Global Networks, Monetary Policy and Trade

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Tariffs and Macroeconomy

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- Unbalanced trade and incomplete markets,
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Analyze macro impact of tariffs — both implemented and threatened — in a world of integrated trade and production.

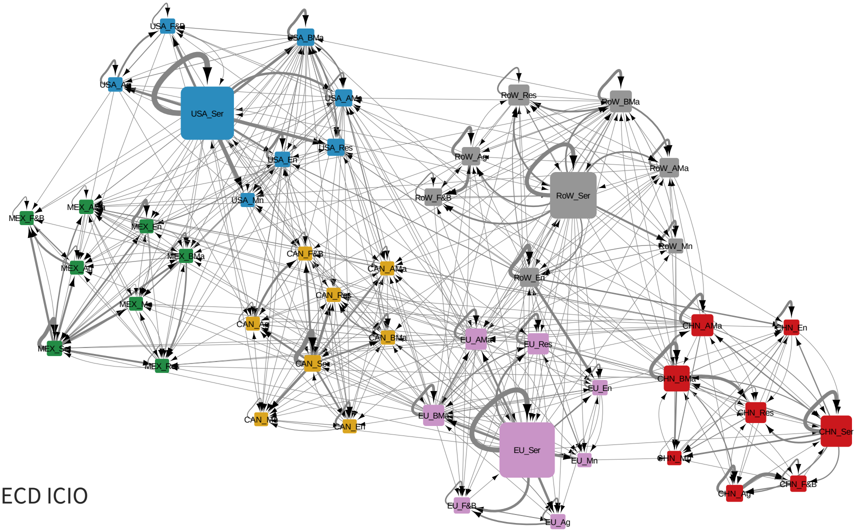
We connect to three large bodies of literatures

- Trade:
  - **Tariffs—a central question since Hume (1758) and Ricardo (1817).**
  - di Giovanni and Levchenko (2010), Johnson and Noguera (2012), Chaney (2014), Dix-Carneiro (2014), Johnson (2014), Koopman et al. (2014), Caliendo and Parro (2015), Adao et al. (2017), Rodríguez-Clare et al. (2020), Dhyne et al. (2021), ...
  - Recent Reviews: Costinot and Rodríguez-Clare (2014), Bernard and Moxnes (2018), Johnson (2018), Antrás and Chor (2022).

- Open Econ Macro:
  - Erceg et al. (2018), Barattieri et al. (2021), Monacelli (2025), Bergin and Corsetti (2023), Bianchi and Coulibaly (2025), Cuba-Borda et al. (2025), Ho et al. (2022), Itskhoki and Mukhin (2025), Auclert et al. (2025), Werning et al. (2025), Qiu et al. (2025).

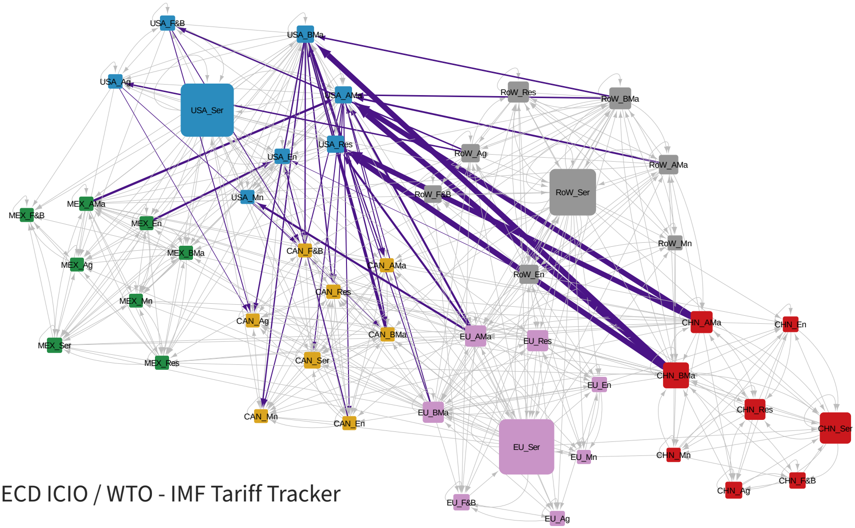
- Production Networks:
  - Rubbo (2023), Baqaee and Farhi (2024), Baqaee and Farhi (2022), Long and Plosser (1983), Foerster et al. (2011), Acemoglu et al. (2012), Atalay (2017), di Giovanni et al. (2023), Silva (2024), Liu (2019), Pasten et al. (2020, 2024), Bigio and La'o (2020), La'O and Tahbaz-Salehi (2022), Foerster et al. (2022), Vom Lehn and Winberry (2022), Huo et al. (2025).

Can global trade and supply chains be re-wired?



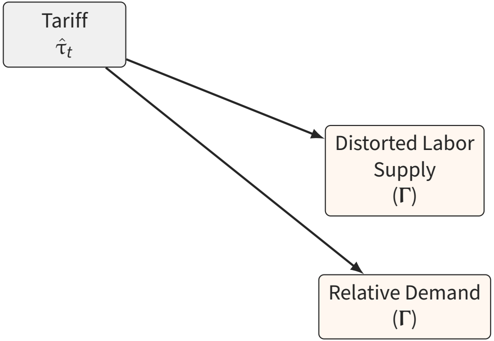
Source: OECD ICIO

2025 Tariffs



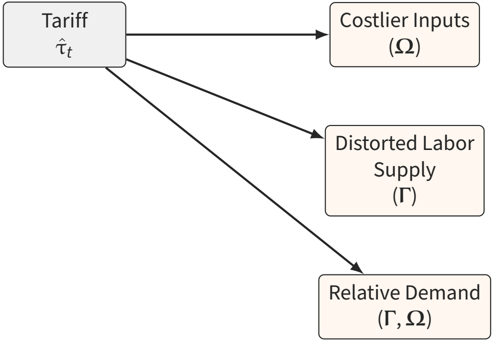
Source: OECD ICIO / WTO - IMF Tariff Tracker

Primitives in Nested CES



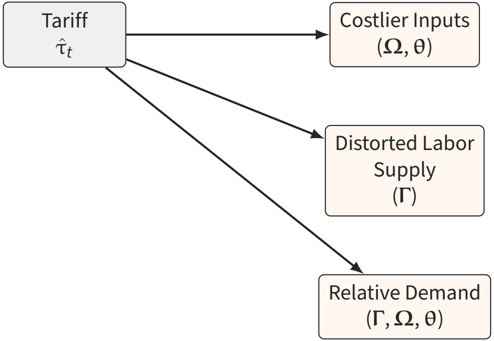
Consumption shares Γ: ToT gains vs. distorted labor supply

Primitives in Nested CES



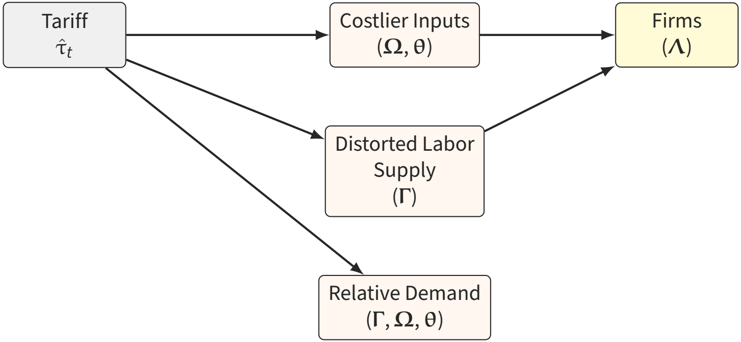
I-O matrix $\mathbf{\Omega}$: ToT gains vs. higher marginal cost propagated by network

Primitives in Nested CES



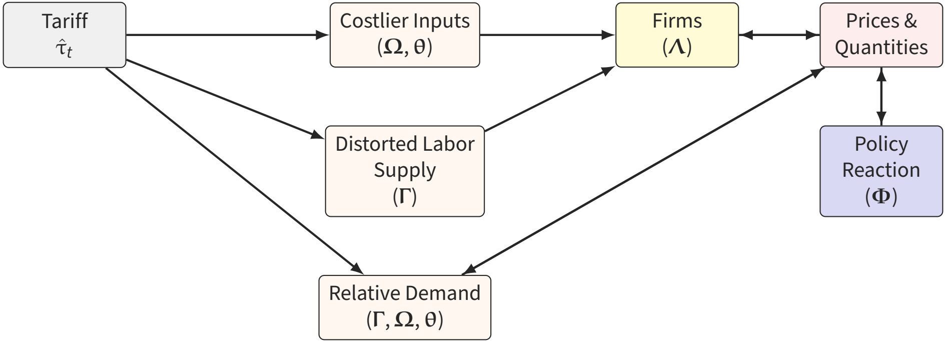
EoS θ: high $\theta \Rightarrow$ easy to substitute $\Rightarrow \hat{Y}_t \uparrow$ or low $\theta \Rightarrow$ complements & bottlenecks, $\rightarrow \hat{Y}_t \downarrow$

Primitives in Nested CES



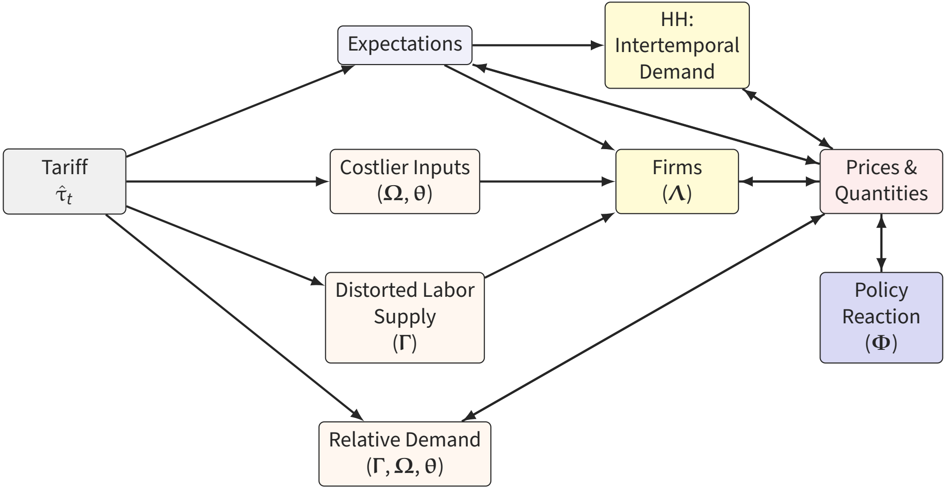
Stickiness $\mathbf{\Lambda}$: $\Lambda\downarrow$ flattens NKPC, smaller π_t^C.

