Causal Inference for Asset Pricing

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- Quantitative demand systems
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  How to interpret estimates? Implicit assumptions on spillovers?

- Quantitative demand systems
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  Which results are robust outside of these models and which are specific to these structures?

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Elasticity matrix: sensitivity of demand to prices

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Elasticity matrix: sensitivity of demand to prices

- Defined in any theory
- Could be log, levels, shares, changes or not, ...
- Flipside: price impact, how do shifts in demand affect prices?

An Example: CalPERS and Corporate Bonds

- Prices have moved **and no other news**. CalPERS adjusts its bond portfolio:

	Price change	Change in position
1. 10-yr Ford	+ 5%	sell 200
2. 10-yr GM	+ 2%	sell 100
3. 5-yr First Solar	- 1%	buy 100
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$$\Delta D_1 = \underbrace{\mathcal{E}_{11}\Delta P_1}_{\text{became more expensive}} + \underbrace{\mathcal{E}_{12}\Delta P_2}_{\text{substitutes from GM}} + \underbrace{\sum_{k\geq 3} \mathcal{E}_{1k}\Delta P_k}_{\text{substitutes from First Solar, ...}}$$

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$\rightarrow$ **Stuck without additional assumptions**

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Two broad paths:

- *Structural approach:* choose a microfoundation and estimate the corresponding model
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  $\rightarrow$ *Portfolio choice, not asset choice!*

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Two broad paths:

- *Structural approach:* choose a microfoundation and estimate the corresponding model
- *Causal inference:* impose elementary restriction keeping as much flexibility on mechanism as possible while letting the data speak
- **Our assumption: homogeneous substitution conditional on observables**
  - When the price of a given bond moves, CalPERS changes positions in other bonds based on their observables (e.g. duration, greenness) only

Homogeneous Substitution Conditional on Observables

	Initial position	Price change	New position
1. 10-yr Ford	1,500	+ 5%	$\downarrow$ 1,300
2. 10-yr GM	1,500	+ 2%	$\downarrow$ 1,400
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$\vdots$	$\vdots$	$\vdots$	$\vdots$

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$\rightarrow$ *comparing assets with same observables differences out substitution*

Formal Setup

- **Homogeneous substitution conditional on observables X**

$$\boxed{\mathcal{E}_{il} = \mathcal{E}_{jl} \text{ if } X_i = X_j} \quad \text{for all } i, j \in \mathcal{S}, \text{ and } l \neq i, j,$$

  - If price of 3rd asset move, response of demand for 2 assets with same observables is the same
  - Parametrize linearly: $\mathcal{E}_{il} = \mathcal{E}_{\text{cross}}(X_i, X_l) = X_i' \boldsymbol{\mathcal{E}}_X X_l$

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- **Decomposition of demand elasticity:**

$$\begin{aligned} \boldsymbol{\mathcal{E}} &= \text{relative elasticity} + \text{substitution} \\ &= \text{diagonal matrix} + X \underbrace{\boldsymbol{\mathcal{E}}_X}_{K \times K} X' \end{aligned}$$

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- **Decomposition of demand elasticity:**

$$\boldsymbol{\mathcal{E}} = \text{relative elasticity} + \text{substitution}$$
$$= \underbrace{\hat{\mathcal{E}}}_{\text{scalar}} I + X \underbrace{\boldsymbol{\mathcal{E}}_X}_{K \times K} X'$$

  - Assume constant relative elasticity $\hat{\mathcal{E}}$ for simplicity, relax in the paper

Many Potential Mechanisms

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- *Risk based motives*: care about portfolio-level factor exposure, so X_i are factor loadings or characteristics that proxy for them
  - Markowitz: $D = \frac{1}{\gamma}\Sigma^{-1}(\mu - P) \Rightarrow \boldsymbol{\mathcal{E}} = -\frac{1}{\gamma}\Sigma^{-1}$
  - If Σ has factor structure: idio risk drives $\hat{\mathcal{E}}$, factor risk drives $\boldsymbol{\mathcal{E}}_X$

