The Past and Future of U.S. Structural Change: Compositional Accounting and Forecasting

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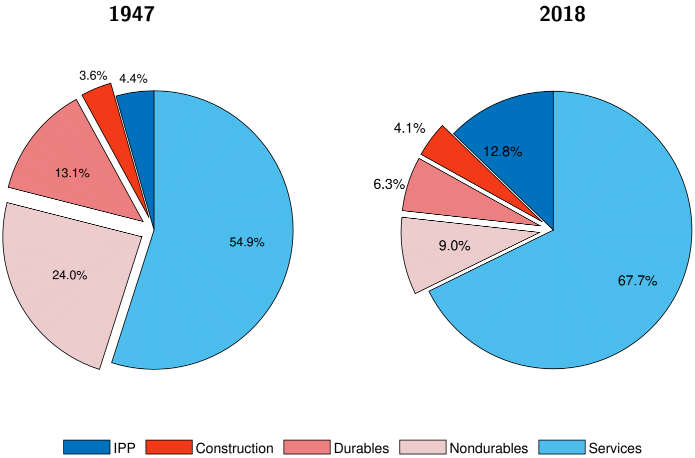
Changes in the sectoral composition of U.S. GDP

- 5-sector Decomposition of U.S. GDP. (Annual 1947-2018, KLEMS, BEA, vLW (2021))
- Goods:
  1. Durable Goods: (motor vehicles, electronics ...)
  2. Nondurable Goods: (food, ...)
  3. Construction: (structures, ...)
- Services:
  4. Intellectual Property Products (IPP) (software, system designs, technical services ...)
  5. Other Services (Health, Education, ...)



Two Questions

- Why did the composition of U.S. economic activity change?
  - Lots of work: Greenwood/Hercowitz/Krusell (1997), Ngai/Pissarides (2007), Herrendorf/Rogerson/Valentinyi (2013,2014,2021), Choi/Foerster (2017), Garcia-Santana/Pijoan-Mas/Villacorta (2021), Comin/Mestieri/Lashkari (2021), Foerster/Hornstein/Sarte/Watson (2022), Gaggl/Gorry/vom Lehn (2023), Boppart/Kiernan/Krusell/Malmberg (2023), Casal/Caunedo (2024), ...
- What will the composition of economic activity be in 2038, 2058, ... ?
  - Less work: Aitcheson (1986), Müeller and Watson (2016, 2018 2020), ...

Two Questions

- Why did the composition of U.S. economic activity change?

  1. Reduced Form Approach: Explores the historical evolution of sectoral shares of GDP in terms of changing constituent subshares (e.g., consumption shares, intermediate input shares, etc.)

  2. Structural Approach: Explores structural change in terms of fundamental supply and demand forces, e.g., sector-specific TFP vs. preferences for particular goods.

- What will the composition of economic activity be in 2038 (or 2058, or ...)?

  1. Reduced-Form: requires transformation of the data to ensure adding up constraints on shares are met.

  2. A Multisector Model: eliminates the need for ad-hoc data transformation in reduced-form forecasting. Structural forecasts depend on forecasts of fundamentals, e.g., TFP, etc.

What is a Trend?

- Structural change is a low-frequency issue, need appropriate econometric tools: Müeller and Watson (2016, 2018, 2020)

- Suppose we have T (scalar) observations, $\boldsymbol{x}_{1:T} = (x_1, ..., x_T)'$.

- **Defining the Long-Run Trend:**

  - Let $\psi_j(s) = \cos(j\pi s)$ on $s \in [0, 1]$ with period $2/j$.

  - Let $\psi(s) = [\psi_1(s), ..., \psi_q(s)]$ denote a vector of these functions with periods 2 through $2/q$.

  - Let $\boldsymbol{\psi}_T$ the $T \times q$ matrix with t^{th} row, $\psi\left(\frac{t-\frac{1}{2}}{T}\right)'$. Note: $\boldsymbol{\psi}_T$ is a matrix of periodic components that capture periodicities greater than $2T/q$.

  - Let $\boldsymbol{W}_T = [\mathbf{1}_T, \boldsymbol{\psi}_T]$ where $\mathbf{1}_T$ is a $T \times 1$ vector of ones.

What is a Trend?

- The OLS coefficients (or cosine transforms) from regressing $\boldsymbol{x}_{1:T}$ on $\boldsymbol{W}_T$ are

$$\widehat{\boldsymbol{B}}_T = \left(\boldsymbol{W}_T'\boldsymbol{W}_T\right)^{-1}\boldsymbol{W}_T'\boldsymbol{x}_{1:T}.$$

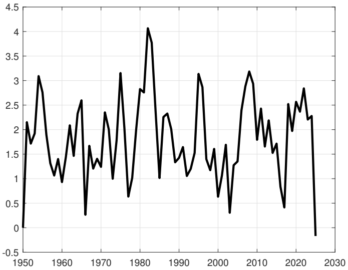
- We define the low frequency trend of x_t as the $T \times 1$ vector, $\widehat{\boldsymbol{x}}_{1:T}$, given by

$$\boxed{\widehat{\boldsymbol{x}}_{1:T} = \boldsymbol{W}_T\widehat{\boldsymbol{B}}_T,}$$

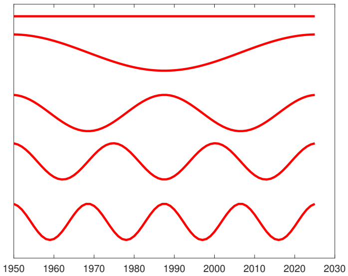
  Since the regressors, $\boldsymbol{W}_T$, are deterministic functions, the stochastic process, $\widehat{\boldsymbol{x}}_{1:T}$, is entirely characterized by the $q+1$ random variables in $\widehat{\boldsymbol{B}}_T$.

- With annual data from 1947 to 2018, $T = 72$. Then $q = 7$ describes long-run variations greater than 20 years.

