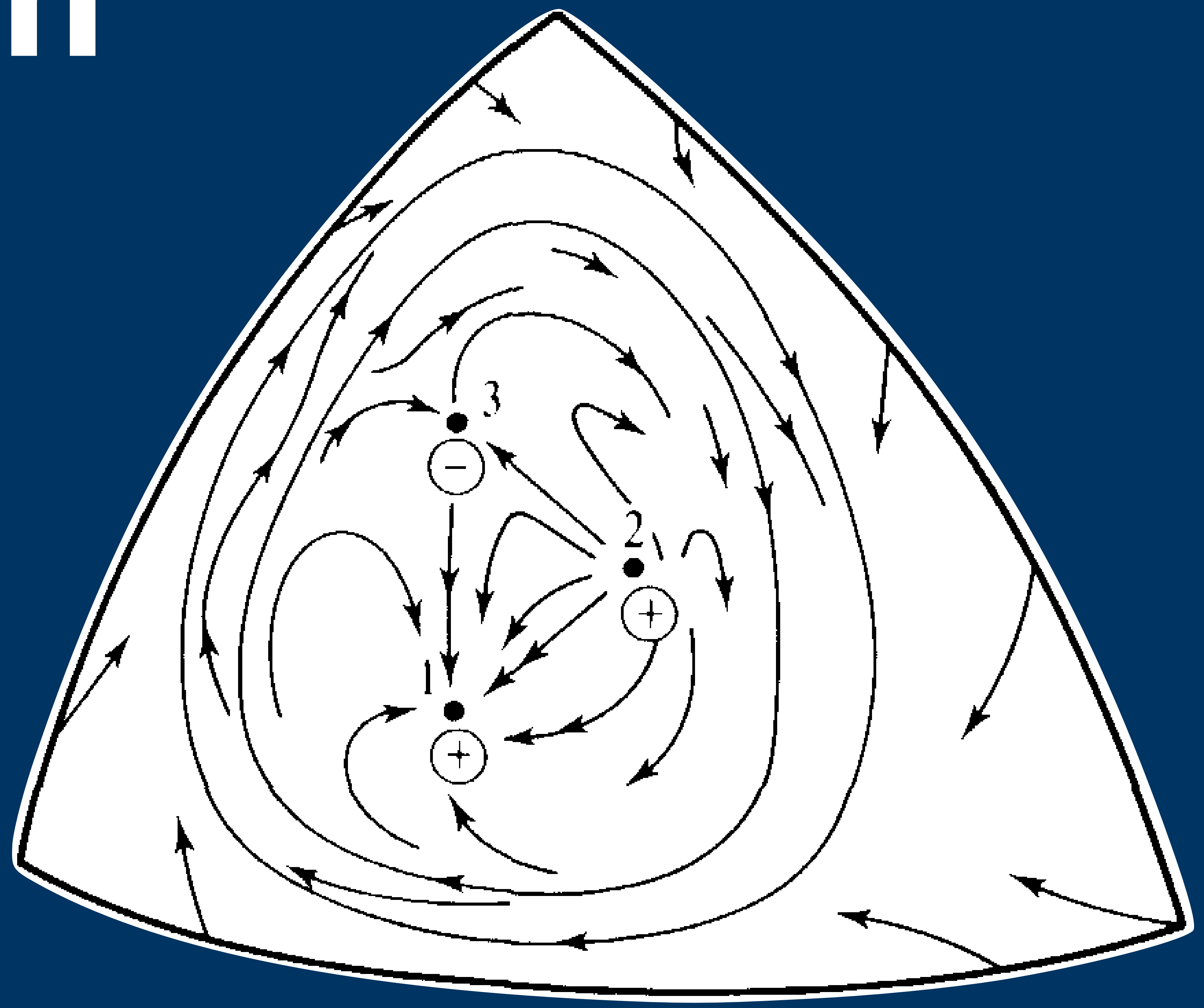
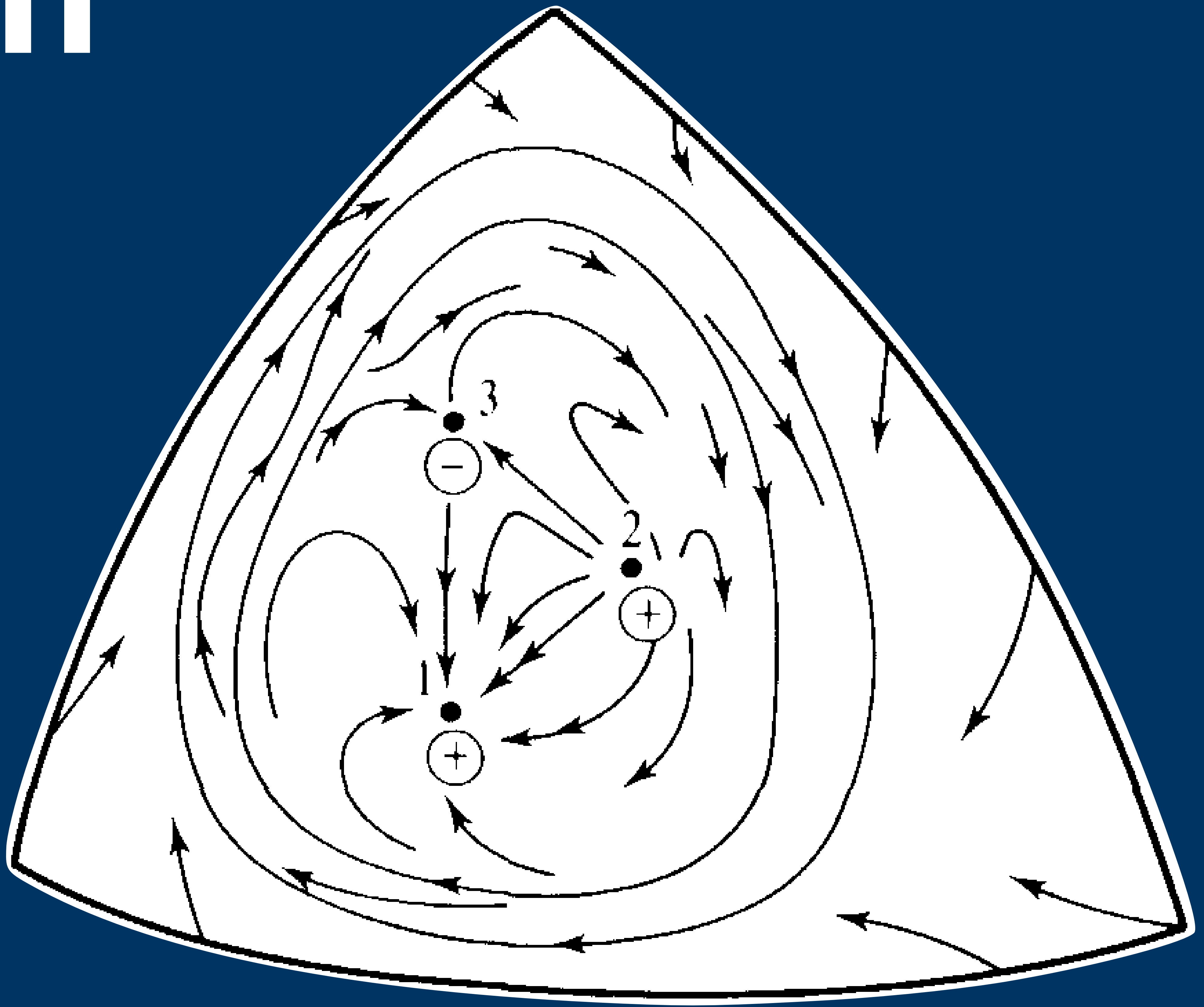
Tâtonnement and Price Setting in General Equilibrium



NBER SI 2025

Tâtonnement and Price Setting in General Equilibrium

Guido Lorenzoni and Iván Werning



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Tâtonnement History

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- Leon Walras…
  - “Our task is **very simple**: we need only show that the upward and downward movements of prices solve the system of equations of offer and demand by a process of **tâtonnement**” [feeling one's way toward the equilibrium?]
  - Samuelson →**ad hoc** equation (disequilibrium)
  - Economic question of stability:
    important and interesting…
    … but no real model!

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- **Our paper:** … revisit question…
  … but with **equilibrium model!**

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  - existence ✓

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  - uniqueness/multiplicity ✓

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$$\dot{p}_t = F(z(p_t))$$


Samuelson
Arrow
Hurwicz
Scarf
Negishi
Hahn
Fisher
Uzawa
Smale
McKenzie

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- … but **deep conceptual problems**…
  - who changes prices? are they reasonable? (alternatives proposed)
  - consumers and producers optimize quantities freely given prices…
    … but if markets don't clear, they cannot, so why is demand curve right object?
  - Static (not forward looking), Rational expectations…? Assets and money?

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- So… at… ar…

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Struggles to find fix…

…but mostly failed.

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This Paper… Micro → Macro

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  - h agents, general heterogeneous preferences
  - f firms, general technology, input-output networks and more

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  - Optimal price setting + Calvo frictions

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- **Dynamics + Market Power** → very general NK GE model
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- **Analysis…**
  - Dynamic equilibrium → path for prices, given initial prices
  - Steady state of dynamic = Walrasian equilibrium of static GE
  - **No disequilibrium!**

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- **Subtle role of monetary policy**
  - we find simple policies that always works
  - Taylor rules with wrong price index may fail: create instability (not indeterminacy)

Related Literature

- **Tâtonnment GE literature (Huge)**
  Samuelson, Arrow-Hurwitz, Nerlove, Uzawa, Negishi, Scarf, Smale, Hahn, Fisher, Iwai, …

- **Macro NK models + N sectors (Healthy, Growing)**
  Carlstrom-Fuerst-Ghironi, Rubbo, Lorenzoni-Werning, Afrouzi-Bhattarai, …


Static Walrasian GE
Static Monopolistic Monopsonistic GE
Dynamic Monopolistic Monopsonistic GE
Analysis of Stability

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Static GE Model

- Importance of generality in GE
- Primitives…
  - n goods (goods and factors, many labor etc.)
  - h household types, general preferences
  - f firms, general technologies (networks, etc.)

$$x = (x_1, \ldots, x_N) \geq 0$$
$$y = (y_1, \ldots, y_N) \geq 0$$

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Samuelson's ad hoc proposal to capture Walras' idea…

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Will come back to this…

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Analysis
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or

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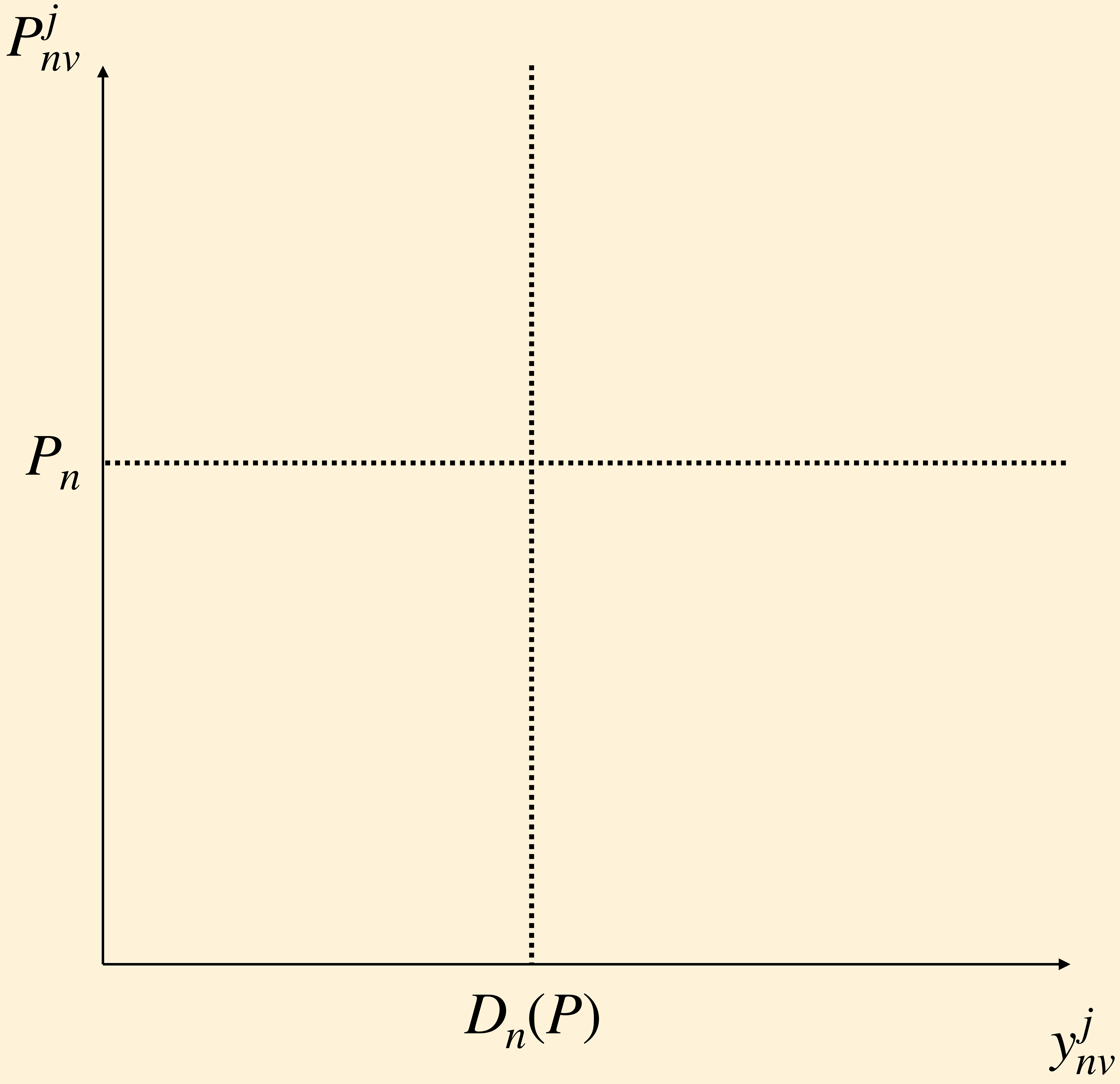
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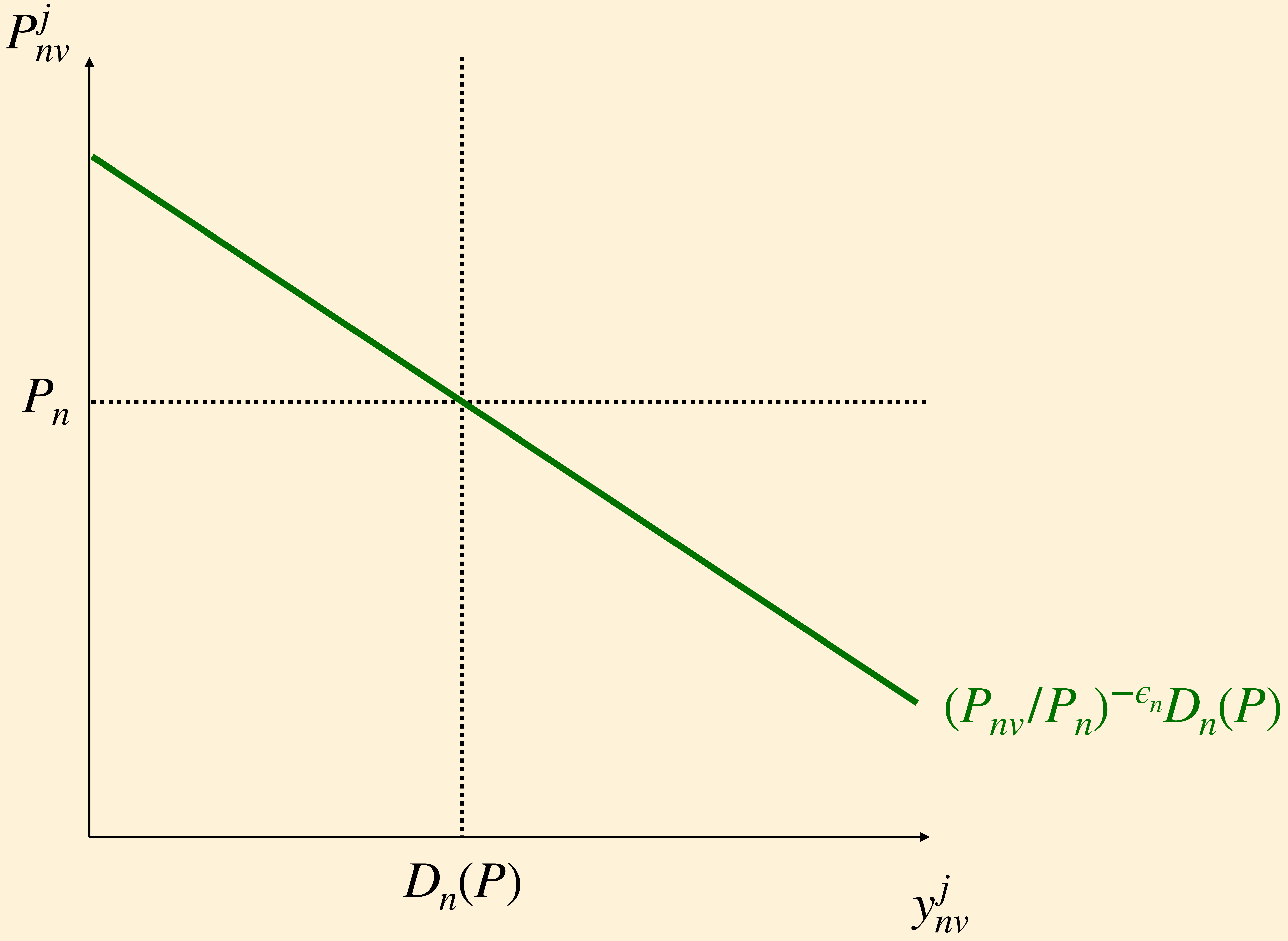