Many Potential Mechanisms

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- *Risk based motives*: care about portfolio-level factor exposure, so X_i are factor loadings or characteristics that proxy for them
- *Non-risk motives*: X_i is asset weight in this objective

$$\max_D \quad D'(\mu - P) - \frac{\gamma}{2} D'\Sigma D - \frac{\kappa}{2}\left(D'X^{(1)}\right)^2$$

$$\text{such that} \quad D'X^{(2)} \leq \Theta$$

  - Binding constraints (leverage), regulatory score (capital ratio), or stakeholders pressure (greenness)

Cross-Sectional Identification

- **Data-Generating-Process:** Elasticity matrix $\mathcal{E}$ + *homogeneous substitution conditional on observable* X

$$\Delta \mathbf{D} = \mathcal{E} \Delta \mathbf{P} + \epsilon$$

  - Demand shift ϵ correlated with prices: Ford is more expensive because the new F150 is amazing, change in CalPERS financial health, ...

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- $\boxed{\textbf{Proposition 1}}$ Under our assumption, and the *usual exclusion and relevance restrictions*, the IV estimator identifies the **relative elasticity** $\widehat{\mathcal{E}} = \boldsymbol{\mathcal{E}}_{ii} - \boldsymbol{\mathcal{E}}_{ji}$ for $X_i = X_j$

$$\Delta D_i = \widehat{\mathcal{E}}\Delta P_i + \theta' X_i + e_i$$
$$\Delta P_i = \lambda Z_i + \eta' X_i + u_i$$

  with Z_i instrument for prices ($Z_i \perp \epsilon_i | X_i$)

  – E.g.: Fed buys some bonds but not others

Absorbing Substitution

- Key step: coefficient on observables θ absorbs substitution from other assets

$$
\begin{aligned}
\Delta D_i &= \mathcal{E}_{ii} \Delta P_i + \sum_{j \neq i} X_i' \mathcal{E}_X X_j \Delta P_j + \epsilon_i \\
&= \left(\mathcal{E}_{ii} - X_i' \mathcal{E}_X X_i\right) \Delta P_i + \sum_j X_i' \mathcal{E}_X X_j \Delta P_j + \epsilon_i \\
&= \underbrace{\left(\mathcal{E}_{ii} - X_i' \mathcal{E}_X X_i\right)}_{\text{relative elasticity}} \Delta P_i + X_i' \underbrace{\sum_j \mathcal{E}_X X_j \Delta P_j}_{\text{constant across assets, absorbed in } \theta} + \epsilon_i
\end{aligned}
$$

- **Relative elasticity:** difference between own-price and cross-price elasticity for assets with same observables
  - How does the relative demand for Ford and GM respond to their relative price?
  - In large cross-sections with substantial idiosyncratic risk $\approx$ own-price elasticity

Substitution and Its Estimation

Estimating substitution $\mathcal{E}_X$ crucial for many questions:

- How does CalPERS adjust its portfolio when the price of all bonds drops?
- Will CalPERS maintain its green tilt if green bonds become very expensive relative to brown bonds?

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$\boxed{\textbf{Proposition 2}}$ **Impossible to identify substitution with the cross-section alone**

$$\Delta D_i = \widehat{\mathcal{E}} \Delta P_i + X_i' \underbrace{\sum_j \mathcal{E}_X X_j \Delta P_j + \epsilon_i}_{\textbf{BOTH absorbed in } \theta}$$

- Coefficient on X_i measures both substitution and shift in demand for observable
  - Does CalPERS reduce its green tilt because of expensive green bonds or weaker environmental priorities?

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- **Classic strategy:** construct portfolios sorted on observables, and measure their price and demand (= portfolio tilt)

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$$\Delta P_{agg} = \frac{1}{N}\sum_i \Delta P_i, \qquad \Delta D_{agg} = \frac{1}{N}\sum_i \Delta D_i$$

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- **Proposition 3** Regressing portfolio tilt on portfolio price with **time series instruments** identifies substitution $\mathcal{E}_X$

$$\Delta D_{agg,t} = \bar{\mathcal{E}}_{agg} \Delta P_{agg,t} + \bar{\mathcal{E}}_X \Delta P_{X,t} + \epsilon_{agg,t}$$
$$\Delta D_{X,t} = \tilde{\mathcal{E}}_{agg} \Delta P_{agg,t} + \tilde{\mathcal{E}}_X \Delta P_{X,t} + \epsilon_{X,t}$$

  - Effectively only K assets = portfolios
  - E.g. Fed does more or less QE and operation twist over time

Summary

Homogeneous substitution conditional on observables X:

$$\begin{aligned}\mathcal{E} &= \text{relative elasticity} + \text{substitution} \\ &= \hat{\mathcal{E}} I + X \boldsymbol{\mathcal{E}}_X X'\end{aligned}$$

Consistent with many motives: risk, constraints, non-pecuniary preferences, irrational, ...