Primitives in Nested CES



Policy $\boldsymbol{\Phi}$: response to $\pi_t^C \Rightarrow$ tariffs contract demand

Primitives in Nested CES



Tariffs lead to intertemporal tradeoffs $\Leftrightarrow$ expectations

5-Equation Global New Keynesian Representation

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NKIS+TR: $$\sigma(\mathbb{E}_t \hat{\boldsymbol{c}}_{t+1} - \hat{\boldsymbol{c}}_t) = \underbrace{\Phi \boldsymbol{\pi}_t^C}_{\hat{\boldsymbol{i}}_t} - \mathbb{E}_t \boldsymbol{\pi}_{t+1}^C$$

5-Equation Global New Keynesian Representation

Expectations

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5-Equation Global New Keynesian Representation

Policy **Expectations**

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UIP+TR: $\mathbb{E}_t \hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t = \underbrace{\tilde{\mathbf{\Phi}}\boldsymbol{\pi}_t^C}_{\hat{i}_t - \hat{i}_t^*}$

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UIP+TR: $\mathbb{E}_t\hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t = \tilde{\boldsymbol{\Phi}}\boldsymbol{\pi}_t^C$

CPI: $\underbrace{\hat{\boldsymbol{P}}_t^C}_{\text{Consumer Prices}} = \underbrace{\boldsymbol{\Gamma}}_{\text{Consumption Weights}} \underbrace{\hat{\boldsymbol{P}}_t^P}_{\text{Producer Prices}} + \boldsymbol{L}_{\mathcal{E}}^C \underbrace{\hat{\mathcal{E}}_t}_{\text{ER}} + \boldsymbol{L}_{\tau}^C \underbrace{\hat{\tau}_t}_{\text{Tariff}}$

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NKPC: $\underbrace{\hat{\boldsymbol{P}}_t^P}_{\text{Producer Prices}} = \underbrace{\Psi_\Lambda}_{\text{Leontief Inverse}} \left[\hat{\boldsymbol{P}}_{t-1}^P + \underbrace{\Lambda}_{\text{Stickiness}} \underbrace{\left(\alpha \underbrace{(\hat{\boldsymbol{P}}_t^C + \sigma \hat{\boldsymbol{C}}_t)}_{\boldsymbol{W}_t} + \boldsymbol{L}_{\mathcal{E}}^P \hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^P \hat{\tau}_t \right)}_{\text{Transformed MC}} + \beta \mathbb{E}_t \hat{\boldsymbol{P}}_{t+1}^P \right]$

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NKIS+TR: $\sigma(\mathbb{E}_t \hat{\boldsymbol{C}}_{t+1} - \hat{\boldsymbol{C}}_t) = \Phi \pi^C_t - \mathbb{E}_t \pi^C_{t+1}$

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Labor Supply Distortion

CPI: $\hat{\boldsymbol{P}}^C_t = \Gamma \hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^C_{\mathcal{E}} \hat{\mathcal{E}}_t + \boldsymbol{L}^C_{\tau} \hat{\tau}_t$

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NKIS+TR: $\sigma(\mathbb{E}_t\hat{C}_{t+1} - \hat{C}_t) = \Phi\pi_t^C - \mathbb{E}_t\pi_{t+1}^C$

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CPI: $\hat{P}_t^C = \Gamma\,\hat{P}_t^P + L_{\mathcal{E}}^C\hat{\mathcal{E}}_t + L_\tau^C\hat{\tau}_t$

Labor Supply Distortion

Costlier Inputs

NKPC: $$\underbrace{\hat{P}_t^P}_{\text{Producer Prices}} = \underbrace{\Psi_\Lambda}_{\text{Leontief Inverse}}\left[\hat{P}_{t-1}^P + \underbrace{\Lambda}_{\text{Stickiness}}\underbrace{\left(\alpha\,\underbrace{(\hat{P}_t^C + \sigma\hat{C}_t)}_{W_t} + L_{\mathcal{E}}^P\hat{\mathcal{E}}_t + L_\tau^P\hat{\tau}_t\right)}_{\text{MC without } P_t^P} + \beta\mathbb{E}_t\hat{P}_{t+1}^P\right]$$

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BoP: $\beta \underbrace{\hat{V}_t}_{\text{Debt}} = \underbrace{\hat{V}_{t-1}}_{\text{Debt Dynamics}} + \underbrace{\Xi_2 \hat{\boldsymbol{C}}_t + \Xi_3 \hat{\boldsymbol{P}}^P_t + \Xi_4 \hat{\mathcal{E}}_t + \Xi_5 \hat{\tau}_t}_{\text{NX Response to AD \& ToT}}$

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Relative Demand

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+ inflation definition

Shock Propagation: The Anatomy of Leontief Inverse

- Under flexible prices, the Standard Leontief Inverse:

$$\boldsymbol{\Psi} = [\boldsymbol{I} - \boldsymbol{\Omega}]^{-1}.$$

- Under fixed nominal demand:

$$\boldsymbol{\Psi}_{\Lambda} = \Big[\boldsymbol{I}(1 + \underbrace{\beta}_{\text{Discount F.}}) + \underbrace{\boldsymbol{\Lambda}}_{\text{Stickiness}} (\boldsymbol{I} - \boldsymbol{\Omega})\Big]^{-1}.$$

- Under a Taylor rule:

$$\boldsymbol{\Psi}_{\phi} = \Big[\boldsymbol{I}(1 + \beta) + \boldsymbol{\Lambda}\big[\boldsymbol{I} - \boldsymbol{\Omega} + \underbrace{\boldsymbol{\alpha}}_{\substack{\text{Labor}\\ \text{Shares}}} (\underbrace{\boldsymbol{\Phi}}_{\substack{\text{Central Bank}\\ \text{Sensitivity}}} - \boldsymbol{I}) \underbrace{\boldsymbol{\Gamma}}_{\substack{\text{Consumption}\\ \text{Shares}}}\big]\Big]^{-1}.$$

- Solving DGE yields: $\boldsymbol{\Psi}_{\phi} \Rightarrow \boldsymbol{\Psi}_{\phi}^{NKOE}$

Analytical Solution: Decomposing the Impact on Inflation

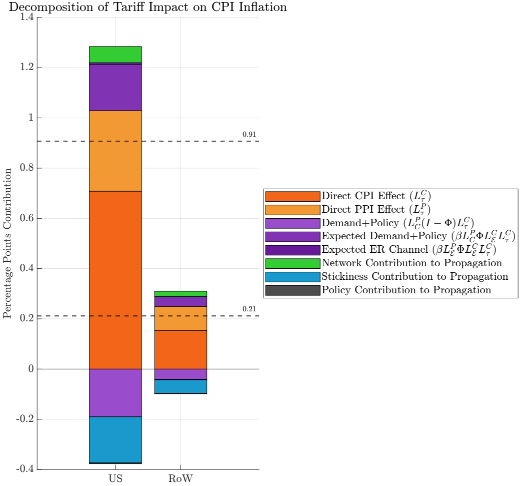
U.S. and RoW, 10% reciprocal tariffs.

Based on analytical solution, the impact of a one-time tariff on CPI inflation is:

$$\frac{\partial \pi_t^C}{\partial \tau_t} = \underbrace{L_\tau^C}_{\text{Direct CPI Effect}} + \underbrace{\Gamma L_\tau^P}_{\text{Direct PPI Effect}}$$

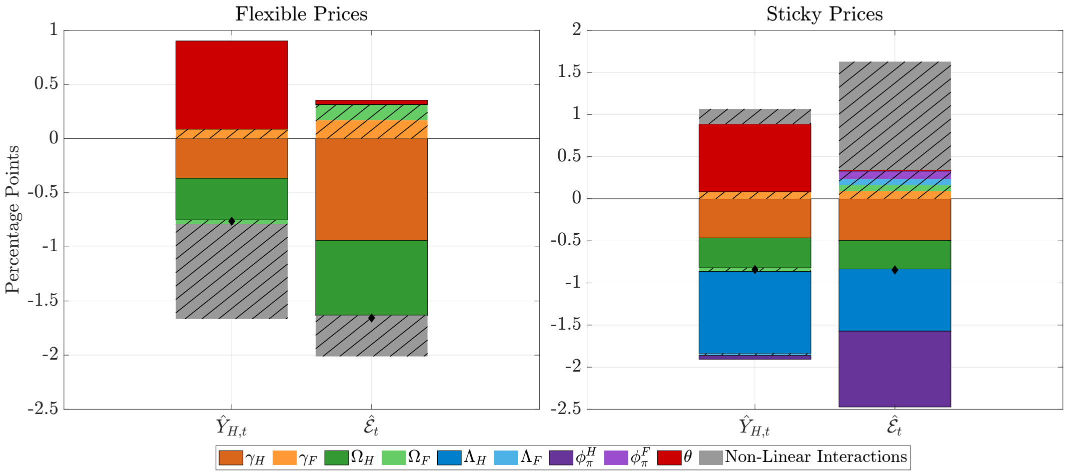
$$+ \underbrace{\Gamma\,\alpha\,(I-\Phi)\,L_\tau^C}_{\text{Demand+Policy}} + \underbrace{\beta\,\Gamma\,\alpha\,\Phi\,\tilde{L}_E^C\,L_\tau^C}_{\text{Expected Demand+Policy}}$$

$$+ \underbrace{\beta\,\Gamma\,L_E^P\,\Phi\,\tilde{L}_E^C\,L_\tau^C}_{\text{Expected ER Channel}} + \underbrace{\Gamma\left(\Psi_\phi^{NKOE}\,\Lambda - I\right)Z}_{\text{Propagation via Network \& Stickiness}}$$



Why global GE networks over SOE?

Flexible Prices vs. Sticky Prices



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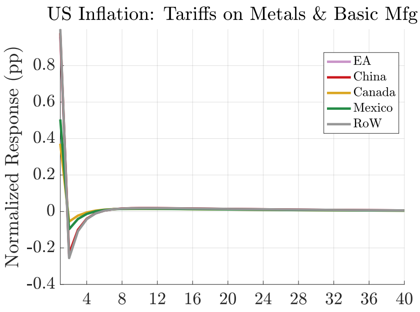

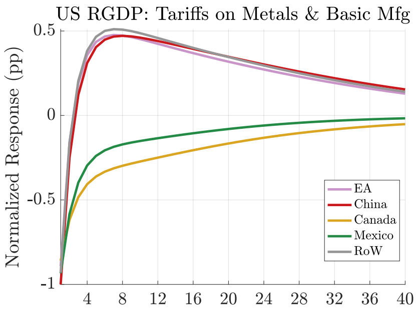

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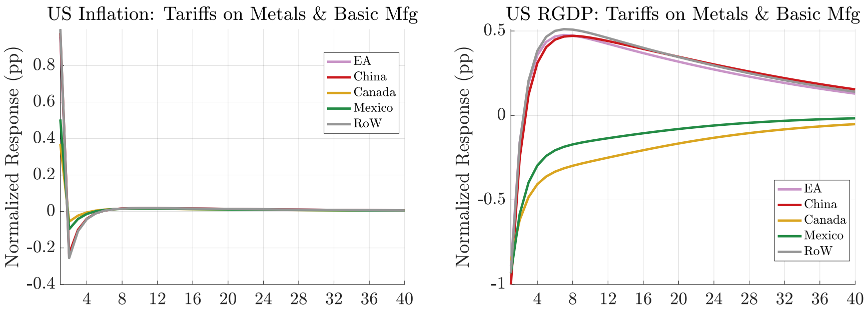
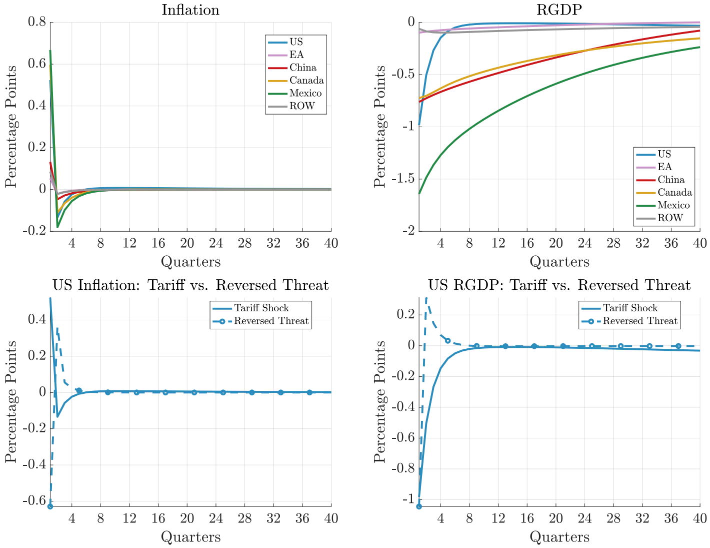


- International risk sharing mutes network impact ($\psi \uparrow$: financial autarky)

$$\frac{\partial \hat{\boldsymbol{P}}_t^P}{\partial \tau_t} = \left[(\boldsymbol{\Psi}_\phi \boldsymbol{\Lambda})^{-1} + \boldsymbol{\Theta}_1 \right]^{-1} \left[\boldsymbol{\Theta}_2 - \left(\boldsymbol{L}_{\mathbf{E}}^P \frac{\partial \hat{V}_t}{\partial \hat{\tau}_t} \right) \psi \right]$$

2025: Tariff Threats and Trade War

- Implemented tariffs, country-sector.
- Symmetric retaliation.
- Near-permanent shock.
- **Reversed threat:**
  - U.S. announces future tariffs
  - Retaliation is anticipated
  - At t = 2 no tariffs implemented.



