Estimating Demand with Recentered Instruments

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Introduction

Demand for differentiated products is key to many economic analyses

- IO: welfare effects of mergers or new products; conduct testing
- Trade: welfare effects of tariffs; gains from trade or internal migration

Modern demand systems tractably model rich substitution patterns:

- IO: mixed logit (Berry, Levinsohn, Pakes '95), nested logit
- Trade: nested CES (Hottman, Redding, Weinstein '18), mixed CES (Adao, Costinot, Donaldson '17)

But finding credible IVs to estimate these models' parameters can be hard

- We propose a new IV construction under weaker assumptions

Mixed/Nested Logit Overview

Markets m with products $j \in \mathcal{J}_m$ and an outside good $j = 0$

- Researcher observes prices p_{jm} and product characteristics x_{jm}

+ market shares s_{jm} arising from random utility maximization: Details

$$s_{jm} = S_j(\boldsymbol{\delta}_m; \boldsymbol{\sigma}, \boldsymbol{x}_m^{(1)}, \boldsymbol{p}_m) \qquad \text{for } \delta_{jm} = \alpha p_{jm} + \beta' x_{jm} + \xi_{jm}$$

ξ_{jm} captures unobserved product characteristics and consumer tastes, α governs own-price elasticities, $\boldsymbol{\sigma}$ governs substitution patterns

Berry '94, Berry et al. '95 famously show the model can be inverted:

$$\mathcal{D}_j(\boldsymbol{s}_m; \boldsymbol{\sigma}, \boldsymbol{x}_m^{(1)}, \boldsymbol{p}_m) = \delta_{jm}$$

Yields moment conditions given suitable instruments...

Endogeneity and Conventional IVs

E.g. in nested logit:

$$\log(s_{jm}/s_{0m}) - \sigma \log(s_{jm}/\sum_{k\in \text{nest}(j)} s_{km}) = \delta_{jm} = \alpha p_{jm} + \xi_{jm}$$

- Endogeneity #1: p_{jm} is likely correlated with ξ_{jm}
  - Natural IV: exogenous cost shocks g_{jm} (assume available)
- Endogeneity #2: $\boldsymbol{s}_m$ is correlated with ξ_{jm} even if prices are random

Standard IVs for σ: fn's of rivals' characteristics (BLP '95, Gandhi-Houde '20)

- E.g. # of rivals w/ characteristics close to j (nested logit: in j's nest)
- Usually **relevant**: e.g. larger nests have smaller within-nest shares
- Excluded from utility
- But **validity** requires characteristics to be econometrically exogenous
  - Violated e.g. if firms introduce more products in nests for which consumers have a higher preference ξ

Our Instruments

Fn's of cost shocks and characteristics of rival products that are "recentered": by construction uncorrelated with any fn's of characteristics

- E.g. average cost shock of products with characteristics similar to x_{jm} (nested logit: products in j's nest)
- If cost shocks are exogenous, **valid** even with endogenous characteristics, thanks to recentering (Borusyak-Hull 2023)
- **Relevant:** e.g. favorable cost shocks of rivals reduce s_{jm}
  - Choose the fn optimally, a la Chamberlain (BLP '99, Borusyak-Hull '25)
  - Bonus: varies across markets even if the choice set does not (Nevo '01)
- **Intuition**: Think of product substitution as spillover effects
  - Nested logit $\equiv$ peer effects model: $\log \frac{s_{jm}}{s_{0m}} = \alpha p_{jm} + \sigma \log \frac{s_{jm}}{s_{\text{nest}(j)m}} + \xi_{jm}$
  - Peer groups can be endogenous if shocks to peers are exogenous

Related Literatures

Demand estimation without exogenous characteristics:

- *Restricting unobservables:* Sweeting '13, Moon et al. '18
- *Modeling product entry:* Fan '13, Petrin et al. '22
  - We leave unobservables and entry unrestricted
- *Using IVs orthogonal to characteristics:* Ackerberg and Crawford '09
  - We propose a way to construct such IVs by reusing cost shocks

Identifying models with shift-share and other "formula" IVs:

- *Linear models:* Borusyak, Hull, Jaravel '22, '25a,b; Borusyak, Hull '23, '25
- *Linearized models:* Adao, Arkolakis, Esposito '25, Borusyak, Dix-Carneiro, Kovak '23
  - We work with a nonlinear model directly
- *Recentered IVs yield robustness to misspecification:* Andrews et al. '25
  - We show a different advantage and propose a specific IV construction