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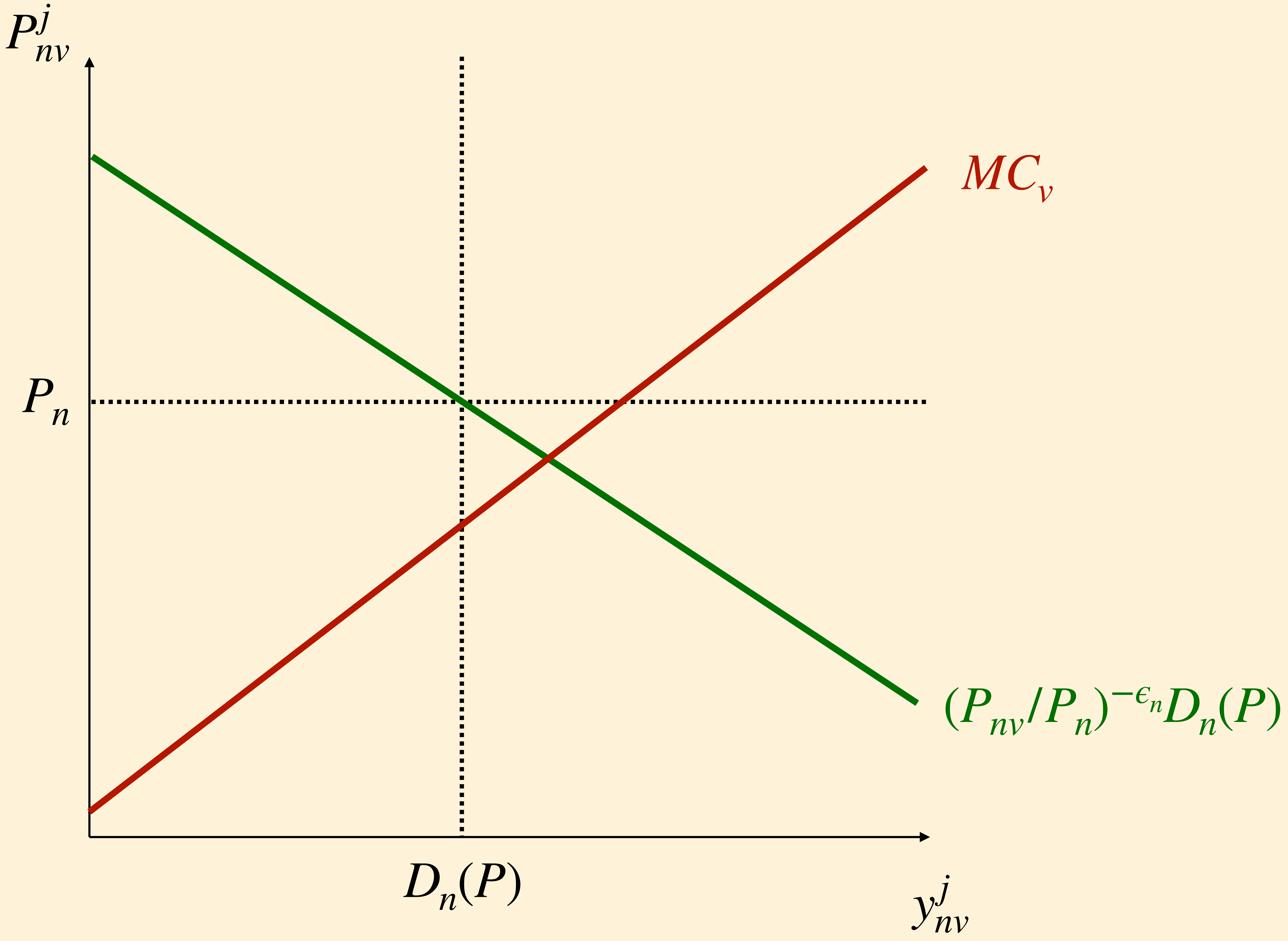
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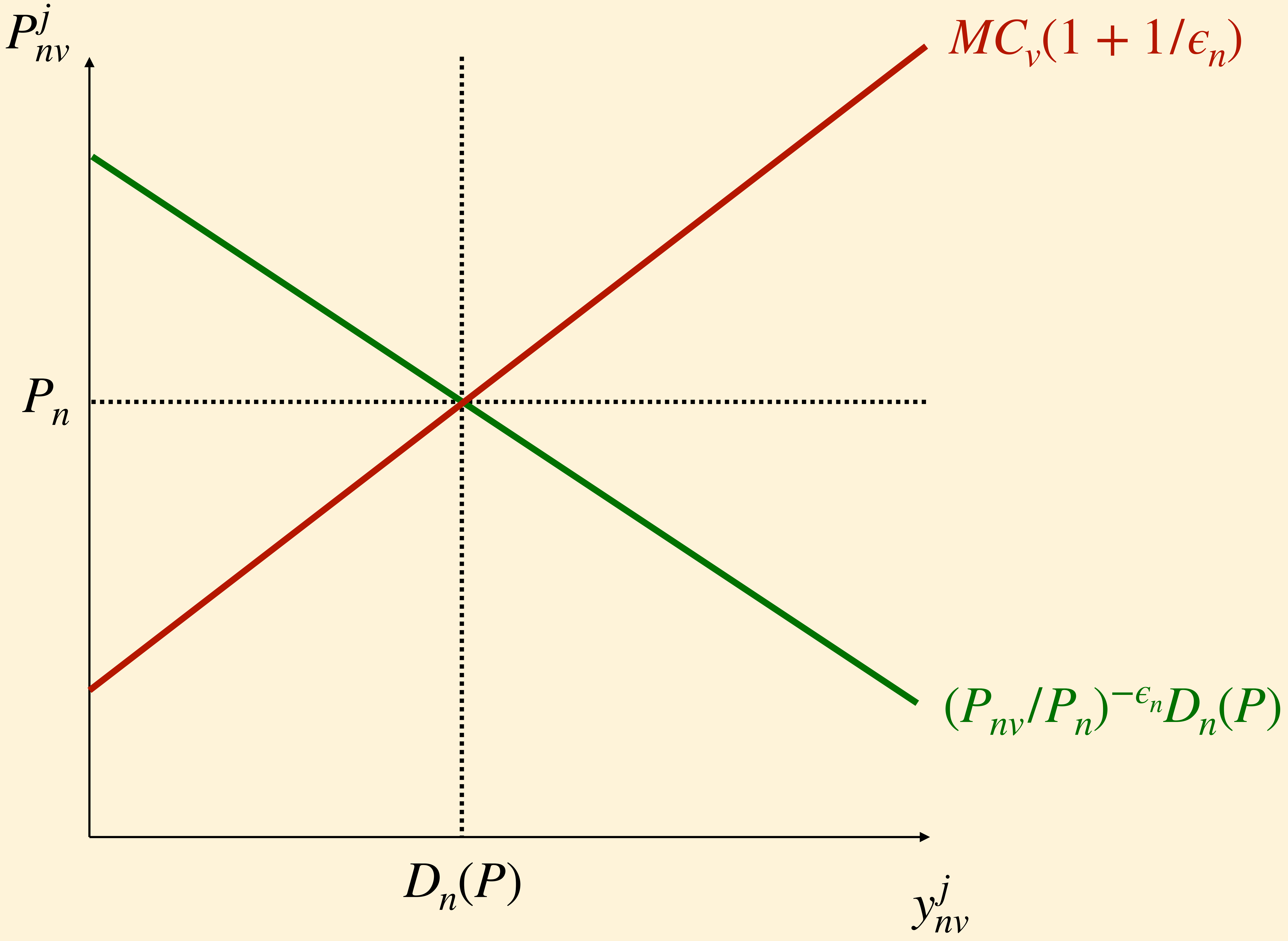
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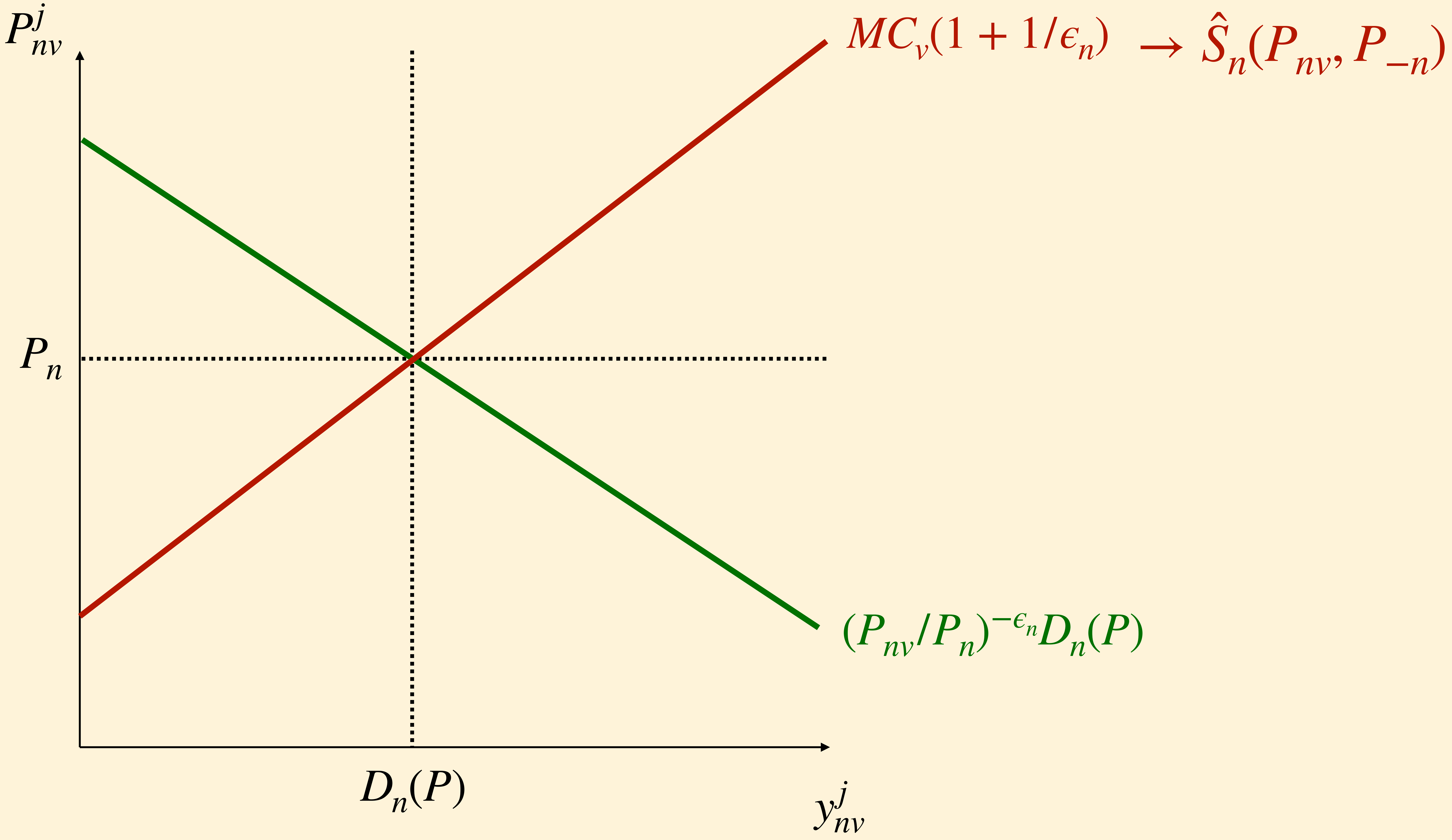
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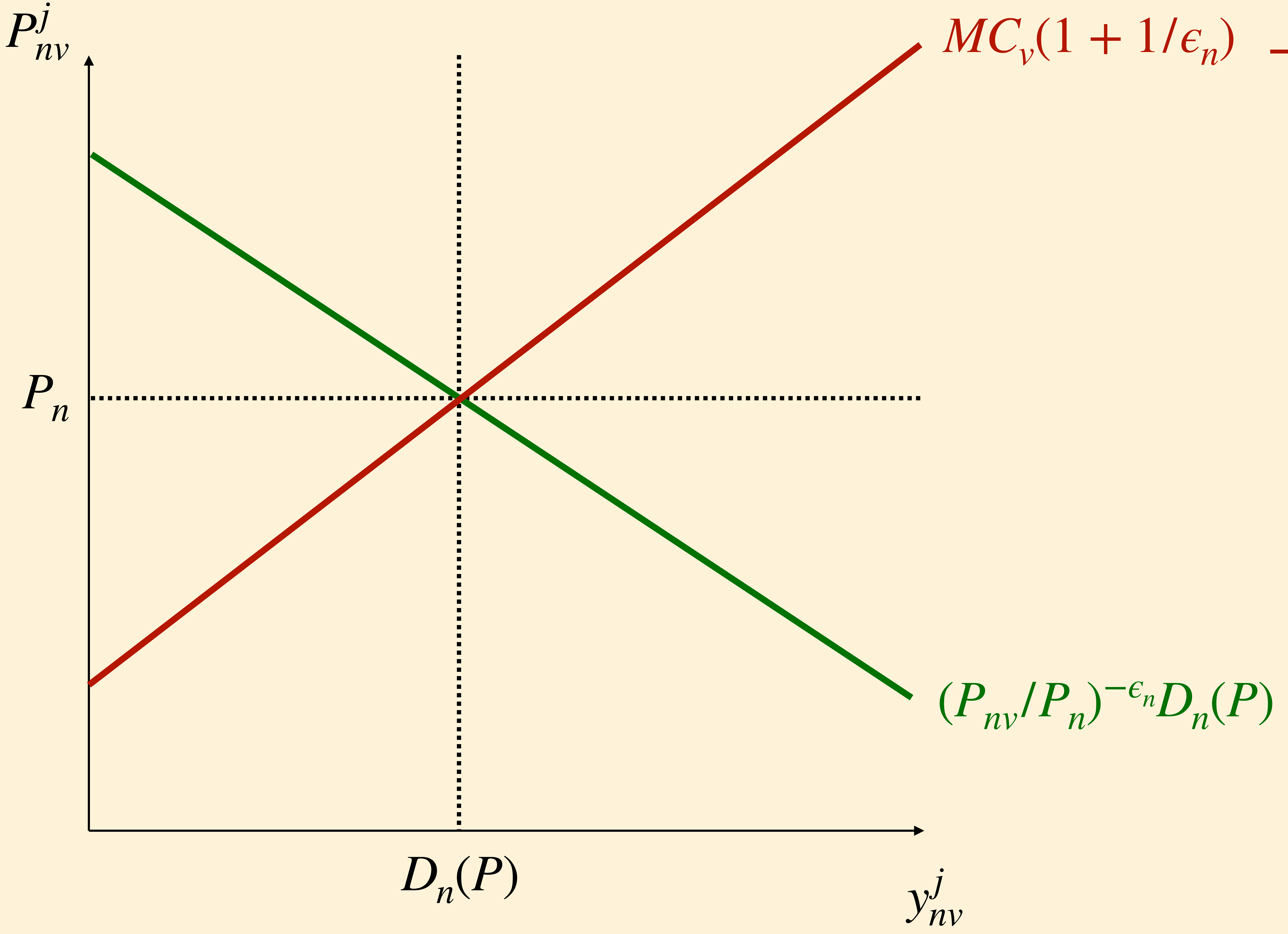

