

Estimating Production Functions with Expectations Data

Agnes Norris Keiller ¹ Áureo de Paula ²
John Van Reenen ³

¹LSE

²UCL, IFS and CeMMAP

³LSE and NBER

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Outline

Motivation

Methodology: Model, Identification and Estimation

Monte Carlo Simulation

Application

Discussion

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- y_{it} = log(output) of firm i at time t
- l_{it} = log(labor), k_{it} = log(capital), u_{it} = error term

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- ▶ Such data enables us to develop a new production function estimator 

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 - Proxy variables (control functions): e.g., Olley and Pakes (1996, OP); Levinsohn & Petrin (2003, LP); Akerberg, Caves & Fraser, (2015, ACF)

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- 5. NPR brings in exploit **new data** on firm expectations of output and inputs [Card's Maxim]
 - These expectations reveal what a firm *believes* it's productivity will be next period, thus helping reveal what unobserved productivity is *this* period
 - Hence, use functions of observed expectations to get unbiased estimates of production function

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- Expectations data allow one to relax assumptions of firm input decision rules required by proxy variable estimators

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3. Application

- UK firm-level panel data 2010-2020 (MES + ABI data)
- NPR particularly useful when choosing inputs optimally hard (e.g., restaurants around Brexit/Covid period). An “estimator for turbulent times”.
- NPR TFP estimates produces more sensible dynamic moments between initial TFP and subsequent growth

▶ Advantages of NPR

- Relax strong assumptions on optimization of factor inputs common to all proxy methods (generates monotonicity of input with productivity to allow invertibility)
 - ▶ So NPR better under oligopoly; unmeasured input prices, turbulence, etc.
- Can estimate NPR with a single cross section of data.
 - ▶ Need ≥ 2 periods for proxy methods and ≥ 3 periods for dynamic panel estimation
 - ▶ Add expectations questions to standard surveys (e.g. WB Enterprise Survey) to help deal with unobserved heterogeneity

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▶ Disadvantages of NPR

- Require explicit firm-level expectations data
- Some (testable) assumptions over this data (biases, measurement error, etc.)

Some Existing Literature

- ▶ **History:** Von Thuenen (1820s); Tolley et al. (1924); Cobb and Douglas (1928); de Loecker & Syverson (2021);
- ▶ **Proxy Methods:** Olley & Pakes (1996); Levinsohn & Petrin (2003); Akerberg, Caves & Fraser (2015) **Dynamic Panel Data Methods**; Bond & Soderbom (2005); Blundell & Bond (2000); Anderson & Hsiao (1981); Arellano & Bond (2001);
- ▶ **Mark-Ups:** de Loecker and Warzynski (2012); Akerberg and de Loecker (2025); de Ridder et al. (2024)
- ▶ **Extensions:** Gandhi et al., (2020); Orr (2022); Doraszelski-Jaumandreu (2013); Valmari (2023); Demirer (2022); Doraszelski & Jaumandreu (2018); Raval (2023)
- ▶ **Expectations:** Dominitz & Manski (1997); Altig et al., (2022); Bloom et al., (2025); Gennaioli et al., (2016); Arellano et al. (2024); Born et al., (2022)

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The object of interest

- ▶ Consider a general production function of the following form

$$y_{it} = f(k_{it}, l_{it}; \beta) + \omega_{it} + \epsilon_{it}$$

- ▶ $\omega_{it} \equiv$ idiosyncratic productivity known by the firm when deciding period t input and investment
- ▶ ϵ_{it} unanticipated mean-zero disturbances
 - measurement error (or productivity shocks unknown by the firm when making period t decisions)

Dynamics

- ▶ ω_{it} follows a Markov process

$$\omega_{it} = \mathbb{E}[\omega_{it} | \omega_{it-1}] + \xi_{it} = \mathbf{g}(\omega_{it-1}) + \xi_{it}$$

Dynamics

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$$\omega_{it} = \mathbb{E}[\omega_{it} | \omega_{it-1}] + \xi_{it} = g(\omega_{it-1}) + \xi_{it}$$

- ▶ Capital evolves according to

$$K_{it} = (1 - \delta)K_{it-1} + I_{it-1}$$

where δ = the depreciation rate; I_{it-1} = investment

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 - OP: firms' investment policy $\rightarrow \omega = \Phi^{OP}(i_{it}, k_{it})$
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- ▶ In NPR, we assume nothing about the invertibility of firms' decisions.