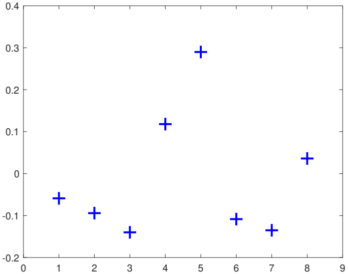
Hypothetical Time Series



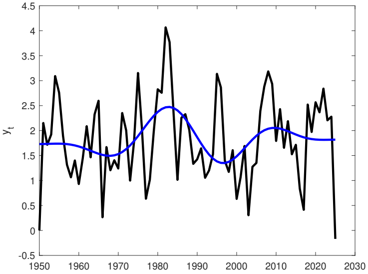
Low-Frequency Regressors



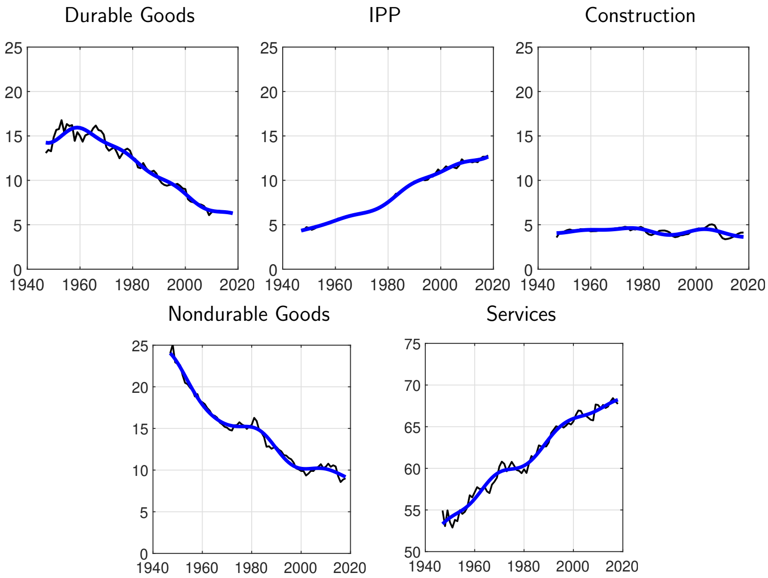
Low-Frequency Regression Coefficients



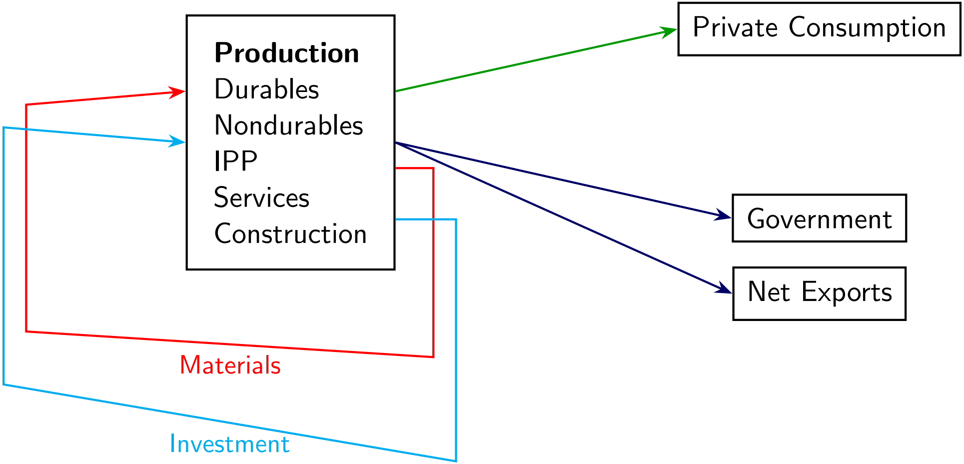
Actual and Low-Frequency Fitted Values



Trend Value Added Shares

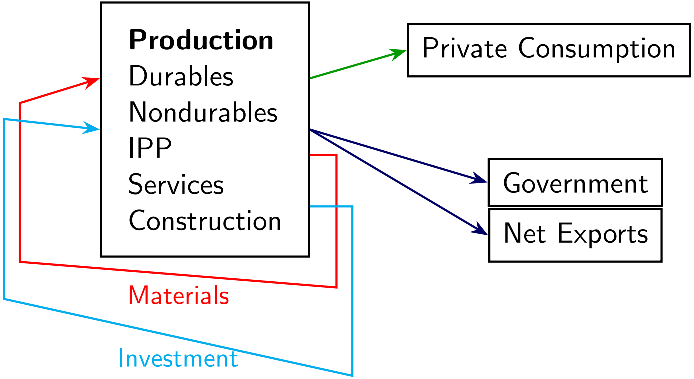


Production and Uses



Production and Uses

- Reduced-Form: Uses a direct parameterization of channels/ arrows.
- Multisector Model: Explains the channels/ arrows in terms of fundamentals of technology and preferences.



Compositional Accounting

Y_j : Gross Output/Production

C_j : Consumption

M_{ji} : Materials purchased by i from j

X_{ji} : Investment purchased by i from j

G_j : Government Purchases

NX_j : Net Exports

$$\boxed{\begin{aligned} &M_j = \sum_i M_{ij} : \text{Materials used by } j \\ &X_j = \sum_i X_{ij} : \text{Investment used by } j \\ &V_j = Y_j - M_j : \text{Value Added in } j \\ &GDP = \sum_j V_j \end{aligned}}$$

Compositional Accounting

Sectors: j $(or\ i) = 1, ...n$ $(n = 5)$

Y_j : Gross Output/Production

C_j : Consumption

M_{ji} : Materials purchased by i from j

X_{ji} : Investment purchased by i from j

G_j : Government Purchases

NX_j : Net Exports

Accounting Identity:

$$\boxed{Y_j = C_j + \sum_i M_{ji} + \sum_i X_{ji} + G_j + NX_j}$$

$M_j = \sum_i M_{ij}$: Materials used by j

$X_j = \sum_i X_{ij}$: Investment used by j

$V_j = Y_j - M_j$: Value Added in j

Computing Shares from Accounting Identity

Sectors: $j \ (or \ i) = 1, \dots N \ (= 5)$

Y_j : Gross Output/Production

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$M_j = \sum_i M_{ij}$: Materials used by j

$X_j = \sum_i X_{ij}$: Investment used by j

$V_j = Y_j - M_j$: Value Added in j

Accounting Identity:

$$\boxed{\gamma_j = V_j / Y_j}$$

$$\boxed{Y_j} = C_j + \textstyle\sum_i M_{ji} + \sum_i X_{ji} + G_j + NX_j$$

$$\downarrow$$

$$\boxed{\frac{V_j}{\gamma_j}} = C_j + \textstyle\sum_i M_{ji} + \sum_i X_{ji} + G_j + NX_j$$

Computing Shares from Accounting Identity

Sectors: j $(or\ i) = 1, \ldots N$ $(= 5)$

Y_j : Gross Output/Production

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$X_j = \sum_i X_{ij}$: Investment used by j

$V_j = Y_j - M_j$: Value Added in j

Accounting Identity:

$$\boxed{\theta_j = C_j / C}$$

$$Y_j = \boxed{C_j} + \sum_i M_{ji} + \sum_i X_{ji} + G_j + NX_j$$

$$\frac{V_j}{\gamma_j} = \boxed{\theta_j C} + \sum_i M_{ji} + \sum_i X_{ji} + G_j + NX_j$$

Computing Shares from Accounting Identity

Sectors: $j\ (or\ i) = 1, \ldots N\ (= 5)$

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$X_j = \sum_i X_{ij}$: Investment used by j

$V_j = Y_j - M_j$: Value Added in j

Accounting Identity:

$$\boxed{\phi_{ji} = M_{ji} / M_i}$$

$$Y_j = C_j + \boxed{\sum_i M_{ji}} + \sum_i X_{ji} + G_j + NX_j$$

$$\frac{V_j}{\gamma_j} = \theta_j C + \boxed{\sum_i \phi_{ji}(1-\gamma_i)\frac{V_i}{\gamma_i}} + \sum_i X_{ji} + G_j + NX_j$$

