is given by:

$$\Delta \log L_j = \theta \left(s_1 \Delta \log a_1 + s_2 \Delta \log a_2 \right) - \frac{1}{\rho - 1} s_{-j} \Delta \log \left(\frac{a_j}{a_{-j}} \right). \tag{7}$$

Equation (7), which we refer to as the scale-substitution equation, decomposes the impact of shocks to the cost of external finance on loan-level lending into two intuitive components, capturing scale and substitution effects. The first term on the right hand side of the equation, $\theta \left(s_1 \Delta \log a_1 + s_2 \Delta \log a_2 \right)$, is a scale effect which captures the impact of the weighted average of the external finance shocks experienced by the banks lending to a given firm on individual bank lending. The second term on the right hand side of equation (7), $\frac{1}{\rho-1} s_2 \Delta \log \left(\frac{a_1}{a_2} \right)$, reflects the substitution effect in lending, in that when the relative cost of lending from bank 1 increases, borrowing from bank 1 declines according to the elasticity of substitution $\frac{1}{\rho-1}$.[13]

Another instructive way to write the scale-substitution equation in (7) is

$$\Delta \log L_j = \left(\theta s_j - \frac{1}{\rho - 1} s_{-j} \right) \Delta \log a_j + \left(\theta s_{-j} + \frac{1}{\rho - 1} s_{-j} \right) \Delta \log a_{-j}. \tag{8}$$

This formulation shows the two effects of a bank-specific loan supply shock on loan-level lending. In particular, a change in a_j (holding a_{-j} constant) affects lending via two channels: a scale effect, given by θs_j, reflecting the change in the cost of external finance, and a substitution effect, $-\frac{1}{\rho-1} s_{-j}$, reflecting the firm's tilting of borrowing from the affected bank (j) to the unaffected bank ($-j$). Note that as the share of lending from bank j rises, the impact of loan supply shocks to bank j becomes dominated by the scale effect. Finally, the equation also shows how lending by bank j is influenced by a negative supply shock in bank $-j$: the scale effect (θs_{-j}) pushes bank j lending down, while the substitution effect ($\frac{1}{\rho-1} s_{-j}$) pushes it up.

Comparing magnitudes It is useful to compare the magnitudes of the three elasticities discussed above: (1) the elasticity of substitution, $\frac{1}{\rho-1}$, (2) the scale elasticity, θ, and (3) the loan-level lending elasticity,

[13] The case of N banks is analogous, where, similar to equation (7), the impact of shocks to external finance on the lending of bank j is given by:

$$\Delta \log L_j = \theta \left(\sum_{k=1}^{N} s_k \Delta \log a_k \right) - \frac{1}{\rho - 1} \sum_{k=1}^{N} s_k \Delta \log \left(\frac{a_j}{a_k} \right).$$

Once again, the impact of shocks to the cost of external finance on loan-level lending can be decomposed into scale and substitution effects; scale effects are driven by the impact of the average supply shock (across banks) on lending, while substitution effects are driven by how differential changes to the cost of external finance between lenders cause the firm to reallocate borrowing from one bank to the other.

$\theta s_j - \frac{1}{\rho-1}s_{-j}$. To do this, note that equation (4) implies that the elasticity of the marginal cost of external finance, $\eta_{C',L}$, is simply equal to $\rho - 1$, which, together with (6), implies that

$$\theta = \frac{1}{\left(\frac{1+C'}{C'}\right)\eta_{R',L} - (\rho-1)}. \tag{9}$$

Given that $\eta_{R',L} < 0$ and $C' > 0$, it is easy to show that:

$$-\frac{1}{\rho-1} < \theta s_j - \frac{1}{\rho-1}s_{-j} < \theta < 0. \tag{10}$$

That is, in absolute value, the elasticity of substitution is larger than the loan-level lending elasticity, which is larger than the scale elasticity. Indeed, theoretically, these three values can be quite different from each other.[14] For example, as ρ approaches one – i.e., the cost function is close to linear – the elasticity of substitution $\frac{1}{\rho-1}$ goes to infinity, while θ will depend on the PV function and on the relative importance of marginal cost (and can even tend to zero).

2.3. Specific Supply Shifters versus Total Loan Supply Changes

Up to this point we have considered the impact of bank-level loan supply shocks (a_j), which reflect changes in the cost at which banks provide financing to their borrowing firms. These shocks, however, are not directly observable in empirical analysis. Consider, therefore, an observable variable w_j, which shifts loan supply in bank j. Examples of such observable shifters analyzed in the literature include such variables as bank-level deposit flows (Khwaja and Mian (2008)), exposure to the interbank lending market on the eve of a financial crisis (Iyer et al. (2014)), credit line drawdowns (Greenwald, Krainer and Paul (forthcoming)), etc. Using a linear projection, we can relate the total change in cost of external finance provided by the bank, $\Delta \log a_j$, to the bank-level shifter, w_j:

$$\Delta \log a_j = b_0 + b_1 w_j + \chi_j, \tag{11}$$

where without loss of generality, we assume that the transmission coefficient b_1 is positive (so that a higher supply shifter is associated with a higher cost of financing). In what follows, we refer to the bank-level loan-supply shifter, w_j, as a *specific* loan supply shifter to emphasize the fact that many such potential shifters exist. Indeed, these other shifters are captured by χ_j in equation (11).[15] This is in contrast to variation in the bank-level cost of external finance, a_j, which captures the *total* change in loan supply at the bank (and is hence unique over a given time period for any given bank). Note also that to more easily relate the analysis

[14] In the model, the only case where the three values coincide is when the PV function, R, is linear.

[15] By the properties of linear projection, χ_j is uncorrelated with w_j and captures the other supply shifters orthogonalized to w_j.

to prior empirical work, we assume without loss of generality that it is the level of w, rather than changes in w, that is correlated with the loan supply shock, $\Delta \log a_j$.

Analogously to equation (5), it is then easy to show that to the first order the response of total firm borrowing (i.e., across both banks) to the loan supply shifter satisfies:

$$\Delta \log L = \text{constant} + b_1 \theta \left(s_1 w_1 + s_2 w_2\right) + \tilde{\chi}, \quad \text{with } \tilde{\chi} = \theta \left(s_1 \chi_1 + s_2 \chi_2\right). \tag{12}$$

That is, the change in total lending to the firm responds to the weighted average of the loan-supply shifter across the two banks lending to the firm ($s_1 w_1 + s_2 w_2$), according to the scale effect elasticity θ, and the transmission coefficient b_1.

Similarly, analogously to equation (8), it is also straightforward to show that the response of loan-level lending of bank j to the specific loan supply shifter is given by:

$$\Delta \log L_j = \text{constant} + \left(b_1 \theta s_j - \frac{b_1}{\rho - 1} s_{-j}\right) w_j + \left(b_1 \theta s_{-j} + \frac{b_1}{\rho - 1} s_{-j}\right) w_{-j} + \tilde{\chi}_j; \tag{13}$$

with $\tilde{\chi}_j = \tilde{\chi} + \frac{1}{(\rho-1)} s_{-j} \left(\chi_{-j} - \chi_j\right)$.

2.4. Taking Stock

Taken together, equations (5), (8), (12), and (13) describe the four objects of interest in understanding the impact of loan supply shocks on lending.

Equation (5) describes the impact of total supply shocks on total firm-level borrowing across both banks. This effect is given by $\theta \left(s_1 \Delta \log a_1 + s_2 \Delta \log a_2\right)$.

Equation (8) describes the impact of total supply shocks on loan-level lending, which includes both the direct effect, $\left(\theta s_j - \frac{1}{\rho-1} s_{-j}\right) \Delta \log a_j$, and the cross effect, $\left(\theta s_{-j} + \frac{1}{\rho-1} s_{-j}\right) \Delta \log a_{-j}$.

Equation (12) describes the impact of a specific supply shifter, w, on total firm-level borrowing. This is given by $b_1 \theta \left(s_1 w_1 + s_2 w_2\right)$.

Finally, equation (13) describes the impact of a specific supply shifter w on loan-level lending, which, similar to equation (8), includes both the direct effect, $\left(b_1 \theta s_j - \frac{b_1}{\rho-1} s_{-j}\right) w_j$, and the cross effect, $\left(b_1 \theta s_{-j} + \frac{b_1}{\rho-1} s_{-j}\right) w_{-j}$.

3. Connecting the Model to Data

Our immediate goal is to relate the model to empirical analysis commonly performed in the literature. To this end, we begin by introducing random components and loan demand shifters to the model described above. We assume that the cost function for firm i in borrowing an amount L_{ijt} from bank j at time t can be written as:

$$c_j\left(L_{ijt}\right) = a_{jt} u_{ijt} L_{ijt}^{\rho}, \tag{14}$$

where a_{jt} is the bank-level cost shifter, and u_{ijt} captures a random time-t bank-firm-level component of borrowing costs.

Next, we allow the profitability of investment to change over time and across firms. In particular, we assume that the present value function is of the form

$$R_{it}(L_{it}) = B_{it}\widetilde{R}(L_{it}),$$

where L_{it} is firm i's total borrowing (and investment) across both its banks, B_{it} is a time-varying, firm-specific variable that shifts investment opportunities, and $\widetilde{R}$ is a time-constant investment function. This formulation thus allows for cross-sectional and time-series variation in firm-level demand for loans, and in particular, correlation between investment opportunities, B_{it}, and borrowing costs, a_{jt}.

Log-linearizing the first-order conditions, we show in the appendix that the empirical counterpart of equation (5), showing the response of total firm lending across both banks to the changes in the cost of external finance, is given by:

$$\Delta \log L_i = \text{constant} + \underbrace{\Delta \log \tilde{B}_i}_{x^*_{d,i}} + \underbrace{\theta\left(s_{i1}\Delta \log a_1 + s_{i2}\Delta \log a_2\right)}_{x^*_{s,i}} + \tilde{\nu}_i,\ \tilde{\nu}_i = s_{i1}\nu_{i1} + s_{i2}\nu_{i2} \tag{15}$$

where in this regression, $\Delta \log \tilde{B}_i$ is a manipulation of the shock to investment opportunities, B, and ν_{ij} is an error term which is linear in the $\log u_{ij}$ terms.[16] Equation (15) decomposes the change in firm borrowing into shifts due to demand and shifts due to supply effects. The demand component is given by $x^*_{d,i} := \Delta \log \tilde{B}_i$, whereas the supply component is given by $x^*_{s,i} := \theta\left(s_{i1}\Delta \log a_1 + s_{i2}\Delta \log a_2\right)$, i.e., the weighted average of the loan supply shifts multiplied by the scale elasticity, θ. Note that given that θ is negative, a negative cost shock amounts to a positive loan supply shock, $x^*_{s,i}$.

Similarly, the empirical counterpart of the scale-substitution equation in (8), which describes how lending by bank j to firm i is affected by the total supply shocks $\Delta \log a_j$ is given by:

$$\Delta \log L_{ij} = \text{constant} + \underbrace{\Delta \log \tilde{B}_i}_{x^*_{d,i}} + \underbrace{\left(s_{i,j}\theta - s_{i,-j}\frac{1}{\rho - 1}\right)\Delta \log a_j}_{x^*_{s,i,j}} + \underbrace{\left(s_{i,-j}\theta + s_{i,-j}\frac{1}{\rho - 1}\right)\Delta \log a_{-j}}_{x^*_{s,i,-j}} + \nu_{ij}. \tag{16}$$

[16] Specifically, without loss of generality, we assume that $E\left(\log u_{ijt}\right) = 0$ and that $\Delta \log \tilde{B}_i$ is the deviation from the cross-sectional mean of $-\frac{R_i'(L_i)}{G_i'(L_i)}\theta \Delta \log B_i$. Further, $\nu_{i,j} = \frac{1}{(\rho-1)}\left(s_{i,j}\left(1+\theta(\rho-1)\right) - 1\right)\Delta \log u_{i,j} + s_{i,-j}\frac{1}{(\rho-1)}\left(1+\theta(\rho-1)\right)\Delta \log u_{i,-j}$.