Takeaways

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   0.2-0.5 pp inflation; 0.5-1 percent output decline
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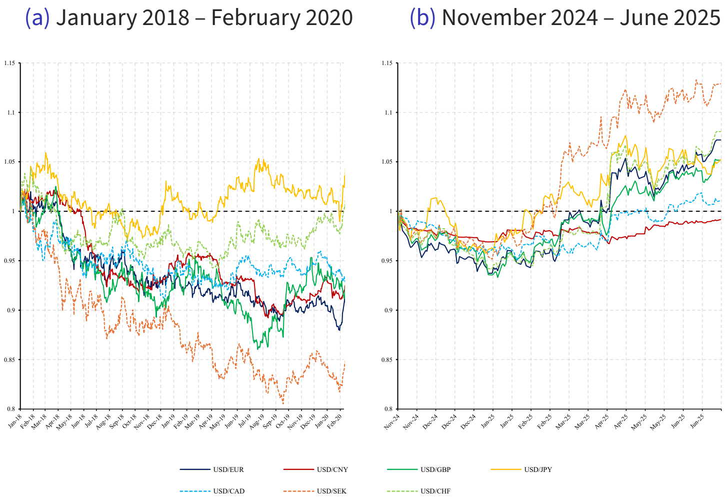
Open economy macro needs to reckon with the networked reality of global trade.

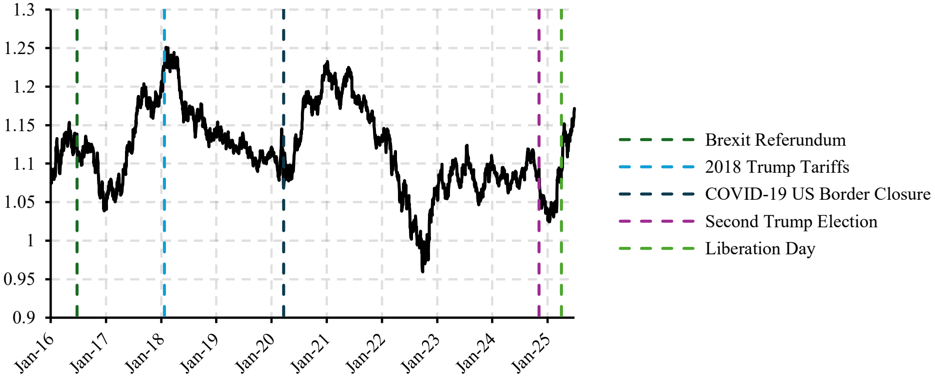
- With and w/o networks, inflation-output trade-off differ: overestimation of inflation and underestimation of unemployment.

Appendix

USD Exchange Rates against to Major Currencies, following 2018 tariff war and 2025 Inaugurations



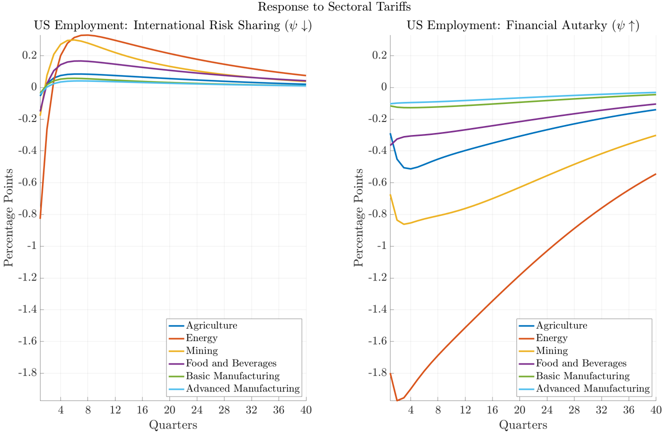
USD - Euro Exchange Rate 2016-2025



NOTE: USD Euro Exchange Rate from 2015 to 2025. The vertical lines indicate different events. Source: Bloomberg.

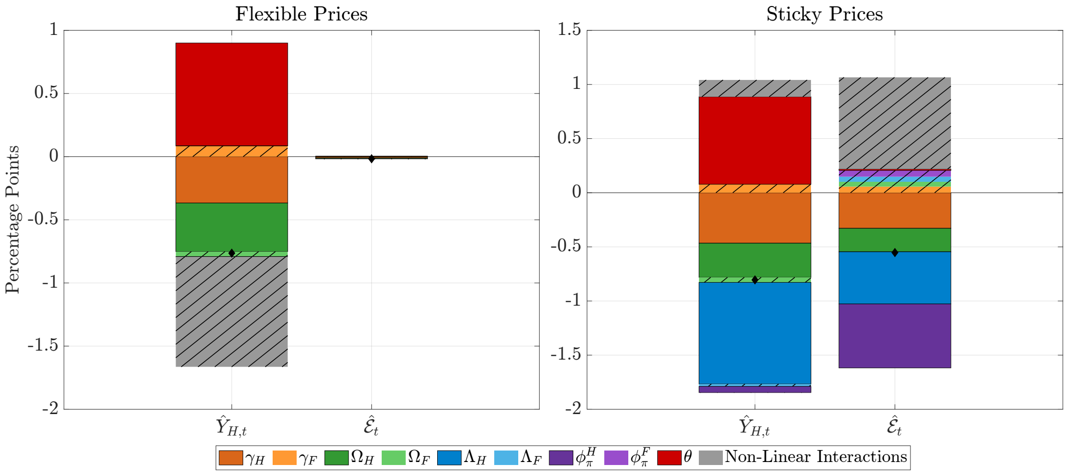
Network and Intertemporal smoothing

- Macro impact of US tariffs on Chinese rare earths vs. Chinese cars is different

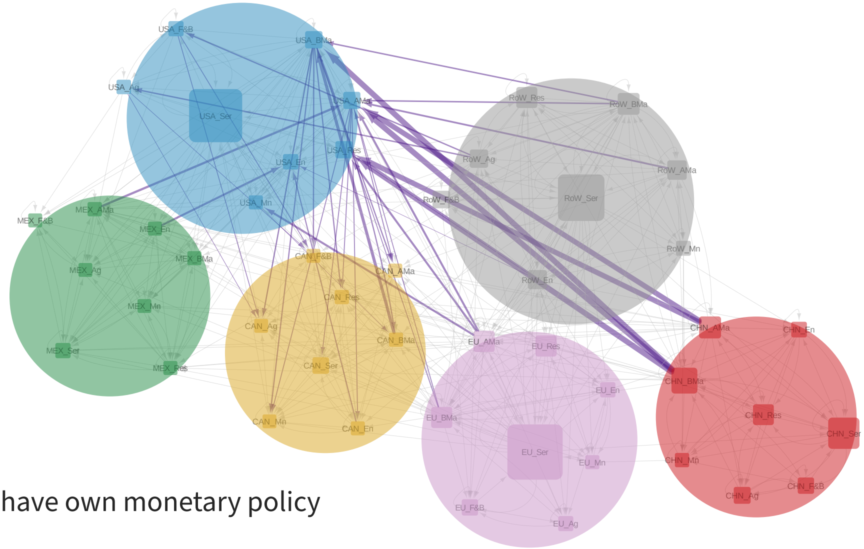


Contribution of Primitives to Macro Aggregates

Flexible Prices vs. Sticky Prices

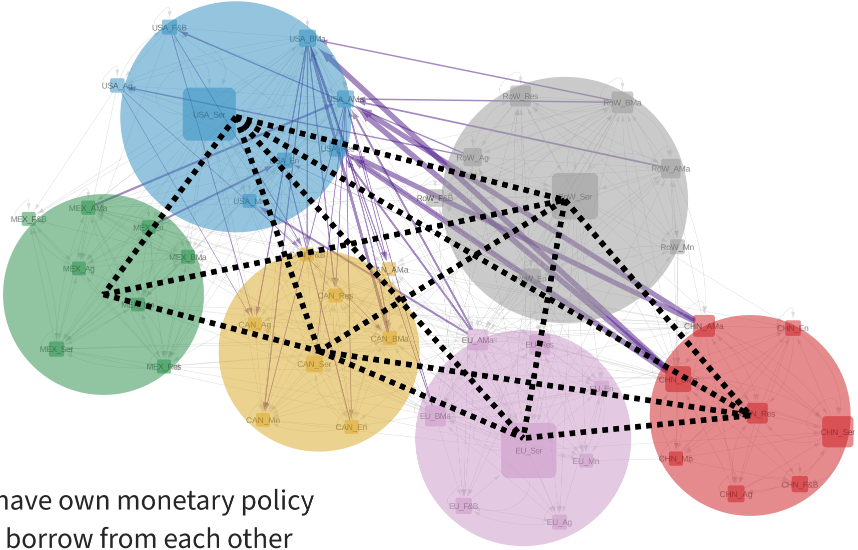


Country-Sector Linkages and Heterogeneity



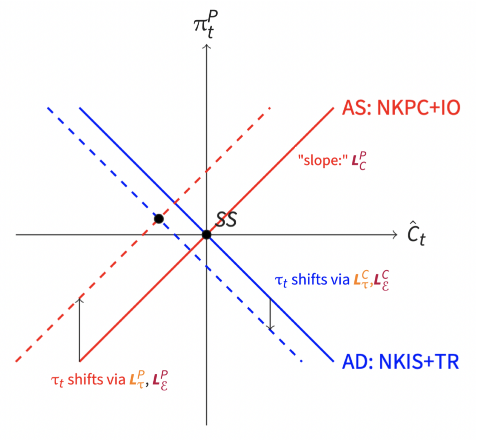
Countries have own monetary policy

Country-Sector Linkages and Heterogeneity



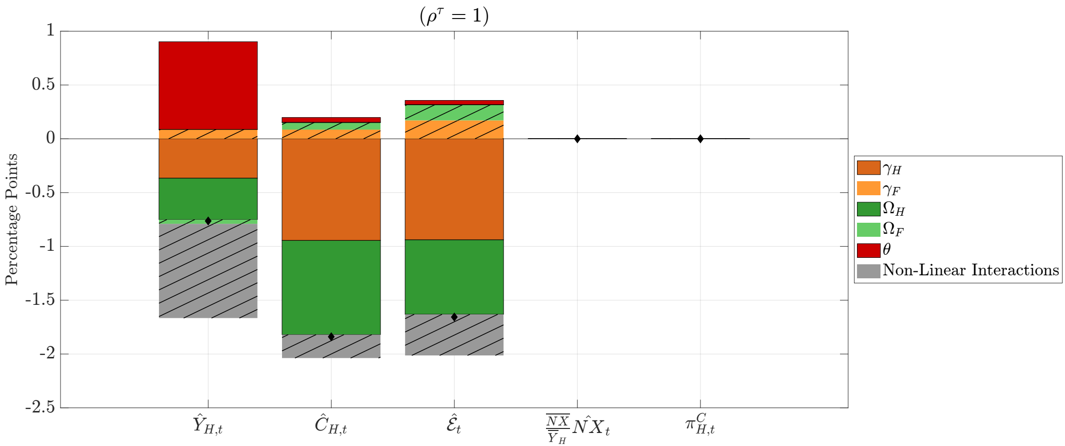
Countries have own monetary policy
And they borrow from each other