Expectations

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- ▶ Firm's expectations, \mathbb{E}_{it} are **data**. Don't need to be rational (see App A)
- ▶ If firms' expectations align with the true production technology

$$\begin{aligned}\mathbb{E}_{it}[y_{it+1} | \Omega_{it}] &= \int f(k_{it+1}, l_{it+1}; \beta) dF_{it}(l_{it+1}) \\ &\quad + \mathbb{E}_{it}[\omega_{it+1} | \Omega_{it}] + \mathbb{E}_{it}[\epsilon_{it+1} | \Omega_{it}] \\ &= \int f(k_{it+1}, l_{it+1}; \beta) dF_{it}(l_{it+1}) + g(\omega_{it})\end{aligned}\tag{1}$$

- $F_{it}(l_{it+1}) \equiv$ firm i 's subjective prob distribution over next-period labour input
- $\mathbb{E}_{it}[\xi_{it+1} | \Omega_{it}] = \mathbb{E}_{it}[\epsilon_{it+1} | \Omega_{it}] = 0$

More

Recovering ω_{it}

- ▶ Rearranging equation (1) for $g(\omega_{it})$ gives:

$$g(\omega_{it}) = \mathbb{E}_{it}[y_{it+1} | \Omega_{it}] - \int f(k_{it+1}, l_{it+1}; \beta) dF_{it}(l_{it+1}) \quad (2)$$

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- ▶ Monotonicity assumption means RHS of equation (2) is strictly increasing in ω_{it}

$$\begin{aligned} \omega_{it} &= g^{-1} \left(\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \int f(k_{it+1}, l_{it+1}; \beta) dF_{it}(l_{it+1}) \right) \\ &\equiv \Psi \left(\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \int f(k_{it+1}, l_{it+1}; \beta) dF_{it}(l_{it+1}) \right) \end{aligned}$$

Identification

- ▶ Combining

$$\omega_{it} = \Psi \left(\mathbb{E}_{it}[y_{it+1} | \Omega_{it}] - \int f(k_{it+1}, l_{it+1}; \beta) dF_{it}(l_{it+1}) \right)$$

with

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⇒ a moment condition we can use to recover β

$$\begin{aligned} 0 &= \mathbb{E}[\epsilon_{it} | \Omega_{it}] = \mathbb{E}[y_{it} - f(k_{it}, l_{it}; \beta) - \omega_{it} | \Omega_{it}] \\ &= \mathbb{E} \left[y_{it} - f(k_{it}, l_{it}; \beta) - \Psi \left(\mathbb{E}_{it}[y_{it+1} | \Omega_{it}] - \int f(k_{it+1}, l_{it+1}; \beta) dF_{it}(l_{it+1}) \right) | \Omega_{it} \right] \end{aligned}$$

Example

Cobb-Douglas production and $\omega_{it} = \rho\omega_{it-1} + \xi_{it}$ implies

$$\begin{aligned}y_{it} &= \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \epsilon_{it} \\ &= \frac{\rho - 1}{\rho} \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \frac{1}{\rho} \mathbb{E}_{it}[y_{it+1} | \Omega_{it}] - \frac{\beta_k}{\rho} k_{it+1} - \frac{\beta_l}{\rho} \mathbb{E}_{it}[l_{it+1} | \Omega_{it}] + e_{it}\end{aligned}$$

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- ▶ ...but how does one get measurements on $\mathbb{E}_{it}[y_{it+1} | \Omega_{it}]$ and $\mathbb{E}_{it}[l_{it+1} | \Omega_{it}]$?

Yes, Virginia, there is a . . . Management and Expectations Survey!

Looking ahead to the 2021 calendar year, what is the approximate turnover you would anticipate for this business in the following scenarios?

Lowest turnover

Report to the nearest £1,000. For example, £1,357,689 would be reported as £1,358,000

£ 2,800,000

Low turnover

£ 4,200,000

Medium turnover

£ 5,000,000

High turnover

£ 6,300,000

Highest turnover

£ 7,500,000

For the approximate turnover values you have just given for 2021, how likely do you think each scenario is to occur?