Computing Shares from Accounting Identity

Sectors: j (or i) $= 1, \ldots N$ $(= 5)$

Y_j : Gross Output/Production

C_j : Consumption

M_{ji} : Materials purchased by i from j

X_{ji} : Investment purchased by i from j

G_j : Government Purchases

NX_j : Net Exports

$$M_j = \sum_i M_{ij} \text{ : Materials used by } j$$

$$X_j = \sum_i X_{ij} \text{ : Investment used by } j$$

$$V_j = Y_j - M_j \text{ : Value Added in } j$$

Accounting Identity:

$$\boxed{\omega_{ji} = X_{ji} / X_i, \quad \psi_i^x = X_i / V_i}$$

$$Y_j = C_j + \sum_i M_{ji} + \boxed{\sum_i X_{ji}} + G_j + NX_j$$

$$\frac{V_j}{\gamma_j} = \theta_j C + \sum_i \phi_{ji}(1 - \gamma_i)\frac{V_i}{\gamma_i} + \boxed{\sum_i \omega_{ji} \psi_i^x V_i} + G_j + NX_j$$

Computing Shares from Accounting Identity

Sectors: $j\ (or\ i) = 1, \ldots N\ (= 5)$

Y_j : Gross Output/Production

C_j : Consumption

M_{ji} : Materials purchased by i from j

X_{ji} : Investment purchased by i from j

G_j : Government Purchases

NX_j : Net Exports

$M_j = \sum_i M_{ij}$: Materials used by j

$X_j = \sum_i X_{ij}$: Investment used by j

$V_j = Y_j - M_j$: Value Added in j

Accounting Identity:

$$\boxed{\psi_j^g = G_j / V_j, \quad \psi_j^{nx} = NX_j / V_j}$$

$$Y_j = C_j + \textstyle\sum_i M_{ji} + \sum_i X_{ji} + \underbrace{G_j + NX_j}$$

$$\frac{V_j}{\gamma_j} = \theta_j C + \sum_i \phi_{ji}(1-\gamma_i)\frac{V_i}{\gamma_i} + \sum_i \omega_{ji}\psi_i^x V_i + \boxed{\psi_j^g V_j + \psi_j^{nx} V_j}$$

Compositional Accounting

$$\boxed{\begin{aligned}&\mathbf{\Phi} = [\phi]_{ij} \text{ with } \sum_i \phi_{ij} = 1 \ \forall j\\ &\mathbf{\Omega} = [\omega]_{ij} \text{ with } \sum_i \omega_{ij} = 1 \ \forall j\\ &\boldsymbol{V} = (V_1, \cdots, V_N)'\end{aligned}}$$

$$\frac{V_j}{\gamma_j} = \theta_j C + \sum_i \phi_{ji}(1-\gamma_i)\frac{V_i}{\gamma_i} + \sum_i \omega_{ji}\psi_i^x V_i + \psi_j^g V_j + \psi_j^{nx} V_j$$

$$\mathbf{\Gamma}\boldsymbol{V} = \boldsymbol{\theta} C + \mathbf{\Phi}\left((\boldsymbol{I} - \mathbf{\Gamma})\,\mathbf{\Gamma}^{-1}\boldsymbol{V} + \mathbf{\Omega}\mathbf{\Psi}^x\boldsymbol{V} + \mathbf{\Psi}^g\boldsymbol{V} + \mathbf{\Psi}^{nx}\boldsymbol{V}\right.$$

$$\boldsymbol{s}^v = \frac{\boldsymbol{\eta}}{\mathbf{1}'\boldsymbol{\eta}}$$

$$\text{where } \boldsymbol{\eta} = \frac{\boldsymbol{V}}{C} = \left[(\boldsymbol{I} - \mathbf{\Phi}[\boldsymbol{I} - \mathbf{\Gamma}])\,\mathbf{\Gamma}^{-1} - \mathbf{\Omega}\mathbf{\Psi}^x - \mathbf{\Psi}^g - \mathbf{\Psi}^{nx}\right]^{-1}\boldsymbol{\theta}$$

$$\boxed{s_j^v = \mathcal{S}_j\left(\boldsymbol{\theta}, \mathbf{\Gamma}, \mathbf{\Psi}^x, vec(\mathbf{\Phi}), vec(\mathbf{\Omega}), \mathbf{\Psi}^g, \mathbf{\Psi}^{nx}\right)}$$

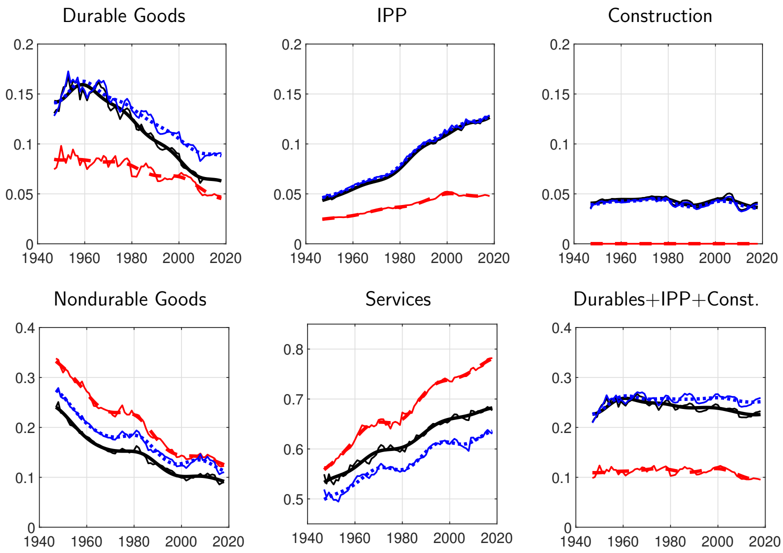
Compositional Accounting

- Last slide, with time subscripts

$$s_{jt}^{v} = \mathcal{S}_j \left(\boldsymbol{\theta}_t, \boldsymbol{\Gamma}_t, \boldsymbol{\Psi}_t^{x}, vec(\boldsymbol{\Phi}_t), vec(\boldsymbol{\Omega}_t), \boldsymbol{\Psi}_t^{g}, \boldsymbol{\Psi}_t^{nx} \right)$$

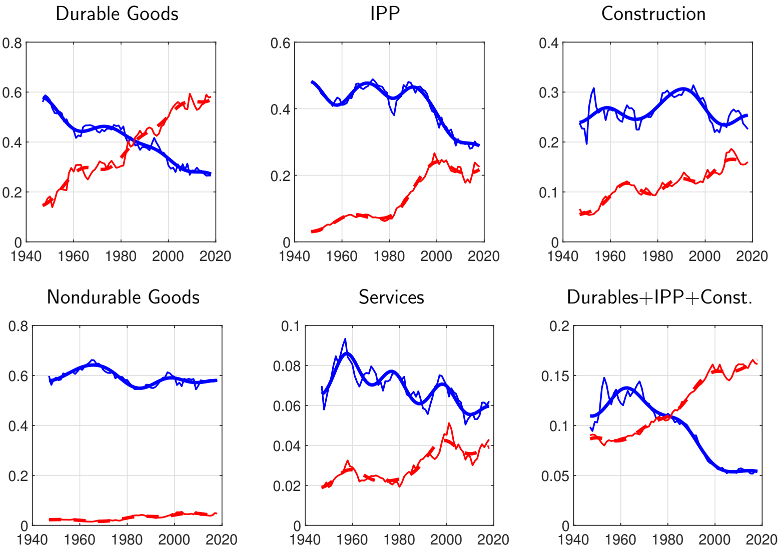
- With $n = 5$, $\mathcal{S}_j$ is a function of $2n^2 + 5n$ constituent sub-shares, $\theta_{jt}, \gamma_{jt}, \phi_{ijt}$, etc., or 75 interrelated compositional changes for each j.