Equation (16) decomposes the change in bank j's lending to the firm into four components: (1) the demand component, $x^*_{d,i}$; (2) a component capturing how bank j lending to the firm is influenced by the total loan supply shock in bank j, given by $x^*_{s,i,j} := \left(s_{i,j}\theta - s_{i,-j}\frac{1}{\rho-1}\right)\Delta\log a_j$; (3) a component capturing how bank j lending to the firm is influenced by the total loan supply shock in bank $-j$, given by $x^*_{s,i,-j} := \left(s_{i,-j}\theta + s_{i,-j}\frac{1}{\rho-1}\right)\Delta\log a_{-j}$; and (4) a component capturing changes in borrowing costs at the bank-firm level, $\nu_{i,j}$.

Finally, in analyzing the lending impact of the specific loan supply shifter w (as opposed to the total supply shocks, a), the empirical counterparts of equations (12) and (13) can easily be shown to be:

$$\Delta\log L_i = \text{constant} + \Delta\log\tilde{B}_i + b_1\theta\left(s_{i1}w_1 + s_{i2}w_2\right) + \tilde{\nu}_i + \tilde{\chi}_i, \tag{17}$$

and

$$\Delta\log L_{i,j} = \text{constant} + \Delta\log\tilde{B}_i + \left(b_1\theta s_{i,j} - \frac{b_1}{\rho-1}s_{i,-j}\right)w_j + \left(b_1\theta s_{i,-j} + \frac{b_1}{\rho-1}s_{i,-j}\right)w_{-j} + \nu_{i,j} + \tilde{\chi}_{i,j}, \tag{18}$$

with $\tilde{\chi}_{i,j}$ defined as in (13).

4. Common Estimation Methods through the Lens of the Model

In this section, we discuss the interpretation of two estimators commonly used in the literature – the KM estimator and lending regressions with bank and firm fixed effects as in AW. The common theme in both is the use of firm fixed effects to capture unobserved firm-level variation in the demand for credit. For each method, we begin with a short discussion of the econometric framework and the interpretation of the estimators, and then proceed to analyze each of the estimators through the lens of our model.

4.1. Loan Regressions with Firm Fixed Effects through the Lens of the Model

Consider the following canonical KM specification in a multi-firm, multi-bank environment. Assume that between two periods, t and $t+1$, banks are affected by loan supply shocks that are potentially correlated with loan demand shocks to their portfolio firms.[17] Following KM, a common method of estimating the impact of loan supply shocks on bank lending is to run a regression of the form

$$\Delta\log L_{ij} = \beta_{KM}w_j + \delta_i + \varepsilon_{ij}, \tag{19}$$

[17] The extension from two to multiple periods is trivial.

where w_j is an observable, bank-level shifter of loan supply in bank j and δ_i is a firm-level fixed effect. The identifying assumption to obtain a consistent estimate of β_{KM} is that w_j is uncorrelated with ε_{ij} – i.e., conditional on the firm fixed effects, bank-level credit supply shocks, w_j, are uncorrelated with unobserved loan demand or supply shifters at the bank-firm level. This assumption allows for firm and bank matching – generating correlation between bank supply shocks and firm demand shocks, so long as the demand shocks are at the firm level. (We discuss violations of this assumption in Section 6.4.)

The main coefficient of interest, β_{KM}, is commonly interpreted in the literature as measuring the loan-level effect of the supply shifter, i.e. the impact of the loan supply shifter on a bank's lending to the firm. A standard sentence used in interpreting the coefficient is of the form, "a one standard deviation increase in the [loan supply shifter] reduces bank lending by x%". This interpretation is inaccurate, though. Taking the difference in equation (19) between the two banks yields

$$\Delta\log\left(\frac{L_{i2}}{L_{i1}}\right) = \beta_{KM}(w_2 - w_1) + (\varepsilon_{i2} - \varepsilon_{i1}). \tag{20}$$

Equation (20) shows that β_{KM} identifies the *elasticity of substitution* of borrowing between banks with respect to the supply shifter, w. It measures how relative borrowing from two lenders responds when changes in their relative supply conditions make credit from one lender more expensive than credit from the other. This is distinct from the loan-level effect, which measures the change in lending by a given bank to the firm induced by a change in that bank's supply shifter.

To see this, consider the case where only bank 2 experiences a shift in the loan supply so that the cost of borrowing from this bank rises. The coefficient β_{KM} captures how this shift translates into a change in the relative amounts of borrowing between bank 1 and bank 2: because the relative cost of borrowing from bank 2 rises, the firm substitutes borrowing from bank 2 to bank 1. To the extent that such substitution exists, the effect on the relative amounts of borrowing from the two banks (captured by β_{KM}) will generally be different from the impact that the supply shifter has on borrowing from the bank experiencing the supply shift.

To connect the KM methodology to the model, we return to the maximization problem in (1), and write the first-order condition with respect to the level of borrowing from bank j:

$$B_{it}\widetilde{R}'(L_{it}) = 1 + \rho a_{jt} u_{ijt} L_{ijt}^{\rho-1} \tag{21}$$

Taking logs of this first-order condition, rearranging, using equation (11), and taking first differences yields that for a specific supply shifter, w_j:

$$\Delta\log L_{ij} = \text{constant} + \frac{1}{(\rho-1)}\Delta\log(B_i\widetilde{R}'(L_i) - 1) - \frac{b_1}{(\rho-1)}w_j - \frac{1}{(\rho-1)}(\Delta\log u_{ij} + \chi_j). \tag{22}$$

There is a natural correspondence between equation (22) and the KM regression in equation (19). Similar to the KM assumption, we assume that the error term in (22), $\Delta \log u_{ij}$, is uncorrelated with the credit supply shock, w_j. The error term χ_j is uncorrelated with w_j by construction as equation (11) is a linear projection. Further, the first term on the right hand side of (22), i.e., $\Delta \log(B_i \widetilde{R}'(L_i) - 1)$, varies at the firm level and so will be absorbed by the firm fixed effects in the KM regression in (19). Taking all this into account, running the KM regression in (19), and maintaining the usual assumption that w_j is uncorrelated with ε_{ij}, implies that:

$$\beta_{KM} = -\frac{b_1}{\rho - 1}. \tag{23}$$

Put differently, the KM coefficient measures the *elasticity of substitution in borrowing between banks*, $\frac{1}{\rho-1}$, which is then scaled by b_1, the transmission coefficient between the credit supply shifter (w_j) and the bank-level cost of external finance (a_j).

What is the relation between the scaled elasticity of substitution, $\beta_{KM} = -\frac{b_1}{\rho-1}$, the loan-level and the firm-level responses to the loan-supply shifter? As shown in equation (12), the impact of the specific supply shifter, w_j, on total firm borrowing is given by the elasticity $b_1\theta$, whereas equation (13) shows that the impact of the specific supply shifter on loan-level lending to the firm is given by $b_1\theta s_j - \frac{b_1}{\rho-1}s_{-j}$. Equation (10) allows us to rank the relative magnitudes of these three elasticities. Since $b_1 > 0$:

$$\underbrace{-\frac{b_1}{\rho - 1}}_{\beta_{KM}} < \underbrace{b_1\theta s_j - \frac{b_1}{\rho - 1}s_{-j}}_{\text{loan-level effect}} < \underbrace{b_1\theta}_{\substack{\text{total}\\ \text{firm-level}\\ \text{effect}}} < 0 \tag{24}$$

Thus, while informative of the existence of a loan-level effect, $\beta_{KM} = -\frac{b_1}{\rho-1}$, overestimates the elasticity of loan-level lending to the loan-supply shifter ($b_1\theta s_j - \frac{b_1}{\rho-1}s_{-j}$), which is itself greater (in absolute value) than the response of total firm-level lending to the loan-supply shifter ($b_1\theta$).[18]

4.2. Loan Regressions with Bank and Firm Fixed Effects through the Lens of the Model

In its essence, the bank and firm fixed effect strategy employs a regression of the following nature:

$$\Delta \log L_{ij} = \phi_i + \zeta_j + \upsilon_{ij}, \tag{25}$$

where $\Delta \log L_{ij}$ is the log change in lending from bank j to firm i between t and $t+1$, ζ_j is a vector of

[18] This result is different from that obtained in the theoretical model in KM. This is because the KM theoretical model does not allow for substitution across banks by borrowers.

bank fixed effects, ϕ_i is a vector of firm fixed effects, and υ_{ij} is a loan-specific error term.[19] The fixed effects are identified using banks that lend to more than one firm and using firms that borrow from more than one bank. In this specification, the bank fixed effect is interpreted as capturing the change in lending due to the total, idiosyncratic bank supply shock (i.e., the total shock not common among all banks), and similarly, the firm fixed effect is interpreted as capturing the change in lending due to the total, idiosyncratic firm demand shock (see, for example, Amiti, McGuire and Weinstein (2017), Amiti and Weinstein (2018), Alfaro, García-Santana and Moral-Benito (2021)).

However, associating the firm and bank fixed effects with loan demand and loan supply shocks, respectively, is inaccurate. Intuitively, the firm fixed effects capture all firm-level variation in changes in borrowing. As such, in addition to the firm-level demand shock, these fixed effects will also capture the impact on borrowing of an average supply shock across the firm's banks, i.e. the scale effect. Because the firm fixed effects conflate loan supply and loan demand effects, standard applications using these bank and firm fixed effects – for example to measure the real effects of loan supply shocks – will be incorrect. That is, such regressions will not provide estimates of the impact of loan supply shocks on real outcomes.

To formally illustrate these arguments, we return to the first-order condition in (21). Log-linearizing once again, we obtain a variant of equation (22) for total supply shocks ($\Delta \log a_j$) rather than supply shifters (w_j):

$$\Delta \log L_{ij} = \frac{1}{(\rho-1)}\Delta \log(B_i \widetilde{R}'(L_i) - 1) - \frac{1}{(\rho-1)}\Delta \log a_j - \frac{1}{(\rho-1)}\Delta \log u_{ij}. \tag{26}$$

Comparing this equation to the bank and firm fixed effects regression in (25), it is readily seen that the vector of bank fixed effects corresponds to the set of scaled bank-level cost shocks, $-\frac{1}{(\rho-1)}\Delta \log a_j$, where the scaling parameter is the elasticity of substitution $\frac{1}{(\rho-1)}$.[20] By the same argument, the firm fixed effect corresponds to $\frac{1}{(\rho-1)}\Delta \log(B_i \widetilde{R}'(L_i) - 1)$. Importantly, in addition to the demand shock, this term also incorporates the scale effect – the firm-level response to the loan supply shocks through the optimal choice of L_i. As such, the firm fixed effect incorporates not just loan demand shocks, but also some of the response to loan supply shocks.

So how can one measure the loan supply shock? To get at this question it is useful to decompose the change in loans, $\Delta \log L_{ij}$, into demand and supply effects, taking into account the supply effect which

[19] There are many variants of this regression in the literature. Amiti and Weinstein (2018) conduct a comprehensive analysis that formalizes how different variants of this regression – using percent change instead of log difference, using WLS instead of OLS, and careful treatment of entry and exit of bank-firm lending relationships – have different aggregation properties. Furthermore, running this regression in practice requires taking a stand on the normalization of the fixed effect, but this normalization is immaterial for the conceptual point we make here.

[20] The elasticity of substitution scaling factor is explained by the fact that the bank fixed effects are estimated in the presence of firm fixed effects, and so they are identified only from within-firm substitution between banks.

shows up in $\frac{1}{(\rho-1)}\Delta\log(B_i\widetilde{R}'(L_i)-1)$. Rearranging the scale substitution equation in (16) yields:

$$\Delta\log L_{ij} = \text{constant} + \underbrace{\Delta\log\tilde{B}_i + \overbrace{\left(\theta+\frac{1}{\rho-1}\right)(s_{i,j}\Delta\log a_j + s_{i,-j}\Delta\log a_{-j})}^{\tau_i}}_{\phi_i} + \underbrace{\left(-\frac{1}{\rho-1}\Delta\log a_j\right)}_{\zeta_j} + \nu_{ij}. \tag{27}$$

This equation introduces a new object, τ_i, which is only a function of the loan supply shocks, $\Delta\log a_j$, and hence is a supply-driven component. This component, which is proportional to the scale effect, is symmetric across the two banks – i.e., it changes at the firm level. A direct result of equation (27) is that the bank and firm fixed effects can be written as:

$$\zeta_j = x^*_{s,i,j} + x^*_{s,i,-j} - \tau_i \tag{28}$$

$$\phi_i = x^*_{d,i} + \tau_i. \tag{29}$$

Thus, the bank fixed effect ζ_j is simply the sum of $x^*_{s,i,j}$, the supply component capturing how bank j's lending to the firm is influenced by the supply shock to bank j, and $x^*_{s,i,-j}$, the supply component capturing how bank j's lending to the firm is influenced by the supply shock to bank $-j$, net of the firm-level supply driven term τ_i. By the same token, the firm fixed effect is a sum of two components: the demand component, $x^*_{d,i}$, and the firm-level term, τ_i.

