Redistricting with Endogenous Candidates

Paola Moscariello

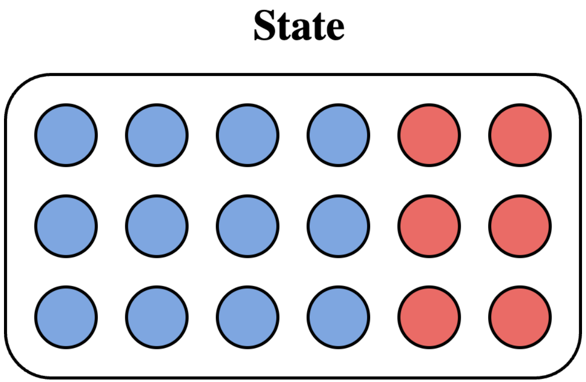
Yale University

Introduction

- In the US, congressional district boundaries drawn by political partisans
- Partisan gerrymandering: "the practice of dividing a geographic area into electoral districts, often of highly irregular shape, to give one political party an unfair advantage by diluting the opposition's voting strength"
- This paper: gerrymandering when district composition affects candidates' policy positions, and thus voting behavior, at the district level

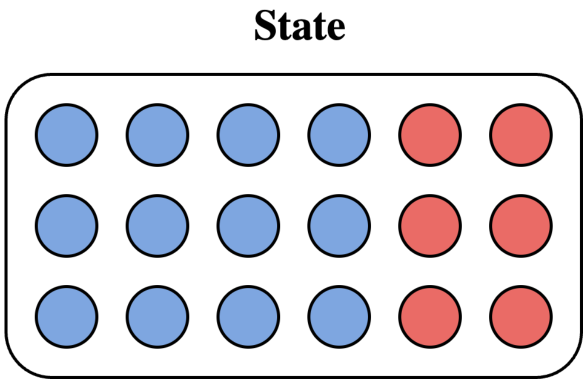
Recap: Standard Redistricting

Recap: Standard Redistricting

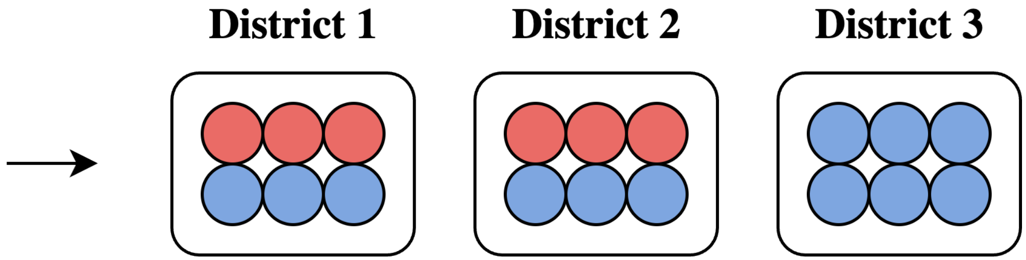


$\frac{1}{3}$ Republicans and $\frac{2}{3}$ Democrats

Recap: Standard Redistricting

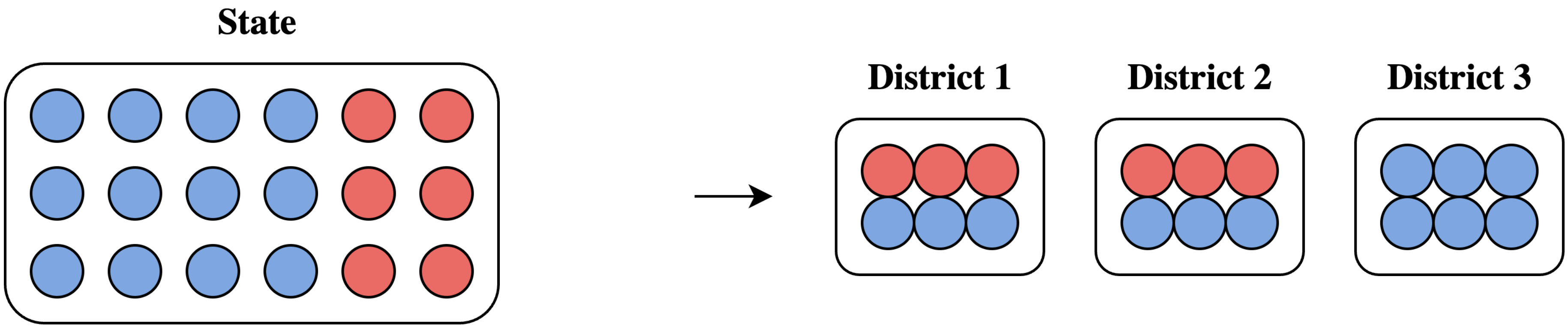


$\frac{1}{3}$ Republicans and $\frac{2}{3}$ Democrats



“Pack-and-crack”

Recap: Standard Redistricting

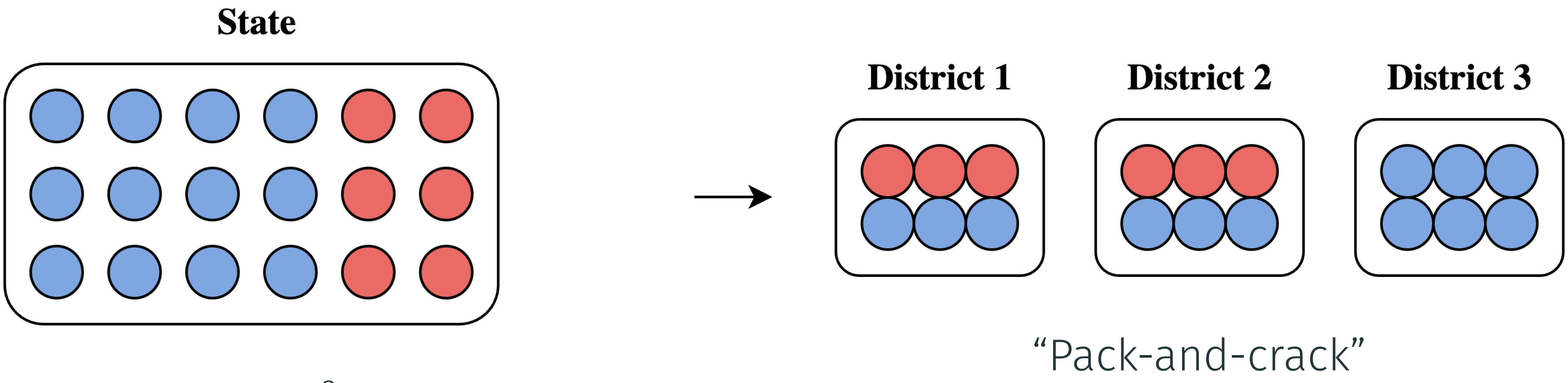


"Pack-and-crack"

$\frac{1}{3}$ Republicans and $\frac{2}{3}$ Democrats

- Considers party affiliation but not preference intensities
- Underlying assumption: voting behavior is fixed

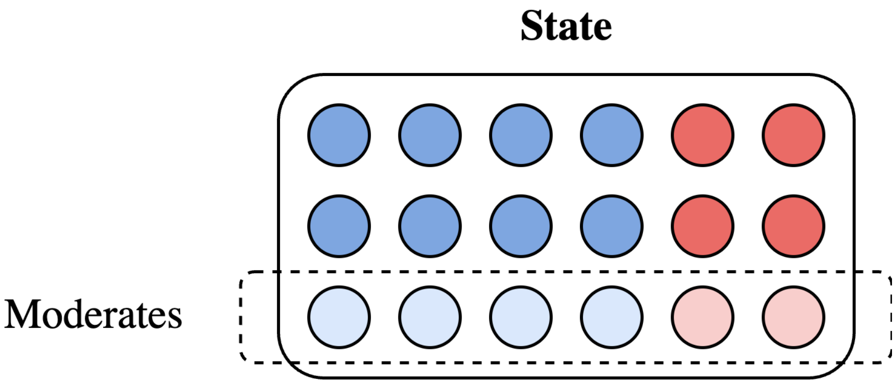
Recap: Standard Redistricting



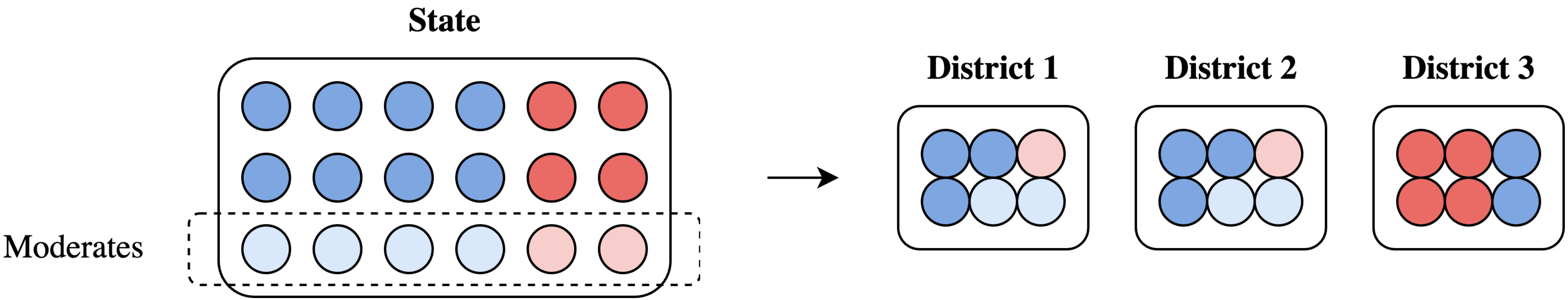
$\frac{1}{3}$ Republicans and $\frac{2}{3}$ Democrats

- Considers party affiliation but not preference intensities
- Underlying assumption: voting behavior is fixed
- Behavior might be endogenous to districts through primary elections...

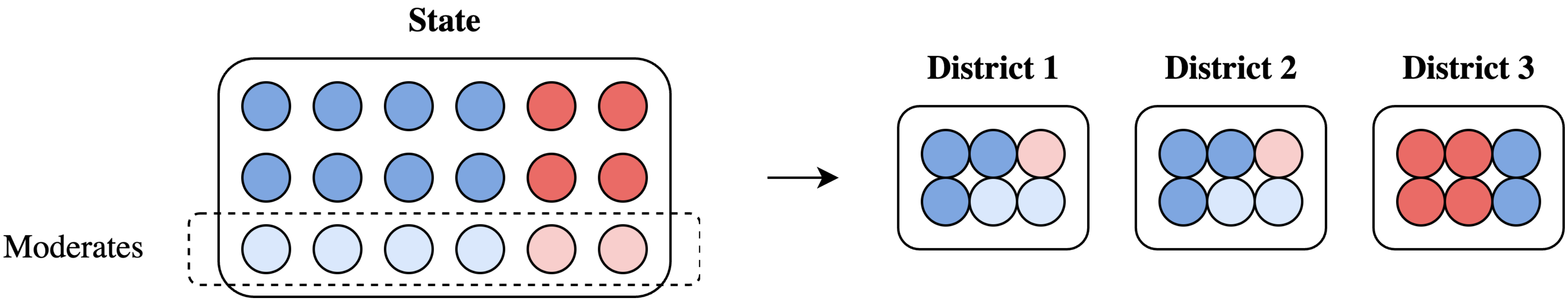
Taking Electoral Incentives into Account



Taking Electoral Incentives into Account

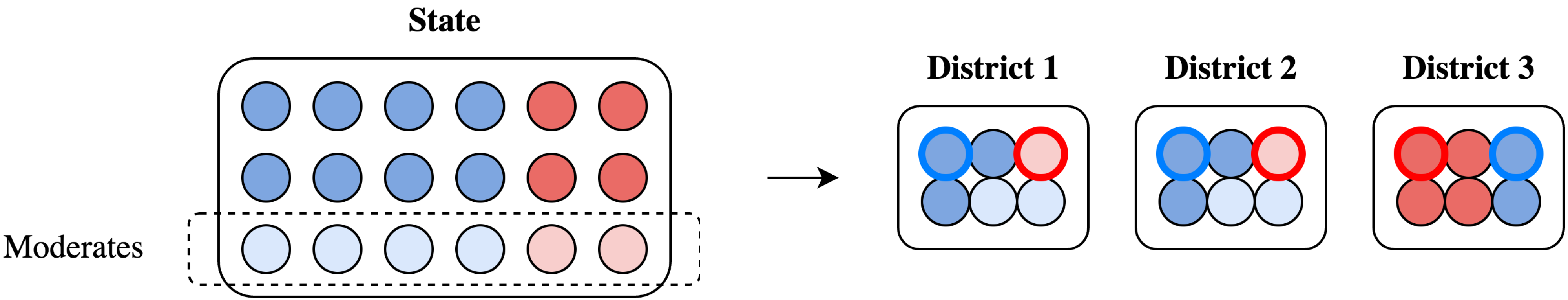


Taking Electoral Incentives into Account



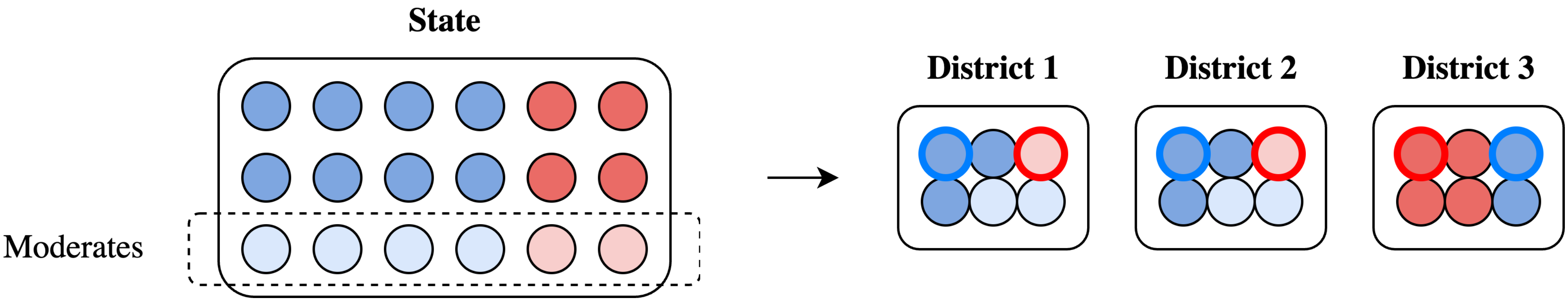
- Rep/Dem candidate is moderate or extremist, depending on party majority