Your answers should add up to 100%

Likelihood of lowest turnover occurring

5 %

Likelihood of low turnover occurring

10 %

Likelihood of medium turnover occurring

60 %

Likelihood of high turnover occurring

20 %

Likelihood of highest turnover occurring

5 %

Administrative Data

- ▶ **ABI/ABS**: Representative sample of all private sector industries. Panel data on standard production variables (like US ASM, but covers non-manufacturing)
- ▶ **MES 2017**: Expectations on sales revenue (turnover), employment, capital expenditure, intermediates expenditure
- ▶ **MES 2020**: Expectations on sales & employment
- ▶ Table 3 documents 8 other datasets with necessary NPR info (e.g., US SBU).

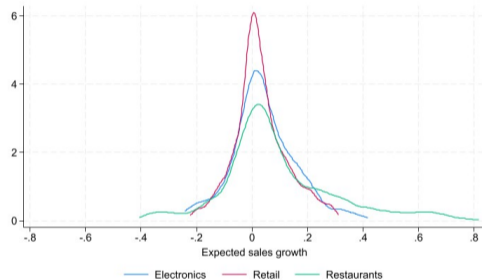
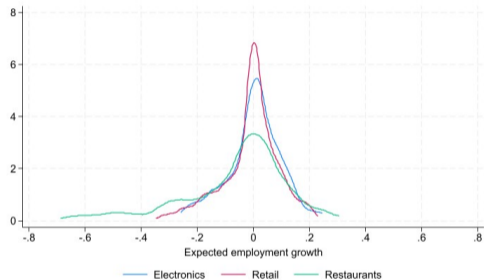
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- ▶ To get what we need
 - Convert scenario responses into 5 points on a CDF (or 1-CDF)
 - Estimate the parameters of a lognormal distribution to fit these points via least-squares minimization (cf. Dominitz & Manski, 1997)

Predictions and Outcomes

- ▶ Use match between expectations and future outcomes (Table 7)
- ▶ On average firms reasonably good at forecasting future sales & employment (see Bloom et al., 2026)
- ▶ Forecasts not significantly different from outcomes (e.g., employment forecast error between -4% and +2%)
- ▶ But some heterogeneity, especially for restaurants [Table 7](#)
- ▶ Focusing just on growth predictions.....

Restaurants more uncertain about growth than other industries we focus on



Note: SD of expected log employment growth are 0.09 for Restaurants, 0.05 for Retail and 0.04 for Electronics.
Obs are: 462 for Restaurants, 2084 for Retail and 472 for Electronics.

Estimation

Under a more general law of motion, Cobb-Douglas production implies

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \Psi (\mathbb{E}_{it}[y_{it+1} | \Omega_{it}] - \beta_k k_{it+1} - \beta_l \mathbb{E}_{it}[l_{it+1} | \Omega_{it}]) + e_{it}$$

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- Impose constraints on the 1st derivatives of the smooth function that comprises Ψ (Pya & Wood, 2015)

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- Impose constraints on the 1st derivatives of the smooth function that comprises Ψ (Pya & Wood, 2015)

Problem 2: Ψ 's argument is a function of the linear parameters

- Use an iterative 'backfitting' algorithm (Ichimura & Todd, 2007)

Estimation Protocol

Adapting the [Friedman and Stuetzle \(1981\)](#) algorithm to our setting:

1. Initialize the parameter vector at $\hat{\theta}_0 = (\hat{\beta}_{00}, \hat{\beta}_{k0}, \hat{\beta}_{l0})$
 2. For iteration j , calculate $Z_{ij} = \mathbb{E}_{it}[y_{it+1} | \Omega_{it}] - \hat{\beta}_{0j-1} - \hat{\beta}_{kj-1} k_{it+1} - \hat{\beta}_{lj-1} l_{it+1}$
 3. Fit the model $y_{it} = \beta_0 - \beta_k k_{it} - \beta_l l_{it} + \Psi(Z_{ij}) + \epsilon_{it} + v_{it}$ using the shape constrained estimator of [Pya and Wood \(2015\)](#) to obtain $\hat{\theta}_j$
 4. Calculate the Euclidean distance between $\hat{\theta}_j$ and $\hat{\theta}_{j-1}$. If the distance is below some tolerance level, stop and treat $\hat{\theta}_j$ as the model's parameter estimates. If not then set $j \leftarrow j + 1$ and repeat from step 2
-
- ▶ For the remainder of these slides, this algorithm is referred to as **n p r**
 - ▶ Convergence proof of NPR in Appendix B (draws on [Dominitz and Sherman \(2005\)](#))