Outline

1. Introduction ✓

2. Theoretical Results
   - Identification assumption
   - IV construction
   - Special cases
   - Estimation and asymptotics
   - Extensions

3. Monte Carlo Simulations

Exogenous Cost Shocks

Observe supply-side shocks g_{jm}: to input costs, productivity, taxes/subsidies, firm ownership

Assumption 1: $\mathbb{E}[\xi_{jm} \mid \boldsymbol{g}_m, \boldsymbol{x}_m, \boldsymbol{q}_m] = \mathbb{E}[\xi_{jm} \mid \boldsymbol{x}_m, \boldsymbol{q}_m]$
where optional q_m collects other data, e.g. lagged prices and shares

- $\boldsymbol{g}_m$ should not affect product entry or consumer preferences, or correlate with any variables affecting these
- E.g. exchange rate fluctuations when estimating the demand for cars
- Relative to standard assumption $\mathbb{E}[\xi_{jm} \mid \boldsymbol{g}_m, \boldsymbol{x}_m] = 0$, allows observed characteristics to correlate with own & rivals' ξ

Optimal IV (extending Borusyak and Hull '25)

Lemma: $\mathbb{E}[z_{jm}\xi_{jm}] = 0$ follows from Assumption 1 if and only if z_{jm} consists of recentered formula IVs:

$$z_{jm} = f_{jm}(\boldsymbol{g}_m, \boldsymbol{x}_m, \boldsymbol{q}_m) \quad \text{such that} \quad \mathbb{E}[f_{jm}(\boldsymbol{g}_m, \boldsymbol{x}_m, \boldsymbol{q}_m) \mid \boldsymbol{x}_m, \boldsymbol{q}_m] = 0$$

Proposition: For $\nabla_{jm} = \frac{\partial \xi_{jm}}{\partial \theta}$, the asymptotically optimal IV is

$$(z^*_{jm})_{j,m} = \mathbb{E}\left[\xi\xi' \mid \boldsymbol{x}, \boldsymbol{q}\right]^{-1}(\mathbb{E}[\nabla \mid \boldsymbol{g}, \boldsymbol{x}, \boldsymbol{q}] - \mathbb{E}[\nabla \mid \boldsymbol{x}, \boldsymbol{q}])$$

Our proposed IV approximates the numerator of z^*_{jm}:

1) Approximate $\mathbb{E}[\nabla_{jm} \mid \boldsymbol{g}, \boldsymbol{x}, \boldsymbol{q}]$ by $\hat{\nabla}_{jm}(\boldsymbol{g}_m, \boldsymbol{x}_m, \boldsymbol{q}_m)$
2) Recenter it: $z_{jm} = \hat{\nabla}_{jm} - \mu_{jm}$ for $\mu_{jm} = \mathbb{E}\left[\hat{\nabla}_{jm}(\boldsymbol{g}_m, \boldsymbol{x}_m, \boldsymbol{q}_m) \mid \boldsymbol{x}_m, \boldsymbol{q}_m\right]$

$\hookrightarrow$ z_{jm} = unexpected part of ∇_{jm} due to $\boldsymbol{g}_m$

Our IV Construction: For α and β

Recall $\xi_{jm} = \mathscr{D}_j\left(\boldsymbol{s}_m; \boldsymbol{\sigma}, \boldsymbol{x}_m^{(1)}, \boldsymbol{p}_m\right) - \alpha p_{jm} - \beta' x_{jm}$

For α: predicted response of $\frac{\partial}{\partial\alpha}\xi_{jm} = -p_{jm}$ to shocks

- Just use g_{jm}: implicit pass-through model $p_{jm} = \pi g_{jm} + \omega_{jm}$
- Can improve power by adding a supply side

Can't identify β using cost shocks: they don't affect $\frac{\partial}{\partial\beta}\xi_{jm} = -x_{jm}$

- But cross-price elasticities do not involve β (Ackerberg-Crawford '09)
- Same for counterfactuals that hold x_{jm} fixed, even new products
- Can control for x_{jm} for efficiency, like pre-period covariates in an RCT

