Nonlinear Micro Income Processes with Macro Shocks

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Micro Data and Macro Models
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Motivation

- Large macro literature quantifying how aggregate shocks affect individual outcomes:
  - Transmission of monetary policy is uneven across households (Holm, Paul, Tischbirek, 2021).
  - Household heterogeneity influences impact of recessions (Krueger, Mitman, Perri, 2016).
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- Large empirical literature has uncovered key facts about income risk:
  - Different degrees of persistence (Hall, Mishkin, 1982; Blundell, Pistaferri, Preston, 2008).
  - Non-Gaussian shocks (Geweke, Keane, 2000; Guvenen, Karahan, Ozcan, Song, 2021).
  - Nonlinear persistence (Arellano, Blundell, Bonhomme, 2017).
  - Business cycles (Storesletten, Telmer, Yaron , 2004; Guvenen, Ozcan, Song, 2014).

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  - Business cycles (Storesletten, Telmer, Yaron , 2004; Guvenen, Ozcan, Song, 2014).

➡ This paper: Framework to integrate nonlinear income processes with aggregate shocks.

Our framework

- Nonlinear micro income process with macro business cycle state:

$$\eta_{it} = Q_\eta(\eta_{i,t-1}, Z_t, Z_{t-1}, u_{it}),$$
$$Z_t = Q_Z(Z_{t-1}, V_t),$$

- u_{it} and V_t are micro and macro shocks.

- η_{it} and Z_t are potentially unobserved, linked to micro and macro data.

Our framework

- We recover Z_t from external aggregate time series via a factor model.
- We recover η_{it} from panel data on household income using a flexible ABB-style micro income process with Z_t as additional arguments.
- We adopt a time series of panels approach:
  - Collection of short panels evolving over time concurrently with the macro data.
  - In the spirit of Storesletten, Telmer, Yaron (2004).
- Tools for identification, estimation, impulse response analysis and risk quantification.

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2. How do macro and micro shocks propagate over time?
   - Impulse response functions to macro and micro shocks.
3. How much income risk do macro and micro shocks imply?
   - Compensating-variation estimates of the cost of risk.

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   - Persistence goes up for low-η/good micro shocks, down for high-η/bad micro shocks.
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   - Short-lived responses to macro shocks + persistent responses to micro shocks.
   - Distributional responses to macro shocks are U-shaped in the micro ranks.

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3. How much income risk do macro and micro shocks imply?
   - Large cost of business cycle risk stems from nonlinear micro impact of macro shocks.

Selected literature

- Income dynamics (with and without business cycles): Storesletten, Telmer, Yaron (2004), Guvenen, Ozkan, Song (2014), Arellano, Blundell, Bonhomme (2017), Guvenen, McKay, Ryan (2022), Guvenen, Pistaferri, Violante (2022), GRID project, ...

- Estimation of heterogeneous agents models using micro data: Arellano, Bonhomme (2017), Liu, Plagborg-Møller (2023), Fernández-Villaverde, Hurtado, Nuño (2023), ...

- Econometrics of macro-micro data: Tobin (1950), Chetty (1968), Maddala (1971), Hahn, Kuersteiner, Mazzocco (2020), Chang, Chen, Schorfheide (2024), Almuzara, Sancibrián (2025), ...

- Inequality and business cycles, welfare cost of fluctuations: Krusell, Smith (1998), Bhandari, Evans, Golosov, Sargent (2021), Lucas (1987, 2003), ...

Outline

1. Framework
   - Model & objects of interest
   - Identification & estimation
2. Macro shocks and nonlinear dynamics
3. Macro/micro impulse responses
4. Macro/micro risk quantification
5. Conclusion

Framework

Model & objects of interest

Model: nonlinear income process

- Persistent-transitory decomposition:

$$y_{it} = \eta_{it} + \varepsilon_{it}.$$

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  u_{it} serially independent $U(0,1)$ rv's, independent of $\eta_{i1}, \varepsilon_{i\tau}$.

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Model: nonlinear income process

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$$y_{it} = \eta_{it} + \varepsilon_{it}.$$

- Markovian persistent component with macro state Z_t:

$$\eta_{it} = Q_\eta(\eta_{i,t-1}, Z_t, Z_{t-1}, u_{it}),$$

u_{it} serially independent $U(0,1)$ rv's, independent of $\eta_{it_0}, \varepsilon_{i\tau}, Z_\tau$.

- Initial condition and transitory component:

$$\eta_{it_0} = Q_{\text{init},t_0}(\nu_{it_0}),$$
$$\varepsilon_{it} = Q_{\varepsilon,t}(v_{it}).$$

Model: business cycle indicator

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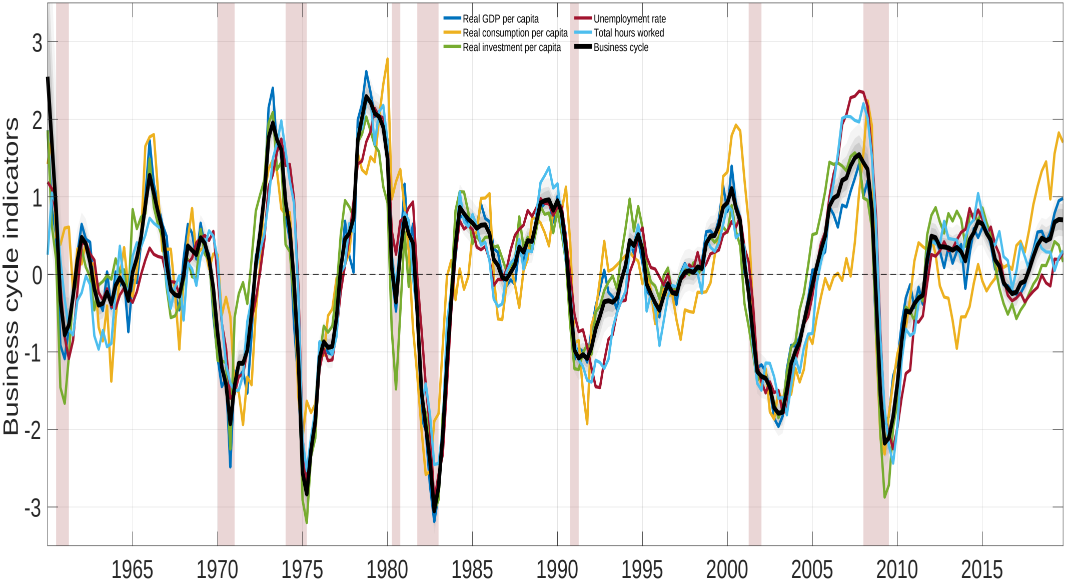
- Markovian business cycle state:

$$Z_t = Q_Z(Z_{t-1}, V_t) = \Phi Z_{t-1} + V_t$$

  V_t serially independent $N(0, \Sigma)$ rv's, independent of E_t.

- $E_{jt} = \phi_j E_{j,t-1} + e_{jt}$ mutually independent with $e_{jt} \sim N(0, \sigma_j^2)$.

Model: macro state in US data



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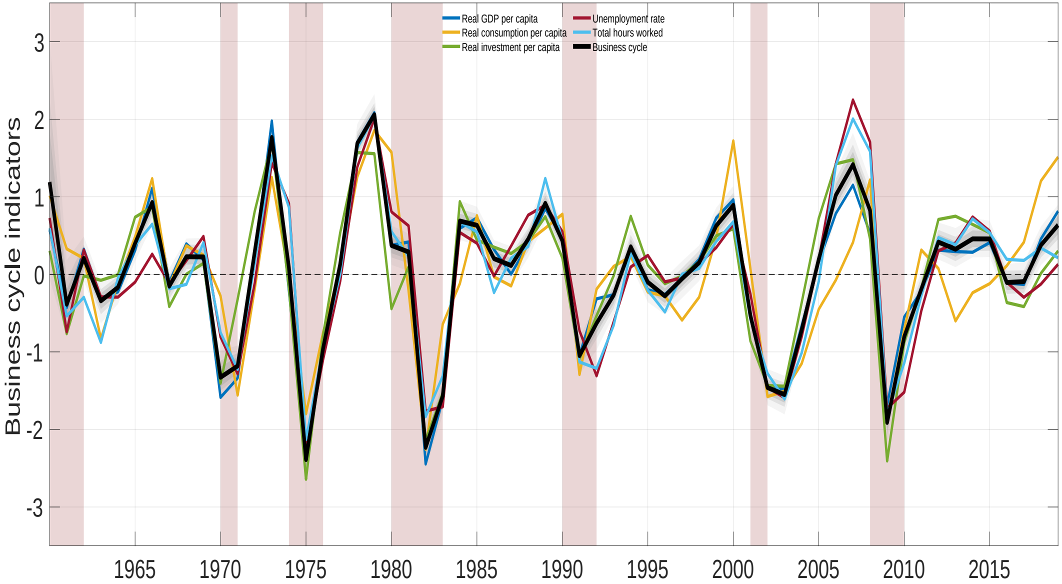

Objects of interest

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- Persistence:

$$\rho(u_{it}, \eta_{i,t-1}, Z_t, Z_{t-1}) = \frac{\partial Q_\eta(\eta_{i,t-1}, Z_t, Z_{t-1}, u_{it})}{\partial \eta_{i,t-1}}.$$

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- Skewness:

$$\text{sk}(\eta, Z_t, Z_{t-1}) = \frac{Q_\eta(\eta, Z_t, Z_{t-1}, 0.9) + Q_\eta(\eta, Z_t, Z_{t-1}, 0.1) - 2Q_\eta(\eta, Z_t, Z_{t-1}, 0.5)}{Q_\eta(\eta, Z_t, Z_{t-1}, 0.9) - Q_\eta(\eta, Z_t, Z_{t-1}, 0.1)}.$$

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- More: macro/micro IRFs + macro/micro risk quantification.