Visualizing Our Approach

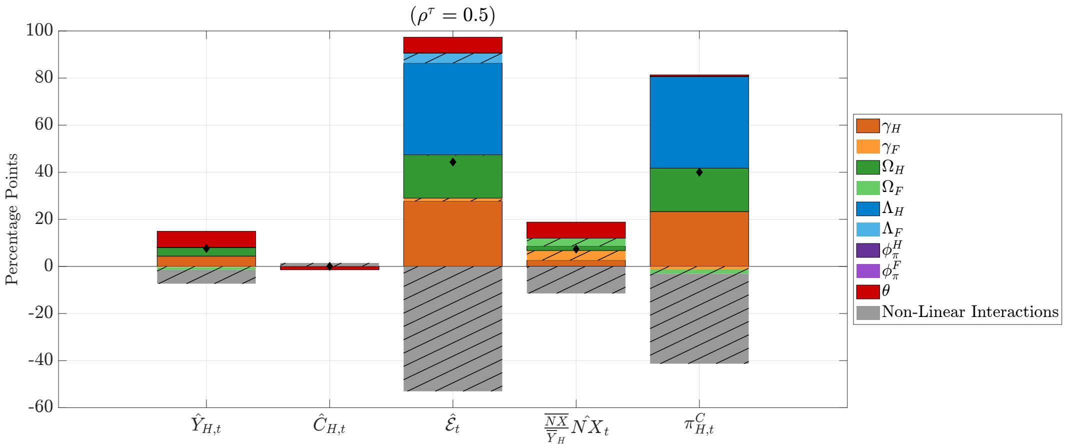


DGE impact of tariffs will depend on direct impact (L_τ^C & L_τ^P) and indirect reallocation via (L_C^P, $L_\mathbf{E}^C$ & $L_\mathbf{E}^P$)

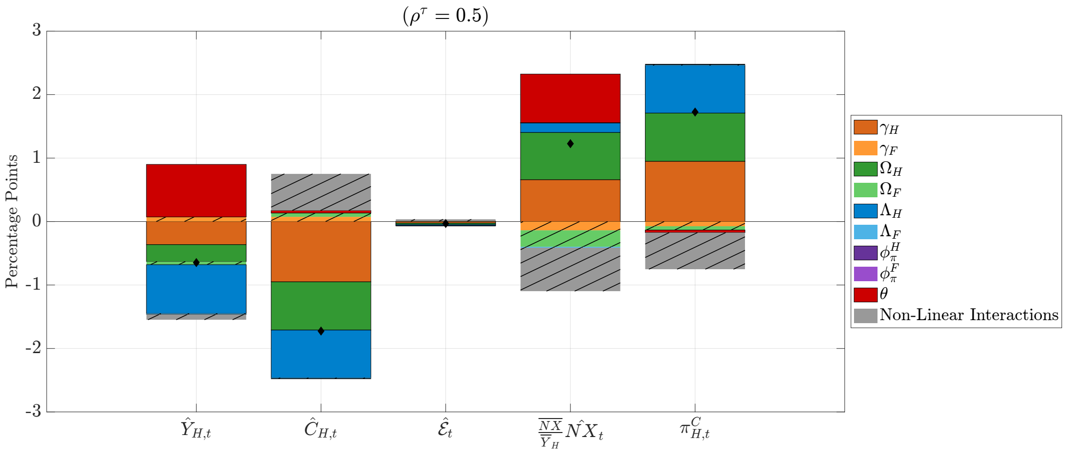
Contribution of Primitives to Macro Aggregates Under Flexible Prices



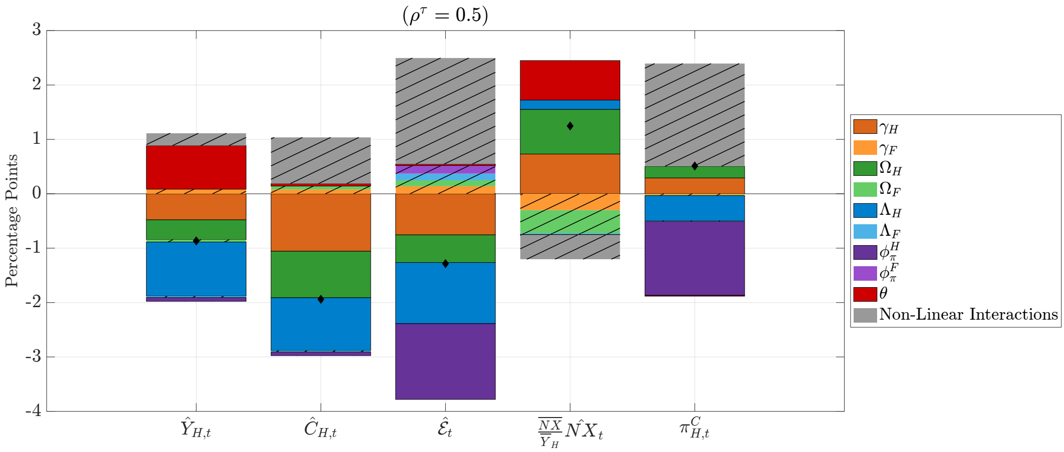
Contribution of Primitives to Macro Aggregates Under Real Rate Rule



Contribution of Primitives to Macro Aggregates Under Fixed Nominal Demand

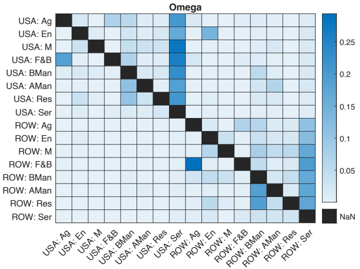


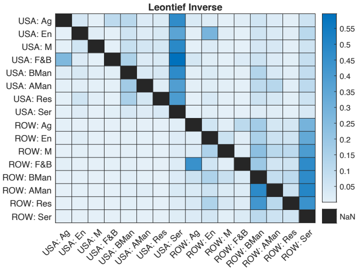
Contribution of Primitives to Macro Aggregates Under a Taylor Rule

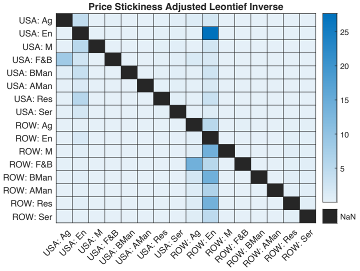


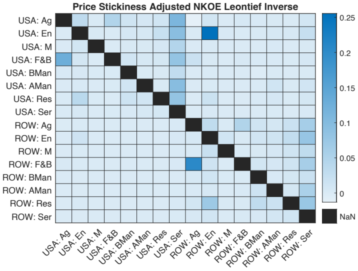
Why Networks?

Impact of Heterogeneity on Inflation: Price Stickiness vs. Monetary Policy

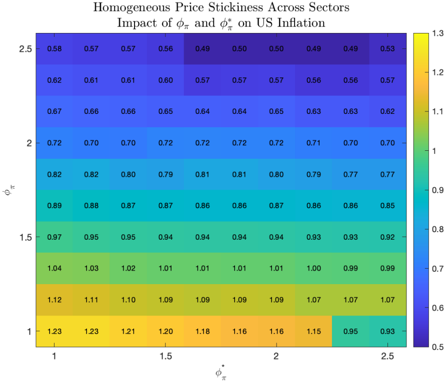


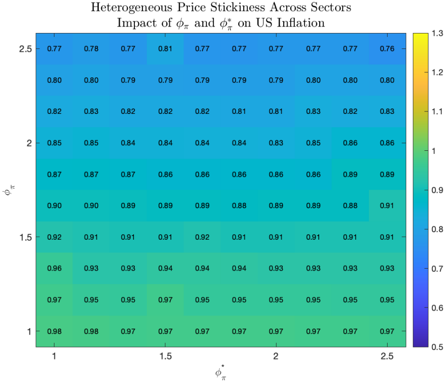






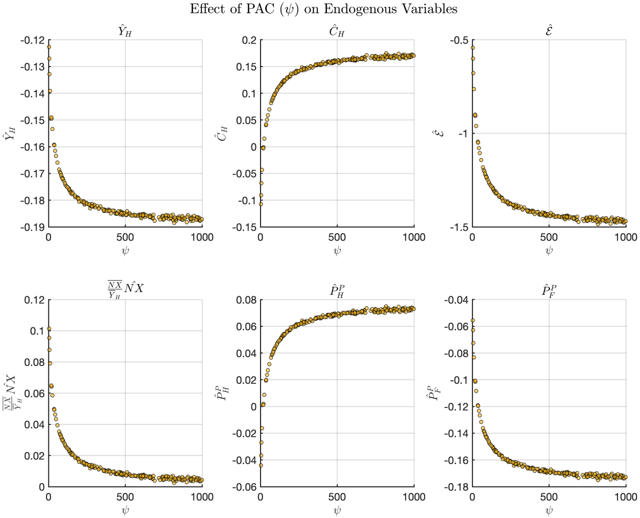
Impact of Heterogeneity on Inflation: Price Stickiness vs. Monetary Policy



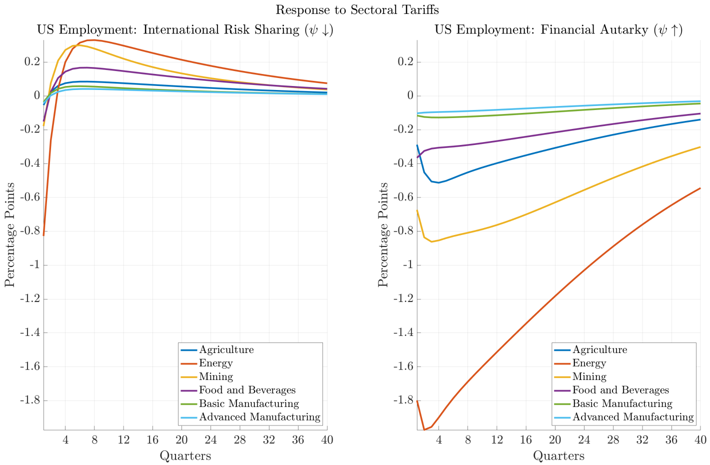


Network amplifies or soothes inflation depending on sectoral price stickiness regardless of endogenous monetary policy response: A numerical example with 10% reciprocal tariff.

Numerical Second Derivative: $\frac{\partial^2 \hat{\boldsymbol{P}}_t^P}{\partial \tau_t \, \partial \psi}$



Sectoral Shocks and International Risk Sharing



Quantitative Model

Sector Statistics for USA (%)

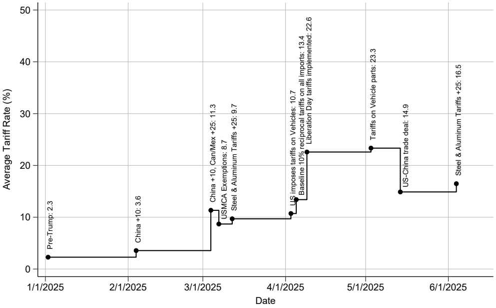
Industry	Output Share	VA Share	Consumption Share	Output Home Share	Consumption Home Share	Intermediate Home Share
Agriculture	1.3	0.9	0.6	87.2	88.5	89.3
Energy	3.0	2.0	1.5	85.7	89.4	75.0
Mining	0.5	0.5	0.5	91.2	98.5	89.9
Food and Beverages	2.6	1.2	3.1	94.0	91.2	91.7
Basic Manufacturing	6.6	4.7	4.1	87.6	66.0	82.5
Advanced Manufacturing	6.2	5.1	8.2	81.7	67.0	66.9
Residential Services	6.4	6.1	7.7	99.9	99.9	99.5
Services	73.4	79.4	74.3	95.3	96.7	96.2

SOURCE: OECD ICIO for year 2019.