$\rightarrow \hat{S}_n(P_{nv}, P_{-n})$

symmetric equilibrium $P_{nv} = P_n$

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$$\epsilon_n \to \infty$$

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Subsidies
$\tau_n = -1/\epsilon_n$

$P = P_W$

Monopolistic GE
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😎🔥

“As if” competitive…

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"As if" competitive…

$$\Pi^f(P) \equiv \max P \cdot (y^f - x^f) + P^f_{nv}(y^f_{nv} - x^f_{nv})$$

$$(x^f, y^f, x^f_{nv}, y^f_{nv}) \in Y^f$$

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Static
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Analysis
of
Stability

Preview

- Add: dynamics + forward looking + price setting a la Calvo
  $\rightarrow$ very general New Keynesian GE Model

- Focus: adjustment of **vector of spot prices** P_t set by private agents (endogenous)

- Financial market…
  - insurance for “Calvo fairy”
  - saving and borrowing at central bank interest rate

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s.t. Calvo friction

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**Analysis
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Stability**

Analysis

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Samuelson's equation!

log-linearized…

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Why set $P^*_{nt} = \bar{P}_{nt}$?…

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↓

$$\left(Q_t = \hat{Q}_t e^{-\rho t}\right)$$

(Lagrangian)

$$L^j = \mu^j \int_0^\infty e^{-\rho t}[\frac{1}{\mu^j} U^j(x_t^j, y_t^j, x_{nvt}^j, y_{nvt}^j) + \hat{Q}_t P_t \cdot (x_t^j - y_t^j) + \hat{Q}_t P_{nvt}^j (x_{nvt}^j - y_{nvt}^j)]\,dt$$

↑

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$$\max_{x,y} \rightarrow V^j(P_t, \hat{Q}_t, x_{nvt}, y_{nvt})$$

(indirect utility)

↑

$$\max \int_0^\infty e^{-\rho t} U^h(x_t^h, y_t^h, x_{nvt}^h, y_{nvt}^h)\, dt$$

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- For now set $\hat{Q}_t = 1...$

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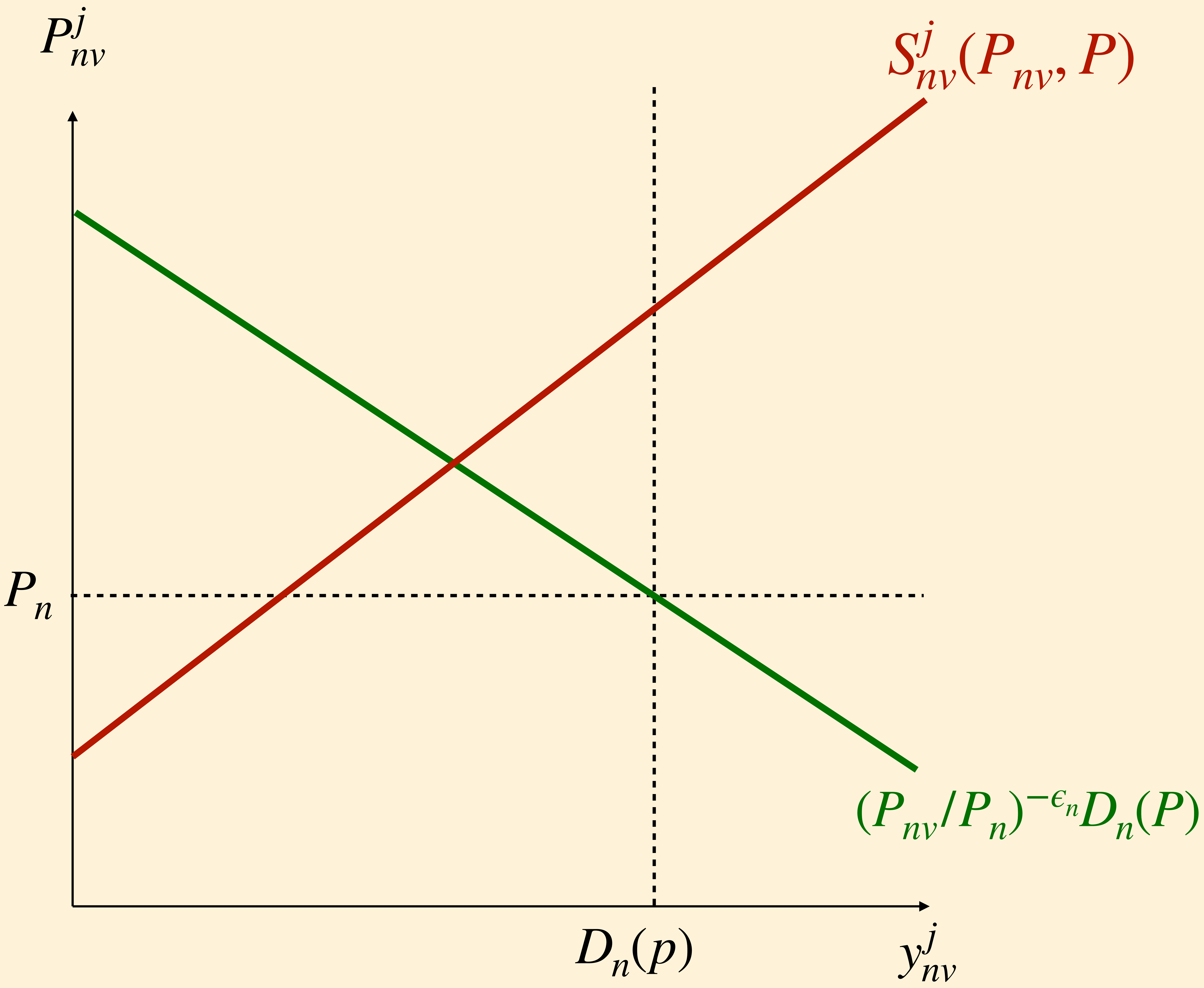
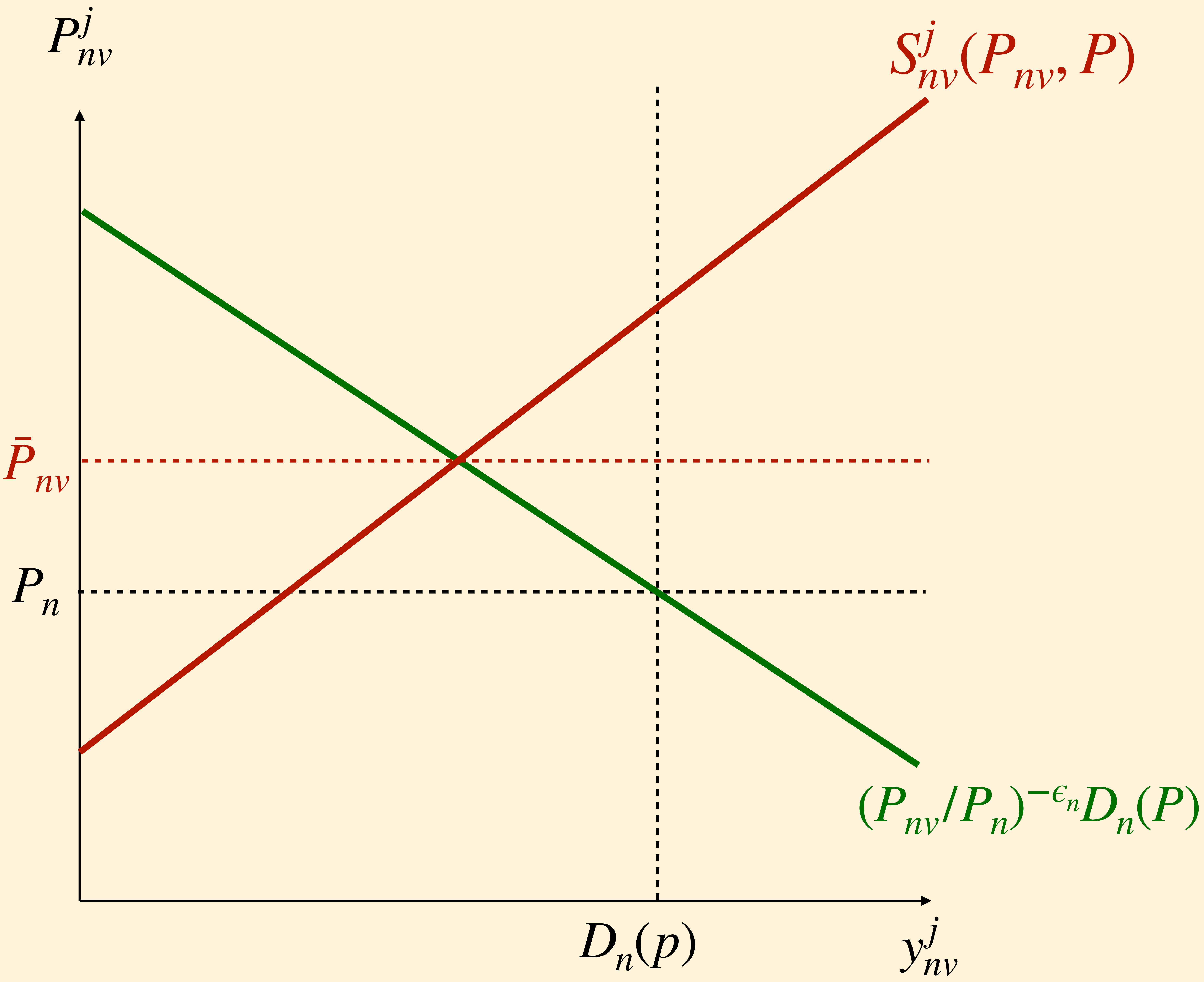
(Monopsonistic case: similar)



Similar to Static…

… but do not impose symmetry.