Framework

Identification & estimation

Identification: time series of panels

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  where $\mathcal{I}_t$ is a set of cross-sectional indexes of size N_t.

- S fixed to balance identification with credibly representative samples:
  - In between repeated cross-sections and longitudinal panels.
  - Precursor: Storesletten, Telmer, Yaron (2004).

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  ➡ $Q_{\eta,t}(\eta, u)$ is identified for each t by ABB techniques ($S \geq 4$).

Identification: large N, T with small S

- **Final step:** Pin down $Q_\eta(\eta, \tilde{Z}, Z, u)$ by linking $Q_{\eta,t}(\eta, u)$ to $\{Z_t\}$.
  - $Q_{\eta,t}(\eta, u)$ is equivalent to the time-t transition probability "matrix" $F_{\eta,t}(\tilde{\eta}|\eta)$.
  - Regressing $F_{\eta,t}(\tilde{\eta}|\eta)$ nonparametrically on $w_t^S = \{W_{t+s}\}_{s=0}^{S-1}$,

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  - Under injectivity conditions, we can solve this equation for $Q_\eta(\eta, \tilde{Z}, Z, u)$.

- Key is the concurrence of time series of aggregates and panels:
  - Many short income histories subject to recessions/expansions over many cycles.

Data: time series of panels

- PSID:
  - Interviewed an initial sample representative of US households in 1968.
  - Thereafter, kept track of initial households and offspring + refresher/immigrant samples.
  - Interviews are annual between 1968 to 1997, biennial between 1999 to 2019.

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  - Each panel has $S = 4$, made biennial for comparability (but we use all years).
  - $y =$ log income net of education/family size/state of residence/time trends/etc.
    - Male earnings: labor income of representative person (age 25-to-60/male/married).
    - Disposable income: labor income of representative person and spouse + transfers - taxes.
  - We include age as additional argument in $Q_\eta, Q_{\text{init},t_0}$ but show results averaged over age.

Estimation: flexible parametric model

- Flexible specification of Q_η:

$$Q_\eta(\eta, Z_t, Z_{t-1}, u) = \psi(\eta)'\Theta(u)\varphi(Z_t, Z_{t-1}) = \sum_j \sum_k \psi_j(\eta)\theta_{jk}(u)\varphi_k(Z_t, Z_{t-1}),$$

  - $\psi(\cdot), \varphi(\cdot,\cdot)$: vectors of known basis functions (e.g., orthogonal polynomials).
  - $\Theta(\cdot)$: matrix of linear splines switching to exponential in the tails $\implies$ parameters θ.

- Flexible time-varying specification of $Q_{\varepsilon,t}$ and Q_{init,t_0}.

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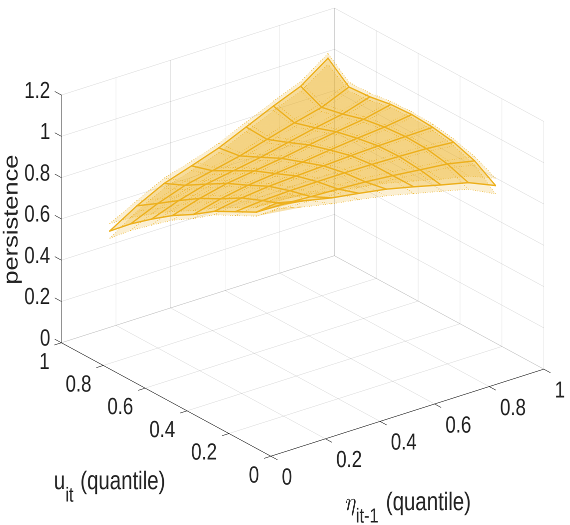
- We study the econometrics of this class of models:
  - Simulation-based estimation algorithm. ▸ Pseudo stochastic EM
  - Asymptotic approximations. ▸ Large-sample properties
  - Bootstrap inference accounting for cross-sectional + unit-level dependence. ▸ Bootstrap

- ▸ Empirical specification and model fit

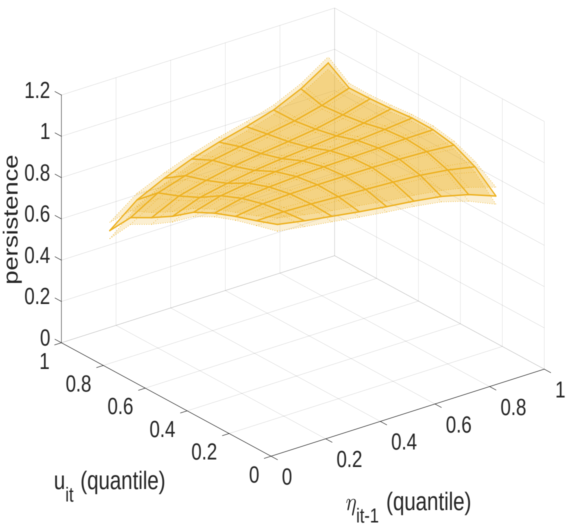
Macro shocks and nonlinear dynamics

Nonlinear persistence is macro and micro state-dependent: $\rho(u, \eta, Z_t, Z_{t-1})$

(a) Male earnings



(b) Disposable income



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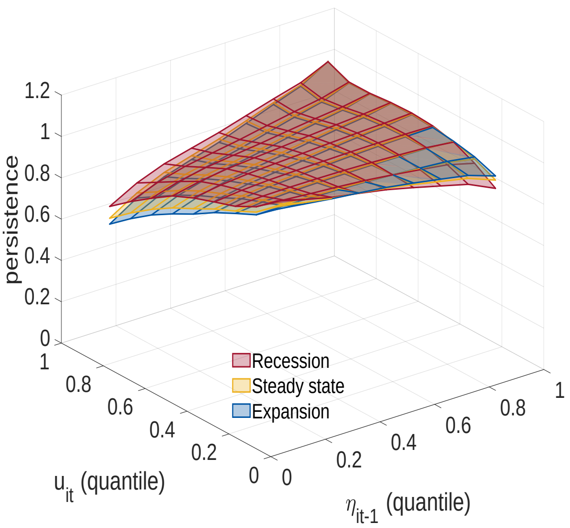

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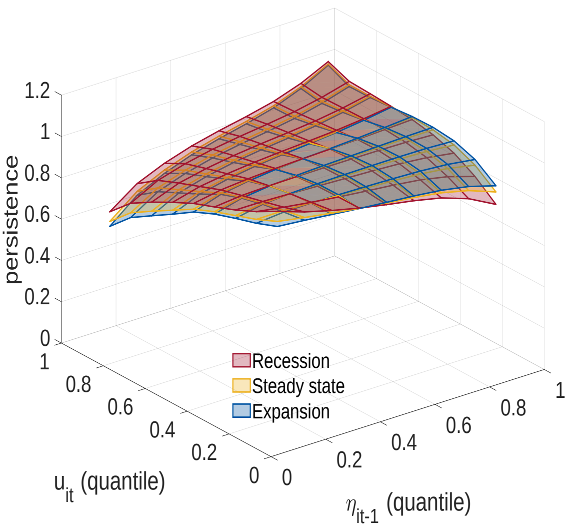

Differences in persistence between recessions and expansions

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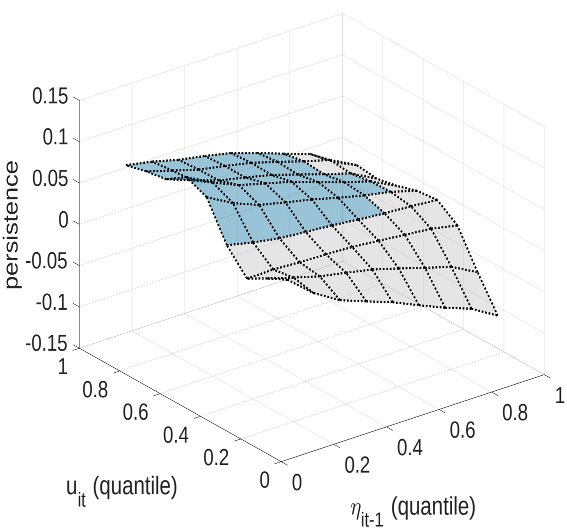