Our IV Construction: For σ

1) Pick preliminary parameter estimates $\check{\alpha}, \check{\sigma}$ (needn't be consistent)
   - E.g. using characteristic-based IVs
2) Construct a "no-shock scenario" $(\check{\boldsymbol{p}}_m, \check{\boldsymbol{s}}_m)$ as a fn of $(\boldsymbol{x}_m, \boldsymbol{q}_m)$
   - E.g. pre-period prices and shares, if available
3) Construct eq'm changes due to cost shocks as a fn of $(\boldsymbol{g}_m, \boldsymbol{x}_m, \boldsymbol{q}_m)$:
   - Prices: $\hat{p}_{jm} - \check{p}_{jm} = \check{\pi} g_{jm}$ for pass-through $\check{\pi}$
   - Mean utilities: $\hat{\delta}_{jm} - \check{\delta}_{jm} = \check{\alpha}\check{\pi}_{jm} g_{jm}$
   - Shares: $\hat{s}_{jm} - \check{s}_{jm} = S_j(\hat{\boldsymbol{\delta}}_m; \check{\sigma}, \boldsymbol{x}_m^{(1)}, \hat{\boldsymbol{p}}_m) - S_j(\check{\boldsymbol{\delta}}_m; \check{\sigma}, \boldsymbol{x}_m^{(1)}, \check{\boldsymbol{p}}_m)$
   - $\hat{\nabla}_{jm} - \check{\nabla}_{jm} = \frac{\partial}{\partial \sigma}\mathcal{D}_j(\hat{\boldsymbol{s}}_m; \check{\sigma}, \boldsymbol{x}_m^{(1)}, \hat{\boldsymbol{p}}_m) - \frac{\partial}{\partial \sigma}\mathcal{D}_j(\check{\boldsymbol{s}}_m; \check{\sigma}, \boldsymbol{x}_m^{(1)}, \check{\boldsymbol{p}}_m)$
   - Or 1st-order approximation: $\hat{\nabla}_{jm} - \check{\nabla}_{jm} \approx \sum_{k \in \mathcal{J}_m} w_{jk}(\boldsymbol{x}_m, \boldsymbol{q}_m) \cdot g_{km}$
4) Recenter $\hat{\nabla}_{jm}$: set $z_{jm} = \hat{\nabla}_{jm} - \mathbb{E}\left[\hat{\nabla}_{jm} \mid \boldsymbol{x}_m, \boldsymbol{q}_m\right]$. How?

How to Recenter $\hat{\nabla}_{jm}$? (see Borusyak and Hull '23)

Compute $\mu_{jm} = \mathbb{E}\left[\hat{\nabla}_{jm} \mid \boldsymbol{x}_m, \boldsymbol{q}_m\right]$ without nonparametric regression:

- Use formula for $\hat{\nabla}_{jm}$ + view cost shocks as a natural experiment, i.e. a draw from some "serendipitous randomized trial" (DiNardo '04)

Example: leveraging exchange rates of the country of car production

- Assume a random walk for each currency
- Define g_{jm} as exchange rate increments, $\mathbb{E}\left[g_{jm} \mid \boldsymbol{x}_m, \boldsymbol{q}_m\right] = 0$
- Shift-share IVs don't need recentering:

$$\mu_{jm} = \mathbb{E}\left[\sum_{k \in \mathscr{J}_m} w_{jk}(\boldsymbol{x}_m, \boldsymbol{q}_m) \cdot g_{km} \mid \boldsymbol{x}_m, \boldsymbol{q}_m\right] = 0$$

- For nonlinear IVs, draw many counterfactuals $\boldsymbol{g}_m^{(c)}$ as permutations of increments over time for each currency and compute

$$\mu_{jm} = \frac{1}{C} \sum_{c} \hat{\nabla}_{jm}(\boldsymbol{g}_m^{(c)}, \boldsymbol{x}_m, \boldsymbol{q}_m)$$

Special Cases

- In nested logit, $\partial \xi_{jm} / \partial \sigma = \log(s_{jm}/s_{\text{nest}(j)m}) \implies$ Shares IV

$$z_{jm} \propto g_{jm} - \sum_{k \in \text{nest}(j)} \frac{\check{s}_{km}}{\check{s}_{\text{nest}(j)m}} g_{km}$$

  - Increased prices of rivals in the nest raise $\log(s_{jm}/s_{0m}) \implies \sigma > 0$