Taking Electoral Incentives into Account



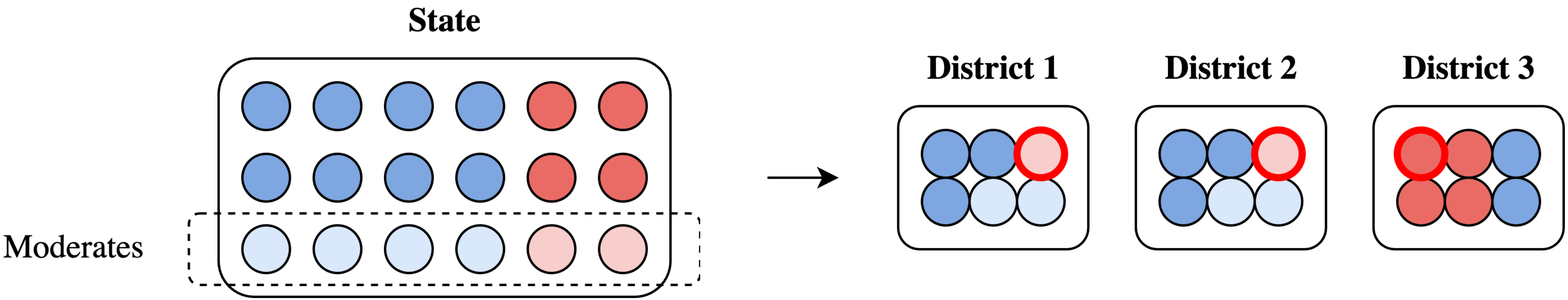
- Rep/Dem candidate is moderate or extremist, depending on party majority

Taking Electoral Incentives into Account



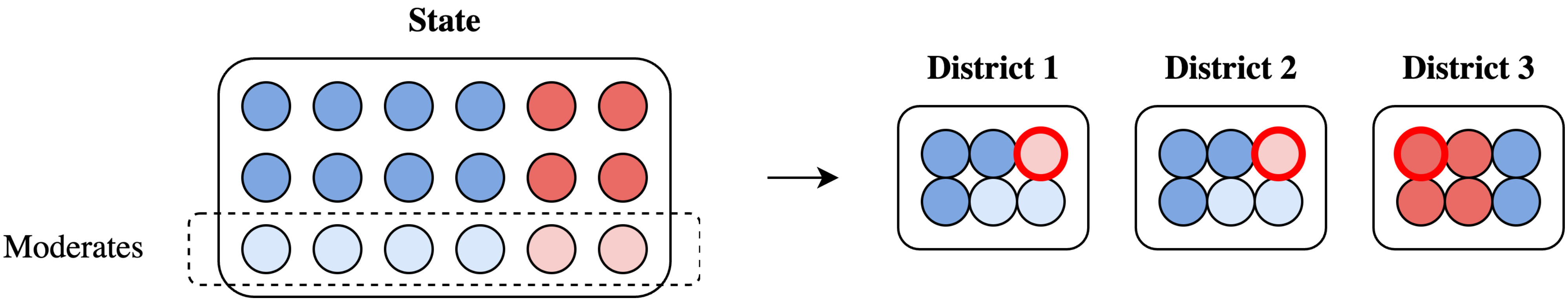
- Rep/Dem candidate is moderate or extremist, depending on party majority
- Moderate voters prefer a moderate of *either* party to an extremist of their own party

Taking Electoral Incentives into Account



- Rep/Dem candidate is moderate or extremist, depending on party majority
- Moderate voters prefer a moderate of *either* party to an extremist of their own party

Taking Electoral Incentives into Account



- Rep/Dem candidate is moderate or extremist, depending on party majority
- Moderate voters prefer a moderate of *either* party to an extremist of their own party
- Gerrymandering is even more powerful: win ALL districts

The Model: Redistricting with Primary Elections

The Agents

- Single-peaked voters' preferences, ideal points distribution $\phi \in \Delta([\underline{v}, \overline{v}])$
  - Strictly increasing and continuous CDF
  - Median $v^m = 0$
- Two parties: Republican and Democratic
- Voter affiliation:

The Agents

- Single-peaked voters' preferences, ideal points distribution $\phi \in \Delta([\underline{v}, \overline{v}])$
  - Strictly increasing and continuous CDF
  - Median $v^m = 0$
- Two parties: Republican and Democratic
- Voter affiliation:

$$v < k \to D \qquad\qquad v \geqslant k \to R$$

The Agents

- Single-peaked voters' preferences, ideal points distribution $\phi \in \Delta([\underline{v}, \overline{v}])$
  - Strictly increasing and continuous CDF
  - Median $v^m = 0$
- Two parties: Republican and Democratic
- Voter affiliation:

$$v < k \rightarrow D \qquad v \geqslant k \rightarrow R$$

- Designer allocates voters into districts to maximize districts won by Republicans

Districts and Redistricting Plans

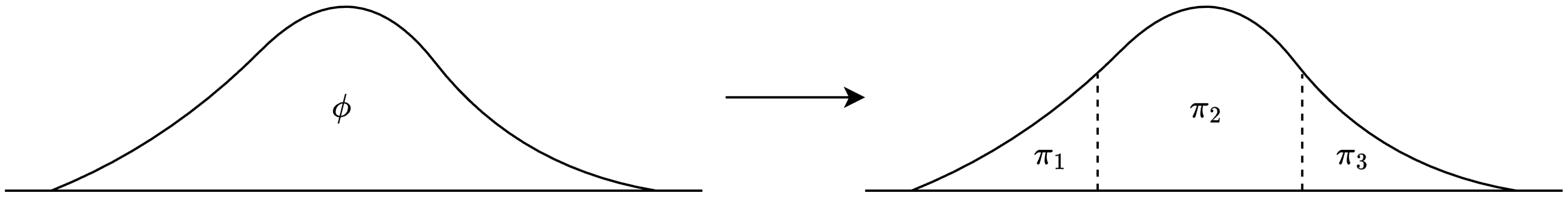
- A district is a distribution of preferences $\pi \in \Delta(\mathbb{R})$
- A redistricting plan is a distribution of districts $\mathcal{H} \in \Delta(\Delta(\mathbb{R}))$ s.t.

$$\int \pi d\mathcal{H}(\pi) = \phi \quad \rightarrow \quad \text{Feasibility}$$

Districts and Redistricting Plans

- A district is a distribution of preferences $\pi \in \Delta(\mathbb{R})$
- A redistricting plan is a distribution of districts $\mathcal{H} \in \Delta(\Delta(\mathbb{R}))$ s.t.

$$\int \pi d\mathcal{H}(\pi) = \phi \quad \rightarrow \quad \text{Feasibility}$$



$$\pi_1\mathcal{H}(\pi_1) + \pi_2\mathcal{H}(\pi_2) + \pi_3\mathcal{H}(\pi_3) = \phi$$

Elections in District π

What determines whether a district is won?

- Two-stage elections: Party Primaries and General Elections
- In the first stage:
  - voters $v < k$ select D with position $c_{\pi,D}$
  - voters $v \geqslant k$ select R with position $c_{\pi,R}$
- In the second stage all voters select $c_{\pi} \in \{c_{\pi,D}, c_{\pi,R}\}$

For this talk: Assume primary candidates position at party medians

Microfoundation

Elections in District π

What determines whether a district is won?

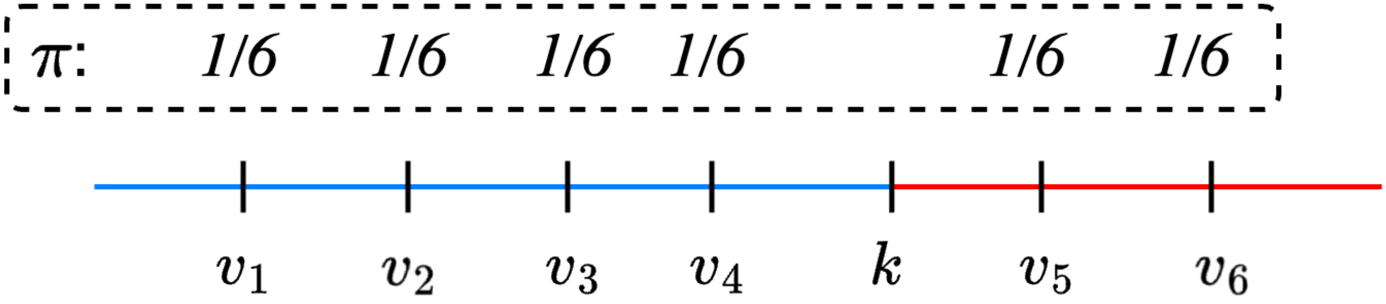
- Two-stage elections: Party Primaries and General Elections
- In the first stage:
  - voters $v < k$ select D with position $c_{\pi,D}$
  - voters $v \geqslant k$ select R with position $c_{\pi,R}$
- In the second stage all voters select $c_{\pi} \in \{c_{\pi,D}, c_{\pi,R}\}$

Need candidates to respond to party base

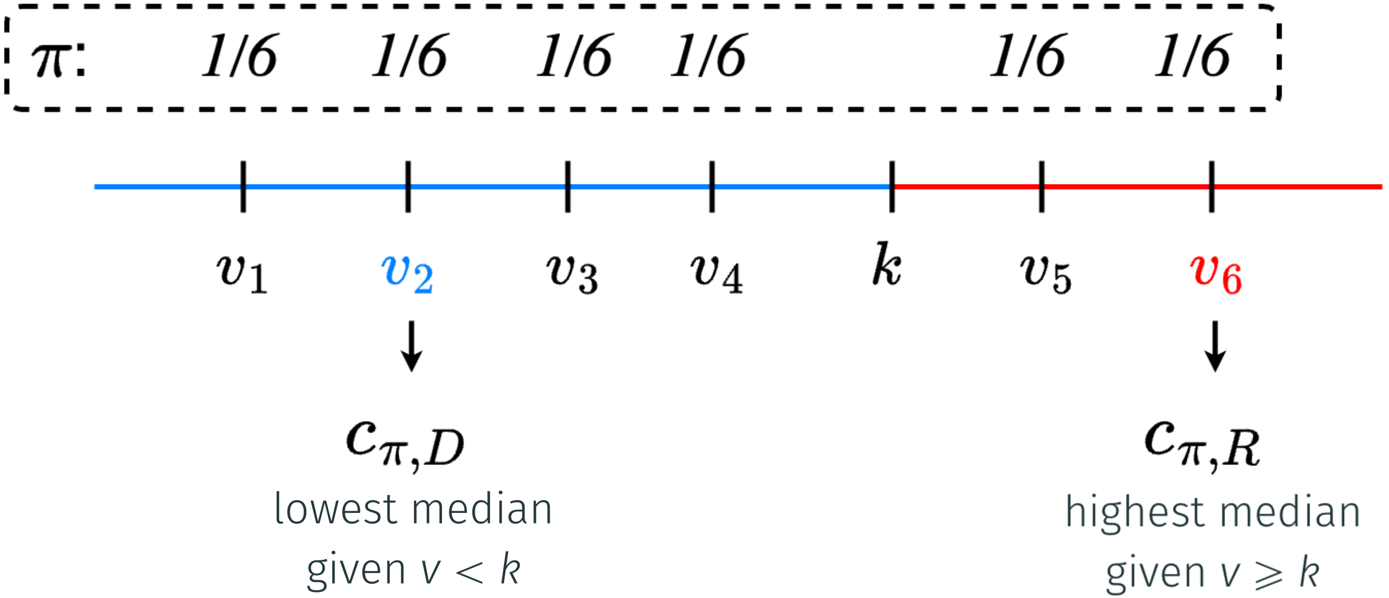
For this talk: Assume primary candidates position at party medians