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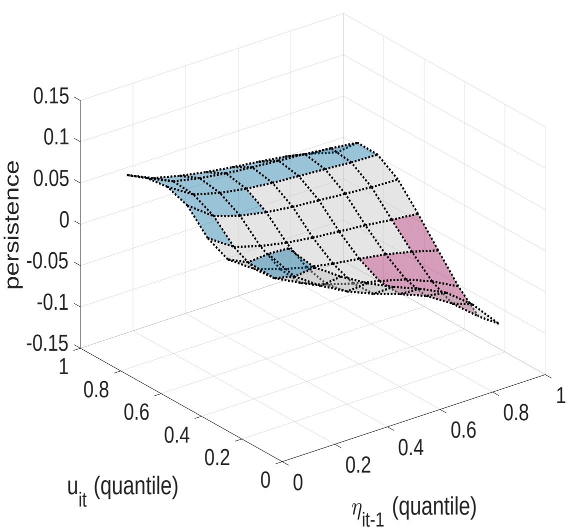
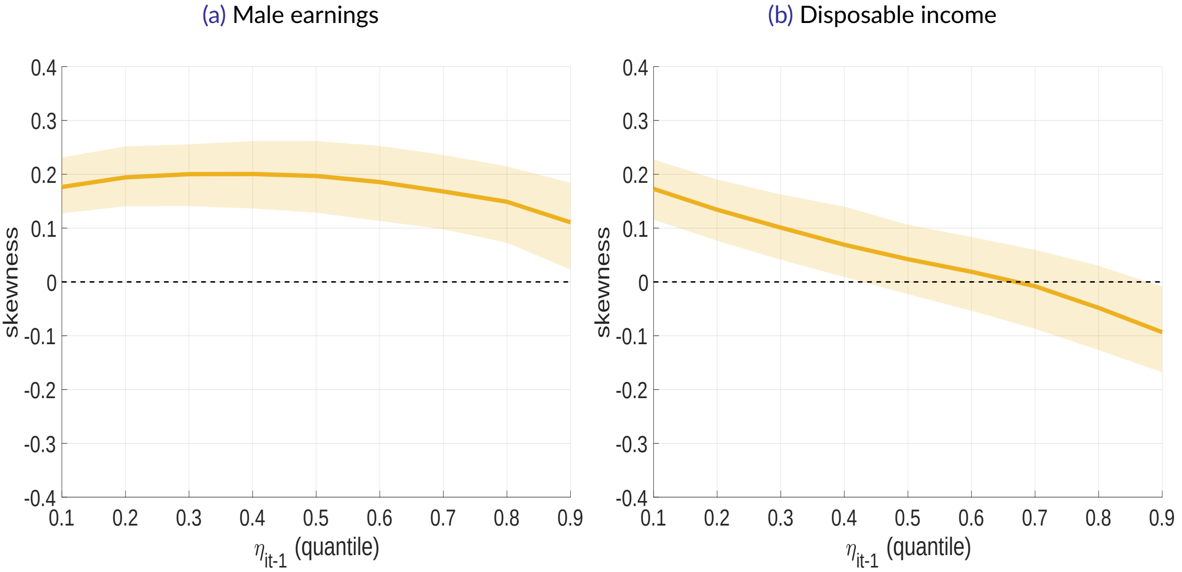


A tale of two skewnesses: sk(η, Z_t, Z_{t-1})



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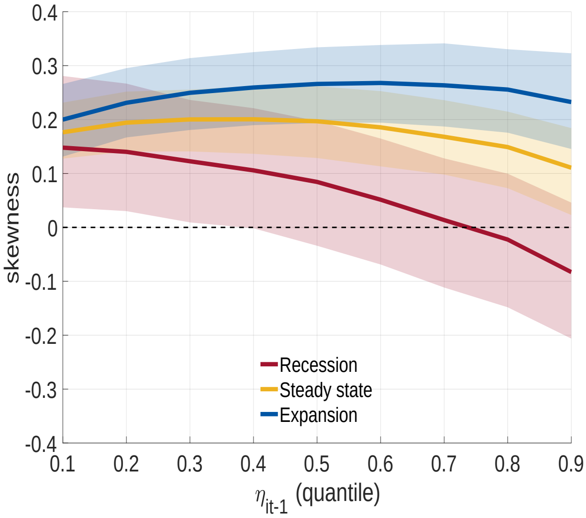

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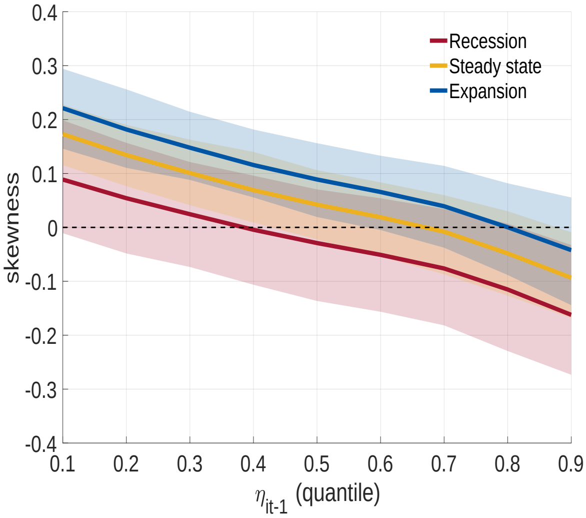

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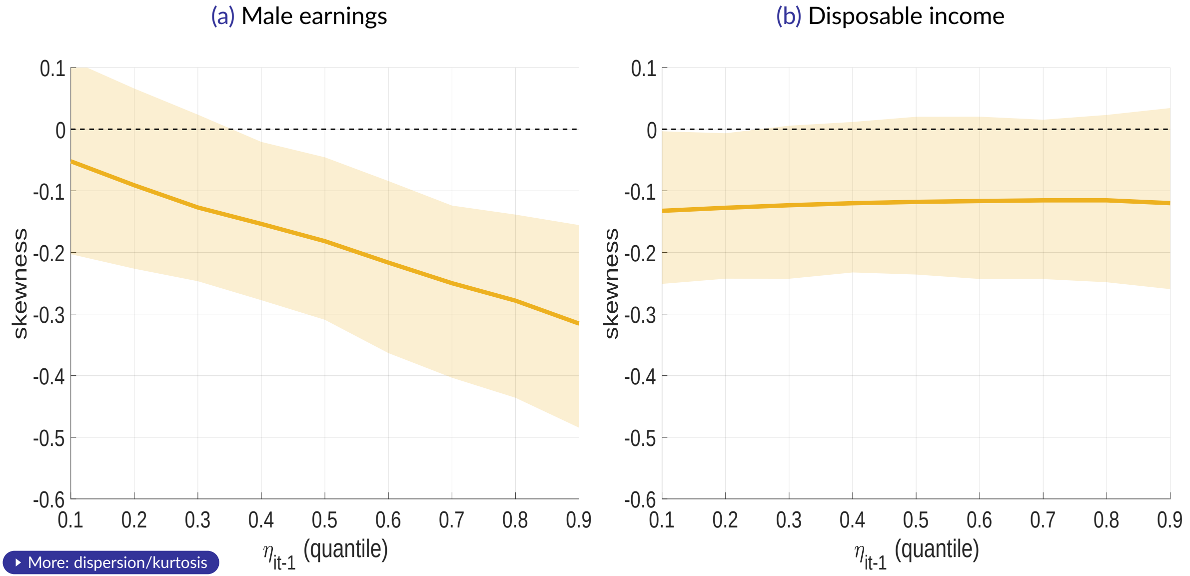
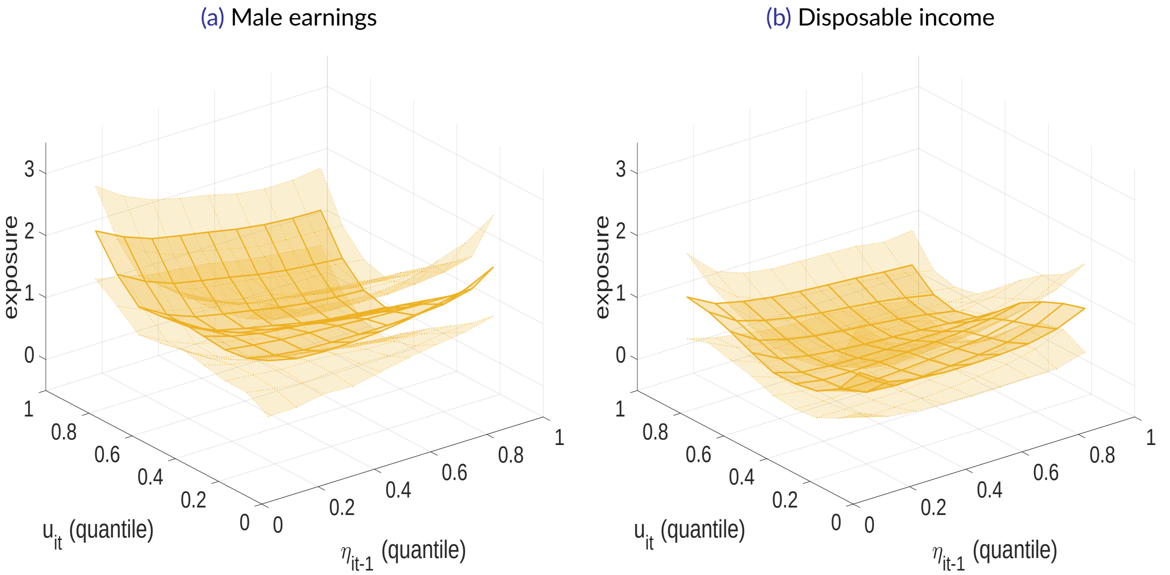


▸ More: dispersion/kurtosis

Exposures to aggregate shocks are countercyclical: $\beta(u, \eta, Z_t, Z_{t-1})$