- In the "local to logit" approximation to mixed logit, i.e. $\check{\sigma} \approx 0$ (Salanie-Wolak '22, here for a non-price random coefficient)

$$z_{jm} \approx x_{jm} \cdot \sum_{k \in \mathcal{J}_m} \check{s}_{km} (x_{km} - \bar{x}_m) g_{km}, \qquad \bar{x}_m = \sum_{k \in \mathcal{J}_m} \check{s}_{km} x_{km}$$

  - Increased prices of rivals with higher-than-average x_{km} raise market shares of products with high $x_{jm} \implies \sigma > 0$

Extensions

- Improve IV power by leveraging an auxiliary supply model Details
- Incorporate observed consumer characteristics
- Use input shocks with estimated mapping from inputs to products
- Identify β using exogenous shocks to non-price characteristics
- Alternative demand models: nested/mixed CES, Hotelling model (Houde 2012), "principles of differentiation" (Bresnahan et al. 1997), etc.
- Nonparametric identification: straightforward without a random coefficient on price Details Conjecture

Outline

1. Introduction ✓

2. Theoretical Results ✓

3. Monte Carlo Simulations

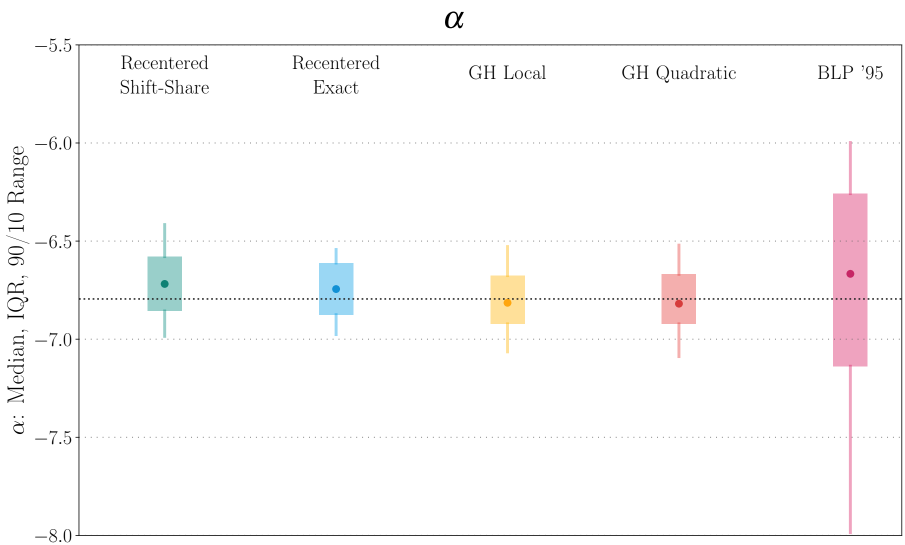
- Power comparison with exogenous characteristics
- Bias comparison with endogenous characteristics

Monte Carlo I: Exogenous Characteristics

DGP based on Gandhi and Houde (2020), 100 simulations: Details

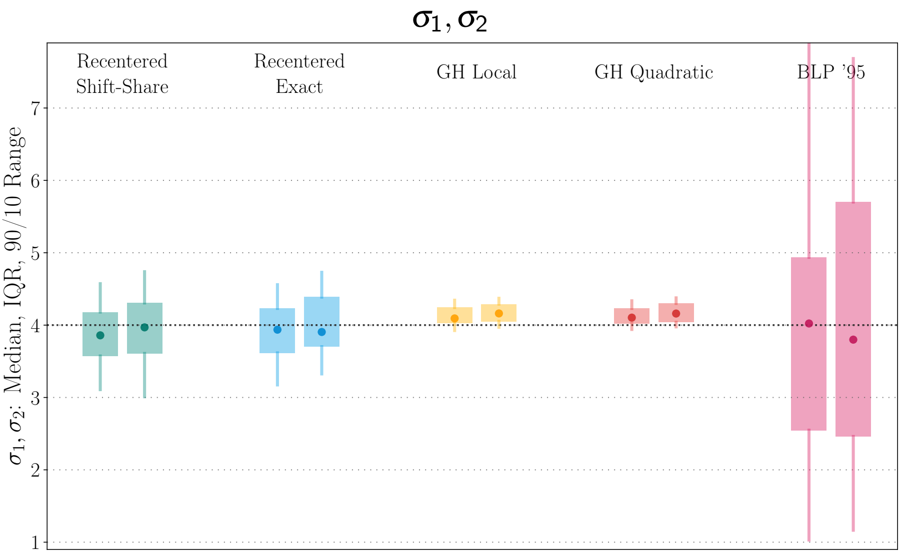
- 100 regional markets r over time periods $t = 1, 2$; $m = (r, t)$
- 15 products per market with 2 observed time-invariant characteristics and random coefficients on them
- Shocks g_{jr2} affect costs in period 2
- Estimate $(\alpha, \sigma_1, \sigma_2)$ with our exact and shift-share IVs
  (continuously updating, in differences)
- Compare with Gandhi-Houde local and quadratic IVs and BLP '95 IV
  (in period 2)