Some Extensions

- ▶ Measurement Error in expectations data (cf. ?)
 - Extend NPR using Evdokimov & Zeleneev (2025)
 - “EZ-NPR” performs well in Monte Carlos and our application
- ▶ Biased Expectations (Appendix D)
 - OK if output expectation biases reflected in input biases
 - Productivity expectations bias can be handled by extensions to NPR with panel of expectations
 - But MES data shows little systematic bias (Tab 9) & expectations panel limited
- ▶ Different specifications of production function (e.g., Translog; Value Added instead of gross output)

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Monte Carlo Setup

- ▶ We follow ACF Monte Carlo setup: [Details](#)
 - y a Leontief composite of m and a 'value added' function of l and k .
 - ω follows an AR(1) process.
 - Investment subject to a firm-specific convex adjustment cost.
 - Firms face a common, time-invariant wage cost.
 - Optimization error in labor, l (as in ACF) but also in materials, m & investment, i .
- ▶ Firms' optimal decisions and expectations have an analytical solution.
- ▶ Simulate 1,000 firms over 100 periods, use data from last 10.

- ▶ Compare OLS\LP\ACF\NPR across various DGPs.

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Optimization error in labor choice / (Table 1 in paper)

	$\beta_l = 0.6$				$\beta_k = 0.4$				N runs
	Mean	Median	S.D.	MSE	Mean	Median	S.D.	MSE	
NPR	0.600	0.600	0.003	0.000	0.400	0.400	0.005	0.000	500
OLS LEVELS	0.919	0.919	0.002	0.102	0.098	0.098	0.004	0.091	500
OP	0.840	0.840	0.004	0.057	0.161	0.161	0.008	0.057	500
LP	0.600	0.600	0.003	0.000	0.400	0.400	0.014	0.000	500
ACF	0.823	0.601	1.043	1.135	0.161	0.401	1.125	1.320	500
ACF $ (\beta_l, \beta_k \in (0, 1))$	0.600	0.600	0.009	0.000	0.401	0.401	0.016	0.000	478

Number of replications given in the 'N runs' column. ACF results from initialization at $\beta_{l0} = 0.5$ and $\beta_{k0} = 0.5$.

Optimization error in labor choice / (Table 1 in paper)

	$\beta_l = 0.6$				$\beta_k = 0.4$				N runs
	Mean	Median	S.D.	MSE	Mean	Median	S.D.	MSE	
NPR	0.600	0.600	0.003	0.000	0.400	0.400	0.005	0.000	500
OLS LEVELS	0.919	0.919	0.002	0.102	0.098	0.098	0.004	0.091	500
OP	0.840	0.840	0.004	0.057	0.161	0.161	0.008	0.057	500
LP	0.600	0.600	0.003	0.000	0.400	0.400	0.014	0.000	500
ACF	0.823	0.601	1.043	1.135	0.161	0.401	1.125	1.320	500
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LP	0.600	0.600	0.003	0.000	0.400	0.400	0.014	0.000	500
ACF	0.823	0.601	1.043	1.135	0.161	0.401	1.125	1.320	500
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Optimization error in all inputs (l, m, i)

	$\beta_l = 0.6$				$\beta_k = 0.4$				N runs
	Mean	Median	S.D.	MSE	Mean	Median	S.D.	MSE	
NPR	0.600	0.600	0.006	0.000	0.399	0.400	0.034	0.001	500
OLS LEVELS	0.919	0.919	0.002	0.102	0.097	0.097	0.004	0.092	500
OP	0.897	0.897	0.003	0.088	0.104	0.105	0.011	0.088	500
LP	0.304	0.304	0.006	0.088	0.763	0.763	0.020	0.132	500
ACF	0.383	0.347	0.401	0.207	0.664	0.701	0.441	0.264	500
ACF $ (\beta_l, \beta_k \in (0, 1))$	0.347	0.347	0.028	0.065	0.704	0.701	0.035	0.093	496

Number of replications given in the 'N runs' column. ACF results from initialization at $\beta_{l0} = 0.5$ and $\beta_{k0} = 0.5$.