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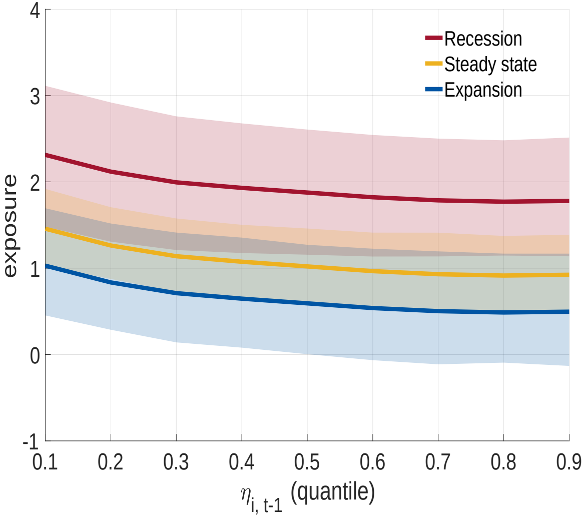

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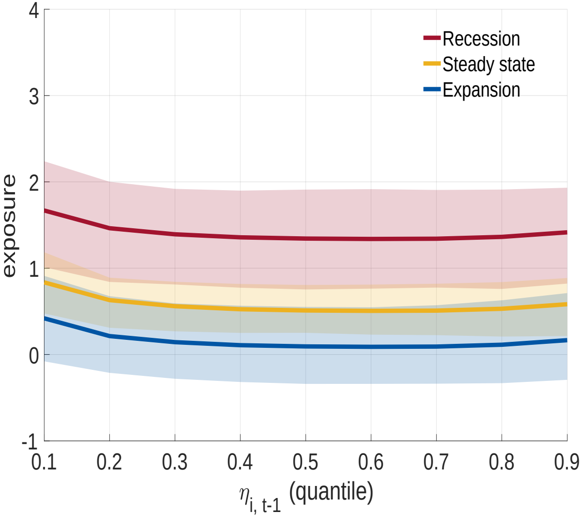

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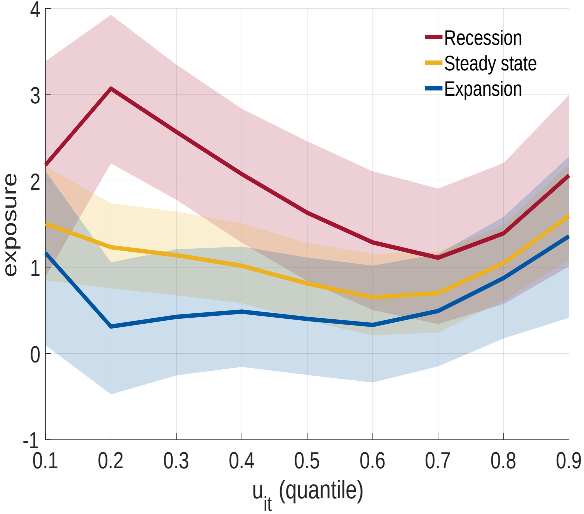

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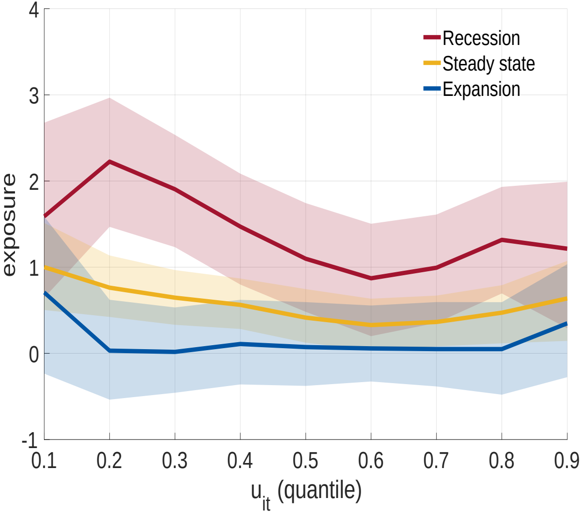

Macro/micro impulse responses

Nonlinear macro/micro IRFs

- We extend to our nonlinear macro/micro setup the idea in Gallant, Rossi, Tauchen (1993):
  - Fix initial state benchmark value, perturb it and track the evolution of outcomes.

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  - Fix initial state benchmark value, perturb it and track the evolution of outcomes.

- Macro impulse responses. Perturbation $g(Z_t^b) = g(Z_t^b + \Delta(\delta)) - \delta$:

$$\mathrm{IRF}_{\eta Z}(h,\delta) = \frac{\mathrm{E}\left[\eta_{i,t+h} \,\middle|\, \eta_{i,t-1}, Z_t^b + \Delta(\delta), Z_{t-1}\right] - \mathrm{E}\left[\eta_{i,t+h} \,\middle|\, \eta_{i,t-1}, Z_t^b, Z_{t-1}\right]}{\delta}.$$

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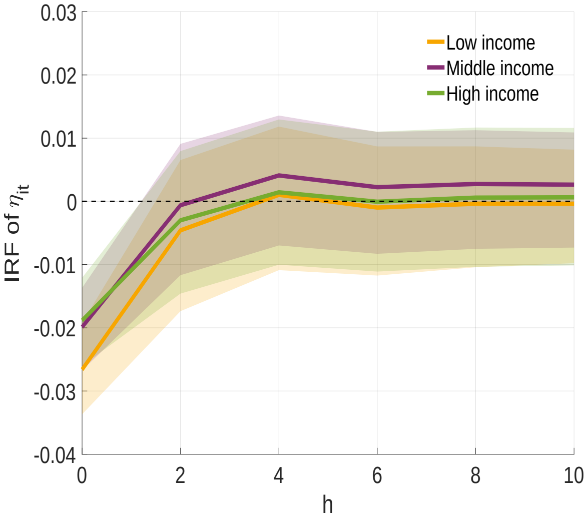
- Micro impulse responses. Perturbation $g(\eta_{i,t-1}^b) = g(\eta_{i,t-1}^b + \Delta(\delta)) - \delta$:

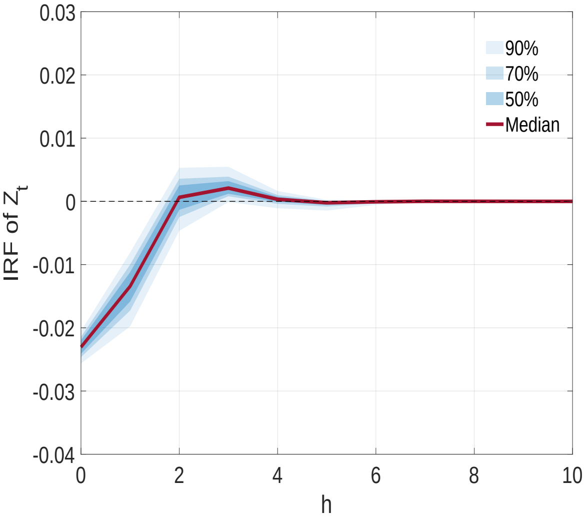
$$\mathrm{IRF}_{\eta\eta}(h,\delta) = \frac{\mathrm{E}\left[\eta_{i,t+h} \,\middle|\, \eta_{i,t-1}^b + \Delta(\delta), Z_t, Z_{t-1}\right] - \mathrm{E}\left[\eta_{i,t+h} \,\middle|\, \eta_{i,t-1}^b, Z_t, Z_{t-1}\right]}{\delta}.$$

Responses to macro shocks are short-lived

(a) Male earnings



(b) GDP



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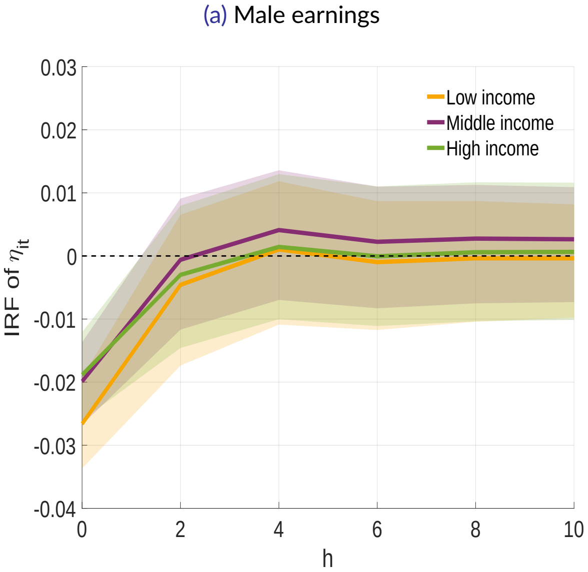
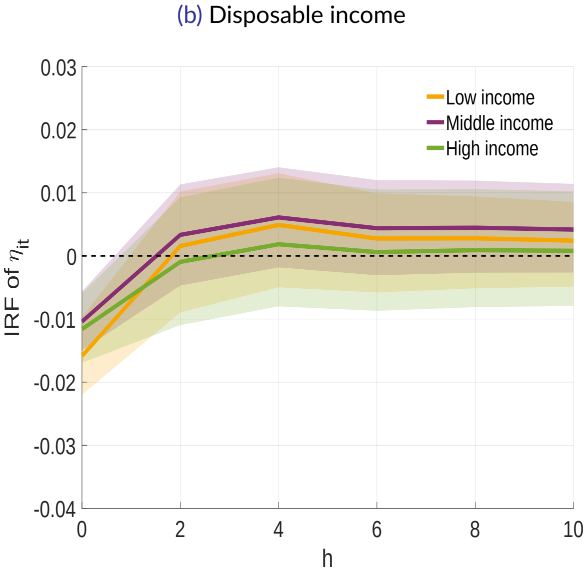
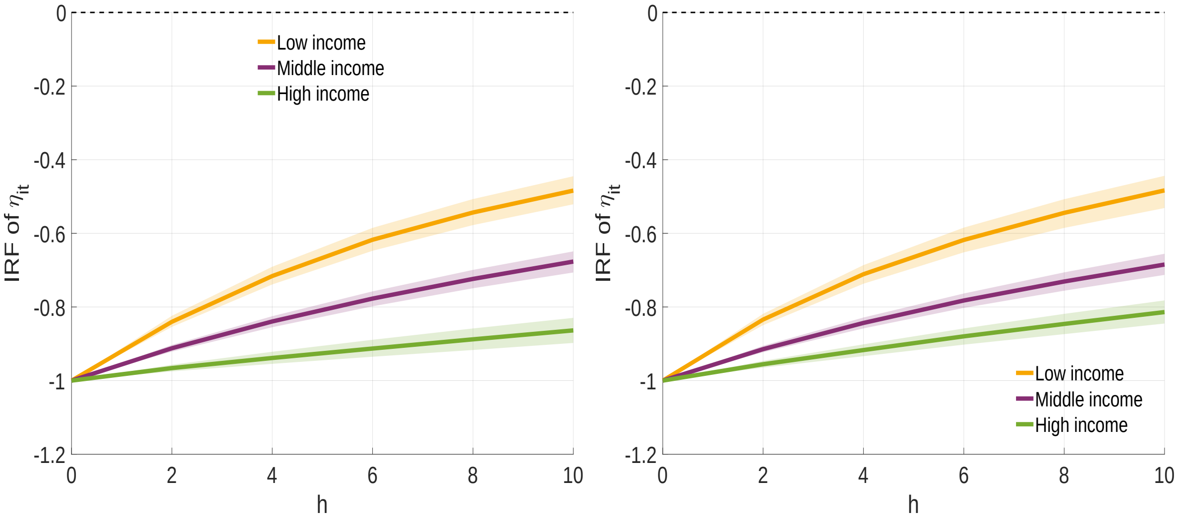