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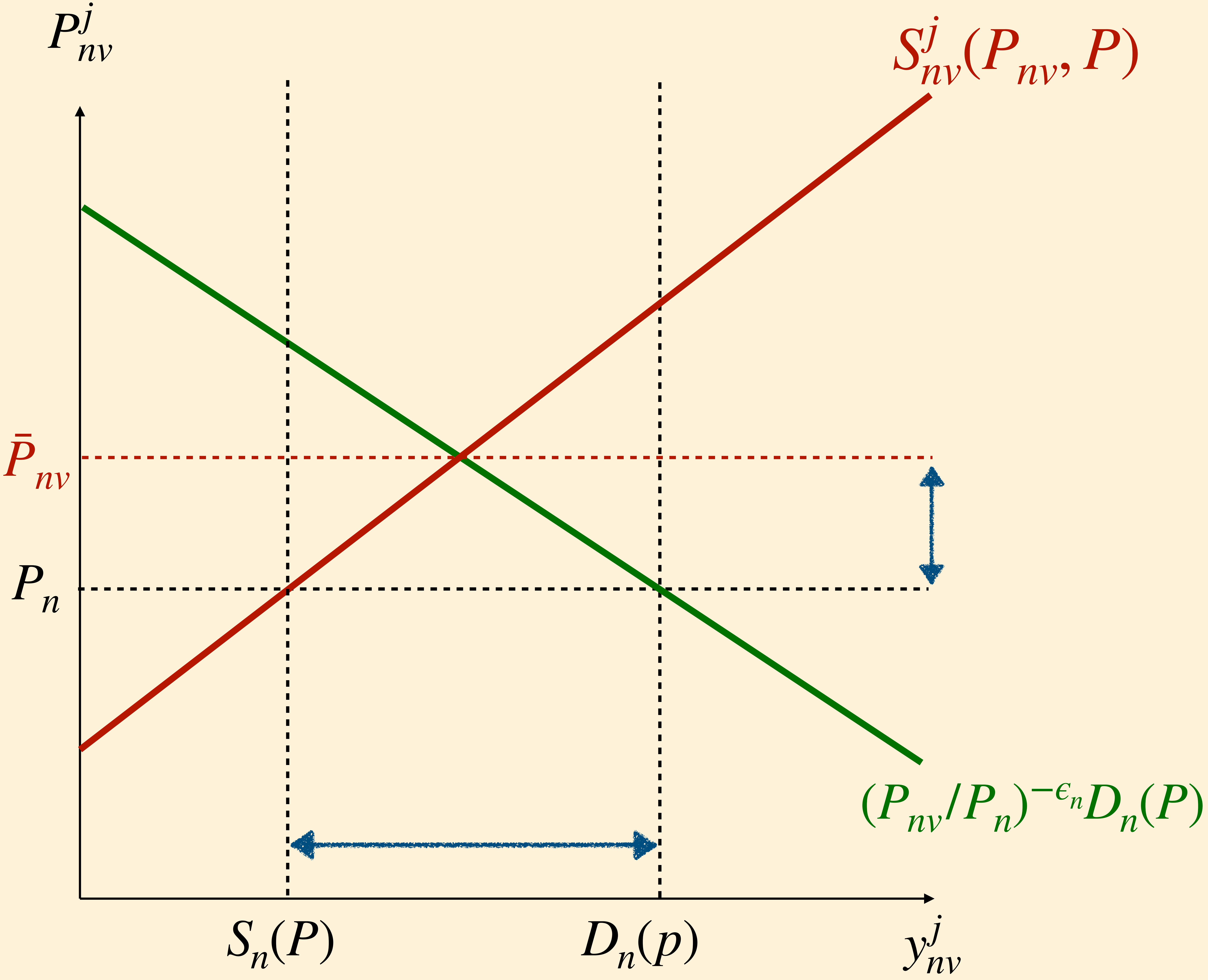

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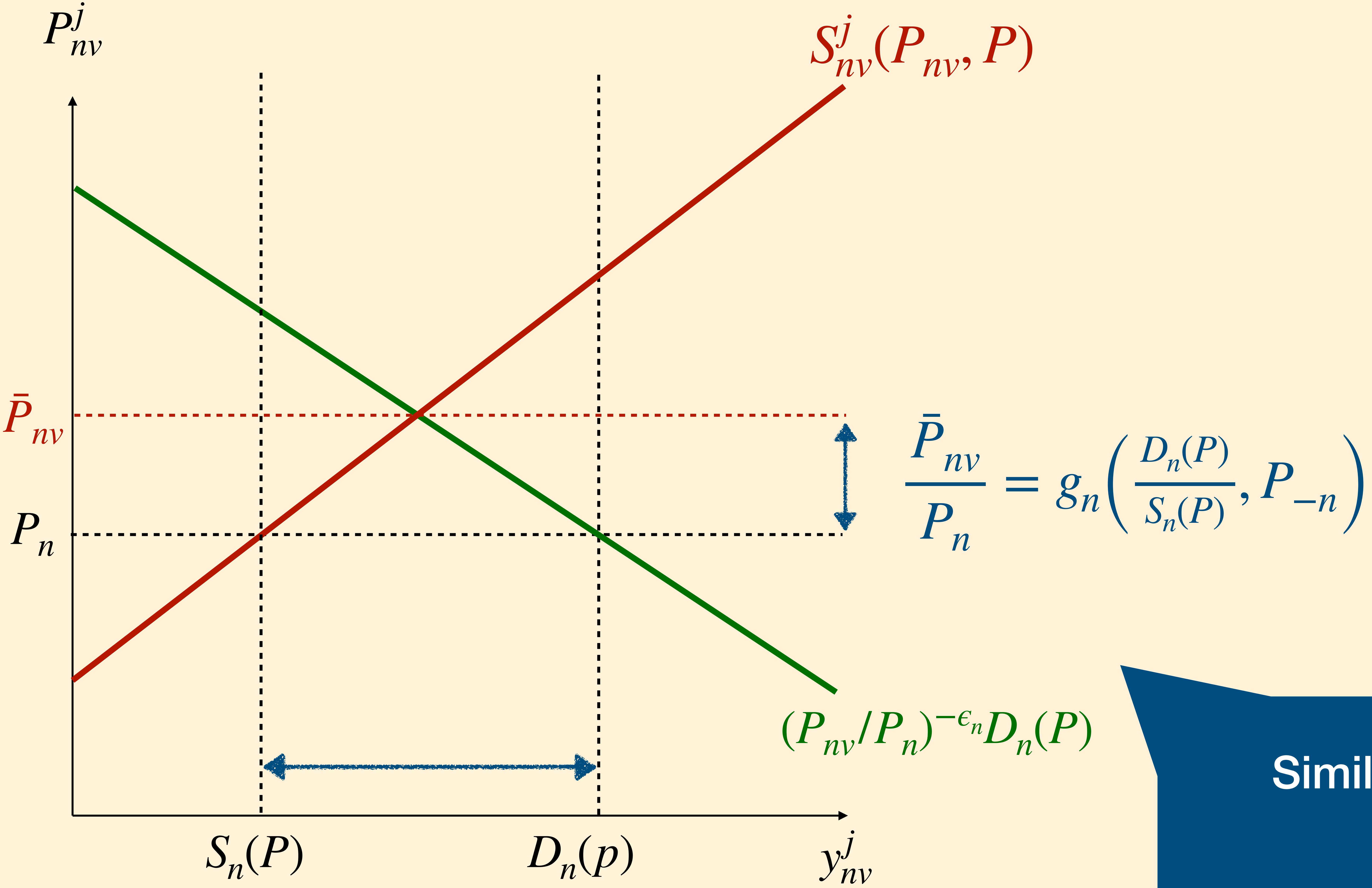

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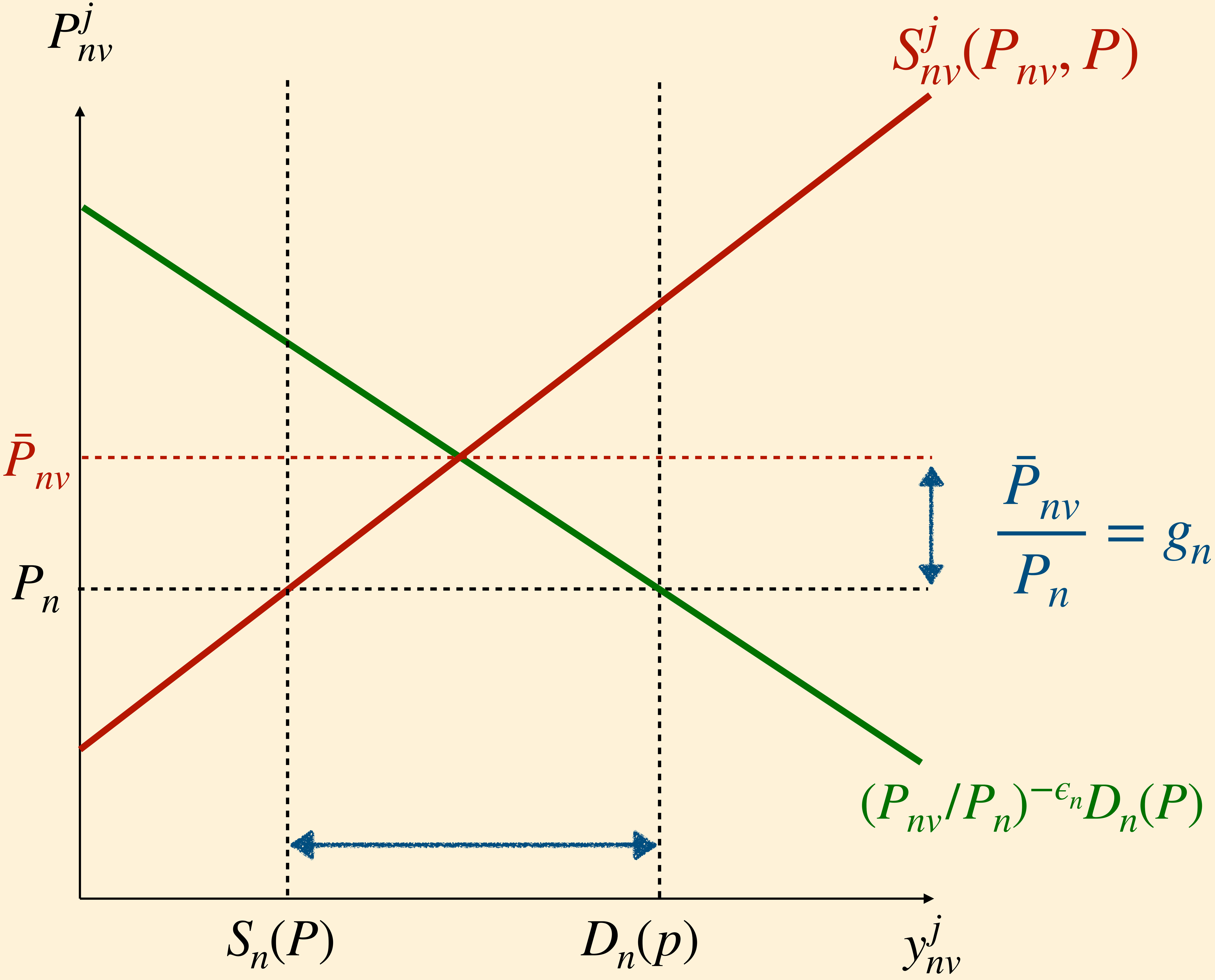

$$\frac{\bar{P}_{nv}}{P_n} = g_n\left(\frac{D_n(P)}{S_n(P)}, P_{-n}\right) \begin{cases} > 1 & \text{if } D_n(P) > S_n(P) \\ = 1 & \text{if } D_n(P) = S_n(P) \\ < 1 & \text{if } D_n(P) < S_n(P) \end{cases}$$

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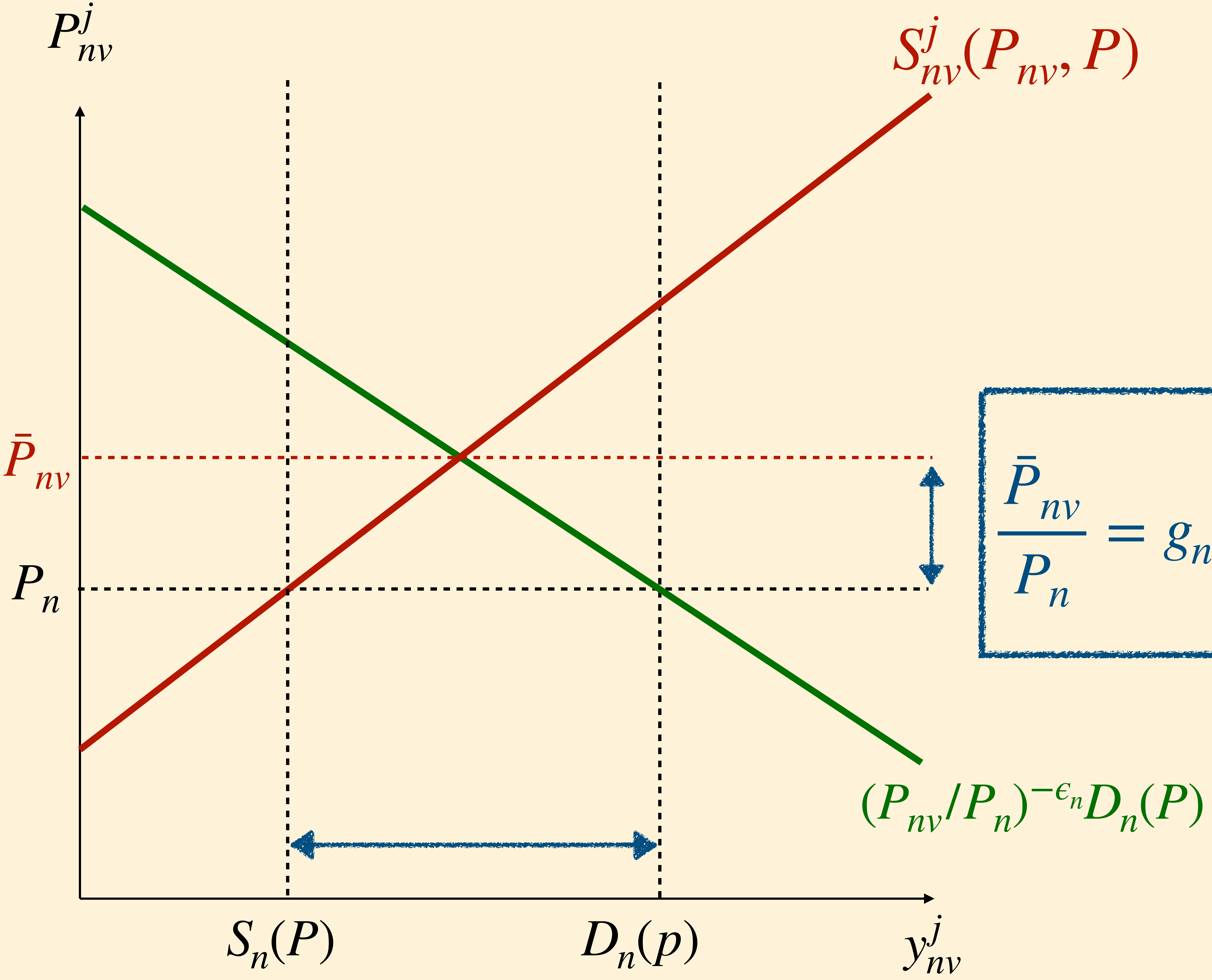