Typical applications in the literature that use the bank and firm fixed effect model proceed by using the bank fixed effects to calculate a firm-level loan-supply shock measure. They do so by calculating for each firm the share-weighted average of the bank fixed effects, with shares calculated over the banks lending to the firm. Thus, the literature calculates for each firm i: $\bar{\zeta}_i := s_{1,i}\zeta_1 + s_{2,i}\zeta_2$. This measure, however, does not map to the loan supply shock at the firm level. Indeed, equation (15) shows that the supply shock at the firm level amounts to $x^*_{s,i} = \theta(s_{1,i}\Delta\log a_1 + s_{2,i}\Delta\log a_2)$, which is easy to show is different than $\bar{\zeta}_i$. In fact, similar to the loan-level case, $\bar{\zeta}_i$ is missing the firm-level term, τ_i:

$$\bar{\zeta}_i = x^*_{s,i} - \tau_i.$$

Finally, given that the bank fixed effects correspond to $\zeta_j = -\frac{1}{\rho-1}\Delta\log a_j$, we have that at the firm level:

$$\bar{\zeta}_i = -\frac{1}{\rho-1}(s_{1,i}\Delta\log a_1 + s_{2,i}\Delta\log a_2).$$

Note that since $-\frac{1}{\rho-1} < \theta < 0$ (see equation (10)), the actual supply-driven change in lending, $x^*_{s,i}$, is smaller in magnitude (i.e., less negative) than the erroneously calculated supply driven change in lending, $\bar{\zeta}_i$.

4.2.1. Applications of Regressions with Both Bank and Firm Fixed Effects

The fact that the firm fixed effects from regressions with both bank and firm fixed effects incorporate supply effects, and that the bank fixed effects are off by a scaling factor implies that the two most common applications of the fixed effect specifications commonly employed in the literature yield biased results. First, the aggregation (either to the bank-level or to the economy-level) of lending changes into supply-driven versus demand-driven changes misattributes firm-level supply changes to the demand channel rather than to the supply channel.

Second, the literature estimates real-effect regressions, relating firm-level outcomes (such as employment changes) to the firm fixed effects, ϕ_i, and the share-weighted average of bank fixed effects, $\bar{\zeta}_i$, interpreting the coefficient on the latter as capturing the impact of the supply-driven change in lending on the real effect being examined. Given that, as shown above, ϕ_i and $\bar{\zeta}_i$ do not correspond to supply- and demand-driven changes in lending, these regressions will generally yield biased estimates for the impact of loan supply shocks on real outcomes. To formally see this, note that given (15), the correctly specified real effect regression is of the form:

$$y_i = \gamma_0^* + \gamma_s^* x_{s,i}^* + \gamma_d^* x_{d,i}^* + \varphi_i^* \tag{30}$$

where y_i is the firm-level real outcome (e.g., investment or change in employment), while $x_{s,i}^*$ and $x_{d,i}^*$ are, respectively, the supply- and demand-driven changes in lending as derived above.

However, instead of the independent variables in (30), the literature uses on the right-hand side the share-weighted average bank fixed effect, $\bar{\zeta}_i$ and the firm fixed effect ϕ_i. What is the result of running this regression? We have shown above that $\bar{\zeta}_i = x_{s,i}^* - \tau_i$ and $\phi_i = x_{d,i}^* + \tau_i$, which when plugged into (30) yields:

$$y_i = \gamma_0^* + \gamma_s^* \bar{\zeta}_i + \gamma_d^* \phi_i + \varphi_i^* + \tau_i \left(\gamma_s^* - \gamma_d^*\right).$$

In this equation, the composite error $\varphi_i^* + \tau_i \left(\gamma_s^* - \gamma_d^*\right)$ is correlated with the explanatory variables $\bar{\zeta}_i$ and ϕ_i through the object τ_i. As such, a regression of y_i on $\bar{\zeta}_i$ and ϕ_i would generally yield biased results for γ_s^* and for γ_d^*.

5. Estimating the Impact of Loan Supply Shocks

In this section we lay out the methodology for estimating the four objects of interest in analyzing loan supply shocks. In particular, in the next subsection we show how to estimate the impact of a specific supply shifter on total firm-level borrowing and loan-level lending. In the following subsection we then show how to estimate the impact of total supply shocks (i.e., not driven by a specific supply shifter) on total firm-level borrowing and loan-level lending. For ease of exposition, we focus on the case of two banks per firm, relegating the general N-bank case to the appendix.

5.1. The Impact of Specific Supply Shifters

We are after the following two objects: first, $b_1\theta$, which, given equation (12), is the elasticity of total firm-level lending to the weighted average of the loan supply shifter experienced by the banks lending to the firm (i.e., the scale effect); and second, $\left(b_1\theta s_j - \frac{b_1}{\rho-1}s_{-j}\right)$, which, given equation (13), is the elasticity of bank j's lending with respect to the specific loan supply shifter, w_j. As shown above, the KM regression consistently estimates the scaled elasticity of substitution, $\frac{b_1}{1-\rho}$. Thus, because bank lending shares are observable, estimating the two desired elasticities boils down to estimating the scale elasticity $b_1\theta$.

To make progress, consider the empirical counterpart of the scale-substitution equation by rearranging equation (18) in the following manner:

$$\Delta\log L_{i,j} = \text{constant} + b_1\theta\underbrace{(s_{i,1}w_1 + s_{i,2}w_2)}_{x_{i,1}} - \frac{b_1}{(\rho-1)}\underbrace{s_{i,-j}\left(w_j - w_{-j}\right)}_{x_{i,j,2}} + \underbrace{\Delta\log\tilde{B}_i + \nu_{i,j} + \tilde{\chi}_{i,j}}_{e_{i,j}} \tag{31}$$

Note that if we could consistently estimate equation (31), we could recover the desired scale elasticity $b_1\theta$. Of course, the model in equation (31) cannot be estimated using OLS: the firm-level demand shocks, $\tilde{B}_i$, which are potentially correlated with the supply shocks, w_j, are not observable and are part of the error term, $e_{i,j}$. Instead, what is estimatable is the following *scale-substitution regression*:

$$\Delta\log L_{i,j} = d_0 + d_1 x_{i,1} + d_2 x_{i,j,2} + r_{i,j}. \tag{32}$$

Equation (32) is a linear projection $\Delta\log L_{i,j}$ on $x_{i,1}$ and $x_{i,j,2}$. To the extent that the error term $e_{i,j}$, which includes the demand shock, is uncorrelated with the scale and substitution variables, $x_{i,1}$ and $x_{i,j,2}$, the OLS estimate for d_1 in (32) is consistent for the elasticity of interest, $b_1\theta$. However, when the demand shock *is* correlated with the loan supply shocks – the standard concern in the literature – equation (32) suffers from classical omitted variable bias. For example, as is intuitive, when adverse loan supply shocks are associated with decreased loan demand ($cov\left(w_{j(i)}, \Delta\log\tilde{B}_i\right) < 0$), then $\hat{d}_1$ from the empirical scale-substitution regression (32) would be downward biased – i.e., more negative than the true loan supply effect, $b_1\theta$. In what follows we provide a method to estimate $b_1\theta$.

5.1.1. A New Estimator for Estimating the Lending Elasticities of a Specific Loan Supply Shifter

We show in the appendix that under the following two assumptions we can recover a consistent estimate for the loan supply effect $b_1\theta$. The first assumption is that $E\left[\Delta\log u_{ij} + \chi_j \middle| w_j, w_{-j}, s_{i,j}\right] = 0$, i.e., the non-demand-driven loan-level idiosyncratic shocks are mean independent of the supply shifters and the shares. This assumption is in the same spirit as the typical assumption made in the literature: the non-demand-driven idiosyncratic shocks are uncorrelated with the bank supply shifters, w_j, and the shares (of course, the supply shifters can still be correlated with the demand shocks). The second assumption is that $E\left(w_{i,j}\Delta\log\tilde{B}_i \,\middle|\, s_{i,j}\right) = E\left(w_{i,j}\Delta\log\tilde{B}_i\right)$, which implies that the correlation of supply and demand shocks does not change with the shares, s_i, j. In other words, knowing the loan share $s_{i,j}$ does not add additional information regarding the covariance between the demand and supply shocks.

Under these identification assumptions, combining estimates from the scale-substitution regression (32) and the KM regression yields the following consistent estimator for $b_1\theta$:

$$\widehat{b_1\theta} = \widehat{d}_1 - \frac{1}{\widehat{\delta}_{x_1,x_2}}(\widehat{\beta_{KM}} - \widehat{d}_2), \tag{33}$$

where $\widehat{\delta}_{x_1,x_2}$ is the regression coefficient from the univariate regression of $x_{i,1}$ on $x_{i,j,2}$. We sketch here the idea for the proof of equation (33), providing the formal derivation in the appendix. The estimator exploits the fact that the substitution effect can be estimated in two ways. The scale-substitution regression delivers one estimate, which is biased by supply-demand correlation ($\widehat{d}_2$). The KM regression, by contrast, provides a consistent estimate of the same substitution effect because firm fixed effects purge firm-level demand ($\widehat{\beta_{KM}}$). The difference between these two estimates thus isolates the bias term, allowing us to back out the covariance between loan supply and demand. This, in turn, allows us to correct the biased estimate of the scale effect in the scale-substitution regression ($\widehat{d}_1$), thereby obtaining a measure of the impact of the loan supply shock on total firm borrowing.

Slightly more formally, we show that under the two identifying assumptions $x_{i,j,2}$ is not correlated with the demand shock $\Delta\log\tilde{B}_i$, and hence the error term $e_{i,j}$ is not correlated with $x_{i,j,2}$. The bias in d_1 therefore stems only from the correlation between $e_{i,j}$ and $x_{i,1}$. At the same time, the bias in d_2 is determined by the correlation between $e_{i,j}$ and $x_{i,1}$ and the correlation between $x_{i,1}$ and $x_{i,j,2}$. We can therefore use the difference between the consistent β_{KM} estimator and the biased d_2 estimator to recover the covariance between $x_{i,1}$ and $e_{i,j}$, and use it to debias d_1 to obtain an estimate of $b_1\theta$. As such, the obtained estimator in equation (33) relies on the difference between β_{KM} and d_2 and the regression coefficient $\widehat{\delta}_{x_1,x_2}$, which measures the correlation between $x_{i,j,2}$ and $x_{i,1}$.[21]

To empirically implement the estimator for $b_1\theta$ one runs the standard KM regression, the scale-substitution regression in equation (32), along with the regression of x_1 on x_2, combining the coefficients according to

[21] A technical requirement for the estimator in (33) is that $x_{i,1}$ and $x_{i,j,2}$ are correlated, i.e., that $\delta_{x_1,x_2} \neq 0$. This requirement is generally satisfied given the way in which the shares $s_{i,j}$ enter the definitions of both variables.

equation (33).

In sum, this process results in (1) a consistent estimator of the scale elasticity $b_1\theta$, and (2) by using the KM regression to obtain $-\frac{b_1}{\rho-1}$, a consistent estimate of $\left(b_1\theta s_{i,j} - \frac{b_1}{\rho-1}s_{i,-j}\right)$, the elasticity of firm i's borrowing from bank j with respect to bank j's specific loan supply shifter, w_j. Standard errors can be obtained using bootstrap.