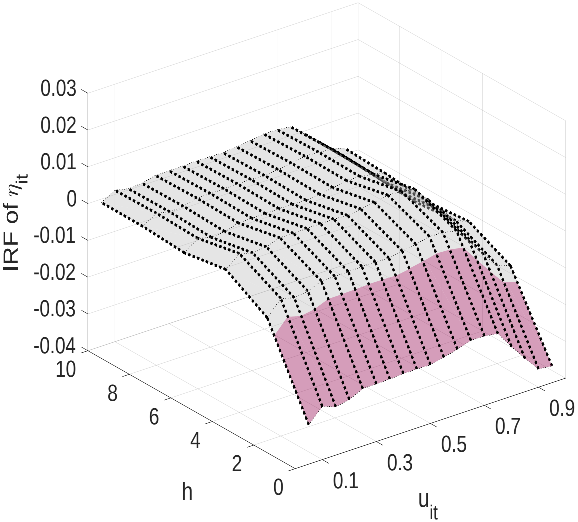
Responses to micro shocks decay slowly

(a) Male earnings　　　(b) Disposable income

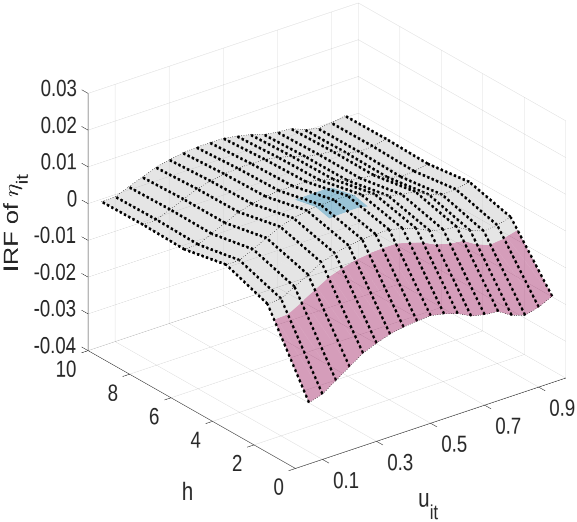


Distributional responses to macro shocks are U-shaped

(a) Male earnings (low)

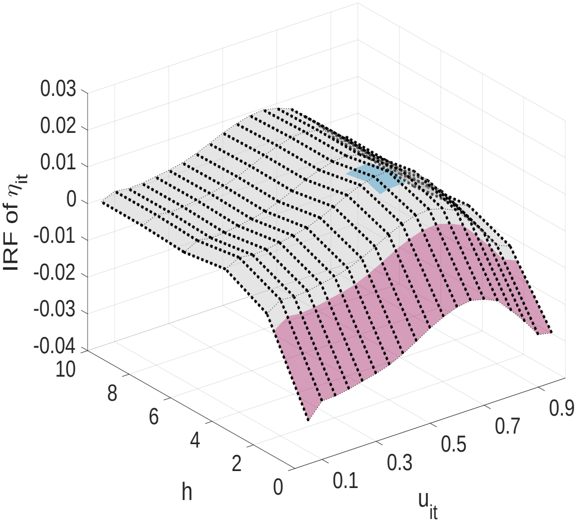


(b) Disposable income (low)

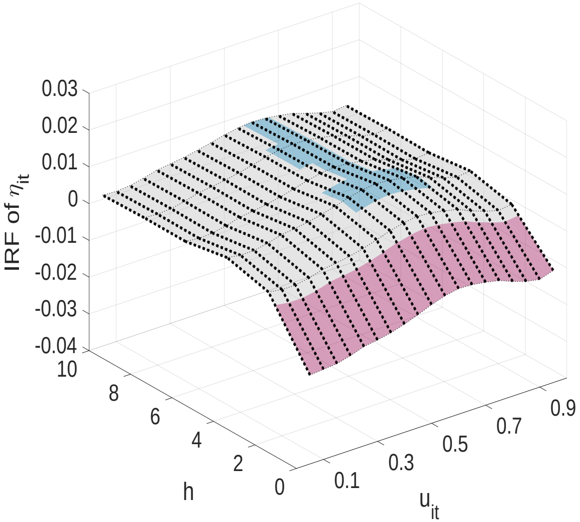


Distributional responses to macro shocks are U-shaped

(a) Male earnings (middle)

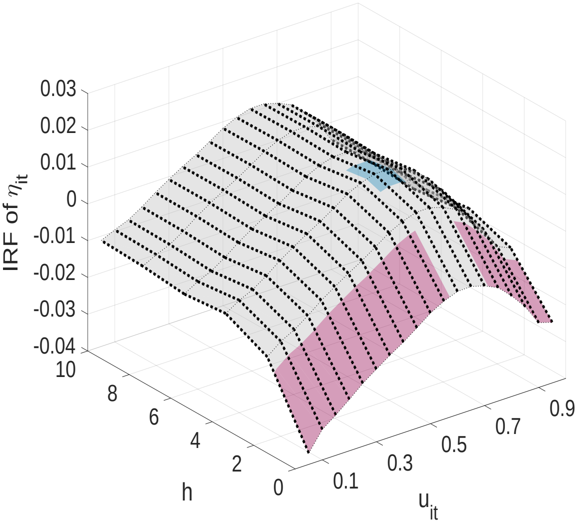


(b) Disposable income (middle)

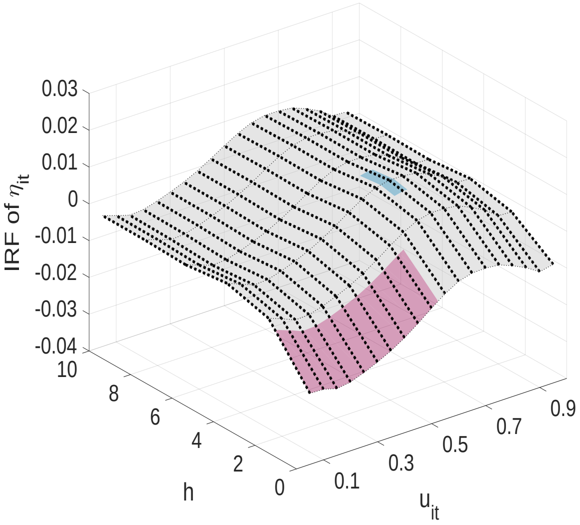


Distributional responses to macro shocks are U-shaped

(a) Male earnings (high)



(b) Disposable income (high)



Macro/micro risk quantification

Cost of business cycle risk

- Cost of macro/micro sources of income risk: find CV such that

$$\mathrm{E}\left[\sum_{h=1}^{H}\delta^h U\Big((1-\mathrm{CV})e^{\eta_{i,t+h}}\Big)\;\Bigg|\;\text{no shocks}, \eta_{it}, Z_t\right] = \mathrm{E}\left[\sum_{h=1}^{H}\delta^h U\Big(e^{\eta_{i,t+h}}\Big)\;\Bigg|\;\eta_{it}, Z_t\right].$$

Below we set $U(y) = y^{1-\gamma}/(1-\gamma)$ with $\gamma = 3$ and $\delta = (0.96)^2$.