Contributions from: $\boldsymbol{\theta}$ (red), $\boldsymbol{\theta} + (\boldsymbol{\Phi}, \boldsymbol{\Omega}, \boldsymbol{\Gamma})$ (blue), Value-Added Shares (black)



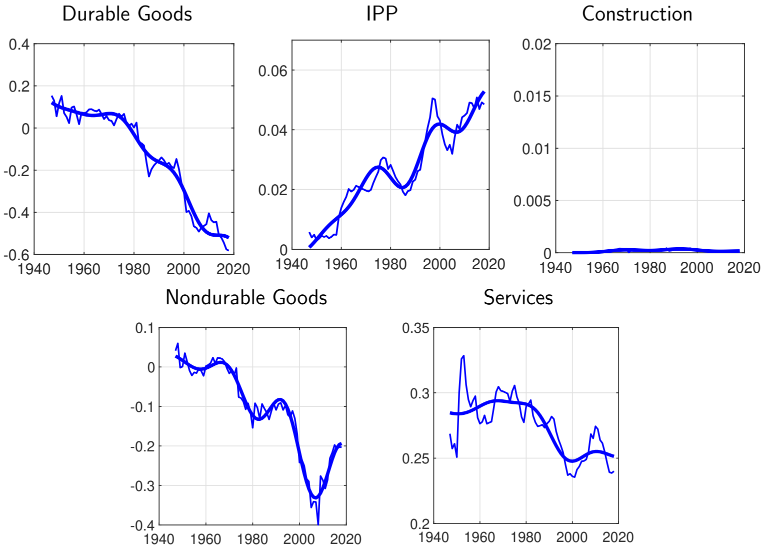
- Nondurables and Services produce for final consumption, Construction, Durables, and IPP supply inputs for production.
- Significant gap between $\boldsymbol{\theta}_t$ and $\boldsymbol{s}_t^V$.
- Change in $\boldsymbol{s}_t^V$ in Services and Nondurable goods reflect shifts in $\boldsymbol{\theta}_t$ and cancel out!
- Change in $\boldsymbol{s}_t^V$ in Durable Goods and IPP reflect shifts in $\boldsymbol{\Phi}$, $\boldsymbol{\Omega}$, and cancel out!
- $\boldsymbol{s}_t^V$ in input-producing sectors constant throughout.

Investment and Materials Spending Shares, Durables (blue) vs. IPP (red)



- Investment spending shares generally increase over time, intermediate input spending shares generally decline.
- Change in investment and intermediate input spending shares almost exactly offset!

International Trade and Government



Reduced-Form Trend Compositional Forecasting

- Suppose we partition x_t into in-sample and out-of-sample observations, $\boldsymbol{x}_{1:T} = (x_1, \ldots, x_T)'$ and $\boldsymbol{x}_{T+1:T+h} = (x_{T+1}, \ldots, x_{T+h})'$.

- In this case,

$$\sqrt{T}\begin{pmatrix} \boldsymbol{B}_T \\ \boldsymbol{B}_{T+h} \end{pmatrix} \sim^a \mathcal{N}(0, \boldsymbol{\Omega}) \text{ where } \boldsymbol{\Omega} = \sigma_e^2 \begin{bmatrix} \widetilde{\boldsymbol{W}}'_{T+h} \\ \boldsymbol{W}'_{T+h} \end{bmatrix} \left[\kappa \mathbf{1}_{T+h} \mathbf{1}'_{T+h} + \boldsymbol{I}_{T+h}\right] \begin{bmatrix} \widetilde{\boldsymbol{W}}'_{T+h} \\ \boldsymbol{W}'_{T+h} \end{bmatrix}'.$$

- Partition $\boldsymbol{\Omega}$ conformably with $\begin{pmatrix} \boldsymbol{B}_T \\ \boldsymbol{B}_{T+h} \end{pmatrix}$ such that $\boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{\Omega}_{\widetilde{W}\widetilde{W}} & \boldsymbol{\Omega}_{\widetilde{W}W} \\ \boldsymbol{\Omega}_{W\widetilde{W}} & \boldsymbol{\Omega}_{WW} \end{bmatrix}$.

- Then, we have

$$\sqrt{T}\left(\boldsymbol{B}_{T+h} | \boldsymbol{B}_T\right) \sim^a \mathcal{N}\left(\boldsymbol{\Omega}_{W\widetilde{W}} \boldsymbol{\Omega}_{\widetilde{W}\widetilde{W}}^{-1} \boldsymbol{B}_T, \boldsymbol{\Omega}_{WW} - \boldsymbol{\Omega}_{W\widetilde{W}} \boldsymbol{\Omega}_{\widetilde{W}\widetilde{W}}^{-1} \boldsymbol{\Omega}_{\widetilde{W}W}\right).$$

One Last Step: The Adding-Up Constraint

- s_{jt} : value added share of sector j in GDP $\Rightarrow$ $s_j \in [0,1]$ $\forall j$ and $\sum_{j=1}^{N} = 1$.
- x_{jt} : $\log \frac{s_{jt}}{s_{Nt}}$ $j = 1, \cdots, N-1$, so $s_{jt} = F(\boldsymbol{x}_t)$ with

$$F(\boldsymbol{x}_t) = \frac{e^{x_{jt}}}{1 + \sum_{i=1}^{N} e_{it}^{x}}.$$

- $\widehat{x}_{jt}$: Trend of x_{jt}.
- Construct $\widehat{x}_{jT+h|T}$ using method on previous slide.
- Set $\widehat{s}_{jT+h|T} = F(\widehat{x}_{jT+h|T})$

Reduced-Form Forecasts

Sector	Durable Goods	IPP	Const.	Nondurable Goods	Services
Levels					
Actual 1947	14.2	4.3	4.1	24.1	53.3
Actual 2018	6.3	12.7	3.6	9.2	68.2
Forecast for 2038	4.7	16.1	3.3	6.6	69.2
	[3.7,5.9]	[14.9,17.3]	[2.5,4.5]	[5.5,7.9]	[66.9,70.7]
Changes					
Actual 1947-2018	-8.0	8.3	-0.4	-14.9	14.9
20 Year Hist Avg	-2.2	2.3	-0.1	-4.1	4.1
Forecast 2018-2038	-1.5	3.5	-0.3	-2.6	0.9
	[-2.4,-0.3]	[2.3,4.5]	[-1.1,0.7]	[-3.6,-1.4]	[-1.1,2.5]

Notes: The forecasts are the means of the associated predictive distributions. Values in brackets are equal-tailed 67% prediction intervals.