Effective Tariff Rates

(a) Historic and Estimated, (%)

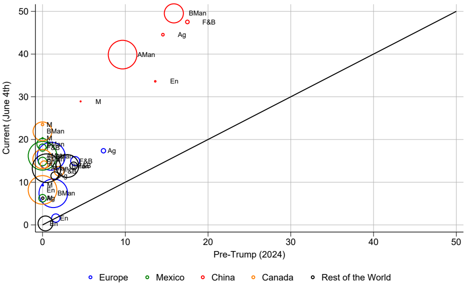
(b) Since January 1, 2025, (%)



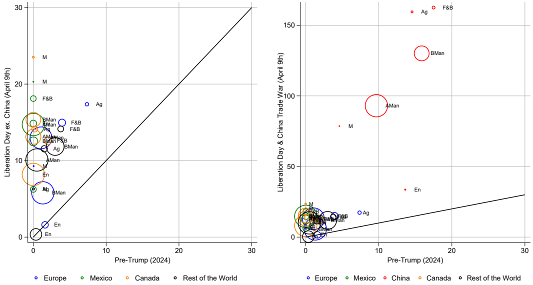
SOURCE: (a) Yale Budget Lab (b) WTO - IMF Tariff Tracker.

Effective Country-Sector Level Tariff Rates

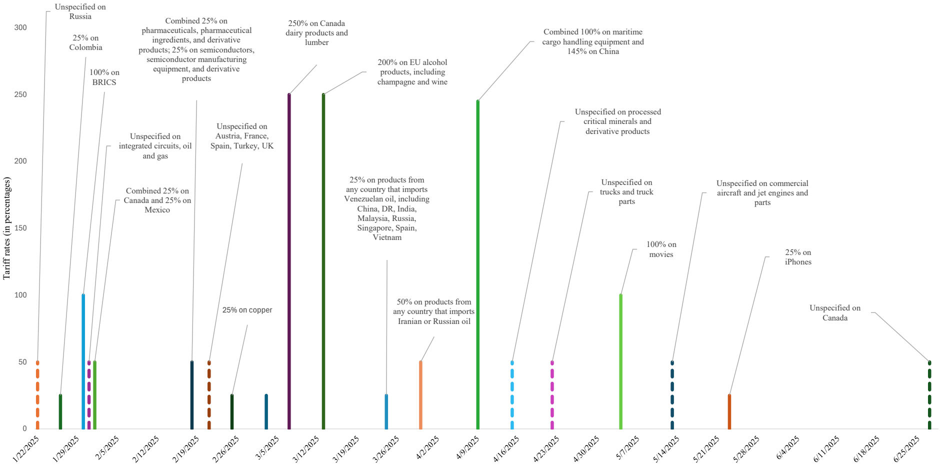
(a) As of June 4, 2025 (%)



(b) As of the "Liberation Day", (%)

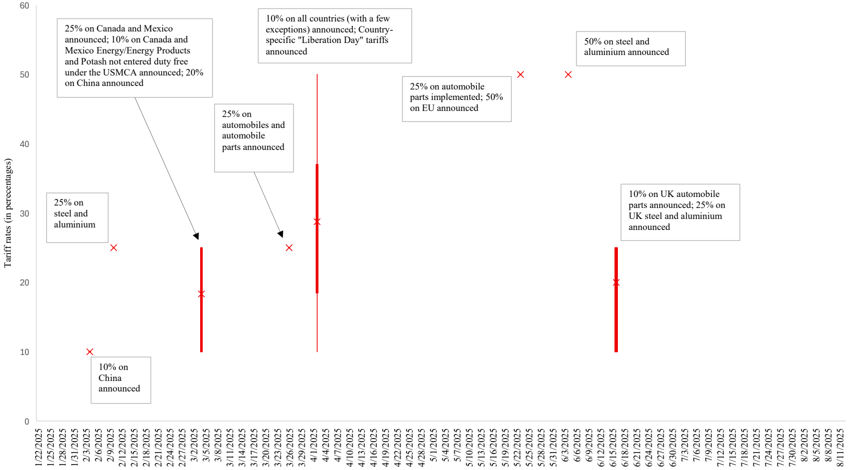


Tariff Threats - not implemented and future implementation uncertain



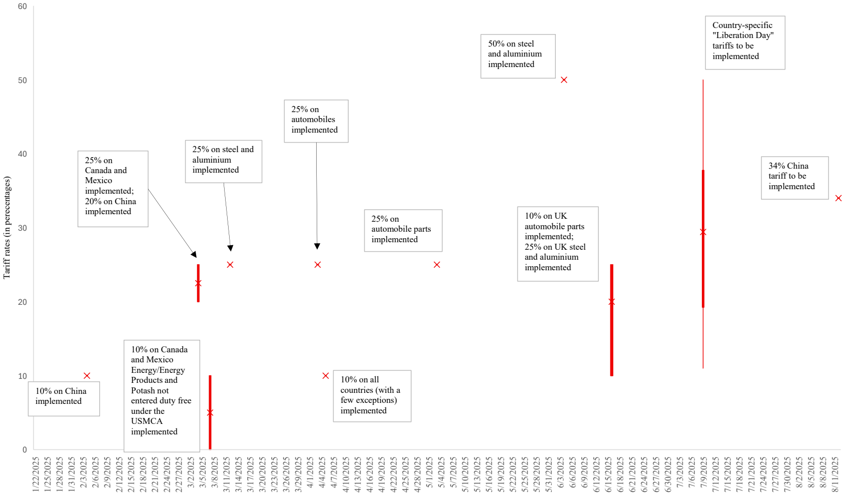
SOURCE: Trade Compliance Resource Hub Trump 2.0 Tariff Tracker.

Tariff Announcements



SOURCE: Trade Compliance Resource Hub Trump 2.0 Tariff Tracker.

Tariffs - Implemented (and to be Implemented)



Source: Trade Compliance Resource Hub Trump 2.0 Tariff Tracker.

Calibration

- IRFs computed non-linearly with MIT shocks (perfect foresight)
- Global I–O structure: 2019 OECD ICIO
- 2019 treated as steady state
  - Permanent capital account wedge Unpleasant SS Arithmetic

Calibration Parameters

Parameter	**Explanation**	**Value**	**Source**
σ	Intertemporal EoS	2	e.g., Itskhoki and Mukhin (2021)
η	Elasticity of Labor	1	e.g., Itskhoki and Mukhin (2021)
ψ	Reactivity of UIP to Debt	0.001 – 0.0001	Standard
ρ_m^n	Inertia in Taylor Rule for $n \neq US$	0.95	Clarida et al. (2000)
ρ_m^{US}	Inertia in Taylor Rule for U.S.	0.82	Carvalho et al. (2021)
ϕ_π^{US}	Weight on inflation in Taylor Rule for U.S.	1.29	Carvalho et al. (2021)
λ_n	Sector specific price rigidities		Nakamura and Steinsson (2008)
θ^P	EoS between intermediates and VA	0.6	Atalay (2017)
θ_h^C	Intratemporal EoS of consumption among sectors	0.6	Calibrated for consistency
θ_h^P	EoS among intermediate inputs	0.001 – 0.2	Baqaee and Farhi (2019); Boehm et al. (2019)
θ_{li}^C	Sector level consumption bundle EoS	0.6	di Giovanni et al. (2023)
θ_{li}^P	Sector level input bundle EoS	0.6	di Giovanni et al. (2023)

Note: "EoS" = elasticity of substitution. Inflation coefficients calibrated via $\phi_\pi^n = \frac{1-\rho_m^n}{\overline{\pi}_n^C}$.

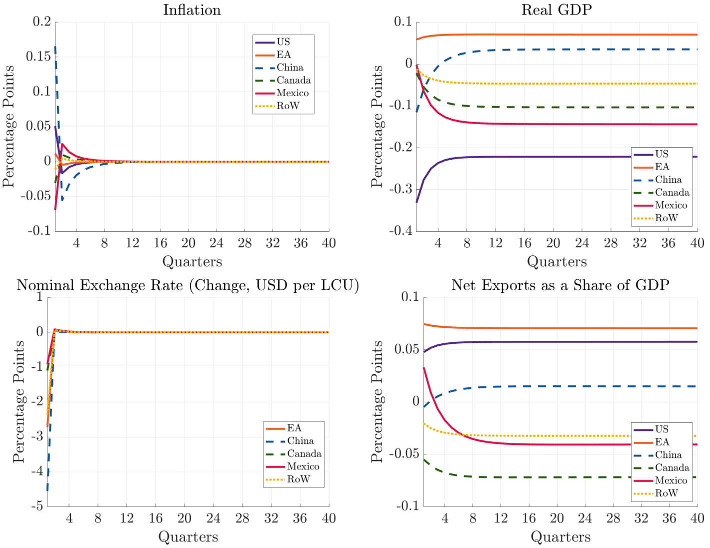
Benchmarking: 2018's Trump Tariffs

Scenario:

- Implemented tariffs from Fajgelbaum et al. (2020) (25% tariffs by U.S. on China in 2018 + Washer, Solar, Aluminum, and Iron & Steel Tariffs).
- No retaliation.
- Near-permanent shock ($\rho^{\tau} = 0.95$, $\phi_y = 0$).

Barbiero and Stein (2025) estimate 0.1 to 0.2pp increase in $\pi^{C}_{US,t}$ $\rightarrow$ model predicts 0.07pp.

USD appreciated by ~6% from June 2018-December 2018- model predicts ~4%.

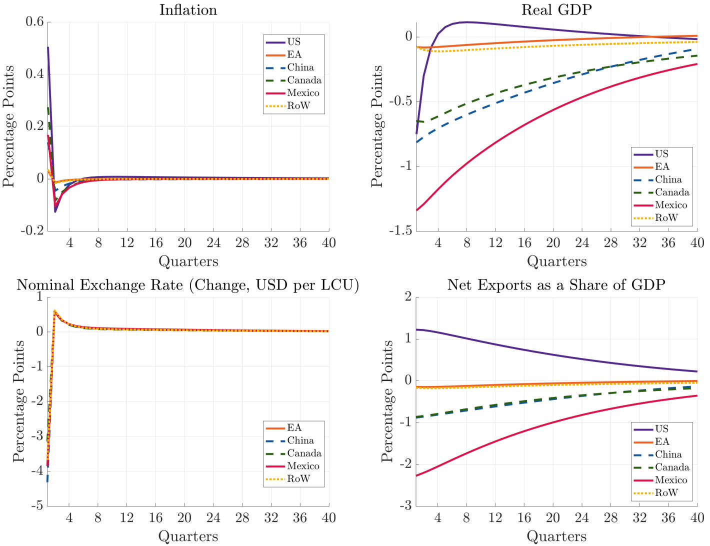


2025 Current Tariffs (June 4, 2025)

Case 2: 2025 Tariffs

Scenario:

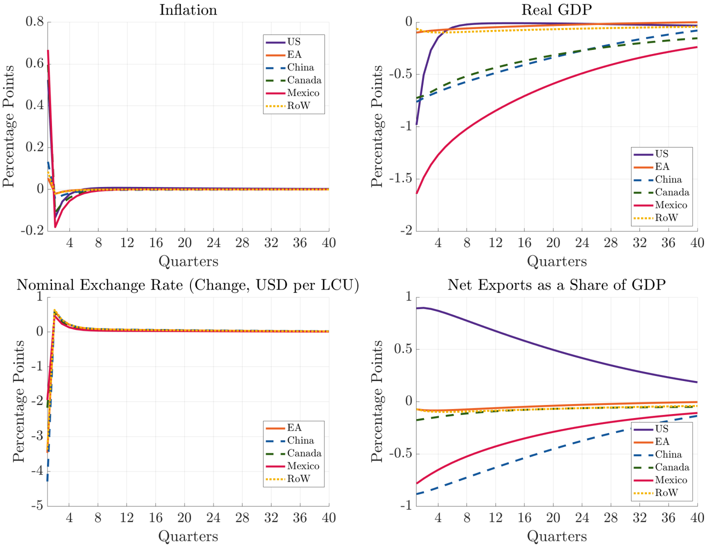
- US tariffs:
  - Actual implemented tariffs at country-sector level as of June 4, 2025 from WTO-IMF Tariff Tracker.
- No retaliation.
- Near-permanent shock ($\rho^{\tau} = 0.95$, $\phi_y = 0.1$).