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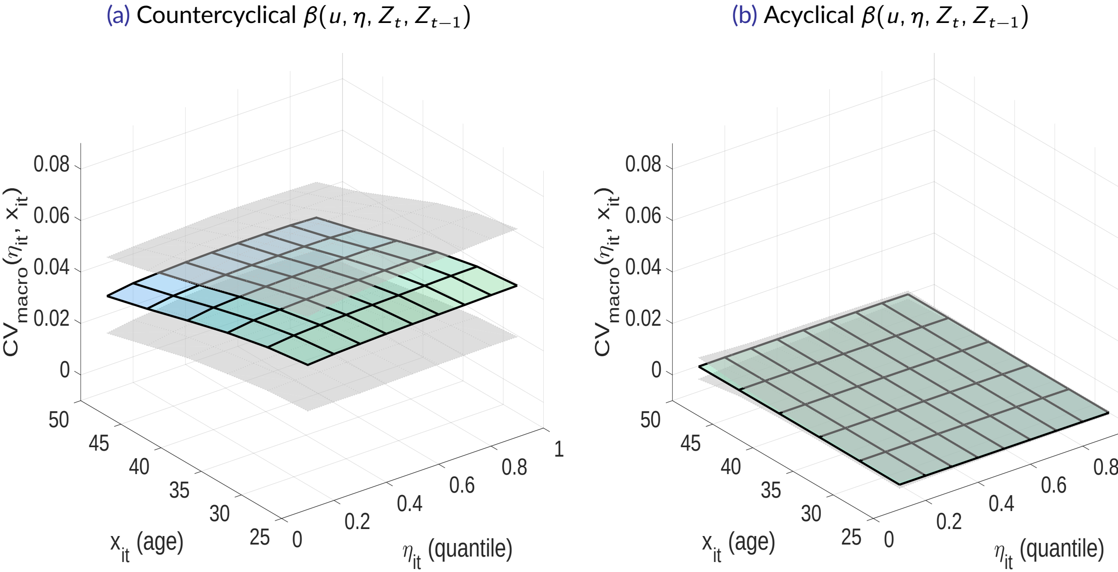
- One focus of the literature is curvature in preferences:
  - Typically need high risk-aversion to obtain even minimal costs of fluctuations.

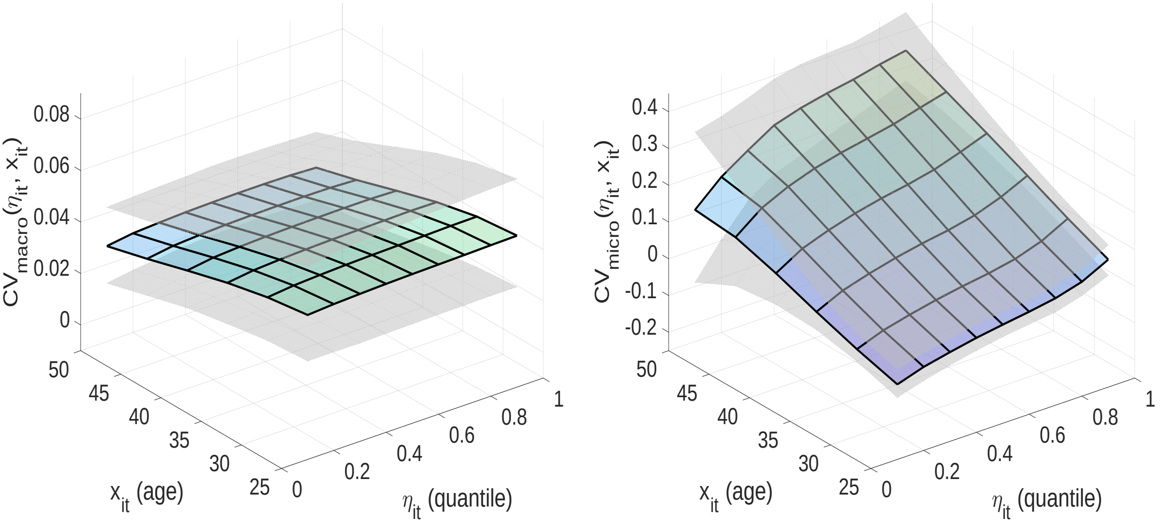
- Extra channel. Interaction between marginal utility and macro nonlinearities:
  - Countercyclical $\beta(u, \eta, Z_t, Z_{t-1})$ can generate large costs of risk! ▸ Small-noise expansion

Nonlinear exposures amplify cost of macro risk



Macro risk is comparable to micro risk for young/low-η units

(a) Macro risk (b) Micro risk



Conclusion

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- Nonlinear framework to study transmission of aggregate/idiosyncratic shocks to income, leveraging macro and micro data and identification:
  - Our setup extends to multiple macro/micro state variables and richer feedback channels.
  - Tools to build reduced forms for heterogeneous agents models with macro shocks.
- We document aggregate state-dependence in persistence, skewness, exposures.
- Cyclicality in micro exposures to macro shocks matters for welfare calculations:
  - More general interest in measuring exposure of consumption and different forms of wealth to aggregate shocks $\implies$ our framework offers an avenue to potentially do this.

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  - More general interest in measuring exposure of consumption and different forms of wealth to aggregate shocks $\Longrightarrow$ our framework offers an avenue to potentially do this.

Thank you!

Backup slides

Backup: assumptions

Assumptions

1. Macro states:
   a. Z_t, E_t satisfy model in slide 8 with $V_t, e_t \overset{\text{iid}}{\sim} N(0,1)$.
   b. There are omitted aggregate shocks $\{G_t\}$ i.i.d. over t, independent of $\{Z_t, E_t\}$.
2. Micro processes with macro states:
   a. $\{\eta_{it}, \varepsilon_{it}\}$ i.i.d. across i given $\{Z_t, E_t, G_t\} \implies$ exchangeable given $\{Z_t, E_t\}$.
   b. $\eta_{it}, \varepsilon_{it}$ satisfy model in slide 7 with $u_{it}, v_{it} \overset{\text{iid}}{\sim} U(0,1)$ conditional on $\{Z_t\}$.
3. Atomicity:
   - Micro shocks u_{it}, v_{it} are independent of macro shocks $\{V_t, e_t, G_t\}$.

Back

Backup: pseudo stochastic EM algorithm

Notation: $\overline{Z} = \{Z_t\}_{t=0}^{T+S}$, $\overline{W} = \{W_t\}_{t=1}^{T+S}$, $\bar{y}_{it}^S = \{y_{i,t+s}\}_{s=0}^{S-1}$, $\bar{Z}_t^S = \{Z_{t+s}\}_{s=0}^{S-1}$.

Estimation algorithm

Initialize $\widehat{\theta}^{(0)}$, $\{\widehat{\delta}_t^{(0)}\}_{t=1}^{T}$. For $j = 1, ..., J$, iterate between the following:

1. Pseudo-Stochastic E step:
   - (i) draw $\overline{Z}(j) = \{Z_t(j)\}_{t=0}^{T+S}$ from the macro posterior $f\left(\overline{Z}\middle|\overline{W}, \widehat{\lambda}\right)$,
   - (ii) independently over units i and subpanels t, draw $\bar{\eta}_{it}^S(j) = \{\eta_{i,t+s}(j)\}_{s=0}^{S-1}$ from the micro posterior $f\left(\bar{\eta}_{it}^S\middle|\bar{y}_{it}^S, \bar{Z}^S(j), \widehat{\theta}^{(j-1)}, \widehat{\delta}_t^{(j-1)}\right)$.
2. Pseudo M step:
   - (i) update parameters to $\widehat{\theta}^{(j)}$ and $\{\widehat{\delta}_t^{(j)}\}_{t=1}^{T}$ by quantile and exponential regressions treating $\left\{\{\{\eta_{i,t+s}(j), y_{i,t+s}, x_{i,t+s}, Z_{t+s}(j)\}_{s=0}^{S-1}\}_{i\in\mathcal{I}_t}\right\}_{t=1}^{T}$ as data.

For some $\mu \in (0,1)$, set $\widehat{\theta} = (\mu J)^{-1}\sum_{j=(1-\mu)J}^{J}\widehat{\theta}^{(j)}$ and $\widehat{\delta}_t = \sum_{j=(1-\mu)J}^{J}\widehat{\delta}_t^{(j)}$.

Back

Backup: large-sample properties

- Plug-in estimator of summaries: $\widehat{\gamma} = \gamma(\widehat{\theta})$ for some smooth function γ.

- Estimator $\widehat{\theta}$ sets to zero the sample counterpart to doubly-integrated moments:

$$\mathrm{E}\left[\int \left(\frac{1}{N_t} \sum_{i \in \mathcal{I}_t} \int m_\theta(\theta_0; \bar{y}_{it}^S, \bar{\eta}^S, \bar{Z}_t^S) f\left(\bar{\eta}^S \middle| \bar{y}_{it}^S, \bar{Z}_t^S, \theta_0, \delta_t\right) d\bar{\eta}^S \right) f\left(\bar{Z}^S \middle| \overline{W}, \lambda_0\right) d\bar{Z}^S \right] = 0$$

  where m_θ collects quantile/exponential regression orthogonality conditions.
  - Taylor expansions of moment conditions around $(\theta_0, \lambda_0) \implies$ asymptotic distribution.

Asymptotic approximations

As $N, T \to \infty$, $\sqrt{T}(\widehat{\theta} - \theta_0) \xrightarrow{\mathrm{d}} N\left(0, \Sigma_{\theta_0}\right)$ and $\sqrt{T}(\widehat{\gamma} - \gamma_0) \xrightarrow{\mathrm{d}} N\left(0, J_{\theta_0} \Sigma_{\theta_0} J_{\theta_0}'\right)$ with Σ_{θ_0} symmetric, positive semi-definite and J_{θ_0} the Jacobian of γ evaluated at θ_0.