Price Coefficient (Exogenous Characteristics)



Nonlinear Parameters (Exogenous Characteristics)

More



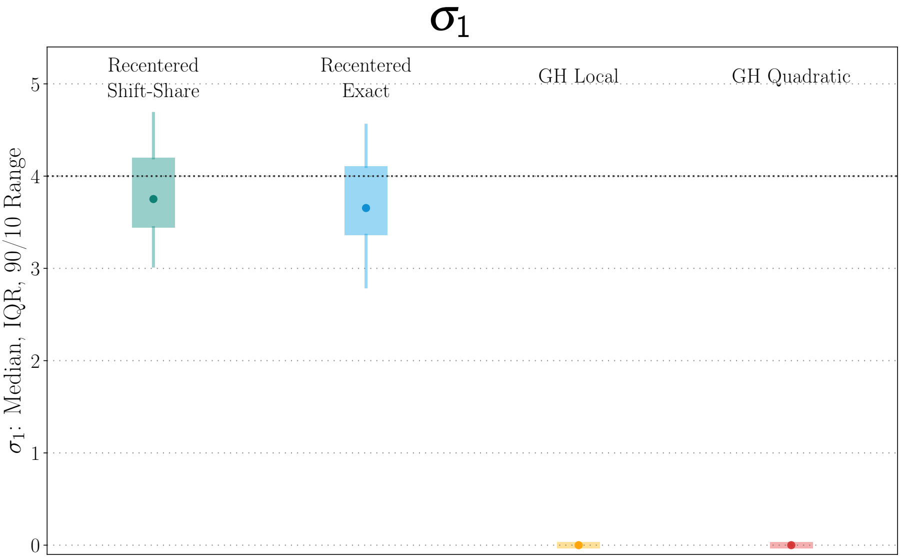
Monte Carlo II: Endogenous Characteristics

Assume each region has a "bliss point" B_r for the first characteristic

- Products near B_r are more popular (subtract $3(x_{jr1} - B_r)^2$ from ξ_{jrt})
- More entry near the bliss point ($x_{jr1} \sim N(B_r, 1)$)

$\Rightarrow$ GH instruments are invalid: popular products are in the dense part of the local distribution of characteristics

Characteristic-Based IVs Have Strong Bias



Conclusion

A new IV construction for nested/mixed logit and similar demand models

- Leverages product characteristics but does not require their exogeneity
- Inspired by thinking of product substitution as spillovers

Open questions:

- Usefulness with microdata (e.g., individual-level choice data)?
  - For MLE or "micro-BLP" GMM estimation of mixed logit
- Usefulness beyond demand, e.g. for estimation of games?
  - Key: IV-GMM estimation

In-progress applications:

- Demand for automobiles, with exchange rate shocks
- Demand for contraceptives, with price cap shocks

Thank You!

Appendix

Random Utility Model ◂ Back

Consumers max:

$$u_{ijm} = \delta_{jm} + \eta_{i0} p_{jm} + \eta_i' x_{jm}^{(1)} + \varepsilon_{ijm}, \quad \delta_{jm} = \alpha p_{jm} + x_{jm}' \beta + \xi_{jm}$$

for observed characteristics $x_{jm} = (x_{jm}^{(1)}, x_{jm}^{(2)})$ and random coefficients $\eta_{i\ell}$

- Assume $(\eta_{i\ell})_\ell \overset{iid}{\sim} \mathscr{P}(\cdot;\sigma)$ for known $\mathscr{P}$ (e.g. Gaussian)
- Nested logit corresponds to $x_{jm}^{(1)}$ = nest dummies and a special distribution of η_i