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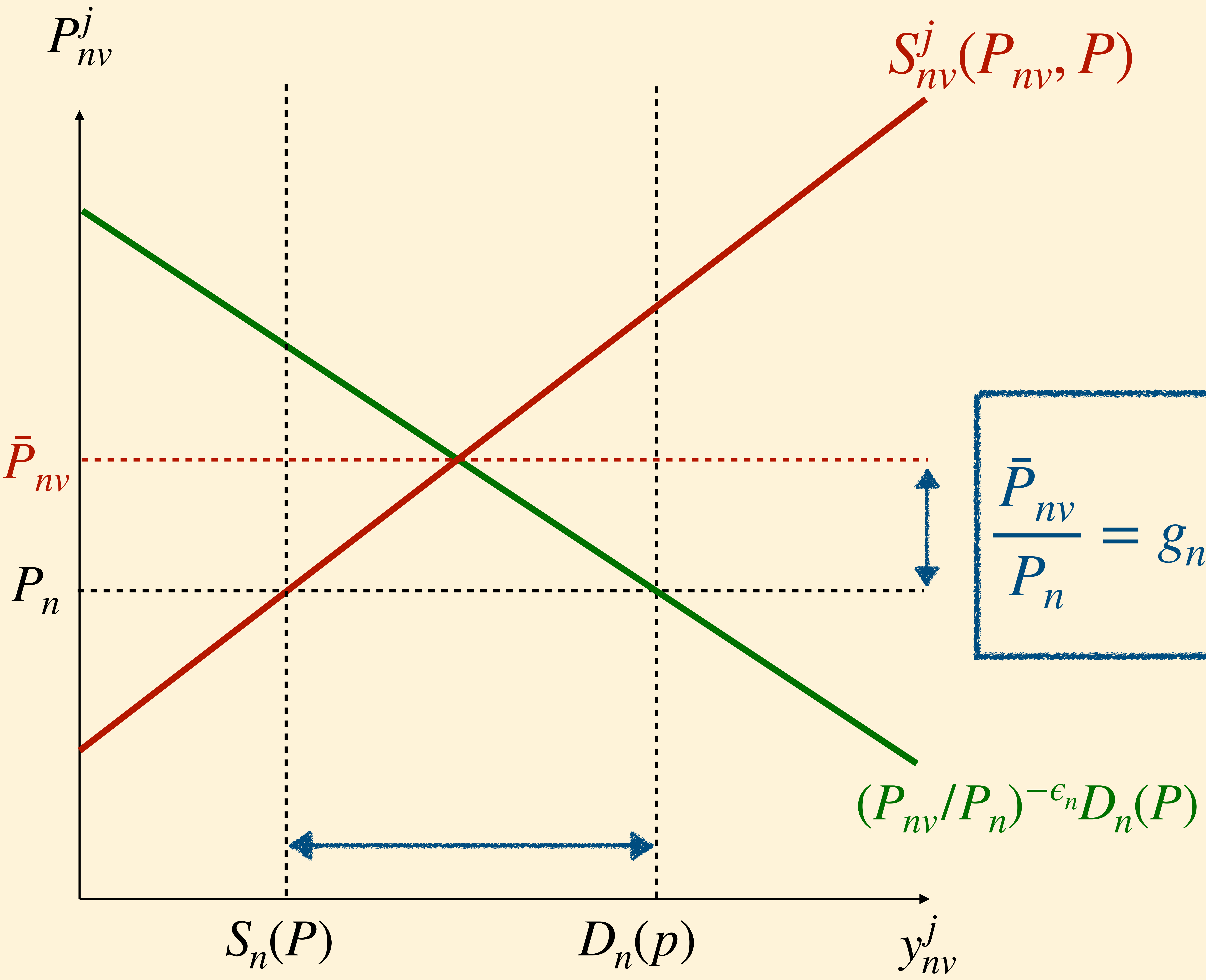

$$\bar{p}_{nv} - p_n = \frac{1}{\epsilon_n + \epsilon_n^s}(d_n(p) - s_n(p))$$

log-linearized

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- Equation has Samuelson form…
  - … **but** demand and supply are **Frisch**, not Marshallian…
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  - converge for *any* household multipliers μ^h
  - multipliers pinned down by budget constraints…
    … by Walras' Law → only $H-1$ independent budget constraints…
    … one degree of indeterminacy!
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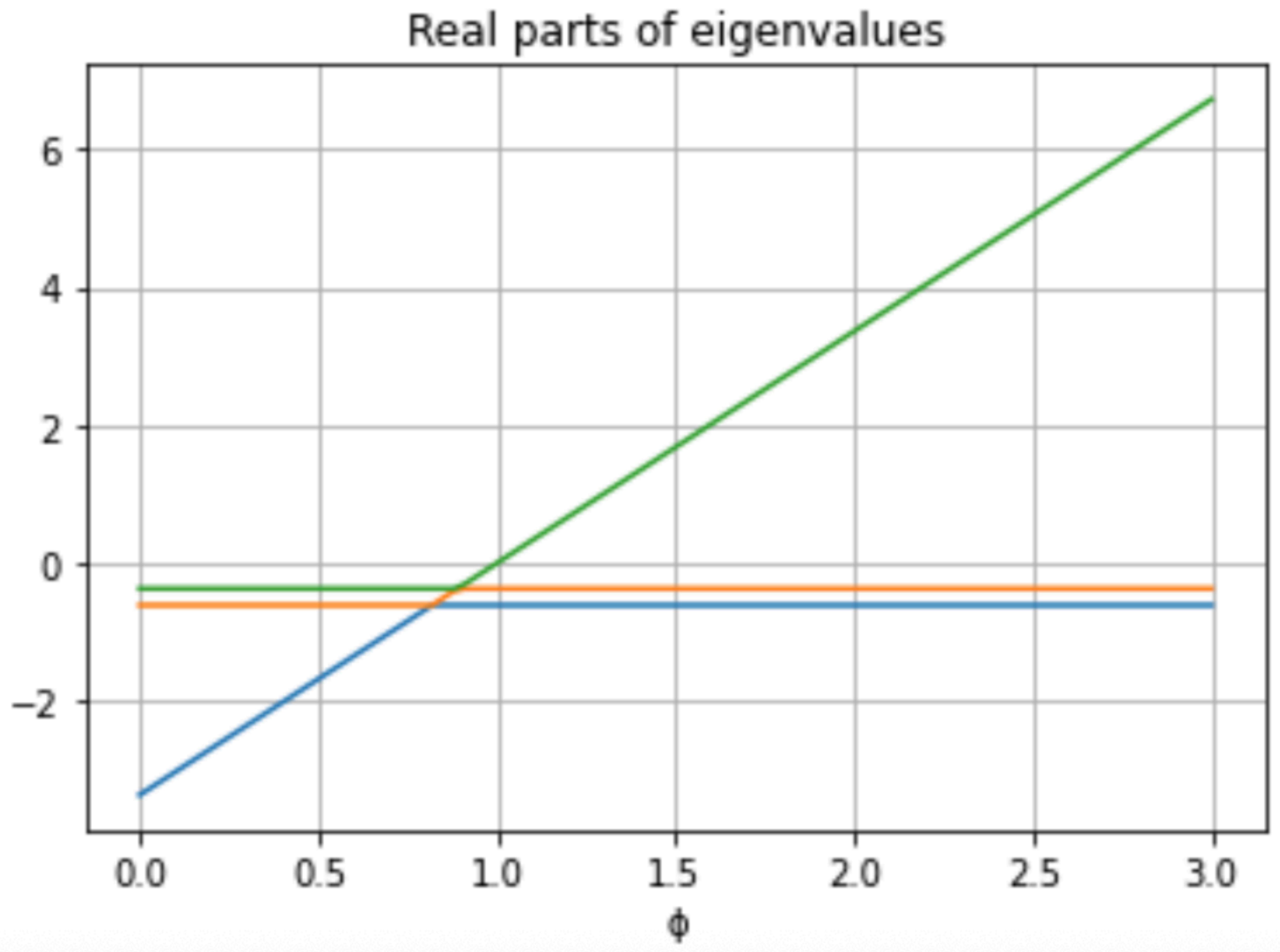
- We show Taylor principle for arbitrary or CPI ω fails to work…
  - less than $N - 1$ stable
  - complex roots

Example 1: Three good economy

```
plt.xlabel("ϕ")

print(vl)
```

```
[0.149198 0.504783 0.346019]
```



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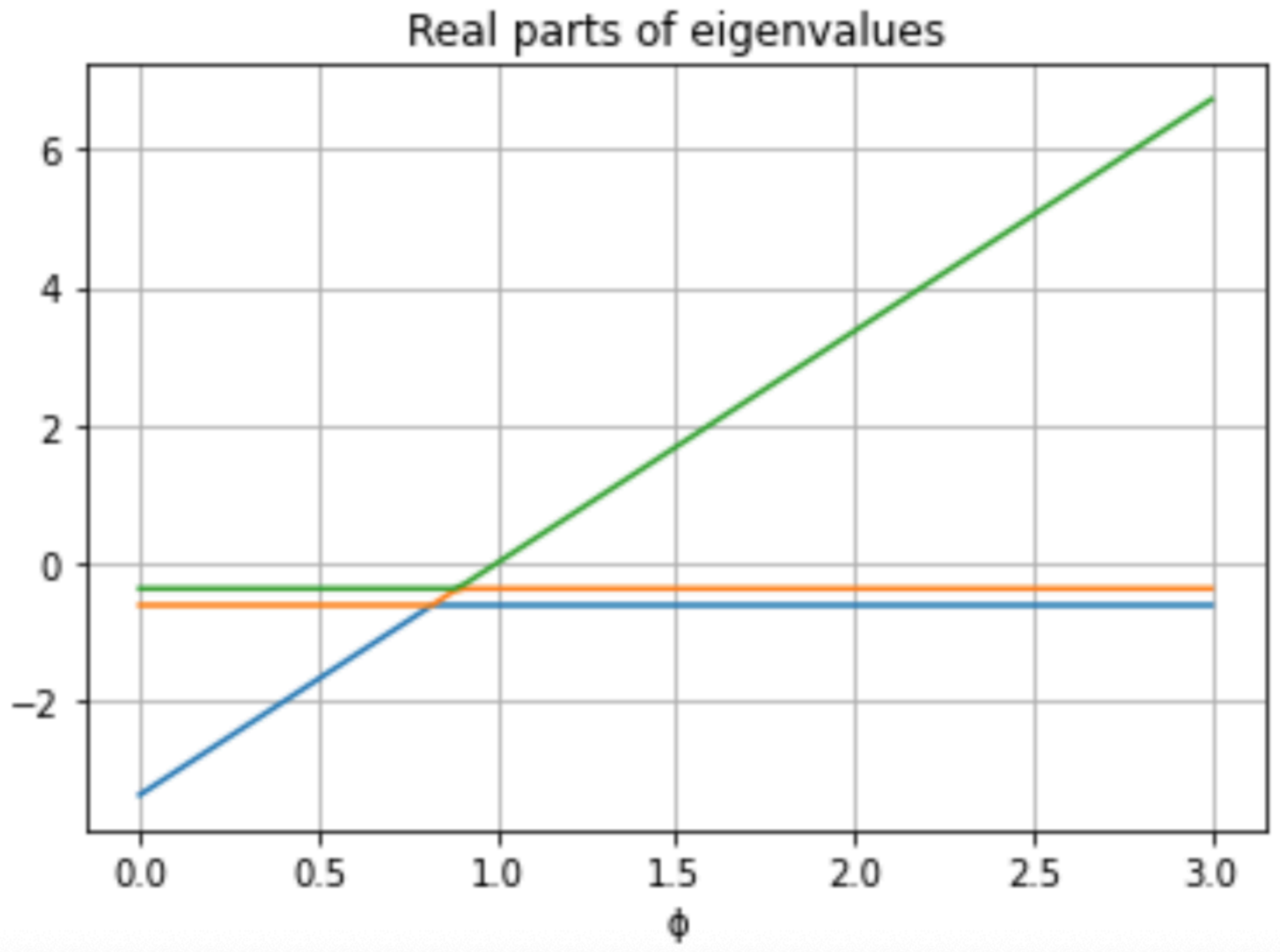

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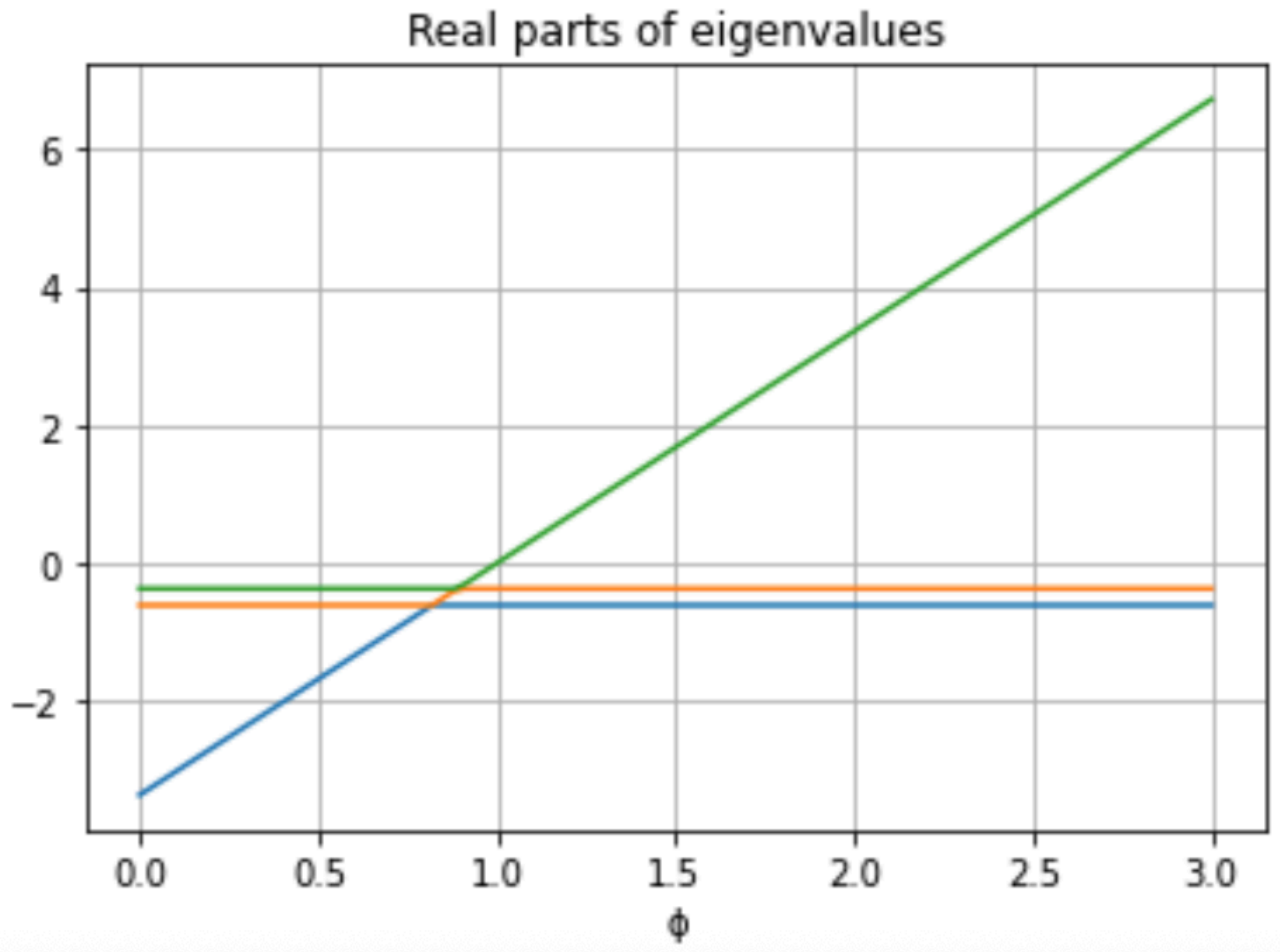