Back

Backup: bootstrap

- Omitted aggregate factors:
  - Common shocks $G_t = (G_{\eta,t}, G_{\varepsilon,t}, G_{\text{init},t}) \overset{\text{iid}}{\sim} U(0,1)$ drive cross-sectional dependence.
  - Micro ranks generated via Gaussian copula:

$$u_{it} = \Phi\left(c_\eta \Phi^{-1}(G_{\eta,t}) + \sqrt{1-c_\eta^2}\,\Phi^{-1}(\tilde{u}_{it})\right), \text{ similarly for } v_{it}, \nu_{i,t_0}.$$

  - Parameters $c_\eta, c_\varepsilon, c_{\text{init}}$ estimated by sample mean of copula-inverted ranks over EM draws.

- Unit overlap:
  - For individuals in consecutive odd/even panels, idiosyncratic ranks follow

$$\left(\Phi^{-1}(\tilde{u}_{it}) \quad \Phi^{-1}(\tilde{u}_{it'})\right)' \sim N\left(0, \begin{pmatrix} 1 & d_\eta \\ d_\eta & 1 \end{pmatrix}\right), \text{ similarly for } \tilde{v}_{it}, \tilde{\nu}_{i,t_0}.$$

  - Parameters $d_\eta, d_\varepsilon, d_{\text{init}}$ estimated alongside $c_\eta, c_\varepsilon, c_{\text{init}}$ within EM.

Backup: bootstrap

Bootstrap approach

Given estimates of $(c_\eta, c_\varepsilon, c_{\text{init}}, d_\eta, d_\varepsilon, d_{\text{init}})$, do the following:

1. Simulate the time series of aggregate factors $\{G_{\eta,t}, G_{\varepsilon,t}, G_{\text{init},t}\}_{t=1}^{T}$.
2. For each unit i determine the first (t_0) and last (t_1) period in the dataset. Next,
   - i. draw the path of idiosyncratic shocks $\{\tilde{u}_{it}, \tilde{v}_{it}, \tilde{\nu}_{it}\}_{t_0 \le t \le t_1}$ imposing the correlations d_η, d_ε and d_{init} across consecutive periods;
   - ii. combine aggregate and idiosyncratic factors to obtain $\{u_{it}, v_{it}, \nu_{it}\}_{t_0 \le t \le t_1}$ imposing the cross-sectional dependence implied by c_η, c_ε and c_{init};
   - iii. for the first two periods, use $Q_{\text{init},t}$ and ν_{it} to generate η_{it};
   - iv. for every other period, use Q_η and u_{it} to generate η_{it};
   - v. for all periods, use $Q_{\varepsilon,t}$ and v_{it} to generate ε_{it};
   - vi. form $y_{it} = \eta_{it} + \varepsilon_{it}$ for all $t_0 \le t \le t_1$.
3. Assign the data to the appropriate unit and time cell.

▸ Back

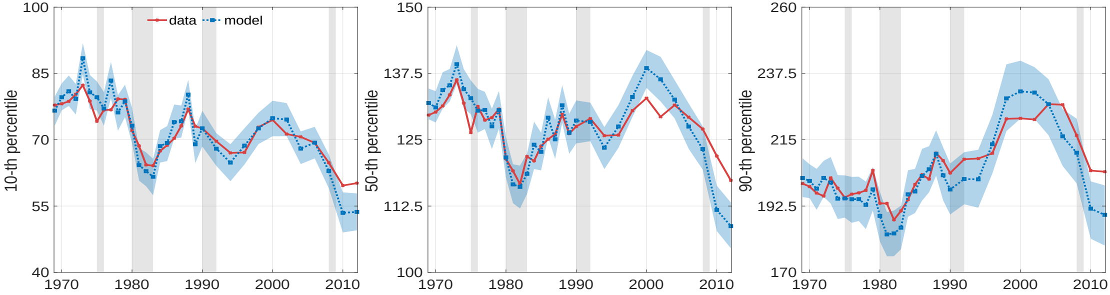
Backup: empirical specification

- Specification of Q_η:
  - ψ is third-order Hermite polyn on η $\times$ second-order Hermite polyn on age x.
  - φ is second-order Hermite polyn on (Z_t, Z_{t-1}) but with restrictions:
    - linear term excluded from (η, x) interactions, quadratic term only in the intercept.
  - Grid on rank space $L = 11$.

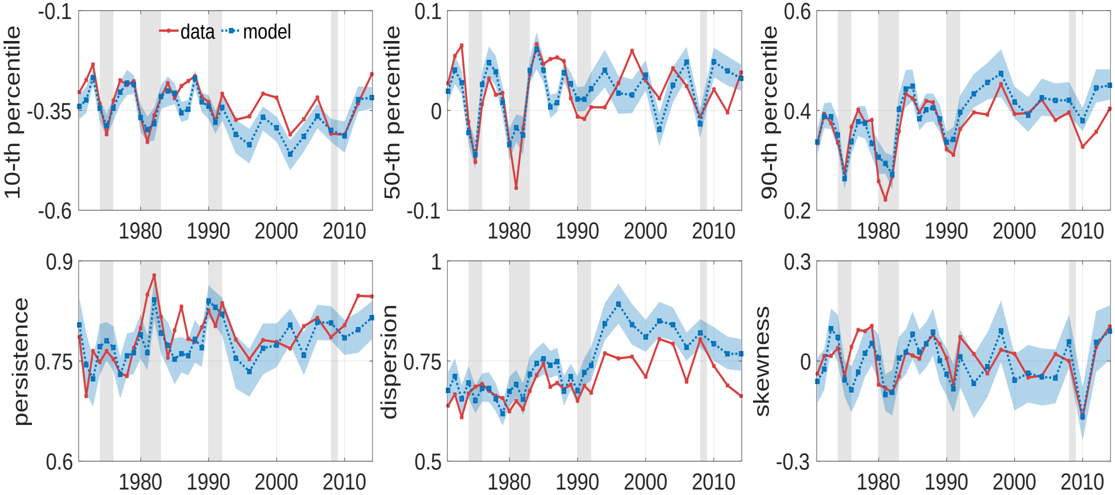
- Specification of $Q_{\varepsilon,t}$:
  - Time effects + $L = 11$.

- Specification of Q_{init,t_0}:
  - Time effects + second-order Hermite polyn on age x + $L = 11$.

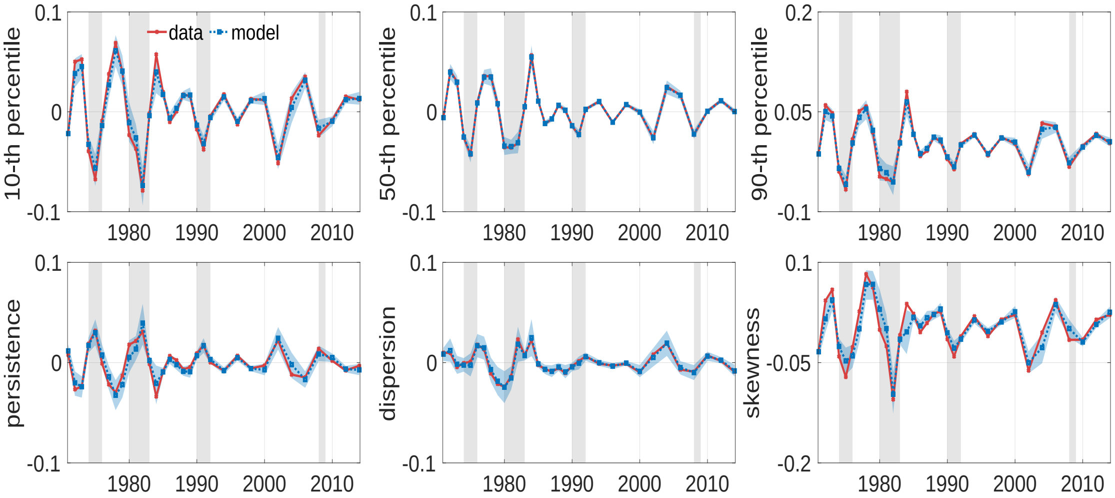
Backup: model fit (income level, thousands of 2016 US$)



Backup: model fit (biennial income growth)

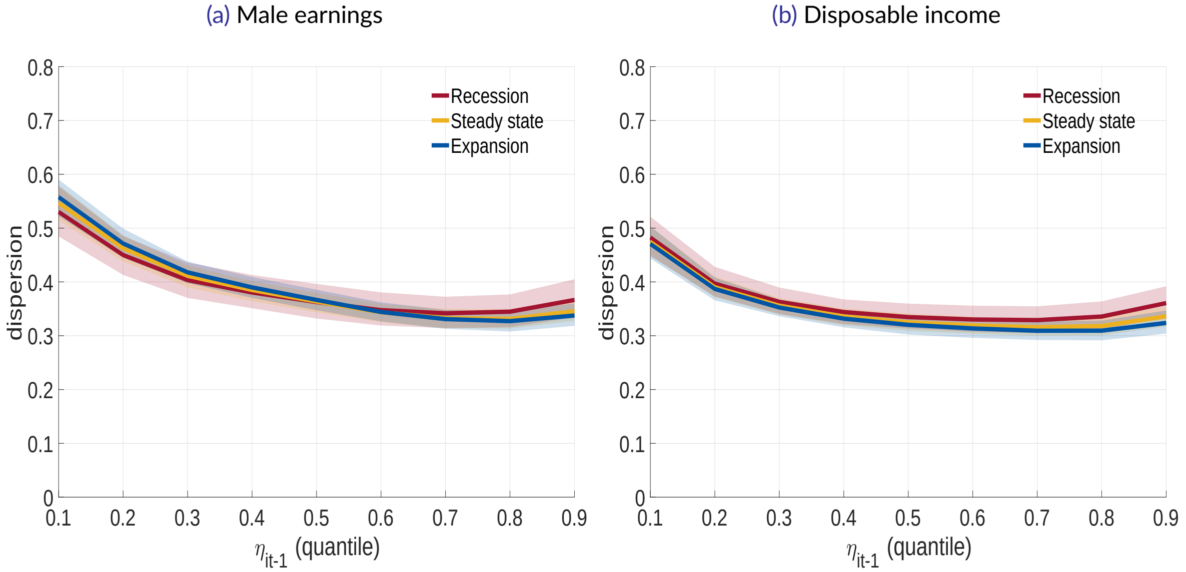


Backup: model fit (biennial income growth, projection on aggregate state)