Microfoundation

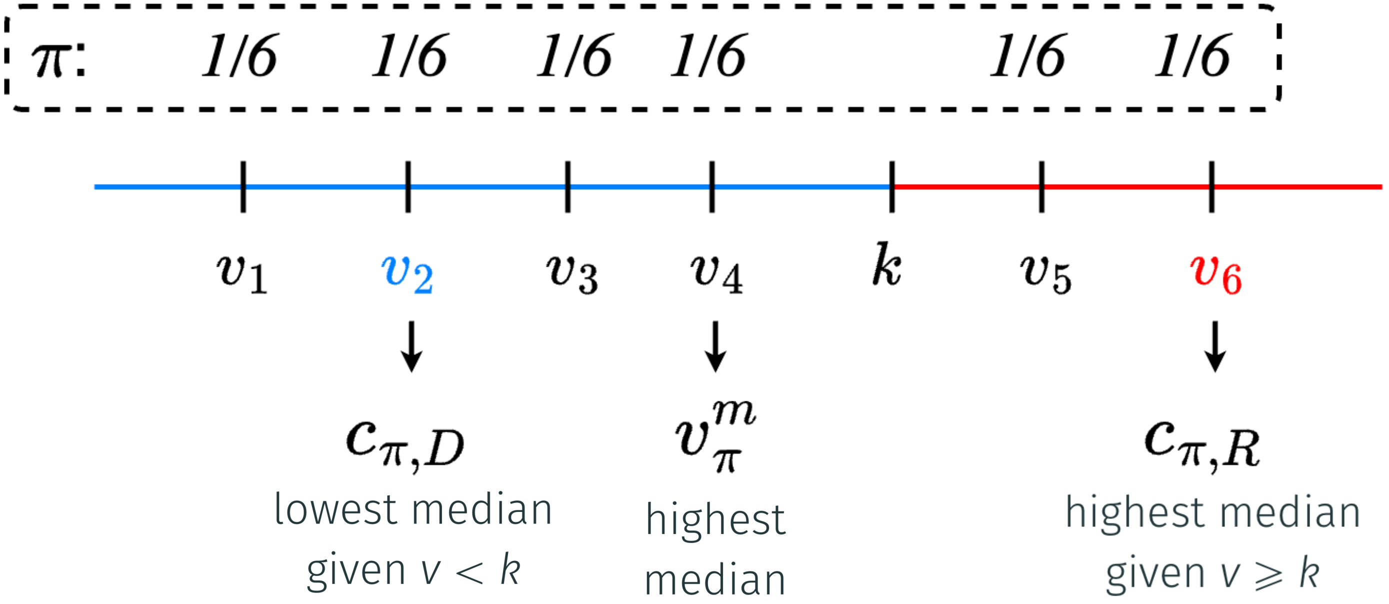
Elections in District π



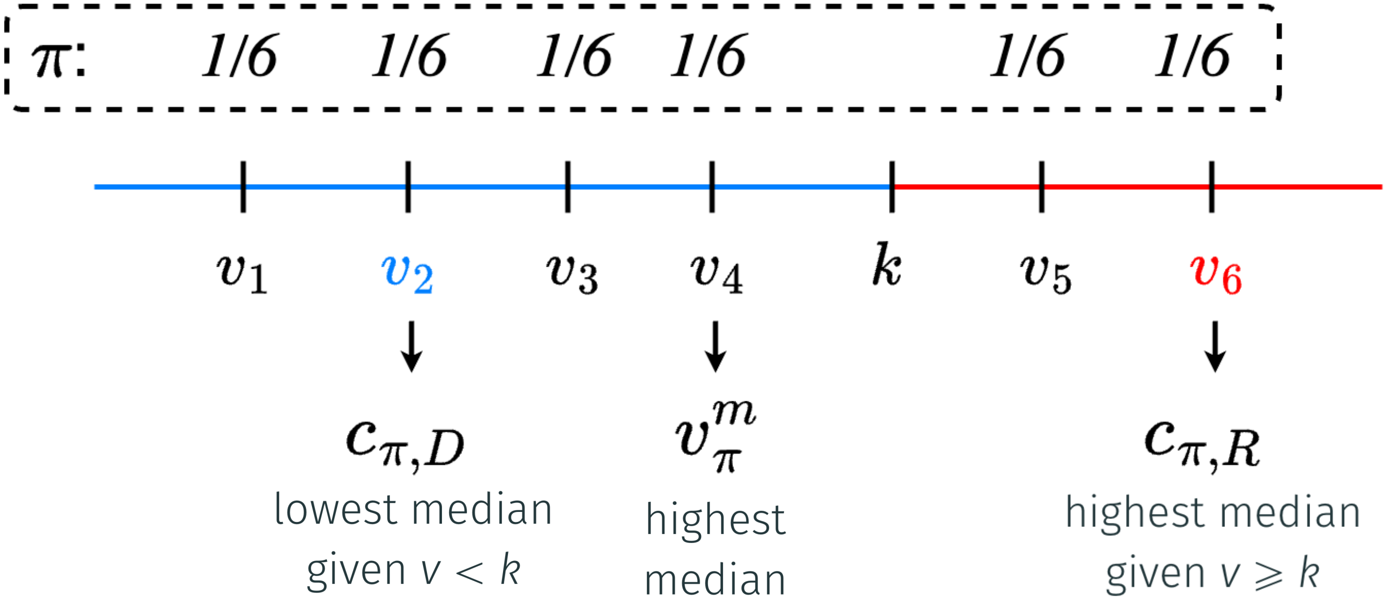
Elections in District π



Elections in District π



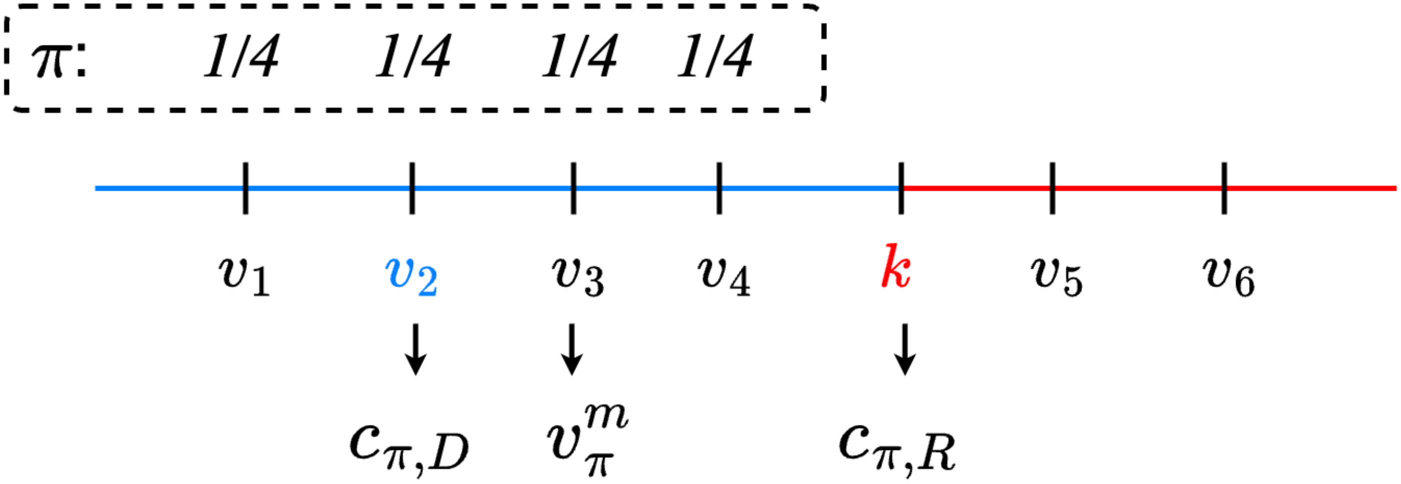
Elections in District π



Disclaimer: Tie-breaking "in favor" of Republicans. Does not matter given continuum of voters

Elections in District π

- If there are no Republicans in π...



- Same for Democrats

Allowing for Uncertainty

- Redistricting happens every 10 years
- Designer faces uncertainty about voters' preferences
- Aggregate shock: All voters experience common shock $\omega \to v - \omega$
- $\omega \sim \gamma \in \Delta(\mathbb{R})$, with CDF G increasing and continuous on $[2(\underline{v} - \overline{v}), 2(\overline{v} - \underline{v})]$
- $\pi \to \pi^{\omega}$

Timing

The Redistricter's Problem

- Designer's problem is:

$$\max_{\mathcal{H}\in\Delta(\Delta([\underline{v},\overline{v}]))} \int\int \mathbb{1}\left(v^{m}_{\pi^\omega} - \frac{c_{\pi^\omega,D} + c_{\pi^\omega,R}}{2} \geqslant 0\right) d\mathcal{H}(\pi) d\gamma(\omega) \tag{RP}$$

$$s.t. \int \pi d\mathcal{H}(\pi) = \phi$$

The Redistricter's Problem

- Designer's problem is:

$$\max_{\mathcal{H}\in\Delta(\Delta([\underline{v},\overline{v}]))} \int\int \mathbb{1}\left(v^m_{\pi^\omega} - \frac{c_{\pi^\omega,D} + c_{\pi^\omega,R}}{2} \geqslant 0\right) d\mathcal{H}(\pi) d\gamma(\omega) \tag{RP}$$

$$\text{s.t.} \int \pi d\mathcal{H}(\pi) = \phi$$

How do we solve it?

Solution

Solution

Mapping to Optimal Transport Problem

Focus on Subset of Feasible Plans

Proposition 1

Any district π in an optimal plan is such that $\mathbf{supp}(\pi) = \{v', v''\}$, with:

$$v' \geqslant 0 \geqslant v''$$

$$\pi(\{v'\}) = \pi(\{v''\})$$

$\rightarrow$ Matching voters above the median with voters below the median

Proof

Focus on Subset of Feasible Plans

Proposition 1

Any district π in an optimal plan is such that $\textbf{supp}(\pi) = \{v', v''\}$, *with:*

$$v' \geqslant 0 \geqslant v''$$
$$\pi(\{v'\}) = \pi(\{v''\})$$

$\rightarrow$ Matching voters above the median with voters below the median

Proof

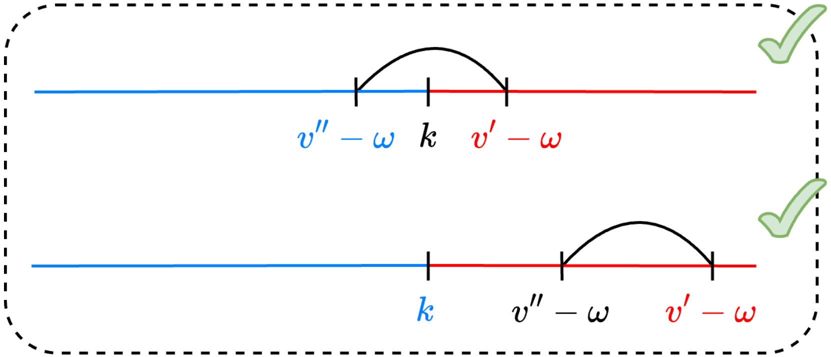
- Define $\phi' = \phi(\cdot|v \geqslant 0)$ and $\phi'' = \phi(\cdot|v \leqslant 0)$
- $\mathcal{M}(\phi', \phi'')$ set of joint distributions over v', v'' with marginals ϕ', ϕ''
- Designer's problem becomes...