5.2. The Impact of Total Supply Shocks

In this section we estimate the impact of total loan supply shocks – as opposed to the impact of a specific supply shifter – on each bank's lending to the firm and on total firm lending. Equation (5) shows that the impact of total supply shocks on total firm lending across the two banks is given by $x^*_{s,i} = \theta s_1 \Delta\log a_1 + \theta s_2 \Delta\log a_2$. Further, as shown in equation (8), the impact of a total supply shock to bank j on borrowing from bank j is given by $x^*_{s,i,j} = \left(\theta s_j - \frac{1}{\rho-1}s_{-j}\right)\Delta\log a_j$. Note that these two objects are not elasticities of lending to the total supply shocks, but rather the impact of the total supply shocks, $\Delta\log a_j$. It is natural to look at the impact of the total supply shock, rather than the elasticity, since the total supply shocks themselves are unobservable.

We estimate these two objects by combining estimates from the bank and firm fixed effects regression in (25) together with estimates of the lending elasticity of specific supply shocks discussed in Section 5.1 in the following manner. As shown in equation (27), since the loan shares $s_{i,j}$ are observed, the only thing remaining to recover in order to estimate $x^*_{s,i}$ and $x^*_{s,i,j}$ is $\theta\Delta\log a_j$. This latter object is proportional to the bank fixed effect, ζ_j, but is off by a scaling factor of $-\theta(\rho-1)$. We thus need to rescale the bank fixed effects. This can be done by using the estimates obtained from a specific loan supply shock, $b_1\theta$ and $\beta_{KM} = -\frac{b_1}{\rho-1}$, as discussed in Section 5.1, and dividing one by the other. In particular, we have that

$$\theta\Delta\log a_j = \underbrace{-\frac{1}{(\rho-1)}\Delta\log a_j}_{\zeta_j}\frac{b_1\theta}{\beta_{KM}}. \tag{34}$$

Having estimated $\theta\Delta\log a_j$ and $-\frac{1}{\rho-1}\Delta\log a_j$, we can then easily estimate the two objects of interest $\theta s_1\Delta\log a_1 + \theta s_2\Delta\log a_2$ and $\left(\theta s_j - \frac{1}{\rho-1}s_{-j}\right)\Delta\log a_j$, which measure the impact of total loan supply shocks on total firm-level and loan-level lending, respectively.

In sum, by combining results from the regression with bank and firm fixed effects together with information from a specific, bank-level shock to loan supply, w_j, we can estimate the change in lending to firms as a result of the sum total of loan supply shocks experienced by banks (and not just due to the specific loan supply shock captured by w_j).

5.2.1. Applications Using Total Supply Shocks

In the prior section we have shown how our framework can be used to recover the change in total lending to firm i due to all loan supply shocks experienced by the banks that lend to it. This was captured by the variable $x^*_{s,i} = \theta s_1 \Delta \log a_1 + \theta s_2 \Delta \log a_2$. It is then straightforward to recover the change in total lending to the firm driven by changes in demand. Indeed, as shown in (27), the change in lending due to demand factors is given by $x^*_{d,i} = \phi_i - \tau_i$, and from the discussion above $\tau_i = x^*_{s,i} - \bar{\zeta}_i$. We thus have both demand and supply-side effects.

Real Effect Regressions As discussed in section 4.2.1, a typical use of regressions with both bank and firm fixed effects is to analyze how changes in loan supply affect real outcomes (such as employment changes, investment, etc.). The standard approach in the literature is to proxy for supply and demand shocks with the bank and firm fixed effects, respectively. As we have shown in section 4.2.1, the firm fixed effect captures both loan demand and loan supply-side effects, which implies that the impact of loan supply shocks is mismeasured. However, with the decomposition of firm-level lending into loan demand and loan supply effects provided above, it is straightforward to analyze how total loan supply shocks affect various firm-level real outcomes, by running regression (30) using estimates for $x^*_{d,i}$ and $x^*_{s,i}$.

Decomposing Aggregate Lending to Demand and Supply Next, we use our framework to decompose the bank-level changes in lending into demand and supply effects. Following Amiti and Weinstein (2018), we have that the firm and bank fixed effects can be used to decompose the change in bank-level lending in the following manner:[22]

$$\overline{D_j} = \zeta_j + \sum_{i \in I_j} q_{ij} \phi_i, \tag{35}$$

where $\overline{D_j}$ is the average change in bank j's loans to all the firms it lends to; ζ_j and ϕ_i are the bank and firm fixed effects, respectively, obtained from the firm and bank fixed effect regression; $q_{ij} = \frac{L_{ij,t-1}}{\sum_{i \in I_j} L_{ij,t-1}}$ is the share of lending to firm i out of total lending by bank j, and I_j is the set of firms borrowing from bank j.

To obtain a decomposition of the change in lending into demand and supply effects, the following calcu-

[22]To obtain the exact aggregation results from Amiti and Weinstein (2018), note that the set of ζ_j and ϕ_i should be obtained from an OLS regression where the dependent variable is the percent change in loans rather than their log difference. Furthermore, to obtain aggregate growth in bank j's lending (rather than the average), one needs to run the regression with WLS. Because we assume no entry or exit, the Amiti and Weinstein (2018) corrections for such events are not relevant.

lation needs to be applied to the bank and firm fixed effects:

$$
\begin{aligned}
\bar{D}_j &= \zeta_j + \sum_{i \in I_j} q_{ij} \phi_i \\
&= \zeta_j + \sum_{i \in I_j} q_{ij} \left(x^*_{d,i} + \tau_i \right) \\
&= \sum_{i \in I_j} q_{ij} (\tau_i + \zeta_j) + \sum_{i \in I_j} q_{ij} x^*_{d,i} \\
&= \underbrace{\sum_{i \in I_j} q_{ij} x^*_{s,i,j}}_{\text{Own Supply}} + \underbrace{\sum_{i \in I_j} q_{ij} x^*_{s,i,-j}}_{\text{Peer Supply}} + \underbrace{\sum_{i \in I_j} q_{ij} x^*_{d,i}}_{\text{Demand}}.
\end{aligned} \tag{36}
$$

The first line is the Amiti and Weinstein (2018) decomposition into firm and bank fixed effects, while the subsequent lines are algebraic manipulations directly from equation (28). Equation (36) provides the decomposition of bank-level lending into supply and demand elements, where the fact that the firm fixed effects also include loan supply effects (as captured by τ_i; see equation (27)) has been accounted for. The loan supply component is divided into two: (1) The "own supply" element which captures how bank j's total lending is influenced by supply shocks experienced by bank j itself, and (2) The "peer supply" element which captures how bank j's total lending is influenced by supply shocks experienced by bank j's "co-lenders" – i.e., the banks that lend to the firms to which bank j lends.[23]

Finally, one can also decompose the average loan growth in the economy into demand and supply effects relying on $\bar{D} = \sum_j z_j \bar{D}_j$, where $z_j = \frac{\sum_{i \in I_j} L_{ij,t-1}}{\sum_j \sum_{i \in I_j} L_{ij,t-1}}$.

6. Extensions

6.1. Testing for the Correlation between Supply and Demand Shocks

To the extent that loan supply and loan demand are uncorrelated, the empirical question of the impact of loan supply shocks on firm-level outcomes is greatly simplified. Indeed, if the two are uncorrelated, to estimate the scale effect, $b_1\theta$, one can simply run the firm-level regression in equation (17), $\Delta \log L_i = \beta_0 + \beta_1 x_{i,1} + e_i$, with $x_{i,1} = s_{i1}w_1 + s_{i2}w_2$ and $e_i = \Delta \log \tilde{B}_i + \tilde{v}_i + \tilde{\chi}_i$. Furthermore, one can run similar regressions with other firm-level outcomes on the left-hand side to study real effects of loan supply shocks. It is thus useful to provide a test that examines whether loan supply and demand shocks are correlated.

[23] Note, that as is clear from (36), the corrected decomposition inherits the aggregation properties shown in Amiti and Weinstein (2018).

As a starting point, note that one can run the firm-level regression if $cov(x_{i,1}, e_i) = 0$. As we show in the appendix, maintaining the identification assumption that $E\left[\Delta \log u_{ij} + \chi_j \,\middle|\, w_j, w_{-j}, s_{i,j}\right] = 0$, this covariance of interest, $cov(x_{i,1}, e_i)$ equals the covariance of $x_{i,1}$ with the error term in the *loan*-level regression, $cov(x_{i,1}, e_{i,j})$. As discussed in Section 5.1.1, part of the estimation process to obtain a consistent estimator of $b_1\theta$ involves obtaining an estimate of $cov(x_{i,1}, e_{i,j})$. Indeed, as shown in the proof for equation (33), this is given by:

$$\widehat{cov(x_1, e_{i,j})} = \frac{\widehat{\sigma}^2_{x_1} - \widehat{\sigma}^2_{x_2}\widehat{\delta}^2_{x_1,x_2}}{\widehat{\delta}_{x_1,x_2}}(\widehat{\beta}_{KM} - \widehat{d}_2), \tag{37}$$

where $\widehat{\sigma}_1$ is the standard deviation of $x_{i,1}$, and $\widehat{\sigma}_2$ is the standard deviation of $x_{i,j,2}$.

Thus, to test whether it is possible to obtain a consistent estimate of $b_1\theta$ by running the firm-level regression, one can test the null that $\widehat{cov(x_1, e_{i,j})} = 0$ using equation (37). Failure to reject the null suggests that the firm-level regression provides a consistent estimator of the scale effect, $b_1\theta$.

The test in equation (37) is different from an alternative test employed in the literature. This alternative test involves running the KM regression both with and without firm fixed effects, and comparing the resultant coefficients on the supply shifter, $w_{i,j}$. The literature has interpreted a small difference in the coefficients $\widehat{\beta}_{KM} - \widehat{\beta}_{KM,NoFE}$ as indicative of a lack of correlation between the supply and demand shocks, which would imply that the firm level regression can be used.

However, this procedure does not provide a valid test for the correlation between demand and supply shocks. To see this, it is instructive to examine the alternative test through the lens of our model. As discussed above, the firm-level regression can be used to estimate the scale effect when $cov(x_{i,1}, e_i) = 0$. The question is thus: what does the difference $\widehat{\beta}_{KM} - \widehat{\beta}_{KM,NoFE}$ indicate regarding $cov(x_{i,1}, e_i)$? As we show in the appendix, this difference does not consistently estimate $cov(x_{i,1}, e_i)$, and, as such, a small difference $\widehat{\beta}_{KM} - \widehat{\beta}_{KM,NoFE}$ does not imply that this covariance term is small.

To see the intuition for this, it is useful to rewrite the scale-substitution equation (18) as follows:

$$\Delta \log L_{i,j} = \text{constant} - \frac{b_1}{\rho - 1} w_j + \underbrace{b_1\left(\theta + \frac{1}{\rho - 1}\right)(s_{i,j}w_j + s_{i,-j}w_{-j}) + \Delta \log \tilde{B}_i + \nu_{i,j} + \tilde{\chi}_{i,j}}_{e^{NFE}_{i,j}}. \tag{38}$$

As is shown in this equation, in addition to the demand shocks $\Delta \log \tilde{B}_i$, the error term in the no fixed effect KM regression includes another firm level variable which is proportional to the scale effect. As we show in the appendix, this implies that the difference in population regression coefficients, $\beta_{KM,NoFE} - \beta_{KM}$ is given by:[24]

$$\beta_{KM,NoFE}-\beta_{KM}=\frac{cov(w_j,e_{i,j}^{NFE})}{var(w_j)}=\frac{cov\left(w_j,\Delta\log\tilde{B}_i\right)}{var(w_j)}+b_1\left(\theta+\frac{1}{\rho-1}\right)\frac{cov\left(w_j,(s_{i,j}w_j+s_{i,-j}w_{-j})\right)}{var(w_j)}. \tag{39}$$

The first equality in (39) shows, the difference $\beta_{KM,NoFE}-\beta_{KM}$ is informative about $cov(w_j,e_{i,j}^{NFE})$ rather than about the covariance of interest $cov(w_j,e_i)$. As we show in the appendix, these two objects are quite different.