Estimator and Asymptotics ◂ Back

Given initial $\check{\theta} = (\check{\alpha}, \check{\sigma})$ and $\check{\pi}$, we estimate θ using moment conditions:

$$\mathbb{E}\left[z_{jm}(\check{\theta}, \check{\pi}) \cdot \left(\mathscr{D}_j(\boldsymbol{s}_m; \boldsymbol{\sigma}, \boldsymbol{x}_m^{(1)}, \boldsymbol{p}_m) - \alpha p_{jm} - \mathscr{B}_j(\boldsymbol{x}_m, \boldsymbol{q}_m; \gamma, \check{\theta})\right)\right] = 0$$

where $\mathscr{B}_j$ generalizes non-causal controls, e.g. $\mathscr{B}_j = \xi_{jm}^{\text{pre}}$ (estimation in differences) or $\mathscr{B}_j = \gamma' x_{jm}$, with $\hat{\gamma}(\theta)$ minimizing residual sum of squares

Initial estimates $\check{\theta}$ can come from conventional characteristic-based IVs

- Better: "continuously updating" $z_{jm}(\theta, \check{\pi})$

Consistency, asymptotic normality, and inference:

- When markets are *iid*, follow from standard GMM results
- With non-*iid* markets but if z_{jm} is a shift-share, extend shock-level asymptotics of Adao, Kolesar, Morales (2019), Borusyak, Hull, Jaravel (2022)
    - E.g. if all regions are affected by the same exchange rate shocks

Nested Logit Shares ◂ Back

Let $\delta_{jm} = \alpha p_{jm} + \xi_{jm}$ and $D_{nm} = \sum_{j \in \mathscr{J}_m} d_{jn} \exp(\delta_{jm}/(1-\sigma))$. Then nested logit shares satisfy:

$$\frac{s_{jm}}{s_{n(j)m}} = \frac{\exp(\delta_{jm}/(1-\sigma))}{D_{n(j)m}},$$
$$s_{nm} = \frac{D_{nm}^{1-\sigma}}{1+\sum_{n'} D_{nm}^{1-\sigma}},$$
$$s_{0m} = \frac{1}{1+\sum_{n'} D_{nm}^{1-\sigma}}$$

Manipulating these terms yields share inversion:

$$\log(s_{jm}/s_{0m}) = \alpha p_{jm} + \sigma \log\left(s_{jm}/s_{n(j)m}\right) + \xi_{jm}$$

IV Construction for Nested Logit ◂ Back

Exact prediction: For $\hat{p}_{jm} = \check{\pi}_0 + \check{\pi} g_{jm}$ and $\delta_{jm} = \check{\alpha}\hat{p}_{jm}$:

$$\widehat{\log}\frac{s_{jm}}{s_{n(j)m}} = \frac{\check{\alpha}}{1-\check{\sigma}}(\check{\pi}_0 + \check{\pi} g_{jm}) - \log \sum_{k \in \mathscr{J}_m} d_{kn} \exp\left(\frac{\check{\alpha}}{1-\check{\sigma}}(\check{\pi}_0 + \check{\pi} g_{jm})\right)$$

$$= \frac{\check{\alpha}}{1-\check{\sigma}}\check{\pi} g_{jm} - \log \sum_{k \in \mathscr{J}_m} d_{kn} \exp\left(\frac{\check{\alpha}}{1-\check{\sigma}}\check{\pi} g_{jm}\right).$$

First-order approximation around $g_{km} = \mu_g$:

$$\widehat{\log}\frac{s_{jm}}{s_{n(j)m}} \approx \frac{\check{\alpha}}{1-\check{\sigma}}\check{\pi}\mu_g - \log \sum_{k \in \mathscr{J}_m} d_{kn} \exp\left(\frac{\check{\alpha}}{1-\check{\sigma}}\check{\pi}\mu_g\right)$$

$$+ \frac{\check{\alpha}}{1-\check{\sigma}}\check{\pi}(g_{jm} - \mu_g) - \frac{\sum_{k \in \mathscr{J}_m} d_{kn} \exp\left(\frac{\check{\alpha}}{1-\check{\sigma}}\check{\pi}\mu_g\right) \frac{\check{\alpha}}{1-\check{\sigma}}\check{\pi}(g_{jm} - \mu_g)}{\sum_{k \in \mathscr{J}_m} d_{kn} \exp\left(\frac{\check{\alpha}}{1-\check{\sigma}}\check{\pi}\mu_g\right)}$$

$$= -\log N_{n(j)m} + \frac{\check{\alpha}}{1-\check{\sigma}}\check{\pi}\left(g_{jm} - \frac{1}{N_{n(j)m}} \sum_{k \in \mathscr{J}_m} d_{kn} g_{jm}\right).$$