Optimization error in all inputs (l, m, i)

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	Mean	Median	S.D.	MSE	Mean	Median	S.D.	MSE	
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OLS LEVELS	0.919	0.919	0.002	0.102	0.097	0.097	0.004	0.092	500
OP	0.897	0.897	0.003	0.088	0.104	0.105	0.011	0.088	500
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Number of replications given in the 'N runs' column. ACF results from initialization at $\beta_{l0} = 0.5$ and $\beta_{k0} = 0.5$.

Monte Carlo summary

1. NPR robust to optimization errors
 - (Outperforms OLS\LP\ACF as optimization error $\sigma \uparrow$)
2. When optimization error on labor only, NPR, LP and ACF do well – though ACF is at times numerically unstable, see footnote 16 in ACF or [Kim, Luo, and Su \(2019\)](#).
3. When optimization errors affect multiple inputs, NPR remains adequate but other estimators deteriorate as they rely on input choices to proxy for ω .

Outline

Motivation

Methodology: Model, Identification and Estimation

Monte Carlo Simulation

Application

Discussion

Production function estimates (Table 8): Restaurants

Table: Restaurants

	NPR	OLS LVLS	OLS FD	OLS FE	OP	LP	ACF
β_l	0.82 (0.07)	0.81 (0.06)	0.63 (0.11)	0.68 (0.10)	0.69 (0.05)	0.69 (0.05)	1.06 (0.15)
β_k	0.26 (0.07)	0.21 (0.06)	-0.01 (0.06)	0.02 (0.05)	0.09 (0.09)	0.25 (0.14)	0.05 (0.14)
$\beta_l + \beta_k$	1.08	1.01	0.62	0.70	0.78	0.94	1.10
CRS	0.00	0.33	0.00	0.00	0.03	0.71	0.18
N firms	430	430	392	389	430	430	430

Restaurants have big optimization errors in materials

Production function estimates (Table 8): Electronics Manufacturing

Table: Electronics

	NPR	OLS LVLS	OLS FD	OLS FE	OP	LP	ACF
β_l	0.90 (0.14)	0.86 (0.05)	0.45 (0.12)	0.47 (0.10)	0.60 (0.05)	0.62 (0.07)	0.79 (0.13)
β_k	0.21 (0.12)	0.23 (0.04)	0.04 (0.03)	0.04 (0.03)	0.26 (0.15)	0.34 (0.08)	0.23 (0.09)
$\beta_l + \beta_k$	1.11	1.09	0.49	0.51	0.86	0.96	1.02
CRS	0.00	0.00	0.00	0.00	0.36	0.66	0.88
N firms	422	422	411	400	422	422	422

Production function estimates (Table 8): Retail

Table: Retail

	NPR	OLS LVL	OLS FD	OLS FE	OP	LP	ACF
β_l	0.80*** (0.11)	0.75*** (0.03)	0.46*** (0.09)	0.52*** (0.07)	0.63*** (0.02)	0.55*** (0.04)	0.66*** (0.11)
β_k	0.16*** (0.05)	0.25*** (0.03)	-0.02 (0.02)	-0.02 (0.02)	0.01 (0.08)	0.16*** (0.05)	0.17* (0.09)
$\beta_l + \beta_k$	0.96	1.00	0.43	0.49	0.64	0.71	0.83
CRS	0.00	0.80	0.00	0.00	0.00	0.00	0.15
N firms	1853	1853	1807	1763	1853	1853	1853