Further, the second equality in (39) provides intuition for why finding $\widehat{\beta}_{KM,NoFE}-\widehat{\beta}_{KM}=0$ actually suggests that the firm-level lending regression will tend to *overestimate* the impact of the supply shock. To see this, note that the covariance between the loan- and firm-level supply shocks, $cov\left(w_j,(s_{i,j}w_j+s_{i,-j}w_{-j})\right)$, will tend to be positive, implying that finding that $\widehat{\beta}_{KM}-\widehat{\beta}_{KM,NoFE}=0$ must be accounted for by $cov\left(w_j,\Delta\log\tilde{B}_i\right)<0$.[25] This implies a *positive* correlation of demand and supply shocks (because high w_j corresponds to a negative supply shock). As such, a finding that $\widehat{\beta}_{KM}-\widehat{\beta}_{KM,NoFE}=0$ suggests that the firm-level regressions tend to overestimate the impact of loan supply shocks.

6.2. Relation to the Estimator in Jiménez et al. (2020)

Jiménez et al. (2020) develops an estimator for the elasticity on total firm lending – i.e., for what we denote by $b_1\theta$.[26] However, as we discuss here, this estimator does not identify this elasticity. Similar to our approach, the method in Jiménez et al. (2020) seeks to estimate the covariance between supply and demand shocks in order to capture the total firm-level lending effect. In particular, their method involves running the KM regression with and without firm fixed effects and using the difference in the coefficients on the supply shifter from the two regressions, $\widehat{\beta}_{KM}-\widehat{\beta}_{KM,NoFE}$, to recover a covariance term which they use in order to correct the firm-level borrowing regression. But, as discussed in Section 6.1, and using the notation from there, the difference $\widehat{\beta}_{KM}-\widehat{\beta}_{KM,NoFE}$ can be used to recover the covariance $cov(w_j,e_{i,j}^{NFE})$, which is different from the covariance term required to correct the firm-level regression, given by $cov(x_{i,1},e_i)$.

Specifically, correcting the firm-level regression involves removing from the firm-level regression coefficient the bias $\frac{cov(x_{i,1},e_i)}{var(x_{i,1})}$. Rather than performing this correction with an estimate of $cov(x_{i,1},e_i)$, Jiménez et al. (2020) does so with an estimate of $cov(x_{i,1},e_{i,j}^{NFE})$ obtained by calculating $\widehat{\beta}_{KM}-\widehat{\beta}_{KM,NoFE}$. As shown

[24] The second equality in (39) is derived by applying the identification assumption $E\left[\Delta\log u_{ij}+\chi_j\,\middle|\,w_j,w_{-j},s_{i,j}\right]=0$. Without this assumption, the equation includes a third term given by $\frac{cov(w_j,v_{i,j}+\tilde{\chi}_{i,j})}{var(w_j)}$.

[25] $b_1\left(\theta+\frac{1}{\rho-1}\right)>0$ from (24).

[26] Jiménez et al. (2020) do not distinguish between what we call specific loan supply shifters (w) and total loan supply shocks (a). However, they apply their framework to Spanish banks' exposure to real estate, which corresponds to a specific loan supply shifter.

in Section 6.1, these two covariances are different.

6.3. Credit Rationing and Binding Constraints

The analysis so far has modeled bank supply shocks as shifts in banks' lending-cost schedules and has focused on interior borrowing choices. An alternative possibility is that bank shocks operate through the degree of credit rationing: they tighten or relax the maximum amount of credit that a bank is willing to extend.

It is useful to consider an extreme benchmark in which the firm is constrained at both of its lending banks. In this case, realized borrowing from each bank is determined by the corresponding credit limit, rather than by an interior optimality condition. A loan-supply shock to bank j therefore shifts the constraint faced by the firm at that bank. Since the firm is already constrained at both banks, the shock does not induce a reallocation of borrowing across lenders: a tightening of the constraint at bank 1 changes borrowing from bank 1, but does not change borrowing from bank 2. The cross-lender interaction terms that discipline substitution in the interior model are therefore absent.

In the pure rationing benchmark, it can easily be shown that this implies that the distinction between the substitution effect, the loan-level effect, and the scale effect collapses. Each response is governed by the same underlying object: the effect of the bank-level rationing shock on the binding credit limit. Consequently, applying the estimation procedure developed above would deliver consistent estimates of this same object across the different specifications. As such, empirically finding that these estimates differ – for example, that β_{KM} differs from $b_1\theta$ – implies that the data are not consistent with the extreme rationing model in which all relevant lending relationships are simultaneously binding. Such a finding points instead to the presence of an interior margin of adjustment.

In practice, bank shocks may affect both the marginal cost of funds and the degree of credit rationing. Intuitively, in such an environment, the estimated effect of a bank supply shifter (w_j) on borrowing will combine an interior cost-shift component with a rationing component. Separating these two channels requires additional structure and is beyond the scope of this paper.

6.4. Specialization and Sorting

A central concern in the literature on estimating the impact of loan supply shocks is that firms do not randomly match to banks. It is therefore useful to clarify which forms of sorting are accommodated by the framework and which are not. The classic concern is sorting between weak banks and weak firms: banks exposed to negative supply shocks may be disproportionately matched with firms facing negative demand shocks. This is precisely the demand–supply correlation that the KM approach was designed to address through within-firm comparisons across lending relationships. Our analysis allows for this form of sorting as well.

A different issue arises when banks specialize in particular loan products or activities. Such specialization does not by itself invalidate the framework, provided that loans from different banks are perfect substitutes in the firm's revenue function. In that case, a dollar borrowed from one bank may not be identical to a dollar borrowed from another, but each can be converted into the same effective borrowing input.

Complications arise when firms engage in multiple activities, lenders specialize across activities, and either loans from different lenders are not perfect substitutes across those activities or shocks to credit demand occur at the activity level. For example, term loans and credit lines may serve distinct roles in the firm's production, investment, or liquidity-management problem, and the activities financed by these instruments may be exposed to different shocks. If activity-level demand shocks are correlated with the shocks to banks that specialize in financing those activities, firm fixed effects no longer absorb all relevant demand variation. This identification problem, and its implications for within-firm estimators in the spirit of KM, is studied in Paravisini, Rappoport and Schnabl (2023).[27] Extending our framework to allow for non-fungible credit, activity-specific demand shocks, and bank specialization in particular activities is beyond the scope of the paper, but represents a natural direction for future research.

7. Application: The 2011 Spanish Debt Crisis

7.1. Background and Data

In this section, we apply our framework to the 2011 Spanish sovereign debt crisis. The bursting of the housing bubble in the late 2000s in Spain burdened Spanish banks with distressed real estate assets, triggering financial instability (see, for example, Baudino, Herrera and Restoy (2023)). We apply our methodology to this setting to measure the impact of loan supply shocks and contrast our results with those that would be generated with common empirical strategies used in the existing literature.

We employ a detailed dataset from the Bank of Spain's confidential credit register (CIR), which includes loan-level credit registry data in Spain, combined with firm-level financial information available from the Spanish Mercantile Register. We use the degree of banks' exposure as of the end of 2011 to real estate loans

[27]As Paravisini, Rappoport and Schnabl (2023) shows, even in the presence of such specialization, if conditional on the firm fixed effects, the activity-level demand shocks are uncorrelated with the loan supply shifter then the KM identification assumption still holds.

as a "specific supply shifter" to proxy for the severity of lending constraints. Our analysis proceeds in two steps: first, we estimate the impact of loan supply shocks resulting from this specific real-estate-exposure supply shifter, and then, we estimate the impact of the total loan supply shock affecting banks. In doing so, we analyze both lending and real effects.

To match the analysis to our model and empirical framework, we consider firms with two or more lenders in 2011 and 2012 that also have data on firm-level characteristics. Table 1 presents the descriptive statistics. There are a total of 42,295 firms. The average decline in lending between 2011 and 2012 is approximately 15%. In our analysis, we use real estate exposure of banks as the loan supply shifter. Real estate exposure is measured as the ratio of a bank's total real estate lending to its total lending. The sample is comprised of 119 banks, which on average have a real estate exposure of 57%.

7.2. The Impact of Real-Estate Exposure

Table 2 presents the estimates for the impact of real estate exposure obtained using our empirical framework and compares them to results obtained using the standard empirical techniques in the literature. The first row in the table reports the result of a standard KM lending regression, which uses the log change in lending at the bank-firm level between 2011 and 2012 as the dependent variable, the real estate exposure as the loan supply shifter, and which includes firm fixed effects to control for unobservable loan demand. As our framework shows, the KM coefficient on the real estate exposure (-0.221) captures only the substitution effect $-\frac{1}{\rho-1}$, scaled by b_1.

Rows 2 and 3 of Table 2 show the results of the scale-substitution regression, equation (32). The coefficients of interest d_1 and d_2 are -0.07 and -0.216, respectively. These coefficients are potentially biased for the scale and substitution effects since the demand shocks are not accounted for in the regression. We then apply equation (33) and obtain the estimate of $b_1\theta$, shown in the fourth row of Table 2, which equals -0.093. Because the difference between the biased and unbiased estimates of the substitution effect (d_2 and β_{KM}, respectively) is small, the estimated covariance term, $cov(x_{i,j,1}, e_{i,j})$, which captures the correlation between demand and supply shocks, is close to zero and statistically not different from it (p-value = 0.491). This explains why the estimated scale effect, $b_1\theta$, is close to the biased scale effect d_1. In fact, because we cannot reject that the covariance term is equal to zero, we can also run the firm-level lending regression as explained in section 6.1 to consistently estimate $b_1\theta$. When we do so, the resultant value is -0.057 (s.e. 0.011). This estimate is not statistically different from the estimate above obtained using our estimation procedure (p-value = 0.27).

Finally, the last part of Table 2 reports the estimate of the average *loan*-level elasticity to be -0.149.[28] As shown by the theoretical discussion, indeed this value is smaller (in absolute value) compared to β_{KM}, the latter of which is typically treated as the loan-level effect. Thus, if β_{KM} is used as an estimate of the loan level effect, one overestimates the loan-level effect by about 50%. Because the loan-level effect varies by the borrowing shares ($s_{i,j}$), the average loan-level effect masks a good deal of heterogeneity: at the 10th percentile of the distribution, the loan-level effect is -0.173 while at the 90th percentile it is -0.125.

7.3. The Impact of Total Supply Shocks on Real Outcomes

We now turn to estimating the impact of total supply shocks on lending and real outcomes. We start with the estimation of a lending regression with bank and firm fixed effects (equation (25)). Using our estimates of $b_1\theta$ and β_{KM}, along with equation (34), we recover the total supply shocks experienced by different banks, $x^*_{s,j}$. Figure 1 compares the distribution of supply shocks at the bank level ($x^*_{s,j}$) to the distribution of bank fixed effects (ζ_j). Given our theoretical framework, which implies that true supply shocks are a scaled version of the bank fixed effects (equation (34)), the distribution of supply shocks in the figure is a compressed version of the distribution of bank fixed effects. This highlights that the bank fixed effects overstate the extent of total supply shocks experienced by banks.

Next, following the discussion in 5.2.1 we recover estimates for the firm-level supply and demand shocks, $x^*_{s,i}$ and $x^*_{d,i}$. This allows us to examine the impact of supply shocks on real outcomes including investment, employment, and value added. As discussed in section 4.2.1, the regression of real outcomes on the estimated bank and firm fixed effects from the lending regression delivers biased estimates for the impact of supply shocks. The results in Table 3 demonstrate that in our sample, the difference between the biased and corrected estimates is large. In particular, from column (1), a one standard deviation change in averaged bank FE ($\bar{\zeta}_i$) is associated with a 0.66 percent increase in long-term assets growth. Correcting for the bias, column (2) shows that a one standard deviation increase in total supply ($x^*_{s_i}$) is associated with only a 0.32 percent increase in long-term assets growth – approximately half the magnitude of the biased coefficient.[29] Columns (3) to (6) reveal similar patterns for employment and for value added growth, with the corrected coefficients different from the biased coefficients using the bank and firm fixed effects.

[28] Because the loan-level elasticity depends on loan shares, we first calculate the average loan-level effect at the bank level, using loan shares across firms as weights, and then report the mean, 10th percentile, and 90th percentile of this distribution across banks.

[29] Recall that a more positive $x^*_{s_i}$ reflects a larger positive supply shock.