Case 3: 2025 Potential All-Out Trade War

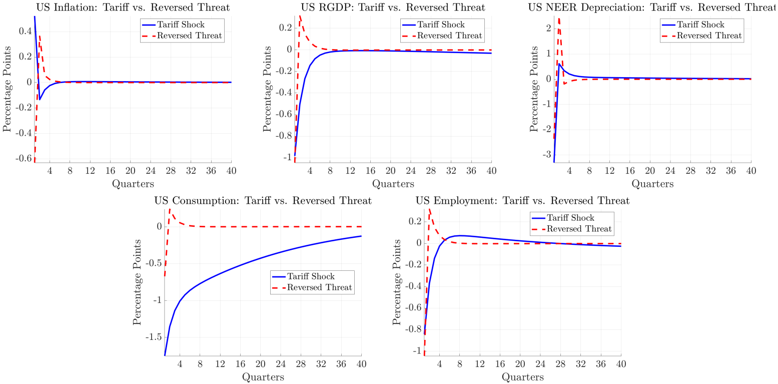
Scenario:

- US tariffs as Case 2.
- Symmetric retaliation by all partners.
- Near-permanent shock ($\rho^{\tau} = 0.95$, $\phi_y = 0.1$).



Case 4: Tariff Threats for Geopolitical Reasons

- U.S. announces future tariffs
- Retaliation is anticipated
- At $t = 2$ no tariffs implemented.



Model Primitives

Model Overview

Model combines

1. New Keynesian model with Rotemberg costs

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2. "Full" open economy $\rightarrow$ N-country DGE
   - Portfolio Adjustment Costs (PAC)
   - Producer Currency Pricing (PCP) and tariffs: $$P^C_{n,mj,t} = \mathbf{E}_{n,m,t}\, P^P_{mj,t}\,(1 + \tau_{n,mj,t})$$

Model Overview

Model combines

1. New Keynesian model with Rotemberg costs

2. "Full" open economy $\rightarrow$ N-country DGE
   - Portfolio Adjustment Costs (PAC)
   - Producer Currency Pricing (PCP) and tariffs: $P^{C}_{n,mj,t} = \mathbf{E}_{n,m,t} \, P^{P}_{mj,t} \, (1 + \tau_{n,mj,t})$

3. Production network with full Input-Output (IO) matrix
   - *n* is consuming country, *i* is consuming sector, *m* is producing country, *j* is producing sector
   - Both consumption goods and intermediate inputs are nested CES
     - German cars+American cars+ Japanese cars $\rightarrow C_t^{cars}$
     - $C_t^{cars} + C_t^{food} \rightarrow C_t$

Household's Problem

- The household maximizes the present value of lifetime utility:

$$\max_{\{C_{n,t},L_{n,t},B^{US}_{n,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{n,t}^{1-\sigma}}{1-\sigma} - \chi \frac{L_{n,t}^{1+\gamma}}{1+\gamma} \right]$$

s.t.

$$P_{n,t}C_{n,t} + T_{ni,t} - B_{n,t} - \mathbf{E}^{US}_{n,t}B^{US}_{n,t} + \mathbf{E}^{US}_{n,t}\psi(B^{US}_{n,t}) \leq$$
$$W_{n,t}L_{n,t} + \sum_i \Pi_{ni,t} - (1+i_{n,t-1})B_{n,t-1} - \mathbf{E}^{US}_{n,t}(1+i^{US}_{n,t-1})B^{US}_{n,t-1}$$

Linearized Model

Analytical Solution

- To provide intuition, we linearize the model:
  - Assuming portfolio adjustment costs are ≈ 0.
  - Adopting Golosov and Lucas (2007) preferences with $\sigma = 1$ and $\gamma = 0$.

Analytical Solution

- To provide intuition, we linearize the model:
  - Assuming portfolio adjustment costs are ≈ 0.
  - Adopting Golosov and Lucas (2007) preferences with $\sigma = 1$ and $\gamma = 0$.
- Inter-Country Input-Output Matrix, $\mathbf{\Omega}$, relates all country-sector pairs to each other.
- Leontief Inverse, $\mathbf{\Psi} = [\boldsymbol{I} - \mathbf{\Omega}]^{-1} = \sum_{k=0}^{\infty} \mathbf{\Omega}^k$, combines all direct and indirect linkages.
- "Loading" notation $\rightarrow$ exposure of superscript to subscript
  - $\boldsymbol{L}^C_\tau$ captures how τ_t "loads" onto CPI equation
    $\rightarrow$ tariffs levied on 5% of consumption basket
  - Similarly $\boldsymbol{L}^C_{\hat{\mathbf{E}}}$ $\rightarrow$ consumption basket is exposed to a given bilateral exchange rate

Household's Problem

The household in country n maximizes the present value of lifetime utility:

$$\max_{\{C_{n,t},L_{n,t},B^{US}_{n,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{n,t}^{1-\sigma}}{1-\sigma} - \chi \frac{L_{n,t}^{1+\eta}}{1+\eta} \right]$$

subject to

$$P^C_{n,t} C_{n,t} + T_{n,t} - B_{n,t} - \mathbf{E}^{US}_{n,t} B^{US}_{n,t} + \mathbf{E}^{US}_{n,t} P^{US}_{n,t} \psi(B^{US}_{n,t}/P^{US}_{n,t}) \leq$$
$$W_{n,t} L_{n,t} + \sum_i \Pi_{ni,t} - (1 + i_{n,t-1}) B_{n,t-1} - \mathbf{E}^{US}_{n,t} (1 + i^{US}_{n,t-1}) B^{US}_{n,t-1}$$

Household's Problem

Standard first-order conditions $\forall n \in N, \forall t$:

$$1 = \beta E_t \left[\left(\frac{C_{n,t+1}}{C_{n,t}} \right)^{-\sigma} \frac{P^C_{n,t}}{P^C_{n,t+1}} (1 + i_{n,t}) \right] \forall n \in N, \forall t \quad \text{(Euler Equation)}, \tag{1}$$

$$\frac{1 + i_{n,t}}{1 + i^{US}_{n,t}} = E_t \left[\frac{\mathbf{E}_{n,t+1}}{\mathbf{E}_{n,t}} \right] \frac{1}{1 - \psi'(B^{US}_{n,t}/P^{US}_{n,t})} \quad \text{(UIP) } n \in N - 1 \tag{2}$$

$$\frac{W_{n,t}}{P_{n,t}} = \chi L^{\psi}_{n,t} C^{\sigma}_{n,t} \ \forall n \in N, \forall t \quad \text{(Labor-Cons. tradeoff)} \tag{3}$$

Exchange Rate:

$$\mathbf{E}_{n,m,t} = \frac{\mathbf{E}^{US}_{n,t}}{\mathbf{E}^{US}_{m,t}} \ \forall n \neq m \ \& \ m \neq US \ n, m \in N \tag{4}$$

$$\mathbf{E}_{n,n,t} = 1 \ \forall n \in N \tag{5}$$

PCP

Producer's price goods in their currency. The price for end-users converts that price with the exchange rate and importers pay tariffs.

$$P_{n,mj,t} = \mathbf{E}_{n,m,t} P_{mj,t} (1 + \tau_{n,mj,t}) \tag{6}$$

where $\mathbf{E}_{nm,t}$ is the bilateral exchange rate and τ_t are tariffs.

Firm's Problem

- CES Production:

$$Y_{ni,t} = A_{ni,t}\left[\alpha_{ni}^{1/\theta^P} L_{ni,t}^{\frac{\theta^P-1}{\theta^P}} + (1-\alpha_{ni})^{1/\theta^P}(X_{ni,t})^{\frac{\theta^P-1}{\theta^P}}\right]^{\frac{\theta^P}{\theta^P-1}} \forall n \in N, \forall i \in J,$$

Firm's Problem

- CES Production:

$$Y_{ni,t} = A_{ni,t}\left[\alpha_{ni}^{1/\theta^P} L_{ni,t}^{\frac{\theta^P-1}{\theta^P}} + (1-\alpha_{ni})^{1/\theta^P}(X_{ni,t})^{\frac{\theta^P-1}{\theta^P}}\right]^{\frac{\theta^P}{\theta^P-1}} \forall n \in N, \forall i \in J,$$

- Rotemberg setup:

$$P^f_{ni,t} = \arg\max_{P^f_{ni,t}} \mathbb{E}_t\left[\sum_{T=t}^{\infty} \mathrm{SDF}_{t,T}\left[Y^f_{ni,T}(P^f_{ni,T})\left(P^f_{ni,T} - MC_{ni,T}\right) - \frac{\delta_{ni}}{2}\left(\frac{P^f_{ni,T}}{P^f_{ni,T-1}} - 1\right)^2 Y_{ni,T}P_{ni,T}\right]\right]$$

- This yields the New Keynesian Phillips Curve in terms of MC:

$$\left(\Pi_{ni,t}-1\right)\Pi_{ni,t} = \frac{\theta^R}{\delta_{ni}}\left(\frac{MC_{ni,t}}{P_{ni,t}} - \frac{\theta^R-1}{\theta^R}\right) + \beta\mathbb{E}_t\left[\left(\Pi_{ni,t+1}-1\right)\Pi_{ni,t+1}\right]$$

Balance of Payments and NIIP

- Evolution of each country n's net international position:

$$\sum_{m\in \mathbf{N}} \sum_{j\in \mathbf{J}} \left(\frac{P_{n,mj,t}}{1+\tau_{n,mi,t}} C_{n,mj,t} \right) + \sum_{m\in \mathbf{N}} \sum_{i\in \mathbf{J}} \sum_{j\in \mathbf{J}} \left(\frac{P_{n,mj,t}}{1+\tau_{n,mi,t}} X_{ni,mj,t} \right) + \mathbf{E}_{n,t}(1+i^{US}_{n,t-1})B^{US}_{n,t-1}$$
$$+ \mathbf{E}_{n,t} P^{US}_{n,t} \psi(B^{US}_{n,t}/P^{US}_{n,t}) = \sum_{i\in \mathbf{J}} P_{ni,t} Y_{ni,t} + \mathbf{E}_{n,t} B^{US}_{n,t} \quad \forall n \in N-1$$

to account for tariffs canceling out we divide $P_{n,mj,t}$ by $1+\tau_{n,mj,t}$.