Reduced-Form Forecasts

Sectors	Durables	IPP	Construction	Nondurables	Services
Forecast Changes					
Direct	-1.5	3.5	-0.3	-2.6	0.9
Bottom Up	-1.6	3.5	-0.4	-2.6	1.1
By Component:					
$\boldsymbol{\theta}_t$	-0.4	0.3	0.0	-1.3	1.4
γ_t	0.0	0.3	0.2	0.1	-0.7
$\boldsymbol{\Phi}_t$	-0.3	1.0	-0.4	-0.1	-0.1
$\boldsymbol{\Omega}_t$	-0.5	1.6	-0.1	-1.2	0.2
ψ_t^X	0.0	0.2	-0.2	0.1	-0.1
ψ_t^{NX}	-0.5	0.1	0.0	-0.3	0.6
ψ_t^G	0.0	0.1	0.0	0.1	-0.2

Notes: Adding up is subject to rounding and non-linearities in compositional identities.

A Model for Explaining and Forecasting Structural Change

- Historical changes in value-added shares arise from:
  - Changes in consumption shares, production linkages (input shares, materials and investment), Net Exports
- Changes in consumption shares:
  - Important for Nondurables and Services, but largely offsetting
  - Not so much for Durables, IPP and Construction
- Changes in production input shares:
  - Important for Durables and IPP, but largely offsetting
  - Not so much for Services, Construction, and Nondurables

A Model for Explaining and Forecasting Structural Change

- Preferences: Important for changes in consumption shares
  - Sectoral preference shocks: Θ_{jt}
- Technology: Important for changes in input shares
  - TFP: z_{jt}
  - Input-Biased Technical Change (IBTC): z_{jt}^{m}, z_{jt}^{x}
- These driving processes also lead to endogenous changes in prices, income ...
- Exogenous variations in Net Export demand, Government demand

- Historical changes in value-added shares arise from:
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Modeling Structural Change

- Subset of sectors whose goods are assembled using Cobb-Douglas aggregators, $\bar{\mathcal{N}}$, subset of sectors whose goods are assembled using CES aggregators, $\widetilde{\mathcal{N}}$
- These two subsets cover all sectors, $\mathcal{N} = \bar{\mathcal{N}} \cup \widetilde{\mathcal{N}}$, and they are disjoint, $\bar{\mathcal{N}} \cap \widetilde{\mathcal{N}} = \varnothing$
- Many shares are constant $\Rightarrow$ the 'spirit' of the canonical model remains.
- Two key assumptions:
  - Trend shifts in allocations arise from sequential steady states
  - We abstract from balanced growth

Technology

- Gross output

$$y_{j,t} = \mathcal{Y}_j\left(v_{j,t}, m_{j,t}\right) = \left(\frac{v_{j,t}}{\gamma_j}\right)^{\gamma_j}\left(\frac{m_{j,t}}{1-\gamma_j}\right)^{1-\gamma_j}, \;\; \gamma_j \in (0,1), \;\; \forall j \in \mathcal{N}$$

- Value added

$$v_{j,t} = \mathcal{V}_j(k_{j,t}, \ell_{j,t}) = z_{j,t}\left(\frac{k_{j,t}}{\alpha_j}\right)^{\alpha_j}\left(\frac{\ell_{j,t}}{1-\alpha_j}\right)^{1-\alpha_j}, \;\; \alpha_j \in (0,1), \;\; \forall j \in \mathcal{N}$$

Technology

- Materials

$$m_{j,t} = \mathcal{M}_j(\widetilde{m}_{j,t}, \bar{m}_{j,t}) = \left(\frac{\widetilde{m}_{j,t}}{\rho_j^m}\right)^{\rho_j^m} \left(\frac{\bar{m}_{j,t}}{1-\rho_j^m}\right)^{1-\rho_j^m}, \quad \rho_j^m \in (0,1)\ \forall j \in \mathcal{N}$$

- Materials sub-bundles

$$\widetilde{m}_{j,t} = \sum_{i \in \widetilde{\mathcal{N}}_j^m} \left[z_{ij,t}^m m_{ij,t}^{\frac{\epsilon_j^m - 1}{\epsilon_j^m}} \right]^{\frac{\epsilon_j^m}{\epsilon_j^m - 1}}, \quad \sum_{i \in \widetilde{\mathcal{N}}_j^m} z_{ij,t}^m = 1,\ \epsilon_j^m \in (0, \infty)$$

$$\bar{m}_{j,t} = \prod_{i \in \bar{\mathcal{N}}_j^m} \left(\frac{m_{ij,t}}{\zeta_{ij}^m}\right)^{\zeta_{ij}^m}, \quad \sum_{i \in \bar{\mathcal{N}}_j^m} \zeta_{ij}^m = 1$$

- Capital accumulation

$$k_{j,t+1} = x_{j,t} + (1-\delta_j)k_{j,t}$$

- Investment

$$x_{j,t} = \mathcal{X}(\widetilde{x}_{j,t}, \bar{x}_{j,t}) = \left(\frac{\widetilde{x}_{j,t}}{\rho^x_j}\right)^{\rho^x_j} \left(\frac{\bar{x}_{j,t}}{1-\rho^x_j}\right)^{1-\rho^x_j}, \quad \rho^x_j \in (0,1)\ \forall j \in \mathcal{N}$$

- Investment sub-bundles

$$\widetilde{x}_{j,t} = \sum_{i \in \widetilde{\mathcal{N}}^x_j} \left[z^x_{ij,t} x_{ij,t}^{\frac{\epsilon^x_j - 1}{\epsilon^x_j}} \right]^{\frac{\epsilon^x_j}{\epsilon^x_j - 1}}, \sum_{i \in \widetilde{\mathcal{N}}^x_j} z^x_{ij,t} = 1,\ \epsilon^x_j \in (0,\infty)$$

$$\bar{x}_{j,t} = \prod_{i \in \bar{\mathcal{N}}^x_j} \left(\frac{x_{ij,t}}{\zeta^x_{ij}}\right)^{\zeta^x_{ij}}, \sum_{i \in \bar{\mathcal{N}}^x_j} \zeta^x_{ij} = 1$$

Preferences

- Two consumption bundles, $\bar{c}_t$, and $\widetilde{c}_t$:
- $\widetilde{c}_t$ is bundled according to a Comin, et al. (2021) non-homothetic aggregator (defined implicitly by)

$$\sum_{j \in \widetilde{\mathcal{N}}^c} \Theta_{j,t}{}^{1/\sigma} \left[c_{j,t} / (\widetilde{c}_t)^{\epsilon_j^c} \right]^{(\sigma-1)/\sigma} = 1$$

- $\bar{c}$ is bundled using a Cobb-Douglas aggregator,

$$\bar{c}_t = \prod_{j \in \bar{\mathcal{N}}^c} \left(\frac{c_{j,t}}{\zeta_j} \right)^{\zeta_j}, \sum_{j \in \bar{\mathcal{N}}^c} \theta_j = 1$$

Expenditure Functions

- The non-homothetic bundle,

$$\widetilde{e}_t = \mathcal{E}(p^y_t, \widetilde{c}_t) = \left[\sum_{j \in \widetilde{\mathcal{N}^c}} \Theta_{j,t} (p^y_{j,t})^{1-\sigma} (\widetilde{c}_t)^{\epsilon^c_j (1-\sigma)} \right]^{\frac{1}{1-\sigma}}$$

- The Cobb-Douglas bundle,

$$\bar{e}_t = p^{\bar{c}}_t \bar{c}_t \text{ where } p^{\bar{c}}_t = \prod_{j \in \bar{\mathcal{N}}^c} (p^y_{j,t})^{\zeta^c_j}$$