8. Conclusion

This paper revisits one of the central empirical challenges in banking and macro-finance: how to estimate the impact of loan supply shocks on lending and real outcomes. We show that commonly used approaches do not, in general, recover the economic objects they are often interpreted to measure. The Khwaja and Mian (2008) estimator is informative about whether banks transmit shocks to firms, but its magnitude reflects substitution across lenders rather than the loan-level effect of a supply shock. Furthermore, approaches that interpret bank and firm fixed effects as supply and demand shocks in the spirit of Amiti and Weinstein (2018) misattribute part of the supply shock to firm fixed effects, leading to biased estimates of the impact of credit supply on borrowing and real outcomes.

Using a model with standard ingredients, we derive estimators that recover the relevant objects of interest: the effect of specific supply shifters on loan-level lending, their effect on total firm borrowing, and the impact of total bank supply shocks on credit and real activity. The methodology also provides a test for whether loan supply and loan demand are correlated.

Applying the framework to the 2011 Spanish debt crisis, we find that the KM approach overestimates the loan-level effect. We further find that the effect on total firm borrowing is approximately half of the loan-level effect. Finally, methods using bank fixed effects and firm fixed effects (in the spirit of AW) overestimate the real effects of total loan supply shocks.

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Table 1: Descriptive Statistics

	mean	s.d.
Panel A: Loan-level		
Lending ($L_{i,j}$, thousand euros)	267	655
$\Delta \log L_{i,j}$	-0.151	0.442
Real estate exposure (w_j)	0.566	0.113
Share from each bank ($s_{i,j}$)	0.405	0.268
Panel B: Firm-level outcomes, levels		
Long-term assets (thousand euros)	1,277	3,706
Employment	10.833	23.128
Value added (thousand euros)	470	1,260
Panel C: Firm-level outcomes, changes		
Δlog long-term assets	-0.056	0.280
Δ employment	-0.038	0.355
Δlog value added	-0.117	0.520
Number of firms	42,295	
Number of banks	119	
Unique bank-firm observations	104,423	

Notes: Lending, long-term assets, employment and value added, as well as their change measures, are winsorized at the 1% level. Change measures are log differences, except for employment, where we use percentage changes.

Table 2: Impact of Loan Supply Shock: Spanish Debt Crisis

	Interpretation	Source	Estimate (s.e.)
β_{KM}	Substitution effect $(-\frac{b_1}{\rho-1})$	Standard KM reg	-0.221 (0.015)
d_1	Biased scale effect	scale-substitution	-0.070 (0.016)
d_2	Biased substitution effect	scale-substitution	-0.216 (0.015)
Total firm-level borrowing effect			
$b_1\theta$		applying section 5.1.1	-0.093 (0.034)
Loan-level effect			
Mean		applying section 5.1.1	-0.149 (0.022)
P_{10}		applying section 5.1.1	-0.173 (0.017)
P_{90}		applying section 5.1.1	-0.125 (0.027)

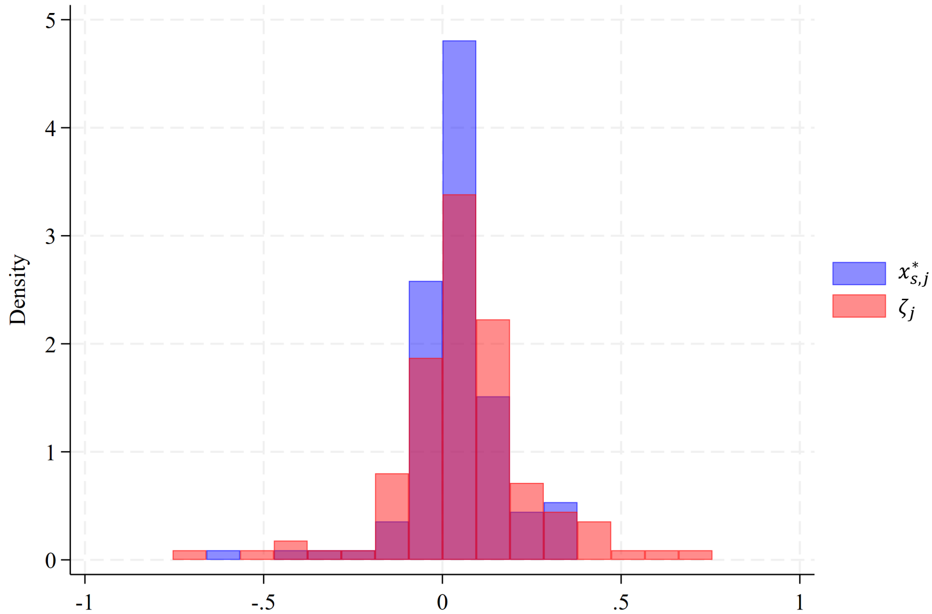
Notes: Standard errors are clustered at the firm level, and are calculated using block bootstrap.

Table 3: Real Effect Regressions

	Long-Term Assets Growth		Employment Growth		Value Added Growth	
	(1)	(2)	(3)	(4)	(5)	(6)
	Bank FE $(\bar{\zeta}_i)$	Supply Shock $(x^*_{s,i})$	Bank FE $(\bar{\zeta}_i)$	Supply Shock $(x^*_{s,i})$	Bank FE $(\bar{\zeta}_i)$	Supply Shock $(x^*_{s,i})$
Total Supply Shock $\times 100$	0.660	0.322	0.010	-0.194	0.611	0.401
(s.e.)	(0.130)	(0.208)	(0.178)	(0.211)	(0.272)	(0.290)
Observations	41,683		38,124		37,975	

Notes: The table reports regressions of firm-level outcomes on supply shocks. In columns (1) and (2), the dependent variable is the log change in long-term assets. In columns (3) and (4), the dependent variable is the percentage change in employment. In columns (5) and (6), the dependent variable is the log change in value added. In columns (1), (3), and (5), the explanatory variable is the bank fixed effect $(\bar{\zeta}_i)$, controlling also for the firm fixed effect (ϕ_i). In columns (2), (4), and (6), the explanatory variable is the supply shock $(x^*_{s,i})$, controlling also for the demand shock $(x^*_{d,i})$. Standard errors are clustered at the firm level, and are calculated using block bootstrap.

Figure 1: Bank-Level Total Supply Shock Distribution



Online Appendix
Not for Publication

A. Proofs

A.1. Section 2 Proofs

A.1.1. Cost Minimization

Cost minimization problem is given by equation (3) in the paper:

$$C(L) \;:=\; \min_{L_1,L_2} \left\{ a_1 L_1^{\rho} + a_2 L_2^{\rho} \;\middle|\; L_1 + L_2 = L \right\}, \quad \rho > 1. \tag{3}$$

Since $L_1 + L_2 = L$, minimize the one–variable objective:

$$f(L_1) \;:=\; a_1 L_1^{\rho} + a_2 (L - L_1)^{\rho} \quad \text{over } L_1 \in [0, L].$$

First-order condition.

$$a_1 L_1^{\rho-1} = a_2 (L - L_1)^{\rho-1}.$$

Rearranging:

$$\frac{L_1}{L - L_1} = \left(\frac{a_1}{a_2}\right)^{\frac{1}{1-\rho}} \iff \frac{L_1}{L_2} = \left(\frac{a_1}{a_2}\right)^{\frac{1}{1-\rho}}. \tag{A.1}$$

Solving for the optimizer:

$$\frac{L_1}{L_2} = \frac{a_1^{\frac{1}{1-\rho}}}{a_2^{\frac{1}{1-\rho}}} \implies L_1 = \frac{a_1^{\frac{1}{1-\rho}}}{a_1^{\frac{1}{1-\rho}} + a_2^{\frac{1}{1-\rho}}} L, \qquad L_2 = \frac{a_2^{\frac{1}{1-\rho}}}{a_1^{\frac{1}{1-\rho}} + a_2^{\frac{1}{1-\rho}}} L.$$

Value at the minimizer. Plugging this in, we get:

$$a_1 L_1^{\rho} = a_1 \left(\frac{a_1^{\frac{1}{1-\rho}}}{a_1^{\frac{1}{1-\rho}} + a_2^{\frac{1}{1-\rho}}} L \right)^{\rho} = L^{\rho} \frac{a_1^{1+\frac{\rho}{1-\rho}}}{\left(a_1^{\frac{1}{1-\rho}} + a_2^{\frac{1}{1-\rho}}\right)^{\rho}} = L^{\rho} \frac{a_1^{\frac{1}{1-\rho}}}{\left(a_1^{\frac{1}{1-\rho}} + a_2^{\frac{1}{1-\rho}}\right)^{\rho}},$$

and similarly,

$$a_2 L_2^{\rho} = L^{\rho} \frac{a_2^{\frac{1}{1-\rho}}}{\left(a_1^{\frac{1}{1-\rho}} + a_2^{\frac{1}{1-\rho}}\right)^{\rho}}.$$

Adding both terms:

$$C(L) = a_1 L_1^{\rho} + a_2 L_2^{\rho} = L^{\rho} \frac{a_1^{\frac{1}{1-\rho}} + a_2^{\frac{1}{1-\rho}}}{\left(a_1^{\frac{1}{1-\rho}} + a_2^{\frac{1}{1-\rho}}\right)^{\rho}} = L^{\rho} \left(a_1^{\frac{1}{1-\rho}} + a_2^{\frac{1}{1-\rho}} \right)^{1-\rho}.$$

Therefore,

$$C(L) = \kappa_C L^\rho, \qquad \kappa_C = \left(a_1^{\frac{1}{1-\rho}} + a_2^{\frac{1}{1-\rho}}\right)^{1-\rho}.$$

which is equation (4) in the paper.

A.1.2. Elasticity of Substitution Between Banks

First-order condition of the cost minimization problem gives:

$$\frac{L_1}{L_2} = \left(\frac{a_1}{a_2}\right)^{\frac{1}{1-\rho}}, \qquad \rho > 1.$$

Taking logs on both sides:

$$\log\left(\frac{L_1}{L_2}\right) = -\frac{1}{\rho - 1}\log\left(\frac{a_1}{a_2}\right).$$

A.1.3. Impact on Total Firm Borrowing

The firm solves:

$$\max_L \underbrace{R(L) - L}_{G(L)} - C(L) \quad \text{with} \quad C(L) = \kappa_C L^\rho,\ \kappa_C = \left(a_1^{\frac{1}{1-\rho}} + a_2^{\frac{1}{1-\rho}}\right)^{1-\rho},$$

where $L = L_1 + L_2$ and $s_j = \frac{L_j}{L}$.

The first-order condition yields $G'(L) = C'(L)$. Taking logs and differencing between time t and $t+1$:

$$\Delta \log G'(L) = \Delta \log C'(L) \tag{A.2}$$

Taking the first-order approximation of the left hand side of (A.2):

$$\Delta \log G'(L) \approx \eta_{G',L} \Delta \log L. \tag{A.3}$$

Similarly, for the right hand side of (A.2):

$$\Delta \log C'(L) = \Delta \log \kappa_C + \underbrace{(\rho - 1)}_{\eta_{C',L}} \Delta \log L.$$

Moreover,

$$\Delta \log \kappa_C = \Delta \log \left(a_1^{\frac{1}{1-\rho}} + a_2^{\frac{1}{1-\rho}} \right)^{1-\rho} \approx (1-\rho) \sum_{j=1}^{2} \frac{a_j^{\frac{1}{1-\rho}}}{a_1^{\frac{1}{1-\rho}} + a_2^{\frac{1}{1-\rho}}} \Delta \log \left(a_j^{\frac{1}{1-\rho}} \right)$$

$$= \sum_{j=1}^{2} \frac{a_j^{\frac{1}{1-\rho}}}{a_1^{\frac{1}{1-\rho}} + a_2^{\frac{1}{1-\rho}}} \Delta \log (a_j) = \sum_{j=1}^{2} s_j \Delta \log (a_j)$$

since $s_j = \frac{L_j}{L} = \frac{a_j^{\frac{1}{1-\rho}}}{a_1^{\frac{1}{1-\rho}} + a_2^{\frac{1}{1-\rho}}}$ from (A.1).