Definitions, Market Clearing and Policy

Definitions, market clearing conditions and policy:

$$B_t^{US} = \sum_{m}^{N-1} B_{m,t}^{US}$$

$$Y_{ni,t} = \sum_{n \in \mathbf{N}} C_{m,ni,t} + \sum_{m \in \mathbf{N}} \sum_{j \in \mathbf{J}} X_{mj,ni,t}$$

$$L_{n,t} = \sum_{i \in J} L_{ni,t}$$

$$\Pi_{n,t} = \frac{P_{n,t}}{P_{n,t-1}}$$

$$1 + i_{n,t} = \left(\Pi_{n,t}\right)^{\phi_\pi} e^{\hat{M}_{n,t}} \quad \forall n \in N$$

Trade Deficit and Exchange Rate

- Impact on trade deficit depends on:
  - Short run vs Long run
  - Transitionary vs Permanent Shock
- In the long run (under flexible prices) and permanent shocks, initial net asset position of countries do not change
- In the short run (under sticky prices) and temporary shocks, tariffs can improve net position at a cost

Intratemporal Demand Structure

Nested CES

$$C_{n,t} = \left[\sum_{i \in J} \Gamma_{n,i}^{\frac{1}{\theta_h^C}} C_{n,i,t}^{\frac{\theta_h^C - 1}{\theta_h^C}} \right]^{\frac{\theta_h^C}{\theta_h^C - 1}} . \tag{7}$$

Relative Demand:

$$C_{n,i,t} = \left[\sum_{m \in N} \Gamma_{n,i,mi}^{\frac{1}{\theta_{l,i}^C}} C_{n,i,mi,t}^{\frac{\theta_{l,i}^C - 1}{\theta_{l,i}^C}} \right]^{\frac{\theta_{l,i}^C}{\theta_{l,i}^C - 1}} . \tag{8}$$

Consumption Prices and Allocations

Prices:

$$P^C_{n,t} = \left[\sum_{i \in J} \Gamma_{n,i} (P^C_{n,i,t})^{1-\theta^C_h}\right]^{\frac{1}{1-\theta^C_h}}$$

$$P^C_{n,i,t} = \left[\sum_{m \in N} \Gamma_{n,i,mi} P_{n,mi,t}^{1-\theta^C_{l,i}}\right]^{\frac{1}{1-\theta^C_{l,i}}}$$

Allocations:

$$C_{n,i,t} = \Gamma_{n,i} \left(\frac{P^C_{n,i,t}}{P^C_{n,t}}\right)^{-\theta^C_h} C_{n,t} \tag{9}$$

$$C_{n,mi,t} = \Gamma_{n,i,mi} \left(\frac{P_{n,mi,t}}{P^C_{n,i,t}}\right)^{-\theta^C_{l,i}} C_{n,i,t} \tag{10}$$

PCP

Producer's price goods in their currency. The price for end-users converts that price with the exchange rate and importers pay tariffs.

$$P_{n,mi,t} = \mathbf{E}_{nm,t} P_{mj,t}(1 + \tau_{n,m,t}) \tag{11}$$

where $\mathbf{E}_{nm,t}$ is the bilateral exchange rate and τ_t are tariffs.

Supply

Production is also Nested CES:

$$Y_{ni,t} = A_{ni,t}\left[\alpha_{ni}^{1/\theta^P} L_{ni,t}^{\frac{\theta^P-1}{\theta^P}} + (1-\alpha_{ni})^{1/\theta^P}(X_{ni,t})^{\frac{\theta^P-1}{\theta^P}}\right]^{\frac{\theta^P}{\theta^P-1}} \quad \forall n \in N, \forall i \in J, \tag{12}$$

Marginal cost minimization problem:

$$MC_{ni,t} = \min_{\{X_{ni,j,t}, L_{ni,t}\}} W_t L_{ni,t} + P^X_{ni,t} X_{ni,t} \quad \text{s.t.} \quad Y_{ni,t} = 1.$$

Intermediate Good Bundles

Intermediate goods from different countries are first bundled into a country-industry-good bundle:

$$X_{ni,j,t} = \left[\sum_{m\in N} \Omega_{ni,j,mj}^{\frac{1}{\theta^P_{l,j}}} X_{ni,mj,t}^{\frac{\theta^P_{l,j}-1}{\theta^P_{l,j}}} \right]^{\frac{\theta^P_{l,j}}{\theta^P_{l,j}-1}} , \quad X_{ni,mj,t} = \Omega_{ni,j,mj} \left(\frac{P_{n,mj,t}}{P^X_{ni,j,t}} \right)^{-\theta^P_{l,j}} X_{ni,j,t} \tag{13}$$

The intermediate bundle is constructed as follows:

$$X_{ni,t} = \left[\sum_{j\in J} \Omega_{ni,j}^{\frac{1}{\theta^P_h}} X_{ni,j,t}^{\frac{\theta^P_h-1}{\theta^P_h}} \right]^{\frac{\theta^P_h}{\theta^P_h-1}} , \quad \frac{X_{ni,j,t}}{X_{ni,t}} = \Omega_{ni,j} \left(\frac{P^X_{ni,j,t}}{P^X_{ni,t}} \right)^{-\theta^P_h} \forall\, j \in J \tag{14}$$

Marginal Cost

The marginal cost $MC_{ni,t}$ problem yields:

$$\frac{X_{ni,t}}{L_{ni,t}} = \frac{(1-\alpha_{ni})}{\alpha_{ni}} \left(\frac{W_t}{P^X_{ni,t}} \right)^{\theta^P} \tag{15}$$

$$MC_{ni,t} = \frac{1}{A_{ni,t}} \left[\alpha_{ni} W_t^{1-\theta^P} + (1-\alpha_{ni}) \left(\sum_j \Omega_{ni,j} (P^X_{ni,j,t})^{1-\theta^P_h} \right)^{\frac{1-\theta^P}{1-\theta^P_h}} \right]^{\frac{1}{1-\theta^P}} \tag{16}$$

Rotemberg Costs

Representative firm f in sector i of country n solves the following problem Rotemberg setup:

$$P^f_{ni,t} = \arg\max_{P^f_{ni,t}} \mathbb{E}_t \left[\sum_{T=t}^{\infty} \text{SDF}_{t,T} \left[Y^f_{ni,T}(P^f_{ni,T}) \left(P^f_{ni,T} - MC_{ni,T} \right) - \frac{\delta_{ni}}{2} \left(\frac{P^f_{ni,T}}{P^f_{ni,T-1}} - 1 \right)^2 Y_{ni,T} P_{ni,T} \right] \right]$$

This yields the New Keynesian Phillips Curve expressed in terms of real marginal costs:

$$\left(\Pi_{ni,t} - 1\right) \Pi_{ni,t} = \frac{\theta^R}{\delta_{ni}} \left(\frac{MC_{ni,t}}{P_{ni,t}} - \frac{\theta^R - 1}{\theta^R} \right) + \beta \mathbb{E}_t \left[\left(\Pi_{ni,t+1} - 1 \right) \Pi_{ni,t+1} \right] \tag{17}$$

Balance of Payments and NIIP

We track each country's net international position's evolution as follows:

$$\sum_{m\in \mathbf{N}} \sum_{j\in \mathbf{J}} \left(\frac{P_{n,mj,t}}{1+\tau_{n,mi,t}} C_{n,mj,t} \right) + \sum_{m\in \mathbf{N}} \sum_{i\in \mathbf{J}} \sum_{j\in \mathbf{J}} \left(\frac{P_{n,mj,t}}{1+\tau_{n,mi,t}} X_{ni,mj,t} \right) + \mathbf{E}_{n,t}(1+i^{US}_{n,t-1})B^{US}_{n,t-1}$$
$$+ \mathbf{E}_{n,t} P^{US}_{n,t} \psi(B^{US}_{n,t}/P^{US}_{n,t}) = \sum_{i\in \mathbf{J}} P_{ni,t} Y_{ni,t} + \mathbf{E}_{n,t} B^{US}_{n,t} \quad \forall n \in N-1 \tag{18}$$

where import tariffs cancel out because they are both a cost and a revenue at the country-level.

Definitions, Market Clearing and Policy

Definitions, market clearing conditions and policy:

$$B_t^{US} = \sum_{m}^{N-1} B_{m,t}^{US} \tag{19}$$

$$Y_{ni,t} = \sum_{n\in \mathbf{N}} C_{m,ni,t} + \sum_{m\in \mathbf{N}} \sum_{j\in \mathbf{J}} X_{mj,ni,t} \tag{20}$$

$$L_{n,t} = \sum_{i\in J} L_{ni,t} \tag{21}$$

$$\Pi_{n,t} = \frac{P_{n,t}}{P_{n,t-1}} \tag{22}$$

$$1 + i_{n,t} = \left(\Pi_{n,t}\right)^{\phi_\pi} e^{\hat{M}_{n,t}} \quad \forall n \in N \tag{23}$$

Equilibrium

Definition 1

A non-linear competitive equilibrium for the model is a sequence of 11 endogenous variables $\{C_{nt}, C_{ni,t}, C_{n,mj,t}, X_{ni,mj,t}, X_{ni,j,t}, X_{ni,t}, Y_{ni,t}, L_{ni,t}, L_{n,t}, MC_{ni,t}, B^{US}_{n,t}\}_{t=0}^{\infty}$ *and 11 prices* $\{P_{ni,t}, P_{n,mi,t}, P^{C}_{n,t}, P^{C}_{ni,t}, P^{X}_{ni,t}, P^{X}_{ni,j,t}, \Pi_{n,t}, \Pi_{ni,t}, \mathbf{E}_{n,t}, i_{n,t}, W_{n,t}\}_{t=0}^{\infty}$ *given exogenous processes* $\{\tau_t, A_{ni,t}, \hat{M}_{n,t}\}_{t=0}^{\infty}$ *such that equations* (1)-(23) *hold for all countries and time periods.*

Steps for Analytical Solution

- Reduce the system to fewer equations
  - Solve out endogenous variables in NKPC
  - Apply method of undetermined coefficients at matrix scale

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  1. (Today) **Fix nominal demand** with exogenous policy: $\overline{M}_t = \hat{P}_t + \hat{C}_t$
     - Fixes nominal exchange rate so tariffs have no impact via NER or nominal demand.

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     - Best to capture tariffs as a demand and supply shock with impact on NER and RER

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  3. (Appendix) Adopt real rate rule a la HANK literature with $\phi_\pi \to 1$ where $i_t = \phi_\pi E_{t+1}\pi_{t+1} + \bar{r}_t$
     - Fixes real exchange rate and aggregate consumption
     - Implicitly similar to SOE

Solution With Fixed Nominal Demand

- When policy fixes both $\hat{W}_t$ and $\hat{\mathbf{E}}_t$ NKPC for PPI becomes:

$$\hat{\boldsymbol{P}}^P_t = \underbrace{\boldsymbol{\Psi}_{\Lambda}}_{\text{Propagation}} \left[\underbrace{\hat{\boldsymbol{P}}^P_{t-1}}_{\substack{\text{Impact of}\\ \text{lagged prices}}} + \boldsymbol{\Lambda} \left(\underbrace{(\boldsymbol{I} - \boldsymbol{\Omega})}_{\substack{\text{Policy impact}\\ \text{via Wages and ER}}} \hat{\boldsymbol{M}}_t + \underbrace{\boldsymbol{L}^P_{\tau} \hat{\boldsymbol{\tau}}_t}_{\text{Tariff incidence}} \right) + \beta \underbrace{\mathbb{E}_t \hat{\boldsymbol{P}}^P_{t+1}}_{\substack{\text{Forward-looking}\\ \text{behavior}}} \right]$$

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- When policy fixes both $\hat{W}_t$ and $\hat{\mathbf{E}}_t$ NKPC for PPI becomes:

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- **Solving the model** with the method of undetermined coefficients, we find:

$$\boldsymbol{\pi}_t^C = \left(\underbrace{\boldsymbol{\Gamma}\, \boldsymbol{\Psi}_\Lambda^{\text{NKOE}} \boldsymbol{\Lambda}}_{\substack{\text{NKPC}\\ \text{propagation}}} \underbrace{(\boldsymbol{I} - \boldsymbol{\Omega})}_{\substack{\text{via Wages and}\\ \text{via ER for producers}}} + \underbrace{(\boldsymbol{I} - \boldsymbol{\Gamma})}_{\text{via ER for consumers}} \right) \hat{\boldsymbol{M}}_t$$

$$+ \left(\underbrace{\boldsymbol{\Gamma}\, \boldsymbol{\Psi}_\Lambda^{\text{NKOE}} \boldsymbol{\Lambda}}_{\substack{\text{NKPC}\\ \text{propagation}}} \underbrace{\boldsymbol{L}_\tau^P}_{\substack{\text{Tariff incidence}\\ \text{for Producers}}} + \underbrace{\boldsymbol{L}_\tau^C}_{\substack{\text{Tariff incidence}\\ \text{for consumers}}} \right) \hat{\boldsymbol{\tau}}_t \underbrace{+ \boldsymbol{\Gamma} \left(\boldsymbol{\Psi}_\Lambda^{\text{NKOE}} - \boldsymbol{I} \right) \hat{\boldsymbol{P}}_{t-1}^P}_{\text{Impact of lagged prices}}$$

Solution Under $\phi_\pi \to 1$

- Forwarding the Euler equation, assuming $\lim_{t\to\infty} \hat{C}_t = 0$ as HANK literature does:

$$\hat{C}_t = -E_t \sum_{j=0}^{\infty} \left[\phi_\pi \pi_{t+j} - \pi_{t+j+1}\right]$$

- Taking the limit of $\phi_\pi \to 1$ turns NKIS+TR into downward sloping AD curve:

$$\hat{C}_t = -\pi_t \tag{24}$$

Quantitative model (e.g. $\phi_\pi = 1.001$) confirms replacing (24) with NKIS is identical

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- Similarly forwarding the UIP condition yields:

$$\hat{\mathbf{E}}_t = P^C_{t-1} - P^{*C}_{t-1} \tag{25}$$

Solution Under $\phi_\pi \to 1$

- Plugging in (24) and (25) we now have a system of two equations and two unknowns $\{\hat{\boldsymbol{P}}^P_t, \hat{\boldsymbol{P}}^C_t\}_{t=0}^{\infty}$ for a given sequence of $\{\tau_t\}_{t=0}^{\infty}$:

$$\hat{\boldsymbol{P}}^P_t = \boldsymbol{\Psi}_\phi \left[\hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{\Lambda} \left((\boldsymbol{L}^P_C + \mathbf{L}^P_{\mathbf{E}}) \boldsymbol{\Phi} \boldsymbol{P}^C_{t-1} + \left[\boldsymbol{L}^P_C (\mathbf{I} - \boldsymbol{\Phi}) \boldsymbol{L}^C_\tau + \mathbf{L}^P_\tau \right] \tau_t \right) + \beta \mathbb{E} t \hat{\boldsymbol{P}}^P_{t+1} \right] \tag{26}$$

$$\hat{\boldsymbol{P}}^C_t = \beta \cdot \boldsymbol{P}^P_t + \boldsymbol{\Gamma} \hat{\boldsymbol{P}}^C_{t-1} + \boldsymbol{L}^C_\tau \tau_t \tag{27}$$

- The *stickiness and policy-adjusted Leontief Inverse*.

$$\boldsymbol{\Psi}_\phi = \left[\boldsymbol{I}(1+\beta) - \boldsymbol{\Lambda} \left[\boldsymbol{\Omega} - \boldsymbol{I} + \boldsymbol{L}^P_C (\boldsymbol{I} - \boldsymbol{\Phi}) \boldsymbol{\Gamma} \right] \right]^{-1}$$

- Solution once again obtained with method of undetermined coefficients

Impact of Tariffs on Inflation Under $\phi_\pi \to 1$

Proposition 1

The impact of a one-time tariff on CPI inflation under $\phi_\pi \to 1$ is

$$\frac{\partial \pi_t^C}{\partial \tau_t} = \Gamma \Psi_\phi^{NKOE} \Lambda \left[\boldsymbol{L}_\tau^P + \left(\boldsymbol{L}_C^P (\mathbf{I} - \boldsymbol{\Phi}) + \beta (\boldsymbol{L}_C^P + \boldsymbol{L}_{\mathbf{E}}^P) \boldsymbol{\Phi} \tilde{\boldsymbol{L}}_{\mathbf{E}}^C \right) \boldsymbol{L}_\tau^C \right] + \boldsymbol{L}_\tau^C \tag{28}$$

Corollary 2

Under flexible prices (efficient allocation) the impact is the following direct effect:

$$\frac{\partial \boldsymbol{\pi}_t^{flex^C}}{\partial \tau_t} = \beta \Psi \Lambda^{-1} \boldsymbol{L}_\tau^P + \boldsymbol{L}_\tau^C \tag{29}$$

and the difference between (28) *and* (29) *yields the allocative efficiency term.*

Impact of Tariffs on Inflation Under $\phi_\pi \to 1$

- The allocative efficiency term depends on price stickiness via Λ, expectations via β and home bias via Γ.
- Reallocation operates via demand and exchange rate channels: L_C^P, $L_{\mathbf{E}}^C$ & $L_{\mathbf{E}}^P$.
- When tariffs are imposed on all imports, they serve as a combination of cost-push shock and aggregate demand shocks. Models without any imported inputs would miss cost-push component.
- For tariffs to be inflationary, the cost-push aspect via PPI needs to overpower the demand shock aspect. L_τ^C, L_τ^P,L_C^P, $L_{\mathbf{E}}^C$ and $L_{\mathbf{E}}^P$ terms can serve as ex-ante sufficient statistic.

Taking Stock and Two Modeling Questions

- **Intuition of model:** Tariffs 1) directly impact CPI and PPI, 2) indirectly impact via demand, 3) indirectly impact via exchange rate
  - $\pi_t^C \uparrow \rightarrow \hat{i} - \hat{i}_t^* \uparrow \rightarrow \hat{\mathbf{E}}_t \downarrow$
  - Higher prices $\rightarrow$ consumption switching $\hat{\mathbf{E}}_t \downarrow$

- Raises question

Taking Stock and Two Modeling Questions

- **Intuition of model:** Tariffs 1) directly impact CPI and PPI, 2) indirectly impact via demand, 3) indirectly impact via exchange rate
  - $\pi_t^C \uparrow \rightarrow \hat{i} - \hat{i}_t^* \uparrow \rightarrow \hat{\mathbf{E}}_t \downarrow$
  - Higher prices $\rightarrow$ consumption switching $\hat{\mathbf{E}}_t \downarrow$

- Raises question
  1. Why not just use a small open economy (SOE)?
     - By construction SOE misses loadings from RoW
     - Implicitly makes $\hat{C}_t$ exogenous
  2. Why use full IO matrix? Why not intermediates?
     - Shape of IO matrix matters more than just for quantitative precision

Impact of Tariffs on Inflation in Global Networks

Proposition 3

Based on analytical solution, the impact of a one-time tariff on CPI inflation is

$$\frac{\partial \pi_t^C}{\partial \tau_t} = \Gamma \tilde{\Psi}_{\phi}^{NKOE} \Lambda \left[\mathbf{L}_{\tau}^{P} + \left(\mathbf{L}_{C}^{P} (\mathbf{I} - \Phi) + \beta (\alpha + \mathbf{L}_{\mathbf{E}}^{P}) \Phi \mathbf{L}_{\mathbf{E}}^{C} \right) \mathbf{L}_{\tau}^{C} \right] + \mathbf{L}_{\tau}^{C} \qquad (30)$$

where $\tilde{\Psi}_{\phi}^{NKOE}$ $\rightarrow$ *stickiness- and policy-adjusted NKOE Leontief inverse &* Φ $\rightarrow$ *Taylor rule coefficients.*

Impact of Tariffs on Inflation in Global Networks

Proposition 3

Based on analytical solution, the impact of a one-time tariff on CPI inflation is

$$\frac{\partial \pi_t^C}{\partial \tau_t} = \Gamma \tilde{\Psi}_\phi^{NKOE} \Lambda \left[\boldsymbol{L}_\tau^P + \left(\boldsymbol{L}_C^P (\mathbf{I} - \Phi) + \beta(\alpha + \boldsymbol{L}_{\mathbf{E}}^P)\Phi \boldsymbol{L}_{\mathbf{E}}^C \right) \boldsymbol{L}_\tau^C \right] + \boldsymbol{L}_\tau^C \tag{30}$$

where $\tilde{\Psi}_\phi^{NKOE}$ $\rightarrow$ stickiness- and policy-adjusted NKOE Leontief inverse & Φ $\rightarrow$ Taylor rule coefficients.

Rearranging Equation (30) yields the following decomposition:

$$\frac{\partial \pi_t^C}{\partial \tau_t} = \underbrace{\boldsymbol{L}_\tau^C}_{\text{Direct CPI effect}} + \underbrace{\Gamma \boldsymbol{L}_\tau^P}_{\text{Direct PPI effect}} + \underbrace{\Gamma \boldsymbol{L}_C^P (\mathbf{I} - \Phi) \boldsymbol{L}_\tau^C}_{\text{Demand channel}} + \underbrace{\beta \Gamma \alpha \Phi \boldsymbol{L}_{\mathbf{E}}^C \boldsymbol{L}_\tau^C}_{\text{Expected demand channel}}$$

$$+ \underbrace{\beta \Gamma \boldsymbol{L}_{\mathbf{E}}^P \Phi \boldsymbol{L}_{\mathbf{E}}^C \boldsymbol{L}_\tau^C}_{\text{Expected ER channel}} + \underbrace{\Gamma (\tilde{\Psi}_\phi^{NKOE} \Lambda - \mathbf{I}) \boldsymbol{Z}}_{\text{Network Propagation}} \tag{31}$$

Deriving the Backus Smith Condition

- Recall Euler equations and modified UIP condition:

$$\sigma\left(E_t \Delta c_{t+1}\right) = \hat{i}_t - E_t \pi_{t+1}$$
$$\sigma\left(E_t c^*_{t+1}\right) = \hat{i}^*_t - E_t \pi^*_{t+1}$$
$$\hat{i}_t - \hat{i}^*_t = E_t \Delta \hat{\mathbf{E}}_{t+1} + \psi_t$$

- Subtract the second from the first and substitute out $\hat{i}_t - \hat{i}^*_t$:

$$\sigma\left(E_t \Delta c_{t+1} - E_t \Delta^* c_{t+1}\right) = \underbrace{E_t \Delta \hat{\mathbf{E}}_{t+1} + E_t \pi^*_{t+1} - E_t \pi_{t+1}}_{E_t \Delta q_{t+1}} + \psi_t$$

- Moving from bilateral to multi-country stage

$$\sigma\left(E_t \Delta c_{t+1} - E_t \Delta^* c_{t+1}\right) = E_t \Delta q_{t+1} + \left(\psi_t - \psi^*_t\right)$$

Unpleasant Steady State Arithmetic

Let B_t be nominal debt. The simplified flow budget constraint is:

$$P_t C_t - P_t Y_t + (1 + i_{t-1}) B_{t-1} = B_t$$

At steady state:

$$\bar{P}\bar{C} - \bar{P}\bar{Y} + (1 + \bar{i})\bar{B} = \bar{B}$$

$$\bar{P}\bar{C} - \bar{P}\bar{Y} = -\bar{i}\bar{B}$$

Our model has consumption (incl. intermediate input) and output data, so for the model to be closed steady-state debt is calculated within our system. Then if there is a current deficit at the steady state ($\bar{P}\bar{C} > 0$), since $\bar{i} > 0$, it must be that the model-consistent net debt is negative $\bar{B} < 0$.

Fixing the Unpleasant Steady State Arithmetic

As a solution we can introduce a permanent wedge in real USD terms:

$$P_t C_t - P_t Y_t + i_{t-1} B_{t-1} + \overline{KA}\, E_t\, P_t = B_t - B_{t-1}$$

At steady state:

$$\bar{P}\bar{C} - \bar{P}\bar{Y} + \overline{KA} = -\bar{i}\bar{B}$$