Incorporating a Supply Model ◂ Back

Assume constant MC c_{jm} + Nash-Bertrand:

$$\boldsymbol{p}_m^* = \boldsymbol{c}_m - \left(\boldsymbol{H}_m \odot \frac{d}{d\boldsymbol{p}_m'} \boldsymbol{s}_m(\boldsymbol{p}_m^*) \right)^{-1} \boldsymbol{s}_m(\boldsymbol{p}_m^*)$$

1. Given $(\check{\alpha}, \check{\sigma})$ solve for c_{jm}. Regress $c_{jm} = \hat{\pi}\tilde{g}_{jm} + \text{error}_{jm}$
2. Solve for costs $\check{c}_{jm}$ corresponding to $(\check{\boldsymbol{p}}_m, \check{\boldsymbol{s}}_m)$
3. Predict costs $\hat{c}_{jm} = \check{c}_{jm} + \hat{\pi}\tilde{g}_{jm}$
4. Predict prices $\hat{\boldsymbol{p}}_m = \boldsymbol{p}_m^*(\hat{\boldsymbol{c}}_m)$ (or via 1st-order approximation in g_{km})
5. Use $\hat{\boldsymbol{p}}$ to construct recentered IVs

Nonparametric Identification ◂ Back

Demand model w/index restriction $p_{jm} + \xi_{jm} = \mathscr{D}_j(\boldsymbol{s_m}, \boldsymbol{x_m})$ for *unknown* $\mathscr{D}_j$ (generalizes mixed logit with no random coefficient on price)

- Assume (i) shock exogeneity: $\mathbb{E}[\xi_{jm} \mid \boldsymbol{g_m}, \boldsymbol{x_m}] = \mathbb{E}[\xi_{jm} \mid \boldsymbol{x_m}]$, (ii) completeness: $\mathbb{E}[h(\boldsymbol{s_m}, \boldsymbol{x_m}) \mid \boldsymbol{g_m}, \boldsymbol{x_m}] \overset{a.s.}{=} 0 \implies h(\boldsymbol{s_m}, \boldsymbol{x_m}) \overset{a.s.}{=} 0$
- Then $\mathscr{D}_j$ and ξ_{jm} are identified up to an additive function $\beta(\boldsymbol{x_m})$; enough for counterfactuals that hold x_{jm} fixed

Nonparametric Identification: A Conjecture ◂ Back

- Nonparametric demand with an index restriction on a non-price characteristic: $x_{jm} + \xi_{jm} = \mathscr{D}_j(\boldsymbol{p}_m, \boldsymbol{s}_m)$ (fixing other characteristics)
- Berry and Haile (2014): if $\mathbb{E}[x_{jm} + \xi_{jm} \mid \boldsymbol{x}_m, \boldsymbol{g}_m] = x_{jm}$, NPIV identifies $\mathscr{D}_j(\cdot)$ under completeness of $(\boldsymbol{p}_m, \boldsymbol{s}_m) \mid (\boldsymbol{g}_m, \boldsymbol{x}_m)$
  - Conclusion: not enough to have exogenous $\boldsymbol{g}_m$; also need exogenous $\boldsymbol{x}_m$
  - But interactions of endogenous $\boldsymbol{x}_m$ & cost shocks are valid IVs!
- Conjecture (Borusyak, Chen, Hull in progress): If $\boldsymbol{g}_m \perp\!\!\!\perp \boldsymbol{\xi}_m \mid \boldsymbol{x}_m$, then $\mathscr{D}(\cdot)$ is identified up to invertible transformations $\boldsymbol{r}(\cdot) \Longrightarrow$ "conditional demand" is point-identified
  - So far proved assuming $\boldsymbol{p}_m \perp\!\!\!\perp \boldsymbol{x}_m \mid (\boldsymbol{\delta}_m, \boldsymbol{g}_m)$ (index restriction on prices, includes the case of exogenous prices)
  - In general, $\mathbb{E}[\mathscr{D}(\boldsymbol{p}_m, \boldsymbol{s}_m) \mid \boldsymbol{g}_m, \boldsymbol{x}_m] = \boldsymbol{h}(\boldsymbol{x}_m)$ for some $\boldsymbol{h}(\cdot)$
  - Given $\boldsymbol{h}(\cdot)$, completeness implies at most one $\boldsymbol{f}(\cdot)$ via NPIV
  - This equation is satisified by $\mathscr{D}(\cdot)$ but also $\boldsymbol{r}(\mathscr{D}(\cdot))$ for any $\boldsymbol{r}(\cdot)$
  - For any $\boldsymbol{h}(\cdot)$ such that $\exists \boldsymbol{r} : \mathbb{E}[\boldsymbol{r}(\boldsymbol{x}_m + \boldsymbol{\xi}_m) \mid \boldsymbol{x}_m] = \boldsymbol{h}(\boldsymbol{x}_m)$ a.s., there is no other solution. *But we don't know about other $\boldsymbol{h}(\cdot)$ functions yet*