Thus:

$$\Delta \log C'(L) \approx s_1 \Delta \log a_1 + s_2 \Delta \log a_2 + \eta_{C',L} \Delta \log L. \tag{A.4}$$

Plugging in (A.3) and (A.4) into (A.2), we arrive at:

$$\eta_{G',L} \Delta \log L \approx s_1 \Delta \log a_1 + s_2 \Delta \log a_2 + \eta_{C',L} \Delta \log L$$

Rearranging:

$$\Delta \log L \approx \frac{1}{\eta_{G',L} - \eta_{C',L}} \left(s_1 \Delta \log a_1 + s_2 \Delta \log a_2 \right). \tag{A.5}$$

Defining $\theta = \frac{1}{\eta_{G',L} - \eta_{C',L}}$, we arrive at equation (5) in the paper.

Also notice that:

$$\eta_{G',L} = \frac{\partial G'(L)}{\partial L} \frac{L}{G'(L)} = \frac{\partial R'(L)}{\partial L} \frac{L}{R'(L)-1} = \frac{R'(L)}{R'(L)-1} \underbrace{\left(\frac{L}{R'(L)} \frac{\partial R'(L)}{\partial L} \right)}_{\eta_{R',L}}.$$

At the optimum $R'(L) = C'(L) + 1$. Therefore:

$$\eta_{G',L} = \frac{1+C'}{C'} \eta_{R',L}.$$

Hence, we can express θ also as:

$$\theta = \frac{1}{\left(\frac{1+C'}{C'} \eta_{R',L} - \eta_{C',L} \right)}.$$

This is equation (6) in the paper.

A.1.4. Impact on Loan Level Borrowing

By definition:

$$L_j = s_j L, \text{ where } j \in \{1,2\} \text{ and } s_j + s_{-j} = 1.$$

Taking logs and time difference between t and $t+1$:

$$\Delta \log L_j = \Delta \log s_j + \Delta \log L. \tag{A.6}$$

Since we know the first-order approximation of $\Delta \log L$ (see (A.5)), we only need to find first-order approximation of $\Delta \log s_j$.

From (A.1), we know that:

$$s_j = \frac{a_j^{\frac{1}{1-\rho}}}{a_j^{\frac{1}{1-\rho}} + a_{-j}^{\frac{1}{1-\rho}}}.$$

Taking logs and differentiating:

$$d\log s_j = d\log(a_j^{\frac{1}{1-\rho}}) - d\log(a_j^{\frac{1}{1-\rho}} + a_{-j}^{\frac{1}{1-\rho}}). \tag{A.7}$$

Right-hand side terms of (A.7):

$$d\log(a_j^{\frac{1}{1-\rho}}) = \frac{1}{1-\rho} d\log(a_j)$$

$$\begin{aligned} d\log(a_j^{\frac{1}{1-\rho}} + a_{-j}^{\frac{1}{1-\rho}}) &= \frac{1}{1-\rho}\left(\frac{a_j^{\frac{1}{1-\rho}} d\log(a_j) + a_{-j}^{\frac{1}{1-\rho}} d\log(a_{-j})}{a_j^{\frac{1}{1-\rho}} + a_{-j}^{\frac{1}{1-\rho}}}\right) \\ &= \frac{1}{1-\rho}\left(s_j d\log(a_j) + s_{-j} d\log(a_{-j})\right) \end{aligned}$$

Then:

$$\begin{aligned} d\log s_j &= \frac{1}{1-\rho} d\log(a_j) - \frac{1}{1-\rho}\left(s_j d\log(a_j) + s_{-j} d\log(a_{-j})\right) \\ &= \frac{1}{1-\rho} \underbrace{(1-s_j)}_{s_{-j}} d\log(a_j) - \frac{1}{1-\rho} s_{-j} d\log(a_{-j}) \\ &= -\frac{1}{\rho-1} s_{-j} d\log\left(\frac{a_j}{a_{-j}}\right) \end{aligned}$$

Taking the first-order approximation:

$$\Delta \log s_j \approx -\frac{1}{\rho-1} s_{-j} \Delta \log\left(\frac{a_j}{a_{-j}}\right). \tag{A.8}$$

Substituting (A.8) and (A.5) into (A.6), we arrive at:

$$\Delta \log L_j \approx \theta(s_1 \Delta \log a_1 + s_2 \Delta \log a_2) - \frac{1}{\rho-1} s_{-j} \Delta \log\left(\frac{a_j}{a_{-j}}\right),$$

which is equation (7) in the paper.

A.2. Section 3 Proofs

The cost minimization problem of firm i at time t is:

$$C_{it}(L_{it}) := \min_{L_{i1t},L_{i2t}} \left\{ a_{1t}u_{i1t}L_{i1t}^{\rho} + a_{2t}u_{i2t}L_{i2t}^{\rho} \,\middle|\, L_{i1t} + L_{i2t} = L_{it} \right\}, \qquad \rho > 1.$$

Solving the minimization problem as in Section A.1.1, we will arrive at:

$$C_{it}(L_{it}) = \kappa_{C,it}L_{it}^{\rho} \quad \text{where} \quad \kappa_{C,it} = \left(\sum_{j=1}^{2} (a_{jt}u_{ijt})^{\frac{1}{1-\rho}} \right)^{1-\rho}.$$

Moreover, the bank shares at the optimum satisfies:

$$s_{ijt} = \frac{L_{ijt}}{L_{it}} = \frac{(a_{jt}u_{ijt})^{\frac{1}{1-\rho}}}{\left((a_{jt}u_{ijt})^{\frac{1}{1-\rho}} + (a_{-jt}u_{i-jt})^{\frac{1}{1-\rho}} \right)}. \tag{A.9}$$

Firm i then maximizes:

$$\max_{L_{it}} \underbrace{B_{it}\tilde{R}(L_{it}) - L_{it}}_{G_{it}(L_{it},B_{it})} - C_{it}(L_{it}) \quad \text{where} \quad C_{it}(L_{it}) = \kappa_{C,it}L_{it}^{\rho}.$$

First-order condition yields:

$$\frac{\partial G_{it}(L_{it},B_{it})}{\partial L_{it}} = C'_{it}(L_{it}).$$

With some abuse of notation, we denote $\frac{\partial G_{it}}{\partial L_{it}}$ with G'_{it}. Taking logs and time difference between t and $t+1$:

$$\Delta \log G'_{it} = \Delta \log C'_{it}(L_{it}). \tag{A.10}$$

log linearizing the left-hand side of A.10, and recalling that G'_{it} is a function of L_{it} and B_{it} yields:

$$\Delta \log G'_{it} \approx \eta_{G',L}\Delta \log L_{it} + \eta_{G',B}\Delta \log B_{it}. \tag{A.11}$$

Next, we log-linearize the right-hand side of A.10. Start from:

$$C'_{it}(L_{it}) = \rho\,\kappa_{C,it}L_{it}^{\rho-1}.$$

Taking differential:

$$d\log C'_{it}(L_{it}) = d\log \kappa_{C,it} + \underbrace{(\rho-1)}_{\eta_{C',L}} d\log L_{it}.$$

Using the same algebra as in Section A.1.1, we can find $d\log \kappa_{C,it}$ as:

$$d\log \kappa_{C,it} = \sum_{j=1}^{2} s_{ijt}\, d\log(a_{jt}u_{ijt}) = \sum_{j=1}^{2} s_{ijt}\, d\log a_{jt} + \sum_{j=1}^{2} s_{ijt}\, d\log u_{ijt}.$$

Taking the first-order approximation using time t as the point of linearization:

$$\Delta \log C'_{it}(L_{it}) \approx (s_{i1t}\Delta \log a_{1t} + s_{i2t}\Delta \log a_{2t}) + (s_{i1t}\Delta \log u_{i1t} + s_{i2t}\Delta \log u_{i2t}) + \eta_{C',L}\Delta \log L_{it}. \tag{A.12}$$

Substituting A.11 and A.12 into A.10 and rearranging, we arrive at:

$$(\eta_{G',L} - \eta_{C',L})\Delta \log L_{it} \approx s_{i1t}\Delta \log a_{1t} + s_{i2t}\Delta \log a_{2t} + s_{i1t}\Delta \log u_{i1t} + s_{i2t}\Delta \log u_{i2t} - \eta_{G',B}\Delta \log B_{it}.$$

Defining $\theta = \frac{1}{\eta_{G',L} - \eta_{C',L}}$ as before, we get:

$$\Delta \log L_{it} \approx \theta(s_{i1t}\Delta \log a_{1t} + s_{i2t}\Delta \log a_{2t}) + \underbrace{\theta(s_{i1t}\Delta \log u_{i1t} + s_{i2t}\Delta \log u_{i2t})}_{\tilde{v}_{it}} - \theta\eta_{G',B}\Delta \log B_{it}.$$

Without loss of generality, assume that $\mathbb{E}(\log u_{ijt}) = 0$, and define the constant as the cross-sectional mean of $-\theta\eta_{G',B}\Delta \log B_{kt}$,i.e. constant$_t = \frac{1}{N}\sum_{k=1}^{N}\left(-\theta\eta_{G',B}\Delta \log B_{kt}\right)$. Define $\Delta \log \tilde{B}_{it} = -\theta\eta_{G',B}\Delta \log B_{it} -$ constant$_t$, then:

$$\Delta \log L_{it} = \text{constant}_t + \underbrace{\Delta \log \tilde{B}_{it}}_{x^\star_{d,i}} + \underbrace{\theta(s_{i1t}\Delta \log a_{1t} + s_{i2t}\Delta \log a_{2t})}_{x^\star_{s,i}} + \tilde{v}_{it}. \tag{A.13}$$

which is equation (15) in the paper.

Finally, note that $\eta_{G',B} = \frac{R'_{it}(L_{it})}{G'_{it}(L_{it})}$, as $G'_{it} = B_{it}\tilde{R}'(L_{it}) - 1$ (and $B_{it}\tilde{R}'(L_{it}) = R'(L_{it})$).

Next, we have that by definition:

$$L_{ijt} = s_{ijt}L_{it}.$$

Taking logs and differencing between time t and $t+1$:

$$\Delta \log L_{ijt} = \Delta \log s_{ijt} + \Delta \log L_{it}. \tag{A.14}$$

Since we know the first-order approximation of $\Delta \log L_{it}$ (see A.13), we only need to find the first-order linear approximation of $\Delta \log s_{ijt}$.

From A.9, we know that:

$$s_{ijt} = \frac{(a_{jt}u_{ijt})^{\frac{1}{1-\rho}}}{(a_{1t}u_{i1t})^{\frac{1}{1-\rho}} + (a_{2t}u_{i2t})^{\frac{1}{1-\rho}}}, \qquad \rho > 1,$$

Taking logs and totally differentiating:

$$
\begin{aligned}
d\log s_{ijt} &= d\log\left((a_{jt}u_{ijt})^{\frac{1}{1-\rho}}\right) \; - \; d\log\left((a_{1t}u_{i1t})^{\frac{1}{1-\rho}} + (a_{2t}u_{i2t})^{\frac{1}{1-\rho}}\right) \\
&= \frac{1}{1-\rho}\, d\log(a_{jt}u_{ijt}) \; - \; \frac{\frac{1}{1-\rho}(a_{jt}u_{ijt})^{\frac{1}{1-\rho}}\, d\log(a_{jt}u_{ijt}) + \frac{1}{1-\rho}(a_{-jt}u_{i,-jt})^{\frac{1}{1-\rho}}\, d\log(a_{-jt}u_{i,-jt})}{(a_{jt}u_{ijt})^{\frac{1}{1-\rho}} + (a_{-jt}u_{i,-jt})^{\frac{1}{1-\rho}}} \\
&= \frac{1}{1-\rho}\left[\left(1-s_{ijt})\right) d\log(a_{jt}u_{ijt}) \; - \; s_{i,-jt}\, d\log(a_{-jt}u_{i,-jt})\right] \\
&= \frac{s_{i,-jt}}{1-\rho}\left[d\log(a_{jt}u_{ijt}) - d\log(a_{-jt}u_{i,-jt})\right] \\
&= -\frac{s_{i,-jt}}{\rho-1}\, d\log\left(\frac{a_{jt}u_{ijt}}{a_{-jt}u_{i,-jt}}\right).
\end{aligned}
$$

Taking the first-order approximation:

$$
\Delta\log s_{ijt} \;\approx\; -\frac{s_{i,-jt}}{\rho-1}\left[\Delta\log a_{jt} - \Delta\log a_{-jt} + \Delta\log u_{ijt} - \Delta\log u_{i,-jt}\right]. \tag{A.15}
$$

Substituting A.15 and A.13 into A.14:

$$
\begin{aligned}
\Delta\log L_{ijt} \approx \text{constant}_t + \Delta\log\tilde{B}_{it} \; + \; \theta\left(s_{i1t}\Delta\log a_{1t} + s_{i2t}\Delta\log a_{2t}\right) \; - \; \frac{s_{i,-jt}}{\rho-1}\left[\Delta\log a_{jt} - \Delta\log a_{-jt}\right] \\
+ \; \underbrace{\tilde{v}_{it} \; - \; \frac{s_{i,-jt}}{\rho-1}\left(\Delta\log u_{ijt} - \Delta\log u_{i,-jt}\right)}_{v_{ijt}}.
\end{aligned}
$$

which, collecting terms, is equation (16) in the paper.