Identification:

- Relative elasticity: compare similar assets = cross-sectional IV controlling for X
- Substitution: demand for portfolios based on X = time-series portfolio level instruments

What About Logit?

- Koijen Yogo (2019), Koijen Richmond Yogo (2024):
  - $\exists$ factor models where volatility and expected returns depend non-linearly of prices which yield asset demand in the logit form
  - Logit has non-zero substitution and can be estimated from the cross-section alone

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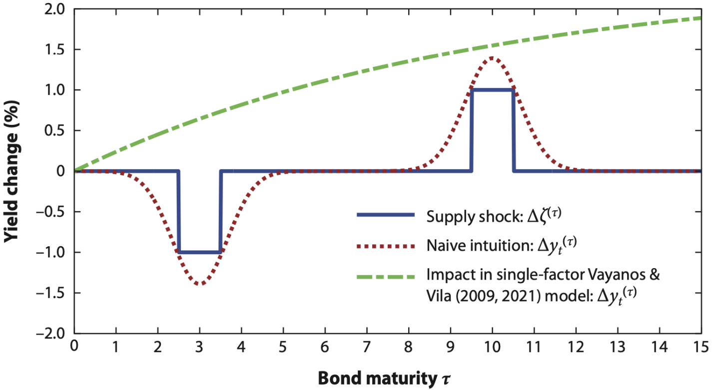
- Logit satisfies our assumption, and its parameter can be robustly interpreted as relative elasticity
- Logit strongly restricts substitution: **an arbitrary factor model is not equivalent to logit**
  - Logit: when the price of any bond $\uparrow$, CalPERS replaces it proportionally to its existing portfolio
  - Factor model: CalPERS replaces it disproportionately with bonds loading on similar factors

Group-based substitution vs Factor Models

- Nested logit (Fang 2023, Koijen Yogo 2024): symmetric groups based on values of observables $\rightarrow$ can use the cross-section of groups to estimate substitution
  - Predict strong local effect and diffuse effect across all other groups
  - Sharply different from factor model with exposure depending on observable (see Cochrane 2008, Vayanos Vila 2021)



Takeaways

- To draw causal inference about demand elasticity, need:
  - A simple assumption: **homogeneous substitution conditional on observables**
    - CalPERS substitutes based on duration and greenness
  - (Standard) source of exogenous variation
    - Fed randomly buys more of some bonds than others, Fed surprisingly engages in QE

- Relative elasticity for similar assets: cross-sectional IV
  - Ford vs GM?
- Substitution = demand for portfolios: time-series IV
  - Green vs brown? Aggregate price?

- Standard structural models of demand rule out most factor-style substitution

Why Causal Inference in Asset Pricing?

- Causal inference particularly valuable when:
  - existing theories are far from the data
  - it is challenging to understand all sources of variations simultaneously
- First step towards better economic theory

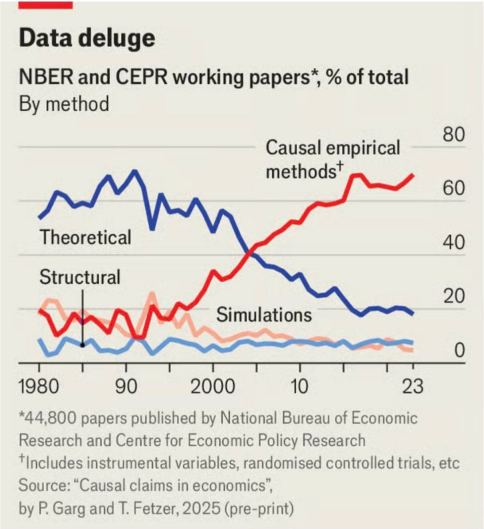


CHART: THE ECONOMIST