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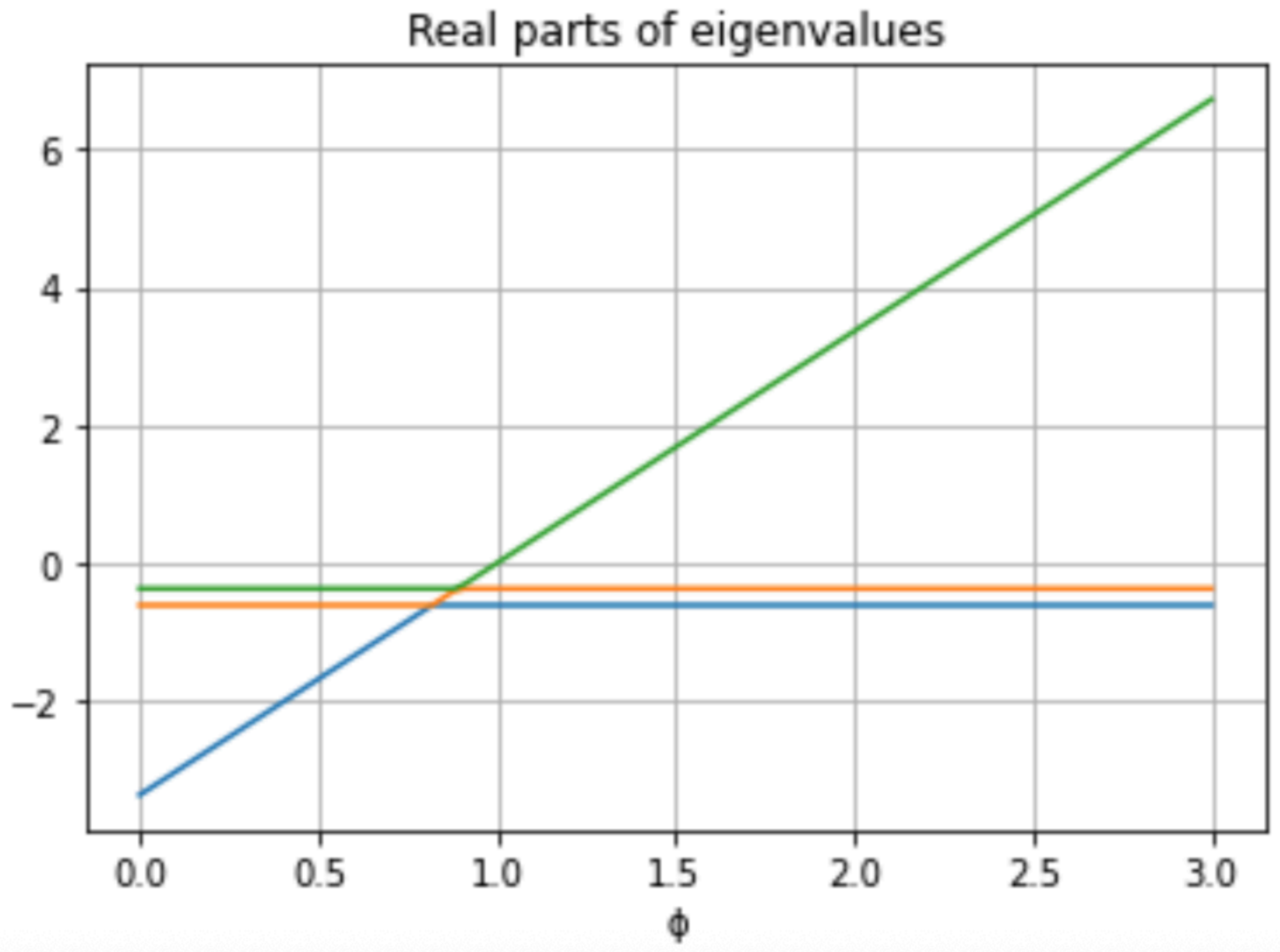

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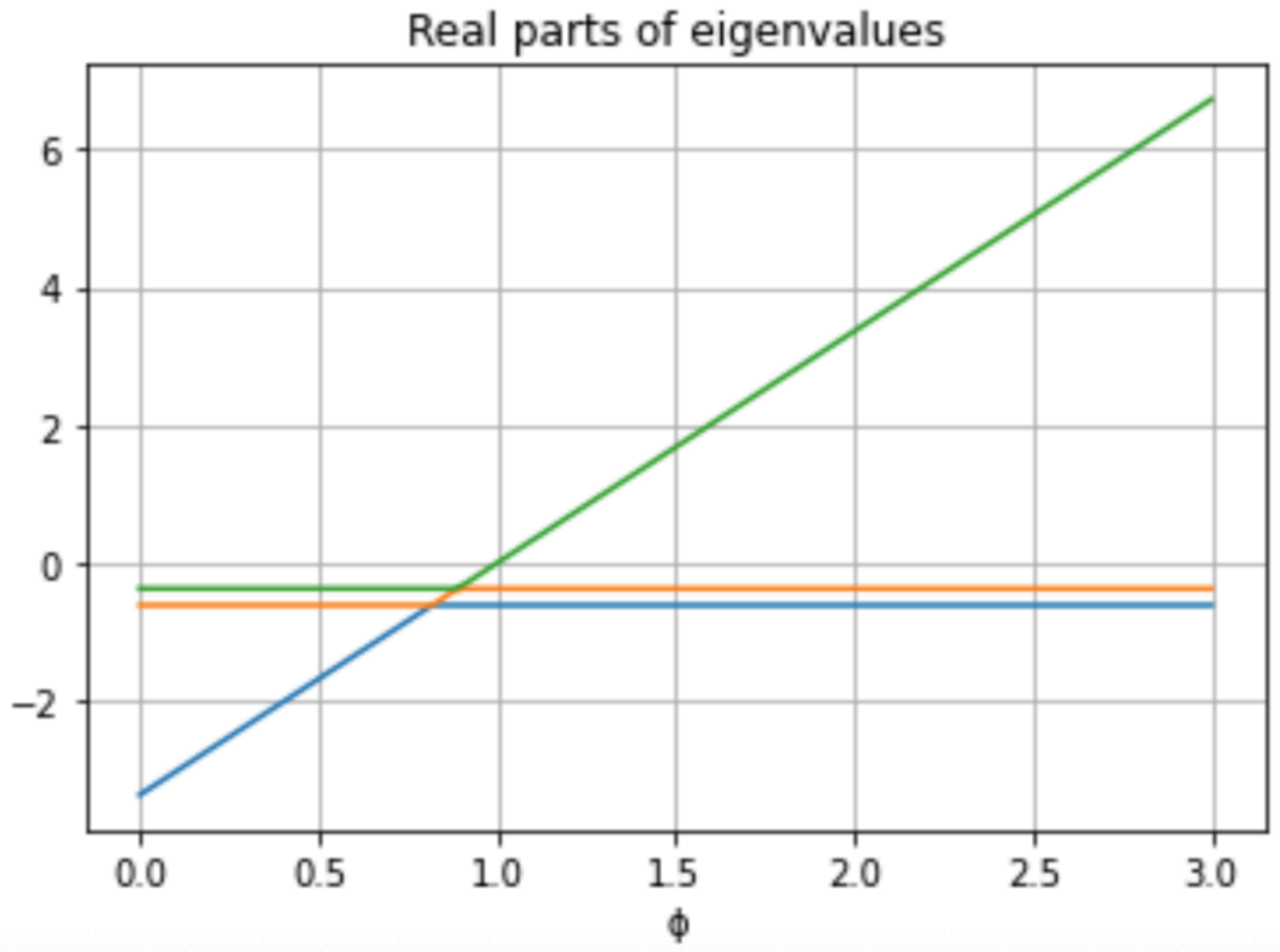

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- Taylor rule with weights = left eigenvector
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  - other rises and crosses 0 at $\phi = 1$! 😎

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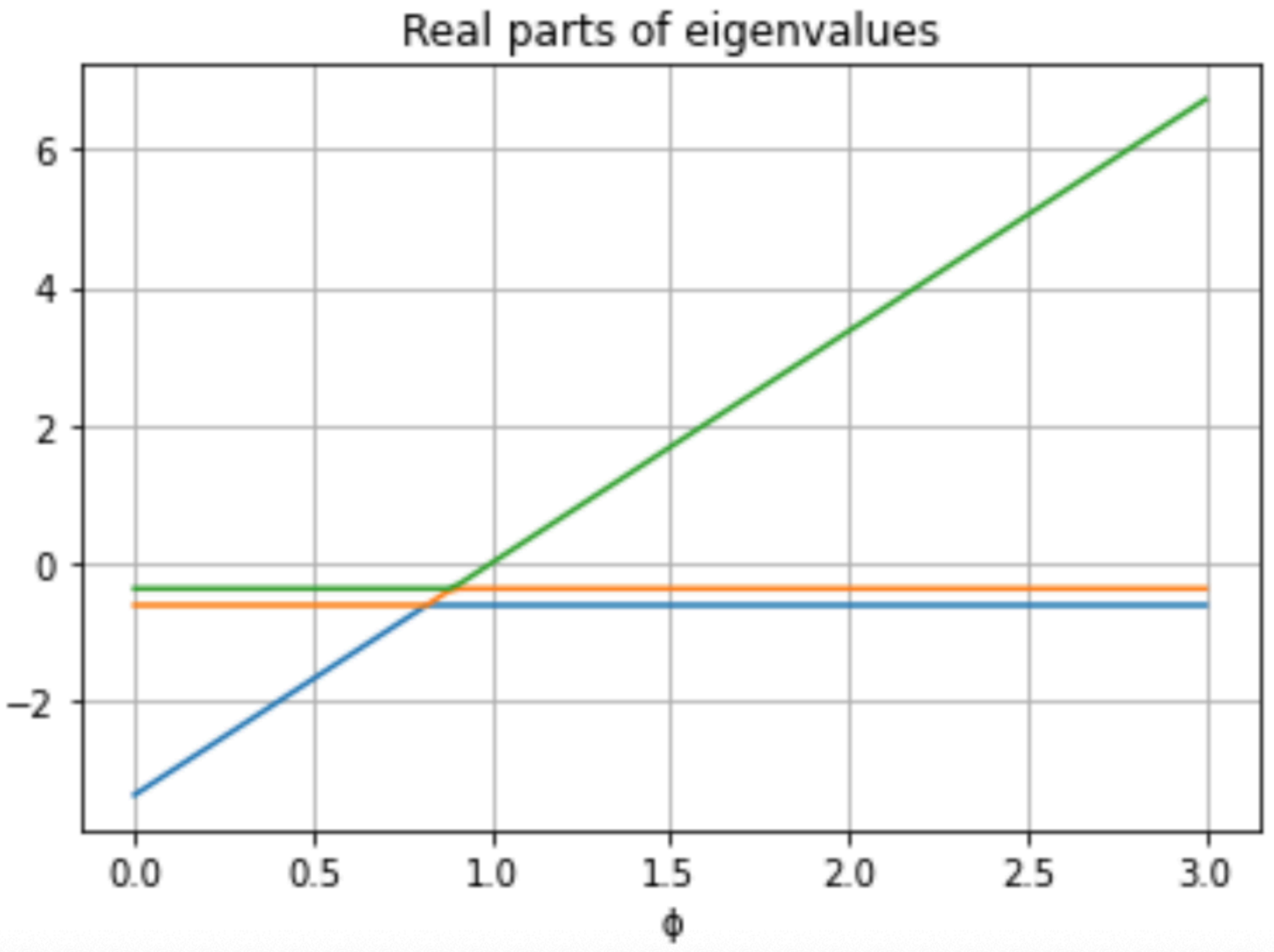
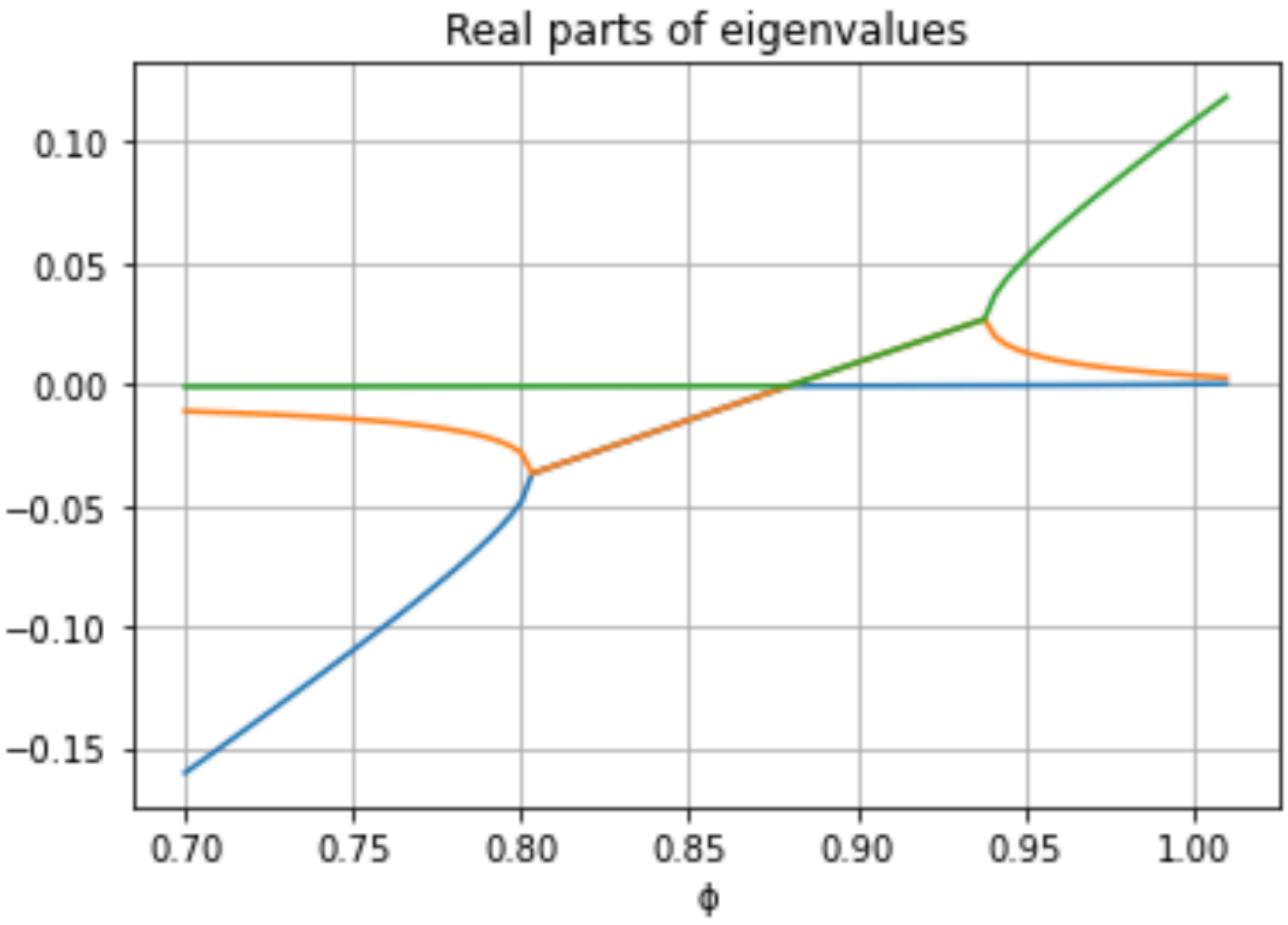