A.3. Section 5 Proofs

A.3.1. Derivation of the Estimator of the Scale Elasticity $b_1\theta$

First, we prove that $\mathbb{E}[x_{i,j,2}e_{i,j}] = 0$.

By definition, $e_{i,j} = \Delta\log\tilde{B}_i + v_{i,j} + \tilde{\chi}_{i,j}$ and so we begin by showing that $\mathbb{E}[x_{i,j,2}\Delta\log\tilde{B}_i] = 0$:

$$
\begin{aligned}
\mathbb{E}[x_{i,j,2}\Delta\log\tilde{B}_i] &= \mathbb{E}(s_{i,-j}\,(w_{i,j} - w_{i,-j})\,\Delta\log\tilde{B}_i) \\
&= \mathbb{E}\left(((1-s_{i,j})\,w_{i,j} - s_{i,-j}w_{i,-j})\,\Delta\log\tilde{B}_i\right) \\
&= \mathbb{E}\left((w_{i,j} - (s_{i,j}w_{i,j} + s_{i,-j}w_{i,-j}))\,\Delta\log\tilde{B}_i\right) \\
&= \mathbb{E}(w_{i,j}\Delta\log\tilde{B}_i) - \mathbb{E}\left((s_{i,j}w_{i,j} + s_{i,-j}w_{i,-j})\,\Delta\log\tilde{B}_i\right) \\
&= \mathbb{E}(w_{i,j}\Delta\log\tilde{B}_i) - \mathbb{E}\left(s_{i,j}w_{i,j}\Delta\log\tilde{B}_i\right) - \mathbb{E}\left(s_{i,-j}w_{i,-j}\Delta\log\tilde{B}_i\right) \\
&= \mathbb{E}(w_{i,j}\Delta\log\tilde{B}_i) - 2\mathbb{E}\left(s_{i,j}w_{i,j}\Delta\log\tilde{B}_i\right),
\end{aligned}
$$

where the first equality stems from the definition of $x_{i,j,2}$, and the last equality stems from symmetry.

Given our assumption that $\mathbb{E}\left(w_{i,j}\Delta\log\tilde{B}_i\,\middle|\,s_{i,j}\right)=\mathbb{E}\left(w_{i,j}\Delta\log\tilde{B}_i\right)$, we have:

$$\begin{aligned}2\mathbb{E}\left(s_{i,j}w_{i,j}\Delta\log\tilde{B}_i\right)&=2\mathbb{E}\left[\mathbb{E}\left(s_{i,j}w_{i,j}\Delta\log\tilde{B}_i\,\middle|\,s_{i,j}\right)\right]\\&=2\mathbb{E}\left[s_{i,j}\mathbb{E}\left(w_{i,j}\Delta\log\tilde{B}_i\,\middle|\,s_{i,j}\right)\right]\\&=2\mathbb{E}\left[s_{i,j}\mathbb{E}\left(w_{i,j}\Delta\log\tilde{B}_i\right)\right]\\&=2\mathbb{E}\left(s_{i,j}\right)\mathbb{E}\left(w_{i,j}\Delta\log\tilde{B}_i\right)=\mathbb{E}\left(w_{i,j}\Delta\log\tilde{B}_i\right)\end{aligned}$$

where the last equality stems from $\mathbb{E}(s_{i,j})=1/2$. We thus have that $\mathbb{E}[x_{i,j,2}\Delta\log\tilde{B}_i]=0$.

Next, we show that $\mathbb{E}[x_{i,j,2}(\nu_{i,j}+\tilde{\chi}_{i,j})]=0$. To see this define $\Omega:=1+\theta(\rho-1)$. Plugging in the definitions of $\nu_{ij}=\frac{1}{\rho-1}\left[(s_{ij}\Omega-1)\,\Delta logu_{ij}+s_{i,-j}\Omega\Delta logu_{i,-j}\right]$, $\tilde{\chi}_i=\theta\left(s_{i,j}\chi_j+s_{i,-j}\chi_{-j}\right)$, and $\tilde{\chi}_{i,j}=\tilde{\chi}_i+\frac{1}{(\rho-1)}s_{i,-j}\left(\chi_{-j}-\chi_j\right)$, we obtain:

$$\begin{aligned}\mathbb{E}(x_{i,2}(\nu_{i,j}+\tilde{\chi}_{i,j}))&=\mathbb{E}\left(x_{i,j,2}\left(\frac{1}{\rho-1}\left[(s_{ij}\Omega-1)\,\Delta logu_{ij}+s_{i,-j}\Omega\Delta logu_{i,-j}\right]+\theta\left(s_{i,j}\chi_j+s_{i,-j}\chi_{-j}\right)+\frac{1}{(\rho-1)}s_{i,-j}\left(\chi_{-j}-\chi_j\right)\right)\right)\\&=\frac{1}{(\rho-1)}\mathbb{E}\left(x_{i,j,2}\left((s_{ij}\Omega-1)\,\Delta logu_{ij}+s_{i,-j}\Omega\Delta logu_{i,-j}+(\rho-1)\,\theta\left(s_{i,j}\chi_j+s_{i,-j}\chi_{-j}\right)+s_{i,-j}\left(\chi_{-j}-\chi_j\right)\right)\right)\\&=\frac{1}{(\rho-1)}\mathbb{E}\left(x_{i,j,2}\left((s_{ij}\Omega-1)\,\Delta logu_{ij}+s_{i,-j}\Omega\Delta logu_{i,-j}+(\rho-1)\,\theta s_{i,j}\chi_j-s_{i,-j}\chi_j+(\rho-1)\,\theta s_{i,-j}\chi_{-j}+s_{i,-j}\chi_{-j}\right)\right)\\&=\frac{1}{(\rho-1)}\mathbb{E}\left(x_{i,j,2}\left((s_{ij}\Omega-1)\,\Delta logu_{ij}+s_{i,-j}\Omega\Delta logu_{i,-j}+\Omega s_{i,j}\chi_j-\chi_j+\Omega s_{i,-j}\chi_{-j}\right)\right)\\&=\frac{1}{(\rho-1)}\mathbb{E}\left(x_{i,j,2}\left((s_{ij}\Omega-1)\,(\Delta logu_{ij}+\chi_j)+s_{i,-j}\Omega\,(\Delta logu_{i,-j}+\chi_{-j})\right)\right)\\&=\frac{1}{(\rho-1)}\mathbb{E}\left(x_{i,j,2}\left(s_{ij}\Omega\,(\Delta logu_{ij}+\chi_j)+s_{i,-j}\Omega\,(\Delta logu_{i,-j}+\chi_{-j})-(\Delta logu_{ij}+\chi_j)\right)\right)\\&=\frac{\Omega}{(\rho-1)}\mathbb{E}\left(x_{i,j,2}s_{ij}\,(\Delta logu_{ij}+\chi_j)\right)+\frac{\Omega}{(\rho-1)}\mathbb{E}\left(x_{i,j,2}s_{i,-j}\,(\Delta logu_{i,-j}+\chi_{-j})\right)-\frac{1}{(\rho-1)}\mathbb{E}\left(x_{i,j,2}\,(\Delta logu_{ij}+\chi_j)\right)\end{aligned}$$

Using the assumption that $E\left[\Delta logu_{ij}+\chi_j\,\middle|\,w_j,w_{-j},s_{i,j}\right]=0$, we show that all three expectations in the line above are zero.

Start from the first term, $\mathbb{E}\left(x_{i,j,2}s_{ij}(\Delta logu_{ij}+\chi_j)\right)$:

$$\begin{aligned}\mathbb{E}\left(x_{i,j,2}s_{ij}(\Delta logu_{ij}+\chi_j)\right)&=\mathbb{E}\left(s_{i,-j}\left(w_j-w_{-j}\right)s_{ij}(\Delta logu_{ij}+\chi_j)\right)\\&=\mathbb{E}\left(s_{ij}\left(1-s_{i,j}\right)\left(w_j-w_{-j}\right)\left(\Delta logu_{ij}+\chi_j\right)\right)\\&=\mathbb{E}\left[\mathbb{E}\left(s_{ij}\left(1-s_{i,j}\right)\left(w_j-w_{-j}\right)\left(\Delta logu_{ij}+\chi_j\right)|w_j,w_{-j},s_{i,j}\right)\right]\\&=\mathbb{E}\left[s_{ij}\left(1-s_{i,j}\right)\left(w_j-w_{-j}\right)\mathbb{E}\left(\Delta logu_{ij}+\chi_j|w_j,w_{-j},s_{i,j}\right)\right]=0\end{aligned}$$

The second and third expectations are zero by an analogous proof.

Thus we've shown that $\mathbb{E}[x_{i,j,2}e_{i,j}]=\mathbb{E}[x_{i,j,2}(\Delta\log\tilde{B}_i+\nu_{i,j}+\tilde{\chi}_{i,j})]=0$. Furthermore, because $e_{i,j}$ is the error term in regression (31), which includes a constant, this also implies that $cov(x_{i,j,2},e_{i,j})=0$.

Next, we turn to the derivation of equation (33) in the paper. Define the linear projection:

$$e_{i,j} = \pi_0 + \pi_1 x_1 + \pi_2 x_2 + r_{i,j} \tag{A.16}$$

Plugging (A.16) into (31) and collecting terms, yields:

$$d_2 = \beta_{KM} + \pi_2.$$

From the definition of multivariate regression (and applying to the case of 2 variables):

$$\pi_2 = \frac{var(x_1)cov(x_2, e_{i,j}) - cov(x_1, x_2)cov(x_1, e_{i,j})}{var(x_1)var(x_2) - cov(x_1, x_2)^2}.$$

Since $cov(x_2, e_{i,j}) = 0$, we have:

$$\begin{aligned} d_2 &= \beta_{KM} - \frac{cov(x_1, x_2)cov(x_1, e_{i,j})}{var(x_1)var(x_2) - cov(x_1, x_2)^2}, \\ cov(x_1, e_{i,j}) &= \frac{\left[var(x_1)var(x_2) - cov(x_1, x_2)^2\right]}{cov(x_1, x_2)}(\beta_{KM} - d_2). \end{aligned}$$

In an analogous manner:

$$d_1 = b_1\theta + \pi_1,$$

where as before (using $cov(x_2, e_{i,j}) = 0$):

$$d_1 = b_1\theta + \frac{var(x_2)cov(x_1, e_{i,j})}{var(x_1)var(x_2) - cov(x_1, x_2)^2}$$

Finally, plugging in the expression for $cov(x_1, e_{i,j})$:

$$\begin{aligned} d_1 &= b_1\theta + \frac{var(x_2)}{var(x_1)var(x_2) - cov(x_1, x_2)^2}\frac{\left[var(x_1)var(x_2) - cov(x_1, x_2)^2\right]}{cov(x_1, x_2)}(\beta_{KM} - d_2) \\ &= b_1\theta + \frac{var(x_2)}{cov(x_1, x_2)}(\beta_{KM} - d_2) \\ &= b_1\theta + \frac{1}{\delta_{x_1,x_2}}(\beta_{KM} - d_2) \end{aligned}$$

where δ_{x_1,x_2} is the population regression coefficient of x_1 on x_2. Replacing population values with estimates, a consistent estimator for $b_1\theta$ is thus given by equation (33) in the paper:

$$\widehat{b_1\theta} = \widehat{d}_1 - \frac{1}{\widehat{\delta}_{x_1,x_2}}(\widehat{\beta_{KM}} - \widehat{d}_2).$$