Reduction to Optimal Transport Problem

$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int \int \qquad d\tau(v', v'') d\gamma(\omega)$$

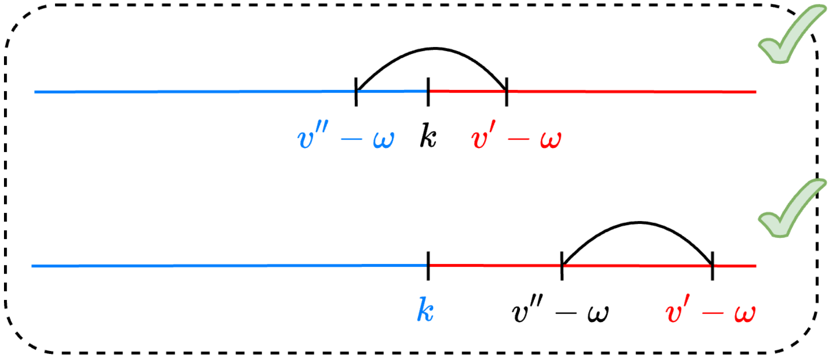
Reduction to Optimal Transport Problem

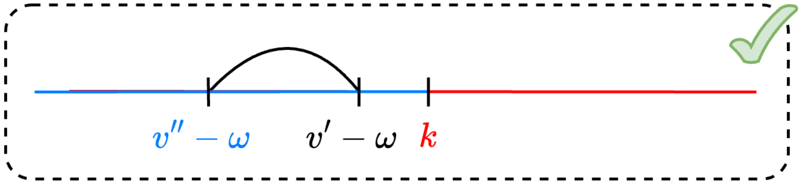
$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int\int \underbrace{\mathbb{1}(v' - \omega \geqslant k)}_{\text{50\% Rep voters}} + \qquad d\tau(v', v'') d\gamma(\omega)$$



Reduction to Optimal Transport Problem

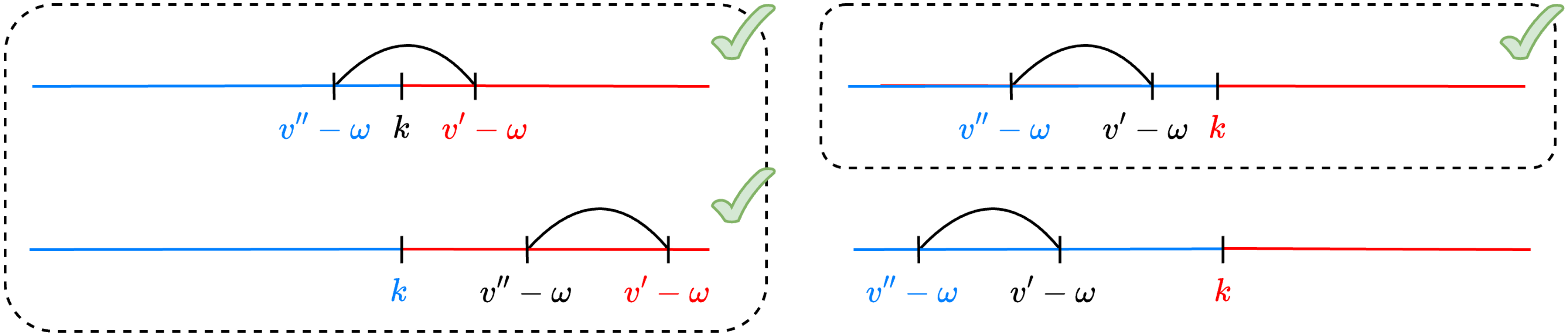
$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int\int \underbrace{\mathbb{1}(v' - \omega \geqslant k)}_{\text{50\% Rep voters}} + \underbrace{\mathbb{1}(v' - \omega < k)\mathbb{1}\left(v' - \omega - \frac{v'' - \omega}{2} \geqslant \frac{k}{2}\right)}_{\text{100\% Dem voters but 50\% vote for R}} d\tau(v', v'')d\gamma(\omega)$$





Reduction to Optimal Transport Problem

$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int\int \underbrace{\mathbb{1}(v' - \omega \geqslant k)}_{\text{50\% Rep voters}} + \underbrace{\mathbb{1}(v' - \omega < k)\mathbb{1}\left(v' - \omega - \frac{v'' - \omega}{2} \geqslant \frac{k}{2}\right)}_{\text{100\% Dem voters but 50\% vote for R}} d\tau(v', v'')d\gamma(\omega)$$



Reduction to Optimal Transport Problem

$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int \int \mathbb{1}(v' - \omega \geqslant k) + \mathbb{1}(v' - \omega < k) \mathbb{1} \left(v' - \omega - \frac{v'' - \omega}{2} \geqslant k \right) d\tau(v', v'') d\gamma(\omega)$$

Reduction to Optimal Transport Problem

$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int \int \mathbb{1}(v' - \omega \geqslant k) + \mathbb{1}(v' - \omega < k)\mathbb{1}\left(v' - \omega - \frac{v'' - \omega}{2} \geqslant k\right) d\gamma(\omega) d\tau(v', v'')$$

Reduction to Optimal Transport Problem

$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int \int \mathbb{1}(v' - \omega \geqslant k) + \mathbb{1}(v' - \omega < k) \mathbb{1}\left(v' - \omega - \frac{v'' - \omega}{2} \geqslant k\right) d\gamma(\omega) d\tau(v', v'')$$

- After some algebra...

$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int G\left(2v' - v'' - k\right) d\tau(v', v'') \quad \text{(OTP)}$$

Reduction to Optimal Transport Problem

$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int \int \mathbb{1}(v' - \omega \geqslant k) + \mathbb{1}(v' - \omega < k)\mathbb{1}\left(v' - \omega - \frac{v'' - \omega}{2} \geqslant k\right) d\gamma(\omega) d\tau(v', v'')$$

- After some algebra...

$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int G\left(2v' - v'' - k\right) d\tau(v', v'') \qquad \text{(OTP)}$$

- This is an optimal transport problem

Theorem 1: Recap

$$\max_{\mathcal{H}\in\Delta(\Delta([\underline{v},\overline{v}]))} \int\int \mathbb{1}\left(v^m_{\pi^\omega} - \frac{c_{\pi^\omega,D} + c_{\pi^\omega,R}}{2} \geqslant 0\right) d\mathcal{H}(\pi) d\gamma(\omega) \tag{RP}$$

$$s.t. \int \pi d\mathcal{H}(\pi) = \phi$$

Theorem 1: Recap

$$\max_{\mathcal{H}\in\Delta(\Delta([\underline{v},\overline{v}]))} \int\int \mathbb{1}\left(v^m_{\pi^\omega} - \frac{c_{\pi^\omega,D}+c_{\pi^\omega,R}}{2} \geqslant 0\right) d\mathcal{H}(\pi)d\gamma(\omega) \tag{RP}$$

$$s.t. \int \pi d\mathcal{H}(\pi) = \phi$$

$$\downarrow$$

$$\max_{\tau\in\mathcal{M}(\phi',\phi'')} \int G\left(2v' - v'' - k\right) d\tau(v', v'') \tag{OTP}$$

Theorem 1: Recap

$$\max_{\mathcal{H}\in\Delta(\Delta([\underline{v},\overline{v}]))} \int\int \mathbb{1}\left(v^m_{\pi^\omega} - \frac{c_{\pi^\omega,D} + c_{\pi^\omega,R}}{2} \geqslant 0\right) d\mathcal{H}(\pi)d\gamma(\omega) \tag{RP}$$

$$\text{s.t.} \int \pi d\mathcal{H}(\pi) = \phi$$

Theorem 1

RP is equivalent to OTP

$$\max_{\tau\in\mathcal{M}(\phi',\phi'')} \int G\left(2v' - v'' - k\right) d\tau(v', v'') \tag{OTP}$$

Solution

Solution to Optimal Transport Problem

A Closer Look at OTP

$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int G\left(2v' - v'' - k\right) d\tau(v', v'') \tag{OTP}$$

A Closer Look at OTP

$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int G\left(2v' - v'' - k\right) d\tau(v', v'') \tag{OTP}$$

- $G(2v' - v'' - k)$ = probability of winning district with v', v''

A Closer Look at OTP

$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int \underbrace{G\left(2v' - v'' - k\right)}_{\text{“surplus” from } v', v''} d\tau(v', v'') \qquad \text{(OTP)}$$

- $G(2v' - v'' - k)$ = probability of winning district with v', v''

A Closer Look at OTP

$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int \underbrace{G\left(2v' - v'' - k\right)}_{\text{“surplus” from } v', v''} d\tau(v', v'') \tag{OTP}$$

- $G(2v' - v'' - k) =$ probability of winning district with v', v''
- Surplus increasing in $v' - v''$ $\rightarrow$ try to max distance bw v' and v''

A Closer Look at OTP

$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int \underbrace{G\left(2v' - v'' - k\right)}_{\text{"surplus" from } v', v''} d\tau(v', v'') \tag{OTP}$$

- $G(2v' - v'' - k) =$ probability of winning district with v', v''
- Surplus increasing in $v' - v''$ $\rightarrow$ try to max distance bw v' and v''

Trade off:

Option 1 Some district with high $v' - v''$ and some districts with low $v' - v''$

Option 2 All districts with moderate $v' - v''$

A Closer Look at OTP

$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int \underbrace{G\left(2v' - v'' - k\right)}_{\text{“surplus” from } v', v''} d\tau(v', v'') \tag{OTP}$$

- $G(2v' - v'' - k)$ = probability of winning district with v', v''
- Surplus increasing in $v' - v''$ $\rightarrow$ try to max distance bw v' and v''

Trade off:

Option 1 Some district with high $v' - v''$ and some districts with low $v' - v''$

Option 2 All districts with moderate $v' - v''$

$\rightarrow$ Depends on concavity of G

Benchmark Cases

Option 1 Some districts with high $v' - v''$ and some districts with low $v' - v''$

Option 2 All districts with moderate $v' - v''$

G strictly concave $\rightarrow$

Benchmark Cases

Option 1 Some districts with high $v' - v''$ and some districts with low $v' - v''$

Option 2 All districts with moderate $v' - v''$

G strictly concave $\rightarrow$

Marginal benefit of increasing $v' - v''$ in a district where it's low

$>$

Marginal cost of decreasing $v' - v''$ in a district where it's high

Benchmark Cases

Option 1 Some districts with high $v' - v''$ and some districts with low $v' - v''$

Option 2 All districts with moderate $v' - v''$

G strictly concave $\rightarrow$

Option 2 $\rightarrow$

Marginal benefit of increasing $v' - v''$ in a district where it's low

$>$

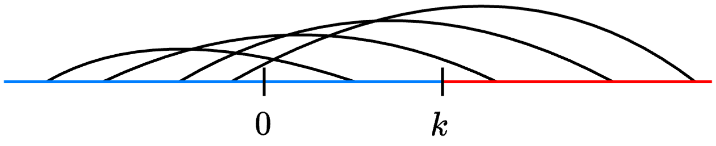
Marginal cost of decreasing $v' - v''$ in a district where it's high

Benchmark Cases

Option 1 Some districts with high $v' - v''$ and some districts with low $v' - v''$

Option 2 All districts with moderate $v' - v''$

G strictly concave $\rightarrow$ **Option 2** $\rightarrow$



positive assortative matching

Marginal benefit of increasing $v' - v''$ in a district where it's low

>

Marginal cost of decreasing $v' - v''$ in a district where it's high

Benchmark Cases

Option 1 Some districts with high $v' - v''$ and some districts with low $v' - v''$

Option 2 All districts with moderate $v' - v''$

G strictly convex $\rightarrow$

Benchmark Cases

Option 1 Some districts with high $v' - v''$ and some districts with low $v' - v''$

Option 2 All districts with moderate $v' - v''$

G strictly convex $\rightarrow$

Marginal benefit of increasing $v' - v''$ in a district where it's high

$>$

Marginal cost of decreasing $v' - v''$ in a district where it's low

Benchmark Cases

Option 1 Some districts with high $v' - v''$ and some districts with low $v' - v''$

Option 2 All districts with moderate $v' - v''$

G strictly convex $\rightarrow$

Option 1 $\rightarrow$

Marginal benefit of increasing $v' - v''$ in a district where it's high

$>$

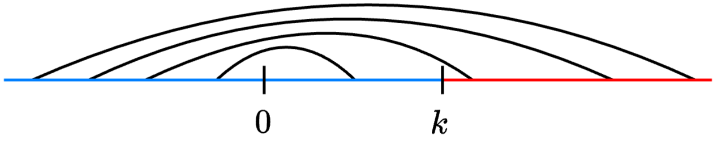
Marginal cost of decreasing $v' - v''$ in a district where it's low

Benchmark Cases

Option 1 Some districts with high $v' - v''$ and some districts with low $v' - v''$

Option 2 All districts with moderate $v' - v''$

G strictly convex $\rightarrow$ **Option 1** $\rightarrow$



negative assortative matching

Marginal benefit of increasing $v' - v''$ in a district where it's high

$>$

Marginal cost of decreasing $v' - v''$ in a district where it's low

Theorem 2: S-shaped and Symmetric Shock

G S-shaped $\rightarrow$

Theorem 2: S-shaped and Symmetric Shock

$$\text{concave if } \overbrace{2v' - v'' - k > 0}^{\text{District won when } \omega=0} \rightarrow$$

G S-shaped $\rightarrow$

Theorem 2: S-shaped and Symmetric Shock

$$\text{concave if } \overbrace{2v' - v'' - k > 0}^{\text{District won when } \omega=0} \rightarrow$$

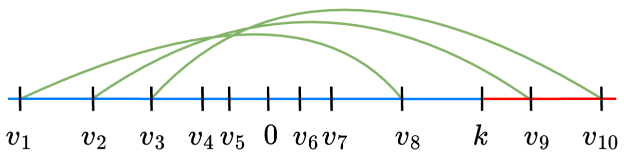
G S-shaped $\rightarrow$

$$\text{convex if } \underbrace{2v' - v'' - k < 0}_{\text{District lost when } \omega=0} \rightarrow$$

Theorem 2: S-shaped and Symmetric Shock

G S-shaped $\rightarrow$

concave if $\overbrace{2v' - v'' - k > 0}^{\text{District won when } \omega=0}$ $\rightarrow$

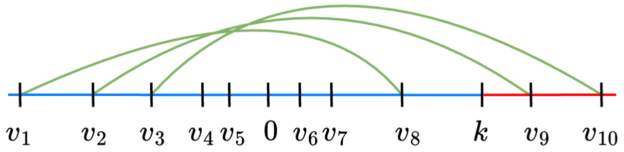


convex if $\underbrace{2v' - v'' - k < 0}_{\text{District lost when } \omega=0}$ $\rightarrow$