Stronger Dynamic Moments using NPR

Table: 10: Initial TFP and future firm performance

	NPR	ACF	OLS	Mean	N
Electronics					
Exit	-0.016*** (0.004)	-0.017*** (0.004)	-0.016*** (0.004)	0.01	5362
$l_{t+2} - l_{t+1}$ Jobs growth (1-yr ahead)	0.010** (0.005)	0.008* (0.005)	0.009** (0.005)	0.01	2201
$l_{t+5} - l_{t+1}$ Jobs growth (4-yr ahead)	0.044** (0.021)	0.03 (0.020)	0.041** (0.020)	0.025	1398
Retail					
Exit	-0.014*** (0.001)	-0.014*** (0.001)	-0.014*** (0.001)	0.024	57630
$l_{t+2} - l_{t+1}$ Jobs growth (1-yr ahead)	0.003*** (0.001)	0.003*** (0.001)	0.003*** (0.001)	0.026	13485
$l_{t+5} - l_{t+1}$ Jobs growth (4-yr ahead)	0.007* (0.004)	0.005 (0.004)	0.008** (0.004)	0.095	8331

Dynamic Moments: Associations of growth and TFP

Table: Initial TFP and future firm performance - Restaurants

	NPR	ACF	OLS	Mean	N
	Restaurants				
Exit	-0.007*** (0.002)	-0.003 (0.002)	-0.011*** (0.002)	0.038	12513
$l_{t+2} - l_{t+1}$ Jobs growth (1-yr ahead)	0.021*** (0.007)	0.012* (0.007)	0.016** (0.007)	0.054	2643
$l_{t+5} - l_{t+1}$ Jobs growth (4-yr ahead)	0.036 (0.031)	0.004 (0.028)	0.005 (0.030)	0.225	1581

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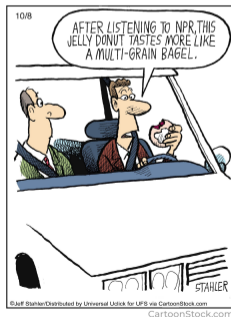
Conclusions

- ▶ Subjective expectations data offer a new way of production function estimation: NPR
 - Relax assumptions on firms choosing factor inputs optimally
 - Allows estimation in cross section
- ▶ Performs well in Monte Carlo and in application to UK MES data
- ▶ Improved dynamic moments
- ▶ Developing downloadable material for new surveys and estimation package



Conclusions

- ▶ Subjective expectations data offer a new way of production function estimation: NPR
 - Relax assumptions on firms choosing factor inputs optimally
 - Allows estimation in cross section
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Appendix

- ▶ Setting $\beta_0 = 0$ for simplicity,

$$y_{it} = \mathbb{E}(y_{it+1}|\Omega_{it})/\rho + \beta_K k_{it} - (\beta_K/\rho)k_{it+1} + \beta_L l_{it} - (\beta_L/\rho)\mathbb{E}(l_{it+1}|\Omega_{it}) + \epsilon_{it}.$$

Without data on expectations but under rational expectations one could use instead:

$y_{it+1} = \mathbb{E}(y_{it+1}|\Omega_{it}) + \zeta_{it+1}^y$ and $l_{it+1} = \mathbb{E}(l_{it+1}|\Omega_{it}) + \zeta_{it+1}^l$. Then

$$y_{it} = (1/\rho)y_{it+1} + \beta_K k_{it} - (\beta_K/\rho)k_{it+1} + \beta_L l_{it} - (\beta_L/\rho)l_{it+1} + e_{it},$$

where $e_{it} = \epsilon_{it} - (1/\rho)\zeta_{it+1}^y + (1/\rho)\beta_L\zeta_{it+1}^l$. (Both y_{it+1} and l_{it+1} are endogenous regressors.)

- ▶ Blundell and Bond (2000) quasi-difference the production function to obtain:

$$y_{it+1} = \rho y_{it} + \beta_K k_{it+1} - \rho\beta_K k_{it} + \beta_L l_{it+1} - \rho\beta_L l_{it} + u_{it+1},$$

where $u_{it+1} = \xi_{it+1} + \rho\epsilon_{it+1} - \rho\epsilon_{it}$. Equivalently,

$$y_{it} = (1/\rho)y_{it+1} + \beta_K k_{it} - (\beta_K/\rho)k_{it+1} + \beta_L l_{it} - (\beta_L/\rho)l_{it+1} + v_{it+1},$$

where $v_{it+1} = -(1/\rho)u_{it+1}$. (Both y_{it+1} and l_{it+1} are endogenous regressors.)