- Total consumption expenditures,

$$e_t = \widetilde{e}_t + \bar{e}_t$$

The Household Problem

$$\max \sum_{t=0}^{\infty} \beta^t \left[(\bar{c}_t)^{\rho_t^c} (\tilde{c}_t)^{1-\rho_t^c} - \sum_{j \in \mathcal{N}} \frac{\varphi_{j,t} \ell_{j,t}^{1+\gamma_\ell}}{1+\gamma_\ell} \right]$$

subject to

$$p_t^{\bar{c}} \bar{c}_t + \mathcal{E}(p_t^y, \tilde{c}_t) + \sum_{j \in \mathcal{N}} p_{j,t}^x \left[k_{j,t+1} - (1-\delta_j) k_{j,t} \right] = \sum_{j \in \mathcal{N}} w_{j,t} \ell_{j,t} + \sum_{j \in \mathcal{N}} u_{j,t} k_{j,t}$$

Steady State Equilibrium

Steady State Equilibrium: *For all $i, j \in \mathcal{N}$, given $z_j, z_{ij}^m, z_{ij}^x, \Theta_j$, and φ_j, a steady state equilibrium (SSE) is defined by a set of sectoral prices, $\{p_j^y, p_j^v, p_j^m, p_j^x, u_j, w_j\}$, and quantities, $\{c_j, x_{ij}, x_j, m_{ij}, m_j, \ell_j, k_j, v_j, y_j\}$, such that, taking prices as given, firms maximize profits, households maximize utility and all markets clear.*

- Conditional on a guess for gross output prices, $p^y = (p_1^y, ..., p^n)$, obtain all other price indices, production, factor demands and consumption demands.
- Adjust p^y until the market for gross output (y_j) clears in every sector $(\forall j \in \mathcal{N})$.
- Low frequency trends are interpreted as sequential steady states (FHSW, 2022)

Modeling Structural Change

Parameters	Driving Processes
$\gamma_j, \rho_j^m, \rho_j^x, \zeta_{ij}^m, \zeta_{ij}^x, \alpha_j, \delta_j, \gamma_\ell$	$z_{jt}, z_{ij,t}^m, z_{ij,t}^x, \Theta_{jt}, \psi_{jt}$

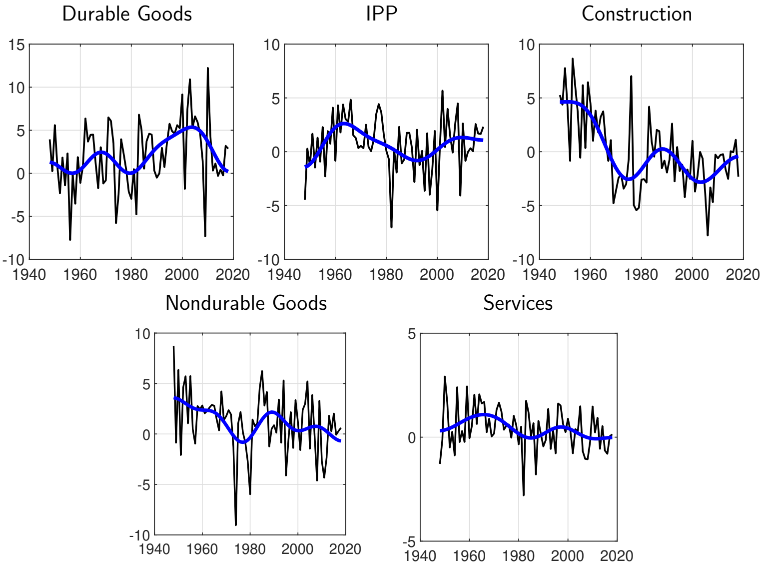
$+$ Competitive Equilibrium

Consumption Shares, Input Cost Shares $\Rightarrow$ GDP Shares

Quantifying the Model

- Choosing parameter values: 3 approaches
  - Observed consumption or input expenditure shares
  - Low-frequency estimates
  - Estimates from others' work (e.g., elasticity of labor supply)

Sectoral Total Factor Productivity Growth (Percent)

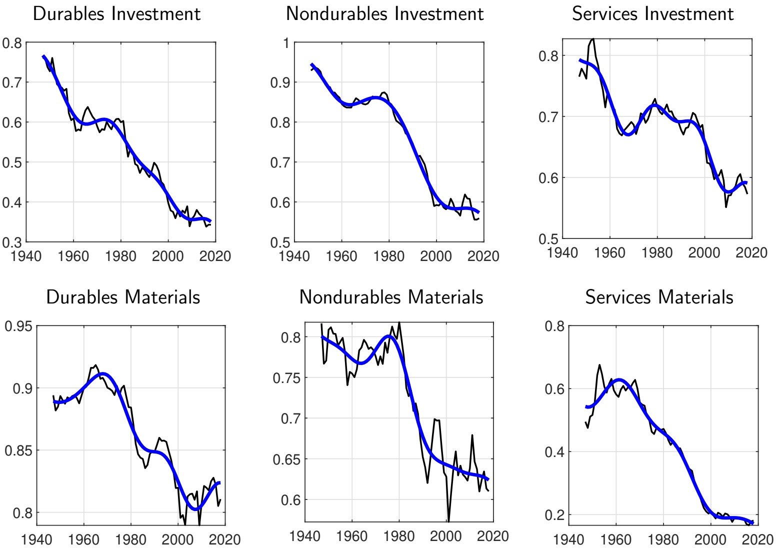


Estimates of Production Elasticities

Sectors	Durables	IPP	Construction	Nondurables	Services
ϵ^x Post. Mean	1.26	0.69	1.43	1.01	1.08
ϵ^x Post. Median	1.14	0.52	1.24	0.84	0.93
	[0.46,2.04]	[0.17,1.20]	[0.37,2.47]	[0.26,1.73]	[0.32,1.81]
ϵ^m Post. Mean	1.69	1.12	2.03	0.86	0.69
ϵ^m Post. Median	1.59	1.03	2.01	0.76	0.51
	[0.69,2.61]	[0.37,1.79]	[1.08,2.92]	[0.27,1.43]	[0.14,1.21]

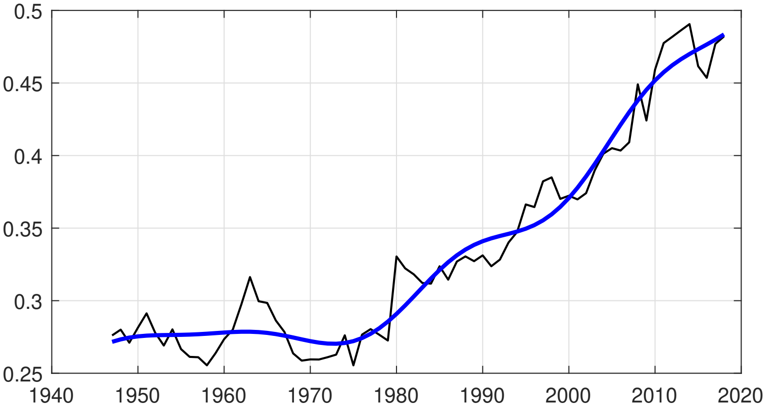
- $$\log \frac{p^y_{i,t} m_{ij,t} / p^m_{j,t} m_{j,t}}{p^y_{i',t} m_{i'j,t} / p^m_{j,t} m_{j,t}} = \epsilon^m_j \log \frac{z^m_{ij,t}}{z^m_{i'j,t}} + (1 - \epsilon^m_j) \log \frac{p^y_{i,t}}{p^y_{i',t}}$$
- Use $\Delta \left(\frac{z_{i,t}}{z_{i',t}} \right)$ as an instrument, only low-frequency variations in the data (periods $\geqslant 20$ years), Bayes and Frequentist methods

Input-Biased Technical Change



- Relative productivity of Durable Goods inputs in selected sectors, z_{DUR} with $z_{DUR} + z_{IPP} = 1$.
- The relative productivity of Durables in production falls steadily across all sectors over the post-war period.
- e.g., In the Services sector, Durables materials are 1.9 times as productive as IPP in 1950, but only 1/4 as productive by the end of the sample.