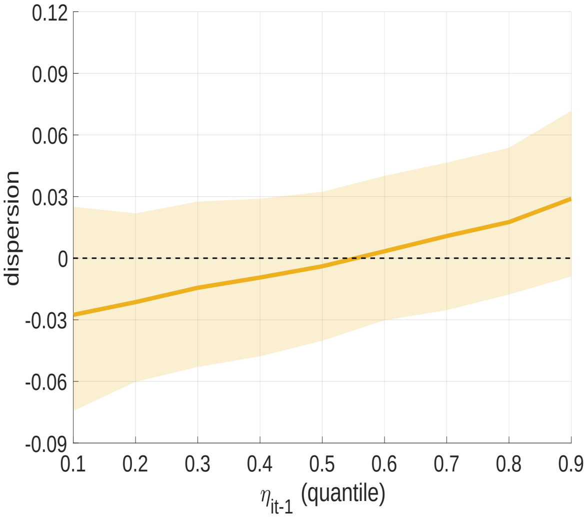
Back

Backup: dispersion

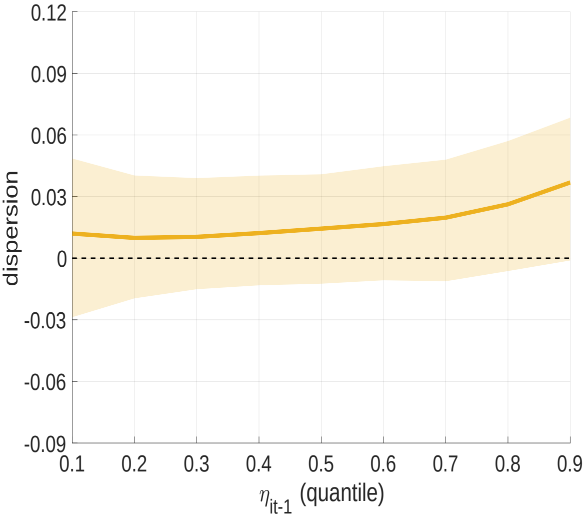


Backup: dispersion (difference between recessions and expansions)

(a) Male earnings

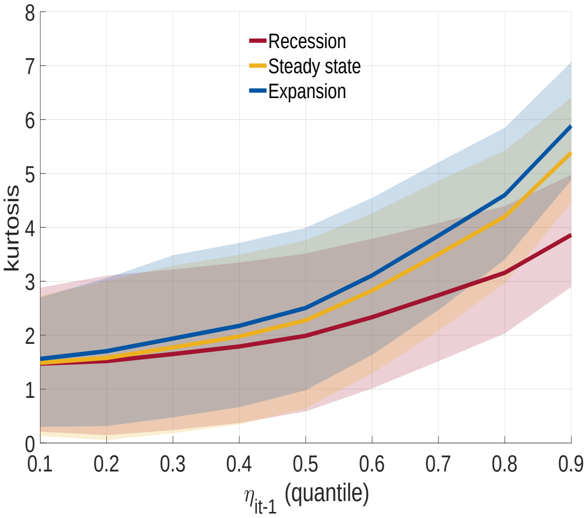


(b) Disposable income

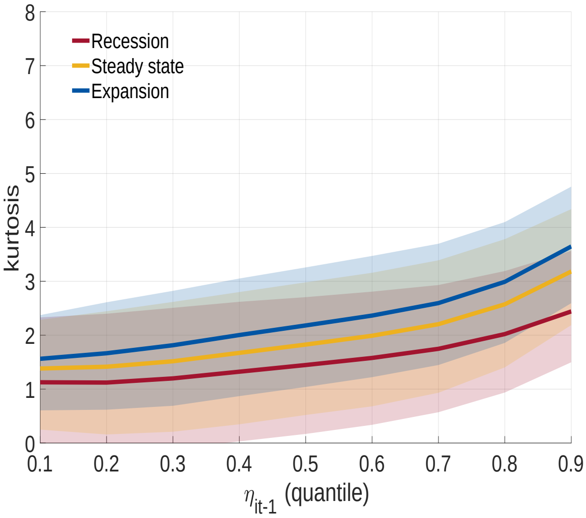


Backup: kurtosis

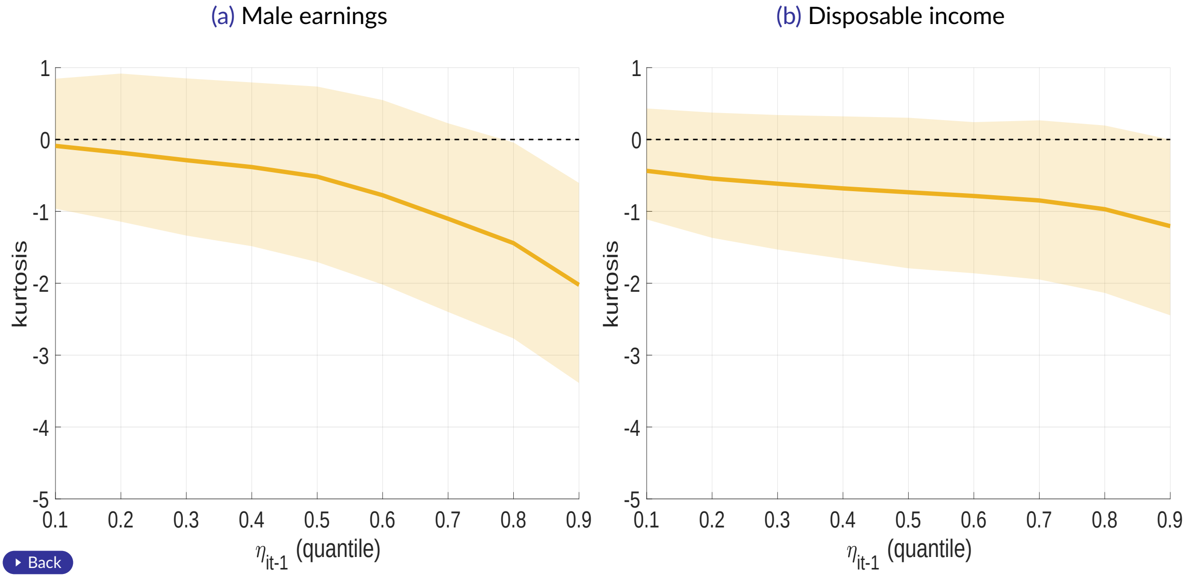
(a) Male earnings



(b) Disposable income



Backup: kurtosis (difference between recessions and expansions)



▸ Back

Risk calculation: small-noise approximation

- Small-noise second-order Taylor expansion of compensating variation:
  - Given η_{it}, Z_t, write $\eta_{i,t+h} = \bar{\eta}_{it}^h(u_{it}^h, V_t^h)$ with $u_{it}^h = \{u_{i,t+\ell}\}_{\ell=1}^h$, $V_t^h = \{V_{t+\ell}\}_{\ell=1}^h$.
  - Recenter u_{it} around zero and scale u_{it}^h, V_t^h by σ_u, σ_V.
  - Curvature is determined by derivatives of $\tilde{U}(y) = U(e^y)$.

- Compensating variation for macro risk (for $\sigma_u = 0$ and $\sigma_V \to 0$):

$$\mathrm{CV}_{\text{macro}} \approx -\frac{\sum_{h=1}^H \delta^h \sum_{\ell=1}^h \left(\tilde{U}''(\bar{\eta}_{it}^h(0,0)) \left[\frac{\partial \bar{\eta}_{it}^h(0,0)}{\partial V_{t+\ell}} \right]^2 + \tilde{U}'(\bar{\eta}_{it}^h(0,0)) \left[\frac{\partial^2 \bar{\eta}_{it}^h(0,0)}{\partial V_{t+\ell}^2} \right] \right)}{\sum_{h=1}^H \delta^h \tilde{U}'(\bar{\eta}_{it}^h(0,0))}.$$

- Under log-utility, $\tilde{U}'(y) = 1$ and $\tilde{U}''(y) = 0$:

$$\mathrm{CV}_{\text{macro}} \approx -\sum_{h=1}^H \underbrace{\frac{\delta^{h-1}(1-\delta)}{(1-\delta^H)}}_{>0,\text{ sum to 1}} \sum_{\ell=1}^h \underbrace{\left[\frac{\partial^2 \bar{\eta}_{it}^h(0,0)}{\partial V_{t+\ell}^2} \right]}_{<0 \text{ for countercyclical } \beta}.$$

▸ Back