Example 2: Three good economy again



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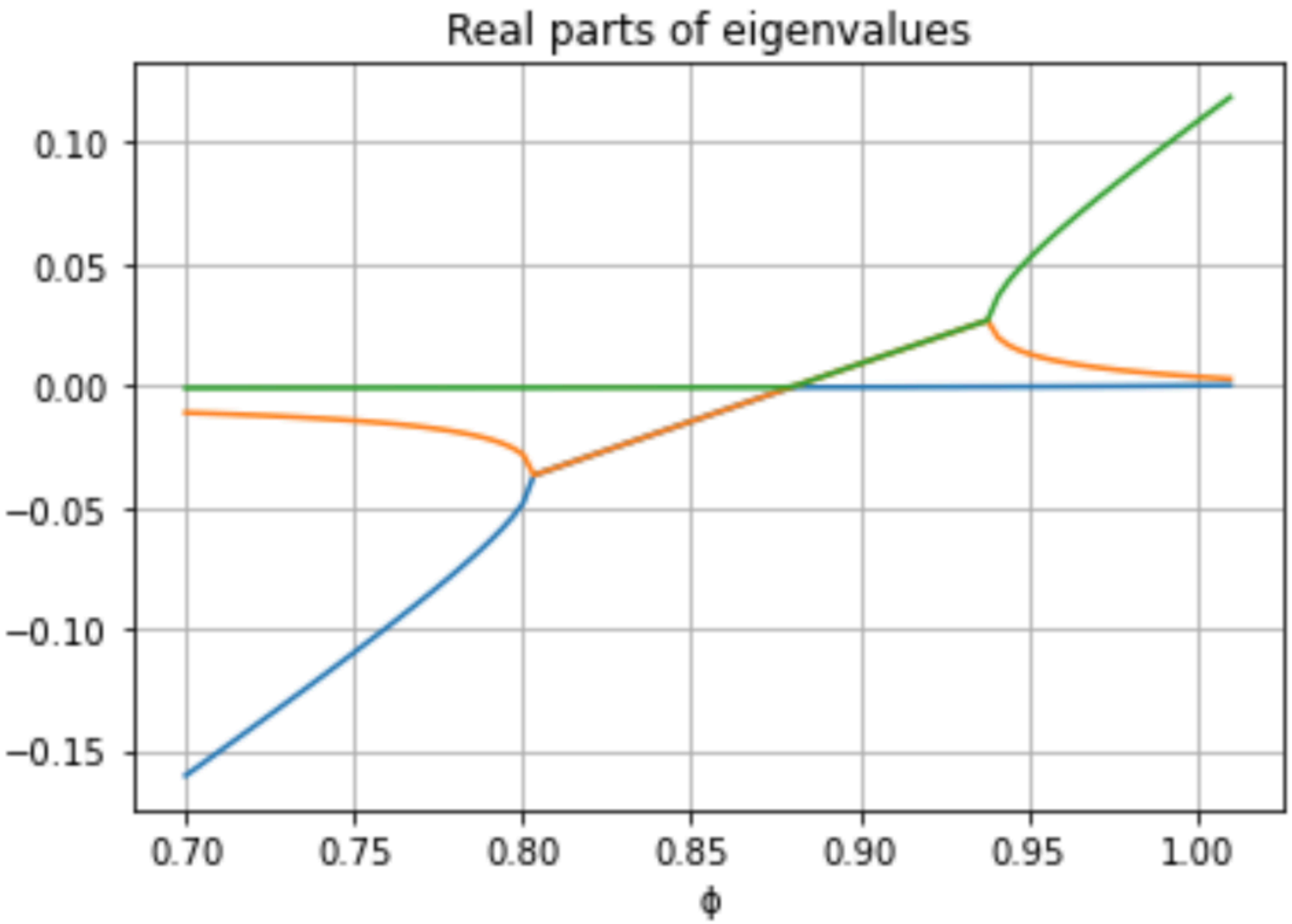

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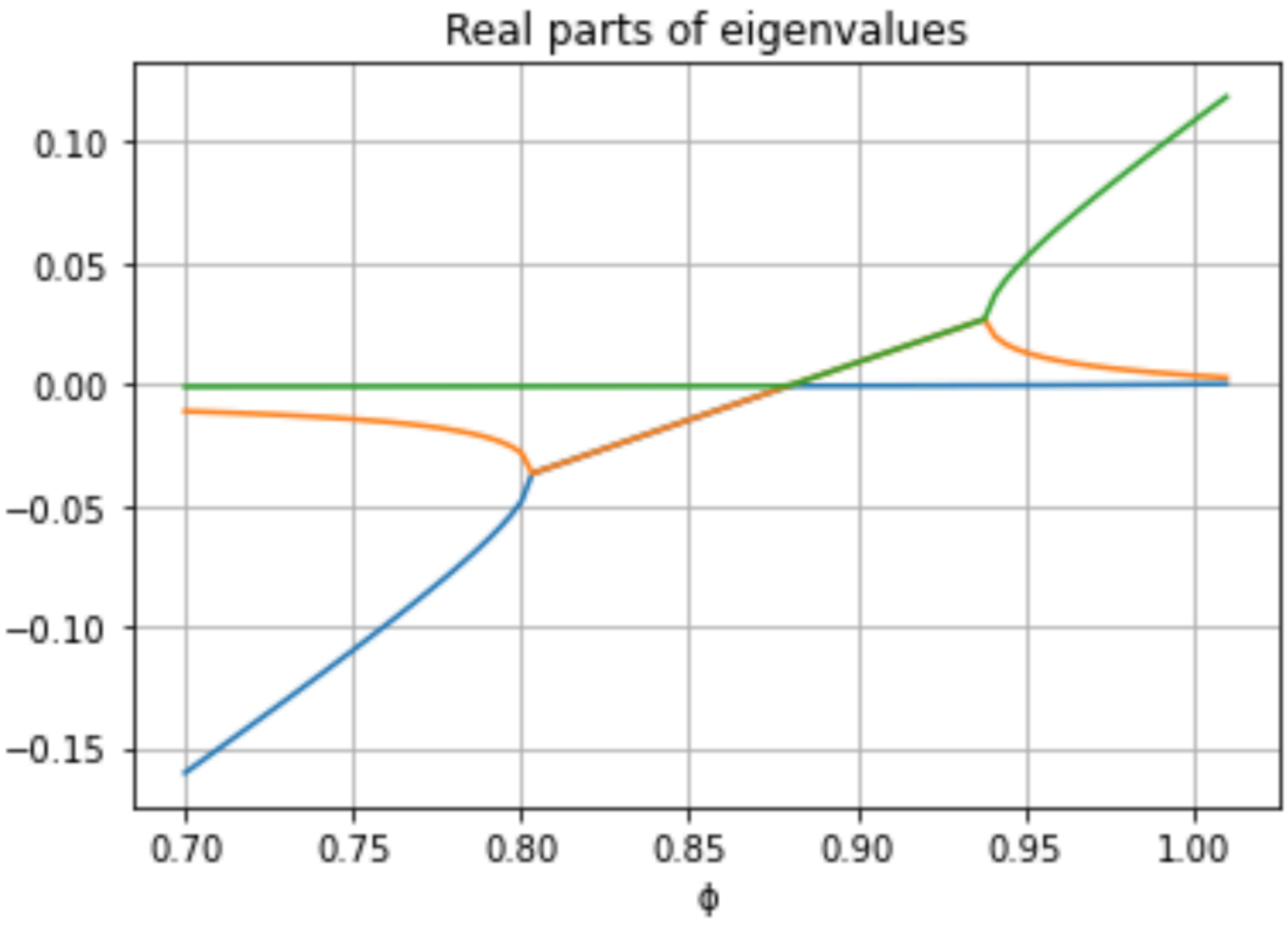

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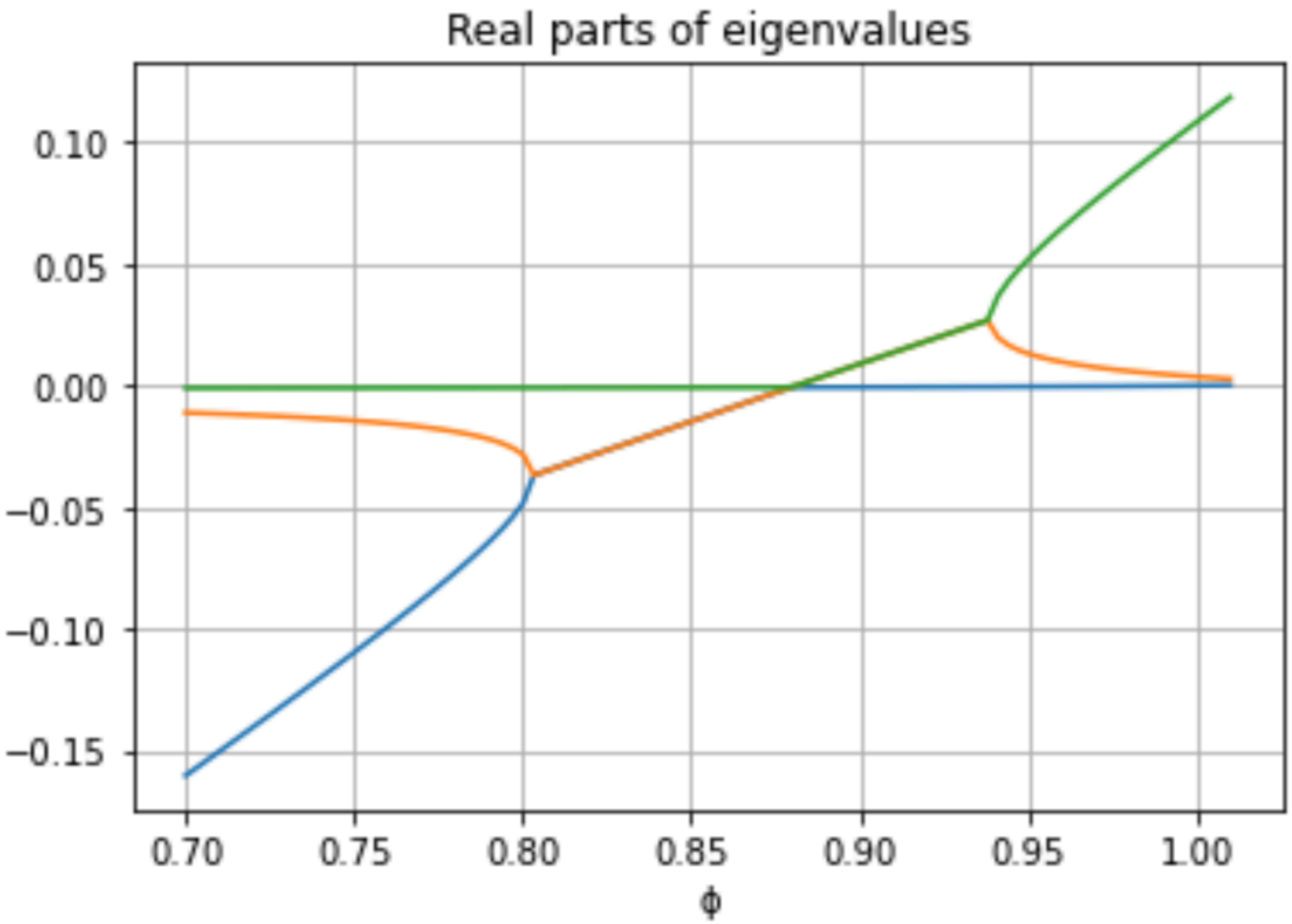