Monte Carlo setup

▶ Following ACF

- $Y = \min\{K_{it}^{\beta_k} L_{it}^{\beta_l} e^{\omega_{it}}, M_{it}\} e^{\epsilon_{it}}$ ($\beta_l = 0.6; \beta_k = 0.4; \epsilon_{it} \sim (0, 0.1)$)
- $\omega_{it} = \rho\omega_{it-1} + \xi_{it}$ ($\rho = 0.7; \text{std dev}(\omega_{it}) = 0.3$)
- $c_i(l_{it}) = \frac{\phi_l}{2} l_{it}^2$ ($1/\phi_l \sim \ln \mathcal{N}(0, 0.6)$)

- optimization error: $X = X_{it}^* e^{\xi_{it}^x}$ for $X = L, M, I$ and $\xi_{it}^x \sim \mathcal{N}(0, 0.37)$.

- ▶ Set $K_{i0} = 0$, simulate the model for 100 periods, use data from the last 10

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Monte Carlo setup

▶ Following ACF

- $Y = \min\{K_{it}^{\beta_k} L_{it}^{\beta_l} e^{\omega_{it}}, M_{it}\} e^{\epsilon_{it}}$ ($\beta_l = 0.6; \beta_k = 0.4; \epsilon_{it} \sim (0, 0.1)$)
- $\omega_{it} = \rho\omega_{it-1} + \xi_{it}$ ($\rho = 0.7; \text{std dev}(\omega_{it}) = 0.3$)
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Production function estimates: Electronics Manufacturing

Table: Sample: Electronics

	NPR	OLS	OLS FD	OLS FE	OP	LP	ACF
β_l	0.90 (0.14)	0.86 (0.05)	0.45 (0.12)	0.47 (0.10)	0.60 (0.05)	0.62 (0.07)	0.79 (0.13)
β_k	0.21 (0.12)	0.23 (0.04)	0.04 (0.03)	0.04 (0.03)	0.26 (0.15)	0.34 (0.08)	0.23 (0.09)
$\beta_l + \beta_k$	1.11	1.09	0.49	0.51	0.86	0.96	1.02
CRS	0.00	0.00	0.00	0.00	0.36	0.66	0.88
N obs.	472	917	458	895	917	917	917
N firms	422	422	411	400	422	422	422

Back

Some Expectations Formation Processes (Pesaran-Weale, 2006)

▶ Rational Expectations

- $\mathbb{E}_{it}[\omega_{it}|\Omega_{it-1}] = \mathbb{E}[\omega_{it}|\Omega_{it-1}] = 0$
- $\mathbb{E}_{it}[\xi_{it+1}|\Omega_{it}] = \mathbb{E}[\xi_{it+1}|\Omega_{it}] = 0$

▶ Static Expectations (Keynes, 1936)

- $\mathbb{E}_{it}[\mathbf{x}_{it+1}|\Omega_{it}] = \mathbf{x}_{it}$
- $\mathbb{E}_{it}[\omega_{it+1}|\Omega_{it}] = \tilde{g}(\omega_{it})$ where $\tilde{g}(\cdot)$ different from $g(\cdot)$.

▶ “Return to normality” models

- $\mathbb{E}_{it}[\omega_{it+1}|\Omega_{it}] = (1 - \lambda)\omega_{it} + \lambda\omega_i^*$
- Include fixed effect (either for firm or industry)

▶ Adaptive Expectations

- $\mathbb{E}_{it}[\mathbf{x}_{it+1}|\Omega_{it}] = \mathbb{E}_{it-1}[\mathbf{x}_{it}|\Omega_{it-1}] + \Gamma_i(\mathbf{x}_{it} - \mathbb{E}_{it-1}[\mathbf{x}_{it}|\Omega_{it-1}])$
- Koyck model.
- Have an extra term in $\Psi(\cdot)$: $(1-\gamma)\{\mathbb{E}_{it-1}[y_{it}|\Omega_{it-1}] - \int f(k_{it}, l_{it}; \beta) dF_{it-1}(l_{it})\}$
- Need an extra year of expectations data