Theorem 2: S-shaped and Symmetric Shock

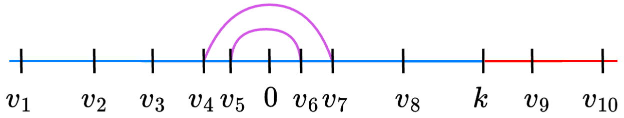
District won when $\omega=0$

concave if $\overbrace{2v' - v'' - k > 0}$ →



G S-shaped →

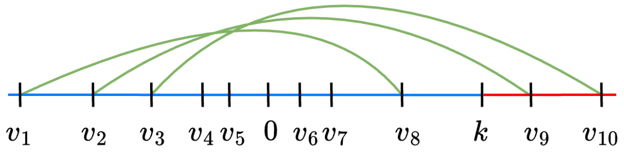
convex if $\underbrace{2v' - v'' - k < 0}$ →



District lost when $\omega=0$

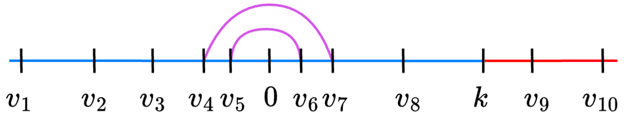
Theorem 2: S-shaped and Symmetric Shock

concave if $\overbrace{2v' - v'' - k}^{\text{District won when } \omega=0} > 0 \rightarrow$

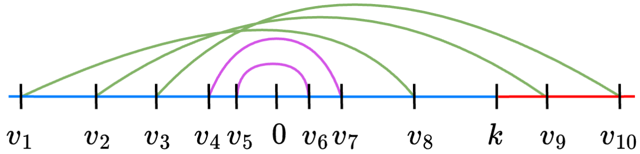


G S-shaped $\rightarrow$

convex if $\underbrace{2v' - v'' - k}_{\text{District lost when } \omega=0} < 0 \rightarrow$

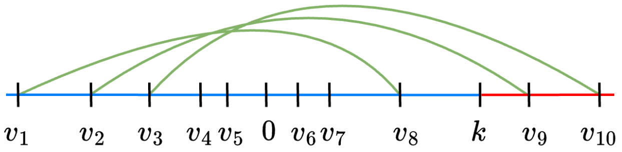


- Solution is a combination of positive and negative assortative matching



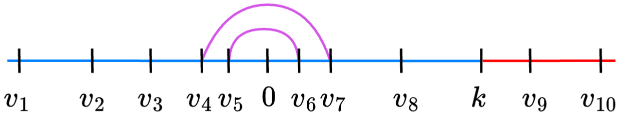
Theorem 2: S-shaped and Symmetric Shock

$$\text{concave if } \overbrace{2v' - v'' - k > 0}^{\text{District won when } \omega=0} \rightarrow$$

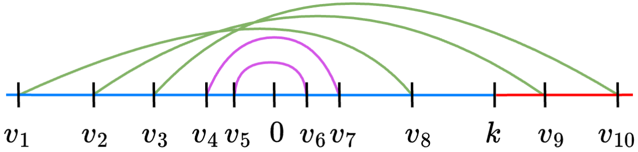


G S-shaped $\rightarrow$

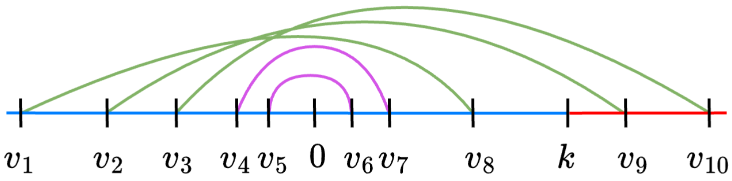
$$\text{convex if } \underbrace{2v' - v'' - k < 0}_{\text{District lost when } \omega=0} \rightarrow$$



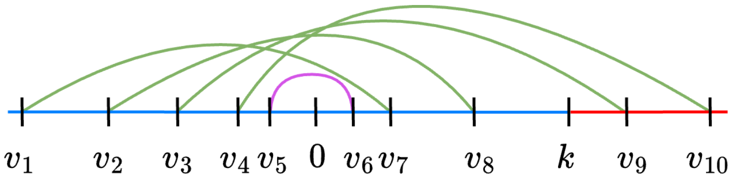
- Solution is a combination of positive and negative assortative matching
- …But which one?



Theorem 2: S-shaped and Symmetric Shock

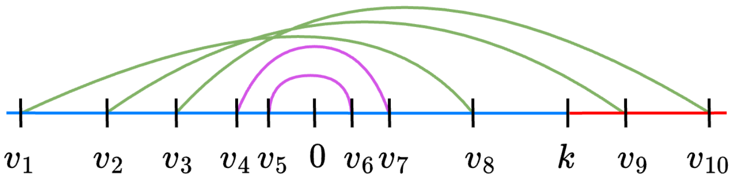


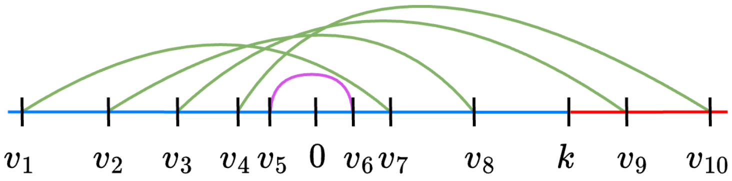
Option 1 Fewer positive matches, but with higher distance each



Option 2 More positive matches, but with lower distance each

Theorem 2: S-shaped and Symmetric Shock





Fewer won when ω=0

Option 1 Fewer positive matches, but with higher distance each

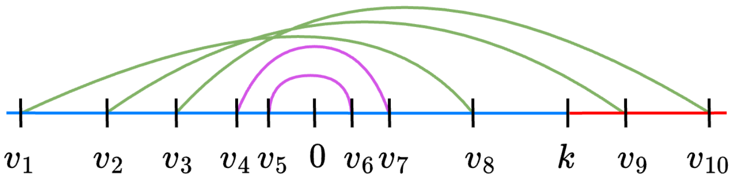
safer when bad ω

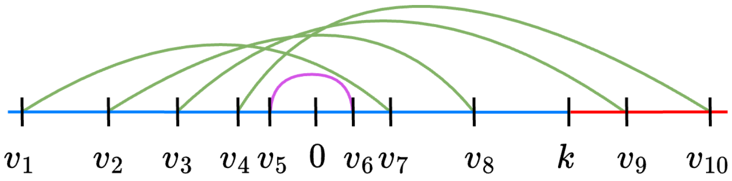
More won when ω=0

Option 2 More positive matches, but with lower distance each

riskier when bad ω

Theorem 2: S-shaped and Symmetric Shock





Fewer won when $\omega=0$

Option 1 Fewer positive matches, but with higher distance each

safer when bad ω

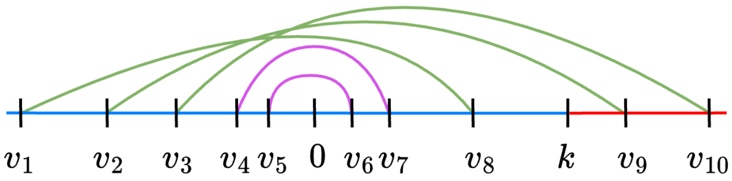
More won when $\omega=0$

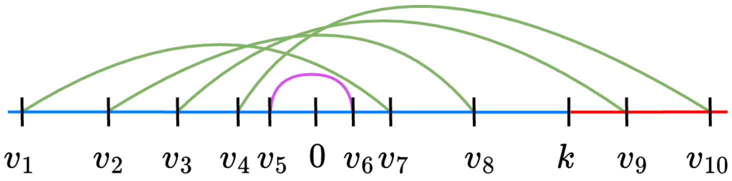
Option 2 More positive matches, but with lower distance each

riskier when bad ω

G symmetric → **Option 2**

Theorem 2: S-shaped and Symmetric Shock





Option 1 Fewer positive matches, but with higher distance each
(Fewer won when $\omega=0$; higher distance: safer when bad ω)

Option 2 More positive matches, but with lower distance each
(More won when $\omega=0$; lower distance: riskier when bad ω)

G symmetric $\rightarrow$ **Option 2**

Intuition: OTP linear in number of districts won + symmetry $\rightarrow$ As if shock doesn't matter

Theorem 2: S-shaped and Symmetric Shock

Theorem 2

There is a unique solution τ to OTP. Moreover, τ is such that:

- $\tau = \alpha\tau^{+} + (1-\alpha)\tau^{-}$ *for appropriate:*
  - $\alpha \in [0,1]$
  - τ^{+} *positive assortative*
  - τ^{-} *negative assortative*
- *There is* $v', v'' \in \text{supp}(\tau^{+})$ *such that* $(1-\alpha)(2v' - v'' - k) = 0$

Theorem 2: S-shaped and Symmetric Shock

Theorem 2

There is a unique solution τ to OTP. Moreover, τ is such that:

- $\tau = \alpha\tau^+ + (1-\alpha)\tau^-$ *for appropriate:*
  - $\alpha \in [0, 1]$
  - τ^+ *positive assortative*
  - τ^- *negative assortative*
- *There is* $v', v'' \in \mathbf{supp}(\tau^+)$ *such that* $(1-\alpha)(2v' - v'' - k) = 0$

FOC: Either $\tau = \tau^+$ or $2v' - v'' - k \geqslant 0$ binding for at least one v', v''

Theorem 2: S-shaped and Symmetric Shock

Theorem 2

There is a unique solution τ to OTP. Moreover, τ is such that:

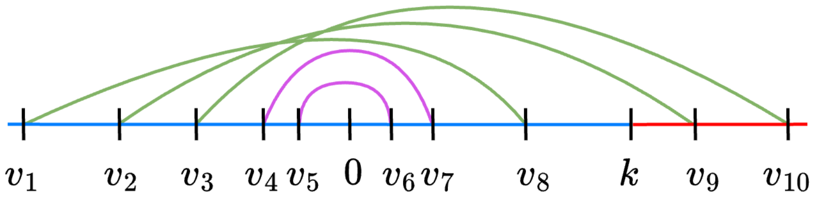
- $\tau = \alpha\tau^+ + (1-\alpha)\tau^-$ *for appropriate:*
  - $\alpha \in [0,1]$
  - τ^+ *positive assortative*
  - τ^- *negative assortative*
- *There is* $v', v'' \in \mathbf{supp}(\tau^+)$ *such that* $(1-\alpha)(2v' - v'' - k) = 0$

FOC: Either $\tau = \tau^+$ or $2v' - v'' - k \geqslant 0$ binding for at least one v', v''

Intuitively: Solution must “maximize” positive assortative matches

Comparative Statics and Implications for the U.S. House

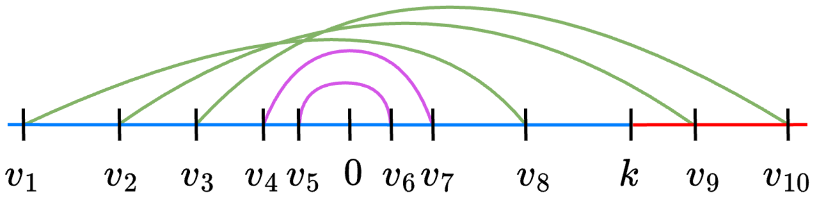
Some Comparative Statics



There is a unique solution τ to OTP:

- $\tau = \alpha\tau^+ + (1-\alpha)\tau^-$
- There is $v', v'' \in \text{supp}(\tau^+)$ such that $(1-\alpha)(2v' - v'' - k) = 0$

Some Comparative Statics

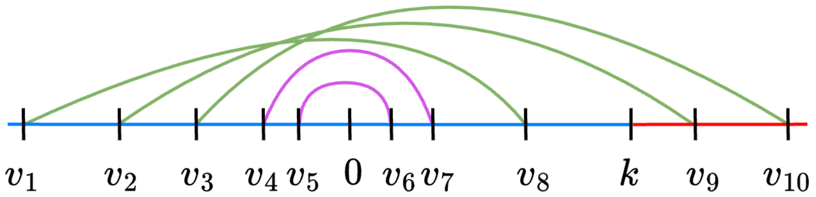


There is a unique solution τ to OTP:

- $\tau = \alpha\tau^+ + (1-\alpha)\tau^-$
- There is $v', v'' \in \text{supp}(\tau^+)$ such that $(1-\alpha)(2v' - v'' - k) = 0$

- More Republican voters in ϕ → More positive matches → Better for designer

Some Comparative Statics

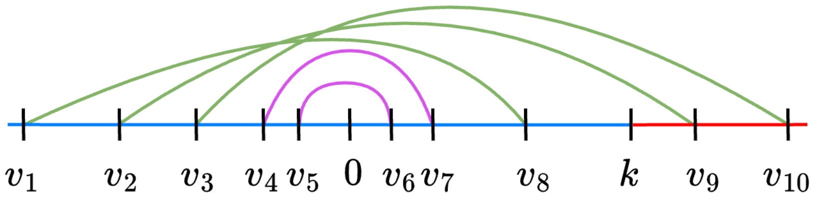


There is a unique solution τ to OTP:

- $\tau = \alpha\tau^+ + (1-\alpha)\tau^-$
- There is $v', v'' \in \text{supp}(\tau^+)$ such that $(1-\alpha)(2v' - v'' - k) = 0$

- More Republican voters in ϕ $\rightarrow$ More positive matches $\rightarrow$ Better for designer
- Median preserving spread of ϕ $\rightarrow$ More positive matches $\rightarrow$ Better for designer

Some Comparative Statics



There is a unique solution τ to OTP:

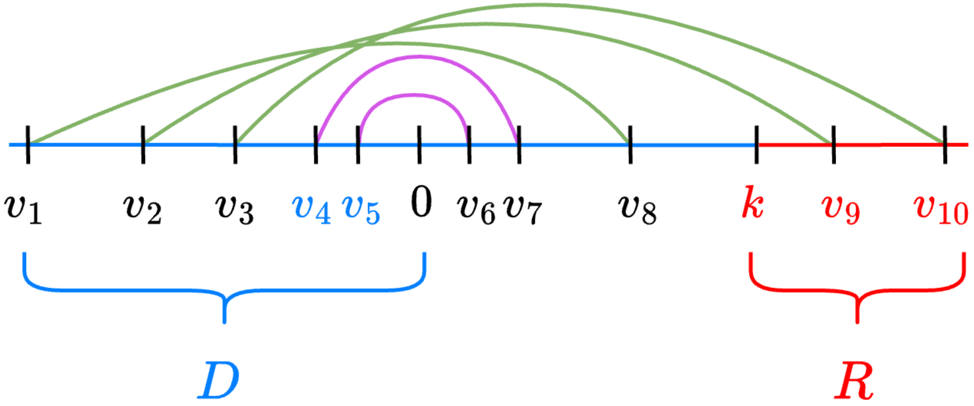
- $\tau = \alpha\tau^+ + (1 - \alpha)\tau^-$
- There is $v', v'' \in \mathbf{supp}(\tau^+)$ such that $(1 - \alpha)(2v' - v'' - k) = 0$

- More Republican voters in ϕ → More positive matches → Better for designer
- Median preserving spread of ϕ → More positive matches → Better for designer

Corollary 1

If ϕ is "spread out" enough, the designer wins ALL districts

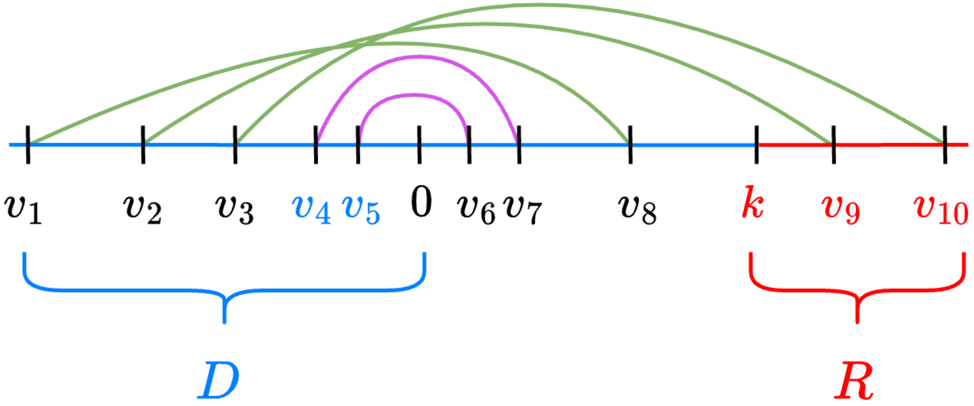
Polarization in the U.S. House



Proposition 2

Distribution of representatives has a gap in $[-\omega, k]$

Polarization in the U.S. House



Proposition 2

Distribution of representatives has a gap in $[-\omega, k]$

Dilution of moderate voters' power

Extensions

Extensions

Primary Elections

- Convex combination of party medians and median → same characterization
- Different party quantile q → "three-wise matching" (in the paper)

Objective Function

- Designer has policy preferences → pack supporters
- Majoritarian objective → non-linear OT → More negative assortment

Other

- Idiosyncratic uncertainty → means instead of medians
- Exogenous/Endogenous turnout

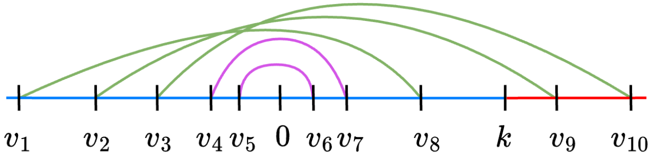
Conclusions

Recap

Optimal redistricting with endogenous candidates' positions

- Designer's objective depends on districts' medians AND conditional medians

→ optimal transport characterization:



- Standard gerrymandering can backfire
- Designer exploits extreme opponents to turn moderate opponents into supporters

Some Implications:

More spread out electorate
→ stronger gerrymandering

Polarized U.S. House

For Future Research

- Empirical follow-up: "Supply Responses to Redistricting" (with Herman and Longuet-Marx)
  - Preliminary findings: candidates respond to district composition

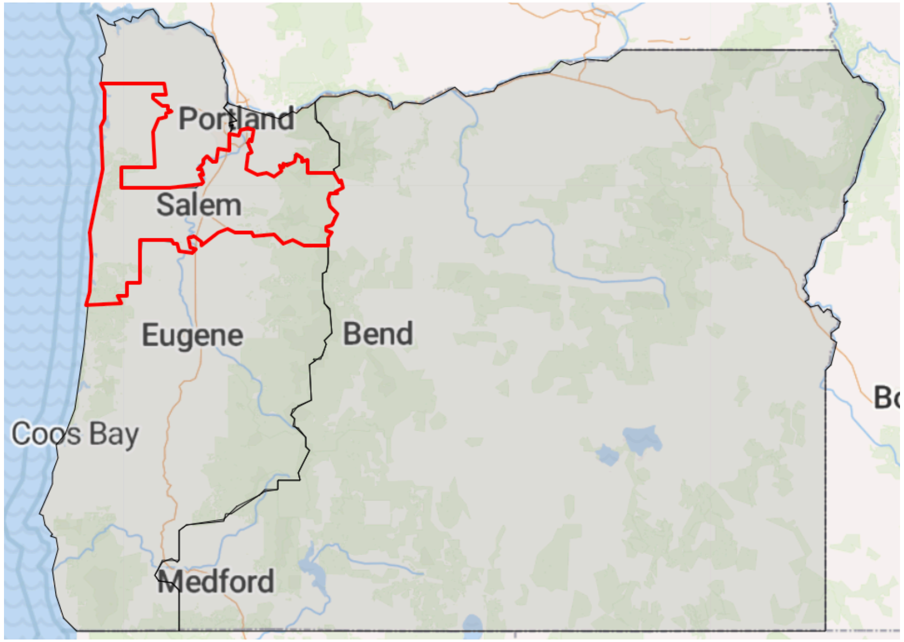
Application to other contexts

- Agenda setting in groups
- Information design with dependence on more than one statistic
  - Multiple signals
  - Behavioral preferences

Extra

Case Study: Oregon 5th Congressional District

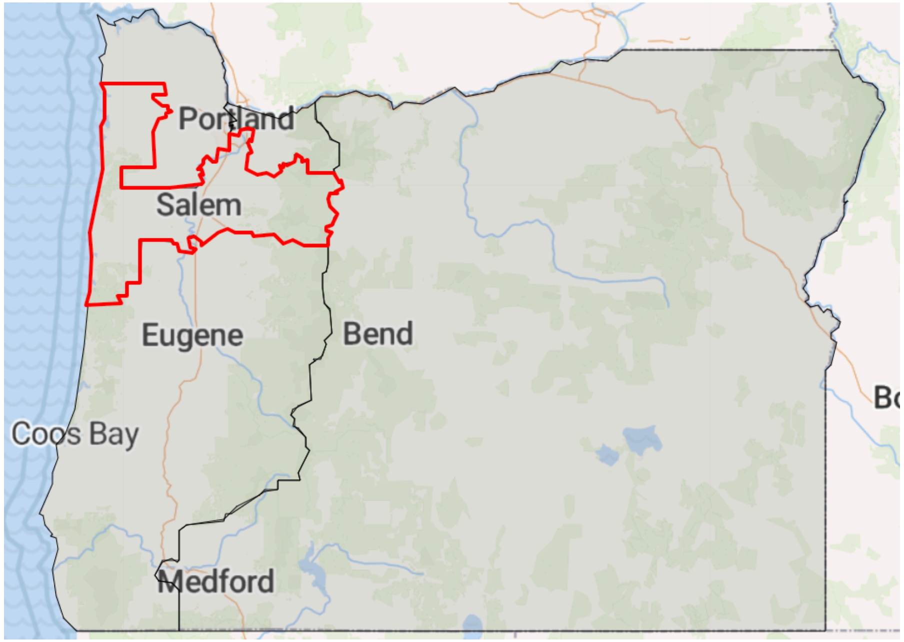
Until January 2, 2023



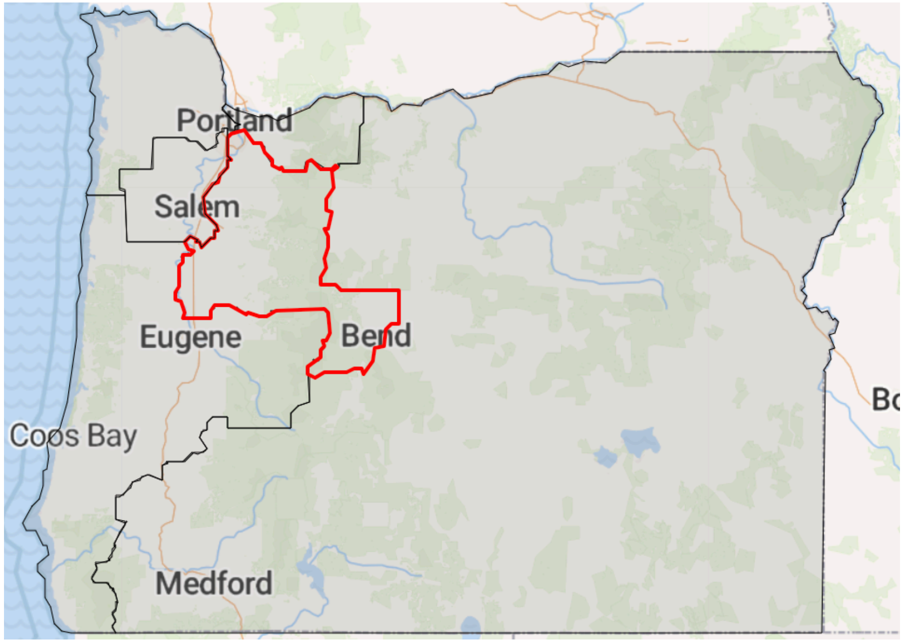
- Democratic gerrymandering
- 53.6% vs 43.9%
- 7th term incumbent *Kurt Shrader*
- Moderate Democrat, endorsed by Biden