Monte Carlo I: Details

Back

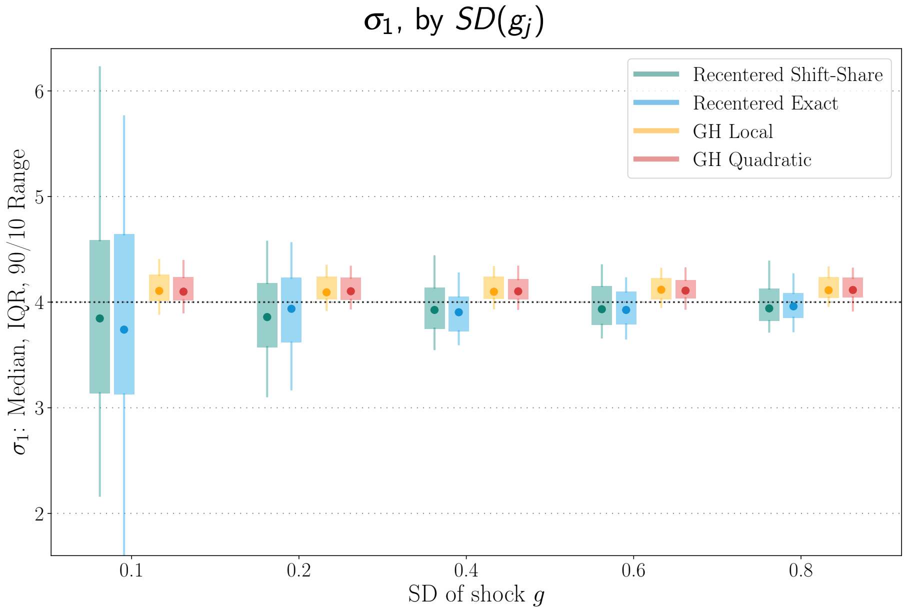
DGP:

- $x_{jr\ell} \sim N(0,1)$, $\xi_{jr1} \sim N(0,1)$, $\xi_{jr2} = 0.5\xi_{jr1} + \sqrt{1-0.5^2} \cdot N(0,1)$
- Costs: $c_{jrt} = \gamma' x_{jrt} + g_{jrt} + \omega_{jrt}$, $\omega_{jr1} \sim N(0,1)$, $g_{jr1} = 0$, $\omega_{jr2} = 0.9c_{jr1} + \sqrt{1-0.9^2} \cdot N(0,1)$, $g_{jr2} \sim N(0,0.04)$
- Prices set by simultaneous Nash-Bertrand

Estimate (α, σ) via alternative moment conditions:

- $\mathbb{E}[\Delta\xi_{jr} \cdot (g_{jr}, z_{jr})] = 0$ where z_{jm} is continuously-updating recentered IV from first-order approximation or exact prediction
- $\mathbb{E}\left[\xi_{jr2} \cdot (g_{jr}, x_{jr}, z^C_{jr})\right] = 0$ where z^C_{jr} is BLP or Differentiation IVs

Recentered IVs Benefit from Cost Shock Variation



GH IVs Benefit from Characteristic Variation

◂ Back