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Identification

Notice that:

$$\mathbb{E}[y_{it}|z_{it}] = \beta^\top \mathbb{E}[x_{it}|z_{it}] + h(z_{it}) \Rightarrow y_{it} - \mathbb{E}[y_{it}|z_{it}] = \beta^\top (x_{it} - \mathbb{E}[x_{it}|z_{it}]) + \epsilon_{it}$$

$\Rightarrow \beta$ is identified as long as $\mathbb{E}[(x_{it} - \mathbb{E}[x_{it}|z_{it}])(x_{it} - \mathbb{E}[x_{it}|z_{it}])^\top]$ is non-singular.

[This condition] prevents any element of X from being a.s. perfectly predictable by Z in the least squares sense. (...) Notice that (nonlinear) functional relations among X elements are not ruled out. Notice also that identification may be possible even if X uniquely defines Z , when the converse is not true. Robinson (ECTA, 1988, p.940)

Once we have β the non-parametric regression of $y_{it} - \beta_k k_{it} - \beta_l l_{it}$ on $\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \beta_k k_{it+1} - \beta_l \mathbb{E}_{it}[l_{it+1}|\Omega_{it}]$ identifies h (and, consequently, Ψ and g).

Identification

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Once we have β the non-parametric regression of $y_{it} - \beta_k k_{it} - \beta_l l_{it}$ on $\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \beta_k k_{it+1} - \beta_l \mathbb{E}_{it}[l_{it+1}|\Omega_{it}]$ identifies h (and, consequently, Ψ and g).

Identification

This holds when $f(\cdot)$ is linear in parameters (e.g., Cobb-Douglas, translog).

More generally (Theorem 1 in paper),

Assume that $g(\cdot)$ is strictly monotonic and $\mathbb{E}[\epsilon_{it}|\Omega_{it}] = 0$. Let $x_{it} = (k_{it}, l_{it})$ and $z_{it} = (\mathbb{E}_{it}[y_{it+1}|\Omega_{it}], k_{it+1}, F_{it}(\cdot))$ and denote by $\theta_0 = (\beta_0, g_0)$ the data generating parameters. Then, if

$$f(x_{it}; \beta_0) - \mathbb{E}[f(x_{it}; \beta_0)|z_{it}] \neq f(x_{it}; \beta) - \mathbb{E}[f(x_{it}; \beta)|z_{it}]$$

with positive probability for any $\beta \neq \beta_0$, the parameter vector $\theta_0 = (\beta_0, g_0)$ is identified.

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Predictions and Outcomes

Table: 7: Log Deviation Between Expected Levels and Outcomes

	Mean	S.D.	p50	N obs.	N firms
	Electronics				
Turnover	-0.01	0.29	-0.04	221	205
Employment	0.02	0.18	0.02	221	205
	Retail				
Turnover	-0.05	0.25	-0.03	733	659
Employment	0.00	0.21	0.01	735	661
	Restaurants				
Turnover	-0.18	0.48	-0.13	170	150
Employment	-0.04	0.44	0.03	170	150

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Restaurants: more optimization errors for intermediates?

Table: MES vs. ABS: 2017 and 2020

	Mean	S.D.	p50	N obs.	N firms	N p50 obs.	N p50 firms
MES (2017 & 2020)							
Turnover	0.06	0.37	0.00	8230	7373	50	49
Employment	-0.01	0.26	0.00	8266	7404	50	50
Capex	0.15	1.74	0.00	7501	6757	50	50
Intermediates	0.09	1.43	0.11	8157	7312	50	50
MES Restaurants (2017 & 2020)							
Turnover	-0.00	0.34	-0.00	235	209	50	48
Employment	-0.03	0.38	0.03	236	209	50	49
Capex	0.62	2.06	0.07	205	180	50	47
Intermediates	0.19	1.03	0.30	236	209	50	48
MES Electronics (2017 & 2020)							
Turnover	0.00	0.17	-0.00	284	252	50	48
Employment	0.02	0.17	0.00	284	252	50	49
Capex	-0.04	1.58	0.01	260	233	50	48
Intermediates	-0.15	1.41	0.02	282	250	50	49

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