Case Study: Oregon 5^{th} Congressional District

Until January 2, 2023



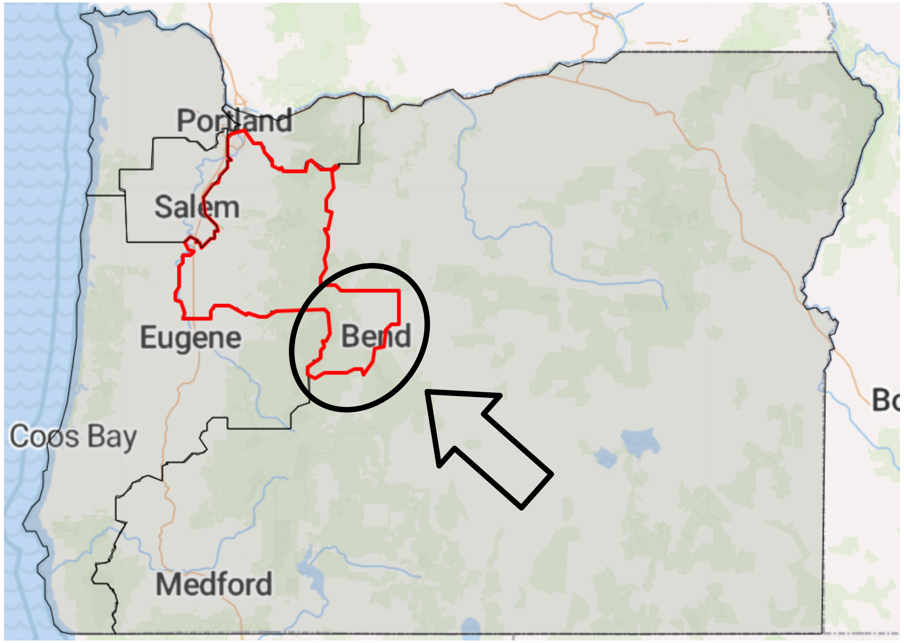
From January 3, 2023



Case Study: Oregon 5th Congressional District

From January 3, 2023

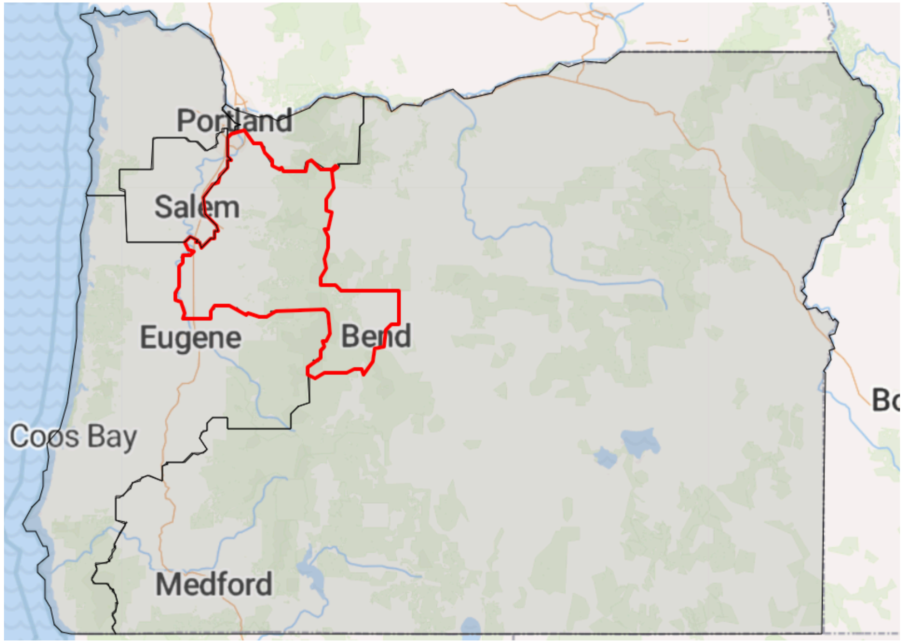
- Democratic gerrymandering
- 53.2% vs 44.4%
- Progressive Democrat *Jamie McLeod-Skinner* has 40 points advantage in Deschute county over incumbent



Case Study: Oregon 5th Congressional District

- Shrader loses the Democratic primary to McLeod-Skinner
- McLeod-Skinner loses general elections to moderate Republican *Lori Chavez-DeRemer*

From January 3, 2023

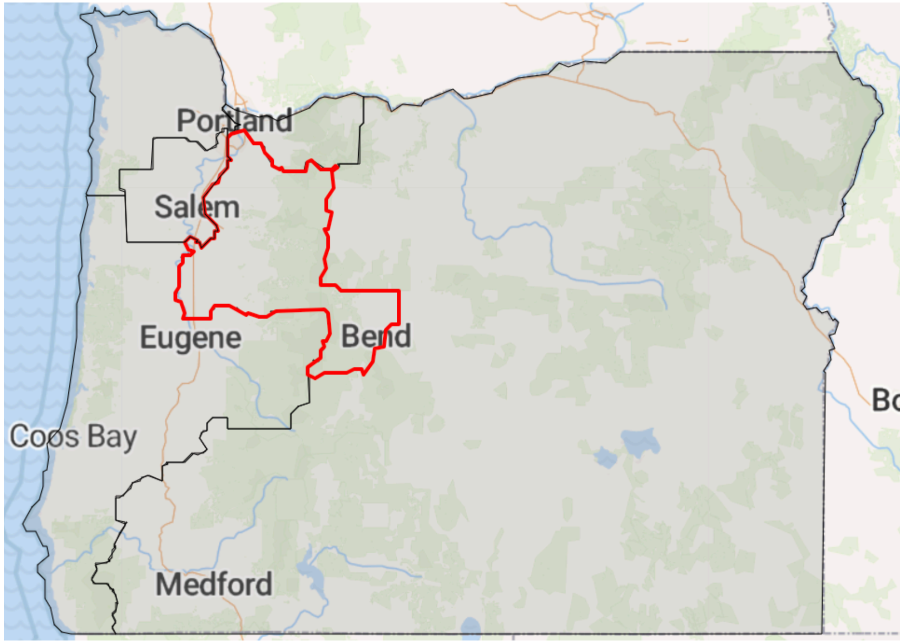


Case Study: Oregon 5^{th} Congressional District

Democrats ate their own [...] and now a standout Republican candidate will face-off against a far-too-liberal activist in Jamie McLeod-Skinner.

- Dan Conston for The Washington Post

From January 3, 2023

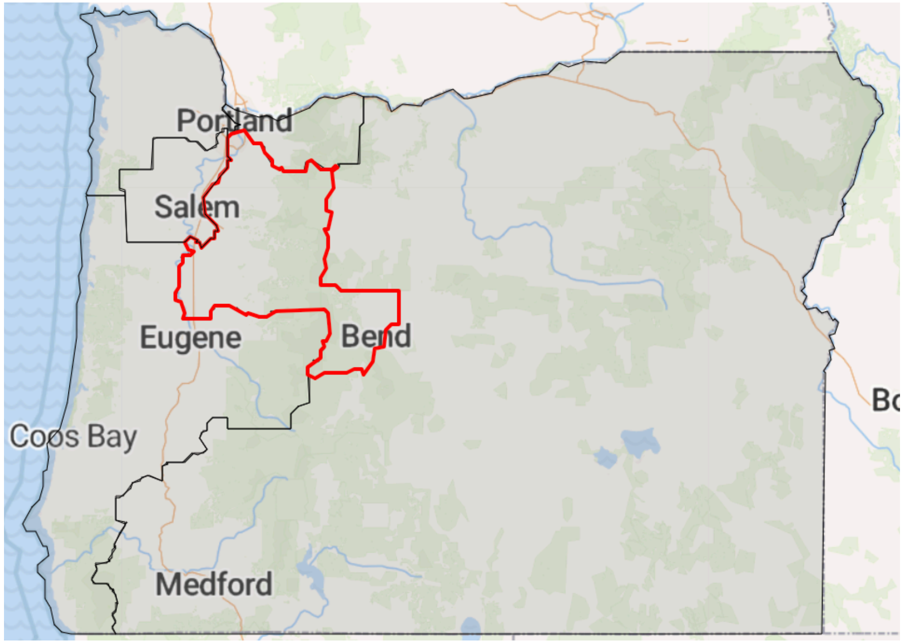


Case Study: Oregon 5th Congressional District

From January 3, 2023

Democrats ate their own [...] and now a standout Republican candidate will face-off against a far-too-liberal activist in Jamie McLeod-Skinner.

- Dan Conston for The Washington Post



Is gerrymandering is less powerful than we thought?

Elections in District π: Primaries

- Primary positions:

$$(c_{\pi,D}, c_{\pi,R}) = \begin{cases} (v^m_{\pi,D}, k) & \textit{if } \operatorname{supp} \pi \subseteq (-\infty, 0] \\ (k, v^m_{\pi,R}) & \textit{if } \operatorname{supp} \pi \subseteq [0, \infty) \\ (v^m_{\pi,D}, v^m_{\pi,R}) & \textit{otherwise} \end{cases}$$

- $v^m_{\pi,D}$ the lowest median of π conditional on $v < 0$
- $v^m_{\pi,R}$ the highest median of π conditional on $v \geqslant 0$

Elections in District π: General Elections

- General elections: Voters vote for the candidate closer to them
- v_π^m highest median of π. Then:

$$c_P = \begin{cases} c_{\pi,D} & \textit{if} \;\; v_\pi^m < \frac{c_{\pi,R}+c_{\pi,D}}{2} \\ c_{\pi,R} & \textit{if} \;\; v_\pi^m \geqslant \frac{c_{\pi,R}+c_{\pi,D}}{2} \end{cases}$$

Elections in District π: Microfoundations

- Single-peaked voters' utilities $u(\cdot, v_i)$, $v_i \in [\underline{v}, \overline{v}]$
  - v_π^m median
  - $v_{\pi,R}^m$ median given $v_i \geqslant k$
  - $v_{\pi,D}^m$ median given $v_i < k$
- Two candidates per party, purely office motivated, commit to policy before elections
- Voters and candidates believe v_π^m to have distribution $H \in \Delta([\underline{v}, \overline{v}])$
  - In addition, candidates know $v_{\pi,R}^m, v_{\pi,D}^m$

Elections in District π: Microfoundations

Primary Elections

- Given position y of opposing nominee, voter t votes for candidate maximizing

$$U(x; y, t) = u(x, t)p(x, y) + u(y, t)(1 - p(x, y))$$

where $p(x, y)$ probability of x winning against y

General Election

- Voters vote for candidate closer to them

Linear setting

Suppose $u(x, v_i) = -|x - v_i|$ and H is uniform on $[\underline{v}, \overline{v}]$

Proposition 3

There exists a unique Nash equilibrium where the Republican and Democratic candidates set positions $c_{\pi,R} = v^m_{\pi,R}$ and $c_{\pi,D} = v^m_{\pi,D}$, respectively.

Convex setting

$$u(x, v_i) = e^{-\alpha|x-v_i|},\ \alpha > 0,\ H \sim \mathcal{N}(v^m_\pi, \sigma)$$

Proposition 4 (Owen and Grofman (2006))

If $\sigma \geqslant \frac{\max\left\{1-e^{1-\alpha(v^m_\pi - v^m_{\pi,D})}, 1-e^{\alpha(v^m_{\pi,R}-v^m_\pi)}\right\}}{\alpha\sqrt{2\pi}}$*, there exists a unique Nash equilibrium where the Republican and Democratic candidates set positions* $c_{\pi,R} = v^m_{\pi,R}$ *and* $c_{\pi,D} = v^m_{\pi,D}$*, respectively.*

Return

Proof Sketch: Step 1

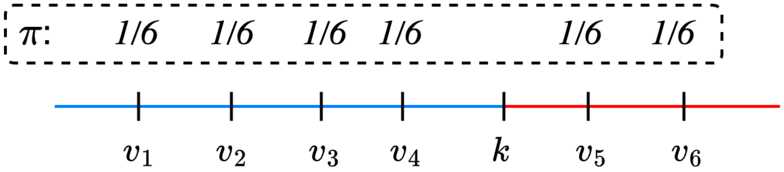
Intuition:

Step 1 For any district π, $\exists\ v', v''$:

$$\text{supp}(\pi) = \{v', v''\}$$
$$\pi(\{v'\}) = \pi(\{v''\})$$

Proof Sketch: Step 1

Intuition:



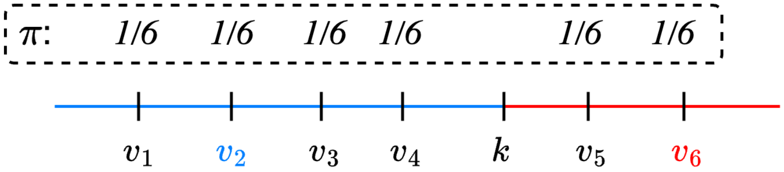
suppose $|\mathbf{supp}(\pi)| > 2$

Step 1 For any district π, $\exists\, v', v''$:

$$\mathbf{supp}(\pi) = \{v', v''\}$$
$$\pi(\{v'\}) = \pi(\{v''\})$$

Proof Sketch: Step 1

Intuition:



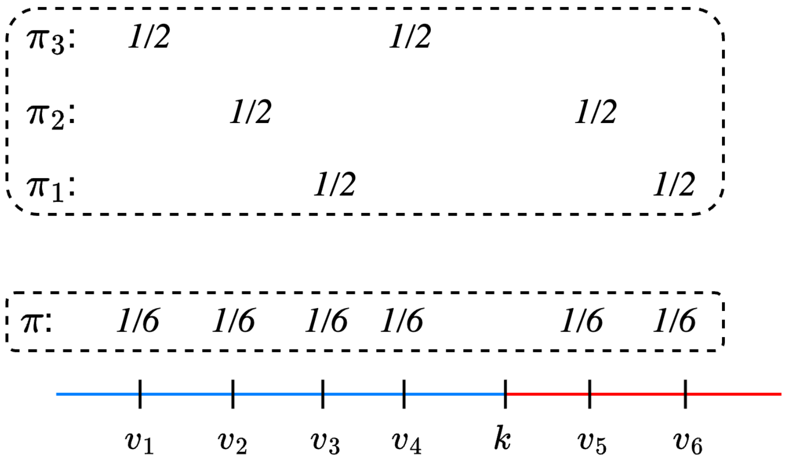
$c_{\pi,R} - v_{\pi}^{m} > v_{\pi}^{m} - c_{\pi,D} \rightarrow \pi$ is lost

Step 1 For any district π, $\exists\, v', v''$:

$$\text{supp}(\pi) = \{v', v''\}$$
$$\pi(\{v'\}) = \pi(\{v''\})$$

Proof Sketch: Step 1

Intuition:



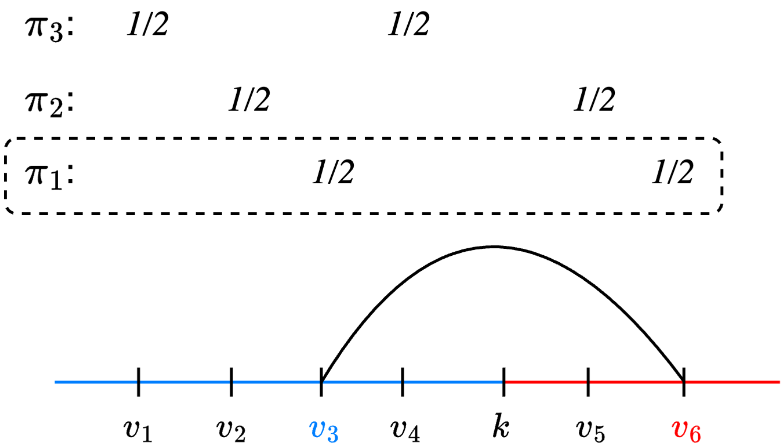
can substitute π with π_1, π_2, π_3

Step 1 For any district π, $\exists\, v', v''$:

$$\text{supp}(\pi) = \{v', v''\}$$
$$\pi(\{v'\}) = \pi(\{v''\})$$

Proof Sketch: Step 1

Intuition:



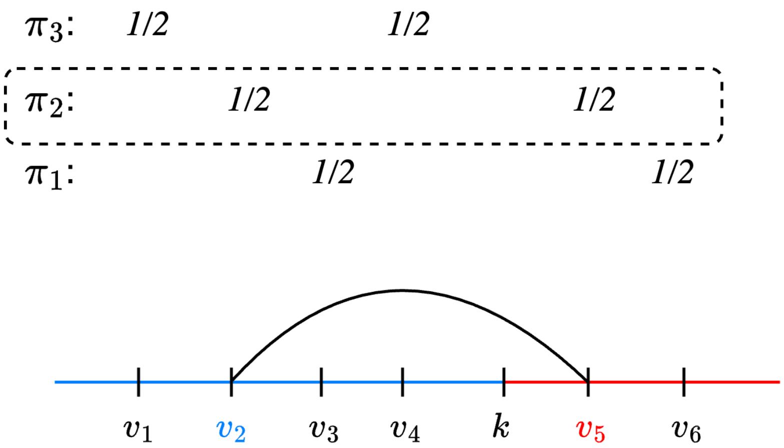
$c_{\pi_1,R} = v^m_{\pi_1} \rightarrow \pi_1$ is won

Step 1 For any district π, $\exists\, v', v''$:

$$\mathrm{supp}(\pi) = \{v', v''\}$$
$$\pi(\{v'\}) = \pi(\{v''\})$$

Proof Sketch: Step 1

Intuition:



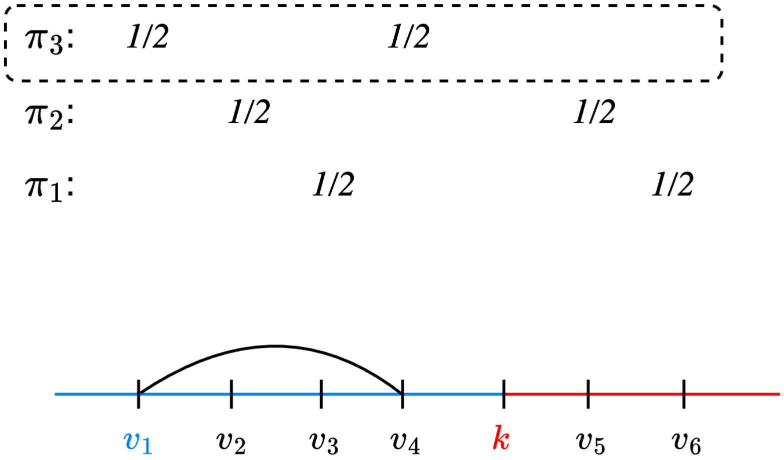
$c_{\pi_2,R} = v^m_{\pi_2} \to \pi_2$ is won

Step 1 For any district π, $\exists\, v', v''$:

$$\text{supp}(\pi) = \{v', v''\}$$
$$\pi(\{v'\}) = \pi(\{v''\})$$

Proof Sketch: Step 1

Intuition:



$c_{\pi_3,R} - V^m_{\pi_3} < V^m_{\pi_3} - c_{\pi_3,D} \rightarrow \pi_3$ is won

Step 1 For any district π, $\exists\, v', v''$:

$$\text{supp}(\pi) = \{v', v''\}$$
$$\pi(\{v'\}) = \pi(\{v''\})$$

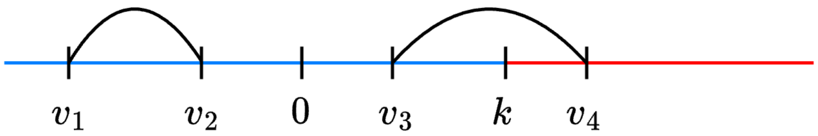
Proof Sketch: Step 2

Intuition:

Step 2 $v' \geqslant 0 \geqslant v''$

Proof Sketch: Step 2

Intuition:

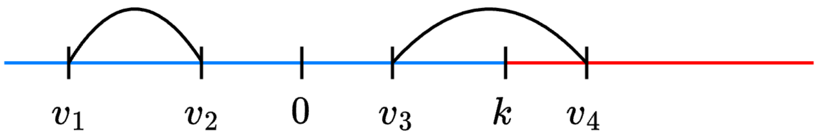


Only district with v_3, v_4 is won

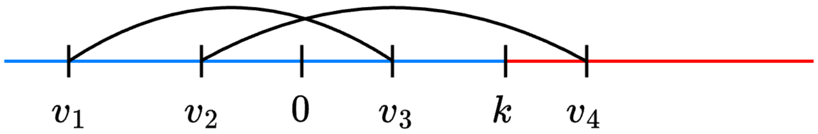
Step 2 $v' \geqslant 0 \geqslant v''$

Proof Sketch: Step 2

Intuition:



Only district with v_3, v_4 is won



Both districts are won

Step 2 $v' \geqslant 0 \geqslant v''$

Return

Reduction to Optimal Transport Problem

- Define $\phi' = \phi(\cdot|v \geqslant v^m)$ and $\phi'' = \phi(\cdot|v \leqslant v^m)$
- $\mathcal{M}(\phi', \phi'')$ set of joint distributions over v', v'' with marginals ϕ', ϕ''
- Designer's problem becomes:

$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int \int \mathbb{1}(v' - \omega \geqslant k) + \mathbb{1}(v' - \omega < k)\mathbb{1}\left(v' - \omega - \frac{v'' - \omega}{2} \geqslant k\right) d\gamma(\omega) d\tau(v', v'')$$

Reduction to Optimal Transport Problem

- Define $\phi' = \phi(\cdot|v \geqslant v^m)$ and $\phi'' = \phi(\cdot|v \leqslant v^m)$
- $\mathcal{M}(\phi', \phi'')$ set of joint distributions over v', v'' with marginals ϕ', ϕ''
- Designer's problem becomes:

$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int G(v' - k) + \int \mathbb{1}(v' - \omega < k)\mathbb{1}\left(v' - \omega - \frac{v'' - \omega}{2} \geqslant k\right) d\gamma(\omega) d\tau(v', v'')$$

Reduction to Optimal Transport Problem

- Define $\phi' = \phi(\cdot|v \geqslant v^m)$ and $\phi'' = \phi(\cdot|v \leqslant v^m)$
- $\mathcal{M}(\phi', \phi'')$ set of joint distributions over v', v'' with marginals ϕ', ϕ''
- Designer's problem becomes:

$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int G(v' - k) + G\left(2v' - v'' - k\right) - G(v' - k) d\tau(v', v'')$$

Reduction to Optimal Transport Problem

- Define $\phi' = \phi(\cdot | v \geqslant v^m)$ and $\phi'' = \phi(\cdot | v \leqslant v^m)$
- $\mathcal{M}(\phi', \phi'')$ set of joint distributions over v', v'' with marginals ϕ', ϕ''
- Designer's problem becomes:

$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int G\left(2v' - v'' - k\right) d\tau(v', v'') \tag{OTP}$$

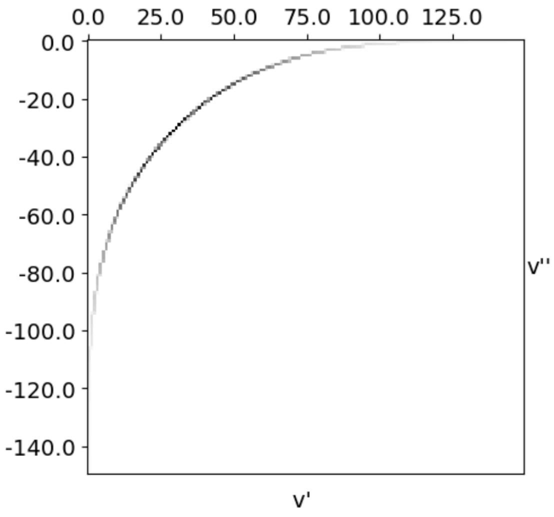
Reduction to Optimal Transport Problem

- Define $\phi' = \phi(\cdot|v \geqslant v^m)$ and $\phi'' = \phi(\cdot|v \leqslant v^m)$
- $\mathcal{M}(\phi', \phi'')$ set of joint distributions over v', v'' with marginals ϕ', ϕ''
- Designer's problem becomes:

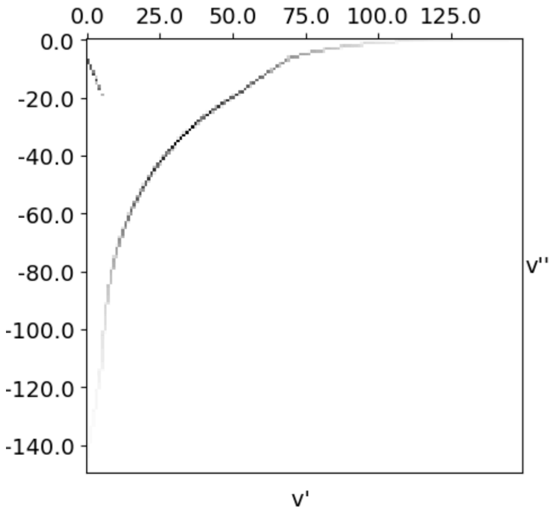
$$\max_{\tau \in \mathcal{M}(\phi', \phi'')} \int G\left(2v' - v'' - k\right) d\tau(v', v'') \tag{OTP}$$

Return

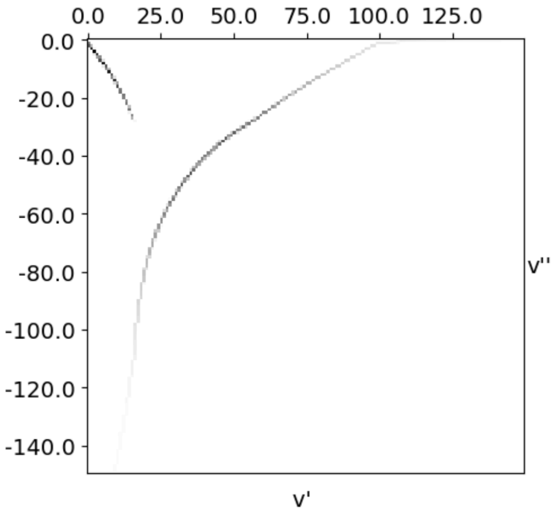
Example with Normal Shock



(a) k=60



(b) k=75



(c) k=100

Solution for simulated F normal, G normal

References

Chiappori, P.-A., R. J. McCann, and L. P. Nesheim (2010): "Hedonic price equilibria, stable matching, and optimal transport: equivalence, topology, and uniqueness," *Economic Theory*, 42, 317–354.

Friedman, J. N. and R. T. Holden (2008): "Optimal gerrymandering: sometimes pack, but never crack," *American Economic Review*, 98, 113–44.

Gul, F. and W. Pesendorfer (2010): "Strategic redistricting," *American Economic Review*, 100, 1616–41.

Kamenica, E. and M. Gentzkow (2011): "Bayesian persuasion," *American Economic Review*, 101, 2590–2615.

References II

Kantorovich, L. V. (1942): "On the translocation of masses," in *Dokl. Akad. Nauk. USSR (NS)*, vol. 37, 199–201.

Kolotilin, A. and A. Wolitzky (2024): "The economics of partisan gerrymandering," .

Monge, G. (1781): "Mémoire sur la théorie des déblais et des remblais," *Mem. Math. Phys. Acad. Royale Sci.*, 666–704.

Owen, G. and B. Grofman (1988): "Optimal partisan gerrymandering," *Political Geography Quarterly*, 7, 5–22.

——— (2006): "Two-stage electoral competition in two-party contests: persistent divergence of party positions," *Social Choice and Welfare*, 26, 547–569.

Santambrogio, F. (2015): "Optimal transport for applied mathematicians," *Birkäuser, NY*, 55, 94.

Yang, K. H. and A. K. Zentefis (2024): "Monotone Function Intervals: Theory and Applications," *American Economic Review*, 114, 2239–2270.