Exogenous Shift in Preferences

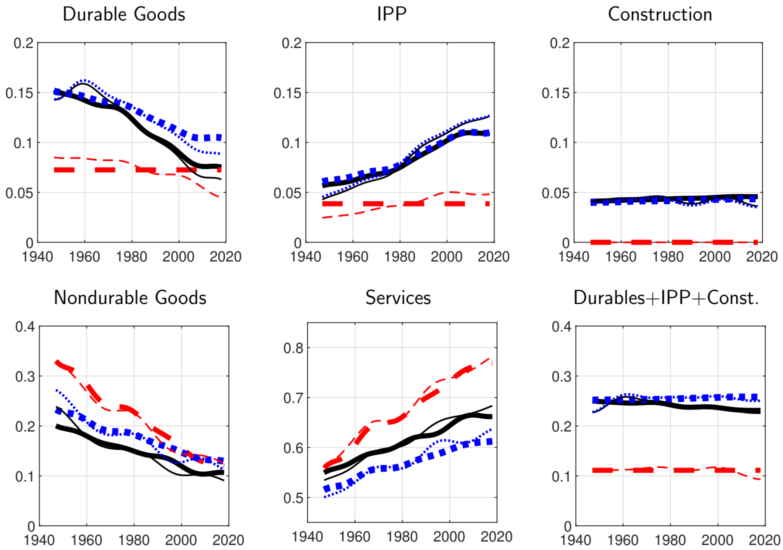


- Exogenous preference for Services relative to Nondurables, Θ_{SVC}.
- Preference for Services relative to Nondurables is relatively stable from 1940 to 1970.
- So compositional shifts in consumption towards Services during that period must stem from other forces through their effects on relative prices and income.
- Beginning in the 1970s, inherent shifts in preferences towards Services complement endogenous changes in prices and income.

$$\frac{p^y_{j,t} c_{j,t}}{\tilde{e}_t} = \Theta_{j,t} \left(\frac{p^y_{j,t}}{p^y_{b,t}} \right)^{1-\sigma} \left(\frac{\tilde{e}_t}{p^y_{b,t}} \right)^{(1-\sigma)(\epsilon^c_j - 1)} \left(\frac{p^y_{b,t} c_{b,t}}{\tilde{e}_t} \right)^{\epsilon^c_j},$$

Structural Change: Model vs. Data

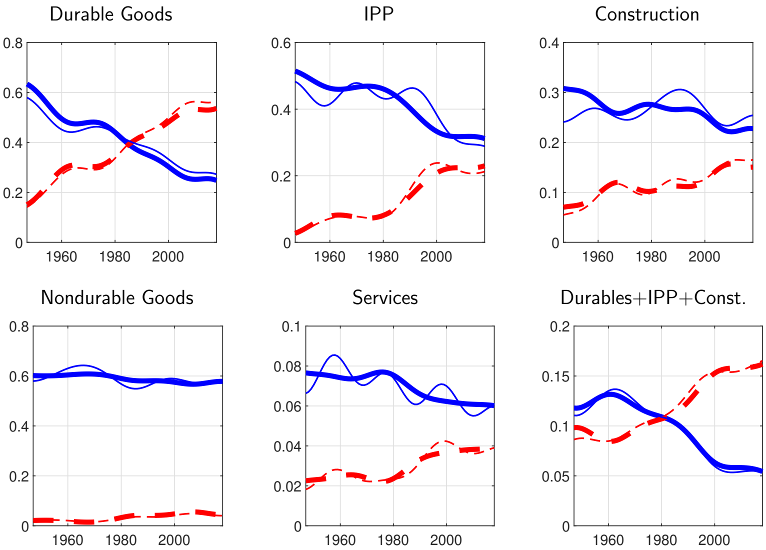
Contributions from: $\boldsymbol{\theta}$ (red), $\boldsymbol{\theta} + (\boldsymbol{\Phi}, \boldsymbol{\Omega}, \boldsymbol{\Gamma})$ (blue), Value-Added Shares (black)



- The baseline model captures essentially all the trend variations in sectoral value added shares *and* constituent sub-shares.
- Model-generated consumption shares are constant by assumption in input-producing sectors, not quite so in the data.
- In Nondurables and Services, the model matches trend variations in consumption shares almost exactly.

Trend Input Cost Shares: Model vs. Data

Investment and Materials Spending Shares, Durables (blue) vs. IPP (red)

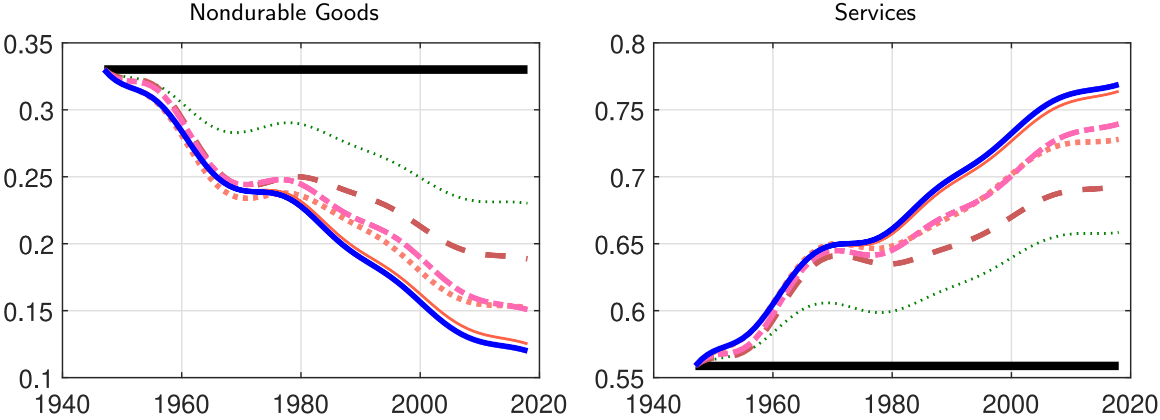


- Trend variations in input shares of Durables and IPP exactly offset in the model by assumption.
- This is essentially true of the data as well, and across both material and investment inputs.