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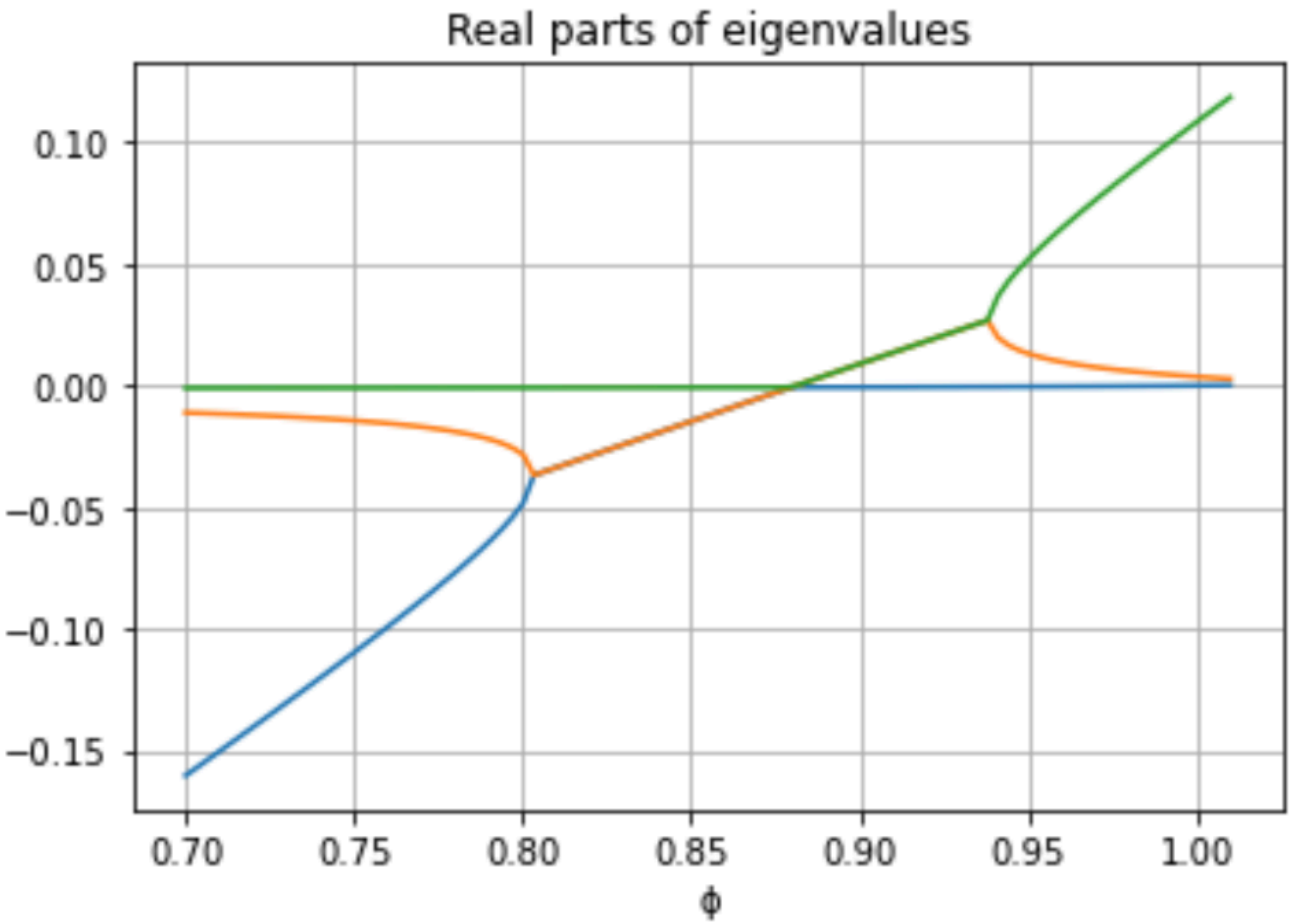

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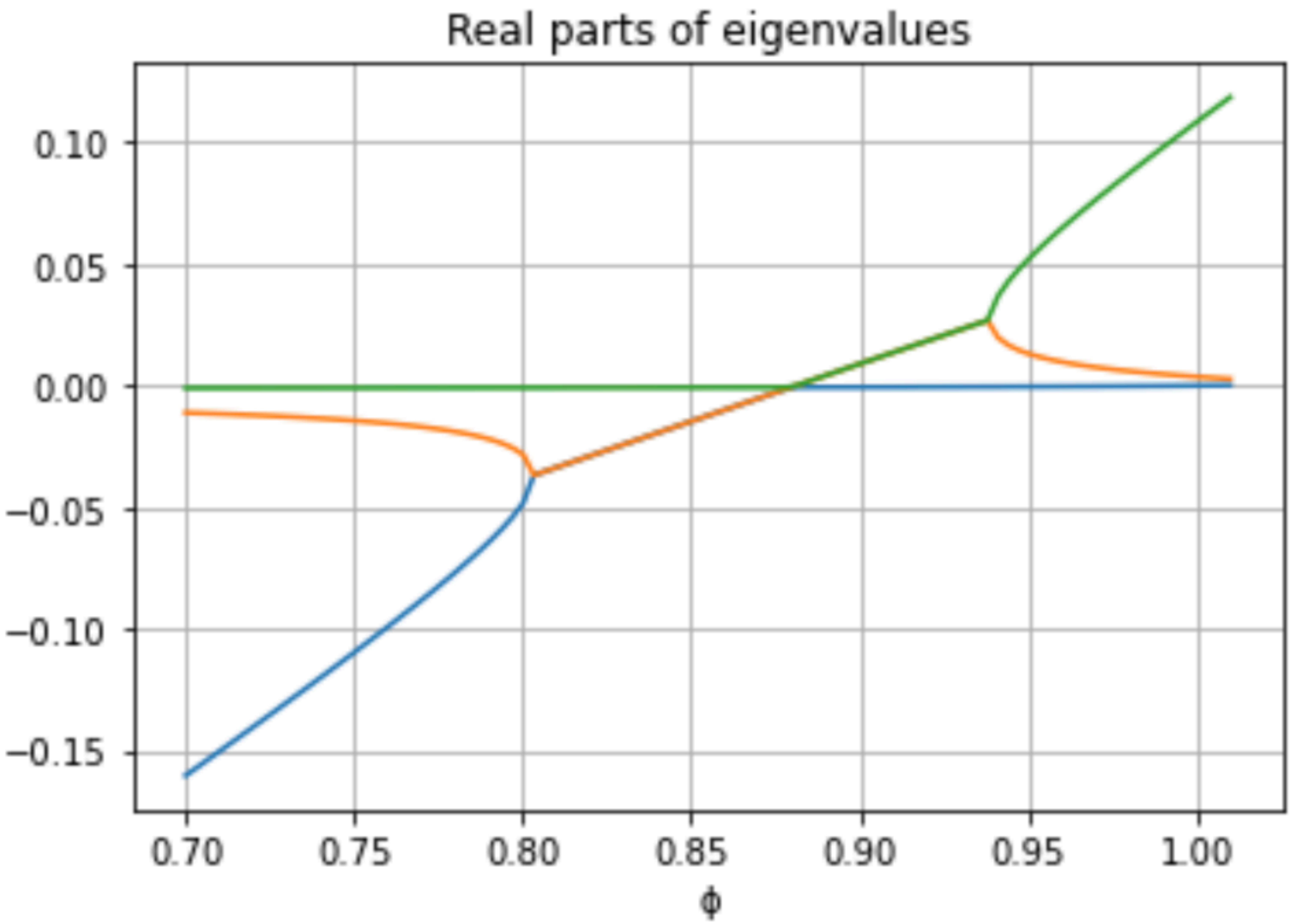

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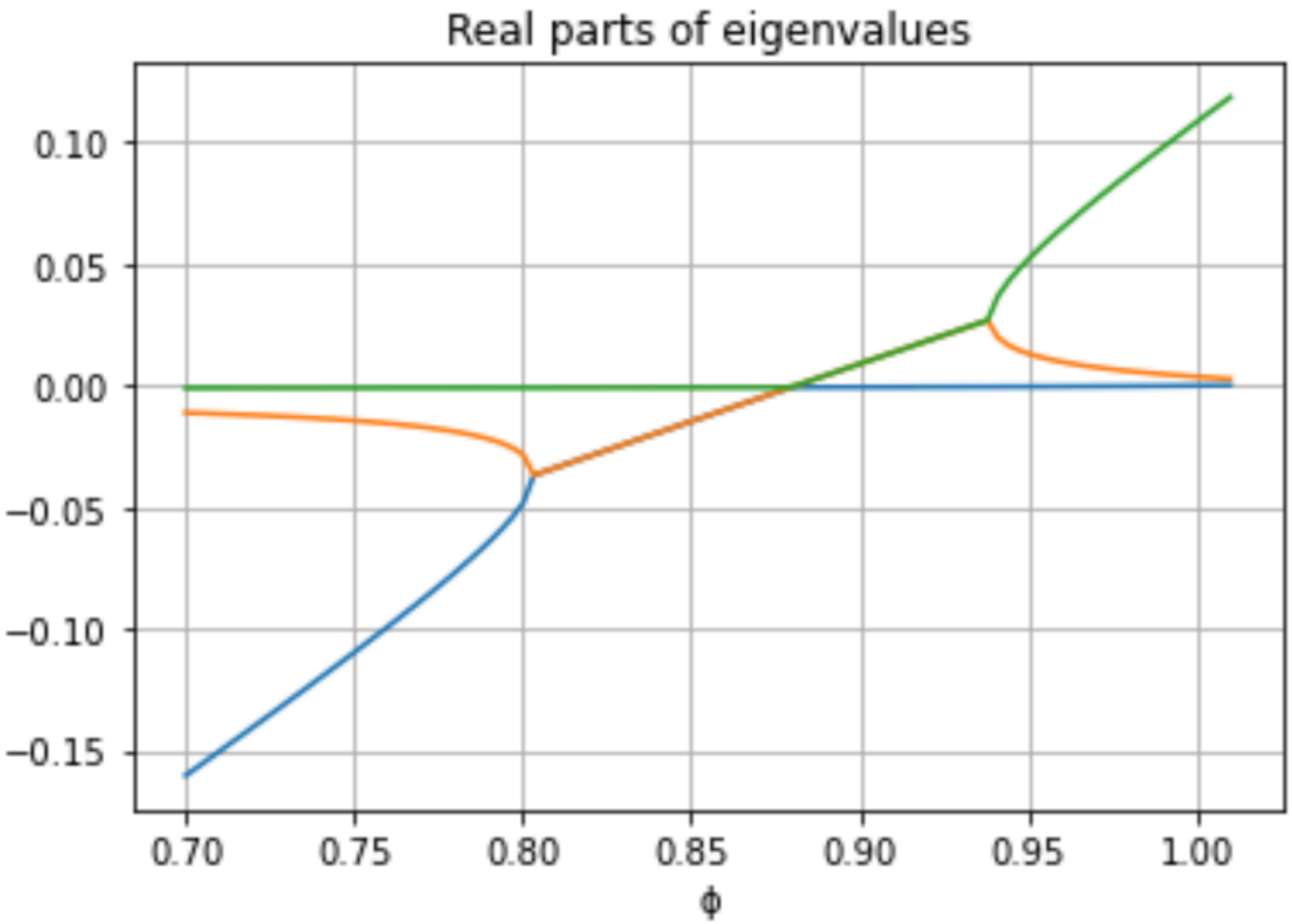

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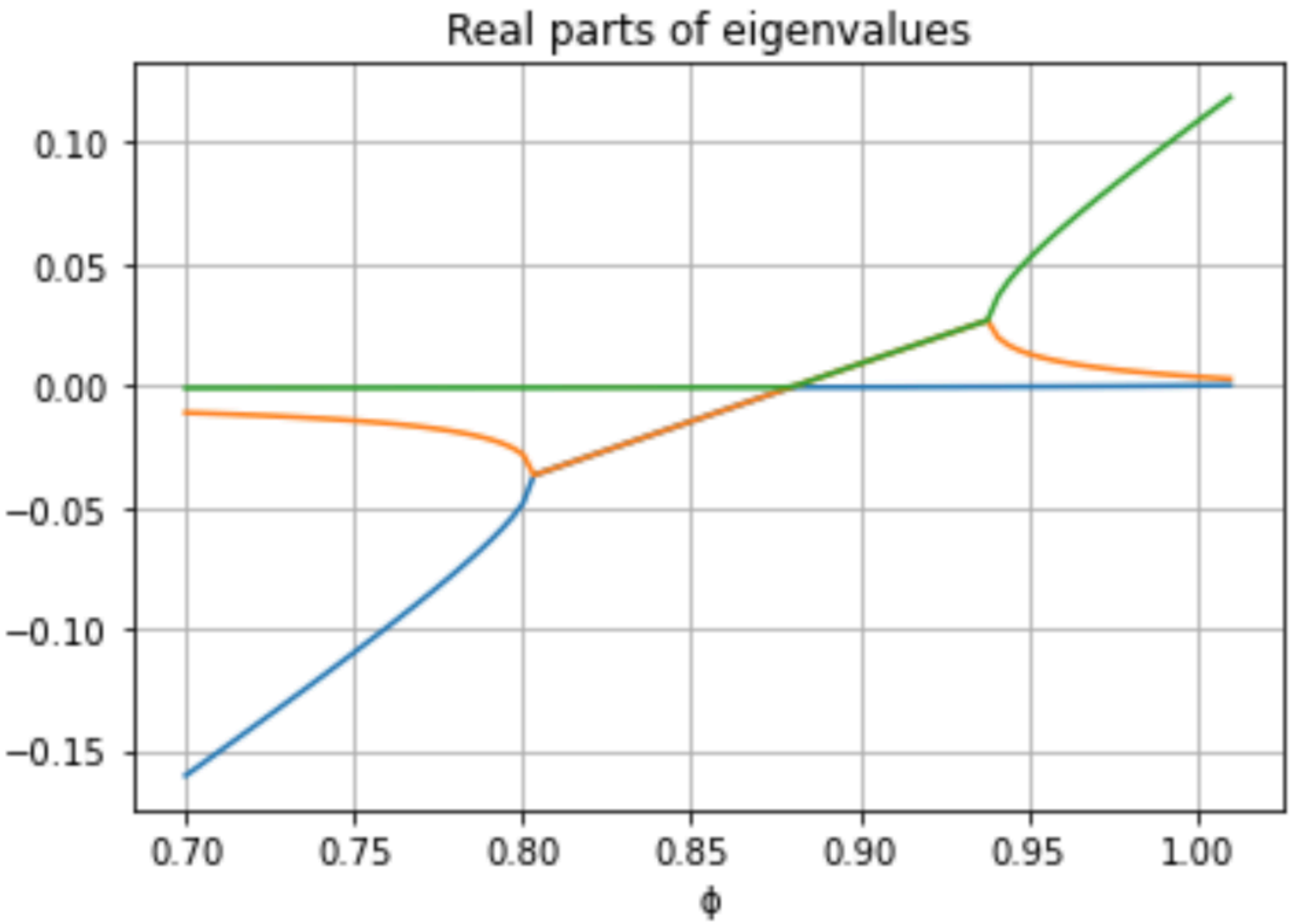
- No ϕ with two negative and one positive eigenvalues! 😢



Analysis

1. Calvo pricing: $\dot{P}_n/P_n = f_n(P_n^*/P_N)$ ✓ $f_n(z) = \frac{\lambda_n}{1-\epsilon_n}(z^{1-\epsilon_n} - 1)$
2. Study flexible $\bar{P}_{nt}$ best response to P_t… ✓

$$\bar{P}_n/P_n = g_n(D_n(P)/S_n(P), P)$$

3. Study dynamics setting $P_{nt}^* = \bar{P}_{nt}$… ✓

$$\dot{P}_n/P_n = h_n(D_n(P)/S_n(P), P)$$

Samuelson's equation!

log-linearized…

$$\dot{p}_n = \alpha_n(d_n(P) - s_n(P))$$

Why set $P_{nt}^* = \bar{P}_{nt}$?…

a. for $\lambda_n/\rho \to 0$ then $P_{nt}^* \to \bar{P}_{nt}$ (reset →flex response) ✓

b. always dominates local dynamics!

4. Main Result: globally stable! Why? ✓

5. Monetary Policy → Determinacy ✓

Conclusions

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  - *equilibrium price setting*
  - *forward-looking, rational expectations*
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- **Main Results…**
  - justify study of Samuelson's ad hoc equation…
  - … but Frisch demands, not Marshallian
  - … as a result: stability can always obtain
  - … monetary policy plays subtle role


THANK
YOU!!