Compositional Accounting - Structural Model

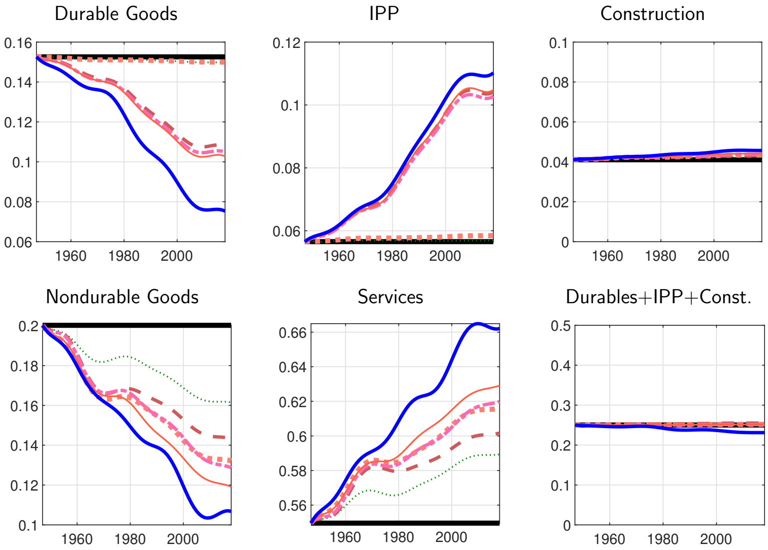
Cumulative Decomposition of Sectoral Shares

TFP Homothetic Preferences (thin dots), TFP Non-Homothetic Preferences (thick dots),
IBTC (dash), Preference Shifts (dash-dot), Labor Supply Shifts (thin), Net Exports (thick)



- Trend changes in sectoral TFP, and their effects on relative prices and income, explain essentially all of the secular shifts in the consumption shares of Nondurables and Services until about 1980 (thin dots), and about 3/4 of thereafter (thick dots).
- Labor supply shocks matter little.

TFP Homothetic Preferences (thin dots), TFP Non-Homothetic Preferences (thick dots), IBTC (dash), Preference Shifts (dash-dot), Labor Supply Shifts (thin), Net Exports (thick)



- IBTC explains the bulk of the trend changes in value-added shares of Durables and IPP
- TFP essentially explains trend changes in the consumption shares of Nondurables and Services and, therefore, most of the trend changes in their value added shares
- However, Net Exports are also important for Durables, Nondurables, and Services

Long-Run Forecasts from the Structural Model

- Compute the forecast distribution of exogenous processes, $z_{jt}, z^{m}_{ij,t}, z^{x}_{ij,t}, \Theta_{jt}, \psi_{jt}$, as explained earlier.
- Use the structural model to compute the implied forecast distribution of equilibrium value-added shares in GDP.
  - We ignore uncertainty in model parameters.

Forecasted Changes in Trend Sectoral Shares of GDP

2018 to 2038 (Percentage Points)

Sectors	Durables	IPP	Construction	Nondurables	Services
Historical 20 Year Avg	-2.2	2.3	-0.1	-4.1	4.1
Reduced Form	-1.5	3.5	-0.3	-2.6	0.9
Forecast 2018-2038	[-2.4,-0.4]	[2.4,4.5]	[-1.1,0.8]	[-3.6,-1.4]	[-1.2,2.5]
Structural Model	-1.4	1.5	0.0	-1.3	1.2
Forecast 2018-2038	[-2.1,-0.8]	[0.6,2.4]	[0.0,0.1]	[-2.0,-0.6]	[0.1,2.2]
By Process:					
TFP ($z_{j,t}$)	0.0	0.0	0.0	-0.7	0.7
IBTC ($z^m_{ij,t}, z^x_{ij,t}$)	-0.9	1.4	0.0	-0.2	-0.2
Preferences ($\Theta_{j,t}$)	-0.1	0.0	0.0	-0.3	0.4
Labor Supply ($\varphi_{j,t}$)	-0.1	0.1	0.0	-0.3	0.3
G and NX ($\Psi_{j,t}$)	-0.5	0.2	0.0	-0.3	0.6

- **Result 1: Forecasts suggest a slowdown in structural change**

Forecasted Changes in Trend Sectoral Shares of GDP

2018 to 2038 (Percentage Points)

Sectors	Durables	IPP	Construction	Nondurables	Services
Historical 20 Year Avg	-2.2	2.3	-0.1	-4.1	4.1
Reduced Form	-1.5	3.5	-0.3	-2.6	0.9
Forecast 2018-2038	[-2.4,-0.4]	[2.4,4.5]	[-1.1,0.8]	[-3.6,-1.4]	[-1.2,2.5]
Structural Model	-1.4	1.5	0.0	-1.3	1.2
Forecast 2018-2038	[-2.1,-0.8]	[0.6,2.4]	[0.0,0.1]	[-2.0,-0.6]	[0.1,2.2]
By Process:					
TFP ($z_{j,t}$)	0.0	0.0	0.0	-0.7	0.7
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Preferences ($\Theta_{j,t}$)	-0.1	0.0	0.0	-0.3	0.4
Labor Supply ($\varphi_{j,t}$)	-0.1	0.1	0.0	-0.3	0.3
G and NX ($\Psi_{j,t}$)	-0.5	0.2	0.0	-0.3	0.6

- **Result 2: Structural forecasts incorporate historical features of the data and constraints**

Forecasted Changes in Trend Sectoral Shares of GDP

2018 to 2038 (Percentage Points)

Sectors	Durables	IPP	Construction	Nondurables	Services
Historical 20 Year Avg	-2.2	2.3	-0.1	-4.1	4.1
Reduced Form	-1.5	3.5	-0.3	-2.6	0.9
Forecast 2018-2038	[-2.4,-0.4]	[2.4,4.5]	[-1.1,0.8]	[-3.6,-1.4]	[-1.2,2.5]
Structural Model	-1.4	1.5	0.0	-1.3	1.2
Forecast 2018-2038	[-2.1,-0.8]	[0.6,2.4]	[0.0,0.1]	[-2.0,-0.6]	[0.1,2.2]
By Process:					
TFP ($z_{j,t}$)	0.0	0.0	0.0	-0.7	0.7
IBTC ($z^m_{ij,t}, z^x_{ij,t}$)	-0.9	1.4	0.0	-0.2	-0.2
Preferences ($\Theta_{j,t}$)	-0.1	0.0	0.0	-0.3	0.4
Labor Supply ($\varphi_{j,t}$)	-0.1	0.1	0.0	-0.3	0.3
G and NX ($\Psi_{j,t}$)	-0.5	0.2	0.0	-0.3	0.6

- **Result 3: Main sources of forecast change is as expected from model**

Conclusions

- Structural Change reflects changes in production (shifts in the kinds of inputs used), and changes in consumption patterns
  - Shift in input use accounts for the rise of IPP as a share of GDP, almost exactly displacing Durables as the dominant source of inputs over the post war
  - The shift in consumption towards Services accounts for its increase as a share of GDP, almost exactly offset by a corresponding fall in Nondurables consumption
- IBTC explains the shift in input use away from Durables towards IPP. Net exports also played a role in the decline of Durables.
- TFP explains more than $1/2$ the shift in consumption towards Services and away from Nondurables, and almost all of this shift until 1980
- Provide framework for compositional forecasting without (or with fewer) ad-hoc assumptions